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Pricing and investment by a two-sided monopoly platform*

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Abstract

This paper analyzes a monopoly platform's joint pricing and investment decisions in the canonical two-sided market model of Armstrong (2006). Participants are heterogeneous in outside options and derive both stand-alone benefits from joining the platform, and network benefits from interacting with the opposite side. The platform sets participation prices on both sides and chooses investments that enhance user experience. We characterize monopoly distortions in participation, pricing, and investment relative to a social planner.

Taking investment as given, the monopoly outcome features under-participation on both sides, yet participation prices need not transparently reflect these participation distortions. We show that at least one participation price is excessively high relative to the social optimum. Equivalently, while one side's participation price may be inefficiently low, participation prices that are too low on both sides are impossible.

When investment enhances network benefits, marginal returns are proportional to interaction volume; since the planner induces greater participation and therefore more interactions, the monopoly underinvests on both sides. By contrast, when investment enhances stand-alone benefits, marginal returns scale with own-side participation, so investment distortions may be asymmetric across sides, although overinvestment on both sides is ruled out.

An application to app platforms, with user-side device pricing and developer-side commissions on in-app purchases, yields sharp predictions for device price, commission and investment distortions, as well as for the effects of commission caps on buyer and seller surplus.

JEL Classification: D42, L12, L14, L40

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1 Introduction

Digital platforms intermediate between distinct user groups—buyers and sellers, users and developers, consumers and advertisers—and shape market outcomes through both pricing and non-price design choices. Beyond setting access or usage fees, platforms invest in features and infrastructure that affect the value users obtain from joining and transacting on the platform. Such investments may target one side directly (e.g., seller analytics, fulfillment services, or developer tools) or the other (e.g., search and recommendation quality, user-interface design, or matching algorithms). Through cross-group network effects, these investments can alter incentives and outcomes on both sides simultaneously.

This paper analyzes a monopoly platform's joint pricing and investment decisions in the canonical two-sided market model of Armstrong (2006). Participants derive (i) stand-alone benefits from membership and (ii) network benefits from interacting with the opposite side. The platform chooses participation prices on both sides and allocates investment to enhance user experience. We compare the monopoly outcome to the socially optimal allocation and characterize when and how monopoly pricing and investment are distorted.

We study two conceptually distinct investment environments. In the first, investment increases users' interaction benefits (network benefits), as in improvements to search, matching, and compatibility that raise the surplus generated by buyer-seller interactions. In the second, investment increases users' stand-alone benefits, such as cybersecurity, identity verification, or account-recovery systems that raise users' value independently of how many others join. The distinction is important because the marginal return to investment scales differently: with network-benefit investment it is proportional to interaction volume (the product of the participation rates on both sides of the market), whereas with stand-alone investment it is proportional to own-side participation only. This difference is responsible for our distinct results on the direction and symmetry of monopoly investment distortions.

We begin by fixing investments and revisiting the efficiency of monopoly participation and pricing, a question for which the existing literature offers valuable insights but an incomplete characterization of the nature of *joint* distortions that can arise across sides (Weyl, 2010; Tan and Wright, 2018, 2021; Jullien et al., 2021). Our first set of results delivers a characterization of the monopoly participation and pricing distortions that can jointly occur in the Armstrong's framework. In our setting, higher participation on either side raises transaction volume and increases both the platform's and the planner's objective, so participation choices are complements for both decision makers. Because the platform does not internalize participants' surplus, the monopoly outcome features under-participation on both sides relative to the social optimum.

Importantly, even though the monopoly outcome features under-participation on both sides relative to the social optimum, participation prices need not transparently reflect these participation distortions. We show that a monopoly platform sets an excessive participation price on at least one side of the market, so participation prices that are too low on both sides are impossible. At the same time, participation prices may be distorted either symmetrically (too high on both sides) or asymmetrically (too high on one side and too low on the other). For uniformly distributed outside options, we further provide transparent conditions under which asymmetric price distortions arise: they require sufficiently weak cross-side network effects on one side and sufficiently large asymmetry in net stand-alone values across sides.

Methodologically, we obtain these results by moving beyond one-dimensional comparisons of side-specific monopoly and planner first-order conditions and adopting a multidimensional optimization approach. We proceed in two steps. First, we formulate the platform's problem directly in participation levels and use the sign of the planner's welfare gradient evaluated at the private optimum to characterize participation distortions. Second, we work in price space to characterize pricing distortions, again exploiting the information contained in the multidimensional gradient. This approach clarifies how the two sides interact and yields sharp restrictions on equilibrium distortions, including the impossibility of both sides being underpriced. It also sidesteps equilibrium multiplicity concerns by separating the choice of target participation levels from the implementation of prices that support them, in the spirit of the insulated tariffs approach (see Weyl, 2010; Anderson and Coate, 2005; White and Weyl, 2016; Tan and Wright, 2021; De Cornière et al., 2025; Anderson and Bedre-Defolie, 2025). To support this analysis, we establish conditions under which the user-coordination stage admits a unique and stable equilibrium for any participation prices. These conditions ensure well-behaved demand in the sense that raising the participation price on one side reduces own-side participation and, through cross-group effects, also reduces participation on the other side.

We then endogenize investment. When investment enhances network benefits, the platform's marginal return to investment on either side is proportional to interaction volume. Since the planner induces greater participation and hence more interactions, the planner has stronger investment incentives. Hence, the monopoly always underinvests on both sides. We also study investments that enhance stand-alone benefits. Here the marginal return to investing on a given side increases with own-side participation rather than interaction volume, which gives the platform more scope to tilt investment across sides. As a result, investment distortions may be asymmetric: the platform may overinvest on one side while underinvesting on the other. However, overinvestment on both sides is impossible.

We also apply our framework to app marketplaces, where users access the ecosystem through the device price while developers are monetized via an ad valorem commission on in-app purchases. This pricing architecture is standard in major app stores (see, e.g., Apple, 2026a; Google, 2026). For fixed investment, participation remains inefficiently low on both sides, and the commission rate is always excessive even though the device price may lie above or below its social benchmark. This sharp commission distortion provides a natural basis for policy concerns about developer-side fees and the scope for regulatory constraints. Motivated by recent EU enforcement and the ensuing revisions to Apple's fee schedule (see European Commission, 2021; Apple, 2026c), we then study a canonical intervention, namely, a cap on the commission rate. We characterize how the platform rebalances its pricing structure in response and identify conditions under which a binding commission cap expands user participation and surplus, as well as conditions under which it backfires by reducing user surplus.

We further contribute to the app-platform literature by incorporating investment decisions and analyzing market outcomes both with and without a commission cap. In the unregulated equilibrium, when the platform's investment enhances network benefits for users and developers, such as by increasing in-app purchase demand, the privately optimal investment level is insufficient. When instead the platform's investment raises stand-alone benefits on both sides of the market, such as improving users' privacy or developers' protection against cybersecurity threats, the privately optimal investment is either too low on both sides, or too low on one side and too high on the other.

We find that imposing a locally binding commission cap may lead to more or less investment in in-app purchase demand depending on parameters. More interestingly, imposing a commission cap when the platform also invests may reverse the results obtained when it is not allowed to invest. In particular, there exist circumstances in which the platform reduces investment so much that user surplus falls, while it would have increased had investment remained constant.

The remainder of the paper is organized as follows. Section 3 presents the model and equilibrium concept. Section 4 characterizes monopoly distortions in participation rates and associated participation prices for fixed investment, and provides an explicit uniform-outside-options benchmark. Section 5 endogenizes investment in network benefits. Section 6 studies investment in stand-alone benefits. Section 7 applies the framework to an app-platform environment and analyzes commission-based pricing and the effects of commission caps. Section 8 concludes.

2 Related literature

We study the efficiency properties of a monopoly platform's joint pricing and investment decisions in the canonical two-sided market of Armstrong (2006), where participants enjoy both stand-alone (membership) benefits and cross-group interaction benefits. Our contributions are twofold. First, we contribute to the literature on investment in two-sided markets: we show how the platform's investment incentives hinge on the participation incentives induced by its pricing, and on the margin through which investment operates (enhancing either interaction or stand-alone benefits). Because pricing and investment are intrinsically intertwined in this environment, our analysis also delivers a sharp characterization of inefficient participation and, consequently, inefficient volume of interactions. These distortions are the key determinants of investment incentives. Second, we develop an app-platform application in which the platform charges users through a device price and monetizes developers through an ad valorem commission, and we use our framework to characterize the associated distortions and the effects of commission caps.

We build on the seminal analysis of monopoly platform pricing in two-sided markets in Armstrong (2006), who studies a setting with homogeneous cross-group externalities. In his framework, the platform's optimal price on each side reflects the standard markup term as well as a correction for the cross-group benefits generated by participation on the other side. In the corresponding social optimum, the planner sets prices strictly below marginal cost, accounting only for the external benefits of participation. While Armstrong (2006) characterizes the structural difference between monopoly and socially optimal pricing, he does not explicitly analyze the direction or magnitude of the resulting distortions in participation.

Weyl (2010) advances the literature by allowing for agent heterogeneity in both stand-alone and network benefits. He shows that monopoly distortions reflect two distinct components: a standard markup distortion and a Spence-type (composition) distortion. Rather than working directly with participation prices, Weyl (2010) introduces the notion of *insulating tariffs*, a pricing framework that nests participation and transaction fees and that can be used to address equilibrium multiplicity, the "chicken-and-egg" problem in two-sided markets (Caillaud and Jullien, 2003; Rochet and Tirole, 2006; Jullien et al., 2021). In particular, the platform can be viewed as choosing participation rates on both sides to maximize its objective, and then selecting an insulating tariff that supports these participation levels in equilibrium. We adopt the same logic by solving the platform's problem directly in terms of participation levels. This approach eludes equilibrium-selection concerns and makes the platform's multidimensional optimization problem tractable.

Tan and Wright (2018) point out that Weyl (2010)'s conclusion regarding pricing distortions is incomplete. They argue that Weyl focuses exclusively on comparing first-order conditions, assuming them implicitly evaluated at the same participation levels, hence without accounting for the fact that monopoly and planner outcomes generally induce different equilibrium participation rates. As a result, his analysis captures only the divergence in marginal incentives between a monopoly platform and a social planner, without fully characterizing the resulting pricing distortions. Building on this critique, Tan and Wright (2021) make a key contribution by decomposing the sources of pricing distortion into four distinct components, namely, the markup distortion, the Spence distortion, the displacement distortion, and the scale distortion, and showing that, even in the homogeneous network effects setting of Armstrong (2006), the distortion on a given side can be positive, negative, or zero depending on parameter values. However, their decomposition is inherently one-dimensional, implying that it characterizes the distortion on each side separately rather than jointly through the system of first-order conditions, and therefore does not yield restrictions on which distortions can arise simultaneously across both sides of the market.

Our analysis of pricing and participation distortions in Section 4 extends the existing literature by offering a more complete characterization. While Tan and Wright (2021) decompose pricing distortions into several components, our multidimensional optimization approach uncovers structural restrictions that cannot be obtained from a side-by-side comparison of single first-order conditions. In particular, we show that the monopoly optimum must exhibit under-participation on both sides. Turning to prices, we show that under-participation can be accompanied by a range of pricing patterns, including excessive prices on both sides or asymmetric outcomes in which the platform charges less than the social benchmark on one side and more on the other. Finally, we show that prices below the social benchmark on both sides cannot be sustained.

A central contribution of the paper is to the literature on investment in two-sided markets (Lianos and Motchenkova, 2013; Anderson et al., 2014; Casadesus-Masanell and Llanes, 2015; Angelini et al., 2024; Choi and Jeon, 2023). Relative to our framework, much of this literature studies more restrictive investment technologies and, often, abstracts from the joint welfare implications of pricing-induced participation and investment. While we allow the platform to make separate investment decisions aimed at enhancing the utility of each side of the market individually, a common approach in prior work is to model a single investment in platform quality or performance. Such an aggregate investment typically affects demand or participation in a symmetric way across sides, or it targets only one side of the market. In contrast, our framework accommodates differentiated investments across sides. Moreover, it examines investment that may affect two distinct margins, either enhancing interaction benefits or stand-alone benefits. In addition, our app-platform application contributes to the emerging literature on ad valorem commissions and device pricing by using the same multidimensional logic to characterize commission and device-price distortions and to study commission caps.

Lianos and Motchenkova (2013) examine quality-enhancing investments that raise user utility on both sides of a platform market. Although their model builds on Armstrong (2006), they restrict pricing by allowing the platform to charge only one side of the market, while the other side access freely. They show that a monopoly platform underinvests relative to the socially optimal level. Their conclusion however is obtained under an investment technology that benefits both sides symmetrically, and under a pricing restriction that rules out joint fee-setting across sides.

Anderson et al. (2014) study platform performance investments that benefit users on one side while raising adaptation costs for content providers on the other. This creates a fundamental trade-off: higher performance can attract more users but may simultaneously reduce participation on the supply side. Accordingly, the platform chooses performance by balancing improvements in user experience against the opportunity cost of discouraging developer engagement.

Casadesus-Masanell and Llanes (2015) also examine one-sided investment aimed at improving users' network benefits. Investment is undertaken either by the owner of a proprietary platform or by developers in an open platform, and the paper compares the resulting investment incentives across governance regimes. They show that whether an open monopoly platform invests more or less than a proprietary monopoly platform depends on the sizes of the user and developer bases and on the degree of complementarity between investment and participation.

Choi and Jeon (2023) develop a model of platform design choices, with innovation adoption as a natural application. They examine the misalignment between a monopoly platform's design choices and a second-best social optimum in which a planner selects the desired investments on both the user and advertiser sides, while leaving the platform to set prices, including consumer participation fees and advertiser commission rates. In the absence of price regulation, compared to the planner's allocation, the monopoly platform's choice of design is biased toward users. However, if the platform is forced to provide free access to users, the bias reverses.[1]

Wang and Wright (2023) model a monopoly search platform in which consumers can search either through the platform or directly, with different search costs across the two channels. The platform invests in reducing consumers' search costs, which increases the expected match value for transactions conducted on its interface. They show that when consumers can switch to buying directly after discovering a good match through the platform ("showrooming"), the platform's investment incentives become distorted. Moreover, the direction of the distortion depends on the scope of price-parity clauses: wide price parity can induce overinvestment, whereas narrow parity can induce underinvestment.

Angelini et al. (2024) study a monopolistic platform that simultaneously chooses membership fees and a single, demand-enhancing investment in the presence of seller competition. Their focus is on comparative statics and on how investment interacts with seller competition. In contrast, our analysis is organized around a welfare benchmark for both participation and investment and around an investment technology that allows the platform to tilt investment across sides and across margins, which is central to our characterization of investment distortions. Moreover, their results rely on specific functional-form assumptions, including the Singh and Vives (1984) system of demands and a uniform distribution of outside options.

Our app-platform application adds to a recent literature studying the efficiency of commission rates and commission regulation in two-sided markets (Gomes and Mantovani, 2025; Anderson and Bedre-Defolie, 2025; Jeon and Rey, 2026; Karle et al., 2026; Teh and Wright, 2026). The closest paper is Anderson and Bedre-Defolie (2025), who develop a tractable monopoly app-store model with two-sided heterogeneity and ad valorem commissions. Their heterogeneity operates through user demand and app quality: consumers differ in willingness-to-pay for app quality and apps differ in quality, so network effects operate through the endogenous quality stock of participating apps and the aggregate market value of device-holders. In contrast, our application stays in the spirit of our main model and introduces heterogeneity through agents' outside options (or, equivalently, heterogeneity in stand-alone values). Methodologically, this allows us to characterize

[1]Relatedly, Teh (2022) studies platforms' non-price governance design and the conditions under which private incentives align with welfare, with applications that can be viewed as choices of quality standards or investments in quality.

commission and device-price distortions using the same multidimensional logic as in Section 4, while accommodating a developer-side commission instrument. Relative to Anderson and Bedre-Defolie (2025), who find both the device price and the commission excessively high, our application delivers a richer distortion pattern: the commission remains excessive, but the device price can lie above or below the social benchmark, depending on asymmetries in net stand-alone values across sides and on per-transaction costs, which dampen the monopolist's incentive to facilitate too many interactions.

Other recent contributions emphasize complementary mechanisms. Teh and Wright (2026) show in a general competitive-bottleneck framework that seller-side instruments such as commissions are generally distorted against sellers and harmful to welfare, while Jeon and Rey (2026) show that, under platform competition, commissions exceed the levels maximizing consumer surplus or platforms' joint profit when developers face economies of scope across platforms, and that fostering platform competition or interoperability may paradoxically exacerbate this distortion. On the regulatory side, Gomes and Mantovani (2025) derive an optimal cap under price parity and Karle et al. (2026) show in a dual-channel model that fee caps can have unintended consequences for consumer surplus.

Turning to policy, Anderson and Bedre-Defolie (2025) emphasize that a binding cap on commissions induces the platform to rebalance by raising the device fee; as a consequence, a cap necessarily benefits developers (app surplus increases) and reduces platform profit, but its effect on consumers can be ambiguous and may be negative, depending on the distributions of consumer and app types. Our commission-cap analysis complements theirs by isolating the two forces that govern the effect of a tighter cap: the platform loses commission revenue on each transaction, but may gain from higher developer participation and the resulting increase in network benefits. A notable difference is that, in our setting, buyer heterogeneity only plays a second-order role for the sign of the commission cap's effect on buyer participation: the primary distributional dependence is on the developer side, through the distribution of developers' outside options that governs how developer participation responds to a tighter cap. Finally, when outside options are uniformly distributed, we obtain a sharp benchmark: any locally binding commission cap raises developer surplus and total (buyer-plus-developer) surplus, and the effect on buyer surplus is characterized by an explicit threshold condition.

3 The model

We build on the canonical monopoly two-sided platform model of Armstrong (2006). Specifically, we consider a market where a monopoly platform acts as an intermediary between two groups of participants, whom we refer to as buyers (B) and sellers (S). At $t = 1$, the platform chooses its pricing structure on each side of the market and selects investment efforts directed at each side to enhance participants' experience. At $t = 2$, buyers and sellers decide whether to join the platform and, conditional on participation, interact on the platform under the prevailing terms and obtain their payoffs.

3.1 Agents

Each participant benefits from using the platform itself, and from interacting with the participants on the other side. The membership benefit, or stand-alone value, of a participant from group i is denoted as $v_i > 0$, $i \in \{B, S\}$. This value represents the utility a participant from a group derives from just being a member of the platform and is independent of any interactions with participants from the other group. The value a participant from group i derives from an interaction with a participant from the other group is denoted as $\theta_i(e_i) > 0$, $i \in \{B, S\}$. These network externalities are increasing functions of the investment levels e_i, $i \in \{B, S\}$, which represent the investment effort exerted by the monopoly platform on each side of the market to enhance the participants' experience.[2] To ensure well-behaved maximization problems, we assume that the functions $\theta_i(e_i)$, $i \in \{B, S\}$ are strictly concave.

As in Armstrong (2006), the expected surplus of a single buyer and a single seller who participate in the platform is, respectively, as follows:

$$u_B = v_B + \theta_B(e_B)N_S^e - p_B; \quad u_S = v_S + \theta_S(e_S)N_B^e - p_S,$$

where p_i, $i \in \{B, S\}$ is the participation price an agent who joins the platform must pay in order to enjoy the platform services and interact with the participants on the other side.[3] Here, N_j^e denotes the number of participants on side j that an agent from side i expects to interact with, $i, j \in \{B, S\}$, $i \neq j$.

Agents on each side i have heterogeneous outside options denoted by o_i, $i \in \{B, S\}$. We assume that these outside options are drawn from atomless distribution functions $G_i(\cdot)$ with support $[\underline{o}_i, \bar{o}_i]$, $i \in \{B, S\}$ and $g_i(\cdot)$ is the density function corresponding to $G_i(\cdot)$.[4]

[2] Both stand-alone and network benefits are present in practice, making it important to study a platform's incentives to exert effort in enhancing each. For example, job seekers on *LinkedIn* benefit from the presence of recruiters (a network benefit), but they also join the platform to access training courses and earn certificates that help them develop skills independently (a stand-alone benefit). Similarly, *Tinder* users benefit from the presence of other users, yet some also engage with AI partners or receive feedback to improve their flirting skills, features that provide value even without direct interaction with others. We present the model for the case in which platform effort enhances the benefits users derive from interacting with other users, and analyze such investments in Section 5. In Section 6, we then consider efforts aimed at increasing stand-alone benefits.

[3] We focus on *participation* (subscription) prices. For completeness, Appendix F considers per-transaction fees and shows that the two instruments can be formulated to induce the same participation rates. See also Section 7 where we develop an application where users access through a device price and developers are subject to commissions on in-app purchases.

[4] To be sure, our model does not require the distribution functions of outside options to be identical across sides; instead, $G_B(\cdot)$ and $G_S(\cdot)$ may differ in both functional form and support. It is straightforward to see that heterogeneity in outside options is isomorphic to heterogeneity in stand-alone benefits, which is the setting in Armstrong (2006). The other seminal paper in the literature, Rochet and Tirole (2003), models heterogeneity in users' interaction benefits (see also Weyl, 2010; Rochet and Tirole, 2006; Tan and Wright, 2021). Our modeling choice

We proceed in a way similar to the "insulating tariffs" equilibrium approach of Weyl (2010) and White and Weyl (2016) (see also Anderson and Coate, 2005; Tan and Wright, 2021; De Cornière et al., 2025; Anderson and Bedre-Defolie, 2025). Specifically, we first determine the optimal participation levels of buyers and sellers, and then characterize the prices that implement these participation decisions. On each side of the market, agents only choose to become participants and pay for the services of the platform if their utility exceeds the value of their outside option. Agents who choose not to join the platform make no payment and derive utility equal to their outside options. Hence, the number of participants on each side of the market can be expressed as functions of the realized utilities:

$$N_B = \Pr[u_B \geq o_B] = G_B(u_B); \quad N_S = \Pr[u_S \geq o_S] = G_S(u_S).$$

Because the distribution functions are monotone, for later use, we define utilities as increasing functions of the number of participants, denoted by $u_i = \varphi_i(N_i)$, $i \in \{B, S\}$.

Given participation levels N_B and N_S set by the platform, the fees that implement these participation levels are:

$$p_B = v_B + \theta_B(e_B)N_S - \varphi_B(N_B); \quad p_S = v_S + \theta_S(e_S)N_B - \varphi_S(N_S), \tag{1}$$

where we have assumed that the participants on side i of the market form correct expectations about the participation level of the agents on the other side, i.e. $N_i^e = N_i$, $i \in \{B, S\}$. For a fixed pair of prices (p_B, p_S), a user equilibrium in $t = 2$ is given by the solution to the system of equations:

$$N_B = G_B(v_B + \theta_B(e_B)N_S - p_B), \tag{2}$$

$$N_S = G_S(v_S + \theta_S(e_S)N_B - p_S). \tag{3}$$

Lemma 1 in Appendix A shows that the user equilibrium is stable when $g_B(\cdot)g_S(\cdot)\theta_B\theta_S < 1$, and unique when $\max_{u_i \in \mathbb{R}} \theta_i g_i(u_i) < 1, \ i \in \{B, S\}$.[5]

3.2 Platform

We assume that the platform chooses participation levels and investment efforts to maximize its profits. The platform incurs an operating cost $c_i \geq 0$ for serving a participant on side i, $i \in \{B, S\}$; in addition, facilitating an interaction between a buyer and a seller costs the platform a transaction cost equal to $s \geq 0$. We assume the net stand-alone values are positive, i.e. $v_i - c_i > 0$, $i \in \{B, S\}$,

is driven by our desire to study investment inefficiency in two-sided markets. If participants were heterogeneous in their stand-alone values, or interaction benefits, we would need to model investments that shift probability distribution functions, which is significantly more difficult (see, e.g., Moraga-González and Sun, 2023).

[5]The latter condition is in line with that in Tan and Zhou (2021) and Chica et al. (2025) who establish the existence and uniqueness of a user equilibrium with multiple platforms. However, they do not analyze the stability of the equilibrium.

which ensures agents have incentives to participate. Moreover, we assume that total network effects $\theta_i(e_i) + \theta_j(e_j)$ exceed the transaction cost s.[6]

Efforts to enhance the users' experiences are also costly. The total cost of exerting effort e_i on side i of the market, with $i \in \{B, S\}$, is equal to $C(e_B + e_S)$. We assume that $C(\cdot)$ is an increasing and convex function, with $C(0) = 0$.[7]

The objective function of a profits-maximizing monopoly platform is then given by:

$$\begin{aligned}\Pi^p(N_B, N_S, e_B, e_S) =& N_B(p_B - c_B) + N_S(p_S - c_S) - sN_BN_S - C(e_B + e_S)\\ =& N_B(v_B + \theta_B(e_B)N_S - \varphi_B(N_B) - c_B) + N_S(v_S + \theta_S(e_S)N_B - \varphi_S(N_S) - c_S)\\ &- sN_BN_S - C(e_B + e_S),\end{aligned} \tag{4}$$

where we have used the price equations in the second equality.

3.3 Planner

We study the efficiency of the monopoly platform's allocation by comparing it to the socially optimal one, assuming the planner maximizes the sum of the platform's profits and buyers' and sellers' surpluses. The expression for the platform's profits is above in equation (4). Let $V_B(\cdot)$ and $V_S(\cdot)$ represent the buyers' and sellers' surpluses, respectively.

