TI 2026-058/VII
Tinbergen Institute Discussion Paper

An Allocative Efficiency Rationale for a Universal Basic Income

Robert Dur[1]
Anja Schöttner[2]

1 Erasmus University Rotterdam and Tinbergen Institute

2 Humboldt-Universität zu Berlin, Berlin School of Economics, and CEPR

An Allocative Efficiency Rationale for a Universal Basic Income

Robert Dur* and Anja Schöttner†‡

August 13, 2026

Abstract

This paper shows that allocative efficiency may entail high income tax revenues and a basic income. We consider an economy where people consume three types of goods: market goods, public goods, and social goods. The latter are non-priced goods with positive externalities that are produced by citizens in their leisure time. We show that in the absence of income taxes, work hours are too high and social-goods production is too low as compared to the socially optimal levels. We characterize optimal income taxation and develop a set of testable predictions. One implication of the model is that, as wages grow over time, at some point a basic income becomes part of the optimal policy under economically plausible conditions.

1 Introduction

Traditional economic models of labor supply and income taxation are commonly silent about what people do in their leisure time. Leisure is simply treated as a private good that people consume. In practice, however, many people devote a substantial part of their leisure time to the production of non-priced goods that involve positive externalities for others or for society at large. Think, for instance, about people taking care of their neighbors, their elderly parents, or their children. Think about acts of friendship and love. Or consider voluntary work and community service, as well as many other types of civic engagement. In all these cases, time and energy are devoted to creating something valuable that has no market price and increases other people's well-being. For brevity, we call these goods and services 'social goods'.

In this paper, we enrich an otherwise standard model of labor supply and income taxation to account for the presence of social goods and find that it has important consequences for optimal policy. Indeed, as the title of our paper alludes to, allocative efficiency may entail substantial income tax revenues and a basic income. This contrasts with much of the existing theoretical literature, which primarily evaluates a basic income in terms of redistribution, insurance, and its distortionary effects on labor supply and investment (e.g., Ghatak and Maniquet, 2019; Chen et al., 2021; Daruich and Fernández, 2024; Afa et al., 2026).

We model an economy with three types of goods: market goods (that people acquire by spending their after-tax earnings), public goods (that are provided by the government and are

*Erasmus University Rotterdam, Tinbergen Institute, CESifo Munich, and IZA Bonn. E-mail: dur@ese.eur.nl

†Humboldt-Universität zu Berlin, Berlin School of Economics, and CEPR. E-mail: anja.schoettner@hu-berlin.de

‡Funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) -- Project-ID 280092119 -- TRR 190 Rationality and Competition.

financed through taxes), and social goods (which are non-priced goods with positive externalities that are produced by citizens during their leisure time). Each individual in our economy chooses their work hours and the amount of social goods she produces. These two activities compete for scarce resources such as time and energy. The latter feature of our model gives rise to interesting interdependencies between paid work hours and social goods production.

Our economy suffers from two externality problems. First, if the government imposes an income tax, the classic fiscal externality problem emerges, which means that people work too few hours because they do not take into account that working more means higher tax payments, which in turn finance public goods or allow for lower tax rates. Second, our model contains a novel social externality, which means that people do not take into account the positive externalities that their production of social goods entail. The interaction between these two externalities drives our key results on optimal policy.

We start our analysis by studying people's choices and societal outcomes for any given government policy. We show that the economy can end up in three different situations, depending on the size of the social externality. First, when the social externality is weak, the economy can arrive in a state where social-goods production is too high and work hours are too low as compared to the social optimum. Conversely, it can also happen that the economy ends up in a state where people work too many hours and produce too few social goods. This happens when the strength of the social externality is high, which calls for high social-goods production and, consequently, low work hours. The third and last state is where work hours and social goods are both below their socially optimal levels. This happens when the social externality is of intermediate size. The economy never ends up in a state with both too many work hours and too many social goods.

Next we study optimal government policy. We derive the optimal mix of income tax and lump-sum tax for any given level of public goods. Note that a negative lump-sum tax is a basic income. We obtain four key results.

First, we show that, in the absence of social goods, the government exclusively uses a lump-sum tax to finance public goods and sets the income tax rate to zero. In this way, the government avoids the fiscal externality problem and the first-best outcome is achieved.

Second, in the presence of social goods, the government always prefers to set a positive income tax rate. Two mild conditions are essential for this result: 1) social goods should exhibit positive externalities and 2) paid work hours and social-goods production compete for scarce resources (time, energy). The positive income tax creates a fiscal externality because individuals do not internalize the tax revenues generated by their work. However, it also helps to mitigate the issue of too low production of social goods. By discouraging work hours, the marginal cost of social goods falls, and as a result production of social goods goes up. The optimal level of the income tax rate trades off increased social-goods production and decreased work hours.

Third, the government provides a basic income if the level of public goods the government needs to finance is sufficiently low. The positive income tax rate (implemented to stimulate social-goods production) results in a budget surplus that is transferred back in the form of a reduction in the lump-sum tax. If the level of public goods is sufficiently low, the lump-sum tax becomes negative, implying a basic income. The basic income, via a standard income effect on work hours, further contributes to increasing the amount of social-goods production. Note that the positive tax rate and the basic income do not play a redistributive role in our paper. They are part of optimal government policy for allocative efficiency reasons only.

Fourth and last, we show that under quite plausible assumptions, a basic income may become optimal if labor productivity and thus wages are sufficiently high. Hence, our theory is consistent with the fact that the basic income has so far not been implemented at scale, but at the same time predicts that at some point in time it should be, provided that labor productivity and thus wages keep growing.

Our theory relates to the increased interest in economics in 'civil society' as a third form of governance in the economy in addition to the state and the market (Bowles and Carlin, 2026). Whereas the standard public finance model only includes the state and the market, we add 'civil society', i.e., behavior in families, neighborhoods, communities, and other social movements. Importantly, and well in line with the literature, we assume that the activities people perform in civil society are not priced and cannot be contracted on. Moreover, we assume that they involve positive externalities.

The idea that leisure time is not simply a consumption good but is also used for production goes back to at least Becker (1965) and Gronau (1977). We add to this literature by allowing for positive externalities of outside-of-work production and studying the implications of these externalities for optimal government policy. There is also a small theoretical literature on 'leisure externalities' (which can be positive or negative) and what they mean for optimal tax policy (Pintea, 2010; Escobar-Posada and Monteiro, 2017). A part of this literature considers positional preferences regarding leisure as well as consumption, where the relative strength of these two determine the consequences for optimal tax policy (Frijters and Leigh, 2008; Aronsson and Johansson-Stenman, 2013). Like the literature on positive leisure externalities, our model implies a corrective role for labor income taxation. Our contribution is not this basic corrective-tax logic itself. Rather, our analysis differs from models of leisure externalities in an important respect. In those models, leisure is typically determined residually by a time constraint, so that a reduction in work translates one-for-one into an increase in the externality-generating activity. In our model, social-goods production is a separate, noncontractible choice that cannot be targeted directly. Labor income taxation is therefore an indirect instrument: by discouraging paid work, it lowers the marginal cost of social-goods production, but it need not generate a corresponding increase in the production of social goods. The strength of this response depends on individuals' preferences and on the interaction between work and social-goods production in the cost function. This endogenous and generally imperfect transmission between the two activities creates a genuinely two-dimensional allocation problem. In particular, individuals may work too little while still producing too few social goods, and labor taxation generally cannot correct both distortions fully. The optimal tax therefore depends not only on the magnitude of the social externality but also on how effectively a reduction in work is translated into additional social-goods production.

Our paper also relates to the recent empirical literature on the effects of a basic income on labor supply and out-of-work activities. Bernhard et al. (2025) and Bohmann et al. (2026) report the results of a large-scale field experiment in Germany where randomly selected subjects received a monthly payment of 1,200 euros for three years. In line with our predictions, they find that the basic income reduced work hours (by 1.3 hours per week) and increased social-goods production as measured by improved social life and improved prosocial behavior.[1] Similarly, in a recent field experiment in the US that paid 1,000 dollars per month to randomly selected households for three years, Vivalt et al. (2025) find that the basic income led to a reduction in work hours of 1-2 hours per week and an increase in 'social leisure', but no change in caring for others or community engagement. In a reanalysis of older experiments on the basic income using new data, Riddell and Riddell (2026) find sizeable negative effects on work hours and positive effects on social activities.[2] There is also empirical evidence from both sides of the Atlantic that long working hours are negatively associated with volunteer hours (Kelle et al., 2024; Taniguchi, 2012), both formal and informal volunteering.

The remainder of our paper is structured as follows. The next section describes the model. Section 3 provides the analysis of individual behavior, the social optimum, and optimal government policy. Section 4 concludes.

2 The Model

We consider an economy populated by n identical individuals. The government sets a labor income tax rate $t \in [0, 1)$ and a uniform transfer b, taking public-good provision $g > 0$ as given. A positive transfer is a basic income, whereas a negative transfer is a lump-sum tax. Individuals then choose their work hours $h_i \geq 0$ and their production of social goods $s_i \geq 0$. The gross hourly wage $w > 0$ is exogenous and the same for all individuals. Social-goods production comprises socially valuable activities undertaken outside paid work, such as providing informal care, maintaining social relationships, volunteering, and contributing to community life. We use s_i as an index of the extent to which individual i engages in such activities. What distinguishes social-goods production from other activities outside paid work is that it generates benefits for others as well as for the individual undertaking it.

Individual earnings are verifiable and can therefore be taxed. By contrast, individual social-goods production is nonverifiable.[3] The activities summarized by s_i may be observable to the people directly involved, but their extent and social value cannot be measured and verified by the government at reasonable cost. The government therefore cannot condition an individual's taxes or transfers on s_i. Accordingly, its policy instruments are restricted to the labor income tax rate t and the uniform transfer b.

