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A Subsidization Dilemma: the Effect of Shadow Cost of Public Funds and Consumer Distribution

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Abstract

This article studies subsidy competition in a two-country-two-firm model incorporating bilateral trade, consumer distribution, and a shadow cost of public funds. Governments choose subsidies to maximize domestic or international welfare, with cooperation defined by subsidy choices maximizing international welfare. Under equal consumer bases, the subsidy game is a prisoner's dilemma if and only if the shadow cost is positive. This creates an opportunity for cooperation; without cooperation, however, governments oversubsidize their firms. We find that incentives to cooperate increase with the shadow cost. However, sufficiently unequal consumer distributions can eliminate the prisoner's dilemma, removing the larger country's incentive to cooperate.

Keywords: Bilateral Trade, International Duopoly, Prisoner's Dilemma, Shadow Cost of Public Funds, Subsidy.

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1 Introduction

On June 15th, 2021, the European Union (EU) and the United States (US) announced the truce that ended a 17-year-long subsidy war over aircraft subsidies (European Commission, 2021).[1] Since 2004, the World Trade Organization (WTO) received complaints from both the EU and the US about policies from the other party that violated fair trade (WTO, 2020a, 2020b). Although the WTO imposed multiple sanctions on both the EU and the US (WTO, 2020a, 2020b), it found itself to be a referee without firm authority. Over a period of 17 years, the US and the EU engaged in fierce competition through policies favoring their domestic aircraft industries (respectively Boeing and Airbus) in an effort to increase competitiveness and expand market shares. Although the Trump administration renewed attention to protectionist trade policies through its tariff measures, such policies have a long history. Examples include 17th-century French Colbertism (Minard, 2014), 18th-century British export bounties (Smith, 1776, Book IV, Chapter V), Otto von Bismarck's Schutzzölle (literally "protective tariffs") during the Belle Époque (Zussman, 2002), and the substantial export subsidies of modern China (e.g., see Xiang, Yin, and Zi, 2026), among others. Consistent with Adam Smith's critique of mercantilism (Smith, 1776), such policies can be characterized as beggar-thy-neighbor policies, whereby countries increase domestic welfare through measures that reduce the welfare of other countries.

In light of such protective measures, this article studies government subsidization behavior from both noncooperative and cooperative perspectives, as illustrated by the 17-year Boeing-Airbus subsidy dispute and the policies adopted following the subsequent truce. We model a common market consisting of two countries, each with one domestic firm. By subsidizing its domestic firm, the government of a country can influence the firm's output quantity and potentially achieve a higher domestic surplus, which consists of consumer and producer surplus within the country. Indirectly, by increasing the output of its domestic firm through its subsidy, a government also exerts a negative influence on foreign output. This creates incentives for governments to compete by setting subsidies above the socially optimal level in an attempt to secure larger market shares for their domestic firms and thereby increase domestic surplus.

In our setup, firms choose their production levels and governments can influence these by imposing a per-unit subsidy or tax (i.e., negative subsidy). We define the *domestic-welfare-maximizing subsidy* as the subsidy level that maximizes a country's domestic welfare and the *cosmopolitan subsidy* as the subsidy level chosen by a hypothetical cosmopolitan social planner that maximizes international welfare. The former represents an equilibrium subsidy level, while the latter is the solution to a single maximization problem. *Oversubsidization* occurs when a government provides a subsidy that is larger than the cosmopolitan subsidy. Similarly, *undersubsidization* occurs when the government provides a subsidy that is lower than the cosmopolitan subsidy. The terms over- and undersubsidization draw inspiration from the concept of over- and underinvestment, which describe situations where firms or governments are incentivized to invest more or less for strategic reasons. This concept is well-established in various strands of literature (e.g., industrial organization and corporate

[1] The full declaration can be found here: https://circabc.europa.eu/rest/download/cbb6fc3a-c106-4bf4-885a-45a84baad256.

finance).

A *subsidy game* describes the game played by two countries in which each chooses between its domestic-welfare-maximizing and cosmopolitan subsidy. A subsidy game is referred to as a *subsidization dilemma* if it constitutes a prisoner's dilemma, whereby both countries would be better off if they both choose the cosmopolitan subsidy level than if they both choose the domestic-welfare-maximizing subsidy level. Note that in our context, the existence of a subsidization dilemma is positive as it provides countries with an incentivize to cooperate. This mutually beneficial cooperation increases international welfare. If the subsidy game does not constitute a *subsidization dilemma*, at least one country cannot increase its welfare by providing cosmopolitan subsidies and therefore has no incentive to cooperate.

Typically, subsidies are financed through distortionary taxation. Our model explicitly incorporates a shadow cost of raising public funds to account for this (Laffont & Tirole, 1986). Although Neary (1994) and Leahy and Montagna (2001) consider the role of a shadow cost of raising public funds in a third-country model without domestic consumption, this article, to the best of our knowledge, is the first to model a subsidy game with a shadow cost of raising public funds in a framework featuring domestic consumption and bilateral trade. Additionally, our model considers a consumer population distributed across the two countries, potentially in unequal proportions.

	$\lambda = 0$		$\lambda > 0$	
	Subsidies	Outcome	Subsidies	Outcome
$\theta_1 = \theta_2$	$\widehat{s}_1 = \widehat{s}_2 = s_1^* = s_2^*$	No dilemma	$\widehat{s}_1 = \widehat{s}_2 > s_1^* = s_2^*$	Dilemma
$\theta_1 > \theta_2$	$\widehat{s}_1 > s_1^* = s_2^* > \widehat{s}_2$	No dilemma	$\widehat{s}_1 > s_1^* = s_2^*$ and $\widehat{s}_1 > \widehat{s}_2$	Depends on parameters

Table 1.1: Equilibrium subsidies and the existence of a subsidization dilemma. The parameter λ is the shadow cost of public funds. For $i \in \{1, 2\}$, $\widehat{s}_i$ and s_i^* are the domestic-welfare-maximizing and cosmopolitan subsidies, respectively, and θ_i is the fraction of consumers in country i.

In our analysis, we consider two cases: equal and unequal consumer distributions. The results with respect to the existence of a shadow cost of public funds are summarized in Table 1.1. First, when consumer bases are equal and in the absence of a shadow cost of public funds, over- or undersubsidization by domestic-welfare-maximizing countries does not take place. In this case, price and marginal production cost are equal under the cosmopolitan subsidy level. As a result, there is no incentive for domestic-welfare-maximizing governments to compete by setting a higher subsidy level in order to increase the market share of their domestic firm, as this would yield higher subsidization costs than price gains.

However, when a positive shadow cost of public funds is introduced, we find that domestic-welfare-maximizing countries oversubsidize their domestic firm. In this case, the price exceeds marginal production and subsidization costs under the cosmopolitan subsidy level. This gives domestic-welfare-maximizing governments room to compete in subsidies above the cosmopolitan level such that their domestic firm increases its market share, while still maintaining a positive profit margin. Therefore, the shadow cost of public funds is not merely a cost to consider in subsidization; it actually fosters oversubsidization. Under a positive shadow cost and equal consumer bases, the subsidy game constitutes a subsidization dilemma, where joint oversubsidization is the Nash equilibrium and joint cosmopolitan subsidization is a Pareto improvement over this Nash equilibrium. Oversubsidization is thus costly from a social welfare perspective.

Turning to situations in which the consumer population is unevenly distributed between the two countries, we find the following results. First, governments focused on maximizing domestic welfare set unequal subsidy levels, whereas the cosmopolitan outcome is obtained under equal subsidy rates. The disparity in domestic subsidies introduces additional welfare losses stemming from reduced competition, beyond those already caused by oversubsidization in the symmetric case. Second, if consumer bases are sufficiently unequal, the nature of the subsidy game can change fundamentally: setting domestic-welfare-maximizing subsidies remains the Nash equilibrium, but the country with the larger consumer base is now better off under this equilibrium than under the cosmopolitan subsidy. The subsidy game therefore no longer constitutes a subsidization dilemma, and the country with the larger consumer base has little incentive to cooperate and adopt a common, cosmopolitan optimal subsidy rate.

The article is organized as follows. In Section 2, we discuss the relevant literature. In Section 3, we introduce the model and equilibrium conditions corresponding to the noncooperative and cooperative settings. Section 4 introduces the formal definitions of oversubsidization and the subsidization dilemma. Section 5 covers the case in which both countries have equal consumer bases, whereas Section 6 considers the case in which consumer bases are unequal. Section 7 concludes and proposes directions for further research. The proofs of all theorems, propositions, lemmas, and corollaries are provided in the appendix.

2 Related Literature

Our framework extends Brander and Spencer (1985)—who analyze subsidization behavior in a two-country, two-firm model—by incorporating a shadow cost of public funds, and by modeling bilateral trade. We also go beyond Brander and Spencer (1985) and related works (e.g., Eaton and Grossman, 1986) by explicitly accounting for asymmetries in consumer bases. Brander and Spencer (1985) find that governments can increase domestic welfare by jointly reducing subsidy levels below the Nash equilibrium. To obtain this result, they assume no bilateral trade: domestically produced goods are consumed either domestically or in a third country. We show that, given symmetric consumer bases, a positive shadow cost of public funds is necessary and sufficient to obtain the same result as Brander and Spencer (1985) that in the Nash equilibrium governments subsidize at levels exceeding the social optimum.

Krugman (1987, 1994) builds on the Brander-Spencer model by constructing illustrative examples of two-by-two bimatrix games that describe changes in a firm's strategy when its government provides a subsidy. Specifically, if one government subsidizes its domestic firm, that firm gains a significant advantage over its competitors, potentially driving them out of the market. Our analytical model formalizes and extends these illustrative examples into a subsidy counterpart of the terms-of-trade prisoner's dilemma with tariffs, in which international cooperation is the only way to escape the dilemma (e.g., Bagwell and Staiger, 2011).

In line with Laffont and Tirole (1986), we assume that subsidies are financed through distortionary taxation, implying that each unit of public funds raised entails an additional welfare cost. This is known as the shadow cost of public funds. Laffont and Pouyet (2004) extend this model to an international setting with competing regulators, which shows more similarities with our model. However, while Laffont and Tirole (1986) and Laffont and Pouyet (2004) focus on regulation of firms with a contract theory approach, our article studies how governments use subsidies to stimulate firms toward a desired production level.

Relatively little literature incorporates a shadow cost of raising public funds into international trade models with subsidies. Notable exceptions are Neary (1994) and Leahy and Montagna (2001), who find that subsidizing domestic firms is optimal from a domestic-welfare-maximizing perspective when the shadow cost of raising public funds is sufficiently low. Although our framework shares several similarities with these studies, it goes beyond them by not only considering the optimality of positive domestic subsidies, but also the existence and magnitude of oversubsidization associated with a shadow cost of raising public funds, and the implications in the context of a subsidy game.

Earlier work in the new trade theory literature focusing on asymmetries, such as Neary (1994), Melitz (2003), and Yomogida (2008), considers supply-side asymmetries, such as transportation costs, asymmetric cost functions, and differences in the number of firms. By contrast, we focus on demand-side asymmetry by allowing for a consumer population that is unevenly distributed across the two countries. To the best of our knowledge, this demand-side asymmetry has not previously been considered in this literature.

A recently applied study related to our work is Böhm, Hohmann, Raabe, and Riedel (2025), who employ a difference-in-differences approach to show that exports of South African firms decline when foreign competitors in the same product category benefit from corporate tax cuts. They further document that these competitive pressures translate into reductions in profits and employment at affected South African exporters, underscoring that corporate tax policy in one country can impose significant externalities on abroad. These results are in line with our finding that one country, by increasing its subsidy, reduces welfare in the other country (i.e., beggar-thy-neighbor).

3 The Model

We first present the model in Section 3.1. Section 3.2 then describes the production equilibrium of the firms. Lastly, Sections 3.3 and 3.4 characterize the subsidy setting of a domestic-welfare-maximizing government and that of a cosmopolitan social planner, respectively.

3.1 The International Duopoly Model

We consider a model of *international duopoly* in which two countries or groups of countries each have a single domestic firm, and these two firms compete with one another in a shared market. Together, these two firms form a duopoly, meaning that the entire consumer market is served exclusively by them. The set of two countries or groups of countries is denoted by $N = \{1, 2\}$. For ease of exposition, we refer to them as country 1 and country 2 throughout the article. Because each firm is inherently tied to a specific country, firm 1 is associated with country 1 and firm 2 with country 2.

An international duopoly is represented by a tuple (c, p, λ, θ) on $N = \{1, 2\}$ in which $c\colon \mathbb{R}_+ \to \mathbb{R}_+$ is a *production cost function*, $p\colon \mathbb{R}_+ \to \mathbb{R}_+$ is a *market price function*, $\lambda \in \mathbb{R}_+$ is a *shadow cost of public funds*, and $\theta = (\theta_1, \theta_2) \in \mathbb{R}_+^{\{1,2\}}$ with $\theta_1 + \theta_2 = 1$ is a vector for which each coordinate θ_i represents the fraction of consumers living in country $i \in \{1, 2\}$.

The following assumptions apply to the international duopoly (c, p, λ, θ). The shadow cost of public funds λ is identical across both countries. The production cost function c is continuous and twice differentiable, and is identical for both firms. The market is homogeneous and the price function p is continuous, twice differentiable, and identical for both firms. Furthermore, the production cost function is strictly increasing (i.e., $c' > 0$), and the market price function is strictly decreasing in output (i.e., $p' < 0$). Finally, we make the standard assumption $p(0) > c'(0)$, which implies that production is initially profitable without subsidies.

We assume Cournot competition, meaning that each firm decides on its optimal output quantity. For each firm $i \in \{1, 2\}$, an arbitrary output quantity is denoted by $\tilde{q}_i \in \mathbb{R}_+$, whereas firm i's optimal output quantity is denoted by q_i.[2] Total market output is denoted by $\tilde{Q} = \tilde{q}_1 + \tilde{q}_2$, whereas the optimal total market output is denoted by $Q = q_1 + q_2$. Because output quantities are determined by firms as part of their decision-making process, governments do not directly control these quantities. Nevertheless, each country can offer its domestic firm a per-unit subsidy, which influences the firm's output quantity. For each country i, $s_i \in \mathbb{R}$ denotes the subsidy provided to its domestic firm. Note that a negative subsidy is interpreted as a per-unit tax.

3.2 Production Equilibrium

In this section, we analyze the firms' behavior and the resulting *production equilibrium*. In line with Brander and Spencer (1985), each of the two firms maximizes its own profit.

Let $i, j \in \{1, 2\}$ with $i \neq j$. The profit function of firm i is given by

$$\pi_i(\widetilde{q}_i, \widetilde{q}_j, s_i) = \widetilde{q}_i p(\widetilde{Q}) - c(\widetilde{q}_i) + \widetilde{q}_i s_i,$$

where $\tilde{q}_j$ is the output quantity of firm j and s_i is the per-unit subsidy that firm i receives from country i.

To maximize the profit of firm i, we take the first derivative of π_i with respect to q_i and set it equal to zero. The first-order condition, which is given by

$$p(\widetilde{Q}) + \widetilde{q}_i p'(\widetilde{Q}) + s_i = c'(\widetilde{q}_i), \tag{3.1}$$

implies that the optimal output quantity q_i is determined by equating marginal revenue and marginal cost. To guarantee a unique production equilibrium, we assume that marginal

[2] Although it is conventional to denote optimal output quantities by q_i^*, we instead write q_i, because the firm's optimization problem is secondary in our analysis.

profits decrease with the firm's own and foreign output, with the former effect dominating (see Tirole, 1988, Chapter 5.7.1.2). That is, for $i, j \in \{1, 2\}$ with $i \neq j$, we assume that[3]

$$\frac{\partial^2 \pi_i(\widetilde{q}_i, \widetilde{q}_j, s_i)}{\partial \widetilde{q}_i^2} < \frac{\partial^2 \pi_i(\widetilde{q}_i, \widetilde{q}_j, s_i)}{\partial \widetilde{q}_i \partial \widetilde{q}_j} < 0. \tag{3.2}$$

Specifically, the optimal output quantity for firm 1 is given by $q_1(\widetilde{q}_2, s_1)$, while for firm 2 it is given by $q_2(\widetilde{q}_1, s_2)$. The optimal output quantity q_1 depends on $\widetilde{q}_2$; in particular, it depends on the optimal output quantity q_2. Consequently, the optimal output quantity q_1 follows from solving

$$p(q_1 + q_2(q_1, s_2)) + q_1 p'(q_1 + q_2(q_1, s_2)) + s_1 = c'(q_1).$$

And, by applying the exact same arguments, the optimal quantity q_2 follows from

$$p(q_1(q_2, s_1) + q_2) + q_2 p'(q_1(q_2, s_1) + q_2) + s_2 = c'(q_2).$$

Hence, the production equilibrium depends on the per-unit subsidies chosen by both countries. Let $q_1(s_1, s_2)$ and $q_2(s_1, s_2)$ denote the corresponding equilibrium outputs.

