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Mergers and R&D Investment: A Unified Approach*

José L. Moraga-González[†] Evgenia Motchenkova[‡]

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Abstract

We investigate the impact of mergers on R&D incentives within a framework of R&D competition where effort can influence both the probability of innovation and the payoff conditional on success. Our framework nests the results of two classes of existing models and reveals assumptions that are restrictive. In models where R&D effort increases the probability of innovation but does not directly affect the payoff upon success, we show that the assumption of zero payoff upon innovation failure is restrictive. In models where R&D effort influences the payoff conditional on success, but not the probability of success itself, the assumption of deterministic innovation success (i.e., a success probability of one) is similarly restrictive. Across both modeling approaches, we offer a novel insight: the shape of investment costs, and by implication the pre-merger level of innovation, can be pivotal in determining whether a merger strengthens or weakens firms' incentives to invest in R&D. In an extensions section, we further examine the role of R&D input and output synergies, firm asymmetries, as well as the implications for consumer surplus.

JEL Classification: K21, L13, L40

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1 Introduction

The impact of mergers on innovation has been an important concern for antitrust authorities for at least two decades. For example, before the acquisition of *Sun Microsystems* by *Oracle* was cleared in the US and the EU in 2010, the European Commission investigated innovation concerns in areas such as cloud computing, database management, and open-source software development. Another example is the acquisition of *Monsanto* by *Bayer* in 2018, which underwent extensive antitrust investigation due to concerns about its potential impact on research and development of genetically modified seeds, agricultural chemicals, and digital farming technologies. A third case in point is the acquisition of *Celgene Corporation* by *Bristol-Myers Squibb*. Before the merger was approved in 2019, the Federal Trade Commission and the European Commission conducted thorough investigations to assess the risk of innovation loss resulting from discontinuation, delay, or redirection of overlapping drug development pipelines.

This paper studies the implications of mergers for R&D within a general two-stage model of R&D and product-market competition, in which firms first invest in R&D and then compete in the marketplace. A higher R&D effort may either increase a firm's innovation success probability, a firm's payoff conditional on innovation success, or both. Conceptually, our reduced-form approach is flexible enough to accommodate a variety of product-market environments, including different competitive regimes and the possibility that firms offer multiple products. In the bulk of our analysis, however, we focus on standard single-product models to illustrate our main results on when mergers increase or decrease R&D investment.

Our framework nests two broad classes of existing models. The first class assumes a stochastic R&D process where effort raises the probability of innovation success, while the size of the innovation (a cost reduction, a quality improvement, or a new product) is independent of effort. Most contributions in this class (Federico, Langus and Valletti, 2017, 2018; Denicolò and Polo, 2018; Jullien and Lefouili, 2020) study R&D for entry, where firms earn positive payoffs only if the project succeeds, with the exception of Federico et al. (2018), who allow for some profits upon failure. The second class assumes a deterministic R&D process where any effort level leads to innovation success with probability one, and R&D only affects the magnitude of the cost reduction or quality improvement (Motta and Tarantino, 2021, Section 3.1; Bourreau, Jullien and Lefouili, 2025, online Appendix).[1] For a recent comprehensive survey of this literature, see Lefouili and Madio (2025).

The main contribution of our paper is a more general characterization of the conditions under which a merger increases or decreases R&D investment. In doing so, we show that the existing literature offers only a partial view of the problem, largely because it relies on two

[1]The main models in Motta and Tarantino (2021) and Bourreau et al. (2025) are *one-stage* games where firms choose prices (or quantities) and R&D efforts simultaneously. Our two-stage game approach is different and captures, arguably, more realistic scenarios where R&D is a long-run variable and prices (or quantities) are short-run.

restrictive modelling assumptions and on a narrow set of market structures, functional forms, or numerical simulations. Moreover, we demonstrate that our framework can be readily extended to incorporate both R&D input and output synergies, allow for firm asymmetries, as well as to assess the effects of mergers on consumer surplus.

Specifically, we first show that, in models of stochastic R&D where investment effort increases a firm's probability of innovation but does not directly affect its payoff conditional on success, the assumption that firms earn positive profits only upon successful innovation plays a crucial role (cf. Federico et al., 2017; Denicolò and Polo, 2018; Jullien and Lefouili, 2020). If firms can earn positive profits even in the event of innovation failure by entering or remaining in the market with the *status quo* cost or quality (like in Federico et al., 2018; see also Mukherjee, 2022), then the pre-merger level of innovation (and thus the shape of the success probability and R&D cost functions) may become a key determinant of whether a merger increases or decreases R&D.

Secondly, we show that, in models where R&D effort influences firms' payoffs conditional on innovation success, e.g. because the degree to which a firm lowers its marginal cost or improves its product quality depends on that effort, the common assumption that innovation success occurs with probability one is restrictive (cf. Motta and Tarantino, 2021, Section 3.1; Bourreau et al., 2025, online Appendix). When success is uncertain, the pre-merger level of innovation (and thus the shape of the R&D cost function) may again play a pivotal role in determining whether a merger enhances or dampens R&D incentives.

Finally, our broader framework highlights the difficulty of simultaneously modelling the impact of R&D effort on both the probability of innovation and the payoff conditional on success. We derive a general condition that characterizes whether a merger raises or lowers R&D investment. The complexity of this condition may help explain why the existing literature has typically focused on only one of these two margins at a time.

Whether a merged entity has stronger or weaker incentives to invest than a stand-alone firm hinges on comparing the pre-merger Arrow *replacement effect* with its post-merger counterpart, adjusted for the externality on the partner firm.[2] We identify three key channels through which a merger modifies the incentives for R&D investment relative to a stand-alone firm: (i) the anticipation of post-merger price coordination; (ii) the internalization of an innovation externality arising from the increased probability of innovation success; and (iii) the internalization of an innovation externality stemming from the introduction of superior products or the production of goods at lower marginal costs. The key to our new results lies in recognizing that, when innovation outcomes are stochastic, the magnitude of these replacement effects generally differs across

[2] In Arrow's (1962) original paper, the innovator obtains a patent and becomes a monopolist, so the replacement effect refers to the disincentive to innovate arising from the existence of pre-innovation profits. In our setting, we informally use the term replacement effect to refer to the difference between post- and pre-innovation profits for both the duopoly firm and the post-merger entity.

two distinct states of the world: conditional on the partner firm's innovation effort succeeding and conditional on it failing. As a consequence, innovation incentives aggregate the effects across these two states, with weights that depend on the steepness of the R&D cost function. In many relevant cases, the underlying forces are positive in one state and negative in the other, so that the relative weights attached to success and failure are critical in determining the overall effect of a merger on R&D. Neglecting this two-state structure overlooks a central source of forces that drive our findings.

We now describe in more detail the results we obtain for the different classes of models outlined above. The first class of models refers to models where R&D effort only impacts the likelihood of innovation success, while keeping the payoff conditional on innovation success independent of R&D effort. In this class of models, we identify four possible outcomes. In the first outcome, the replacement effect post-merger net of the externality on the partner firm is smaller than pre-merger, both conditional on partner success and failure. In such a case, the incentives to innovate are stronger post-merger than pre-merger and, hence, a merger definitely spurs innovation. We observe this outcome in a logit model of price competition with differentiated products and quality-enhancing innovation, provided that demand is not very price sensitive. This outcome also arises in a Hotelling model with quality-enhancing innovation and price competition (see e.g. Gilbert and Katz, 2022). This result that a merger may unambiguously lead to more innovation generalizes that in Jullien and Lefouili (2020), where it is assumed that failure to innovate results in zero profits.

The second outcome arises when the opposite configuration holds: the post-merger replacement effect net of the externality on the partner firm is larger than its pre-merger counterpart, both conditional on partner success and on partner failure. In such a case, a merging firm has weaker incentives to invest than an individual firm pre-merger, so the merger unambiguously reduces the partner firms' incentives to invest in innovation. This outcome occurs, for instance, in the logit model with quality-enhancing innovation when demand is sufficiently price sensitive, as well as in the standard logit model when innovation is meant to reduce marginal costs. The same result arises in a market for horizontally differentiated products with the Singh and Vives (1984) system of demands and cost-reducing innovation, regardless of whether firms compete in prices or in quantities. We also obtain this outcome in a market for vertically differentiated products with the quality-augmented demand system of Sutton (1997, 2001), quality-enhancing innovation and Cournot competition, provided that the quality difference between the low and high variants is sufficiently large. Finally, we show that it also arises in a horizontally differentiated market with Singh and Vives (1984) demands, demand-enhancing innovation and Cournot competition. This result—that a merger may lead to lower R&D investment—generalizes the findings of Federico et al. (2017) and Denicolò and Polo (2018), although, again, in their models a failure to innovate results in zero profits.

The third type of outcome arises when the merged entity invests more than a stand-alone firm in the pre-merger market if and only if the pre-merger level of innovation is sufficiently low, or, equivalently, the marginal cost of R&D effort is high enough. This occurs when the merged entity's incentive to innovate is too weak relative to that of the individual firm conditional on partner success, but too strong conditional on partner failure. In such a case, the "failure" effect dominates when the marginal cost of effort is high, because the pre-merger level of innovation is then low and partner failure is relatively likely. This outcome arises in the standard logit framework when innovation increases quality, and also when it reduces marginal costs, provided that demand is sufficiently price sensitive. We also observe it in Sutton's model of Cournot competition with quality-differentiated products mentioned above, provided that the quality gap between the low- and high-quality variants is small enough.

The last type of outcome is the mirror image of the one just described: the merged entity invests more than a stand-alone firm in the pre-merger market if and only if the pre-merger level of innovation is sufficiently high, or, equivalently, the marginal cost of effort is low enough. As before, this outcome arises from conflicting incentives, depending on whether the partner's R&D effort is successful or not. We show that this case arises naturally in the Mussa and Rosen (1978) model of vertical product differentiation with quality-enhancing innovation and price competition (see also Motta, 1993).

The second class of models considers settings in which R&D effort affects the payoff conditional on innovation success, typically because the degree to which firms reduce marginal costs or improve product quality is proportional to effort, while the probability of innovation success remains independent of effort. In the literature, these models assume a fully deterministic R&D process, in which any investment results in guaranteed innovation (cf. Motta and Tarantino, 2021; Bourreau et al., 2025). We relax this assumption by allowing the R&D process to have a non-zero probability of failure. This leads to a new class of models which, fundamentally, yields results similar to those discussed for the previous class. The underlying reason is that the expected payoff expression retains the same structure: it remains a weighted average of the payoffs conditional on the partner's success and failure, net of investment costs. Accordingly, as discussed above, the merged entity's incentive to invest depends on how the *marginal* replacement effect net of the post-merger externality on the partner compares to the corresponding *marginal* replacement effect in the pre-merger setting under the two scenarios: conditional on partner failure and conditional on partner success. Crucially, whether this marginal effect is smaller or larger may differ between these two scenarios: it may be smaller in one case (e.g., conditional on partner failure), leading to stronger incentives, but larger (leading to weaker incentives) in the other (conditional on partner success). We show that, in the standard model of price competition with the Singh and Vives (1984) demand system and cost-reducing innovation, these two incentives may either align or diverge. In contrast to Motta and Tarantino (2021, Section 3.1), we demonstrate that the magnitude of the probability of innovation success is crucial. When this probability is sufficiently high (as in Motta and Tarantino, where it is set equal to one), the merged entity's incentives to invest, both conditional on either partner's success or failure, are strictly lower than those of an individual firm before the merger. Consequently, the merger leads to a reduction in innovation. However, when the probability of success is low, the merged entity's incentives to invest conditional on partner failure can exceed those of a stand-alone firm. In such cases, this effect dominates provided that investment costs are sufficiently low, leading to an increase in innovation following the merger.

In an extensions section, we analyze the impact of R&D synergies, firm asymmetries, and the consumer surplus effects of mergers. We first look at the role of R&D output synergies in shaping innovation incentives. Inspired by Farrell and Shapiro (1990), we assume that innovations developed by one division of the merged entity can be leveraged across the other division to drive broader organizational benefits for the merged entity. We show that these synergies do sometimes enhance the incentives to increase R&D of the post-merger entity, but not always, in particular when partner success is highly likely. The reason why R&D output synergies may disincentivize the merged entity to invest in R&D is that they create free-riding incentives between the divisions of the merged entity. We then examine the role of R&D input synergies. Inspired by the Research Joint Ventures (RJV) literature (e.g. d'Aspremont and Jacquemin, 1988; Kamien, Muller and Zhang, 1992; Suzumura, 1992), we model R&D input synergies as R&D spillover effects across the divisions of the merged entity. We show that these synergies increase the likelihood that mergers result in higher R&D in all circumstances.

