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Road Networks and Tropical Deforestation

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Abstract

We analyze a dynamic spatial model of deforestation in which roads facilitate agricultural expansion. The model's key innovation is that road construction costs decrease with the existing road stock, creating positive network externalities that are specific to infrastructure: each road built serves as a stepping stone for further road construction. Under open access, myopic producers fail to internalize that roads built today facilitate future road construction. As a result, the pace of deforestation is faster under a forward-looking regime of private property than open access, reversing a standard result from resource economics. We calibrate the model to the Brazilian Amazon, where cattle ranching is the primary driver of deforestation, and calculate the international transfer required to incentivize Brazil to implement a carbon tax that completely stops deforestation. We find that Brazil's extensive existing road network increases this required transfer, demonstrating how infrastructure creates "deforestation lock-in" that makes conservation progressively more expensive.

JEL codes: Q15, Q23, Q24, Q54, R12, R13.
Keywords: deforestation, agriculture, climate change, property rights, roads.

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1 Introduction

Between 2010 and 2020 the world lost 98 million hectares of tropical rainforest, an area roughly twice the size of France (FAO, 2020).[1] Deforestation is costly, not least because each year it is responsible for about 10 percent of global CO_2 emissions, with an even higher percentage for other greenhouse gases (Harris et al., 2012; Baccini et al., 2012).[2] In 1950, rainforests covered 14 percent of the earth's land surface area. Today only 6 percent remains, and if the current rate of tropical deforestation continues, some rainforests may disappear in less than a century (Tyukavina et al., 2018; Vidal, 2017).[3]

The driving force behind tropical deforestation is the desire to make room for farming, cattle ranching, timber production, and mining. These activities, however, would be impossible without the construction of vast road networks that enable producers to access forest reserves at reasonable cost. Because a large share of soybeans, beef, and timber is typically destined for international markets (cf. Hoang and Kanemoto, 2021), this creates a demand for road construction to connect rainforest production locations and ports. Thus, while roads enable international and intranational trade, and hence promote economic development, they also facilitate deforestation.[4] [5]

Against this backdrop of ongoing tropical deforestation mediated by roads, we analyze a dynamic, spatial economic model with endogenous road construction and deforestation under different property rights regimes. While conversion of rainforests into farmland generates a negative environmental externality through carbon emissions and, possibly, lost ecosystem services, the novelty of our framework lies in recognizing that road construction itself also creates positive network externalities. Specifically, myopic producers fail to internalize that roads built today constitute an input for future road construction, thereby creating a dynamic externality. This stylized framework is used to address several questions. First, how do road networks and deforestation evolve over time under different property rights regimes? Second, after calibrating the model to Brazilian Amazon data, how do counterfactual

[1]Globally, net forest loss was about half that amount, due to substantial reforestation in some parts of the world especially China (FAO, 2020).

[2]On top of its importance for global warming, deforestation, especially in the tropical biome, also negatively affects biodiversity and diminishes wildlife populations, due to habitat destruction.

[3]The situation may be more dire, however, than a linear extrapolation of current trends suggests. Boulton et al. (2022) find that due to climate change and deforestation three-quarters of the Amazon rainforest has lost resilience since the early 2000s, which may put it dangerously close to a threshold of rainforest dieback, where the forest will turn into savanna.

[4]New roads do not necessarily increase welfare as (1) planning of new road projects is often guided by inaccurate or incomplete (cost-benefit) assessments and (2) development of other basic infrastructure such as sanitation is often lagging behind road construction (cf. Van Dijck, 2014).

[5]Roads and other infrastructure also drive forest fragmentation: by 2100 half of all tropical forest area is predicted to be less than 100 meters away from a forest boundary (Fischer et al., 2021).

carbon taxes affect deforestation dynamics? Third, do existing road networks create carbon lock-in effects—long-lasting commitments to carbon-intensive infrastructure and land-use patterns that make it difficult and costly to transition to lower-emission alternatives—by lowering the costs of future road construction and land conversion?

Our spatial model starts from the premise that roads are a necessary condition to gain access to forest reserves. There exists a two-dimensional plane with a dimensionless city or economic core in the middle, which is surrounded by a rainforest that takes the shape of a circle. To expand production, agricultural producers and loggers can invest in new roads to gain access to land that was previously unreachable. In our spatial setting, the road network is assumed to expand in the form of a circle that grows from the city into the rainforest.[6] Each period the quantity of new forest reserves the producers (or road developers) gain access to is proportional to the quantity of roads built in that period. Once the road is completed, producers can then start agricultural production or rent the land to farmers.[7] [8]

In our framework aggregate road production depends not only on the input of labor, but also on the size or total length of the existing local road network around the city: the longer the existing road network (in kilometers) is today, the larger the potential expansion of roads tomorrow. In cost terms, this means that the costs of road construction decline with the size of the road network. This dependence of road construction costs on the stock of roads arises from two assumptions. First, the construction of a new road always starts from the end of an existing road, such that each road built acts as a stepping stone − borrowing terminology from Harstad (2020) − for building more roads. Second, there are diminishing returns to each road project such that workers become more productive if they can work on multiple roads simultaneously.

We start out by characterizing outcomes with exogenous government policies under two different property rights regimes. In the first case, property rights to road construction and land conversion are allocated to a representative (but regu-

[6]In Supplementary Appendix B.2 we explain that this particular road construction process can result from a set of reasonable assumptions on a discrete grid where producers constructing a single road segment face diminishing returns to labor, thus making it more efficient to work on multiple roads simultaneously in all possible directions rather than extending roads in one direction. We compare the discrete and continuous space grids in Supplementary Appendix B.3.

[7]For simplicity, we assume initially that marginal costs of transportation from forest reserves to the market are zero. See Anderson et al. (2016) for a stylized model of deforestation in the tradition of the Von Thünen model in which transportation costs play a role in driving deforestation patterns and the location of conservation zones. Section 5 discusses an extension of our model with transportation costs.

[8]Our analysis abstracts from both technological change and heterogeneity in agricultural productivity. In practice, agricultural productivity evolves over time and varies substantially across regions due to differences in soil and climatic conditions, as well as the adoption of new technologies (e.g., Bustos et al. (2016)). These forces can generate heterogeneous responses in land use and sectoral allocation that are not captured in our framework.

lated) firm or monopolist, who decides on the optimal path of road construction and deforestation to maximize the present discounted value of income from agricultural production.[9] We also consider the alternative of open access. In this second scenario there is no initial allocation of property rights over road construction nor over land. There exists a large number of potential developers who can enter freely into road construction, and the government grants property rights to land to whomever completes a road. Van Dijck (2014) describes how during the 1970s government programs in Ecuador, Peru and especially Brazil encouraged deforestation as a requirement to obtain property rights to land.

Under open access, myopic decision-making guides road construction, giving rise to two market failures: first, with scarce land (i.e., a situation in which all land is eventually developed) and convex development costs, producers rush to develop although society would benefit from smoother development over time (i.e., a common pool resource problem); second, road producers do not internalize positive network externalities, as free entry into the road market causes them to ignore that roads produced today will increase the potential for road construction and exploitation tomorrow, creating a cascade effect that continues until the road network covers the entire forest.

Comparing the open access regime to private property, we find that when land is abundant (i.e., some land remains undeveloped in equilibrium), the pace of deforestation is unambiguously higher under private property than under open access. This reflects the fact that the positive road network externality is internalized by the forward-looking monopolist under private property, but not by the myopic road constructors under open access. This result flips conventional wisdom from classic resource economics, where private property typically prevents overexploitation (as in fishery management, see e.g. Gordon (1954), Hardin (1968)). However, with scarce land, the dynamics change because the common pool resource problem emerges. Whether the overall pace of deforestation is then faster or slower under open access than under private property depends on which effect dominates: the common pool problem that accelerates development under open access, or the network externality that speeds up development under private property. We show how these two counteracting forces shape the evolution of optimal taxes/subsidies on road construction over time, also accounting for a climate externality of deforestation.

We then calibrate our model to the Brazilian Amazon, where cattle ranching is the dominant driver of deforestation. In this quantitative application we focus on foreign demand instead of domestic demand for Brazilian beef. Our calibrated model serves two key purposes: first, we evaluate the impacts of hypothetical carbon taxes

[9] There are rules regarding the property rights that are granted to the road developer. These are explained in Supplementary Appendix B.

on deforestation rates; second, we analyze a scenario where developed nations offer Brazil a lump-sum payment in exchange for implementing an Amazon carbon tax that stops deforestation.

We find that a relatively modest carbon tax equal to Brazil's domestic cost of carbon (DCC) of about 2.96 USD/tCO2, or 54 thousand USD/km^2, is capable of reducing Brazil's cumulative projected deforestation by 40 percent. A tax of a little over 7 USD/tCO2, or about 135 thousand USD/km^2, eliminates deforestation in the Amazon altogether. Our results confirm the efficacy of relatively modest carbon taxes in limiting deforestation (cf. Souza-Rodrigues, 2019; Araujo et al., 2024).

Carbon taxes are particularly effective in this context because the trade-off between cattle ranching and conservation strongly favors conservation. Tropical rainforests store massive amounts of carbon, while cattle ranching on cleared land generates modest profits.[10] Consequently, we find that the minimum compensation developed nations would need to offer Brazil to halt deforestation entirely is small: approximately \$2.5 billion in our baseline scenario (and \$7 billion with a 5% rather than a 10% discount rate). Beyond the baseline transfer amount, our analysis also reveals the substantial cost of Brazil's existing road network.

Our quantitative analysis also highlights a lock-in mechanism. Because the road stock lowers the cost of additional expansion, an already-developed network raises the opportunity cost of conservation. When comparing a highly developed scenario with the undeveloped baseline, the transfer needed to halt deforestation goes up by 34% under open access and by 72% under private property.

Our contribution is best understood as complementary to the recent quantitative spatial models of infrastructure-mediated deforestation (Araujo et al., 2023; Gollin and Wolfersberger, 2024) and studies on optimal transport networks (Fajgelbaum and Schaal, 2020). Those frameworks incorporate rich spatial detail and general equilibrium effects—such as market access, comparative advantage, and spatial sorting—but employ static frameworks. By contrast, our model deliberately abstracts from spatial heterogeneity and general equilibrium market-access forces to focus on a dynamic mechanism through which infrastructure can accelerate frontier expansion.

This paper also contributes to the literature by developing a dynamic model of road investment, land conversion, and deforestation that generates realistic baseline deforestation projections. Existing structural and simulation models often feature implausibly high business-as-usual deforestation rates. Our model, calibrated to the Brazilian Amazon using a rich variety of data, including global beef demand

[10]Low profit margins (in our parametrized model) result from several factors: global beef demand has limits, ranchers are price takers rather than price setters, and the large stock of available rainforest in the Brazilian Amazon means supply can easily expand. In practice, ranching profitability in the Amazon is partly sustained by weak enforcement of environmental regulations (Balboni et al., 2023), which our model does not capture.

projections and satellite-derived time-series data on road density and temporal road dynamics, produces near-term projected deforestation rates comparable to historically observed rates. We demonstrate that carbon taxes are remarkably effective in our calibrated model. This realistic baseline enables credible counterfactual policy analysis, including potential comparisons between supply-side interventions (like road taxes) and demand-side policies (like meat taxes). The framework could be extended to incorporate time-inconsistent preferences or alternative drivers of road construction such as timber harvesting and connectivity motives (see Section 5).

Related literature. A growing scientific literature has established roads as a major proximate driver of habitat loss, overhunting of wildlife, and environmental degradation (Van Dijck, 2014; Alamgir et al., 2017; Laurance et al., 2014; Ibisch et al., 2016). In the Amazon rainforest, nearly 95 percent of all deforestation takes place within 5.5 km of roads or 1 km of rivers (cf. Barber et al., 2014).[11] Roads, however, may also have a positive role to play in preventing deforestation. First, they provide ecotourists with access to exotic natural environments, potentially boosting conservation. Second, they may also provide natural and strategic opportunities for tax collection or monitoring. Indeed, the World Bank recommends to collect environmental forestry taxes at choke points, i.e., locations where "illegally and informally sourced (or otherwise difficult-to-tax) goods enter a formal market structure" (Heine et al., 2021, p. 25). Such locations include timber aggregation points, border crossings and international ports.

The literature on the economics of deforestation is vast. Early empirical work, summarized by Angelsen and Kaimowitz (1999), Bulte and van Kooten (2000), and Barbier and Burgess (2001), among others, analyzed how country-level factors like GDP, population growth, and infrastructure access are related to deforestation using cross-country regressions. In a more recent review, Balboni et al. (2023) emphasize that contemporary analysis of tropical deforestation must grapple with multiple externalities, including soil erosion, biodiversity loss and climate change. This marks a shift from the simpler frameworks used in 20th century studies that revolved around optimal resource extraction and forest yield management. The problem is to find the right instruments to combat the large number of externalities. In this paper, we focus on policies that address both the network externalities in road construction and the climate externalities from the deforestation that roads enable.

Balboni et al. (2023) also argue that economic analysis has only recently started

[11] There are many ways of measuring deforestation and detecting new roads. For deforestation, remote sensing is the most widely used method nowadays. Examples are DETER (Real-Time Deforesation Detection System) in Brazil and the Global Forest Watch. As for roads, machine learning algorithms have recently successfully been applied to high-resolution satellite imagery to automate the detection and mapping of roads in the Brazilian Amazon (cf. Botelho Jr et al., 2022).

to catch up with the fast-moving land use changes revealed by satellite data with the aim of quantifying the forces that drive deforestation. This requires models that encompass growing pressures for land use change, national and international externalities, insecure property rights, and political economy challenges. Our quantitative model deals with these aspects.

Like we do, recent studies find that modest carbon or land taxes can substantially reduce Amazon deforestation. Souza-Rodrigues (2019) estimates that a land tax of 42.5 thousand USD per km^2 would reduce deforestation by 64%, while Araujo et al. (2024) find that carbon taxes of \$2.5-\$10 per tCO_2 would achieve 60-95% of the socially efficient reduction. Assunção et al. (2023) show that transfers of \$15-20 per tCO_2 would not only prevent the Amazon from crossing critical tipping points but would also trigger large-scale forest restoration. However, while our analysis indicates that a finite carbon tax (approximately $7/tCO_2$) can fully halt deforestation, both Souza-Rodrigues (2019) and Araujo et al. (2024) find that some deforestation persists even under high taxes. This difference arises from their use of logistic/logit functional forms, implying that only conversion can only be driven to zero asymptotically, as well as from firm heterogeneity, which ensures that some highly productive firms remain profitable.

Our work relates most directly to the literature on infrastructure-mediated deforestation, particularly recent contributions examining general equilibrium effects. Araujo et al. (2023) show that roads and railways affect deforestation far beyond their immediate vicinity by altering market access throughout the infrastructure network. Ignoring these general equilibrium effects underestimates deforestation impacts by 25% on average. Similarly, Gollin and Wolfersberger (2024) use a quantitative trade model to demonstrate that Brazil's road network expansion since the 1990's explains approximately 10% of total Amazon deforestation. While these studies incorporate rich spatial detail and general equilibrium effects, they employ static frameworks. In contrast, our paper examines how road network externalities shape deforestation dynamics. Our dynamic approach builds on earlier work by Hartwick et al. (2001), who model deforestation in a small open economy with road investment, but without the network externality that is central to our framework.

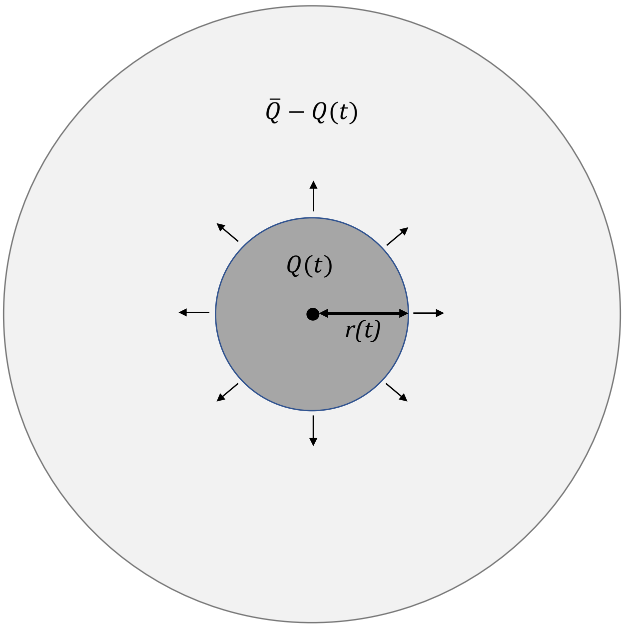
Outline. The paper is structured as follows. Section 2 presents our model and describes the main analytical results. Section 3 presents a quantitative application of the model, focusing on cattle ranching in the Brazilian Amazon. Section 4 examines the quantitative effects of a counterfactual carbon tax, conducts a sensitivity analysis, and quantifies the effects of existing roads on transfers required to stop deforestation. Section 5 discusses potential avenues for future research, including transportation costs and time-inconsistent preferences. Section 6 concludes.

2 The model

2.1 Set-up

There exists a dimensionless city or economic core in two-dimensional space, which is surrounded by a circular forest with surface area $\bar{Q}$, as shown in Figure 1 by the black dot and the grey-shaded circular area, respectively. The forest can be converted into agricultural land. However, it first needs to be made accessible by building a road network. We start from a situation in which already part of the forest is covered by roads. The dark-shaded road covered area (RCA) at time t is denoted by $Q(t)$, with radius $r(t)$ so that $r(t) = \sqrt{Q(t)/\pi}$. Hence, at each instant of time the light-shaded remaining pristine forest area uncovered by the road network is given by $\bar{Q} - Q(t)$.

Figure 1: Forest with road network.



Notes: The total forest area initially given is $\bar{Q}$. The road covered area (RCA) at time t is denoted by $Q(t)$ (the dark-shaded part of the circular area). The remaining size of the forest at this instant of time is $\bar{Q} - Q(t)$ (the light-shaded part of the circular area). Radial roads can be constructed to increase the RCA, as indicated by the arrows.

The rate of increase in the RCA at time t is given by the following production function:

$$q(t) = f\big(u(t), 2\pi r(t)\big) = f\Big(u(t), 2\sqrt{\pi Q(t)}\Big). \tag{1}$$

We make the following assumptions about the characteristics of the production function:

Assumption 1. *Let $q = f(u,s)$ with $s(Q) = 2\pi r = 2\sqrt{\pi Q}$. Assume f is C^2 and satisfies*

$$f(0,s) = 0, \quad f(u,0) = 0, \quad f_1 > 0, \quad f_2 \geq 0, \quad f_{11} \leq 0, \quad f_{22} \leq 0, \quad f_{12} \geq 0,$$

together with the Inada conditions $\lim_{u\downarrow 0} f_1 = \infty$, $\lim_{u\uparrow\infty} f_1 = 0$, and analogously for f_2. In addition, f is jointly concave *in (u,s).*

The circumference of the RCA, is a measure for the mass (or "number") of roads at which constructors can work as indicated by the centrifugal arrows in Figure 1.[12] Alternatively, $q(t)$ can also be interpreted, up to a constant, as the rate of increase in the length of the road network (in kilometers, km).[13]

Assumption 1 implies that there are diminishing returns to construction workers for a given RCA. Intuitively, keeping constant the mass of existing roads from which constructors can extend the road network, the productivity of the marginal construction worker is decreasing in the amount of workers due to a *stepping on toes* effect. Similarly, there are diminishing returns to increasing the mass of roads at which constructors can work for a given number of workers. Furthermore, construction workers and the RCA are complementary: a larger RCA means a larger road network and hence a higher marginal product of construction workers due to less stepping on toes.

Constructors earn a constant and exogenously given wage rate ω. The constant average costs of converting forest into agricultural land are given by κ. Due to discounting, an area that has been made accessible by roads will immediately be converted into agricultural land. Hence, the area of agricultural land is given by Q. We define the reduced-form development cost $C(Q,q)$ as induced by the primitive technology f through the cost-minimization problem

$$C(Q,q) \equiv \min_{u\geq 0} \left\{ \omega u + \kappa q : \; f\left(u, 2\sqrt{\pi Q}\right) \geq q \right\}. \tag{2}$$

We interpret $C(Q,q)$ as the reduced-form cost of expanding the road-covered area and converting the unlocked area into agricultural land at rate q when the existing network has size Q, induced by the primitive technology (1) and wage ω through (2).

[12]The appendix presents an alternative framework of road construction on a discrete grid in a two-dimensional space in which the road covered area is represented by a rectangle. This provides a rationalization for the model of circular road expansion in continuous space presented here.

[13]The rate of road construction $m(t)$ is a flow in km, while $q(t)$ is a flow in km^2. To relate the two concepts, assume $m(t) = dq(t)$, where d is the density of roads required to gain access to a certain quantity of land (in km/km^2). Based on Barber et al. (2014), who find that 95% of deforestation takes place up to a distance of 5.5 km from a road, and limiting ourselves to land perpendicular to the road, $d = 1/11$ appears to be a reasonable minimum value. In practice densities tend to be higher. See Section 3.2. By extension this implies that the total length of the road network (in km) is given by $l(t) = dQ(t)$. Thus, $Q(t)$ (in km^2) is not only the RCA (an area measure), but it is also proportional to the length of the road network.

Under Assumption 1, this construction implies the sign restrictions on the *first- and second-order* derivatives of C (in particular $C_q > 0$, $C_{qq} \geq 0$, $C_Q \leq 0$, $C_{qQ} \leq 0$, and $C_{QQ} \geq 0$); see Lemma 1.

Lemma 1. *Let C be defined by (2) and suppose Assumption 1 holds. Then for all $q > 0$ and $Q > 0$:*

(i) $C(Q, 0) = 0$ *for all* $Q > 0$, *and* $C(Q, q) \to \infty$ *as* $Q \downarrow 0$ *for any fixed* $q > 0$.

(ii) $C_q > 0$ *and* $C_Q \leq 0$.

(iii) $C_{qq} \geq 0$ *and* $C_{qQ} = C_{Qq} \leq 0$.

(iv) $C_{QQ} \geq 0$.

Proof. See Appendix A. □

These properties imply that there are no costs if no new roads are built, costs are increasing and convex in the road construction rate q, and decreasing and convex in the road network Q. Also, the higher is Q the smaller is the marginal cost of road construction. This implies that, in a decentralized equilibrium, there is a positive *stock* externality of increasing the road network.

To preserve the clean phase-diagram geometry used below, we additionally impose the following assumption.