Let

$$\bar{s} = \frac{1}{n} \sum_{j=1}^{n} s_j$$

[1]Spending time with friends and family went up, as does volunteering. Subjects also experience stronger purpose in life, better mental health, and higher social satisfaction. Not all of these results are statistically significant, though, likely because of small sample size (107 treated subjects).

[2]It is important to note that tax rates were held constant in these field experiments, which mutes the overall effect on work hours and social goods if the substitution effect dominates the income effect of a net wage change on labor supply, see Proposition 1 below.

[3]Some components of social-goods production, such as formal volunteering or professional care, may be partially observable and supported through targeted policies. We interpret s_i as the component of socially valuable activity that cannot be measured and verified sufficiently well to serve as the basis for individual payments.

denote average social-goods production in society. Individual i's utility is

$$U_i = u(c_i) + \beta\frac{g}{n} + \gamma v(s_i) + \mu\bar{s} - K(h_i, s_i),$$

where

$$c_i = (1-t)wh_i + b$$

denotes disposable income, which is entirely consumed. The function $u(\cdot)$ captures utility from consumption, while $v(\cdot)$ captures the intrinsic utility that individuals derive from producing social goods. We assume that both functions are twice continuously differentiable, increasing, and concave: $u'(c) > 0$, $u''(c) \leq 0$, $v'(s) > 0$, and $v''(s) \leq 0$.

The parameters $\gamma \geq 0$ and $\mu \geq 0$ capture two distinct benefits of social-goods production. The term $\gamma v(s_i)$ represents the intrinsic satisfaction individual i derives from producing social goods. Because $v(\cdot)$ is concave, additional social-goods production generates diminishing marginal intrinsic utility. By contrast, the term $\mu\bar{s}$ captures the benefit that each individual receives from society-wide social-goods production. We assume that this external benefit is linear, so that each additional unit of average social-goods production generates a constant marginal benefit μ. Thus, γ measures the private motivation to produce social goods, whereas μ measures their external social benefit. Finally, $\beta \geq 0$ measures the value individuals attach to public-good provision.

The function $K(h_i, s_i)$ represents an individual's personal cost associated with work and social-goods production. We assume that $K(\cdot,\cdot)$ is twice continuously differentiable and weakly increasing, with $K_h(0,0) = K_s(0,0) = 0$, and that its Hessian is positive definite:

$$K_{hh}(h_i, s_i) > 0, \qquad K_{ss}(h_i, s_i) > 0, \qquad K_{hh}(h_i, s_i)K_{ss}(h_i, s_i) - K_{hs}(h_i, s_i)^2 > 0.$$

Thus, K is strictly convex in (h_i, s_i). We further assume that

$$K_{hs}(h_i, s_i) > 0.$$

Thus, work and social-goods production compete for scarce resources such as time and energy: an increase in either activity raises the marginal cost of the other. Rather than imposing a literal time constraint, the model captures this interaction through the cost function.

The government's budget constraint is

$$g + bn = twhn,$$

where h denotes work hours per individual in a symmetric allocation. Public-good provision g is exogenously fixed because our analysis focuses on the allocation of resources between work and social-goods production. Although g does not directly affect the marginal allocation decisions studied below, financing this expenditure requirement is central to the government's budget constraint.

We treat individuals as atomistic. When choosing h_i and s_i, each individual therefore takes per-capita public-good provision g/n and average social-goods production $\bar{s}$ as given. This assumption can be interpreted as the limiting case of a large population. Individuals consequently internalize neither the additional tax revenue generated by their work nor the external benefit generated by their own social-goods production.

Because individuals are identical, we focus on symmetric allocations in which $h_i = h$ and $s_i = s$ for all i. Suppressing individual subscripts, utility in a symmetric allocation can therefore be written as

$$U = u((1-t)wh + b) + \beta \frac{g}{n} + \gamma v(s) + \mu \bar{s} - K(h, s).$$

In a symmetric allocation, $\bar{s} = s$. Importantly, however, an individual takes $\bar{s}$ as given when choosing their own social-goods production.

3 Analysis

3.1 Individual behavior

We start by analyzing individual behavior. Each individual chooses work hours h and social-goods production s to maximize utility. The assumptions on u, v, and K imply that the individual's objective function is strictly concave. We assume that an interior optimum exists; it is therefore unique. Let h^I and s^I denote the work hours and social-goods production chosen by the individual, respectively. The first-order conditions for an interior optimum are

$$\begin{aligned} u'(c^I)(1-t)w - K_h(h^I, s^I) &= 0, \\ \gamma v'(s^I) - K_s(h^I, s^I) &= 0, \end{aligned}$$

where $c^I = (1-t)wh^I + b$ denotes the corresponding disposable income. Holding b and t constant, and thus abstracting from the government's budget constraint, we obtain several comparative-static results.[4] A stronger intrinsic valuation of producing social goods, reflected by an increase in γ, increases social-goods production and reduces work hours. The intuition is straightforward: individuals devote more time and energy to activities they value intrinsically, leaving less time and energy for work.

The effect of a change in the wage, w, is more subtle. A higher wage increases the return to work, creating a substitution effect toward work. At the same time, because utility from consumption is concave, a higher wage also makes individuals richer, generating an income effect that encourages them to work less and produce more social goods. The overall effects of the wage on work hours and social-goods production therefore depend on the relative strength of these two effects. If the substitution effect dominates, which is the case if $u'(c^I) + (1-t)wh^I u''(c^I) > 0$, work hours increase and social-goods production decreases with w. If $u'(c^I) + (1-t)wh^I u''(c^I) < 0$ and thus the income effect dominates, work hours decrease and social-goods production increases with w.

Our first result concerns the effects of income taxation and the uniform transfer on labor supply and social-goods production.

Lemma 1. *(i) Suppose that $u''(c^I) < 0$. Holding the labor income tax rate constant, an increase in the uniform transfer, b, strictly reduces work hours and strictly increases*

[4] All proofs are provided in the Appendix.

social-goods production.

(ii) Suppose that $u'(c^I) + (1-t)wh^I u''(c^I) > 0$. Holding the uniform transfer constant, an increase in the labor income tax rate, t, strictly reduces work hours and strictly increases social-goods production.

If the utility from consumption is strictly concave, an increase in non-labor income, b, lowers the marginal utility of additional labor income and thereby reduces the incentive to work. Because work and social-goods production compete for time and energy, the resulting decline in work hours encourages social-goods production.

A higher labor income tax lowers the net wage, giving rise to two opposing effects. The substitution effect makes social-goods production relatively more attractive than work, reducing work hours and increasing social-goods production. By contrast, the associated reduction in disposable income creates an income effect that induces individuals to work more to compensate for part of the income loss. Part (ii) of the lemma assumes that the substitution effect dominates the income effect. Under this condition, work hours decline and social-goods production increases following an increase in the tax rate. In contrast, if $u'(c^I) + (1-t)wh^I u''(c^I) < 0$, work hours increase and social-goods production decrease with the tax rate.

3.2 Social optimum

We next determine the socially optimal levels of work hours and social-goods production. The social planner chooses h and s to maximize the sum of individual utilities,

$$nU = n\left[u((1-t)wh + b) + \beta\frac{g}{n} + \gamma v(s) + \mu\overline{s} - K(h,s)\right],$$

subject to the government budget constraint. Because the social planner chooses the same levels of h and s for all individuals, $\overline{s} = s$. Moreover, using the government budget constraint, $g + bn = twhn$, we can substitute

$$t = \frac{g+bn}{whn}.$$

Hence,

$$nU = n\left[u\left(\left(1 - \frac{g+bn}{whn}\right)wh + b\right) + \beta\frac{g}{n} + \gamma v(s) + \mu s - K(h,s)\right].$$

The argument of $u(\cdot)$ simplifies to $wh - \frac{g}{n}$. Thus, the uniform transfer drops out because it is a pure transfer that—dictated by the government budget constraint—needs to be financed by an equivalent amount of taxes, and the planner's objective becomes

$$nU = n\left[u\left(wh - \frac{g}{n}\right) + \beta\frac{g}{n} + \gamma v(s) + \mu s - K(h,s)\right].$$

Unlike individuals, the social planner takes both the fiscal and social externalities into account. The planner chooses work hours and social-goods production for all individuals simultaneously and therefore internalizes their effects on the government budget and average social-goods production.

The planner's objective is strictly concave. We assume that an interior solution exists; it is therefore unique. Let h^P and s^P denote the work hours and social-goods production chosen by the social planner, respectively. The first-order conditions are

$$u'(c^P)w - K_h(h^P, s^P) = 0,$$
$$\gamma v'(s^P) + \mu - K_s(h^P, s^P) = 0,$$

where $c^P = wh^P - \frac{g}{n}$. Compared with individual decision making, there are two differences. First, the marginal benefit of an additional work hour is based on the gross wage rather than the net wage. This reflects the internalization of the fiscal externality: taxes paid by individuals are revenues available to the government. Second, the social planner internalizes the positive externality from social-goods production, as reflected by the term μ in the second first-order condition.

Let us consider first the case that is standard in the literature, the case without social-goods production (i.e., $\gamma = \mu = 0$, implying $s = 0$). Under a balanced government budget, consumption at any allocation is $c = wh - \frac{g}{n}$. For any strictly positive tax rate, $t > 0$, the planner chooses more work hours than individuals, $h^P > h^I$. To see this, evaluate the planner's marginal objective at the individual allocation. Using the individual's first-order condition,

$$K_h(h^I, 0) = u'(c^I)(1-t)w,$$

we obtain

$$u'(c^I)w - K_h(h^I, 0) = u'(c^I)tw > 0$$

for all $t > 0$. The planner therefore benefits from increasing work hours above h^I. This difference reflects the classic fiscal externality: individuals do not internalize the tax revenue generated by their work.