Our framework extends naturally to mergers between asymmetric firms. In this case, we study the sign of the merged entity's R&D gradient evaluated at the pre-merger asymmetric equilibrium, which determines the direction of change in each division's investment. This analysis uncovers a rich set of possibilities: depending on parameters, a merger may increase or decrease the R&D effort of each division, and total R&D can rise even if one division cuts its effort (a "partial killer acquisition"), or fall even if one division invests more. This further underscores how crucial the pre-merger innovation profile may be for assessing the impact of mergers on R&D when firms are asymmetric.

Finally, we explore the impact of merger activity on consumer surplus. A merger induces a shift in the distribution of consumer surplus, driven not only by changes in consumer-surplus levels (through the price effects of the merger) but also by changes in the probabilities with which different innovation outcomes occur. When a merger results in a reduction of R&D, consumer surplus unambiguously falls. By contrast, when a merger increases R&D, the net effect on consumer surplus depends on the balance between the innovation and price effects of the merger. In single-product settings without synergies, the micro-founded examples provide little support for a positive effect of mergers on consumer surplus, even when consolidation raises

R&D investment. The intuition is that, in these environments, the price effects of the merger are typically strong enough to offset the gains from higher innovation effort. With R&D output synergies, consumer surplus can increase because successful innovations by one unit benefit the other unit of the merged entity. This outcome is, for example, observed in the Singh and Vives quantity-competition models. With R&D input synergies, it is more likely that mergers result in substantially higher R&D by effectively lowering the marginal cost of effort. This strengthens the innovation effect and, in turn, increases the scope for consumer-surplus gains. Finally, we show that consumer surplus may increase in multi-product environments where the overlap between the merging firms' products is limited, as illustrated by a logit example with additional non-rival products. In such cases, the adverse price effects of the merger are confined to the overlapping product segment, whereas the innovation effects apply more broadly across the firms' product portfolios.

2 Related literature

Our work is related to the growing theoretical literature on the impact of mergers on innovation. As mentioned above, this literature can be divided into two broad classes. The first comprises models in which the R&D process is uncertain: firms succeed in R&D with a probability that depends on their R&D effort, while the magnitude of the innovation (cost reduction, quality improvement, or new product) is independent of effort. Federico et al. (2017), Denicolò and Polo (2018), and Jullien and Lefouili (2020) fall into this class and study how horizontal mergers affect the incentives to develop a new product. Federico et al. (2017) and Denicolò and Polo (2018) assume homogeneous products and show that a merger between symmetric duopolists reduces R&D effort; by contrast, Jullien and Lefouili (2020) identify conditions under which a merger can spur innovation in a Hotelling model with horizontal product differentiation.[3]

These papers assume that a firm that fails to develop the new product receives a zero payoff. We relax this assumption and formulate a more general model that nests their results. More importantly, we show that a richer set of outcomes may arise: depending on the shape of the success probability and the investment cost functions, a merger may either increase or decrease R&D effort. Mukherjee (2022) and Federico et al. (2018) also relax the zero-payoff assumption. Mukherjee (2022) shows that, in a quantity-competition model with the Singh and Vives (1984) demand system and cost-reducing innovation, a merger may enhance innovation incentives, but this result is driven by cost synergies; later in the paper, we show that, in the absence of such synergies, this outcome cannot occur in that model. Federico et al. (2018) develop a more

[3]Denicolò and Polo (2018) further show that a merger can lead to more innovation when the returns to R&D decrease moderately (or when the probability of failure is log-concave in R&D investment). In that case, the merged entity optimally concentrates R&D in a single lab, internalising the cost of innovation duplication that arises under competition.

general two-stage oligopoly framework for the analysis of stochastic innovation and, due to analytical intractability, rely on numerical simulations based on price competition with Sutton-type, nested-logit, and CES demand systems with quality-enhancing innovation, finding that mergers always reduce innovation incentives. Relative to these contributions, we provide an analytical characterization of the possible merger outcomes in Proposition 2, show that the range of outcomes is broader, and offer micro-founded examples for each case. In this sense, our paper delivers a more complete view of the impact of mergers on R&D incentives.

The second broad class of models considers settings where R&D effort affects the payoff conditional on innovation, while the probability of innovation success is independent of effort. Following the seminal contribution of d'Aspremont and Jacquemin (1988), further developed by Suzumura (1992) and Kamien, Muller and Zhang (1992), these models typically describe settings in which firms invest in R&D to lower marginal costs and compete in the product market. Motta and Tarantino (2021) analyze the impact of mergers on process innovation in this framework. Starting from a simultaneous-moves model where prices and investments are chosen together, they show that, in the absence of efficiency gains, mergers reduce cost-reducing R&D, while increases in R&D are possible with quality-enhancing investments for certain demand specifications. In their Section 3.1, they extend the analysis to a two-stage setup and, for two specific demand structures (based on Shubik-Levitan and Salop), report simulation evidence that investment post-merger is lower than pre-merger. Also relevant is Bourreau et al. (2025), who provide a detailed characterization of the effects of mergers on R&D in a model of deterministic innovation and highlight the roles of the demand-expansion and innovation-diversion effects. As in Motta and Tarantino (2021), they first study a simultaneous choice of prices and R&D and show that mergers can increase R&D with demand-enhancing innovation but not with cost-reducing innovation. In an extension, they consider a sequential setup in which firms invest in demand-enhancing R&D before competing in prices, and argue that the incentives to raise R&D after a merger can be stronger than in the simultaneous-move case. A common assumption in this line of work is that the R&D process is deterministic in the sense that effort leads to innovation success with probability one. We relax this assumption and show in Proposition 3 that the likelihood of success may be crucial for the nature of the results; fixing this probability to one, therefore, yields an incomplete view of the impact of mergers on innovation.

Finally, our work is related to the literature on the relationship between competition intensity and innovation. This literature shows that the Arrow replacement effect for a monopolist need not be larger than for a firm facing competition, and our main results build on this insight. For instance, Chen and Schwartz (2013) study incentives to introduce new products and show that the gain from such an innovation can be larger for a monopolist than for a firm facing competition from sellers of the old product.[4] Greenstein and Ramey (1998) analyze process innovation

[4]Their framework differs from ours in that there is only one product on which innovation can take place: a successful innovation brings a new product to the market and does not affect the quality of the initial product.

when new products are vertically differentiated from older ones and show that, under certain conditions, competition and monopoly in the old product market can yield identical returns from innovation, while a monopolist threatened by entry may have strictly stronger incentives to innovate. Relatedly, Aghion et al. (2005) document an inverted-U relationship between product-market competition and innovation, arising from the contrast between an "escape-competition" effect (competition raising the incremental profit from innovation for leaders) and a "catching-up" effect (competition weakening the incentives of laggard firms). Our framework provides a unifying perspective on these forces by explicitly distinguishing two states of the world: when rival innovation succeeds, firms have incentives to catch up, whereas when rival innovation fails, they have incentives to escape ahead. By tracing how mergers reshape innovation incentives in each of these two states, and how these state-contingent effects are weighted in equilibrium, our analysis helps rationalise why different empirical studies (Aghion et al., 2005; Hashmi, 2013; Correa and Ornaghi, 2014; Beneito et al., 2017) find decreasing, increasing, or inverted-U patterns in the relationship between competition and innovation.

The remainder of the paper is structured as follows. Section 3 presents the general model, identifies the key externalities, and compares the optimal investment in the merger scenario with that in the non-cooperative equilibrium. Section 4.1 zooms in on the conditions under which mergers spur or discourage innovation when the probability of success is endogenous but innovation outcomes are fixed, while Section 4.2 provides analogous analysis for models in which innovation outcomes are endogenous but the success probability is fixed. Section 5 analyses synergies, asymmetries, and the effects of mergers on consumer surplus. Section 6 concludes. Proofs and detailed examples illustrating our results are collected in the Appendix.

3 The model and preliminary intuition

3.1 Model description and assumptions

We consider a duopoly market with symmetric firms, which we index by i and j.[5] Firms interact in the market during two stages. In the first stage, firms invest in R&D. Let x_i and x_j the amounts firms i and j put in R&D. In the second stage, upon observing the outcomes of their R&D investments, firms compete in the market.

In the innovation stage, we assume that if a firm, say i, invests in R&D an amount $x_i > 0$, then it costs the firm $C(x_i)$, with $C' > 0$ and $C'' > 0$. Investment need not result in innovation success and we denote by $\beta(x_i) \in (0, 1]$ the probability of successful innovation, with $\beta' > 0$ and $\beta'' < 0$.[6] In the competition stage, firms compete in the market to sell their products. To keep the model as general as possible, we remain agnostic about the exact nature of competition in the

[5] We examine the impact of firms' asymmetries in Section 5.2.

[6] Further, we assume that $\beta(0) = 0$ and $\lim_{x \to \infty} \beta(x) = 1$.

second stage and formulate reduced-form payoffs corresponding to the possible subgames that ensue after the innovation stage is over.[7] The following table describes the possible subgames and the notation we use to denote the corresponding firms' payoffs:

		Firm j	
		$Success(s)$	$Failure(f)$
Firm i	$Success(s)$	$\pi_i^{ss}(x_i; x_j)$, $\pi_j^{ss}(x_j; x_i)$	$\pi_i^{sf}(x_i)$, $\pi_j^{fs}(x_i)$
	$Failure(f)$	$\pi_i^{fs}(x_j)$, $\pi_j^{sf}(x_j)$	π_i^{ff}, π_j^{ff}

Table 1: Firms' conditional payoffs under product competition

Note that the super-indices that describe a given subgame are ordered by player. For example, in the subgame where firm i's innovation is successful while firm j's is not, we index the payoff corresponding to firm i by "sf" to indicate that this is the payoff of a successful innovator competing with a failing one, and, likewise, the payoff corresponding to firm j is indexed by "fs". Hence, the first entry of the super-index refers to the innovation outcome of the player in question and the second entry to the innovation outcome of the rival player.

An important aspect of this formulation is that, provided the firms' R&D projects are successful, it allows for the conditional payoffs to depend on firms' investment levels. This is often the case in models of cost-reducing and product innovations. To ensure firms have incentives to invest in R&D and that their problem is strictly concave, we next make some natural assumptions on these conditional payoffs.[8]

Assumption 1. Firms' conditional payoffs.

i. Conditional payoffs of a firm i rank as follows: $\pi_i^{ss}(x_i, x_j) \geq \pi_i^{fs}(x_j)$ and/or $\pi_i^{sf}(x_i) \geq \pi_i^{ff}(x_j)$, with one of them being a strict inequality.

ii. Firm i's conditional payoffs $\pi_i^{ss}(\cdot)$ and $\pi_i^{sf}(\cdot)$ are continuous in x_i and slope as follows: $\frac{\partial \pi_i^{ss}(\cdot)}{\partial x_i} \geq 0$ and/or $\frac{\partial \pi_i^{sf}(\cdot)}{\partial x_i} \geq 0$, with one of them being a strict inequality.

iii. $\beta(x)$ is sufficiently concave, and $C(x)$ sufficiently convex.

Assumptions 1(i) and (ii) are relatively weak, only requiring that innovations increase some of the conditional payoffs of the firms. They hold in all the examples we use to illustrate our results (see Appendix B). Assumption 1(iii) ensures the strict concavity of a firm's payoff, which guarantees the existence of a Nash equilibrium in pure strategies.

[7]Later in the paper, to illustrate our main Propositions, we use various examples based on single-product firms engaging in quantity- or price-setting games. However, an advantage of the reduced-form approach is that our model can capture under-researched situations where, for example, firms sell multiple products (see, in particular, Section 5.3).

[8]As an example, suppose single-product firms and price competition in the second-stage. Then, anticipating the second-stage Nash equilibrium of firms whose R&D investments are successful, i.e. $\mathbf{p}(\mathbf{x}) = (p_i(x_i; x_j), p_j(x_i; x_j))$, the first-stage payoff is $\pi_i^{ss}(x_i; \mathbf{p}(\mathbf{x}))$, which we write more shortly as $\pi_i^{ss}(x_i; x_j)$.