Assumption 2. *The reduced-form cost function C is C^3 on $\mathbb{R}_{++} \times (0, \bar{Q})$ and satisfies*

$$
\begin{aligned}
C_{QQq} = C_{qQQ} \geq 0 \text{ and } C_{Qqq} = C_{qqQ} \leq 0, \\
C_{qq}(Q, q) > 0 \text{ for all } q > 0, Q \in (0, \bar{Q}), \\
\lim_{Q \downarrow 0} \left(C_Q - qC_{qQ} \right) = \infty \text{ for all } q > 0.
\end{aligned}
$$

Output per unit of agricultural land is constant and by appropriate choice of units equal to the amount of land. A representative consumer living in the economic core derives utility from the consumption of agricultural goods according to a strictly concave utility function $U(Q)$. The unit variable cost of agricultural production (growing crops or raising pasture-fed animals) is constant and is given by ν. These assumptions can be summarized as follows:

Assumption 3. *The instantaneous net benefit from agricultural land equals*

$$
B(Q) \equiv U(Q) - \nu Q, \tag{3}
$$

where U is C^2 on $\mathbb{R}_{++}$ and satisfies $U' > 0$ and $U'' < 0$.

Two implicit simplifying assumptions deserve discussion. First, we treat all land as homogeneous, with constant output per hectare and constant unit variable costs. In reality, productivity varies sharply across space, and land closer to existing infrastructure is typically more valuable due to lower transportation costs (cf. Souza-Rodrigues, 2019). We discuss transportation costs as a source of land heterogeneity in Section 5. Second, we assume constant productivity over time, abstracting from the substantial agricultural TFP growth observed in Brazil (cf. Bustos et al., 2016). Rising productivity would amplify the incentive to deforest and make conservation more costly, reinforcing the case for early policy intervention.

2.2 Functional forms

In our numerical analysis in Sections 3 and 4 we use the following Cobb-Douglas production function:

$$f(u,s) = u^{\alpha}\,(2\pi r)^{\beta} = u^{\alpha}\left(2\sqrt{\pi Q}\right)^{\beta},\ \text{with } \alpha,\beta \in (0,1), \text{and } \alpha+\beta \leq 1. \tag{4}$$

The implied cost function is given by

$$C(Q,q) = \omega\,\left(2\sqrt{\pi}\right)^{-\frac{\beta}{\alpha}} q^{\frac{1}{\alpha}} Q^{-\frac{1}{2}\frac{\beta}{\alpha}} + \kappa q. \tag{5}$$

Assumptions 1 and 2 are satisfied in the Cobb-Douglas case. Furthermore, for the numerical analysis we use the quadratic utility function

$$U(Q) = \gamma Q - \frac{1}{2}\delta Q^2, \tag{6}$$

where γ and δ are positive constants. We take $\bar{Q} < \gamma/\delta$ to ensure that Assumption 3 is satisfied as well.

We summarize this as follows:

Assumption 4. *The cost and utility functions are given by* (5) *and* (6)*, respectively, with* $\alpha,\beta \in (0,1)$ *and* $\alpha+\beta \leq 1$*. Moreover, we take* $\bar{Q} < \gamma/\delta$ *to ensure strictly increasing utility.*

2.3 First-best

In the first-best the social net benefits are maximized by choosing a time path of the road construction rate subject to the conditions outlined above. In mathematical terms the problem can be written as follows

$$\max_{\{q(t)\}} \int_0^T e^{-\rho t}\left[B(Q(t)) - C(Q(t),q(t))\right] dt + e^{-\rho T}\frac{B(Q(T))}{\rho}, \tag{7}$$

subject to

$$\dot{Q}(t) = q(t), \tag{8}$$
$$Q(0) = Q_0 \in (0, \bar{Q}), \tag{9}$$
$$q(t) \geq 0. \tag{10}$$

where T denotes the endogenous time at which developing the forest stops. The Hamiltonian reads

$$\mathcal{H}(t, q, Q) = e^{-\rho t} \left[B(Q) - C(Q, q) \right] + \lambda q.$$

Suppose there exists an interior solution. Then the following conditions hold:

$$\frac{d\mathcal{H}}{dq} = -e^{-\rho t} C_q(Q(t), q(t)) + \lambda(t) = 0, \tag{11}$$
$$\dot{\lambda}(t) = -\frac{d\mathcal{H}}{dQ} = -e^{-\rho t} \left[B'(Q(t)) - C_Q(Q(t), q(t)) \right]. \tag{12}$$

Furthermore, the transversality condition

$$\left(\lambda(T) - e^{-\rho T} \frac{B'(Q(T))}{\rho} \right) [\bar{Q} - Q(T)] = 0 \tag{13}$$

should be satisfied. The static condition (11) states that for an interior solution, i.e., $q(t) > 0$, the social planner increases the amount of roads $q(t)$ built at time t to the point where the present value of the marginal cost of agricultural land expansion, which is the sum of the marginal cost of land conversion and road building, equals the present value of the shadow price of the RCA, $\lambda(t)$. The dynamic condition (12) is an asset pricing equation requiring that the sum of the present value of the capital gain and the net marginal benefits of arable land is equal to zero. This ensures that the return to investment in arable land equals the discount rate. The transversality condition (13) says that when development stops, either the RCA should be equal to the entire available area, $\bar{Q}$, or the shadow price of the RCA equals the net marginal utility of agricultural land, both in present value terms.

It is straightforward to derive the resulting system of differential equations in $Q(t)$ and $q(t)$, for $q(t) > 0$. The differential equation for Q is (8). The one for q is

$$\dot{q} = \frac{1}{C_{qq}(Q, q)} [C_Q(Q, q) + \rho C_q(Q, q) - C_{qQ}(Q, q) q - B'(Q)]. \tag{14}$$

The final road construction rate equals $q(T) = 0$, which can be seen as follows. Formally, the maximized Hamiltonian should be continuous at time T, which implies that the Hamiltonian just before time T should be equal to minus the time derivative of the scrap value at time T, that is, $\mathcal{H}(T^-) = e^{-\rho T}\left(U(Q(T)) - \nu Q(T)\right)$. This implies

$$B(Q(T)) - C(Q(T), q(T^-)) + C_q(Q(T), q(T^-))q(T^-) = B(Q(T)),$$

where we substituted for $\lambda(T^-)$ by using equation (11). It follows that $q(T^-) = 0$ due to the strict convexity of the cost function. Intuitively, if $q(T^-) > 0$, marginal cost jumps down to zero at T, implying that welfare could be increased by postponing some development until $t > T$.

An important role in the analysis of the first-best as well as of the solution under open access (to be discussed below) is played by $\hat{Q}$, which is defined by

$$\frac{B'(\hat{Q})}{\rho} = \kappa. \tag{15}$$

Note that $\kappa = C_q(Q, 0)$, i.e., κ equals the marginal development cost when $q = 0$. Moreover, if along a first-best $Q(t) = \hat{Q}$ for all $t > T$ for some $T > 0$, total incremental welfare from T onward, taking account of $q(t) = 0$ for all $t > T$, is

$$e^{\rho T}\int_T^\infty e^{-\rho t}B'(\hat{Q})dt = \frac{B'(\hat{Q})}{\rho}.$$

Two possibilities arise. If $\kappa > B'(\bar{Q})/\rho$, then $\hat{Q} < \bar{Q}$ and from (11) and (13) it follows that it is socially optimal to stop developing once the arable land reaches $\hat{Q}$: the marginal development cost exceeds the marginal welfare gain. We say that available land is *abundant*. If the inequality is reversed so that $\hat{Q} > \bar{Q}$ then it is socially optimal to only stop developing once Q reaches $\bar{Q}$. Available land is said to be *scarce*.

In conclusion, conditions (8) and (14) constitute a system of differential equations in q and Q. The boundary conditions are $Q(0) = Q_0$, $q(T) = 0$ and $Q(T) = \min\{\bar{Q}, \hat{Q}\}$. We characterize the solution using phase diagrams. Define

$$\Omega(Q, q) \equiv C_Q + \rho C_q - C_{qQ}q - B'(Q). \tag{16}$$

For $q > 0$, define the $\dot{q} = 0$ isocline in the (Q, q)-plane by $q = h(Q)$, i.e. $\Omega(Q, h(Q)) = 0$. Proposition 1 describes the characteristics of the $\dot{q} = 0$ isocline.

Proposition 1. *Consider the first-best system* (8) *and* (14) *under Assumptions 1 and 2.*

(i) Suppose $C_Q = 0$. Then: $h'(Q) < 0$ for all $Q \in (0, \hat{Q})$. Moreover, $\lim_{Q\uparrow\hat{Q}} h(Q) = 0$.

(ii) Suppose $C_Q < 0$. Then: $\lim_{Q\downarrow 0} h(Q) = 0 = lim_{Q\uparrow\hat{Q}} h(Q) = 0$.

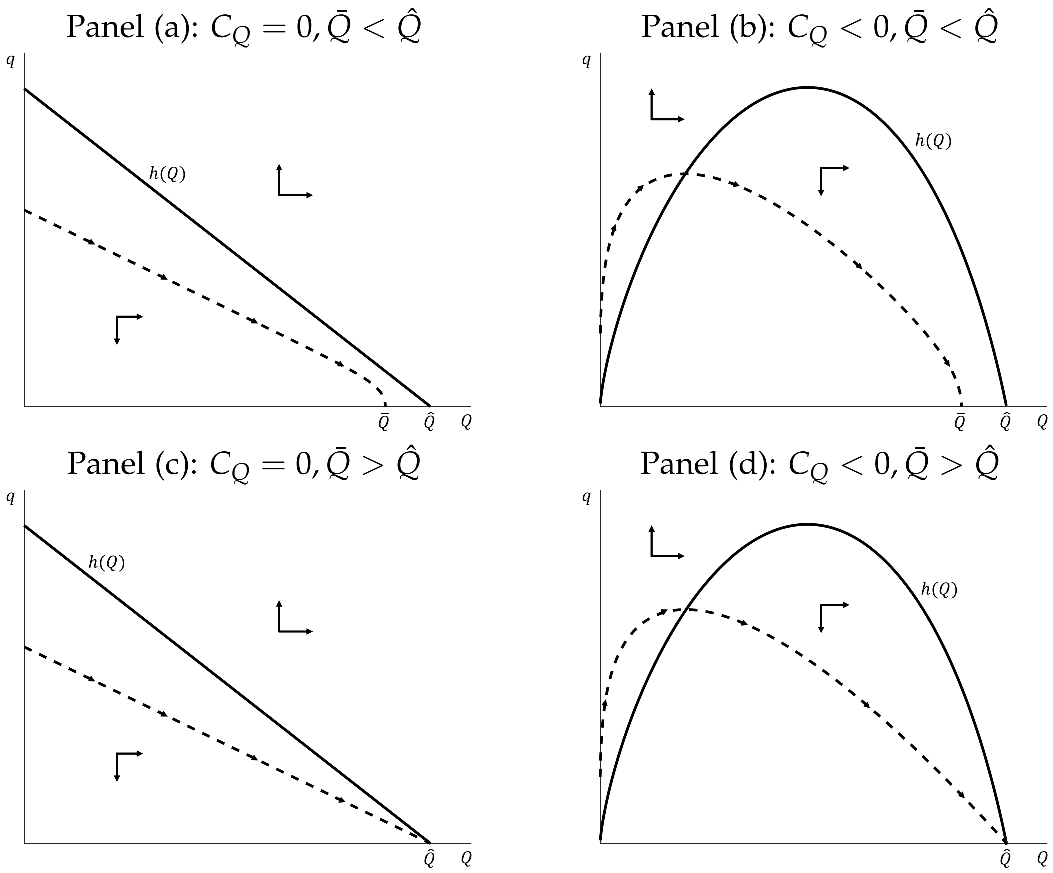
Proof. See Appendix A. □

For the case with Cobb-Douglas production and quadratic utility as discussed in Section 2.2, we can narrow part (ii) down to:

Proposition 2. *Consider the first-best system* (8) *and* (14) *under Assumption 4. Suppose* $C_Q < 0$. *Then:* $h(Q)$ *is inverted U-shaped with* $\lim_{Q\downarrow 0} h(Q) = \lim_{Q\uparrow \hat{Q}} h(Q) = 0$.

Proof. See Appendix A. □

Figure 2: Phase diagrams for the first-best.



Notes: The solid curves are the $\dot{q} = 0$ isoclines. The dashed curves represent the optimal trajectories. The horizontal (vertical) arrows indicate the direction of movement of Q (q) over time.

Figure 2 shows phase diagrams in the (Q, q)-plane for the case with Cobb-Douglas production and quadratic utility (see Assumption 4). The figure depicts two different parameter configurations: with and without costs depending on the size of the existing road network. The optimal trajectories are indicated by the dashed curves. They start near the vertical axis and end at $\bar{Q}$ (panels (a) and (b)) or at $\hat{Q}$ (panels (c) and (d)) on the horizontal axis. Obviously, the RCA increases with time as long as the rate of road construction q is positive. The shape of the time path of the road construction rate depends on the cost function. To understand the different possibilities, note that (12) implies the following no-arbitrage condition for the current value of the shadow price of the RCA, $\tilde{\lambda} = e^{\rho t}\lambda$:

$$\rho = \frac{\dot{\tilde{\lambda}}}{\tilde{\lambda}} + \frac{U'(Q(t)) - \nu - C_Q(Q(t), q(t))}{\tilde{\lambda}}, \tag{17}$$

$$\tilde{\lambda}(t) = \int_t^T \left[U'(Q(\tau)) - \nu - C_Q(Q(\tau), q(\tau))\right] e^{-\rho(\tau - t)} d\tau + e^{-\rho(T-t)} \frac{B'(Q(T))}{\rho}. \tag{18}$$

In determining the optimal path of q the social planner is guided by the asset pricing equation (17), which says that the rate of return to cleared land given by the right-hand side of the equation, should equal the rate of return on alternative investment projects, ρ. The rate of return on the right-hand side consists of the capital gain plus the sum of the net agricultural yield and the effect of extending the road network on future road construction costs. Hence, the shadow price of the RCA grows at a rate smaller than ρ, implying that the present value of the shadow price of the RCA declines over time, as required by (12). Hence, the present value of the marginal cost of road construction must also decrease with time, according to (11).

If the existing road network has no effect on the marginal cost of the construction of new roads, i.e., if $C_Q = 0$, it follows from (11) and (18) that the road construction rate must decline monotonically over time. Mathematically, this explains the shape of the optimal trajectories in panel (a) of Figure 2. A more intuitive explanation is that the social planner wants to smooth development and road construction over time, due to the convexity of the cost function in q. Still, early development is preferred over later development, as a stream of agricultural yields becomes available from the moment of development onward. Hence, development starts at a high level and gradually declines over time. The decrease over time is more pronounced if the discount rate is smaller, as this makes the discounted future stream of agricultural yields more valuable.

However, if extension of the current road network lowers future road construction costs, i.e., if $C_Q < 0$, the time path for q may be non-monotonic: if the initial RCA is small enough, extending the road network considerably lowers future construction costs, implying that the construction rate initially increases over time. This effect becomes weaker as the RCA increases over time and the remaining amount of roads to be constructed in the future goes down. Eventually, the road construction rate becomes declining over time. This explains the non-monotonicity of the optimal trajectories in panels (b) and (d) of Figure 2.[14]

2.4 Public ownership

Let us now consider the case in which a benevolent government owns the pristine forest. The government has the ability to develop parts of the forest (by clearing them and making them accessible by road) to maximize social welfare. Once an area has been developed, the government sells the land to a large number of small farmers, without exercising its market power. The constructed road network will remain in public ownership.

The government takes the time path of the land price, $p(t)$, as given and chooses the time path of the rate of increase of the RCA, $q(t)$, and the time at which developing the forest stops, T, in order to maximize

$$\int_0^T [p(t)q(t) - C(Q(t), q(t))]\, e^{-\rho t} dt, \tag{19}$$

subject to (8)-(10). The Hamiltonian reads

$$\mathcal{H}(t, q, Q) = e^{-\rho t} [pq - C(Q, q)] + \mu q.$$

Suppose again that there exists an interior solution. Then the following conditions hold:

$$C_q(Q(t), q(t)) = p(t) + e^{\rho t}\mu(t), \tag{20}$$

$$e^{-\rho t} C_Q(Q(t), q(t)) = \dot{\mu}(t), \tag{21}$$

where μ is the present value of the shadow price of the road network (excluding agricultural yields). Therefore, the marginal cost of road construction is equal to the price of land augmented with the value of having more roads. The dynamic condition (21) has the same interpretation as before. The transversality condition reads:

$$\mu(T)(\bar{Q} - Q(T)) = 0, \tag{22}$$

which requires that either the present value of the shadow price of the road network or the area of pristine forest must vanish. Finally, development stops when the rate of profits (current and future) falls to zero, that is,

$$\mathcal{H}(T) = e^{-\rho T}[p(T)q(T) - C(Q(T), q(T))] + \mu(T)q(T) = 0. \tag{23}$$

[14] Note that the $\dot{q} = 0$ isocline intersects the horizontal axis at $Q > 0$ because marginal utility is strictly declining in Q. This implies that marginal benefits of more arable land may become negative if Q becomes too large.

Intuitively, the gains from continuing development for an additional period are given by current profits (first two terms) plus future profits i.e., the change in the value of the RCA (third term). This gain of continuation should be zero when T is chosen optimally. Together with (20), (23) implies $q(T) = 0$, as in the first-best.

The price of agricultural land satisfies the following no-arbitrage condition:

$$\dot{p}(t) + B'(Q(t)) = \rho p(t), \tag{24}$$

where the left-hand side captures the benefits of holding a marginal additional unit of land (which consist of a capital gain and the net marginal utility from agricultural consumption) and the right-hand side equals the interest cost of holding a marginal additional unit of land. By integrating this no-arbitrage condition, we find that the land price equals the present value of future net agricultural revenues:

$$p(t) = \int_t^\infty B'(Q(\tau))e^{-\rho(\tau-t)}d\tau. \tag{25}$$

Together, (8), (20), (21) and (24) imply a differential equation for the road construction rate, which is identical to (14). The boundary conditions of the system of differential equations in q and Q are identical to those of the first-best as well. Hence, the case with public ownership of the pristine forest and the constructed road network coincides with the first-best.

2.5 Open access

In this section we consider the case where no concession is needed to build roads and cut trees. In addition, the technology to carry out these activities is available to everyone. The main objective of these activities is to gain land for agriculture, whether it is crop production or cattle ranching.[15] [16] Once the land has been developed, property rights to agricultural land are granted to the firm that has developed the land.

Formally, there is a large number of profit-maximizing firms, which all ignore the shadow value of the RCA and the pristine forest. Moreover, these firms take the price path of agricultural land as given. As a prelude to the subsequent policy analysis, we assume that at each instant of time they receive a subsidy $\sigma(t)$ per kilometer of

[15]Rainforest soils are often nutrient-poor, especially in the Amazon, which hampers sustainable agricultural production. Farmers resort to various measures to deal with low soil quality. For starters, burning the forest creates a layer of nutrient-rich material on an otherwise poor soil. Second, some crops fare longer on cleared forest land. Third, in later stages fertilizer may be needed to keep crop production profitable. Fourth, at some point, ranchers can convert the land into pasture land by planting drought-resistant grasses. This pasture land may support a limited number of cattle.

[16]Our framework could also be applied to mining, with κ as mine development cost (ignoring mineral exhaustibility issues).

constructed roads. Hence, at each instant of time t the representative firm solves

$$\max_{q(t)}\{p(t)q(t) - C(Q(t), q(t)) + \sigma(t)q(t)\}. \tag{26}$$

The necessary condition for $q(t) > 0$ reads

$$p(t) - C_q(Q(t), q(t)) + \sigma(t) = 0. \tag{27}$$

The price of agricultural land still satisfies (24), because the subsidy applies to new roads, and not to agricultural activities. By taking the time derivative of (27) and using (24), we get

$$\dot{q} = \frac{1}{C_{qq}(Q,q)}[\dot{\sigma} - \rho\sigma + \rho C_q(Q,q) - C_{qQ}(Q,q)q - B'(Q)]. \tag{28}$$

Hence, once again we have a system of differential equations in q and Q, given by (8) and (28). The boundary conditions for Q are similar to those prevailing in the previous two scenarios: $Q(0) = Q_0$ and $Q(T) = \min\{\bar{Q}, \hat{Q}\}$. Indeed, arable land area can never go beyond $\bar{Q}$ nor beyond the critical area where profits vanish. However, in the absence of the subsidy equation (28) differs from (14), as the term C_Q is no longer present. The reason is that firms operating under open access ignore that the roads they build to unlock their property reduce the costs to further extend the road network in order to unlock other areas.

If subsidies are in place, the government can restore the first-best by setting the subsidy equal to the first-best rate $\sigma^*(t) = e^{\rho t}\mu(t)$, where μ is as introduced in the previous subsection. From (20)-(21) and (24), this yields

$$\dot{\sigma}^*(t) - \rho\sigma^*(t) = C_Q(Q^*(t), q^*(t)), \tag{29}$$

with

$$\sigma^*(T) = C_q(\min\{\bar{Q}, \hat{Q}\}, q^*(T)) - \frac{B'(\min\{\bar{Q}, \hat{Q}\})}{\rho}, \tag{30}$$

where $Q^*(t)$ and $q^*(t)$ are the first-best time paths.

In the remainder of this section, we consider open access without a subsidy. The end condition for q depends on whether land is scarce or abundant. If land is scarce, i.e., $\bar{Q} < \hat{Q}$, then $Q(T) = \bar{Q}$, and $q(T^-)$ follows from the optimality condition (27) and the price of land (25),

$$\frac{B'(\bar{Q})}{\rho} = C_q(\bar{Q}, q(T^-)).$$

In this case development goes on until $\bar{Q}$ is reached. Then

$$\frac{B'(\bar{Q})}{\rho} > C_q(\bar{Q}, 0),$$

and therefore $q(T) > 0$ in order to maximize profits. If, on the other hand, land is abundant we have $\bar{Q} > \hat{Q}$ so that $Q(T) = \hat{Q}$. Using the definition of $\hat{Q}$ from (15), it then follows that $q(T) = 0$ under open access, as in the first-best. The reason is that in this case development continues as long as the net marginal utility of additional arable land is larger than the lowest possible marginal cost of development, which materializes at $q = 0$.

2.6 Comparing open access and the first-best

In this section, we compare the outcome under open access with the first-best. Define $\widetilde{\Omega}(Q, q) \equiv \rho C_q - C_{qQ}q - B'(Q)$ and let $q = k(Q)$ denote the open-access $\dot{q} = 0$ isocline for $q > 0$. Recall that the $\dot{q} = 0$ isocline for $q > 0$ in the first-best is defined as $q = h(Q)$.

Proposition 3. *Consider the first-best system* (8) *and* (14)*, and the open-access system* (8) *and* (28) *with $\sigma = 0$ under Assumptions 1 and 2. Let $q = m(Q)$ and $q = n(Q)$ denote the (interior) equilibrium paths in the (Q, q)-plane under the first-best and open access, respectively.*

(i) The qualitative shape statements of Proposition 1 apply to the open-access isocline $k(Q)$ as well.

