Counter intuitive results in a Simple Model of Wage Negotiations*

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Abstract

Short-term contracts and exogenous productivity growth are introduced in a simple wage bargaining model. The equilibrium utilities corresponding to militant union behaviour are independent of the contract length. The wage dynamics are linear if strike is credible (low wage shares) and nonlinear otherwise (high wage shares). The model can admit two steady state wage shares. The one under strike is not credible exceeds the one under strike is credible. A wage decrease can occur if strike is credible, but never when strike is not credible. In the limit as time between bargaining rounds vanishes only the first paradox survives.

Key-words: wage bargaining, wage dynamics, chaos, strike

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1. Introduction

The strategic wage bargaining model in Fernandez and Glazer (1991), Haller (1991) and Haller and Holden (1990) is extended to allow for multiple wage contract renegotiations and productivity growth, by making the following more realistic assumptions. First, the parties can only agree upon short-term contracts. Second, contracts are incomplete in two different ways, namely a contract specifies a nominal wage that remains constant until it expires (i.e. wages cannot be contingent upon productivity growth) and future behaviour after the expiration date cannot be included in the contract. Third, the last expired contract remains in place until it is replaced by a new one. The latter is in accordance with the common practice in many Western countries, where it is forbidden by law that a firm unilaterally lowers wages after the expiration of a central agreement without the workers' consent, e.g. Holden (1997).

In Fernandez and Glazer (1991), Haller (1991) and Haller and Holden (1990) it is shown that wage increases can only occur in case the union's threat of going on strike is credible. Furthermore, in Fernandez and Glazer (1991) a brief remark is made with respect to short-term contracts: The union's minimum and maximum equilibrium[1] utility is not affected by assuming short-term contracts instead of everlasting contracts. The strategies that support this maximum equilibrium utility mimic everlasting contracts by having immediate agreement upon a wage increase equal to the wage increase in the model with everlasting contracts in the first short-term contract and, after the expiration of this first contract, all future negotiations are immediately concluded with a short-term contract that features *no* wage increase. Our interpretation of these strategies is as follows: The union is very militant in the first contract's negotiations by exploiting the threat of strike, while it behaves very peacefully in all

[1] By equilibrium we mean subgame perfect equilibrium.

future contracts' negotiations by refraining from this threat.

Since some unions are notorious for their aggressive behaviour in wage contracting it is more interesting to adapt the maximum equilibrium wage strategies such that the union uses the threat of strike whenever this threat is credible and only refrains from this threat in case it is not credible. The aim of our analysis is to investigate only this particular 'militant union' equilibrium (MUE).[2] By doing so, the credibility issue is incorporated into a dynamic context. Since both parties take into consideration how the current contract will affect future contracts' negotiations more interesting wage dynamics result than in Fernandez and Glazer (1991).

First, we show that the union's MUE utility coincides with its maximum equilibrium utility in Fernandez and Glazer (1991) after a minor modification for productivity growth. Thus, it is without loss of generality to assume everlasting contracts in deriving the present value of the stream of wages the union is able to subtract from the firm. Furthermore, the union's threat of strike is only credible if the *wage share* does not exceed some threshold. The union's MUE utility depends upon whether or not strike is credible.

Second, the MUE wage dynamics are derived from the union's MUE utility. In these dynamics it is crucial whether or not strike is credible during the current contract's negotiations and whether or not strike will be credible at the next contract's negotiations. This implies four cases. Each case induces a (possibly empty) domain on the interval of wage shares and each of these domains has its own wage dynamics. If strike is credible at both the current and the next contract's negotiations, then the wage dynamics are linear and easy to handle, while implicit[3] and nonlinear dynamics result if strike is not credible at both the current and the next contract's negotia-

[2] As in Fernandez and Glazer (1991) and Haller and Holden (1990) it is possible to derive inefficient equilibria which feature strike. Since doing so is by now a routine exercise, it is omitted.

[3] An implicit difference equation is defined as $f(x_{t+1}) = g(x_t)$, with f a noninvertible function.

tions. The other two domains imply a switch from strike is currently credible into strike is not credible at the next contract's negotiations or vice versa. The 'just one wage increase ever' equilibrium in Fernandez and Glazer (1991) and the MUE yield different paths of wages. Since both these equilibria yield the same present value but different wage dynamics this implies that the distribution of wages over time must differ. Rational parties are indifferent between these two paths of wages. Psychologically, however, the MUE path may be enjoyed because roughly speaking wages that keep up with profits which might be considered 'fair'.

Does the MUE admit a unique steady state wage share? For a large class of parameter values the answer is affirmative. However, we also derive sufficient conditions under which the linear and nonlinear wage dynamics both admit one steady state and that there is monotonic convergence toward each steady state within its domain of attraction. The steady state wage share on the domain where the union fails strike as a credible weapon is higher than the steady state wage share on the domain where strike is credible. Furthermore, the corresponding wage increase if strike is not credible is larger than the wage increase if strike is credible. These results are counter intuitive and quite different from the results in the literature.

Furthermore, we derive a sufficient condition for wage decreases to occur. Wage decreases never occur if the union fails strike as a credible weapon in the current and next contract's negotiations, but can occur either if strike is credible and the wage share is sufficiently high, or if strike is not currently credible but will be credible at the next contract's negotiations. Since credibility of strike is associated with wage increases this result is also counter intuitive. Wage decreases can only last for a short number of wage contracts and after that wages are forever increasing. The intuition is as follows. Wage decreases in the short run redistribute wages (and profits) over time in such a way that the long-run MUE wages must overtake the long-run wages corresponding to the 'just one wage increase ever' equilibrium path.

Finally, following Binmore, Rubinstein and Wolinsky (1986), we let the time between proposals vanish and we show that in this limit strike is always credible for the union independent of the wage share. In this limit only the 'linear' wage dynamics survive, there is monotonic convergence to the steady state wage share of these dynamics and the union is unable to grasp the entire surplus. As before, wage decreases can occur even in this limit for sufficiently high wage shares.

This paper is organized as follows. In section 3 the union's MUE utility and the MUE wage dynamics are derived. In section 4 the steady states and evolution of wage shares are analyzed. This section also contains the counter intuitive results. The limit behaviour of the MUE as the time between bargaining rounds vanishes is investigated in section 5. Section 6 concludes the paper, while the next section introduces the model.

2. The Model

Our wage bargaining model is basically the wage bargaining model in Fernandez and Glazer (1991), Haller (1991) and Haller and Holden (1990) in which the assumption of an everlasting contract is dropped and exogenous productivity growth is included. Time is discrete and time $t \in N$. In order to establish notation we define T, $0 < T < \infty$, as the contract length and $\pi \Leftrightarrow 1$, $\pi > 1$, as the growth rate of productivity corresponding to a learning-by-doing technology. The initial revenue generated by the firm is normalized to 1, meaning π^t is the revenue at time t. A holdout[4] is

[4] A holdout means that workers engage in production under the terms of the last expired contract.

assumed to be Pareto efficient,[5] with per period payoff w_t for the union at time t (w_t is the wage specified by the last contract's wage which is either still valid or expired at time t) and the firm's profit at time t is equal to $\pi^t \Leftrightarrow w_t$. Each party's payoff at time t corresponding to strike is normalized to 0. The union is only allowed to strike at time t if at time t the last agreed upon contract has expired. If one of the parties receives the infinite sequence of payoffs $\langle x_t \rangle_{t=0}^{\infty}$, where x_t is the payoff at time t, consistent with our description given above, then the *normalized* payoff to this party is simply $(1 \Leftrightarrow \delta) \sum_{t=0}^{\infty} \delta^t x_t$, $\delta \in (0,1)$, where δ denotes the common discount factor.[6] We assume that $\pi < \delta^{-1}$ in order to ensure that the present value at time t of the discounted stream of revenues from time t onward $(1 \Leftrightarrow \delta) \sum_{\tau=t}^{\infty} \delta^{\tau - t} \pi^{\tau} = \frac{1-\delta}{1-\delta\pi} \cdot \pi^t$ is well defined. From section 3 onwards we will use $F_t = \frac{1-\delta}{1-\delta\pi} \cdot \pi^t$ as short hand notation for this present value and we will call $\frac{w_t}{F_t}$ the wage share.

The bargaining process is identical to Fernandez and Glazer (1991). At bargaining round t, t even, the union demands a wage and at round t, t odd, the firm offers a wage. In case a proposed wage is rejected, the union can either strike for one round or holdout. An agreed upon (wage) contract w lasts T rounds, after which bargaining starts over again. There are no negotiations or strikes during the validation time of a contract. Note that w influences future negotiations by specifying the disagreement payoff in these negotiations. It is necessary to specify which party restarts the negotiations after the expiration of each contract. In order to make the calculations less tedious we impose that T is even, which means that if a party proposes a wage that is accepted in round t, then this party is the proposing party at the restart of the negotiations at $t + T$.

Finally, we stress the importance for including productivity growth. Suppose it

[5] A discussion of inefficient holdouts is postponed to the concluding remarks.

