Social (In)Stability, Distributive Conflicts, and Investment in Poor and Rich Economies *

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Abstract

A recently much debated issue is why observed investment and growth rates in poor countries are lower than traditional theory predicts. Empirical evidence suggests that social and political instability is a major reason for the divergence between poor and rich countries. However, there is still the unsolved puzzle that the relationship between development and investment rates is not monotonic but follows a hump-shaped pattern. The empirical evidence shows that although very poor economies have very low investment rates there are 'intermediately' developed economies that exhibit extremely high investment rates. This paper shows - within the framework of a simple game theoretic model - that if property rights over produced wealth are not perfectly secure very poor countries are in an instability and inefficiency trap. There exists no redistribution schedule sustaining social stability. However, intermediately productive economies can exhibit investment rates higher than those of high productive economies. Hence, the results of the model predict the empirically observed hump-shaped relationship. Furthermore, the results also support the hypothesis that inequality and investment rates are negatively correlated.

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1 Introduction

Social and political instability is a major determinant explaining the observed divergence in growth rates between poor and rich countries. Poor countries' investment rates are lower, the frequency of social conflict is higher, they exhibit more political instability, and thus have lower growth rates than rich countries. This is the conclusion that can be drawn from several recent empirical studies investigating possible reasons for the non-convergence of poor and rich countries. Alesina, Özler, Roubini, and Swagel (1996) investigate the relationship between political instability and growth for a cross-section of 113 countries for the period 1950-1982. They find that a high degree of instability is significantly correlated with low growth. Alesina and Perotti (1996) and Devereux and Wen (1998) provide further evidence that supports this hypothesis. Svenson (1998) also identifies a link between political stability and investment. Hence, the cross-country evidence suggest that low developed countries are prone to political instability and, thus, have lower investment rates. However, this is only half of the truth, because not all poor countries exhibit low investment rates. It is rather the case that only very poor economies show persistently very low growth rates whereas economies in an intermediate income range grow very fast. In particular, there is evidence that it is always " ... a subset of the lower-income countries (that) grows faster than *any* high-income country does" (Olson 1996, p. 20) and that "within the group of poorer countries (...) some have done very well and others very badly" (Temple 1999, p. 116). Thus, the empirical relationship between the wealth of a nation and investment rates is most probably hump-shaped, as depicted in Figure 1 (This figure is reproduced from

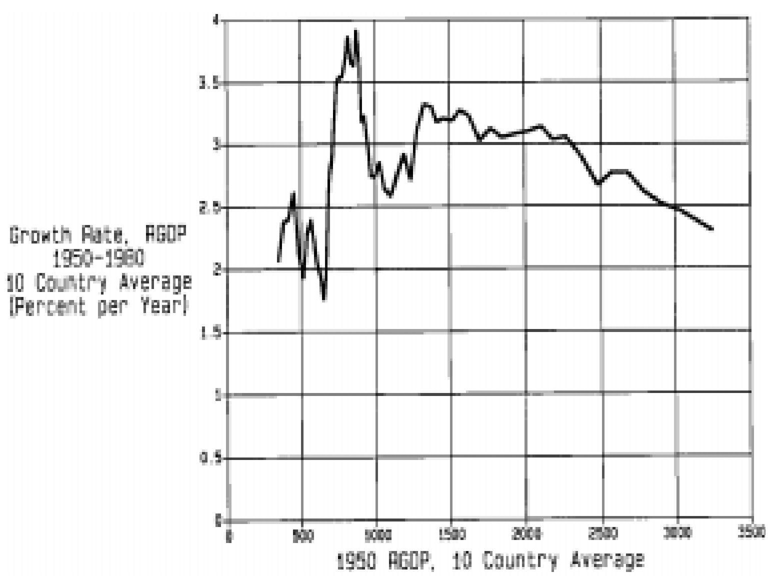

Figure 1: Growth rate, RGDP (1950-80) vs. 1950 RGDP 10-country moving average, 72 countries ranked by 1950 RGDP (Reproduction of Figure 2 in Baumol and Wolff (1988))

Figure 2 in Baumol and Wolff (1988); for further evidence see also De Long (1988) and Figure 1 in Temple (1999)). Very poor countries invest very little, some 'intermediate' countries have very high investment rates, and in rich countries investment rates are lower again.

This paper offers a simple game theoretic model that provides arguments why this hump-shaped relationship is observed. The model builds upon the idea that, ex ante, claims to property are never fully secure. There always exists the possibility that organized social groups can capture, or at least try to capture, a larger share of the produced output. The means for doing that can be any kind of redistributive 'policy', ranging from direct appropriation, like theft or even revolts, to more subtle political influence, like lobbying or bribery. Resource owners being aware of this may therefore choose to allocate some of their resources to unproductive activities increasing the security of their claims. Such activities can range from simply installing electronic alarm systems to employing private safe guards to counter lobbying or counter bribery. In any case, the fear of redistributive activities can lead to disincentives to invest and, therefore, to lower productive investment than in a world of perfectly secure property rights. Furthermore, as will be shown below, it can lead to the observed hump-shaped relationship between income levels and investment across countries.

The model developed in this paper is the most simple framework within which the interplay of development, redistributive activities, and investment can be analyzed. The model is used to determine the allocation of resources between productive and unproductive investment at given development levels - in terms of productivity - of an economy. In the presented model social stability of an economy is exogenously influenced by the economy's productivity but endogenously determined by investment rates and the behavior of economic agents in a redistribution game.[1] There are two social groups in the economy: a few resource owners ('capitalists') and many non-resource owners ('the poor'). Capitalists possess a basic resource they can allocate between production of consumables and activities increasing the security of their property claims to the consumption goods during redistributive activities. The 'security-activity' is modeled with the help of a defense technology. Investing more in this technology increases the amount capitalists can capture for themselves in case of redistributive pressure or a social conflict. The total wealth produced, however, decreases with defense investment. Hence, the resource owner faces a trade-off between increasing total - an thereby potentially his own - wealth and making the claims to this wealth more secure. After the investment decision a simple game of redistribution takes place. The capitalists offer a distribution schedule and the poor can either accept the proposed schedule or reject it and engage in (extralegal) redistributive activities. An important ingredient of the model is the existence of a threshold level of the poor. It is assumed that a distribution schedule giving an income less than this level is always rejected and leads to a cocial conflict sure.

The main result of the analysis shows that very poor, low productive economies are caught in an inefficiency and instability trap. There exists no distribution schedule leading to social stability. This, in turn, leads to high investment in unproductive defense activities, and thereby to large inefficiencies. At some stage of development, however,

[1] In the presented model the state plays no role. This choice is made to make the mechanisms which drive the main results as transparent as possible. Introducing the state and analyzing the consequences of governmental redistributive policies for social stability and investment rates is an interesting question, however, and shortly discussed at the end of the paper.

productive investment increases discontinuously. From this point on a redistribution schedule sustaining social stability exists. The characteristics of such economies are high productive investment levels and a relative low income for the non-resource owners. In particular, in such intermediate economies the poor receive an income which matches their threshold level. As development improves further redistributive pressure increases leading to a decrease in productive investment again. However, productive investment in highly developed and stable economies never falls short of productive investment in poor and unstable economies. Hence, productive investment is higher in intermediately developed economies than in highly developed economies than in low developed economies. Thus, the results obtained predict the empirically observed hump-shaped relationship between development and investment.

A further result is that in the course of development inequality follows more or less a U-shape. This is in contrast to the famous and much debated Kuznets (1955) inverted U-hypothesis, but along the line of theoretical results found by other authors with different models (see e.g. Bannerjee and Newman (1998)). The model, however, predicts very well the observed, although also much debated, negative correlation between inequality and investment rates (see e.g. Perotti (1996)). In particular, it is shown that very high productive investment go together with low inequality.

The model presented in this paper is related and partly builds upon the work of Hirshleifer (1991a, 1991b, 1995) and Skaperdas (1991, 1992). These authors also analyze models of the allocation of resources among appropriative and productive investments. They analyze cases where all produced wealth is in a common pool from which all agents try to appropriate. But, as assumed in this paper, in most cases the producer has an initial - though not completely secure - claim to the produced consumables. Furthermore, in contrast to the model presented in this paper, in the models of Hirshleifer and Skaperdas the decision to actually engage in appropriative activities is not explicitly modeled. They assume, that whenever at least one party invests in the conflict technology a conflict will occur. Also in contrast to the mentioned papers the present paper deals with the question of investment in defense rather than in predatory activities and it is the party with no access to a conflict technology that decides to engage in appropriative activities or not. The work of Grossman (1994) and Grossman and Kim (1995) is also related to this paper. They analyze models of production and appropriation with one-sided or (potential) asymmetric initial claims. Grossman (1994) focuses on the possibility of 'voluntary' land reform by the landlord as optimal response to the threat of extralegal appropriation of land rents. Grossman and Kim (1995) analyze the allocation of resources between appropriative and productive activities, emphasizing the distinction between offensive and defensive weapons. However, like the other papers, and in contrast to the present paper, they do not highlight the possible interaction between development and resource allocation.

Probably most closely related to the present work are the papers by Benhabib and Rustichini (1996) and Falkinger (1999). Benhabib and Rustichini (1996) analyze the relationship between development and growth within a dynamic game framework where returns to investment are subject to appropriation. They show in their model that, depending on specific aspects of the economy like preferences and technology, inefficiencies - in the sense of under-investment - can set in at low or high levels of wealth. In particular, they obtain a result suggesting that poor countries may indeed accumulate at lower rates than rich ones. However, they do not obtain the empirically observed hump-shaped relationship between development and investment. Falkinger (1999) describes a two-class world where the poor have the possibility to express their discontent with a given income distribution by threatening to disrupt the economy. The game theoretic set-up of his model is similar to the model presented here, however, also differs in several respects. In particular, he does not take into account the possible influence of investment in defense activities by the rich on the wealth of an economy. Similar to one result of the presented model he also finds that for lower levels of development a stable income distribution may not exist.[2]

The rest of the paper is organized as follows. First the redistribution game and the capitalist's investment decision are described and analyzed. This is followed by the main results about social instability and the hump-shaped relationship between development and investment rates. Thereafter the results concerning the relationship between development and the agents' welfare and between inequality and investment are presented. Conclusions and possible extensions are set out in chapter 3. All formal proofs not contained in the main text are given in the appendix.

2 Investment, Distributive Conflict, and Appropriation

Consider the following one-period model of distributive conflict, investment and appropriative activities. There are two types of risk-neutral economic agents: a few resource owners and many non-resource owners. Each collective consists of homogeneous agents and is assumed to act as one man. For convenience the collective of resource owners is sometimes called 'capitalist' and the collective of non-resource owners 'the poor'. The resource owners are endowed with a perfectly divisible basic resource of size one. The resource as such is of no direct use for any of the economic agents and the poor do not have the knowledge and/or technology to use it as a productive input. In this sense the ownership rights to the basic resource are perfectly secure. However, the consumables produced with this resource are subject to appropriation by the poor. Appropriative activities occur only if no mutual agreement about the redistribution of wealth between the resource owners and the poor can be reached. The possibility of redistribution and appropriation arises after production took place.

In a first step the capitalist has to decide how to allocate his endowment between productive and defense investment. If he devotes c to defense activities the share devoted to production is given by $1-c$. The poor are endowed only with labor, which is supplied inelastically. Production is carried out with the help of the basic resource and labor, both are necessary for production. The production function is denoted by $q(c)$, where c is the amount devoted to defense activities, and satisfies $q'(c) < 0$ and $q''(c) \leq 0$. Hence, with respect to *productive* investment the basic resource has a positive and non-increasing marginal product. Without loss of generality the production function is

[2] The literature on non-binding contracts and the hold-up problem deals also with a similar problem as the present paper does (see, e.g., Grout (1984).

normalized such that, $q(0) = 1$ and $q(1) = 0$.[3] The produced output also depends on a development parameter, denoted by A. It reflects the maximal possible wealth or the productivity of the economy. Total production is therefore given by $Aq(c)$.