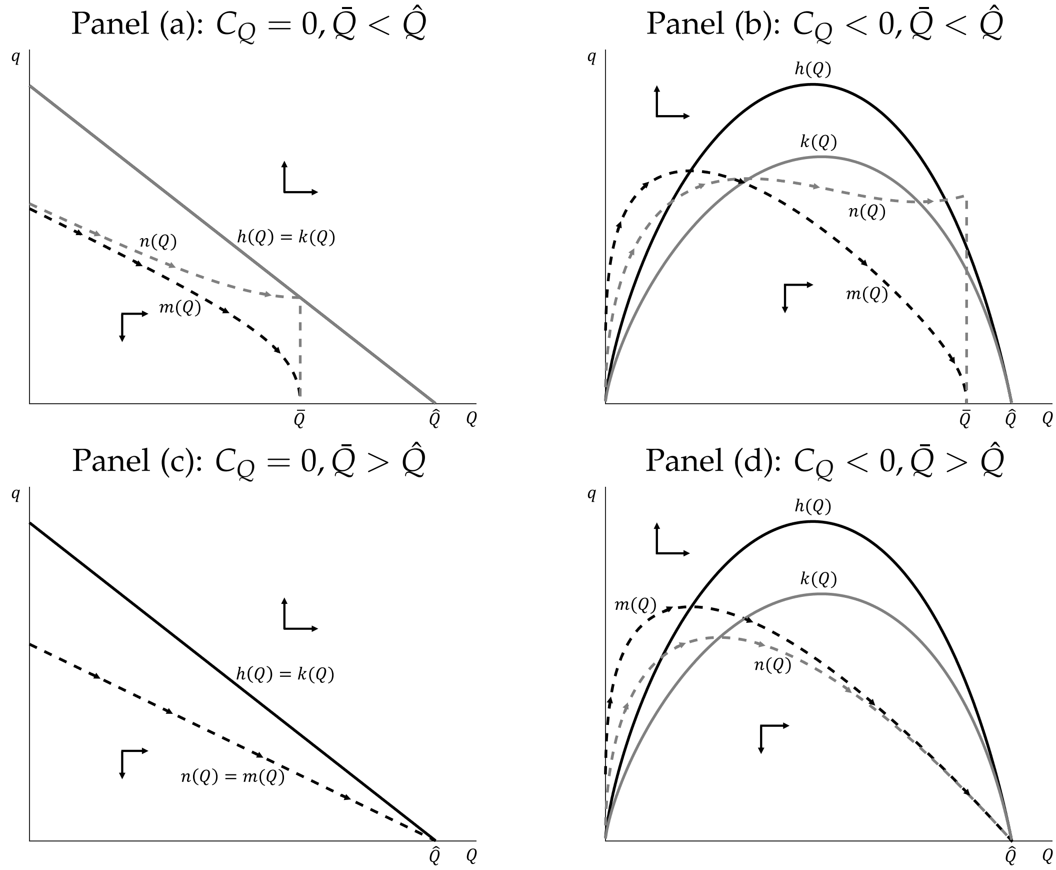
(ii) Suppose land is scarce, so that the terminal condition is $Q(T) = \bar{Q}$ and $n(\bar{Q}) > 0$. If $C_Q = 0$, then $k(Q) = h(Q)$ and $n(Q) > m(Q)$ for all $Q \in (Q_0, \bar{Q})$. If $C_Q < 0$, then $m(Q)$ and $n(Q)$ intersect at most once.

(iii) Suppose land is abundant, so that the terminal condition is $Q(T) = \hat{Q} < \bar{Q}$ and $m(\hat{Q}) = n(\hat{Q}) = 0$. If $C_Q = 0$, then $m(Q) = n(Q)$ for all $Q \in (Q_0, \hat{Q})$. If $C_Q < 0$, then $m(Q) \geq n(Q)$ for all $Q \in (Q_0, \hat{Q})$.

Proof. See Appendix A. □

Figure 3 shows phase diagrams for the case with Cobb-Douglas production and quadratic utility (see Assumption 4). Let us first consider the case where development costs are independent of the existing road network ($C_Q = 0$). Panels (a) and (c) relate to this case, depicting equilibrium paths for the first-best and for the open access regime. In this case the first-best and open access isoclines for q, i.e., $h(Q)$ and $k(Q)$, coincide, and the equilibrium paths of q, i.e., $m(Q)$ and $n(Q)$, are declining

Figure 3: Phase diagrams for the first-best and open access.



Notes: The solid curves are the $\dot{q} = 0$ isoclines. The dashed curves indicate the equilibrium paths. The black (grey) curves correspond to the first-best (open access).

in the quantity of land already converted. This occurs because the costs of road construction are convex ($C_{qq} \geq 0$) and because the marginal utility of consumption and thus the price of land is declining in Q, thereby weakening the incentive for more land conversion and road construction over time. This motive is present in both the first-best and under open access.

In case of land abundance, we have $q(T) = 0$ in both scenarios. The boundary conditions for Q are also identical under open access and the first-best. Then open access yields the same outcome as the first-best. If land is abundant and cannot offer a positive return in the long-run, a farsighted planner will not be able to do better than the myopic producers under open access. This is panel (c). However, if land is scarce (see panel (a)), the farsighted social planner maximizes social net benefits by choosing a relatively slow pace of road construction, thus smoothing investment costs intertemporally. Road construction is therefore unambiguously larger under open access than in the first-best, $n(Q) > m(Q)$, which implies that total deforestation always occurs sooner under open access than in the first-best.

If development costs decrease with the size of the existing road network ($C_Q < 0$), the isocline for q in the open access regime lies below the one for the first-best due to the extra term C_Q in the differential equation for the first-best. See panels (b) and (d) of Figure 3. Several outcomes are possible. Interestingly, when land is abundant (panel (d)), the pace of road construction and development is always faster in the first-best than under open access, since the road network spillover is internalized by the social planner, which speeds up development relative to open access.

When land is scarce, the two optimal paths may intersect, as in panel (b) of Figure 3. In this case, deforestation under open access may initially be *smaller* than in the first-best. For some $Q < \bar{Q}$, however, it will again be larger. The reason is that initially the road network spillover effect may dominate, whereas later on the scarcity effect becomes dominant. It is also possible that the optimal paths under open access and first-best do not intersect: in this case the path under open access will always be above the first-best.

Panel (b) also shows that there may be an intermediate phase under open access during which q is falling over time, before it starts to increase again in the final phase. During this intermediate phase, the declining marginal utility of agricultural goods dominates the decreasing costs of road construction over time.

The black curves in Figure 4 show the evolution of road construction (panels (a) and (c)) and the road network (panels (b) and (d)) over time for the case in which the costs of development depend negatively on the existing road network ($C_Q < 0$). The top and bottom panels depict the case with and without land scarcity, respectively. The solid time paths correspond to the first-best and the dashed ones to the outcome under open access. In the case with land scarcity, the time paths of road construction and the road network differ markedly between the first-best and the open access regime.

In the case without scarcity, the shapes of the time paths are more similar. The reason is that firms under open access not only ignore the positive road externality, but also fail to smooth development over time in order to lower costs, because this means that some other firm will then develop the area instead. In the case of land scarcity, this causes a downward jump in the development rate at the stopping time. Hence, the deviation from the first-best is especially salient if land is scarce.

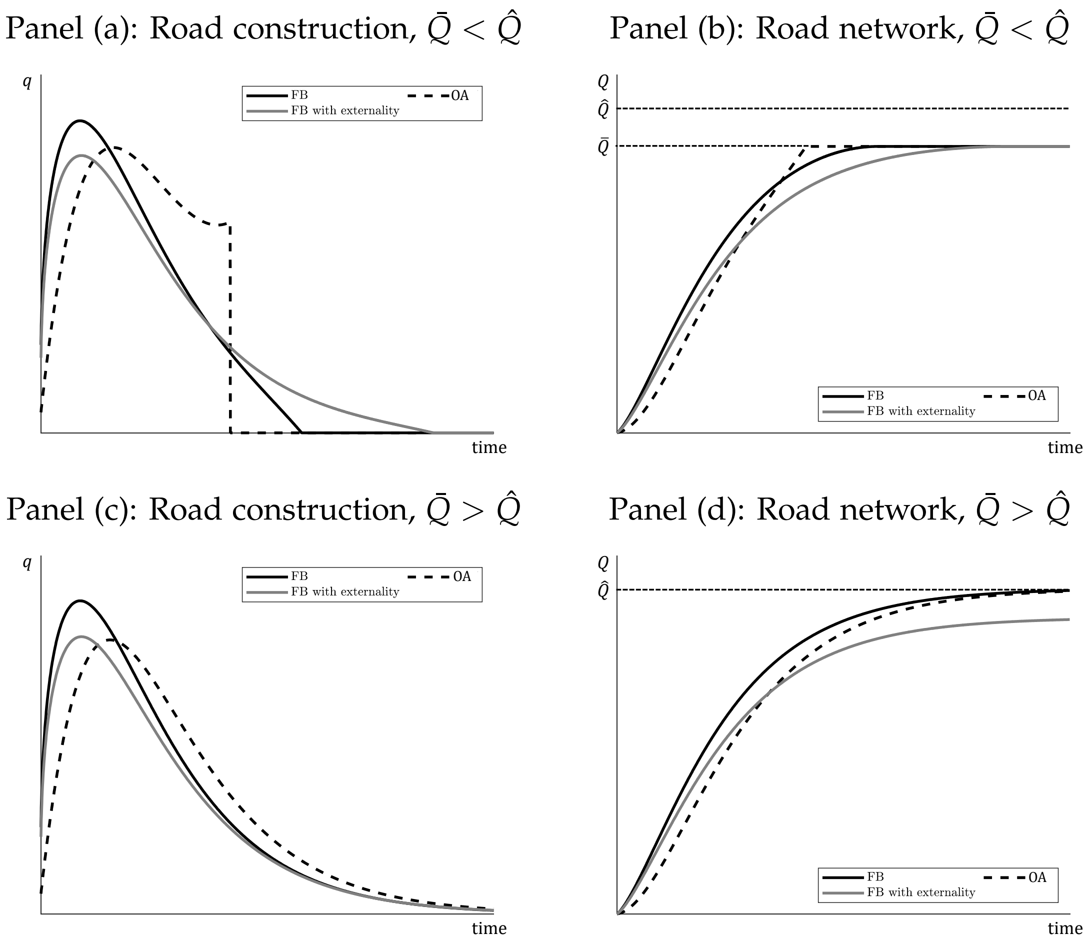
Panels (a) and (c) demonstrate that the rate of land conversion under open access is initially lower but ultimately higher compared to the first-best. In the case of scarcity, this pattern emerges because the positive road-building externality initially dominates, whereas the motive to smooth development over time becomes more influential later. In the case of abundance, the absence of a scarcity effect simplifies the explanation for the catch-up phase: given that long-run development is equivalent in both scenarios, open access must eventually exhibit a higher rate of land conversion. This occurs as the first-best experiences diminishing returns sooner, leading to a decelerating in its rate of land conversion.

2.7 Optimal policies

We now introduce an external climate cost linked to rainforest conversion to our model. This cost, along with the road network spillover, is internalized in the first-best but not under open access. Technically, it is similar to an increase in the land-conversion cost parameter from κ to $\kappa + \vartheta$ in the social optimum, where ϑ denotes the constant marginal climate damage (i.e., social cost of carbon) of converting one unit of forest to pasture land. We demonstrate how an optimal regulatory instrument—a subsidy or tax on road construction under open access—evolves over time. This instrument, designed so that producers fully internalize both the road network and climate externalities, ensures that the outcome under open access coincides with the first-best.

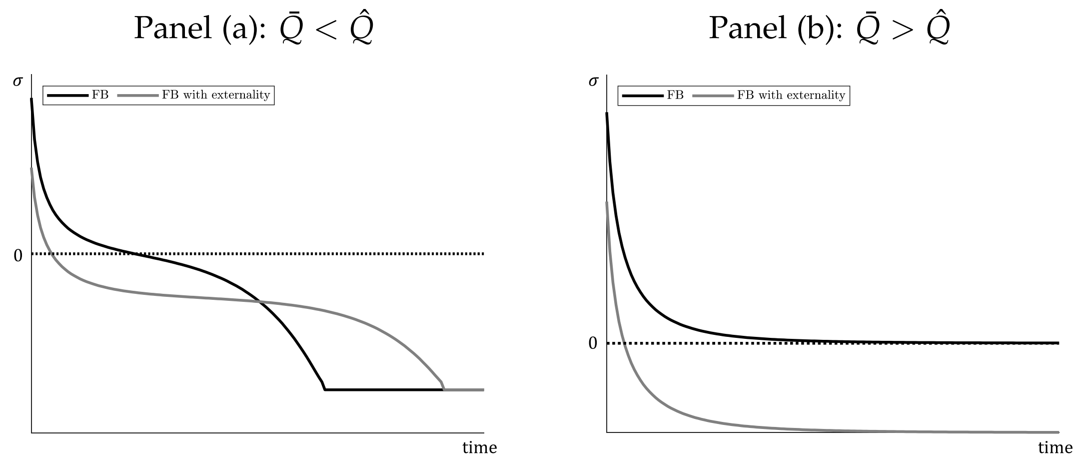
Figure 4 and 5 depict model outcomes in a specific case to illustrate the qualitative effects of the climate externality on first-best road construction and on optimal policies. Figure 4 shows the effects on road construction. The black curves correspond to the benchmark case without the climate externality. The grey curves represent the case in which development causes a constant climate externality per km of road constructed (and hence per km^2 of land deforested). The road network externality is present in both scenarios. In the case with land scarcity (i.e., all forest will ultimately be cleared), the climate externality calls for less road construction in the first-best initially, but more later on (see panel (a)). Moreover, panel (b) shows that although

Figure 4: Time paths of road construction and the road network.



Notes: The black solid (dashed) curves correspond to the time paths in the first-best (open access) without the climate externality. The grey curves correspond to the first-best with the climate externality.

Figure 5: First-best road-building subsidy



Notes: The grey (black) curves correspond to the case with (without) the climate externality. The dash-dotted line indicates the level where the subsidy rate is zero: above (below) this line the subsidy rate is positive (negative).

cumulative development does not change, the transition to $\bar{Q}$ is slower because the rate of land conversion in the first-best goes down due to the climate externality.

Panels (c) and (d) show the scenario with land abundance, implying that part of the forest will remain unexploited. In this case, the climate externality lowers the first-best road construction throughout (panel (c)). Furthermore, the climate externality reduces cumulative development (panel (d)).

Figure 5 shows the time paths of the first-best subsidy rate on road construction in the cases with (panel (a)) and without scarcity (panel (b)). The black curves correspond to the case without the climate externality. Panel (a) clearly shows that the first-best subsidy starts at a positive rate, declines over time and becomes negative before the road network covers the entire forest. The reason is that the positive network externality dominates initially, whereas the market failure under open access due to land scarcity becomes dominant later on. In panel (b), the scarcity effect is not present, which explains why the subsidy rate remains positive. Note that in this case the subsidy rate converges to zero, as road construction and hence the road network spillover effect converge to zero as well.

The grey curves correspond to the case with the climate externality. In both cases, the climate externality decreases the initial subsidy rate. However, in the case with land scarcity there is also an intermediate period during which the climate externality actually calls for a higher subsidy rate. The reason is that the externality slows forest conversion, thereby extending the duration of the positive road network spillover thus temporarily justifying a higher subsidy rate.

The climate externality increases the long-run tax in the case of land abundance, but not in the case of land scarcity. This difference arises because the impact of climate costs on the incentive to convert land varies between the two scenarios. In the case of land scarcity, the marginal benefits of land are so high that the additional external climate cost does not alter the incentive to convert all available land (although it does lower producer surplus). In contrast, in the scenario of land abundance, a permanent tax driving up the price of land and agricultural goods becomes necessary to prevent excessive land conversion.

2.8 A two-country setting

To accommodate our application centered around beef export demand growth in Section 3, let us now present a stylized two-country version of our framework, featuring North and Brazil. North represents a coalition of countries, organized through either a supranational entity like the EU, or an international organization like the World Bank. Consumers in North enjoy utility from consuming imported Brazilian beef. Brazil fully exports its beef production to North. The beef market is perfectly competitive, implying that the decentralized market outcome in this two-region setting coincides with the first-best described in Section 2.3 (and thus with the case of public ownership in Section 2.4) as long as land property rights to pristine forest in Brazil are enforced. If these property rights are not enforced, the outcome in the current two-country setting coincides with the open access regime discussed in Section 2.5. The coincidence between the decentralized market equilibrium in a perfectly competitive two-country setting and the benchmark model in Sections 2.3-2.5 prevails provided the governments of both North and Brazil act as price-takers in the beef market, i.e., act non-strategically.[17]

Although we abstract from strategic behavior, we allow for a climate externality from rainforest conversion, as in Section 2.7. In particular, we assume that rainforest conversion in Brazil negatively affects North's welfare, captured by the constant ϑ. Hence, North has an incentive to pay Brazil for protection of its rainforest. We assume that North offers Brazil a one-time lump sum transfer $F = \overline{F}$ if the Brazilian regulator implements a constant and permanent carbon tax $\tau = \overline{\tau} > 0$ on pristine forest land, and offers $F = 0$ otherwise (more details below).[18]

[17]This implies, for example, that Brazil, despite controlling a significant share of the global beef market, refrains from restricting ranching to influence world market prices. Such strategic behavior, which could emerge in a dynamic game between North and Brazil, might face challenges under WTO rules, which generally discourage export restrictions designed to manipulate terms of trade.

[18]We abstract from issues of time consistency. For example, after receiving the transfer Brazil would have an incentive to abandon the carbon tax. To ensure Brazil's continued participation, North would have to disburse the transfer in annual tranches.

In this setting, welfare of North is given by:

$$J_N(\tau) = \int_0^\infty e^{-\rho t} \left[U(Q(t)) - p(Q(t))Q(t) - \vartheta q(t) \right] dt - F(\tau). \tag{31}$$

where $F(\tau)$ denotes the dependence of the lump-sum transfer on the tax choice made by Brazil. Welfare in Brazil depends on revenues from beef exports to North plus transfers from North:

$$J_B(\tau) = \int_0^\infty e^{-\rho t} [p(Q(t))Q(t) - \nu Q(t) - C(Q(t), q(t)) - \tau q(t) + G(q(t), \tau)] dt + F(\tau). \tag{32}$$

where $G(q(t), \tau) \equiv \tau q(t)$ denotes tax revenues of the Brazilian government. A few remarks are in order. First, apart from climate damages the global welfare function, $J_W = J_N(\tau) + J_B(\tau)$, is identical to the social welfare function in the benchmark model:

$$J_W = \int_0^\infty e^{-\rho t} \left[U(Q(t)) - \nu Q - C(Q(t), q(t)) - \vartheta q(t) \right] dt, \tag{33}$$

where we have used $G(q(t), \tau) = \tau q(t)$. Indeed, with $\vartheta = 0$, this expression is identical to the maximand in (7), as $Q(t)$ is constant and $q(t) = 0$ for $t \geq T$. This implies that in the global first-best the social planner equates the carbon tax to the marginal damage, $\tau = \vartheta$, as in Section 2.7. Second, without climate damages (i.e., with $\vartheta = 0$), the solution implemented by the Brazilian regulator would coincide with the first-best described in Proposition 1, if this regulator, as assumed, does not try to influence the terms-of-trade.

Variables corresponding to the solution chosen by the Brazilian planner when $\tau = 0$ are labeled with an asterisk (*), and variables corresponding to the Brazilian planner solution when $\tau = \overline{\tau}$ with a double asterisk (**). Then using (32) with $G(q(t), \tau) \equiv \tau q(t)$, the required transfer needs to ensure that the incentive compatibility constraint (ICC) holds, that is, $J_B(\overline{\tau}) \geq J_B(0)$. The minimum required transfer that satisfies the ICC equals

$$\begin{aligned} \overline{F} = & \int_0^\infty e^{-\rho t} [p(Q^*(t))Q^*(t) - \nu Q^*(t) - C(Q^*(t), q^*(t))] dt \\ - & \int_0^\infty e^{-\rho t} [p(Q^{**}(t))Q^{**}(t) - \nu Q^{**}(t) - C(Q^{**}(t), q^{**}(t))] dt. \end{aligned} \tag{34}$$

3 From Theory to Application: Parameterizing the Model for the Brazilian Amazon

We now present a quantitative application of our model, focusing on cattle ranching in the Brazilian Amazon.[19] This application allows us to evaluate the impact of counterfactual policies — specifically carbon taxes and road taxes — on deforestation rates and cattle rancher profits under various assumptions regarding the size of the existing road network in the Amazon.

With the functional forms given in Section 2.2, the differential equation for road construction (14) in the first-best reads

$$\frac{\dot{q}}{q} = \frac{\beta q}{2Q} + \frac{\alpha}{1-\alpha}\rho\left(1 - \frac{\alpha}{A}\left[\frac{(\gamma - \nu - \delta Q)}{\rho} - \kappa\right] q^{\frac{\alpha-1}{\alpha}} Q^{\frac{1}{2}\frac{\beta}{\alpha}}\right).$$

Under open access without policies, (28) becomes

$$\frac{\dot{q}}{q} = \frac{1}{1-\alpha}\frac{\beta q}{2Q} + \frac{\alpha}{1-\alpha}\rho\left(1 - \frac{\alpha}{A}\left(\frac{\gamma - \nu - \delta Q}{\rho} - \kappa\right) q^{\frac{\alpha-1}{\alpha}} Q^{\frac{1}{2}\frac{\beta}{\alpha}}\right).$$

In the case of abundance, from (15) the total amount of deforestation is given by

$$\hat{Q} = (\gamma - \nu - \rho\kappa)/\delta. \tag{35}$$

We consider the two-country specification of the model discussed in Section 2.8 and explore both a baseline scenario (with no policy intervention) and alternative policy scenarios under two distinct property rights regimes.

First, we model the Brazilian government as owner of the pristine forest (as detailed in Section 2.4). While our theoretical model assumed beef was produced for domestic consumers, in this quantitative application we focus on foreign demand for Brazilian beef. This distinction does not alter the model solution as long as the Brazilian government acts as a price-taker in the international beef market, as discussed in Section 2.8.

Second, we examine an open access property rights regime (described in Section 2.5), where cattle ranchers make myopic profit-maximizing decisions regarding road construction and land clearing. For parameterization, we combine values from existing literature with parameters calibrated to reflect key observations from Brazilian data at the national, sectoral, and municipal levels. Table 2 provides a comprehensive

[19]While our approach more closely resembles a parameterization than a calibration (as we do not jointly calibrate multiple free parameters to match specific data moments), our baseline simulation predicts deforestation rates over the medium-run that are comparable to historical averages. This suggests our method of individually selecting parameters to match specific data points yields results comparable to a formal calibration.

overview of all parameters values and their sources.