[6] A common discount factor avoids the problems reported in Bolt (1995).

is absent, i.e. $\pi = 1$. As pointed out in Fernandez and Glazer (1991), strike is not credible iff $\delta^2 < w_t$, with w_t the last contract's wage. Without going into details,[7] we mention that MUE strategies imply that *i)* only a finite number of wage increases can take place and *ii)* the last contract featuring a wage increase specifies a wage in between δ^2 and $1 \Leftrightarrow \delta (1 \Leftrightarrow \delta) < 1$. Hence, no productivity growth and MUE strategies impose a wage ceiling and once strike ceases to be credible it remains not credible forever.

However, many Western economies feature increasing wages over time. The above shows that to explain ongoing wage increases, productivity growth should be an essential ingredient of the model. The intuition is simple. With productivity growth the strategies associated with militant union behaviour are as follows. The wage share $\frac{w_t}{F_t}$ measures the relative size of the current wage, there is a threshold θ, $\theta \in [0, 1]$,[8] for which strike is credible iff $\frac{w_t}{F_t} \leq \theta$. If the militant union fails strike as a credible option, i.e. $\frac{w_t}{F_t} > \theta$, then it has to resort to holdout. However, due to productivity growth, i.e. $F_{t+\tau}$ increases in τ, the wage share will drop below θ within finite time. At that time, strike becomes a credible option again which will be fully utilized by the militant union.

3. The Militant Union Equilibrium

In this section we derive the union's MUE utility and the associated wage dynamics. As in Fernandez and Glazer (1991), Haller (1991) and Haller and Holden (1990) the union's minimum equilibrium *utility* is w_t, because the union can secure this payoff

[7] The following statements can be derived similarly as in the proofs of propositions 4.1 and 4.2 after substitution of $\pi = 1$ and a minor modification due to $l(w_t, 1) = \infty$ in case $w_t > \delta^2$. Furthermore, $l(w_t, 1) = \infty$ implies that $[R_1, R_2]$ in proposition 4.2 is empty.

[8] Actually, θ is the level for all t that are even, while the level for t odd is simply $\pi\theta$.

simply by holding out forever and never proposing nor agreeing upon wage contracts that prescribe a wage below w_t. If the union does not carry out the threat of a stutter strike in case $\frac{w_t}{F_t} \leq \theta$, then an immediate switch to the minimum wage equilibrium strategies occurs (no punishment is necessary for $\frac{w_t}{F_t} > \theta$). The latter is only needed in the derivation of θ and will be neglected in the rest of the analysis.

First, let the function $V_i^j(w_t, F_t)$, $i, j = 1, 2$, denote party i's equilibrium continuation payoff at the *start* of round t and from this round onward when party j is the proposing party as function of the state variables w_t and F_t. The militant union strategies feature immediate agreement at the expiration date of an old contract and, therefore, this equilibrium is Pareto efficient. Since $V_U^j(w_t, F_t)$ is the normalized discounted value of an infinite stream of wages and $V_F^j(w_t, F_t)$ is the firm's value for the stream of revenues minus the discounted stream of wages (with the same discount factor) we have that $V_U^j(w_t, F_t) + V_F^j(w_t, F_t) = F_t$, $j = U, F$. Thus, the firm's value functions $V_F^j(w_t, F_t) = F_t \Leftrightarrow V_U^j(w_t, F_t)$ can be ignored.

Second, if the wage share exceeds the threshold θ, then stutter strike is not credible at t, t even. However, due to the productivity growth, going on strike will become credible at $t + l(w_t, F_t)$, where $l(w_t, F_t)$ is the smallest integer that solves

$$l(w_t, F_t) = \arg\min_{l \in \mathrm{N}} l, \text{ s.t. } \frac{w_t}{\pi^l F_t} \leq \theta.$$

Note that it is without loss of generality to assume that $t + l(w_t, F_t) > 0$ is even, because in case $t + l(w_t, F_t)$ is odd the strategies prescribe a round of holdout in case of disagreement in this round and standard results from bargaining theory, e.g. Rubinstein (1982), imply that the union obtains its continuation payoff, i.e.

$$V_U^F\left(w_{t+l}, \pi^l F_t\right) = (1 \Leftrightarrow \delta)\, w_{t+l} + \delta V_U^U\left(w_{t+l}, \pi^{l+1} F_t\right),$$

where l denotes $l(w_t, F_t)$, and it is as if strike becomes credible at $t + l(w_t, F_t) + 1$. For computational convenience the real number $l(w_t, F_t) = (\ln \pi)^{-1}\left(\ln \frac{w_t}{F_t} \Leftrightarrow \ln \theta\right)$ is used throughout the analysis as if it were an even integer. It is sometimes useful to write $l\left(w_t, F_t\right) = 0$ in case $\frac{w_t}{F_t} \leq \theta$ at t even.

The following proposition characterizes the union's MUE utility and the threshold θ. All proofs are deferred to the appendix.

Proposition 3.1. *The functions $V_U^U\left(w_t, F_t\right)$ and $V_U^F\left(w_t, F_t\right)$ are given by*

$$V_U^U\left(w_t, F_t\right) = \begin{cases} \frac{\delta}{1+\delta} w_t + \frac{1}{1+\delta\pi} F_t, & \frac{w_t}{F_t} \leq \frac{\delta^2\pi^2(1+\delta)}{1+\delta\pi}, \\ w_t + (1 \Leftrightarrow \delta\pi)\left(\frac{(1+\delta\pi)}{\delta^2\pi^2(1+\delta)} \cdot \frac{w_t}{F_t}\right)^{1+\frac{\ln\delta}{\ln\pi}} F_t, & \frac{w_t}{F_t} > \frac{\delta^2\pi^2(1+\delta)}{1+\delta\pi}, \end{cases}$$

and

$$V_U^F\left(w_t, F_t\right) = \begin{cases} \frac{1}{1+\delta} w_t + \frac{\delta\pi}{1+\delta\pi} F_t, & \frac{w_t}{F_t} \leq \frac{\delta^2\pi^3(1+\delta)}{1+\delta\pi}, \\ w_t + (1 \Leftrightarrow \delta\pi)\left(\frac{(1+\delta\pi)}{\delta^2\pi^2(1+\delta)} \cdot \frac{w_t}{F_t}\right)^{2+\frac{\ln\delta}{\ln\pi}} F_t, & \frac{w_t}{F_t} > \frac{\delta^2\pi^3(1+\delta)}{1+\delta\pi}. \end{cases}$$

Moreover, $V_U^U\left(w_t, F_t\right) \geq w_t$ and $V_U^F\left(w_t, F_t\right) \geq w_t$.

The threshold $\theta = \frac{\delta^2\pi^2(1+\delta)}{1+\delta\pi} \in \left(\delta^2, 1\right)$ is increasing in π, $\pi \in \left(1, \delta^{-1}\right)$. Thus, productivity growth relaxes the credibility constraint. Furthermore, the expressions are independent of the contract length and, therefore, also hold for everlasting contracts, i.e. $T = \infty$. Consequently, the maximum wage equilibrium in Fernandez and Glazer (1991) and the militant union strategies only differ with respect to the distribution of per-round utilities over time if compared with each other.

We continue this section by deriving the wage dynamics under MUE strategies. Since T is even the MUE strategies induce an infinite sequence of consecutive contracts that are all proposed by the union and, therefore, we restrict attention to the wage proposed by the union. Basically, the wage w proposed by the union is the solution of

$$\left(1 \Leftrightarrow \delta^T\right) w + \delta^T V_U^U\left(w, \pi^T F_t\right) = V_U^U\left(w_t, F_t\right). \tag{3.1}$$

Since there are two different expressions for the function on each side there are four different cases. One remark is in place. In the next proposition only the wage is given that will be proposed in case the union did not deviate in the past, because otherwise the union's minimum equilibrium utility strategies prescribe $w(w_t, F_t) = w_t$.

Proposition 3.2. *At round t, t even, the union proposes $w(w_t, F_t)$ given by*

$$\begin{cases} \frac{\delta}{1+\delta-\delta^T} w_t + \frac{\left(1-\delta^T\pi^T\right)(1+\delta)}{\left(1+\delta-\delta^T\right)(1+\delta\pi)} F_t, & \textit{if } \frac{w_t}{F_t} \leq \min\{\theta^*, \theta\}, \\ w^*, & \textit{if } \frac{w_t}{F_t} \in (\theta^*, \theta], \\ \frac{1+\delta}{1+\delta-\delta^T} w_t + \frac{1+\delta}{1+\delta-\delta^T} \left[(1 \Leftrightarrow \delta\pi)\left(\frac{w_t}{\theta F_t}\right)^{1+\frac{\ln\delta}{\ln\pi}} \Leftrightarrow \frac{\delta^T\pi^T}{1+\delta\pi}\right] F_t, & \textit{if } \frac{w_t}{F_t} \in (\theta, 1] \cap [R_1, R_2], \\ \widetilde{w}, & \textit{if } \frac{w_t}{F_t} \in (\theta, R_1) \cup (R_2, 1]. \end{cases}$$

where $w^* > w_t$ corresponds to the largest of the two real roots of

$$\frac{w^*}{F_t} + \delta^T\pi^T (1 \Leftrightarrow \delta\pi)\left(\frac{w^*}{\theta\pi^T F_t}\right)^{1+\frac{\ln\delta}{\ln\pi}} \Leftrightarrow \frac{\delta}{1+\delta}\frac{w_t}{F_t} \Leftrightarrow \frac{1}{1+\delta\pi} = 0, \tag{3.2}$$