Brander and Spencer (1985) show that an increase in a country's per-unit subsidy to its domestic firm raises that firm's optimal output while reducing the optimal output of its foreign competitor. This occurs because the subsidy provides the domestic firm with a competitive advantage. Formally, for all $i, j \in \{1, 2\}$ with $i \neq j$, it holds that

$$\frac{\partial q_i(s_1, s_2)}{\partial s_i} > 0 \quad \text{and} \quad \frac{\partial q_j(s_1, s_2)}{\partial s_i} < 0. \tag{3.3}$$

Furthermore, Brander and Spencer (1985) find that the total equilibrium output increases with the subsidy of each country. That is, the rise in domestic firm output outweighs the decline in foreign firm output. Formally, it holds that

$$\frac{\partial Q}{\partial s_1} > 0 \quad \text{and} \quad \frac{\partial Q}{\partial s_2} > 0.$$

3.3 Domestic Subsidy Equilibrium

We now turn to the equilibrium conditions for the countries that take $q_1(s_1, s_2)$ and $q_2(s_1, s_2)$ as given. For ease of exposition, we omit the function arguments (s_1, s_2) from $q_1(s_1, s_2)$ and $q_2(s_1, s_2)$. We introduce an objective in which each government maximizes domestic welfare, defined as the sum of domestic producer and consumer surplus. The domestic welfare levels of countries 1 and 2 are denoted by $w_1(s_1, s_2)$ and $w_2(s_1, s_2)$, respectively. Let $i \in \{1, 2\}$. Then, the domestic welfare function of country i is given by

$$w_i(s_1, s_2) = \underbrace{\theta_i \left(\int_0^Q p(\widetilde{Q}) d\widetilde{Q} - Qp(Q) \right) - (1 + \lambda) s_i q_i}_{\text{domestic consumer surplus}} + \underbrace{p(Q) q_i - c(q_i) + s_i q_i}_{\text{domestic producer surplus}}$$

[3] We impose this condition on the objective function rather than adopting assumptions on the price and production cost functions (e.g., convex production costs) as this choice ensures a weakly larger admissible set of functions.

$$= \underbrace{\theta_i U}_{(1)} + \underbrace{p(Q)q_i - c(q_i)}_{(2)} - \underbrace{\lambda s_i q_i}_{(3)}, \tag{3.4}$$

in which $U = \int_0^Q p(\widetilde{Q})d\widetilde{Q} - Qp(Q)$. In expression (3.4), (1) captures domestic in-market consumer surplus, while (2) represents domestic profits. The cost of financing the subsidy to the domestic firm is given by (3), where λ is the shadow cost of raising public funds. Via the market price, which is determined by the total market output, a country's welfare is intricately linked to the economic policies of the other nation and hence a function of both its domestic subsidy and the foreign subsidy.

The domestic optimization problem of country i is given by

$$\max_{s_i \in \mathbb{R}_+} w_i(s_1, s_2). \tag{3.5}$$

The first-order condition, given by

$$\theta_i \frac{\partial U}{\partial s_i} + p(Q)\frac{\partial q_i}{\partial s_i} + q_i p'(Q)\frac{\partial Q}{\partial s_i} = \lambda \left(q_i + s_i \frac{\partial q_i}{\partial s_i} \right) + c'(q_i)\frac{\partial q_i}{\partial s_i}, \tag{3.6}$$

implies equality between the marginal benefits and costs of increasing the subsidy from a domestic point of view. The marginal benefits consist of the marginal gains in domestic consumer surplus and domestic profits. The marginal costs comprise the marginal increase in the subsidization and production costs incurred due to the same subsidy increment.

To find the domestic subsidy equilibrium, one solves the first-order conditions (3.6) for both countries simultaneously. The resulting subsidy equilibrium, denoted by $\widehat{s} = (\widehat{s}_1, \widehat{s}_2)$, is such that $\widehat{s}_1$ is the best response to $\widehat{s}_2$ and vice versa. To guarantee a unique domestic equilibrium, we assume that marginal domestic welfare is decreasing in both the domestic and foreign subsidies, with the domestic effect dominating. That is, for $i, j \in \{1, 2\}$ with $i \neq j$, we assume that

$$\frac{\partial^2 w_i(s_1, s_2)}{\partial s_i^2} < \frac{\partial^2 w_i(s_1, s_2)}{\partial s_i \partial s_j} < 0. \tag{3.7}$$

3.4 Cosmopolitan Optimal Subsidies

Next, we consider the setting in which governments jointly maximize international welfare. For this purpose, we introduce the concept of a *cosmopolitan social planner*,[4] that can be interpreted as a hypothetical world government concerned with international total welfare. International total welfare is defined as the sum of the two individual welfare functions:

$$\begin{aligned} W(s_1, s_2) &= w_1(s_1, s_2) + w_2(s_1, s_2) \\ &= \int_0^Q p(\widetilde{Q})d\widetilde{Q} - \sum_{i \in \{1,2\}} \left(c(q_i) + \lambda s_i q_i \right). \end{aligned} \tag{3.8}$$

[4]This term refers to the philosophical and political tradition of the Cosmopolitan movement, which criticizes the notion of competing nation-states and envisions a unified global community (Beck, 2006).

The expression consists of total gross consumer surplus minus production cost and the shadow cost of financing the subsidy. The cosmopolitan planner maximizes international total welfare with respect to both s_1 and s_2. The cosmopolitan problem is defined as

$$\max_{s_1,s_2\in\mathbb{R}_+} W(s_1,s_2). \tag{3.9}$$

For ease of exposition, we set $G=\int_0^Q p(\widetilde{Q})d\widetilde{Q}$.[5] The resulting first-order conditions,

$$\frac{\partial G}{\partial s_1}=\lambda\left(q_1+s_1\frac{\partial q_1}{\partial s_1}\right)+\lambda s_2\frac{\partial q_2}{\partial s_1}+c'(q_1)\frac{\partial q_1}{\partial s_1}+c'(q_2)\frac{\partial q_2}{\partial s_1} \tag{3.10}$$

and

$$\frac{\partial G}{\partial s_2}=\lambda\left(q_2+s_2\frac{\partial q_2}{\partial s_2}\right)+\lambda s_1\frac{\partial q_1}{\partial s_2}+c'(q_2)\frac{\partial q_2}{\partial s_2}+c'(q_1)\frac{\partial q_1}{\partial s_2}, \tag{3.11}$$

imply equality between the marginal benefits and costs of increasing the subsidy from a cosmopolitan point of view. In the first-order conditions (3.10) and (3.11), the term on the benefits side captures the gain in (gross) consumer surplus. On the costs side, the first λ term reflects the increase in domestic shadow costs of subsidization from a rise in the domestic subsidy level, while the second λ term represents the corresponding decrease in foreign shadow costs. The final two terms account for the change in production costs resulting from an increase in the domestic subsidy. To guarantee a unique solution, as later shown in Proposition 3.1, we assume that marginal cosmopolitan welfare declines in both subsidies, with the domestic effect dominating. That is,

$$\frac{\partial^2 W(s_1,s_2)}{\partial s_1^2}<\frac{\partial^2 W(s_1,s_2)}{\partial s_1\partial s_2}<0 \quad\text{and}\quad \frac{\partial^2 W(s_1,s_2)}{\partial s_2^2}<\frac{\partial^2 W(s_1,s_2)}{\partial s_1\partial s_2}<0. \tag{3.12}$$

The solution to the system of first-order conditions (3.10) and (3.11) is denoted by $s^*=(s_1^*,s_2^*)$. This solution maximizes total welfare internationally and yields a socially optimal outcome, although due to the presence of a shadow cost of subsidization, this outcome does not necessarily need to be first-best. When there is no shadow cost of raising public funds, governments can replicate the first-best outcome using subsidies, since transferring funds from consumers to producers to incentivize socially optimal production causes no efficiency losses. If a positive shadow cost is present, some welfare is lost in the subsidization process. The outcome becomes second-best.

The following proposition states that there exists a unique solution to the cosmopolitan problem, which has the property that each country sets the same subsidy level.

Proposition 3.1. *Let (c,p,λ,θ) be an international duopoly. Then, there exists a unique solution $s^*=(s_1^*,s_2^*)$ to the cosmopolitan problem (3.9) and this solution exhibits the property $s_1^*=s_2^*$.*

Since the cosmopolitan planner maximizes international welfare and international welfare is independent of consumer distribution, the cosmopolitan subsidy does not depend on the consumer share of that country. As a result, for any allocation of consumers, the planner optimally utilizes the production capacities of both countries by setting an identical subsidy.

[5] From the definition of U it follows that $G=U+Qp(Q)$.

4 Oversubsidization and the Subsidization Dilemma

The choice for each country between the domestic-welfare-maximizing and the cosmopolitan subsidy can be represented by a bimatrix game, that is, a *subsidy game*.[6] Table 4.1 represents such a subsidy game. From assumptions 3.7, both players choosing the domestic-welfare-maximizing subsidy strategy $\widehat{s}$ is the only Nash equilibrium in any subsidy game.

	s_2^*	$\widehat{s}_2$
s_1^*	$(w_1(s_1^*, s_2^*), w_2(s_1^*, s_2^*))$	$(w_1(s_1^*, \widehat{s}_2), w_2(s_1^*, \widehat{s}_2))$
$\widehat{s}_1$	$(w_1(\widehat{s}_1, s_2^*), w_2(\widehat{s}_1, s_2^*))$	$(w_1(\widehat{s}_1, \widehat{s}_2), w_2(\widehat{s}_1, \widehat{s}_2))$

Table 4.1: Subsidy game for two governments.

When the subsidy game constitutes a prisoner's dilemma, it is referred to as a *subsidization dilemma*. The occurrence of a subsidization dilemma implies that both governments can achieve higher domestic welfare through joint cosmopolitan subsidization, for example by entering into a trade agreement. However, if the foreign country turns out not to be trustworthy and deviates, domestic welfare under the cosmopolitan subsidy is lower than under the domestic-welfare-maximizing subsidy. We define the existence of a subsidization dilemma in Definition 4.1.

Definition 4.1. Let $(\widehat{s}_1, \widehat{s}_2)$ be the domestic equilibrium and (s_1^*, s_2^*) be the cosmopolitan solution for the international duopoly (c, p, λ, θ). Then, the corresponding subsidy game constitutes a *subsidization dilemma* if:

(i) $w_1(\widehat{s}_1, s_2^*) > w_1(s_1^*, s_2^*)$ and $w_2(s_1^*, \widehat{s}_2) > w_2(s_1^*, s_2^*)$;

(ii) $w_1(\widehat{s}_1, \widehat{s}_2) > w_1(s_1^*, \widehat{s}_2)$ and $w_2(\widehat{s}_1, \widehat{s}_2) > w_2(\widehat{s}_1, s_2^*)$;

(iii) $w_1(s_1^*, s_2^*) > w_1(\widehat{s}_1, \widehat{s}_2)$ and $w_2(s_1^*, s_2^*) > w_2(\widehat{s}_1, \widehat{s}_2)$.

In Definition 4.1, conditions (i) and (ii) describe that both governments have incentives to deviate from the cosmopolitan solution toward the domestic solution, such that joint adoption of domestic subsidies $(\widehat{s}_1, \widehat{s}_2)$ is the only Nash equilibrium of the game. Condition (iii) indicates that joint cosmopolitan subsidization (s_1^*, s_2^*) is a strong Pareto improvement over this Nash equilibrium.

The existence of a subsidization dilemma is generally a positive phenomenon in the context of this article. It implies that both countries have incentives to cooperate and set cosmopolitan subsidies, offering hope for an agreement that maximizes international welfare. If a subsidization dilemma does not occur, at least one country has no incentive to cooperate and international welfare maximization through a joint agreement seems further out of reach.

To analyze whether competitive behavior of countries causes excessive or insufficient subsidization, we introduce the terms *oversubsidization* and *undersubsidization*, which describe situations where governments are incentivized to subsidize more or less than the cosmopolitan subsidy.

[6] Krugman (1987) earlier constructed a firm bimatrix entry game under subsidies for illustrative purposes. Neary (1994) was the first to refer to a country's subsidy choice under strategic interaction as a subsidy game.

Definition 4.2. Let $i \in \{1, 2\}$. Then, country i *oversubsidizes* its domestic firm if it sets a subsidy larger than s_i^*, and country i *undersubsidizes* its domestic firm if it sets a subsidy smaller than s_i^*.

Typically, if the subsidy game is not a subsidization dilemma, over- or undersubsidization occurs because there is no mutual benefit in cooperating. If the subsidy game constitutes a dilemma, then there exists a mutually beneficial cooperation opportunity, such that there is more potential to rule out over- or undersubsidization. Especially oversubsidization will play a large role in our analysis, occurring in most cases of welfare-maximizing subsidies in Sections 5 and 6, but also undersubsidization can occur under domestic subsidies in the case of unequal consumer distribution, as will be highlighted in Section 6.

The Boeing-Airbus subsidy dispute illustrates a real-world scenario in which two countries (i.e., the US and the EU) were both oversubsidizing domestic aircraft manufacturers. They found themselves trapped: not increasing subsidies beyond the socially optimal level in response to the other country's actions would harm the international competitiveness of the domestic manufacturer and reduce domestic welfare. As a result, a subsidy war emerged that pushed the US and the EU further away from maximizing international welfare through international cooperation, ultimately harming their domestic welfare as well.

In Sections 5.2 and 6.2, we show that for sufficiently equal consumer bases, joint implementation of the cosmopolitan subsidy results in a strong Pareto improvement, such that cooperative behavior can increase the welfare of both countries.

5 Equal Consumer Distribution

This section considers the case in which both countries have equal consumer bases. After deriving that countries set the same domestic subsidy, we show in Section 5.1 that domestic-welfare-maximizing countries oversubsidize if and only if there is some positive shadow cost of public funds and that the level of oversubsidization increases with a larger shadow cost of public funds. Furthermore, in Section 5.2 we show that the resulting subsidy game is a subsidization dilemma.

The following proposition states that the domestic subsidies of the countries coincide under equal consumer bases.

Proposition 5.1. *Let (c, p, λ, θ) be an international duopoly with $\theta_1 = \theta_2 = 1/2$. Then, $\widehat{s}_1 = \widehat{s}_2$.*

The economic interpretation behind Proposition 5.1 is as follows. Countries are identical when their consumer populations are equal, meaning that their objective functions are identical, which leads to identical subsidy levels.

5.1 Shadow Cost of Public Funds Encourage Oversubsidization

Under equal consumer bases, the domestic first-order condition (3.6) of country $i \in \{1, 2\}$ simplifies to[7]

$$(p(Q) - c'(q_i) - \lambda s_i)\frac{\partial q_i}{\partial s_i} = \lambda q_i \tag{5.1}$$

and the cosmopolitan first-order condition simplifies to

$$(p(Q) - c'(q_i) - \lambda s_i)\frac{\partial Q}{\partial s_i} = \lambda q_i. \tag{5.2}$$

The key difference between (5.1) and (5.2) is that the domestic planner accounts only for the effect of its subsidy on domestic output, whereas the cosmopolitan planner accounts for the effect on total output—including the resulting decline in foreign output. This difference creates a wedge between domestic and cosmopolitan subsidies whenever the shadow cost of public funds is positive, as Theorem 5.1 shows below.