3.2 Pre-merger market equilibrium

The innovation stage payoff of a firm i investing x_i in R&D is given by:

$$\mathbb{E}\pi_i(x_i; x_j) = \beta_i(x_i)\left[\beta_j(x_j)\pi_i^{ss}(x_i, x_j) + (1-\beta_j(x_j))\pi_i^{sf}(x_i)\right] \\ +(1-\beta_i(x_i))\left[\beta_j(x_j)\pi_i^{fs}(x_j) + (1-\beta_j(x_j))\pi_i^{ff}\right] - C(x_i),$$

The bracket in the first line of this expression is firm i's payoff conditional on its innovation project being successful; this payoff depends on the rival's innovation outcome. The bracket in the second line gives firm i's payoff conditional on failing to innovate.

Assuming the equilibrium is interior, it is given by the solution to the system of first-order conditions (FOCs) for profits-maximization:

$$\underbrace{\frac{\partial \beta_i(\cdot)}{\partial x_i}\left[\beta_j(\cdot)\left[\pi_i^{ss}(x_i, x_j) - \pi_i^{fs}(x_j)\right] + (1-\beta_j(\cdot))\left[\pi_i^{sf}(x_i) - \pi_i^{ff}\right]\right]}_{\text{marginal gains from increasing success probability}} \tag{1}$$

$$+\underbrace{\beta_i(\cdot)\left[\beta_j(\cdot)\frac{\partial \pi_i^{ss}(x_i, x_j)}{\partial x_i} + (1-\beta_j(\cdot))\frac{\partial \pi_i^{sf}(x_i)}{\partial x_i}\right]}_{\text{marginal gains from increasing conditional payoffs}} - C'(x_i) = 0, \quad \text{and similarly for firm } j.$$

This FOC says that a firm should continue to increase its R&D investment till the marginal revenue equals the marginal cost of investment. An increase in x_i has two effects on the expected payoff of a firm. On the one hand, it increases the probability of innovation. The first line of this FOC describes this effect, keeping constant conditional payoffs. On the other hand, it increases a firm's payoff conditional on innovation. The second line of this FOC describes this second effect, keeping constant the probability of innovation. For later use, let x^* denote the pre-merger symmetric equilibrium R&D effort.

To the best of our knowledge, the literature has not presented models in which these two effects of increasing innovation effort are in place together. There is a group of papers, namely, Federico et al. (2017, 2018), Denicolò and Polo (2018), Jullien and Lefouili (2020) and Mukherjee (2022), focusing on stochastic R&D in which the first effect is examined but, owing to their assumptions on the constancy of the conditional payoffs, the second effect is shut down. Likewise, there is a second group of papers of deterministic R&D, namely Motta and Tarantino (2021), Bourreau and Jullien (2018), and Bourreau et al. (2025), where success probabilities are exogenously set to 1 and hence the first effect is shut down by construction.

3.3 Mergers

We now examine the impact of mergers on R&D investment. Consider now that firms i and j merge and assume it is optimal for the merged entity to maintain the two research units of the constituent firms. A merger has two important implications. On the one hand, a merger results in the monopolisation of the product market, thereby, as it is by now well known, creating upward pressure on prices. On the other hand, a merger results in the monopolisation of the innovation market. These two effects are related to one another and a complete understanding of the impact of mergers on innovation ought to take both of them into account.

We capture the price effects of mergers by specifying reduced-form conditional payoffs that are higher than pre-merger and to avoid notation confusion we label the monopoly payoffs with the "hat" symbol. Specifically, the conditional payoffs of the divisions within the merged entity are given in the following table:

		Division j	
		Success(s)	*Failure(f)*
Division i	*Success(s)*	$\hat{\pi}_i^{ss}(x_i; x_j), \hat{\pi}_j^{ss}(x_j; x_i)$	$\hat{\pi}_i^{sf}(x_i), \hat{\pi}_j^{fs}(x_i)$
	Failure(f)	$\hat{\pi}_i^{fs}(x_j), \hat{\pi}_j^{sf}(x_j)$	$\hat{\pi}_i^{ff}, \hat{\pi}_j^{ff}$

Table 2: Conditional payoffs for the divisions within the merged entity

In describing the payoffs in Table 2, we assume that the conditional payoffs depend on the divisions' investment levels only in the event of successful innovation. The fact that the merged entity coordinates prices in the product market implies that $\hat{\pi}_i^{ss}(x_i, x_j) \geq \pi_i^{ss}(x_i, x_j)$, $\hat{\pi}_i^{sf}(x_i) \geq \pi_i^{sf}(x_i)$, $\hat{\pi}_i^{fs}(x_j) \geq \pi_i^{fs}(x_j)$, $\hat{\pi}_i^{ff} \geq \pi_i^{ff}$ for firm i and/or j. Further, in addition to assuming also here that $\beta(x)$ is sufficiently concave, and $C(x)$ sufficiently convex, for similar reasons as in the pre-merger market, we make the following assumptions:

Assumption 2. Monopoly conditional payoffs.

i. Conditional payoffs of a division i of the merged entity satisfy: $\hat{\pi}_i^{ss}(x_i, x_j) \geq \hat{\pi}_i^{fs}(x_j)$ and/or $\hat{\pi}_i^{sf}(x_i) \geq \hat{\pi}_i^{ff}$, with one of them being strict inequality.

ii. Division i's conditional payoffs $\hat{\pi}_i^{ss}(\cdot)$ and $\hat{\pi}_i^{sf}(\cdot)$ are continuous and slope as follows: $\frac{\partial \hat{\pi}_i^{ss}(\cdot)}{\partial x_i} \geq 0$ and/or $\frac{\partial \hat{\pi}_i^{sf}(\cdot)}{\partial x_i} \geq 0$, with one of them being strict inequality.

Because the merged entity keeps running the two research labs of the constituent firms, its investment problem consists of choosing investments x_i and x_j to maximize the (joint) payoff:

$$\begin{aligned}\mathbb{E}\pi^m(x_i, x_j) = \beta_i(x_i) &\left[\beta_j(x_j)\hat{\pi}_i^{ss}(x_i, x_j) + (1 - \beta_j(x_j))\hat{\pi}_i^{sf}(x_i)\right] \\ &+ (1 - \beta_i(x_i)) \left[\beta_j(x_j)\hat{\pi}_i^{fs}(x_j) + (1 - \beta_j(x_j))\hat{\pi}_i^{ff}\right] - C(x_i) \\ &+ \beta_j(x_j) \left[\beta_i(x_i)\hat{\pi}_j^{ss}(x_i, x_j) + (1 - \beta_i(x_i))\hat{\pi}_j^{sf}(x_j)\right] \\ &+ (1 - \beta_j(x_j)) \left[\beta_i(x_i)\hat{\pi}_j^{fs}(x_i) + (1 - \beta_i(x_i))\hat{\pi}_j^{ff}\right] - C(x_j)\end{aligned}$$

This payoff is constructed as the sum of the payoffs of the merging parties. Our assumptions imply that the Hessian matrix is negative definite. Hence, assuming that an interior maximum exists, it is given by the solution of the system of FOCs. The FOC for the maximization of the profits of the merged entity with respect to x_i is given by:

$$
\begin{aligned}
FOC_i^m(x_i, x_j) \equiv & \underbrace{\frac{\partial \beta_i(\cdot)}{\partial x_i}\left[\beta_j(\cdot)\left[\hat{\pi}_i^{ss}(x_i, x_j) - \hat{\pi}_i^{fs}(x_j)\right] + (1-\beta_j(\cdot))\left[\hat{\pi}_i^{sf}(x_i) - \hat{\pi}_i^{ff}\right]\right]}_{\text{marginal gains from increasing success probability}} \\
& \underbrace{+\frac{\partial \beta_i(\cdot)}{\partial x_i}\left[\beta_j(\cdot)\left[\hat{\pi}_j^{ss}(x_i, x_j) - \hat{\pi}_j^{sf}(x_j)\right] + (1-\beta_j(\cdot))\left[\hat{\pi}_j^{fs}(x_i) - \hat{\pi}_j^{ff}\right]\right]}_{\text{innovation externality via success probability}} \\
& + \underbrace{\beta_i(\cdot)\left[\beta_j(\cdot)\frac{\partial \hat{\pi}_i^{ss}(x_i, x_j)}{\partial x_i} + (1-\beta_j(\cdot))\frac{\partial \hat{\pi}_i^{sf}(x_i)}{\partial x_i}\right]}_{\text{marginal gains from increasing conditional payoffs}} \\
& + \underbrace{\beta_i(\cdot)\left[\beta_j(\cdot)\frac{\partial \hat{\pi}_j^{ss}(x_i, x_j)}{\partial x_i} + (1-\beta_j(\cdot))\frac{\partial \hat{\pi}_j^{fs}(x_i)}{\partial x_i}\right]}_{\text{innovation externality via second-stage payoffs}} - C'(x_i) = 0, \quad \text{and similarly for } x_j.
\end{aligned}
\tag{2}
$$

A comparison between the FOCs pre-merger and post-merger is central to the understanding of the complexity of the impact of mergers on R&D investment. Moreover, it is key to understand why the different assumptions in the literature have led to distinct results. Comparing the post-merger FOC (2) to the pre-merger one in equation (1) leads to three important observations.

- First, because the divisions of the merged entity coordinate their prices in the market stage, the post-merger FOC involves monopoly rather than duopoly payoffs. This means that when a division of the merged entity chooses its R&D effort, it does so factoring in higher conditional payoffs than pre-merger. This is reflected in the fact that the marginal gains from investing, in lines 1 and 3 of the FOC (2), are analogous to those in the FOC (1) but with competitive payoffs replaced by monopoly payoffs.[9]

- Second, the post-merger FOC reflects the internalization of two innovation externalities that typically act in a negative direction on the incentives to invest.

  - The first is an externality arising because when division i of the merged-entity increases its R&D effort x_i, it increases its success probability, which reduces the returns from investment of its partner j (this is the second line of the FOC (2)).

[9]Higher conditional payoffs are the result of charging higher prices, or equivalently supplying lower quantities, in the second stage. When innovation takes the form of a quality improvement that shifts demand outward, a division of the merged entity evaluates this shift at monopoly prices and quantities rather than at competitive ones. As emphasized by Spence (1975) (see also Gaudin, 2025), the profitability of a given upward shift in demand depends on the output level and the price–cost margin at which the firm operates, so a merger can raise or lower investment incentives depending on whether the demand shift is stronger at low or at high quantities, a point we return to in Section 4.

– The second externality arises because an increase in division i's R&D effort, x_i, reduces the partner division j's second-stage conditional payoffs (see the last line of the FOC (2)).

The standard approach to address the question whether a merger leads to more or less investment compared to the pre-merger equilibrium consists of studying the sign of the FOC (2) evaluated at the pre-merger symmetric equilibrium x^*. This gives:

$$\begin{aligned} FOC_i^m(x^*) =& \frac{\partial \beta_i(x^*)}{\partial x_i}\left[\beta_j(x^*)\left[\hat{\pi}_i^{ss}(x^*,x^*) - \hat{\pi}_i^{fs}(x^*)\right] + (1-\beta_j(x^*))\left[\hat{\pi}_i^{sf}(x^*) - \hat{\pi}_i^{ff}\right]\right] \\ &+ \frac{\partial \beta_i(x^*)}{\partial x_i}\left[\beta_j(x^*)\left[\hat{\pi}_j^{ss}(x^*,x^*) - \hat{\pi}_j^{sf}(x^*)\right] + (1-\beta_j(x^*))\left[\hat{\pi}_j^{fs}(x^*) - \hat{\pi}_j^{ff}\right]\right] \\ &+ \beta_i(x^*)\left[\beta_j(x^*)\frac{\partial \hat{\pi}_i^{ss}(x^*,x^*)}{\partial x_i} + (1-\beta_j(x^*))\frac{\partial \hat{\pi}_i^{sf}(x^*)}{\partial x_i}\right] \\ &+ \beta_i(x^*)\left[\beta_j(x^*)\frac{\partial \hat{\pi}_j^{ss}(x^*,x^*)}{\partial x_i} + (1-\beta_j(x^*))\frac{\partial \hat{\pi}_j^{fs}(x^*)}{\partial x_i}\right] - C'(x^*). \end{aligned} \tag{3}$$

Since the FOC (1) holds (with equality) at the pre-merger market symmetric equilibrium x^*, equation (3) can be simplified to:

$$\begin{aligned} FOC_i^m(x^*) =& \frac{\partial \beta_i(x^*)}{\partial x_i}\left[\beta_j(x^*)\left[\Delta_i^{ss}(x^*,x^*) - \Delta_i^{fs}(x^*)\right] + (1-\beta_j(x^*))\left[\Delta_i^{sf}(x^*) - \Delta^{ff}\right]\right] \\ &+ \frac{\partial \beta_i(x^*)}{\partial x_i}\left[\beta_j(x^*)\left[\hat{\pi}_j^{ss}(x^*,x^*) - \hat{\pi}_j^{sf}(x^*)\right] + (1-\beta_j(x^*))\left[\hat{\pi}_j^{fs}(x^*) - \hat{\pi}_j^{ff}\right]\right] \\ &+ \beta_i(x^*)\left[\beta_j(x^*)\frac{\partial \Delta_i^{ss}(x^*,x^*)}{\partial x_i} + (1-\beta_j(x^*))\frac{\partial \Delta_i^{sf}(x^*)}{\partial x_i}\right] \\ &+ \beta_i(x^*)\left[\beta_j(x^*)\frac{\partial \hat{\pi}_j^{ss}(x^*,x^*)}{\partial x_i} + (1-\beta_j(x^*))\frac{\partial \hat{\pi}_j^{fs}(x^*)}{\partial x_i}\right]. \end{aligned} \tag{4}$$

In this expression, to shorten its length, we write $\Delta_i^{ss}(\cdot) \equiv \hat{\pi}_i^{ss}(\cdot) - \pi_i^{ss}(\cdot)$ for the additional profits accruing to division i in the subgame where both research labs succeed, arising solely from price coordination. Define $\Delta_i^{sf}(\cdot)$, $\Delta_i^{fs}(\cdot)$ and $\Delta_i^{ff}(\cdot)$ analogously.