$\widetilde{w} > w_t$ corresponds to the largest of the two real roots of

$$\frac{\widetilde{w}}{F_t} + \delta^T\pi^T (1 \Leftrightarrow \delta\pi)\left(\frac{\widetilde{w}}{\theta F_t}\right)^{1+\frac{\ln\delta}{\ln\pi}} \Leftrightarrow \frac{w_t}{F_t} \Leftrightarrow (1 \Leftrightarrow \delta\pi)\left(\frac{w_t}{\theta F_t}\right)^{1+\frac{\ln\delta}{\ln\pi}} = 0 \tag{3.3}$$

and $\theta = \frac{\delta^2\pi^2(1+\delta)}{(1+\delta\pi)}$, $\theta^* = \delta^{-1}\left[\left(1+\delta \Leftrightarrow \delta^T\right)\delta^2\pi^{T+2} \Leftrightarrow \left(1 \Leftrightarrow \delta^T\pi^T\right)\right]\frac{1+\delta}{1+\delta\pi}$ and $R_2 > R_1$ are the two positive, real roots of $x + (1 \Leftrightarrow \delta\pi)\left(\theta^{-1}x\right)^{1+\frac{\ln\delta}{\ln\pi}} \Leftrightarrow \pi^T\theta \Leftrightarrow \delta^T\pi^T(1 \Leftrightarrow \delta\pi) = 0$.

The four cases mentioned above partition the interval of wage shares. The first case, $\frac{w_t}{F_t} \leq \min\{\theta^*, \theta\}$, corresponds to strike is both credible in the current and next contract's negotiations. The second case, $\frac{w_t}{F_t} \in [R_1, R_2] \cap (\theta, 1]$, corresponds to the situation in which strike is not credible at time t but strike is credible at the expiration date of $w(w_t, F_t)$. Thus, the militant union has one more contract to go before strike becomes credible. The third case, $\frac{w_t}{F_t} \in (\theta^*, \theta]$, is the transition from strike is credible in the current contract's negotiations to strike is not credible at the next contract's negotiations. So, the militant union has one more contract to go before strike ceases to be credible. The fourth case, $\frac{w_t}{F_t} \in (\theta, R_1) \cup (R_2, 1]$, corresponds to strike is not credible at the current contract's negotiations and it will neither be credible at the expiration date of $w(w_t, F_t) = \widetilde{w}$. Despite the fact that strike is not credible the union negotiates a higher wage $\widetilde{w} > w_t$. The reason is that both parties anticipate a higher wage if they do not agree for the next $l(w_t, F_t)$ rounds and impatience makes the union willing to accept a lower wage increase today in order to avoid the waiting. This result differs from the results in the literature where holdouts are associated with no wage increase. So, militant behaviour guarantees a wage increase even in rounds where strike is not credible.

Based upon the following condition and its complement we distinguish two mutually exclusive cases for which sharper results can be obtained, which are given in the next proposition. This condition is given by

$$\delta^2\pi^2 \leq \delta^{-1}\left(\pi^{T+2}\left(\delta^2 + \delta^3\right) + \delta^T\pi^T\left(1 \Leftrightarrow \delta^2\pi^2\right) \Leftrightarrow 1\right). \tag{3.4}$$

Proposition 3.3. ***1.*** *If condition (3.4) is satisfied, then* $(\theta^*, \theta]$ *and* (θ, R_1) *are both empty and* $[R_1, R_2] \cap (\theta, 1] = \left(\theta, \pi^T\theta\right]$. *Moreover,* $\frac{w_t}{F_t} \leq \pi^T\theta$ *implies* $\frac{w(w_t,F_t)}{\pi^T F_t} \leq \theta$, *and* $w(w_t, F_t)$ *is discontinuous in* $w_t = \pi^T\theta F_t$.

2. *If condition (3.4) does not hold, then* $(\theta^*, \theta]$ *and* (θ, R_1) *are both not empty and* $[R_1, R_2] \cap (\theta, 1] = [R_1, R_2]$.

Thus, under condition (3.4), if $\frac{w_t}{F_t} \leq \theta$, then also $\frac{w(w_t,F_t)}{\pi^T F_t} \leq \theta$. Hence, once strike is credible it remains credible in all future contracts' negotiations. If $\frac{w_t}{F_t}$ is slightly above θ, i.e. $\frac{w_t}{F_t} \in \left(\theta, \pi^T\theta\right]$, then the militant union is one contract away from the situation in which strike will be credible forever. Otherwise, if condition (3.4) does not hold, then for every $\frac{w_t}{F_t} \in (\theta, R_1)$ strike is not credible in both the current and next contract's negotiations. The percentage wage increase for $\frac{w_t}{F_t} \in (\theta, R_1)$ has to be larger than the percentage wage increase for $\frac{w_t}{F_t} \in [R_1, R_2]$, because for the latter (former) the wage share at the expiration date of $w(w_t, F_t)$ is smaller (larger) than θ. Clearly, $w(w_t, F_t)$ is decreasing in $\frac{w_t}{F_t}$ around $\frac{w_t}{F_t} = R_1$. A plausible explanation is the following. The wage increase for $\frac{w_t}{F_t} \in (\theta^*, \theta]$ is relatively large, because strike will not be credible at the next contract's negotiations. Productivity growth and $\frac{w_t}{F_t}$ slightly above θ imply that the union's equilibrium utility is approximately the equilibrium utility at θ, because $l(w_t, F_t) \approx 0$ (read: 2 rounds of bargaining). Therefore, the wage the union negotiates for $\frac{w_t}{F_t}$ slightly above θ is roughly the same wage as at θ. As $\frac{w_t}{F_t}$ approaches R_1 this effect diminishes, i.e. $l(w_t, F_t)$ increases, and the wage negotiated at $\frac{w_t}{F_t} = R_1$ is equal to $w^* = \pi^T \theta F_t \geq R_1 F_t$.[9]

4. Steady State Wage Shares

In this section steady state wage shares and monotonic convergence to such states, under the MUE strategies, are investigated. The wage share $x^* \in [0, 1]$ is a steady state wage share if $\frac{w_t}{F_t} = x^*$ implies that $\frac{w(w_t, F_t)}{F_t} = \pi^T x^*$. So, at time $t + T$, i.e. the expiration date of $w(w_t, F_t)$, the wage share will be equal to the wage share at t. Wage shares that induce a transition from strike is credible at the current contract's negotiations into strike is not credible at the next contract's negotiations or vice versa cannot be steady states. As indicated in proposition 3.3, the wage dynamics depend on condition (3.4) to hold or not to hold. The next two subsections state the long-run results in each of these two cases.

[9] This holds because either R_1 or R_2 equals $\pi^T \theta$ and $R_1 < R_2$, see the proof of proposition 3.2.

4.1. The case condition (3.4) holds

If condition (3.4) holds, then once strike is credible it remains credible forever. The first proposition states that this implies a unique feasible steady state on $[0, \theta]$.

Proposition 4.1. *The linear wage dynamics on $[0, \theta]$ admit a unique steady state wage share $x^L \in [0, \theta]$ iff condition (3.4) is satisfied . Moreover,*

$$x^L = \frac{\left(1 \Leftrightarrow \delta^T \pi^T\right) \theta}{\delta^2 \pi^2 \left(\pi^T \left(1 + \delta \Leftrightarrow \delta^T\right) \Leftrightarrow \delta\right)}$$

and there is monotonic convergence in wage shares to x^L for all $\frac{w_t}{F_t} \leq \pi^T \theta$.

Note that the wage share (and, consequently, also the wage) is forever monotonically increasing if the initial wage share lies below x^L. However, if initially the wage share lies above x^L, then even a militant union which has strike as a credible option at its disposal cannot prevent that the wage share will decrease over time.

The next proposition states necessary and sufficient conditions under which the nonlinear dynamics on $\left[\pi^T \theta, 1\right]$ admit a unique feasible steady state as well as the necessary and sufficient conditions for monotonic convergence in wage shares to it.

Proposition 4.2. *If condition (3.4) is satisfied, then the nonlinear wage dynamics on $\left(\pi^T \theta, 1\right]$ admit a unique steady state wage share $x^{NL} \in \left(\pi^T \theta, 1\right]$ iff*

$$\left(\pi^T \Leftrightarrow 1\right) \delta^2 \pi^2 (1 + \delta) \Leftrightarrow \left(1 \Leftrightarrow \delta^2 \pi^2\right) \left(1 \Leftrightarrow \delta^{2T} \pi^{2T}\right) \delta^T < 0. \tag{4.1}$$

There is monotonic convergence in wage shares towards x^{NL} for all $\frac{w_t}{F_t} \in \left(\pi^T \theta, 1\right]$ iff

$$\left(\pi^T \Leftrightarrow 1\right) \left|1 + \frac{\ln \delta}{\ln \pi}\right| \Leftrightarrow \left(1 \Leftrightarrow \delta^{2T} \pi^{2T}\right) < 0. \tag{4.2}$$

The condition for monotonic convergence is independent of the condition for feasibility. Conditions (4.1) and (4.2) both require that π should not be too high at a given δ and T, or T should not be too large given δ and π. Numerical investigation of these conditions revealed that there exists a generic and non-empty subclass of parameter values for which the conditions for feasibility and monotonic convergence are simultaneously satisfied.