Theorem 5.1. *Let (c, p, λ, θ) be an international duopoly with $\theta_1 = \theta_2 = 1/2$.*

(i) If $\lambda = 0$, then $\hat{s}_1 = s_1^*$ and $\hat{s}_2 = s_2^*$.

(ii) If $\lambda > 0$, then $\hat{s}_1 > s_1^*$ and $\hat{s}_2 > s_2^*$.

Theorem 5.1 shows that if there exists a positive shadow cost of public funds, governments that maximize domestic welfare oversubsidize. This oversubsidization follows from intergovernmental competition to obtain a larger domestic surplus. The incentive is as follows: if a government increases subsidization towards its domestic firm, the firm will produce more and consequently obtains a relatively larger market share, which increases domestic welfare. From a domestic perspective, the welfare gain from increased subsidization is relatively large, as only the resulting increase in domestic output is taken into account. In contrast, from a cosmopolitan perspective, the focus lies on the change in total output. Since an increase in one country's subsidy reduces output of the foreign firm, the global gain is smaller.

Conversely, in the absence of a positive shadow cost of public funds, domestic-welfare-maximizing governments have no incentive to subsidize beyond the cosmopolitan socially optimal level. Under the cosmopolitan subsidy level, the market price in (5.2) is equal to the marginal production costs if $\lambda = 0$. If, for instance, government 1 were to subsidize more, the market price would fall below marginal costs. In that case, gaining a larger market share becomes a disadvantage rather than an advantage, as producing additional units at a negative profit margin reduces overall profits. Hence, when there is no shadow cost of public funds and when consumer bases are equal, governments have no incentive to oversubsidize. Notably, in Section 6.1, we show that under unequal consumer bases, the country with the larger consumer base may be willing to accept a negative profit margin, such that oversubsidization may occur. This occurs because an additional price-channel effect, which is offset in the symmetric setting, generates a net benefit for the largest country through its consumers.

[7] See Appendix A.2.

The following proposition shows that both the domestic and the cosmopolitan subsidies decrease monotonically with the shadow cost of public funds and that these subsidies diverge over the shadow cost.

Proposition 5.2. *Let (c, p, λ, θ) be an international duopoly with $\theta_1 = \theta_2 = \frac{1}{2}$. Then, as λ increases, the domestic subsidies $(\widehat{s}_1, \widehat{s}_2)$ decrease monotonically and converge pointwise to $(\widehat{s}_{\min}, \widehat{s}_{\min})$, while the cosmopolitan subsidies (s_1^*, s_2^*) decrease monotonically and converge pointwise to $(s_{\min}^*, s_{\min}^*)$, where $s_{\min}^* < \widehat{s}_{\min} < 0$. In particular, both subsidies become taxes.*

Proposition 5.2 first establishes that both domestic and cosmopolitan subsidies decrease monotonically with the shadow cost. This follows from the fact that a higher shadow cost implies a stronger penalty on subsidization, and thus a greater incentive for taxation. The subsidies converge point-wise to $\widehat{s}_{min}$ and s_{min}^* respectively.

The subsidy $\widehat{s}_{\min}$ is the level at which a marginal increase in the per-unit tax reduces production by exactly enough to leave domestic tax revenue unchanged. That is, the output loss precisely offsets the higher tax rate. Similarly, $s_{\min}^*$ is the tax level at which total revenue from both firms is unchanged under a marginal increase in the per-unit tax. These bounds represent the maximum tax each government can impose while still keeping both firms in the market. The domestic bound is higher than the cosmopolitan bound. This is because in the cosmopolitan case, the foreign government imposes a higher tax on its own firm, weakening the foreign competitor. This weaker competitive pressure allows the domestic firm to remain active even under a heavier tax burden.

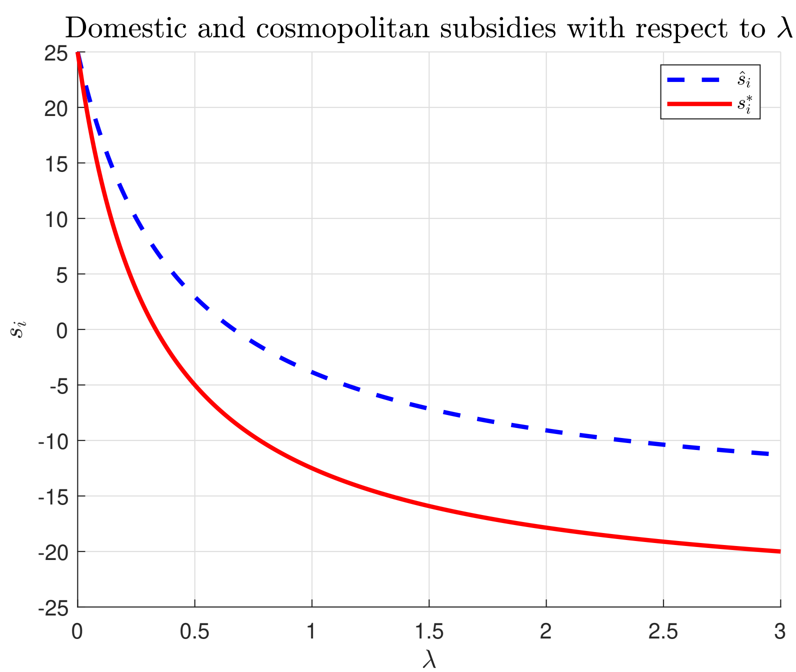

Figure 5.1: Cosmopolitan socially optimal and domestic-welfare-maximizing subsidies. Model specification: $p(Q) = 100 - Q$ and $c(q_i) = 50q_i$, for $i \in \{1, 2\}$.

Figure 5.1 plots the cosmopolitan and the domestic-welfare-maximizing subsidies for varying values of λ. It illustrates that the shadow cost of public funds not merely introduces a penalty on subsidies, but also intensifies the incentives of domestic-welfare-maximizing governments to oversubsidize. Consistent with Theorem 5.1, for $\lambda = 0$ the subsidies are identical. It is noteworthy that at $\lambda = 0$, the cosmopolitan subsidy corresponds to the first-best social optimum: governments can costlessly redistribute surplus from consumers to producers to stimulate first-best production levels. When $\lambda > 0$, however, redistributing surplus carries a cost, making the first-best outcome unattainable. As the shadow cost of public funds rises, it reduces the cosmopolitan subsidy and pushes the second-best solution further from the first-best. Via this mechanism, higher shadow cost drive the cosmopolitan market price above the marginal cost of production and subsidization. As a result, a wedge emerges between the market price and the marginal cost. This provides domestic-welfare-maximizing governments with some leeway to compete for a larger market share by increasing their subsidies while still having a market price that is higher than marginal production and subsidization costs. To maximize domestic welfare, governments exploit part of this available gap, which widens with the shadow cost, to outcompete the other. Consequently, domestic and cosmopolitan subsidies diverge, and oversubsidization increases as the shadow cost of public funds becomes large.

5.2 Equal Consumer Distribution leads to a Subsidization Dilemma

As shown in the previous section, from a domestic point of view countries have incentives to oversubsidize their domestic firm when there is a positive shadow cost of public funds. Since countries are completely symmetric, this unleashes a subsidy war in which they mutually drive up subsidies. As a result, the equilibrium moves away from cosmopolitan social optimality. International cooperation can prevent such a suboptimal equilibrium. For this to be appealing for both countries, the subsidy game must be a subsidization dilemma, in which a Pareto improvement exists over the Nash equilibrium that results from unilateral, domestically motivated subsidies. The following proposition shows that indeed, under equal consumer bases and a positive shadow cost of public funds, a subsidization dilemma arises.

Theorem 5.2. *Let (c, p, λ, θ) be an international duopoly with $\theta_1 = \theta_2 = 1/2$. If $\lambda > 0$, then the subsidy game is a subsidization dilemma.*

Theorem 5.2 implies that in a symmetric international duopoly, in the case of oversubsidization, welfare is lost in both countries. Table 5.1 illustrates this by an example. The unique Nash equilibrium prescribes that both countries opt for a domestic-welfare-maximizing subsidy. However, a joint adoption of cosmopolitan subsidies leads to a Pareto improvement. Given that both countries would benefit from selecting the internationally socially optimal subsidy, there seems from a political point of view to be a relatively fruitful opportunity for international cooperation to effectively address the issue of oversubsidization.

The strategic interaction between the United States and the European Union in the large civil aircraft industry can be interpreted as such a subsidy game where $\theta_1 = \theta_2$. Within the framework of this article, the 1992 bilateral agreement and the 2021 settlement can be interpreted as cooperative arrangements aimed at limiting subsidies to a cosmopolitan

	s_2^*	$\widehat{s}_2$
s_1^*	$(554, 554)$	$(531, 574)$
$\widehat{s}_1$	$(574, 531)$	$(542, 542)$

Table 5.1: Bimatrix subsidy game in symmetric duopoly for $p = 100 - Q$, $c = 50q_i$ (for $i \in \{1, 2\}$), $\theta_1 = \theta_2 = \frac{1}{2}$ and $\lambda = 0.3$.

welfare optimum. By contrast, the 17-year Boeing-Airbus WTO from 2004 to 2021 represents a prolonged period in which both sides reverted to domestically welfare-maximizing subsidy policies, ultimately reducing welfare on both sides of the Atlantic. As the Boeing-Airbus case demonstrates, a single deviation from the cooperative equilibrium can trigger a sustained shift toward non-cooperative behavior. Once mutual trust erodes, both players may remain locked in a subsidy race for an extended period. It is therefore crucial for policymakers to recognize that this interaction may resemble a repeated game. A short-term political gain from deviation can generate multiple subsequent stages of reduced welfare before cooperation is restored.

6 Unequal Consumer Distribution

In this section, we discuss the international duopoly with an unequal division of consumers between the countries. Without loss of generality, we assume that $\theta_1 > \theta_2$. Section 6.1 shows that, under unequal consumer bases, domestic-welfare-maximizing objectives lead to divergent subsidy levels, resulting in a welfare loss. Section 6.2 shows that the corresponding subsidy game only constitutes a subsidization dilemma if consumer bases are sufficiently equal.

6.1 Unequal Consumer Bases Lead to Different Domestic Subsidies

First, we consider the behavior of a country from the domestic point of view. The following theorem establishes that country 1 sets a higher domestic subsidy than country 2, and that domestic subsidies are socially suboptimal regardless of whether a shadow cost of public funds is present.

Theorem 6.1. *Let (c, p, λ, θ) be an international duopoly. Then,*

$$\frac{d\widehat{s}_1}{d\theta_1} > 0 \quad \textit{and} \quad \frac{d\widehat{s}_2}{d\theta_1} < 0,$$

which implies that the equilibrium pair of domestic subsidies $(\widehat{s}_1, \widehat{s}_2)$ satisfies $\widehat{s}_1 > \widehat{s}_2$. This domestic subsidy equilibrium is socially suboptimal for all $\lambda \geq 0$.

First, Theorem 6.1 states that in the domestic-welfare-maximizing setting, as a result of its larger consumer surplus gains from subsidizing, country 1 sets a higher domestic-welfare-maximizing subsidy than country 2. Due to the relatively high subsidy of country 1, country

2 is trapped. Subsidizing its domestic firm results in comparatively little benefit due to its smaller share of consumers. Conversely, imposing too low a subsidy or even a tax on the domestic firm could push it out of the market. Therefore, determining the optimal subsidy entails a trade-off: it must be sufficient to support competitive market participation of its domestic firm, yet not excessive given the limited gains in consumer surplus resulting from the smaller consumer base.

Second, Theorem 6.1 shows that domestic subsidies are socially suboptimal under any distribution of consumers across the two countries, regardless of whether a shadow cost of public funds is present. This follows directly from the disparity in welfare-maximizing subsidy levels across countries: by Proposition 3.1, the cosmopolitan optimum requires equal subsidies.

To interpret the relationship between domestic subsidies $\widehat{s}$ and cosmopolitan subsidies s^*, it is important to understand their connection to the symmetric case discussed in Section 5. As stated in Theorem 5.1, the domestic subsidies $\widehat{s}$ are equal to the cosmopolitan subsidies s^* if $\lambda = 0$ or are elevated above the cosmopolitan level if $\lambda > 0$. Under unequal consumer bases, the domestic-welfare-maximizing subsidy of country 1 is larger than the domestic-welfare-maximizing subsidy in the symmetric case. Therefore, when following a domestic-welfare-maximizing strategy, country 1 is oversubsidizing and the level of oversubsidization becomes larger with more consumer asymmetry, which even holds for $\lambda = 0$. In contrast, country 2 sets its domestic-welfare-maximizing subsidy lower than in the symmetric case. If $\lambda = 0$, this means that the country is under-subsidizing when following a domestic-welfare-maximizing strategy. For $\lambda > 0$, the situation becomes more complex. On the one hand, a smaller consumer base compared to the symmetric case negatively affects the subsidy. However, note that in the symmetric case, the domestic subsidy is set above the cosmopolitan (socially optimal) level, as shown in Theorem 5.1. This implies that there is a range within which the subsidy can decrease before it falls below the cosmopolitan level. Therefore, whether or not the smallest country is over- or under-subsidizing when following a domestic-welfare-maximizing strategy depends largely on the division of consumers. The greater the asymmetry in consumer bases, the stronger the incentive for country 2 to under-subsidize in the domestic-welfare-maximizing case.

It is important to note that the average level of subsidies is not a reliable indicator of their optimality. Also the variation in subsidy levels across countries matters. Differences in subsidy size distort market competition: the greater the disparity, the larger the competitive advantage for the firm receiving the higher subsidy. As competition weakens, achieving the same output level requires relatively higher subsidies. The resulting inefficiency can be measured by the difference in total welfare between two cases: domestic-welfare-maximizing subsidies under equal consumer bases, and domestic-welfare-maximizing subsidies under unequal consumer bases. This welfare gap can be interpreted as the loss arising from reduced competition due to asymmetric subsidies. The following example illustrates this effect.

Example 6.2. Consider an international duopoly with an inverse demand function $p(Q) = 100 - Q$, cost function $c(q_i) = 50q_i$, with $i \in \{1, 2\}$, and $\lambda = 0.3$. We compare welfare under equal and unequal consumer bases. Let us first consider equal fractions of consumers (i.e., $\theta_1 = \theta_2 = \frac{1}{2}$). Using conditions (3.10) and (3.11), we derive the cosmopolitan socially optimal subsidies:

$$s_1^* = s_2^* = 1.32.$$

Using condition (3.1), the total output quantity under cosmopolitan subsidies is $Q = q_1(s_1^*, s_2^*) + q_2(s_1^*, s_2^*) = 34$, giving a total welfare level

$$W(s_1^*, s_2^*) = 1109.$$

Using condition (3.6), the domestic-welfare-maximizing subsidy of both countries is

$$\widehat{s}_1 = \widehat{s}_2 = 8.21.$$

By condition (3.1), the total output quantity under domestic-welfare-maximizing subsidies is $Q = q_1(\widehat{s}_1, \widehat{s}_2) + q_2(\widehat{s}_1, \widehat{s}_2) = 38$. Consequently, the associated total welfare level is

$$W(\widehat{s}_1, \widehat{s}_2) = 1084.$$

Now assume that $\theta_1 = 0.75$ and $\theta_2 = 0.25$. Using once again condition (3.6), we derive the domestic-welfare-maximizing subsidies under unequal consumer bases, denoted by $(\check{s}_1, \check{s}_2)$ in this example to separate from equal consumer bases, to be $\check{s}_1 = 12.09$ and $\check{s}_2 = 4.33$. From condition (3.1) it follows that $q_1(\check{s}_1, \check{s}_2) = 23$ and $q_2(\check{s}_1, \check{s}_2) = 15$. The total welfare in this case is

$$W(\check{s}_1, \check{s}_2) = 1074.$$

This clearly indicates the additional welfare loss from unequal consumer bases. First, we observe the difference between the subsidy levels in the cosmopolitan and domestic-welfare-maximizing case under equal consumer bases, leading to a welfare loss (as described in Section 5). Secondly, we see an additional welfare loss caused by disparity in subsidization level as a result of unequal consumer bases. △

The following proposition zooms in more deeply into the subsidy setting of a domestic-welfare-maximizing country under unequal consumer distributions and identifies a price channel through which the government of country 1 is incentivized to set a larger subsidy and country 2 to set a smaller subsidy than in the symmetric case.

