LUMP SUM MOVING COST AND AGGREGATE OFFICE SPACE USE : THEORY AND EMPIRICAL EVIDENCE FOR THE NETHERLANDS, 1974-1995

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Abstract

When firms decide to change office space use, in many instances this involves relocation. Relocation involves sizable costs to the firm that can to a large extent be characterized as lump sum, i.e. independent of the change in demand. In this paper we propose and solve a model of the demand for office space with lump sum adjustment costs at the firm level. The optimal policy for a firm is a so-called control band policy, or (*s*,*S*)-rule: leave office space use unchanged until the difference between actual office space use and desired office space use exceeds a certain threshold. Desired office space use is defined as the office space use that would result if no frictions were present and conforms to a model were actual office space depends only on relevant current state variables.

Next we go on to investigate the aggregate implications of this lumpy microeconomic behaviour using a stochastic aggregation framework of Bertola and Caballero (1994). The lumpy behaviour at the firm level implies that aggregate demand for office space is a time-varying weighted average of the current and lagged state of the economy. Only a fraction of aggregate demand depends on the current state of the economy. Furthermore, the magnitude of this fraction also depends on the current state of the economy.

We use our model on office space market data for The Netherlands. We find that desired office space use is more volatile than actual office space use and does not track actual office space use very well. Aggregate office space implied by our model tracks actual office space use much closer, indicating that the gap between theoretical desired office space use and actual office space use can be accounted for by lumpy adjustment at the firm level.

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1. INTRODUCTION

When one sets out to model the market for office space, one of the central variables that requires modelling is the demand for office space or the occupied stock of office space. In one of the first attempts Rosen (1984) models the occupied stock of office space as being a function of the employment level in the key service producing industries and the real rental rate. When one projects this aggregate model to the microeconomic level it implies that office space using firms change their use of office space every time their employment or the rent changes. However, we see in practice that firms only infrequently change their use of office space. To remedy this Wheaton (1987) models the net absorption (i.e. the change in occupied stock) as a partial adjustment process: current absorption equals a fraction of the difference between desired office space use and lagged actual office space use. Desired office space use is again modelled as a function of office employment and the real rental rate. Additionally, Wheaton adds office employment growth to account for expectations regarding future office space needs. Hence Wheaton accommodates the gradual change in occupied office space in two ways, the partial adjustment of net absorption and the appearance of office employment growth in desired office space use. However, both mechanisms are incorporated in an ad hoc fashion at the aggregate level. At the firm level it is not realistic to assume that firms adjust gradually to some desired level of office space use since this involves continual adjustment of office space use, something Wheaton claims hardly occurs. Instead he says

> "*It is likely that the long-term leasing structure of the office market reflects a high cost to moving and relocating business.*" [Wheaton, (1987, p. 285)]

In a recent paper Romijn, Hakfoort and Lie (1996) (henceforth referred to as RHL) introduce adjustment costs in a microeconomic model for the use of office space. They motivate these costs primarily as relocation or moving cost for which it is reasonable to assume that these costs are to a large extent unrelated to the amount of office space in use or the change therein. Therefore they model them as being lump sum. This adjustment cost structure implies that the individual firm's office space use is governed by a (*s*,*S*)-rule.[1] Hence office space using firms only infrequently relocate, just as observed in practice. Their empirical findings are based on a cross section survey of individual firms and indicate that at the firm level the lump sum nature of relocation costs matters.

This paper uses the set up of RHL with lump sum adjustment cost at the firm level but instead of the heuristic solution given in RHL, here we solve explicitly an intertemporal maximization programme. Additionally, instead of focusing on the microeconomic implications as did RHL, we investigate the implications the individual firm's behaviour has for aggregate office space use. The data we use concern the Dutch office market for which a consistent dataset has been compiled in Romijn (1996). As main office space using industries we identify the government, banking and insurance, and other commercial services. Output and employment in these sectors are assumed to be a good indicator for office related output and office employment.

The rest of this paper is set up as follows. In section 2 we formulate and solve the model for firm behaviour, which is governed by a so-called control band policy or (*s*,*S*)-rule. The mathematical argumentation in this section is heuristic and not entirely rigorous. For more rigor we refer to the paper by Harrison, Sellke and Taylor (1983) (henceforth referred to as HST).

Obviously when all firms are exactly identical, aggregate behaviour would coincide with individual behaviour. However, we assume that individual firms are faced by stochastic shocks that are only partly shared by other firms. This implies that at every point in time a certain fraction of all firms will relocate whereas others will not. To determine what fraction of firms relocates we have to concern ourselves with distributional issues. In section 3 this problem is addressed using the framework of Bertola and Caballero (1994). This results in a relation between aggregate desired office space and aggregate actual office space use with the gap between them depending on the growth rate of desired office space use.

In section 4 we first calculate aggregate desired office space use which is subsequently used for calculating the aggregate gap and obtaining an

[1]For more on (*s*,*S*)-rules and lump sum adjustment costs, see for instance Blanchard and Fischer (1989, ch. 8), or Caballero and Engel (1991).

estimate for actual office space use. Our estimate for aggregate office space use tracks actual office space use much better than desired office space implying that observed aggregate behaviour can indeed be accounted for by our relocation-cost-cum-stochastic-aggregation model. Finally, section 5 summarizes and concludes.

2. OPTIMAL DEMAND FOR OFFICE SPACE WITH LUMP SUM ADJUSTMENT COSTS

Consider a firm that uses office space in the production process. We want to focus on the demand for office space exercised by this firm. Assume that there exists a desired demand for office space that summarizes all relevant information about the office space use of the firm. This desired demand evolves stochastically over time. We interpret desired office space use as the minimum cost or maximum efficiency office space use for the firm in the sense that when actual office space use equals the desired office space use, the intensity of use of the office space is optimal. Actual office space use may deviate from its desired level. These deviations result in extra costs or loss of efficiency. This implies that in a frictionless environment a cost minimizing firm would like to adjust its demand for office space continually in response to the stochastic fluctuations in desired office space use. However, we assume that in order for the firm to change its demand it has to move to a new location and that this move is costly. Specifically, we assume that the cost of moving is lump sum. This implies that the firm has to balance two types of cost. On the one hand there is the opportunity cost of not adjusting demand for office space to its desired value. On the other hand the more frequently the firm moves the higher will be the moving costs.

