Maintenance of Light Standards, a Case-Study

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September 1996

abstract

This paper discusses several strategies for the maintenance of light standards, where each light standard consists of n independent and identical lamps screwed on a chandelier. The lamps are subject to stochastic failures, and must be correctively replaced if the number of failed lamps reaches a prespecified number m; a norm that is set by the local management to guarantee a minimum luminance. As lamps have an increasing failure rate, and there is a fixed cost of hoisting the chandelier, we propose various variants of the m-failure group replacement rule which have in particular an age-criterion to indicate which of the non-failed lamps must be preventively replaced at the time that the chandelier is lowered for a corrective lamp replacement; we show how the optimal threshold age can be determined. It appears that this modification reduces the long run average maintenance cost of the *Europe Combined Terminals* with approximately 8.3%.

keywords: Energy Consumption, Group Maintenance, Hot and Cold Standby, k-out-of-n System, Lamp Replacement, Light Standards, (Modified) m-Failure Group Replacement, Multiple Components, (Modified) Secant Algorithm.

1. Introduction

The function of a light standard is to provide illumination. The lamps screwed on the chandelier are the components which must enlighten a certain area. Replacement of lamps is generally not easy as either the chandelier has to be lowered (which is the case in this paper) or a tower wagon has to be applied. In both cases a relative high set-up cost is involved. To avoid immediate failure replacements there is usually some redundancy in the sense that there are more components than necessary to provide a minimal required luminance. Because of such a redundancy, one can postpone the replacement of a failed lamp till the number of failed lamps reaches a certain number at which the luminance becomes unacceptably low; as a consequence the cost of hoisting the chandelier can be spread over several failed lamps. When the chandelier is lowered to replace the failed components there is an opportunity to replace also the non-failed components. Replacing the non-failed components may be advantageous when the components have an increasing failure rate. Such a preventive replacement will in general delay the next time that the chandelier is hauled down for a simultaneous lamp-replacement. On the other hand it may be not be very economical to replace the non-failed lamps after a short period of operation as this will throw away a possible substantial amount of remaining life. Thus the age of the non-failed lamps can seriously affect the advantageousness of a preventive replacement.

It should be emphasised that the cost of maintaining a light-standard largely depends on the configuration of the light-standard. Thus it is important to verify whether the light-standard is correctly designed; one has to determine whether and how redundancy should be built in. We consider the situation in which the design of the light-standard is largely fixed; only the number of lamps screwed on the chandelier can be altered. Given the design of the light-standard there are two questions which characterise the lamp replacement problem: first, *at what moment* should the chandelier be lowered for a simultaneous lamp replacement, and second, *which lamps* should be replaced at the time that the chandelier is lowered - is it worthwhile to keep track of the ages of the lamps ?

These questions arise in almost every multi-component maintenance problem where a replacement of one or more components includes a set-up activity which is quite expensive. If there are no costs associated with the failure of one or more components, one has the intention to delay the replacement as this will increase the number of failed components, and thus decrease the set-up cost of hoisting the chandelier per component. However, one has also to take the light-output into account which reduces with the number of non-operating components. Here we assume that the reduction in light-output over the economical life of all lamps is less than the loss in output caused by the failure of one lamp. To keep the light-output reduction within acceptable bounds, one can impose a penalty for every non-operating lamp, or set an upper-bound on the number of failed components; the latter is much easier in practice. The aim is to find a maintenance strategy which preserves a group components against minimal costs without much output reduction.

Good overviews of multi-component maintenance optimisation are given by Cho & Parlar [1991], and Dekker, Van der Duyn Schouten & Wildeman [1996]. Below, some group replacement rules are described which are relevant for the maintenance of light standards.

Okumoto & Elsayed [1983] suggested a corrective block replacement rule which is to renew every T_R time-units *only the failed* components. A variant of this rule is the preventive block replacement rule which is to replace every T_R time-units *all* the components. Sheu [1991] investigated the preventive block replacement rule with immediate failure treatments which requires that the components can be easily reached; for the maintenance of light standards it is generally too expensive to lower a chandelier for a single lamp replacement. However the block replacement rule without immediate failure treatments has the disadvantage that it does not guarantee a minimum luminosity.

To guarantee a minimum luminosity one must follow a control-limit strategy based on the number of failed components. Gertsbakh [1984] showed that under the assumption that (i) the components have exponential lifetimes, (ii) a replacement takes a negligible time, and (iii) the cost structure is $c_0+c_1\cdot k$ for a k-bulb replacement ($c_0,c_1>0$) plus c_R for any non-operating component per unit of time, with c_R greater than c_1 divided by the expected lifetime of the components : the optimal replacement policy is to replace all failed components at the moment when the number of failed components reaches some prescribed number m. Assaf & Shantikumar [1987] considered the situation in which the operational lamps can be switched off; they show that under the assumptions (i),(ii) and (iii) it is senseless to let a number of operational lamps idle as this will not reduce the optimal cost.

The problem with the studies of Okumoto & Elsayed, Gertsbakh, and Assaf & Shantikumar, is that they assume an exponentially distributed lifetime. In general, the lifetime of lamps is not exponentially distributed, mostly it has an increasing failure rate. Besides, it is very hard to determine the cost c_R. It seems much easier to set a norm on the number of failed lamps based on the inconvenience which is experienced when a part of the lamps is out-of-order.

The control-limit rule by which all the failed components are replaced at the time of m failures may not be the most economical rule for the maintenance of light standards, but it is a much applied rule in practice as it is a rather easy task to determine the number m at which the light-output becomes unacceptable. It has only the deficiency that it does not consider the individual ages to indicate the components which could be better preventively replaced when some of the components are correctively replaced. We therefore propose to modify this m-failure group replacement rule with an age-criterion for the replacement of non-failed components. We will study the cost-effectiveness of this rule by simulation, and by an analytical derivation of the expected costs for a renewing-type version of the rule.

The idea to incorporate an age-criterion within the m-failure group replacement rule is taken from the modified block replacement rule of Archibald & Dekker [1994] adapted to multi-component systems. The modified block replacement rule originates from Berg & Epstein [1976] which is to consider a preventive maintenance every t time-intervals at which all the components whose age is greater than or equal to a fixed threshold age are replaced. A contingent redundancy is not taken into account - every component that fails is immediately replaced.