4 Results

Evaluating the sign of the FOC in (4) is, in principle, difficult because there are many terms, some of which have opposite signs. However, the following definitions, which have a straightforward interpretation, allow us to establish our first general result. Let:

$$K_1 \equiv \hat{\pi}_i^{ss} - \hat{\pi}_i^{fs} - \left(\hat{\pi}_j^{sf} - \hat{\pi}_j^{ss}\right) - \left(\pi_i^{ss} - \pi_i^{fs}\right),$$

$$K_2 \equiv \hat{\pi}_i^{sf} - \hat{\pi}_i^{ff} - \left(\hat{\pi}_j^{ff} - \hat{\pi}_j^{fs}\right) - \left(\pi_i^{sf} - \pi_i^{ff}\right),$$

$$K_3(x^*) \equiv \frac{\partial \hat{\pi}_i^{ss}(x^*, x^*)}{\partial x_i} + \frac{\partial \hat{\pi}_j^{ss}(x^*, x^*)}{\partial x_i} - \frac{\partial \pi_i^{ss}(x^*, x^*)}{\partial x_i},$$

$$K_4(x^*) \equiv \frac{\partial \hat{\pi}_i^{sf}(x^*, x^*)}{\partial x_i} + \frac{\partial \hat{\pi}_j^{fs}(x^*, x^*)}{\partial x_i} - \frac{\partial \pi_i^{sf}(x^*, x^*)}{\partial x_i}.$$

Proposition 1. *A merger results in an increase in R&D if and only if:*

$$\frac{\partial \beta_i(x^*)}{\partial x_i} \left[\beta_j K_1 + (1 - \beta_j) K_2\right] + \beta_i \left[\beta_j K_3(x^*) + (1 - \beta_j) K_4(x^*)\right] > 0. \tag{5}$$

Proposition 1 reveals that the incentives of the merged entity to increase or decrease R&D investment are related to the signs of the expressions K_1, K_2, $K_3(x^*)$ and $K_4(x^*)$. These expressions all have a similar interpretation. Specifically, the term K_1 represents the difference between division i's gains from innovation $(\hat{\pi}_i^{ss} - \hat{\pi}_i^{fs})$ net of the externality on the division j $(\hat{\pi}_j^{sf} - \hat{\pi}_j^{ss})$, and firm i's gains from innovation in the pre-merger market $(\pi_i^{ss} - \pi_i^{fs})$, all conditional on partner success. As indicated above, we refer to the gains post- and pre-merger as *replacement effects*. Hence, K_1 can be interpreted as the difference between division i's post-merger replacement effect net of the externality on division j, and firm i's pre-merger replacement effect, conditional on partner success. Similarly, the term K_2 represents the analogous difference, but conditional on partner failure. The terms $K_3(x^*)$ and $K_4(x^*)$ also have a similar interpretation; however, because they involve marginal changes in the conditional payoffs, we refer to them as *marginal replacement effects*.[10]

A straightforward implication of Proposition 1 is the following. Consider a merger in which there are no price effects, for example because prices are regulated, as is the case for some pharmaceutical products. Alternatively, it could be that, while the R&D departments manage to coordinate investments after the merger, the sales departments do not. In such a case, $\hat{\pi}_i^{ss}(\cdot) = \pi_i^{ss}(\cdot)$, $\hat{\pi}_i^{sf}(\cdot) = \pi_i^{sf}(\cdot)$, $\hat{\pi}_i^{fs}(\cdot) = \pi_i^{fs}(\cdot)$ and $\hat{\pi}_i^{ff}(\cdot) = \pi_i^{ff}(\cdot)$, and hence K_1, K_2, $K_3(x^*)$ and $K_4(x^*)$ are all negative. As a result, (5) is strictly negative. Hence, in the absence of gains from price (or quantity) coordination in the market stage, the merged firm would simply adjust its investment in order to internalize the innovation externalities (which are negative), thereby reducing its R&D effort.

More generally, when price effects are present, the sign of (5) is ambiguous and quite difficult to analyse because there are four terms that need to be evaluated and may potentially have

[10] A similar interpretation of K_1 and K_2 appears in Federico et al. (2018). Another way to interpret K_1 and K_2 is in terms of coordination and diversion effects, as in Chen and Schwartz (2013): the difference in replacement effects is analogous to their coordination effect, and the externality on the other division is analogous to their diversion effect. Similarly, $K_3(x^*)$ and $K_4(x^*)$ can be related to a combination of the market-power, demand-expansion, and innovation-diversion effects highlighted in Bourreau et al. (2025). However, because our framework is a two-stage game in which firms first choose R&D effort and then compete, there is an additional strategic effect of firms' investments on profits (details on this later).

different signs. To do this in a didactic way, we now proceed to discuss the two classes of models that have received attention in the literature. The first class of models, which is studied in Section 4.1, refers to models where R&D effort impacts the success probabilities, while keeping the innovation outcomes independent of R&D investment. The second class of models, which is studied in Section 4.2, refers to models where R&D effort impacts the innovation outcomes, while keeping the success probabilities independent of R&D effort. For both classes of models, we show the conditions under which mergers may spur innovation or discourage it.

4.1 Endogenous probability of success

In this section we focus on a class of models in which firms choose their R&D effort to influence the success probability of their projects, while the payoffs conditional on success do not depend on R&D effort. Examples of existing models with these features include Federico et al. (2017, 2018), Denicolò and Polo (2018), Jullien and Lefouili (2020) and Mukherjee (2022). With the exception of Federico et al. (2018) and Mukherjee (2022), these models can be regarded as models of *R&D for entry* because firms obtain positive payoffs only upon project success. Our next result shows that this restriction is important: allowing firms to earn positive profits even when their R&D projects fail, for instance by entering (or remaining in) the market with the *status quo* technology, leads to a much richer set of predictions for the impact of mergers on R&D. Federico et al. (2018) allow for entry upon project failure, but their focus on oligopoly markets impedes a general analytical characterization. In their numerical examples, mergers only reduce R&D, whereas our next result provides a full analytical characterization of all possible outcomes.[11]

Proposition 2. *In markets where R&D effort increases the probability of project success but does not affect the innovation outcome conditional on success:*

(i) If $K_1 \geq 0$ *and* $K_2 \geq 0$*, then* $x^m \geq x^*$ *(with equality if* $K_1 = K_2 = 0$*).*

(ii) If $K_1 \leq 0$ *and* $K_2 \leq 0$*, then* $x^m \leq x^*$ *(with equality if* $K_1 = K_2 = 0$*).*

(iii) If $K_1 < 0$ *and* $K_2 > 0$*, then* $x^m > x^*$ *if and only if* $x^* < \beta^{-1}\left(\frac{K_2}{K_2 - K_1}\right)$.

(iv) If $K_1 > 0$ *and* $K_2 < 0$*, then* $x^m > x^*$ *if and only if* $x^* > \beta^{-1}\left(\frac{K_2}{K_2 - K_1}\right)$.

This proposition makes two relevant points. First, a merger may increase or decrease R&D investment. Second, whether a merger spurs innovation may depend on the pre-merger level of R&D effort. We now elaborate on these two observations.

[11] Mukherjee (2022) also allows firms to earn profits upon project failure, but his study differs from ours because he incorporates synergies in his model, while the focus in this section is on the effects of price and R&D decision-making coordination. We postpone the analysis of synergies to Section 5.

Part *(i)* of Proposition 2 describes a situation in which, regardless of whether the rival's project succeeds or fails, the pre-merger replacement effect is larger than the corresponding post-merger effect net of the externality on the partner. In such situations, a merger leads to an increase in R&D. Part *(ii)* refers to the opposite case. More interestingly, parts *(iii)* and *(iv)* describe environments where the pre-merger level of innovation, and thus the shapes of the success probability and the R&D cost functions, plays a crucial role. Specifically, in part *(iii)*, conditional on the rival's project succeeding, the pre-merger replacement effect is smaller than the post-merger effect net of the externality on the partner, whereas, conditional on the rival's project failing, the opposite holds. In such a case, there are two effects with opposite signs: the first pushes the merged entity to undertake less R&D, the second pushes it to invest more. A merger spurs innovation whenever the second effect dominates. This naturally occurs when the pre-merger level of R&D effort is sufficiently low, because then the probability that the rival's project succeeds is low. Part *(iv)* refers to the reverse situation where, conditional on the success of the rival's project, the pre-merger replacement effect is larger than the post-merger one net of the external effect on the partner, while, conditional on the failure of the rival's project, the opposite holds. In that case, for a merger to spur innovation, the pre-merger level of R&D effort must be sufficiently high; otherwise, the probability that the rival's project succeeds is too low and the latter effect dominates.

Proposition 2 advances the literature by pointing to a broader set of conditions under which mergers may discourage or spur innovation. Before turning to examples that illustrate the various results in Proposition 2, it is useful to clarify what drives the different possible signs of K_1 and K_2 across settings. The key is to decompose these terms into their underlying components. For intuition, we hold fixed the type of product-market interaction in the second stage (e.g. Bertrand or Cournot) and distinguish between two broad classes of innovations: demand- or quality-enhancing innovations, on the one hand, and cost-reducing innovations, on the other. This allows us to isolate how market power, Spence (1975)-type quantity effects, strategic responses, and cannibalisation interact in each class of models to determine the signs of K_1 and K_2.

Quality-enhancing innovations. Consider first the case of a quality-enhancing innovation. To fix ideas, assume that firms sell horizontally differentiated products and compete in prices.[12] In addition, each firm can invest to increase the quality of its product. Let the quality in case of project failure be s_f and the quality in case of success be s_s, with $\Delta s = s_s - s_f > 0$. For any pair of qualities (s_i, s_j), let $(p_i^*(s_i, s_j), p_j^*(s_i, s_j))$ denote the (pre-merger) price equilibrium in the second stage. Anticipating these equilibrium prices, the reduced-form profits of a stand-alone firm i in the first stage are

$$\pi_i(s_i; s_j) = \big(p_i^*(s_i, s_j) - c\big)\, q_i\big(s_i, s_j, p_i^*(s_i, s_j), p_j^*(s_i, s_j)\big).$$

[12]Price competition is just one possible mode of second-stage interaction covered by Proposition 2. If firms competed in quantities instead, similar intuitive arguments could be formulated.

A small increase in the quality of firm i's product then affects its profit through three channels: (i) a direct, positive effect on its own demand; (ii) an indirect effect via its own equilibrium price, which can be ignored by virtue of the envelope theorem; and (iii) an indirect, strategic effect via the rival's equilibrium price, which is negative because a higher s_i induces firm j to cut its price to protect its sales (see also Bourreau et al., 2025). Accordingly, firm i's gain from innovation in the pre-merger market, given by $\pi_i^{ss} - \pi_i^{fs}$ when the partner's project succeeds and by $\pi_i^{sf} - \pi_i^{ff}$ when it fails, can be approximated as:

$$\pi_i^{s-} - \pi_i^{f-} \simeq \Delta s_i \Bigg(\underbrace{(p_i^* - c)\,\frac{\Delta q_i}{\Delta s_i}}_{\text{direct effect} > 0} + \underbrace{(p_i^* - c)\,\underbrace{\frac{\Delta q_i}{\Delta p_j^*}}_{>0}\,\underbrace{\frac{\Delta p_j^*}{\Delta s_i}}_{<0}}_{\text{strategic effect} < 0} \Bigg), \tag{6}$$

where the superscripts "$s-$" and "$f-$" indicate that this approximation holds both when the partner's innovation effort succeeds (states ss or fs) and when it fails (states sf or ff).