Note that, for any steady state x^*, the parties agree upon the wage $w = \pi^T F_t x^*$ if $\frac{w_t}{F_t} = x^*$. But then, the wage increase in x^L is smaller than the wage increase in x^{NL}. Furthermore, if condition (4.2) is satisfied, then the wage share monotonically increases over time on $\left(\pi^T\theta, x^{NL}\right)$ even though strike is not credible. The latter results and $x^L < x^{NL}$ are all counter intuitive, because the higher steady state wage share and the higher wage increase correspond to the situation in which strike is not credible at the current wage share while the lower steady state corresponds to the situation in which the union has strike as a credible threat at its disposal and strike will remain credible forever. These results are quite different from the results in Fernandez and Glazer (1991), Haller (1990) and Haller and Holden (1990).

The next corollary states the necessary and sufficient conditions for x^L to be the unique steady state on $[0,1]$. Monotonic convergence to the x^L from every wage share is automatically ensured in this case .

Corollary 4.3. *If condition (3.4) holds, then the MUE strategies admit a unique steady state wage share iff condition (4.1) does not hold. Moreover, x^L of proposition 4.1 is the unique steady state wage share and there is monotonic convergence in wage shares on $[0,1]$ to x^L.*

Finally, an increasing wage share is unambiguously associated with a wage increase. However, a decreasing wage share implies either a small wage increase or perhaps even a wage decrease. The next proposition states a sufficient condition for a wage *decrease* to occur on a non-empty interval of wage shares.

Proposition 4.4. *Suppose condition (3.4) holds. If* $\frac{1-\delta^T\pi^T}{\delta^2\pi^2\left(1-\delta^T\right)} < 1$, *then* $w\left(w_t, F_t\right) < w_t$ *iff* $\frac{w_t}{F_t} \in \left(\frac{1-\delta^T\pi^T}{\delta^2\pi^2\left(1-\delta^T\right)}\theta, \pi^T\theta\right]$. *Moreover,* $\frac{1-\delta^T\pi^T}{\delta^2\pi^2\left(1-\delta^T\right)}\theta > x^L$.

Numerical investigation revealed that this sufficient condition implies that condition (3.4) is automatically satisfied. Furthermore, there exists a generic and non-empty subclass of parameter values for which the sufficient condition is satisfied. The sufficient condition can be relaxed, because there exists a (possibly empty) interval $\left(\lambda\theta, \pi^T\theta\right]$, for some $\lambda > 1$,[10] for which wage decreases occur in case $\frac{1-\delta^T\pi^T}{\delta^2\pi^2\left(1-\delta^T\right)} \geq 1$.

Note that it is counter intuitive that a wage decrease occurs in or near the region where strike is credible, but not in a region where strike is not credible. The number of consecutive contracts featuring a wage decrease is bounded from above, because the long-run wage approximates $x^L F_t$ and F_t goes to infinity as t goes to infinity. So, after a finite number of contracts, wages exceed the initial level. This implies that for the union the initial wage decreases are compensated by higher wage increases in the future. For the firm it is the other way around. The initial wage decreases compensate for a faster growth of wages in the future. Somehow, initial wage decreases can be regarded as a redistribution of the growing surplus.

4.2. The case condition (3.4) does not hold

If condition (3.4) does not hold, then the linear dynamics do not admit a feasible steady state. Still the wage share monotonically increases on the domain $[0, \theta^*]$ and

[10] The bound $\lambda\theta$ corresponds to one of two roots of the polynomial in the proof of proposition 4.4.

eventually, after a finite number of contracts, the wage share will be larger than θ^*.[11] It is not possible to state an analog of proposition 4.2 for the only candidate steady state x^{NL}. However. Note that the domain $(\theta, R_1) \cup (R_2, 1]$ of nonlinear dynamics now consists of two disjunct intervals, while (3.3) maps the wage share at the expiration date of the new contract to the image $(\theta, 1]$ and, hence, possibly to $[R_1, R_2]$, which implies a drop in the wage share below θ at the expiration date of the next contract. The following example, in which x^{NL} is feasible, is very instructive.

Example Consider $\delta = 0.3$, $\pi = 1.5$ and $T = 2$, for which condition (3.4) does not hold. Then $\theta^* = \Leftrightarrow 0.738 < 0$ (so, $[0, \min\{\theta^*, \theta\}]$ is empty), $\theta = 0.182$, $R_1 = 0.272$, $R_2 = \pi^T \theta = 0.408$ and $x^{NL} = 0.241 \in (\theta, R_1)$. So, x^{NL} is feasible. Starting at $\frac{w_0}{F_0} = 0.5 > R_2$, the wage share at the expiration date of the first contract belongs to (θ, R_1) and from there on the wage share converges to x^{NL}. However, starting at $\frac{w_t}{F_t} = 0.6$, the wage share at the expiration date of the first contract enters the interval $[R_1, R_2]$ and very soon converges to a limit cycle of periodicity two, which is given by $0.307 \in [R_1, R_2]$ and $0.173 \in [\theta^*, \theta]$. The latter can also happen from below: Starting at $\frac{w_0}{F_0} = 0.2$, the wage share overshoots x^{NL} and, by entering the interval $[R_1, R_2]$, starts converging toward this limit cycle. Thus, convergence to a steady state is in some cases even absent, depending upon the initial wage share, and the wage dynamics exhibit chaotic behaviour.

The overall picture is more complicated. Consider first the case in which x^{NL} is feasible. Suppose $\frac{w_t}{F_t} \in [0, \theta]$. Then within a finite number of contracts the wage share lies above θ and it enters either $[R_1, R_2]$ or $(\theta, R_1) \cup (R_2, 1]$. If the former happens,

[11] This follows directly from the proof of proposition 4.1 if $[0, 1]$ is taken as the domain of the linear dynamics and observing that monotonic convergence to x^L on $[0, 1]$ holds even if $x^L > \theta$.

then the dynamics are back in $[0, \theta]$ again. If the wage share enters $(\theta, R_1) \cup (R_2, 1]$, then either there is convergence to x^{NL} or within a finite number of contracts the wage share enters $[R_1, R_2]$ and, consequently, the dynamics return to $[0, \theta]$ again. Due to the nonlinearity of the dynamics and the implicitly given bounds R_1 and R_2 one has to resort to numerical methods in order to obtain results. We did not pursue this line of research.

Finally, consider the second case in which x^{NL} is not feasible. Then convergence to x^{NL} as described in the previous case cannot occur and the most one can obtain are limit cycles (if these exist at all). These limit cycles are not restricted to two-round cycles as in the example above, but may exhibit more complicated chaotic behaviour in which the wage share also stays in $[0, \theta^*]$ and $(\theta, R_1) \cup (R_2, 1]$ for a finite number of consecutive contracts.

5. Limit Results

In this section we follow the literature on strategic bargaining by letting the time between bargaining rounds vanish (e.g. Binmore et al., 1986) and, meanwhile, maintaining a constant level of the contract length measured in real time.

In order to make the analysis precise we define Δ, $\Delta > 0$, as the time between two bargaining rounds, $\delta = e^{-r\Delta}$, where r denotes the interest rate, and $\pi = e^{\rho\Delta}$, where ρ denotes the growth rate of productivity. The assumption $1 < \pi < \delta^{-1}$ implies that $0 < \rho < r$. Furthermore, we define $L > 0$ as the contract length measured in real time. As said before, we keep L constant while Δ goes to 0. Since $L = T\Delta$ we can only keep L constant while Δ vanishes if T adjusts to changes in Δ, which means that $T = L/\Delta$.[12] For simplicity we neglect that $T = L/\Delta$ should be an even integer.

[12]Note that keeping T fixed would mean that $\lim_{\Delta \to 0} L = \lim_{\Delta \to 0} T\Delta = 0$, i.e. the contract

The following theorem states the limit results.

Theorem 5.1. *In the limit, as* Δ *goes to* 0*, strike is credible for every* $\frac{w_t}{F_t} \in [0,1]$*, the union proposes the wage* $\left(2 - e^{-rL}\right)^{-1} \left[w_t + \left(1 - e^{-(r-\rho)L}\right) \frac{r}{r-\rho} e^{\rho t}\right]$ *for every* $\frac{w_t}{F_t} \in [0,1]$ *and its MUE utility is* $\frac{1}{2}\left(w_t + \frac{r}{r-\rho} e^{\rho t}\right)$ *for every* $\frac{w_t}{F_t} \in [0,1]$*. Moreover, there is monotonic convergence in wage shares on* $[0,1]$ *to* $\lim_{\Delta \to 0} x^L = \frac{1-e^{-(r-\rho)L}}{e^{\rho L}(2-e^{-rL})-1} < 1$ *and the wage decreases if* $\frac{w_t}{F_t} \in \left(\frac{1-e^{-(r-\rho)L}}{1-e^{-rL}}, 1\right]$*.*

This theorem implies that the union is unable to grasp the entire surplus, i.e. $\lim_{\Delta \to 0} x^L$ is bounded away from 1. In the limit wage decreases can occur, but, as before, only a limited number of consecutive contracts can feature a wage decrease. Furthermore, the union's limit MUE utility corresponds to the Nash bargaining solution with everlasting contracts and disagreement point $(w_t, 0)$, i.e.

$$\max_{w} \left(w - w_t\right) \left(\frac{r}{r-\rho} e^{\rho t} - w\right) \Rightarrow w = \frac{1}{2}\left(w_t + \frac{r}{r-\rho} e^{\rho t}\right).$$

This means that the limit results can be obtained by a simple two-step procedure. First, compute the union's limit MUE utility by applying this Nash bargaining solution and, second, derive the wage dynamics from the union's limit MUE utility.