The resource owner has also access to a defense technology ϕ. By investing in defense, he improves his position during redistributive activities (for convinience also called 'conflict'). He can retain a fraction $\phi(c)$ of the available consumables in case of appropriative activities by the poor, where c is the amount invested in defense. ϕ is assumed to be twice differentiable with the following properties. By devoting more resources to defense, the resource owner can increase the fraction he can retain, $\phi'(c) > 0$, and the conflict technology exhibits a non-increasing marginal product, i.e., $\phi''(c) \leq 0$.[4] Furthermore, it is assumed that even if the capitalist invests nothing in defense he can always retain some part of the consumables, i.e., $\phi(0) > 0$.

After the resource owner has made his investment decision production takes place. Both parties observe how much has been produced. Thereafter, the game of redistribution and appropriative activities takes place. The parties are bargaining over a redistribution scheme. It is assumed that the resource owner is in a strong position. He not only owns the resource and has access to a defense technology, but he can also make a take-it-or-leave-it offer in the redistribution game. He offers a redistribution scheme, denoted by t, which gives him $tAq(c)$ of the consumables and leaves the poor with $[1-t]Aq(c)$. The poor can either accept the scheme or reject and engage in appropriative activities. Hence, the agents are playing a kind of ultimatum game. However, there are two important departures from the standard ultimatum game. Firstly, the payoffs in case of rejection, i.e. conflict, are not exogenously given, but endogenously determined by the investment decision of the resource owner. Secondly, it is assumed that there exists a lowest acceptable income level b. This has the effect that a proposed redistribution schedule giving the poor an income below this level is always rejected and leads to appropriative activities for sure. b is assumed to be strictly positive but can be arbitrarily nearby zero. To avoid trivialities it is also assumed that in a first best world (i.e., if $c = 0$) this level can always be met (i.e., $b < A$). Since the existence of a lowest acceptable income b is a crucial and non-standard assumption introducing some kind of irrationality an elaboration is in order here. It is a well documented stylized fact from experimental bargaining games that in such games positive but too small offers are rejected quite frequently. The driving force behind this 'anomalous' behavior is the propensity of people to reciprocate. That is, they are ready to punish unfair behavior even if it is costly for them (for an extensive overview of behavior in experimental bargaining games, see Roth (1995)). Another rational for the observed rejection of positive offers is that people care for relative standings and exhibit some sort of inequity aversion (see, e.g., Fehr and Schmidt (1999)). Hence, in this interpretation b can be viewed as an acceptance threshold. Another possibility is to view b as such a low level of income that people offered an income below this level have 'nothing

[3]Since labor is assumed to be always fully employed it is not written as an explicit argument of q.

[4]This assumption is made for simplicity. Some authors, e.g. Skaperdas (1992), assume an S-shaped conflict technology. However, to prove existence of an equilibrium a restriction on the convex part of the function is needed. In the presented model a similar approach would be possible. All results hold if a not too convex S-shaped conflict technology would be assumed.

left to lose', and that as a consequence violent riots , e.g., a hunger revolt, break out. That such riots are not necessarily fully rational actions is quite obvious.

2.1 The Redistribution and Appropriation Game

In this section the redistribution and appropriation game is discussed and analyzed. Figure 2 depicts the structure of this game. The rules of the game require to distinguish between two cases. (i) If the capitalist offers a redistribution schedule t which gives the poor *at least* their threshold level then they can either accept it or go for a conflict (see Figure 2(i)). The produced output - for any previously determined investment level - is

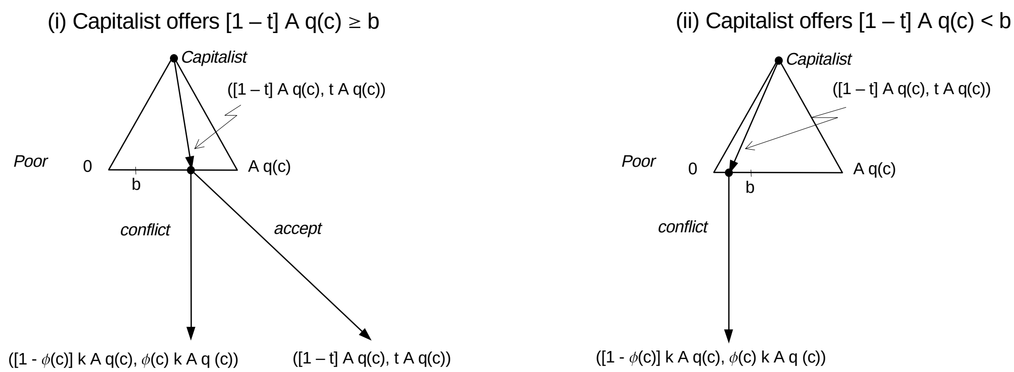

Figure 2: The redistribution and appropriation game

given by $Aq(c)$. A proposal of the resource owner is a pair $([1-t]Aq(c), tAq(c))$, where the first entry is the offer to the poor and the second entry reflects what the capitalist demands for himself. If the poor accept the schedule an agreement is made and the output is split accordingly. If they do not accept, appropriative activities take place and the payoffs are given by $([1-\phi(c)]kAq(c), \phi(c)kAq(c))$, $0 < k < 1$. The parameter k measures the destructiveness ('damage potential') of a conflict, i.e., it is assumed that during a conflict the fraction $1-k$ of the produced consumables is destroyed. (ii) If the capitalist decides to make an offer *strictly below* the threshold value b then the poor will go for a conflict for sure, giving the same payoffs as in (i) after a rejection (see Figure 2(ii)). Note that, even if the rules of the game would allow the poor to choose between conflict and acceptance (in equilibrium) they could not earn more than their conflict payoff. (This follows from the ultimatum character of the game; see also the remarks after Proposition 1, below.) Hence, although the rules of the game prescribe that the poor will choose conflict for sure if they are offered less than b they are not earning less in this case compared to a situation where they would behave completely rational.

Next potential candidates for an equilibrium of the described game are discussed. Suppose, that the capitalist decides to propose a redistribution schedule, which would give the poor less then their threshold level, i.e., $[1-t]Aq(c) < b$. In this case appropriative activities take place for sure. The capitalist will propose such a schedule if and only if he is better off in that case than in the case where he makes an offer of at least

b. Assume for the moment that the poor accept a schedule which gives them b even if they would get the same payoff in case of conflict.[5] Hence, proposing a schedule that gives the poor exactly b gives the resource owner the payoff $Aq(c) - b$. Offering less - thereby provoking a conflict - leads to the payoff $\phi(c)kAq(c)$. Therefore, provoking appropriative activities is better than offering b if and only if the inequality

$$[1 - k\phi(c)]Aq(c) < b \tag{2.1}$$

is satisfied. Since in such a situation actual conflict takes place it is called conflict regime (*C-regime*). Inspecting the above inequality reveals that - for a given b - such a regime will be observed if the wealth of the economy, $Aq(c)$, is that small that the capitalist is not willing to give up at least b to avoid a conflict.
Now consider that the resource owner is ready to give up b to avoid sure appropriative activities, i.e.,

$$b \leq [1 - k\phi(c)]Aq(c). \tag{2.2}$$

The poor will accept such an offer if they cannot credibly threat to engage in appropriative activities. This is the case if b is greater than what they can expect in case of a conflict, that is, if

$$[1 - \phi(c)]kAq(c) \leq b \tag{2.3}$$

holds. Therefore if the inequality chain

$$[1 - \phi(c)]kAq(c) \leq b \leq [1 - k\phi(c)]Aq(c) \tag{2.4}$$

is satisfied,[6] no conflict takes place *and* the poor are held down to their threshold level. Therefore, such a situation is called threshold regime (*T-regime*).
Now consider the case where the economy is rich enough, i.e., where $Aq(c)$ is relatively large, such that

$$b < [1 - \phi(c)]kAq(c) \tag{2.5}$$

holds. In such a situation, on the one hand, it pays the capitalist to propose a redistribution scheme that avoids conflict and, on the other hand, the poor can credibly threat to engage in appropriative activities. In equilibrium, however, no appropriation takes place and the poor receive a payoff above the threshold level. For that reason this situation is called stable or peace regime (*P-regime*).
The above reasoning is summarized in the following proposition:

Proposition 1 EQUILIBRIUM IN THE REDISTRIBUTION AND APPROPRIATION GAME
Let $c < 1$. In the redistribution and appropriation game exists a (almost always) essentially unique subgame perfect equilibrium in pure strategies. The equilibrium is characterized by the following actions and payoffs:

1. *If $[1 - k\phi(c)]Aq(c) < b$ then*
   the capitalist offers a redistribution schedule leaving the poor with a payoff strictly less than the threshold level b. This leads to appropriative activities with payoffs

[5] That in equilibrium such an offer actually is accepted is shown in the proof of Proposition 1.
[6] That this chain is well defined follows from $k < 1$.

- *$U_c(c) := [1-\phi(c)]kAq(c)$ for the poor, and*
- *$\Pi_c(c) := \phi(c)kAq(c)$ for the capitalist.*

2. *If $[1-\phi(c)]kAq(c) \leq b \leq [1-k\phi(c)]Aq(c)$ then the capitalist offers a schedule giving exactly b to the poor. They accept any schedule which gives them at least b. There are no appropriative activities and the payoffs in this situation are*
   - *$U_t(c) := b$ for the poor, and*
   - *$\Pi_t(c) := Aq(c) - b$ for the capitalist.*

3. *If $b < [1-\phi(c)]kAq(c)$ then the capitalist offers the redistribution schedule $t^* = 1 - [1-\phi(c)]k$. The poor accept any schedule t lower or equal t^* and reject any other schedule. There are no appropriative activities and the payoffs are given by*
   - *$U_p(c) := [1-\phi(c)]kAq(c)$ for the poor, and*
   - *$\Pi_p(c) := [1-[1-\phi(c)]k]Aq(c)$ for the capitalist.*

Some comments are in order here. (i) With respect to *1.* of the above proposition one could argue that it does not make sense for the poor to protest against an offer below b because they do not gain from doing so. However, besides the experimental-empirical fact that in bargaining games too low offers are punished by rejection even if it is costly for the responder to do so, in this game rejection of b is - given the rules of the game - by no means irrational. To see this, suppose that the poor would be allowed to choose between acceptance and conflict after an offer smaller b. The capitalist knowing this would (in equilibrium) make the poor indifferent between acceptance and conflict, hence he would offer exactly the conflict payoff U_c. Thus, introducing the threshold value b has only the effect that it allows the poor to punish the capitalist at zero costs. (ii) If $c = 1$ any strategy combination is an equilibrium since nothing is produced in this case. However, since the resource owner can always guarantee something positive for himself by choosing $c < 1$, devoting everything to defense can never be an optimal investment choice. (iii) The described equilibrium is essentially unique since if $[1-k\phi(c)]Aq(c) < b$ holds, all offers below b are payoff equivalent equilibrium offers. (iv) This is true only "almost always" because, in fact, there exists a second subgame perfect equilibrium in pure strategies for the special case where b equals $[1-k\phi(c)]kAq(c)$. To see this suppose the equality holds. Then the poor will always accept a redistribution schedule that gives them exactly the threshold level since the equality implies that the payoff from appropriative activities is strictly smaller than b. The capitalist, however, receives the same payoff by giving up b thereby avoiding a conflict or by offering less and provoking a conflict. The latter case is also an equilibrium because the capitalist is indifferent between the two actions and the poor have nothing to choose if they receive such an offer. Since this second equilibrium does not change the capitalist's payoff the analysis of the investment decision is based on the equilibrium stated in Proposition 1.

2.2 Productive and Defense Investment

Before turning to the capitalist's investment decision some important properties of his payoff functions in the different regimes are discussed. The functions Π_c, Π_t, and Π_p from Proposition 1 are well defined for all investment levels c in the interval $[0,1]$. The assumptions on q and ϕ guarantee that each is twice continuously differentiable in the interior of that interval.

