

# **On the distance dependence of the price elasticity of telecommunications demand; meta-analysis, and alternative theoretical backgrounds**

*Hans Ouwersloot*  
*University Maastricht*

*Piet Rietveld*  
*Free University Amsterdam*

## Abstract

The positive correlation between the absolute price elasticity of telecommunications demand and the distance of the calling relation is well known. In this paper we first present a meta-analysis of existing studies to buttress the distance dependence empirically. The analysis confirms the existence of distance dependence, and gives insight into the size of the effect.

Next we look for various explanations of the distance dependence. We analyse the roles of the functional form of demand functions in conjunction with the dependence of price on distance, and consider whether spatial interaction theory can provide an explanation. One of the interesting findings is that the price effect may explain the distance dependence, but that this explanation is not unequivocal. On the other hand we show that incorporating spatial interaction theory elements in a quite basic utility maximization model of information demand also leads to distance dependent telecommunications demand (keeping prices of calling fixed).

JEL code: L96

Key words: Price Elasticities, Meta-analysis, Telecommunication, Spatial Interaction Models

## Correspondence

Hans Ouwersloot  
University Maastricht  
FdEWB/MW/MMO  
Postbus 616  
6200 MD Maastricht  
the Netherlands  
phone: + 31 43 3883813  
fax: + 31 43 3210265  
email: h.ouwersloot@mw.unimaas.nl

## 1. Introduction

The field of telecommunications demand is well documented, due to the seminal works of Taylor (1980, 1994). One of the most intriguing points which emerges from this work is the "finding that the absolute value of the toll price elasticity increases with length-of-haul" which is "probably (...) the best-established empirical regularity in telecommunications demand" (Taylor, 1994, p. 260). In this paper we study this distance dependence effect, labelling it by PDDAPE: Positive Distance Dependence of the Absolute Price Elasticity<sup>1</sup>. By including the absoluteness of the price elasticity when can safely and briefly speak of greater elasticities when we mean greater in absolute terms of the elasticity.

The purpose of this paper is to seek an explanation for this thus far unexplained phenomenon, making use of spatial interaction modelling theory. In section 2 we critically review the evidence of the PDDAPE effect by a selective literature review, resulting in a meta-analysis of the elasticities found. In Section 3 we provide a critical discussion of explanations of the PDDAPE effect. This section concentrates on two explanations, the first is that the source of the increase in the price elasticity is a price increase, due to a positive relationship between price and distance. The second approach is based on the notion that telecommunication is a heterogeneous good, where the shares of the various types of calls are distance dependent. In this discussion, we adopt a formal approach explicating the mathematical foundation of the arguments. In section 4 we start our own search for an explanation of the PDDAPE effect by incorporating elements from spatial interaction theory. Section 4 makes clear that the functional form plays an important role as it defines two classes of functions for which PDDAPE is, or is not found by construction. Section 5 then addresses the empirical aspects of functional forms. In section 6 we finally analyse a utility maximisation model that incorporates some notions from the spatial interaction literature. We show that under certain conditions which are not very restrictive the PDDAPE effect is found. Section 7 concludes this paper.

---

<sup>1</sup>Suggestions for a more attractive acronym are welcome.

## 2. Empirical Evidence of the Distance Dependence of the Price Elasticity

Despite the attractiveness of a straightforward one-liner like “the longer the haul, the greater the price elasticity” things are not as clear as it seems. In this section we examine a number of studies in the literature reporting price elasticities, followed by a discussion.

In Tables 1 to 3 we have listed elasticities that we have found during our literature survey. Table 1 contains the short distance elasticities, mainly local telephone demand, Table 2 long distance, national elasticities and Table 3 international elasticities. The tables contain the following characteristics: The source, the elasticity, the origin and destination of the call relation, the years to which the data apply, and relevant other information.

A number of entries originate from Taylor (1994). Unfortunately not all studies mentioned there could be verified since many of them are unpublished (except for the appearance in Taylor’s book). Not all studies reported by Taylor, or found by us are included in the tables, for which various reasons exist. In general the following criteria were applied for inclusion here:

- \* Does the study lead to a proper price elasticity?
- \* Is the estimation and the calculation methodologically sound?
- \* Is sufficient background information available?

[INSERT TABLES 1 TO 3 HERE]

Before we will discuss these data, two remarks apply. First, following the Houthakker-Taylor model, most empirical studies of telecommunications demand employ a dynamic model like:

$$\ln Q_t = \alpha \ln Q_{t-1} + \beta \ln P_t + \gamma \ln X \quad (1)$$

where  $Q$  denotes demand,  $P$  the price and  $X$  all other explanatory variables. In this model the short run price elasticity equals  $\beta$ , and the long-run elasticity is  $\beta/(1-\alpha)$ . Since in general  $0 < \alpha < 1$ , the long run elasticity will be larger in absolute value than the short run elasticity.

Studies that are not of the Houthakker-Taylor type report only one elasticity. We have interpreted these as long run elasticities, since in the equation

$$\ln Q = a + b \ln P + c \ln X \quad (1')$$

$b$  and  $c$  can be interpreted as  $\beta/(1-\alpha)$  and  $\gamma/(1-\alpha)$ .

Secondly, note that short and long distance are relative terms. International calls are long distance, but for e.g. the Netherlands, Belgium is also abroad although every place in Belgium is approximately within 500 km from every place in the Netherlands. *Mutatis mutandis* this holds for many (smaller) countries for which international calls - from a viewpoint of distance alone - can be compared to US interstate calls.

From Tables 1 to 3 we can make a number of observations. First the elasticities of local demand are in the order of  $-.3$ . Those for long distance, national demand (Table 2) are in the order of  $-.9$ . Notice that although the majority of elasticities refer to northern American data, European elasticities are very well in line with these results. The apparently diverging result for long distance elasticities in Belgium (Table 2, first entry) in fact confirms the observation that local demand elasticities are approximately  $-.3$ . Belgium is so small that interprovincial calling in this country is comparable to intrastate calling in the US and the latter elasticities are reported in Table 1. The only truly diverging result is Dobell *et al.* (1972) for which no apparent explanation can be given. Also notice the large difference between the long-run and short-run elasticity in this study.