The modified m-failure group replacement rule is an opportunistic maintenance rule; it keeps failed components idling for a certain time until m components are failed, and replaces then all the failed components together with the non-failed components whose age has passed a critical threshold age. It looks a little like the m-failure/ t-age group replacement rule of Ritchken & Wilson [1990] which is to replace all the components at time t or earlier when m components fail beforehand, though the modified m-failure group replacement rule is less restrictive. The only action within the m-failure/ t-age group replacement setting is a complete overhaul which is but a possible action within the modified m-failure group replacement setting. Second, the time between two successive simultaneous replacements is in the m-failure/t-age setting restricted to a maximum length of t time intervals; within the modified m-failure group replacement setting a simultaneous replacement is done only then when there are m components failed. A disadvantage of the modified m-failure group replacement rule is that the components have their own ageing process which is not necessarily renewed at a pointed group replacement time, a system renewal is accidental and not enforced which makes the analysis of the modified m-failure group replacement rule somewhat difficult; an easy method to find the optimal threshold age is by an hybrid simulation optimisation method which is explained in section 4. Because the system time - which is the time between two successive system renewals - is an indispensable variable to analyse the maintenance costs by the renewal-reward theory, we have adopted a renewing type version of the modified m-failure group replacement rule. For this rule, it is possible to determine the optimal threshold value in an analytic way.

We study the cost-efficiency of the modified m-failure group replacement rule when it is applied on a number of light-standards which must enlighten several terminals of the Europe Combined Terminals (ECT) in the Rotterdam harbour. We view the rules in two contexts, in a hot standby context in which all the redundant lamps are switched on, and in a hypothetical cold standby context in which the redundant lamps are switched off till they become necessary to provide sufficient illumination. It will appear that the m-failure group replacement rule is a fairly efficient rule for the maintenance of ECT's light standards.

We think that the m-failure group replacement rule will be an economical rule for many other multi-component systems where the components have an increasing failure rate, and a replacement of one or more components requires a quite expensive set-up activity. The only thing the manager has to do is to set a norm m on the number of failed components[1].

Thus the paper tries to determine the value of considering the individual ages of the components within a case study where (i) the components have an increasing failure rate, (ii) a replacement of a component requires a set-up activity, and (iii) there is a prespecified norm on the number of failed components.

The paper is organised as follows. Section 2 gives ECT's case-description. Section 3 describes the standard and complete m-failure group replacement rule for the maintenance of n components having an independent and identical distributed lifetime. Section 4 elaborates on the modified m-failure group replacement rule - and its renewing type version, and explains how the optimal threshold age is found. Section 5 gives an overview of the results obtained with the various m-failure group replacement rules, and in Section 6 some conclusions are drawn.

2. Problem Formulation

The case concerns the maintenance of light-standards on the terrain of the Rotterdam Container Terminal called *Europe Combined Terminals* (ECT). The terminals are illuminated by 49 light standards with each 15 lamps screwed on a chandelier. The cost of replacing a lamp is DFL.131.50 (=c_0), and the cost of lowering a chandelier is estimated from the time needed to perform the action - the harbour operations have to be stopped during the replacement - and is about DFL.363.00 (=c_1). Thus the cost of a k-bulb replacement is about DFL.363+131.5·k, k=1,..,15. The lamps are identical and do have an independent stochastic lifetime; failures are directly observed by the daily inspection round. The manufacturer of the lamps provided us the following empirical life-time distribution function[2]:

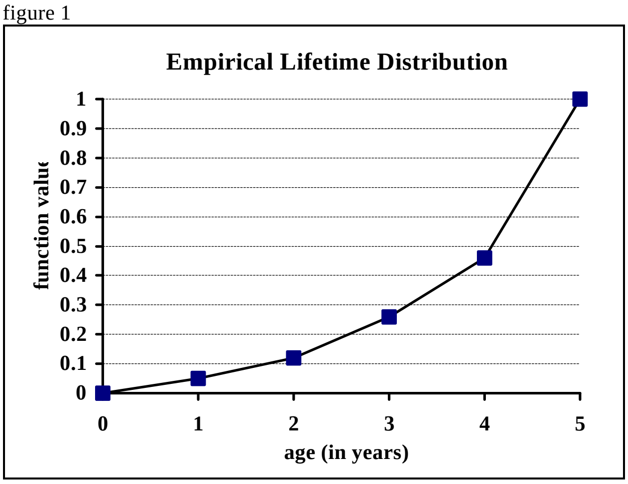
$$F_e(x) = P(\underline{X} \leq x) = \begin{cases} 0.05 \cdot x & , \quad 0 \leq x < 1 \\ 0.07 \cdot x - 0.02 & , \quad 1 \leq x < 2 \\ 0.14 \cdot x - 0.16 & , \quad 2 \leq x < 3 \\ 0.20 \cdot x - 0.34 & , \quad 3 \leq x < 4 \\ 0.54 \cdot x - 1.70 & , \quad 4 \leq x \leq 5 \end{cases} \qquad \text{(see figure 1)}$$

where x is the age - measured in years - of the lamp.

[1] If there are n components then the norm m induces an n-m+1 out of n system, which means that at least n-m+1 out of n components must operate.

[2] The original life-time distribution function was based on burning hours; these hours are converted into calendar-years by using a fixed utilisation factor.

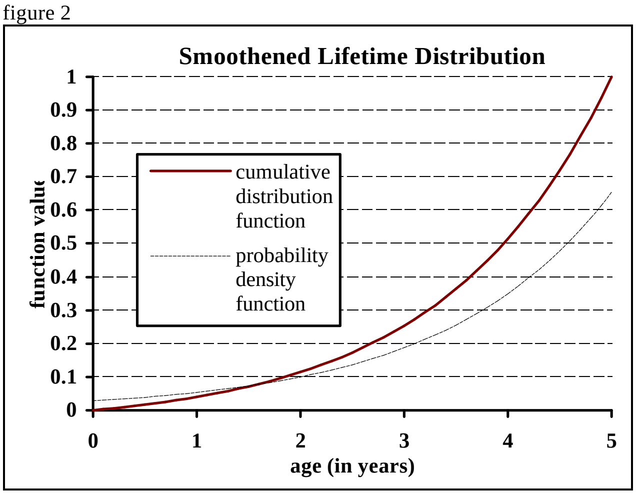
figure 1



From the function $F_e(x)$ it follows that the lamps have a mean life-time of 3.61 year with a variance of 1.48 year. Since the lifetime function F_e is a piece-wise linear interpolation of six data-points the density function $dF_e(x)/dx$ is discontinuous and the failure rate $dF_e(x)/dx \cdot (1-F_e(x))^{-1}$ has multiple extremes.