The payoff function Π_t in the *T-regime* is strictly decreasing with defense investment, because

$$\Pi_t'(c) = Aq'(c) < 0. \tag{2.6}$$

Hence, the *T-regime* profit is maximized at $c_t^* = 0$. Furthermore, since the production function q is concave, Π_t is also concave.

Next consider the capitalist's payoff function in the *P-regime*. The first derivative is given by

$$\begin{aligned} \Pi_p'(c) &= \phi'(c)kAq(c) + [1-k+k\phi(c)]Aq'(c) \\ &= MB_p(c) - MC_p(c), \end{aligned} \tag{2.7}$$

where $MB_p(c) := \phi'(c)kAq(c)$ and $MC_p(c) := -[1-k+k\phi(c)]Aq'(c)$ denote marginal benefits and marginal costs, resp., of an additional unit of conflict investment. The assumptions about the production function and the conflict technology guarantee that the second derivative,

$$\Pi_p''(c) = \left[\phi''(c)q(c) + 2\phi'(c)q'(c) + \phi(c)q''(c)\right]kA + [1-k]Aq''(c), \tag{2.8}$$

is strictly negative for any c. Hence, Π_p is strictly concave. To get an interior maximum it is assumed that Π_p is strictly increasing at $c = 0$,[7] which is equivalent to the assumption that

$$MB_p(0) > MC_p(0). \tag{2.9}$$

Together with $\Pi_p(0) > 0$, $\Pi_p(1) = 0$, and the strict concavity this guarantees a unique interior maximizer, denoted by c_p^*. That is, there exists a unique $c_p^* \in \,]0,1[$ satisfying the first order condition

$$MB_p(c_p^*) - MC_p(c_p^*) = 0. \tag{2.10}$$

Note that the maximizer, c_p^*, is independent of the development parameter A, but may change with damage potential k.

Similar properties hold for the profit function in the *C-regime*. The first derivative is given by

$$\begin{aligned} \Pi_c'(c) &= \phi'(c)kAq(c) + \phi(c)kAq'(c) \\ &= MB_c(c) - MC_c(c), \end{aligned} \tag{2.11}$$

where, again, $MB_c(c) := \phi'(c)kAq(c)$ and $MC_c(c) := -\phi(c)kAq'(c)$ denote marginal benefits and marginal costs, respectively. The second derivative is given by

$$\Pi_c''(c) = \left[\phi''(c)q(c) + 2\phi'(c)q'(c) + \phi(c)q''(c)\right]kA. \tag{2.12}$$

[7] Hence, it is assumed that it always pays to invest at least a little bit in defense. If this condition does not hold optimal investment is given by $c = 0$.

Inspecting (2.12) reveals that Π_c is also strictly concave in c. This together with condition (2.9), $\Pi_c(0) > 0$, and $\Pi_c(1) = 0$ implies that Π_c exhibits a unique interior maximizer, too.[8] Hence, it exists a unique $c_c^* \in]0, 1[$ such that the first order condition

$$MB_c(c_c^*) - MC_c(c_c^*) = 0 \tag{2.13}$$

is met. The maximizer c_c^* is independent of A and k.
Furthermore, $c_p^* < c_c^*$ holds for all values of the exogenous parameters. Inspecting the marginal benefit and marginal cost curves reveals this property. The marginal benefit curve is the same for Π_c and Π_p and strictly decreasing, whereas both marginal cost curves are strictly increasing. Comparing $MC_c(c)$ with $MC_p(c)$ shows that the marginal costs in the *P-regime* are always above the marginal costs in the *C-regime*. Therefore, c_p^* is always strictly smaller than c_c^* (see also Figure 3).

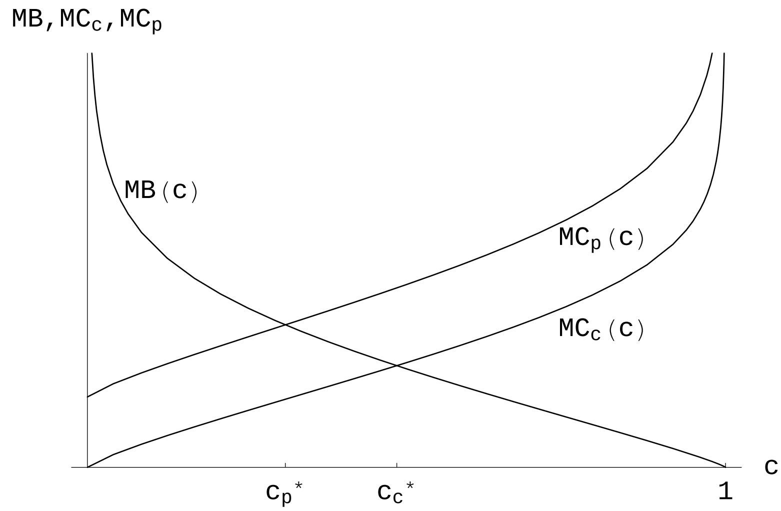

Figure 3: Marginal benefits and marginal costs in *C*- and *P-regime*

In addition, it is easily verified that $\Pi_c(c) < \Pi_p(c)$ for all $c \in [0, 1[$ and that $\Pi_c(c)$ and $\Pi_p(c)$ are both zero for $c = 1$. These results are summarized in the following Lemma.

Lemma 1 PROPERTIES OF THE CAPITALIST'S PROFIT IN THE DIFFERENT REGIMES
Assume that $\Pi_p(c)$ is strictly increasing at $c = 0$, i.e., that the boundary condition (2.9) *holds, then*

- *The C-regime payoff $\Pi_c(c)$ of the resource owner is strictly concave and has a unique interior maximizer c_c^*,*
- *in the T-regime the capitalist's profit $\Pi_t(c)$ is strictly concave, and strictly decreasing with c (i.e., $c_t^* = 0$),*
- *the payoff $\Pi_p(c)$ of the resource owner in the P-regime is strictly concave and has a unique interior maximizer c_p^*, and*

[8]Note that condition 2.9 is sufficient but not necessary.

- $\Pi_c(c) < \Pi_p(c)$ *for all* $c \in [0,1[$*; for* $c = 1$ *the C-regime and the P-regime profits are both zero.*

Furthermore, $c_p^* < c_c^*$*,* c_c^* *is independent of* A *and* k*, whereas* c_p^* *is only independent of the development parameter* A*.*

By considering the optimal investment decision the resource owner faces the following maximization problem:

$$\max_{c\in[0,1]} \Pi(c) = \max_{c\in[0,1]} \begin{cases} \Pi_c(c) & \text{if } [1-k\phi(c)]Aq(c) < b, \\ \Pi_t(c) & \text{if } [1-\phi(c)]kAq(c) \leq b \leq [1-k\phi(c)]Aq(c), \\ \Pi_p(c) & \text{if } b < [1-\phi(c)]kAq(c). \end{cases} \tag{2.14}$$

It is clear from this formulation that the capitalist - by considering the optimal investment level - implicitly also determines the regime resulting as the outcome of the redistribution and appropriation game. Therefore, he has to take into account under which conditions which regime will turn out to be the actual one. Hence, it is necessary to know at which investment levels a transition from one to another regime will take place. Existence and uniqueness of these investment levels and an important property are stated in the following Lemma.

Lemma 2 THE REGIME-SWITCHING INVESTMENT LEVELS
For any $A > b$ *and* $k \in]0,1[$ *there is a unique investment level* $c_{ct}(A,k)$ *where the transition from the C-regime to the T-regime takes place. It is given by*

$$\begin{aligned} [1-k\phi(c_{ct}(A,k))]Aq(c_{ct}(A,k)) &= b \quad \text{if } A > \underline{A}, \\ c_{ct}(A,k) &:= 0 \quad \text{if } A \leq \underline{A}. \end{aligned} \tag{2.15}$$

Similarly, there is a unique investment level $c_{tp}(A,k)$ *where the transition from the T-regime to the P-regime takes place. It is given by*

$$\begin{aligned} [1-\phi(c_{tp}(A,k))]kAq(c_{tp}(A,k)) &= b \quad \text{if } A > A_{min}, \\ c_{tp}(A,k) &:= 0 \quad \text{if } A \leq A_{min}, \end{aligned} \tag{2.16}$$

where, $A_{min} := \frac{b}{[1-\phi(0)]k} > \underline{A} := \frac{b}{1-k\phi(0)} > b$*.*
Furthermore, $c_{tp}(A,k)$ *is strictly increasing with* A *and* k*.*

For the ease of exposition, $c_{ct}(A,k)$ and $c_{tp}(A,k)$ are denoted by $c_{ct}(.)$ and $c_{tp}(.)$, respectively. The exact notation will be used only if it is necessary for the analysis. Figure 4 depicts the defining functions for c_{ct} and c_{tp},

$$\Gamma_{ct}(c) := [1-k\phi(c_{ct}(.))]Aq(c_{ct}(.)) - b, \text{ and}$$

$$\Gamma_{tp}(c) := [1-\phi(c_{tp}(.))]kAq(c_{tp}(.)) - b,$$

respectively. It shows graphically that $\Gamma_{tp} < \Gamma_{ct}$ for any $c < 1$ (and $A \geq \underline{A}$). This immediately implies that $c_{tp}(.)$ is always strictly smaller than $c_{ct}(.) < 1$. Hence, if $c_{tp}(.)$ is positive so is $c_{ct}(.)$. Note, that for values of A between $\underline{A}$ and A_{min}, $c_{ct}(.) > 0$

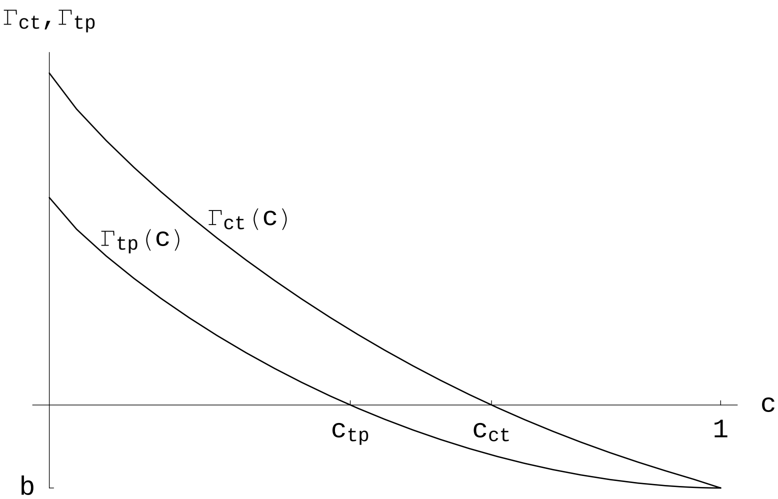

Figure 4: The regime switching investment levels

but $c_{tp}(.) = 0$, and only a transition from the C- to the *T-regime* is feasible, but no change from the T- to the *P-regime*. Furthermore, if A is even smaller than $\underline{A}$, implying $c_{ct}(.) = c_{tp}(.) = 0$, the economy is completely trapped in conflict. In is the case no investment level exists such that the economy can leave the *C-regime*. This already gives a first indication that for very poor economies it may be very hard to leave a state of political instability. However, this does not yet tell that the investment rate is also low, since this depends on the capitalist's decision.

With the above definition and properties of the regime switching investment levels at hand it is now possible to rewrite the capitalist's objective function in the following form:

$$\max_{c\in[0,1]} \Pi(c;.) = \max_{c\in[0,1]} \begin{cases} \Pi_p(c;.) & \text{if} \quad c \in [0, c_{tp}(.)[\,, \\ \Pi_t(c;.) & \text{if} \quad c \in [c_{tp}(.), c_{ct}(.)], \\ \Pi_c(c;.) & \text{if} \quad c \in]c_{ct}(.), 1], \end{cases} \tag{2.17}$$

where the "." stands for the productivity parameter A and damage potential k. In the following section the above formulation is used to analyze the capitalist's optimal investment decision.