After the merger, the integrated firm chooses prices to maximise the sum of division i's and division j's profits. Anticipating the optimal post-merger prices $(\hat{p}_i(s_i, s_j), \hat{p}_j(s_i, s_j))$, the reduced-form profits of the merged entity in the first stage are:

$$\Pi(s_i; s_j) = \big(\hat{p}_i(s_i, s_j) - c\big)\, q_i\big(s_i, s_j, \hat{p}_i(s_i, s_j), \hat{p}_j(s_i, s_j)\big) + \big(\hat{p}_j(s_i, s_j) - c\big)\, q_j\big(s_i, s_j, \hat{p}_i(s_i, s_j), \hat{p}_j(s_i, s_j)\big).$$

By the envelope theorem, a small increase in division i's quality affects the merged entity's profit only through the induced changes in quantities, so that its gain from innovation satisfies:

$$\hat{\pi}^{s-} - \hat{\pi}^{f-} \simeq \Delta s_i \Bigg(\underbrace{(\hat{p}_i - c)\,\frac{\Delta q_i}{\Delta s_i}}_{\text{direct effect} > 0} + \underbrace{(\hat{p}_j - c)\,\frac{\Delta q_j}{\Delta s_i}}_{\text{cannibalisation} < 0} \Bigg). \tag{7}$$

The first term is the direct effect for division i, now evaluated at the post-merger price $\hat{p}_i$; the second term captures the externality on division j, which is typically negative because the quality improvement by i diverts demand away from j.

In both decompositions (6) and (7), the first term is a first-order (direct) effect of the quality improvement, while the remaining terms are second-order (indirect) effects operating through induced changes in prices and the rival's quantity. On the direct side, there are two components. First, there is a market-power effect: because the merged entity typically sets a higher price ($\hat{p}_i > p_i^*$) and therefore has a higher price-cost margin, a given outward shift in demand tends to be more profitable for the merged entity than for a stand-alone firm. Second, there is a Spence (1975)-type quantity effect: the profitability of a given quality-induced shift in demand depends on the output level at which it is evaluated. If the quality improvement shifts demand relatively more at low than at high quantities, then the merged entity, which operates at lower output and higher margin, enjoys a larger direct gain from innovation than a stand-alone firm; if, by contrast, demand shifts relatively more at high quantities, the opposite may occur in which case the direct effect can work against the merger.

On the indirect side, the strategic and cannibalisation effects differ across market structures and states. Pre-merger, a stand-alone firm is subject to a strategic effect: by improving its product, it induces the rival to cut its price, which reduces the profitability of innovation (the second term in (6)). Post-merger, this strategic effect is internalised, but a cannibalisation effect appears: division i's quality improvement diverts business away from division j (the second term in (7)). The balance between these forces depends on whether innovation amounts to catching up or escaping.

Evaluating the sign of K_1 (the catch-up case), which subtracts the pre-merger gains from innovation conditional on partner success ($\pi_i^{ss} - \pi_i^{fs}$) from the post-merger ones ($\hat{\pi}^{ss} - \hat{\pi}^{fs}$), boils down to comparing (6) and (7).[13] In this catch-up case, division i's innovation mainly steals business from the leading partner inside the merged entity, so the cannibalisation effect is relatively strong, while the strategic effect for a stand-alone firm is relatively weak. In this situation, the indirect effects tend to make K_1 negative. Hence, conditional on partner success, if the first-order effects are of similar magnitude, the merged entity is more likely to have weaker incentives to invest than a stand-alone firm.

Evaluating the sign of K_2 (the escape case), which deduces the pre-merger gains from innovation conditional on partner failure ($\pi_i^{sf} - \pi_i^{ff}$) from the post-merger ones ($\hat{\pi}^{sf} - \hat{\pi}^{ff}$), amounts to comparing (6) and (7) conditional on partner failure. In this escape configuration, firm i's innovation allows it to move ahead of a low-quality rival, which induces a strong strategic response. The laggard rival reacts by cutting its price aggressively, sharply reducing the gain from investing. Within the merged entity, however, the cannibalisation effect is comparatively weak, because a large fraction of the additional sales generated by the escaping division comes at the expense of the outside option, rather than the partner division. As a result, the indirect effects tend to make K_2 positive. Hence, conditional on partner failure, if the first-order direct effects are similar, the merged entity is more likely to have stronger incentives to invest than a stand-alone firm.

Cost-reducing innovations. Consider now the case of cost-reducing innovations. As before, assume that firms compete in prices. Initially, firms operate at marginal costs equal to $c_i = c_f$, $i = 1, 2$. An individual firm can invest to decrease its marginal cost from c_f to c_s, with $c_f > c_s \geq 0$. For any pair of marginal costs (c_i, c_j), let $(p_i^*(c_i, c_j), p_j^*(c_i, c_j))$ denote the (pre-merger) price equilibrium in the second stage. Anticipating these equilibrium prices, the reduced-form profits of a stand-alone firm i in the first stage are:

$$\pi_i(c_i; c_j) = p_i^*(c_i, c_j)\, q_i\big(p_i^*(c_i, c_j), p_j^*(c_i, c_j)\big) - C\big(c_i,\, q_i\big(p_i^*(c_i, c_j), p_j^*(c_i, c_j)\big)\big)\,.$$

[13] Here we use shorter notation for the total profit of the merged entity in the various innovation states as follows: $\hat{\pi}^{ss} \equiv \hat{\pi}_i^{ss} + \hat{\pi}_j^{ss}$, $\hat{\pi}^{fs} \equiv \hat{\pi}_i^{fs} + \hat{\pi}_j^{sf}$, $\hat{\pi}^{sf} \equiv \hat{\pi}_i^{sf} + \hat{\pi}_j^{fs}$, $\hat{\pi}^{ff} \equiv \hat{\pi}_i^{ff} + \hat{\pi}_j^{ff}$.

A small decrease in firm i's marginal cost affects its profit through three channels: (i) a direct, positive cost-saving effect; (ii) an indirect effect via its own equilibrium price, which can be ignored by virtue of the envelope theorem; and (iii) an indirect, strategic effect via the rival's equilibrium price, which is negative because a lower marginal cost c_i induces firm j to cut its price to protect its sales (see also Motta and Tarantino, 2021). Accordingly, firm i's gain from innovation in the pre-merger market can be approximated as:

$$\pi_i^{s-} - \pi_i^{f-} \simeq \Delta c_i \Bigg(\underbrace{-\frac{\Delta C(\cdot)}{\Delta c_i}}_{\text{direct effect} > 0} + \underbrace{\left(p_i^* - \frac{\Delta C(\cdot)}{\Delta q_i}\right) \underbrace{\frac{\Delta q_i}{\Delta p_j^*}}_{>0} \underbrace{\frac{\Delta p_j^*}{\Delta c_i}}_{<0}}_{\text{strategic effect} < 0} \Bigg), \tag{8}$$

where the superscripts "$s-$" and "$f-$" have the same meaning as above, i.e. the expression applies both when the partner's project succeeds and when it fails.

After the merger, the integrated firm chooses prices to maximise the sum of division i's and division j's profits. Anticipating the optimal post-merger prices $(\hat{p}_i(c_i, c_j), \hat{p}_j(c_i, c_j))$, the reduced-form profit of the merged entity in the first stage is:

$$\begin{aligned} \Pi(c_i; c_j) = {} & \hat{p}_i(c_i, c_j)\, q_i\big(\hat{p}_i(c_i, c_j), \hat{p}_j(c_i, c_j)\big) + \hat{p}_j(c_i, c_j)\, q_j\big(\hat{p}_i(c_i, c_j), \hat{p}_j(c_i, c_j)\big) \\ & - C\big(c_i,\, q_i\big(p_i^*(c_i, c_j), p_j^*(c_i, c_j)\big)\big) - C\big(c_j,\, q_j\big(p_i^*(c_i, c_j), p_j^*(c_i, c_j)\big)\big)\,. \end{aligned}$$

By the envelope theorem, a small increase in x_i affects the merged entity's profit only through the induced change in its cost, so that:

$$\hat{\pi}^{s-} - \hat{\pi}^{f-} \simeq \Delta c_i \Bigg(\underbrace{-\frac{\Delta C(\cdot)}{\Delta c_i}}_{\text{direct effect} > 0} \Bigg). \tag{9}$$

In both decompositions, the first term is a first-order (direct) effect of the cost reduction, while the remaining term in (8) is a second-order (indirect) strategic effect operating through induced changes in the rival's price.

On the direct side, there is no analogue of the Spence (1975) quantity effect driven by demand shifts. The direct gain from a marginal increase in x_i comes from cost savings and is proportional to the quantity sold. Since a stand-alone firm typically produces a larger quantity than a division of the merged entity in a given state, the direct effect, $-\Delta C(\cdot)/\Delta c_i$, is always larger for the stand-alone firm than for the merged entity. Taken in isolation, this pushes the post-merger gain from cost reduction below the pre-merger one and works against the merger.

On the indirect side, only the pre-merger firm is exposed to a strategic effect: by becoming more efficient, the stand-alone firm induces the rival to cut its price, which reduces the profitability of its cost-reducing effort (the second term in (8)). Post-merger, this strategic effect is internalised and disappears from (9); the merged entity faces only the direct cost-saving effect.

The strength of the strategic effect, however, depends on whether innovation amounts to catching up or escaping: in the catch-up configuration (partner success), the innovation mainly serves to match the rival's marginal cost, so the induced change in the rival's price is limited; in the escape configuration (partner failure), the innovation creates a substantial cost advantage, and the laggard rival has a strong incentive to cut its price aggressively to protect its sales.

Evaluating the sign of K_1 (the catch-up case, i.e. conditional on partner success) amounts to comparing (8) and (9) conditional on partner success: $\hat{\pi}^{ss} - \hat{\pi}^{fs} - (\pi_i^{ss} - \pi_i^{fs})$. In this catch-up configuration, the strategic effect for the stand-alone firm is relatively weak, while the direct cost-saving effect is stronger for the stand-alone firm than for the merged entity because it produces more. As a result, the direct-effect disadvantage of the merged entity dominates, so that K_1 is expected to be negative: conditional on partner success, the merged entity typically has weaker incentives to invest in cost-reducing R&D than a stand-alone firm.

Signing K_2 (the escape case, i.e. conditional on partner failure) implies comparing (8) and (9) conditional on partner failure, i.e. $\hat{\pi}^{sf} - \hat{\pi}^{ff} - (\pi_i^{sf} - \pi_i^{ff})$. In the escape configuration, firm i's innovation allows it to produce more than a high-cost rival, which induces a strong strategic response: the rival reacts by cutting its price aggressively, sharply reducing the stand-alone firm's gain from investing in cost reduction. After the merger, this strategic loss is internalised and no longer appears in (9), while the direct cost-saving effect remains. Hence, even though the stand-alone firm has a stronger direct effect because it produces more, the removal of the large negative strategic term can tilt the balance in favour of the merged entity. As a result, K_2 can turn positive: conditional on partner failure, the internalisation of the strategic effect may dominate the direct-effect disadvantage of the merged entity, so that the merged firm has stronger incentives to invest in cost-reducing innovation than a stand-alone firm.

We now list a series of examples illustrating the different parts of Proposition 2. The complete derivations are provided in Appendix B.

- **Merger leads to higher R&D effort.** Consider a market where firms sell horizontally differentiated products, compete in prices, and undertake quality-enhancing innovations, with demand given by a logit system and price-sensitivity parameter $\kappa = 1/2$ (see Appendix B for details). Initially, firms sell low-quality products and can invest in R&D to increase the probability of offering high quality. We show that there exist parameter values for which both K_1 and K_2 are strictly positive in this setting, so that mergers spur innovation, thereby illustrating Proposition 2*(i)*. A similar result arises in a Hotelling model with price competition and quality-enhancing innovations *á la* Gilbert and Katz (2021). Finally, in this Hotelling model the same conclusion holds when R&D effort is directed at reducing marginal costs rather than improving quality.