We now show that the buyers' and sellers' surpluses depend solely on participation levels. Consider the surplus of the agents on side i of the market, $i \in \{B, S\}$. Agents who join the platform and become participants obtain utility u_i, while agents who opt for the outside option get the value of their outside alternative. Therefore, the total surplus of the agents on side i of the market is given by:

$$V_i(\cdot) = N_iu_i + \int_{u_i}^{\bar{o}_i} zg_i(z)dz = \bar{o}_i - \int_{u_i}^{\bar{o}_i} G_i(z)dz = \bar{o}_i - \int_{\varphi_i(N_i)}^{\bar{o}_i} G_i(z)dz, \tag{5}$$

where $g_i(\cdot)$ is the density function corresponding to $G_i(\cdot)$, the second equality follows from integration by parts, and the third from replacing u_i by $\varphi_i(N_i)$. Hence, from now on, we shall write $V_i(N_i)$ to represent the total surplus of the agents on side i of the market, $i \in \{B, S\}$.[8] Then, the social welfare function can be written as:

$$SW(N_B, N_S, e_B, e_S) = \Pi^p(N_B, N_S, e_B, e_S) + V_B(N_B) + V_S(N_S). \tag{6}$$

[6]This assumption guarantees that allowing for transactions is beneficial for the platform.

[7]This assumption implies that the marginal cost of improving the network experience for one group increases with the level of investment in enhancing the network experience of the other group. In this sense, the platform faces an investment portfolio problem as in Moraga-González et al. (2022), where allocating resources to one side of the market affects the marginal cost of investing in the other.

[8]Further, note that the surplus of the agents on side i of the market does not depend on investment on its side. This is because the platform's problem can be seen as offering a given level of utility to each side of the market. By the expression (1), an increase in investment is offset by an increase in the price to keep the same level of offered utility.

For later use, note that the derivative of the surplus $V_i(N_i)$ with respect to N_i is:

$$\frac{\partial V_i(N_i)}{\partial N_i} = G_i(\varphi_i(N_i))\frac{\partial \varphi_i(N_i)}{\partial N_i} = N_i\frac{\partial \varphi_i(N_i)}{\partial N_i}.$$

4 Participation and pricing decisions

It will become apparent in Sections 5 and 6 that the (in-)efficiency of the monopoly platform's investment decision is intimately linked to the (in-)efficiency of the number of participants on each side as well as the number of interactions facilitated by the platform, i.e., the product of the number of buyers and the number of sellers, $N_B N_S$. Hence, in this section, we compare the monopoly and socially optimal participation of buyers and sellers for arbitrarily given investments. In addition, we characterize the prices that sustain such optimal participation levels.

Fix an arbitrary vector of investments (e_B, e_S) to enhance users' experiences. Later in Section 5, we shall endogenize the choice of such a vector. For simplicity of notation, let in this section be $\theta_B \equiv \theta_B(e_B)$ and $\theta_S \equiv \theta_S(e_S)$, so that we can ignore the dependency of the network effects on investment and analyze participation and pricing distortions in the setting of Armstrong (2006). This setting has received independent attention in Weyl (2010), Tan and Wright (2018), and Tan and Wright (2021). However, to the best of our knowledge, the literature lacks a general characterization of the inefficiency of monopoly participation and pricing decisions. In this section, we provide such a characterization. We show that the monopoly sets participation prices either above the social optimum on both sides, or below the social optimum on one side and above it on the other. Moreover, even when one side faces a below-optimal price, participation is inefficiently low on both sides.

The platform maximizes its profit with respect to the number of participants; hence, the first-order conditions (FOCs) derived from equation (4) are as follows:

$$\frac{\partial \Pi^p(N_B, N_S)}{\partial N_B} = (\theta_B + \theta_S - s)N_S + v_B - c_B - \varphi_B(N_B) - \frac{\partial \varphi_B(N_B)}{\partial N_B}N_B = 0, \tag{7}$$

$$\frac{\partial \Pi^p(N_B, N_S)}{\partial N_S} = (\theta_B + \theta_S - s)\, N_B + v_S - c_S - \varphi_S(N_S) - \frac{\partial \varphi_S(N_S)}{\partial N_S}N_S = 0. \tag{8}$$

To ensure that the solution to the system of FOCs (7)-(8) corresponds to the global maximum, we assume that the monopoly payoff is strictly concave.[9]

Assuming that the monopoly payoff is maximized at an interior solution, which we denote (N_B^*, N_S^*), we can rewrite (7)-(8) in terms of the fees charged to the agents on the two sides of the

[9]For this, it suffices that the distribution functions of outside options have decreasing densities, and $\theta_B + \theta_S - s$ is sufficiently small. The latter condition also ensures the planner's problem is strictly concave (see Appendix B).

market:

$$p_B^* = c_B - (\theta_S - s)N_S^* + \frac{\partial \varphi_B(N_B^*)}{\partial N_B} N_B^*, \tag{9}$$

$$p_S^* = c_S - (\theta_B - s)N_B^* + \frac{\partial \varphi_S(N_S^*)}{\partial N_S} N_S^*. \tag{10}$$

These two expressions for p_B^* and p_S^* are similar to those in Armstrong (2006) and Tan and Wright (2021). Intuitively, the profit-maximizing price charged to group i is equal to the cost of providing the service, c_i, adjusted either upward or downward by the net external benefit to the opposite side j, $(\theta_j - s)N_j$, and adjusted upward by a factor related to the elasticity of the group participation, i.e. $\frac{\partial \varphi_i(N_i)}{\partial N_i} N_i$.

Similarly, the FOCs for the maximization of the social planner's objective are:

$$\frac{\partial SW(N_B, N_S)}{\partial N_B} = (\theta_B + \theta_S - s)N_S + v_B - c_B - \varphi_B(N_B) = 0, \tag{11}$$

$$\frac{\partial SW(N_B, N_S)}{\partial N_S} = (\theta_B + \theta_S - s)\, N_B + v_S - c_S - \varphi_S(N_S) = 0. \tag{12}$$

Assuming the planner's objective is maximized at an interior solution, which we denote as (N_B^o, N_S^o), we can rewrite the FOCs (11)-(12) in terms of the fees it would charge to the agents on each side of the market:

$$p_B^o = c_B - (\theta_S - s)N_S^o, \tag{13}$$

$$p_S^o = c_S - (\theta_B - s)N_B^o. \tag{14}$$

These two expressions for p_B^o and p_S^o are again similar to those in Armstrong (2006) and Tan and Wright (2021). Intuitively, the welfare-maximizing price for group i is equal to the cost of providing the service, c_i, adjusted either upward or downward by the net external benefit to the opposite side j, $(\theta_j - s)N_j$.

To shorten the expressions, let $\gamma \equiv \theta_B + \theta_S$. Inspection of the FOCs corresponding to the monopoly platform's problem (equations (7)–(8)) and the planner's problem (equations (11)–(12)) reveals that, under our maintained assumption $\gamma - s > 0$, participation choices on the two sides are complements for both the platform and the planner. This complementarity reflects that higher participation on either side increases transaction volume $N_B N_S$, which raises surplus for both the platform and the planner. As we show next, this property plays a central role in our results on the efficiency of buyer and seller participation under monopoly.

Proposition 1 *Let $\gamma \equiv \theta_B + \theta_S$ and $\max_{u_i \in \mathbb{R}} \theta_i g_i(u_i) < 1$, $i \in \{B, S\}$. Then, compared to the social optimum:*

(i) Both the buyer and seller sides of the market exhibit under-participation.

(ii) The participation prices that sustain such a configuration are either too high on both sides, or too high on one side and too low on the other. Hence, the two sides of the market are never simultaneously charged participation prices that are too low.

(iii) Furthermore, both prices are excessive if $\theta_i \geq s$, $i \in \{B, S\}$; and for the price on side i to be insufficient, it must be the case that $\theta_i < s$.

Proof. See Appendix C. ■

The proof of part (i) builds on two important observations. First, we show that both components of the gradient vector in (11)-(12) evaluated at the monopoly equilibrium (N_B^*, N_S^*) are positive for all parameter values. Second, because of complementarity, the loci $N_B^o(N_S)$ and $N_S^o(N_B)$ implicitly defined by the FOCs in (11)-(12) are upward sloping. These two observations are illustrated in Figure 1.[10] In this figure we plot iso-welfare curves in the (N_B, N_S)-space. The solid lines represent the FOCs (11) and (12) corresponding to the planner's problem, and the intersection point (in black) represents the social welfare maximizing buyer and seller participation levels, (N_B^o, N_S^o).

Because both components of the gradient vector of the planner's problem evaluated at the monopoly solution are positive, and the loci $N_B^o(N_S)$ and $N_S^o(N_B)$ are upward-sloping, it follows that the monopoly solution must fall in Region I of Figure 1. As a result, we conclude that the private optimum exhibits under-participation on both sides of the market relative to the social optimum. For the chosen parameters, the monopoly equilibrium (N_B^*, N_S^*) is indicated in Figure 1 by the grey point.

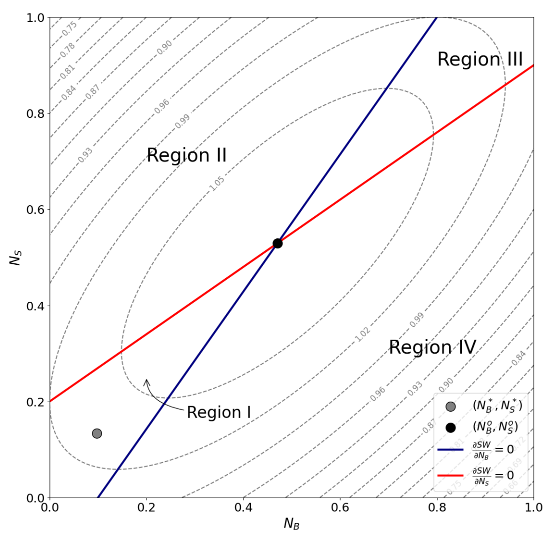


Figure 1: Private and socially optimal numbers of participants.

Turning to the intuition, note that the term $(\gamma - s)N_B N_S$ appears in both the profit and welfare functions. Because network benefits exceed total transaction costs, buyer and seller participation

[10]Figure 1 is constructed assuming outside options are uniformly distributed on the unit interval (see Appendix D). In this figure, the parameters are $s = 0.8, \theta_B = 0.9, \theta_S = 0.6, c_B = 0.2, v_B = 0.3, v_S = 0.9, c_S = 0.7$.

levels are complements: a higher participation level on either side raises transaction volume and hence payoffs. Therefore, both the platform and the planner prefer more interactions, but the planner values them more because it also accounts for participants' surplus: for a fixed number of sellers (buyers), the benefit of an additional buyer (seller) is larger for the social planner than for the platform because the extra terms $\frac{\partial V_i(N_i)}{\partial N_i} = \frac{\partial \varphi_i(N_i)}{\partial N_i} N_i$, $i \in \{B, S\}$ that enter the FOCs (7)–(8) are positive. This implies that, given N_S, the monopoly platform's optimal choice of N_B is always smaller than the planner's; likewise, given N_B, the platform's optimal choice of N_S is smaller than the planner's.

Part (ii) of Proposition 1 shows that participation prices that sustain such a privately optimal participation configuration exceed the socially optimal levels on at least one side of the market. The proof rests on the observation derived from Lemma 1 that, owing to the complementarity of the participation decisions of buyers and sellers, an increase in the participation price charged to one side of the market reduces participation rates on both sides. This, in turn, implies that, when evaluated at the profit-maximizing prices (p_B^*, p_S^*), each component of the planner's gradient vector in the (p_B, p_S)-space is negative for all parameter values. Geometrically, this means that the social optimum lies in the "up-right" quadrant relative to the monopoly outcome (see e.g. Figure 2a below). Consequently, both sides cannot be simultaneously priced strictly below their socially optimal levels. Moreover, only when the loci $p_B^o(p_S)$ and $p_S^o(p_B)$, implicitly defined by the FOCs (44)-(46) in Appendix C, are both upward sloping can we be certain that monopoly prices are excessive on both sides. Otherwise, prices may still be too high on both sides, but also too high on one side and too low on the other, depending on parameters.[11]

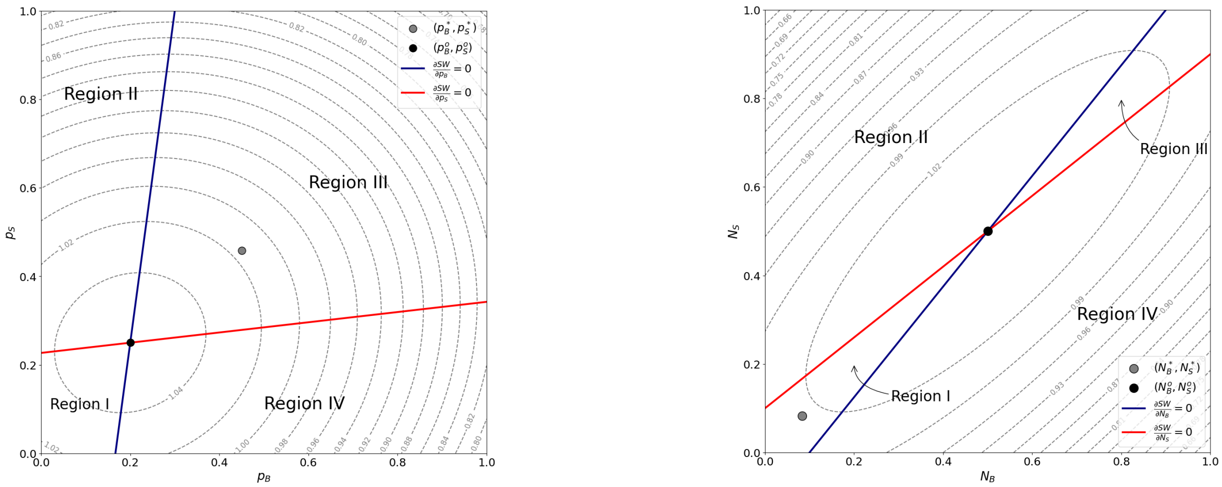
Part (iii) of Proposition 1 first provides sufficient conditions under which the monopoly prices exceed the socially optimal ones on both sides of the market. We note that these conditions hold in settings where per-transaction costs s are equal to zero. This means that models without per-transaction costs can only yield an excessive pricing result on both sides, as e.g. in Armstrong (2006) and Jeon et al. (2025). Further, this part provides a necessary condition for the monopolist to charge too low a price on one side of the market.

Figures 2, 3 and 4 illustrate different aspects of Proposition 1 for the case in which agents' outside options are uniformly distributed on the unit interval. In these figures we plot iso-welfare curves in the (p_B, p_S)-space and in the (N_B, N_S)-space. Consider first Figure 2, which corresponds to a parameter constellation for which s is relatively small.[12] In Figure 2a, the solid lines represent the FOCs with respect to prices p_B and p_S in the welfare-maximizing problem, and the intersection point represents the socially optimal pair of prices (p_B^o, p_S^o). As it can be seen, these lines slope

[11]For example, with uniformly distributed outside options, the pricing decisions of the social planner are substitutes for sufficiently large s and complement otherwise. More concretely, if $s < \frac{\theta_B \theta_S(\theta_B + \theta_S)}{1 + \theta_B \theta_S}$, then the FOCs are upward sloping. Otherwise, for $\frac{\theta_B \theta_S(\theta_B + \theta_S)}{1 + \theta_B \theta_S} < s < \theta_B + \theta_S$ we have downward sloping FOCs.

[12]Figure 2 is constructed assuming outside options are uniformly distributed on the unit interval. In Figure 2, the parameters are $s = 0.1, \theta_B = 0.4, \theta_S = 0.5, c_B = 0.4, v_B = 0.5, v_S = 0.5, c_S = 0.4$.

upward, which means that, from the point of view of the planner, prices on both sides of the market are complements. Because both components of the planer's problem gradient vector are negative, the private optimum (p_B^*, p_S^*), which is indicated by the grey point, must lie in Region III. As a result, monopoly prices are excessive on both sides.



(a) Iso-welfare curves, (p_B, p_S)-space

(b) Iso-welfare curves, (N_B, N_S)-space

Figure 2: Excessive prices on both sides

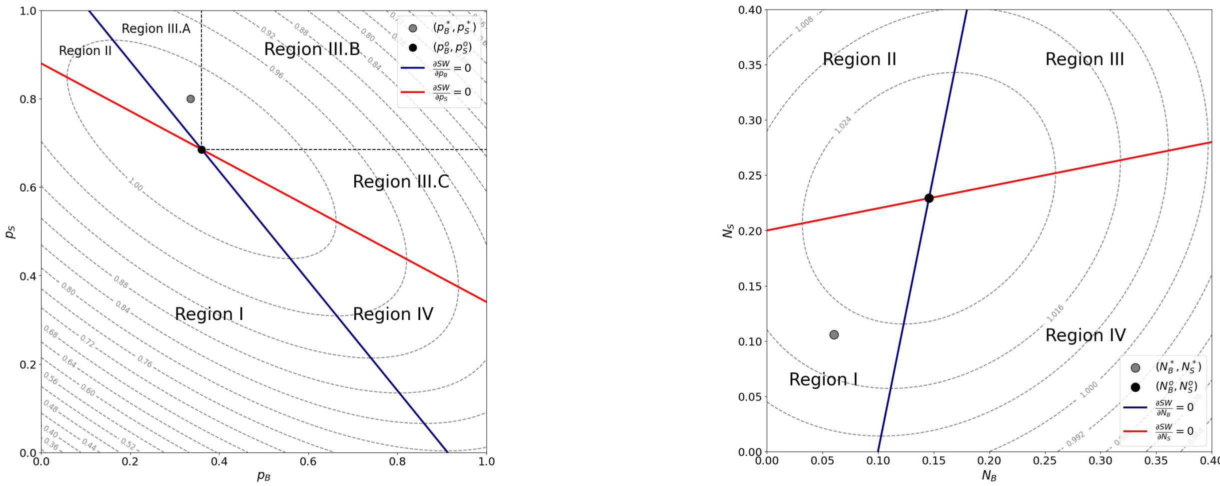
Figures 3 and 4 illustrate that only when the FOCs slope upward can we be certain that monopoly prices are excessive on both sides.[13] In both figures, transaction costs are relatively large and the FOCs with respect to p_B and p_S in the welfare-maximizing problem slope downward. Since both components of the planner's gradient vector evaluated at the private equilibrium are negative, the private optimum must lie in one of the regions labeled III.A, III.B, or III.C.

Figure 3 depicts a case in which sellers have larger net stand-alone benefits than buyers. The optimal monopoly price pair (p_B^*, p_S^*) lies in Region III.A (grey point). This means that, relative to the social optimum, sellers face prices that are too high, while buyers face prices that are too low. Despite this asymmetric price distortion, the monopoly participation levels (N_B^*, N_S^*) shown by the grey point in Region I of Figure 3b are still inefficiently low on both sides.

Figure 4 shows a configuration in which buyers have larger net stand-alone values than sellers. In this case, the private price equilibrium (p_B^*, p_S^*) lies in Region III.C of Figure 4a, where the platform favors sellers by charging them a price below the social optimum, while charging buyers a price that is too high. Despite this, the market continues to exhibit under-participation on both sides, with the monopoly participation levels (N_B^*, N_S^*) falling in Region I of Figure 4b.

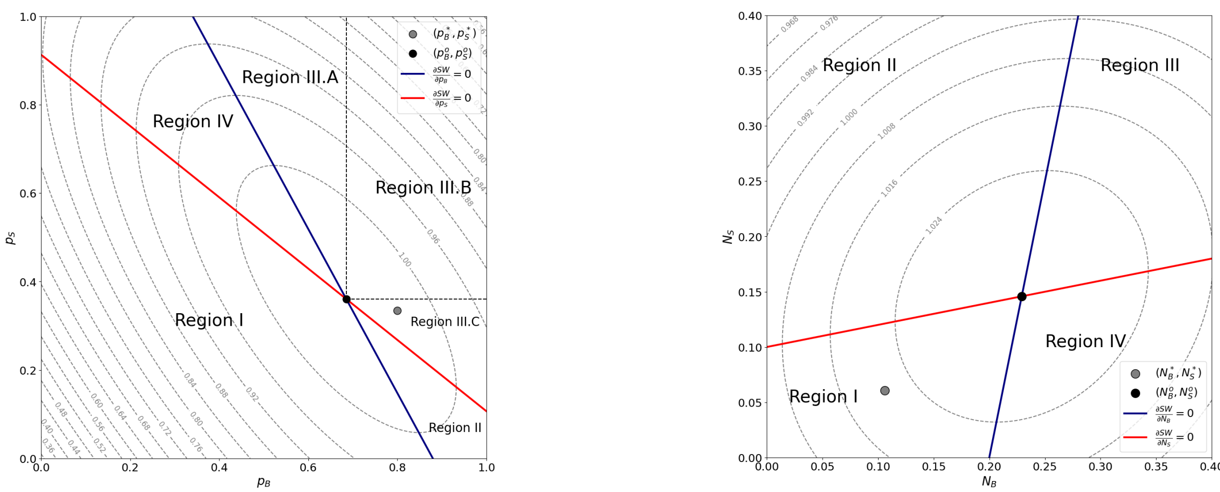
Tan and Wright (2021) compare the FOCs of the monopolist and the planner for a given side of the market and argue that monopoly participation prices may be excessively high, excessively low, or fully efficient (see also Weyl, 2010; Tan and Wright, 2018). However, in a two-sided

[13] In Figure 3, the parameters are $s = 0.8, \theta_B = 0.9, \theta_S = 0.1, c_B = 0.2, v_B = 0.3, v_S = 0.9, c_S = 0.7$. In Figure 4, the parameters are $s = 0.8, \theta_B = 0.1, \theta_S = 0.9, c_B = 0.7, v_B = 0.9, v_S = 0.3, c_S = 0.2$.



(a) Iso-welfare curves, (p_B, p_S)-space (b) Iso-welfare curves, (N_B, N_S)-space

Figure 3: Too low price on side B ($p_B^* < p_B^o$), too high price on side S ($p_S^* > p_S^o$)



(a) Iso-welfare curves, (p_B, p_S)-space (b) Iso-welfare curves, (N_B, N_S)-space

Figure 4: Too low price on side S ($p_S^* < p_S^o$), too high price on side B ($p_B^* > p_B^o$)

market, one-dimensional, side-by-side comparisons of single FOCs are insufficient: the relevant objects are the systems of FOCs in (9)–(10) and (13)–(14) because prices on the two sides are jointly determined. Because of this, a proper comparison of the optimal prices requires to work in price space. Proposition 1 provides a more complete characterization of the direction of pricing distortions and the associated participation distortions in the canonical monopoly two-sided platform model of Armstrong (2006). Adopting an explicitly multidimensional perspective, we show that over-participation on either side and prices that are too low on both sides are impossible. If the monopoly price falls below the social optimum on one side of the market, it must exceed the social optimum on the other. Moreover, participation is consistently too low on both sides, regardless of the pattern of price distortions.

Understanding the exact conditions under which the platform sets too low a price on one side and too high a price on the other (Regions III.A and III.C in Figures 3 and 4) is of particular interest. Unfortunately, for general distribution functions it is not possible to derive closed-form expressions for the boundaries of these regions. To gain further insight into this question, we therefore focus on the canonical case in which outside options are uniformly distributed (see, e.g., Angelini et al., 2024; De Cornière et al., 2025).

Corollary 1 (Corollary to Proposition 1) *Let the outside options o_i be uniformly distributed over the interval $[0,1]$, $i \in \{B, S\}$. Define $\mathcal{R} \equiv \frac{v_B - c_B}{v_S - c_S} > 0$, which captures the asymmetry in net stand-alone benefits across the two sides of the market, and assume the parameters of the model satisfy*

$$0 < \theta_S < s - \frac{(\gamma - s)(1 - (\gamma - s)^2)}{2 + (\gamma - s)^2}.$$

Then, for fixed $v_S - c_S$, we have $p_B^* < p_B^o$ and $p_S^* > p_S^o$ if and only if $\mathcal{R} < \tilde{\mathcal{R}}$, where

$$\tilde{\mathcal{R}} \equiv \frac{(s - \theta_S)(2 + (\gamma - s)^2) - (\gamma - s)(1 - (\gamma - s)^2)}{2(1 - (\gamma - s)^2) - 3(s - \theta_S)(\gamma - s)} \in (0, 1).$$

(And symmetrically, if we fix $v_B - c_B$.)

Proof. See Appendix D.2. ■

Corollary 1 complements Proposition 1 by characterizing when the monopoly price is inefficiently low on one side in the case of uniformly distributed outside options. The key driver is the asymmetry in net stand-alone values, i.e., the difference between $v_B - c_B$ and $v_S - c_S$. An asymmetric pricing distortion, that is, one side priced below and the other above the social optimum, arises only if (i) the cross-side network effects are sufficiently weak on one side and (ii) the net stand-alone benefit on the side that is underpriced is sufficiently small relative to the other side. For example, holding $v_S - c_S$ fixed, the platform sets $p_B^* < p_B^o$ only when θ_S is small relative to s and $v_B - c_B$ small relative to $v_S - c_S$.

Intuitively, when sellers' net stand-alone values are large, it is profitable for the monopoly platform to raise the seller-side price and extract surplus from the sellers. However, increasing the seller-side price reduces seller participation and, therefore, transaction volume. To mitigate this loss, a platform that extracts more from sellers has an incentive to lower the buyer-side price, which increases buyer participation and, via cross-side network effects, partially restores seller demand. The strength of this offsetting effect depends on the sellers' cross-side network effect. When θ_S is large, a given increase in buyer participation generates a strong spillover to sellers, so a modest reduction in the buyer-side price suffices to compensate for the initial decline in seller participation. When θ_S is small, the spillover is weak and the platform must cut the buyer-side price more aggressively to achieve the same recovery in seller participation. In that case, the buyer-side price can fall below the socially optimal level because the platform internalizes profits but not the full marginal welfare effects of shifting participation across sides.