The above problem can be reformulated as a special case of a more general problem studied by HST. They show that the optimal policy is a so called *control band policy* (CBP). This policy entails an optimal demand for office space depending on the state variable (in our case desired office space use) and intervention bands around this optimal demand. As long as actual demand remains within the bands the firm does not move and hence does not adjust demand. When actual demand is on or outside the band it is optimal for the firm to move and adjust its demand to the optimal demand.

Denote by $z(t)$ the logarithm of actual office space use and by $z^d(t)$ the logarithm of desired office space use. We model $z^d(t)$ as a $(-\mu,\sigma)$ Brownian motion, i.e.

$$dz^d(t) = -\mu\, dt + \sigma\, dw(t) \tag{1}$$

with $w(t)$ a standard Brownian motion. For simplicity we model the above-mentioned loss of efficiency due to deviations from actual office space use from desired office space use to be quadratic in the deviation, i.e. $\frac{1}{2}[z(t)-z^d(t)]^2$. The firm now faces the problem of deciding whether or not to move and what demand to exercise if it moves. When the firm moves to a new building it incurs a lump sum moving cost of magnitude γ. Otherwise it remains in the old building without changing its demand for office space.

To solve this problem note first that it is not optimal for the firm to adjust its demand for office space continually as it would then incur the strictly positive moving cost γ at each moment in time making total moving cost infinite. Hence, the firm will only move at discrete intervals. Denote the times at which it moves by T_n, $n \in \{0,1,\ldots\}$, $0 = T_0 < T_1 < \ldots \rightarrow \infty$. Furthermore denote the change in the demand for office space at points in time when the firm moves by ξ_n. Because of the assumption of $T_0 = 0$ we have to allow for $\xi_0 = 0$. Hence, we see that a policy consists of sequence of stopping times $\{T_0,T_1,\ldots\}$ and associated stochastic jumps $\{\xi_0,\xi_1,\ldots\}$. Now define the cost function of moving

$$\phi(\xi) = \begin{cases} 0 & \mathit{for}\ \xi = 0 \\ \gamma & \mathit{for}\ \xi \neq 0 \end{cases} \tag{2}$$

Next, define $N(t) = \sup\{n \geq 0 : T_n \leq t\}$ and $y(t) = \xi_0 + \ldots + \xi_{N(t)}$ the cumulated change in the demand for office space from time zero to time t, $x(t) = -z^d(t)+z(0)$ a (μ,σ) Brownian motion, and $u(t) = z(t)-z^d(t)$ the gap between actual and frictionless demand. The latter variable follows a process that is the sum of two processes: $u(t) = x(t)+y(t)$. Note that $\xi_n = u(T_n)-u(T_n-)$. We see that without any action by the firm the instantaneous rate of cost is given by $\frac{1}{2}x(t)^2$. This may however result in large negative rate of profit so occasionally - at the stopping times - it is profitable for the firm to change its demand by an amount ξ so as to bound losses.

For any feasible policy $\{(T_n,\xi_n)\}$ and initial value $x(0) = x$ define the value function $V(x)$ to be the current value of all expected future cost discounted to the present at rate r. It is given by

$$V(x) = E\left[\int_0^\infty \tfrac{1}{2} u(t)^2 e^{-rt} dt + \sum_{n=0}^{\infty} \phi(\xi_n) e^{-rT_n}\right] \tag{3}$$

The firm will be assumed to choose the CBP $\{(T_n,\xi_n)\}$ that minimizes this value function.

The Value Function

It can be shown by arguments similar to those in HST that an optimal CBP is characterized by a set of numbers (s,S,Q), $s < Q < S$, with Q the optimal demand for office space, and s and S the lower bound and upper bound of the CBP, respectively. The CBP parameters are parameters of the value function. Since the value function does not depend on time, we know that the CBP parameters cannot be functions of time. Additionally, the CBP parameters are values of the state variable for which the value function meets certain criteria: Q is the value of the state variable that maximizes the value function. s and S are the values of the state variable for which the boundary of the control band is reached and the boundary conditions for the value function apply. This implies that the CBP parameters are constants and not function of time or the state variable.

When $u(T_n\text{-})$ reaches the lower bound s the firm will move into a new building and change its demand to Q. Hence, for $n \geq 1$, the jump in u_t will be given by $\xi_n = Q\text{-}s > 0$. Analogously, when $u(T_n\text{-})$ reaches the upper bound S the jump is equal to $\xi_n = Q\text{-}S < 0$. For time zero we have to allow for the possibility of a jump of size zero. Hence, we define

$$\xi_0 = \begin{cases} 0 & \textit{if } s < x < S \\ Q - x & \textit{otherwise} \end{cases} \tag{4}$$

To find an explicit solution for the value function, note that between stopping times by definition no jumps occur and u will remain between the upper and lower intervention band. Hence for values of u between the upper and lower band (or analogously for points in time between two adjacent stopping times), the bellman equation that follows from (3) can be obtained by forgetting about the second term on the RHS of (3) and using Ito's Lemma (see for instance Dixit and Pindyck (1994) ch.3). This of course also holds for the initial value x when $s \leq x \leq S$ and we obtain

$$rV(x) = \tfrac{1}{2}x^2 + \tfrac{1}{2}\sigma^2 V''(x) + \mu V'(x) \quad \textit{for } s \le x \le S \tag{5}$$

Additionally, we have boundary conditions given by

$$\begin{aligned} V(s) &= V(Q) + \gamma \\ V(S) &= V(Q) + \gamma \end{aligned} \tag{6}$$

The solution to differential equation (5) is given by

$$V(x) = Ae^{\alpha x} + Be^{-\beta x} + v_0 + v_1 x + \tfrac{1}{2}v_2 x^2, \quad s \le x \le S \tag{7}$$

with

$$\begin{aligned} \alpha &= [(\mu^2 + 2\rho\sigma^2)^{1/2} - \mu] / \sigma^2 > 0 \\ \beta &= [(\mu^2 + 2\rho\sigma^2)^{1/2} + \mu] / \sigma^2 > 0 \end{aligned} \tag{8}$$

and

$$v_0 = \tfrac{1}{2}\frac{\sigma^2}{r^2} + \frac{\mu^2}{r^3}, \quad v_1 = \frac{\mu}{r^2}, \quad v_2 = \frac{1}{r} \tag{9}$$