3.1 Amazon deforestation and projections of Brazilian beef export demand growth

This section describes how most deforestation in the Brazilian Amazon is linked to cattle ranching, and then presents projections for the future growth of Brazilian beef exports. These serve as inputs to the model calibration in Section 3.2.

Deforestation in the Amazon. The leading proximate driver of deforestation in the Brazilian Amazon is cattle ranching (cf. Kuschnig and Vashold, 2023). Recent data from MapBiomas Project (2023) indicate that between 1985 and 2022 roughly 13 percent of the Amazon biome was converted into farmland (see Table 1). Today, the amount of agricultural land in the Amazon biome spans 0.65 million (M) km^2, an area the size of France. Most of this agricultural land is allocated to pasture (88%), with the remainder divided between cropland (11%) — predominantly occupied by soybeans — and forest plantations (1%).

Global beef trade. According to trade and production data from FAO (2024), the global market for beef has continued to expand rapidly since the early 2000s, with global exports growing by 3.5 (4.5) percent per annum on average between 2002-2022 (2012-2022). Beef exports from Brazil have grown even faster, at 6.7 (6.9) percent between 2002-2022 (2012-2022). As a result, Brazil's share of the global beef export market increased from an average of 4 percent between 1961-2001, during which its share was fairly stable, to 15 percent between 2012-2022.

Under business-as-usual, FAO (2018) expects the global consumption of beef to grow by roughly 16 percent between 2022 and 2035 (i.e., an annual rate of 1.125 percent). This implies an additional demand for 9.52 million metric tons (Mmt) of beef. Assuming half of this will be supplied by exporters (raising the world trade share of beef from 20 percent today to 23.9 percent in 2035) and, in turn, 30 percent of those exports by Brazil, this will imply an additional 1.43 Mmt of beef exports by Brazil in 2035 (from 2.86 Mmt in 2022). This would bring Brazil's share of global exports to 21 percent, up from 18 percent in 2022. This is our baseline scenario.[20]

Considering Brazil's relatively low beef pasture productivity of 6000 kg/km^2 per year leaves ample room for improvement according to Barbero et al. (2021), along with the substantial availability of arable land and the growing competitiveness of

[20] Brazil came close to reaching this share of 21 percent in 2007, but a combination of problems in Brazil's beef industry including high corporate debt and financing issues, as well as weaker demand from Europe and Asia, caused a slump in beef exports that lasted almost a decade.

Table 1: Land use across biomes in Brazil, 1985-2022.

Amazon biome	land use type	area size 1985 (in M km^2)	area size in 2022 (in M km^2)
Amazon	forest	3.8	3.29
	all farming	0.14	0.65
	pasture	-	0.57
	agriculture	-	0.072
	forest plantation	-	0.007
Atlantic forest	forest	0.32	0.30
	all farming	-	0.73
	pasture	0.39	0.30
	agriculture	0.12	0.24
	mosaic of uses	0.21	0.19
	forest plantation	-	0.04
Caatinga	forest	0.53	0.47
	all farming	0.28	0.34
	pasture	-	0.22
	agriculture	-	0.02
	mosaic of uses	-	0.09
Cerrado	forest	0.30	0.25
	native vegetation	1.27	0.95
	all farming	0.67	0.99
	pasture	0.34	0.51
	agriculture	0.04	0.26
	mosaic of uses	-	0.19
	forest plantation	-	0.03
Pampa	forest	0.022	0.023
	grassland + other	0.093	0.065
	all farming	0.056	0.084
	agriculture	-	0.061
	mosaic of uses	-	0.015
	forest plantation	-	0.008
Pantanal	forest	0.029	0.024
	pasture	0.008	0.022
All (Brazil total)	forest	5.82	4.94
	all farming	1.87	2.82
	agriculture	0.19	0.61
	pasture	1.03	1.64
	mosaic of uses	-	0.48
	forest plantation	-	0.09

[a] **Source**: MapBiomas Project - Collection v8.0 of the Annual Land Use Land Cover Maps of Brazil, accessed on september 25 2023 through the link: https://brasil.mapbiomas.org/en/. See also Souza Jr et al. (2020).

the overall agricultural sector, much larger growth in world demand for Brazilian beef seems possible.[21] So for our high-demand scenario we will assume instead that

[21] In 2020-2022 Brazil's Producer Support Estimate (PSE) to farmers in terms of gross farm receipts was equal to 3.1%, down from 7.6% in 2000-2002, and well below the OECD average of 15.2%. See OECD (2023).

Table 2: Model parameter estimates

parameter	description	value (range)	source
d	road density (km/km^2)	1.15	Botelho Jr et al. (2022)
α	labor share road production	0.67	Gao and Kehrig (2017), Kariel and Savagar (2022)
b	beef pasture productivity (kg/km^2/yr)	6000	Barbero et al. (2021)
Q_{2022}	current export demand Brazilian beef 2022 (M km^2)	2.86/6	FAO (2024), authors' calculations
Q_{2035}	additional exp. demand Brazilian beef 2035 (M km^2)	1.43/6	FAO (2018), authors' calculations
p_{avg}	Brazil fed cattle price avg. 2014-2023 ($/kg)	3.20	Cepea/Esalq (2024)
p_{99}	Brazil fed cattle price 99th percentile 2014-2023 ($/kg)	4.586	Cepea/Esalq (2024)
p^w	Brazil bovine FOB export price avg. 2014-2022 ($/$kg$)	5.28	FAO (2024)
r_{avg}	avg. unit revenue of ranching (1000$/km^2/yr)	19.2	authors' calculations
$\gamma = r_{max}$	demand intercept (1000$/km^2/yr)	27.52	authors' calculations
δ	demand slope	34.9	authors' calculations
ρ	rate of time preference	0.10	Araujo et al. (2024)
ν	marginal cost of cattle ranching (1000$/km^2/yr)	10	Oliveira and Couto (2018), authors' calculations
ω	labor cost for d km of road (1000$)	427.9	authors' calculations
$\frac{1}{d}C/q$	road construction costs (1000 $/km)	30.4	Winkler (1998), Dulac (2013), and Collier et al. (2016).
κ	conversion cost forest to pasture (1000 $/km^2)	40	Killeen (2022), Araujo et al. (2024)
ϑ	NPV foregone benefits forest (1000 of 2017 $/km^2)	92-936	Franklin Jr and Pindyck (2018), authors' calculations

Note: Parameter estimates are obtained from various sources and authors' calculations (see source column).

50 percent of the growth in global beef exports will be satisfied by Brazil, resulting in an additional 2.38 Mmt of beef exports by Brazil in 2035. This amounts to an annual growth rate of Brazil's beef exports of 4.8 percent over the next decade, which sits in between the previously mentioned 2012-2022 growth rates for global beef exports (i.e., 4.5%) and Brazilian beef exports (i.e., 6.9%), raising Brazil's share of the global export market to 25.6 percent.

3.2 Model calibration

The following paragraphs detail our model parameterization. Each paragraph is titled with the relevant parameter(s) being discussed, with mathematical symbols shown in brackets. For each parameter, we explain the data sources, estimation approach, and rationale for our chosen values.

Notation. For space considerations, we will consistently use \$ to refer to current USD.

Road costs, road density, and returns to scale [d, α, β]. To fit our stylized model to data on road density and road costs, we assume that the road network constitutes a weighted average of 3 types of roads, namely forest roads, secondary paved roads and highways. Based on Brazilian data, which reveals only 12 percent of all roads are paved, with highways accounting for 1 percent of the total, we select the following weights: 0.88, 0.11, and 0.01.[22] The costs of constructing forest roads range between 10 and 15 thousand \$/km according to Sessions and Sessions (1992) and Winkler (1998); for paved asphalt roads we take the estimate of 56 thousand \$/km from Collier et al. (2016); and finally, for highways, we rely on the work of Dulac (2013), who estimated the average cost of highway construction in Latin America at 1.1 million \$/km of paved-lane based on data from the period 1999-2010. Based on these numbers, we find that average road costs equal 30.4 thousand \$/km.

We base our estimate for the Cobb-Douglas parameter α, that is, the cost share of reproducible factors in road production, such as labor (and capital), on data from the construction sector, because we are not aware of data specific to road construction in emerging markets. At the firm-level, Kariel and Savagar (2022) estimate that in the construction sector α ranges between 0.5 and 0.7 for the UK economy, while at the industry level Gao and Kehrig (2017) find a value of 0.86 for the US economy. For our baseline, we set $\alpha \equiv 0.67$ and $\beta = 1 - \alpha = 0.33$.

[22] To motivate this calibration approach, note that deforestation typically begins with loggers and farmers using unpaved forest roads. These roads, however, often connect to paved roads, which in turn branch off from highways. Additionally, a network of paved roads and highways is essential for transporting agricultural products, such as crops and beef, to markets.

Temporal road dynamics and road labor costs [ω]. Based on recent findings by Botelho Jr et al. (2022), who use AI in the form of deep neural networks to detect unofficial roads in the Brazilian Amazon by screening satellite imagery, we set road density d equal to 1.15 km/km^2. Next, to fully pin down the road cost production function, the last parameter we need to calibrate is ω, the unit labor cost of constructing a road of length d (which will open up one km^2 of land). To determine ω, we assume that the average road construction cost estimate reported in the literature corresponds to the cost incurred when roads are being built at the average historical rate observed in our data. With this assumption, we can use the average road cost identity to solve for ω. Using the average road cost identity from the model, i.e., $\frac{1}{d}\frac{C-\kappa q}{q} = 30.4$, to solve for ω yields:

$$\omega = d30.4 \left(\frac{2\sqrt{\pi Q}}{q} \right)^{\frac{1-\alpha}{\alpha}}. \tag{36}$$

Thus, to find ω given α and d, we need to pick values for q and Q. Here we rely both on (i) aggregate data for the Brazilian Amazon as well as (ii) satellite-derived, municipality-level data presented by Ahmed et al. (2013). Starting with the former, we rely on land use data from Brazil for 1985 and 2022 presented in Table 1. We calculate the hypothetical mid-sample quantity of pasture land in the Amazon, that is, $Q_{avg} = 0.395$ (in M km^2), and the average pace of deforestation over the sample, $q_{avg} = 0.51/37 = 0.0138$ (in M km^2). Substituting these values into (36), this results in $\hat{\omega} = 427.9$. Our calibration implies that when $\frac{q}{2\sqrt{\pi Q}}$, i.e., the pace of road construction relative to the circumference, doubles or quadruples, the unit costs increase by a factor $\sqrt{2}$ and 2 respectively. Finally, satellite-derived, time-series data on municipality-level road construction in the Amazon presented by Ahmed et al. (2013) suggest a somewhat faster pace of road construction than our aggregate data. Using these micro-data instead (see Supplementary Appendix B.1), we find $\hat{\omega} = 327.7$, not too far from our macro estimate.

Initial road network size [Q_0]. By 2022, total farmland area in the Brazilian Amazon, proportional to the size of the region's road network, amounted to 0.65 M km^2 (see Table 1). However, directly equating this area to Q_0 (the initial amount of cleared land) in our model would be problematic for a fundamental reason: our theoretical model assumes expansion from a single center, while actual Amazon deforestation occurs along an 'Arc of Deforestation' involving multiple (urban) centers simultaneously. These centers shift over time as road networks in older areas become saturated and new settlements develop further into the forest. This spatial complexity differs significantly from our simplified single-center circular expansion model. Therefore, we treat Q_0 as a free parameter rather than attempting to derive it directly from the data. In our baseline we set Q_0 close to zero, but in Section 4.2 we analyze the sensitivity of our results using a range of different values to account for different stages of frontier development.

A clarification on the interpretation of Q is in order. In the theoretical model, Q denotes the road-covered area (RCA), with the network externality operating through road construction costs. In the calibration, we proxy Q with total agricultural land, since our model assumes immediate conversion of accessible land. Under this assumption, the RCA and agricultural land are formally equivalent, and the road network length is proportional to Q via the density parameter d (see footnote 14). In the lock-in exercise of Section 4.2, we relax this equivalence by varying $\tilde{Q}$ in the cost function independently of Q_0 in the utility function, effectively introducing an accessible-but-uncleared state. This formulation connects to the extension in Appendix B.4.1, where road construction and land conversion are explicitly decoupled.

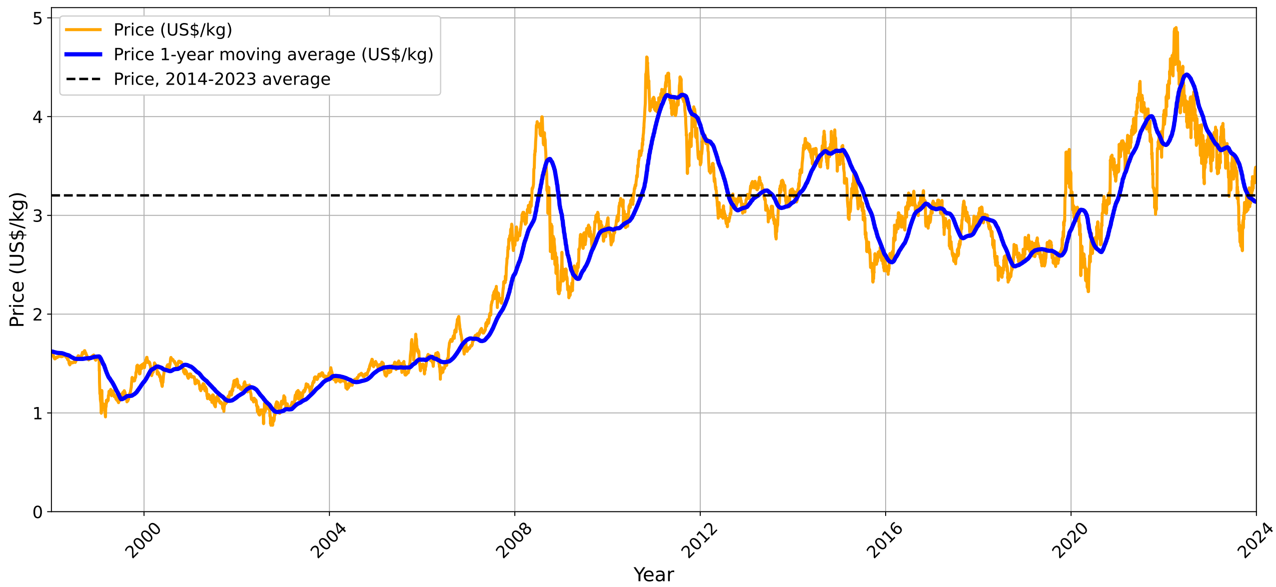
Discount rate [ρ]. Based on observations that the Brazilian Central Bank's federal funds rate has averaged 9.5% between 2014-2023, we set the discount rate at 0.10, which is also the value chosen by Araujo et al. (2024).

Pasture productivity and land conversion costs [b, κ]. Following Barbero et al. (2021), we set beef pasture productivity, b, equal to 6000 kg/km^2/yr. Before forest land can become productive in this way, however, it needs to be cleared. Killeen (2022) estimates that the cost of converting forest to pasture is about 50 thousand \$/km^2. A lower estimate of around 28.7 thousand \$/km^2 is provided by Araujo et al. (2024). We pick a value in between these two estimates and set the cost of land conversion to 40 thousand \$/km^2.

Variable costs of cattle ranching [ν]. We set the marginal cost of cattle ranching at 10 thousand \$/km^2 per year. Our reasoning is as follows. First, we obtained cost estimates from the literature. Oliveira and Couto (2018) studied the profitability of four Brazilian cattle ranching systems. Their calculations suggest that the variable costs of the two most extensive ranching systems are between 3.21-15.77 thousand \$/km^2 per year. For a more intensive ranching system based on feedlots, Sartorello et al. (2018) put the cost at 10.8 thousand \$/km^2/yr. We choose a cost of 10 thousand \$/km^2 per year, at the higher end of what these studies suggest is reasonable, because back-of-the-envelope calculations suggest that for smaller values, cattle ranching would be too profitable compared to what is known from other studies.[23]

[23]For example, Bowman et al. (2012) suggest that average annual profits (revenues minus variable costs) of a mid-size to large-size Brazilian ranch in the Amazon equal 1.2-8 thousand \$/km^2 per

Figure 6: Fed cattle price São Paulo state.



Note: the source of daily cattle price data is Cepea/Esalq (2024).

Fed cattle price data and beef demand [γ, δ, $\hat{Q}$]. Next, we set the parameters of the linear demand curve (see (6)). In our application it represents the additional export demand for Brazilian beef by 2035.[24] To this end, we use our projection of demand for Brazilian beef exports by 2035 and historical fed cattle price data from Brazil (see Figure 6). We start by setting the intercept parameter γ, which we express in 1000 $/km^2 and represents the maximum willingness to pay, equal to the product of the 99th percentile of prices observed over the period 2014-2023, p_{99}= $4.58/kg, and beef pasture productivity b. We will refer to the latter as maximum revenue, r_{max}. Hence, $\gamma = bp_{99} \equiv r_{max}$. While γ is unobserved and undoubtedly higher in reality, it captures the idea that producers in other major exporting countries, e.g., Argentina and the U.S., will step in to prevent prices from rising too much. It also ensures that the profitability of Brazilian cattle ranching does not reach unrealistic magnitudes.

To set the slope parameter δ, we assume the extra demand for Brazilian exports in 2035, Q_{2035}, is demand that will be realized at the average price of Brazilian beef in the last 10 years, p_{avg} = $3.20/kg (see Figure 6), such that $r_{avg} = bp_{avg}$ is the average revenue earned in 1000 $/km^2/yr at that price level. Since p is the fed cattle price as received by Brazilian ranchers, the demand curve represents the demand for Brazilian

year. Their NPV estimate sits at 5.4-30 thousand $/km^2. If we assume a revenue stream of 19.2-27.5 thousand $/km^2 per year, total fixed costs of 40 thousand $/km^2, a marginal cost of 10 thousand $/km^2 per year, and a discount rate of 10 percent, we end up with an annual profit flow and a NPV of 5.2-13.5 thousand $/km^2 per year and 52-135 thousand $/km^2, respectively.

[24]For simplicity of exposition, we model growth in demand as the arrival of a new export market segmented from existing domestic and international markets for Brazilian beef. It is straightforward, however, to start from an existing demand curve which rotates outward from its y-axis intercept γ to model growth in demand.

exports by the meat processing and export industry (which in turn constitutes derived demand from international consumers).[25] Once we have set δ, we can also determine $\hat{Q}$, using the identity $p(\hat{Q}) = r_{min}$ where $r_{min} \equiv \rho\kappa + \nu$.

Social costs of deforestation [ϑ]. Converting rainforest to pasture causes a loss of carbon from plants and soil, which is then released as carbon dioxide into the atmosphere.[26] To calculate the costs of this loss, we take the product of the net loss of CO_2 (tCO_2/km^2) and the average social cost of carbon ($\$/tCO_2$). To start with the former, according to Franklin Jr and Pindyck (2018), rainforest carbon (CO_2) density is at least 10,000 (36,700) tons per square kilometer and we will assume that 50 percent[27] of this is lost when rainforest is converted to pasture.[28]

As for the social cost of carbon (SCC), at the end of 2023 the global average carbon tax was about $\$5/tCO_2$ according to Cui et al. (2023), which we take as our lower bound. For our upper bound of the SCC we take the estimate of $\$51/tCO_2$ used by the Biden administration to inform public policy. This brings the costs of carbon storage losses from conversion to 92-936 thousand $\$/km^2$.[29]

4 Policy counterfactuals: assessing carbon taxes

This section presents results from our calibrated model. We begin with the baseline scenario without policy interventions, followed by an examination of the quantitative

[25]Based on our calculations on data from FAO (2024), the average 2014-2022 Brazilian FOB export price was \$5.28, suggesting that domestic transportation, processing costs, and profit mark-ups on average added about 65 percent to the cattle price.

[26]For several reasons we abstract from livestock emissions. First, beef production and consumption from pasture has deep cultural roots, especially in many countries of the Western Hemisphere, and subsequently public support for taxing this emission source currently seems low (cf. Funke et al. (2022)). Second, the literature has focused on land conversion emissions, so our assumption also eases comparison with earlier work (cf. Araujo et al. (2024); Souza-Rodrigues (2019)). Third, our analysis demonstrates that fully internalizing the social cost of emissions from land conversion through taxation should, in principle, already render deforestation economically unviable. Fourth, including livestock emissions also requires a more elaborate way of modeling livestock production, which would take away from our focus on roads and deforestation.

[27]Our loss estimate probably constitutes a lower bound. One reason is that higher temperatures and droughts are also degrading the carbon sink function of the Amazon ecosystem. Since this loss is hard to quantify we abstract from it.

[28]It is possible that a sizeable portion of future habitat converted will take place in the Cerrado, a tropical savanna that constitutes the second-largest biome in South-America and that borders on the Amazon. Between 1985 and 2022 the total amount of pasture in Brazil increased by 0.61 M km^2, with large increases in the Amazon (+0.57 M km^2) and Cerrado (+0.17 M km^2), among other areas, outweighing losses in the Atlantic forest (-0.09 M km^2). See Table 1. The potential for carbon losses is smaller in the Cerrado, but we ignore this difference since our estimate for carbon losses in the Amazon, as argued, likely constitutes a lower bound.

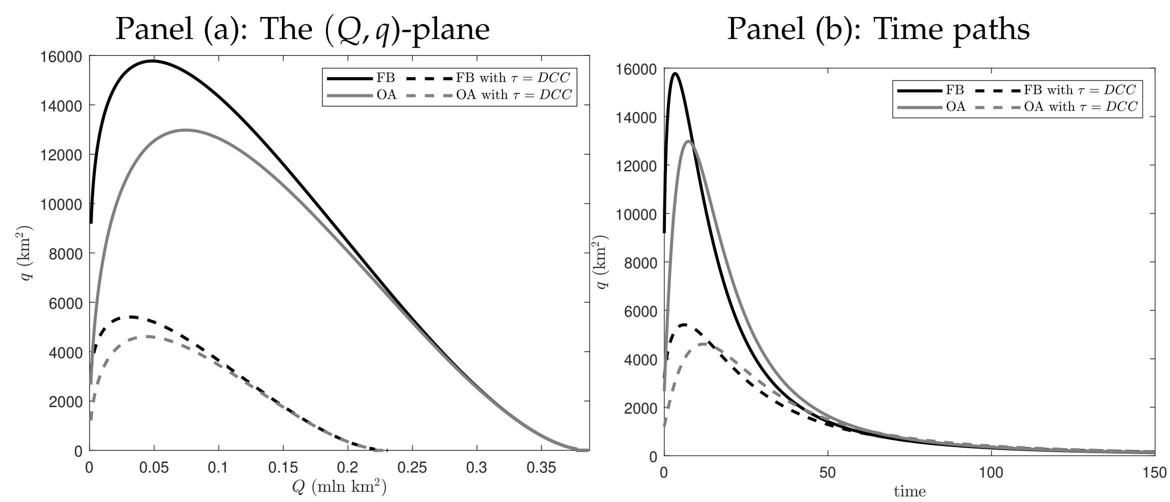
[29]Recent work by Franklin Jr and Pindyck (2018) on deforestation tipping points and Bilal and Känzig (2024) on the impact of global temperature shocks, however, suggests that the SCC could be an order of magnitude higher. Our results in Section 4 show that deforestation already becomes inefficient when carbon taxes are set at relatively low levels.

effects of a counterfactual carbon tax (Section 4.1). Finally, we conduct sensitivity analysis (Section 4.2).