Proposition 6.1. *Let (c, p, λ, θ) be an international duopoly. Then, for $i, j \in \{1, 2\}$ with $i \neq j$, the domestic equilibrium $(\widehat{s}_1, \widehat{s}_2)$ follows implicitly from*

$$\left.(p(Q) - c'(q_i) - \lambda s_i)\frac{\partial q_i}{\partial s_i} + \underbrace{((1-\theta_i)q_i - \theta_i q_j)\frac{\partial p(Q)}{\partial s_i} - \lambda q_i}_{\textit{price channel}}\right|_{s_i = \widehat{s}_i} = 0. \tag{6.1}$$

The optimality condition of Proposition 6.1 incorporates an additional term compared to the equal consumer distribution case in (5.1): the price channel. Noting that $\frac{\partial p(Q)}{\partial s_i}$ is negative, (i.e., when a country raises its subsidy, the resulting increase in total output lowers the market price), this term represents a country's gain in domestic consumer surplus (i.e., $-\theta_i Q \frac{\partial p(Q)}{\partial s_i}$) net of its loss in domestic profits (i.e., $q_i \frac{\partial p(Q)}{\partial s_i}$), arising from the change in the market price induced by an increase in the domestic subsidy. Since all consumers are in countries 1 and 2, this term is zero-sum across the two countries. The price reduction is particularly beneficial for country 1, as it has a larger consumer base. By triggering a price reduction through a subsidy increase, it can compel the foreign firm to serve the market, which is primarily composed of domestic consumers, at a lower price. Consequently, the net effect is that the foreign firm effectively pays for a price reduction that benefits domestic consumers.

Let us, for example, consider the extreme case where $\theta_1 = 1$, such that all consumers live in country 1. In this situation, the price channel for country 2 reduces to the loss in domestic profits caused by the price reduction following an increase in domestic subsidies. In contrast, for country 1, any decline in its own firm's profits through the price channel is exactly offset by an equal increase in domestic consumer surplus. In addition, country 1 gains increased consumer utility from the price reduction of the foreign firm. In effect, firm 2 transfers part of its profits to country 1 through lower prices enjoyed by consumers in country 1.

Another interesting result that can be derived from Proposition 6.1 is that, due to the price channel, it is possible that the country with the largest consumer base has a negative profit margin $p - c(q_i)' - \lambda s_i$, under domestic-welfare-maximizing subsidization. For $i \in \{1, 2\}$, using Proposition 6.1, we derive that this is the case if

$$\left. \frac{\theta_i}{(1-\theta_i) - \frac{\lambda}{\partial p(Q)/\partial s_i}} \right|_{s=\widehat{s}} > \frac{q_i(\widehat{s}_1, \widehat{s}_2)}{q_j(\widehat{s}_1, \widehat{s}_2)}.$$

In this case, country 1 is willing to accept a loss in domestic production—the costs of production and subsidization exceed domestic revenue—as this is offset by cheaper imports from country 2.

6.2 Subsidization Dilemma Exists for Sufficiently Equal Consumer Bases

In this section, we consider the influence of an unequal distribution of consumer between the countries on the existence of a subsidization dilemma. To examine the country-specific welfare implications of domestic and cosmopolitan subsidization, which determine whether the subsidy game constitutes a subsidization dilemma (see Definition 4.1), we decompose domestic welfare as an integral over marginal welfare. For this, we first define the lower bound of this respective integral.

Definition 6.3. Let $\bar{s}_2 \in \mathbb{R}$. Then, we define the function

$$s_{\min}(\bar{s}_2) \equiv \inf \{ s_1 \in \mathbb{R} \mid q_1(s_1, \bar{s}_2) \geq 0,\ q_2(s_1, \bar{s}_2) \geq 0 \},$$

The function $s_{\min}$ describes the lowest subsidy country 1 can set, given country 2's subsidy level, such that both firms produce non-negatively. This lower bound is increasing in the foreign subsidy, as a higher foreign subsidy $\bar{s}_2$ intensifies competition for the domestic firm, requiring a relatively higher domestic subsidy (or equivalently, a smaller tax) for it to remain active.[8]

Next, we decompose the welfare of country 1 as the area under the marginal welfare with respect to its domestic subsidy level. This is illustrated graphically in Figure 6.1. Firstly, the

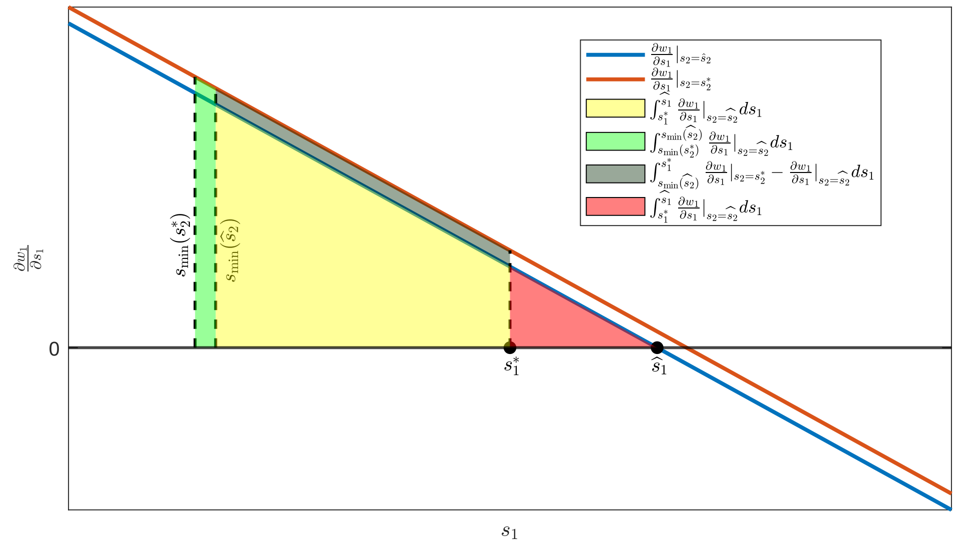

Figure 6.1: Graphical representation of welfare under joint domestic and joint cosmopolitan subsidization. The yellow and red areas together represent the welfare of country 1 under joint domestic subsidization, while the yellow and green areas together represent its welfare under joint cosmopolitan subsidization. See Lemma A.4 for a formal derivation of the corresponding condition.

yellow area represents the welfare of country 1 when it chooses a cosmopolitan subsidy while country 2 chooses the domestic subsidy. Country 1 its subsidy only affects welfare when $s_1 > s_{\min}(\widehat{s}_2)$, as there is no domestic production below this threshold. The total welfare is given by the integral of the marginal values $\frac{\partial w_1}{\partial s_1}\Big|_{s_2=\widehat{s}_2}$ from $s_{\min}(\widehat{s}_2)$ up to the cosmopolitan subsidy s_1^*.

The red area represents the additional welfare country 1 obtains by switching from the cosmopolitan to the domestic subsidy when country 2 also chooses the domestic subsidy. In this sense, it captures the gain from playing the Nash equilibrium given that the other player does the same. Adding it to the yellow area gives the total welfare of country 1 if both countries adopt the domestic subsidy.

Both green areas correspond to the increase in domestic welfare of country 1 when country 2 adopts cosmopolitan subsidies. Changing country 2's subsidy to the cosmopolitan level shifts the minimum subsidy at which country 1's firm produces, altering the segment over which marginal welfare is integrated; this is captured by the light green area. Lastly, when country 2 adopts cosmopolitan subsidies, the marginal welfare of subsidizing increases across all values of s_1 for country 1 (Assumption 3.7). The dark green area represents this additional gain. Adding both green areas to the yellow area yields the total welfare of country 1 if both countries adopt cosmopolitan subsidization. Notably, all colored areas, together with the

[8] See Appendix A.3

white area between the red and blue lines just above the red area, jointly represent the welfare of country 1 if it sets its domestically optimal subsidy while the foreign country sets its cosmopolitan-optimal subsidy.[9]

As a result, determining whether the subsidy game constitutes a subsidization dilemma reduces to comparing the red and green areas. If the red area is larger, country 1 attains higher welfare under joint domestic subsidization, such that joint cosmopolitan subsidization is not a Pareto improvement over the Nash equilibrium of joint domestic subsidies. Consequently, the game would not constitute a subsidization dilemma.

The following corollary derives that there is no subsidization dilemma if the domestic-welfare-maximizing subsidy of country 2 lies below the cosmopolitan subsidy level.

Corollary 6.1. *Let (c, p, λ, θ) be an international duopoly with $\theta_1 > \theta_2$. Then, the subsidy game is not a subsidization dilemma if $s_2^* > \widehat{s}_2$.*

Corollary 6.1 shows that the subsidization dilemma does not arise when country 2 under-subsidizes in the domestic subsidy equilibrium. This is because, in this case, country 2's subsidies are lower in the domestic equilibrium than in the cosmopolitan equilibrium, so that there is no competitive pressure on country 1 via country 2's firm. As a result, country 1 can capture a large market share at a relatively low subsidization cost in the domestic equilibrium, making it prefer this outcome to the cosmopolitan equilibrium, where higher foreign subsidies generate stronger competition for its domestic firm. Although under-subsidization by country 2 is a sufficient condition for the absence of a subsidization dilemma, it is not a necessary condition. Example 6.2 and Table 6.2 present an example where the subsidization dilemma does not occur, while both countries are oversubsidizing.

The following proposition shows under which condition a larger asymmetry in consumer distributions expands the range of scenarios in other model parameters and functions (e.g., λ and c) in which there is no subsidization dilemma.

Proposition 6.2. *Let $\frac{\partial^2 p(Q)}{\partial s_1 \partial s_2} < 0$. Then, the number of scenarios for which the subsidy game is not a subsidization dilemma increases in θ_1.*

Proposition 6.2 states that, ceteris paribus, an increase in θ_1 can transform the subsidy game from a subsidization dilemma into no dilemma. When consumers are unevenly distributed across countries, country 1 benefits more from higher outputs and therefore captures a disproportionate share of the gains from oversubsidization. If the remaining model parameters are sufficiently favorable to country 1 (e.g., cost convexity is not too high), its welfare under domestic subsidization exceeds its welfare under cosmopolitan subsidization. As a result, joint cosmopolitan subsidization is no longer a Pareto improvement over joint domestic subsidization and the subsidy game ceases to be a subsidization dilemma.

The sufficient condition $\frac{\partial^2 p(Q)}{\partial s_1 \partial s_2} < 0$ requires that the price-depressing effect of the domestic subsidy becomes weaker when the foreign subsidy is reduced, which happens if θ_1 increases.

[9]The green areas may also lie within the yellow area, thereby adding negatively to country 1's welfare under joint cosmopolitan subsidization. This occurs if country 2 under-subsidizes, since then $\frac{\partial w_1}{\partial s_1}\big|_{s_2=\widehat{s}_2} > \frac{\partial w_1}{\partial s_1}\big|_{s_2=s_2^*}$, because $s_2^* > \widehat{s}_2$ and, by assumption (3.7), $\frac{\partial^2 w_1}{\partial s_1 \partial s_2} < 0$. Moreover, $s_{\min}(\widehat{s}_2) < s_{\min}(s_2^*)$, since $s_{\min}$ is increasing and $s_2^* > \widehat{s}_2$ (see Lemma A.3).

Since demand is then more inelastic, consumers derive relatively large surplus from consumption, as quantity demanded falls only slightly following a price increase. Raising domestic subsidies allows country 1 to capture this surplus—both directly through consumer surplus, and indirectly through higher producer surplus from the resulting increase in quantity.

We illustrate the results of Proposition 6.2 through a phase diagram in the (λ, θ_1) space that depicts the existence of the subsidization dilemma for the linear model. First, holding λ fixed, greater asymmetry in consumer bases (i.e., a higher θ_1) increases the benefits of oversubsidization for country 1. Thereby, a larger inequality in consumer distribution can shift the subsidy game from the dilemma region to the no-dilemma region, moving from the green to the red area in Figure 6.2. The reverse transition is not possible through an increase in θ_1. Hence, increasing asymmetry in consumer bases reduces the set of configurations under which the subsidization dilemma arises.

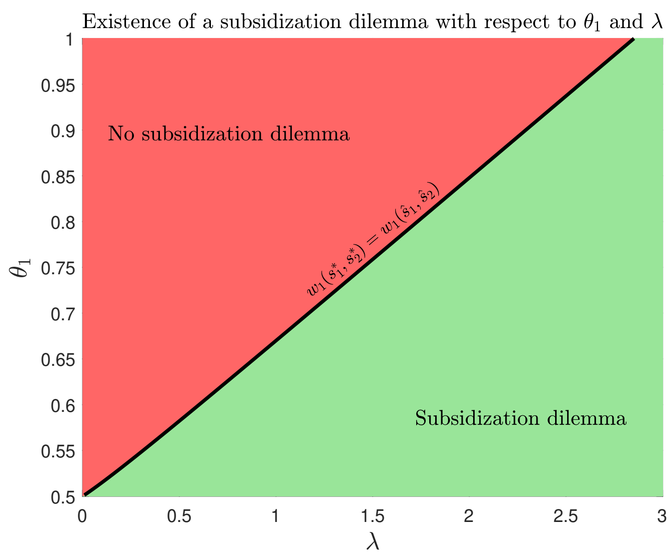

Figure 6.2: Existence of a subsidization dilemma with respect to θ_1 and λ. Model specification: $p(Q) = 100 - Q$ and $c(q_i) = 50q_i$, for $i \in \{1, 2\}$.

Second, holding θ_1 fixed, a larger shadow cost of public funds can push the subsidy game toward the dilemma region, that is, from the red to the green area. This occurs because as λ increases, the subsidization costs increase relative to the consumer surplus gains for country 1, so that the incentive to tax the domestic firm gains the upper hand over the incentive to capture a larger market share. This aligns with Proposition 5.2, where both subsidies, $\widehat{s}_1$ and $\widehat{s}_2$, converge to the same limit, $\widehat{s}_{min}$, as λ increases. Here, the gains from a larger consumer base become negligible as $\lambda \to \infty$, so that the symmetric subsidization strategy from Section 5—where a subsidization dilemma always exists—is effectively mimicked under an unequal consumer distribution. This explains why, as illustrated in Figure 6.2, a larger shadow cost of public funds expands the set of configurations for which the subsidization dilemma arises.

To illustrate more concretely how consumer distributions shape the subsidy game, Tables 6.1 and 6.2 present two bimatrix games that differ only in the distribution of consumers across countries, holding all other parameters fixed. Table 6.1 represents a subsidy game in which consumer bases are sufficiently equal, such that joint cosmopolitan subsidization is a Pareto improvement over joint domestic subsidization (i.e., the green area is larger than the red area in Figure 6.1) and a subsidization dilemma arises. This game falls in the green area of Figure 6.2. In contrast, Table 6.2 shows a subsidy game in which the distribution of consumers is relatively uneven. Unlike the setting in Section 5 and the game in Table 6.1, the asymmetry in consumer bases induces country 1 to gain more welfare from domestic-welfare-maximizing subsidization than from joint cosmopolitan subsidization (i.e., the green area is smaller than the red area in Figure 6.1). As a result, joint adoption of the cosmopolitan subsidy does not constitute a Pareto improvement and the subsidy game is not a subsidization dilemma. This game falls in the red area of Figure 6.2.

	s_2^*	$\widehat{s}_2$
s_1^*	$(583, 525)$	$(548, 550)$
$\widehat{s}_1$	$(591, 511)$	$(571, 522)$

Table 6.1: Subsidy game where $\theta_1 = 0.53$, $\theta_2 = 0.47$, $p(Q) = 100 - Q$, $c(q_i) = 50q_i$, with $i \in \{1, 2\}$, and $\lambda = 0.3$.

	s_2^*	$\widehat{s}_2$
s_1^*	$(699, 410)$	$(693, 413)$
$\widehat{s}_1$	$(742, 343)$	$(740, 345)$

Table 6.2: Subsidy game where $\theta_1 = 0.75$, $\theta_2 = 0.25$, $p(Q) = 100 - Q$, $c(q_i) = 50q_i$, with $i \in \{1, 2\}$, and $\lambda = 0.3$.