The constants A and B can be found by substituting (7)-(9) into the boundary conditions (6). Define $a(y) = e^{\alpha y} - e^{\alpha Q}$ and $b(y) = e^{-\beta y} - e^{-\beta Q}$. We obtain

$$A = \frac{[v_1(Q-s) + \tfrac{1}{2}v_2(Q^2 - s^2) + \gamma]b(S) - [v_1(Q-S) + \tfrac{1}{2}v_2(Q^2 - S^2) + \gamma]b(s)}{a(s)b(S) - a(S)b(s)} \tag{10}$$

$$B = \frac{[v_1(Q-S) + \tfrac{1}{2}v_2(Q^2 - S^2) + \gamma]a(s) - [v_1(Q-s) + \tfrac{1}{2}v_2(Q^2 - s^2) + \gamma]a(S)}{a(s)b(S) - a(S)b(s)} \tag{11}$$

Now we want to extend the value function for values of x outside the control band. To see how, note that when x lies outside the control band, by definition the firm will immediately pay the moving cost γ and jump to Q. Hence,

$$V(x) = V(Q) + \gamma, \quad x \notin [s,S] \tag{12}$$

This completes the characterization of the value function for CBP (s, S, Q), $s < Q < S$, and starting value x.

Optimal Control Band Parameters

Having obtained the value function we can now find the optimal CBP consisting of the set of numbers (s, S, Q), $s < Q < S$. If the firm starts outside the interval control band, i.e. $x \notin [s,S]$, it will immediately jump to Q and follow the CBP (s, S, Q). The total reward from this will be $V(Q) + \gamma$. For Q to be optimal we should have

$$V'(Q) = 0, \quad V''(Q) > 0 \qquad \textbf{(13)}$$

Additionally, by arguments similar to those in section 5 of HST it can be shown that the following conditions hold at the boundaries

$$\begin{aligned} V'(s) &= V'(Q) \\ V'(S) &= V'(Q) \end{aligned} \qquad \textbf{(14)}$$

These can be interpreted as some sort of smooth pasting conditions for problems involving jump processes.

Solving equations (13) and (14) yields the following expressions for parameters of the optimal CBP. The derivative of the value function is given by

$$V'(x) = \begin{cases} \alpha A(s,S,Q)e^{\alpha x} - \beta B(s,S,Q)e^{-\beta x} + v_1 + v_2 x & \textit{for } s \le x \le S \\ 0 & \textit{otherwise} \end{cases} \qquad \textbf{(15)}$$

The roots of (15) yield a system of equations the solutions of which are the optimal control band parameters (s,S,Q). Unfortunately the roots of (15) cannot be determined analytically and hence we have to resort to numerical solutions for reasonable values of the model parameters (μ, σ, γ, r).

3. AGGREGATE DEMAND FOR OFFICE SPACE

We now turn to the aggregate implications of our lump sum adjustment cost micromodel. For aggregation of individual units' actions we rely heavily on Bertola and Caballero [1994, pp. 229-234].

First, the markets for real estate are populated by a large number of units which we approximate by continuum indexed by $i \in [0,1]$. To facilitate the subsequent discussion we introduce some notation. Let $x_i(t)$ denote the value of a variable x for unit i at time t. Additionally, let $\tilde{x}(t)$ denote a random variable with a probability distribution $\phi(x,t)$ identical to that of the cross-section distribution of the $x_i(t)$ (see Caballero and Engel [1991, p.1663] for this construct). Note that the following holds

$$E\tilde{x} = \int_0^1 x_i \, di \qquad \textbf{(16)}$$

Finally, let $x(t)$ denote the associated aggregate.

First consider actual office space use. To aggregate we simply sum over all units, i.e. $Z = \int_0^1 Z_i \, di$. Hence the process followed by the logarithm of aggregate actual office space use $z(t)$ is found as

$$dz(t) = \int_0^1 h_i(t) \, dz_i(t) \, di \tag{17}$$

with $h_i(t)$ unit's i share in aggregate office space use with $\int_0^1 h_i \, di = 1$.

Individual frictionless demand $z_i^d(t)$ follows a process given by

$$dz_i^d(t) = -\mu \, dt + \sigma \, dw_i(t) \tag{18}$$

with $w_i(t)$ a standard brownian motion. To aggregate individual desired office space use we have to make assumptions about how individual uncertainty translates into aggregate uncertainty. We assume that the correlation structure between the individual firms can be described by $E[dw_i(t)dw_j(t)] = \rho^2$, for all $i,j \in [0,1]$, $i \neq j$. This implies that the covariance between an individual shock $dw_i(t)$ and the aggregate shock $dw(t)$, which is given as

$$\rho \, dw(t) = \int_0^1 h_i(t) \, dw_i(t) \, di \tag{19}$$

equals ρ. Hence we can decompose the individual brownian motions $w_i(t)$ into an aggregate and a purely idiosyncratic component $w_{Ii}(t)$ according to

$$dw_i(t) = \rho \, dw(t) + \sqrt{1 - \rho^2} \, dw_{Ii}(t) \tag{20}$$

By construction the idiosyncratic components are uncorrelated among each other and the aggregate shock, and wash out in the aggregate. Using the expression for aggregate uncertainty in (19) and aggregating (18) we obtain

$$dz^d(t) = -\mu \, dt + \sigma \rho \, dw(t) \tag{21}$$

Now define stochastic variables $\tilde{u}$ and $\tilde{h}$ with probability density functions identical to the cross sectional distribution of the u_i and h_i. Since there is no reason to assume that the u_i and h_i vary systematically with each other we assume the opposite, i.e. $\tilde{u}$ and $\tilde{h}$ are independent. Note that the following relations hold

$$\int_0^1 h_i u_i di \;=\; E(\tilde{h}\tilde{u}) \;=\; E(\tilde{h})E(\tilde{u}) \;=\; E(\tilde{u}) \;=\; \int_0^1 u_i di \;\equiv\; u \tag{22}$$

Using these we obtain an expression for the process for actual aggregate office space use given as

$$dz(t) \;=\; dz^{d}(t) \;+\; du(t) \tag{23}$$

and we see that the difference between actual and desired aggregate net absorption ratios differ by the change in the average difference of logged actual and desired office space use at the firm level. To obtain the change in this average we need to track the probability density function $\phi(\tilde{u},t)$ through time.