Our next result shows that, in the presence of social-goods production, individuals may work too much relative to the social optimum when the social externality is sufficiently large.

Proposition 1. *Suppose that the government budget is balanced and that the individual and social planner problems have interior solutions. If there exists $\widehat{s} > s^I$ satisfying*

$$K_h(h^I, \widehat{s}) - K_h(h^I, s^I) = u'(c^I)tw,$$

then there exists a unique threshold $\widehat{\mu} > 0$ at which $h^I = h^P$ and a unique threshold $\widetilde{\mu} \in (0, \widehat{\mu})$ at which $s^I = s^P$. Since the social planner's optimal work hours are strictly decreasing in μ, whereas optimal social-goods production is strictly increasing in μ, the individual and socially efficient allocations compare as follows:

(i) If $\widehat{\mu} < \mu$, then $h^I > h^P$ and $s^I < s^P$.

(ii) If $\widetilde{\mu} < \mu < \widehat{\mu}$, then $h^I < h^P$ and $s^I < s^P$.

(iii) If $\mu < \widetilde{\mu}$, then $h^I < h^P$ and $s^I > s^P$.

The existence of $\widehat{s}$ ensures that there exists a social externality $\widehat{\mu}$ for which the planner chooses the same work hours as individuals but a strictly higher level of social-goods production.[5] For $\mu > \widehat{\mu}$, the planner places even greater value on social-goods production and therefore chooses $h^P < h^I$. Thus, if the social externality is sufficiently large, individuals work more than is socially efficient. Because they do not internalize the benefits their social-goods production creates for others, they devote too few resources to social goods. This keeps their marginal cost of work relatively low and induces them to work too much. By contrast, if the social externality is relatively small ($\mu < \widehat{\mu}$), the fiscal externality dominates: individuals base their work decision on the net wage and do not internalize the tax revenue generated by their work. They therefore work less than is socially efficient.

The threshold $\widetilde{\mu}$ is the social externality for which the planner chooses the same level of social-goods production as individuals, while choosing strictly more work hours. Because work and social-goods production are substitutes in the individual's cost function, the planner's higher work hours increase the marginal cost of social-goods production. If $\mu < \widetilde{\mu}$, the social benefit is too small to compensate for this higher marginal cost, and the planner chooses less social-goods production than individuals. If $\mu > \widetilde{\mu}$, the social benefit is sufficiently large to outweigh the higher marginal cost, and the planner chooses more social-goods production than individuals.

Note that Proposition 1 does not cover the case of a zero tax rate, because in this case no $\widehat{s} > s^I$ satisfying the condition in the proposition exists. The individual's and the planner's first-order conditions imply that, when the tax rate is zero, the individual and socially efficient allocations coincide in the absence of a social externality, that is, when $\mu = 0$. For every $\mu > 0$, we have $h^I > h^P$ and $s^I < s^P$: individuals work too much and produce too little of the social good because they do not internalize the social externality.

Example. Consider the special case in which

$$K(h, s) = \theta(h^2 + s^2 + hs), \qquad u(c) = c, \qquad v(s) = s,$$

where $\theta > 0$. In this case, the cross-partial $K_{hs} = \theta$ is constant, and there is no income effect because utility from consumption is linear. The individual's first-order conditions are

$$(1 - t)w = \theta(2h^I + s^I) \qquad \text{and} \qquad \gamma = \theta(2s^I + h^I).$$

Solving this system gives

$$h^I = \frac{2(1 - t)w - \gamma}{3\theta} \qquad \text{and} \qquad s^I = \frac{2\gamma - (1 - t)w}{3\theta}.$$

The condition for an interior individual allocation is $\frac{(1-t)w}{2} < \gamma < 2(1 - t)w$. The planner's

[5] A sufficient condition for the existence of $\widehat{s}$ is $\lim_{s\to\infty} K_h(h, s) = \infty$ for every $h > 0$. Intuitively, this condition requires that sufficiently high social-goods production eventually makes an additional hour of work arbitrarily costly. Together with $K_{hs} > 0$, this condition implies that $K_h(h^I, \widehat{s}) - K_h(h^I, s^I)$ is continuous and strictly increasing in $\widehat{s}$, starting from zero at $\widehat{s} = s^I$ and becoming arbitrarily large. It therefore reaches the finite fiscal wedge $u'(c^I)tw$ at a unique value of $\widehat{s}$.

first-order conditions are

$$w = \theta(2h^P + s^P) \qquad \text{and} \qquad \gamma + \mu = \theta(2s^P + h^P).$$

The socially efficient allocation is therefore

$$h^P = \frac{2w - \gamma - \mu}{3\theta} \qquad \text{and} \qquad s^P = \frac{2(\gamma + \mu) - w}{3\theta}.$$

The condition for an interior socially optimal allocation is $\frac{w}{2} < \gamma + \mu < 2w$.

Comparing individual and socially efficient work hours yields

$$h^I - h^P = \frac{\mu - 2tw}{3\theta}.$$

Thus, the threshold in Proposition 1 is $\widehat{\mu} = 2tw$. Individuals work too much when the social externality exceeds twice the fiscal wedge, $\mu > 2tw$. Conversely, if $\mu < 2tw$, the fiscal externality dominates and individuals work too little. At $\mu = 2tw$, individual work hours coincide with the socially efficient level.

Comparing individual and socially efficient social-goods production yields

$$s^I - s^P = \frac{tw - 2\mu}{3\theta}.$$

Individuals therefore produce too many social goods if $\mu < tw/2 = \widetilde{\mu}$. In this case, the tax rate is high relative to the social externality. The high tax rate discourages work, and the resulting reduction in work hours lowers the marginal cost of social-goods production because $K_{hs} > 0$. Individuals may consequently produce more social goods than is socially efficient. By contrast, if $\mu > tw/2$, individuals produce too few social goods. In particular, when $t = 0$ and $\mu > 0$, individuals necessarily produce too few social goods because they do not internalize their positive externality.

Figure 1 illustrates the three possible regions for varying μ and t. We restrict attention to the parameter range in which both the individual and socially optimal allocations are interior. In particular, assuming $w/2 < \gamma < 2w$, this requires

$$0 \leq t < 1 - \frac{\gamma}{2w} \qquad \text{and} \qquad 0 \leq \mu < 2w - \gamma.$$

Within this parameter range, under individual decision making, there are: (i) too many work hours and too little social-goods production if $\mu > 2tw$; (ii) too few work hours and too little social-goods production if $tw/2 < \mu < 2tw$; and (iii) too few work hours and too much social-goods production if $\mu < tw/2$. $\square$

3.3 Government decision-making

Now consider a government that sets the tax rate t and the uniform transfer b so as to maximize social welfare, taking into account individuals' subsequent choices of work hours and social-goods

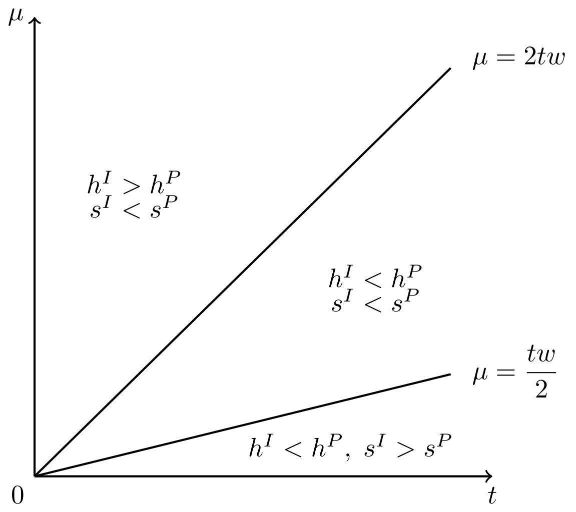

Figure 1: Individual choices relative to the socially optimal allocation for $K(h,s) = \theta(h^2+s^2+hs)$, $u(c) = c$, and $v(s) = s$, within the parameter region in which both allocations are interior.

production, as well as the government's budget constraint. The government's problem thus reads:

$$\begin{aligned} \max_{t,b}\ n & \left[u\left((1-t)wh^I + b\right) + \beta\frac{g}{n} + \gamma v(s^I) + \mu s^I - K(h^I, s^I)\right] \\ \text{s.t.}\quad & u'((1-t)wh^I + b)(1-t)w - K_h(h^I, s^I) = 0 \\ & \gamma v'(s^I) - K_s(h^I, s^I) = 0 \\ & g + bn - twh^I n = 0 \end{aligned}$$

We can use the budget constraint to replace b, so that the problem becomes:

$$\begin{aligned} \max_{t}\quad n & \left[u\left(wh^I - \frac{g}{n}\right) + \beta\frac{g}{n} + \gamma v(s^I) + \mu s^I - K(h^I, s^I)\right] \\ \text{s.t.}\quad & u'\left(wh^I - \frac{g}{n}\right)(1-t)w - K_h(h^I, s^I) = 0 \\ & \gamma v'(s^I) - K_s(h^I, s^I) = 0 \end{aligned}$$

Let $h^I(t)$ and $s^I(t)$ denote the individual's choices along the balanced-budget path, and define

$$c^I(t) = wh^I(t) - \frac{g}{n}.$$

Let t^* denote the government's optimal tax rate. The first-order condition for an interior optimum is

$$\left\{\frac{dh^I(t)}{dt}\left[u'(c^I(t))w - K_h(h^I(t), s^I(t))\right] + \frac{ds^I(t)}{dt}\left[\gamma v'(s^I(t)) + \mu - K_s(h^I(t), s^I(t))\right]\right\}\Bigg|_{t=t^*} = 0.$$

Using the individual's first-order conditions, this simplifies to

$$\left[\frac{dh^I(t)}{dt}u'(c^I(t))tw + \frac{ds^I(t)}{dt}\mu\right]\Bigg|_{t=t^*} = 0. \tag{1}$$

When analyzing individual decision-making, Lemma 1 treats the tax rate and the uniform transfer as independent policy instruments. Under a balanced government budget, however, an increase in the tax rate is accompanied by an endogenous adjustment in the transfer, eliminating the opposing income effect and yielding an unambiguous response. Whether the uniform transfer increases or decreases with the tax rate depends on whether the increase in the tax rate outweighs the resulting reduction in the tax base, $wh^I(t)$.