Because the sequence $F_e(i)-F_e(i-1)$, i=1,2,..,5 increases more than proportional, we smoothened the function F_e by the following exponential function : $F_s(x)=A \cdot B^{x/5}+C$, $0 \le x \le 5$. From the conditions $F_s(0)=0$, and $F_s(5)=1$ it follows that the parameter C must be equal to -A and the parameter B to 1+1/A. Therefore we fitted the following function $F_s(x)=A \cdot [(1+1/A)^{x/5}-1]$, $0 \le x \le 5$. Under the least-squares criterion with intervals of one-tenth length, the optimum parameter-value for A is 0.046. For this parameter the fit has a maximum absolute deviation[3] of 0.054 in the point x=4, and a mean squared deviation[4] of ± 0.00025. The smoothened cumulative lifetime distribution function: $F_s(x)$ and its derivative are displayed in Figure 2.

figure 2

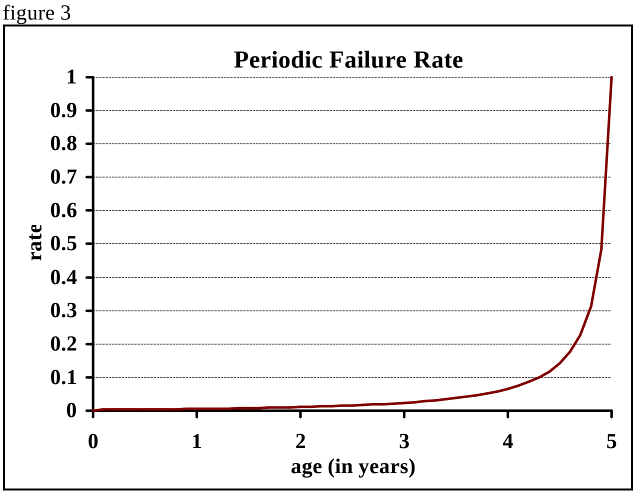


The failure rate $r(x):= dF_s(x)/dx \cdot (1-F_s(x))^{-1}$ is given by $(1/5) \cdot \ln(1+1/A)/\{(1+1/A)^{1-x/5}-1\}$, $x \in [0,5)$, and is strictly increasing, thus the smoothened lifetime-distribution satisfies the IFR-property. Figure 3 shows the periodic failure rate $r_\Delta(x):= (F(x+\Delta x)-F(x))/(1-F(x))$ for $\Delta x=0.1$.

3 The maximum absolute deviation is defined as the maximum absolute difference between the function $F_e(x)$ and $F_s(x)$ over x=0,0.1,...,5.

4 The mean squared deviation is the mean value of the squared difference between the function $F_e(x)$ and $F_s(x)$ over x=0,0.1,...,5.

figure 3



For the years 0-2½ the probability that a functioning lamp will fail within $\Delta x=0.1$ year is smaller than 1.8%. For the years 2½-4 the periodic failure probability increases from 1.8% to 7.5%. If a functioning lamp is older than 4½ then the probability that it will fail within 0.1 year is more than 22%.

The decision to replace one or more lamps can be made instantaneously; we assume that the time necessary to do a k-bulb replacement which includes the time of hoisting the chandelier, can be neglected. The ECT currently replaces all the lamps when there are 4 lamps failed, thus they apply the complete 4-failure group replacement rule. The aim is to find a maintenance rule which leads to a lower long run average maintenance cost per unit of time, given that a corrective replacement is mandatory when the number of failed components reaches the number[5] m=4.

For the sake of completeness, it is mentioned that the light-standards are approximately 3600 hours a year in use, and that the lamps have a wattage of 80 per hour where the cost of electricity is 10½ cents per kilowatt-hour. Thus the average energy cost of an operational lamp is circa DFL.30.24 (= c_2) per year.

3. Standard and Complete m-Failure Group Replacement

The m-failure group replacement rule of Gertsbakh [1984] is a control-limit rule based on the number of failed components. It prescribes replacement of all failed components if and only if the number of failed component reaches the number m. The standard m-failure group replacement rule renews at an m-failure-epoch *only* the failed components, contrary to the complete m-failure group replacement rule which replaces *all* the components.

The following assumptions are made. First, there are n components having an independent and identical lifetime distribution; when m components fail it is mandatory to replace *all* the failed components. Replacement can be done instantaneously; the time necessary to perform a k-bulb replacement, $k \geq 1$, is negligible. The cost of a k-bulb replacement is linear in k : $c_0 + c_1 \cdot k$ for all integers $k \geq 1$. Further, there is full information about the number of failed components, and about the age of any non-failed component.

For the standard m-failure group replacement rule the long run average cost per unit time can be expressed by

$$\lim_{t \to \infty} \frac{R(t)}{t} \tag{3.1}$$

where R(t), $t \geq 0$ is the cumulative cost until time-point t. We use this expression as there are no obvious renewing-points; after replacement of the failed components there are m *new* components and n-m *old* components where each component may have a distinct age. The long run average cost per unit time can be approximated by computing $R(t^\infty)/t^\infty$ for sufficient large t^∞ in a steady-state simulation.