First consider the case where no aggregate shocks are present and all shocks are fully idiosyncratic, i.e. $\rho = 0$, and the cross section density has settled into its steady state. Due to the independence of the different shocks and the fact that the number of units is large, this density $\phi(\tilde{u})$ is identical to the ergodic density of a single $u_i(t)$. The derivation of this density is derived in Appendix 1. It is given as

$$\phi(\tilde{u}) \;=\; \begin{cases} A_1[e^{\theta\tilde{u}} \;-\; e^{\theta s}], & s \;<\; \tilde{u} \;<\; Q \\ A_2[e^{\theta\tilde{u}} \;-\; e^{\theta S}], & Q \;<\; \tilde{u} \;<\; S \end{cases} \tag{24}$$

with $\theta = 2\mu/\sigma^2$, $A_1 = cA_2$, $c = [e^{\theta Q}\text{-}e^{\theta S}][e^{\theta Q}\text{-}e^{\theta s}]^{-1}$, and $A_2 = \text{-}[ce^{\theta s}(Q\text{-}s)\text{-}e^{\theta S}(Q\text{-}S)]^{-1}$. From this it follows that in steady state with uncorrelated shocks $u(t)$ is given as

$$\begin{aligned} E(\tilde{u}) \;=&\; (A_1/\theta)\left[(Q-1)e^{\theta Q} - (s-1)e^{\theta s}\right] \;-\; (A_1/2)e^{\theta s}(Q^2-s^2) \;+ \\ &+\; (A_2/\theta)\left[(S-1)e^{\theta S} - (Q-1)e^{\theta Q}\right] \;-\; (A_2/2)e^{\theta S}(S^2-Q^2) \end{aligned} \tag{25}$$

Now we want to introduce aggregate shocks. When aggregate shocks are present the shocks faced by individual firms are correlated and the steady state cross sectional density can no longer be represented by the ergodic density of a single random walk in a CBP. Instead the cross sectional density is changing at every point in time and will not settle down into a steady state density as long as new aggregate shocks keep arriving. To model aggregate shocks we use the approach used by Bertola and Caballero (1994). They approximate the ongoing aggregate shocks by discrete changes in the drift rate μ. In other words we assume that the realizations of aggregate uncertainty are evenly spread within an observation interval. Bertola and Caballero (1994) motivate this as follows

> '*accumulation over a finite time period of abnormally positive aggregate shocks has roughly the same effect for the cross-sectional distribution as a larger mean rate of growth*' [Bertola and Caballero (1994, p. 232)].[2]

This approximation neglects within period path-dependency and infinite variation of Brownian motions. About this Bertola and Caballero (1994) say

> '*... any empirical importance of these issues is overshadowed by the substantial simplification of the analytical and estimation problem*' [Bertola and Caballero (1994, p. 242)]

In addition to these simplifications note that data on the Dutch office markets is available on an annual basis only. Since this constitutes relatively low frequency data, this motivates another simplification. We assume that the length of the time interval relative to the time-scale at which the infinitesimal processes in our micromodel operate is large. Hence we assume that by the end of a period the effects of the change in the aggregate growth rate at the beginning of the period have petered out and the cross-section distribution has settled into its steady state distribution associated with a drift rate of μ_t.

4. EMPIRICAL IMPLICATIONS AND EVIDENCE

In this section we assess the aggregate empirical implications and importance of microeconomic lumpy adjustment. In order to do so we first have to find an estimate for aggregate desired office space use z^d. Subsequently we calibrate

[2]Note that normally the drift rate affects the parameters of the CBP. However, in this case the changing drift rate is an approximation to an aggregate process. To the individual firms the growth rate remains constant and hence does not affect the CBP parameters. So we let the drift rate vary with unchanging CBP parameters.

the micromodel parameters (μ,σ,γ,r).

Aggregate desired office space use

Consider again the individual firm. The firm takes input prices and the rate of output as given and minimizes cost conditional on input prices and output. We assume that the firm has two inputs, labour L and office space Z (capitals indicate levels whereas lowercase letters indicate logarithms) with wage rate W and rental rate R. Denote output (= value added) by office using sectors by Y. Additionally, we use a unit of output as numeraire so both rent and wage rate are deflated by the output price index for office using sectors.

In a frictionless world cost minimization yields a cost function as a function of output and input prices alone. Denote this cost function by $C(Y,R,W)$. Frictionless conditional factor demand is then given by $Z^d = \partial C/\partial R$ and $L = \partial C/\partial W$. In the presence of relocation cost total costs consist of frictionless cost, costs of relocation and costs associated with deviation of actual office space use from desired office space use. We model the latter as in section 2, i.e. quadratic in the difference of the logarithm of actual and desired office space use. Hence, the rate of total costs are given by

$$C(Y,R,W) \ + \ \kappa\left[\tfrac{1}{2}(z-z^{d})^2 \ + \ \phi(\xi)\right] \tag{26}$$

The present value of expected future cost is then given as

$$E\left[\int_0^\infty C(Y,R,W)e^{-rt}dt\right] \ + \ \kappa E\left[\int_0^\infty \tfrac{1}{2}(z-z^{d})^2 e^{-rt}dt \ + \ \sum_{n=0}^{\infty} \phi(\xi_n)e^{-rT_n}\right] \tag{27}$$

The first term is given to the firm whereas the second term is just κ times the value function of equation (3). Hence, we see that minimizing total cost as given in (27) is equivalent to the cost minimization problem of section 2.

To find an estimate for z^d we now only have to specify a frictionless cost function $C(Y,R,W)$, take its derivative with respect to R and transform to logarithms. What should the functional form of $C(Y,R,W)$ be? To get a clue we graphed the share of office space expenditure in output and the logarithm of the rental rate over the wage rate in Figure 1 (we standardized both series to have zero mean and unit variance to fit into one graph). Obviously these two series have a lot in common which leads us to consider a translog functional form for the frictionless cost function. Hence

$$\begin{aligned}\log C(Y,R,W) &= \alpha_0 + \alpha_1 \log R + (1-\alpha_1)\log W + \\ &+ \tfrac{1}{2}\alpha_2(\log R)^2 - \alpha_2 \log R \log W + \tfrac{1}{2}\alpha_2(\log W)^2 + \\ &+ \alpha_3 \log Y\end{aligned} \qquad \textbf{(28)}$$

Note that in the absence of frictions profit maximization yields $C = (1/\alpha_3)Y$. Usually α_3 is restricted to unity so that $C = Y$, i.e. no profit is made. In our case this cannot be imposed because in addition to frictionless cost C we also have costs associated with the friction. Setting α_3 to unity would then imply that the firm would never make a positive profit and make a loss most of the time. This is clearly inconsistent. Instead we impose that the firm should make a strictly positive profit when it is at its desired demand, i.e. $\alpha_3 > 1$.