Lemma 2. *Along the balanced-budget path, an increase in the tax rate reduces work hours and increases social-goods production. The uniform transfer increases with the tax rate if*

$$h^I(t) + t\frac{dh^I(t)}{dt} > 0,$$

is unchanged if this expression equals zero, and decreases if it is negative.

Lemma 2 implies that the first term in the government's first-order condition, equation (1), is negative and captures the welfare loss associated with the fiscal externality: the individual does not take into account that an additional hour of work generates additional tax revenue. The marginal social value of this revenue is $u'(c^I(t^*))t^*w$. The second term is positive and captures the welfare gain from the increase in social-goods production. The optimal tax balances these two effects.

To characterize the optimal tax rate more explicitly, we totally differentiate the individual's second first-order condition along the balanced-budget path. Suppressing the dependence of h^I and s^I on t, we obtain

$$\gamma v''(s^I)\frac{ds^I}{dt} - \left[K_{sh}(h^I, s^I)\frac{dh^I}{dt} + K_{ss}(h^I, s^I)\frac{ds^I}{dt}\right] = 0.$$

Collecting the terms involving ds^I/dt gives

$$\left[\gamma v''(s^I) - K_{ss}(h^I, s^I)\right]\frac{ds^I}{dt} = K_{sh}(h^I, s^I)\frac{dh^I}{dt}.$$

Hence,

$$\frac{ds^I}{dt} = -\frac{K_{hs}(h^I, s^I)}{K_{ss}(h^I, s^I) - \gamma v''(s^I)}\frac{dh^I}{dt}.$$

Substituting this relationship into the government's first-order condition, we obtain that the optimal tax rate t^* satisfies

$$u'(c^I(t^*))t^*w = \frac{K_{hs}(h^I(t^*), s^I(t^*))}{K_{ss}(h^I(t^*), s^I(t^*)) - \gamma v''(s^I(t^*))}\mu \tag{2}$$

Accordingly, the optimal tax policy equates the utility cost of the tax wedge, $u'(c^I(t^*))t^*w$, with the marginal social benefit generated by the induced reallocation from work toward social-goods production. To see this, note that

$$-\frac{ds^I}{dh^I} = \frac{K_{hs}(h^I, s^I)}{K_{ss}(h^I, s^I) - \gamma v''(s^I)}$$

measures the increase in social-goods production associated with a one-unit reduction in work hours. Multiplying this response by the externality μ gives the right-hand side of equation (2).

Finally, using equation (2) and the government budget constraint, we obtain the associated poll tax or basic income:

$$b^* = t^* w h^I(t^*) - \frac{g}{n} = \frac{K_{hs}(h^I(t^*), s^I(t^*))}{K_{ss}(h^I(t^*), s^I(t^*)) - \gamma v''(s^I(t^*))} \frac{\mu}{u'(c^I(t^*))} h^I(t^*) - \frac{g}{n} \tag{3}$$

Equations (2) and (3) yield our next result.

Proposition 2. *(i) If social-goods production generates no externality ($\mu = 0$), then*

$$t^* = 0 \qquad \textit{and} \qquad b^* = -\frac{g}{n} < 0.$$

Thus, public goods are financed entirely through a poll tax.
(ii) If social-goods production generates a positive externality ($\mu > 0$) and the individual problem has an interior solution, then $t^* > 0$. Moreover, the government provides a basic income, $b^* > 0$, if and only if

$$\mu \frac{K_{hs}(h^I(t^*), s^I(t^*))}{K_{ss}(h^I(t^*), s^I(t^*)) - \gamma v''(s^I(t^*))} \frac{h^I(t^*)}{u'(c^I(t^*))} > \frac{g}{n}.$$

Otherwise, it levies a poll tax, $b^* \leq 0$.

When social-goods production generates no externality, an income tax only distorts individuals' choices: it discourages work and induces greater social-goods production without creating an offsetting social benefit. The government therefore sets the income tax rate to zero and finances public goods through a poll tax. With positive externalities of social-goods production, by contrast, the government uses a positive income tax to encourage individuals to shift time and resources away from work and toward social-goods production. The resulting tax revenue may be sufficient not only to finance public goods but also to provide a basic income. Whether this occurs depends on the level of public goods, the strength of the externality of social goods, and on how strongly taxation redirects individual choices toward social-goods production. In the following, we use different specifications of the model to examine under which circumstances a basic income arises.

Example. For the illustrative example with $K(h, s) = \theta(h^2 + s^2 + hs)$, $u(c) = c$, and $v(s) = s$, the optimal tax rate is particularly simple:

$$t^* = \frac{\mu}{2w}.$$

If $\mu > 0$, the optimal tax rate is positive and increases with the strength of the externality. Substituting the optimal work hours,

$$h^I(t^*) = \frac{2w - \gamma - \mu}{3\theta},$$

into the government's budget constraint gives

$$b^* = t^* w h^I(t^*) - \frac{g}{n} = \frac{\mu(2w - \gamma - \mu)}{6\theta} - \frac{g}{n}.$$

b^* is inverse-U-shaped in the externality μ. A larger externality raises the optimal tax wedge, $t^*w = \mu/2$, but it also reduces optimal work hours. Initially, the higher tax wedge dominates and tax revenue increases. For sufficiently large externalities, however, the reduction in work hours dominates and tax revenue—and therefore b^*—decreases. Thus, the scope for providing a basic income is greatest for intermediate externalities. By contrast, higher wages unambiguously increase b^*, so that a basic income is eventually optimal for sufficiently high wages. Higher wages do not affect the optimal tax wedge, t^*w, but increase optimal work hours, thereby generating higher tax revenues.

At the optimal tax rate, individual work hours coincide with the socially optimal level, $h^I(t^*) = h^P$. However, individuals still produce too few social goods:

$$s^I(t^*) = \frac{4\gamma - 2w + \mu}{6\theta}$$

and therefore

$$s^I(t^*) - s^P = -\frac{\mu}{2\theta} < 0.$$

Thus, the optimal tax implements the socially efficient level of work hours but does not fully correct the underproduction of social goods. □

The positive relationship between b^* and w in the example is derived under linear utility from consumption and therefore abstracts from income effects. With strictly concave utility, a higher wage changes not only the return to work but also the marginal utility of consumption. This affects both individual behavior and the utility cost of the tax wedge, so it is no longer immediate that higher wages generate a basic income or generate a higher poll tax. The following result encompasses both linear and strictly concave utility and shows that, under the other structural assumptions of the example, the positive relationship continues to hold. Moreover, the optimal tax policy continues to implement the socially efficient level of work hours, although social-goods production remains inefficiently low.

Proposition 3. *Suppose that utility from consumption is concave, K_{ss} and K_{hs} are constant, v is linear, and $\mu > 0$. (i) If the individual problem has an interior solution, then the optimal uniform transfer, b^*, is strictly increasing in the wage, $\frac{db^*}{dw} > 0$. (ii) Work hours equal the socially optimal level, $h^I(t^*) = h^P$, whereas social-goods production is inefficiently low, $s^I(t^*) < s^P$.*

For the case considered in the proposition, equation (2) implies that the optimal tax rate satisfies

$$u'(c^I(t^*))t^*w = \frac{K_{hs}}{K_{ss}}\mu.$$

Because K_{hs} and K_{ss} are constant and v is linear, the marginal social benefit of reallocating time from work toward social-goods production, given by the right-hand side, is constant. A higher wage increases consumption along the optimal-policy path, regardless of whether the substitution or income effect on work hours dominates, $\frac{dc^I}{dw} > 0$.[6] Because utility from consumption is strictly concave, the resulting increase in consumption reduces $u'(c^I)$. At higher consumption levels, the utility cost of a given tax wedge is therefore lower. Because the marginal social benefit of

[6]See the proof of Proposition 3.

reallocating time toward social-goods production remains constant, the government optimally responds by increasing the tax wedge $t^* w$. The optimal tax rate t^* itself may nevertheless increase or decrease, because the wage also enters the tax wedge directly.

If the substitution effect dominates, optimal work hours increase with the wage. In this case, both the tax wedge and the tax base increase, so that tax revenue, and hence the optimal uniform transfer,

$$b^* = t^* w h^I(t^*) - \frac{g}{n},$$

unambiguously increases. More surprisingly, b^* also increases when the income effect dominates and optimal work hours decline. Although the decline in work hours reduces the tax base, the increase in the optimal tax wedge more than compensates for this reduction. Thus, optimal tax revenue—and hence the b^* remaining after public-good expenditures—increases with the wage in either case.

Moreover, $h^I(t^*)$ equals the socially optimal level of work hours, whereas social-goods production is inefficiently low. Individuals do not take the external benefit μ into account and therefore devote too few resources to social-goods production. Because work and social-goods production compete for the same resources, this also makes work more attractive to them than it is from a social perspective: at their inefficiently low level of social-goods production, they perceive the marginal cost of working as too low. The income tax corrects this distortion by reducing the private return to work. The tax wedge chosen by the government exactly offsets the extent to which individuals underestimate the marginal cost of work. The government therefore implements the socially optimal level of work hours, even though it does not fully eliminate the underproduction of social goods. This exact result relies on K_{ss} and K_{hs} being constant and on v being linear. Linearity of v ensures that the marginal intrinsic benefit from social-goods production does not change with its level, while constant K_{ss} and K_{hs} make the relevant cost effects independent of the allocation.