For the complete m-failure group replacement rule the long run average cost per unit time can be found by applying renewal theory. Denote by $T^{(m)}$ the time between two successive m-failure

[5] This number indicates when the luminosity becomes unacceptable; we assume that the age of the operational lamps is not an argument for the replacement of the failed lamps although the luminosity of a lamp may decrease when the lamp becomes older.

epochs at which all the components are renewed. The period between two successive m-failure epochs can be regarded as a cycle within the ageing process of all components. During this cycle there is a fixed maintenance cost of $c_0 + c_1 \cdot n$, thus the long run average maintenance cost per unit time is given by

$$\frac{c_0 + c_1 \cdot n}{ET^{(m)}} \tag{3.2}$$

To obtain the long run average cost in which the energy cost is incurred, one must add the following term to the numerator of (3.2):

$$\begin{aligned} &c_2 \cdot \left\{ n \cdot T^{(m)} - \sum_{k=1}^{m-1} \left(ET^{(m)} - ET^{(k)} \right) \right\} \text{ if the redundant bulbs are kept in hot standby} \\ &c_2 \cdot (n - m + 1) \cdot ET^{(m)} \text{ if the redundant bulbs are kept in cold standby.} \end{aligned} \tag{3.3}$$

where $T^{(k)}$, $k = 1,..,m$ is the time till the k-th failure after the complete system renewal, which depends on the operational use of the redundant lamps.

If the redundant lamps are kept in hot standby, then the distribution function $G_k(x)$ of $T^{(k)}$ - which is just the distribution function of the k-th order statistic of the lifetimes of all n components - is

$$G_k(x) = \sum_{i=k}^{n} \binom{n}{i} \{F(x)\}^i \{1 - F(x)\}^{n-i}, \quad k = 1,..,m. \tag{3.4}$$

If the redundant lamps are kept in cold standby then the distribution function $G^k(x)$ of $T^{(k)}$ has a more complicated form.

For convenience, define the function $F_R(x;z)$ as the residual lifetime distribution function of a component that has survived z time-units, thus

$$F_R(x;z) := \frac{F(x+z) - F(z)}{1 - F(z)}, \; x \geq 0, \; z \geq 0 \tag{3.5}$$

and let

$$f_R(x;z) := \frac{F_R(x;z)}{dx}, \; x \geq 0, \; z \geq 0 \tag{3.6}$$

be its probability density function.

Next, number the operational lamps by $1,2,..,n\text{-}m+1$; if a component fails then the lamp which replaces the failed lamp gets the number of the failed lamp. Define the random variable $T^{(k)}(\mathbf{z})$ as the time until the k-th failure from now, given the current ages of the operational components: $\mathbf{z} = (z_1, z_2, .., z_{n-m+1})$, where z_i corresponds to component number i. Observe that $T^{(k)} = T^{(k)}(\mathbf{0})$.

Further let

$$V_k(x;\mathbf{z}) := P(T^{(k)}(\mathbf{z}) \leq x), \; x \geq 0 \tag{3.7}$$

be the probability that the k-th failure will arise within x time-units when the vector of current ages is $\mathbf{z}$,
and let

$$\overline{V}_k(x;\mathbf{z}) := 1 - V_k(x;\mathbf{z}), \; k = 1,2,..,m. \tag{3.8}$$

be the complementary function of $V_k(x;\mathbf{z})$.

Denote by

$$Q(i;y,\mathbf{z}) := \frac{f_R(y;z_i)}{\sum_{w=1}^{n-m+1} f_R(y;z_w)}, \; x \geq 0 \qquad (3.9)$$

the probability that component number i induces the next failure given that this failure occurs after exactly y time-intervals and the vector of current ages is **z**.

Now, observe that

$$G_k(x) = V_k(x;\mathbf{0}) \;, \; k = 1,..,m \qquad (3.10)$$

These functions can be recursively determined the following equation

$$\overline{V}_k(x;\mathbf{z}) := \overline{V}_1(x;\mathbf{z}) \; + \int_0^x \left\{ \sum_{i=1}^{n-m+1} Q(i;y,\mathbf{z}) \cdot \overline{V}_{k\text{-}1}(x-y;\Gamma_{-i}(\mathbf{z}+y\cdot\mathbf{1})) \right\} dV_1(y;\mathbf{z}) \,, \quad k = 2,3,..,m \qquad (3.11)$$

where

$$V_1(x;\mathbf{z}) = 1 - \prod_{i=1}^{n-m+1} (1 - F_R(x;z_i)) \,, \qquad (3.12)$$

and

$$\Gamma_{\text{-}i}(\mathbf{z}) = (z_1,..,z_{i-1},0,z_{i+1},..,z_{n-m+1}) \,. \qquad (3.13)$$

in the case that the redundant lamps are kept in cold standby.

The expected time between a system renewal and the k-th failure epoch after the system renewal can be computed from

$$ET^{(k)} = \int_0^T (1\text{-}G_k(x))\, dx, \quad k = 1,..,m \qquad (3.14)$$

The last integral may be difficult to compute, depending on the lifetime distribution function F. If necessary, the expected value $ET^{(k)}$ can be determined by using a numerical integration method (see for instance Burden & Faires [1989]).

4. Modified m-Failure Group Replacement Rule

The modified m-failure group replacement rule is an opportunistic maintenance rule which states that only the non-failed components with an age greater than or equal to a certain threshold age t_c may be and have to be replaced together with the failed components, at the time that the number of failed components has reached the level m.

Because the ageing process of the components is not necessarily ‘renewed’ at the group replacement times, renewal theory is not appropriate to analyse the long-term expected costs. However, the long run average cost per unit time can be approximated by doing a steady-state simulation for a certain threshold age t_c. Denote by $v(t_c)$ the steady-state approximation of (3.1). Its derivative can be estimated by

$$\Delta \;\; v(t_c) := \frac{v(t_c + \;\;) - v(t_c)}{\;\;} \;\;, \qquad (4.1)$$

for small   > 0. We use this expression for the optimisation[6] of the threshold age t_c in which we circumvent the problem of the derivative evaluation which we should have if we apply the Newton-Raphson method or the Secant method on the derivative of $v(t_c)$.

[6] In fact we are trying to determine the optimal threshold age t_c by doing a perturbation analysis (see for instance Ho & Cao [1991]).

To find the optimum threshold age $v(t_c)$ we apply the Secant method on the function $\Delta\ v(t_c)$ instead of $\frac{d}{dt_c} v(t_c)$. The optimisation algorithm is as follows:

modified secant-algorithm for finding the optimum threshold age t_c

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Initialisation :
Evaluate for two different threshold ages x_0, x_1, the values $v(x_0)$, $v(x_1)$, and the values $v(x_0+\varepsilon)$, $v(x_1+\varepsilon)$
in order to calculate the values $\Delta\ v(x_0)$ and $\Delta\ v(x_1)$.