Conditional desired demand for office space can be found by differentiating frictionless cost with respect to the office space rental rate. Using equation (28) we find

$$S^d = \alpha_3\alpha_1 + \alpha_3\alpha_2\left[\log R - \log W\right] \qquad \textbf{(29)}$$

with S^d the desired share of office space expenditure in output, i.e. $S^d = RZ^d/Y$. Using equation (23) we see that $S^d = S/U$ with S the actual share. From the way in which U is constructed we know that it must be stationary. Figure 1 indicates however that the office expenditure share is not stationary over the sample period. Linearizing the relation between S^d, S and U we see that a co-integrating relation exists between S and S^d. Hence to obtain an estimate of S^d we estimate a co-integrating relation between S and r-w. The results are reported in Table II. The VECM that was ultimately used to calculate the co-integrating relation is of order 2. The deterministic term has an unrestricted constant and a trend component that is restricted in the co-integrating relation. Hence, the estimated co-integrating relation between S and r-w includes a time trend. At a significance level of 10 per cent we find one co-integrating relation. This co-integrating relation is then used to calculate desired frictionless share of office space expenditures S^d. The growth rate of desired frictionless office space use can then be found as $dz^d = ds^d + dy - dr$.

Figure 2 contains the growth rates for actual aggregate office space use and desired office space use. We see that desired office space use is a lot more volatile than actual demand. This is also indicated by the summary statistics in Table I. The growth rate standard deviation of actual office space use is 1.43 per cent per annum, whereas the growth rate standard deviation of desired office space use equals 2.28 per cent per annum.

The cross-correlation reported in Table I show that the contemporaneous correlation between the growth rates of actual and desired aggregate office space use is only .435 leaving ample space for improvement. Additionally, from the cross correlation pattern, we see that actual office space use lags desired office space by approximately one year. This can also be seen in Figure 2.

Calibration of CBP model parameters

In section 2 we assumed that desired office space use to be a $(-\mu,\sigma)$-Brownian Motion. This implies that the increments for desired office space use are identically independently normally distributed. The Jarque-Bera test for normality and the AR(1) parameter and its *t*-value that are also reported in Table I, do not indicate important deviations from these assumptions. Hence, using the figures in Table I, we set μ = -.0334 and $\rho\sigma$ = .0228. Note that this choice of parameters implicitly sets the unit of time to a year.

Next, we need to find a value for the discount rate r. The discount rate only affects the boundaries of the inaction interval (s,S) and the optimal value Q. Moreover, these parameters are not very sensitive to the actual choice of r, so that the precise choice of r is not very critical. We set the discount rate at 5 per cent per annum. This is a reasonable choice that is also frequently employed in the real business cycle literature.

Finally, we need to find values for the correlation between individual and aggregate shocks ρ, and for the lump sum moving cost γ. This is however a bit of a problem since we do not have any information regarding their magnitude. We estimate γ and ρ so that the growth rates of calculated and actual demand for office space resemble each other as much as possible. We do this by minimizing the variance of the difference between calculated and actual demand using a grid search. This yields ρ = .15 and γ = .025. This choice implies that individual shocks correlate relatively weakly with aggregate shocks and that a large share of the shocks faced by individual firms is purely idiosyncratic.

Implied aggregate office space use

Having found values for our micromodel parameters we can now set out to calculate aggregate office space use as implied by our model. Denote its logarithm by $\hat{z}$ and hence its growth rate by $\Delta\hat{z}$. The micromodel parameters are used to calculate values for the CBP bounds (s,S,Q). Next we calculate a value for u_t using equation (25) with θ substituted by $\theta_t = 2\mu_t/\sigma^2 = -2\Delta z_t^d/\sigma^2$. Adding the change in u_t to the growth rate of desired aggregate office space use we obtain the growth rate of fitted aggregate office space use as implied by our model, i.e.

$$\Delta\hat{z}_t = \Delta z_t^d + \Delta u_t \tag{30}$$

We plotted the growth rate of fitted aggregate office space use in Figure 2 together with actual and desired aggregate office space use. We see clearly that fitted aggregate office space use tracks actual office space use much better than desired office space use. This is confirmed by the contemporaneous correlations which equal .821 for the growth rates of actual and fitted office space and only .435 between actual and desired office space use. Additionally, the cross-correlation pattern shows that the time series patterns of the growth rates of actual and fitted office space use coincide since the contemporaneous correlation is the largest cross-correlation and the cross-correlations taper of symmetrically in both directions. Also, just like actual office space use, fitted office space lags desired office space use by about one year.

Let us take a closer look at the relation between $\Delta\hat{z}$ and Δz^d. u_t is calculated as a function f of Δz_t^d with $f(\Delta z_t^d)$ given by equation (25) with θ substituted by $\theta_t = -2\Delta z_t^d/\sigma^2$. Hence we have

$$\Delta\hat{z}_t = \Delta z_t^d + \Delta f(\Delta z_t^d) \tag{31}$$

Now define

$$\omega(\Delta z_t^d, \Delta z_{t-1}^d) = -\frac{\Delta f(\Delta z_t^d)}{\Delta z_t^d - \Delta z_{t-1}^d} \tag{32}$$

and we can rewrite (31) as

$$\Delta\hat{z}_t = \left[1 - \omega(\Delta z_t^d, \Delta z_{t-1}^d)\right]\Delta z_t^d + \left[\omega(\Delta z_t^d, \Delta z_{t-1}^d)\right]\Delta z_{t-1}^d \tag{33}$$

We see that the growth rate of fitted aggregate office space use is a weighted average of Δz_t^d and Δz^d_{t-1} with time-varying weights that depend on the current and lagged state of the economy (i.e. aggregate desired office space use): At any point in time only a fraction of the firms will actually relocate so that only a fraction of actual office space use is determined by current market conditions. Also the fraction of firms that relocates in a certain period will depend on market conditions in that period. If for instance during a time period market conditions remain relatively stable only few firms will relocate making the fraction of actual office space use that depends on current market conditions small. Highly volatile market conditions will induce a lot of firms to relocate implying that a large fraction of actual demand depends on current market conditions.