- **Merger leads to lower R&D effort.** When, in the logit model with quality-enhancing innovations described above, the price-sensitivity parameter is increased to $\kappa = 2$, both K_1 and K_2 become strictly negative, so mergers discourage innovation, thereby illustrating Proposition 2*(ii)*. The same outcome arises in an analogous logit model with cost-reducing innovations with $\kappa = 1$. It also obtains in a market for horizontally differentiated products with the Singh and Vives (1984) demand system and cost-reducing innovation, whether firms compete in prices or quantities. Finally, we find the same pattern in a Cournot model with Sutton's (2001) demand system and quality-enhancing innovations for a sufficiently large gap between low and high quality.

- **Merger leads to higher R&D effort if the marginal cost of effort is high, and to lower R&D effort otherwise.** When, in the logit model with quality-enhancing innovations described above, price sensitivity is set to the standard value $\kappa = 1$, K_1 becomes strictly negative while K_2 is strictly positive. In this case, if R&D costs are sufficiently steep, the pre-merger level of innovation is low enough that a merger spurs innovation; otherwise, a merger discourages it. This illustrates Proposition 2*(iii)*. The same pattern arises in the logit setting with cost-reducing innovations and $\kappa = 2$. We also obtain an instance of Proposition 2*(iii)* in the Cournot model with Sutton's (2001) demand system and quality-enhancing innovation when the quality gap between the low- and high-quality products is sufficiently small.

- **Merger leads to higher R&D effort if the marginal cost of effort is low, and to lower R&D effort otherwise.** Consider a duopoly with price competition and demands derived from Mussa and Rosen's (1978) model. Initially, firms sell low-quality products and can invest to increase the likelihood of offering high quality. In this model, K_1 is strictly positive while K_2 is strictly negative. In such a case, if R&D costs are small, the pre-merger level of innovation is high and a merger spurs innovation; otherwise, a merger discourages it. This illustrates Proposition 2*(iv)*.

Compared to the existing literature, Proposition 2 (and the examples just provided) identify a broader range of environments where mergers can discourage or spur innovation. To the best of our knowledge, there are no existing results where the pre-merger level of innovation is critical to the assessment of the impact of mergers on R&D.

Proposition 2 captures as particular cases two well-known theoretical results in the literature, namely, the negative effect of mergers on R&D in Federico et al. (2017)[14] and the positive effect of mergers on R&D identified in Jullien and Lefouili (2020). In these papers, the pre-merger level of innovation is irrelevant because of the restrictive assumption that firms cannot enter the market upon project failure.

[14] Also observed in Denicolò and Polo's (2018) interior equilibrium.

Corollary (to Proposition 1). *Assume that firms can only operate in the market upon a successful innovation; otherwise, they exit. Then, we have* $K_1 = \hat{\pi}_i^{ss} + \hat{\pi}_j^{ss} - \hat{\pi}_j^{sf} - \pi_i^{ss}$ *and* $K_2 = 0$*. As a result:*

- *If* $K_1 > 0$*, then* $x^m > x^*$*. (For an example, see Jullien and Lefouili, 2020)*
- *If* $K_1 < 0$*, then* $x^m < x^*$*. (For an example, see Federico et al. 2017; and Denicolò and Polo, 2020).*

Corollary to Proposition 1 captures the class of models where R&D is intended for entry, and entry only occurs upon project success. In that case, the conditional payoffs in case of innovation failure are all equal to zero: $\hat{\pi}_i^{fs}(\cdot) = \pi_i^{fs}(\cdot) = 0$, and $\hat{\pi}_i^{ff}(\cdot) = \pi_i^{ff}(\cdot) = 0$, and similarly for firm j. Moreover, $\hat{\pi}_i^{sf}(\cdot) = \pi_i^{sf}(\cdot)$ because a firm's freedom to set price/quantity is not constrained by a rival that fails to innovate. Because of this, $K_2 = 0$ and only the value of K_1 has a bearing on R&D incentives. In the setting of Jullien and Lefouili (2020) with horizontally differentiated products, K_1 can be positive, in which case a merger increases R&D. In Federico et al. (2017) and Denicolò and Polo (2018), K_1 is surely less than zero and a merger decreases R&D.

Proposition 2 also helps rationalise why the empirical literature on competition and innovation has obtained mixed results. As discussed above, K_1 is the difference between the net incentives to catch up of the merged entity and those of a stand-alone firm, whereas K_2 captures the corresponding difference in the incentives to escape competition. The various cases in Proposition 2 show that these incentives can both be higher for the merged entity, both lower, or move in opposite directions (stronger to catch-up while weaker to escape, or vice versa). This richness is helpful to explain why the empirical literature on competition and innovation, pioneered by Aghion et al. (2005), finds different patterns depending on the sample of firms and industries considered. For example, Aghion et al. (2005) document an inverted-U relationship, which is consistent with environments akin to Proposition 2(iv), whereas Hashmi (2013) finds a decreasing relationship, in line with settings where Proposition 2(i) predominates. By contrast, Carrera and Ornaghi (2014) and Beneito et al. (2017) find an increasing relationship between competition and innovation, corresponding to Proposition 2(ii).

4.2 Endogenous innovation outcomes

In this subsection, we focus on the second class of models in which success probabilities are exogenous while innovation outcomes depend on R&D effort. Examples include Motta and Tarantino (2021) and Bourreau et al. (2025). These contributions, however, assume that R&D projects succeed with probability one and therefore rule out innovation failure. Moreover, except for Section 3.1 of Motta and Tarantino (2021), most of their analysis is conducted in one-stage models where firms choose prices and R&D simultaneously. The following proposition shows that relaxing the deterministic-success assumption in a two-stage framework yields a richer set of possible merger effects on R&D.

Proposition 3. *In markets where innovation outcomes are endogenous while R&D success probabilities are exogenous but not necessarily equal to 1 ($\beta(x) = \mu \in (0, 1]$ for all x):*

(i) If $K_3(x^*) > 0$ and $K_4(x^*) > 0$, then $x^m > x^*$.

(ii) If $K_3(x^*) < 0$ and $K_4(x^*) < 0$, then $x^m < x^*$.

(iii) Let $\Phi(x) \equiv \frac{K_4(x)}{K_4(x)-K_3(x)}$. If $\Phi(x)$ is decreasing in x, then:

(a) If $K_3(x^*) < 0$ and $K_4(x^*) > 0$ and $\mu = \Phi(x)$ has a solution, denoted $\tilde{x}$, then $x^m > x^*$ if and only if $x^* < \tilde{x}$. If $\mu = \Phi(x)$ does not have a solution, then $x^m > x^*$.

(b) If $K_3(x^*) > 0$ and $K_4(x^*) < 0$, then $x^m > x^*$ if and only if $x^* > \tilde{x}$. If $\mu = \Phi(x)$ does not have a solution, then $x^m < x^*$.

(iv) If $\Phi(x)$ is increasing in x, then:

(a) If $K_3(x^*) < 0$ and $K_4(x^*) > 0$, and $\mu = \Phi(x)$ has a solution, then $x^m > x^*$ if and only if $x^* > \tilde{x}$. If $\mu = \Phi(x)$ does not have a solution, then $x^m < x^*$.

(b) If $K_3(x^*) > 0$ and $K_4(x^*) < 0$, and $\mu = \Phi(x)$ has a solution in x, then $x^m > x^*$ if and only if $x^* < \tilde{x}$. If $\mu = \Phi(x)$ does not have a solution, then $x^m > x^*$.

This proposition shows that also in models where the extent of cost-reduction or quality improvement is endogenous, while the success probability is exogenous, a merger can lead to an increase or decrease in R&D effort. Moreover, it again highlights the pre-merger level of R&D investment and the difficulty of project success as key determinants of this outcome.

We now provide some additional details on Proposition 3. Part (i) covers situations in which, regardless of whether the rival's project succeeds or fails, the pre-merger marginal replacement effect is stronger than the corresponding post-merger effect net of the external marginal effect on the partner. In such cases, a merger unambiguously increases R&D. Part (ii) describes the opposite situation, where the post-merger marginal replacement effects net of the externality on the partner dominate their pre-merger counterparts, so that a merger unambiguously reduces R&D. Parts (iii) and (iv) describe environments in which the magnitude of the success probability and the pre-merger level of innovation matter for the outcome. When the net post-merger marginal replacement effect is stronger than the pre-merger effect conditional on the rival's project success, but weaker conditional on the rival's project failure (or vice versa), two forces work in opposite directions and the impact of a merger on R&D is a priori ambiguous. Parts (iii)(a) and (iv)(a) cover cases in which the merged entity increases R&D effort when the failure state (i.e. conditional on partner's failure) receives enough weight in expected profits. Conversely, parts (iii)(b) and (iv)(b) describe parameter regions where the merged entity increases R&D effort when the success state (i.e. conditional on partner's success) dominates in expected terms.

Similarly to Proposition 2, Proposition 3 enlarges the set of possible merger effects on R&D, here in models with exogenous success probabilities and endogenous innovation outcomes. In particular, once innovation failure is allowed for, mergers may either spur or discourage R&D, depending on the pre-merger investment level, the success probability, and the relative strength of the pre- and post-merger marginal replacement effects. The following example with single-product firms illustrates parts *(ii)* and *(iv)* of Proposition 3; detailed derivations are provided in Appendix C.

- Consider a duopoly with price competition and the Singh and Vives (1984) demand system. Initially, firms produce at marginal cost c. Firms can invest in R&D to reduce this cost: an investment $C(x)$ lowers marginal cost by x if the project is successful, while failure leaves marginal cost at c. The probability of success is $\mu \in (0, 1]$. In this model, $K_3(x^*)$ is strictly negative, $K_4(x^*)$ is negative when x^* is sufficiently small and positive otherwise, and $\Phi(x)$ is increasing in x. Hence, when R&D costs are sufficiently steep, the pre-merger level of innovation x^* is small and a merger unambiguously discourages innovation, illustrating Proposition 3*(ii)*. By contrast, when R&D costs are flat enough, a merger spurs innovation if the success probability μ is low enough and discourages it otherwise, illustrating Proposition 3*(iv)*.

We now note that some existing results in the literature are special cases of Proposition 3.

Corollary (to Proposition 3). ***[Deterministic R&D]*** *Assume that innovation outcomes are endogenous and that R&D projects are surely successful (i.e.* $\mu = 1$*). Then* $(1 - \mu)K_4(x^*) = 0$ *and hence only* $K_3(x^*)$ *matters. We then have:*

- *If* $K_3(x^*) > 0$*, then* $x^m > x^*$*.*
- *If* $K_3(x^*) < 0$*, then* $x^m < x^*$*. (For an example, see Motta and Tarantino, 2021; section 3.1).*

This corollary captures the class of two-stage models in which the R&D process is deterministic. The two-stage formulations in Motta and Tarantino (2021, Section 3.1) and in the online appendix of Bourreau et al. (2025) fall into this class. Our corollary to Proposition 3 shows that even in this deterministic setting the effect of a merger on R&D is, in general, ambiguous. However, Bourreau et al. (2025) do not provide explicit examples, while the examples examined by Motta and Tarantino (2021), based on Shubik-Levitan and Salop demand systems and cost-reducing innovations, yield the conclusion that mergers reduce R&D.

Our example described above is based on the Singh-Vives demand system, which is close in spirit to Shubik-Levitan, and nevertheless delivers cases in which R&D increases after a merger. The key difference is that we allow for stochastic innovation outcomes. As discussed in Section 4.1, with cost-reducing innovations the strategic effect is much stronger in the escape state (a low-cost innovator facing a high-cost rival) than in the catch-up state (a low-cost innovator facing a similar rival). In our framework with uncertain innovation, this strong strategic effect in the escape state can make the net marginal replacement effect of the merged entity larger than that for the stand-alone firm, i.e. $K_4(x^*)$ can become positive. When the success probability μ is sufficiently low, this state receives enough weight in expected profits for the merger to raise R&D. By contrast, in the deterministic model of Motta and Tarantino (2021), success occurs with probability one, so only the success state matters in expectations and the escape configuration never plays a role. In that case, the positive escape effect that can overturn the standard result in our setting is effectively shut down, and mergers can only reduce cost-reducing R&D, as in their examples. Indeed, if we were to impose deterministic success (set $\mu = 1$) in our Singh-Vives example, we would obtain the same qualitative result (for details see Appendix C).