2.3 Social (In)Stability and (In)Efficiency

The optimal investment decision of the resource owner depends in an important way on the development parameter A and the damage potential k. The most interesting question is how investment and, therefore, efficiency changes with the productivity of an economy. (Recall that the economic situation is the more efficient the lower defense investment c is.) Let for the moment the parameter k be fixed. What is the optimal decision of the capitalist and how does it change with productivity A? This question will now be answered in an informal way with the help of Figures 5(a)-(d). These figures depict the capitalist's objective function for different stages of development. In

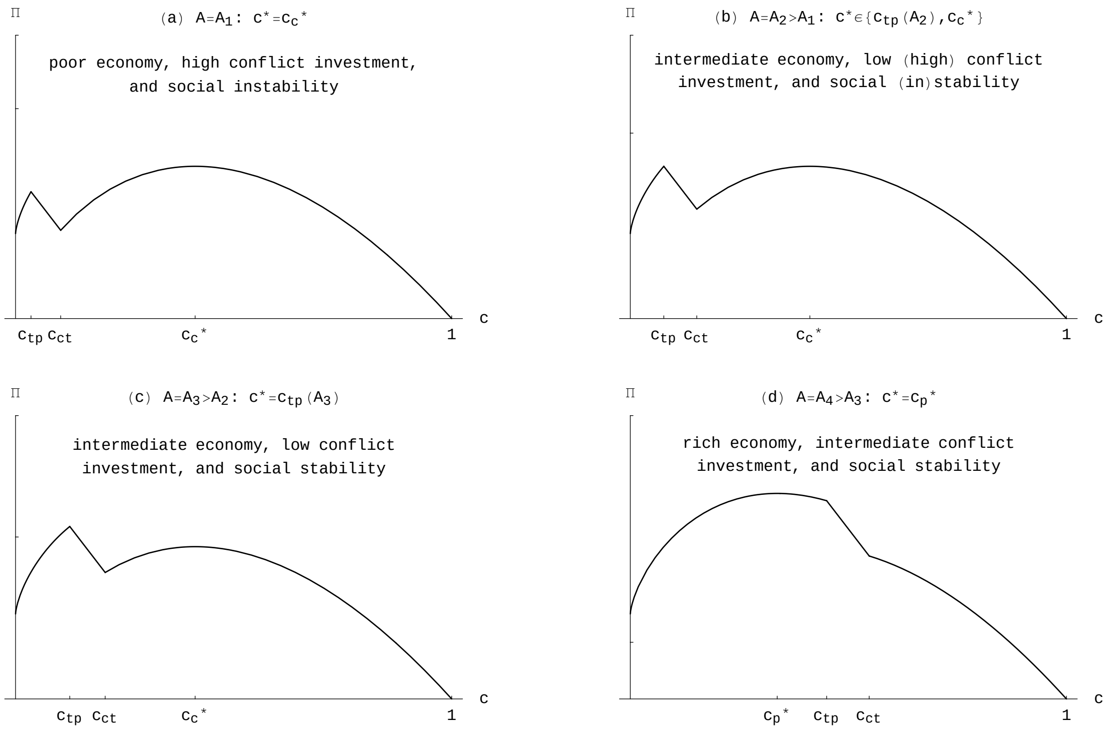

Figure 5: Profits for different productivity levels

all four figures the part of the graph between the defense investment levels of zero and c_{tp} is given by Π_p, the part between c_{tp} and c_{ct} by Π_t, and between c_{ct} and one by Π_c.

In the following a situation will be called socially (or politically) unstable if a conflict cannot be avoided. Figure 5(a) depicts a situation where the economy is very poor (A_1 is very small). In this case the objective function is maximized at the maximizer of Π_c, and the capitalist will therefore choose c_c^*. Hence, in equilibrium the resource owner chooses a defense investment level which provokes a conflict in the redistribution and appropriation game. Note, that in this case there is no way out of the social instability trap. Although, in principle the resource owner could choose to invest nothing or very little in defense and propose a redistribution schedule that gives the poor at least b - in equilibrium - he will not to that. It is in the capitalist's interest to devote c_c^* to defense and to provoke a conflict. Figure 5(b) depicts an economy which is richer ($A_2 > A_1$). Furthermore, A_2 is chosen such that in this situation the resource owner has two optimal choices. Firstly, as in the case depicted in Figure 5(a), he may again choose c_c^* and provoke appropriative activities, or, secondly, he may choose $c_{tp}(A_2,.)$. The second choice not only avoids conflict, but also leads to a more efficient situation since $c_{tp}(A_2,.) < c_c^*$. Increasing the productivity parameter to the level A_3, as depicted in Figure 5(c), leads to the unique optimal investment choice $c_{tp}(A_3,.)$, which is greater than $c_{tp}(A_2,.)$, but still smaller than c_c^*. Hence, in the transition from a poor economy to an economy with an intermediate productivity potential avoidance of conflict and higher efficiency go together. If the economy becomes very rich (A_4 large), the optimal investment is given by c_p^* (see Figure 5(d)). On the one hand this level is smaller than

c_c^* but on the other hand larger than $c_{tp}(A_2, .)$ and $c_{tp}(A_3, .)$.

In summary, what has been shown is, that for very poor economies there is a social instability and inefficiency trap, in the sense that the defense investment, c_c^*, chosen by the resource owner is relatively large, thereby enforcing inefficient outcomes and leading to political instability. If, however, the productivity of the economy is above a particular level inefficiency due to defense investment decreases discontinuously to c_{tp} and the economy becomes more efficient and politically stable. Thereafter, if the economy becomes even more productive, redistributive concerns become more severe and inefficiency due to defense investment steadily increases. This remains so until the economy has reached some breakeven point where optimal defense investment stays at c_p^* in a socially stable situation. These results are summarized in the following proposition.

Proposition 2 Social (In)Stability and (In)Efficiency
Let k be given, and suppose that $\Pi_p(0; .) < \Pi_c(c_c^*; .)$ holds, then

1. *Very poor economies are in a social instability and inefficiency trap, in the sense that there exists a unique $\tilde{A} > b$ such that for all $A \in]b, \tilde{A}[$ the capitalist chooses to devote c_c^* to defense, thereby provoking appropriative activities.*
2. *In intermediate and rich economies no appropriative activities take place and inefficiency is always strictly smaller than in poor economies. However, the richer an intermediate economy the stronger is the pressure towards redistribution. This leads to decreasing productive investment and, therefore, to increasing inefficiency. Formally,*
   - (a) *There exists a unique $\hat{A} > \tilde{A}$ such that for all $A > \hat{A}$ the resource owner invests $c_p^*(.)$ $(c_p^*(.) < c_c^*)$.*
   - (b) *For $A = \tilde{A}$ optimal conflict investment is given by $\{c_{tp}(\tilde{A}, .), c_c^*\}$, whereas for $A \in]\tilde{A}, \hat{A}]$, it equals $c_{tp}(A; .)$ $(c_{tp}(A; .) < c_c^*)$.*

   $c_{tp}(A; .)$ is strictly increasing with A, $c_p^*(.) > c_{tp}(A; .)$ for all $A \in [\tilde{A}, \hat{A}[$, and for $A = \hat{A}$, $c_p^*(.)$ and $c_{tp}(A; .)$ coincide.

Remark: For the above Proposition it is assumed that $\Pi_p(0; .) < \Pi_c(c_c^*; .)$ holds. This is equivalent to $\phi(c_c^*)q(c_c^*) - \phi(0) > [1 - k]/k$ and, therefore, satisfied if k is sufficiently close to one, i.e., if the damage potential of a conflict is not too high, and/or if $\phi(0)$ is sufficiently small. If it is not satisfied the existence of $\tilde{A}$ is not guaranteed and only part two of the above proposition holds. This tells, that for very poor economies, instability can only be avoided if either the damage potential is that high that it never pays for the resource owner to provoke a conflict, or if the defense technology is that effective that the threat of the poor to engage in appropriative actions is worthless even if the resource owner invests nothing in defense. Both instances seem not very realistic.

Figure 6 summarizes the statements of Proposition 2. It depicts the optimal *productive* investment level, $1 - c^*$, in dependence of the productivity parameter A. It nicely shows the hump-shaped relationship between development levels and investment rates shown in Figure 1. For economies at a low level of development inefficiency due to low productive investment, $1 - c_c^*$, is highest, and such economies are caught in an inefficiency and social instability trap. Since the economy is poor the resource owner has no incentive to propose a redistribution schedule that would lead to political stability. This in turn leads to relative high investment devoted to unproductive defense because the capitalist has to be prepared to defend the consumables in case of appropriative actions. Thus, the economy stays poor. This shows that there is a clear link between low productivity, high political instability, and low productive investment. At a particular stage of development the productive potential of the economy becomes large enough such that a distributive schedule that avoiding appropriative actions exists. Investment can now be directed to productive activities. This leads, compared to very poor economies, to huge efficiency gains. This result is in line with the stylized fact of the high investment and growth rates of intermediate-income countries, like, e.g., the "Asian-Tigers". With further development redistributive pressures become more prominent and productive investment decreases down to the level $1 - c_p^*$. The observa-

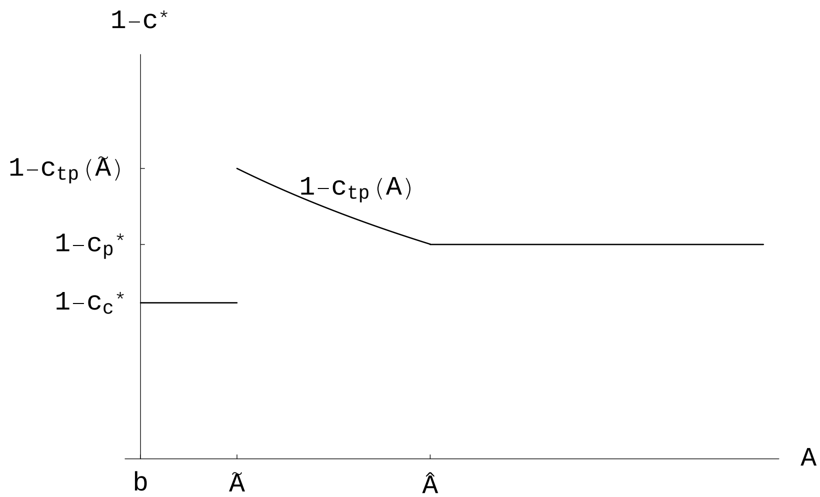

Figure 6: Productive investment and development

tion that over some range inefficiencies increase with the wealth of a stable economy is in line with the argument, put forward by Olson (1982). There, Olson argues that in such economies organized groups exert redistributive pressures.