But suppose that we can approximate it well by a linear function $g(\Delta z^d) = f(-\mu) + f'(-\mu)(\Delta z^d+\mu)$. In that case $\omega(\Delta z_t^d,\Delta z^d_{t-1}) = -f'(-\mu)$ and equation (33) reduces to a simple weighted average of the current and lagged state. Figure 3 contains the graph of the function f for our choice of micromodel parameters (μ = -.0334, ρ = .15, $\rho\sigma$ = .0228, γ = .025). We see that this function is highly non-linear so that generally a linear approximation will not yield satisfactory results. However, the figure also contains the actually observed values for Δz^d and the associated values for u (indicated as ○). We see that these observed value all lie in a relatively narrow margin for which a linear approximation may well be adequate. From the graph we conclude that the non-linearity of f only becomes important for values of the growth rate of desired office space use below -10 per cent and above 15 per cent. Our dataset does not include values of those magnitudes.

The above merits an investigation whether we cannot simply explain the growth rate of actual office space use by a fix-weight average of the current and lagged growth rate of desired office space use. To investigate this we run three simple regressions of the growth rate of actual office space use on (1) the growth rate of desired office space use, (2) the growth rate of desired office space use and lagged growth rate of desired office space use, and (3) the growth rate of fitted office space use. The results are reported in Table III.

The results indicate that regression (1) performs poorly compared to the other two and should be discarded as a model for the use of office space. The statistics of regressions (2) and (3) however do not differ very much although regression (3) performs slightly better on all statistics.[3] We interpret

[3]We have to bear in mind however that the application of the stochastic aggregation model involves the 'estimation' of two additional parameters, i.e. ρ and γ, the sampling variability of which have not been taken into account when comparing models (2) and (3).

the results in Table III as implying that the apparent dependence of the actual aggregate net absorption rate on the current and lagged state as indicated by regression (2) can be accounted for by our relocation-cost-cum-stochastic-aggregation model. Apparently the restrictions that are imposed by our stochastic aggregation model on the relative importance of the current versus the lagged state variable constitute an improvement over a simple model with constant weights, although the degree of improvement is not dramatic. It also confirms our suspicions that the non-linearity of the function *f* in Figure 3 is not very important for the dataset at our disposal.

5. SUMMARY AND CONCLUSION

This paper sets up and solves a model for the demand for office space by individual firms with lumpy adjustment costs and studies its implications for aggregate office space use when both idiosyncratic and aggregate uncertainty are present. Additionally, it provides some empirical evidence for the model using aggregate time series data for the Dutch office market over the period 1974-1995.

The most distinguishing feature of the micromodel is the lump sum adjustment cost. These are motivated by noting that firms in order to adjust their demand for office space in many instances have to relocate and that this entails moving costs that are - at least to a large extent - independent of the amount of office space rented or the change therein. The resulting behaviour by firms is a so-called control band policy or (*s*,*S*)-rule in which a firm only adjusts its demand for office space at discreet points in time when the deviation of actual from desired office space use exceeds a certain threshold. When the deviation is smaller than this threshold, the deviation is said to fall within the inaction interval and the firm will not relocate. This obviously describes an important feature of actual behaviour as firm relocations are generally infrequent whereas business condition change frequently and significantly.

We then go on to investigate the aggregate implications of this lumpy individual behaviour. The rate of relocation is determined by the measure of firms that are in the immediate neighbourhood of their relocation threshold. Hence, we have to find the distribution of firms over the inaction interval. Specifically we need the cross section distribution of the deviation of actual from desired office space use at the firm level. When no aggregate uncertainty is present this distribution will settle into a steady state that equals the ergodic distribution of the process followed by the deviation for a single firm. However in the presence of aggregate uncertainty this convenient relation breaks down since then individual shocks are correlated among each other. Although theoretically it is possible to track the cross sectional distribution over time, no closed form solution exists. We prefer to follow the approach of Bertola and Caballero (1994) who approximate the infinite variation of the aggregate shock by a discrete variation in the mean growth rate of individual shocks. This approach lends itself readily to further analytical and empirical work while preserving the most important features of the model. We find that the logarithm of actual aggregate office space use equals the logarithm of desired aggregate office space use plus the cross sectional mean of the log-deviations at the firm level. The latter depends on the growth rate of desired aggregate office space use, implying that actual office space use is a weighted average of current and lagged desired office space, with weights that are time-varying and dependent on current and lagged desired office space.

We apply the above lumpy-adjustment-cum-stochastic-aggregation model to aggregate time series for the Dutch office market. We find that aggregate desired office space use is much more volatile than actual aggregate office space and does not track actual office space use very well. This implies that for the Dutch office market a simple static model for the demand for office space, which could be compared to the approach taken by Rosen (1984), is not adequate. Calculated aggregate office space use as implied by our model tracks actual office space use much better. This indicates that deviations of desired office space use and actual office space use can be accounted for by lumpy adjustment at the individual unit's level. Remarkably, we find that, for the Dutch office market data, a fix-weight weighted average of current and lagged desired office space use constitutes a good approximation of the time-varying weighted average. This is due to the fact that the variation in the data is too small to make the weights vary very much over time. Hence, we see that the office space use in The Netherlands can be described nearly equally well by some form of the fix-weight partial adjustment approach as taken by Wheaton (1987). However, the partial adjustment model does not apply at the individual firm level and hence it is not clear what economic principles lie at the heart of the partial adjustment model. The model proposed in this paper explicitly looks at microeconomic behaviour and in fact rationalizes the ad hoc partial adjustment assumption at the aggregate level from microeconomic principles. Additionally, it shows the limitations of the partial adjustment approach since the fix-weight partial adjustment approach is an adequate approximation only when the variation in the data is not too large. When the data are more volatile the fix-weight partial adjustment model no longer constitutes an adequate approximation to our model and the advantages of our model should become more apparent.

One way to look into this is to look at a more localized market. The aggregate shocks in our data cover all of The Netherlands and is likely that the shocks observed at the national level smooth out the shocks at regional or local levels. Hence, we expect that at the regional or local level the office markets exhibit much more volatility so that the time-variation of the weights as implied by our model become much more pronounced. Hence, it is interesting to take a look at the office market of Amsterdam for which regional accounts exist and for which the office market is relatively well documented. This is however a topic for future research.

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TABLES

Table I

Summary statistics.