Set $i=1$.

Repeat the following steps until $|x_i - x_{i-1}| < \gamma$ for some small $\gamma > 0$.

step 1. Determine the next threshold age by:

$$x_{i+1} := x_i - \Delta\ v(x_i) \cdot \frac{x_i - x_{i-1}}{\Delta\ v(x_i) - \Delta\ v(x_{i-1})} \tag{4.2}$$

step 2. Evaluate for the threshold age x_{i+1} the value $v(x_{i+1})$ and $v(x_{i+1}+\varepsilon)$, and calculate the value $\Delta\ v(x_{i+1})$.

step 3. Set $i := i+1$.

<u>*remark*</u> : As the function $v(t_c)$ has small distortions caused by simulation, we also investigated the advantage of fitting a quadratic polynomial $p(t_c)$ through the evaluated points $(x_j, v(x_j))$, $j=0,1,..,i$ in each iteration of the modified secant algorithm, so that we can work with the derivative of $p(t_c)$ instead of $\Delta\ v(t_c)$. However it appeared that this modification does not really improve the efficiency of the algorithm, as the fit may have a derivative in the point x_i which is completely different from the derivative of (4.1) in the point x_i, so that the next threshold age x_{i+1} may be far from optimal. ■

Although the modified m-failure group replacement rule is likely to be the best type of rule, it does require keeping track of the ages of the individual components. Besides, it can only be analysed by simulation. Therefore we also looked for another rule which requires less administration and can be evaluated analytically. To this end we considered the system age, i.e. time since the last complete replacement, and defined a control limit policy on that indicator. This policy is to perform a complete replacement if the system age is above a certain threshold value, and else to replace only the failed components. To keep the policy analytically tractable we need that in the case that only the failed components are replaced, all the components are replaced at the next m-failure epoch. This policy is referred to as the *renewing* modified m-failure group replacement rule.

From the renewal-reward theory (see for instance Tijms [1995]) it follows that the long run average cost[7] per unit time (3.1) is given by

$$v(t_c) = \frac{EM(t_c)}{EL(t_c)} \tag{4.3}$$

where $L(t_c)$ is the length of a cycle defined as the time between two successive system renewals, and $M(t_c)$ the maintenance cost that is made during a cycle, when the threshold age is t_c.

It is not difficult to verify that in the case energy cost can be neglected:

$$EM(t_c) = \{c_0 + c_1 \cdot n\} \cdot \{1 - G_m(t_c)\} + \{2 \cdot c_0 + (n+m) \cdot c_1\} \cdot G_m(t_c) \tag{4.4}$$

where G_m has the definition of (3.3) in the case of hot standby, and (3.9) in the case of cold standby.

[7] For the maintenance of light standards $v(t_c)$ is the long run average cost per light standard.

The derivative of (4.3) is given by

$$v'(t_c) = \frac{\left\{\frac{d}{dt_c} EM(t_c)\right\} EL(t_c) - EM(t_c)\left\{\frac{d}{dt_c} EL(t_c)\right\}}{\{EL(t_c)\}^2} \tag{4.5}$$

and the second derivative by

$$v''(t_c) = \frac{\left\{\frac{d^2}{dt_c^{\,2}} EM(t_c)\right\} EL(t_c) - EM(t_c)\left\{\frac{d^2}{dt_c^{\,2}} EL(t_c)\right\} - 2\frac{d}{dt_c} EL(t_c)\, v'(t_c)}{\{EL(t_c)\}^2} \tag{4.6}$$

where

$$\frac{d}{dt_c} EM(t_c) = (c_0 + m \cdot c_1)\frac{d}{dt_c} G_m(t_c)\,, \tag{4.7}$$

$$\frac{d}{dt_c} EL(t_c) = \left\{t_c + \int_0^{T-t_c} y\, dH_m(y|t_c)\right\}\frac{d}{dt_c} G_m(t_c) \;-\; (1 - G_m(t_c)), \tag{4.8}$$

$$\frac{d^2}{dt_c^{\,2}} EM(t_c) = (c_0 + m \cdot c_1)\frac{d^2}{dt_c^{\,2}} G_m(t_c)\,, \tag{4.9}$$

and

$$\begin{aligned}\frac{d^2}{dt_c^{\,2}} EL(t_c) &= \left\{t_c + \int_0^{T-t_c} y\, dH_m(y|t_c)\right\}\frac{d^2}{dt_c^{\,2}} G_m(t_c) \\ &+ \left\{2 + (t_c - T)\cdot h_m(T - t_c|t_c) + \int_0^{T-t_c} y \frac{dh_m(y|t_c)}{dt_c} dy\right\}\frac{d}{dt_c} EG_m(t_c)\end{aligned} \tag{4.10}$$

with

$$h_m(y|t_c) = \frac{dH_m(y|t_c)}{dy} \tag{4.11}$$

<u>*remark*</u> : The formulas (4.7) and (4.9) are without energy cost. If one wants to incorporate the energy cost within the cost-function $v(t_c)$, then one has to add to the numerator of (4.3) the term

$$\left[\begin{aligned}&\left\{n \cdot E\left(T^{(m)} \middle| T^{(m)} > t_c\right) - \sum_{k=1}^{m-1} E\left(T^{(m)} - T^{(k)} \middle| T^{(m)} > t_c\right)\right\}\cdot G_m(t_c) + \\ &\left\{n \cdot \left(E\left(T^{(m)}\right) + E\left(T^{(m)} \middle| T^{(m)} \le t_c\right)\right) - \sum_{k=1}^{m-1}\left(E\left(T^{(m)} - T^{(k)}\right) + E\left(T^{(m)} - T^{(k)} \middle| T^{(m)} \le t_c\right)\right)\right\}\cdot (1 - G_m(t_c))\end{aligned}\right]\cdot c_2 \tag{4.12}$$

if the redundant lamps are kept in hot standby, and the term

$$(n - m + 1)\cdot\left[\left\{E\left(T^{(m)} \middle| T^{(m)} > t_c\right)\right\}\cdot G_m(t_c) + \left\{E\left(T^{(m)}\right) + E\left(T^{(m)} \middle| T^{(m)} \le t_c\right)\right\}\cdot (1 - G_m(t_c))\right]\cdot c_2 \tag{4.13}$$