Summarizing, Proposition 1 and Corollary 1 together deliver a complete characterization of monopoly distortions in participation and pricing. Regardless of parameters, the monopoly outcome features under-participation on both sides relative to the social optimum. At the same time, participation prices need not be excessive on both sides: they can be too high on both sides, or too high on one side and too low on the other. The latter asymmetric pricing pattern arises only when cross-side network effects are sufficiently weak on one side, and net stand-alone benefits are sufficiently asymmetric across sides.

These results also help reconcile our findings with the per-transaction-fee comparison emphasized by Jullien et al. (2021). Under participation prices, the set of possible distortions is richer because the platform can tilt prices across sides while still inducing under-participation on both sides. By contrast, per-transaction fees $f_i \equiv p_i/N_i$ bundle two margins, namely, a price distortion (through p_i) and a participation distortion (through N_i). Because f_i is a ratio, under-participation (a lower N_i) raises p_i/N_i by construction. In our model, this effect always dominates any downward distortion in p_i, so per-transaction fees are always (weakly) excessive, even though participation prices themselves may display asymmetric distortions across sides. For details, see Appendix F.

5 Investments in network benefits

In Section 4, we have characterized monopoly inefficiencies in participation and pricing for an arbitrary (fixed) vector of investments. We now endogenize investments in network effects. Thus, in this section both the monopoly platform and the social planner choose participation levels (N_B, N_S) as well as investment efforts (e_B, e_S) that enhance users' interaction benefits. The key observation is that the marginal return to investment on either side is proportional to the volume of interactions facilitated by the platform, $N_B N_S$. Consequently, distortions in participation translate into distortions in interaction volume, which in turn determine whether the monopoly platform under- or overinvests relative to the social planner.

Consider first the problem of the monopoly platform, whose payoff is still given by (4). The FOCs for the maximization of the monopoly platform's profits with respect to participation rates and investment levels are:

$$\frac{\partial \Pi^p(N_B, N_S, e_B, e_S)}{\partial N_B} = (\gamma(e_B, e_S) - s)N_S + v_B - c_B - \varphi_B(N_B) - \frac{\partial \varphi_B(N_B)}{\partial N_B} N_B = 0,$$
$$\frac{\partial \Pi^p(N_B, N_S, e_B, e_S)}{\partial N_S} = (\gamma(e_B, e_S) - s)\, N_B + v_S - c_S - \varphi_S(N_S) - \frac{\partial \varphi_S(N_S)}{\partial N_S} N_S = 0,$$
$$\frac{\partial \Pi^p(N_B, N_S, e_B, e_S)}{\partial e_B} = N_B N_S \frac{\partial \theta_B(e_B)}{\partial e_B} - C'(e_B + e_S) = 0,$$

$$\frac{\partial \Pi^p(N_B, N_S, e_B, e_S)}{\partial e_S} = N_B N_S \frac{\partial \theta_S(e_S)}{\partial e_S} - C'(e_B + e_S) = 0, \tag{15}$$

where, in line with the notation above, we define $\gamma(e_B, e_S) \equiv \theta_B(e_B) + \theta_S(e_S)$.

The first two FOCs are identical to (7)–(8). Because network externalities are now functions of the investment levels, we additionally impose the FOCs of (4) with respect to the investment efforts (e_B, e_S). Inspection of these FOCs shows that, at an interior optimum, the platform's marginal return from investing to enhance buyers' participation utility must equal the marginal return from investing to enhance sellers' participation utility.

Similarly, the FOCs for the maximization of the planner's objective in (6) are:

$$\begin{aligned}
\frac{\partial SW(N_B, N_S, e_B, e_S)}{\partial N_B} &= (\gamma(e_B, e_S) - s)\, N_S + v_B - c_B - \varphi_B(N_B) = 0, \\
\frac{\partial SW(N_B, N_S, e_B, e_S)}{\partial N_S} &= (\gamma(e_B, e_S) - s)\, N_B + v_S - c_S - \varphi_S(N_S) = 0, \\
\frac{\partial SW(N_B, N_S, e_B, e_S)}{\partial e_B} &= N_B N_S \frac{\partial \theta_B(e_B)}{\partial e_B} - C'(e_B + e_S) = 0, \\
\frac{\partial SW(N_B, N_S, e_B, e_S)}{\partial e_S} &= N_B N_S \frac{\partial \theta_S(e_S)}{\partial e_S} - C'(e_B + e_S) = 0.
\end{aligned} \tag{16}$$

We now present an auxiliary result for our main proposition in this section. It turns out that the monopoly and the socially optimal investment levels are located on the same increasing locus in the (e_B, e_S)-space.

Lemma 2 *Let (e_B^*, e_S^*) and (e_B^o, e_S^o) be the monopoly and socially optimal investment levels. Then, there exists a unique increasing function $y(\cdot)$ such that $e_S^* = y(e_B^*)$ and $e_S^o = y(e_B^o)$.*

Proof. See Appendix C. ■

The reason behind this result is that both the planner and the platform choose their investments in network effects to equalize the marginal returns across sides. The implication is that any investment distortion must occur in the same direction on both sides of the market. Thus, the platform either overinvests on both sides or underinvests on both sides; opposite-signed distortions across sides are impossible.

Our main result combines Proposition 1 and Lemma 2 to provide a general characterization of monopoly inefficiency in investment in our two-sided market:

Proposition 2 *In the private equilibrium, investment in network benefits is inefficiently low on both sides: $e_B^* < e_B^o$ and $e_S^* < e_S^o$.*

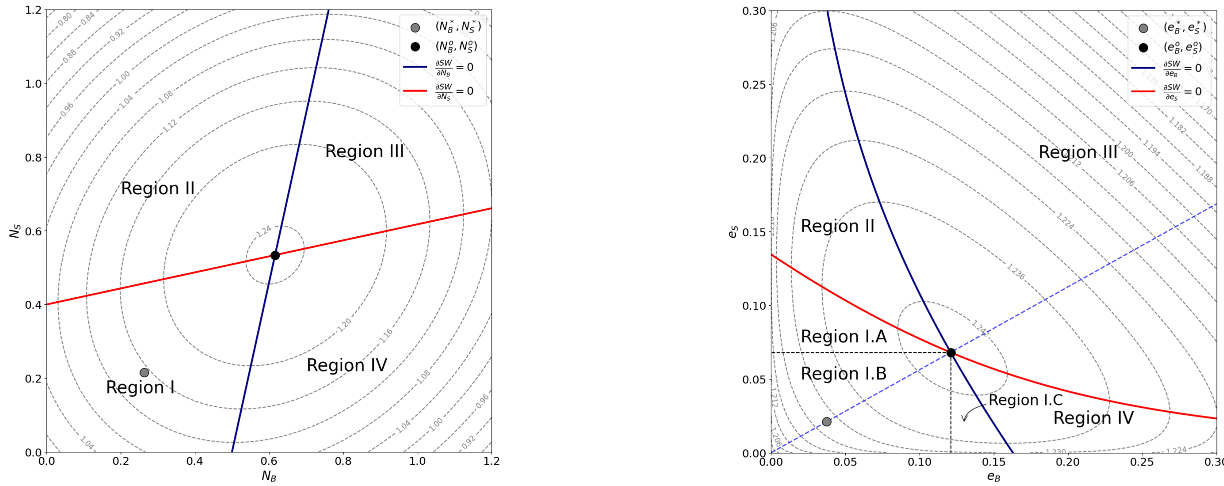
Proof. See Appendix C. ■

This proposition shows that distortions in participation translate into underinvestment on both sides of the market. The idea of the proof is as follows. First, we show that the loci $e_B^o(e_S)$ and $e_S^o(e_B)$ implicitly defined by the planner's FOCs in (16) are downward sloping, implying that investment effort on one side of the market is a substitute for investment effort on the other side. Second, evaluating the planner's investment FOCs at the private optimum (e_B^*, e_S^*) characterizes the sign of the investment distortion in terms of the gap in the volume of interactions: the welfare gradient points toward higher (lower) investment when $\widetilde{N}_B\widetilde{N}_S - N_B^* N_S^* > 0$ (respectively, < 0). Here, $\widetilde{N}_B$ and $\widetilde{N}_S$ are the participation rates the social planner would implement if investment levels were fixed at the private equilibrium, i.e., $\widetilde{N}_B$ and $\widetilde{N}_S$ solve the first two FOCs of the social planner problem after setting investment levels to (e_B^*, e_S^*). Because investment levels (e_B^*, e_S^*) are fixed across the optimization problems of the platform and the planner in this step, we can invoke Proposition 1 and Lemma 2 to obtain the general characterization in Proposition 2.

We illustrate Proposition 2 in Figure 5. In Figure 5a, we plot iso-welfare curves in the (N_B, N_S)-space; the construction of this figure is as above. In Figure 5b, we show iso-welfare curves in the (e_B, e_S)-space. The solid lines represent the planner's FOCs with respect to e_B and e_S, given in (16), and their intersection corresponds to the welfare-maximizing investment levels (e_B^o, e_S^o). The locus identified in Lemma 2 is illustrated by the increasing blue-dashed curve. By Proposition 1, the monopoly exhibits under-participation on both sides, so that $\widetilde{N}_B\widetilde{N}_S > N_B^* N_S^*$. This implies that the last two components of the gradient of the planner's objective evaluated at (e_B^*, e_S^*) are positive, and hence (e_B^*, e_S^*) must lie in Region I.A, I.B, or I.C of Figure 5b. This means that overinvestment could actually arise. However, Lemma 2 pins down the location on the blue-dashed locus, implying that the monopoly underinvests on both sides of the market.[14]

Intuitively, the marginal payoff from raising e_i, $i \in \{B, S\}$, is proportional to the interaction volume $N_i N_j$ in both the platform's and the planner's problems. The difference is that the planner internalizes participants' surplus, so, evaluated at a given (N_B, N_S), it attaches a higher marginal value to investment than the platform. Moreover, as discussed in Section 4, holding investment fixed the planner implements higher participation rates than the platform, and thus facilitates a larger number of interactions. Together, these forces imply that the platform's investment incentives are weaker, yielding underinvestment on both sides relative to the social optimum (Proposition 2). Finally, because investment costs are symmetric across sides and investment incentives are linked through interaction volume, any investment distortion must have the same sign on both sides.

[14] In Figure 5, outside options are uniformly distributed on $[0, 1]$, the investment cost function is $C(e_B + e_S) = \frac{(e_B+e_S)^2}{2}$, and $\theta_B(e_B) = \theta_B\sqrt{e_B}$ and $\theta_S(e_S) = \theta_S\sqrt{e_S}$. Moreover, the parameters are set $s = 0, \theta_B = 0.4, \theta_S = 0.3, c_B = 0, v_B = 0.5, v_S = 0.4, c_S = 0$. Note that level curves represent $SW(N_B^o, N_S^o, e_B, e_S)$. The pair (e_B^o, e_S^o) corresponds to the social optimum, obtained from expressions in (72) (see Appendix D), while (e_B^*, e_S^*) represents the private optimum, derived from expressions in (70) (see Appendix D).



(a) Under-participation on both sides

(b) Underinvestment on both sides

Figure 5: Private and socially optimal participation and investments in network externalities

6 Investments in stand-alone benefits

Both stand-alone and network benefits are present in many platform environments, which makes it important to study the platform's incentives to invest in enhancing each. In Section 5, we have focused on investments that increase the benefits users derive from interacting with other users (i.e., investments that raise network benefits). In this section, we instead study investments aimed at increasing users' stand-alone values.[15]

Using notation analogous to that in Section 3, let the stand-alone utility of an agent from group i be $v_i(e_i)$, $i \in \{B, S\}$, where e_i is the effort the platform exerts to increase it. In line with our earlier assumptions, we assume that $v_i(\cdot)$ is increasing and concave to ensure well-behaved optimization problems. We also let $\gamma \equiv \theta_B + \theta_S$, which in this section is independent of investment. All other features of the model are as described in Section 3.

In this setting, the platform's optimal participation and investment levels are characterized by the following FOCs:

$$\begin{aligned}
\frac{\partial \Pi^p(N_B, N_S, e_B, e_S)}{\partial N_B} &= (\gamma - s)\, N_S + v_B(e_B) - c_B - \varphi_B(N_B) - \frac{\partial \varphi_B(N_B)}{\partial N_B} N_B = 0, \\
\frac{\partial \Pi^p(N_B, N_S, e_B, e_S)}{\partial N_S} &= (\gamma - s)\, N_B + v_S(e_S) - c_S - \varphi_S(N_S) - \frac{\partial \varphi_S(N_S)}{\partial N_S} N_S = 0, \\
\frac{\partial \Pi^p(N_B, N_S, e_B, e_S)}{\partial e_B} &= N_B \frac{\partial v_B(e_B)}{\partial e_B} - C'(e_B + e_S) = 0, \\
\frac{\partial \Pi^p(N_B, N_S, e_B, e_S)}{\partial e_S} &= N_S \frac{\partial v_S(e_S)}{\partial e_S} - C'(e_B + e_S) = 0. \qquad (17)
\end{aligned}$$

[15] A leading example of an investment that enhances stand-alone utility is cybersecurity. A more secure platform experience can increase user trust and satisfaction independently of others' presence on the platform. The relevance of such investment is illustrated by the *Ashley Madison* data breach, which generated substantial reputational harm and litigation risk and placed the company under severe financial strain.

These FOCs show that, for each side $i \in \{B, S\}$, the optimal investment level equates the marginal cost of effort to the marginal benefit from raising stand-alone utility, which increases with participation on that side. Compared with the FOCs in (15), the marginal gain from investing in side i therefore depends only on N_i, rather than on transaction volume $N_B N_S$. This distinction reflects the fact that participation decisions affect investment in stand-alone values directly, rather than via network effects.

The social planner's optimal participation and investment levels are characterized by the following FOCs:

$$\begin{aligned}
\frac{\partial SW(N_B, N_S, e_B, e_S)}{\partial N_B} &= (\gamma - s)\, N_S + v_B(e_B) - c_B - \varphi_B(N_B) = 0, \\
\frac{\partial SW(N_B, N_S, e_B, e_S)}{\partial N_S} &= (\gamma - s)\, N_B + v_S(e_S) - c_S - \varphi_S(N_S) = 0, \\
\frac{\partial SW(N_B, N_S, e_B, e_S)}{\partial e_B} &= N_B \frac{\partial v_B(e_B)}{\partial e_B} - C'(e_B + e_S) = 0, \\
\frac{\partial SW(N_B, N_S, e_B, e_S)}{\partial e_S} &= N_S \frac{\partial v_S(e_S)}{\partial e_S} - C'(e_B + e_S) = 0. \qquad (18)
\end{aligned}$$

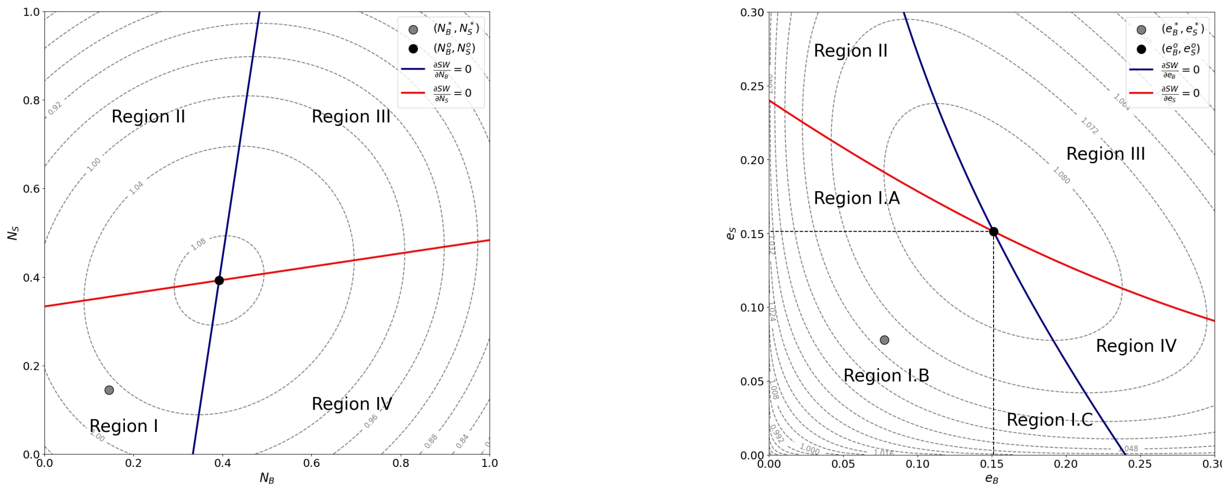
These conditions display the same feature as in the platform's problem: for each $i \in \{B, S\}$, the marginal gain from investment depends only on participation on that side, rather than on transaction volume. This distinction has important implications for the type of investment inefficiencies that may arise.

Proposition 3 *The private equilibrium exhibits underinvestment in stand-alone benefits on at least one side of the market, i.e., either $e_B^* < e_B^o$, or $e_S^* < e_S^o$, or both. Hence, overinvestment on both sides is impossible.*

Proof. See Appendix C. ■

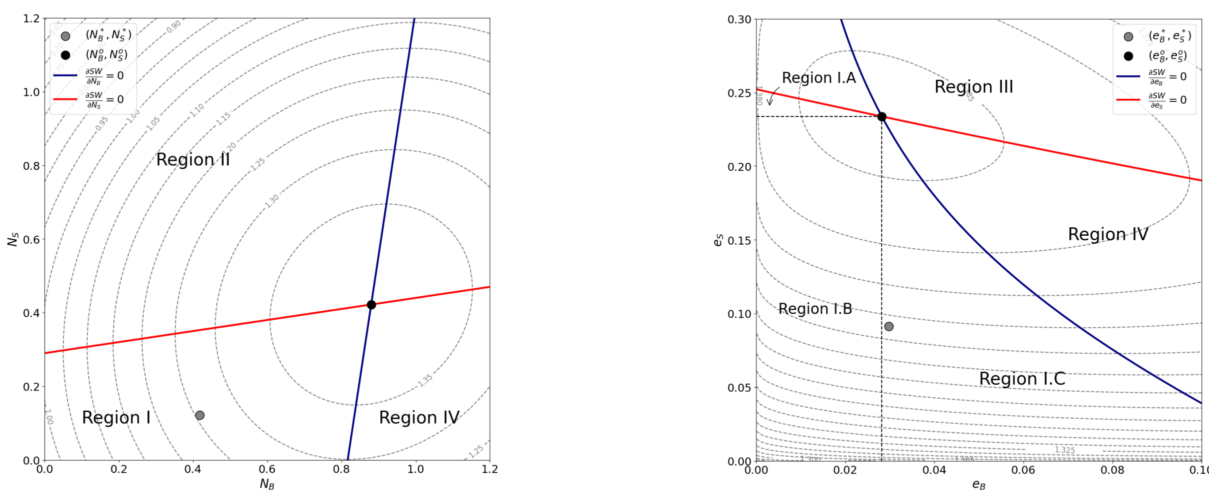
Proposition 3 contrasts with the case of investments that enhance network externalities, where both sides of the market experience underinvestment (Proposition 2). With investments that increase stand-alone benefits, by contrast, distortions may be asymmetric across sides, with overinvestment on one side and underinvestment on the other.

The idea of the proof is as follows. First, we show that, as in the case in which investment enhances network effects, the loci $e_B^o(e_S)$ and $e_S^o(e_B)$ implicitly defined by the planner's investment FOCs in (18) are downward sloping. Second, we note that when we evaluate the planner's investment FOCs at the private optimum (e_B^*, e_S^*), the stand-alone values $v_i(\cdot)$ are the same in the private and social problems. We can therefore invoke Proposition 1, which implies that $\widetilde{N}_B > N_B^*$ and $\widetilde{N}_S > N_S^*$, where $(\widetilde{N}_B, \widetilde{N}_S)$ denotes the participation levels that solve the social planner's problem evaluated at the private optimal investment levels (e_B^*, e_S^*). These inequalities imply that the components of the gradient of the social welfare function with respect to e_B and e_S evaluated at (e_B^*, e_S^*) are both positive. Combined with the downward-sloping loci $e_B^o(e_S)$ and $e_S^o(e_B)$, we conclude that the monopoly platform cannot overinvest in stand-alone benefits on both sides: it either underinvests on both sides, or underinvests on one side while overinvesting on the other.



(a) Under-participation on both sides (b) Underinvestment on both sides

Figure 6: Private and socially optimal participation and investments in stand-alone benefits (symmetric distortions)



(a) Under-participation on both sides (b) Overinvestment on the B side, under on the S

Figure 7: Private and socially optimal participation and investments in stand-alone benefits (asymmetric distortions)

We illustrate Proposition 3 in Figures 6 and 7.[16] These figures are constructed as in Section 5. The graphs on the left show iso-welfare curves in the (N_B, N_S)-space, and the graphs on the right

[16]To construct these figures, we assume that outside options are uniformly distributed on $[0, 1]$, the investment cost function is $C(e_B + e_S) = \frac{(e_B+e_S)^2}{2}$, and the stand-alone benefits functions are $v_B(e_B) = \hat{v}_B + v_B\sqrt{e_B}$ and $v_S(e_S) = \hat{v}_S + v_S\sqrt{e_S}$. The level curves represent combinations of investments (e_B, e_S) that yield the same welfare level; the pair (e_B^o, e_S^o) indicates the social optimum, while (e_B^*, e_S^*) is the private optimum. In Figure 6, we set

show them in the (e_B, e_S)-space. The graphs on the left show our result in Section 4 that the monopoly optimum exhibits under-participation on both sides of the market. The graphs on the right show inefficiency in investment in stand-alone benefits. Since the components of the welfare gradient with respect to (e_B, e_S) evaluated at the private optimum are both positive, the private equilibrium must lie in Region I.A, I.B, or I.C, and cannot lie in Region III (which corresponds to overinvestment on both sides). Accordingly, the private equilibrium may feature underinvestment on both sides (Region I.B) or asymmetric distortions, with overinvestment on the buyers' side and underinvestment on the sellers' side (Region I.A), or the reverse (Region I.C). Figure 6b illustrates underinvestment on both sides, whereas Figure 7b illustrates overinvestment on the buyers' side and underinvestment on the sellers' side.

The situation depicted in Figure 7 deserves further explanation. When the net stand-alone value of buyers is sufficiently high, a monopoly platform has an incentive to raise the buyer-side price to extract surplus relative to the planner. Higher prices, however, reduce buyer participation, so the platform may respond by either increasing investment in buyers' stand-alone benefits, increasing investment on the seller side, or lowering the seller-side price to mitigate the demand loss. The strength of buyer-side network effects and the elasticities of the stand-alone investment functions then govern which margin is most effective. If buyer-side network effects are sufficiently strong, increasing seller participation generates a large positive spillover to buyers, so the platform can restore buyer demand with relatively little seller-side investment provided that the elasticity of the seller-side stand-alone investment function is sufficiently high. If buyer-side network effects are weak (as in Figure 7b), then the spillover is limited and the platform must rely more heavily on buyer-side investment to weaken the demand loss. In that case, if the elasticity of the buyer-side stand-alone investment function is low enough, the platform will invest quite heavily on the buyer-side and may even invest beyond the socially optimal level, leading to overinvestment on the buyers' side.

The result in Proposition 3 contrasts with Proposition 2 in that stand-alone investments can generate asymmetric distortions across sides, with underinvestment on one side and overinvestment on the other. The underlying reason is that the marginal benefits of investment scale differently across the two environments. With investments in network effects, the marginal benefit of raising e_i is proportional to the interaction volume $N_B N_S$, so (given identical investment costs) investment incentives are tightly linked across sides and distortions cannot point in opposite directions. By contrast, with stand-alone investments the marginal benefit of raising e_i is scaled by participation on that side, i.e., it is proportional to N_i. This dependence on participation levels gives the platform more scope to tilt investment toward one side, possibly leading to asymmetric stand-alone investment distortions across sides.

the parameters to the following values: $s = 0$, $\theta_B = 0.075$, $\theta_S = 0.075$, $c_B = 0$, $\hat{v}_B = 0.1$, $v_B = 0.6$, $\hat{v}_S = 0.1$, $v_S = 0.6$, and $c_S = 0$. In Figure 7, parameters are $s = 0$, $\theta_B = 0.075$, $\theta_S = 0.075$, $c_B = 0$, $\hat{v}_B = 0.8$, $v_B = 0.1$, $\hat{v}_S = 0$, $v_S = 0.6$, and $c_S = 0$ (see Appendix E for further details).

7 Application: An app platform model

App platforms operated by tech giants, such as Apple's App Store and Google's Play Store, have evolved into prominent marketplaces that facilitate interactions between app users and app developers. These platforms have been subject to sustained regulatory scrutiny, largely centered on the level and structure of commissions charged on in-app purchases. A salient example is the European Commission's finding that Apple engaged in anticompetitive conduct through its "sky-high" 30% commission on in-app purchases (see European Commission, 2021). In response, Apple revised its fee structure in the EU, including lower commission rates, separate payment-related fees, and a "Core Technology Fee" of 0.50 euro per first annual install above one million (see Apple, 2026c).

This section illustrates how the framework developed in Sections 3–6 can be used to analyze the joint pricing and investment strategies of dominant app marketplaces. We focus on three questions. First, holding investment fixed, we compare the platform's privately optimal participation levels and the associated price distortions to the social optimum in an environment in which the platform monetizes developers through an ad valorem fee on digital sales, while users pay for access to the ecosystem through the device price. This pricing architecture is standard in app marketplaces (see, for example Apple, 2026a; Google, 2026). Second, when investment is endogenous, we study how the platform's incentives to invest depend on the channel through which investment affects user and developer payoffs. Third, we examine a canonical regulatory intervention in this setting, namely, a cap on the commission rate, and characterize how the platform rebalances its pricing structure and investments in response, as well as when such a cap raises or lowers user participation and total surplus on both sides of the market.