If these conditions do not hold, the optimal tax correction need not match the distortion in the individual's work-hours decision exactly. Work hours induced by the optimal policy may then be higher or lower than the socially optimal level. To understand how work hours can exceed their socially optimal level, consider a situation in which $h^I > h^P$ and $s^I < s^P$. This comparison does not necessarily imply that a marginal increase in the tax rate would improve welfare. The government cannot control work hours and social-goods production independently. Raising the tax rate reduces work hours and increases social-goods production simultaneously, and the government chooses the tax rate according to the marginal trade-off along this behavioral response path. In the general model, this trade-off varies with the allocation. In particular,

$$-\frac{ds^I}{dh^I} = \frac{K_{hs}(h^I, s^I)}{K_{ss}(h^I, s^I) - \gamma v''(s^I)}$$

measures how much additional social-goods production is induced by a one-unit reduction in work hours. If this response is relatively weak at the relevant allocation, achieving a further increase in s^I requires a relatively large reduction in work. The welfare gain from additional social-goods production may then be outweighed by the cost of the accompanying reduction in work before work hours have fallen to h^P. The government may therefore optimally choose a policy that induces $h^I > h^P$ and $s^I < s^P$.

To examine the relationship between b^* and w under more general preferences and cost functions, define

$$R(h, s) := \frac{K_{hs}(h, s)}{K_{ss}(h, s) - \gamma v''(s)}.$$

This ratio measures the effectiveness of taxation in reallocating resources from work toward social-goods production. Using equation (3), the optimal uniform transfer satisfies

$$b^* + \frac{g}{n} = \mu R(h^I(t^*), s^I(t^*)) \frac{h^I(t^*)}{u'(c^I(t^*))}.$$

Hence, log-differentiating gives

$$\frac{d}{dw} \ln \left(b^* + \frac{g}{n} \right) = \frac{1}{R} \frac{dR}{dw} + \frac{1}{h^I} \frac{dh^I}{dw} - \frac{u''(c^I)}{u'(c^I)} \frac{dc^I}{dw},$$

where all derivatives are evaluated along the optimal-policy path. It follows that b^* is increasing in w if

$$\frac{1}{R} \frac{dR}{dw} + \frac{1}{h^I} \frac{dh^I}{dw} - \frac{u''(c^I)}{u'(c^I)} \frac{dc^I}{dw} > 0.$$

This condition separates three effects of a wage increase. First, a change in R affects how effectively the income tax encourages social-goods production. Second, a change in work hours affects the tax base from which the transfer is financed. Third, a change in consumption affects marginal utility and thereby the optimal tax rate and the revenue available for the uniform transfer. Thus, a set of sufficient conditions for the optimal uniform transfer to be strictly increasing in the wage is

$$\frac{dR}{dw} \geq 0 \qquad \text{and} \qquad \frac{dh^I}{dw} > 0.$$

The latter condition also implies that consumption increases with the wage. Indeed, along the balanced-budget path, we then have

$$\frac{dc^I}{dw} = h^I + w \frac{dh^I}{dw} > 0.$$

Since $u'' \leq 0$, all three terms in the condition above are then nonnegative, and the work-hours effect is strictly positive. Consequently, the optimal uniform transfer is strictly increasing in the wage.

These sufficient conditions appear economically plausible. A positive total response of work hours arises when the substitution effect dominates the income effect and the adjustment of the optimal tax rate does not overturn this response. Since work and social-goods production compete for scarce resources, $dh^I/dw > 0$ implies $ds^I/dw < 0$. Moreover, it seems plausible that K_{hs} is increasing in h: when work hours are already high, an additional hour of work may crowd out social activities more strongly. Likewise, K_{ss} may be increasing in s, reflecting increasing marginal costs of social-goods production. A wage-induced increase in h^I and decrease in s^I then tends to raise K_{hs} and lower K_{ss}, both of which tend to increase

$$R(h^I, s^I) = \frac{K_{hs}(h^I, s^I)}{K_{ss}(h^I, s^I) - \gamma v''(s^I)}.$$

This conclusion is not automatic because K_{hs} may also vary with s, K_{ss} may also vary with h, and $v''(s)$ may change with s. Provided that these additional effects are not sufficiently strong to offset the increase in K_{hs} caused by higher work hours and the decrease in K_{ss} caused by lower social-goods production, we obtain $dR/dw \geq 0$. Thus, although the positive wage effect on the optimal uniform transfer is not guaranteed in the fully general model, it arises under an economically plausible set of conditions.

The condition $dh^I/dw > 0$ is sufficient but not necessary. At high wage levels, the income effect may dominate the substitution effect, so that work hours decline as the wage increases. If this decline is moderate, such that

$$\frac{dh^I}{dw} > -\frac{h^I}{w},$$

or, in words, the uncompensated wage elasticity of labor supply is higher than -1, consumption continues to increase. The resulting decline in marginal utility then exerts a positive effect on the optimal uniform transfer, which may, together with an increase in R, outweigh the negative effect of lower work hours. By contrast, if work hours decline sufficiently strongly or if the effectiveness of taxation falls substantially, the optimal uniform transfer may decrease with the wage. If the reduction in work hours is so pronounced that consumption also falls, the marginal-utility effect reinforces this negative relationship. Hence, the model allows the relationship between b^* and w to become negative at high wage levels, but does not predict that it must do so.

4 Conclusion

We have enriched the classic model of labor supply and income taxation by relaxing the standard assumption that leisure is just a good that people enjoy. Instead, we have assumed that people may choose to spend part of their leisure time to produce social goods—nonpriced goods that yield positive externalities for other people or for society at large. We have seen that this novel feature has important consequences for optimal policy. Even when a lump-sum tax is available, a social-welfare maximizing government optimally sets a positive income tax, so as to reduce work hours and increase the production of social goods. When the hourly wage becomes sufficiently high (and some mild conditions are satisfied), the lump-sum tax turns into a basic income. Our model is thus consistent with the fact that the basic income has so far not been implemented at scale, but at the same time predicts that at some point it might be implemented, if wages continue to rise as they did in the past decades and if the economically plausible conditions derived in the previous section hold. Our model's predictions are also well in line with the results of randomized experiments on the effects of the basic income on work hours and social activities (Bernhard et al., 2025; Bohmann et al., 2026; Riddell and Riddell, 2026; Vivalt et al., 2025), as discussed in the Introduction.

While our model quite generally captures essential features of labor supply and public policy, we can think of at least two omissions. The first is the demand for public goods. We have simply assumed that the amount of public goods g is exogenously given. Making the amount of public goods an additional policy choice variable of the government may weaken our result that a positive basic income at some point arises as wages continue to grow. Indeed, when public goods are a normal or luxury good, the demand for public goods will increase with economic prosperity (which is measured by the wage in our economy), implying that it becomes less likely that the income tax revenues at some point exceed the expenditures on public goods, which is necessary for the basic income to arise. The evidence on the income elasticity of demand for public goods is quite mixed, however, with most studies finding rather modest elasticities (Ram, 1987; Durevall and Henrekson, 2011) and some even negative (Shelton, 2007), instead of an elasticity greater than one as predicted by Wagner (1890)'s law.

A second omission is the lack of heterogeneity of individuals in our model. Heterogeneity (e.g. of ability or preferences) would generally give rise to a redistributive role for taxation and transfers, in addition to the allocative role studied here. We leave it for further research to explore how the redistributive and allocative roles interact in the determination of optimal taxes and transfers.

References

Afa, D., Füllbrunn, S., Bohn, F., Van Alebeek, R., Pitz, T., and Sickmann, J. (2026). Universal basic income: Consumption, human capital, and labor supply. Technical report, Radboud University Nijmegen.

Aronsson, T. and Johansson-Stenman, O. (2013). Conspicuous leisure: Optimal income taxation when both relative consumption and relative leisure matter. *The Scandinavian Journal of Economics*, 115(1):155–175.

Becker, G. S. (1965). A theory of the allocation of time. *The Economic Journal*, 75(299):493–517.

Bernhard, S., Bohmann, S., Fiedler, S., Kasy, M., Schupp, J., and Schwerter, F. (2025). Basic income and labor supply: Evidence from an RCT in Germany. *CESifo Working Papers No. 11940, CESifo, Munich.*

Bohmann, S., Fiedler, S., Kasy, M., Schupp, J., and Schwerter, F. (2026). Cash transfers, mental health and agency: Evidence from an RCT in Germany. *CESifo Working Papers No. 11989, CESifo, Munich.*

Bowles, S. and Carlin, W. (2026). The governance triangle: Economic interactions in civil society, the state, and the market. *Annual Review of Economics*. Forthcoming.

Chen, Y., Chien, Y., Wen, Y., and Yang, C. (2021). Are unconditional lump-sum transfers a good idea? *Economics Letters*, 209:110088.

Daruich, D. and Fernández, R. (2024). Universal basic income: A dynamic assessment. *American Economic Review*, 114(1):38–88.

Durevall, D. and Henrekson, M. (2011). The futile quest for a grand explanation of long-run government expenditure. *Journal of Public Economics*, 95(7-8):708–722.

Escobar-Posada, R. A. and Monteiro, G. (2017). Optimal tax policy in the presence of productive, consumption, and leisure externalities. *Economics Letters*, 152:62–65.

Frijters, P. and Leigh, A. (2008). Materialism on the march: From conspicuous leisure to conspicuous consumption? *The Journal of Socio-Economics*, 37(5):1937–1945.

Ghatak, M. and Maniquet, F. (2019). Universal basic income: Some theoretical aspects. *Annual Review of Economics*, 11(1):895–928.

Gronau, R. (1977). Leisure, home production, and work—The theory of the allocation of time revisited. *Journal of Political Economy*, 85(6):1099–1123.