if the redundant lamps are kept in cold standby, where the conditional expectations

$$E\left(T^{(k)} \middle| T^{(m)} \le t_c\right) \text{and } E\left(T^{(k)} \middle| T^{(m)} > t_c\right) \tag{4.14}$$

can be calculated from the distributions[8]

$$G_k^{\leftarrow}(x;t_c) = \frac{\int_x^{t_c} \sum_{i=k}^{m-1} \frac{n!}{i!(m-i-1)!(n-m)!} \left[F(x)\right]^i \cdot \left[F(y)-F(x)\right]^{m-i-1} \cdot \left[1-F(y)\right]^{n-m} dG_m(y)}{G_m(t_c)}, \quad x < t_c \tag{4.15}$$

and

$$G_k^{\rightarrow}(x;t_c) = \frac{\int_{\max(x,t_c)}^{5} \sum_{i=k}^{m-1} \frac{n!}{i!(m-i-1)!(n-m)!} \left[F(x)\right]^i \cdot \left[F(y)-F(x)\right]^{m-i-1} \cdot \left[1-F(y)\right]^{n-m} dG_m(y)}{1-G_m(t_c)}, \quad x < 5 \tag{4.16}$$

through

$$E\left(T^{(k)} \middle| T^{(m)} \le t_c\right) = \int_0^{t_c} x dG_k^{\leftarrow}(x;t_c) \tag{4.17}$$

and

$$E\left(T^{(k)} \middle| T^{(m)} > t_c\right) = \int_0^{5} x dG_k^{\rightarrow}(x;t_c)\ . \tag{4.18}$$

In this case the minimisation has to be done by search methods that do not require a derivative, e.g. the golden section search. ∎

The cycle-length $L(t_c)$ has an expected value[9] of

$$EL(t_c) = \int_0^{t_c} \left\{ x + \int_0^{T-x} y\, dH_m(y|x) \right\} dG_m(x) + \int_{t_c}^{T} \left(1 - G_m(x)\right) dx \tag{4.19}$$

where $H_m(y|x)$,$0 \le y \le T-x$ is the distribution function of the m-th order statistic of the lifetimes of all n components given that at time y=0 there are m *just-replaced* components with age 0, and n-m *non-replaced* components with age x. The above formula can be explained as follows : the integral defined on the interval $[0,t_c)$ corresponds to the situation in which m components fail before time t_c , this means that *only* the failed components are replaced, and that a system renewal will occur at the second m-failure epoch. The integral defined on the interval $[t_c ,T]$ corresponds to the situation in which the first m-th failure epoch occurs after time t_c so that the system is completely renewed at the m-th failure epoch.

To find the optimal threshold age t_c one can simply use the Newton-Raphson method for solving the root-finding problem v'(x)=0. This method starts with an initial estimate x_0 and generates a sequence of threshold ages $\{x_i\}$, defined by

$$x_{i+1} = x_i - \frac{v'(x_i)}{v''(x_i)}, \quad i = 0,1,... \tag{4.20}$$

for which one can use (4.13) and (4.14).

However, the second order derivative may be difficult to calculate. To overcome this problem, one may use the Secant method which uses only the first order derivative (4.6) in order to find the root of v'(x)=0; within the Secant method the sequence $\{x_i\}$ is generated by

$$x_{i+1} = x_i - v'(x_i) \cdot \frac{x_i - x_{i-1}}{v'(x_i) - v'(x_{i-1})}, \quad i = 1,2,... \tag{4.21}$$

starting with initial estimates x_0 and x_1.

This means that one need not to do a steady-state simulation to get an estimate of the first- (and possibly the second-)order derivative of v(x) to obtain the next threshold age. As a consequence

[8] The distribution $G_k^{\leftarrow}(x;t_c)$ corresponds to the probability $P(T^{(k)} \le x \mid T^{(m)} \le t_c)$, and the distribution $G_k^{\rightarrow}(x;t_c)$ to the probability $P(T^{(k)} \le x \mid T^{(m)} > t_c)$.

[9] Under the assumption that $2 \cdot m \le n$, so that the second m-failure epoch occurs within the interval (0, T].

the optimisation of the threshold age t_c is a more efficient and robust process for the renewing modified m-failure group replacement rule than for the modified m-failure group replacement rule.

5. Numerical Results

We evaluated the four m-failure group replacement strategies for the maintenance of the ECT light standards, for m=4. First we considered the situation in which the residual lamps are kept in hot standby without energy cost; the results[10] are given in table 5.1.

table 5.1: ***Performance of the various 4-failure group replacement rules in the hot standby context***

	optimal value °	*optimal threshold age*	*administration*	*energy consumption*
standard 4-failure group replacement	DFL. 40,681.-	-	-	190,355 kWh/y
complete 4-failure group replacement	DFL. 40,190.-	-	-	196,574 kWh/y
modified 4-failure group replacement	DFL. 36,848.-	3.02 year**	individual ages	194,774 kWh/y
ren. mod. 4-failure group replacement	DFL. 38,839.-	2.64 year*	system age	196,287 kWh/y

(°) : only maintenance cost; (*) : obtained with the ordinary secant method; (**) : obtained with the modified secant method.

From the above table it follows that it is not economical to replace only the failed lamps at an m-failure epoch. It is also not economical to replace *all* the failed and non-failed components. It appears that the modified 4-failure group replacement rule with a threshold age of 3.02 is 9.4 % cheaper than the standard 4-failure group replacement rule, and 8.3 % cheaper than the complete 4-failure group replacement rule. The renewing modified 4-failure group replacement rule is not as efficient as the modified 4-failure group replacement rule because of the system renewals which brings the performance slightly to that of the complete 4-failure group replacement rule. The optimal threshold ages of the modified 4-failure group replacement rule and the renewing replacement rule differ 4½ months; if the threshold age of 2.64 years is used for the modified 4-failure group replacement rule then the long run average cost becomes Dfl.37,371.-

As a light-standard is expected to fulfil its function for 3600 hours a year, and a lamp uses up 80 watts a hour, the light-standard has an expected energy consumption of at least 12 times 288 kWh/y, that is 169,344 kWh/y for all the 49 light-standards. The standard, complete, modified and renewing modified 4-failure group replacement rule have an expected consumption of respectively 190,355, 196,574, 194,774 kWh and 196,287 kWh a year, due to the fact that the redundant lamps are kept in hot standby. This means that the standard, complete, modified and renewing modified 4-failure group replacement rule have an expected yearly energy cost of DFL.19,987.-, DFL.20,640.-, DFL.20,451.- and DFL.20,610.- respectively, so that the rules have a total expected maintenance and energy cost of DFL.60,668.-, DFL.60,830.-, DFL.57,300.- and DFL.59,449.- respectively.