The results provide a framework for thinking about how regulatory interventions that constrain commissions and hence induce platforms to rebalance their pricing structures may also affect incentives to invest in the marketplace.

Model details. We interpret the platform as an app store that intermediates between users (B) and app developers (S). Users pay a device price p_B to access the ecosystem. Developers pay the platform a commission at rate τ on revenue generated through in-app purchases. Each developer supplies an independent app.

Let $D(\rho)$ denote a developer's in-app purchase demand at in-app price ρ, where in-app purchases are acquisitions of virtual goods or services within an app (e.g., subscriptions or virtual items). We assume developers have zero marginal costs, as is typical for digital goods. Users and developers have heterogeneous outside options as in the main model.

Following Anderson and Bedre-Defolie (2025), we assume the following timing. At $t = 1$, the platform chooses its pricing instruments (p_B, τ) and, when relevant, its investment decisions. At $t = 2$, users and developers decide whether to join the platform. At $t = 3$, conditional on participation, developers choose in-app prices. We first compare the platform's choice of device price and commission rate to the socially optimal policy, under the assumption that the planner cannot directly influence in-app prices. Later, we compare the platform's and the planner's investment incentives. Finally, we study the effects of a cap on the commission rate.

In this environment, network benefits are driven by in-app purchase activity. Let the per-user interaction benefits for users and developers be denoted as:

$$\theta_B(\rho) \equiv \theta_B(D(\rho)), \qquad \theta_S(\rho) \equiv \theta_S(D(\rho)).$$

Here, $\theta_B(\rho)$ can be interpreted as the (per-user) surplus from in-app purchases at price ρ, while $\theta_S(\rho)$ is the (per-user) in-app purchase revenue (or profit) generated at price ρ before the platform's commission is applied.

At $t = 3$, each developer chooses ρ to maximize in-app revenue net of the commission. Since τ enters as a multiplicative factor on in-app revenue, the optimal in-app price is the monopoly price, which we denote by ρ^m, and is independent of both p_B and τ. Accordingly, in what follows all interaction benefit terms are evaluated at ρ^m.

We consider two classes of platform investment, corresponding to the channels studied in Sections 5 and 6:

- *Investments that promote in-app purchase demand.* These investments shift in-app purchase demand itself, for instance by improving payment processing or reducing transaction frictions. Formally, we define:

  $$\theta_B(\epsilon) \equiv \theta_B(D(\rho^m(\epsilon), \epsilon)), \qquad \theta_S(\epsilon) \equiv \theta_S(D(\rho^m(\epsilon), \epsilon)).$$

  This notation highlights that these investments operate through ϵ by shifting up the demand and the corresponding monopoly price so that $\partial\theta_B(\cdot)/\partial\epsilon > 0$ and $\partial\theta_S(\cdot)/\partial\epsilon > 0$.[17]

- *Stand-alone investments.* These investments increase stand-alone benefits v_i on either side. In the app-store context, a natural example on the user side is device quality. For instance, Apple introduced OLED displays with the iPhone X (Apple, 2017), and battery capacity has increased across recent iPhone generations.

  On the developer side, stand-alone investments include tools and services that improve platform reliability, security, and compliance. For example, Apple states that it assists developers with tax obligations in many regions of the world and supports dispute processes to help protect trademarks and copyrights (Apple, 2026d). Apple also regularly releases software updates to address security vulnerabilities (Apple, 2026b).

[17]This framework can easily be used to study investments that favor the sellers at the expense of the buyers, e.g., investments that enable the sellers to price discriminate. Likewise, it can be easily adapted to examine investments that protect the buyers and reduce opportunistic behavior of the sellers, such as payments holdings, reputation systems, or deposit requirements.

7.1 Participation, pricing, and investment inefficiency

The app-platform environment just described is naturally embedded in our baseline framework, with the only difference that the developer-side instrument is an ad valorem commission on in-app purchase revenue rather than a participation fee. We can nevertheless apply the analysis in Sections 3-6 once we rewrite the platform's developer-side revenue in the appropriate form.

Specifically, in the app-platform interpretation, the platform does not charge a developer participation fee directly. Instead, it collects a commission at rate τ on each developer's in-app purchase revenue generated by participating users. Ignoring platform investment for the moment, evaluated at the developer's monopoly in-app price ρ^m, revenue per user is $\theta_S(\rho^m)$ so a participating developer's gross in-app revenue equals $\theta_S(\rho^m)N_B$. The platform therefore earns commission revenue $\tau\,\theta_S(\rho^m)N_B$ per participating developer, and its payoff becomes:

$$\Pi^p(\cdot) = N_B\,(p_B - c_B) \;+\; N_S\big(\tau\,\theta_S(\rho^m)\,N_B - c_S\big) \;-\; s\,N_B N_S.$$

Thus, relative to the baseline model, the only change is that the developer-side payment p_S is replaced by the commission revenue, which is proportional to the number of participating users.

Following the steps in Section 4, we first study the platform's problem of choosing target participation levels (N_B, N_S) and then the prices (p_B, τ) that implement those targets. We then consider the socially optimal participation levels. Importantly, expressed in participation space, buyer and developer surpluses take the same form as in the main model; the only difference is that the developer-side instrument is implemented through a commission on in-app purchase revenue.

We study distortions in two steps, mirroring the main analysis: first in participation space, comparing the privately optimal and socially optimal pair of user and developer participation rates (N_B, N_S); and, subsequently, in price space, characterizing the device price and commission rate that implement those participation choices.

Proposition 4 *For any fixed investment profile, in the app platform model where the platform charges users a device price and developers a commission rate, compared to the social optimum:*

(i) Both sides of the market exhibit under-participation.

(ii) The commission rate is too high, while the device price may be either too low or too high.

Proof. See Appendix C. ■

Part (i) mirrors the baseline model: both sides of the market exhibit under-participation. Participation decisions are complementary, and the planner's objective is locally increasing in each participation level when evaluated at the private optimum. Hence the private equilibrium lies "south-west" of the planner's solution and features under-participation on both sides.

For part (ii), note first that under the stability conditions corresponding to the coordination problem between users and developers, an increase in either p_B or τ reduces participation on both sides. This, in turn, implies that both components of the welfare gradient in the price space, evaluated at the private optimum, are negative. Thus, as in the main model, a configuration in which both instruments are below their socially optimal levels is impossible, whereas a configuration in which one of the instruments is too low is, in principle, possible.

The app-platform-specific conclusion is that the commission rate must be excessive. To see why, note from (62) that the planner's commission satisfies

$$\tau^o = \frac{c_S}{\theta_S(\rho^m)\, N_B^o} - \frac{\theta_B(\rho^m) - s}{\theta_S(\rho^m)},$$

so τ^o falls with N_B^o: a larger user base allows the platform to cover developer-side marginal costs with a smaller per-transaction wedge. A similar dependence on N_B appears in the platform's commission choice in (61):

$$\tau^* = \frac{\dfrac{\partial \varphi_S(N_S^*)}{\partial N_S} N_S^* + c_S}{\theta_S(\rho^m) N_B^*} - \frac{\theta_B(\rho^m) - s}{\theta_S(\rho^m)}.$$

Since the private optimum features too few users, $N_B^* < N_B^o$, this mechanically pushes the privately chosen commission upward. Moreover, relative to the planner, the platform's commission contains an additional "market power" term proportional to $\frac{\partial \varphi_S(N_S^*) N_S^*}{\partial N_S}$, which reflects the platform's incentive to extract surplus on the developer side and has no analogue in the planner's commission formula. Together, these two effects imply $\tau^* > \tau^o$.

By contrast, the device price is not pinned down in sign because it is the platform's re-balancing instrument. Once participation is inefficiently low on both sides, the platform may use a low p_B to expand the user base and thereby raise commission revenue on the developer side. At the same time, the platform has a direct incentive to raise p_B to extract surplus from users. Depending on primitives, either force can dominate, so the device price may be either above or below its socially optimal level, as in the main model.

The framework presented in our paper allows us to extend existing app-platform analyses by incorporating platform investment decisions. We next endogenize investment and show that the qualitative conclusions from Propositions 2 and 3 carry over to the app-platform setting.

Proposition 5 *In the context of the app platform model:*

(i) If the app platform's investment ϵ enhances user and developer network benefits through the promotion of in-app purchase demand, the privately optimal investment level is inefficiently low.

(ii) If the app platform's investments e_B and e_S increase stand-alone benefits on both sides of the market, the privately optimal investment profile is such that investment is inefficiently low on at least one side.

Proof. See Appendix C ■

Part (i) concerns investments that promote in-app purchase demand. In our notation, such an investment ϵ shifts $D(\rho^m(\epsilon), \epsilon)$ and thereby simultaneously raises interaction benefits on both sides, which we abbreviate as $\theta_B(\epsilon)$ and $\theta_S(\epsilon)$. The argument is then the same as in Proposition 2. The marginal return to ϵ is proportional to interaction volume $N_B N_S$, and since the private optimum features under-participation on both sides, the platform underinvests in ϵ relative to the social planner.

Part (ii) follows directly from Proposition 3. The key observation is that stand-alone investments enter utilities only through the stand-alone values $v_B(e_B)$ and $v_S(e_S)$ and therefore affect payoffs through own-side participation, not through interaction volume. In particular, the marginal return to increasing e_i is scaled by the measure of agents on side i, namely N_i. As a result, it cannot be the case that the platform overinvests in stand-alone benefits on both sides. Equivalently, at least one of the stand-alone investment levels is inefficiently low relative to the social planner's choice. The distortion may be asymmetric, so the platform may overinvest on one side while underinvesting on the other.

7.2 Capping commission rates

Proposition 4 has shown that commission rates are always too high in our model. A similar result has been found in the monopoly platform model of Anderson and Bedre-Defolie (2025) and persists under oligopoly competition (see e.g. Teh and Wright, 2026).[18] We now use the app-platform model to study a common regulatory intervention in this context: a cap on the commission rate charged on in-app purchases. Such constraints are central to recent policy discussions and enforcement actions directed at app stores, and they directly relate to the changes in fee schedules discussed above (see European Commission, 2021).

Formally, suppose the platform faces the constraint

$$\tau \leq \bar{\tau}, \tag{19}$$

where $\bar{\tau} \in [0, \tau^*)$ is fixed by the regulator. We study the platform's optimal response to (19), with particular attention to the induced rebalancing of the overall pricing and investment structure.

[18] Teh and Wright (2026) study a competitive bottleneck model and show that the commission rate is always too high even from a buyer-side surplus objective. In their paper the distortion arises because in equilibrium each platform acts to maximize the surplus of its singlehoming buyers and its own profit, while ignoring seller surplus. Since competition on the sellers' side is absent under bottleneck competition, platforms can act as monopolists toward sellers, driving commission rates well above the social optimum.

7.2.1 Commission caps with fixed investment

We start with the case in which investment is fixed. To understand the impact of capping commissions, notice that, for any target participation levels (N_B, N_S), the prices that implement them can be obtained by inverting the system of demands:

$$p_B \;=\; v_B + \theta_B(\rho^m)N_S - \varphi_B(N_B), \qquad \tau \;=\; 1 - \frac{\varphi_S(N_S) - v_S}{\theta_S(\rho^m)\,N_B}. \tag{20}$$

This means that the cap (19) can be seen as a constraint on the participation profiles (N_B, N_S). That is, given $\bar{\tau}$, (N_B, N_S) must satisfy:

$$1 - \frac{\varphi_S(N_S) - v_S}{\theta_S(\rho^m)\,N_B} \;\le\; \bar{\tau} \quad\Longleftrightarrow\quad \theta_S(\rho^m)\,(1-\bar{\tau})\,N_B \;\le\; \varphi_S(N_S) - v_S. \tag{21}$$

Because the platform's objective is locally increasing in participation on both sides, the constraint (21) must be binding. As a result, the platform effectively chooses a single participation rate, say N_B, with developer participation determined by the function:

$$N_S(N_B;\bar{\tau}) \equiv G_S(v_S + \theta_S(\rho^m)\,(1-\bar{\tau})\,N_B).$$

Substituting this into the device-price implementing equation in (20) yields the device price as a function of N_B under the cap:

$$p_B(N_B) = v_B + \theta_B(\rho^m)\,G_S\Big(v_S + \theta_S(\rho^m)(1-\bar{\tau})N_B\Big) - \varphi_B(N_B). \tag{22}$$

Accordingly, the platform's regulated problem can be written as a one-dimensional maximization problem in N_B. That is, the regulated platform chooses N_B to maximize:

$$\begin{aligned}\hat{\Pi}^p(N_B;\bar{\tau}) &\equiv \Pi^p\big(N_B, N_S(N_B;\bar{\tau}), \bar{\tau}\big)\\ &= N_B(p_B - c_B) + N_S(N_B;\bar{\tau})\,(\bar{\tau}\theta_S(\rho^m)N_B - c_S) - sN_BN_S\\ &= N_B(v_B + \theta_B(\rho^m)N_S(N_B;\bar{\tau}) - \varphi_B(N_B) - c_B) + N_S(N_B;\bar{\tau})\,(\bar{\tau}\theta_S(\rho^m)N_B - c_S) - sN_BN_S.\end{aligned}$$

An interior optimum $\hat{N}_B$ must satisfy the first-order condition

$$\frac{\partial \hat{\Pi}^p(N_B;\bar{\tau})}{\partial N_B} = \frac{\partial \Pi^p}{\partial N_B} + \frac{\partial \Pi^p}{\partial N_S}\,\frac{\partial N_S(N_B;\bar{\tau})}{\partial N_B} = 0, \tag{23}$$

where

$$\frac{\partial N_S(N_B;\bar{\tau})}{\partial N_B} = g_S\Big(v_S + \theta_S(\rho^m)(1-\bar{\tau})\,N_B\Big)\,\theta_S(\rho^m)(1-\bar{\tau}) = \frac{\theta_S(\rho^m)(1-\bar{\tau})}{\frac{\partial \varphi_S\left(N_S(N_B;\bar{\tau})\right)}{\partial N_S}} \;>\; 0. \tag{24}$$

Equation (23) defines implicitly a function $\hat{N}_B(\bar{\tau})$. Differentiating (23) implicitly with respect to $\bar{\tau}$ yields

$$\frac{\partial \hat{N}_B}{\partial \bar{\tau}} = -\frac{\frac{\partial}{\partial \bar{\tau}}\left[\frac{\partial \hat{\Pi}^p(N_B;\bar{\tau})}{\partial N_B}\right]}{\frac{\partial}{\partial N_B}\left[\frac{\partial \hat{\Pi}^p(N_B;\bar{\tau})}{\partial N_B}\right]}. \tag{25}$$

Assuming that the regulated objective $\hat{\Pi}^p(N_B;\bar{\tau})$ is strictly concave in N_B, a sufficient condition for tightening the cap (lowering $\bar{\tau}$) to *increase* buyer participation is that it raises the platform's marginal payoff from expanding N_B:

$$\frac{\partial}{\partial\bar{\tau}}\left[\frac{\partial\hat{\Pi}^p(N_B;\bar{\tau})}{\partial N_B}\right] = \frac{\partial}{\partial\bar{\tau}}\left(\frac{\partial\Pi^p}{\partial N_B}\right) + \frac{\partial}{\partial\bar{\tau}}\left(\frac{\partial\Pi^p}{\partial N_S}\,\frac{\partial N_S(N_B;\bar{\tau})}{\partial N_B}\right) < 0. \tag{26}$$

The decomposition in (26) makes clear that a commission cap affects the incentive to expand the buyer side through two channels. The first term captures the *direct* effect on the marginal value of an additional buyer, holding developer participation fixed: a tighter cap reduces commission revenue per interaction and therefore tends to lower $\partial\Pi^p/\partial N_B$. The second term captures the *indirect* effect operating through the binding constraint: expanding N_B also changes the induced developer participation $N_S(N_B;\bar{\tau})$, which feeds back into the buyer-side network-benefit term $\theta_B(\rho^m)N_S$ and into the platform's developer-side margin.

To obtain sharp comparative statics, we next study the effects of a *locally binding* commission cap, i.e., a binding cap $\bar{\tau} < \tau^*$ that is sufficiently close to the unregulated optimal τ^*. The following proposition provides a sufficient condition under which capping commissions just below τ^* increases participation on both sides, and hence raises buyer and developer surplus.

Proposition 6 *For any privately optimal commission rate τ^*, there is a commission cap $\bar{\tau}$ sufficiently close to τ^* for which in the regulated market equilibrium user and developer participation rates increase, and hence user and developer surpluses too, provided that*

$$-N_S^* - c_S\, g_S(u_S^*) + \theta_S(\rho^m)\, N_B^*\,(1-\tau^*)\, g_S(u_S^*)\left(1 - \frac{N_S^*\, g_S'(u_S^*)}{g_S^2(u_S^*)}\right) < 0 \tag{27}$$

where $u_S^* = v_S + \theta_S(\rho^m)\,(1-\tau^*)\,N_B^*$.

When $G_S(\cdot)$ is uniform, this condition holds, so a locally binding commission cap increases user and developer surplus.

Proposition 6 provides a sufficient condition under which a locally binding commission cap increases both user and developer surplus. Although the cap raises developer participation, it does not necessarily increase buyer participation, because the platform may rebalance by raising the device price. This is the tension highlighted in the discussion of (26).

The condition in Proposition 6 admits a transparent interpretation. The first term, $-N_S^*$, is the direct effect of the locally binding commission cap on commission revenue: holding developer participation fixed, the cap reduces the commission revenue earned from in-app purchase activity. The second term, $-c_S\, g_S(u_S^*)$, captures the increase in the marginal cost of serving developers: when the cap induces additional developer entry, the platform incurs extra developer-side costs at rate c_S, and the magnitude of this effect is proportional to the density of marginal developers

$g_S(u_S^*)$. The last term is the participation-feedback effect. Under a locally binding commission cap, developer participation is pinned down by the constraint (21), summarized by the schedule $N_S(N_B; \bar{\tau})$. Hence, the response of N_S to changes in N_B is governed by the distribution of developers' outside options: the factor $g_S(u_S^*)$ measures the mass of developers at the margin (indifferent between joining and not), while $g'_S(u_S^*)$ determines how this marginal responsiveness changes as buyer participation changes.

For the sufficient condition in Proposition 6 to hold, it is useful to separate two cases depending on the sign of the curvature-adjusted density term. If $1 - \frac{N_S^* g'_S(u_S^*)}{g_S^2(u_S^*)} \leq 0$, then the participation-feedback term is non-positive, so all three components are negative and, under the concavity of $\hat{\Pi}^p$, a locally binding commission cap raises $\hat{N}_B$. If instead $1 - \frac{N_S^* g'_S(u_S^*)}{g_S^2(u_S^*)} > 0$, then the participation-feedback term is positive and partially offsets the direct effects. In this case the sign is in principle ambiguous, and a locally binding cap still increases $\hat{N}_B$ provided the feedback is not too strong, i.e., provided that the cap-induced change in the developer-side response at the margin (captured by $g_S(u_S^*)$ and $g'_S(u_S^*)$) does not overturn the direct reduction in marginal commission profitability.

Although Proposition 6 shows that a binding commission cap close to the unregulated optimum can improve both buyer and developer surplus, the uniform-outside-options benchmark permits a sharper and more global characterization. In particular, when outside options are uniformly distributed, we can sign the effect of any binding commission cap on developer surplus and total (user-plus-developer) surplus, and we obtain an explicit threshold that determines when a tighter cap raises or lowers user surplus. Proposition 7 states these results.

Proposition 7 *Let the outside options o_i be uniformly distributed over the interval $[0, 1]$, for $i \in \{B, S\}$.*

(i) Any binding commission cap $\overline{\tau} \in [0, \tau^*)$ increases the sellers' surplus.

(ii) Define the threshold $\tau_B \equiv \frac{\theta_S(\rho^m) - \theta_B(\rho^m) + s}{2\theta_S(\rho^m)}$.

a. If $\tau_B > 0$ (which holds true for any demand function for which revenue is greater than consumer surplus, e.g. linear demand), then a binding commission cap $\overline{\tau} \in (\tau_B, \tau^*)$ increases user surplus, while tighter caps $\overline{\tau} \in [0, \tau_B]$ (weakly) reduce it.

b. If $\tau_B < 0$, then any commission cap $\overline{\tau} \in [0, \tau^*)$ increases user surplus.

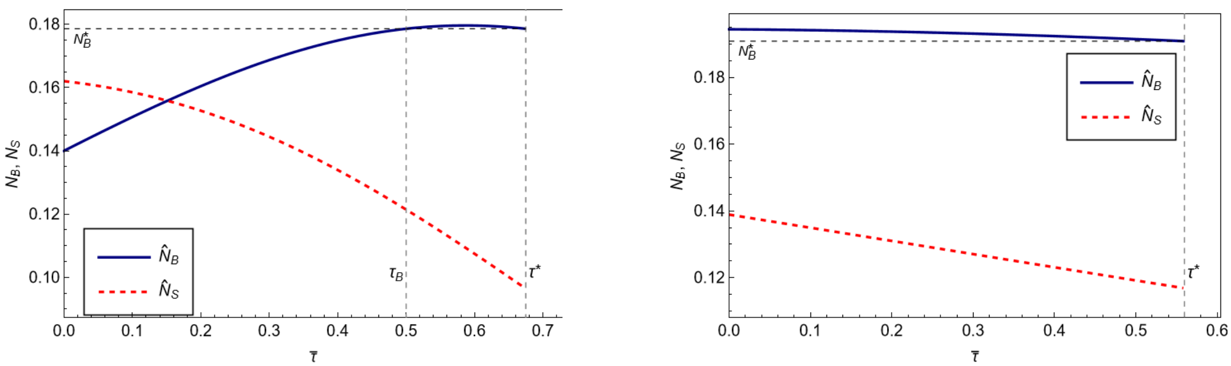
(iii) Any binding commission cap $\overline{\tau} \in [0, \tau^*)$ increases the sum of users' and sellers' surplus (despite perhaps reducing the surplus of the users).

Proof. See Appendix C. ■

We illustrate Proposition 7 in Figures 8a and 8b. Both panels plot $\hat{N}_B(\bar{\tau})$ (solid blue) and $\hat{N}_S(\bar{\tau})$ (dashed red) against the commission cap $\bar{\tau} \in [0, \tau^*]$. The horizontal dashed line indicates the unregulated buyer participation level N_B^*, and the vertical dashed lines mark τ_B and τ^*.

Figure 8a corresponds to a parameter configuration with $\tau_B > 0$, illustrating the inverted-U shape for user participation described in part (ii)(a): as the cap tightens from τ^* downward, user participation first rises, reaching a peak somewhere in between τ_B and τ^*, and then falls. Developer participation increases monotonically throughout, consistent with part (i). The figure thus shows that commission regulation entails a trade-off: caps in the intermediate range (τ_B, τ^*) benefit both sides, while caps tighter than τ_B improve developer surplus at the expense of users.[19]

Figure 8b illustrates the case $\tau_B < 0$, corresponding to part (ii)(b): buyer interaction benefits exceed seller interaction benefits net of transaction costs, so any binding cap raises user participation. Accordingly, $\hat{N}_B$ increases monotonically as the cap tightens from τ^* toward zero, while $\hat{N}_S$ remains above N_S^* throughout. In this configuration, there is no trade-off: regulation is unambiguously beneficial for both sides.[20]



(a) Non-monotonic effect on buyer participation

(b) Cap increases buyer and seller participation

Figure 8: Commission caps and buyer and seller participation

The two figures illustrate that caps are unambiguously beneficial for developers, while their effect on users depends on the balance of network effects: large seller interaction benefits ($\theta_S(\rho^m) \gg \theta_B(\rho^m)$, Figure 8a) create a trade-off in which very tight caps harm users, whereas large buyer interaction benefits ($\theta_B(\rho^m) > \theta_S(\rho^m) + s$, Figure 8b) make regulation beneficial for both buyers and sellers.

7.2.2 Commission caps with endogenous investment

In the previous subsection we characterized how a binding commission cap affects the platform's pricing and the equilibrium participation mix when investment is held fixed. We now extend the analysis to the case in which the platform can also invest in promoting in-app purchase demand. Developers continue to set the monopoly in-app price $\rho^m(\epsilon)$, which is independent of the

[19]Figure 8a is constructed with uniformly distributed outside options $o_i \sim U[0,1]$, where the parameters are $s = 0.1$, $\theta_B = 0.1$, $\theta_S = 0.8$, $c_B = 0.2$, $c_S = 0$, $v_B = 0.48$, $v_S = 0.05$. This setting gives us the unregulated optimum $(N_B^*, N_S^*, \tau^*) = (0.179, 0.096, 0.675)$ and $\tau_B = 0.5$.

[20]Figure 8b is constructed in a way similar to Figure 8a, except for the parameters: $s = 0.3$, $\theta_B = 0.8$, $\theta_S = 0.2$, $c_B = 0.2$, $c_S = 0$, $v_B = 0.5$, $v_S = 0.1$, $\kappa = 1$. These parameters give us the unregulated optimum $(N_B^*, \tau^*, N_S^*) = (0.191, 0.560, 0.117)$ and $\tau_B = -0.75$.

commission rate and the cap, but now depends on investment. Investment therefore affects the platform only by shifting in-app purchase demand and, through it, the interaction benefits on both sides captured, for brevity, by $\theta_B(\epsilon)$ and $\theta_S(\epsilon)$, as defined above.