These results for local and long distance domestic calls confirm the PDDAPE postulate. The results for the international elasticities however show a different picture. We find very large elasticities (well below  $-1$ ) for the US to other countries in the period 1962-1973 (the Lago, and Rea and Lage results). Other studies for the US to other countries show an elasticity of about  $-.4$  for the time span 1976-1990 (Applebe *et al.*, Acton and Vogelsang). Also, for other country pairs a wide range of elasticities is found, in particular the Fiebig and Bewley (1987) study gives an extremely broad range. In any case, a PDDAPE effect is not clearly visible in these international results.

When we consider the development over time of the local and long distance national elasticities, the data in Tables 1 and 2 suggest that elasticities for local demand are relatively stable over time, whereas the long distance national elasticities show a decrease from approximately  $-1$  in the 1960s to  $-.7$  in the 1980s.

These findings suggest that the international price elasticity is in the first place determined by general characteristics (which may include distance) of the (economic) relationship between countries as witnessed by the diversity of elasticities found. Within a country, distance may play a more profound role. However, the difference in elasticity between local and long distance (national) telecommunication seems to diminish over time.

To further investigate the above suggestions we perform a meta-analysis on the data of Tables 1-3. In a definition of Glass (1976) "Meta-analysis refers to the analysis of analysis ... the statistical analysis of a large collection of analysis results from individual studies for the purpose of integrating the findings". He continues: "It connotes a rigorous alternative to the casual, narrative discussions of research studies which typify our attempts to make sense of the rapidly expanding research literature". When a well defined, unique parameter is the object of study (which usually is the case in physics), meta-analysis should be cast in a Bayesian statistics context. For less well defined or non-unique parameters, like elasticities in economics, more simple techniques like regression analysis is more appropriate. In addition, a regression offers the opportunity, not only to meta-analyse the most likely value of the parameter under study, but also to study which external factors influence its magnitude. In our study we apply meta analysis in this latter sense, regressing the reported elasticities on the logarithm of the (estimated) distances and a few control variables. We estimated a separate equation for long-run and short-run elasticities. The result of the regressions are in Table 4.

#### INSERT TABLE 4 ABOUT HERE

The Table exhibits some interesting results. First and foremost there are significant differences between the regressions of the long-run and short-run elasticities. The long-run (LR) estimation shows an acceptable  $R^2$  and interpretable estimates with correct signs and reasonable t-values. The short-run (SR) estimation on the other hand gives far less satisfactory results. The  $R^2$  is reasonable (although the adjusted  $R^2$  is negative!), but the t-values are importantly less than in the LR estimation. In particular the parameter of our greatest interest (distance) is practically identical to null.

The LR estimation confirms the PDDAPE postulate, significantly at a 10% level. The parameter estimate implies that doubling the distance leads to an increase in the absolute LR price elasticity of .06. The time trend is

significant at the 5% level. It unambiguously shows that LR elasticities have reduced in absolute values over the last 30 years or so. This result is the only one that more or less transposes to the SR estimation.

The volume variable is not statistically significant but has the plausible sign. Volume is a dummy parameter with the value 1 if the endogenous variable is measured as the volume of calls in contrast to just the number of calls. In response to a price change consumers can reduce the number of calls, but also in the length of individual calls. Both responses are measured when the dependent variable is measured as the volume of calling, whereas only the former response is measured when the dependent variable is the number of calls. It therefore is plausible that volume measures show a greater absolute elasticity. The North America dummy is also insignificant, but its sign suggests that elasticities are larger in the US and Canada than in the rest of the world. This could be expected since due to the more liberal markets in Northern America consumers are likely to be more responsive to price changes in telecommunication.

The short run (SR) estimation reveals that PDDAPE is not found when the immediate, i.e. short-run, response to price changes is measured. The only other estimate which warrants our attention is for the variable "International". When the calling relation crosses the border, the SR price elasticity is much larger on international relations than on national links.

The discrepancy between the LR and SR estimates deserves more attention. The different magnitudes of the elasticities is a well known and well understood effect. In the long run, more options are available to make adjustments in one's behaviour, so larger impacts (elasticities) are just a natural and common phenomenon. Apparently, the immediate, short run adaptations are equally available for short distance and for long distance relations. On the other hand, in the long run, differences in adapting behaviour are found for short versus long distance telecommunication. Explaining the PDDAPE effect it drives our attention to structural models, instead of immediate responses.

Finally, we discuss two studies not included in Tables 1 to 3, because of their somewhat distinct nature. Hackl and Westlund (1992) (notice that this is another study of these authors than the one included in Table 3) apply moving local regression analysis on a time series of data, investigating the constancy of the price elasticity over time. They conclude that constancy is to be rejected for all three relations included (Sweden to

the US, UK, and Germany). Specifically, the elasticity for Germany gradually increases from a value of approximately 0 by 1976 to -0.80 by 1990, the UK elasticity also gradually increases from -.50 to -1.25 over the same period, whereas the US elasticity decreases from -1.75 in 1976 to -.70 in 1980, stays at this value until 1983, then increases rapidly to approximately 0 by 1986, and more or less remains at this value for the rest of this period.<sup>2</sup> These results are only partially in line with our positive parameter estimate for a time trend in Table 4. The important point of the Hackl and Westlund study however, is that they demonstrate that the often assumed constancy of price elasticities (or parameters), implicit in the many analyses of time series is not necessarily justified.

Madden *et al.* (1993) estimate a range of price elasticities based on a stated preference approach. The elasticities these authors derive in general support the PDDAPE postulate. However, the fact that almost no elasticity is significant and that a number of positive price elasticities are found casts doubt on their results and more in particular on the usefulness of the stated preference approach in this context.

The conclusion from this section is that the PDDAPE effect is confirmed in the many studies that are examined. A distinction can be made between the long run elasticities, which convincingly exhibit the PDDAPE effect, and short run elasticities, for which the PDDAPE effect could not be demonstrated in our meta-analysis. In the next sections we will discuss various explanations of the PDDAPE phenomenon, starting with the explanations that are encountered in the literature in Section 3, and presenting our own ideas on the topic in Sections 4 to 6.

---

<sup>2</sup>Since Hackl and Westlund only present a diagram on this path of the price elasticities, these numbers may be subject to interpretation errors.

### 3. Current explanations

The effort to offer an explanation for the rise in price elasticities with distance is one of the more challenging intellectual endeavours in Taylor's 1994 book. Besides giving his own explanation he points to the suggestion of some researchers that this regularity is just a price effect. As the definition of the price elasticity -  $(\partial q/\partial p) * (p/q)$  - includes the price as a factor, it is just a natural effect that this elasticity rises with price. This combined with the fact that longer distance calls are more costly, leads to the result that the price elasticity rises with distance. In this section we examine both this price argument, and Taylor's explanation.