4.1 The calibrated model

Figure 7 shows the evolution of land conversion with and without a carbon tax in the (Q, q)-plane (panel (a)) and over time (panel (b)). The solid curves correspond to the baseline without a carbon tax, whereas the dashed curves show the outcome with a constant carbon tax in place, equal to Brazil's *domestic cost of carbon*. We will discuss the baseline and the policy scenario in turn.

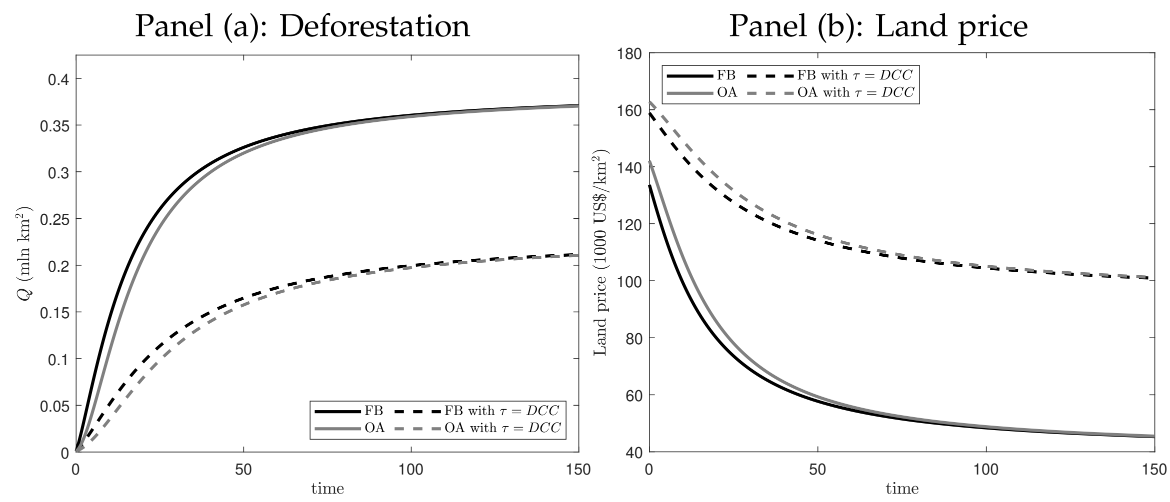
Figure 7: Land conversion with and without a carbon tax



Notes: Black (grey) curves correspond to a first-best (open access) scenario. Solid (dashed) curves correspond to the case without (with) carbon taxation. The carbon tax is set at the rate that is unilaterally optimal for Brazil, i.e., the DCC (see main text).

Baseline. In the 2035 baseline beef demand scenario, we find that deforestation peaks at around 13,000 km^2/yr under open access and around 16,000 km^2/yr in the first-best, and in both the first-best and under open access does not drop below 8,000 km^2/yr for the next 20 years. See the solid curves in Figure 7. The difference between first-best and open access suggests the road network spillover is quantitatively important. The simulated, near-future deforestation rates of our baseline scenario are close to the deforestation rates observed in the Brazilian Amazon in recent years; with a low of 4,570 km^2 in 2012, a recent high of 13,000 km^2 in 2021, and an average of 8,500 (15,000) km^2 between 2013-2022 (2000-2012). Thus our model, calibrated using future beef demand scenarios, generates projected deforestation rates comparable to historically observed levels, suggesting its predictions are plausible.

Figure 8: Deforestation and the land price



Notes: Black (grey) lines correspond to a first-best (open access) scenario. Solid (dashed) lines correspond to the case without (with) carbon taxation. The carbon tax is set at the rate that is unilaterally optimal for Brazil, i.e., the DCC (see main text).

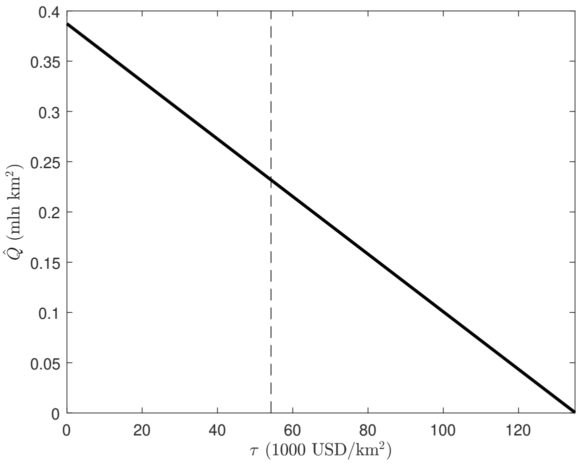
Figure 8 shows the evolution of deforestation (panel (a)) and the land price (panel (b)). The cumulative amount of deforestation in our normal beef demand scenario converges to around 390,000 km^2 in 150 years — an area roughly nine times the size of the Netherlands or Denmark — of which about half occurs in the first 20 years. high beef prices initially cause land prices to be high, but in the long run when demand saturates, they converge to the marginal cost of converting land.

Carbon tax. Next, we use the calibrated model to quantify the effect of a counterfactual carbon tax, which is equivalent to a land-clearing fine in our model. As before, in the first-best a planner, e.g., the Brazilian government, internalizes the road network spillover, while under open access myopic cattle ranchers ignore this externality. Farmers now pay a constant and exogenous carbon tax τ per unit land, such that their cost of land conversion equals $\kappa + \tau$. The optimization problem is thus qualitatively unaffected by the imposition of the tax.

We consider a scenario in which the Brazilian government only accounts for the *Domestic Cost of Carbon* (DCC) from deforestation. The DCC is inherently lower than the (global) Social Cost of Carbon (SCC) because it only considers climate damages to Brazil, not the entire world. Using our previous SCC estimate of \$51 and research by Ricke et al. (2018), which suggests Brazil bears about 5.8% of global climate costs, we conservatively estimate Brazil's DCC at 2.96 USD/tCO$_2$, which corresponds to 54 thousand USD/km^2 if half of the stored carbon is released as a result of deforestation.

The dashed curves in Figures 7 and 8 show that the carbon tax considerably lowers deforestation and increases the land price. The reason is that deforestation becomes more expensive. This reduces the long-run availability of farmland and hence drives up its profitability and therefore its price.

Figure 9: The effect of a carbon tax on long-run deforestation



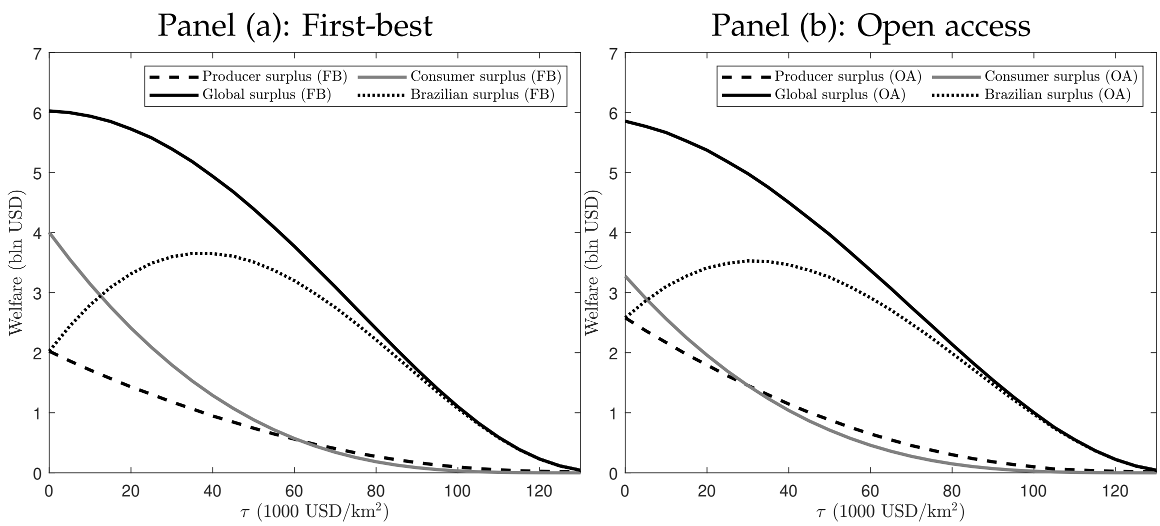
Notes: The dashed vertical line indicates the carbon tax rate that is unilaterally optimal for Brazil, i.e., the DCC (see main text). $\hat{Q}$ on the vertical axis is the cumulative amount of deforestation.

Figure 9 shows that the total amount of deforestation, $\hat{Q}$, declines linearly as the land tax increases, owing to the linear demand function and the constant marginal cost when q goes to zero (see (35), with κ replaced by $\kappa + \tau$). The land tax required to substantially decrease deforestation under this beef demand scenario is relatively small. A land tax of 67.6 thousand \$/km^2 (or, equivalently, a carbon tax of 3.68 \$/tCO_2) is enough to cut total deforestation almost in half, while one of 135.2 thousand \$/km^2 (7.37 \$/tCO_2) eliminates deforestation in the Amazon altogether. As recent estimates of the global social cost of carbon are typically at least one order of magnitude higher (cf. the Biden administration's estimate of 51 \$/tCO_2), this implies that even if only a relatively small fraction of the damages are internalized, further land conversion for beef production at current prices and productivity levels would become unprofitable in the Amazon. A carbon tax equal to Brazil's DCC level of 54 thousand USD/km^2, $\tau = DCC$, indicated by the dashed vertical line, would be sufficient to reduce Brazil's projected cumulative future deforestation by 40 percent.

Our results align with recent studies demonstrating that a relatively small land or carbon tax can have a sizeable impact on Amazon deforestation. Souza-Rodrigues (2019) used 2006 Census data on existing Amazon farms to estimate an aggregate demand curve for deforestation, similar to our Figure 9. His research found that in the long-run a land tax of 42.5 thousand USD per km^2 would boost the existing overall forest coverage of 44% to 80%, effectively reducing deforestation by 64%. Araujo et al. (2024) develop a dynamic discrete choice model where farmers maximize profits by choosing land use, accounting for conversion costs and returns from alternative uses.

Using location-specific carbon stock data, they estimate the effects of hypothetical carbon taxes. They found that taxes of \$2.5, \$5, and \$10 per ton of CO_2 stored in the forest would achieve 60%, 81%, and 95% of the socially efficient reduction in deforestation, respectively. Based on our calculations, these carbon taxes correspond to land conversion taxes of 46, 92, and 183 thousand USD per km^2. Both studies, like ours, demonstrate that modest economic incentives can lead to substantial decreases in Amazon deforestation, supporting the efficacy of such policy interventions.

Figure 10: Welfare effects of a carbon tax



Notes: The graphs show the impacts of a carbon tax on the different welfare components for both the first-best (panel (a)) and open access (panel (b)) for different values of the carbon tax..

Welfare. Figure 10 examines the effect of a carbon tax on different welfare components: consumer surplus (in North), producer surplus (in Brazil, net of carbon tax revenues), global surplus, and Brazil's surplus (i.e., the sum of producer surplus and carbon tax revenues), all in present discounted value terms and all ignoring climate damages.

Panel (a) shows the first-best outcome and panel (b) the outcome under open access. The figure shows that the carbon tax lowers consumer surplus, producer surplus, and global surplus. Brazil's surplus reaches its maximum under a positive carbon tax, which restricts beef supply and raises consumer prices, benefiting Brazil through an improved terms of trade. Our calibration shows that this surplus peak occurs at a modest carbon tax of approximately 40 thousand \$/km^2 (or \$2.18/tCO2), which is less than the DCC. The initial amount of agricultural land is negligible, which explains why all welfare components converge to almost zero if the tax converges to the prohibitive rate of 135.2 thousand \$/km^2.

When comparing panels (a) and (b), it stands out that the outcomes in the first-best and under open access are quite similar. Brazil's surplus without a carbon tax is lower in the first-best than under open access, as deforestation under open access is relatively slower initially resulting in a path with higher consumer prices.

Our sensitivity analysis in the next section shows that the differences between open access and the first-best can be substantial, however. Especially if the discount rate is low, the road network spillover is strong, and if land is scarce instead of abundant.

4.2 Sensitivity analysis

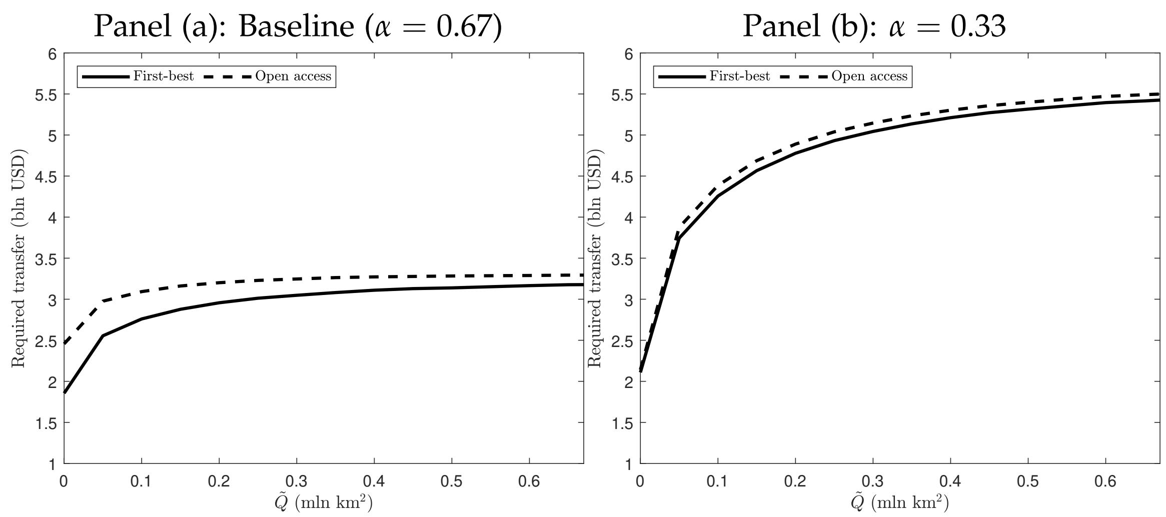
In this section we calculate how the different components of welfare respond to changes in various key parameter values. We also study how the change in Brazil's surplus resulting from a carbon tax is affected by the initial size of the road network. This thought experiment is used to study the possibility of a deforestation lock-in (i.e., the costs to stop deforestation increase in the size of the existing road network).

The initial size of the road network. Our baseline scenario considered a small initial road network (see Section 3.2). Let us now consider how welfare and its various components change when we increase the size of the initial road network, while keeping agricultural production and demand unchanged. The rationale behind this thought experiment is to model a deforestation lock-in effect: if the existing road network is larger, it should make deforestation ceteris paribus more profitable for a *given* level of export demand for Brazilian beef. To do so, we vary $\tilde{Q}$ in the cost function $C(Q_0 + \tilde{Q}, q(0))$, while keeping Q_0, and hence $U'(Q_0)$, constant.

Figure 11 plots the incentive-compatible transfer that North must pay Brazil to implement a carbon tax rate that halts deforestation completely (i.e., 135.2 thousand $\$/\text{km}^2$ (7.37 $\$/\text{tCO}_2$). In calculating this transfer, we assume that Brazil ignores climate damages and we use the equilibrium without a carbon tax, with surplus $J_B(0)$, as outside option (in accordance with (34)).[30] The figure shows that a larger initial road network increases the incentive-compatible transfer that stops deforestation. The reason is that development costs decrease with road network size, leading to a development path with more immediate land conversion, higher producer surplus, and greater surplus for Brazil (producer surplus plus carbon tax revenues). Therefore, North must also offer Brazil a larger transfer to compensate for the greater welfare loss of stopping deforestation. Furthermore, panels (a) and (b) show that the effect

[30]Note that the surplus loss from halting deforestation completely, and hence the minimum required transfer, would be larger if the government of Brazil would account for the effect of the carbon tax on the terms-of-trade, as it would then use the equilibrium with the positive carbon tax rate that maximizes Brazil's surplus as outside option, see Figure 10.

Figure 11: Required transfer to stop deforestation



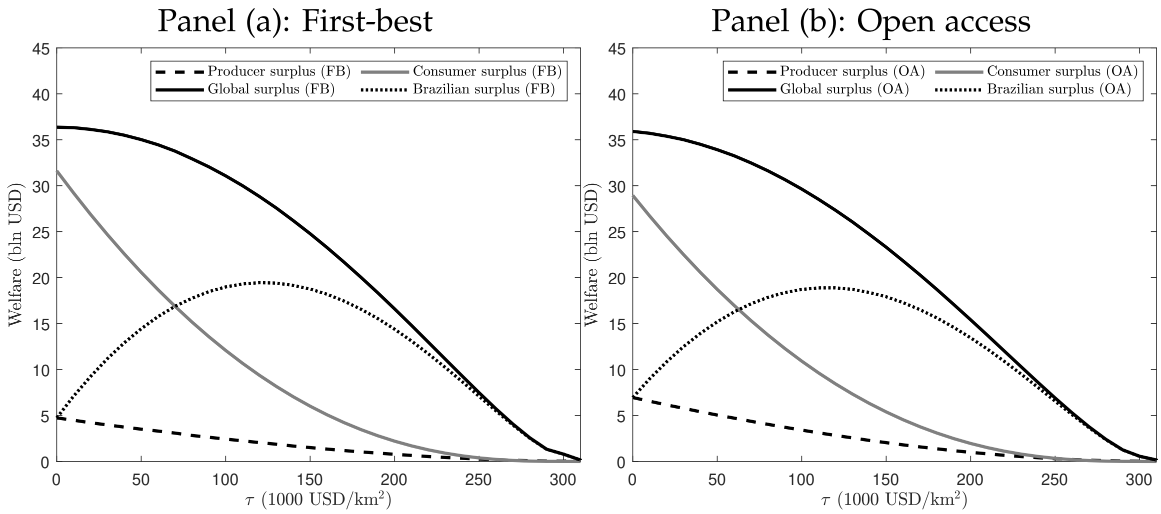
Notes: The graphs show the effect of varying the initial Q in the cost function while keeping the initial Q in the utility function constant. Hence, the initial cost function and initial utility function read $C(Q_0 + \tilde{Q}, q(0))$ and $U(Q_0)$, where Q_0 denotes the RCA from the benchmark model. In calculating the transfer, we assume that Brazil ignores climate damages and use the equilibrium without a carbon tax, with surplus $J_B(0)$, as outside option.

of the initial road network size becomes more pronounced if the importance of the road network in the cost function goes up (we have imposed $\beta = 1 - \alpha$) or, stated otherwise, if the input of labor in road construction is subject to stronger diminishing returns.

Thus, panel (a) of Figure 11 reveals how existing road infrastructure creates a "lock-in" effect that increases the cost of conservation. Comparing a highly developed scenario ($\tilde{Q} = 0.65$ mln km^2), corresponding to the "all farming" category in Table 1 to an undeveloped baseline ($\tilde{Q} = 0$), we find that the required transfer is 34% higher under open access and 72% higher under private property. This lock-in effect becomes even more pronounced when road infrastructure plays a larger role in reducing development costs: as shown in panel (b), the required transfer increases by over 150% between the undeveloped and highly developed scenarios, both in the first-best and under open access.

While Brazil's Amazon development cannot be reversed, our results highlight a potential deforestation "lock-in" effect: existing infrastructure makes future conservation increasingly expensive. This suggests that North may find it more cost-effective to prevent deforestation in countries with lower infrastructure density, such as Papua New Guinea and the Democratic Republic of Congo (DRC), rather than paying the higher costs required to halt further deforestation in Brazil's already-developed Amazon

Figure 12: Welfare effects of a carbon tax ($\rho = 0.05$)



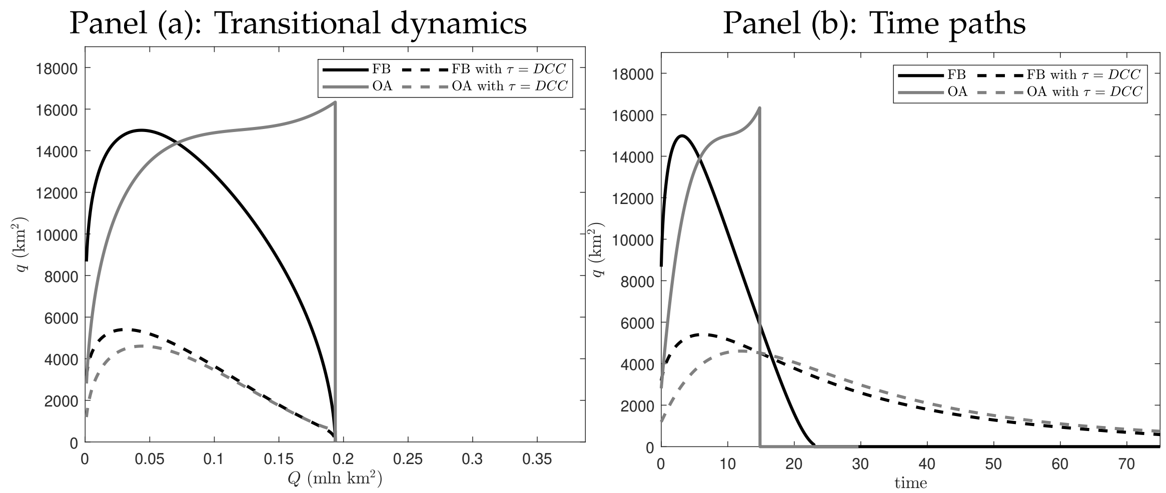
Notes: The graphs show the impacts of a carbon tax on welfare and its sub-components for both the first-best (panel (a)) and open access (panel (b)) for different values of the carbon tax. Vertical dashed line represents the DCC (in 1000 USD/km^2) for Brazil.

The discount rate. Figure 12 shows that welfare levels become significantly larger if the discount rate is reduced from $\rho = 0.1$ to $\rho = 0.05$ (compare with Figure 10). The welfare effects of a carbon tax are more pronounced as well. The reason for the differences with the baseline is that the stream of future agricultural profits and climate damages is valued more in present value terms.