Under a commission cap $\bar{\tau} < \tau^*$, the platform now chooses buyer participation N_B and investment ϵ, while developer participation is pinned down by the binding cap locus $N_S(N_B, \epsilon; \bar{\tau})$. The platform's regulated profit maximization problem is then:

$$\max_{N_B,\epsilon} \left\{ \hat{\Pi}^p(N_B, \epsilon) \equiv \Pi^p(N_B, N_S(N_B, \epsilon; \bar{\tau}), \epsilon; \bar{\tau}) \right\} \tag{28}$$

Assuming the SOCs hold, the FOCs with respect to N_B and ϵ jointly characterize the regulated equilibrium $(\hat{N}_B(\bar{\tau}), \hat{\epsilon}(\bar{\tau}))$. To trace how a stricter cap affects the equilibrium, we totally differentiate the system of FOCs with respect to $\bar{\tau}$ and apply Cramer's rule, which yields:

$$\frac{d\hat{N}_B}{d\bar{\tau}} = -\frac{1}{\det(\mathcal{H})} \left(\frac{\partial^2 \Pi^p}{\partial N_B \partial \bar{\tau}} \frac{\partial^2 \Pi^p}{\partial \epsilon^2} - \frac{\partial^2 \Pi^p}{\partial \epsilon \partial \bar{\tau}} \frac{\partial^2 \Pi^p}{\partial N_B \partial \epsilon} \right), \tag{29}$$

$$\frac{d\hat{\epsilon}}{d\bar{\tau}} = -\frac{1}{\det(\mathcal{H})} \left(\frac{\partial^2 \Pi^p}{\partial \epsilon \partial \bar{\tau}} \frac{\partial^2 \Pi^p}{\partial N_B^2} - \frac{\partial^2 \Pi^p}{\partial N_B \partial \bar{\tau}} \frac{\partial^2 \Pi^p}{\partial \epsilon \partial N_B} \right), \tag{30}$$

where $\det(\mathcal{H}) \equiv \frac{\partial^2 \Pi^p}{\partial N_B^2} \frac{\partial^2 \Pi^p}{\partial \epsilon^2} - \left(\frac{\partial^2 \Pi^p}{\partial N_B \partial \epsilon} \right)^2$ denotes the determinant of the Hessian matrix of the regulated optimization problem in (28). Under the SOCs for a maximum, $\det(\mathcal{H}) > 0$.

Equations (29)–(30) show that a commission cap affects outcomes through two margins: it changes (i) the platform's marginal incentive to expand user participation and (ii) its marginal incentive to invest in promoting in-app purchase demand. As a result, the responses of $\hat{N}_B$ and $\hat{\epsilon}$ to a tighter cap need not have the same sign.

To obtain sharper comparative statics, we follow the same local approach as in Proposition 6 and study the impact of imposing a locally binding cap, i.e., a cap $\bar{\tau}$ set marginally below the privately optimal rate τ^*. Evaluating (29)–(30) at the unregulated optimum yields that imposing a locally binding cap raises user participation if and only if

$$\left. \left(\frac{\partial^2 \hat{\Pi}^p}{\partial N_B \, \partial \bar{\tau}} \frac{\partial^2 \hat{\Pi}^p}{\partial \epsilon^2} - \frac{\partial^2 \hat{\Pi}^p}{\partial \epsilon \, \partial \bar{\tau}} \frac{\partial^2 \hat{\Pi}^p}{\partial N_B \, \partial \epsilon} \right) \right|_{\bar{\tau}=\tau^*} > 0, \tag{31}$$

and it raises investment if and only if

$$\left. \left(\frac{\partial^2 \hat{\Pi}^p}{\partial \epsilon \, \partial \bar{\tau}} \frac{\partial^2 \hat{\Pi}^p}{\partial N_B^2} - \frac{\partial^2 \hat{\Pi}^p}{\partial N_B \, \partial \bar{\tau}} \frac{\partial^2 \hat{\Pi}^p}{\partial \epsilon \, \partial N_B} \right) \right|_{\bar{\tau}=\tau^*} > 0. \tag{32}$$

Taken together, (31)–(32) imply that a locally binding cap may (i) raise both N_B and ϵ, (ii) lower both, or (iii) generate a trade-off whereby user participation increases while investment falls, or vice versa.

Conditions (31)–(32) are general and difficult to sign because the associated cross-effects need not have the same sign. Accordingly, we proceed in two steps. First, we put forward a robustness check showing that the participation results from Section 7.2.1 carry over when the investment margin is only marginally active. We then impose additional structure (uniform outside options) to obtain sharper conclusions on the investment response.

Remark 1 *Suppose the investment cost is sufficiently convex. Then, locally around the unregulated optimum, the regulated problem is well approximated by the fixed-investment case analyzed in Section 7.2.1. In particular, the effect of imposing a locally binding cap $\bar{\tau} < \tau^*$ on user participation is governed by the condition in Proposition 6 evaluated at the non-regulated optimal investment.*

The remark above shows that, when the investment margin is only marginally active because the cost function is very convex, endogenizing ϵ does not alter the insight derived under fixed investment. We now turn to the question whether a locally binding cap strengthens or weakens the platform's incentive to invest. In general, the sign of the investment response cannot be pinned down without additional structure, because the cap affects both the direct return to investment (through interaction benefits) and the indirect return (through the induced participation adjustments along the binding-cap locus). Accordingly, we specialize to the uniform benchmark, which yields transparent sufficient conditions.

Proposition 8 *Assume that, on both sides of the market, outside options are uniformly distributed over the interval [0,1]. Fix a privately optimal commission rate τ^*, and consider a locally binding cap $\bar{\tau} < \tau^*$ sufficiently close to the privately optimal commission rate τ^*. Then the platform's equilibrium investment level increases (i.e., $d\hat{\epsilon}/d\bar{\tau} < 0$) under either of the following conditions:*

(i) $\tau^* > \frac{1}{2}$*; or*

(ii) $\tau^* < \frac{1}{2}$, $v_S \to 0$, $c_S \to 0$, *and* $\frac{\theta_B'(\epsilon^*)}{\theta_S'(\epsilon^*)} > 1 - 2\tau^* > 0$.

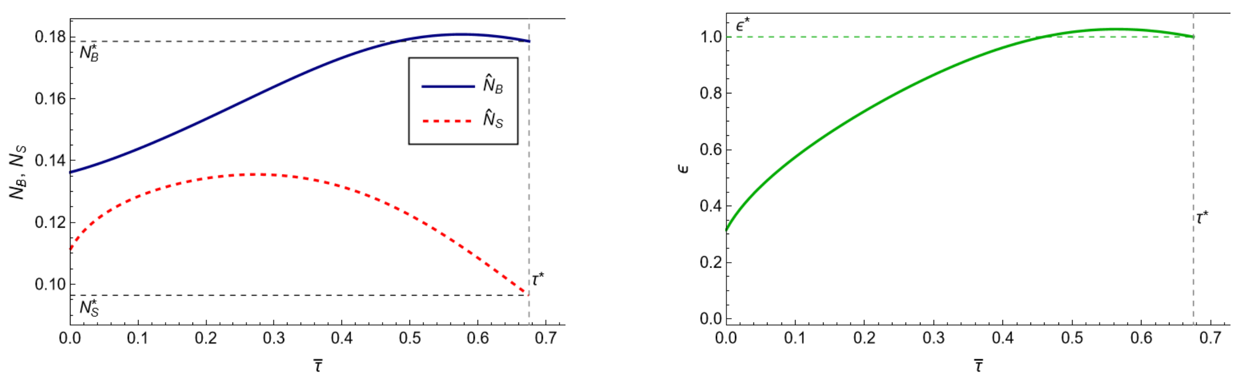
Proof. See Appendix C. ■

Proposition 8 isolates when a commission cap strengthens the platform's incentive to invest in demand-enhancing features. The key mechanism is that, once the cap reduces the platform's ability to extract developer-side revenue per interaction, the platform may find it profitable to restore the profitability of attracting users by increasing ϵ, which raises interaction benefits on both sides. Under uniform outside options, this effect can be signed locally: when $\tau^* > \frac{1}{2}$ the cap increases investment, whereas for $\tau^* < \frac{1}{2}$ the investment response depends on the relative sensitivity of user- and developer-side interaction benefits to ϵ (captured by $\theta_B'(\epsilon^*)/\theta_S'(\epsilon^*)$) and on the importance of the developer-side participation margin (captured by the limiting case $v_S, c_S \to 0$).

Finally, combining Proposition 6 with Proposition 8 yields a simple sufficient condition for a locally binding cap to increase both participation and investment when outside options are uniformly distributed: it suffices that the investment cost function is sufficiently convex, and that either (i) or (ii) in Proposition 8 holds.

We illustrate Proposition 8 using the same two parameter configurations as in Section 7.2.1, so that the fixed-investment and endogenous-investment cases are comparable. Specifically, for each configuration we keep all parameters identical to those in Figure 8 and calibrate the investment cost function so that the unregulated equilibrium delivers $\hat{\epsilon} \approx 1$, which is the assumed investment level.[21] This calibration ensures that any difference between the graphs illustrating Propositions 7 and 8 is attributable solely to the platform's ability to adjust investment in response to the cap.

Figure 9 displays the impact of endogenizing investment for the parameters underlying Figure 8a. The right panel shows the equilibrium investment $\hat{\epsilon}(\bar{\tau})$, with the horizontal dashed line indicating the unregulated level $\epsilon^* = 1$. For caps close to τ^*, the platform raises investment above ϵ^*, but as the cap tightens further, investment eventually falls below the unregulated level. Near the unregulated equilibrium, the participation panel closely mirrors Figure 8a: both $\hat{N}_B$ and $\hat{N}_S$ rise as the cap tightens. What is striking, however, is that $\hat{N}_S$ eventually turns down under sufficiently stringent caps, driven by the severe decline in investment. As $\hat{\epsilon}$ falls, interaction benefits erode, and the platform can no longer sustain developer participation despite the lower commission rate.



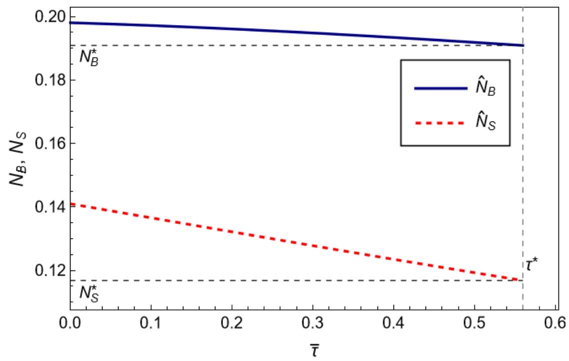
(a) Non-monotonic effect on N_B and N_S (b) Non-monotonic effect on investment

Figure 9: Commission caps with endogenous investment (I).

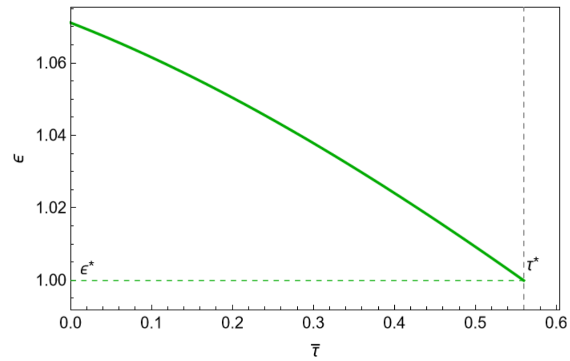
Figure 10 presents the results for the parameters underlying Figure 8b. In this case, any binding cap raises investment above ϵ^*. This response reinforces the participation effects discussed under fixed investment: the rise in $\hat{N}_B$ is amplified, as higher investment increases interaction benefits on both sides and makes the platform more attractive to users and developers.

Taken together, the two figures illustrate that endogenizing investment may, but need not, overturn the qualitative conclusions of Proposition 7. Near the unregulated equilibrium, the direction of the participation effects is preserved. For more stringent caps, however, the investment

[21]We use a cubic cost function $C(\epsilon) = \frac{\kappa}{3}\epsilon^3$. For the parameter configuration underlying Figure 8a, we choose $\kappa = 0.00775$, yielding $\epsilon^* = 1$, $N_B^* = 0.179$, $\tau^* = 0.675$. For the configuration underlying Figure 8b, $\kappa = 0.01115$, which gives $\epsilon^* = 1$, $N_B^* = 0.191$, $\tau^* = 0.560$.



(a) Cap increases N_B and N_S



(b) Cap increases investment

Figure 10: Commission caps with endogenous investment (II).

channel introduces an additional force that can reduce user and developer participation beyond what the fixed-investment analysis would predict.

8 Concluding remarks

In this paper, we examine how a monopoly two-sided platform jointly determines participation pricing and investment. Building on the seminal contribution of Armstrong (2006), we study an environment in which agents on both sides are heterogeneous in outside options and derive both stand-alone benefits and interaction (network) benefits.

Our analysis advances the price theory of Weyl (2010) and Tan and Wright (2021) in two respects. First, we provide a more complete characterization of the inefficiency of monopoly pricing. While Tan and Wright (2018) suggest that participation prices may be excessively high, excessively low, or fully efficient within the framework of Armstrong (2006), we show that at least one side's participation price is excessively high, and that excessively high prices on both sides cannot occur. Second, we show that participation distortions need not straightforwardly follow from price distortions: even when participation prices are distorted asymmetrically—too high on one side and too low on the other—both sides remain inefficiently under-served.

Methodologically, we obtain these results by formulating the platform's problem directly in terms of participation levels and exploiting the associated welfare and profit gradients to characterize distortions. Under our maintained assumptions, participation decisions are complementary across sides: higher participation on either side raises transaction volume and increases both the platform's and the planner's objective. This complementarity yields a sharp participation prediction: relative to the social optimum, the monopoly outcome features under-participation on both sides. For the case of uniformly distributed outside options, we further characterize when asymmetric price distortions arise: they occur when cross-side spillovers are sufficiently weak on one side and net stand-alone values are sufficiently asymmetric across sides.

While our main analysis focuses on participation (subscription) prices, Appendix F considers per-transaction fees and clarifies how the two instruments relate. These results also help reconcile our findings with the per-transaction-fee comparison emphasized by Jullien et al. (2021). Because per-transaction fees $f_i \equiv p_i/N_i$ bundle participation and price distortions, under-participation mechanically raises p_i/N_i and, in our model, dominates any downward distortion in p_i. As a result, per-transaction fees are always (weakly) excessive, even when participation prices display asymmetric distortions.

From a policy perspective, the analysis cautions against inferring efficiency from participation prices alone. Distortions in participation can be masked by price distortions: even when both sides experience under-participation relative to the social optimum, participation prices may be too high on both sides or too low on one side and too high on the other. A below-optimal participation price on one side may therefore give the impression that only the opposite side is harmed, while in fact both sides suffer from under-participation. In that sense, per-transaction fees may provide a more reliable summary measure of market power in our environment, since they incorporate participation distortions and remain (weakly) excessive.

We then endogenize investment and consider two types of platform effort: investment that enhances network benefits and investment that enhances stand-alone benefits. When the platform invests to enhance network benefits, the marginal return to investment on either side is proportional to the interaction volume $N_B N_S$. Since the planner induces higher participation and therefore more interactions, it has stronger investment incentives, implying that the monopoly platform underinvests on both sides. In contrast, when investment enhances stand-alone values, marginal returns scale with own-side participation, which gives the platform more scope to tilt investment toward one side. Investment distortions can therefore be asymmetric, although overinvestment on both sides is ruled out.

We also apply the framework to an app-platform environment in which users access the ecosystem through the device price while developers are monetized via an ad valorem commission on in-app purchases. In this setting, the model delivers a sharp prediction: for fixed investment, the commission rate is always excessive even though the device price may lie above or below its social benchmark. This provides a natural basis for policy attention. We therefore studied a common intervention in these markets, namely, a cap on the commission rate, and characterized how the platform rebalances its pricing and investment structure in response. The analysis identifies conditions under which a locally binding commission cap expands user and developer participation and surplus, as well as conditions under which it backfires for example because it reduces investment and user surplus as a consequence.

Appendices

A Stability and uniqueness of a user equilibrium

Lemma 1 *For any fixed p_B and p_S, if the network effects satisfy*

$$\max_{u_i \in \mathbb{R}} \theta_i(e_i) g_i(u_i) < 1, \text{for all } \ i \in \{B, S\},$$

there exists a unique user equilibrium $(N_B^*, N_S^*) \in [0,1]^2$, and the equilibrium is stable.

Proof. Stability. Given a pair of prices (p_B, p_S), the users' demands are given by the solution to the system of equations (2)-(3), or equivalently:

$$D_B(N_B, N_S) \equiv G_B(v_B + \theta_B(e_B)N_S - p_B) - N_B = 0, \tag{33}$$

$$D_S(N_B, N_S) \equiv G_S(v_S + \theta_S(e_S)N_B - p_S) - N_S = 0. \tag{34}$$

Let (N_B^*, N_S^*) be a user equilibrium. Following Vives (1999), the user equilibrium is locally stable if near this point, the slope of the function $N_S(N_B)$ implicitly given by (33) exceeds that of the function given by (34) in the (N_B, N_S)-space.

Implicit differentiation gives:

$$\left.\frac{\partial N_S}{\partial N_B}\right|_{N_B^*} = -\left.\frac{\partial D_B/\partial N_B}{\partial D_B/\partial N_S}\right|_{N_B^*} = \frac{1}{g_B(u_B^*)\theta_B}, \quad \left.\frac{\partial N_S}{\partial N_B}\right|_{N_S^*} -\left.\frac{\partial D_S/\partial N_B}{\partial D_S/\partial N_S}\right|_{N_B^*} = g_S(u_S^*)\theta_S.$$

Hence, the local stability condition is:

$$\frac{1}{g_B(u_B^*)\theta_B} > g_S(u_S^*)\theta_S \quad \Leftrightarrow \quad \theta_B\theta_S g_B(u_B^*) g_S(u_S^*) < 1,$$

which is satisfied given the condition in the lemma.

Uniqueness. To prove uniqueness, we invoke the Contraction Mapping Theorem, which states that a contraction mapping on a complete metric space has a unique fixed point. First, note that (N_B, N_S) lies in $[0,1]^2$, which is a closed subset of the complete metric space $\mathbb{R}^2$ and is its therefore complete. Hence, the Contraction Mapping Theorem may be applied.

Following Vives (1999) and Chica et al. (2025), a sufficient condition for the system of equations (33)-(34) to be a contraction is:

$$\frac{\partial D_i}{\partial N_i} + \frac{\partial D_i}{\partial N_j} < 0, \qquad i \in \{B, S\}, \ i \neq j.$$

Taking the derivatives in (33)-(34), this condition amounts to:

$$\theta_i(e_i) g_i(u_i) < 1, \qquad i \in \{B, S\}.$$

This must hold for all u_i, $i \in \{B, S\}$, which is guaranteed under the condition in the lemma. ■

B Second-order conditions

Verification of the SOCs requires analyzing the matrix of the second-order derivatives for both the monopoly's and the planner's optimization problems.

The Hessian matrix corresponding to the platform's profit maximization problem is given by:

$$\mathcal{H}\Big(\Pi^p(N_B, N_S)\Big) = \begin{bmatrix} -2\varphi_B'(N_B) - \varphi_B''(N_B)N_B & \gamma - s \\ \gamma - s & -2\varphi_S'(N_S) - \varphi_S''(N_S)N_S \end{bmatrix},$$

where, recall, $\gamma = \theta_B + \theta_S$.

To ensure a global maximum, the Hessian must be negative definite. Define the leading principal minors as:

$$D_1 = -\,2\varphi_B'(N_B) - \varphi_B''(N_B)N_B,$$
$$D_2 = \left(-2\varphi_B'(N_B) - \varphi_B''(N_B)N_B\right)\left(-2\varphi_S'(N_S) - \varphi_S''(N_S)N_S\right) - (\gamma - s)^2.$$

Negative definiteness requires:

$$\frac{\varphi_B''(N_B)N_B}{\varphi_B'(N_B)} > -2,$$
$$(\gamma - s)^2 < \left(-2\varphi_B'(N_B) - \varphi_B''(N_B)N_B\right)\left(-2\varphi_S'(N_S) - \varphi_S''(N_S)N_S\right).$$

Similarly, the Hessian matrix for the planner's optimization is:

$$\mathcal{H}\Big(SW(N_B, N_S)\Big) = \begin{bmatrix} -\varphi_B'(N_B) & \gamma - s \\ \gamma - s & -\varphi_S'(N_S) \end{bmatrix}.$$

Since $-\varphi_i'(N_i) < 0$ always holds, the planner's objective is strictly concave provided $(\gamma - s)^2 < \varphi_B'(N_B)\varphi_S'(N_S)$.

C Proofs

Proof of Proposition 1.

Part (i). Participation (in-)efficiency. Recall the FOCs corresponding to the monopoly problem:

$$\frac{\partial \Pi^p(N_B, N_S)}{\partial N_B} = (\gamma - s)\, N_S + v_B - c_B - \varphi_B(N_B) - \frac{\partial \varphi_B(N_B)}{\partial N_B} N_B = 0, \tag{35}$$

$$\frac{\partial \Pi^p(N_B, N_S)}{\partial N_S} = (\gamma - s)\, N_B + v_S - c_S - \varphi_S(N_S) - \frac{\partial \varphi_S(N_S)}{\partial N_S} N_S = 0. \tag{36}$$

Likewise, the FOCs of the planner's problem are:

$$\frac{\partial SW(N_B, N_S)}{\partial N_B} = (\gamma - s)\, N_S + v_B - c_B - \varphi_B(N_B) = 0, \tag{37}$$

$$\frac{\partial SW(N_B, N_S)}{\partial N_S} = (\gamma - s)\, N_B + v_S - c_S - \varphi_S(N_S) = 0. \tag{38}$$

Equation (37) implicitly defines a function $N_B^o(N_S)$ whose slope is given by:

$$\frac{\partial N_S}{\partial N_B} = -\frac{-\dfrac{\partial \varphi_B(N_B)}{\partial N_B}}{\gamma - s}. \tag{39}$$

Similarly, equation (38) implicitly defines a function $N_S^o(N_B)$ whose slope is given by:

$$\frac{\partial N_S}{\partial N_B} = -\frac{\gamma - s}{-\dfrac{\partial \varphi_S(N_S)}{\partial N_S}} . \tag{40}$$

Since $\gamma > s$ and $\frac{\partial \varphi_i(\cdot)}{\partial N_i} > 0$, we conclude that the loci $N_B^o(N_S)$ and $N_S^o(N_B)$ are increasing in the (N_B, N_S)-space, see Figure 1.

Next, we show that both components of the gradient vector $\left(\frac{\partial SW(N_B,N_S)}{\partial N_B}, \frac{\partial SW(N_B,N_S)}{\partial N_S}\right)$ evaluated at (N_B^*, N_S^*) are positive for all parameter values. Rearranging (37) and evaluating at (N_B^*, N_S^*) we get

$$\frac{\partial SW(N_B,N_S)}{\partial N_B}\Big|_{N_B^*,N_S^*} = (\gamma - s)\, N_S^* + v_B - c_B - \varphi_B(N_B^*) = \frac{\partial \Pi^p(N_B,N_S)}{\partial N_B}\Big|_{N_B^*,N_S^*} + \frac{\partial \varphi_B(N_B^*)}{\partial N_B} N_B^*.$$

Similarly, rearranging (38) and evaluating at (N_B^*, N_S^*) we obtain

$$\frac{\partial SW(N_B,N_S)}{\partial N_S}\Big|_{N_B^*,N_S^*} = (\gamma - s)\, N_B^* + v_S - c_S - \varphi_S(N_S^*) = \frac{\partial \Pi^p(N_B,N_S)}{\partial N_S}\Big|_{N_B^*,N_S^*} + \frac{\partial \varphi_S(N_S^*)}{\partial N_S} N_S^*.$$

Note that $\frac{\partial \varphi_i(\cdot)}{\partial N_i} > 0$ and $\frac{\partial \Pi^p(N_B,N_S)}{\partial N_B}\Big|_{N_B^*,N_S^*} = \frac{\partial \Pi^p(N_B,N_S)}{\partial N_S}\Big|_{N_B^*,N_S^*} = 0$, imply that $\frac{\partial SW(N_B,N_S)}{\partial N_B}\Big|_{N_B^*,N_S^*} > 0$ and $\frac{\partial SW(N_B,N_S)}{\partial N_S}\Big|_{N_B^*,N_S^*} > 0$. The fact that the two components of the planner's gradient vector evaluated at the monopoly equilibrium are positive implies that the latter must lie on Region I of Figure 1. Hence, the private optimum exhibits under-participation on both sides relative to the social optimum.

Part (ii). Pricing (in-)efficiency. We first establish that, under the condition in Lemma 1, which ensures the stability and uniqueness of a user equilibrium, the system of demands $N_i(p_i, p_j)$, $i \in \{B, S\}$ that solves (33)-(34) is such that $\frac{\partial N_i}{\partial p_i} < 0$ and $\frac{\partial N_j}{\partial p_i} < 0$. In fact, by totally differentiating the system (33)-(34), we can readily obtain the derivatives:

$$\frac{\partial N_i}{\partial p_i} = \frac{g_i(u_i)}{g_i(u_i)g_j(u_j)\theta_i\theta_j - 1}, \quad \frac{\partial N_j}{\partial p_i} = \frac{g_j(u_j)g_i(u_i)\theta_j}{g_i(u_i)g_j(u_j)\theta_i\theta_j - 1}, \quad i, j \in \{B, S\}. \tag{41}$$

Because the stability condition requires $\theta_i\theta_j g_i(u_i)g_j(u_j) < 1$, we have $\frac{\partial N_i}{\partial p_i} < 0$ and $\frac{\partial N_j}{\partial p_i} < 0$.