#### 3.1 The price argument

To examine the price argument in detail, consider the general demand function for calls:

$$q = q(x, L, p)$$

where  $q$  is the demand for calls,  $p$  is the price of calls which may depend on distance,  $L$  is distance and  $x$  is a vector of other variables and the parameters in the demand function. The price elasticity of demand is given by

$$\varepsilon_{pq} = \frac{\partial q}{\partial p} \frac{p}{q}$$

The price elasticity depends on both  $L$  and  $p$ . Differentiating  $\varepsilon_{pq}$  with respect to distance leads to, under the assumption that the effect of distance is only found via the price  $p$  means that we have to consider:

$$\frac{\partial \varepsilon_{pq}}{\partial L} = \frac{\partial \varepsilon_{pq}}{\partial p} * \frac{\partial p}{\partial L} \quad (2)$$

Also, the implicit argument is that  $\partial p/\partial L > 0$ . Thus for PDDAPE we only have to examine the sign of  $\partial \varepsilon_{pq}/\partial p$ . Working this out gives:

$$\frac{\partial \varepsilon_{pq}}{\partial p} = \frac{p}{q} \frac{\partial^2 q}{\partial p^2} + \frac{1}{p} (\varepsilon_{pq} (1 - \varepsilon_{pq})) \quad (3)$$

Assuming that the price elasticity itself is negative the second term on the right hand side is negative. So a sufficient condition for PDDAPE being explained by the price argument is that the second derivative of the demand function with respect to price is not positive.



Theory does not give much guidance in this area of second derivatives. Examining some common forms of the demand function shows that things are not very clear. First, a linear demand function  $q = a + bp$ , ( $b < 0$ ) gives a second derivative equal to zero, for which PDDAPE thus holds. A double log-linear form,  $\ln q = a + b \ln p$ , ( $b < 0$ ), gives a constant price elasticity equal to  $b$ , which is negative, but does not change over the entire range of prices. Hence the price argument for PDDAPE is ruled out. Finally, assuming a hyperbolic form,  $q = a + b/p$ , ( $b > 0$ ), the second derivative is positive - viz.  $2b/p^3$  - and further examination reveals a positive price dependence of the price elasticity (assuming a positive parameter  $a$ ) ruling out PDDAPE.

Our conclusion is that the price may account for the PDDAPE effect, but that this is not necessarily the case: it critically depends on the form of the demand function. Some forms lead to PDDAPE caused by a mere price effect, for others a price increase actually results in the opposite of PDDAPE: in absolute terms the price elasticity decreases with a rising price. Unless we have convincing evidence that certain shapes of the demand function are systematically more probable than other shapes, the price effect does not help to explain PDDAPE. The issue of the dependence on the functional form is continued in section 4.2.

### 3.2 Taylor's explanation

Taylor (1994, p. 260) rejects the price effect argument on an empirical basis, but his argument is not entirely convincing. Taylor discusses an empirical study by Gatto *et al.* (1988) who report, based on data from 1984-1987, a long-haul toll calls elasticity of -0.723. This is - according to Taylor - very close to the same elasticity found in pre-1980 studies. As real prices were significantly lower in the 1984-1987 period, a lower elasticity (in absolute value) should be expected on basis of the price-effect argument instead of the reported comparable elasticity.

However, examining the pre-1980 price elasticities Taylor refers to (given in appendix 1, Table 5 in his 1994-book) reveals that these latter elasticities are generally (in absolute terms) larger than -0.723. Hence, the observed elasticities do not support Taylor's argument. Nevertheless, his rejection brings Taylor to develop his own 'model' to explain PDDAPE, which we discuss now.

The foundations for Taylor's argument are provided by statistical information from a report by Meade from 1985<sup>3</sup>. The key point in Taylor's argument is to define different categories of calls according to the value callers attach to them. Put simply, Taylor argues that some calls generate relatively much value, such as those for business, or price information of products, while others generate relatively little value, e.g. those to relatives and friends. High value calls are less price sensitive than low value calls, and since the statistical evidence of Meade shows that these high value calls are predominantly made at short distances, the distance dependence of the price elasticity is established<sup>4</sup>.

Besides by statistical evidence Taylor's argument is also buttressed by the theory of the communities of interest (Taylor, 1994, p. 23), i.e. the group of people to which the caller belongs. In modern terminology one would speak of personal networks. Indeed, persons may be assumed to be involved in a number of distinct networks, for example private, personal business, business etc. Taylor uses the concept to make the assumption that various networks will be concentrated at different distances, i.e. can be characterized as either local or remote.

Careful examination of Taylor's argument reveals that it consists of various independent steps:

- 1) At a given distance the demand for calls consists of the demand for calls of two distinct types;
- 2) The distinction exists in the value, the user attaches to the calls. The categories are high value calls and low value calls;
- 3) High value calls have a lower absolute price elasticity than low value calls;
- 4) The share of high value calls decreases when the distance increases;
- 5) Hence the absolute price elasticity increases with distance.

The first and second step are primarily conceptual. The third step is the crucial one, and will be discussed more thoroughly below. The fourth step is evidenced by Meade's data. Finally the fifth step is one that can easily be proved. To prove it, assume that  $q = q^h + q^l$ , respectively high and

---

<sup>3</sup>Unfortunately this report is unavailable to us, but Taylor gives sufficient information to appreciate his arguments.

<sup>4</sup>It should be noted that Taylor constructs his argument much more carefully than the abbreviated version here suggests. The present representation however, captures the main points.

low value calls. Then

$$\varepsilon_{pq} = \frac{\partial q}{\partial p} \frac{p}{q} = \frac{\partial q^h}{\partial p} \frac{p}{q^h} \frac{q^h}{q} + \frac{\partial q^l}{\partial p} \frac{p}{q^l} \frac{q^l}{q} = \varepsilon_{pq}^h \frac{q^h}{q} + \varepsilon_{pq}^l \frac{q^l}{q}$$

which shows that the overall price elasticity is a weighted average of the elasticities of the disaggregated demands. Therefore, given the correctness of steps 3 and 4, this proves step 5.

Consider now step 3: high value calls have lower absolute price elasticities than low value calls. This seems intuitively plausible but can we buttress it in a microeconomic framework?