2.4 Welfare, Inequality, and Development

The preceding section analyzed the relation between development, political instability, and investment. In this section the relation between the productivity of an economy and the associated investment decisions and the welfare of the economic agents is analyzed. In particular, it will be asked how total and individual welfare are influenced by the productivity of the economy. Furthermore, the implications of productivity on income distribution are addressed. This also allows to draw conclusions about a

possible correlation between investment rates and inequality.
Individual equilibrium welfare for the capitalist is given by

$$\Pi(A,k) = \begin{cases} \phi(c_c^*)kAq(c_c^*) & \text{if } A \in]b, \tilde{A}], \\ Aq(c_{tp}(A,k)) - b & \text{if } A \in [\tilde{A}, \hat{A}], \\ \left[1 - \left[1 - \phi(c_p^*(k))\right] k\right] Aq(c_p^*(k)) & \text{if } A > \hat{A}, \end{cases} \tag{2.18}$$

and for the poor it is

$$U(A,k) = \begin{cases} [1 - \phi(c_c^*)]kAq(c_c^*) & \text{if } A \in]b, \tilde{A}], \\ b & \text{if } A \in [\tilde{A}, \hat{A}], \\ \left[1 - \phi(c_p^*(k))\right] kAq(c_p^*(k)) & \text{if } A > \hat{A}. \end{cases} \tag{2.19}$$

Total (equilibrium) welfare is defined to be the sum of individual welfare and given by

$$W(A,k) = \begin{cases} kAq(c_c^*) & \text{if } A \in]b, \tilde{A}], \\ Aq(c_{tp}(A,k)) & \text{if } A \in [\tilde{A}, \hat{A}], \\ Aq(c_p^*(k)) & \text{if } A > \hat{A}. \end{cases} \tag{2.20}$$

Note first that the capitalist's welfare is continuous in A (see Lemma A.2 in the appendix) and that for poor ($A < \tilde{A}$) as well as rich ($A > \hat{A}$) economies the optimal investment levels, c_c^* and $c_p^*(k)$, respectively, are independent of the development parameter A. Hence, increasing A unambiguously leads to welfare gains for both economic agents in very poor and very rich economies. Furthermore, since $c_p^*(k)$ is strictly smaller than c_c^* and the capitalist's profit in the *P-regime* is (for any given investment level) always larger than in the *C-regime* (Lemma 1), individual (and, thus, total welfare) is higher in rich economies than in poor. For intermediate economies ($\tilde{A} \le A \le \hat{A}$), however, things are different, because optimal defense investment, $c_{tp}(A,k)$, is increasing with A (Lemma 2). Consider the case where the productivity of the economy is exactly $\tilde{A}$. At this development level there are two optimal investment choices, c_c^* and $c_{tp}(\tilde{A},k)$. Continuity of the capitalist's welfare together with $c_{tp}(\tilde{A},k) < c_c^*$ imply that total welfare and welfare of the poor is strictly greater at $c_{tp}(\tilde{A},k)$ than at c_c^*. Furthermore, in intermediate economies the *T-regime* is prevalent which means that, independent of the productivity of the economy, the poor are held down to their threshold level. Hence, welfare of the poor stays constant as productivity increases. This, however, does not necessarily imply that the capitalist's welfare (and thus total welfare) is monotonically increasing with the development of an intermediate economy. The reason is that increasing A not only increases the potential total wealth but also decreases optimal productive investment $1-c_{tp}(A,k)$. In general, it is not possible to say at which productivity levels which of the two effects is stronger. What can be said, however, is that the capitalist's welfare cannot decrease everywhere, since on the one hand $\Pi_t(c_{tp}(\hat{A},k),\hat{A},k) = \Pi_p(c_p^*(k),\hat{A},k)$ and on the other hand $\Pi_p(c_p^*(k),\hat{A},k) > \Pi_c(c_c^*,\tilde{A},k) = \Pi_t(c_{tp}(\tilde{A},k),\tilde{A},k)$ (by continuity and Lemma 1). Hence, overall (that is, going from $\tilde{A}$ to $\hat{A}$) the capitalist's welfare increases with productivity in intermediate economies. To summarize:

Proposition 3 Development and Welfare
(i) In poor and rich economies, $A < \tilde{A}$ and $A > \hat{A}$, resp., individual welfare of all agents and, thus, total welfare are strictly increasing with the productivity of the economy. Furthermore, total and individual welfare is higher in rich economies than in poor economies.
(ii) In intermediate economies, $\tilde{A} < A \leq \hat{A}$, welfare of the poor stays constant at the threshold level b, which is larger than their welfare in poor economies but smaller than in rich economies. Furthermore, in such economies, if productivity is increased from $\tilde{A}$ to $\hat{A}$, the welfare of the capitalist and, thus, total welfare are 'on average' increasing, but may decrease somewhere.

Although, formally speaking, the analyzed economy consists only of two agents, properly interpreted it can be viewed as an economy consisting of only a few capitalists and many people owing no resources. In such an economy the inverse of the welfare of the poor relative to total welfare, i.e., W/U, can be viewed as a natural measure of inequality. Let $\Delta := W/U$, then two questions can be answered by using Δ as a proxy for inequality. (i) Does Δ follow the famous and much debated Kuznets inverted U as productivity of the economy increases, and (ii) does the model predict the empirically observed negative correlation between inequality and investment rates? Figure 7 depicts the evolution of Δ in dependence of the productivity of the economy for different values of the damage potential k. Figure 7(a) shows it for rather small values of k, i.e., a high damage potential of a conflict, and Figure 7(b) for a value of k nearby one, i.e., for a low damage potential of a conflict. It is obvious from the figures that the

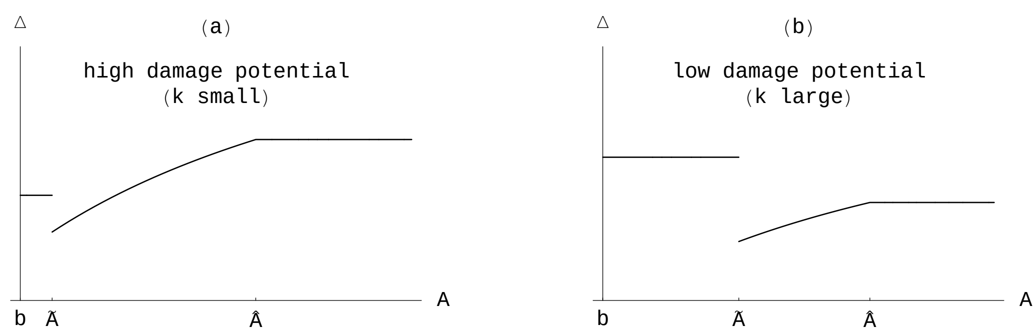

Figure 7: Inequality and development

evolution of inequality predicted by the model hardly resembles the inverted U. The process rather has a shape turning Kuznets on his head. This finding is similar to a result obtained by Bannerjee and Newman (1998) in a different model. They are able to generate a whole range of patterns for the evolution of income distribution during the process of modernization among which there is also a U-pattern.

One feature which is independent of the value of k is that inequality is smallest at the point of transition from poor to intermediate economies, $\tilde{A}$. Recall, that in this productivity range of an economy productive investment, $1 - c^*$, is highest. Furthermore, for low damage potential the shape of Δ follows the inverse of the pattern of optimal productive investment over the whole range of possible development levels (compare to Figure 6). Hence, it almost perfectly replicates the empirically observed (but also much debated) negative correlation between inequality and investment rates. For high damage potential the correlation is not that clear-cut. It only holds when going from the poor end of intermediate economies to rich economies, but shows a positive correlation when comparing very poor with rich economies. The results depicted in Figure 7 are summarized more formally in the next proposition. Observe first, that the measure of inequality, Δ, is given by

$$\Delta(A,k) = \begin{cases} [1-\phi(c_c^*)]^{-1} & \text{if } A \in]b, \tilde{A}], \\ b^{-1} A q(c_{tp}(A,k)) & \text{if } A \in [\tilde{A}, \hat{A}], \\ \left[[1-\phi(c_p^*(k))]\, k\right]^{-1} & \text{if } A > \hat{A}. \end{cases} \tag{2.21}$$

Proposition 4 Development and Inequality
(i) If the damage potential of a conflict is sufficiently low, i.e., if k is sufficiently nearby one, then inequality is larger in poor economies, then in rich economies, then in intermediate economies. More formally, $\Delta(A_1,k) > \Delta(A_2,k) > \Delta(A_3,k)$ *for* $A_1 \in]b, \tilde{A}[$, $A_2 > \hat{A}$, *and* $A_3 \in]\tilde{A}, \hat{A}[$.
(ii) If the damage potential of a conflict is high, i.e., if k is small enough, then inequality is higher in rich economies, then on the rich end of intermediate economies, then in poor economies, then on the poor end of intermediate economies. Formally, $\Delta(A_1,k) > \Delta(A_2,k) > \Delta(A_3,k) > \Delta(A_4,k)$ *for* $A_1 > \hat{A}$, $A_2 \in]\tilde{A}, \hat{A}[$, $A_3 \in]b, \tilde{A}[$, *and* $A_4 \in]\tilde{A}, \hat{A}[$, *with* $A_4 < B < A_2$ $(B \in]\tilde{A}, \hat{A}[)$.
(iii) For any given damage potential of a conflict inequality in intermediate economies has a tendency to increase as the economy develops, in the sense that $\Delta(A,k)$ *is continuous in A, lowest at* $\tilde{A}$ *and highest at* $\hat{A}$.

3 Conclusions and Discussion

In this paper a simple game theoretic model with insecure claims to produced consumables was developed. It allowed to analyze the interaction between an economy's productivity, its impact on social stability, and the resulting investment rates in productive activities. The main result is that the model predicts the empirically observed hump-shaped relationship between development and investment. Low developed, low productive economies are caught in an inefficiency and social instability trap. In such economies no redistribution schedule guaranteeing social stability exists. Since the claims to property are insecure resource owners have an incentive to devote part of their resources to unproductive defense investment. In poor economies optimal *productive* investment is small leading to large inefficiencies and social instability. At some level of development optimal investment in productive activities increases discontinuously leading to huge efficiency gains and a stable social environment in intermediately developed economies. Thereafter, with further economic development redistributive pressure increases leading to smaller productive investment, again. However, productive investment in rich economies never falls short of productive investment in very poor economies. In summary, productive investment is lowest in very poor economies and highest in economies with an intermediate level of development. The investment rates in rich economies are somewhere in-between.

The paper also analyzed the relation between economic and individual welfare, inequality and the productivity of an economy. The analysis showed that overall total and individual welfare increases with the productivity of an economy. However, for intermediately developed economies there is also the possibility that total and the resource owner's welfare decrease over some range of development. With respect to inequality the model predicts that in the course of development inequality follows basically a U-shape, turning Kuznets inverted U hypothesis on its head. However, it also predicts the empirically observed negative correlation between inequality and development.

In the presented model the state plays no role. This choice was made to leave the model simple and make the forces driving the investment decisions as transparent as possible. It is quite clear that in reality the state plays a crucial role and that extending the model by introducing a government can be a worthwhile task. Two directions for governmental policy are immediate. (i) The government could provide security to claims and thereby reduce private defense investment. Since publicly provided security has to be financed by taxes the question of relative efficiency between public and private defense investment could be analyzed in such a model. (ii) Governmental policy could try to implement more efficient outcomes with the help of redistribution policy. If there is a publicly enforced distribution schedule giving both parties at least the income they would receive without governmental intervention the economy could save private defense investment and reach a more efficient stage. In particular in may be interesting to see if very poor countries are able to leave the inefficiency and social instability trap with the help of redistribution policy.

Another important extension of the present analyses would be the development of a dynamic model in which agents have repeated interactions and resource owners can spent part of their earnings to accumulate capital. In such a model it could be analyzed if the instability and inefficiency trap also occurs in a dynamic environment and identify conditions under which poor economies are able to leave this trap. It could also identify conditions under which the hump-shaped relationship between development levels and investment rates found in the present paper are translated to a hump-shaped relationship between levels of development and growth rates.

Finally, it would be interesting to extend the analysis to more than two agents. This would allow, for example, to analyze a more realistic case where the defense technology is not exogenously given, but where the resource owner has to hire people defending his property in case of conflict. In such a framework also the questions of coalition formation and possible collusion between different social groups in the society, their impact on investment rates and the interaction with development levels could be analyzed.

A Appendix: Proofs

A.1 Proof of Proposition 1

Here the existence of the subgame perfect equilibrium stated in Proposition 1 is proved. For convenience the content of Proposition 1 is restated in Proposition A.1 here.