Land scarcity. In the baseline scenario a large part of the Amazon rainforest remains unexploited as the price of agricultural goods declines with the amount of forest converted to pasture. This implies that long-run deforestation exhibits similar dynamics in the first-best and under open access, with deforestation asymptotically converging to zero in both cases, as the profitability of development gradually vanishes over time. Here we examine a scenario with land scarcity (see Section 2.3) in which, for some reason, only half of the deforested area in the baseline model can actually be developed. Figure 13 shows that this implies a markedly different deforestation trajectory in the medium run. In the first-best, deforestation is smoothed over a long time horizon due to convex development costs. Under open access, however, a deforestation race materializes which becomes more rapid over time due to the declining costs as a result of the expanding road network.

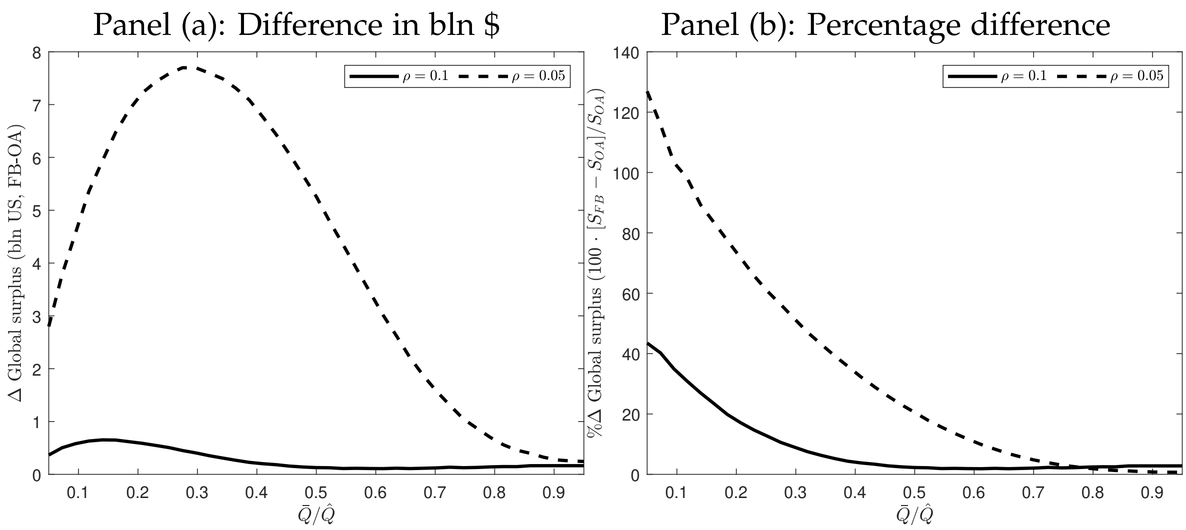
Figure 14 illustrates the global surplus difference between the first-best scenario and the open access regime, assuming a discount rate of $\rho = 0.1$ (solid curves) and $\rho = 0.05$ (dashed curves). Panel (a) presents the absolute difference in global surplus

Figure 13: The effects of a carbon tax on land conversion: the case of land scarcity



Notes: Black (grey) lines correspond to a first-best (open access) scenario. Solid (dashed) lines correspond to the case without (with) carbon taxation. Carbon tax is set at rate that is unilaterally optimal for Brazil (see text).

Figure 14: Difference in surplus between FB and OA: the effect of land scarcity



Notes: The graphs show the difference in global surplus between the first-best and the open access regime, at various total rainforest sizes, $\bar{Q}$, as a share of $\hat{Q}$, the amount that is economically viable to develop. In panel (b), we use S_{FB} (S_{OA} to denote global surplus in the first-best (under open access), respectively.

(in billion \$), whereas panel (b) shows the difference as a percentage of the global surplus under open access. In both panels, the horizontal axis represents the size of the rainforest available for development ($\bar{Q}$) as a share of the economically viable area ($\hat{Q}$). As the area available for development decreases, land scarcity becomes more acute. This leads to a larger percentage difference in surplus between the first-best and open access regime, as shown in panel (b). The absolute surplus difference shown in panel (a) is non-monotonic, peaking at approximately 7.5 billion \$ in the case with $\rho = 0.05$ (dashed curve). This is because, when the area available for development becomes small, the surpluses in both scenarios diminish. If the size of the rainforest is larger than the economically viable area, the difference in global surplus between the first-best and the open access regime is small (less than 1 bln \$, as shown in Figures 10 and 12 for the case with $\rho = 0.1$ and $\rho = 0.05$, respectively).

5 Discussion

Our framework suggests several extensions that we develop in Supplementary Appendices B.4-B.7. First, we relax the assumption that road construction and land conversion occur simultaneously by introducing "pioneers" — loggers or government agencies — who build roads for timber harvesting and connectivity purposes, while agricultural producers convert accessible land at a potentially different pace (Supplementary Appendix B.4). This extension can rationalize the finding by Botelho Jr et al. (2022) that the road network in unconverted parts of the Amazon is more extensive than previously recognized, and it introduces an additional externality requiring a separate policy instrument. Second, we show how transportation costs that increase linearly with distance from the economic core can be incorporated into the model as a convex cost of aggregate production (Supplementary Appendix B.5). Third, we explore how time-inconsistent preferences à la Harstad (2020) interact with the road network externality: because today's road investments lower tomorrow's construction costs, incumbent governments have a strategic incentive to subsidize infrastructure to constrain their successors, an effect that climate externalities may partially offset (Supplementary Appendix B.6). Finally, we discuss how weak state capacity may prevent governments from deploying the full set of instruments needed to implement the first-best, forcing trade-offs between internalizing the road network externality and limiting the climate externality from deforestation (Supplementary Appendix B.7).

6 Conclusion

In their survey of the economics literature on deforestation, Balboni et al. (2023) argue that economic analysis and theory have only recently begun to catch up with the rapid land use changes revealed by satellite data, with the goal of quantifying the forces that drive deforestation. This requires models that encompass growing pressures for land use change, national and international externalities, insecure property rights, and political economy challenges.

Our paper addresses this challenge by proposing a spatial model of road investment, land conversion for agriculture, and deforestation. Building on elementary geometric and economic principles, the model's key innovation is that road costs decrease with the size of the existing road stock. This stepping-stone externality is specific to infrastructure networks: each new road must connect to an existing one, creating path-dependent expansion dynamics that static spatial models cannot capture. Under open access, myopic producers fail to internalize this dynamic externality, and consequently the pace of deforestation is faster under private property than under open access—reversing a standard result from resource economics.

A counteracting climate externality, however, calls for less and slower deforestation, both under private property and open access. Calibrating our model to the Brazilian Amazon, we find that relatively small carbon taxes can fully stop the conversion of forest to pasture, effectively halting the expansion of cattle ranching into forested areas that is driven by the growing export demand for Brazilian beef. Furthermore, the calibration reveals that existing road networks create lock-in effects that make halting deforestation more costly as they would require larger international transfers to Brazil. Given these lock-in effects, we suggest that conservation-minded NGOs and international donors may be able to reduce deforestation more efficiently in the Democratic Republic of Congo and Papua New Guinea, two rainforest-rich countries with sparse infrastructure networks, than in Brazil.

Our analysis has important limitations. The model treats all land as homogeneous and abstracts from agricultural productivity growth, spatial sorting, and the general equilibrium effects emphasized by recent quantitative trade models of deforestation. It also focuses on carbon taxes as the primary policy instrument, while in practice spatially targeted policies such as protected areas and payment-for-ecosystem-services programs may be more cost-effective. Nonetheless, the lock-in mechanism we identify—whereby existing infrastructure makes future conservation progressively more expensive—is likely to be robust to these extensions.

The relationship between roads and tropical deforestation offers many avenues for future research. As momentum builds to reconstruct Brazil's BR-319 highway, our proposed extension examining road construction driven not only by agricultural interests but also by mobility, timber, and mining concerns appears particularly relevant. In this framework, road taxation or subsidization becomes distinct from taxing land clearing, creating multiple policy instruments. While subsidizing roads may advance development goals like mobility, it also makes forests accessible to illegal loggers and land speculators, raising monitoring costs and creating a trade-off between development and conservation. This trade-off becomes more complex still under time-inconsistent preferences, a realistic feature of political systems where presidents or other leaders with different environmental priorities alternate in power. We leave an exploration of this and other promising research directions for future work.

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Appendix

A Mathematical proofs

Proof of Lemma 1

Proof. Fix $Q > 0$ and write $s = s(Q) = 2\sqrt{\pi Q}$. For $q > 0$, monotonicity in u implies the constraint binds at the optimum. Hence there exists a (unique) labor requirement $u = \varphi(q,s)$ satisfying $q = f(\varphi(q,s),s)$, and

$$C(Q,q) = \omega\, \varphi\big(q, s(Q)\big) + \kappa q. \tag{A.1}$$

(i) If $q = 0$, choose $u = 0$ to obtain $C(Q,0) = 0$. If $Q \downarrow 0$, then $s(Q) \downarrow 0$ and $f(u,0) = 0$ for all u, so no finite u can produce $q > 0$; hence $C(Q,q) \to \infty$.

(ii) Differentiate $q = f(\varphi(q,s),s)$ w.r.t. q (holding s fixed): $1 = f_1\, \varphi_q \quad \Rightarrow \quad \varphi_q = \frac{1}{f_1} > 0$. Thus $C_q = \omega\varphi_q + \kappa = \omega/f_1 + \kappa > 0$. Differentiate $q = f(\varphi(q,s),s)$ w.r.t. s (holding q fixed): $0 = f_1\, \varphi_s + f_2$. This yields $\varphi_s = -\frac{f_2}{f_1} \leq 0$. Since $s'(Q) > 0$, we get $C_Q = \omega\varphi_s s'(Q) \leq 0$.

(iii) Differentiate $\varphi_q = 1/f_1$ w.r.t. q: $\varphi_{qq} = -\frac{f_{11}}{f_1^3} \geq 0$ (since $f_{11} \leq 0$), so $C_{qq} = \omega\varphi_{qq} \geq 0$. Next differentiate $\varphi_q = 1/f_1$ w.r.t. s: $\varphi_{qs} = -\frac{f_{11}\varphi_s + f_{12}}{f_1^2} = \frac{f_{11}f_2 - f_{12}f_1}{f_1^3} \leq 0$, using $\varphi_s = -f_2/f_1$, $f_{11} \leq 0$, $f_2 \geq 0$, $f_{12} \geq 0$, and $f_1 > 0$. Hence $C_{qQ} = \omega\varphi_{qs}s'(Q) \leq 0$.

(iv) Differentiate $0 = f_1\varphi_s + f_2$ w.r.t. s: $0 = f_{11}\varphi_s^2 + f_1\varphi_{ss} + 2f_{12}\varphi_s + f_{22}$. This yields $\varphi_{ss} = -\frac{f_{11}\varphi_s^2 + 2f_{12}\varphi_s + f_{22}}{f_1}$. Joint concavity of f implies the quadratic form in the numerator is non-positive, hence $\varphi_{ss} \geq 0$. Therefore $C_{QQ} = \omega\left(\varphi_{ss}(s')^2 + \varphi_s s''\right) \geq 0$, because $\varphi_{ss} \geq 0$, $s'(Q) > 0$, $\varphi_s \leq 0$, and $s''(Q) < 0$ for $s(Q) = 2\sqrt{\pi Q}$. □

Proof of Proposition 1

Proof. To prove existence, uniqueness, and differentiability, fix $Q \in (0, \hat{Q})$. Recall that the first-best $\dot{q} = 0$ isocline for $q > 0$ is defined by $\Omega(Q, h(Q)) = 0$, where $\Omega(Q,q) \equiv C_Q(Q,q) + \rho C_q(Q,q) - C_{qQ}(Q,q)q - B'(Q)$.

Since $C(Q,0) = 0$ for all $Q > 0$ (Lemma 1(i)), it follows that $C_Q(Q,0) = 0$. Moreover, $C_q(Q,0) = \kappa$. Hence $\Omega(Q,0) = C_Q(Q,0) + \rho C_q(Q,0) - B'(Q) = \rho\kappa - B'(Q)$. By the definition of $\hat{Q}$, we have $B'(\hat{Q}) = \rho\kappa$, and since B' is strictly decreasing, it follows that $\Omega(Q,0) < 0$ for all $Q \in (0, \hat{Q})$.

On the other hand, for fixed $Q > 0$, note that $\Omega_q(Q,q) = \rho C_{qq}(Q,q) - C_{qqQ}(Q,q)q$. By Assumption 2, $C_{qq}(Q,q) > 0$ and $C_{qqQ}(Q,q) \leq 0$ for all $q > 0$ and $Q \in (0,\bar{Q})$. Hence $\Omega_q(Q,q) = \rho C_{qq}(Q,q) - C_{qqQ}(Q,q)q > 0$ for all $q > 0$. Therefore, for each fixed $Q \in (0,\hat{Q})$, the function $q \mapsto \Omega(Q,q)$ is strictly increasing on $\mathbb{R}_{++}$, implying that $\Omega(Q,q) \to \infty$ as $q \to \infty$. Hence, there exists a unique $q = h(Q) > 0$ such that $\Omega(Q,h(Q)) = 0$.

Since Ω is continuously differentiable and $\Omega_q(Q,h(Q)) > 0$ for all $Q \in (0,\hat{Q})$, the Implicit Function Theorem implies that h is continuously differentiable on $(0,\hat{Q})$, with

$$h'(Q) = -\frac{\Omega_Q(Q,h(Q))}{\Omega_q(Q,h(Q))}.$$

(i) If $C_Q = 0$, then also $C_{qQ} = 0$ and $C_{qQQ} = 0$. Hence $\Omega_Q(Q,q) = -B''(Q)$. Since $U''(Q) < 0$, it follows that $-B''(Q) > 0$. Therefore,

$$h'(Q) = -\frac{\Omega_Q}{\Omega_q} < 0$$

for all Q such that $h(Q) > 0$. Moreover, as $Q \uparrow \hat{Q}$, $\Omega(\hat{Q},0) = \rho\kappa - B'(\hat{Q}) = 0$. Hence, $\lim_{Q\uparrow\hat{Q}} h(Q) = 0$.

(ii) Take $C_Q < 0$. As $Q \downarrow 0$, due to Assumption 2 we get $C_Q(Q,q) - qC_{qQ}(Q,q) \to \infty$ for any $q > 0$. Hence, $\Omega(Q,h(Q)) = 0$ requires $q = h(Q) \to 0$. As $Q \uparrow \hat{Q}$, we have $\Omega(\hat{Q},0) = C_Q(\hat{Q},0) + \rho\kappa - B'(\hat{Q}) = C_Q(\hat{Q},0) = 0$. Hence, $\lim_{Q\uparrow\hat{Q}} h(Q) = 0$.

□

Proof of Proposition 2

Proof. Under Assumption 4, the reduced-form cost function can be written as

$$C(Q,q) = Aq^{1/\alpha}Q^{-\theta} + \kappa q, \qquad \theta \equiv \frac{\beta}{2\alpha} > 0,$$

with $A \equiv \omega\left(2\sqrt{\pi}\right)^{-\frac{\beta}{\alpha}}$. Moreover, $B'(Q) = \gamma - \delta Q - \nu$, and the first-best isocline is defined by $\Omega(Q,h(Q)) = 0$, where $\Omega(Q,q) = C_Q(Q,q) + \rho C_q(Q,q) - C_{qQ}(Q,q)q - B'(Q)$. Now $C_Q = -\theta A q^{1/\alpha}Q^{-\theta-1}$, $C_q = \frac{A}{\alpha}q^{1/\alpha-1}Q^{-\theta} + \kappa$, and $C_{qQ} = -\frac{\theta A}{\alpha}q^{1/\alpha-1}Q^{-\theta-1}$. Substituting into $\Omega(Q,h(Q)) = 0$ and rearranging yields

$$B'(Q) - \rho\kappa = AQ^{-\theta}h(Q)^{1/\alpha-1}\left[\frac{\rho}{\alpha} + \theta\left(\frac{1}{\alpha} - 1\right)\frac{h(Q)}{Q}\right]. \tag{A.2}$$

Since $B'(\hat{Q}) = \rho\kappa$ by definition of $\hat{Q}$, and $B'(Q) = \gamma - \delta Q - \nu$, we have $B'(Q) - \rho\kappa = \delta(\hat{Q} - Q)$. Hence (A.2) becomes

$$\delta(\hat{Q} - Q) = AQ^{-\theta}h(Q)^{1/\alpha - 1}\left[\frac{\rho}{\alpha} + \theta\left(\frac{1}{\alpha} - 1\right)\frac{h(Q)}{Q}\right]. \tag{A.3}$$

Define $x(Q) \equiv h(Q)/Q$, $p \equiv (1-\alpha)/\alpha > 0$, $a \equiv \rho/\alpha > 0$, and $b \equiv \theta(1/\alpha - 1) > 0$. Then (A.3) can be written as

$$x(Q)^p(a + bx(Q)) = \frac{\delta}{A}(\hat{Q} - Q)Q^{-\lambda}, \qquad \lambda \equiv p - \theta = \frac{2(1-\alpha) - \beta}{2\alpha} > 0, \tag{A.4}$$

where $\lambda > 0$ follows from $\alpha + \beta \leq 1$. Let $f(x) \equiv x^p(a + bx)$ and $r(Q) \equiv (\hat{Q} - Q)Q^{-\lambda}$. Then (A.4) reads

$$f(x(Q)) = \frac{\delta}{A}r(Q). \tag{A.5}$$

Now $f'(x) = x^{p-1}(pa + (p+1)bx) > 0$ for all $x > 0$, while $r'(Q) = -Q^{-\lambda-1}[Q + \lambda(\hat{Q} - Q)] < 0$ for all $Q \in (0, \hat{Q})$. Hence f is strictly increasing and r is strictly decreasing, so (A.5) implies that $x(Q)$ is strictly decreasing on $(0, \hat{Q})$.

We next establish the boundary behavior. As $Q \downarrow 0$, the right-hand side of (A.4) tends to $+\infty$ because $\lambda > 0$, so $x(Q) \to \infty$ since f is strictly increasing and unbounded on $\mathbb{R}_{++}$. Returning to (A.3), we obtain $\delta(\hat{Q} - Q) \geq \frac{A\rho}{\alpha}Q^{-\theta}h(Q)^{1/\alpha - 1}$, and therefore $h(Q)^{1/\alpha - 1} \leq \frac{\alpha\delta\hat{Q}}{A\rho}Q^{\theta}$. Because $1/\alpha - 1 > 0$ and $\theta > 0$, it follows that

$$h(Q) \to 0 \qquad \text{as } Q \downarrow 0. \tag{A.6}$$

As $Q \uparrow \hat{Q}$, the right-hand side of (A.4) converges to zero, so $f(x(Q)) \to 0$, hence $x(Q) \to 0$. Therefore

$$h(Q) = Qx(Q) \to 0 \qquad \text{as } Q \uparrow \hat{Q}. \tag{A.7}$$

Finally, since $h(Q) = Qx(Q)$, we have

$$h'(Q) = x(Q) + Qx'(Q). \tag{A.8}$$

Differentiating (A.5) gives

$$x'(Q) = \frac{\delta}{A}\frac{r'(Q)}{f'(x(Q))} < 0. \tag{A.9}$$

Now define

$$L(Q) \equiv -Q\frac{r'(Q)}{r(Q)} = \frac{Q + \lambda(\hat{Q} - Q)}{\hat{Q} - Q}, \qquad M(x) \equiv x\frac{f'(x)}{f(x)} = \frac{pa + (p+1)bx}{a + bx}.$$

Using (A.5) and (A.9), we obtain $Qx'(Q) = -L(Q)x(Q)/M(x(Q))$, and thus

$$h'(Q) = x(Q)\left[1 - \frac{L(Q)}{M(x(Q))}\right]. \tag{A.10}$$

Now $L'(Q) = \hat{Q}/(\hat{Q} - Q)^2 > 0$, so L is strictly increasing, and $M'(x) = ab/(a + bx)^2 > 0$, so M is strictly increasing in x. Since $x(Q)$ is strictly decreasing, $M(x(Q))$ is strictly decreasing in Q. Hence the ratio $L(Q)/M(x(Q))$ is strictly increasing on $(0, \hat{Q})$.

As $Q \downarrow 0$, we have $x(Q) \to \infty$, so $L(Q) \to \lambda$ and $M(x(Q)) \to p + 1 = 1/\alpha$. Since $\lambda = p - \theta < p + 1 = 1/\alpha$, it follows that $L(Q)/M(x(Q)) < 1$ for Q sufficiently small, and therefore $h'(Q) > 0$ near $Q = 0$ by (A.10). As $Q \uparrow \hat{Q}$, we have $L(Q) \to \infty$ while $x(Q) \to 0$, so $M(x(Q)) \to p < \infty$. Hence $L(Q)/M(x(Q)) > 1$ for Q sufficiently close to $\hat{Q}$, and therefore $h'(Q) < 0$ near $\hat{Q}$.

Because $L(Q)/M(x(Q))$ is continuous and strictly increasing, there exists a unique $Q^* \in (0, \hat{Q})$ such that $L(Q^*)/M(x(Q^*)) = 1$. By (A.10), $h'(Q) > 0$ for $Q < Q^*$ and $h'(Q) < 0$ for $Q > Q^*$. Hence $h(Q)$ is inverted U-shaped on $(0, \hat{Q})$. Together with (A.6) and (A.7), this proves the proposition. □

Proof of Proposition 3

Proof. We first rewrite both systems as differential equations in the (Q, q)-plane. Since $\dot{Q} = q > 0$ along any interior path, we have $dq/dQ = \dot{q}/\dot{Q}$. Hence along the first-best path, $m'(Q) = F(Q, m(Q))$ with $F(Q, q) \equiv \Omega(Q, q)/(qC_{qq}(Q, q))$, while along the open-access path, $n'(Q) = G(Q, n(Q))$ with $G(Q, q) \equiv \tilde{\Omega}(Q, q)/(qC_{qq}(Q, q))$, where $\Omega(Q, q) = C_Q(Q, q) + \rho C_q(Q, q) - qC_{qQ}(Q, q) - B'(Q)$ and $\tilde{\Omega}(Q, q) = \rho C_q(Q, q) - qC_{qQ}(Q, q) - B'(Q)$. Since $\Omega(Q, q) = \tilde{\Omega}(Q, q) + C_Q(Q, q)$, it follows that whenever $C_Q < 0$,

$$F(Q, q) < G(Q, q) \qquad \text{for all } q > 0. \tag{A.11}$$

(i) Suppose first that $C_Q = 0$. Then also $C_{qQ} = 0$, so $\tilde{\Omega}_Q(Q, q) = -B''(Q) > 0$, because $U''(Q) < 0$. Therefore $k'(Q) = -\tilde{\Omega}_Q/\tilde{\Omega}_q < 0$ for all $Q \in (0, \hat{Q})$. Moreover, $\tilde{\Omega}(\hat{Q}, 0) = \rho\kappa - B'(\hat{Q}) = 0$, and therefore $\lim_{Q\uparrow\hat{Q}} k(Q) = 0$.