A natural thought is to lower the energy consumption by screwing only 14, 13 or 12 lamps onto the chandelier, so that the number of redundant lamps is limited to respectively 2, 1 and 0 lamps. One may also wonder what would happen with the total expected maintenance and energy cost if the chandelier is extended with two extra lampholders and there are 16 or 17 lamps screwed on the chandelier. In table 5.2 you see the performance of the various group replacement rules when the chandelier holds 12, 13, 14, 15, 16 or 17 lamps.

Table 5.2: ***Performance in the case of a 12 out of 15-k system, for k= -2,-1, .., 3, with hot standby.***

system	*group replacement rule*	*optimal value °*	*optimal threshold age***	*energy consumption*	*total cost •*
12 out of 12	*standard 1-failure group replacement*	DFL. 80,506.-	-	169,344 kWh/y	DFL. 98,287.-
	complete 1-failure group replacement	DFL. 72,018.-	-	169,344 kWh/y	DFL. 89,799.-
	modified 1-failure group replacement	DFL. 50,067.-	2.29 year	169,344 kWh/y	DFL. 67,848.-
	ren. mod. 1-failure group replacement	DFL. 55,865.-	2.34 year	169,344 kWh/y	DFL. 73,646.-
12 out of 13	*standard 2-failure group replacement*	DFL. 53,076.-	-	176,380 kWh/y	DFL. 71,596.-
	complete 2-failure group replacement	DFL. 49,503.-	-	178,010 kWh/y	DFL. 68,194.-
	modified 2-failure group replacement	DFL. 40,928.-	2.72 year	177,519 kWh/y	DFL. 59,567.-
	ren. mod. 2-failure group replacement	DFL. 43,430.-	2.66 year	177,793 kWh/y	DFL. 62,098.-
12 out of 14	*standard 3-failure group replacement*	DFL. 44,510.-	-	183,354 kWh/y	DFL. 63,762.-
	complete 3-failure group replacement	DFL. 42,877.-	-	187,153 kWh/y	DFL. 62,528.-
	modified 3-failure group replacement	DFL. 37,971.-	2.87 year	185,989 kWh/y	DFL. 57,500.-
	ren. mod. 3-failure group replacement	DFL. 40,073.-	2.65 year	186,893 kWh/y	DFL. 59,697.-
12 out of 15	*standard 4-failure group replacement*	DFL. 40,681.-	-	190,355 kWh/y	DFL. 60,668.-
	complete 4-failure group replacement	DFL. 40,190.-	-	196,574 kWh/y	DFL. 60,830.-

[10] For the standard and modified 4-failure group replacement rule we performed a ‘steady-state’ simulation of 24,000 years.

	modified 4-failure group replacement	DFL. 36,848.-	3.02 year	194,774 kWh/y	DFL. 57,299.-
	ren. mod. 4-failure group replacement	DFL. 38,857.-	2.68 year	196,189 kWh/y	DFL. 59,457.-
12 out of 16	*standard 5-failure group replacement*	DFL. 38.660.-	-	197,005 kWh/y	DFL. 59,345.-
	complete 5-failure group replacement	DFL. 39,067.-	-	206,152 kWh/y	DFL. 60,173.-
	modified 5-failure group replacement	DFL. 36,534.-	3.17 year	203,532 kWh/y	DFL. 57,905.-
	ren. mod. 5-failure group replacement	DFL. 38,523.-	2.64 year	205,842 kWh/y	DFL. 60,136.-
12 out of 17	*standard 6-failure group replacement*	DFL. 37,834.-	-	204,915 kWh/y	DFL. 59,350.-
	complete 6-failure group replacement	DFL. 38,739.-	-	215,876 kWh/y	DFL. 61,406.-
	modified 6-failure group replacement	DFL. 36,643.-	3.38 year	212,086 kWh/y	DFL. 58,912.-
	ren. mod. 6-failure group replacement	DFL. 38,560.-	2.79 year	215,579 kWh/y	DFL. 61,196.-

(°) : only maintenance cost; (•) : maintenance and energy cost; (**) : obtained with the modified secant method.

It appears that the maintenance cost decreases substantially if the number of lamps screwed on the chandelier increases and the redundancy is not too large. Given that the chandelier has 15 lampholders, the optimal number of lamps screwed on the chandelier is 15 for all the four group replacement rules, even when an energy cost of 10½ cents per kilowatt-hour is incurred[11]; thus it appears to be beneficial to have some redundancy. The operational use of the light-standard is therefore justified. However, if the standard or complete m-failure group replacement rule is applied and there were two extra lampholders, then it would be optimal to use the extra lampholders; the long run average maintenance cost would decrease with another 7.0% when the standard replacement rule is applied, and 3.6% if the complete replacement rule is applied.

Observe that the modified replacement rule performs considerably better than the standard and complete replacement rule when there are slightly more lamps then needed to provide the minimum luminosity. It seems that the use of an age criterion is especially remunerative when there is a scant redundancy so that the chandelier must be frequently lowered for the replacement of the failed lamps. When the chandelier is often lowered, there are a lot of opportunities to replace a non-failed lamp preventively, which makes the decision to replace a non-failed component or not, more time-dependent.

We have also considered the situation in which the residual lamps are kept in cold standby, where the chandelier holds 15 lamps; the results are given in table 5.3.

table 5.3 : ***Performance of the various 4-failure group replacement rules in the cold standby context***

	optimal value °	*optimal threshold age*	*administration*	*total cost •*
standard 4-failure group replacement	DFL. 36,166.-	-	-	DFL. 53,947.-
complete 4-failure group replacement	DFL. 37,348.-	-	-	DFL. 55,129.-
modified 4-failure group replacement	DFL. 32,596.-	3.00 year**	individual ages	DFL. 50,377.-

(°) : only maintenance cost; (•) : maintenance and energy cost; (**) : obtained with the modified secant method.