Finally, the threshold θ goes to 1 as the time between bargaining rounds vanishes, i.e. $\lim_{\Delta \to 0} \theta = 1$. The reason is that the equilibrium condition for strike to be credible is given by $w_t \leq \left(1 - e^{-r\Delta}\right) \cdot 0 + e^{-r\Delta} V_U^F\left(w_t, e^{\rho\Delta} F_t\right)$, t even. Since $V_U^F\left(w_t, \pi F_t\right) > w_t$ for all $\Delta > 0$ this condition requires $e^{-r\Delta}$ sufficiently close to 1, which can be accomplished for $\Delta > 0$ sufficiently close to 0. This equilibrium condition easily generalizes to a large class of extended models, provided w_t is replaced by the union's minimum equilibrium utility in such model.

length measured in real time vanishes. Then, in the limit, parties would constantly negotiate new contracts which expire instantaneously at the moment of conclusion. Clearly, this is unrealistic.

6. Concluding Remarks

In this paper we have extended the wage bargaining model by Fernandez and Glazer (1991), Haller (1991) and Haller and Holden (1990), by allowing for finite contract length and productivity growth. By doing so, credibility over time of the strike threat enters the analysis. However, the credibility issue vanishes as time between bargaining rounds goes to zero and matters simplify greatly in the limit. The results then hint that a large class of extended models can be analyzed using the Nash bargaining solution with the appropriate disagreement points. For instance one could think of inefficient holdouts, e.g. Holden (1996) and Moene (1988), or introducing competition among strategic options for the union, such as holdout, work-to-rule and strike, e.g. Houba and Bolt (1997). Inefficient holdouts only require a minor modification of the union's minimum equilibrium utility.[13] Competition among strategic options would yield that all actions are credible in the limit and that strike is the union's most effective action among the credible actions.

Another extension is to allow for endogenous contract length, meaning that the parties negotiate a wage and an expiration date of the contract. Then the union's (extended) MUE utilities coincide with those of proposition 3.1. The reason is that the union's (extended) MUE utilities solve a recursive relation that is independent of both the contract's wage and the contract's length. Consequently, the contract agreed upon is no longer uniquely specified, but if the parties agree upon a particular contract's length then proposition 3.2 states the associated contract's wage. As in the current proposition,the contract's wage and the contract's length are negatively related and both parties are indifferent. Thus, this extended model is to simple in

[13]In Holden (1997) the Nash bargaining solution is applied in a model with one-year contracts. However, the assumption that (inefficient) holdouts determine the disagreement point avoids the credibility issue raised in our analysis. Implicitly, wage decreases can occur in Holden (1997), because workers have an outside option and conditions for a binding outside option are derived.

order to offer a theory with predictive power about endogenous contract length.

The results of section 5 show that the apparently minor extensions lead to major changes in the dynamics of the wage level and the wage share. Moreover, one soon meets the boundaries of where analytical results for the evolution of wages can be obtained. For instance, π in our model is fixed, i.e. not influenced by investment decisions of the firm. It would be interesting to incorporate these decisions in the model, thereby making the parameter π endogenous. Another approach could be to let π be a known function, fluctuating over time, representing business cycle effects. Within the simple setup of this paper all one can hope for is analytical solutions for the value functions.

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7. Appendix

Proof of proposition 3.1

Suppose there is a threshold $\theta < 1$ such that at t is even strike is credible iff $\frac{w_t}{F_t} \leq \theta$. If $\frac{w_t}{F_t} > \theta$, then strike becomes credible at $t + l$ "even". There are two cases to be considered.

Case 1 $\frac{w_t}{F_t} > \theta$. Then $t + l$ is the first round strike will be credible again and, hence, $V_U^U\left(w_{t+l}, F_{t+l}\right)$ corresponds to the function at which strike is credible. Until $t + l$ strike is not credible and w_t is the union's disagreement payoff. The firm's problem at $\tau + 1$, $\tau = t + l \Leftrightarrow 2, t + l \Leftrightarrow 4, \ldots, t$ (τ even), is given by

$$F_{\tau+1} \Leftrightarrow V_U^F\left(w_{\tau+1}, F_{\tau+1}\right) = \max_{w_F} F_{\tau+1} \Leftrightarrow \left(1 \Leftrightarrow \delta^T\right) w_F \Leftrightarrow \delta^T V_U^F\left(w_F, \pi^T F_{\tau+1}\right),$$

$$\text{s.t.} \quad \left(1 \Leftrightarrow \delta^T\right) w_F + \delta^T V_U^F\left(w_F, \pi^T F_{\tau+1}\right) = \left(1 \Leftrightarrow \delta\right) w_{\tau+1} + \delta V_U^U\left(w_{\tau+1}, \pi F_{\tau+1}\right).$$

Substitution of the constraint into the objective function and rewriting yields

$$V_U^F\left(w_{\tau+1}, F_{\tau+1}\right) = \left(1 \Leftrightarrow \delta\right) w_{\tau+1} + \delta V_U^U\left(w_{\tau+1}, \pi F_{\tau+1}\right). \tag{7.1}$$

Similarly, the union's problem at τ given by

$$V_U^U\left(w_\tau, F_\tau\right) = \max_{w_U} \left(1 \Leftrightarrow \delta^T\right) w_U + \delta^T V_U^U\left(w_U, F_{\tau+T}\right),$$

$$\text{s.t.} \quad F_\tau \Leftrightarrow \left(1 \Leftrightarrow \delta^T\right) w_U \Leftrightarrow \delta^T V_U^U\left(w_U, F_{\tau+T}\right) = F_\tau \Leftrightarrow \left(1 \Leftrightarrow \delta\right) w_\tau \Leftrightarrow \delta V_U^F\left(w_\tau, \pi F_\tau\right)$$

yields

$$V_U^U(w_\tau, F_\tau) = (1 \Leftrightarrow \delta) w_\tau + \delta V_U^F(w_\tau, \pi F_\tau). \tag{7.2}$$

Furthermore, if the parties would not agree at τ then at $\tau+1$ we have that $w_{\tau+1} = w_\tau$ and $F_{\tau+1} = \pi F_\tau$. Making use of these two equalities and substitution of (7.1) into (7.2) yields the recursive relation

$$V_U^U(w_\tau, F_\tau) = \left(1 \Leftrightarrow \delta^2\right) w_\tau + \delta^2 V_U^U\left(w_\tau, \pi^2 F_\tau\right), \ \tau = t + l \Leftrightarrow 2, t + l \Leftrightarrow 4, \ldots, t.$$

Solving the recursion yields

$$V_U^U(w_t, F_t) = \left(1 \Leftrightarrow \delta^l\right) w_t + \delta^l V_U^U(w_t, F_{t+l}), \text{ for } \frac{w_t}{F_t} > \theta \tag{7.3}$$

and $V_U^U(w_t, F_{t+l})$ refers to case 2, i.e. $\frac{w_t}{F_{t+l}} \leq \theta$.