	Δz	Δz^d	$\Delta \hat{z}$
Sample	75-95	75-95	76-95
Number of obs.	21	21	20
Mean	.0310	.0334	.0339
Standard error	.0143	.0228	.0169
Skewness	-.591	-.691	-.466
Kurtosis	2.54	4.31	2.80
Jarque-Bera	1.41	3.18	.760
Probability	.495	.0204	.684
AR(1) parameter	.397	.0338	.640
t-value	1.77	.143	3.46
cross-correlations			
k	corr(Δz_t,Δz^d_{t+k})	corr(Δz_t,$\Delta \hat{z}_{t+k}$)	corr($\Delta \hat{z}_t$,Δz^d_{t+k})
-3	.049	-.194	.097
-2	.231	.187	.237
-1	.700	.589	.833
0	.435	.821	.590
1	.315	.517	.285
2	-.118	.206	-.039
3	-.081	-.168	-.278

Table II
Test results for co-integration in office space-rent wage ratio-employment-system.

This table contains the results of the Johansen's trace test for co-integration. The associated vector error correction model (VECM) is of order 2 with unrestricted constant and linear trend restricted in the co-integration relation, i.e.

$$\Delta X_t = \mu_0 + \alpha(\mu_1 t + \beta' X_{t-1}) + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + u_t,\ u_t \sim IIN(0,\Omega),$$

with $X_t = (S,r\text{-}w)_t'$. For a detailed description of the tests and the issues involved see Johansen (1995).

Johansen co-integration likelihood ratio test statistics			
eigenvalues	0.561	0.357	
trace test	24.0*	8.40	
One co-integration vector normalized co-integrating relation β (standard error)			
S	*r-w*	trend (1966=1)	constant
1	-.0609 (.00368)	.000551 (6.6E-5)	-.426

* Significant at 10 per cent using critical values reported in Johansen (1995, Ch. 15.3, Table 15.4).

Table III
Office space use growth rate regressions

Dependent variable: Δz
Sample: 1976-1995

independent (x)	(1) Δz^d	(2) Δz^d	(3) $\Delta \hat{z}$
constant	.0220 (4.03)	.00768 (1.66)	.00713 (1.62)
x_t	.272 (2.03)	.257 (2.90)	.710 (6.09)
x_{t-1}	-	.436 (4.92)	-
R^2	.186	.664	.673
$\bar{R}^2$	.141	.625	.655
AIC	-8.50	-9.29	-9.42
SC	-8.40	-9.14	-9.32

$\bar{R}^2$: adjusted R^2
AIC : Akaike Information Criterion
SC : Schwartz Criterion

FIGURES

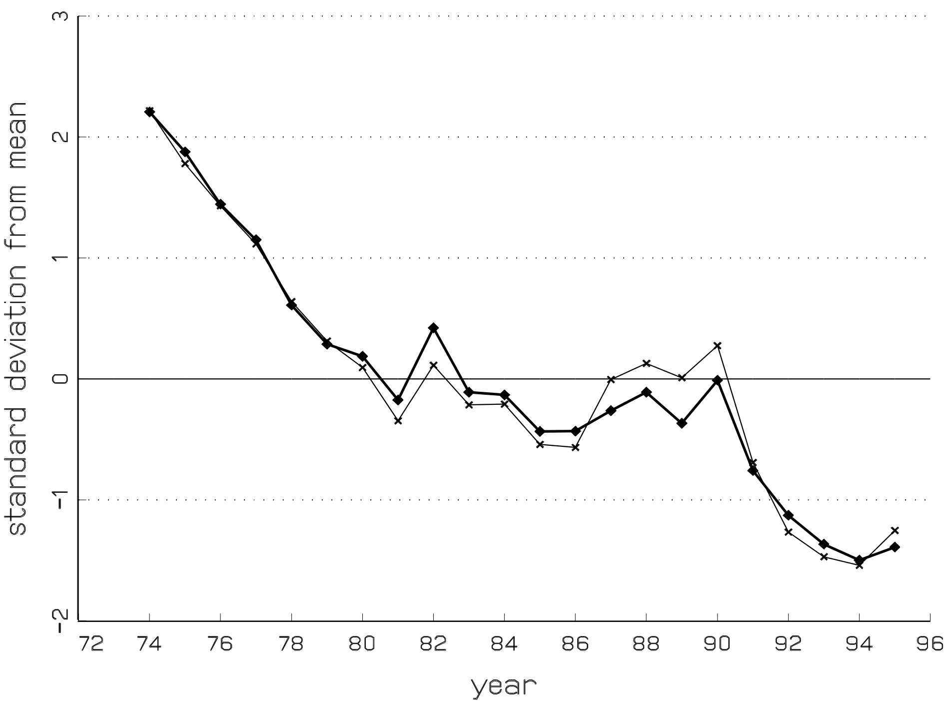

Figure 1
The share of office space expenditure in output (♦) and the logarithm of the rental rate over the wage rate (×) (both series have been standardized to fit into one graph).

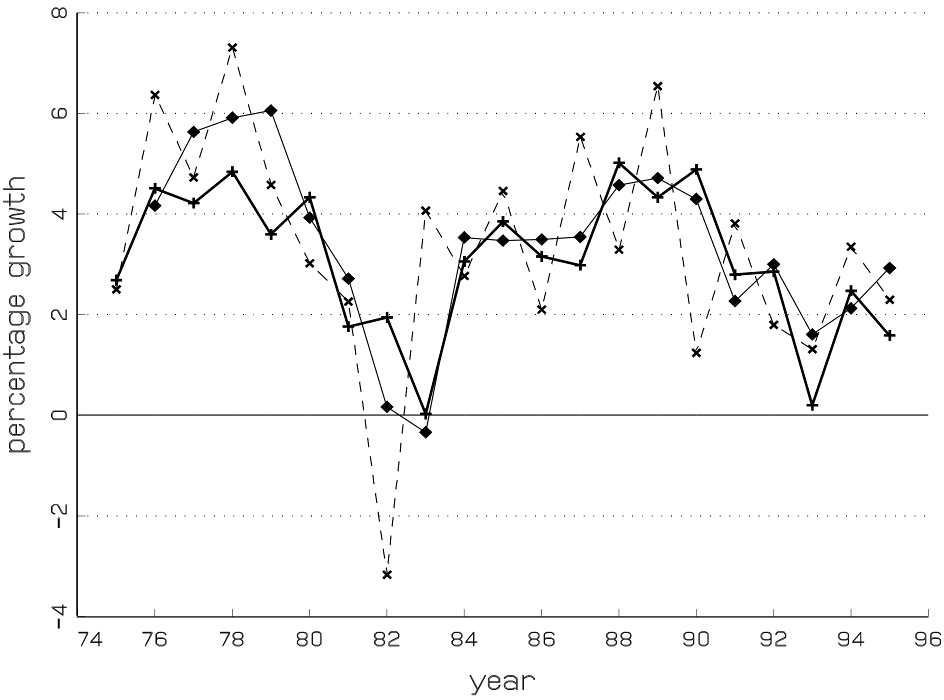

Figure 2
Growth rates of actual occupied stock (+), desired occupied stock (×), and fitted occupied stock (♦).