Proposition A.1 Equilibrium in the Redistribution and Appropriation Game
Let $c < 1$. In the redistribution and appropriation game exists a (almost always) essentially unique subgame perfect equilibrium in pure strategies. The equilibrium is characterized by the following actions and payoffs:

1. *If $[1 - k\phi(c)]Aq(c) < b$ then the capitalist offers a redistribution schedule leaving the poor with a payoff strictly less than the threshold level b. This leads to appropriative activities with payoffs*
   - *$U_c(c) := [1 - \phi(c)]kAq(c)$ for the poor, and*
   - *$\Pi_c(c) := \phi(c)kAq(c)$ for the capitalist.*
2. *If $[1 - \phi(c)]kAq(c) \leq b \leq [1 - k\phi(c)]Aq(c)$ then the capitalist offers a schedule giving exactly b to the poor. They accept any schedule which gives them at least b. There are no appropriative activities and the payoffs in this situation are*
   - *$U_t(c) := b$ for the poor, and*
   - *$\Pi_t(c) := Aq(c) - b$ for the capitalist.*
3. *If $b < [1 - \phi(c)]kAq(c)$ then the capitalist offers the redistribution schedule $t^* = 1 - [1 - \phi(c)]k$. The poor accept any schedule t lower or equal t^* and reject any other schedule. There are no appropriative activities and the payoffs are given by*
   - *$U_p(c) := [1 - \phi(c)]kAq(c)$ for the poor, and*
   - *$\Pi_p(c) := [1 - [1 - \phi(c)]k]Aq(c)$ for the capitalist.*

Proof:
Case 1: Suppose that $[1 - k\phi(c)]Aq(c) < b$ holds. It has to be shown that it is optimal for the capitalist to propose a redistribution schedule leaving the poor with less than b. If he actually makes such a proposal the payoffs are given by $U_c(c)$ and $\Pi_c(c)$. Suppose, that he proposes $[1-t]Aq(c) \geq b > [1-k\phi(c)]Aq(c)$. This implies a profit of $tAq(c) < \phi(c)kAq(c) = \Pi_c(c)$ for the capitalist *if* the poor accept. That they will accept follows from $[1-t]Aq(c) > [1-k\phi(c)]Aq(c) > [1 - \phi(c)]kAq(c) = U_c(c)$. Hence, proposing $[1 - t]Aq(c) \geq b$ would make the capitalist strictly worse off and he will not do that.

Case 2: Suppose that $[1 - \phi(c)]kAq(c) \leq b \leq [1 - k\phi(c)]Aq(c)$ holds. It will be shown that offering b and acceptance of such an offer by the poor is an equilibrium. Suppose the poor choose $conflict$ instead of *accept*. This gives $U_c(c) = [1 - \phi(c)]kAq(c)$ to the poor. Hence, from the left hand side of the above inequality it follows that the poor are not better off by choosing $conflict$. Therefore, accepting any offer greater or equal to b is a best response. For the capitalist it follows that offering more than b is worse than offering exactly the threshold level. Now suppose he offers strictly less than b. This leads to conflict giving him a profit of $\Pi_c(c) = \phi(c)kAq(c)$. However, since the right hand side of the above inequality is equivalent to $\Pi_t(c) = Aq(c) - b \geq \phi(c)kAq(c) = \Pi_c(c)$, offering less than the threshold level makes

the capitalist not better off. It follows that the strategies proposed in Proposition 1 form an equilibrium.
It remains to show that the equilibrium is unique as long as $[1-\phi(c)]kAq(c) \leq b < [1-k\phi(c)]Aq(c)$ holds. Suppose first that the left-hand side of the inequality is strict. By the arguments given above $U_c(c) < b$ holds. Hence, it is always strictly better for the poor to accept b than to go for a conflict. Furthermore, $\Pi_c < \Pi_t$ holds. Hence, the unique best action for the capitalist is to offer exactly b. Now suppose the left-hand side of the inequality holds with equality, i.e., $b = [1-\phi(c)]kAq(c)$. In this case the poor are indifferent between acceptance of b and choosing *conflict*. It will be shown that choosing *conflict* cannot be an equilibrium action. Suppose, the poor actually choose *conflict*. In that case the capitalist would receive the payoff Π_c. However, from the above arguments it follows that the capitalist is *strictly* better off by proposing b instead of risking a conflict. Thus, $\Pi_t > \Pi_c$ and an $\epsilon > 0$ can be found such that the capitalist can make an offer $b + \epsilon$ which is surely accepted by the poor and makes him better off, i.e., $\Pi_t - \epsilon > \Pi_c$. Therefore to choose *conflict* instead of accepting such a b cannot be part of an equilibrium.

Case 3: Let $b < [1-\phi(c)]kAq(c)$. In this case the game has the typical ultimatum game character. The capitalist will make an offer which makes the poor indifferent between accepting and rejecting, and in equilibrium they will accept. Suppose the capitalist makes a proposal such that the poor receive $[1-t]Aq(c) = [1-\phi(c)]kAq(c)$ if they accept. Acceptance would give $[1-[1-\phi(c)]k]Aq(c)$ to the capitalist, and $[1-\phi(c)]kAq(c) > b$ to the poor. Hence, in any subgame perfect equilibrium the poor will accept any offer greater or equal $U_p(c) = [1-\phi(c)]kAq(c)$ and reject any other offer. Therefore, the capitalist will make exactly such a proposal leaving him with $\Pi_p(c)$. The uniqueness of this equilibrium follows from the ultimatum character of the game. ■

A.2 Proof of Lemma 2

For convenience Lemma 2 is restated as Lemma A.1, below.

Lemma A.1
For any $A > b$ and $k \in]0,1[$ there is a unique investment level $c_{ct}(A,k)$ where the transition from the C-regime to the T-regime takes place. It is given by

$$\begin{aligned} [1-k\phi(c_{ct}(A,k))]Aq(c_{ct}(A,k)) &= b \quad \text{if } A > \underline{A}, \\ c_{ct}(A,k) &:= 0 \quad \text{if } A \leq \underline{A}, \end{aligned} \tag{A.1}$$

Similarly, there is a unique investment level $c_{tp}(A,k)$ where the transition from the T-regime to the P-regime takes place. It is given by

$$\begin{aligned} [1-\phi(c_{tp}(A,k))]kAq(c_{tp}(A,k)) &= b \quad \text{if } A > A_{min}, \\ c_{tp}(A,k) &:= 0 \quad \text{if } A \leq A_{min}, \end{aligned} \tag{A.2}$$

where, $A_{min} := \frac{b}{[1-\phi(0)]k} > \underline{A} := \frac{b}{1-k\phi(0)} > b$.

Furthermore, $c_{tp}(A,k)$ is strictly increasing with A and k.

Proof: First consider the case where the regime changes from Threshold to Peace, and define the function $\Gamma_{tp} :]A_{min}, \infty[\times]0,1[\times [0,1] \to J$, where J is an interval on the real line and $A_{min} := \frac{b}{[1-\phi(0)]k} > b$, such that

$$\Gamma_{tp}(A,k;c) := [1-\phi(c)]kAq(c) - b. \tag{A.3}$$

From

$$\Gamma_{tp}(A,k;0) = [1-\phi(0)]kA - b > 0,$$
$$\Gamma_{tp}(A,k;1) = -b < 0, \text{ and}$$
$$\frac{\partial \Gamma_{tp}}{\partial c}(A,k;\gamma) = -\phi'(\gamma)kAq(\gamma) + [1-\phi(\gamma)]kAq'(\gamma) < 0,$$

and the implicit function theorem it follows that there exists a unique and continuously differentiable function c_{tp} defined over the parameters A and k such that

$$\Gamma_{tp}(A,k;c_{tp}(A,k)) = 0.$$

Hence, this function defines for any given pair (A,k) the unique defense investment level where the transition from the *T*- to the *P-regime* takes place.
In an equivalent way define the function $\Gamma_{ct} :]\underline{A}, \infty[\times]0,1[\times [0,1] \to J$, where $\underline{A} := \frac{b}{1-k\phi(0)} > b$, for the change from the *C-regime* to the *T-regime* by

$$\Gamma_{ct}(A,k;c) := [1-k\phi(c)]Aq(c) - b. \tag{A.4}$$

Again, the properties

$$\Gamma_{ct}(A,k;0) = [1-k\phi(0)]A - b > 0,$$
$$\Gamma_{ct}(A,k;1) = -b < 0,$$
$$\frac{\partial \Gamma_{ct}}{\partial c}(A,k;\gamma) = -k\phi'(\gamma)Aq(\gamma) + [1-k\phi(\gamma)]Aq'(\gamma) < 0,$$

and the implicit function theorem imply that there exists a unique and continuously differentiable function c_{ct} defined over the parameter pair (A,k) such that

$$\Gamma_{ct}(A,k;c_{ct}(A,k)) = 0.$$

Next the stated properties of $c_{tp}(A,k)$ will be proved. $c_{tp}(A,k)$ is defined by

$$\Gamma_{tp}(A,k;c_{tp}) := [1-\phi(c_{tp})]kAq(c_{tp}) - b = 0,$$

and

$$\begin{aligned}
\frac{\partial \Gamma_{tp}}{\partial c}(\alpha,\kappa;\gamma) &= \kappa[1-\phi(\gamma)]Aq'(\gamma) - \kappa\phi'(\gamma)Aq(\gamma) < 0,\\
\frac{\partial \Gamma_{tp}}{\partial A}(\alpha,\kappa;\gamma) &= \kappa[1-\phi(\gamma)]q(\gamma) > 0,\\
\frac{\partial \Gamma_{tp}}{\partial k}(\alpha,\kappa;\gamma) &= [1-\phi(\gamma)]\alpha q(\gamma) > 0,
\end{aligned}$$

hold. By the rule of implicit differentiation it follows, therefore,

$$\frac{dc_{tp}}{dA}(\alpha,\kappa) > 0, \ \frac{dc_{tp}}{dk}(\alpha,\kappa) > 0.$$

■

A.3 Proof of Proposition 2

First continuity and differentiability of the capitalist's objective function is proved.

Lemma A.2
With respect to c the capitalist's objective function is continuous on $[0,1]$ *and twice differentiable on* $]0,1[-(c_{tp}(.)\cup c_{ct}(.))$ *for any given parameter values in the relevant sets.*

Proof: The second statement follows from the properties of $q(c)$ and $\phi(c)$. It is, therefore, sufficient to show that $\Pi(c;.)$ is continuous at $c_{tp}(.)$ and $c_{ct}(.)$.

$$\begin{aligned}\lim_{c\uparrow c_{tp}(.)}\Pi(c;.) &= \lim_{c\uparrow c_{tp}(.)}[Aq(c)-[1-\phi(c)]kAq(c)]\\ \text{`by continuity of } q \text{ and } c\text{`} &= Aq(c_{tp}(.))-[1-\phi(c_{tp}(.))]kAq(c_{tp}(.))\\ \text{`by definition of } c_{tp}(.)\text{`} &= Aq(c_{tp}(.))-b\\ &= \Pi_t(c_{tp}(.);.)\\ &= \Pi(c_{tp}(.);.),\end{aligned}$$

and

$$\begin{aligned}\lim_{c\downarrow c_{ct}(.)}\Pi(c;.) &= \lim_{c\downarrow c_{ct}(.)}\phi(c)kAq(c)\\ \text{`by continuity of } q \text{ and } c\text{`} &= \phi(c_{ct}(.))kAq(c_{ct}(.))\\ \text{`by definition of } c_{ct}(.)\text{`} &= Aq(c_{ct}(.))-b\\ &= \Pi_t(c_{ct}(.);.)\\ &= \Pi(c_{ct}(.);.).\end{aligned}$$

■

For convenience the content of Proposition 2 is restated in Proposition A.2.

Proposition A.2 SOCIAL (IN)STABILITY AND (IN)EFFICIENCY
Let k be given, and suppose that $\Pi_p(0;.)<\Pi_c(c_c^*;.)$ *holds, then*

1. *Very poor economies are in a social instability and inefficiency trap, in the sense that there exists a unique* $\tilde{A}>b$ *such that for all* $A\in\,]b,\tilde{A}[$ *the capitalist chooses to devote* c_c^* *to defense, thereby provoking appropriative activities.*
2. *In intermediate and rich economies no appropriative activities take place and inefficiency is always strictly smaller than in poor economies. However, the richer an intermediate economy the stronger is the pressure towards redistribution. This leads to decreasing productive investment and, therefore, to increasing inefficiency. Formally,*
   (a) *There exists a unique* $\hat{A}>\tilde{A}$ *such that for all* $A>\hat{A}$ *the resource owner invests* $c_p^*(.)$ $(c_p^*(.)<c_c^*)$.
   (b) *For* $A=\tilde{A}$ *optimal conflict investment is given by* $\{c_{tp}(\tilde{A},.),c_c^*\}$, *whereas for* $A\in\,]\tilde{A},\hat{A}]$, *it equals* $c_{tp}(A;.)$ $(c_{tp}(A;.)<c_c^*)$.