Suppose a consumer has to allocate his income  $M$  among two goods, high value calls (H) and low value calls (L) which have the same price  $p$ . We ignore the consumption of other goods for ease of presentation. Their inclusion would not significantly change the argument. His budget constraint is thus:

$$p(H + L) = M$$

In this model, an increase in the price  $p$ , is equivalent to a decrease in income  $M$ , so considering the effect of a price change on the consumption of H and L is equivalent to considering the income expansion paths. Microeconomic theory tells that an income expansion path bends towards the good which is called the luxury good, when income increases, see Figure 1. The other good is then a necessity. In the high value calls / low value calls case, it seems reasonable to assume that the high value calls are the necessities and the low value calls are the luxury (when you can afford to make low value calls, you are wealthy enough to afford this luxury).

[INSERT FIGURE 1 ABOUT HERE]

So assume that the income expansion path bends towards the low value calls. The curve describing the combination of L and H consumed at various prices can be approximated by  $L = H^\alpha$ , with  $\alpha > 1$ . This proves to be a sufficient assumption for Taylor's third step. With  $L = H^\alpha$ , we have:

$$\frac{\partial L}{\partial p} = \alpha H^{\alpha-1} \frac{\partial H}{\partial p} \leftrightarrow$$

$$\frac{\partial L}{\partial p} \frac{p}{H^\alpha} = \alpha \frac{p}{H} \frac{\partial H}{\partial p} \leftrightarrow$$

$$\varepsilon_{pL} = \alpha \varepsilon_{pH}$$

Consequently, since  $\alpha > 1$  the (absolute) price elasticity of low value calls is larger than the price elasticity of high value calls. From this we can derive that a (first) test of the appropriateness of Taylor's argument is whether high value calls have a lower income elasticity than the low value calls, which is a sufficient condition for the income expansion path to bend towards the low value calls.

Finally we turn to Taylor's fourth step which says that the share of high value calls decreases with distance. His argument is based on the empirical study of Meade and the notion that "the GTE survey, as well as common sense, suggests that the community of interest for local calls, again for most people, will consist of friends and relatives and people and acquaintances associated with work, shopping, and recreation. Friends and relatives become progressively more important in relative terms as the communities of interest become increasingly more distant " (p. 263). Friends and relatives are the networks for which calls have relatively low value in Taylor's approach.

Common sense seems to be the basis of Taylor's fourth step. No reasons of income, prices or costs are used to argue why 'friends and relatives become more important' in relative terms, at greater distances. This is an observation on the spatial structure of our activities and communication pattern, for which little theoretical justification has been given. It may be correct, but it is not easy to tell why. In fact, an economic analysis that takes the previous (third) step into account makes clear that the share of high value calls should become greater at larger distances. The only assumption that is needed for this is that the price of a call increases with increasing distances. This point is already suggested in Figure 1, realizing that an increase in the price implies a move along the income expansion path into the direction of the origin. Clearly in such a case the share of high value calls increases. But a more general proof can be constructed.

Assume that the demand for high value and low value calls is  $H$  and  $L$  respectively. Both are a function of the price  $p$ . Consider the demand for  $H$  and  $L$  at two distances, associated with two distinct prices  $p_n$  for short distance calls (near) and  $p_f$  for long distance calls (far). Using a first order Taylor approximation for the demand for high value-long distance calls we find:

$$H(p_f) = H(p_n) + (p_f - p_n) \frac{\partial H}{\partial p}$$

where the derivative is evaluated at  $p_n$ . For low value calls a similar expression can be derived:

$$L(p_f) = L(p_n) + (p_f - p_n) \frac{\partial L}{\partial p}$$

where the derivative is again evaluated at  $p_n$ .

Now, consider the ratio of high over low value calls at both prices. Applying the above approximations, it is straightforward to show that Taylor's assumptions 3): high value calls have a lower absolute price elasticity than low value calls; and 4) the share of high value calls is smaller at far locations than at near locations lead to a contradiction. Starting from assumption 4) we find

$$\frac{H(p_f)}{L(p_f)} = \frac{H(p_n) + (p_f - p_n) \frac{\partial H}{\partial p}}{L(p_n) + (p_f - p_n) \frac{\partial L}{\partial p}} < \frac{H(p_n)}{L(p_n)} \Leftrightarrow$$

$$1 + (p_f - p_n) \frac{\partial H}{\partial p} \frac{1}{H} < 1 + (p_f - p_n) \frac{\partial L}{\partial p} \frac{1}{L} \Leftrightarrow$$

$$\varepsilon_{pH} < \varepsilon_{pL}$$

which is in contradiction with assumption 3). Thus, using the economics approach alone and assuming that an increase in the price of calls is the only effect of distance on the demand for calling, this leads to a contradiction between the third and fourth step in Taylor's explanation of the PDDAPE effect. Other arguments are needed to underpin the fourth step in Taylor's explanation.

Our conclusion is that Taylor has made an interesting contribution in explaining the PDDAPE effect. The third step is liable to testing, and also the fourth step is in need of more systematic empirical research, which should incorporate other disciplines than economics alone. Note that Taylor's approach is based on the assumption that a telephone call is not a homogeneous good (he distinguishes between high value and low value calls). In section 7 we will give an alternative explanation of the PDDAPE effect where such an assumption is not needed.

### 3.3 Two further considerations

In this subsection we introduce two further notions that deserve to be taken into account when studying the PDDAPE effect.

The first notion is actually a counterargument for the PDDAPE effect, i.e. an argument for a negative distance dependence of the absolute price elasticity. This argument is that for short distance calls an alternative, personal visit, is more easily available, either as a purposed visit or as a meeting by chance. Given the availability of a substitute, a larger absolute price elasticity could be expected for short distance calls. This argument makes the observed regularity of increasing price elasticities with distance even more puzzling, and reinforces the need for an explanation. We will refer to this as the substitution argument.

The second notion is that interaction of any kind simply diminishes with distance. Usually this is attributed to a price effect, and hence a simple matter of demand, but also an autonomous distance effect is recognized (the tyranny of space). Therefore, besides the already discussed price argument, this autonomous effect, to which we will refer as the diminishing interaction effect (DI), deserves our attention. In a demand function, DI is primarily captured by a negative distance dependence of the intercept. The implication of this effect for PDDAPE will be investigated in Section 5.