An important implication is that, from a political perspective, insufficiently symmetric consumer bases make it more challenging to secure cooperation between both countries in reaching international agreements in which both provide subsidies that maximize total welfare internationally. From this perspective, the emergence of equally sized trading partners (e.g., the creation of the EU) might be beneficial not only for these emerging players, through greater bargaining power, but also for society as a whole, as it may trigger the existence of a subsidization dilemma, and thus the existence of a mutually beneficial cooperation opportunity.

7 Concluding remarks

This article examines the underlying causes of oversubsidization in an international setting. We consider two types of regulatory authorities: a domestic-welfare-maximizing government and a cosmopolitan social planner, both of which use subsidies to incentivize domestic firms to produce in accordance with the authority's objectives. The domestic-welfare-maximizing governments represent regulatory authorities that are pursuing national interests over the common good, which aligns with the trend of neomercantilist policies recently. The cosmopolitan social planner is rather hypothetical in today's world, resembling a world government or, with some imagination, a WTO endowed with the power to prevent distortionary protective measures by national governments. It is dedicated to maximize global welfare.

We show that, under equal consumer bases, there is no oversubsidization in the absence of a shadow cost of public funds, and that oversubsidization rises with the shadow cost. The subsidy game played by the governments constitutes a subsidization dilemma with the Nash equilibrium occurring when each government sets a domestic-welfare-maximizing subsidy. However, if both governments set the cosmopolitan socially optimal subsidy, this represents a Pareto improvement over the Nash equilibrium, providing both countries with an incentive to cooperate and reduce subsidies to the socially optimal level.

When consumer bases are unequal, governments that focus on maximizing domestic welfare set unequal subsidies, whereas the cosmopolitan socially optimal outcome requires equal subsidy rates. This disparity in subsidies reduces competition between firms, generating an additional welfare loss on top of the welfare loss from oversubsidization in the symmetric case. If consumer bases are sufficiently asymmetric, the nature of the subsidy game changes: joint domestic-welfare-maximizing subsidization remains the Nash equilibrium, but it also becomes Pareto efficient. As a result, the subsidy game is no longer a subsidization dilemma, which from a political perspective makes it less likely that governments will reach agreements to end distortionary subsidies as this is not mutually beneficial.

Regarding future research, a natural extension of our work would be to explore asymmetries between the countries in terms of the shadow cost of public funds. Such an extension could be particularly insightful for understanding the strategic interactions related to setting subsidies and tariffs between developed and developing countries, as the latter generally have significantly higher values for the shadow cost of public funds (Laffont, 2005). Future research could also focus on developing an investment game within a dynamic framework with demand uncertainty. Another extension to make the model more comprehensive in terms of firm behavior could involve incorporating the possibility for firms to relocate to another country if subsidy levels there are more attractive. Lastly, further research could use this model concept to consider a tariff war instead of a subsidy war. This focus is particularly relevant given the current geopolitical climate.

References

Bagwell, K., & Staiger, R. W. (2011). What do trade negotiators negotiate about? empirical evidence from the world trade organization. *American Economic Review*, *101*(4), 1238–1273.

Beck, U. (2006). *Cosmopolitan Vision.* Polity.

Böhm, T., Hohmann, A., Raabe, R., & Riedel, N. (2025). *Corporate taxes and export competition* (No. 38/25). WIDER Working Paper.

Brander, J. A., & Spencer, B. J. (1985, 2). Export subsidies and international market share rivalry. *Journal of International Economics*, *18*(1-2), 83–100.

Eaton, J., & Grossman, G. M. (1986, 5). Optimal Trade and Industrial Policy under Oligopoly. *The Quarterly Journal of Economics*, *101*(2), 383.

European Commission. (2021, 6). *EU and US take decisive step to end aircraft dispute.* Retrieved from `https://ec.europa.eu/commission/presscorner/detail/en/IP_21_3001`

Krugman, P. (1987, 11). Is free trade passé? *The Journal of Economic Perspectives*, *1*(2), 131–144.

Krugman, P. (1994). *Peddling Prosperity.* National Geographic Books.

Laffont, J. J. (2005). *Regulation and development.* Cambridge University Press.

Laffont, J. J., & Pouyet, J. (2004, 1). The subsidiarity bias in regulation. *Journal of Public Economics*, *88*(1-2), 255–283.

Laffont, J. J., & Tirole, J. (1986). Using cost observation to regulate firms. *Journal of Political Economy*, *94*(3), 614–641.

Leahy, D., & Montagna, C. (2001, 7). Strategic Trade Policy with Heterogeneous Costs. *Bulletin of Economic Research*, *53*(3), 177–182.

Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, *71*(6), 1695–1725.

Minard, P. (2014). *La fortune du colbertisme: état et industrie dans la france des lumières.* Fayard.

Neary, J. (1994, 11). Cost asymmetries in international subsidy games: Should governments help winners or losers? *Journal of International Economics*, *37*(3-4), 197–218.

Smith, A. (1776, 3). An inquiry into the nature and causes of the wealth of nations. In (chap. III (part II)).

Tirole, J. (1988). *The theory of industrial organization.* MIT press.

WTO. (2020a, 9). *DS316: European communities and certain member States - measures affecting trade in large civil aircraft.* Retrieved from `https://www.wto.org/english/tratop_e/dispu_e/cases_e/ds316_e.htm`

WTO. (2020b, 9). *DS353: United States - Measures affecting trade in large civil aircraft - Second complaint.* Retrieved from `https://www.wto.org/english/tratop_e/dispu_e/cases_e/ds353_e.htm`

Xiang, S., Yin, X., & Zi, Y. (2026, January). *Two decades of chinese industrial subsidies.* Retrieved from `https://cepr.org/voxeu/columns/two-decades-chinese-industrial-subsidies`

Yomogida, M. (2008, 1). Competition, technology, and trade in oligopolistic industries. *International Review of Economics and Finance*, *17*(1), 127–137.

Zussman, A. (2002). The rise of german protectionism in the 1870s: A macroeconomic perspective. *Stanford Institute for Economic Policy Research, Discussion Paper.(Letöltve: 2017. szeptember 2., http://siepr. stanford. edu/sites/default/files/publications/01-19_0. pdf).*

A Proofs

Lemma A.1

Lemma A.1. *Let $s_1, s_2 \in \mathbb{R}$. Then, the production equilibria $q(s_1, s_2) = (q_1(s_1, s_2), q_2(s_1, s_2))$ and $q(s_2, s_1) = (q_1(s_2, s_1), q_2(s_2, s_1))$ exhibit the following symmetry property:*

$$q_1(s_1, s_2) = q_2(s_2, s_1) \quad \text{and} \quad q_2(s_1, s_2) = q_1(s_2, s_1).$$

Proof. Without loss of generality, we prove that $q_1(s_1, s_2) = q_2(s_2, s_1)$.

For each firm $i \in \{1, 2\}$, let $b_i(\tilde{q}_j, s_i) \equiv \arg\max_{\tilde{q}_i} \pi_i(\tilde{q}_i, \tilde{q}_j, s_i)$ denote the best-response function for a fixed $\tilde{q}_j \in \mathbb{R}_+$ and s_1. For each $\tilde{q}_j$ and s_i, the best-response function $b_i(\tilde{q}_j, s_i)$ is defined implicitely by

$$p(b_i + \tilde{q}_j) + b_i p'(b_i + \tilde{q}_j) + s_1 = c'(b_i).$$

The best-response function is well-defined because the profit function of each firm is strictly concave by (3.2), implying that there exist a unique maximizer.

Because the firms face identical demand and cost functions, the best-response functions are identical:

$$b_1(q, s) = b_2(q, s) \equiv b(q, s), \quad \forall q \in \mathbb{R}_+,\ s \in \mathbb{R}.$$

Fix an arbitrary subsidy pair $(s_1, s_2) \in \mathbb{R} \times \mathbb{R}$. The production equilibrium is characterized by the following fixed point problem:

$$\begin{aligned} q_1(s_1, s_2) &= b(q_2(s_1, s_2), s_1) \\ &= b(b(q_1(s_1, s_2), s_2), s_1), \end{aligned} \tag{A.1}$$

and for the swapped subsidies (s_2, s_1), the production equilibrium is characterized by

$$\begin{aligned} q_2(s_2, s_1) &= b(q_1(s_2, s_1), s_1) \\ &= b(b(q_2(s_2, s_1), s_2), s_1). \end{aligned} \tag{A.2}$$

Consider the metric space $(\mathbb{R}, d)$, where $d(x, y) = |x - y|$. Since $\mathbb{R}$ is complete under the absolute-value metric, this is a complete metric space. Then, we define the composed operator

$$T : \mathbb{R} \to \mathbb{R}, \qquad T(x) \equiv b(b(x, s_2), s_1).$$

Then (A.1) and (A.2) can be rewritten as

$$\begin{aligned} q_1(s_1, s_2) &= T(q_1(s_1, s_2)), \\ q_2(s_2, s_1) &= T(q_2(s_2, s_1)), \end{aligned}$$

so both are fixed points of the same operator T on $(\mathbb{R}, |\cdot|)$.

By the implicit function theorem,

$$b_i'(q_j, s_i) = -\frac{\frac{\partial^2 \pi_i}{\partial \tilde{q}_i \partial \tilde{q}_j}}{\frac{\partial^2 \pi_i}{\partial \tilde{q}_i^2}}.$$

From (3.2), both the numerator and the denominator are negative, with the denominator larger in magnitude. Hence $-1 < b_i'(q_j, s_i) < 0$.

The derivative of the composed map is

$$T'_{s_1,s_2}(x) = b'(b(x, s_2), s_1)\, b'(x, s_2).$$

Each factor has a magnitude strictly less than 1, so

$$|T'(x)| < 1 \quad \text{for all } x \in \mathbb{R}.$$

Let $c \equiv sup_{x\in\mathbb{R}}|T'(x)| < 1$. Then, by the mean value theorem, for all $x, y \in \mathbb{R}$, it holds that

$$|T(x) - T(y)| \leq c|x - y|.$$

Thus, T is a contraction mapping on the complete metric space $(\mathbb{R}, d)$. By the Banach Fixed Point Theorem, this implies that T has a unique fixed point. Since both $q_1(s_1, s_2)$ and $q_2(s_2, s_1)$ are fixed points of T, we must have that $q_1(s_1, s_2) = q_2(s_2, s_1)$.

□

Proof of Proposition 3.1

Proof. Let (s_1^*, s_2^*) be a solution to the cosmopolitan problem. We first show that this solution is unique. Next, we show that the cosmopolitan welfare function (3.8) is symmetric in s_1 and s_2. Finally, we use the former and the latter to prove by contradiction that $s_1^* = s_2^*$.

Let $H_W(s_1, s_2)$ be the Hessian matrix of $W(s_1, s_2)$. Then,

$$\det\left[H_W(s_1, s_2)\right] = \frac{\partial^2 W(s_1, s_2)}{\partial s_1^2}\,\frac{\partial^2 W(s_1, s_2)}{\partial s_2^2} - \left(\frac{\partial^2 W(s_1, s_2)}{\partial s_1 \partial s_2}\right)^2 > 0,$$

where the inequality follows from (3.12). Since $\frac{\partial^2 W(s_1,s_2)}{\partial s_1^2} < 0$ holds by condition (3.12), it follows that the Hessian is negative definite. Therefore, the function $W(s_1, s_2)$ is strictly concave and has a unique maximizer $s^* = (s_1^*, s_2^*)$.

The cosmopolitan welfare function can be written as

$$\begin{aligned} W(s_1, s_2) = &\int_0^{q_1(s_1,s_2)+q_2(s_1,s_2)} p(\tilde{Q})\, d\tilde{Q} - \big(c(q_1(s_1, s_2)) + \lambda s_1 q_1(s_1, s_2)\big) \\ &- \big(c(q_2(s_1, s_2)) + \lambda s_1 q_2(s_1, s_2)\big) \\ = &\int_0^{q_2(s_2,s_1)+q_1(s_2,s_1)} p(\tilde{Q})\, d\tilde{Q} - \big(c(q_2(s_2, s_1)) + \lambda s_2 q_2(s_2, s_1)\big) \end{aligned}$$

$$
\begin{aligned}
&- \big(c(q_1(s_2, s_1)) + \lambda s_1 q_1(s_2, s_1)\big) \\
&= W(s_2, s_1),
\end{aligned}
$$

where both equalities follow from $q_1(s_1, s_2) = q_2(s_2, s_1)$ (see Lemma A.1). Hence, it follows that

$$
W(s_1, s_2) = W(s_2, s_1) \quad \forall s_1, s_2 \in \mathbb{R}. \tag{A.3}
$$

Assume that $s_1^* \neq s_2^*$. By symmetry of the cosmopolitan welfare function (A.3), it holds that

$$
W(s_1^*, s_2^*) = W(s_2^*, s_1^*). \tag{A.4}
$$

Furthermore, uniqueness of the optimal solution solution $s^* = (s_1^*, s_2^*)$ implies that, for all $s_1, s_2 \in \mathbb{R}$ with $s_1 \neq s_1^*$ and $s_2 \neq s_2^*$,

$$
W(s_1^*, s_2^*) > W(s_1, s_2).
$$

In particular, because $s_1^* \neq s_2^*$, it follows that

$$
W(s_1^*, s_2^*) > W(s_2^*, s_1^*),
$$

which contradicts (A.4). $\square$

Proof of Proposition 5.1

Proof. Let $(\widehat{s}_1, \widehat{s}_2)$ be a domestic subsidy equilibrium stemming from the domestic problems (3.5). We will first show that this equilibrium is unique. Then we show that the domestic welfare function (3.4) is symmetric in s_1 and s_2. Lastly, we use the former and the latter to prove by contradiction that $\widehat{s}_1 = \widehat{s}_2$.

First, as shown by Tirole (1988), Chapter 5.7.1.2, condition (3.7) guarantees a unique domestic subsidy equilibrium for the game arising from the domestic problems in (3.5).