   $c_{tp}(A;.)$ *is strictly increasing with A,* $c_p^*(.)>c_{tp}(A;.)$ *for all* $A\in[\tilde{A},\hat{A}[$, *and for* $A=\hat{A}$, $c_p^*(.)$ *and* $c_{tp}(A;.)$ *coincide.*

Proof: The proof is done in three steps. First, the existence of a unique $\tilde{A}$, second that of a unique $\hat{A}$, and third the statements about the optimal choice of c will be proven.

Step 1. It will be proven that there is a unique $\tilde{A} > b$ such that $\Pi_c(c_c^*; \tilde{A}, .) = \Pi_t(c_{tp}(\tilde{A}, .); \tilde{A}, .)$ holds. For that, rewrite the above condition to $\Pi_c(c_c^*; \tilde{A}, .)/\tilde{A} = \Pi_t(c_{tp}(\tilde{A}, .); \tilde{A}, .)/\tilde{A}$, and define

$$\tilde{F}(A) := \Pi_c(c_c^*; A, .)/A - \Pi_t(c_{tp}(A, .), A, .)/A. \tag{A.5}$$

It holds that $c_{tp}(b, .) = 0$ (by the definition of c_{tp} and $b < A_{min}$) and $\Pi_t(c_{tp}(A, .); A, .) = \Pi_p(c_{tp}(A, .); A, .)$ (by Lemma A.2). Therefore,

$$\tilde{F}(A_{min}) = \Pi_c(c_c^*; b, .)/b - \Pi_p(0, b, .)/b > 0$$

(by assumption). Now let an $\hat{A} > b$ be such that $\Pi_t(c_{tp}(\hat{A}, .); \hat{A}, .) = \Pi_p(c_p^*(.); \hat{A}, .)$. (That such an $\hat{A}$ actually exists will be proven in *Step 2*). Then,

$$\begin{aligned} \tilde{F}(\hat{A}) &= \Pi_c(c_c^*; \hat{A}, .)/\hat{A} - \Pi_t(c_{tp}(\hat{A}, .), \hat{A}, .)/\hat{A} \\ &= \Pi_c(c_c^*; \hat{A}, .)/\hat{A} - \Pi_p(c_p^*(.); \hat{A}, .)/\hat{A} < 0. \end{aligned}$$

$\tilde{F}$ is continuous in A, hence there is a $\tilde{A} \in]b, \hat{A}[$ such that $\tilde{F}(\tilde{A}) = 0$. To prove uniqueness, suppose to the contrary that there exists a $\check{A} \in]b, \hat{A}[$ with $\tilde{F}(\check{A}) = 0$ and $\check{A} \neq \tilde{A}$, and note that $\Pi_c(c; A, .)/A = \Pi_c(c; A', .)/A'$ for any $A, A' > 0$. Hence,

$$\begin{aligned} \tilde{F}(\tilde{A}) &= \tilde{F}(\check{A}) \\ &\Leftrightarrow \\ \Pi_c(c_c^*; \tilde{A}, .)/\tilde{A} - \Pi_t(c_{tp}(\tilde{A}, .); \tilde{A}, .)/\tilde{A} &= \Pi_c(c_c^*; \check{A}, .)/\check{A} - \Pi_t(c_{tp}(\check{A}, .); \check{A}, .)/\check{A} \\ &\Leftrightarrow \\ \Pi_t(c_{tp}(\tilde{A}, .); \tilde{A}, .)/\tilde{A} &= \Pi_t(c_{tp}(\check{A}, .); \check{A}, .)/\check{A} \\ &\Leftrightarrow \text{ `by continuity'} \\ \Pi_p(c_{tp}(\tilde{A}, .); \tilde{A}, .)/\tilde{A} &= \Pi_p(c_{tp}(\check{A}, .); \check{A}, .)/\check{A} \end{aligned}$$

Define now $\pi_p(c) := [1 - [1 - \phi(c)]k]q(c)$. The last equation above is then equivalent to

$$\pi_p(c_{tp}(\tilde{A})) = \pi_p(c_{tp}(\check{A})). \tag{A.6}$$

It will shown that (A.6) cannot hold true. Like Π_p the function π_p attains its maximum at $c_p^*(.)$. Furthermore, $c_{tp}(\tilde{A}) \neq c_{tp}(\check{A})$, and with $\tilde{A}, \check{A} < \hat{A}$ also $c_{tp}(\tilde{A}), c_{tp}(\check{A}) < c_{tp}(\hat{A})$ (see Lemma 2). In the course of *Step 2* it will be shown that $c_{tp}(\hat{A}) = c_p^*$, and therefore, since the function $\pi_p(c)$ is strictly increasing to the left of $c_p^*(.)$ this contradicts A.6, which proves *Step 1.*

Step 2. Now it will be shown that there exists a unique $\hat{A}$ such that $\Pi_t(c_{tp}(\hat{A}, .); \hat{A}, .) = \Pi_p(c_p^*(.); \hat{A}, .)$. Note, that existence of such an $\hat{A}$ implies (by *Step 1*) that $\hat{A} > \tilde{A}$. Define

$$\hat{F}(A) := \Pi_t(c_p^*(.); A, .) - \Pi_p(c_p^*(.); A, .). \tag{A.7}$$

It follows

$$\begin{aligned} \hat{F}(A_{min}) &= \Pi_t(c_p^*(.); A_{min}, .) - \Pi_p(c_p^*(.); A_{min}, .) \\ \text{`}0 < c_p^*(.) \Rightarrow \Pi_t(0, .) > \Pi_t(c_p^*(.), .)\text{`} &< \Pi_t(0; A_{min}, .) - \Pi_p(c_p^*(.); A_{min}, .) \\ \text{`} - \Pi_p(c_p^*(.), .) < -\Pi_p(0, .)\text{`} &< \Pi_t(0; A_{min}, .) - \Pi_p(0; A_{min}, .) \\ \text{`}c_{tp}(A_{min}, .) = 0\text{`} &= \Pi_t(c_{tp}(A_{min}, .); A_{min}, .) - \Pi_p(c_{tp}(A_{min}, .); A_{min}, .) \\ \text{`definition of } c_{tp}(A, .)\text{`} &= 0. \end{aligned}$$

Hence, $\hat{F}(A_{min}) < 0$. On the other hand

$$lim_{A \to \infty} \hat{F}(A) = +\infty.$$

Therefore, since $\hat{F}(A) = Aq(c_p^*(.))[1 - \phi(c_p^*(.))]k - b$ strictly increases with A, there is a unique $\hat{A} > A_{min}$ such that $\Pi_t(c_p^*(.), \hat{A}, .) = \Pi_p(c_p^*(.); \hat{A}, .)$. That is, (by continuity) there exists a unique $\hat{A} > A_{min}$ such that $c_{tp}(\hat{A}, .) = c_p^*(.)$ and $\Pi_t(c_{tp}(\hat{A}, .); \hat{A}, .) = \Pi_t(c_p^*(.), \hat{A}, .) = \Pi_p(c_p^*(.); \hat{A}, .)$. This proves *Step 2* and the remaining parts of *Step 1*.

Before turning to the final step concerning the optimal defense investments some Lemmas have to be proven.

Lemma A.3
Let $\Pi_p(0; .) < \Pi_c(c_c^*; .)$. *If* $A \in]0, \tilde{A}[$ *then* $\Pi_t(c_{tp}(A, .); A, .) < \Pi_c(c_c^*; A, .)$.

Proof:

$$\begin{aligned} \Pi_t(c_{tp}(A_{min}, .); A_{min}, .) &= \\ \text{`continuity of } \Pi(c; .)\text{'} &= \Pi_p(c_{tp}(A_{min}, .); A_{min}, .) \\ \text{`}c_{tp}(A_{min}, .) = 0\text{'} &= \Pi_p(0; A_{min}, .) \\ &< \Pi_c(c_c^*; A_{min}, .). \end{aligned}$$

In *Step 1* it has been shown that the only A where the equality $\Pi_t(c_{tp}(A, .); A, .) = \Pi_c(c_c^*; A, .)$ holds is $\tilde{A}$. This together with the continuity of $\Pi_t - \Pi_c$ in A and the above inequality proves the statement. ■

Lemma A.4
For $A \in]0, \tilde{A}[$ *the inequality* $c_{ct}(A, .) < c_c^*$ *holds.*

Proof: Continuity of $\Pi(c; .)$ guarantees $\Pi_t(c_{tp}(A, .); .) = \Pi_p(c_{tp}(A, .); .)$, and since $\Pi_p(c; .) > \Pi_c(c; .)$ (see Lemma 1), the inequality

$$\Pi_t(c_{tp}(A, .); .) > \Pi_c(c_{tp}(A, .); .) \quad (*)$$

holds.
Since $A < \tilde{A}$ it follows by Lemma A.3 that $\Pi_t(c_{tp}(A, .); A, .) < \Pi_c(c_c^*; A, .)$. Together with $c_{tp}(A, .) < c_c^*$ (by $c_{tp}(A, .) < c_p^*(.)$ for $A < \hat{A}$; see *Step 2* and the monotonic behavior of c_{tp} in A), $c_p^*(.) < c_c^*$ (see Lemma 1), and the fact that Π_t is strictly decreasing in c it follows that

$$\Pi_t(c_c^*; A, .) < \Pi_c(c_c^*; A, .) \quad (**).$$

$\Pi_c(c; .)$ is strictly increasing and $\Pi_t(c; .)$ is strictly decreasing on $[c_{tp}(A, .), c_c^*]$. By continuity of $\Pi(c; .)$ it follows that $\Pi_t(c_{ct}(A, .); A, .) = \Pi_c(c_{ct}(A, .); A, .)$. This equation also defines $c_{ct}(A, .)$. Hence, $(*)$ and $(**)$ imply that $c_{ct}(A, .) < c_c^*$. ■

Lemma A.5
If $A \in]\tilde{A}, \hat{A}]$ *then* $\Pi_t(c_{tp}(A, .); A, .) > \Pi_c(c_c^*; A, .)$.

Proof: $\Pi_t(c_{tp}(\hat{A},.);\hat{A},.) = \Pi_p(c_p^*(.);\hat{A},.)$ (by *Step 2*), and $\Pi_p(c_p^*(.);\hat{A},.) > \Pi_c(c_c^*;\hat{A},.)$ (by Lemma 1); hence $\Pi_t(c_{tp}(\hat{A},.);\hat{A},.) > \Pi_c(c_c^*;\hat{A},.)$. Furthermore, in *Step 1* it has been shown that there is a unique $\tilde{A} \in]b,\hat{A}]$ such that $\Pi_t(c_{tp}(\tilde{A},.);\tilde{A},.) = \Pi_c(c_c^*;\tilde{A},.)$. This together with the above inequality and the continuity of Π_t and Π_c proves the statement. ■

Now the remaining part of Proposition 2 will be proven.

Step 3. Observe first that it has already been shown that $c_{tp}(\tilde{A},.) < c_{tp}(\hat{A},.) = c_p^*(.) < c_c^*$ holds, and that $c_{tp}(A,.)$ is strictly increasing in A.