Next, let (p_B^*, p_S^*) be the global maximizer of the monopoly's payoff $\Pi^p(N_B(p_B, p_S), N_S(p_B, p_S))$. Assuming the SOCs hold and that the maximum is interior, (p_B^*, p_S^*) must satisfy the FOCs:

$$\begin{aligned}
\frac{\partial \Pi^p(p_B^*, p_S^*)}{\partial p_B} &= \frac{\partial \Pi^p(N_B^*, N_S^*)}{\partial N_B}\frac{\partial N_B(p_B^*, p_S^*)}{\partial p_B} + \frac{\partial \Pi^p(N_B^*, N_S^*)}{\partial N_S}\frac{\partial N_S(p_B^*, p_S^*)}{\partial p_B} = 0, \\
\frac{\partial \Pi^p(p_B^*, p_S^*)}{\partial p_S} &= \frac{\partial \Pi^p(N_B^*, N_S^*)}{\partial N_B}\frac{\partial N_B(p_B^*, p_S^*)}{\partial p_S} + \frac{\partial \Pi^p(N_B^*, N_S^*)}{\partial N_S}\frac{\partial N_S(p_B^*, p_S^*)}{\partial p_S} = 0.
\end{aligned} \tag{42}$$

Consider now the welfare maximization problem:

$$\max_{p_B, p_S} SW(p_B, p_S) = \max_{p_B, p_S} \left\{ \Pi^p(p_B, p_S) + V_B(p_B, p_S) + V_S(p_B, p_S) \right\}. \tag{43}$$

The FOCs are:

$$\begin{aligned} \frac{\partial SW(p_B, p_S)}{\partial p_B} &= \left(\frac{\partial \Pi^p(N_B, N_S)}{\partial N_B} \frac{\partial N_B(p_B, p_S)}{\partial p_B} + \frac{\partial \Pi^p(N_B, N_S)}{\partial N_S} \frac{\partial N_S(p_B, p_S)}{\partial p_B} \right) \\ &+ \left(\frac{\partial V_B(N_B)}{\partial N_B} \frac{\partial N_B(p_B, p_S)}{\partial p_B} + \frac{\partial V_S(N_S)}{\partial N_S} \frac{\partial N_S(p_B, p_S)}{\partial p_B} \right) = 0, \end{aligned} \tag{44}$$

(45)

$$\begin{aligned} \frac{\partial SW(p_B, p_S)}{\partial p_S} &= \left(\frac{\partial \Pi^p(N_B, N_S)}{\partial N_B} \frac{\partial N_B(p_B, p_S)}{\partial p_S} + \frac{\partial \Pi^p(N_B, N_S)}{\partial N_S} \frac{\partial N_S(p_B, p_S)}{\partial p_S} \right) \\ &+ \left(\frac{\partial V_B(N_B)}{\partial N_B} \frac{\partial N_B(p_B, p_S)}{\partial p_S} + \frac{\partial V_S(N_S)}{\partial N_S} \frac{\partial N_S(p_B, p_S)}{\partial p_S} \right) = 0, \end{aligned} \tag{46}$$

where, to shorten the expressions, we have not explicitly written the dependency of $N_i(\cdot)$, $i \in \{B, S\}$, on prices everywhere.

Evaluating the gradient vector $\left(\frac{\partial SW(p_B, p_S)}{\partial p_B}, \frac{\partial SW(p_B, p_S)}{\partial p_S} \right)$ at the private optimum (p_B^*, p_S^*) gives:

$$\left. \frac{\partial SW(p_B, p_S)}{\partial p_B} \right|_{p_B^*, p_S^*} = \left. \left(\frac{\partial V_B(N_B)}{\partial N_B} \frac{\partial N_B(p_B, p_S)}{\partial p_B} + \frac{\partial V_S(N_S)}{\partial N_S} \frac{\partial N_S(p_B, p_S)}{\partial p_B} \right) \right|_{p_B^*, p_S^*},$$

$$\left. \frac{\partial SW(p_B, p_S)}{\partial p_S} \right|_{p_B^*, p_S^*} = \left. \left(\frac{\partial V_B(N_B)}{\partial N_B} \frac{\partial N_B(p_B, p_S)}{\partial p_S} + \frac{\partial V_S(N_S)}{\partial N_S} \frac{\partial N_S(p_B, p_S)}{\partial p_S} \right) \right|_{p_B^*, p_S^*}.$$

Recall that $\left. \frac{\partial V_i}{\partial N_i} \right|_{N_B^*, N_S^*} = \frac{\partial \varphi_i(N_i^*)}{\partial N_i} N_i^* > 0$ while $\frac{\partial N_i}{\partial p_i} < 0$ and $\frac{\partial N_j}{\partial p_i} < 0$. As a result, we get $\left. \frac{\partial SW(p_B, p_S)}{\partial p_B} \right|_{p_B^*, p_S^*} < 0$ and $\left. \frac{\partial SW(p_B, p_S)}{\partial p_S} \right|_{p_B^*, p_S^*} < 0$.

The fact that both components of the gradient vector of the planner's objective evaluated at the monopolist's optimal prices are negative implies that a configuration in which both sides are simultaneously priced strictly below their socially optimal levels is impossible. Moreover, we can conclude that only when the loci $p_B^o(p_S)$ and $p_S^o(p_B)$ are both upward sloping can we be sure that monopoly prices are excessive on both sides. Otherwise, prices may still be too high on both sides, or too high on one side and too low on the other.

Part (iii). The result follows from taking the difference between equations (9)–(10) and equations (13)–(14):

$$p_i^* - p_i^o = (\theta_j - s)(N_j^o - N_j^*) + \frac{\partial \varphi_i(N_i^*)}{\partial N_i} N_i^*, \; i, j = \{B, S\}, \; i \neq j.$$

Because $N_i^o > N_i^*$ (part (i) of Proposition 1), it is clear that $\theta_j > s$ implies $p_i^* > p_i^o$, while for $p_i^* < p_i^o$, it is necessary that $\theta_j < s$. ■

Proof of Lemma 2. Recall the FOCs for investments in (15) are given by

$$N_B^* N_S^* \frac{\partial \theta_B(e_B^*)}{\partial e_B} = N_B^* N_S^* \frac{\partial \theta_S(e_S^*)}{\partial e_S} = C'(e_B^* + e_S^*). \tag{47}$$

The FOCs for investments in (16) are given by

$$N_B^o N_S^o \frac{\partial \theta_B(e_B^o)}{\partial e_B} = N_B^o N_S^o \frac{\partial \theta_S(e_S^o)}{\partial e_S} = C'(e_B^o + e_S^o). \tag{48}$$

Note that (47) and (48) imply

$$\frac{\partial \theta_B(e_B^*)}{\partial e_B} - \frac{\partial \theta_S(e_S^*)}{\partial e_S} = \frac{\partial \theta_B(e_B^o)}{\partial e_B} - \frac{\partial \theta_S(e_S^o)}{\partial e_S} = 0.$$

Define the monotone functions $h_1(e_B) = \frac{\partial \theta_B(\cdot)}{\partial e_B}$ and $h_2(e_S) = \frac{\partial \theta_S(\cdot)}{\partial e_S}$. Note that (e_B^*, e_S^*) and (e_B^o, e_S^o) both satisfy $h_1(e_B) - h_2(e_S) = 0$. As a result, both (e_B^*, e_S^*) and (e_B^o, e_S^o) are on the same locus $L = \{(e_B, e_S) : h_1(e_B) - h_2(e_S) = 0\}$. Define this locus as the function $e_S = y(e_B)$ where $y(\cdot) \equiv h_2^{-1}(h_1(\cdot))$, as per the lemma.

Moreover, the slope of this locus is

$$y'(\cdot) = \frac{\partial e_S}{\partial e_B} = -\frac{h_1'(e_B)}{-h_2'(e_S)} = -\frac{\frac{\partial^2 \theta_B(e_B)}{\partial e_B^2}}{-\frac{\partial^2 \theta_S(e_S)}{\partial e_S^2}} > 0, \tag{49}$$

where the sign follows from the concavity of the functions $\theta_i(\cdot)$, $i \in \{B, S\}$. ■

Proof of Proposition 2. Note that the last two expressions in (16) implicitly define loci $e_B^o(e_S)$ and $e_S^o(e_B)$, whose slopes can be determined through implicit differentiation and are given by (50) and (51), respectively:

$$\frac{\partial e_S}{\partial e_B} = -\frac{N_B N_S \frac{\partial^2 \theta_B(e_B)}{\partial e_B^2} - C''(e_B + e_S)}{-C''(e_B + e_S)}, \tag{50}$$

$$\frac{\partial e_S}{\partial e_B} = -\frac{-C''(e_B + e_S)}{N_B N_S \frac{\partial^2 \theta_S(e_S)}{\partial e_S^2} - C''(e_B + e_S)}. \tag{51}$$

By assumption, the cost function $C(e_B + e_S)$ is convex, implying that $C''(e_B + e_S) > 0$. Further, since $\theta_i(e_i)$ is concave, it follows that $\frac{\partial^2 \theta_B(e_B)}{\partial e_B^2} < 0$ and $\frac{\partial^2 \theta_S(e_S)}{\partial e_S^2} < 0$. Hence, we conclude the loci $e_B^o(e_S)$ and $e_S^o(e_B)$ are decreasing in (e_B, e_S)-space (see Figure 5b).

Next, we show that the gradient of the social welfare function evaluated at the private optimum (e_B^*, e_S^*) can be either positive, when $\tilde{N}_B \tilde{N}_S - N_B^* N_S^* > 0$, or negative when the opposite holds.[22] The

[22] Here $\tilde{N}_B \tilde{N}_S$ is the transaction volume that maximizes the social planner's objective evaluated at the privately optimal investment levels (e_B^*, e_S^*) while $N_B^* N_S^*$ is the privately optimal transaction volume.

outcome depends on parameter values. For this we evaluate $\frac{\partial SW(N_B,N_S,e_B,e_S)}{\partial e_B}$ and $\frac{\partial SW(N_B,N_S,e_B,e_S)}{\partial e_S}$ in (16) at the private optimum (e_B^*, e_S^*):

$$\frac{\partial SW(N_B, N_S, e_B, e_S)}{\partial e_B}\Big|_{e_B^*,e_S^*} = \tilde{N}_B\tilde{N}_S\frac{\partial \theta_B(e_B^*)}{\partial e_B} - C'(e_B^* + e_S^*) = \left(\tilde{N}_B\tilde{N}_S - N_B^*N_S^*\right)\frac{\partial \theta_B(e_B^*)}{\partial e_B}, \tag{52}$$

$$\frac{\partial SW(N_B, N_S, e_B, e_S)}{\partial e_S}\Big|_{e_B^*,e_S^*} = \tilde{N}_B\tilde{N}_S\frac{\partial \theta_S(e_S^*)}{\partial e_S} - C'(e_B^* + e_S^*) = \left(\tilde{N}_B\tilde{N}_S - N_B^*N_S^*\right)\frac{\partial \theta_S(e_S^*)}{\partial e_S}. \tag{53}$$

Hence, the comparison boils down to

$$\text{sign}\left(\frac{\partial SW(N_B, N_S, e_B^*, e_S^*)}{\partial e_B}\right) \equiv \text{sign}\left(\frac{\partial SW(N_B, N_S, e_B^*, e_S^*)}{\partial e_S}\right) \equiv \text{sign}(\tilde{N}_B\tilde{N}_S - N_B^*N_S^*).$$

When evaluated at (e_B^*, e_S^*), we restrict the social planner's investment levels to match those of the private optimum. This implies that for $\gamma(e_B^*, e_S^*) > s$, under-participation always occurs, since in that case we have $\tilde{N}_B\tilde{N}_S - N_B^*N_S^* > 0$. This in turn implies that when $\gamma(e_B^*, e_S^*) > s$, the point (e_B^*, e_S^*) lies in Region I of Figure 5b and, hence, combined with Lemma 2 implies underinvestment. ■

Proof of Proposition 3. The last two conditions in (18) implicitly define the curves $e_B^o(e_S)$ and $e_S^o(e_B)$. Using implicit differentiation, their slopes are given by:

$$\frac{\partial e_S}{\partial e_B} = -\frac{N_B\frac{\partial^2 v_B(e_B)}{\partial e_B^2} - C''(e_B + e_S)}{-C''(e_B + e_S)}, \tag{54}$$

$$\frac{\partial e_S}{\partial e_B} = -\frac{-C''(e_B + e_S)}{N_S\frac{\partial^2 v_S(e_S)}{\partial e_S^2} - C''(e_B + e_S)}. \tag{55}$$

Because the cost function $C(e_B + e_S)$ is convex and the stand-alone value functions $v_B(e_B)$ and $v_S(e_S)$ are concave, both slopes are negative. This means that both curves are downward sloping in the (e_B, e_S)-space, so that investments on the buyer and seller sides are substitutes: an increase in investment on one side tends to reduce investment on the other side.

We now evaluate the social planner's FOCs with respect to e_B and e_S in (18) at the private optimum (e_B^*, e_S^*):

$$\frac{\partial SW(N_B, N_S, e_B, e_S)}{\partial e_B}\Big|_{e_B^*,e_S^*} = \tilde{N}_B\frac{\partial v_B(e_B^*)}{\partial e_B} - C'(e_B^* + e_S^*) = \left(\tilde{N}_B - N_B^*\right)\frac{\partial v_B(e_B^*)}{\partial e_B}, \tag{56}$$

$$\frac{\partial SW(N_B, N_S, e_B, e_S)}{\partial e_S}\Big|_{e_B^*,e_S^*} = \tilde{N}_S\frac{\partial v_S(e_S^*)}{\partial e_S} - C'(e_B^* + e_S^*) = \left(\tilde{N}_S - N_S^*\right)\frac{\partial v_S(e_S^*)}{\partial e_S}, \tag{57}$$

where $(\tilde{N}_B, \tilde{N}_S)$ denotes the participation levels that solve the social planner's problem after fixing investment levels to (e_B^*, e_S^*).

Inspection of these derivatives reveals that the sign of the gradient vector with respect to investments e_B and e_S, evaluated at the private equilibrium, is determined by the sign of $\tilde{N}_i - N_i^*$, $i \in \{B, S\}$, i.e., the difference between the participation levels the social planner and the monopolist would implement at the private investment levels (e_B^*, e_S^*). Formally:

$$\text{sign}\left(\frac{\partial SW(N_B, N_S, e_B^*, e_S^*)}{\partial e_B}\right) \equiv \text{sign}(\tilde{N}_B - N_B^*), \quad \text{sign}\left(\frac{\partial SW(N_B, N_S, e_B^*, e_S^*)}{\partial e_S}\right) \equiv \text{sign}(\tilde{N}_S - N_S^*).$$

Now, because the stand-alone values $v_B(e_B^*)$ and $v_S(e_S^*)$ are equal across the monopolist's and the social planner's optimization problems, we can invoke Proposition 1 to determine the above signs. Proposition 1 implies that for $\gamma > s$ both sides of the market exhibit under-participation relative to the social optimum. This, combined with the fact that the loci $e_B^o(e_S)$ and $e_S^o(e_B)$ are downward sloping, ensures that Regions II, III and IV in Figures 6b and 7b are not attainable. Overinvestment on both sides is therefore impossible. However, the market may exhibit underinvestment on both sides, or underinvestment on one side accompanied by overinvestment on the other. ∎

Proof of Proposition 4. Recall that for any fixed (e_B, e_S), the expected surplus of a user and a developer who join the app marketplace is given by

$$u_B = v_B + \theta_B(\rho)\, N_S - p_B, \quad u_S = v_S + (1-\tau)\,\theta_S(\rho)\, N_B, \tag{58}$$

where $\theta_B(\rho)$ captures the (per-developer) user surplus generated by in-app purchases at in-app price ρ, and $\theta_S(\rho)$ captures the (per-user) developer revenue from in-app sales at price ρ.

We proceed backwards. At $t = 3$, given the pricing strategy set by the app platform (p_B, τ) and the participation rates on the platform (N_B, N_S), each app developer maximizes its payoff from in-app sales: $\max_\rho (v_S + (1-\tau)\,\rho\, D(\rho)\, N_B)$. This yields a monopoly in-app price ρ^m, which is independent of the platform's pricing decisions.

At $t = 2$, given the pricing strategy set by the app platform (p_B, τ), users and developers play a coordination game. As in the main model, the stability and uniqueness condition is derived from Lemma 1 as:

$$\max\left\{\max_{u_B \in \mathbb{R}} \theta_B(\rho^m) g_B(u_B), \max_{u_S \in \mathbb{R}} \theta_S(\rho^m)(1-\tau) g_S(u_S)\right\} < 1, \tag{59}$$

Similar to (41) in the proof of Proposition 1 this condition implies that an increase in the device price reduces participation on both sides, i.e., $\frac{\partial N_B}{\partial p_B} < 0$ and $\frac{\partial N_S}{\partial p_B} < 0$. Also, an increase in the commission rate reduces participation on both sides, i.e., $\frac{\partial N_B}{\partial \tau} < 0$ and $\frac{\partial N_S}{\partial \tau} < 0$.

At $t = 1$, the platform chooses user and developer participation rates to maximize:

$$\Pi^p = N_B(p_B - c_B) + N_S\,(\tau\,\theta_S(\rho^m)\, N_B - c_S) - s\, N_S N_B.$$

As in the baseline model, we can obtain the prices that implement a given participation allocation (N_B, N_S) by inverting the system of demands:

$$p_B(\cdot) = v_B + \theta_B(\rho^m)\, N_S - \varphi_B(N_B), \quad \tau(\cdot) = 1 - \frac{\varphi_S(N_S) - v_S}{\theta_S(\rho^m)\, N_B}.$$

Substituting these price instruments in the expression for platform profits gives:

$$\Pi^p = N_B\big(v_B + \theta_B(\rho^m) N_S - \varphi_B(N_B) - c_B\big) \; + \; N_S\big(v_S + N_B\theta_S(\rho^m) - \varphi_S(N_S) - c_S\big) - sN_BN_S. \tag{60}$$

The FOCs with respect to N_B and N_S are essentially the same as in the baseline model. Denoting the optimal user and developer participation levels by (N_B^*, N_S^*), the optimal monopoly device price and commission rate are given by:

$$p_B^* = c_B - (\theta_S(\rho^m) - s)N_S^* + \frac{\partial \varphi_B(N_B^*)}{\partial N_B} N_B^*, \qquad \tau^* = \frac{\frac{\partial \varphi_S(N_S^*)}{\partial N_S} N_S^* + c_S}{\theta_S(\rho^m) N_B^*} - \frac{\theta_B(\rho^m) - s}{\theta_S(\rho^m)}. \tag{61}$$

The planner's objective function is the same as in Section 3, namely $SW = \Pi^p + V_B + V_S$. Moreover, the expressions for user and developer surplus remain $V_i = \bar{o}_i - \int_{\varphi_i(N_i)}^{\bar{o}_i} G_i(z)\, dz$, $i \in \{B, S\}$. The planner's FOCs are then similar to those in Section 4 and implicitly define functions $N_B^o(N_S)$ and $N_S^o(N_B)$ in participation space. These functions are upward sloping, since their slopes satisfy respectively:

$$\frac{\partial N_S}{\partial N_B} = \frac{\frac{\partial \varphi_B(N_B)}{\partial N_B}}{\theta_B(\rho^m) + \theta_S(\rho^m) - s} > 0, \qquad \frac{\partial N_S}{\partial N_B} = \frac{\theta_B(\rho^m) + \theta_S(\rho^m) - s}{\frac{\partial \varphi_S(N_S)}{\partial N_S}} > 0.$$

Denoting the socially optimal participation rates by (N_B^o, N_S^o), the socially optimal device price and commission rate are therefore given by:

$$p_B^o = c_B - (\theta_S(\rho^m) - s)N_S^o, \qquad \tau^o = \frac{c_S}{\theta_S(\rho^m) N_B^o} - \frac{\theta_B(\rho^m) - s}{\theta_S(\rho^m)}. \tag{62}$$

Since the planner's objective function coincides with that in the main model, we have $\frac{\partial SW}{\partial N_i}\Big|_{N_i^*, N_j^*} = \frac{\partial \varphi_i(N_i^*)}{\partial N_i} N_i^* > 0$, which implies that the private optimum must lie in the south-west region of the (N_B, N_S) space. Combined with the fact that both FOCs of the planner are upward sloping, this south-west region is a subset of the quadrant $\{(N_B, N_S) \mid 0 \leq N_B \leq N_B^o,\ 0 \leq N_S \leq N_S^o\}$. Thus, there is under-participation on both sides.

The welfare maximization problem can be equivalently expressed in terms of device prices and commission rates: $\max_{p_B, \tau} SW(p_B, \tau)$. As in the main model, evaluating the planner's FOCs at the private optimum (p_B^*, τ^*) and combining with the stability condition yields

$$\frac{\partial SW(p_B, \tau)}{\partial p_B}\Big|_{(p_B^*, \tau^*)} < 0, \quad \text{and} \quad \frac{\partial SW(p_B, \tau)}{\partial \tau}\Big|_{(p_B^*, \tau^*)} < 0.$$

This implies that both components of the gradient of the planner's objective, evaluated at the monopolist's price pair in the (p_B, τ)-space, are negative. It follows that prices below their socially optimal levels on both sides are not possible. This is also consistent with Proposition 1, the private pricing structure involves either both instruments being too high, or one too high while the other too low.

In the context of the app marketplace, a stronger conclusion can be made: the commission rate is always excessive. Taking the difference between (61) and (62) gives the distortions in the device price and commission rate, respectively, as follows:

$$p_B^* - p_B^o = (\theta_S(\rho^m) - s)(N_S^o - N_S^*) + \frac{\partial \varphi_B(N_B^*)}{\partial N_B} N_B^*,$$

$$\tau^* - \tau^o = \frac{c_S}{\theta_S(\rho^m)} \left(\frac{1}{N_B^*} - \frac{1}{N_B^o} \right) + \frac{\frac{\partial \varphi_S(N_S^*)}{\partial N_S} N_S^*}{\theta_S(\rho^m) N_B^*}.$$

As in Proposition 1, the monopolist's device price p_B^* may lie above or below its socially optimal level p_B^o, depending on the relative magnitudes of the scale distortion and the markup. However, the fact that $N_i^o > N_i^*$ for $i \in \{B, S\}$, together with the additional "market power" term, implies that $\tau^* > \tau^o$ always holds. Hence, the commission rate is unambiguously excessive. ■

Proof of Proposition 5. For part (i), first recall the notation, $\theta_B(\epsilon) \equiv \theta_B(D(\rho^m(\epsilon), \epsilon))$ and $\theta_S(\epsilon) \equiv \theta_S(D(\rho^m(\epsilon), \epsilon))$. Then, FOCs of the app platform and the planner with respect to the investment level ϵ are given by:

$$\frac{\partial \Pi^p}{\partial \epsilon} = \left(\frac{\partial \theta_B}{\partial \epsilon} + \frac{\partial \theta_S}{\partial \epsilon} \right) N_B N_S - C'(\epsilon) = 0,$$

$$\frac{\partial SW}{\partial \epsilon} = \left(\frac{\partial \theta_B}{\partial \epsilon} + \frac{\partial \theta_S}{\partial \epsilon} \right) N_B N_S - C'(\epsilon) = 0.$$

Evaluating the planner's FOC at the private investment level ϵ^* gives

$$\left. \frac{\partial SW}{\partial \epsilon} \right|_{\epsilon^*} = \left(\frac{\partial \theta_B(\epsilon^*)}{\partial \epsilon} + \frac{\partial \theta_S(\epsilon^*)}{\partial \epsilon} \right) (\tilde{N}_B \tilde{N}_S - N_B^* N_S^*). \tag{63}$$

Here, $(\tilde{N}_B, \tilde{N}_S)$ denotes the participation levels that solve the social planner's problem after fixing the investment level.

Because both users and developers benefit from larger in-app purchase demand, teh first parenthesis is positive. Hence, the sign of the interaction distortion governs that of the investment distortion. By Proposition 4, for any fixed investment level, we have $\tilde{N}_B \tilde{N}_S > N_B^* N_S^*$. This implies $\frac{\partial SW}{\partial \epsilon}\big|_{\epsilon^*} > 0$. Hence, the investment level chosen by the platform is inefficiently low relative to the social optimum.

The proof of part (ii) follows directly from Proposition 3. ■

Proof of Proposition 6. The app platform maximizes its profit subject to the cap on commission rates:

$$\Pi^p = N_B(p_B - c_B) + N_S\Big(\bar{\tau}\,\theta_S(\rho^m)\,N_B - c_S\Big) - s\,N_S N_B,$$
$$\text{subject to}\quad N_S = G_S\big(v_S + \big(1-\bar{\tau}\big)\,\theta_S(\rho^m)\,N_B\big).$$

Plugging the constraint into the objective function gives: $\hat{\Pi}^p(N_B) \equiv \Pi^p\big(N_B,\, N_S(N_B;\bar{\tau})\big)$. Therefore, the FOC is given by:

$$\frac{\partial\hat{\Pi}^p(N_B;\bar{\tau})}{\partial N_B} = \frac{\partial \Pi^p}{\partial N_B} + \frac{\partial \Pi^p}{\partial N_S}\frac{\partial N_S(N_B;\bar{\tau})}{\partial N_B} = 0.$$

Differentiating this FOC with respect to $\bar{\tau}$ gives:

$$\frac{\partial^2\hat{\Pi}^p(N_B;\bar{\tau})}{\partial N_B \partial\bar{\tau}} = (\theta_B + \bar{\tau}\theta_S - s)\left(\frac{\partial N_S}{\partial \bar{\tau}} + \frac{\partial^2 N_S}{\partial N_B \partial\bar{\tau}}N_B\right) - c_S\frac{\partial^2 N_S}{\partial N_B\partial\bar{\tau}} + \frac{\partial N_S}{\partial N_B}\theta_S N_B + N_S\theta_S \quad (64)$$

Note that:

$$\frac{\partial N_S}{\partial N_B} = g_S(\cdot)\,\theta_S(\rho^m)\,(1-\bar{\tau}), \qquad \frac{\partial N_S}{\partial \bar{\tau}} = -\,g_S(\cdot)\,\theta_S(\rho^m)\,N_B,$$

$$\frac{\partial^2 N_S}{\partial N_B\,\partial\bar{\tau}} = -\,\theta_S(\rho^m)\,g_S(\cdot) - \big(\theta_S(\rho^m)\big)^2(1-\bar{\tau})\,N_B\,g'_S(\cdot),$$

where $g_S(\cdot) = G'_S(\cdot)$ and $g'_S(\cdot) = G''_S(\cdot)$.