### 3.4 Summary

Two arguments have been reviewed that have been suggested to explain the observed PDDAPE effect. The arguments that were mentioned are:

- 1) A simple *price effect*. It may explain PDDAPE, but it depends on the functional form of the demand function;
- 2) The existence of various networks - or communities of interest -, i.e. Taylor's point that calls within a region can be considered as at least two functionally distinct goods. More generally this involves the *heterogeneity*

*of calls*, i.e. a distinction between high-value and low-value calls, being of different importance for short and long distances.

Other relevant aspects of the subject are:

- the *substitution* argument, i.e. between telecommunication and spatial interaction;
- the autonomous *diminishing interaction* effect, due to distance dependence, the DI effect.

In the next sections we will exploit these arguments to further investigate the sufficient and necessary conditions for the PDDAPE effect. We will refrain however from the price argument, since we want to concentrate on the "other" explanations.

#### 4. PDDAPE explained by spatial interaction theory

In the previous section we analysed the Marshallian demand function

$$q = q(x, L, p)$$

to find conditions for the price effect to explain the PDDAPE phenomenon. For simplicity we assumed  $\partial q / \partial L = 0$ . In the present section we want to study the PDDAPE effect being explained by other factors than price and hence we now assume  $\partial p / \partial L = 0$  in equation (3) and take a look at PDDAPE by studying the effect of distance on demand directly. In particular we assume that  $\partial q / \partial L < 0$  as a result of spatial interaction theory, which basically says that the level of interaction *ceteris paribus*, diminishes with distance. Our analysis starts from the following expression:

$$\frac{\partial \epsilon_{pq}}{\partial L} = \frac{p}{q} \left[ \frac{\partial^2 q}{\partial p \partial L} - \frac{1}{q} \frac{\partial q}{\partial p} \frac{\partial q}{\partial L} \right] \quad (4)$$

This expression is particularly useful for further analysis, distinguishing two cases.

First, assume that the demand function is *multiplicative* separable in price and distance, i.e.

$$q = q(L, p) = h(L) * g(p) \quad (5)$$

where  $h$  does not depend on  $p$ , and  $g$  does not depend on  $L$ . In this case we assume that  $h'(L) < 0$  to make this form consistent with spatial interaction theory. Applying formula (4) we find:

$$\frac{\partial \epsilon_{pq}}{\partial L} = \frac{p}{q} \left[ h'(L) * g'(p) - \frac{h(L) * g'(p) * h'(L) * g(p)}{h(L) * g(p)} \right] = 0 \quad (6)$$

which shows that this type of demand function leads to distance independence of the price elasticity. Hence, PDDAPE can not be explained when the demand function is multiplicative separable, i.e. when it can be written in the form (5).

Second, assume that the demand function is *additive* separable in price and distance, i.e.

$$q = q(L, p) = h(L) + g(p) \quad (7)$$



where again  $h$  does not depend on  $p$  (and  $h'(L) < 0$ ), and  $g$  does not depend on  $L$  (and of course  $g'(p) < 0$ ). Again applying expression (4) leads to the following expression for the partial derivative of the price elasticity with respect to distance

$$\frac{\partial \varepsilon_{pq}}{\partial L} = \frac{p}{q} \left[ -\frac{1}{q} \frac{\partial g}{\partial p} \frac{\partial h}{\partial L} \right] \quad (8)$$

Since both partial derivatives within the square brackets are assumed negative the PDDAPE effect is unequivocally found for this type of additive separable functions.

These are two important results as they cover a wide range of functional forms. In particular, the double-logarithmic form is essentially a multiplicative separable function, and hence PDDAPE cannot be found there. This is the more interesting, since gravity type models of demand, that follow from spatial interaction theory (see e.g. Erlander and Stewart, 1990), are of the double log type with distance included as a separate variable, not included in the parameter of price. The semi-logarithmic and linear demand functions are of the additive separable type and for these functions we find that PDDAPE is always found, given of course the assumption of an autonomous negative distance dependence of demand ( $h'(L) < 0$ ).

The interpretation of the additive separable demand function gives better understanding of the PDDAPE effect. This interpretation is that the demand for calls from one to another region depends on an autonomous (i.e. price independent) amount of calls, which reflects the level of interaction between the two regions. This part of demand for calls is predominantly given by the derived demand property of calls. Therefore, this autonomous amount depends on distance only as the spatial interaction is mainly determined by distance. The total demand for calls also depends on the price, but the reaction parameters to the price (as captured in the  $g(p)$  function) are the same for all regions. That means, the absolute reaction to price changes is independent of the type of relationship between the two regions, and in particular independent of the distance. Notice that for this argument we consider calls as a homogeneous good.

With this interpretation we can appreciate the result that additive separable functions exhibit PDDAPE. The absolute price effect is assumed the same for all regions, but the base, relative to which this effect is measured in

calculating the elasticity depends on the region. In particular, the more remote regions have a lower base amount of calls, given a lower level of spatial interaction, and therefore the reaction to price changes in relative terms is greater.

This argument is consistent with the results presented in section 2 for the elasticities in international demand. Recall that we found very mixed results concerning price elasticities, with no (clear) relation to distance. This can be appreciated by the linear demand hypothesis, where spatial interaction is in fact the determining factor. Research on international trade and communication reveals that substantial barrier effects exist, related to borders (Rietveld *et al.* (1993) and Bröcker and Rohweder (1990)). Thus, distance alone is not a good indication of spatial interaction in this case.

We conclude that the functional form plays a crucial role in both the price argument (section 3.1) and spatial interaction theory as explanation of PDDAPE. Therefore in the next section we discuss the functional form of demand more elaborately, approaching the issue from both the theoretical and the empirical side.

## 5. About the functional form of the demand function

From the theory of consumer demand it follows that for one equation models that do not involve prices of all commodities, not a single restriction<sup>5</sup> on the equation is maintained, giving researchers the freedom to specify any form of demand function (Deaton and Muellbauer, 1980). Therefore we can approach the specification issue from an empirical point of view. That means, we may specify demand functions flexibly so that the data can decide which functional form is most appropriate. Such flexibility can in part be achieved by using Box-Cox transformations for the price and/or demand variables. A few studies exist that explore this approach and we discuss them below. But first we consider the Box-Cox transformed demand functions in some more detail.

The Box-Cox transformation of a variable is defined as:

$$BC(x) = (x^\lambda - 1)/\lambda$$

Then, when  $\lambda = 1$  this means that essentially no transformation is performed and  $\lambda = -1$ , implies taking the inverse of the original variable, and changing its sign. An interesting property of this transformation is that when  $\lambda$  approaches 0, the transformation results in taking the log<sup>6</sup>. Incorporating  $\lambda$  in, for example, a maximum likelihood estimation gives the opportunity for testing the appropriateness of the linear and semi-log forms (or the double log form).