Step 3.1. Let $A \in]b,\tilde{A}[$ and fixed. For $A < \tilde{A}$ it holds that $c_{tp}(A,.) < c_{tp}(\tilde{A},.) < c_p^*(.)$ and

$$\sup_{c\in[0,c_{tp}(A,.)[} \Pi_p(c;A,.) = \Pi_p(c_{tp}(A,.);A,.),$$

because Π_p strictly increases on $[0,c_p^*(.)[$. Furthermore, since Π_t is strictly decreasing in c on the whole interval $[0,1]$,

$$\max_{c\in[c_{tp}(A,.),c_{ct}(A,.)]} \Pi_t(c;A,.) = \Pi_t(c_{tp}(A,.);A,.),$$

and

$$\max_{c\in]c_{ct}(A,.),1]} \Pi_c(c;A,.) = \Pi_c(c_c^*;A,.),$$

because $c_{ct}(A,.) < c_c^*$ (see Lemma A.4). By continuity of Π in c and Lemma A.3

$$\Pi_p(c_{tp}(A,.);A,.) = \Pi_t(c_{tp}(A,.);A,.) < \Pi_c(c_c^*;A,.)$$

holds. Therefore, if $A \in]b,\tilde{A}[$ then

$$\arg\max_{c\in[0,1]}\Pi(c;A,.) = c_c^*.$$

This - together with *Step 1* establishes part *1.* of Proposition 2.

Step 3.2. Let $A = \tilde{A}$. By the definition of $\tilde{A}$,

$$\Pi_t(c_{tp}(\tilde{A},.);\tilde{A},.) = \Pi_c(c_c^*;\tilde{A},.).$$

Otherwise the same arguments as in *Step 3.1* apply. Hence, for $A = \tilde{A}$,

$$\arg\max_{c\in[0,1]}\Pi(c;A,.) = \{c_{tp}(\tilde{A},.), c_c^*\}.$$

This proves the first part of *2.(b)* of Proposition 2.

Step 3.3. Let $A \in]\tilde{A},\hat{A}]$, then for $A < \hat{A}$,

$$\sup_{c\in[0,c_{tp}(A,.)[} \Pi_p(c;A,.) = \Pi_p(c_{tp}(A,.);A,.)$$

and for $A = \hat{A}$,

$$\sup_{c\in[0,c_{tp}(A,.)[} \Pi_p(c;A,.) = \Pi_p(c_p^*(.);A,.),$$

since $c_{tp}(A,.) < c_{tp}(\hat{A},.) = c_p^*(.)$. Furthermore,

$$\max_{c\in[c_{tp}(A,.),c_{ct}(A,.)]} \Pi_t(c;A,.) = \Pi_t(c_{tp}(A,.);A,.) = \Pi_p(c_{tp}(A,.);A,.),$$

and

$$\max_{c\in]c_{ct}(A,.),1]} \Pi_c(c;A,.) \leq \Pi_c(c_c^*;A,.).$$

By Lemma A.5,

$$\Pi_t(c_{tp}(A,.);A,.) > \Pi_c(c_c^*;A,.)$$

and it follows

$$\arg\max_{c\in[0,1]}\Pi(c;A,.) = c_{tp}(A,.),\ \text{with}\ c_{tp}(\hat{A},.) = c_p^*(.).$$

This proves the second part of *2.(b)* of Proposition 2.

Step 3.4. Let $A > \hat{A}$, then $c_{tp}(A,.) > c_{tp}(\hat{A},.) = c_p^*(.)$. Hence,

$$\max_{c\in[0,c_{tp}(A,.)[}\Pi_p(c;A,.) = \Pi_p(c_p^*(.);A,.),$$

$$\max_{c\in[c_{tp}(A,.),c_{ct}(A,.)]}\Pi_t(c;A,.) = \Pi_t(c_{tp}(A,.);A,.) = \Pi_p(c_{tp}(A,.);A,.) < \Pi_p(c_p^*(.);A,.),$$

and

$$\max_{c\in]c_{ct}(A,.),1]}\Pi_c(c;A,.) \le \Pi_c(c_c^*;A,.) < \Pi_p(c_p^*(.);A,.).$$

Therefore,

$$\arg\max_{c\in[0,1]}\Pi(c;A,.) = c_p^*.$$

This final step - together with *Step 2* proves part *2.(a)* of Proposition 2. ■

A.4 Proof of Proposition 4

As for the other proofs, for convenience, Proposition 4 is restated as Proposition A.3, below.

Proposition A.3 Development and Inequality
(i) If the damage potential of a conflict is sufficiently low, i.e., if k is sufficiently nearby one, then inequality is larger in poor economies, then in rich economies, then in intermediate economies. More formally, $\Delta(A_1,k) > \Delta(A_2,k) > \Delta(A_3,k)$ for $A_1 \in]b,\tilde{A}[$, $A_2 > \hat{A}$, and $A_3 \in]\tilde{A},\hat{A}[$.
(ii) If the damage potential of a conflict is high, i.e., if k is small enough, then inequality is higher in rich economies, then on the rich end of intermediate economies, then in poor economies, then on the poor end of intermediate economies. Formally, $\Delta(A_1,k) > \Delta(A_2,k) > \Delta(A_3,k) > \Delta(A_4,k)$ for $A_1 > \hat{A}$, $A_2 \in]\tilde{A},\hat{A}[$, $A_3 \in]b,\tilde{A}[$, and $A_4 \in]\tilde{A},\hat{A}[$, with $A_4 < B < A_2$ ($B \in]\tilde{A},\hat{A}[$).
(iii) For any given damage potential of a conflict inequality in intermediate economies has a tendency to increase as the economy develops, in the sense that $\Delta(A,k)$ it is continuous in A, and lowest at $\tilde{A}$ and highest at $\hat{A}$.

Proof: The inequality measure Δ is defined by

$$\Delta(A,k) = \begin{cases} \Delta_{poor} := [1-\phi(c_c^*)]^{-1} & \text{if}\ \ A \in]b,\tilde{A}], \\ \Delta_{int} := b^{-1}Aq(c_{tp}(A,k)) & \text{if}\ \ A \in [\tilde{A},\hat{A}], \\ \Delta_{rich} := \left[\left[1-\phi(c_p^*(k))\right]k\right]^{-1} & \text{if}\ \ A > \hat{A}. \end{cases} \tag{A.8}$$

Note that $\Delta_{int}(A,k)$ exhibits the same behavior as the capitalists equilibrium profit in the domain $]\tilde{A},\hat{A}[$. Hence, to prove the above statements it suffices to show the following. (i) There exists a $\bar{k}$ sufficiently near by one such that $\Delta_{poor}(A_s,k) > \Delta_{rich}(A_l,k)$ and $\Delta_{poor}(A_s,k) \le \Delta_{rich}(A_l,k)$ for $\bar{k} < k < 1$ and $0 < k \le \bar{k}$, respectively, and A_s and A_l in the respective domains, (ii) that $\Delta_{int}(\tilde{A},k) < \Delta_{poor}(\tilde{A},k)$ for all k between zero and one, and (iii) that

$\Delta_{int}(\tilde{A}, k) < \Delta_{rich}(A, k)$ for all k in $]0, 1[$ and A in the respective domain.
(i) Note first that $c_p^*(k)$ is strictly increasing in k, because of

$$\begin{aligned} \text{sign}\frac{dc_p^*}{dk}(k) &= \text{sign}\frac{\partial^2 \Pi_p}{\partial c \partial k}(c_p^*, ., k) \\ &= \text{sign}\left[\frac{\partial \phi}{\partial c}(c_p^*) - [1 - \phi(c_p^*)]q'(c_p^*)\right] A > 0. \end{aligned} \tag{A.9}$$

$\Delta_{poor}(A_s, k) > \Delta_{rich}(A_l, k)$ is equivalent to $[1 - \phi(c_p^*(k))]k > 1 - \phi(c_c^*)$. Since, $1 - \phi(c_c^*) < 1 - \phi(c_p^*(k))$ ($c_c^* > c_p^*(k)$) and $c_p^*(k)$ is strictly increasing in k there must be a $\bar{k}$ between zero and one such that for $k > \bar{k}$ the stated inequality holds (and is reversed for $k \leq \bar{k}$).
(ii) Simple algebra and using the fact that equilibrium profits are continuous at $\tilde{A}$ shows that $\Delta_{int}(\tilde{A}, k) < \Delta_{poor}(\tilde{A}, k)$ which is equivalent to $[1 - \phi(c_c^*)]\tilde{A}q(c_{tp}(\tilde{A}, k)) < b$ holds true as long as $k < 1$.
(iii) Suppose the statement $\Delta_{int}(\tilde{A}, k) < \Delta_{rich}(., k)$ does not hold. That is, suppose there is some k such that $b \leq [1 - \phi(c_p^*(k))]k\tilde{A}q(c_{tp}(\tilde{A}, k))$ holds. With the help of the fact that $c_p^* > c_{tp}(\tilde{A}, k)$ this implies $b < [1 - \phi(c_{tp}(\tilde{A}, k))]k\tilde{A}q(c_{tp}(\tilde{A}, k))$. However, by the definition of $c_{tp}(A, k)$ the second term has to be exactly b, thus a contradiction. ■

References

Alesina, A., S. Özler, N. Roubini, and P. Swagel (1996), "Political Instability and Economic Growth", *Journal of Economic Growth*, 1, 189-211.

Alesina, A., and R. Perotti (1996), "Income distribution, political stability, and investment", *European Economic Review*, 40, 1203-1228.

Banerjee, A.V., and A.F. Newman (1998), "Information, the Dual Economy, and Development", *Review of Economic Studies*, 65, 631-653.

Baumol, W.J., and E.N. Wolff (1988), "Productivity Growth, Convergence, and Welfare: Reply", *American Economic Review*, 78, 5, 1155-1159.

Benhabib, J., and A. Rustichini (1996), "Social Conflict and Growth", *Journal of Economic Growth*, 1, 125-142.

De Long, J.B. (1988), "Productivity Growth, Convergence, and Welfare: Comment", *American Economic Review*, 78, 5, 1138-1154.

Devereux, M.B., and J.F. Wen (1998), "Political instability, capital taxation, and growth", *European Economic Review*, 42, 1635-1651.

Falkinger, J. (1999), "Social instability and redistribution of income", *European Journal of Political Economy*, 15, 35-51.

Fehr, E., and K. M. Schmidt (1999), "A Theory of Fairness, Competition and Cooperation", *Quarterly Journal of Economics*, 114, 3, 817-868.

Grossman, H.I. (1994), "Production, Appropriation, and Land Reform", *American Economic Review*, 84, 3, 705-712.

Grossman, H.I., and M. Kim (1995), "Swords or Plowshares? A Theory of the Security of Claims to Property", *Journal of Political Economy*, 103, 6, 1275-1288.

Grout, P.A. (1994), "Investment and wages in the absence of binding contracts: A Nash bargaining approach", *Econometrica*, 52, 2, 449-460.

Hirshleifer, J. (1991a), "The Technology of Conflict as an Economic Activity", *American Economic Review*, 81, 2, 130-134.

Hirshleifer, J. (1991b), "The Paradox of Power", *Economics and Politics*, 3, 3, 177-200.

Hirshleifer, J. (1995), "Anarchy and Its Breakdown", *Journal of Political Economy*, 103, 1, 26-52.

KUZNETS, S. (1955), "Economic Growth and Income Inequality", *American Economic Review*, 45, 1-28.

OLSON, M. (1982), *The Rise and Decline of Nations*, New Haven, CT: Yale University Press.

OLSON, M., JR. (1996), "Big Bills Left on the Sidewalk: Why Some Nations are Rich and Others Poor", *Journal of Economic Perspectives*, 10, 2, 3-24.

PEROTTI, R. (1996), "Growth, Income Distribution, and Democracy: What the Data Say", *Journal of Economic Growth*, 1, 149-187.

SKAPERDAS, S. (1991), "Conflict and Attitudes Toward Risk", *American Economic Review*, 81, 2, 116-120.

SKAPERDAS, S. (1992), "Cooperation, Conflict, and Power in the Absence of Property Rights", *American Economic Review*, 82, 4, 720-739.

SVENSSON, J. (1998), "Investment, property rights and political instability: Theory and Evidence", *European Economic Review*, 42, 1317-1341.

TEMPLE, J. (1999), "The New Growth Evidence", *Journal of Economic Literature*, 37, 2, 112-156.