Plugging these derivatives into the above expression and evaluating it at τ^* gives:

$$\begin{aligned}\left.\frac{\partial^2\hat{\Pi}^p(N_B;\tau)}{\partial N_B\partial\tau}\right|_{\bar{\tau}=\tau^*} &= (\theta_B(\rho^m) + \tau^*\theta_S(\rho^m) - s)\Big(-2\theta_S(\rho^m)\,N_B\,g_S(\cdot) - \big(\theta_S(\rho^m)\big)^2(1-\tau^*)\,N_B^2\,g'_S(\cdot)\Big)\\ &+ c_S\Big(\theta_S(\rho^m)\,g_S(\cdot) + \big(\theta_S(\rho^m)\big)^2(1-\tau^*)\,N_B\,g'_S(\cdot)\Big)\\ &+ \big(\theta_S(\rho^m)\big)^2(1-\tau^*)\,N_B\,g_S(\cdot) + \theta_S(\rho^m)\,N_S.\end{aligned}$$

Using (61) we get:

$$\theta_B(\rho^m) + \tau^*\theta_S(\rho^m) - s = \frac{\frac{\partial\varphi_S(N_S^*)}{\partial N_S}N_S^* + c_S}{N_B^*},$$

and recall that

$$\frac{\partial\varphi_S(N_S^*)}{\partial N_S} = \frac{1}{g_S(v_S + \theta_S(\rho^m)(1-\tau^*)N_B)}.$$

Using this in the above derivative, it can be simplified to:

$$\left.\frac{\partial^2\hat{\Pi}^p(N_B;\bar{\tau})}{\partial N_B\partial\bar{\tau}}\right|_{\bar{\tau}=\tau^*} = \theta_S(\rho^m)\left[-\,N_S^* -\, c_S\,g(\cdot) + \theta_S(\rho^m)\,N_B^*\,(1-\tau^*)\,g_S(\cdot)\left(1 - \frac{N_S^*\,g'_S(\cdot)}{g_S^2(\cdot)}\right)\right].$$

The sign of this expression depends on the distribution of outside options.

For the uniform distribution we have $g_S(\cdot) = 1$ and $g'_S(\cdot) = 0$. Then it simplifies to:

$$\left.\frac{\partial^2 \hat{\Pi}^p(N_B;\bar{\tau})}{\partial N_B \partial \bar{\tau}}\right|_{\bar{\tau}=\tau^*} = \theta_S(\rho^m)\left[-N_S^* - c_S + \theta_S(\rho^m) N_B^*(1-\tau^*)\right] = -\theta_S(\rho^m)\left(v_S + c_S\right) < 0.$$

This shows that for the uniform distribution capping commissions a little bit surely increases the welfare of users and developers. ■

Proof of Proposition 7. Assume that the outside options o_i are uniformly distributed over the interval $[0,1]$, for $i \in \{B,S\}$. Further assume $\theta_i(\rho^m) < 1$ for $i \in \{B,S\}$ which ensures the stability and uniqueness condition (59) holds for all $\tau \in [0,1]$.

The platform maximizes its profit subject to the constraint on commission rates $\tau \leq \bar{\tau}$:

$$\begin{aligned} \Pi^p &= N_B(p_B - c_B) + N_S\Big(\bar{\tau}\,\theta_S(\rho^m)\,N_B - c_S\Big) - s\,N_S N_B, \\ \text{subject to} \quad N_S &= v_S + \big(1-\bar{\tau}\big)\,\theta_S(\rho^m)\,N_B, \end{aligned}$$

which yields the regulated number of users as

$$\hat{N}_B(\bar{\tau}) = \frac{v_B - c_B + v_S\Big(\theta_B(\rho^m) - s + \bar{\tau}\theta_S(\rho^m)\Big) - (1-\bar{\tau})c_S\theta_S(\rho^m)}{2\Big(1 + (1-\bar{\tau})\theta_S(\rho^m)\left(s - \theta_B(\rho^m) - \bar{\tau}\theta_S(\rho^m)\right)\Big)}, \tag{65}$$

and the regulated number of developers as

$$\hat{N}_S(\bar{\tau}) = v_S + (1-\bar{\tau})\theta_S(\rho^m)\cdot\frac{v_B - c_B + v_S\Big(\theta_B(\rho^m) - s + \bar{\tau}\theta_S(\rho^m)\Big) - (1-\bar{\tau})c_S\theta_S(\rho^m)}{2\Big(1 + (1-\bar{\tau})\theta_S(\rho^m)\left(s - \theta_B(\rho^m) - \bar{\tau}\theta_S(\rho^m)\right)\Big)}. \tag{66}$$

Note that the unregulated numbers of users and developers under the uniform distribution are, respectively, given by (see Appendix D):

$$\begin{aligned} N_B^* &= \frac{\Big(\theta_B(\rho^m) + \theta_S(\rho^m) - s\Big)(v_S - c_S) + 2(v_B - c_B)}{4 - \Big(\theta_B(\rho^m) + \theta_S(\rho^m) - s\Big)^2}, \\ N_S^* &= \frac{\Big(\theta_B(\rho^m) + \theta_S(\rho^m) - s\Big)(v_B - c_B) + 2(v_S - c_S)}{4 - \Big(\theta_B(\rho^m) + \theta_S(\rho^m) - s\Big)^2}. \end{aligned}$$

Setting $\hat{N}_S = N_S^*$ and solving for the corresponding commission rates yields

$$\tau^* = 1 - \frac{N_S^* - v_S}{\theta_S(\rho^m) N_B^*} \qquad \text{and} \qquad \tau_S = \frac{\theta_S(\rho^m)\big(\theta_B(\rho^m) + \theta_S(\rho^m) - s\big) - 2}{\theta_S(\rho^m)\big(\theta_B(\rho^m) + \theta_S(\rho^m) - s\big)}.$$

The global maximum condition requires $\theta_B(\rho^m) + \theta_S(\rho^m) - s \in (0,1)$, and combined with the stability and uniqueness condition $\theta_i(\rho^m) < 1$ for $i \in \{B, S\}$, this implies $\tau_S < 0$. Since $\tau^* \in [0,1]$, we have $\tau_S < \tau^*$, so developer participation increases for any $\bar{\tau} \in (\tau_S, \tau^*)$. Because $[0, \tau^*) \subset (\tau_S, \tau^*)$, any commission cap $\bar{\tau} \in [0, \tau^*)$ unambiguously raises developers participation and, consequently, developers surplus. This concludes part (i) of Proposition 7.

Next, we equate $\hat{N}_B$ to N_B^* to solve for τ^* and $\tau_B = \frac{\theta_S(\rho^m) - \theta_B(\rho^m) + s}{2\theta_S(\rho^m)}$. Their difference is given by $\tau^* - \tau_B = \frac{v_S + c_S}{2\theta_S(\rho^m) N_B^*} > 0$. This implies that, for any interior $\tau^* \in [0,1]$, there exists an interval (τ_B, τ^*) for which any commission cap $\bar{\tau} \in (\tau_B, \tau^*)$ increases users participation, and hence users surplus. Further reductions in $\bar{\tau}$ beyond this interval become counterproductive since user participation falls below the level attained without a commission cap. Moreover, if $\tau_B < 0$, any commission cap $\bar{\tau} \in [0, \tau^*)$ increases user participation rates. This concludes part (ii) of Proposition 7.

The threshold for the sum of users and developers participation rates can be determined similarly. Setting $\hat{N}_B + \hat{N}_S$ equal to $N_B^* + N_S^*$ yields the solutions τ^* and $\tau_{BS} = \frac{\theta_S(\rho^m) - 1}{\theta_S(\rho^m)} < 0$. The sign of τ_{BS} follows from the stability and uniqueness condition. Consequently, $\tau_{BS} < \tau^*$, and any commission cap $\bar{\tau} \in [0, \tau^*)$ satisfies (τ_{BS}, τ^*). This proves part (iii) of Proposition 7, which implies that any cap $\bar{\tau} \in [0, \tau^*)$ increases the sum of users and developers surplus. ■

Proof of Remark 1. The proof makes use of the expression for $\frac{d\hat{N}_B}{d\bar{\tau}}$ given in (29):

$$\frac{d\hat{N}_B}{d\bar{\tau}} = -\frac{1}{\det(\mathcal{H})}\left(\frac{\partial^2 \hat{\Pi}^p}{\partial N_B \partial \bar{\tau}}\frac{\partial^2 \hat{\Pi}^p}{\partial \epsilon^2} - \frac{\partial^2 \hat{\Pi}^p}{\partial \epsilon \partial \bar{\tau}}\frac{\partial^2 \hat{\Pi}^p}{\partial N_B \partial \epsilon}\right). \tag{67}$$

Since the curvature $C''(\epsilon)$ appears only in $\frac{\partial^2 \hat{\Pi}^p}{\partial \epsilon^2}$, a sufficiently large convexity of the investment cost implies that the first term in the parenthesis dominates the second, so that

$$\frac{d\hat{N}_B}{d\bar{\tau}} \approx -\frac{1}{\det(\mathcal{H})}\frac{\partial^2 \hat{\Pi}^p}{\partial N_B \partial \bar{\tau}}\frac{\partial^2 \hat{\Pi}^p}{\partial \epsilon^2}, \text{which yields } \operatorname{sign}\left(\frac{d\hat{N}_B}{d\bar{\tau}}\right) \equiv \operatorname{sign}\left(\frac{\partial^2 \hat{\Pi}^p}{\partial N_B \partial \bar{\tau}}\right),$$

since $\frac{\partial^2 \hat{\Pi}^p}{\partial \epsilon^2} < 0$ by the condition for a global maximum.

For a commission cap locally below τ^* to increase buyer participation, it is necessary that $\left.\frac{d\hat{N}_B}{d\bar{\tau}}\right|_{\tau^*} < 0$. By the proof of Proposition 6, this requires that the condition (27) evaluated at ϵ^* is satisfied. This completes the proof of Remark 1. ■

Proof of Proposition 8. Next, we derive the conditions in Proposition 8. From (30):

$$\frac{d\hat{\epsilon}}{d\bar{\tau}} = -\frac{1}{\det(\mathcal{H})}\left(\frac{\partial^2 \hat{\Pi}^p}{\partial \epsilon \partial \bar{\tau}}\frac{\partial^2 \hat{\Pi}^p}{\partial N_B^2} - \frac{\partial^2 \hat{\Pi}^p}{\partial N_B \partial \bar{\tau}}\frac{\partial^2 \hat{\Pi}^p}{\partial \epsilon \partial N_B}\right). \tag{68}$$

Note that the regulated platform maximizes the profit expression:

$$\begin{aligned}\hat{\Pi}^p(N_B, \epsilon; \bar{\tau}) = {} & N_B(v_B + \theta_B(\epsilon) N_S(N_B, \epsilon; \bar{\tau}) - \varphi_B(N_B) - c_B) + N_S(N_B, \epsilon; \bar{\tau})\left(\bar{\tau}\theta_S(\epsilon) N_B - c_S\right) \\ & - s N_B N_S(N_B, \epsilon; \bar{\tau}) - C(\epsilon), \\ \text{where} \quad & N_S(N_B, \epsilon; \bar{\tau}) = G_S\big(v_S + \left(1 - \bar{\tau}\right)\theta_S(\epsilon)\, N_B\big).\end{aligned}$$

Assuming the outside options o_i are uniformly distributed over the interval $[0,1]$, for $i \in \{B, S\}$, we can derive:

$$\begin{aligned}
\frac{\partial^2 \hat{\Pi}^p}{\partial \epsilon \partial \bar{\tau}}\Big|_{\tau^*} &= -N_B^{*2}\theta_S(\epsilon^*)(\theta_B'(\epsilon^*) + (2\tau^* - 1)\theta_S'(\epsilon^*)) \gtreqless 0, \\
\frac{\partial^2 \hat{\Pi}^p}{\partial N_B^2}\Big|_{\tau^*} &< 0 \text{ by the SOC}, \\
\frac{\partial^2 \hat{\Pi}^p}{\partial N_B \partial \bar{\tau}}\Big|_{\tau^*} &= -\theta_S'(\epsilon^*)(v_S + c_S) < 0, \\
\frac{\partial^2 \hat{\Pi}^p}{\partial \epsilon \partial N_B}\Big|_{\tau^*} &= \Big(N_S^* + N_B^*(1-\tau^*)\theta_S(\epsilon^*)\Big)(\theta_B'(\epsilon^*) + \tau^*\theta_S'(\epsilon^*)) + (1-\tau^*)\theta_S'(\epsilon^*)N_S^* \\
&\quad + (1-\tau^*)N_B^*\theta_S'(\epsilon^*)(\theta_B(\epsilon^*) + \tau^*\theta_S(\epsilon^*)) > 0.
\end{aligned}$$

First, consider any unregulated commission rate $\tau^* > \frac{1}{2}$. This implies $\frac{\partial^2 \hat{\Pi}^p}{\partial \epsilon \partial \bar{\tau}}\Big|_{\tau^*} < 0$, which yields: $\frac{d\hat{\epsilon}}{d\bar{\tau}}\Big|_{\tau^*} < 0$. Hence, any commission cap locally below τ^* increases the investment level. This completes the proof of Proposition 8(i).

Next, suppose $v_S \to 0$ and $c_S \to 0$. Under these conditions, it follows that $\frac{\partial^2 \hat{\Pi}^p}{\partial N_B \partial \bar{\tau}} \to 0$ and hence

$$\frac{d\hat{\epsilon}}{d\bar{\tau}}\Big|_{\tau^*} \approx -\frac{1}{\det(\mathcal{H})}\frac{\partial^2 \hat{\Pi}^p}{\partial \epsilon \partial \bar{\tau}}\frac{\partial^2 \hat{\Pi}^p}{\partial N_B^2}, \text{ which yields } \operatorname{sign}\left(\frac{d\hat{\epsilon}}{d\bar{\tau}}\Big|_{\tau^*}\right) = \operatorname{sign}\left(-\left(\theta_B'(\epsilon^*) + (2\tau^* - 1)\theta_S'(\epsilon^*)\right)\right).$$

Therefore, $\frac{d\hat{\epsilon}}{d\bar{\tau}}\Big|_{\tau^*} < 0$ if and only if $\frac{\theta_B'(\epsilon)}{\theta_S'(\epsilon)} > 1 - 2\tau^*$, as stated in Proposition 8(ii). Note that this condition requires $\tau^* < \dfrac{1}{2}$; otherwise, the conclusion of Proposition 8(i) applies directly. ■

D Uniform distribution example

In this section, we provide details of the derivations needed to construct the figures that illustrate our Propositions 1 and 2.

We assume that outside options o_i are drawn from a uniform distribution $U[0,1]$. Further, we assume that network effects are given by the functional forms $\theta_B(e_B) = \theta_B\sqrt{e_B}$ and $\theta_S(y) = \theta_S\sqrt{e_S}$, and that investment costs are quadratic $C(e_B + e_S) = \frac{1}{2}(e_B + e_S)^2$.

Under these assumptions, the demand system is given by the solution to:

$$\begin{aligned}
N_B &= \Pr[u_B \geq o_B] = v_B + \theta_B\sqrt{e_B}N_S - p_B, \\
N_S &= \Pr[u_S \geq o_S] = v_S + \theta_S\sqrt{e_S}N_B - p_S.
\end{aligned}$$

and the prices in (1) satisfy:

$$p_B = v_B + \theta_B\sqrt{e_B}N_S - N_B, \quad p_S = v_S + \theta_S\sqrt{e_S}N_B - N_S.$$

For a monopoly platform, the first-order conditions are:

$$\begin{aligned}
\frac{\partial \Pi^p(N_B, N_S, e_B, e_S)}{\partial N_B} &= (\theta_B\sqrt{e_B} + \theta_S\sqrt{e_S} - s)N_S + v_B - c_B - 2N_B = 0, \\
\frac{\partial \Pi^p(N_B, N_S, e_B, e_S)}{\partial N_S} &= (\theta_B\sqrt{e_B} + \theta_S\sqrt{e_S} - s)N_B + v_S - c_S - 2N_S = 0, \\
\frac{\partial \Pi^p(N_B, N_S, e_B, e_S)}{\partial e_B} &= \frac{N_B N_S \theta_B}{2\sqrt{e_B}} - (e_B + e_S) = 0, \\
\frac{\partial \Pi^p(N_B, N_S, e_B, e_S)}{\partial e_S} &= \frac{N_B N_S \theta_S}{2\sqrt{e_S}} - (e_B + e_S) = 0.
\end{aligned} \tag{69}$$

These equations can be arranged as follows:

$$\begin{aligned}
N_B^*(e_B, e_S) &= \frac{(\theta_B\sqrt{e_B} + \theta_S\sqrt{e_S} - s)(v_S - c_S) + 2(v_B - c_B)}{4 - (\theta_B\sqrt{e_B} + \theta_S\sqrt{e_S} - s)^2}, \\
N_S^*(e_B, e_S) &= \frac{(\theta_B\sqrt{e_B} + \theta_S\sqrt{e_S} - s)(v_B - c_B) + 2(v_S - c_S)}{4 - (\theta_B\sqrt{e_B} + \theta_S\sqrt{e_S} - s)^2}, \\
e_B^*(N_B, N_S) &= \theta_B^2 \sqrt[3/2]{\frac{N_B N_S}{2(\theta_S^2 + \theta_B^2)}}, \\
e_S^*(N_B, N_S) &= \theta_S^2 \sqrt[3/2]{\frac{N_B N_S}{2(\theta_S^2 + \theta_B^2)}}.
\end{aligned} \tag{70}$$

Similarly, the first-order conditions for the social planner are:

$$\begin{aligned}
\frac{\partial SW(N_B, N_S, e_B, e_S)}{\partial N_B} &= (\theta_B\sqrt{e_B} + \theta_S\sqrt{e_S} - s)N_S + v_B - c_B - N_B = 0, \\
\frac{\partial SW(N_B, N_S, e_B, e_S)}{\partial N_S} &= (\theta_B\sqrt{e_B} + \theta_S\sqrt{e_S} - s)N_B + v_S - c_S - N_S = 0, \\
\frac{\partial SW(N_B, N_S, e_B, e_S)}{\partial e_B} &= \frac{N_B N_S \theta_B}{2\sqrt{e_B}} - (e_B + e_S) = 0, \\
\frac{\partial SW(N_B, N_S, e_B, e_S)}{\partial e_S} &= \frac{N_B N_S \theta_S}{2\sqrt{e_S}} - (e_B + e_S) = 0.
\end{aligned} \tag{71}$$

These conditions can be rewritten as follows:

$$\begin{aligned}
N_B^o(e_B, e_S) &= \frac{(\theta_B\sqrt{e_B} + \theta_S\sqrt{e_S} - s)(v_S - c_S) + (v_B - c_B)}{1 - (\theta_B\sqrt{e_B} + \theta_S\sqrt{e_S} - s)^2}, \\
N_S^o(e_B, e_S) &= \frac{(\theta_B\sqrt{e_B} + \theta_S\sqrt{e_S} - s)(v_B - c_B) + (v_S - c_S)}{1 - (\theta_B\sqrt{e_B} + \theta_S\sqrt{e_S} - s)^2}, \\
e_B^o(N_B, N_S) &= \theta_B^2 \sqrt[3/2]{\frac{N_B N_S}{2(\theta_S^2 + \theta_B^2)}}, \\
e_S^o(N_B, N_S) &= \theta_S^2 \sqrt[3/2]{\frac{N_B N_S}{2(\theta_S^2 + \theta_B^2)}}.
\end{aligned} \tag{72}$$

D.1 Participation (in-)efficiency

Recall that $\gamma = \theta_B\sqrt{e_B} + \theta_S\sqrt{e_S}$, where e_B and e_S are arbitrary investment levels. We first show that for the second-order conditions to hold, it must be the case that $\gamma - s$ is sufficiently small. To see this, note that the Hessian matrices corresponding to the platform and planner's optimization problems are:

$$\mathcal{H}\Big(\Pi^p(N_B, N_S)\Big) = \begin{bmatrix} -2 & \gamma - s \\ \gamma - s & -2 \end{bmatrix}, \quad \mathcal{H}\Big(SW(N_B, N_S)\Big) = \begin{bmatrix} -1 & \gamma - s \\ \gamma - s & -1 \end{bmatrix}.$$

For the monopoly problem, the Hessian matrix is negative definite provided that $(\theta_B+\theta_S-s)^2-4 < 0$. For the social planner, it suffices that $(\theta_B + \theta_S - s)^2 - 1 < 0$. The latter is a more stringent condition and ensures that the monopoly's and planner's choice of participation levels in (70) and (72) are strictly positive. Assuming an interior solution, distortions in the number of participants are given by:

$$\Delta N_B = N_B^* - N_B^o = \frac{-\left(2 + (\gamma - s)^2\right)(v_B - c_B) - 3(\gamma - s)(v_S - c_S)}{(4 - (\gamma - s)^2)(1 - (\gamma - s)^2)}, \tag{73}$$

$$\Delta N_S = N_S^* - N_S^o = \frac{-\left(2 + (\gamma - s)^2\right)(v_S - c_S) - 3(\gamma - s)(v_B - c_B)}{(4 - (\gamma - s)^2)(1 - (\gamma - s)^2)}. \tag{74}$$

Note that these equations can be rewritten as:

$$\Delta N_B = -2\mathcal{K}_1 - (\gamma - s)\mathcal{K}_2,$$
$$\Delta N_S = -2\mathcal{K}_2 - (\gamma - s)\mathcal{K}_1,$$

where $\mathcal{K}_1 = \frac{(v_B - c_B) + (\gamma - s)(v_S - c_S)}{(4-(\gamma-s)^2)(1-(\gamma-s)^2)} > 0$ and $\mathcal{K}_2 = \frac{(\gamma-s)(v_B - c_B) + (v_S - c_S)}{(4-(\gamma-s)^2)(1-(\gamma-s)^2)} > 0$. As a result, because $\gamma > s$, both ΔN_B and ΔN_S are negative, which confirms Proposition 1 part (i).

D.2 Pricing (in-)efficiency

We now turn to pricing distortions, as characterized in Proposition 1, part (ii). We first solve for the privately and socially optimal participation prices under uniform outside options and then derive the conditions that delineate the regions in which pricing inefficiencies are symmetric or asymmetric.

For the uniform distribution, and for fixed investment levels, the system (1) becomes:

$$p_B = v_B + \theta_B N_S - N_B, \quad p_S = v_S + \theta_S N_B - N_S.$$

To solve for profit or welfare maximization, we need to express the system in terms of N_B and N_S as functions of p_B and p_S:

$$N_B(p_B, p_S) = \frac{\theta_B p_S - \theta_B v_S + p_B - v_B}{\theta_B\theta_S - 1}, \qquad N_S(p_B, p_S) = \frac{\theta_S p_B - \theta_S v_B + p_S - v_S}{\theta_B\theta_S - 1}.$$

Note that these demands are stable when $\theta_B\theta_S < 1$, which is in line with the general stability condition identified in Lemma 1 in Appendix A.

For the monopoly platform, the first-order conditions are:

$$\frac{\partial \Pi^p}{\partial p_B} = \frac{s(2(v_B - p_B)\theta_S + (v_S - p_S)(1+\theta_B\theta_S)) + (1-\theta_B\theta_S)(\theta_B(v_S - p_S) + \theta_S(c_S - p_S) - c_B + 2p_B - v_B)}{(1-\theta_B\theta_S)^2} = 0,$$

$$\frac{\partial \Pi^p}{\partial p_S} = \frac{s(2(v_S - p_S)\theta_B + (v_B - p_B)(1+\theta_B\theta_S)) + (1-\theta_B\theta_S)(\theta_S(v_B - p_B) + \theta_B(c_B - p_B) - c_S + 2p_S - v_S)}{(1-\theta_B\theta_S)^2} = 0.$$

These first-order conditions define functions $p_B^*(p_S)$ and $p_S^*(p_B)$ whose slopes in the (p_B, p_S)-space are respectively given by:

$$\frac{\partial p_S}{\partial p_B} = -\frac{2s\theta_S + 2(1-\theta_B\theta_S)}{\theta_B + \theta_S + s - \theta_B\theta_S\left(\theta_S + \theta_B - s\right)} = -\frac{2s\theta_S + 2(1-\theta_B\theta_S)}{(\theta_B + \theta_S - s)(1-\theta_B\theta_S) + 2s},$$

$$\frac{\partial p_S}{\partial p_B} = -\frac{\theta_B + \theta_S + s - \theta_B\theta_S\left(\theta_S + \theta_B - s\right)}{2s\theta_B + 2(1-\theta_B\theta_S)} = -\frac{(\theta_B + \theta_S - s)(1-\theta_B\theta_S) + 2s}{2s\theta_B + 2(1-\theta_B\theta_S)}.$$

It is easy to see that these slopes are negative, since the stability condition implies $\theta_B\theta_S < 1$ and, by assumption, $\theta_B + \theta_S - s > 0$.