Kelle, N., Kausmann, C., and Simonson, J. (2024). Time to volunteer: Changing determinants and correlates for time contributions to voluntary activities. *Voluntas: International Journal of Voluntary and Nonprofit Organizations*, 35(6):1219–1233.

Pintea, M. I. (2010). Leisure externalities: Implications for growth and welfare. *Journal of Macroeconomics*, 32(4):1025–1040.

Ram, R. (1987). Wagner's hypothesis in time-series and cross-section perspectives: Evidence from "real" data for 115 countries. *The Review of Economics and Statistics*, pages 194–204.

Riddell, C. and Riddell, W. C. (2026). What did we learn from the North American income maintenance experiments? New data and evidence on two-parent families. *Journal of Public Economics*, 259:105658.

Shelton, C. A. (2007). The size and composition of government expenditure. *Journal of Public Economics*, 91(11):2230–2260.

Taniguchi, H. (2012). The determinants of formal and informal volunteering: Evidence from the American Time Use Survey. *Voluntas: International Journal of Voluntary and Nonprofit Organizations*, 23(4):920–939.

Vivalt, E., Rhodes, E., Bartik, A. W., Broockman, D. E., Krause, P., and Miller, S. (2025). The employment effects of a guaranteed income: Experimental evidence from two US states. *Quarterly Journal of Economics*. Forthcoming.

Wagner, A. (1890). *Finanzwissenschaft*. C.F. Winter, Leipzig.

Appendix

Individual behavior: Derivation of $\frac{dh^I}{d\gamma}$ and $\frac{ds^I}{d\gamma}$. Totally differentiating the individual's first-order conditions,

$$u'(c^I)(1-t)w - K_h(h^I, s^I) = 0,$$
$$\gamma v'(s^I) - K_s(h^I, s^I) = 0.$$

with respect to γ yields

$$(1-t)^2 w^2 u''(c^I)\frac{dh^I}{d\gamma} - K_{hh}\frac{dh^I}{d\gamma} - K_{hs}\frac{ds^I}{d\gamma} = 0$$
$$v'(s^I) + \gamma v''(s^I)\frac{ds^I}{d\gamma} - K_{sh}\frac{dh^I}{d\gamma} - K_{ss}\frac{ds^I}{d\gamma} = 0.$$

Using symmetry, $K_{sh} = K_{hs}$, we obtain

$$\begin{pmatrix} \widetilde{K}_{hh} & K_{hs} \\ K_{hs} & \widetilde{K}_{ss} \end{pmatrix} \begin{pmatrix} \frac{dh^I}{d\gamma} \\ \frac{ds^I}{d\gamma} \end{pmatrix} = \begin{pmatrix} 0 \\ v'(s^I) \end{pmatrix},$$

where the first matrix on the left-hand side is the negative of the Hessian of the individual's objective function. Because the Hessian of K is positive definite and $u'' \leq 0$ and $v'' \leq 0$, this matrix is positive definite. Hence,

$$\widetilde{K}_{hh} := K_{hh} - (1-t)^2 w^2 u''(c^I) > 0, \quad \widetilde{K}_{ss} := K_{ss} - \gamma v''(s^I) > 0,$$

and

$$\widetilde{\Delta} := \widetilde{K}_{hh}\widetilde{K}_{ss} - K_{hs}^2 > 0.$$

Applying Cramer's rule gives

$$\frac{dh^I}{d\gamma} = -\frac{K_{hs}v'(s^I)}{\widetilde{\Delta}},$$
$$\frac{ds^I}{d\gamma} = \frac{\widetilde{K}_{hh}v'(s^I)}{\widetilde{\Delta}}.$$

Since $v'(s^I) > 0$, $K_{hs} > 0$, $\widetilde{K}_{hh} > 0$, and $\widetilde{\Delta} > 0$, it follows that $\frac{dh^I}{d\gamma} < 0$ and $\frac{ds^I}{d\gamma} > 0$.

Individual behavior: Derivation of $\frac{dh^I}{dw}$ and $\frac{ds^I}{dw}$. After a wage increase, disposable income changes according to

$$\frac{dc^I}{dw} = (1-t)\left(h^I + w\frac{dh^I}{dw}\right).$$

Totally differentiating the first first-order condition with respect to w yields

$$(1-t)u'(c^I) + (1-t)wu''(c^I)\frac{dc^I}{dw} - K_{hh}\frac{dh^I}{dw} - K_{hs}\frac{ds^I}{dw} = 0.$$

Substituting for dc^I/dw gives

$$(1-t)u'(c^I) + (1-t)^2 w u''(c^I)\left(h^I + w\frac{dh^I}{dw}\right) - K_{hh}\frac{dh^I}{dw} - K_{hs}\frac{ds^I}{dw} = 0.$$

Rearranging terms, we obtain

$$\left[K_{hh} - (1-t)^2 w^2 u''(c^I)\right]\frac{dh^I}{dw} + K_{hs}\frac{ds^I}{dw} = (1-t)\left[u'(c^I) + (1-t)wh^I u''(c^I)\right]. \qquad (4)$$

Totally differentiating the second first-order condition with respect to w yields

$$\gamma v''(s^I)\frac{ds^I}{dw} - K_{sh}\frac{dh^I}{dw} - K_{ss}\frac{ds^I}{dw} = 0.$$

Using symmetry, $K_{sh} = K_{hs}$, this can be rewritten as

$$K_{hs}\frac{dh^I}{dw} + \left[K_{ss} - \gamma v''(s^I)\right]\frac{ds^I}{dw} = 0. \qquad (5)$$

As above,

$$\widetilde{K}_{hh} := K_{hh} - (1-t)^2 w^2 u''(c^I)$$

and

$$\widetilde{K}_{ss} := K_{ss} - \gamma v''(s^I).$$

Equations (4) and (5) can then be written in matrix form as

$$\begin{pmatrix} \widetilde{K}_{hh} & K_{hs} \\ K_{hs} & \widetilde{K}_{ss} \end{pmatrix}\begin{pmatrix} \dfrac{dh^I}{dw} \\ \dfrac{ds^I}{dw} \end{pmatrix} = \begin{pmatrix} (1-t)\left[u'(c^I) + (1-t)wh^I u''(c^I)\right] \\ 0 \end{pmatrix}.$$

As before, let

$$\widetilde{\Delta} := \widetilde{K}_{hh}\widetilde{K}_{ss} - K_{hs}^2.$$

By Cramer's rule,

$$\frac{dh^I}{dw} = \frac{(1-t)\left[u'(c^I) + (1-t)wh^I u''(c^I)\right]\widetilde{K}_{ss}}{\widetilde{\Delta}}, \qquad (6)$$

$$\frac{ds^I}{dw} = -\frac{(1-t)\left[u'(c^I) + (1-t)wh^I u''(c^I)\right]K_{hs}}{\widetilde{\Delta}}. \qquad (7)$$

Because $K_{hs} > 0$, $\widetilde{K}_{ss} > 0$, and $\widetilde{\Delta} > 0$, work hours and social-goods production always move in opposite directions in response to a wage increase. Their direction is determined by the sign of

$$u'(c^I) + (1-t)wh^I u''(c^I).$$

If $u'(c^I) + (1-t)wh^I u''(c^I) > 0$, the substitution effect dominates the income effect, and hence $\frac{dh^I}{dw} > 0$ and $\frac{ds^I}{dw} < 0$. If instead $u'(c^I) + (1-t)wh^I u''(c^I) < 0$, the income effect dominates the substitution effect, and hence $\frac{dh^I}{dw} < 0$ and $\frac{ds^I}{dw} > 0$.

Proof of Lemma 1. (i) We consider the effect of an increase in the uniform transfer b, holding the tax rate t fixed. Since work hours respond to the uniform transfer, consumption changes according to

$$\frac{dc^I}{db} = (1-t)w\frac{dh^I}{db} + 1.$$

Totally differentiating the individual's first first-order condition with respect to b yields

$$(1-t)wu''(c^I)\left[1 + (1-t)w\frac{dh^I}{db}\right] - K_{hh}\frac{dh^I}{db} - K_{hs}\frac{ds^I}{db} = 0.$$

Rearranging gives

$$\left[K_{hh} - (1-t)^2 w^2 u''(c^I)\right]\frac{dh^I}{db} + K_{hs}\frac{ds^I}{db} = (1-t)wu''(c^I). \tag{8}$$

Totally differentiating the individual's second first-order condition with respect to b yields

$$K_{hs}\frac{dh^I}{db} + \left[K_{ss} - \gamma v''(s^I)\right]\frac{ds^I}{db} = 0. \tag{9}$$

Using the definitions

$$\widetilde{K}_{hh} := K_{hh} - (1-t)^2 w^2 u''(c^I), \qquad \widetilde{K}_{ss} := K_{ss} - \gamma v''(s^I),$$

the system can be written as

$$\begin{pmatrix} \widetilde{K}_{hh} & K_{hs} \\ K_{hs} & \widetilde{K}_{ss} \end{pmatrix}\begin{pmatrix} \frac{dh^I}{db} \\ \frac{ds^I}{db} \end{pmatrix} = \begin{pmatrix} (1-t)wu''(c^I) \\ 0 \end{pmatrix}.$$

Let

$$\widetilde{\Delta} := \widetilde{K}_{hh}\widetilde{K}_{ss} - K_{hs}^2 > 0.$$

By Cramer's rule,

$$\frac{dh^I}{db} = \frac{(1-t)wu''(c^I)\widetilde{K}_{ss}}{\widetilde{\Delta}},$$
$$\frac{ds^I}{db} = -\frac{(1-t)wu''(c^I)K_{hs}}{\widetilde{\Delta}}.$$

Since $u''(c^I) \leq 0$, $K_{hs} > 0$, $\widetilde{K}_{ss} > 0$, and $\widetilde{\Delta} > 0$, it follows that

$$\frac{dh^I}{db} \leq 0 \qquad \text{and} \qquad \frac{ds^I}{db} \geq 0,$$

where the inequalities are strict if $u''(c^I) < 0$.