Hackl and Westlund (1992) estimate three telecommunications demand equations for Sweden, including  $\lambda$ . For this parameter they find values of -.30 for Germany, .08 for UK and .30 for the USA. Hackl and Westlund conclude that the double log form  $\lambda=0$  is acceptable and only has to be rejected for the USA data, but that the linear form has to be rejected for all three models. Fiebig and Bewley (1987) analyse time series of telephone calls from Australia to ten other countries, and conclude that in most cases the null hypothesis of a double log demand function cannot be rejected. The linear demand function is to be rejected again.

So these studies give support to the idea that demand functions are of the double log form. It makes the double-log function the prime candidate for

---

<sup>5</sup>Systems of demand equations theoretically have to obey restrictions of homogeneity, adding up and symmetry. For single equations, adding up and symmetry are not relevant, as these are across equation s restrictions, while homogeneity can only be tested when all prices of all commodities are included.

<sup>6</sup>This results follows as a simple application of the rule d'Hôspital.

the true demand function<sup>7</sup>. In section 4 we found that the double log demand function does not allow for PDDAPE being explained by spatial interaction theory, when this explanation is exclusively interpreted to be working via the constant term. In other words, in a double-log demand, PDDAPE can only be found when the parameter of the  $\log(p)$  term is distance dependent. In the next section we concentrate on this point, and we show there that using a utility maximization model incorporating the basic notions of spatial interaction theory indeed makes it likely that PDDAPE can be found as it influences the parameter of the price variable in a demand equation.

---

<sup>7</sup>Note that a double log demand function implies that the price elasticity does not depend on distance in the context of the price argument (Section 3.1). This would rule out the price argument as a foundation for PDDAPE.

## 6. PDDAPE explained in a simple utility maximisation model

In the following model we integrate four elements from spatial interaction and demand modelling. The model describes a representative consumer who maximizes his consumption of information, constrained by a budget available for this purpose. There are two ways to obtain information, the first is by telephone contacts, the second by visits. For the organization of the visit a fixed number of telephone calls are required. Thus the total demand for calls consists of those calls made for information acquisition, and those calls necessary for arranging a visit. It is assumed that the price of a call is independent of distance, but the price of a visit rises with distance. This first assumption is not realistic but by making this assumption we can make clear that PDDAPE can be found as a result of spatial interaction theory that stresses the role of spatial interaction diminishing with increasing distance.

The model to be solved can accordingly be formulated as:

$$\text{Max } U(I) = U(I(T_i, V)) = T_i^\alpha V^{1-\alpha} \quad (9)$$

$$\text{s.t. } T = T_i + fV \quad (10)$$

$$p_t T + p_v V \leq M \quad (11)$$

In (9) we assume a simple Cobb-Douglas utility function with constant returns to scale, and  $0 < \alpha < 1$ . The consumer derives utility from information (I) which is generated by either telephonic contacts for this purpose ( $T_i$ ) or by visits (V). Thus, for generating information, visits and calls are substitutes, although less than perfect. (10) shows that the total demand for telephone calls (T) is composed of those calls made for generating information, and those that are required for organizing visits.  $f$  is the number of calls needed to organize a visit. The budget constraint (11) should be interpreted as the budget available to the consumer for the acquisition of information. We assume that only  $p_v$  depends on distance, not  $p_t$ . Thus  $\partial p_v / \partial L > 0$  and  $\partial p_t / \partial L = 0$ .

Substituting (10) into (11) makes this problem a standard Cobb-Douglas type of utility maximization model in  $T_i$  and V for which the resulting demand functions are well known. So we have

$$T_i = \alpha M / p_t \quad (12)$$

$$V = (1-\alpha) M / (p_v + fp_t) \quad (13)$$

Substituting (12) and (13) into (10) gives the demand for telephone calls:

$$T = M (fp_t + \alpha p_v) / p_t (p_v + fp_t) \quad (14)$$

We now seek to verify PDDAPE by calculating the own price elasticity and examine its distance dependence, based on the demand function (14). First we rewrite (14) by taking logs:

$$\ln T = \ln M + \ln (fp_t + \alpha p_v) - \ln p_t - \ln (p_v + fp_t) \quad (15)$$

Then we calculate the own price elasticity of T, which is less complex when we use the expression:

$$\varepsilon_{Tp_t} = \frac{\partial \ln T}{\partial p_t} p_t \quad (16)$$

After some algebra the price elasticity is found to be

$$\varepsilon_{Tp_t} = fp_t \left[ \frac{1}{fp_t + \alpha p_v} - \frac{1}{fp_t + p_v} \right] - 1 \quad (17)$$

Next, we take the partial derivative of this elasticity with respect to distance and find:

$$\frac{\partial \varepsilon_{Tp_t}}{\partial L} = \frac{fp_t(1-\alpha)}{(fp_t + \alpha p_v)^2 (fp_t + p_v)^2} [f^2 p_t^2 - \alpha p_v^2] \frac{\partial p_v}{\partial L} \quad (18)$$

From (18) it is immediately clear that when  $\alpha p_v^2 > (fp_t)^2$ , this partial derivative is negative, and hence PDDAPE is found. Unless we have very small values for  $\alpha$  and/or very high values for  $f$  this seems naturally the case<sup>8</sup>.

The intuition behind this result can be explained as follows. The demand for calls T consists of the sum of the informative calls  $T_i$  and organisatory calls  $fV$ .  $T_i$  has a constant elasticity of -1 with respect to  $p_t$ , the elasticity of  $fV$  is  $-fp_t/(p_v + fp_t)$  which is less than 1 in absolute terms. The elasticity of T is a weighted average of these two elasticities, where the weights are the budget shares  $T_i/T$  and  $fV/T$ . In a Cobb-Douglas utility maximization problem it is well known that the budget shares remain constant when

---

<sup>8</sup>Note that  $p_v$  is typically much larger than  $p_t$ . Thus, when for example  $p_v = 40$ ,  $p_t = 1$ ,  $\alpha = 0.2$  and  $f = 2$ , the first term within brackets (4) is much smaller than the second term (320).

prices change. Indeed, inspection of (12) and (13) shows that the shares of  $T_i$  and  $V + fV$  remain constant. Now, what happens when  $p_v$  rises, due to an increase in distance?