Case 2 $\frac{w_t}{F_t} \leq \theta$ at t even. Then going on strike is credible. The union's problem (7.2) at t, t is even, is different and is now given by

$$V_U^U(w_t, F_t) = \max_{w_U} \left(1 \Leftrightarrow \delta^T\right) w_U + \delta^T V_U^U\left(w_U, \pi^T F_t\right)$$

$$\text{s.t. } F_t \Leftrightarrow \left(1 \Leftrightarrow \delta^T\right) w_U \Leftrightarrow \delta^T V_U^U\left(w_U, \pi^T F_t\right) = \delta \pi F_t \Leftrightarrow \delta V_U^F(w_t, \pi F_t),$$

$$V_U^U\left(w_U, \pi^T F_t\right) = \left(1 \Leftrightarrow \delta^{l\left(w_U, \pi^T F_t\right)}\right) w_U + \delta^{l\left(w, \pi^T F_t\right)} V_U^U\left(w_U, F_{t+l(w, \pi^T F_t)}\right).$$

The second constraint comes from case 1 and is necessary in order to take into account w_U's such that $\frac{w_U}{\pi^T F_t} > \theta$ at $t+T$. For $\frac{w_U}{\pi^T F_t} \leq \theta$ we take $l\left(w_U, \pi^T F_t\right) = 0$ and then this constraint is superfluous. As before, substitution of the first constraint into the objective function yields

$$V_U^U(w_t, F_t) = (1 \Leftrightarrow \delta\pi) F_t + \delta V_U^F(w_t, \pi F_t). \tag{7.4}$$

The firm's problem at $t+1$, t is even, does not change and, hence, equation (7.1) is also valid in this case for $\tau + 1 = t + 1$. If the parties would not agree at t, t is even, then at $t+1$ (odd) we have that $w_{t+1} = w_t$ and $F_{t+1} = \pi F_t$. Making use of these two equalities and substitution of (7.1) (for $\tau + 1 = t + 1$) into (7.4) yields

$$V_U^U\left(w_t, F_t\right) = \left(1 \Leftrightarrow \delta\pi\right) F_t + \delta\left(1 \Leftrightarrow \delta\right) w_t + \delta^2 V_U^U\left(w_t, \pi^2 F_t\right), \text{ for } \frac{w_t}{F_t} \leq \theta, \tag{7.5}$$

It is easy to verify that

$$V_U^U\left(w_t, F_t\right) = \frac{\delta}{1+\delta} w_t + \frac{1}{1+\delta\pi} F_t, \text{ for } \frac{w_t}{F_t} \leq \theta,$$

is a solution of (7.5). Then the expression in (7.3) becomes

$$V_U^U\left(w_t, F_t\right) = \frac{1+\delta \Leftrightarrow \delta^l}{1+\delta} w_t + \frac{\delta^l \pi^l}{1+\delta\pi} F_t, \text{ for } \frac{w_t}{F_t} > \theta. \tag{7.6}$$

Substitution of $l = \ln\left(\frac{w_t}{\theta F_t}\right) / \ln\pi$ and making use of $a^l = e^{l \ln a} = \left(\frac{w_t}{\theta F_t}\right)^{\ln a / \ln \pi}$ yields

$$V_U^U\left(w_t, F_t\right) = w_t \Leftrightarrow \frac{\left(\frac{w_t}{\theta F_t}\right)^{\frac{\ln\delta}{\ln\pi}}}{1+\delta} \cdot \frac{w_t}{\theta F_t} \cdot \theta F_t + \frac{\left(\frac{w_t}{\theta F_t}\right)^{1+\frac{\ln\delta}{\ln\pi}}}{1+\delta\pi} F_t, \text{ for } \frac{w_t}{F_t} > \theta,$$

which yields the stated expression. Similar arguments yield $V_U^F\left(w_t, F_t\right)$ at round t is odd. It is easy to verify that $V_U^U\left(w_t, F_t\right) \geq w_t$ and $V_U^F\left(w_t, F_t\right) \geq w_t$.

Finally, strike at t, t even, is credible iff $w_t \leq \left(1 \Leftrightarrow \delta\right) \cdot 0 + \delta V_U^F\left(w_t, \pi F_t\right)$. Solving $\widehat{w}$ in $\widehat{w} = \delta V_U^F\left(\widehat{w}, \pi F_t\right)$ yields $\widehat{w} = \delta^2 \pi^2 \frac{1+\delta}{1+\delta\pi} F_t$. Hence, $\theta = \frac{\widehat{w}}{F_t} = \delta^2\pi^2 \frac{1+\delta}{1+\delta\pi} < 1$, where θ is the threshold postulated at the beginning of the proof. QED

Proof of proposition 3.2

Four different cases in (3.1) have to be distinguished.

First, if $\frac{w_t}{F_t} \leq \theta$ and $\frac{w}{\pi^T F_t} \leq \theta$, then w solves (3.1) iff

$$\left(1 \Leftrightarrow \delta^T\right) w + \delta^T \left(\frac{\delta}{1+\delta} w + \frac{1}{1+\delta\pi} \pi^T F_t\right) = \frac{\delta}{1+\delta} w_t + \frac{1}{1+\delta\pi} F_t,$$

which yields the solution

$$w = \frac{\delta}{1+\delta \Leftrightarrow \delta^T} w_t + \frac{\left(1 \Leftrightarrow \delta^T \pi^T\right)\left(1+\delta\right)}{\left(1+\delta \Leftrightarrow \delta^T\right)\left(1+\delta\pi\right)} F_t.$$

The condition $\frac{w}{F_t} \leq \pi^T \theta$ can be rewritten as

$$\frac{w_t}{F_t} \leq \delta^{-1} \frac{1+\delta}{1+\delta\pi} \left(\delta^T \pi^T \left(1 \Leftrightarrow \delta^2 \pi^2 \right) + \pi^{T+2} \left(\delta^2 + \delta^3 \right) \Leftrightarrow 1 \right) \equiv \theta^*.$$

Combining the two conditions $\frac{w_t}{F_t} \leq \theta$ and $\frac{w}{F_t} \leq \pi^T \theta$ yields $\frac{w_t}{F_t} \leq \min \{\theta^*, \theta\}$.

Second, if $\frac{w_t}{F_t} > \theta$ and $\frac{w}{\pi^T F_t} \leq \theta$, then w solves (3.1) iff

$$\left(1 \Leftrightarrow \delta^T\right) w + \delta^T \left(\frac{\delta}{1+\delta} w + \frac{1}{1+\delta\pi} \pi^T F_t \right) = w_t + (1 \Leftrightarrow \delta\pi) \left(\frac{w_t}{\theta F_t} \right)^{1+\frac{\ln \delta}{\ln \pi}} F_t.$$

Solving for w yields

$$w = \frac{1+\delta}{1+\delta \Leftrightarrow \delta^T} w_t + \frac{1+\delta}{1+\delta \Leftrightarrow \delta^T} \left((1 \Leftrightarrow \delta\pi) \left(\frac{w_t}{\theta F_t} \right)^{1+\frac{\ln \delta}{\ln \pi}} \Leftrightarrow \frac{\delta^T \pi^T}{1+\delta\pi} \right) F_t.$$

The condition $\frac{w}{F_t} \leq \pi^T \theta$ can be rewritten as

$$\frac{w_t}{F_t} + (1 \Leftrightarrow \delta\pi) \left(\frac{w_t}{\theta F_t} \right)^{1+\frac{\ln \delta}{\ln \pi}} \Leftrightarrow \pi^T \theta \Leftrightarrow \delta^T \pi^T (1 \Leftrightarrow \delta\pi) \leq 0. \tag{7.7}$$

The polynomial is decreasing for $\frac{w_t}{F_t}$ small and increasing for $\frac{w_t}{F_t}$ large, because $1+\frac{\ln \delta}{\ln \pi} < 0$ implies $\lim_{x \to 0} x^{1+\frac{\ln \delta}{\ln \pi}} = +\infty$ and $\lim_{x \to \infty} x^{1+\frac{\ln \delta}{\ln \pi}} = 0$. Furthermore, $\frac{w_t}{F_t} = \pi^T \theta$ is one real root, because $\left(\pi^T\right)^{1+\frac{\ln \delta}{\ln \pi}} = \delta^T \pi^T$. Therefore, there exist two positive, real roots, i.e. $R_1 < R_2$. Combining $\frac{w_t}{F_t} > \theta$ and $\frac{w}{F_t} \leq \pi^T \theta$ yields $\frac{w_t}{F_t} \in [R_1, R_2] \cap (\theta, 1] = \emptyset$.

Third, if $\frac{w_t}{F_t} > \theta$ and $\frac{w}{\pi^T F_t} > \theta$, then w solves (3.1) iff

$$\left(1 \Leftrightarrow \delta^T\right) w + \delta^T \left(w + (1 \Leftrightarrow \delta\pi) \left(\frac{w}{\theta F_t} \right)^{1+\frac{\ln \delta}{\ln \pi}} \pi^T F_t \right) = w_t + (1 \Leftrightarrow \delta\pi) \left(\frac{w_t}{\theta F_t} \right)^{1+\frac{\ln \delta}{\ln \pi}} F_t,$$

which can be rewritten as stated in (3.3). Similar as before this polynomial, with $\frac{w}{F_t}$ as the variable, has two positive, real roots. Existence follows after substitution of $\frac{w}{F_t} = \frac{w_t}{F_t}$, which yields

$$\Leftrightarrow \left(1 \Leftrightarrow \delta^T \pi^T\right) (1 \Leftrightarrow \delta\pi) \left(\frac{w_t}{\theta F_t} \right)^{1+\frac{\ln \delta}{\ln \pi}} < 0.$$

Hence, the smallest positive, real root is smaller than $\frac{w_t}{F_t}$ and the largest positive, real root exceeds $\frac{w_t}{F_t}$. The smallest positive, real root fails $\frac{w}{F_t} > \pi^T \theta$ iff (3.3) is non-positive in $\frac{w}{F_t} = \pi^T\theta$ for all $\frac{w_t}{F_t} \in \frac{w_t}{F_t} \in (\theta, R_1) \cup (R_2, 1]$. Substitution of $\frac{w}{F_t} = \pi^T\theta$ in (3.3) yields

$$\pi^T\theta + (\delta\pi)^{2T}(1 \Leftrightarrow \delta\pi) \Leftrightarrow \frac{w_t}{F_t} \Leftrightarrow (1 \Leftrightarrow \delta\pi)\left(\frac{w_t}{\theta F_t}\right)^{1+\frac{\ln\delta}{\ln\pi}} \leq 0,$$

or, equivalently,

$$\frac{w_t}{F_t} + (1 \Leftrightarrow \delta\pi)\left(\frac{w_t}{\theta F_t}\right)^{1+\frac{\ln\delta}{\ln\pi}} \Leftrightarrow \pi^T\theta \Leftrightarrow (\delta\pi)^{2T}(1 \Leftrightarrow \delta\pi) \geq 0. \tag{7.8}$$

The left hand side (LHS) of (7.8) is larger than the LHS of (7.7) and (7.7) holds for all $\frac{w_t}{F_t} \in (\theta, R_1) \cup (R_2, 1]$. Hence, (7.8) also holds on $(\theta, R_1) \cup (R_2, 1]$.