(ii) We consider an increase in t, holding b fixed. Since work hours respond to the tax rate, consumption changes according to

$$\frac{dc^I}{dt} = -wh^I + (1-t)w\frac{dh^I}{dt}.$$

Totally differentiating the individual's first first-order condition with respect to t yields

$$(1-t)wu''(c^I)\left[-wh^I + (1-t)w\frac{dh^I}{dt}\right] - wu'(c^I) - K_{hh}\frac{dh^I}{dt} - K_{hs}\frac{ds^I}{dt} = 0.$$

Rearranging gives

$$\left[K_{hh} - (1-t)^2w^2u''(c^I)\right]\frac{dh^I}{dt} + K_{hs}\frac{ds^I}{dt} = -w\left[u'(c^I) + (1-t)wh^Iu''(c^I)\right]. \tag{10}$$

Totally differentiating the individual's second first-order condition with respect to t yields

$$K_{hs}\frac{dh^I}{dt} + \left[K_{ss} - \gamma v''(s^I)\right]\frac{ds^I}{dt} = 0. \tag{11}$$

As above, let

$$\widetilde{K}_{hh} := K_{hh} - (1-t)^2w^2u''(c^I), \qquad \widetilde{K}_{ss} := K_{ss} - \gamma v''(s^I),$$

and

$$\widetilde{\Delta} := \widetilde{K}_{hh}\widetilde{K}_{ss} - K_{hs}^2.$$

Equations (10) and (11) can then be written as

$$\begin{pmatrix} \widetilde{K}_{hh} & K_{hs} \\ K_{hs} & \widetilde{K}_{ss} \end{pmatrix}\begin{pmatrix} \frac{dh^I}{dt} \\ \frac{ds^I}{dt} \end{pmatrix} = \begin{pmatrix} -w\left[u'(c^I) + (1-t)wh^Iu''(c^I)\right] \\ 0 \end{pmatrix}.$$

By Cramer's rule,

$$\frac{dh^I}{dt} = -\frac{w\left[u'(c^I) + (1-t)wh^Iu''(c^I)\right]\widetilde{K}_{ss}}{\widetilde{\Delta}}, \tag{12}$$

$$\frac{ds^I}{dt} = \frac{w\left[u'(c^I) + (1-t)wh^Iu''(c^I)\right]K_{hs}}{\widetilde{\Delta}}. \tag{13}$$

Thus, if $u'(c^I) + (1-t)wh^Iu''(c^I) > 0$, then $\frac{dh^I}{dt} < 0$ and $\frac{ds^I}{dt} > 0$. If instead $u'(c^I) + (1-t)wh^Iu''(c^I) < 0$, the signs are reversed. □

Proof of Proposition 1. Under a balanced government budget, consumption at any symmetric allocation is

$$c = wh - \frac{g}{n}.$$

The individual allocation (h^I, s^I) satisfies

$$u'(c^I)(1-t)w - K_h(h^I, s^I) = 0, \tag{14}$$

$$\gamma v'(s^I) - K_s(h^I, s^I) = 0, \tag{15}$$

where $c^I = wh^I - g/n$. Because individuals do not internalize the social externality, (h^I, s^I) does not depend on μ.

Let $\widehat{s} > s^I$ satisfy

$$K_h(h^I, \widehat{s}) - K_h(h^I, s^I) = u'(c^I)tw.$$

Since $K_{hs} > 0$, the left-hand side is strictly increasing in $\widehat{s}$. Hence, if such a value $\widehat{s}$ exists, it is unique. Combining its definition with (14) gives

$$K_h(h^I, \widehat{s}) = K_h(h^I, s^I) + u'(c^I)tw = u'(c^I)(1-t)w + u'(c^I)tw = u'(c^I)w.$$

It follows that $(h^I, \widehat{s})$ satisfies the planner's first-order condition for work hours.

Now define

$$\widehat{\mu} = K_s(h^I, \widehat{s}) - \gamma v'(\widehat{s}).$$

By construction,

$$\gamma v'(\widehat{s}) + \widehat{\mu} - K_s(h^I, \widehat{s}) = 0,$$

so $(h^I, \widehat{s})$ also satisfies the planner's first-order condition for social-goods production when $\mu = \widehat{\mu}$. Because the planner's objective is strictly concave, these conditions are sufficient and characterize its unique optimum. Therefore, $h^P(\widehat{\mu}) = h^I$ and $s^P(\widehat{\mu}) = \widehat{s}$.

Notice also that $\widehat{\mu} > 0$. Using (15), we can write

$$\widehat{\mu} = K_s(h^I, \widehat{s}) - K_s(h^I, s^I) + \gamma\big[v'(s^I) - v'(\widehat{s})\big].$$

The first term is strictly positive because $\widehat{s} > s^I$ and $K_{ss} > 0$, while the second term is nonnegative because v is concave.

It remains to establish how the planner's work-hours choice varies with μ. The planner's first-order conditions are

$$u'(c^P)w - K_h(h^P, s^P) = 0, \tag{16}$$

$$\gamma v'(s^P) + \mu - K_s(h^P, s^P) = 0, \tag{17}$$

where $c^P = wh^P - g/n$. Totally differentiating (16)–(17) with respect to μ gives

$$\begin{pmatrix} A^P & K_{hs} \\ K_{hs} & D^P \end{pmatrix} \begin{pmatrix} dh^P/d\mu \\ ds^P/d\mu \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

where all derivatives are evaluated at (h^P, s^P) and

$$A^P = K_{hh} - w^2 u''(c^P) > 0, \qquad D^P = K_{ss} - \gamma v''(s^P) > 0.$$

Let

$$\Delta^P = A^P D^P - K_{hs}^2.$$

Because $u'' \leq 0$ and $v'' \leq 0$,

$$A^P \geq K_{hh}, \qquad D^P \geq K_{ss}.$$

Hence, the positive definiteness of the Hessian of K implies

$$\Delta^P = A^P D^P - K_{hs}^2 \geq K_{hh} K_{ss} - K_{hs}^2 > 0.$$

By Cramer's rule,

$$\frac{dh^P}{d\mu} = -\frac{K_{hs}}{\Delta^P} < 0.$$

Thus, the planner's optimal work hours are strictly decreasing in μ, whereas the individual's work hours are independent of μ. Since $h^P(\widehat{\mu}) = h^I$, it follows that $h^I > h^P \iff \mu > \widehat{\mu}$. The strict monotonicity of $h^P(\mu)$ also establishes the uniqueness of the threshold.

By Cramer's rule, we also obtain

$$\frac{ds^P}{d\mu} = \frac{A^P}{\Delta^P} > 0.$$

Thus, the planner's social-goods production is strictly increasing in μ.

We next establish the existence and uniqueness of a threshold $\widetilde{\mu} \in (0, \widehat{\mu})$ at which $s^P = s^I$. At $\mu = \widehat{\mu}$, we have already shown that $s^P(\widehat{\mu}) = \widehat{s} > s^I$. It remains to compare $s^P(0)$ and s^I. Since $h^P(\widehat{\mu}) = h^I$, $\widehat{\mu} > 0$, and $h^P(\mu)$ is strictly decreasing in μ, it follows that $h^P(0) > h^I$.

When $\mu = 0$, the planner's first-order condition for social-goods production coincides with the individual's corresponding first-order condition:

$$\gamma v'(s) - K_s(h, s) = 0.$$

This condition implicitly defines social-goods production as a function of work hours, $s = s(h)$. By the implicit function theorem,

$$\frac{ds}{dh} = -\frac{K_{hs}(h, s)}{K_{ss}(h, s) - \gamma v''(s)} < 0.$$

Thus, among allocations satisfying this first-order condition, higher work hours are associated with lower social-goods production. Since both $(h^P(0), s^P(0))$ and (h^I, s^I) satisfy the condition and $h^P(0) > h^I$, it follows that $s^P(0) < s^I$. We have thus shown that

$$s^P(0) < s^I \qquad \text{and} \qquad s^P(\widehat{\mu}) = \widehat{s} > s^I.$$

Because $s^P(\mu)$ is continuous and strictly increasing in μ, there exists a unique $\widetilde{\mu} \in (0, \widehat{\mu})$ such that $s^P(\widetilde{\mu}) = s^I$. Consequently, $s^P(\mu) > s^I \iff \mu > \widetilde{\mu}$.

Combining the two threshold comparisons completes the proof. $\square$

Proof of Lemma 2. Along the balanced-budget path, the government budget constraint implies

$$b(t) = twh^I(t) - \frac{g}{n}. \tag{18}$$

Hence, equilibrium consumption is

$$c^I(t) = (1-t)wh^I(t) + b(t) = wh^I(t) - \frac{g}{n},$$

and therefore

$$\frac{dc^I}{dt} = w\frac{dh^I}{dt}.$$

Recall that the individual's first-order conditions are

$$u'(c^I)(1-t)w - K_h(h^I, s^I) = 0, \tag{19}$$

$$\gamma v'(s^I) - K_s(h^I, s^I) = 0. \tag{20}$$

Totally differentiating (19) along the balanced-budget path yields

$$u''(c^I)(1-t)w^2\frac{dh^I}{dt} - wu'(c^I) - K_{hh}(h^I, s^I)\frac{dh^I}{dt} - K_{hs}(h^I, s^I)\frac{ds^I}{dt} = 0.$$

Thus,

$$A\frac{dh^I}{dt} + K_{hs}(h^I, s^I)\frac{ds^I}{dt} = -wu'(c^I), \qquad A \equiv K_{hh}(h^I, s^I) - (1-t)w^2u''(c^I) > 0. \tag{21}$$

Similarly, totally differentiating (20) gives

$$K_{hs}(h^I, s^I)\frac{dh^I}{dt} + D\frac{ds^I}{dt} = 0, \qquad D \equiv K_{ss}(h^I, s^I) - \gamma v''(s^I) > 0. \tag{22}$$

Equations (21)–(22) can be written as

$$\begin{pmatrix} A & K_{hs} \\ K_{hs} & D \end{pmatrix} \begin{pmatrix} dh^I/dt \\ ds^I/dt \end{pmatrix} = \begin{pmatrix} -wu'(c^I) \\ 0 \end{pmatrix}.$$

Let

$$\Delta \equiv AD - K_{hs}^2.$$

Because $u'' \leq 0$, $v'' \leq 0$, $t \leq 1$,

$$A \geq K_{hh}, \qquad D \geq K_{ss},$$

and therefore

$$\Delta \geq K_{hh}K_{ss} - K_{hs}^2 > 0,$$

where the latter inequality follows because the Hessian of K is positive definite.