When  $p_v$  rises, the demand for  $T_i$  remains constant. The money spent on  $V + fV$  remains also constant, but since  $p_v$  rises a larger share of this amount is spent on  $V$ , and necessarily less organisatory calls are made. Hence, while the number of informative calls is constant, the number of organisatory calls decreases. Consequently, the share of informative calls in the total number of calls rises with distance. This means that the share of calls with the larger (absolute) price elasticity (i.e.  $T_i$ ) rises. This would unequivocally imply that the absolute price elasticity would rise *if* the price elasticity of the organisatory calls ( $fV$ ) with respect to  $p_t$  would be constant. This is not the case however, since  $p_v$  also appears in the denominator of this elasticity. Thus, the elasticity of  $fV$  with respect to  $p_t$  decreases in absolute term when  $p_v$  increases. So, we have two conflicting effects when  $p_v$  increases: the *share* of the calls with the larger elasticity rises, but the *magnitude* of the smaller elasticity decreases. The numerical example given in footnote 7 shows that the former effect is clearly dominant for plausible values, implying a strong support for the PDDAPE case.

## 7. Conclusions

A positive distance dependence of the price elasticity of demand is common knowledge in telecommunication research. In this paper we reexamined the validity of the statement and considered existing and new explanations for it.

In our review of the literature we found that PDDAPE is in general found for telephone demand within a country. International demand however, does not in general exhibit the PDDAPE effect. Apparently other effects, such as a mass effect are stronger. Further our review suggests that the elasticities for local and long distance national calls are converging over time. The elasticity for local calls rises (absolutely), the long distance elasticity falls somewhat.

Applying a meta-analysis to the surveyed studies revealed that the PDDAPE effect is found for long-run price elasticities, but that no such effect appears for short-run elasticities. This implies that explanations for the effect should be found in structural parameters and models.

According to many researchers, the PDDAPE effect is merely a price effect. We showed that this may be the case, but that it depends on the functional form of the demand function. Taylor (1994) introduces a heterogeneous good concept (“high value calls” versus “low value calls”) to explain PDDAPE. His explanation appears to depend critically on the assumption that the share of high value calls rises with distance. Some of his assumptions can be questioned although statistics support them.

Returning to the topic of functional forms of demand functions we discussed empirical studies that investigate the functional form by exploiting Box-Cox transformations of the data. The result of these studies is that a linear demand function is not at all supported by the data, while the double log form finds reasonable support. The double log form however, excludes any effect of the distance, unless it is accounted for in the parameter of the price term.

So, spatial interaction theory does not help directly in explaining PDDAPE. The discussion made clear however, that the solution is most likely to be found by explaining why the parameter of the price term is distance dependent. In the final section we studied a basic utility maximization model, incorporating notions from spatial interaction theory, and we showed that in many instances, i.e. for almost all reasonable



parameter values, PDDAPE is found.

To sum up: The PDDAPE effect is found for domestic telephone demand, for which various explanations exist (price, heterogeneous goods, spatial interaction theory). However, in none of these three cases a *general* proof of the positive distance dependence of the absolute price elasticity is given. In all cases additional assumptions are needed before such a proof can be given. These additional assumptions relate to:

- functional form of the demand function (price argument);
- share of high value calls at longer distance (heterogeneity argument);
- trade off between visits and calls (spatial interaction argument).

Further empirical research into these topics would be most welcome. Especially the heterogeneity of telecommunication services, and the relationship between spatial interaction and telecommunication are topics that deserve more attention in telecommunications demand studies.

**References**

Acton, J.P. and I. Vogelsang, (1992), Telephone Demand over the Atlantic: Evidence from Country Pair Data, The Journal of Industrial Economics, Vol. 40, pp. 305-323.

Bröcker, J. and H.C. Rohweder (1990), Barriers to International Trade, Annals of Regional Science, Vol. 24, pp. 289-306.

Chang, H.S. and Y. Hsing (1991), The Demand for Residential Electricity: New Evidence on Time Varying Elasticities, Applied Economics, Vol. 23, 1251-1256.

Davis, B.E., G.J. Caccappolo and M.A. Chaudry (1973), An econometric planning model for American Telephone and Telegraph Company, Bell Journal of Economics and Management Science, Vol 4, p. 29-56

Deaton, A. and J. Muellbauer (1980), Economics and Consumer Behavior, Cambridge University Press, New York.

Dobell, A.R., L.D. Taylor, L. Waverman, T.H. Liu and M.D.G. Copeland (1972), Telephone communications in Canada: demand, production, and investment decisions, Bell Journal of Economics and Management Science, Vol. 3, pp 175-219.

Duncan, G.M. and D.M. Perry (1994), IntraLata toll demand modeling: a dynamic analysis of revenue and usage data, Information Economics and Policy, Vol. 6, p. 163-178.

Erlander, S. and N.F. Stewart (1990), The Gravity Model in Transportation Analysis - Theory and Extensions, VSP, Utrecht.

Fiebig, D.G. and R. Bewley (1987), International Telecommunications forecasting: an investigation of alternative functional forms, Applied Economics, vol. 19, p. 949-960.

Glass, G.V. (1976), Primary, Secondary, and Meta-Analysis of Research, Educational Researcher, vol. 5, p. 3-8.

Hackl, P. and A.H. Westlund (1992), Demand for International Telecommunication: Time Varying Price Elasticities, Paper presented at the CRDE/Journal of Econometrics Conference on Recent Developments in the Econometrics of Structural Change, Montreal.

Hackl, P. and A.H. Westlund (1995), On price elasticities of international telecommunications demand, Information Economics and Policy, Vol. 7, p. 27-36.

Kling, J.P. and S.S. van der Ploeg (1990), Estimating local call elasticities with a model of stochastic class of service and usage choice, in DeFontenay A.F., M.H. Shugard and D.S. Sibley (eds), Telecommunications Demand Modelling, Elsevier Science, Amsterdam.

Lago, A.M. (1970), Demand Forecasting of International Telecommunications and their Policy Implications, Journal of Industrial Economics, Vol. 19, p. 6-21

Larson, A.C., D.E. Lehman and D.L. Weisman (1990), A General Theory of Point-to-Point Long Distance Demand, in DeFontenay A.F., M.H. Shugard and D.S. Sibley (eds), Telecommunications Demand Modelling, Elsevier Science, Amsterdam.