Now suppose that $C_Q < 0$. As $Q \downarrow 0$, Assumption 2 implies that for any fixed $q > 0$, $C_Q(Q, q) - qC_{qQ}(Q, q) \to \infty$. Since Lemma 1(ii) gives $C_Q(Q, q) \leq 0$, it follows that $-qC_{qQ}(Q,q) = (C_Q(Q,q) - qC_{qQ}(Q,q)) - C_Q(Q,q) \to \infty$. Hence, for every fixed $q > 0$, $\tilde{\Omega}(Q,q) = \rho C_q(Q,q) - qC_{qQ}(Q,q) - B'(Q) \to \infty$ as $Q \downarrow 0$. Therefore the identity $\tilde{\Omega}(Q,k(Q)) = 0$ requires $\lim_{Q\downarrow 0} k(Q) = 0$. Finally, $\tilde{\Omega}(\hat{Q},0) = \rho\kappa - B'(\hat{Q}) = 0$, so $\lim_{Q\uparrow\hat{Q}} k(Q) = 0$.

(ii) Suppose first that $C_Q \equiv 0$. Then the first-best and open-access paths solve the same differential equation in the (Q,q)-plane, namely $m'(Q) = G(Q,m(Q))$ and $n'(Q) = G(Q,n(Q))$, but their terminal conditions differ: $m(\bar{Q}) = 0$ and $n(\bar{Q}) > 0$. Hence the two paths cannot intersect on $(Q_0,\bar{Q})$, for otherwise uniqueness of solutions for the same ODE would imply coincidence. Since $n(\bar{Q}) > m(\bar{Q})$, it follows that $n(Q) > m(Q)$ for all $Q \in (Q_0,\bar{Q})$.

Now suppose that $C_Q < 0$. Let $D(Q) \equiv n(Q) - m(Q)$, and assume that the two paths intersect at some interior point Q_c, i.e. $m(Q_c) = n(Q_c) = q_c > 0$. Then $D'(Q_c) = n'(Q_c) - m'(Q_c) = G(Q_c,q_c) - F(Q_c,q_c) > 0$ by (A.11). Therefore any interior intersection is an upward crossing of D, meaning that $n - m$ can only cross zero from negative to positive as Q increases. This implies that there can be at most one interior intersection: if there were two, continuity would force one of them to be a downward crossing, contradicting $D'(Q_c) > 0$ at every intersection.

(iii) Suppose first that $C_Q \equiv 0$. Then the first-best and open-access systems, including the terminal condition $Q(T) = \hat{Q}$, $m(\hat{Q}) = n(\hat{Q}) = 0$, coincide. Hence $m(Q) = n(Q)$ for all $Q \in (Q_0,\hat{Q})$.

Now suppose that $C_Q < 0$. Again let $D(Q) \equiv n(Q) - m(Q)$. At any interior intersection Q_c with $m(Q_c) = n(Q_c) = q_c > 0$, we have $D'(Q_c) = G(Q_c,q_c) - F(Q_c,q_c) > 0$ by (A.11), so every interior intersection is an upward crossing of D. At the terminal point, however, $D(\hat{Q}) = n(\hat{Q}) - m(\hat{Q}) = 0$. We claim that $D(Q) \leq 0$ for all $Q \in (Q_0,\hat{Q})$, equivalently, $m(Q) \geq n(Q)$ for all $Q \in (Q_0,\hat{Q})$. Suppose instead that there exists some $Q_1 \in (Q_0,\hat{Q})$ with $D(Q_1) > 0$. Since $D(\hat{Q}) = 0$ and D is continuous, there must then exist an interior point $Q_c \in (Q_1,\hat{Q})$ where $D(Q_c) = 0$ and D crosses zero from positive to nonpositive as Q increases. That would be a downward crossing, contradicting the fact that every interior crossing satisfies $D'(Q_c) > 0$. Hence no such Q_1 exists, and therefore $D(Q) \leq 0$ for all $Q \in (Q_0,\hat{Q})$. Equivalently, $m(Q) \geq n(Q)$ for all $Q \in (Q_0,\hat{Q})$. □

B Supplementary Appendix (not for publication)

B.1 Temporal road dynamics and model calibration

Ahmed et al. [2013] study the temporal dynamics of road building in a large number of municipalities in the Brazilian Amazon for the period 2003-2007. Using variation in road construction both within and between municipalities, they find that the annual pace of road construction at the municipal level, q (in km/yr/km^2), exhibits an inverted U-shaped relation with the total road stock, Q (in km^2). This in turn implies that the evolution of the road network as a function of time takes on the form of a sigmoid, or S-shape. We utilize their data to get a sense of the average q/Q ratio,

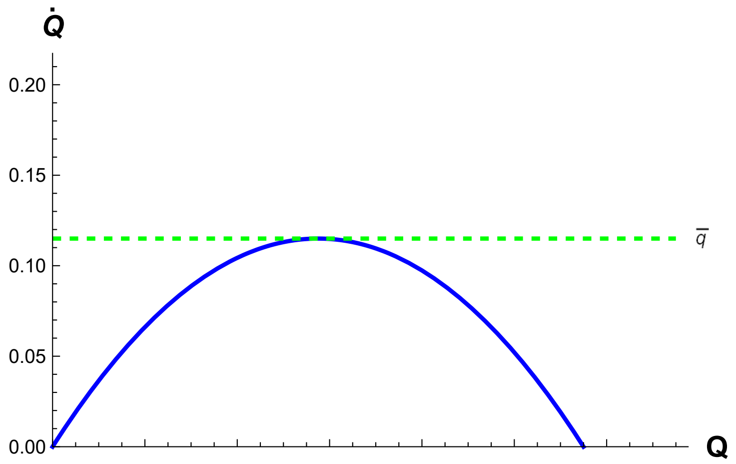


Figure C.1: dynamics of road construction.

that is, the pace of construction relative to the size of the existing network in the Brazilian Amazon (or, said otherwise, the average growth rate of the road network, $\dot{Q}/Q$), as this is a crucial data moment for calibrating the road cost function. To this end, we postulate that the road construction data in their paper can be approximated by logistic growth:

$$\dot{Q} = q = gQ(1 - Q/\overline{Q}) \tag{C.1}$$

The inferred relationship between q and Q is plotted in Figure C.1. Next, the solution to the differential equation (C.1) reads:

$$Q(t) = \frac{e^{gt}Q_0}{1 + \frac{Q_0}{\overline{Q}}\left(e^{gt} - 1\right)} \tag{C.2}$$

Starting in $Q_0 = \alpha\overline{Q}$ with α being a small number, say 0.01, we can then calculate the average ratio q/Q between time 0 and time T, that is, $\frac{1}{T}\left(\int_0^T \frac{q}{Q}dt\right)$, where T is the time at which growth of the road network has almost come to a stop, that is, T

solves $Q(T) = (1-\alpha)\overline{Q}$. Owing to the symmetry of the calculation, with starting and end point both at absolute distance $\left(\frac{1}{2}-\alpha\right)\overline{Q}$ from the center $\frac{1}{2}\overline{Q}$, we find that the average q/Q ratio equals $\frac{1}{2}g$, i.e., the value at the inflection point.

To solve for g, and thus the average q/Q ratio, we first determine $\overline{q}$, that is, the peak road construction level that takes place halfway at $Q = \frac{1}{2}\overline{Q}$. From Figure 4 in Ahmed et al. [2013] we deduce that $\overline{q}$ is approximately $\frac{1}{9}\overline{Q}$ for their 90th percentile regression (implying $g = 4/9$). Furthermore, on page 935 the authors state that on average it takes 75 years for a road network to reach maximum density. Based on this statement, we then solve for g such that the road network grows from $0.01\overline{Q}$ to $0.99\overline{Q}$ in 75 years. This gives $g \approx 0.12$. Putting these two results together, it implies that that the average ratio q/Q is situated between 0.06 and 2/9 (while g is between 0.12 and 4/9).

Using $g = 0.12$ and $Q_{avg} = 0.395$ we can then deduce that $q_{avg} = 0.06 \cdot 0.395$. Using these quantities in the average road cost identity, we can solve for the ω that is commensurate with these costs and this pace of road construction, i.e., $\omega = 1.15 \cdot 30.4 \cdot \left(\frac{2\sqrt{\pi \cdot 0.395}}{0.06 \cdot 0.395}\right)^{\frac{0.33}{0.67}} = 327.7$.

B.2 Road construction: a view in discrete space and time

According to the continuous-space road production function presented in the main text, production depends positively on two inputs, namely the number of construction workers and the mass (or "number") of road projects to be simultaneously worked on. In the short-run, producers can change the input of labor but the number of road projects is fixed. The latter puts a natural brake on the pace of road construction and deforestation in the early stages of road expansion. Conversely, at a later stage the larger number of roads ceteris paribus speeds up the overall pace of road construction and deforestation. The goal of this section is to provide a rationalization for this road production function by representing roads as diagonals in squares (or slots) on a discrete grid.

Rings of potential roads. To motivate our approach in the main text, we consider rings of potential roads around a (circle-shaped) city center (Figure C.2). For the time being it is assumed that the rings extend to infinity, meaning there are no physical barriers to exploiting the forest surrounding the city. We attach numbers to slots, starting in the north-east and going clockwise. So, in Figure C.2, the first ring consists of the slots whose numbers start with 1. Then 1.1 is north-east of the city, 1.2 south-east, 1.3 south-west and 1.4 north-west. Similarly we construct a set of 2's just around the first 'circle' and number them accordingly. We extend the figure up to a fourth ring of roads (whose construction is shown as incomplete in Figure C.2).

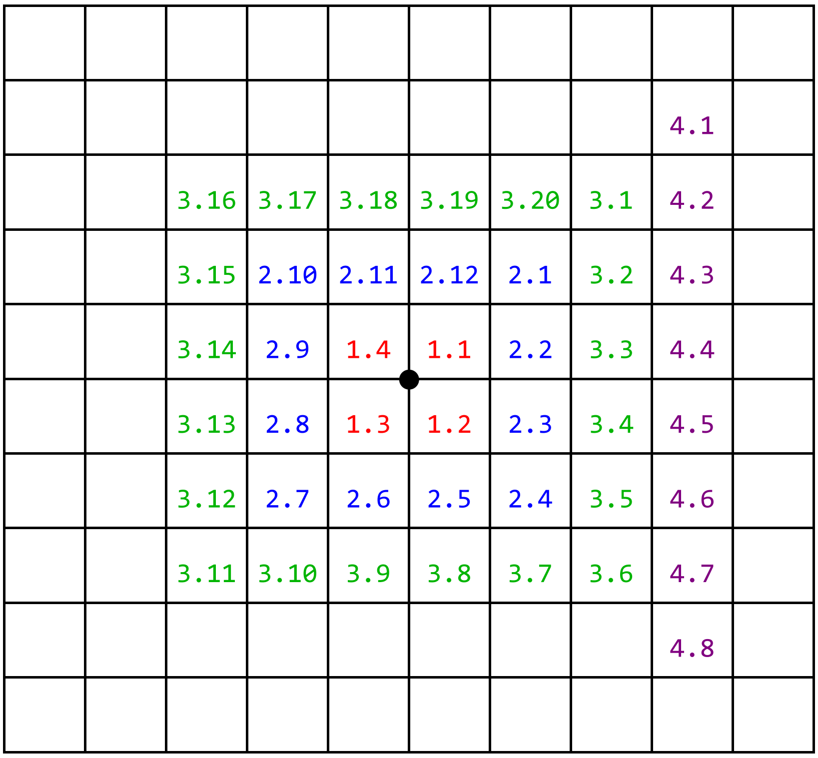


Figure C.2: Discrete grid with three road construction rounds completed.

Roads are *diagonals* in the slots, which are built by producers (e.g., farmers or loggers) with the aim of getting access to and utilizing potentially all resources within a given slot. By Q we denote the total number of slots that have been made accessible by the presence of a road, that is, the road covered area (RCA). A regulator requires that roads either fully connect south west and north east in the slot, or north west and south east. So, in 1.1. the road is into north-east direction, whereas in 2.2. it goes into south-east direction. Also, one cannot start building in a ring without a connection to a previous ring. For example, roads in 3.1 and 3.2 are possible only if there is a road in 1.1 and in 2.1. Hence, each new road connects to a road constructed in the previous round, which in turn connects to a road constructed in the round before that, etc.

We assume that adding a road in the same circle is strictly preferred over adding a road in the next circle. So, if, for example, slot 1.2 is still empty, but 1.1 is not, then building a road in 1.2 is preferred over building a road in 2.1. This could be, for example, because (i) the market for agricultural goods is not only further from 2.1 than 1.2 but also because (ii) construction workers would have to travel further too, so that priority is given to 1.2. This assumption allows for the possibility of not finishing a full circle, but also gives priority to completing the circle before one starts with a new circle.

Next, we introduce some notation to relate the potential expansion of roads in each ring to the existing number of roads. Starting from the center, the total number of roads x constructed up to and including round n increases quadratically with each round of road construction n:

$$x = 4n^2, \tag{C.3}$$

The increase in the number of roads m from round n to round $n+1$ equals:

$$m(n) = x_{t+1} - x_t = 4(2n+1) = 8n + 4. \tag{C.4}$$

Combining (C.3) and (C.4), we obtain the number of new roads that can be constructed as a function of the existing number of roads:

$$m = 4\left(\sqrt{x} + 1\right) \tag{C.5}$$

Property rights. The increase of the road covered area q (in km^2) then equals the number of new roads times the property right A (in km^2) per new road, i.e.,

$$q \equiv \Delta Q = m \cdot A. \tag{C.6}$$

With producers able to access resources on both sides of a road, it seems reasonable to think that the size of the property right will depend on road length l (in *km*) and slot size, which we take as constant and given. We consider three cases with an analytical solution, and consider two that do not appear to have a closed-form solution. In the first case, the producer obtains the right to exploit the entire square described by the diagonal road of length l, that is, $A_1(l) = \frac{1}{2}l^2$.

Before presenting the next cases, consider the following requirements for profitable resource extraction:

- Resources need to be within a distance d (in km) from a road and they can only be towed to a road that is perpendicular to the resource ("perpendicularity requirement"). The regulator does not allow producers to access resources that are within distance d but outside its slot.[31]

- Due to natural transportation barriers or insufficient incentives to exploit remote resources, only resources up to a maximum distance d (in km) from a road are extracted ("distance requirement").

- The road-to-area ratio or road density (in km/km^2) must be at least $1/d$ to avoid congestion in the local road network ("metabolic requirement").

[31]Without this assumption, access would become contested. One can defend this assumption by noting that resource access is therefore granted to the producer/road closest to the resource.

For the second case, we assume that road producers, as in the first case, obtain the right to exploit the entire square described by the diagonal road. However, to this we add the perpendicularity requirement, which implies that resources beyond distance d from the main road are left in situ. In that case, it can be shown that $A_2(l) = 2dl - 2d^2 = al - b$, i.e., there will be forest fragmentation with some parts of the forest unused.

For the third case, we allow producers to build additional roads to extract resources that are de facto inaccessible from the main diagonal road (which is relevant for squares whose diagonal length exceeds $2d$), but the overall network must meet the perpendicularity and metabolic requirements. As it turns out, the latter naturally implies $A_3 = dl$, where l now refers to the total length of the road network in the square. For example, if the area of a slot equals one then $d = \sqrt{2}$ and $A_3 = \sqrt{2}l$. Figure C.3 illustrates the optimal local road network configuration that emerges in this case for squares with diagonal length $2d$, $4d$, $6d$ and $8d$. For example, in panel (b) land in the slots 4 and 13 is completely out of reach from the main diagonal that runs from north-west to south-east, while 3, 8, 9, and 14 are partially out or each. This can be remedied by building a road from north-east to south-west.

For the fourth case, we start from the third case and drop the metabolic requirement leaving only the perpendicularity requirement. For this case, it turns out that for squares with diagonals up to $6d$, the size of the property right is again given by $A_3 = dl$. For larger squares we have not been able to find a closed-form solution to $A(l)$, but the returns per unit road appear to exceed d for these cases.

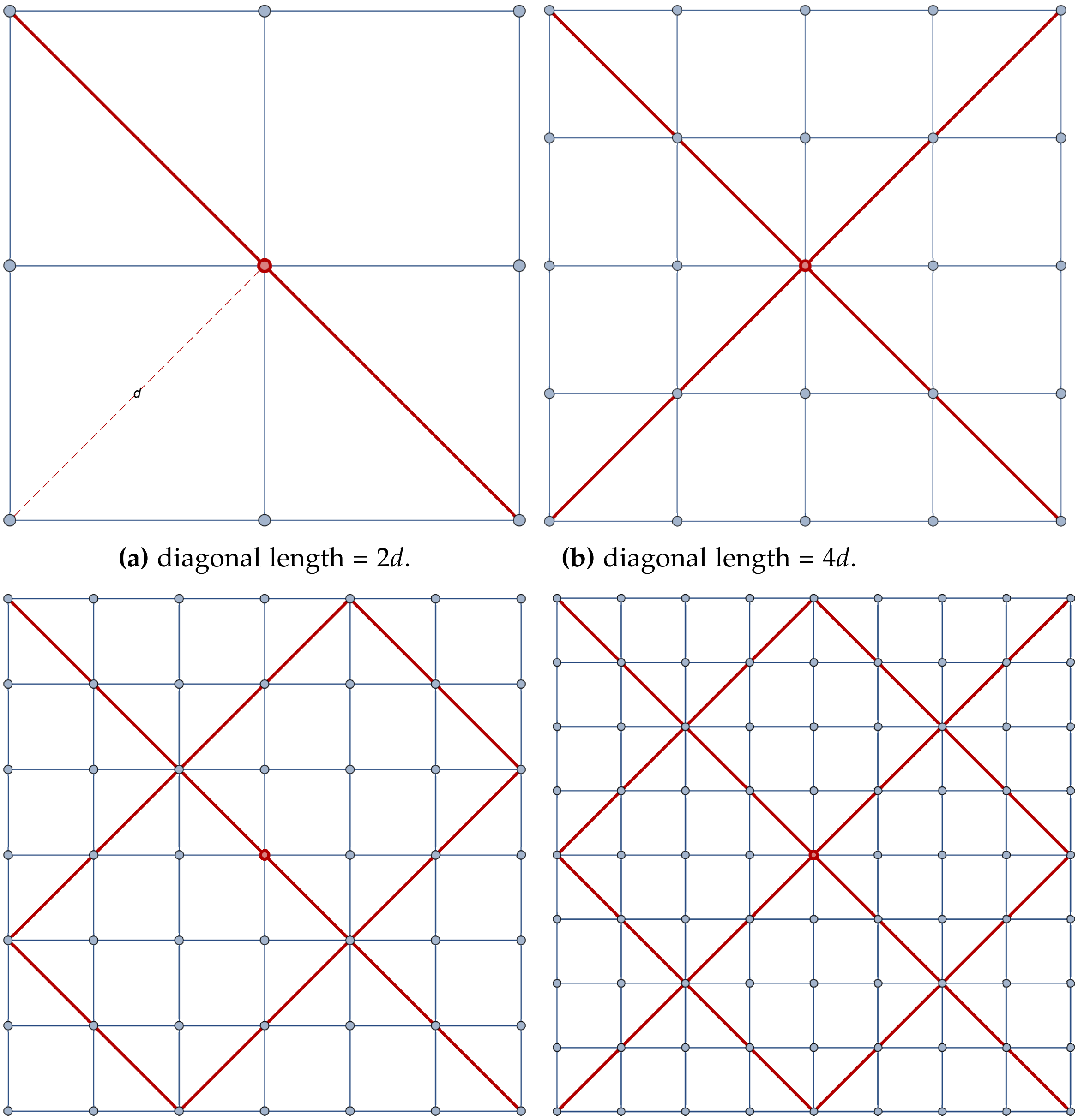
Fifth and finally, consider a case that drops the "perpendicularity requirement" and "metabolic requirement" and replaces it with the weaker "distance requirement". Figure C.4 illustrates this fifth case. For cases with diagonal length $2d$, $4d$, $6d$, and $8d$, we have found that the return per unit road (in km^2/km) is equal to 1, 3/4, 9/7, and 11/16, respectively. With this configuration, it appears we get a form of increasing returns to scale, with $A(l)$ situated between the first quadratic case and the second linear case, but we have not been able to establish a closed-form solution to $A(l)$.

Next, using (C.5) and (C.6) and taking the third case as our leading example, the increase in the road covered area can then be written as:

$$q = 4\left(\sqrt{x} + 1\right) \cdot d \cdot l \tag{C.7}$$

Road production. Agricultural producers and loggers hire u road constructors to work on the predetermined total of m new roads, which according to a decreasing-return-to-scale production function lead to m new roads of length l:

$$l = B\left(\frac{u}{m}\right)^{\alpha} \quad , \quad 0 < \alpha < 1 \tag{C.8}$$



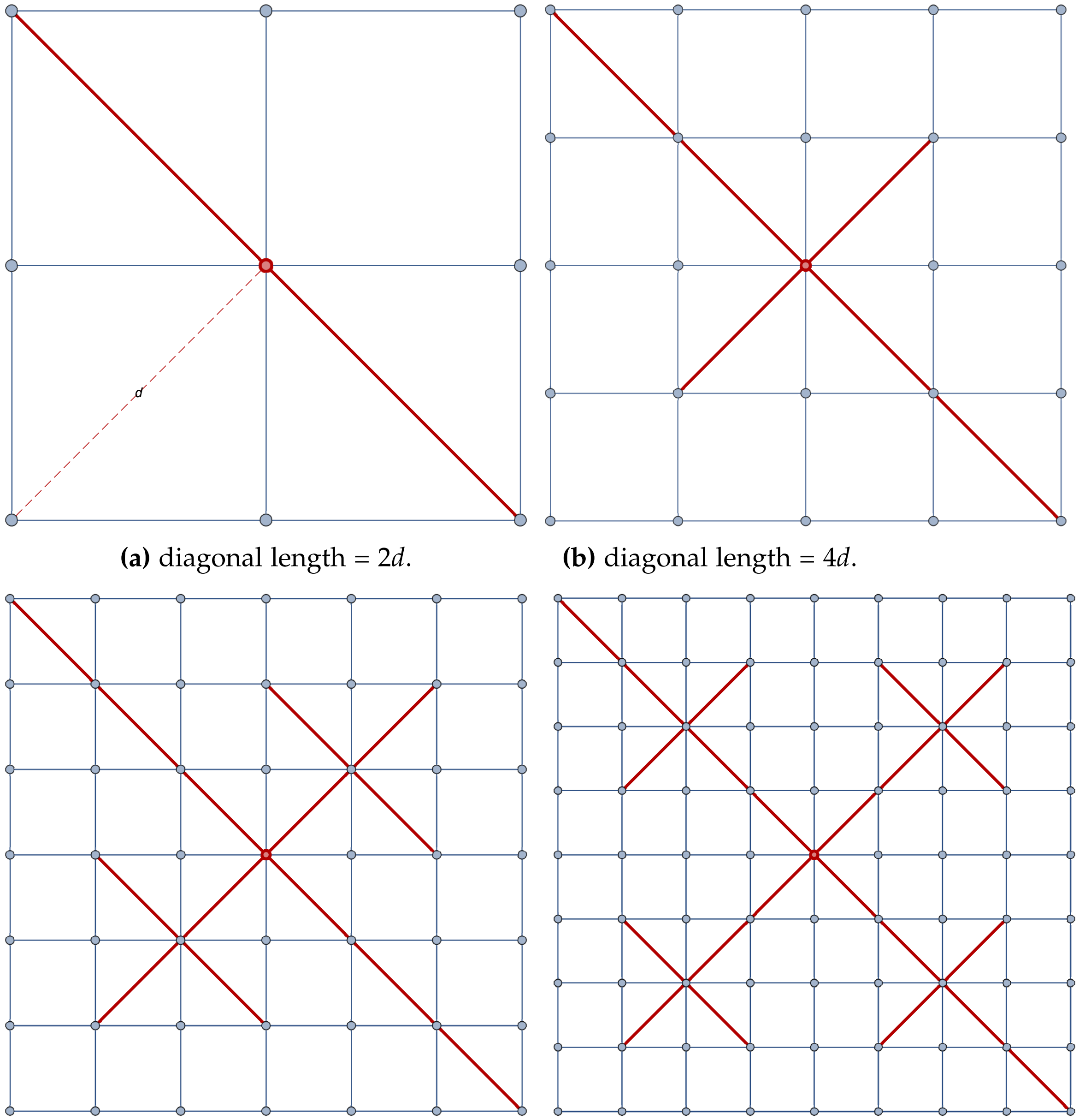
(a) diagonal length = 2d.