Next, for $\theta_1 = \theta_2 = 1/2$, the domestic welfare functions (3.8) can be written as

$$
\begin{aligned}
w_1(s_1, s_2) =& \frac{1}{2}\left[\int_0^{Q(s_1,s_2)} p(\widetilde{Q})d\widetilde{Q} - (Q(s_1, s_2))p(Q(s_1, s_2))\right] + p(Q(s_1, s_2))q_1(s_1, s_2) \\
&- c(q_1(s_1, s_2)) - \lambda s_1 q_1(s_1, s_2) \\
=& \frac{1}{2}\left[\int_0^{Q(s_2,s_1)} p(\widetilde{Q})d\widetilde{Q} - (Q(s_2, s_1))p(Q(s_2, s_1))\right] + p(Q(s_2, s_1))q_2(s_2, s_1) \\
&- c(q_2(s_2, s_1)) - \lambda s_2 q_2(s_2, s_1) \\
=& w_2(s_2, s_1),
\end{aligned}
$$

where we use $q_1(s_1, s_2) = q_2(s_2, s_1)$ from Lemma (A.1), which also implies $Q(s_1, s_2) = q_1(s_1, s_2) + q_2(s_1, s_2) = q_2(s_2, s_1) + q_1(s_2, s_1) = Q(s_2, s_1)$. As a result, the domestic welfare functions are symmetric:

$$
w_1(s_1, s_2) = w_2(s_2, s_1) \quad \forall s_1, s_2 \in \mathbb{R}. \tag{A.5}
$$

Lastly, we prove by contradiction that $\widehat{s}_1 = \widehat{s}_2$. Assume that $\widehat{s}_1 \neq \widehat{s}_2$. If $(\widehat{s}_1, \widehat{s}_2)$ is an domestic equilibrium, then for all $s_1, s_2 \in \mathbb{R}$ with $s_1 \neq \widehat{s}_1$ and $s_2 \neq \widehat{s}_2$, it holds that

$$w_1(\widehat{s}_1, \widehat{s}_2) \geq w_1(s_1, \widehat{s}_2) \quad \text{and} \quad w_2(\widehat{s}_1, \widehat{s}_2) \geq w_2(\widehat{s}_1, s_2).$$

Using (A.5), it must then also hold that

$$w_2(\widehat{s}_2, \widehat{s}_1) \geq w_2(\widehat{s}_2, s_1) \quad \text{and} \quad w_1(\widehat{s}_2, \widehat{s}_1) \geq w_1(s_2, \widehat{s}_1),$$

which implies that $(\widehat{s}_2, \widehat{s}_1)$ is also an equilibrium. This contradicts the uniqueness of the equilibrium. □

Lemma A.2

Lemma A.2. (c, p, λ, θ) *be an international duopoly with* $\theta_1 = \theta_2 = \frac{1}{2}$. *Then, the domestic equilibrium* $(s_1, s_2) = (\widehat{s}_1, \widehat{s}_2)$ *solves the system*

$$\begin{cases} (p(Q) - c'(q_1) - \lambda s_1)\frac{\partial q_1}{\partial s_1} - \lambda q_1 = 0 \\ (p(Q) - c'(q_2) - \lambda s_2)\frac{\partial q_2}{\partial s_2} - \lambda q_2 = 0, \end{cases} \tag{A.6}$$

and the cosmopolitan solution $(s_1, s_2) = (s_1^*, s_2^*)$ *solves the system*

$$\begin{cases} (p(Q) - c'(q_1) - \lambda s_1)\frac{\partial Q}{\partial s_1} - \lambda q_1 = 0 \\ (p(Q) - c'(q_2) - \lambda s_2)\frac{\partial Q}{\partial s_2} - \lambda q_2 = 0. \end{cases} \tag{A.7}$$

Proof. Recall the first-order condition of the domestic problem (3.6). Without loss of generality, we consider country 1 to rewrite the first-order conditions. The first-order condition can be rewritten to

$$\begin{aligned}
\frac{\partial w_1}{\partial s_1} &= \frac{1}{2}\frac{\partial U}{\partial s_1} + \frac{\partial p(Q)q_1}{\partial s_1} - \frac{\partial c(q_1)}{\partial s_1} - \lambda\left(q_1 + s_1\frac{\partial q_1}{\partial s_1}\right) \\
&= \frac{1}{2}\left(\frac{\partial G}{\partial s_1} - \frac{\partial p(Q)q_1}{\partial s_1} - \frac{\partial p(Q)q_2}{\partial s_1}\right) + \frac{\partial p(Q)q_1}{\partial s_1} - \frac{\partial c(q_1)}{\partial s_1} - \lambda\left(q_1 + s_1\frac{\partial q_1}{\partial s_1}\right) \\
&= \frac{1}{2}\frac{\partial G}{\partial s_1} + \frac{1}{2}\frac{\partial p(Q)q_1}{\partial s_1} - \frac{1}{2}\frac{\partial p(Q)q_2}{\partial s_1} - \frac{\partial c(q_1)}{\partial s_1} - \lambda\left(q_1 + s_1\frac{\partial q_1}{\partial s_1}\right) \\
&= \frac{1}{2}\frac{\partial G}{\partial Q}\frac{\partial Q}{\partial s_1} + \frac{1}{2}\frac{\partial p(Q)q_1}{\partial s_1} - \frac{1}{2}\frac{\partial p(Q)q_2}{\partial s_1} - c'(q_1)\frac{\partial q_1}{\partial s_1} - \lambda\left(q_1 + s_1\frac{\partial q_1}{\partial s_1}\right) \\
&= \frac{1}{2}p(Q)\left(\frac{\partial q_1}{\partial s_1} + \frac{\partial q_2}{\partial s_1}\right) + \frac{1}{2}q_1\frac{\partial p(Q)}{\partial s_1} + \frac{1}{2}p(Q)\frac{\partial q_1}{\partial s_1} - \frac{1}{2}q_2\frac{\partial p(Q)}{\partial s_1} - \frac{1}{2}p(Q)\frac{\partial q_2}{\partial s_1} \\
&\quad - c'(q_1)\frac{\partial q_1}{\partial s_1} - \lambda\left(q_1 + s_1\frac{\partial q_1}{\partial s_1}\right) \\
&= p(Q)\frac{\partial q_1}{\partial s_1} + \frac{1}{2}(q_1 - q_2)\frac{\partial p(Q)}{\partial s_1} - c'(q_1)\frac{\partial q_1}{\partial s_1} - \lambda\left(q_1 + s_1\frac{\partial q_1}{\partial s_1}\right).
\end{aligned}$$

From Lemma (A.1) it follows that $q_1(s_1, s_2) = q_2(s_2, s_1)$. Since Proposition 5.1 shows that $\widehat{s}_1 = \widehat{s}_2$ when $\theta_1 = \theta_2$, it follows that $q_1 = q_2$. Consequently, in equilibrium, it holds that

$$\begin{aligned}\left.\frac{\partial w_1}{\partial s_1}\right|_{s_1=\widehat{s}_1} &= p(Q)\frac{\partial q_1}{\partial s_1} - c'(q_1)\frac{\partial q_1}{\partial s_1} - \lambda\left.\left(q_1 + s_1\frac{\partial q_1}{\partial s_1}\right)\right|_{s=\widehat{s}} \\ &= (p(Q) - c'(q_1) - \lambda s_1)\frac{\partial q_1}{\partial s_1} - \lambda q_1\Big|_{s=\widehat{s}}.\end{aligned}$$

Let us now consider the first-order condition of the cosmopolitan social planner with respect to s_1:

$$\begin{aligned}\frac{\partial W}{\partial s_1} &= \frac{\partial G}{\partial s_1} - \frac{\partial c(q_1)}{\partial s_1} - \frac{\partial c(q_2)}{\partial s_1} - \lambda\left(q_1 + s_1\frac{\partial q_1}{\partial s_1}\right) - \lambda s_2\frac{\partial q_2}{\partial s_1} \\ &= p(Q)\frac{\partial q_1}{\partial s_1} - \frac{\partial c(q_1)}{\partial s_1} + p(Q)\frac{\partial q_2}{\partial s_1} - \frac{\partial c(q_2)}{\partial s_1} - \lambda\left(q_1 + s_1\frac{\partial q_1}{\partial s_1}\right) - \lambda s_2\frac{\partial q_2}{\partial s_1} \\ &= p(Q)\frac{\partial q_1}{\partial s_1} - c'(q_1)\frac{\partial q_1}{\partial s_1} + p(Q)\frac{\partial q_2}{\partial s_1} - c'(q_1)\frac{\partial q_2}{\partial s_1} - \lambda\left(q_1 + s_1\frac{\partial q_1}{\partial s_1}\right) - \lambda s_2\frac{\partial q_2}{\partial s_1}.\end{aligned}$$

From Proposition 3.1 it follows that $q_1 = q_2$ holds in the cosmopolitan solution, which implies that $c'(q_1) = c'(q_2)$. This yields

$$\begin{aligned}\left.\frac{\partial W}{\partial s_1}\right|_{s=s^*} &= p(Q)\left(\frac{\partial q_1}{\partial s_1} + \frac{\partial q_2}{\partial s_1}\right) - c'(q_1)\left(\frac{\partial q_1}{\partial s_1} + \frac{\partial q_2}{\partial s_1}\right) - \lambda s_1\left(\frac{\partial q_1}{\partial s_1} + \frac{\partial q_2}{\partial s_1}\right) - \lambda q_1\Big|_{s=s^*} \\ &= (p(Q) - c'(q_1) - \lambda s_1)\frac{\partial Q}{\partial s_1} - \lambda q_1\Big|_{s=s^*}.\end{aligned}$$

□

Proof of Theorem 5.1

Proof. (i) If $\lambda = 0$, the system in Lemma A.2 that solves the domestic and cosmopolitan subsidy levels can be written as

$$\begin{cases} p(Q) - c'(q_1)|_{s=\widehat{s}} = 0 \\ p(Q) - c'(q_2)|_{s=\widehat{s}} = 0. \end{cases} \quad \text{and} \quad \begin{cases} p(Q) - c'(q_1)|_{s=s^*} = 0 \\ p(Q) - c'(q_2)|_{s=s^*} = 0, \end{cases} \tag{A.8}$$

where we use that $\frac{\partial q_1}{\partial s_1}, \frac{\partial q_2}{\partial s_2}, \frac{\partial Q}{\partial s_1}, \frac{\partial Q}{\partial s_2} > 0$ as stated in (3.3). Since both the domestic and cosmopolitan subsidies are unique, it follows that $(\widehat{s}_1, \widehat{s}_2) = (s_1^*, s_2^*)$

(ii) Assume that $\lambda > 0$. Then, according to Lemma A.2, the domestic and cosmopolitan subsidy levels can be implicitly derived from

$$\begin{cases} (p(Q) - c'(q_1) - \lambda s_1)\dfrac{\partial q_1}{\partial s_1} - \lambda q_1\big|_{s=\widehat{s}} = 0 \\ (p(Q) - c'(q_2) - \lambda s_2)\dfrac{\partial q_2}{\partial s_2} - \lambda q_2\big|_{s=\widehat{s}} = 0 \end{cases} \quad \text{and} \quad \begin{cases} (p(Q) - c'(q_1) - \lambda s_1)\dfrac{\partial Q}{\partial s_1} - \lambda q_1\big|_{s=s^*} = 0 \\ (p(Q) - c'(q_2) - \lambda s_2)\dfrac{\partial Q}{\partial s_2} - \lambda q_2\big|_{s=s^*} = 0. \end{cases}$$

From condition (3.3) it follows that $\partial q_1/\partial s_1 > \partial Q/\partial s_1 > 0$. This implies that $\partial W/\partial s_1|_{s=\widehat{s}} < 0$ and $\partial W/\partial s_2|_{s=\widehat{s}} < 0$. Proposition 3.1 states that W has a unique maximum with respect to s_1 and s_2. Consequently, it follows that $\widehat{s}_1 > s_1^*$ and $\widehat{s}_2 > s_2^*$. □

Proof of Proposition 5.2

Proof. Without loss of generality, let $i = 1$.
We first show pointwise convergence of $\widehat{s}_1$ and s_1^* towards $\widehat{s}_{min}$ and s_{min}^*, respectively. Then, we derive that the subsidies decrease monotonically towards these points.

We rewrite equilibrium condition (A.6) from the domestic subsidy to

$$\left(\frac{p(Q) - c'(q_1)}{\lambda} - s_1\right)\frac{\partial q_1}{\partial s_1} - q_1\bigg|_{s=\widehat{s}_1} = 0.$$

Let $\widehat{s}_{min} = \lim_{\lambda\to\infty}\widehat{s}_1$ Then, $\widehat{s}_{min}$ follows implicitly from

$$-\,q_1 - s_1\frac{\partial q_1}{\partial s_1}\bigg|_{s=\widehat{s}_{min}} = 0.$$

As q_1 is positive for a firm that participates in the market and $\frac{\partial q_1}{\partial s_1} > 0$ by (3.3), it must hold that $\widehat{s}_{min} < 0$. Similarly, we rewrite the equilibrium condition (A.7) from the domestic subsidy to

$$\left(\frac{p(Q) - c'(q_1)}{\lambda} - s_1\right)\frac{\partial Q}{\partial s_1} - q_1\bigg|_{s=s_1^*} = 0.$$

Let $s_{min}^* = \lim_{\lambda\to\infty} s_1^*$. Then, s_{min}^* follows implicitly from

$$-\,q_i - s_i\frac{\partial Q}{\partial s_i}\bigg|_{s=s_{min}^*} = 0.$$

Recall from Theorem 5.1 that the left-hand side of this expression is the first-order condition of $\lim_{\lambda\to\infty} W$. Let us evaluate this first-order condition at $\widehat{s}_{min}$:

$$\begin{aligned}-\,q_1 - s_1\frac{\partial Q}{\partial s_1}\bigg|_{s=\widehat{s}_{min}} &= -\,q_1 - s_1\frac{\partial q_1}{\partial s_1} - s_1\frac{\partial q_2}{\partial s_1}\bigg|_{s=\widehat{s}_{min}}\\ &= -\,s_1\frac{\partial q_2}{\partial s_1}\bigg|_{s=\widehat{s}_{min}}\\ &< 0.\end{aligned}$$

By Proposition 3.1, W has a unique maximum with respect to s_1. Therefore, because $\lim_{\lambda\to\infty}\partial W/\partial s_1|_{s_1=\widehat{s}_1} < 0$, this maximum must lie at a smaller subsidy level which implies that $s_{min}^* < \widehat{s}_{min}$.

To show that the converge towards these asymptotes is monotonic, we first take the total derivative of condition 3.6, evaluated at $s = \widehat{s}$, with respect to λ:

$$\frac{\partial w_1}{\partial s_1}\bigg|_{s=\widehat{s}}\frac{d}{d\lambda} = -q_1 - s_1\frac{\partial q_1}{\partial s_1}\bigg|_{s=\widehat{s}} + \frac{\partial^2 w_1}{\partial s_1^2}\bigg|_{s=\widehat{s}}\frac{d\widehat{s}_1}{d\lambda} + \frac{\partial^2 w_1}{\partial s_1\partial s_2}\bigg|_{s=\widehat{s}}\frac{d\widehat{s}_2}{d\lambda} = 0.$$

Proposition 5.1 states that for $\theta_1 = \theta_2$, it holds that $\widehat{s}_1 = \widehat{s}_2$. Symmetry in first-order condition (3.6) for country 1 and 2 implies we can write

$$\frac{d\widehat{s}_1}{d\lambda} = \frac{q_1 + s_1\frac{\partial q_1}{\partial s_1}}{\partial^2 w_1/\partial s_1^2 + \partial^2 w_1/\partial s_1\partial s_2}\bigg|_{s=\widehat{s}}. \tag{A.9}$$

Using conditions (A.6) and (3.1), the numerator can be written as

$$\begin{aligned} q_1 + s_1 \frac{\partial q_1}{\partial s_1}\bigg|_{s=\widehat{s}} &= \frac{p(Q) - c'(q_1)}{\lambda} \frac{\partial q_1}{\partial s_1}\bigg|_{s=\widehat{s}} \\ &= \frac{-q_1 p'(Q) - s_1}{\lambda} \frac{\partial q_1}{\partial s_1}\bigg|_{s=\widehat{s}} \end{aligned}$$

when $\lambda > 0$. If $\widehat{s}_1 \geq 0$, then, by (3.3), the left-hand side of this equation must be greater than zero. If $\widehat{s}_1 < 0$, then again by (3.3) and because prices decrease over quantity the lower right-hand side of the equation must be greater than zero, which, via the equality implies that the left-hand side is positive. Thus, the nominator of (A.9) is positive when $\lambda > 0$. Additionally, condition (3.7) implies that the denominator in (A.9) is negative. Thus, it follows that

$$\frac{d\widehat{s}_1}{d\lambda} < 0,$$

for $\lambda \in (0, \infty)$. Equation (A.8) in Theorem 5.1 states that the optimal domestic subsidy for $\lambda = 0$, denoted by s_{max}, solves $p(Q) - c'(q_i)|_{s_1 = s_{max}} = 0$. Let us now evaluate the equilibrium condition for a positive shadow cost (A.6) at s_{max}:

$$\frac{\partial w_1}{\partial s_1}\bigg|_{s_1 = s_{max}} = -\lambda s_1 \frac{\partial q_1}{\partial s_1} - \lambda q_1 < 0.$$

Since Proposition 5.1 shows the uniqueness of a solution, this implies that solution must satisfy $\widehat{s}_1 < s_{max}$ if $\lambda > 0$. With this, we can conclude that $\widehat{s}_1$ decreases monotonically over λ to $\widehat{s}_{min}$.