5 Extensions

5.1 Synergies

Our analysis has shown that mergers can either increase or reduce investment incentives. While performing the analysis, we restricted ourselves to settings in which there are no synergies. In this section, we relax this assumption. We consider two settings. In the first setting, inspired by Farrell and Shapiro (1990), we allow innovations developed by one division of the merged entity to be leveraged across the other division to drive broader organizational benefits. We refer to these synergies as *R&D output synergies*. In the second setting, inspired by the RJV literature (e.g d'Aspremont and Jacquemin, 1988; Kamien et al., 1992; Suzumura, 1992), we allow investment effort in one division to spill over into the other division. We call these synergies *R&D input synergies*.

5.1.1 R&D output synergies

Following Farrell and Shapiro (1990), assume that innovations from one division of the merged entity can be applied to the other division, thereby creating broader organizational benefits. Moreover, assume that the success probability of a firm's R&D project is endogenous, while innovation outcomes conditional on success do not depend on R&D efforts.

The existence of R&D output synergies means that Table 2 gets replaced by the following table of payoffs:

		Division j	
		$Success(s)$	$Failure(f)$
Division i	$Success(s)$	$\hat{\pi}_i^{ss}, \hat{\pi}_j^{ss}$	$\hat{\pi}_i^{ss}, \hat{\pi}_j^{ss}$
	$Failure(f)$	$\hat{\pi}_i^{ss}, \hat{\pi}_j^{ss}$	$\hat{\pi}_i^{ff}, \hat{\pi}_j^{ff}$

Table 3: Conditional payoffs of the divisions within the merged entity with synergies

As it is clear from the table, the crucial difference is that only two joint payoffs realizations become relevant now, namely, $\hat{\pi}_i^{ss} + \hat{\pi}_j^{ss}$ and $\hat{\pi}_i^{ff} + \hat{\pi}_j^{ff}$. The reason is that a discovery by one division of the merged entity is passed through to the other division.

With synergies, the relevant FOC for the maximization of the merged entity's profits evaluated at the pre-merger equilibrium becomes:

$$FOC_i^m(x^*) = \frac{\partial \beta_i(x^*)}{\partial x_i}\left[\beta_j D_1 + (1-\beta_j)D_2\right] > 0,$$

where

$$D_1 \equiv -\left(\pi_i^{ss} - \pi_i^{fs}\right)$$
$$D_2 \equiv \hat{\pi}_i^{ss} - \hat{\pi}_i^{ff} - \left(\hat{\pi}_j^{ff} - \hat{\pi}_j^{ss}\right) - \left(\pi_i^{sf} - \pi_i^{ff}\right) = 2\left(\hat{\pi}_i^{ss} - \hat{\pi}_i^{ff}\right) - \left(\pi_i^{sf} - \pi_i^{ff}\right),$$

where we have used symmetry. Because $D_1 < 0$, with R&D output synergies, Proposition 1 is modified as follows:

Proposition 4 (R&D output synergies)**.** *Consider markets where R&D effort increases the probability of project success but does not affect the innovation outcomes.*

If a successful innovation can be leveraged across the various divisions of the merged entity, a merger results in an increase in R&D if and only if:

$$FOC_i^m(x^*) = \frac{\partial \beta_i(x^*)}{\partial x_i}\left[\beta_j D_1 + (1-\beta_j)D_2\right] > 0.$$

Hence:

(i) If $D_2 \leq 0$*, then* $x^m < x^*$

(ii) If $D_2 > 0$*, then* $x^m > x^*$ *if and only if* $x^* < \beta^{-1}\left(\frac{D_2}{D_2 - D_1}\right)$.

Surprisingly, because $D_1 < 0$, R&D output synergies need not enhance the incentives to increase R&D of the post-merger entity, in particular when partner success is highly likely.

In Appendix B, we show that Proposition 4(i) holds in the Singh and Vives (1984) model with cost-reducing innovation and Cournot competition provided that the initial marginal cost is sufficiently low. Proposition 4(ii) holds in the Hotelling model with vertically differentiated products, and in the Singh and Vives (1984) model with cost-reducing innovation and Cournot competition when the initial marginal cost is sufficiently large.[15]

[15]Proposition 4(i) also holds in the Singh and Vives (1984) model with demand-enhancing innovation and

5.1.2 R&D input synergies

Inspired by the RJV literature (e.g. d'Aspremont and Jacquemin (1988), Kamien et al. (1992)), suppose that after merger the "effective" R&D effort of a division i is

$$\overline{x}_i = x_i + \lambda x_j$$

where $\lambda \in (0,1)$ is a "spillover" effect across the divisions of the merged entity.

The new FOC at the pre-merger market symmetric equilibrium x^* is:

$$\begin{aligned} FOC_i^m(x^*) &= \frac{\partial \beta_i(x^*)}{\partial x_i}\left[\beta_j K_1 + (1-\beta_j)K_2\right] + \frac{\partial \beta_j(x^*)}{\partial x_i}\left[\beta_i S_1 + (1-\beta_i)S_2\right] \\ &= \frac{\partial \beta_i(x^*)}{\partial x_i}\left[\beta_j (K_1 + \lambda S_1) + (1-\beta_j)(K_2 + \lambda S_2)\right] \end{aligned}$$

where

$$\begin{aligned} K_1 &\equiv \hat{\pi}_i^{ss} - \hat{\pi}_i^{fs} - \left(\hat{\pi}_j^{sf} - \hat{\pi}_j^{ss}\right) - \left(\pi_i^{ss} - \pi_i^{fs}\right) \\ K_2 &\equiv \hat{\pi}_i^{sf} - \hat{\pi}_i^{ff} - \left(\hat{\pi}_j^{ff} - \hat{\pi}_j^{fs}\right) - \left(\pi_i^{sf} - \pi_i^{ff}\right) \\ S_1 &\equiv \hat{\pi}_j^{ss} - \hat{\pi}_j^{fs} - \left(\hat{\pi}_i^{sf} - \hat{\pi}_i^{ss}\right) \\ S_2 &\equiv \hat{\pi}_j^{sf} - \hat{\pi}_j^{ff} - \left(\hat{\pi}_i^{ff} - \hat{\pi}_i^{fs}\right) \end{aligned}$$

Proposition 5 (R&D input synergies)**.** *In markets where R&D effort increases the probability of project success but does not affect the innovation outcomes, with R&D input synergies of size* λ*, Proposition 1 holds exactly the same but with* $\tilde{K}_1 \equiv K_1 + \lambda S_1$ *and* $\tilde{K}_2 \equiv K_2 + \lambda S_2$ *replacing* K_1 *and* K_2*.*

Hence, because

$$\begin{aligned} S_1 &= \hat{\pi}_i^{ss} + \hat{\pi}_j^{ss} - (\hat{\pi}_j^{fs} + \hat{\pi}_i^{sf}) > 0 \\ S_2 &= \hat{\pi}_i^{fs} + \hat{\pi}_j^{sf} - (\hat{\pi}_j^{ff} + \hat{\pi}_i^{ff}) > 0, \end{aligned}$$

and contrary to the case of R&D output synergies, we can state that R&D input synergies increase the likelihood that mergers result in higher R&D in all circumstances.

5.2 Asymmetric firms

The above propositions have focused on situations in which the firms are symmetric. The purpose of this subsection is to extend our results to the case of asymmetric firms. Again, we focus on the

Cournot competition provided that the marginal cost is sufficiently large. Moreover, Proposition 4(ii) also holds in the Singh and Vives (1984) model with cost-reducing innovation and Bertrand competition. Details can be obtained form the authors upon request.

setting where the success probabilities of a firm's R&D project are endogenous, while innovation outcomes conditional on success do not depend on R&D efforts.

With asymmetric players, the system of the FOCs of the merged entity is given by:

$$\begin{aligned} FOC_i^m(x_i, x_j) \equiv & \frac{\partial \beta_i(x_i)}{\partial x_i}\left[\beta_j(x_j)\left[\hat{\pi}_i^{ss} - \hat{\pi}_i^{fs}\right] + (1-\beta_j(x_j))\left[\hat{\pi}_i^{sf} - \hat{\pi}_i^{ff}\right]\right] \\ & + \frac{\partial \beta_i(x_i)}{\partial x_i}\left[\beta_j(x_j)\left[\hat{\pi}_j^{ss} - \hat{\pi}_j^{sf}\right] + (1-\beta_j(x_j))\left[\hat{\pi}_j^{fs} - \hat{\pi}_j^{ff}\right]\right] \\ & - C'(x_i) = 0, \quad \text{and similarly for } x_j. \end{aligned} \tag{10}$$

These two equations implicitly define the loci $x_i^m(x_j)$ and $x_j^m(x_i)$ in the $(x_i, x_j)-$space. The intersection of these two loci gives the post-merger asymmetric investment levels (x_i^m, x_j^m).

The slope of the functions $x_i^m(x_j)$ and $x_j^m(x_i)$ implicitly defined by (10) can be obtained by implicit differentiation. For the function $x_i^m(x_j)$ we have:

$$\frac{\partial x_j^m}{\partial x_i} = -\frac{\beta_i''(x_i)\left[\beta_j(x_j)\left(\hat{\pi}_i^{ss} - \hat{\pi}_i^{fs} + \hat{\pi}_j^{ss} - \hat{\pi}_j^{sf}\right) + (1-\beta_j(x_j))\left(\hat{\pi}_i^{sf} - \hat{\pi}_i^{ff} + \hat{\pi}_j^{fs} - \hat{\pi}_j^{ff}\right)\right] - C''(x_i)}{\beta_i'(x_i)\beta_j'(x_j)\left[\hat{\pi}_i^{ss} - \hat{\pi}_i^{fs} - \left(\hat{\pi}_i^{sf} - \hat{\pi}_i^{ff}\right) + \hat{\pi}_j^{ss} - \hat{\pi}_j^{sf} - \left(\hat{\pi}_j^{fs} - \hat{\pi}_j^{ff}\right)\right]}, \tag{11}$$

and similarly for $x_j^m(x_i)$. The numerator of this expression is negative because $\beta_i''(x_i) < 0$ and $C''(x_j) > 0$. Hence, because $\beta_i'(x_i) > 0$ and $\beta_j'(x_j) > 0$, the sign of the derivative in (11) depends on the sign of the expression in the denominator:

$$\Omega \equiv \hat{\pi}_i^{ss} - \hat{\pi}_i^{fs} - (\hat{\pi}_i^{sf} - \hat{\pi}_i^{ff}) + \hat{\pi}_j^{ss} - \hat{\pi}_j^{sf} - (\hat{\pi}_j^{fs} - \hat{\pi}_j^{ff}) = \hat{\pi}^{ss} - \hat{\pi}^{fs} - (\hat{\pi}^{sf} - \hat{\pi}^{ff})$$

The same applies to the sign of $\partial x_i^m / \partial x_j$.

Therefore, $x_i^m(x_j)$ and $x_j^m(x_i)$ are upward sloping when $\Omega > 0$ and downward sloping when $\Omega < 0$. In other words, when the incremental gain that the merged entity derives from the success of one division's R&D effort conditional on the other division's success is larger than the corresponding gain conditional on the other division's failure, the loci $x_i^m(x_j)$ and $x_j^m(x_i)$ are increasing, indicating that the two divisions' investment decisions are complements. Conversely, when the gain from a division's R&D success is higher conditional on the other division's failure than on its success, the loci $x_i^m(x_j)$ and $x_j^m(x_i)$ are decreasing, implying that investment decisions are substitutes.[16] A similar condition but for the case of competing firms is presented in Aoki and Spiegel (2009), who employ a related model of stochastic innovation to analyze firms' incentives to innovate and apply for a patent.[17]

In order to address the question whether a merger leads to more or less investment compared to the pre-merger equilibrium we analyze the sign of the gradient vector of the merged entity

[16]Note that in the micro-founded examples presented above we have $\hat{\pi}^{ss} - \hat{\pi}^{fs} - (\hat{\pi}^{sf} - \hat{\pi}^{ff}) < 0$ so that investment efforts of the two divisions are substitutes.