(b) diagonal length = 4d.

(c) diagonal length = 6d.

(d) diagonal length = 8d.

Figure C.3: Local road networks drawn in solid red for four different squares sizes to illustrate the case where producers can build additional roads, subject to the *perpendicularity and metabolic requirements*, to extract resources inaccessible from the main diagonal. Closed-form solution with $A_3(l) = dl$.



(a) diagonal length = $2d$.

(b) diagonal length = $4d$.

(c) diagonal length = $6d$.

(d) diagonal length = $8d$.

Figure C.4: Local road networks drawn in solid red for four different squares sizes to illustrate the case where producers can build additional roads, subject to the *distance requirement*, to extract resources inaccessible from the main diagonal.

such that the increase of the road covered area equals:

$$q_1 = m \cdot A_1(l) = \frac{1}{2}B^2 m^{1-2\alpha} u^{2\alpha} \tag{C.9}$$

in the first case, and

$$q_2 = m \cdot A_2(l) = aBm^{1-\alpha}u^{\alpha} - bm \tag{C.10}$$

and

$$q_3 = m \cdot A_3(l) = dBm^{1-\alpha}u^{\alpha} \tag{C.11}$$

in the second and third case respectively. In contrast to two and three, the first case imposes stricter conditions on the road length production function (C.8), that is, it requires $\alpha < \frac{1}{2}$, otherwise the increase in the road covered area features increasing returns to scale with respect to the input of labor, i.e., $\frac{dq_1/q_1}{du/u} = 2\alpha \geq 1$ for $\alpha \geq \frac{1}{2}$. The key lesson that emerges from this overview of possible cases is that limitations on either (i) the de-facto magnitude of the property right associated with l and/or (ii) the degree of returns to scale in road length production are required to ensure that q is subject to diminishing returns to labor. Such limitations seem reasonable, also in light of the fact that in reality we observe very granular road networks in, for example, the Amazon rainforest.

Next, we continue our work in continuous space and time, which will simplify our analysis.

B.3 Road production in continuous space and time

The case of a growing square. Consider a square in continuous space with inradius R, area size $Q = 4R^2$, and perimeter $P = 8R$. If the inradius expands with ΔR, the area of the square will grow by the following quantity:

$$\Delta Q = 4(R + \Delta R)^2 - 4R^2 = 8R\Delta R + 4(\Delta R)^2 \tag{C.12}$$

Next, assume that the inradius expands with $d \cdot l$ units per delta unit of time, $\Delta R = d \cdot l \cdot \Delta t$. Then the marginal change in the size of the square over a marginal unit of time equals the product of the square's perimeter $8R$ and the road length $d \cdot l$:

$$\widetilde{q} \equiv lim_{\Delta t \to 0}\left(\frac{\Delta Q}{\Delta t}\right) = lim_{\Delta t \to 0}\left(8Rdl + 4(dl)^2\Delta t\right) = 8Rdl = 4\sqrt{Q} \cdot d \cdot l \tag{C.13}$$

By comparing (C.7) to (C.13), the similarity between the expressions for q in discrete time and $\tilde{q}$ in continuous time can then be seen immediately.[32] [33] Using (C.8) and interpreting the perimeter $\tilde{m} \equiv 8R = 4\sqrt{Q}$ as the "number" of roads that can be worked on, we can write the change in the road covered area as:

$$\tilde{q} = dB\tilde{m}^{1-\alpha}u^{\alpha} \tag{C.14}$$

which again makes clear the similarities between the growing square in discrete space and continuous space.

Finally, we note that expanding the area of a square in continuous space by increasing its inradius is the closest equivalent to growth in discrete space, where we added an extra "ring" of slots to expand it in all directions (see Fig C.2). To see this, observe that the derivative of the area of the square with respect to the inradius equals the perimeter, i.e.,

$$\frac{dQ}{dR} = Q' = (4R^2)' = 8R = P\,, \tag{C.15}$$

whereas if we would have expressed the area Q in the conventional way using the length of the sides, a, we find, instead, that $\frac{dQ}{da} = (a^2)' = 2a \neq P(a) = 4a$. A geometric interpretation for this difference is that by increasing the length of the sides a we would be adding new slots to only two rather than four sides of the discrete-space square. This means it would not be expanding uniformly in all directions.

The case of a growing circle. The reader may have guessed that it is also perfectly feasible to start from the assumption of an expanding circle rather than a square. Therefore let us consider a circle in continuous space. If the radius expands with ΔR, the circle area will grow by the following amount:

$$\Delta Q = \pi(R + \Delta R)^2 - \pi R^2 = 2\pi R\Delta R + \pi(\Delta R)^2 \tag{C.16}$$

As with the continuous-space square, let us assume that the radius expands with $d \cdot l$ units per delta unit of time, $\Delta R = d \cdot l \cdot \Delta t$. Then the marginal change in the size of

[32] Also note the similarities between the expressions for Q and P for the continuous space square on the one hand, and those for x and m from (C.3) and (C.4) for the discrete space square on the other hand. For example, compare $Q = 4R^2$ of the continuous square with $x = 4n^2$ of the discrete square.

[33] The absence of a grid structure in continuous space implies grid expansion can be fully controlled by choosing l, whereas in discrete space both the number of rings and slot-size constitute choice variables. The parameter d, which captures road density (in km/km^2), brings back the grid structure (and therefore the relation between road length and property right size) in a reduced form manner.

the circle over a marginal unit of time equals:

$$\widehat{q} \equiv lim_{\Delta t \to 0}\left(\frac{\Delta Q}{\Delta t}\right) = 2\pi R \cdot d \cdot l = 2\sqrt{\pi}\sqrt{Q} \cdot d \cdot l \tag{C.17}$$

Using (C.8) once more and defining $\widehat{m} \equiv 2\pi R$ as the number of roads to be worked on, we can write the expansion of the circle-shaped road covered area as:

$$\widehat{q} = dB2\pi R\left(\frac{u}{2\pi R}\right)^{\alpha} = dB(2\pi R)^{1-\alpha}u^{\alpha} = dB\widehat{m}^{1-\alpha}u^{\alpha} \tag{C.18}$$

Note that the only difference between the expressions for the circle and the square is a multiplicative factor, that is, $\widehat{q}/\widetilde{q} = (\sqrt{\pi/4})^{1-\alpha}$.[34]

B.4 Does deforestation incentivize road construction? Or do roads cause deforestation?

We have implicitly assumed that road construction and land clearing are coupled and thus occur simultaneously. In our model road construction is a means to an end, i.e., land conversion. However, recent evidence from Brazil challenges this assumption. Botelho Jr et al. [2022] find that 39 percent of the remaining Amazon rainforest is already covered by roads or lies within 10 kilometers of one, suggesting that the road network extends well beyond the converted area. In Section B.4.1 we outline a model that can explain why the expansion of the road network proceeds at a faster pace than land conversion, by introducing "pioneers" whose road construction is motivated by selective logging and connectivity rather than land clearing.

This extension has three noteworthy implications. First, when net marginal benefits of land clearing decline rapidly enough, the equilibrium features a regime in which land conversion lags behind road construction — rationalizing the empirical pattern documented by Botelho Jr et al. [2022]. In this regime, the transitional dynamics of logging are shaped by road network spillovers, whereas the dynamics of land clearing are not, since there is ample accessible land to convert. Depending on parameter values, this regime may be preceded by one in which road construction and land conversion are coupled. Second, as pioneers do not internalize that the roads they construct will be used by agricultural producers, the extension features an additional externality: pioneers construct roads too slowly if the social benefits of agriculture are positive, and too fast if the social benefits are negative. Third, a single instrument no longer suffices to implement the first-best; both road taxes/subsidies and carbon/land taxes are required.

[34] The ratio $\frac{\pi}{4} < 1$ has in fact a geometric interpretation as it represents the surface ratio of a circle relative to the square to which it is inscribed in.

B.4.1 Extension: benefits from road mobility, selective logging, and conservation

To explain this puzzle, we therefore expand our model in two directions. First, we assume that due to its larger spatial footprint land conversion is a more costly and thus slower process than road building (which we will capture via quadratic conversion costs). Second, we will distinguishing between two actors, agricultural producers and pioneers, that drive the process of deforestation. Agricultural producers convert accessible forest land into agricultural land. Pioneers have the exclusive ability to build roads through the forest and do so with two aims: (i) to selectively log timber (without converting the forest to agricultural land) and (ii) to increase connectivity across space to realize benefits from mobility and trade. If the first motive is dominant, pioneers can best be thought of as logging companies (perhaps more relevant for South-East Asia). If the second motive dominates, pioneers can best be understood as government authorities that expand road capacity on behalf of their constituency to improve accessibility and connectivity (perhaps more relevant for Sub-Saharan Africa).

In this new setting let us continue to use Q to denote the stock of land made accessible by virtue of the road network (or the road covered area). Accessible land is used either for agriculture, denoted A, or conserved as rainforest X, that is,

$$Q(t) = X(t) + A(t). \tag{C.19}$$

This implies that the total amount of conserved rainforest equals the sum of all inaccessible land plus accessible, yet-conserved land, that is, $\overline{Q} - Q + X = \overline{Q} - A$.

Pioneers obtain a payment m per km of road after completion (and thus payment dm, where d is the road density in km/km^2, per unit area of land made accessible).[35] Their selective logging yields an effective stumpage value of bv per unit area, where v is the timber value per unit are and $b \in [0, 1)$ is the fraction per unit forest that is logged.[36] Pioneers also internalize (perhaps through government taxes) a fraction β of the environmental costs ϑ associated with logging (representing the loss of carbon sinks and other ecosystem services, biodiversity, and homes of indigenous tribes). Total net gains per unit area unlocked by the pioneers equals the sum of mobility gains, stumpage value, and environmental costs, that is, $g \equiv dm + b(v - \beta\vartheta) > 0$.

[35]The payment can be interpreted as the net present value of either (i) toll revenue, (ii) government subsidy streams to operate and maintain roads, and/or (iii) net benefits from increased mobility as internalized by the authorities.

[36]Only certain tree species and trees of a certain size are typically logged. In the Amazon, perhaps as little as 5 percent of all trees qualifies whereas in Borneo -in which now virtually all of the primary forest has been logged- as much as 30 percent of all trees is suitable for timber production (Bulte and van Kooten [2000]).

Farmers can convert forest into agricultural land:

$$\dot{A}(t) = h(t), \tag{C.20}$$

where $h(t)$ is the rate (or speed) of conversion at time t. The following inequality constraint must also hold:

$$X(t) = Q(t) - A(t) \geq 0 \tag{C.21}$$

Farmers incur total conversion costs of:

$$k_0 h(t) + \frac{1}{2} k_1 h(t)^2, \tag{C.22}$$

where

$$k_0 \equiv (1-b)\kappa + b\underline{\kappa} + \beta(1-b)\vartheta > 0 \tag{C.23}$$

measures the minimum marginal cost of conversion, with $\kappa > \underline{\kappa} \geq 0$. Pioneers, by selectively logging the forest ($b > 0$), lower conversion costs for farmers coming after them (which explains the first two terms of k_0), but they de-facto also lower the remaining environmental damage that can be "inflicted" by the farmers (who, like pioneers, internalize some of the environment costs), which explains the third and last term of k_0. Finally, speedy conversion increases conversion costs, as captured by the second term in eq. (C.22). Note that we redefine $C(Q,q) \equiv Aq^{\frac{1}{\alpha}} Q^{-\frac{1}{2}\frac{\beta}{\alpha}}$ implying that the term κq is no longer subsumed into $C(Q,q)$ since we now model clearing costs separately as $k_0 h + \frac{1}{2} k_1 h^2$, which reduces to κh in case $k_1 = b = \beta = 0$.

In the first-best the social net benefits are maximized by choosing q and h subject to the conditions outlined above. The problem can be written as follows

$$\max \int_0^\infty e^{-\rho t} \left[U(A(t)) + gq(t) - C(Q(t), q(t)) - k_0 h(t) - \frac{1}{2} k_1 h(t)^2 \right] dt,$$

subject to

$$\begin{aligned} \dot{Q}(t) &= q(t), & \text{(C.24)} \\ \dot{A}(t) &= h(t), & \text{(C.25)} \\ h(t) &\geq 0 & \text{(C.26)} \\ Q(t) &\leq \bar{Q},\ Q(t) - A(t) \geq 0. & \text{(C.27)} \end{aligned}$$

The Hamiltonian reads

$$\mathcal{H} = e^{-\rho t}\left[U(A) + gq - C(Q,q) - k_0 h - \frac{1}{2}k_1 h^2\right] + \lambda_q q + \lambda_a h + \gamma_h h + \gamma_x(Q - A).$$

where λ_q and λ_a are the costate variables associated with the state equations for Q and A, and γ_h and γ_x are the Lagrange multiplier associated with the inequality constraints on h and $Q - A$.

Suppose there exists a solution. Then the following necessary conditions hold:

$$q : -e^{-\rho t}(g - C_q(Q,q)) = \lambda_q \tag{C.28}$$

$$h : e^{-\rho t}(k_0 + k_1 h) = \lambda_a + \gamma_h \tag{C.29}$$

$$Q : \dot{\lambda_q} = e^{-\rho t} C_Q(Q,q)) - \gamma_x \tag{C.30}$$

$$A : \dot{\lambda_a} = -e^{-\rho t} U'(A) + \gamma_x \tag{C.31}$$

In the transition to the steady-state, we conjecture two regimes are possible (with the first regime, if it occurs, always taking place before the second regime):

- road construction and land conversion happen at the same rate during $0 \leq t \leq T_1$.
- land conversion lags behind road construction during $[T_1 \leq t \leq T_2]$

We leave it to future work to analyze the dynamics of these regimes, the transition between them, and the precise conditions that need to be met for sequential occurrence of the two regimes.

B.5 Transportation costs and other convexities

Our benchmark model captures the notion that roads are a necessary condition to gain access to rainforest resources. However, agricultural crops or cattle also need to be transported to processing facilities or slaughterhouses, respectively. If we assume those are located in the model's city center and that transportation costs increase linearly in distance from the city center, we can write those transportation costs as:

$$Z = \int_0^r [\zeta s \cdot 2\pi s]\, ds = \frac{2}{3}\zeta\pi r^3, \tag{C.32}$$

where ζ is the transportation cost per unit distance per unit product. Note that since we chose units such that output equals the quantity of land, the quantity of food or meat to be transported from distance s to the city center equals the circumference at that distance, $2\pi s$. Thus, (C.32) can be written as a function of Q, making this is a straightforward extension of the model:

$$Z(Q) = \frac{2}{3}\zeta\pi^{-\frac{1}{2}}Q^{\frac{3}{2}}.$$

Transportation costs are convex because the larger the quantity of meat/food produced, the larger the average transportation costs. Those costs increase less than quadratically, however, since as we move further from the city the quantity of land "found" grows with every unit increase of distance, keeping costs in check. These costs could be incorporated in the marginal costs of agricultural production, which would then become

$$\nu + Z'(Q) = \nu + \zeta\sqrt{\frac{Q}{\pi}}.$$

Next to downward-sloping demand and transportation costs, other convexities can be incorporated too. It's plausible, for example, that the costs of clearing land and the pace of road expansion will at some stage also be hampered by the growing scarcity of the required capital stock, such as bulldozers, chainsaws, cranes, and logging trucks. Convex costs of land clearing due to capital scarcity would be similar to the mechanism in the neoclassical investment model.

B.6 Time-inconsistent preferences and road construction

Time inconsistency is another issue that deserves attention. Harstad [2020] shows that time inconsistency may prevail in policy making if a government that is in power in a certain period of time cannot be sure that it will be still in power one or more periods ahead. Although each agent in power may employ an identical constant rate of time preference, the consequence can then be that discounting by the agents is hyperbolic, giving rise to dynamic inconsistency in the sense that a future government may take other actions than the action preferred by the current government in power.

Harstad [2020] shows that this observation is relevant in the context of climate change, in particular when it comes to investment in hierarchies of "brown" or "green" technologies, where the cost of investing in one technology is influenced by another complementary technology, further upstream. In this context, time inconsistent preferences create a strategic incentive for the current government to subsidize clean technologies (or tax dirty technologies) to induce tomorrow's government to invest more (or less).

Interestingly, complementarities also play a role in our framework, where today's road investments lower the cost of future investments in roads, i.e., $C_{qQ} < 0$. Ignoring climate externalities, this implies the current government has a strategic incentive to subsidize road construction today to induce the next government to invest more. In other words, (upstream) infrastructure investment constitutes a natural commitment device to (somewhat) tie the hands of future governments. This strategic motive for road subsidies will be greater the stronger the dependence of construction costs on the existing size of the road network. It will also be larger in the early stages of the road network when investment is considered "more upstream" and therefore $|C_{qQ}|$ is larger.[37] Finally, negative climate externalities associated with land conversion may alter the optimal road subsidy/tax [cf. Harstad, 2020]. It is a topic for further research to incorporate these thoughts formally into our model.

While our current model abstracts from these political economy considerations, incorporating time-inconsistent preferences would enrich the analysis. Future work could examine how the interaction between road network externalities and political turnover affects optimal infrastructure policy, particularly when accounting for climate externalities that may partially offset the strategic incentive to overbuild.

B.7 Weak state capacity and rainforest management

In our benchmark model we assumed that all accessible forest land was immediately converted to pasture. In practice, however, countries may decide to impose reserve requirements that stipulate how accessible land can be used. In 1965 Brazil created the Forest Code, a law that requires farmers to maintain at least 50 to 80 percent of their private land as native vegetation. Like carbon taxes, reserve requirements can internalize the negative climate externality from conversion. In reality, a country's ability to implement and monitor both road subsidies and reserve requirements will depend on its state capacity. A state's ability to incentivize or lead road construction falls under what Besley and Persson [2011] and Besley et al. [2021] refer to as 'fiscal state capacity' and 'collective state capacity', whereas a state's ability to administer and monitor a rainforest reserve requirement falls under 'legal state capacity'. Proper rainforest management requires strong capacity in all three dimensions.

Using the parlance of our model, suppose the government decides on the magnitudes of (i) the reserve requirement and (ii) the road construction subsidy in order to maximize the present discounted value of agricultural revenues and forest conservation benefits minus the costs of road construction and forest monitoring costs.

[37] Brazilian policymakers have recently resurrected plans to repave highway BR-319, that connects the largest city of central Amazonia, Manaus, with the current arc of deforestation. This road was opened by the then military government of Brazil in 1973 but soon deteriorated due to lack of maintenance. The previous Brazilian president Bolsanaro (2019-2022) pledged to rebuild BR-319, and the environmental agency IBAMA granted a preliminary license in 2022. As of late 2024, reconstruction plans have continued to advance with political support from the Lula administration, but the project remains mired in legal challenges and environmental controversies. A central highway like BR-319 is a good example of an upstream investment that can affect future governments' decisions regarding road construction and environmental protection (see Section B.7).

In weak states it is conceivable that the government would not be able to use all instruments. In addition, monitoring reserve requirements and controlling illegal deforestation may be more costly in weak states. A strong state would be able to implement the first-best, but a weak state would not. Consider, for example, a state with weak legal state capacity but strong fiscal state capacity. Unable to administer and enforce the reserve requirements, it could use road subsidies to not only internalize the positive road network externality, but also to combat the negative externality from deforestation. As a consequence, the road subsidy will be too small from a first-best point of view, or may even become a tax, in order to trade-off these conflicting objectives.[38]

When reserve requirement laws depend on enforcement, the cost of enforcement could be modeled similar to Harstad and Mideksa [2017], in which a portion b of all accessible land X (that is, land covered with roads) is monitored sufficiently to deter illegal deforestation at a total flow cost of $g(p)bX$. Here $g(p)$ is the enforcement cost per unit land, which will be higher if land prices and thus agricultural prices are higher since these increase the incentives to cheat on the reserve requirement. In this setting, a government with time-inconsistent preferences would recognize roads as a form of brown capital, see Harstad [2020], since roads, once finalized, indefinitely increase accessible forest X and thus commit future governments to higher monitoring costs. This would give rise to a strategic motive to tax road construction today.

More broadly, the focus on carbon taxes in our quantitative analysis in Section 4 reflects the model's structure rather than a claim that carbon taxes are the only or the best instrument available. In practice, spatially targeted policies—such as protected areas, payment-for-ecosystem-services programs (e.g., Bolsa Floresta), and strategic routing of infrastructure to minimize habitat fragmentation [Laurance et al., 2014]—may be more cost-effective precisely because deforestation pressure varies across space. Our model's spatial homogeneity cannot capture these advantages, which constitutes an important limitation. That said, our framework does highlight a complementary insight: regardless of the instrument chosen, the cost of conservation is amplified by existing road networks through the lock-in mechanism. This suggests that spatially targeted policies should prioritize areas with low existing road density, consistent with our finding that conservation transfers are lower for undeveloped frontiers.

[38]This would be especially damaging if roads come with mobility benefits, i.e., benefits independent of resource access, such that in weak states there is not only too little road development, but also too much deforestation.