By Cramer's rule,

$$\frac{dh^I}{dt} = -\frac{wu'(c^I)D}{\Delta} < 0$$

and

$$\frac{ds^I}{dt} = \frac{wu'(c^I)K_{hs}(h^I, s^I)}{\Delta} > 0,$$

where the second inequality uses $K_{hs}(h^I, s^I) > 0$. Thus, along the balanced-budget path, an increase in the income tax rate reduces work hours and increases social-goods production.

Finally, differentiating (18) with respect to t yields

$$\frac{db(t)}{dt} = w\left(h^I(t) + t\frac{dh^I(t)}{dt}\right).$$

Hence, the uniform transfer increases with the tax rate if

$$h^I(t) + t\frac{dh^I(t)}{dt} > 0,$$

remains unchanged if this expression equals zero, and decreases if it is negative. This proves the second part of the lemma. □

Proof of Proposition 2. The proposition directly follows from equations (2) and (3) derived in the main text. □

Proof of Proposition 3. Suppose that v is linear, so that $v'' = 0$, that K_{ss} and K_{hs} are constant, and that $\mu > 0$. (i) We show that the poll tax or basic income chosen under the optimal government policy, b^*, is strictly increasing in the wage. Define

$$R := \frac{K_{hs}}{K_{ss}} > 0.$$

From (2), the optimal tax rate satisfies

$$u'(c^I(t^*))t^*w = \mu R.$$

Combining this condition with the individual's first-order conditions,

$$u'(c^I(t^*))(1-t^*)w = K_h(h^I(t^*), s^I(t^*)) \qquad \text{and} \qquad \gamma = K_s(h^I(t^*), s^I(t^*)),$$

we obtain the following conditions characterizing the allocation under the optimal government policy:

$$wu'(c^I(t^*)) = K_h(h^I(t^*), s^I(t^*)) + \mu R, \tag{23}$$

$$\gamma = K_s(h^I(t^*), s^I(t^*)). \tag{24}$$

We first determine how the optimally induced allocation responds to an increase in w. In what follows, dh^I/dw and ds^I/dw denote total derivatives along the optimal-policy path and thus include the adjustment of the optimal tax rate t^* to the wage. Since

$$\frac{dc^I}{dw} = h^I + w\frac{dh^I}{dw},$$

total differentiation of (23) gives

$$u'(c^I) + wu''(c^I)\left(h^I + w\frac{dh^I}{dw}\right) = K_{hh}\frac{dh^I}{dw} + K_{hs}\frac{ds^I}{dw}.$$

Rearranging,

$$\left[K_{hh} - w^2u''(c^I)\right]\frac{dh^I}{dw} + K_{hs}\frac{ds^I}{dw} = u'(c^I) + wh^Iu''(c^I).$$

Total differentiation of (24) gives

$$K_{hs}\frac{dh^I}{dw} + K_{ss}\frac{ds^I}{dw} = 0,$$

and hence

$$\frac{ds^I}{dw} = -\frac{K_{hs}}{K_{ss}}\frac{dh^I}{dw} = -R\frac{dh^I}{dw}.$$

Substitution yields

$$\frac{dh^I}{dw} = \frac{u'(c^I) + wh^Iu''(c^I)}{D},$$

where

$$D := K_{hh} - \frac{K_{hs}^2}{K_{ss}} - w^2u''(c^I).$$

Positive definiteness of the Hessian of K implies

$$K_{hh} - \frac{K_{hs}^2}{K_{ss}} = \frac{K_{hh}K_{ss} - K_{hs}^2}{K_{ss}} > 0.$$

Since $u''(c^I) \leq 0$, it follows that $D > 0$. Notice that the sign of dh^I/dw is nevertheless ambiguous because the income effect may imply

$$u'(c^I) + wh^Iu''(c^I) < 0.$$

Using equation (3),

$$b^* = t^*wh^I(t^*) - \frac{g}{n} = \frac{\mu Rh^I(t^*)}{u'(c^I(t^*))} - \frac{g}{n}.$$

Differentiating with respect to w, taking into account the optimal adjustment of t, gives

$$\frac{db^*}{dw} = \frac{\mu R}{u'(c^I)}\left[\frac{dh^I}{dw} - h^I\frac{u''(c^I)}{u'(c^I)}\frac{dc^I}{dw}\right].$$

Using the expression for dh^I/dw, the response of consumption can first be written as

$$\begin{aligned}\frac{dc^I}{dw} &= h^I + w\frac{dh^I}{dw} \\ &= h^I + w\frac{u'(c^I) + wh^I u''(c^I)}{D} \\ &= \frac{h^I\left[K_{hh} - \frac{K_{hs}^2}{K_{ss}} - w^2 u''(c^I)\right] + wu'(c^I) + w^2 h^I u''(c^I)}{D} \\ &= \frac{h^I\left(K_{hh} - \frac{K_{hs}^2}{K_{ss}}\right) + wu'(c^I)}{D}.\end{aligned}$$

Substituting the expressions for dh^I/dw and dc^I/dw, we obtain

$$\begin{aligned}\frac{db^*}{dw} &= \frac{\mu R}{u'(c^I)D}\left[u'(c^I) + wh^I u''(c^I) - h^I\frac{u''(c^I)}{u'(c^I)}\left\{h^I\left(K_{hh} - \frac{K_{hs}^2}{K_{ss}}\right) + wu'(c^I)\right\}\right] \\ &= \frac{\mu R}{u'(c^I)D}\left[u'(c^I) + wh^I u''(c^I) - (h^I)^2\frac{u''(c^I)}{u'(c^I)}\left(K_{hh} - \frac{K_{hs}^2}{K_{ss}}\right) - wh^I u''(c^I)\right] \\ &= \frac{\mu R}{u'(c^I)D}\left[u'(c^I) - (h^I)^2\frac{u''(c^I)}{u'(c^I)}\left(K_{hh} - \frac{K_{hs}^2}{K_{ss}}\right)\right].\end{aligned}$$

Because $u'(c^I) > 0$, $u''(c^I) \leq 0$, and positive definiteness of the Hessian of K implies

$$K_{hh} - \frac{K_{hs}^2}{K_{ss}} > 0,$$

it follows that

$$\frac{db^*}{dw} > 0.$$

(ii) We now compare the allocation induced by the optimal government policy, $(h^I(t^*), s^I(t^*))$, with the socially efficient allocation, (h^P, s^P). From equations (23) and (24), the allocation induced by the optimal government policy satisfies

$$wu'(c^I(t^*)) = K_h(h^I(t^*), s^I(t^*)) + \mu R, \tag{25}$$

$$\gamma = K_s(h^I(t^*), s^I(t^*)), \tag{26}$$

where

$$R = \frac{K_{hs}}{K_{ss}} \qquad \text{and} \qquad c^I(t^*) = wh^I(t^*) - \frac{g}{n}.$$

Because $v(s) = s$, the social planner's first-order conditions are

$$u'(c^P)w = K_h(h^P, s^P), \tag{27}$$

$$\gamma + \mu = K_s(h^P, s^P), \tag{28}$$

where

$$c^P = wh^P - \frac{g}{n}.$$

Define

$$\widehat{s} := s^I(t^*) + \frac{\mu}{K_{ss}}.$$

Because K_{ss} is constant, we have

$$\begin{aligned}
K_s(h^I(t^*), \widehat{s}) &= K_s(h^I(t^*), s^I(t^*)) + K_{ss}\left(\widehat{s} - s^I(t^*)\right) \\
&= \gamma + K_{ss}\frac{\mu}{K_{ss}} \\
&= \gamma + \mu.
\end{aligned}$$

Thus, $(h^I(t^*), \widehat{s})$ satisfies the social planner's first-order condition for social-goods production.

Moreover, because K_{hs} is constant,

$$\begin{aligned}
K_h(h^I(t^*), \widehat{s}) &= K_h(h^I(t^*), s^I(t^*)) + K_{hs}\left(\widehat{s} - s^I(t^*)\right) \\
&= K_h(h^I(t^*), s^I(t^*)) + \mu\frac{K_{hs}}{K_{ss}} \\
&= K_h(h^I(t^*), s^I(t^*)) + \mu R \\
&= wu'(c^I(t^*)),
\end{aligned}$$

where the last equality follows from (25). Because consumption along the balanced-budget path depends only on work hours, the pair $(h^I(t^*), \widehat{s})$ also satisfies the social planner's first-order condition for work hours.

Hence, $(h^I(t^*), \widehat{s})$ satisfies both first-order conditions of the social planner. Strict concavity of the social planner's objective function implies that the socially efficient allocation is unique. Therefore,

$$h^I(t^*) = h^P \qquad \text{and} \qquad s^P = \widehat{s} = s^I(t^*) + \frac{\mu}{K_{ss}}.$$

It follows that $s^I(t^*) < s^P$. $\square$