Fourth, $\frac{w_t}{F_t} < \theta$ and $\frac{w}{\pi^T F_t} > \theta$. In that case w solves (3.1) iff

$$\left(1 \Leftrightarrow \delta^T\right) w + \delta^T\left(w + (1 \Leftrightarrow \delta\pi)\left(\frac{w}{\theta\pi^T F_t}\right)^{1+\frac{\ln\delta}{\ln\pi}} \pi^T F_t\right) = \frac{\delta}{1+\delta} w_t + \frac{1}{1+\delta\pi} F_t,$$

which yields (3.2). Similarly as before, (3.2) has two positive, real roots. The smallest root is not feasible, i.e. $\frac{w}{F_t} < \pi^T\theta$, iff the LHS of (3.2) is negative at $\frac{w}{F_t} = \pi^T\theta$. The latter is true iff $\frac{w_t}{F_t} > \theta^*$. Thus, $\frac{w}{F_t}$ is the largest root of (3.2) for all $\frac{w_t}{F_t} \in (\theta^*, \theta)$.QED

Proof of proposition 3.3

1. First, $\min\{\theta^*, \theta\} = \theta$ iff condition (3.4) holds. Next, (7.7) holds for $\frac{w_t}{F_t} = \theta$, because $\left(1 \Leftrightarrow \pi^T\right)\delta^2\pi^2\frac{1+\delta}{1+\delta\pi} + (1 \Leftrightarrow \delta\pi)\left(1 \Leftrightarrow \delta^T\pi^T\right) \leq 0$ is equivalent to the condition stated in the proposition. Hence, $R_1 \leq \theta$ and $R_2 = \pi^T\theta$ in (the proof of) proposition 3.2 imply $\frac{w_t}{F_t} \in (\theta, 1] \cap [R_1, R_2] = \left(\theta, \pi^T\theta\right)$. Finally, the discontinuity at $\frac{w_t}{F_t} = \pi^T\theta$ follows from observing that $\lim_{\frac{w_t}{F_t}\downarrow\pi^T\theta} \widetilde{w} > \lim_{\frac{w_t}{F_t}\downarrow\pi^T\theta} \frac{w_t}{F_t} = \pi^T\theta$ and the latter is, by construction, equal to the proposed wage for $\frac{w_t}{F_t} = \pi^T\theta$.

2. First, $\min\{\theta^*, \theta\} = \theta^* < \theta$ iff condition (3.4) does not hold. Next, from the proof of proposition 3.3.1 it follows that (7.7) in the proof of proposition 3.2 does not hold if the condition (3.4) does not hold for $\frac{w_t}{F_t} = \theta$. Since, $\pi^T\theta$ is one of the two real, positive roots it necessarily follows both R_1 and R_2 are larger than θ. QED

Proof of proposition 4.1

First, the k-th wage share x_k as function of the $(k \Leftrightarrow 1)$-th wage share x_{k-1} under linear dynamics is given by $x_k = \pi^{-T}[ax_{k-1} + b]$, where $a = \frac{\delta}{1+\delta-\delta^T} < 1$ and $b = \frac{\left(1-\delta^T\pi^T\right)}{\delta^2\pi^2\left(1+\delta-\delta^T\right)}\theta$. Then $x^* = \frac{\left(1-\delta^T\pi^T\right)}{\delta^2\pi^2\left(\pi^T\left(1+\delta-\delta^T\right)-\delta\right)}\theta$ solves $\pi^T x^* = ax_{k-1} + b$. Second, $x^*\frac{\left(1-\delta^T\pi^T\right)(1+\delta)}{\left(\pi^T\left(1+\delta-\delta^T\right)-\delta\right)(1+\delta\pi)} \leq \frac{\delta^2\pi^2(1+\delta)}{(1+\delta\pi)}$, i.e. $x^* \leq \theta$, is equivalent to condition (3.4). But then $\min\{\theta^*, \theta\} = \theta$ and the linear dynamics are valid on $[0, \theta]$. Third, monotonic convergence to x^* means that $x_k \leq x_{k-1}$ for all $x_{k-1} > x^*$ and $x_k \geq x_{k-1}$ for all $x_{k-1} < x^*$ for all k. Suppose $x_{k-1} > x^*$. Then $x_k \leq x_{k-1}$ follows from

$$\frac{x_k}{x_{k-1}} = \pi^{-T}a + \frac{\pi^{-T}b}{x_{k-1}} \leq \pi^{-T}a + \frac{\pi^{-T}b}{x^*} = \frac{1}{x^*}\left(\pi^{-T}ax^* + \pi^{-T}b\right) = \frac{x^*}{x^*} = 1$$

Similar arguments apply for $x_{k-1} < x^*$. Since the dynamics for $\left(\theta, \pi^T\theta\right]$ imply a transition to $[0, \theta]$ at the expiration date of the current contract there is also monotonic convergence to x^* from this interval. QED

Proof of proposition 4.2

We first derive the necessary and sufficient conditions for existence of a feasible steady state x^*, which also are sufficient for uniqueness. Substitution of $\frac{w_t}{F_t} = x^*$ and $\frac{\widetilde{w}}{F_t} = \pi^T x^*$ into (3.3) and rewriting yields

$$\left[\left(\pi^T \Leftrightarrow 1\right)\theta \Leftrightarrow (1 \Leftrightarrow \delta\pi)\left(1 \Leftrightarrow \delta^{2T}\pi^{2T}\right)\left(\theta^{-1}x^*\right)^{\frac{\ln\delta}{\ln\pi}}\right]\theta^{-1}x^* = 0.$$

Thus, there exist two roots: the infeasible root $x^* = 0 < \pi^T\theta$ and the unique, positive, real root of the term between square brackets. The latter root is unique, because the term increases in x^*, $\lim_{x^*\to 0}\left(\theta^{-1}x^*\right)^{\frac{\ln\delta}{\ln\pi}} = \infty$, and $\lim_{x^*\to\infty}\left(\theta^{-1}x^*\right)^{\frac{\ln\delta}{\ln\pi}} = 0$. This root is feasible iff $x^* > \pi^T\theta$. A necessary and sufficient condition for this to be the case is that the left hand side of (3.3) is negative at $\pi^T\theta$:

$$\left(\pi^T \Leftrightarrow 1\right)\theta \Leftrightarrow (1 \Leftrightarrow \delta\pi)\left(1 \Leftrightarrow \delta^{2T}\pi^{2T}\right)\left(\theta^{-1}\pi^T\theta\right)^{\frac{\ln\delta}{\ln\pi}} < 0,$$

which is equivalent to (4.1) stated in the proposition.

Next, the necessary and sufficient condition for monotonic convergence is derived. Without loss of generality, assume $[0, 1]$ is the domain of the nonlinear dynamics. Two cases have to be distinguished.

Case 1 $\frac{w_t}{F_t} > x^*$. Monotonic convergence from above to x^* is shown iff $x^* < \frac{\widetilde{w}}{\pi^T F_t} < \frac{w_t}{F_t}$ for all $\frac{w_t}{F_t} > x^*$. First, $\frac{\widetilde{w}}{F_t} < \pi^T \frac{w_t}{F_t}$ is proved as follows: $\widetilde{w} > w_t$ is the largest root of (3.3). Substitution $\frac{\widetilde{w}}{F_t} = \pi^T \frac{w_t}{F_t}$ in the left hand side of (3.3) yields

$$\left[\left(\pi^T \Leftrightarrow 1\right)\theta \Leftrightarrow (1 \Leftrightarrow \delta\pi)\left(1 \Leftrightarrow \delta^{2T}\pi^{2T}\right)\left(\frac{w_t}{\theta F_t}\right)^{\frac{\ln\delta}{\ln\pi}}\right]\frac{w_t}{\theta F_t} > 0,$$

because the term between square brackets is increasing in $\frac{w_t}{F_t}$ and it is zero for $x^* < \frac{w_t}{F_t}$. Thus, $\frac{\widetilde{w}}{\pi^T F_t} < \frac{w_t}{F_t}$ for all $\frac{w_t}{F_t} > x^*$.