Bear in mind that no cost of light-output reduction is incurred so that the replacement rules yield a lower long run average cost in the cold standby context than in the hot standby context. The standard and complete 4-failure group replacement rules are again dominated by the modified 4-failure group replacement rule, with a saving - without energy cost - of 9.9 % and 12.7 %, respectively.

[11] The optimal threshold ages may slightly change if the objective function : 'minimise the long run average maintenance cost' is replaced by : 'minimise the long run average maintenance and energy cost'; however the change is not greater than 0.11 year. Further, the change in the total cost is at most DFL. 11.-, so that the optimal number of lamps screwed on the chandelier remains 15.

table 5.4 : ***Light standard - statistics***

	standby	*# lamps*[1]	*frequency*[2]	*total lamps*[3]	*yearly cost*[4]
standard 4-failure group replacement	hot	4.0	0.91	3.64	DFL. 830.-
complete 4-failure group replacement	hot	15.0	0.36	5.40	DFL. 820.-
modified 4-failure group replacement with a threshold age of 3.02 year	hot	7.9	0.53	4.19	DFL. 752.-
ren. mod. 4-failure group replacement with a threshold age of 2.64 year	hot	12.0	0.40	4.80	DFL. 793.-
standard 4-failure group replacement	cold	4.0	0.83	3.32	DFL. 738.-
complete 4-failure group replacement	cold	15.0	0.33	4.95	DFL. 762.-
modified 4-failure group replacement with a threshold age of 3.00 year	cold	7.0	0.53	3.71	DFL. 665.-

elucidation

[1] : average number of lamps that is replaced at a 4-failure-epoch
[2] : average number of times that the chandelier is lowered during a year
[3] : average number of lamps that is replaced during a year
[4] : average cost per year of maintaining a light standard, without energy cost

Table 5.4 shows that the number of times at which a 4-failure occurs depends partly on the number of lamps which are preventively replaced at a 4-failure epoch. The standard 4-failure group replacement rule induces the lowest maintenance cost at a 4-failure epoch ($c_0+4 \cdot c_1$) as it replaces only the failed lamps, but has a 4-failure frequency which is almost three times as high as that of the complete 4-failure group replacement rule, which makes the rule the most expensive 4-failure group replacement rule. The complete 4-failure group replacement rule is somewhat better than the standard 4-failure group replacement rule, but is also a costly rule as it replaces not only the failed lamps but also the non-failed lamps at a 4-failure epoch, regardless of their expected residual lifetime. Sometimes the lamps are replaced after a short time of operation.

The modified 4-failure group replacement rule with a threshold age of 3 years prescribes a simultaneous replacement of all failed lamps, together with the non-failed lamps with a periodic failure rate above the 2.5 %, and reduces effectively the long run average cost of maintaining a light standard by replacing the lamps which are failed, or are likely to fail soon. Under this rule, the long run average cost of maintaining a light standard is about DFL.750.- per year if the redundant lamps are kept in hot standby, and DFL.665.- per year if the redundant lamps are kept in cold standby. With energy cost the long run average cost is DFL.1,167.- in the hot standby context and DFL.1,028.- in the hypothetical cold standby context.

When we look at the average number of lamps that is replaced during a year, we see that the current strategy (the complete 4-failure group replacement rule) consumes a lot of lamps, if it is compared with the standard and modified 4-failure group replacement rule. Knowing that there are 49 light-standards, the complete replacement rule uses about 86.2 lamps per year more than the standard replacement rule, and 59.3 lamps more than the complete replacement rule, so that the complete replacement rule is a quite unfriendly maintenance rule for the environment.

6. Conclusions

In this paper we considered several group replacement policies for a case in which (i) replacement of the failed components is mandatory when there are 4 components failed, (ii) the components have an increasing failure rate, and (iii) there is a substantial set-up cost associated with the replacement of one or more components. No cost is incurred for individual failures.

It appeared that both the standard and complete 4-failure group replacement rule which are to replace at an m-failure epoch *only the failed* components, and *all* the components respectively, are somewhat constrained in the sense that they do not distinguish between components which should be on economic grounds preventively replaced and components which should not be preventively replaced.

Therefore we defined two types of policies: the modified, and the renewing modified m-failure group replacement rule; both rules use an age-criterion to determine which of the non-failed components should be at an m-failure epoch preventively replaced. The policies differ in the level of information needed; the modified group replacement rule requires that all the individual ages of the components are being tracked, while the renewing modified group replacement rule needs only the system age which is the time between two complete replacements. Another difference is that the first rule can be only evaluated by simulation, while the latter can be evaluated analytically.

The numerical results showed significant relative differences in costs; compared to the complete 4-failure group replacement rule which is currently applied by the ECT, the modified 4-failure group replacement rule leads to a saving of 8.3%, while the renewing modified group replacement rule leads to a saving of 3.4%. It should be emphasised that the redundant lamps are kept in hot standby; if the lamps were kept in cold standby then the saving would be more considerable.

It appeared that the age-criterion becomes more important when there is a small redundancy as this means that there are a lot of opportunities to replace a component preventively, so that the decision whether to replace a component or not becomes more time-dependent. Further, it appeared that the correct level of redundancy depends on the maintenance policy applied. If we consider the complete replacement rule then the ECT would be better off if they had chandeliers containing 16 or more lampholders instead of the current chandeliers, which have 15 lampholders. However, if the ECT applies the (renewing) modified replacement rule, then a redundancy of 3 lamps can be justified, as this leads to the lowest long run average maintenance and energy cost.

Compared to the annual returns of the *Europe Combined Terminals* a cost reduction of DFL.3,342.- per year by applying the modified 4-failure group replacement rule instead of the complete replacement rule is a minor saving, and because of the effort needed to register the ages of the components, the ECT is not very encouraged to change its policy. But from an ecological point of view it may be interesting to reduce the consumption of lamps, as the complete replacement rule replaces about 59.3 lamps per year more than the modified replacement rule. Bethink further that there may be some instances for which the modified m-failure group replacement rule yields a more substantial saving; for those instances it may be worthwhile enough to administrate the individual ages.

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