The solution of the system of the FOCs gives

$$\begin{aligned} p_B^* &= -\frac{c_S\left(-\theta_S + s + \theta_B\right) + \left(-\theta_S + s - \theta_B\right)\left(v_B\left(s - \theta_S\right) - c_B\theta_B\right) - v_S\left(-\theta_S + s + \theta_B\right) - 2\left(c_B + v_B\right)}{4 - (\theta_B + \theta_S - s)^2}, \\ p_S^* &= -\frac{c_B\left(-\theta_B + s + \theta_S\right) + \left(-\theta_B + s - \theta_S\right)\left(v_S\left(s - \theta_B\right) - c_S\theta_S\right) - v_B\left(-\theta_B + s + \theta_S\right) - 2\left(c_S + v_S\right)}{4 - (\theta_B + \theta_S - s)^2}. \end{aligned} \tag{75}$$

To ensure that these prices are an optimum, we verify the second-order conditions. The Hessian matrix corresponding to the monopoly optimization problem is:

$$\mathcal{H}\left(\Pi^p\right) = \begin{bmatrix} -\frac{(2s\theta_S + 2(1-\theta_B\theta_S))}{\left(\theta_B\theta_S - 1\right)^2} & -\frac{\theta_B + \theta_S + s - \theta_B\theta_S\left(\theta_S + \theta_B - s\right)}{\left(\theta_B\theta_S - 1\right)^2} \\ -\frac{\theta_B + \theta_S + s - \theta_B\theta_S\left(\theta_S + \theta_B - s\right)}{\left(\theta_B\theta_S - 1\right)^2} & -\frac{(2s\theta_B + 2(1-\theta_B\theta_S))}{\left(\theta_B\theta_S - 1\right)^2} \end{bmatrix}.$$

Observe that the leading principal minors, denoted D_1 and D_2 are:

$D_1 = -\frac{(2s\theta_S + 2(1-\theta_B\theta_S))}{\left(\theta_B\theta_S - 1\right)^2} < 0$ due to stability condition $\theta_B\theta_S < 1$,

$D_2 = -\frac{(\theta_B + \theta_S - s)^2 - 4}{(\theta_S\theta_B - 1)^2} > 0$ provided that $(\theta_B + \theta_S - s)^2 - 4 < 0$, which is identical to the condition on the second leading principal minor identified in Section D.1.

We now move to the social planner's objective function. The FOCs with respect to prices are:

$$\frac{\partial SW}{\partial p_B} = \frac{\partial \Pi^p}{\partial p_B} + \frac{p_B - v_B + \theta_B p_S - \theta_B v_S}{\left(\theta_B\theta_S - 1\right)^2} + \theta_S\frac{p_S - v_S + \theta_S p_B - \theta_S v_B}{\left(\theta_B\theta_S - 1\right)^2} = 0,$$

$$\frac{\partial SW}{\partial p_S} = \frac{\partial \Pi^p}{\partial p_S} + \frac{p_S - v_S + \theta_S p_B - \theta_S v_B}{(\theta_B\theta_S - 1)^2} + \theta_B \frac{p_B - v_B + \theta_B p_S - \theta_B v_S}{(\theta_B\theta_S - 1)^2} = 0.$$

As before, these FOCs define implicitly functions $p_B^o(p_S)$ and $p_S^o(p_B)$ whose slopes in the (p_B, p_S)-space are given by:

$$\frac{\partial p_S}{\partial p_B} = \frac{-(1-\theta_B\theta_S) - \theta_S(2s - \theta_S - \theta_B)}{s - \theta_B\theta_S(\theta_B + \theta_S - s)},$$

$$\frac{\partial p_S}{\partial p_B} = \frac{s - \theta_B\theta_S(\theta_B + \theta_S - s)}{-(1-\theta_B\theta_S) - \theta_B(2s - \theta_S - \theta_B)}.$$

Observe that stability and uniqueness conditions $\theta_i < 1$ for $i \in \{B, S\}$ ensures that the expression $-(1-\theta_B\theta_S) - \theta_S(2s - \theta_S - \theta_B) < 0$. Then, sign $\left(\frac{\partial p_S}{\partial p_B}\right) \equiv$ sign $(\theta_B\theta_S(\theta_B + \theta_S - s) - s)$, which is positive if and only if $s < \frac{\theta_B\theta_S(\theta_B+\theta_S)}{1+\theta_B\theta_S}$.

Solving the system of the FOCs for prices gives:

$$p_B^o = \frac{c_B(\theta_B(\theta_B + \theta_S - s) - 1) + (s - \theta_S)(c_S - v_S - v_B(\theta_B + \theta_S - s))}{(\theta_B + \theta_S - s)^2 - 1},$$
$$p_S^o = \frac{c_S(\theta_S(\theta_B + \theta_S - s) - 1) + (s - \theta_B)(c_B - v_B - v_S(\theta_S + \theta_B - s))}{(\theta_B + \theta_S - s)^2 - 1}. \quad (76)$$

To ensure these prices are a social optimum, we verify that the SOCs hold. The Hessian matrix corresponding to the planner's optimization problem is:

$$\mathcal{H}(SW) = \begin{bmatrix} \frac{-(1-\theta_B\theta_S) - \theta_S(2s - \theta_S - \theta_B)}{(\theta_B\theta_S - 1)^2} & -\frac{s - \theta_B\theta_S(\theta_B + \theta_S - s)}{(\theta_B\theta_S - 1)^2} \\ -\frac{s - \theta_B\theta_S(\theta_B + \theta_S - s)}{(\theta_B\theta_S - 1)^2} & \frac{-(1-\theta_B\theta_S) - \theta_B(2s - \theta_S - \theta_B)}{(\theta_B\theta_S - 1)^2} \end{bmatrix}.$$

The leading principal minors are:

$D_1 = \frac{-(1-\theta_B\theta_S)-\theta_S(2s-\theta_S-\theta_B)}{(\theta_B\theta_S-1)^2} < 0$ due to the stability and uniqueness conditions.

$D_2 = -\frac{(\theta_B+\theta_S-s)^2-1}{(\theta_b\theta_s-1)^2} > 0$ when $(\theta_B + \theta_S - s)^2 - 1 < 0$, which is identical to the condition on the leading principal minor identified in Section D.1.

To compare the monopoly prices with socially optimal ones, one could deduct the explicit expression for p_B^o in (76) from p_B^* in (75). However, it is more instructive to do it by taking the difference between the equations in (9) and (13):

$$p_B^* - p_B^o = (\theta_S - s)(N_S^o - N_S^*) + N_B^*. \quad (77)$$

and prove Corollary 1.

Proof of Corollary 1. Dividing both sides of (77) by $v_S - c_S$ and denoting $\mathcal{R} \equiv \frac{v_B - c_B}{v_S - c_S} > 0$, we obtain:

$$\frac{p_B^* - p_B^o}{v_S - c_S} = (\theta_S - s)\left(\frac{(\gamma - s)\mathcal{R} + 1}{1 - (\gamma - s)^2} - \frac{(\gamma - s)\mathcal{R} + 2}{4 - (\gamma - s)^2}\right) + \frac{(\gamma - s) + 2\mathcal{R}}{4 - (\gamma - s)^2},$$

For fixed $v_S - c_S$, $\frac{p_B^* - p_B^o}{v_S - c_S} < 0$ implies

$$\mathcal{R}\Big(3(\theta_S - s)(\gamma - s) + 2 - 2(\gamma - s)^2\Big) + 2(\theta_S - s) + (\theta_S - s)(\gamma - s)^2 + (\gamma - s) - (\gamma - s)^3 < 0.$$

Let us define

$$M \equiv 3(\theta_S - s)(\gamma - s) + 2 - 2(\gamma - s)^2 = (\theta_S - s)^2 - \theta_B(\theta_S - s) - 2\theta_B^2 + 2 = x^2 - \theta_B x - 2\theta_B^2 + 2,$$

where $x = \theta_S - s$. Note that $\theta_B + \theta_S - s \in (0, 1)$ due to conditions ensuring global maximum, then $x \in (-\theta_B, 1 - \theta_B)$.

The derivative $M'(x) = 2x - \theta_B$ gives us a minimum point at $x^* = \frac{\theta_B}{2}$. Because $\theta_B > 0$, we consider two cases. First, if $x^* > 1 - \theta_B$, then $\min M(x) = M(1 - \theta_B) = 3(1 - \theta_B) > 0$ due to $\theta_B < 1$. Second, if $x^* \in (0, 1 - \theta_B)$, then $x^* < 1 - \theta_B$ and $\min M(x) = M(x^*) = 2 - \frac{9}{4}\theta_B^2$. Because $x^* < 1 - \theta_B$, $\theta_B < \frac{2}{3}$, and hence $M(x^*) > 2 - \frac{9}{4} \cdot \frac{4}{9} = 1 > 0$.

From the above analysis, $M > 0$, and therefore, $p_B^* < p_B^o$ if and if only:

$$\mathcal{R} < \tilde{\mathcal{R}} := -\frac{(\theta_S - s)(2 + (\gamma - s)^2) + (\gamma - s)(1 - (\gamma - s)^2)}{3(\theta_S - s)(\gamma - s) + 2(1 - (\gamma - s)^2)}.$$

We can solve for s as a function of $\mathcal{R}$. We get:

$$\frac{-2\theta_B(\theta_B + \theta_S) - \sqrt{-8\theta_B^2 + (9\theta_B^2 - 8)\,\mathcal{R}^2 - 2\theta_B\mathcal{R} + 9} - R(\theta_B - 2\theta_S) + 3}{2(\mathcal{R} - \theta_B)}.$$

The critical threshold $\tilde{\mathcal{R}} < 1$ because $\tilde{\mathcal{R}} - 1 = \frac{-(2-(\gamma-s))(1+\gamma-s)(1-\theta_B)}{M} < 0$. Additionally, note that $\mathcal{R} \in (0, \infty)$, then the interval $(0, \tilde{\mathcal{R}})$ is non-empty if $\theta_S < s - \frac{(\gamma-s)(1-(\gamma-s)^2)}{2+(\gamma-s)^2}$. Otherwise, it is empty and $p_B^* > p_B^o$ always holds. ■

E Investments in stand-alone benefits

E.1 Uniform distribution example

Here we assume that stand-alone values follow the functional forms $v_B(e_B) = \hat{v}_B + v_B\sqrt{e_B}$ and $v_S(e_S) = \hat{v}_S + v_S\sqrt{e_S}$. The rest of the parametrizations are the same as before.

For a monopoly platform, the first-order conditions are:

$$\begin{aligned}
\frac{\partial \Pi^p(N_B, N_S, e_B, e_S)}{\partial N_B} &= (\gamma - s)N_S + \hat{v}_B + v_B\sqrt{e_B} - c_B - 2N_B = 0, \\
\frac{\partial \Pi^p(N_B, N_S, e_B, e_S)}{\partial N_S} &= (\gamma - s)N_B + \hat{v}_S + v_S\sqrt{e_S} - c_S - 2N_S = 0, \\
\frac{\partial \Pi^p(N_B, N_S, e_B, e_S)}{\partial e_B} &= \frac{N_B v_B}{2\sqrt{e_B}} - (e_B + e_S) = 0, \\
\frac{\partial \Pi^p(N_B, N_S, e_B, e_S)}{\partial e_S} &= \frac{N_S v_S}{2\sqrt{e_S}} - (e_B + e_S) = 0. \qquad (78)
\end{aligned}$$

Solving these equations yields:

$$\begin{aligned}
N_B^*(e_B, e_S) &= \frac{(\gamma - s)(\hat{v}_S + v_S\sqrt{e_S} - c_S) + 2(\hat{v}_B + v_B\sqrt{e_B} - c_B)}{4 - (\gamma - s)^2},\\
N_S^*(e_B, e_S) &= \frac{(\gamma - s)(\hat{v}_B + v_B\sqrt{e_B} - c_B) + 2(\hat{v}_S + v_S\sqrt{e_S} - c_S)}{4 - (\gamma - s)^2},\\
e_B^*(N_B, N_S) &= \sqrt[3/2]{\frac{N_B^3 v_B^3}{2(N_B^2 v_B^2 + N_S^2 v_S^2)}},\\
e_S^*(N_B, N_S) &= \sqrt[3/2]{\frac{N_S^3 v_S^3}{2(N_B^2 v_B^2 + N_S^2 v_S^2)}}.
\end{aligned} \tag{79}$$

Similarly, the first-order conditions for the social planner are:

$$\begin{aligned}
\frac{\partial SW(N_B, N_S, e_B, e_S)}{\partial N_B} &= (\gamma - s)N_S + \hat{v}_B + v_B\sqrt{e_B} - c_B - N_B = 0,\\
\frac{\partial SW(N_B, N_S, e_B, e_S)}{\partial N_S} &= (\gamma - s)N_B + \hat{v}_S + v_S\sqrt{e_S} - c_S - N_S = 0,\\
\frac{\partial SW(N_B, N_S, e_B, e_S)}{\partial e_B} &= \frac{N_B v_B}{2\sqrt{e_B}} - (e_B + e_S) = 0,\\
\frac{\partial SW(N_B, N_S, e_B, e_S)}{\partial e_S} &= \frac{N_S v_S}{2\sqrt{e_S}} - (e_B + e_S) = 0.
\end{aligned} \tag{80}$$

These conditions then solve:

$$\begin{aligned}
N_B^o(e_B, e_S) &= \frac{(\gamma - s)(\hat{v}_S + v_S\sqrt{e_S} - c_S) + (\hat{v}_B + v_B\sqrt{e_B} - c_B)}{1 - (\gamma - s)^2},\\
N_S^o(e_B, e_S) &= \frac{(\gamma - s)(\hat{v}_B + v_B\sqrt{e_B} - c_B) + (\hat{v}_S + v_S\sqrt{e_S} - c_S)}{1 - (\gamma - s)^2},\\
e_B^o(N_B, N_S) &= \sqrt[3/2]{\frac{N_B^3 v_B^3}{2(N_B^2 v_B^2 + N_S^2 v_S^2)}},\\
e_S^o(N_B, N_S) &= \sqrt[3/2]{\frac{N_S^3 v_S^3}{2(N_B^2 v_B^2 + N_S^2 v_S^2)}}.
\end{aligned} \tag{81}$$

E.2 Truncated exponential distribution example

Another example we consider is that outside options o_i drawn from a truncated exponential distribution $G_i(o_i) \equiv \frac{1-e^{-\lambda o_i}}{1-e^{-\lambda \bar{o}_i}}$ with support $[0, 10]$ and parameter $\lambda = 1$. The rest of the parametrizations are the same as above.

In this setting, system (1) becomes:

$$p_B = \hat{v}_B + \sqrt{e_B} v_B + N_S \theta_B + \ln(1 - (1 - e^{-10})N_B), \qquad p_S = \hat{v}_S + \sqrt{e_S} v_S + N_B \theta_S + \ln(1 - (1 - e^{-10})N_S).$$

For a monopoly platform, the first-order conditions are:

$$\frac{\partial \Pi^p(N_B,N_S,e_B,e_S)}{\partial N_B} = (\gamma - s)N_S + \hat{v}_B + v_B\sqrt{e_B} - c_B + \ln(1-(1-e^{-10})N_B) - \frac{N_B(1-e^{-10})}{1-N_B(1-e^{-10})} = 0,$$

$$\frac{\partial \Pi^p(N_B,N_S,e_B,e_S)}{\partial N_S} = (\gamma - s)N_B + \hat{v}_S + v_S\sqrt{e_S} - c_S + \ln(1-(1-e^{-10})N_S) - \frac{N_S(1-e^{-10})}{1-N_S(1-e^{-10})} = 0,$$

$$\frac{\partial \Pi^p(N_B,N_S,e_B,e_S)}{\partial e_B} = \frac{N_B v_B}{2\sqrt{e_B}} - (e_B+e_S) = 0,$$

$$\frac{\partial \Pi^p(N_B,N_S,e_B,e_S)}{\partial e_S} = \frac{N_S v_S}{2\sqrt{e_S}} - (e_B+e_S) = 0.$$

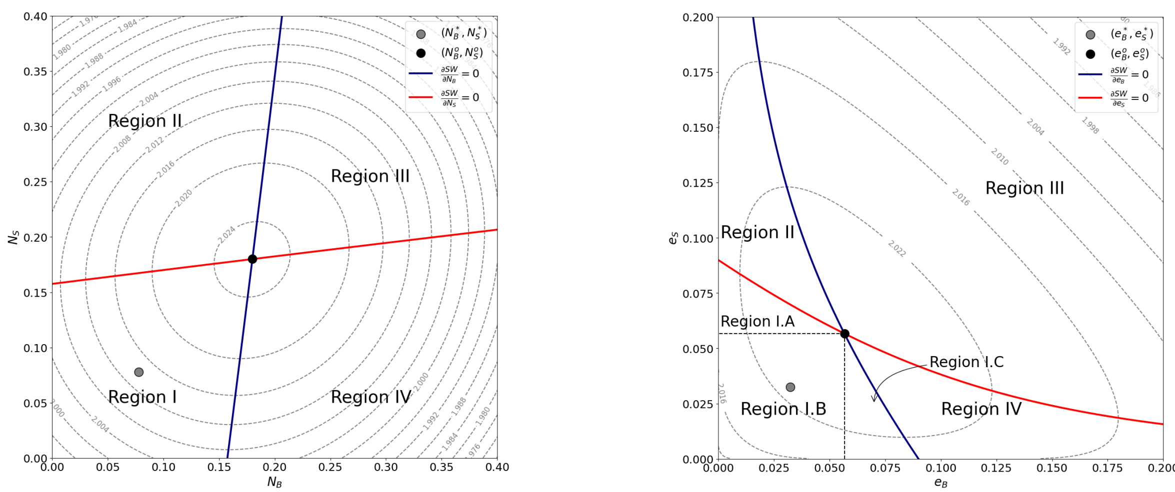
For the social planner, the first-order conditions are:

$$\frac{\partial SW(N_B,N_S,e_B,e_S)}{\partial N_B} = (\gamma - s)N_S + \hat{v}_B + v_B\sqrt{e_B} - c_B + \ln(1-(1-e^{-10})N_B) = 0,$$

$$\frac{\partial SW(N_B,N_S,e_B,e_S)}{\partial N_S} = (\gamma - s)N_B + \hat{v}_S + v_S\sqrt{e_S} - c_S + \ln(1-(1-e^{-10})N_S) = 0,$$

$$\frac{\partial SW(N_B,N_S,e_B,e_S)}{\partial e_B} = \frac{N_B v_B}{2\sqrt{e_B}} - (e_B+e_S) = 0,$$

$$\frac{\partial SW(N_B,N_S,e_B,e_S)}{\partial e_S} = \frac{N_S v_S}{2\sqrt{e_S}} - (e_B+e_S) = 0.$$

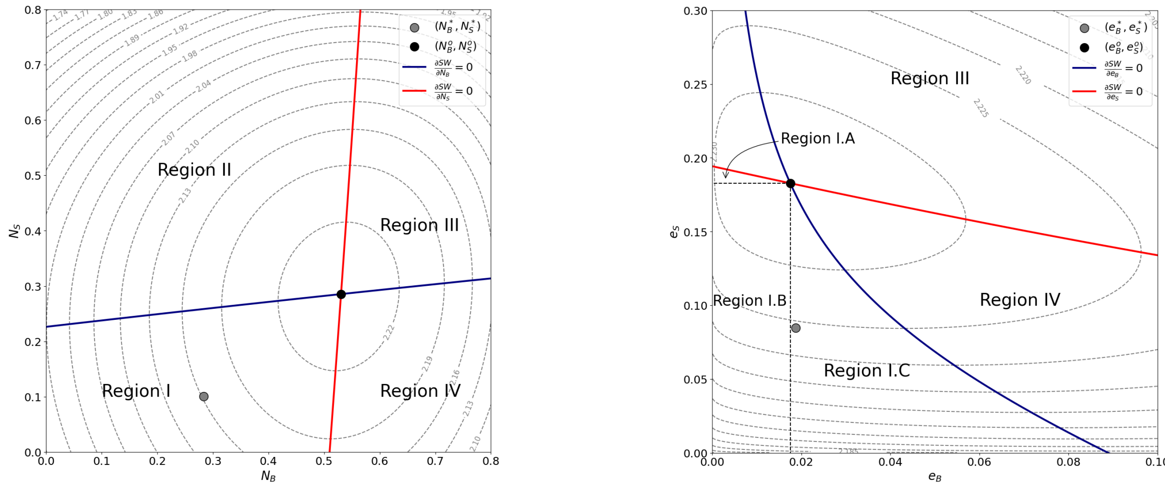


(a) Under-participation on both sides

(b) Underinvestment on both sides

Figure 11: Private and socially optimal participation and investments in stand-alone benefits under truncated exponential distributions (symmetric distortions)

Figures 11 and 12 illustrate the private and socially optimal participation and investment levels in the example where outside options follow truncated exponential distributions. The level curves represent combinations of investments (e_B, e_S) that yield the same welfare level; the pair (e_B^o, e_S^o) indicates the social optimum, while (e_B^*, e_S^*) is the private optimum. In Figure 11, we set the parameters to the following values: $s = 0.05$, $\theta_B = 0.1$, $\theta_S = 0.1$, $c_B = 0$, $\hat{v}_B = 0.1$, $v_B = 0.3$, $\hat{v}_S = 0.1$, $v_S = 0.3$, and $c_S = 0$. In Figure 12, parameters are $s = 0.05$, $\theta_B = 0.1$, $\theta_S = 0.1$, $c_B = 0$,



(a) Under-participation on both sides (b) Overinvestment on the B side, under on the S

Figure 12: Private and socially optimal participation and investments in stand-alone benefits under truncated exponential distributions (asymmetric distortions)

$\hat{v}_B = 0.7$, $v_B = 0.1$, $\hat{v}_S = 0$, $v_S = 0.6$, and $c_S = 0$. The above derivations and corresponding figures of the truncated exponential distribution support Proposition 3 and confirm the robustness of our findings.

F Per-transaction fees

This subsection shows that, under the model with heterogeneous outside options, the result in Proposition 1(i) continues to hold regardless of whether per-transaction fees or participation prices are used, whereas Proposition 1(ii) is weaker under participation prices compared to transaction fees.

Under transaction fees, an agent's utility from joining the platform is:

$$u_B = v_B + \big(\theta_B(e_B) - f_B\big)N_S, \qquad u_S = v_S + \big(\theta_S(e_S) - f_S\big)N_B.$$

The implied transaction fees are therefore:

$$f_B = \frac{v_B + \theta_B(e_B)N_S - \varphi_B(N_B)}{N_S}, \qquad f_S = \frac{v_S + \theta_S(e_S)N_B - \varphi_S(N_S)}{N_B}. \tag{82}$$

Substituting these expressions into the profit function yields:

$$\begin{aligned}
\Pi^p(N_B, N_S) &= N_B N_S(f_B + f_S) - c_B N_B - c_S N_S - sN_B N_S - C(e_B + e_S) \\
&= N_B N_S \left(\frac{v_B + \theta_B(e_B)N_S - \varphi_B(N_B)}{N_S} + \frac{v_S + \theta_S(e_S)N_B - \varphi_S(N_S)}{N_B} \right) \\
&\quad - sN_B N_S - c_B N_B - c_S N_S - C(e_B + e_S) \\
&= N_B\big(\underbrace{v_B + \theta_B(e_B)N_S - \varphi_B(N_B)}_{\text{subscription fee}} - c_B\big) + N_S\big(\underbrace{v_S + \theta_S(e_S)N_B - \varphi_S(N_S)}_{\text{subscription fee}} - c_S\big) \\
&\quad - sN_B N_S - C(e_B + e_S) \\
&= N_B(p_B - c_B) + N_S(p_S - c_S) - sN_B N_S - C(e_B + e_S) \\
&= \text{Equation (4)}.
\end{aligned}$$

The above derivation shows that solving the optimization problem in terms of participation rates leads to the same conclusion regardless of whether participation prices or per-transaction fees are used. Therefore, for fixed e_B, e_S, Proposition 1(i) holds for the per-transaction fees approach that both sides of the market exhibit under-participation.

However, when considering distortions in per-transaction fees, the stability condition changes, leading to a different conclusion from Proposition 1(ii). Indeed, as noted by Jullien et al. (2021), per-transaction fees are unambiguously higher than the social optimum. This can be confirmed directly from Proposition 1 without using the gradient vector in the (f_B, f_S)-space.

For fixed e_B, e_S, the system in (82) can be rewritten as $f_i = \frac{v_i - \varphi_i(N_i)}{N_j} + \theta_i$ for $i \in \{B, S\}$. By Proposition 1, there is under-participation on both sides, i.e., $N_B^o > N_B^*$ and $N_S^o > N_S^*$. Since $\varphi_i(\cdot)$ is increasing, this implies $v_i - \varphi_i(N_i^*) > v_i - \varphi_i(N_i^o)$, and hence $f_i^* > f_i^o$ for all $i \in \{B, S\}$.

While this confirms the observation in Jullien et al. (2021), our analysis further identifies the factors that lead to sufficiently high per-transaction fees. Specifically, the distortion in per-transaction fees can be decomposed into two effects: one arising from participation price distortions and one from participation rate distortions, because $f_i = \frac{p_i}{N_i}$. From Proposition 1 and Corollary 1 on participation prices, there may be cases where participation prices are too low on one side, but per-transaction fees are never too low. This implies that the effect of participation rate distortions always offsets the effect of participation price distortions. This observation was unclear in the literature on the analysis of distortions in per-transaction fees.

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