Madden, G., H. Bloch and D. Hensher (1993), Australian telephone network subscription and calling demands: evidence from a stated preference experiment, Information Economics and Policy, Vol. 5, p. 207-230.

Rea, J.D. and G.M. Lage (1978), Estimates of Demand Elasticities for International Telecommunications Services, Journal of Industrial Economics, Vol. 26, p. 363-381.

Rietveld, P. J. Van Nierop and H. Ouwersloot (1993), Barriers to International Telecommunications: what are your ten most frequently called countries?, Sistemi Urbani, Vol. 2/3, p. 171-187.

Taylor, L.D. (1980), Telecommunications Demand: a survey and a critique, Ballinger, Cambridge, MA.

Taylor, L.D. (1994) Telecommunications Demand in Theory and Practice, Kluwer, Dordrecht.

Author(s)	Estimate(s) of elasticity	S/L run	Country /region	Period	Estimation Issues
Waverman*	-0.27 -0.38	S L	Local within Sweden	1949-1969	double log
Bell system*	-.03 to -.44	L	31 US states		double log AR, a.o.
Feldman*	-.31 to -.75 (R)** -.38 to -1.31 (B)	L	US states		double log Dep. on def. of y
Stuntebeck*	-.05 to -.11 -.21 to -.27 -.24 to -.59	L	US states 37.5 - 75 km 75 - 150 km 150 - 300 km	1968-1973	double log Dep. On way of pooling
Davis <i>et al.</i> (1973)	-.21 -.31	S L	US Bell system	1961-1971	double log AR
Bell Canada*	-.28 to -.32 -.37 to -.39	L	< 225 km (Peak/Off Peak) > 225 km (Peak/Off Peak)	1974-1983	double log
Zona & Jacob*	-.15 (Local) -.47 (IntraLata) -.41 (InterLata)	L	US	1987	Almost Ideal Demand System
Dobell <i>et al.</i> (1972)	-0.23 -0.70	S L	Local, Canada	1952-1967	Linear AR
Duncan and Perry (1994)	-0.16 -0.37	S L	California IntraLata	1986-1990	Double log AR
Larson <i>et al.</i> (1990)	-.32 -.76	S L	IntraLata 9 city pairs	1985	Double log
Kling and Van der Ploeg (1990)	-.17	L	Michigan local	1984-1985	Log linear

**Table 1** A sample of short distance price elasticities (local and intrastate).

\*Study reported in Taylor (1994)

(R) = Residential, (B) = Business

Author(s)	Estimate(s) of elasticity	S/L run	Country/ region	Period	Estimation Issues
Deschamps*	-.24	L	Belgium (national, including local)	1961-1969	double log
Larsen and McCleary*	-1.01 (R) -.98 (B)	L	USA interstate	1966/1968	double log
Waverman*	-1.16 (R) -1.20 (B) -1.35 (B) -.72 -1.12 -.51 -1.08	L S L S L S L	Canada Canada Great Britain Sweden		double log except GB: Linear AR Var. Def of y
Davis <i>et al.</i> (1973)	-.88 -1.03	S L	US Bell system	1962-1971	double log AR
Applebe*	-.36 to -.59 -.48 to -.70 -.73 to -.75	L	Canada < 1000 km 1000-2400 km > 2400 km	1977 - 1986	double log Normal and discount rate elasticities
Gatto <i>et al.</i> *	-.72	L	US	1984-1987	double log
Dobell <i>et al.</i> (1972)	-0.11 -2.57	S L	Canada, interprovincial	1952-1987	Linear AR

**Table 2** A sample of long distance elasticities, national.

\*Study reported in Taylor (1994)

(R) = Residential, (B) = Business

Author(s)	Estimate(s) of elasticity	S/L run	Country/ region	Period	Estimation Issues
Applebe <i>et al</i> *	-0.43 to -0.45 -0.49 to -0.53	L	Canada - US < 1200 km > 1200 km	1977-1986	double log normal/ discount rate
Fiebig & Bewley (1987)	-0.25 -2.19 -1.53 -0.53 -0.6 -0.32 0.27 -0.03 -0.61 -0.75 -0.34 to -1.47 -0.88 to -2.47 -2.41 to -1.47 -1.14 to -0.40 -0.78 to -0.43 -0.26 to -0.59 -2.03 to -0.56 -0.64 to -0.18 -0.61 to -0.74 -3.36 to -1.05	L           S	Australia to: Papua New G. New Zealand Hong Kong Japan USA Canada Greece Italy West Germany Great Britain Papua New G. New Zealand Hong Kong Japan USA Canada Greece Italy West Germany Great Britain	1966 - 1982	Box Cox form, unconstrained $\lambda$ For each year a different short run elasticity can be calculated, the elasticities for 1966 and 1982 are given
Acton & Vogelsang (1992)	-0.36 -0.49 -0.26 -0.28	L	US to Europe Europe to US US to E E to US	1979 - 1986	double log Marginal prices  Average prices
Lago (1970)	-1.69	L	US to 23 European countries	1962-1964	double log
Rea and Lage (1978)	-1.72	L	US to 37 countries	1969-1973	double log
Hackl and Westlund (1995)	-0.96 -0.98 -0.37 -0.98 -1.18 -0.79 -0.12 -0.49 -0.26 -0.39 -0.51 -0.30	L       S	Sweden to USA " to UK " to Germany " to Denmark " to Norway " to Finland USA UK Germany Denmark Norway Finland	1976-1990	double log AR

**Table 3** A sample of long distance, international elasticities.

\*Study reported in Taylor (1994)

(R) = Residential, (B) = Business

---

Dependent variable: price elasticity of telecommunication demand

Variable	Long run elasticities		Short run elasticities	
	Est	St. Error	Est.	St. Error
Constant	-0.16	0.38	-1.15	1.15
Log distance	-0.088	0.052*	0.022	0.119
Year - 1957	0.022	0.008**	0.036	0.027
International	-0.060	0.219	-0.905	0.812
Business	-0.136	0.240	-0.492	0.732
North american	-0.269	0.204	-0.247	0.563
Volume	-0.208	0.156	0.458	0.752
	$R^2 = 0.23$		$R^2 = 0.20$	
	N = 62		N = 26	

\* Indicates significant at 10% level.

\*\* Indicates significant at 5% level.

---

**Table 4** Meta-analysis of price elasticities

**Figure 1** Income expansion path of two goods, a luxury and a necessity