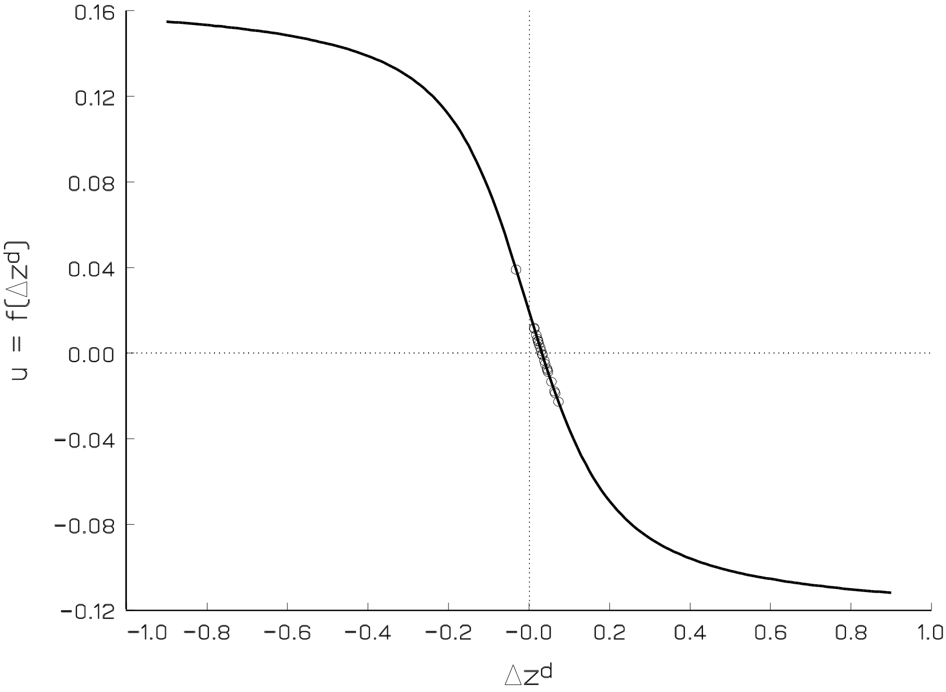

Figure 3
Functional relation $u = f(\Delta z^d)$ (——) and observed values (○) for estimated micromodel parameters (μ = -.0334, ρ = .15, $\rho\sigma$ = .0228, γ = .025).

APPENDIX

In this appendix we derive the steady state distribution a (μ,σ) Brownian motion u in a (s,S,Q) control band. Approximate the continuous time process for u by a discrete time-discrete state process with time jumps Δt and state jumps Δh. At any point in time this approximating process can jump from u at time t_0 to $u+\Delta h$ or $u-\Delta h$ at time $t_0+\Delta t$ with probabilities p and q respectively. To make sure that this discrete process converges to the actual continuous time process we impose the following

$$\Delta h = \sigma\sqrt{\Delta t} \qquad\qquad (34)$$
$$p = \tfrac{1}{2}[1+\frac{\mu}{\sigma}\sqrt{\Delta t}\,], \quad q = \tfrac{1}{2}[1-\frac{\mu}{\sigma}\sqrt{\Delta t}\,]$$

The density function ϕ_u can be found by solving the following difference equation (using steady state occupancy rates)

$$\phi(u) = p\phi(u-\Delta h) + q\phi(u+\Delta h)\ , \quad u \neq Q \qquad\qquad (35)$$

Expanding this expression around u, dividing by Δh^2 and letting $\Delta h \to 0$, we obtain (see Dixit and Pindyck [1994, pp.83-84])

$$\phi^{\prime\prime}(u) = \theta\phi^{\prime}(u)\ , \quad \theta = \frac{2\mu}{\sigma^2},\ u \neq 0 \qquad\qquad (36)$$

Equation (36) is an ordinary differential equation with general solution

$$\phi(u) = \begin{cases} A_1 e^{\theta u} + B_1, & s < u < Q \\ A_2 e^{\theta u} + B_2, & Q < u < S \end{cases} \qquad\qquad (37)$$

The full solution to (36) can be found by substituting equation (37) into the boundary conditions. The boundary conditions are given by

$$\lim_{u\uparrow Q}\phi(u) = \lim_{u\downarrow Q}\phi(u) \equiv \phi(Q) \qquad\qquad (38)$$

$$\phi(s) = \phi(S) = 0 \qquad\qquad (39)$$

Note that ϕ is continuous at Q but not (necessarily) differentiable. The boundary conditions in (39) imply $B_1 = -A_1\,e^{\theta s}$ and $B_2 = -A_2\,e^{\theta S}$ which yields

$$\phi(u) = \begin{cases} A_1[e^{\theta u} - e^{\theta s}], & s < u < Q \\ A_2[e^{\theta u} - e^{\theta S}], & Q < u < S \end{cases} \qquad\qquad (40)$$

Note that, as $s < Q < S$ and a proper density should be nonnegative, it follows that $A_1 \geq 0$ and $A_2 \leq 0$ when $\theta > 0$ and vice versa when $\theta < 0$. Substituting equation (24), (40) we obtain

$$\frac{A_1}{A_2} = \frac{e^{\theta Q} - e^{\theta S}}{e^{\theta Q} - e^{\theta s}} < 0 \tag{41}$$

which we rewrite as $A_1 = cA_2$ with c defined as the RHS of (41).

The final condition we use to determine the constants of the differential equation is that the integral of any proper density function over its support should equal unity. This yields

$$\begin{gathered}\int_s^S \phi(u)du = \int_s^Q cA_2(e^{\theta u} - e^{\theta s})du + \int_Q^S A_2(e^{\theta u} - e^{\theta S})du = 1 \quad \Rightarrow \\ A_2 = -\left[ce^{\theta s}(Q-s) + e^{\theta S}(S-Q)\right]^{-1}\end{gathered} \tag{42}$$

which completes the characterization of the steady state density function of $u(t)$. Finally, figure 1 contains a simulated density function for 100,000 replications, with $\mu = .1$, $\sigma = 1$, $s = -10$, $Q = -5$, $S = 5$. We see that the density consists of two exponential distributions with most probability mass to the right of Q due to the positive drift term.