Similarly, taking the total derivative of condition 3.10, evaluated at $s = s^*$, with respect to λ and gives

$$\frac{\partial W}{\partial s_1}\bigg|_{s=s^*} \frac{d}{d\lambda} = -q_1 - s_1 \frac{\partial q_1}{\partial s_1}\bigg|_{s=s^*} - s_2 \frac{\partial q_2}{\partial s_1}\bigg|_{s=s^*} + \frac{\partial^2 W}{\partial s_1^2}\bigg|_{s=s^*} \frac{ds_1^*}{d\lambda} + \frac{\partial^2 W}{\partial s_1 \partial s_2}\bigg|_{s=s^*} \frac{ds_2^*}{d\lambda} = 0.$$

Proposition 3.1 states that $s_1^* = s_2^*$. Symmetry in first-order conditions (3.10) and (3.11) implies we can write

$$\frac{ds_1^*}{d\lambda} = \frac{q_1 + s_1 \frac{\partial Q}{\partial s_1}}{\partial^2 W / \partial s_1^2 + \partial^2 W / \partial s_1 \partial s_2}\bigg|_{s=s^*}. \tag{A.10}$$

Using conditions (A.7) and (3.1), the numerator can be written as

$$\begin{aligned} q_1 + s_1 \frac{\partial Q}{\partial s_1}\bigg|_{s=s^*} &= \frac{p(Q) - c'(q_1)}{\lambda} \frac{\partial Q}{\partial s_1}\bigg|_{s=s^*} \\ &= \frac{-q_1 p'(Q) - s_1}{\lambda} \frac{\partial Q}{\partial s_1}\bigg|_{s=s^*}. \end{aligned}$$

If $s_1^* \geq 0$, then, by (3.3), the left-hand side of this equation must be greater than zero. If $s_1^* < 0$, then again by (3.3) and because prices decrease over quantity the lower right-hand side of the equation must be greater than zero, which, via the equality implies that the left-hand side is positive. Thus, the nominator of (A.10) is positive for any feasible cosmopolitan subsidy. Additionally, condition (3.12) implies that the denominator in (A.10) is negative. Thus, it follows that

$$\frac{ds_1^*}{d\lambda} < 0,$$

for $\lambda \in (0, \infty)$. Equation (A.8) in Theorem 5.1 states that the optimal cosmopolitan subsidy for $\lambda = 0$, denoted by s_{max}, solves $p(Q) - c'(q_i)|_{s1=s_{max}} = 0$. Let us now evaluate the equilibrium condition for a positive shadow cost (A.7) at s_{max}:

$$\left.\frac{\partial W}{\partial s_1}\right|_{s_1=s_{max}} = -\lambda s_1 \frac{\partial Q}{\partial s_1} - \lambda q_1 < 0.$$

Since Proposition 3.1 shows the uniqueness of a solution, this implies that solution must satisfy $s_1^* < s_{max}$ if $\lambda > 0$. With this, we can conclude that s_1^* decreases monotonically over λ to s_{min}^*. □

Proof of Theorem 5.2

Proof. In order to prove that the governments are involved in a subsidization dilemma, we check the properties of a subsidization dilemma from definition 4.1. Without loss of generality, let $i = 1$ and $j = 2$.

Firstly, consider the case in which $\lambda = 0$. From Theorem 5.1 it follows that

$$\widehat{s}_1 = \widehat{s}_2 = s_1^* = s_2^*.$$

Conditions (3.7) and (3.12) imply these subsidy levels are the unique solutions to the domestic and cosmopolitan objectives of both governments. Consequently, all welfare levels in the two-by-two bimatrix game are the same such that there is no subsidization dilemma.

Next, let us consider the case in which $\lambda > 0$. Then, Theorem 5.1 states that

$$\widehat{s}_1 = \widehat{s}_2 > s_1^* = s_2^*.$$

We now verify that the three properties of a subsidization dilemma, as stated in Definition 4.1, are satisfied.

(i) From the first-order condition in the Nash equilibrium (3.6), we have

$$\frac{\partial w_1(\widehat{s}_1, \widehat{s}_2)}{\partial s_1} = 0.$$

By Theorem 5.1, $s_2^* < \widehat{s}_2$. Given the assumption $\partial^2 w_1/\partial s_1 \partial s_2 < 0$ (3.7), the marginal return to s_1 increases as s_2 decreases. This gives

$$\frac{\partial w_1(\widehat{s}_1, s_2^*)}{\partial s_1} > 0.$$

Also from Theorem 5.1, $s_1^* < \widehat{s}_1$. Since w_1 is concave in s_1 (3.7), it holds that

$$\frac{\partial w_1(s_1^*, s_2^*)}{\partial s_1} > \frac{\partial w_1(\widehat{s}_1, s_2^*)}{\partial s_1} > 0.$$

With the concavity, this implies that $w_1(s_1, s_2^*)$ is strictly increasing on the interval $s_1 \in [s_1^*, \widehat{s}_1]$. Therefore:[10]

$$w_1(\widehat{s}_1, s_2^*) > w_1(s_1^*, s_2^*).$$

(ii) Proposition 5.1 states that $(s_1, s_2) = (\widehat{s}_1, \widehat{s}_2)$ is a unique equilibrium of domestic-welfare-maximizing subsidies. As $\widehat{s}_1 > s_1^*$, it follows that

$$w_1(\widehat{s}_1, \widehat{s}_2) > w_1(s_1^*, \widehat{s}_2)$$

(iii) Proposition 3.1 states that (s_1^*, s_2^*) is the unique total welfare maximizer. This implies that

$$W(s_1^*, s_2^*) > W(\widehat{s}_1, \widehat{s}_2) \tag{A.11}$$

Proposition 3.1 and 5.1 also state that $s_1^* = s_2^*$ and $\widehat{s}_1 = \widehat{s}_2$, where the latter requires $\theta_1 = \theta_2$. In (A.5) we have shown that this implies that the domestic welfare functions 3.4 are symmetric. It follows that

$$w_1(s_1^*, s_2^*) = w_2(s_1^*, s_2^*) \quad \text{and} \quad w_1(\widehat{s}_1, \widehat{s}_2) = w_2(\widehat{s}_1, \widehat{s}_2).$$

Plugging this in (A.11), where we use that $W(s_1, s_2) = w_1(s_1, s_2) + w_2(s_1, s_2)$, gives

$$w_1(s_1^*, s_2^*) > w_1(\widehat{s}_1, \widehat{s}_2).$$

□

Proof of Theorem 6.1

Proof. Taking the total derivative of condition 3.6, evaluated at $s = \widehat{s}$, with respect to θ_1 gives

$$\left.\frac{\partial w_1}{\partial s_1}\right|_{s=\widehat{s}} \frac{d}{d\theta_1} = \left.\frac{\partial U}{\partial s_1}\right|_{s=\widehat{s}} + \left.\frac{\partial^2 w_1}{\partial s_1^2}\right|_{s=\widehat{s}} \frac{d\widehat{s}_1}{d\theta_1} + \left.\frac{\partial^2 w_1}{\partial s_1 s_2}\right|_{s=\widehat{s}} \frac{d\widehat{s}_2}{d\theta_1} = 0,$$

which can be rewritten as:

$$\frac{d\widehat{s}_1}{d\theta_1} = \left.\frac{-\partial U/\partial s_1 - (\partial^2 w_1/\partial s_1 \partial s_2)(d\widehat{s}_2/d\theta_1)}{\partial^2 w_1/\partial s_1^2}\right|_{s=\widehat{s}}.$$

[10]In fact, since $\partial w_1(\widehat{s}_1, s_2^*)/\partial s_1 > 0$, country 1's best response to s_2^* exceeds $\widehat{s}_1$. But the inequality $w_1(\widehat{s}_1, s_2^*) > w_1(s_1^*, s_2^*)$ is already sufficient for part (i) of Definition 4.1.

Similarly, using $\theta_1 = 1 - \theta_2$, we find for government 2 that

$$\frac{\partial w_2}{\partial s_2}\bigg|_{s=\widehat{s}} \frac{d}{d\theta_1} = -\frac{\partial U}{\partial s_2}\bigg|_{s=\widehat{s}} + \frac{\partial^2 w_2}{\partial s_2^2}\bigg|_{s=\widehat{s}} \frac{d\widehat{s}_2}{d\theta_1} + \frac{\partial^2 w_2}{\partial s_1 s_2}\bigg|_{s=\widehat{s}} \frac{d\widehat{s}_1}{d\theta_1} = 0,$$

which can be rewritten to

$$\frac{d\widehat{s}_2}{d\theta_1} = \frac{\partial U/\partial s_2 - (\partial^2 w_2/\partial s_1 \partial s_2)(d\widehat{s}_1/d\theta_1)}{\partial^2 w_2/\partial s_2^2}\bigg|_{s=\widehat{s}} = 0. \tag{A.12}$$

Substituting $d\widehat{s}_2/d\theta_1$ in the expression for $d\widehat{s}_1/d\theta_1$ yields

$$\begin{aligned}\frac{d\widehat{s}_1}{d\theta_1} &= -\frac{\partial U/\partial s_1}{\partial^2 w_1/\partial s_1^2} - \frac{(\partial^2 w_1/\partial s_1 \partial s_2)}{\partial^2 w_1/\partial s_1^2} \frac{\partial U/\partial s_2}{\partial^2 w_2/\partial s_2^2} + \frac{(\partial^2 w_1/\partial s_1 \partial s_2)}{\partial^2 w_1/\partial s_1^2} \frac{\partial^2 w_2/\partial s_1 \partial s_2}{\partial^2 w_2/\partial s_2^2} \frac{d\widehat{s}_1}{d\theta_1}\bigg|_{s=\widehat{s}} \\ &= \left(-\frac{\partial U/\partial s_1}{\partial^2 w_1/\partial s_1^2} - \frac{(\partial^2 w_1/\partial s_1 \partial s_2)}{\partial^2 w_1/\partial s_1^2} \frac{\partial U/\partial s_2)}{\partial^2 w_2/\partial s_2^2}\right) \Big/ \left(1 - \frac{(\partial^2 w_1/\partial s_1 \partial s_2)}{\partial^2 w_1/\partial s_1^2} \frac{\partial^2 w_2/\partial s_1 \partial s_2}{\partial^2 w_2/\partial s_2^2}\right)\bigg|_{s=\widehat{s}},\end{aligned} \tag{A.13}$$

where

$$\begin{aligned}\frac{\partial U}{\partial s_1} =& \frac{\partial G}{\partial s_1} - \frac{\partial Q p(Q)}{\partial s_1} \\ =& p(Q)\frac{\partial q_1}{\partial s_1} + p(Q)\frac{\partial q_2}{\partial s_1} - p(Q)\frac{\partial q_1}{\partial s_1} - q_1\frac{\partial p(Q)}{\partial s_1} - p(Q)\frac{\partial q_2}{\partial s_1} - q_2\frac{\partial p(Q)}{\partial s_1} \\ =& -q_1 p'(Q)\frac{\partial Q}{\partial s_1} - q_2 p'(Q)\frac{\partial Q}{\partial s_1} \\ =& -Qp'(Q)\frac{\partial Q}{\partial s_1}.\end{aligned}$$

As the market price decreases with quantity, and quantity increases with the subsidy (as shown in (3.3)), it follows that $\partial U/\partial s_1 > 0$. Then, by condition(3.7), it follows that

$$\frac{d\widehat{s}_1}{d\theta_1} > 0 \qquad \text{and} \qquad \frac{d\widehat{s}_2}{d\theta_1} < 0. \tag{A.14}$$

Recall from Proposition 3.1 that there is a unique cosmopolitan solution with $s_1^* = s_2^*$ for all $\theta_1 \in [0,1]$. Since $d\widehat{s}_1/d\theta_1 > 0$ and $d\widehat{s}_2/d\theta_1 < 0$, it follows that $\widehat{s}_1 \neq \widehat{s}_2$ when $\theta_1 \neq \theta_2$, which cannot be socially optimal. □

Proof of Proposition 6.1

Proof. Without loss of generality, let $i = 1$ and $j = 2$.
(i) Recall the first order condition in the domestic-welfare-maximizing case from equation (3.6). We can rewrite this to

$$\frac{\partial w_1}{\partial s_1} = \theta_1 \frac{\partial U}{\partial s_1} + \frac{\partial p(Q) q_1}{\partial s_1} - \frac{\partial c(q_1)}{\partial s_1} - \lambda(q_1 + s_1 \frac{\partial q_1}{\partial s_1})$$

$$
\begin{aligned}
&= \theta_1(\frac{\partial G}{\partial s_1} - \frac{\partial p(Q)q_1}{\partial s_1} - \frac{\partial p(Q)q_2}{\partial s_1}) + \frac{\partial p(Q)q_1}{\partial s_1} - \frac{\partial c(q_1)}{\partial s_1} - \lambda(q_1 + s_1\frac{\partial q_1}{\partial s_1}) \\
&= \theta_1\frac{\partial G}{\partial s_1} + (1-\theta_1)\frac{\partial p(Q)q_1}{\partial s_1} - \theta_1\frac{\partial p(Q)q_2}{\partial s_1} - \frac{\partial c(q_1)}{\partial s_1} - \lambda(q_1 + s_1\frac{\partial q_1}{\partial s_1}) \\
&= \theta_1\frac{\partial G}{\partial Q}\ \frac{\partial Q}{\partial s_1} + (1-\theta_1)\frac{\partial p(Q)q_1}{\partial s_1} - \theta_1\frac{\partial p(Q)q_2}{\partial s_1} - \frac{\partial c(q_1)}{\partial s_1} - \lambda(q_1 + s_1\frac{\partial q_1}{\partial s_1}) \\
&= \theta_1 p(Q)(\frac{\partial q_1}{\partial s_1} + \frac{\partial q_2}{\partial s_1}) + (1-\theta_1)q_1\frac{\partial p(Q)}{\partial s_1} + (1-\theta_1)p(Q)\frac{\partial q_1}{\partial s_1} - \theta_1 q_2\frac{\partial p(Q)}{\partial s_1} \\
&\quad - \theta_1 p(Q)\partial q_2/\partial s_1 - \frac{\partial c(q_1)}{\partial s_1} - \lambda(q_1 + s_1\frac{\partial q_1}{\partial s_1}) \\
&= p(Q)\frac{\partial q_1}{\partial s_1} + ((1-\theta_1)q_1 - \theta_1 q_2)\frac{\partial p(Q)}{\partial s_1} - \frac{\partial c(q_1)}{\partial s_1} - \lambda(q_1 + s_1\frac{\partial q_1}{\partial s_1}) \\
&= (p - \partial c/\partial q_1 - \lambda s_1)\frac{\partial q_1}{\partial s_1} + ((1-\theta_1)q_1 - \theta_1 q_2)\frac{\partial p}{\partial s_1} - \lambda q_1.
\end{aligned}
$$

□

Lemma A.3

Lemma A.3. *The function $s_{\min}(\bar{s}_2)$ is increasing in $\bar{s}_2$.*

Proof. Let $\bar{s}_2 \in \mathbb{R}$. We consider two cases.

(i) Suppose

$$0 \le q_1(s_{\min}(\bar{s}_2), \bar{s}_2) \le q_2(s_{\min}(\bar{s}_2), \bar{s}_2).$$

From (3.3), we have

$$\frac{\partial q_1(s_1, s_2)}{\partial s_1} > 0,$$

so $q_1(s_1, s_2)$ is strictly increasing in s_1 for fixed s_2.

By definition, $s_{\min}(\bar{s}_2)$ (6.3) denotes the smallest subsidy level that ensures non-negative production given $\bar{s}_2$. Hence, it satisfies

$$q_1\big(s_{\min}(\bar{s}_2), \bar{s}_2\big) = 0.$$

Now consider an increase in the foreign subsidy to

$$\bar{s}_2 + \epsilon, \quad \epsilon > 0.$$

Because

$$\frac{\partial q_1(s_1, s_2)}{\partial s_2} < 0$$

by (3.3), we have

$$q_1(s_{\min}(\bar{s}_2), \bar{s}_2 + \epsilon) < 0.$$

To restore $q_1 = 0$, it must be that

$$s_{\min}(\bar{s}_2 + \epsilon) > s_{\min}(\bar{s}_2),$$

showing that $s_{\min}$ is increasing.