Second, we prove that $\frac{\widetilde{w}}{F_t} > \pi^T x^*$ for all $\frac{w_t}{F_t} > x^*$. Since $\frac{\widetilde{w}}{F_t} > \frac{w_t}{F_t} > x^*$ a necessary and sufficient condition for $\frac{\widetilde{w}}{F_t} > \pi^T x^*$ is that the left hand side of (3.3) should be negative in $\frac{\widetilde{w}}{F_t} = \pi^T x^*$ for all $\frac{w_t}{F_t} > x^*$, i.e. for all $\frac{w_t}{F_t} > x^*$:

$$\pi^T x^* + \delta^T \pi^T (1 \Leftrightarrow \delta\pi)\left(\frac{\pi^T x^*}{\theta}\right)^{1+\frac{\ln\delta}{\ln\pi}} \Leftrightarrow \frac{w_t}{F_t} \Leftrightarrow (1 \Leftrightarrow \delta\pi)\left(\frac{w_t}{\theta F_t}\right)^{1+\frac{\ln\delta}{\ln\pi}} < 0.$$

Since the left hand side (LHS) is equal to 0 for $\frac{w_t}{F_t} = x^*$ (by definition of x^*) a necessary condition for the latter to hold is that the derivative of the LHS with respect to $\frac{w_t}{F_t}$ is negative in $\frac{w_t}{F_t}$ on $(x^*, x^* + \varepsilon)$ for some $\varepsilon > 0$, i.e.

$$\Leftrightarrow 1 \Leftrightarrow (1 \Leftrightarrow \delta\pi)\left(1 + \frac{\ln\delta}{\ln\pi}\right)\theta^{-1-\frac{\ln\delta}{\ln\pi}}\left(\frac{w_t}{F_t}\right)^{\frac{\ln\delta}{\ln\pi}} < 0,\ \frac{w_t}{F_t} \in (x^*, x^* + \varepsilon)\,.$$

Since this derivative is decreasing in $\frac{w_t}{F_t}$ it is negative iff

$$\frac{w_t}{F_t} > x^{**} := \theta\left(\frac{\theta}{\Leftrightarrow\left(1 + \frac{\ln\delta}{\ln\pi}\right)(1 \Leftrightarrow \delta\pi)}\right)^{\frac{\ln\pi}{\ln\delta}}.$$

Therefore, the derivative is negative in $\frac{w_t}{F_t}$ on $(x^*, x^* + \varepsilon)$ for all $\varepsilon > 0$ iff $x^{**} \leq x^*$. So, the necessary condition is also sufficient. Since x^* is the unique, positive and real root of

$$\left(\pi^T \Leftrightarrow 1\right)\theta \Leftrightarrow (1 \Leftrightarrow \delta\pi)\left(1 \Leftrightarrow \delta^{2T}\pi^{2T}\right)\left(\theta^{-1}x\right)^{\frac{\ln\delta}{\ln\pi}} = 0$$

and the LHS is increasing in x we automatically have: $x^* > x^{**}$ iff the LHS is negative in x^{**}. Substitution of $x = x^{**}$ yields $\theta\left(\pi^T \Leftrightarrow 1 + \frac{1-\delta^{2T}\pi^{2T}}{1+\frac{\ln\delta}{\ln\pi}}\right) < 0$, which yields the condition stated in the proposition.

Case 2 $\frac{w_t}{F_t} < x^*$. First, for increasing wage shares it should hold that $\frac{\widetilde{w}}{F_t} > \pi^T\frac{w_t}{F_t}$. Substitution of $\frac{\widetilde{w}}{F_t} = \pi^T\frac{w_t}{F_t}$ in the LHS of (3.3) yields

$$\left[\left(\pi^T \Leftrightarrow 1\right)\theta \Leftrightarrow (1 \Leftrightarrow \delta\pi)\left(1 \Leftrightarrow \delta^{2T}\pi^{2T}\right)\left(\theta^{-1}\cdot\frac{w_t}{F_t}\right)^{\frac{\ln\delta}{\ln\pi}}\right]\theta^{-1}\frac{w_t}{F_t} < 0$$

because the polynomial between square brackets is increasing in $\frac{w_t}{F_t}$ and it is zero for $\frac{w_t}{F_t} = x^*$. Thus, $\frac{\widetilde{w}}{F_t} > \pi^T\frac{w_t}{F_t}$ for all $\frac{w_t}{F_t} < x^*$. Second, it should hold that $\frac{\widetilde{w}}{F_t} < \pi^T x^*$ for all $\frac{w_t}{F_t} \in \left(\pi^T\theta, x^*\right)$. Similar reasoning as in case 1 requires that the LHS of (3.3) should be positive in $\frac{\widetilde{w}}{F_t} = \pi^T x^*$. As in case 1, the derivative with respect to $\frac{w_t}{F_t}$ should be negative for $\frac{w_t}{F_t} \in (x^* \Leftrightarrow \varepsilon, x^*)$ for some $\varepsilon > 0$. Proceeding as in case 1 yields the necessary and sufficient condition $\pi^T \Leftrightarrow 1 + \frac{\left(1-\delta^{2T}\pi^{2T}\right)}{1+\frac{\ln\delta}{\ln\pi}} < 0$.

Because (3.4) holds, $(\theta, R_1) \cup (R_2, 1] = \left(\pi^T\theta, 1\right]$ and the unique condition derived in case 1 and 2 ensures monotonic convergence towards x^* for $x^* \in \left(\pi^T\theta, 1\right]$. QED

Proof of corollary 4.3

Uniqueness of x^L in proposition 4.1 follows from proposition 4.2. Monotonic convergence follows from the proof of proposition 4.2, because $x^{NL} \leq \pi^T\theta$ and $\frac{w_t}{F_t} > \pi^T\theta$ in case 1 imply $\frac{\widetilde{w}}{\pi^T F_t} < \frac{w_t}{F_t}$. Hence, within finite time, the wage share drops below $\pi^T\theta$. QED

Proof of proposition 4.4

Propositions 3.2 and 3.3 imply a wage increase if $\frac{w_t}{F_t} \in \left(\pi^T, 1\right]$. Thus, only $\frac{w_t}{F_t} \leq \theta$ and $\frac{w_t}{F_t} \in \left(\theta, \pi^T\theta\right)$ have to be investigated. First, $\frac{w_t}{F_t} \leq \min\{\theta^*, \theta\} = \theta$. Then $w < w_t$

iff $\frac{w_t}{F_t} > \frac{\left(1-\delta^T\pi^T\right)}{\delta^2\pi^2\left(1-\delta^T\right)}\theta$. The interval $\left(\frac{\left(1-\delta^T\pi^T\right)}{\delta^2\pi^2\left(1-\delta^T\right)}\theta, \theta\right]$ is non-empty iff $\frac{\left(1-\delta^T\pi^T\right)}{\delta^2\pi^2\left(1-\delta^T\right)} < 1$. It is easy to check that $\frac{\left(1-\delta^T\pi^T\right)}{\delta^2\pi^2\left(1-\delta^T\right)}\theta > x^L$. Second, $\frac{w_t}{F_t} \in \left(\theta, \pi^T\theta\right)$. Then $w < w_t$ iff

$$\delta^{T+2}\pi^2\frac{w_t}{\theta F_t} + \left(1 \Leftrightarrow \delta^2\pi^2\right)\left(\frac{w_t}{\theta F_t}\right)^{1+\frac{\ln\delta}{\ln\pi}} \Leftrightarrow \delta^T\pi^T < 0.$$

Similar to the proof of proposition 3.2 it follows that $\frac{w_t}{F_t} = \pi^T\theta$ is one of the two positive, real roots. This condition holds in $\frac{w_t}{F_t} = \theta$, i.e. the smallest root is smaller than θ, iff $\frac{1-\delta^T\pi^T}{\delta^2\pi^2\left(1-\delta^T\right)} < 1$. In that case $\pi^T\theta$ is necessarily the largest root and the polynomial is negative for all $\frac{w_t}{F_t} \in \left(\theta, \pi^T\theta\right]$. QED

Proof theorem 5.1

First, in the limit, condition (3.4) always holds, because

$$\lim_{\Delta\to 0}\delta^2\pi^2 \le \lim_{\Delta\to 0}\delta^{-1}\left(\pi^{T+2}\left(\delta^2+\delta^3\right)+\delta^T\pi^T\left(1 \Leftrightarrow \delta^2\pi^2\right)\Leftrightarrow 1\right) \Leftrightarrow 1 \le \left(2e^{\rho L}\Leftrightarrow 1\right)$$

and that the latter inequality holds for all $L \geq 0$ and $\rho \geq 0$. Second, $\lim_{\Delta\to 0}\min\{\theta, \theta^*\} = \min\{1, 1\} = 1$. Next, application of l'Hôpital's rule yields $\lim_{\Delta\to 0} F_t = \frac{r}{r-\rho}e^{\rho t}$. Then the limit expressions stated in the theorem follow trivially from propositions 3.1, 3.2, 4.1, 4.4 and corollary 4.3. Furthermore, $\lim_{\Delta\to 0} x^L = \frac{1-e^{-(r-\rho)L}}{e^{\rho L}(2-e^{-rL})-1} < 1$ iff $\rho L > 0$. The latter is assumed. Similarly, $\lim_{\Delta\to 0} x^L < \frac{1-e^{-(r-\rho)L}}{1-e^{-rL}} < 1$ iff $\rho L > 0$. QED