(ii) Suppose

$$q_1(s_{\min}(\bar{s}_2), \bar{s}_2) > q_2(s_{\min}(\bar{s}_2), \bar{s}_2) \geq 0.$$

However, since

$$\frac{\partial q_1(s_1, s_2)}{\partial s_1} > 0, \quad \frac{\partial q_2(s_1, s_2)}{\partial s_1} < 0$$

by (3.3), there would exist a subsidy $s_1 < s_{\min}(\bar{s}_2)$ such that $q_1(s_1, \bar{s}_2) \geq 0$ and $q_2(s_1, \bar{s}_2) \geq 0$, contradicting the definition

$$s_{\min}(\bar{s}_2) \equiv \inf \{s_1 \in \mathbb{R} \mid q_i(s_1, \bar{s}_2) \geq 0, \text{ for } i = 1, 2\}.$$

Hence this case is not possible. □

Lemma A.4

Lemma A.4. *Let (c, p, λ, θ) be an international duopoly with $\theta_1 > \theta_2$. Then, the subsidy game is a subsidization dilemma if*

$$\int_{s_1^*}^{\widehat{s}_1} \frac{\partial w_1}{\partial s_1}\bigg|_{s_2=\widehat{s}_2} ds_1 - \int_{s_{min}(s_2^*)}^{s_{min}(\widehat{s}_2)} \frac{\partial w_1}{\partial s_1}\bigg|_{s_2=s_2^*} ds_1 - \left(\int_{s_{min}(\widehat{s}_2)}^{s_1^*} \frac{\partial w_1}{\partial s_1}\bigg|_{s_2=s_2^*} - \frac{\partial w_1}{\partial s_1}\bigg|_{s_2=\widehat{s}_2} ds_1 \right) < 0.$$

and

$$\int_{s_2^*}^{\widehat{s}_2} \frac{\partial w_2}{\partial s_2}\bigg|_{s_1=\widehat{s}_1} ds_2 - \int_{s_{min}(s_1^*)}^{s_{min}(\widehat{s}_1)} \frac{\partial w_2}{\partial s_2}\bigg|_{s_1=s_1^*} ds_2 - \left(\int_{s_{min}(\widehat{s}_1)}^{s_2^*} \frac{\partial w_2}{\partial s_2}\bigg|_{s_1=s_1^*} - \frac{\partial w_2}{\partial s_2}\bigg|_{s_1=\widehat{s}_1} ds_2 \right) < 0.$$

Proof. To verify whether the subsidy game is a subsidization dilemma, we need to check whether the conditions for a subsidization dilemma in Definition 4.1 are satisfied. The proof of Theorem 5.2 shows that condition (ii) of Definition 4.1 is satisfied for any $\theta_1 \in [0, 1]$. Additionally, it shows that condition (i) of Definition 4.1 holds if $\widehat{s}_2 \geq s_2^*$. Because Corollary 6.1 states that condition (iii) of Definition 4.1 is never satisfied if $\widehat{s}_2 < s_2^*$, when checking whether the subsidy game constitutes a subsidization dilemma under asymmetric consumer bases, it is sufficient to examine the third condition of Definition 4.1. That is, it is sufficient to check whether joint cosmopolitan subsidization is a Pareto improvement over joint domestic subsidization.

Let us without loss of generality consider country 1. We can write the welfare of country 1 under domestic subsidies as

$$w_1(\widehat{s}_1, \widehat{s}_2) = \int_{s_{min}(\widehat{s}_2)}^{\widehat{s}_1} \frac{\partial w_1}{\partial s_1}\bigg|_{s_2=\widehat{s}_2} ds_1$$

$$= \int_{s_{min}(\widehat{s}_2)}^{s_1^*} \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=\widehat{s}_2} ds_1 + \int_{s_1^*}^{\widehat{s}_1} \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=\widehat{s}_2} ds_1.$$

and the welfare of country 1 under cosmopolitan subsidies as

$$\begin{aligned}
w_1(s_1^*, s_2^*) &= \int_{s_{min}(s_2^*)}^{s_1^*} \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=s_2^*} ds_1 \\
&= \int_{s_{min}(s_2^*)}^{s_{min}(\widehat{s}_2)} \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=s_2^*} ds_1 + \int_{s_{min}(\widehat{s}_2)}^{s_1^*} \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=\widehat{s}_2} ds_1 \\
&\quad + \int_{s_{min}(\widehat{s}_2)}^{s_1^*} \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=s_2^*} - \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=\widehat{s}_2} ds_1,
\end{aligned}$$

such that country 1 prefers joint domestic over joint cosmopolitan subsidization if

$$\begin{aligned}
w_1(\widehat{s}_1, \widehat{s}_2) - w_1(s_1^*, s_2^*) = &\int_{s_1^*}^{\widehat{s}_1} \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=\widehat{s}_2} ds_1 - \int_{s_{min}(s_2^*)}^{s_{min}(\widehat{s}_2)} \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=s_2^*} ds_1 \\
&- \left(\int_{s_{min}(\widehat{s}_2)}^{s_1^*} \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=s_2^*} - \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=\widehat{s}_2} ds_1 \right) > 0.
\end{aligned}$$

□

Proof of Corollary 6.1

Proof. Recall the conditions for a subsidization dilemma given in Lemma A.4. In this proof, we will show that for country 1 the first term is positive and both subtracted terms are negative if country 2 under-subsidizes, such that the condition on country 1 for a subsidization dilemma does not hold.

Let country 2 under-subsidize. That is, $\widehat{s}_2 < s_2^*$. Then,

1. $\int_{s_1^*}^{\widehat{s}_1} \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=\widehat{s}_2} ds_1 > 0$**:** Since $\frac{\partial w_1}{\partial s_1}$ is decreasing in s_1 by (3.7) and $\widehat{s}_1$ is the unique solution to $\left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=\widehat{s}_2} = 0$, it follows that $\left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=\widehat{s}_2} > 0$ for all $s_1 < \widehat{s}_1$. Given $s_1^* < \widehat{s}_1$, the integrand is strictly positive on $(s_1^*, \widehat{s}_1)$, implying that the integral is strictly positive.

2. $\int_{s_{min}(s_2^*)}^{s_{min}(\widehat{s}_2)} \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=\widehat{s}_2} ds_1 < 0$**:** Since s_{min} is decreasing (see Lemma A.3), it follows that $s_{min}(\widehat{s}_2) < s_{min}(s_2^*)$. Moreover, $\left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=\widehat{s}_2} > 0$ for all $s_1 < \widehat{s}_1$, as shown in the part for the first term. Therefore, the integral is strictly negative due to the reversed integration limits.

3. $\int_{s_{min}(\widehat{s}_2)}^{s_1^*} \frac{\partial w_1}{\partial s_1}|_{s_2=s_2^*} - \frac{\partial w_1}{\partial s_1}|_{s_2=\widehat{s}_2} ds_1 < 0$**:** Note that $\frac{\partial w_1}{\partial s_1}|_{s_2=\widehat{s}_2} < \frac{\partial w_1}{\partial s_1}|_{s_2=s_2^*}$ since $\frac{\partial w_1}{\partial s_1}$ is decreasing in s_1 by (3.7). Additionally, as $s_1^* > s_{min}(\widehat{s}_2)$, the integral is strictly negative.

□

Proof of Proposition 6.2

Proof. Recall the condition for a subsidization dilemma of country 1 given in Lemma A.4:

$$\int_{s_1^*}^{\widehat{s}_1} \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=\widehat{s}_2} ds_1 - \left[\int_{s_{min}(s_2^*)}^{s_{min}(\widehat{s}_2)} \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=s_2^*} ds_1 + \left(\int_{s_{min}(\widehat{s}_2)}^{s_1^*} \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=s_2^*} - \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=\widehat{s}_2} ds_1\right)\right] < 0$$

In this proof, we will show that for country 1, the first term increases with θ_1, while the subtracted terms decrease in θ_1, such that the entire left-hand-side increases with θ_1

(1) Recall the first term:

$$\int_{s_1^*}^{\widehat{s}_1} \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=\widehat{s}_2} ds_1.$$

Taking the partial derivative of $\partial w_1/\partial s_1$ (as written in (6.1)) with respect to θ_1 yields

$$\frac{\partial}{\partial \theta_1}\left(\frac{\partial w_1}{\partial s_1}\right) = -Q\frac{\partial p(Q)}{\partial s_1}.$$

As $q_1 > 0$ for $s_1 \in (s_1^*, \widehat{s}_1)$ and $\partial p(Q)/\partial s_1 < 0$, it follows that the curve of $\partial w_1/\partial s_1$ comes to lie strictly higher if θ_1 increases. Additionally, note that $\frac{d\widehat{s}_1}{d\theta_1} > 0$ (A.14) adds a segment with positive values of $\partial w_1/\partial s_1$. Therefore, the entire term (i.e., the red area in Figure 6.1) increases with θ_1.

(2) Recall the subtracted terms

$$\int_{s_{min}(s_2^*)}^{s_{min}(\widehat{s}_2)} \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=s_2^*} ds_1 + \left(\int_{s_{min}(\widehat{s}_2)}^{s_1^*} \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=s_2^*} - \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=\widehat{s}_2} ds_1\right) \equiv M$$

Using the Leibniz rule, we derive

$$\frac{d}{d\theta_1}\int_{s_{min}(s_2^*)}^{s_{min}(\widehat{s}_2)} \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=s_2^*} ds_1 = \frac{\partial w_1}{\partial s_1}\frac{ds_{min}(\widehat{s}_2)}{d\widehat{s}_2}\left.\frac{d\widehat{s}_2}{d\theta_1}\right|_{(s_1,s_2)=(s_{min}(\widehat{s}_2),s_2^*)}$$

and

$$\begin{aligned}\frac{d}{d\theta}\int_{s_{\min}(\widehat{s}_2)}^{s_1^*}\left[\left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=s_2^*} - \left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=\widehat{s}_2}\right]ds_1 = &-\frac{d\widehat{s}_2}{d\theta}\Bigg\{\frac{ds_{\min}(\widehat{s}_2)}{d\widehat{s}_2}\Bigg[\left.\frac{\partial w_1}{\partial s_1}\right|_{(s_1,s_2)=(s_{\min}(\widehat{s}_2),s_2^*)} \\ &- \left.\frac{\partial w_1}{\partial s_1}\right|_{(s_1,s_2)=(s_{\min}(\widehat{s}_2),\widehat{s}_2)}\Bigg] \\ &+ \int_{s_{\min}(\widehat{s}_2)}^{s_1^*}\left.\frac{\partial^2 w_1}{\partial s_1 \partial s_2}\right|_{s_2=\widehat{s}_2} ds_1\Bigg\} \\ &+ \int_{s_{\min}(\widehat{s}_2)}^{s_1^*}\left[\frac{\partial}{\partial\theta_1}\left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=s_2^*} - \frac{\partial}{\partial\theta_1}\left.\frac{\partial w_1}{\partial s_1}\right|_{s_2=\widehat{s}_2}\right]ds_1\end{aligned}$$

Combining and using that $\partial^2 w_1/\partial s_1 \partial\theta_1 = -Q(\partial p/\partial s_1)$ yields

$$\frac{d}{d\theta_1}M = \frac{d\widehat{s}_2}{d\theta_1}\left(\frac{ds_{min}(\widehat{s}_2)}{d\widehat{s}_2}\left.\frac{\partial w_1}{\partial s_1}\right|_{(s_1,s_2)=(s_{\min}(\widehat{s}_2),\widehat{s}_2)} - \int_{s_{\min}(\widehat{s}_2)}^{s_1^*}\left.\frac{\partial^2 w_1}{\partial s_1 \partial s_2}\right|_{s_2=\widehat{s}_2} ds_1\right)$$
$$+\int_{s_{\min}(\widehat{s}_2)}^{s_1^*}\left[-Q\left.\frac{\partial p(Q)}{ds_1}\right|_{s_2=s_2^*} + Q\left.\frac{\partial p(Q)}{ds_1}\right|_{s_2=\widehat{s}_2}\right]ds_1$$

In this expression, $d\widehat{s}_2/d\theta_1$ is negative by (A.14), $ds_{min}(\widehat{s}_2)/d\widehat{s}_2$ is positive by Lemma A.3, $\partial w_1/\partial s_1|_{(s_1,s_2)=(s_{min}(\widehat{s}_2),s_2^*)}$ is positive as $s_{min}(\widehat{s}_2) < \widehat{s}_1$ and $s_2^* < \widehat{s}_2$, $\partial^2 w_1/\partial s_1\partial s_2|_{s_2=\widehat{s}_2}$ is negative by assumption (3.7) and $-Q(\partial p/\partial s_1)$ is increasing in s_2 by the assumption $\partial^2 p(Q)/\partial s_1 \partial s_2 < 0$. As a result, the derivative is negative and the subtracted terms decrease as θ_1 increases. □

Derivation of the Linear Model

Let $p(Q) = a - bQ$ and $c(q_i) = cq_i$ for $i, j \in \{1, 2\}$, where $i \neq j$ and $a, b, c \in \mathbb{R}_+$. Solving the firm FOC (3.1) yields

$$a - 2bq_i - bq_j - c + s_i = 0$$
$$q_i = \frac{(a-c) + 2s_i - s_j}{3b},$$

where

$$\frac{\partial q_i}{\partial s_i} = \frac{2}{3b}, \qquad \frac{\partial q_j}{\partial s_i} = -\frac{1}{3b}, \qquad \frac{\partial p}{\partial s_i} = -\frac{1}{3}.$$

The domestic FOC (6.1) can be written as

$$\frac{2}{3b}\big[bq_i - (1+\lambda)s_i\big] - \frac{(1-\theta_i)q_i - \theta_i q_j}{3} - \lambda q_i = 0$$
$$(1 + \theta_i - 3\lambda)\, bq_i + \theta_i\, bq_j = 2(1+\lambda)s_i,$$

where we use the domestic FOC (3.1), to substitute directly:

$$p - c - \lambda s_i = a - b(q_i + q_j) - c - \lambda s_i = 2bq_i + bq_j - s_i - bq_i - bq_j = bq_i - (1+\lambda)s_i,$$

Substituting $bq_i = \frac{(a-c)+2s_i-s_j}{3}$ and $bq_j = \frac{(a-c)+2s_j-s_i}{3}$ and rearranging:

$$(1 + 2\theta_i - 3\lambda)(a-c) + (\theta_i - 1 + 3\lambda)\, s_j = (4 + 12\lambda - \theta_i)\, s_i$$
$$\widehat{s}_i = \frac{A_i + \gamma_i\, s_j}{\alpha_i}$$

where $\alpha_i \equiv 4 + 12\lambda - \theta_i$, $\gamma_i \equiv \theta_i - 1 + 3\lambda$, $A_i \equiv (1 + 2\theta_i - 3\lambda)(a-c)$. Stacking the two best responses and solving for $(\widehat{s}_1, \widehat{s}_2)$:

$$\alpha_1 \widehat{s}_1 = A_1 + \gamma_1 \widehat{s}_2$$

$$\begin{aligned}
\alpha_2 \widehat{s}_2 &= A_2 + \gamma_2 \widehat{s}_1 \\
(\alpha_1\alpha_2 - \gamma_1\gamma_2)\, \widehat{s}_1 &= \alpha_2 A_1 + \gamma_1 A_2 \\
\widehat{s}_1 &= \frac{\alpha_2 A_1 + \gamma_1 A_2}{\alpha_1\alpha_2 - \gamma_1\gamma_2}, \qquad \widehat{s}_2 = \frac{\alpha_1 A_2 + \gamma_2 A_1}{\alpha_1\alpha_2 - \gamma_1\gamma_2}.
\end{aligned}$$

Plugging in the linear model in the cosmopolitan FOC of derived in Lemma A.2:

$$\begin{aligned}
\left(a - b \cdot \frac{2(a-c) + s_1 + s_2}{3b} - c - \lambda s_1\right) \frac{1}{3b} - \lambda \frac{(a-c) + 2s_1 - s_2}{3b} &= 0 \\
3(a-c) - 2(a-c) - s_1 - s_2 - 3\lambda s_1 - 3\lambda(a-c) - 6\lambda s_1 + 3\lambda s_2 &= 0 \\
(1-3\lambda)(a-c) - (1+9\lambda)s_1 + (3\lambda - 1)s_2 &= 0 \\
(1-3\lambda)(a-c) &= (1+9\lambda)s_1 - (3\lambda-1)s_2.
\end{aligned}$$

Imposing symmetry $s_1^* = s_2^* = s^*$ (Proposition 3.1) yields

$$\begin{aligned}
(1-3\lambda)(a-c) &= [(1+9\lambda) - (3\lambda - 1)]\, s_1^* = (2+6\lambda)\, s_1^* \\
s_1^* &= \frac{(1-3\lambda)(a-c)}{2(1+3\lambda)}.
\end{aligned}$$