[17]Federico et al. (2018) also mention this condition in the appendix.

evaluated at the pre-merger asymmetric equilibrium, which we denote as (x_i^*, x_j^*). This gradient vector is given by:

$$FOC_i^m(x_i^*, x_j^*) = \frac{\partial \beta_i(x_i^*)}{\partial x_i}\left[\beta_j(x_j^*)K_i^1 + (1-\beta_j(x_j^*))K_i^2\right], \quad (12)$$
$$FOC_j^m(x_i^*, x_j^*) = \frac{\partial \beta_j(x_j^*)}{\partial x_j}\left[\beta_i(x_i^*)K_j^1 + (1-\beta_i(x_i^*))K_j^2\right].$$

where K_i^1 and K_i^2 are defined as before: $K_i^1 = \hat{\pi}_i^{ss} - \hat{\pi}_i^{fs} - \left(\hat{\pi}_j^{sf} - \hat{\pi}_j^{ss}\right) - \left(\pi_i^{ss} - \pi_i^{fs}\right)$ and $K_i^2 = \hat{\pi}_i^{sf} - \hat{\pi}_i^{ff} - \left(\hat{\pi}_j^{ff} - \hat{\pi}_j^{fs}\right) - \left(\pi_i^{sf} - \pi_i^{ff}\right)$, and K_j^1 and K_j^2 have similar expressions but exchanging the subindex i for j.

Suppose, to start with, that $\Omega > 0$ so that the R&D investments of the merged entity's divisions are complements. We represent this case in the graphs of Figure 1, where we plot in the (x_i, x_j)-space the isoprofit curves of the merged entity, the FOCs given in (10), and the merged entity's optimal choice of investment efforts (x_i^m, x_j^m). The decreasing dashed line represents the loci of (x_i, x_j) combinations satisfying that total investment equals total investment of the merged entity, i.e., $x_i + x_j = x_i^m + x_j^m$. In this case where R&D investments of the merged entity's divisions are complements, Proposition 2 is modified as follows.

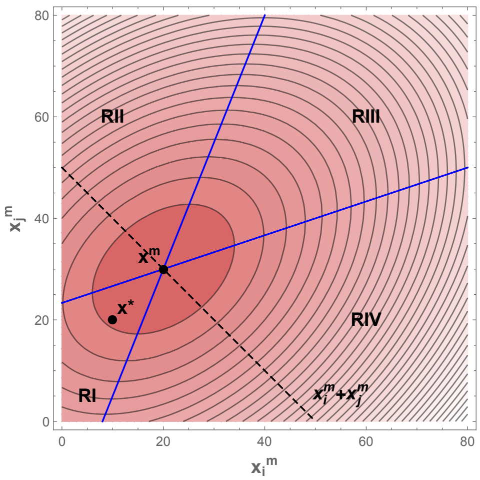
There are 16 possible alternative sign combinations the profits differences K_i^1, K_i^2, K_j^1 and K_j^2 may take. Except for two of them, for any of these sign combinations, it is possible that a merger results in an increase in R&D or a decrease in R&D.

(i) If $K_i^1, K_i^2, K_j^1, K_j^2 \geq 0$, then each of the divisions of the merged entity will do more R&D than the corresponding stand-alone firms, i.e. $x_i^m \geq x_i^*$ and $x_j^m \geq x_j^*$. This situation is represented in Figure 1a. All the K's being positive imply that the two components of the merged entity's gradient vector evaluated at the pre-merger equilibrium are positive. As a result, the pre-merger equilibrium pair of R&D investments must lie in the area marked by the label "RI".[18]

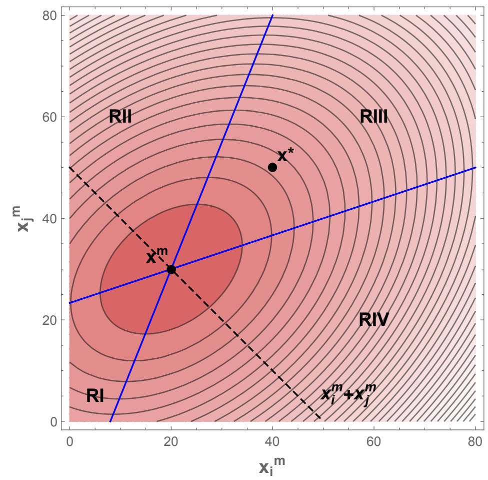
(ii) If $K_i^1, K_i^2, K_j^1, K_j^2 \leq 0$, then each of the divisions of the merged entity will do less R&D than the corresponding stand-alone firms, i.e. $x_i^m \leq x_i^*$ and $x_j^m \leq x_j^*$. This situation is the opposite to that in (i) because with all the K's being negative the two components of the merged entity's gradient vector evaluated at the pre-merger equilibrium are negative. Hence, the pre-merger equilibrium pair of R&D investments must lie in the area marked by the label "RIII". This case is represented in Figure 1b.[19]

[18]The first component of the merged entity's gradient vector evaluated at the pre-merger equilibrium is also positive in the following cases: (a) $K_i^1 \geq 0$ and $K_i^2 \leq 0$ but $x_i^* > \hat{x}_i$, where $\hat{x}_i = \beta_i^{-1}\left(\frac{K_j^2}{K_j^2 - K_j^1}\right)$. (b) $K_i^1 \leq 0$ and $K_i^2 \geq 0$ but $x_i^* < \hat{x}_i$. Likewise, the second component of the merged entity's gradient vector evaluated at the pre-merger equilibrium is also positive when: (a) $K_j^1 \geq 0$ and $K_j^2 \leq 0$ but $x_j^* > \hat{x}_j$, where $\hat{x}_j = \beta_j^{-1}\left(\frac{K_i^2}{K_i^2 - K_i^1}\right)$. (b) $K_j^1 \leq 0$ and $K_j^2 \geq 0$ but $x_j^* < \hat{x}_j$. This means that there are even more cases in which a merger may result in an increase in R&D.

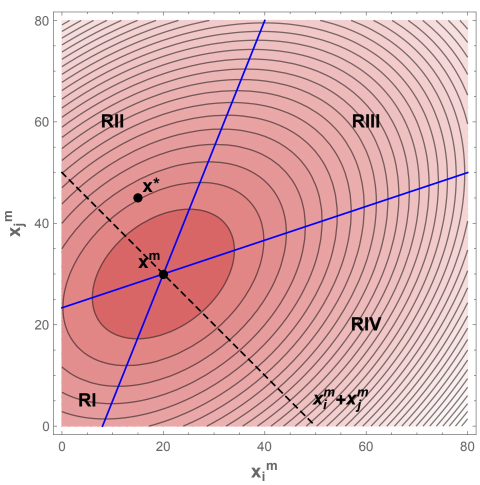
[19]Similarly to case (i) above, there are more cases in which a merger may result in an decrease in R&D.



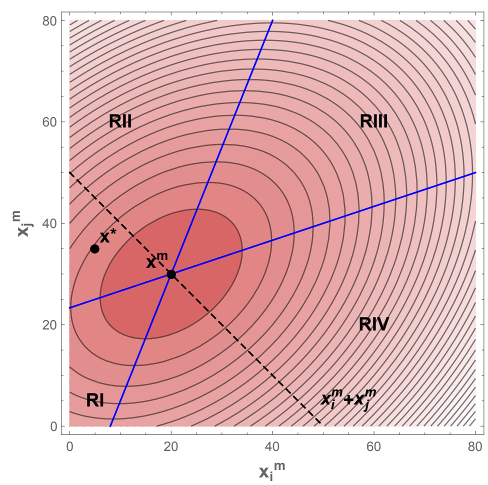
(a) Both divisions increase R&D



(b) Both divisions reduce R&D



(c) Division i reduces and j increases R&D, total increases



(d) Division i reduces and j increases R&D, total decreases

Figure 1: Mergers of asymmetric firms

The cases (i) and (ii) refer to situations in which signs of the REs-differences K_i^1, K_i^2, K_j^1 and K_j^2 combine in such a way that the two components of the merged entity's gradient vector evaluated at the pre-merger equilibrium have the same sign. However, there are plenty of other situations for which the components of the merged entity's gradient vector evaluated at the pre-merger equilibrium may have different sign. We now discuss one such situation and show what outcomes are possible.

(iii) Suppose $K_i^1, K_i^2 \geq 0$ while $K_j^1, K_j^2 \leq 0$. In such a case, the first component of the merged entity's gradient vector evaluated at the pre-merger equilibrium is positive, while the second component is negative. As a result, the pre-merger equilibrium pair of R&D investments must lie in region "RII" of the graphs in Figure 1. This implies that either both divisions of the merged entity increase R&D, both decrease it, or one increase it and the other reduce it. In the latter case, it is ambiguous what happens to total investment, it may increase or decrease. In Figure 1c we show a case in which total investment decreases, while Figure 1d depicts a situation where total investment increases.[20]

The above discussion pertains to the case in which the R&D investments of the merged entity's divisions are complements. However, it may be the case that $\Omega < 0$ and so the R&D investments of the divisions of the merged entity are substitutes. In such situation, a similar analysis can be performed. We omit it to save space.

This discussion shows that the result articulated in Proposition 2, which stresses that the pre-merger level of R&D effort may be crucial for evaluating the effects of a merger on R&D becomes even more relevant when firms are asymmetric. Furthermore, even "partial killer acquisitions" in the sense that one of the divisions' effort is reduced can result in an increase in total R&D effort (Figure 1d).

5.3 Consumer Surplus

Our analysis has shown that mergers may either enhance or diminish investment incentives, and that an increase in R&D does not require the presence of synergies. Because a merger also entails adverse price effects, it is immediate that, when a merger reduces R&D, consumer surplus must fall. By contrast, when a merger raises R&D, consumer surplus may increase if the resulting shift toward better innovation outcomes is strong enough to dominate the negative price effects. In what follows, we provide a high-level analysis of the impact of mergers on consumer surplus.

Consider the pre-merger symmetric market equilibrium and denote the levels of consumer surplus attained in the equilibria corresponding to the four possible subgames described above by CS^{ss}, CS^{sf}, CS^{fs} and CS^{ff}:

		Firm j	
		$Success(s)$	$Failure(f)$
Firm i	$Success(s)$	CS^{ss}	CS^{sf}
	$Failure(f)$	CS^{fs}	CS^{ff}

Table 4: Pre-merger consumer surplus outcomes

That is, When both firms' R&D projects are successful (unsuccessful), product-market competition generates consumer surplus CS^{ss} (CS^{ff}). Similarly, when one firm's R&D project succeeds and the other fails, consumer surplus equals CS^{sf} or CS^{fs}, depending on which firm is successful.

With symmetric firms, it is natural to assume that $CS^{ss} \geq CS^{sf} = CS^{fs} \geq CS^{ff}$ where the equality $CS^{sf} = CS^{fs}$. Then, the expected pre-merger level of consumer surplus is:

$$\mathbb{E}CS(x^*) = [\beta(x^*)]^2 \, CS^{ss} + 2\beta(x^*) \, [1 - \beta(x^*)] \, CS^{sf} + [1 - \beta(x^*)]^2 \, CS^{ff}. \tag{13}$$

[20]As above, there are 8 additional cases in which the first component of the merged entity's gradient vector evaluated at the pre-merger equilibrium is positive and the second is negative.

Consider now the level of consumer surplus after a merger. A merger induces a shift in the distribution of consumer surplus. This shift reflects not only changes in the consumer-surplus levels themselves, driven by the price effects of the merger, but also changes in the probabilities with which the different innovation outcomes occur. Denoting post-merger consumer surplus in each state by $\hat{CS}^{ss}$, $\hat{CS}^{sf}$, $\hat{CS}^{fs}$ and $\hat{CS}^{ff}$, the expected post-merger level of consumer surplus is:

$$\mathbb{E}CS^m(x^m) = [\beta(x^m)]^2 \hat{CS}^{ss} + 2\beta(x^m)[1-\beta(x^m)] \hat{CS}^{sf} + [1-\beta(x^m)]^2 \hat{CS}^{ff}, \tag{14}$$

where, again due to symmetry, we have assumed $\hat{CS}^{sf} = \hat{CS}^{fs}$.

A direct comparison between (13) and (14) is not straightforward, because both the probabilities and the consumer-surplus levels change after the merger. In particular, we cannot rely on first-order stochastic dominance (FOSD), since the cumulative distribution functions (CDFs) of pre- and post-merger consumer surplus typically cross. However, the notion of second-order stochastic dominance (SOSD) can be used to derive a condition under which consumer surplus decreases after a merger:

Proposition 6. *(i) If $\beta((x^m)) < \beta(x^*)$, consumer surplus decreases after a merger.*

(ii) Even if $\beta(x^m) > \beta(x^*)$, consumer surplus decreases after a merger if the consumer surplus CDF pre-merger dominates in the sense of the SOSD criterion that of the consumer surplus CDF post-merger, that is,

$$\begin{aligned}&(1-\beta(x^m))^2(CS^{ff} - \hat{CS}^{ff}) + [1-\beta(x^m)^2 - (1-\beta(x^*))^2](CS^{sf} - \hat{CS}^{sf})\\ &+ \beta(x^*)^2(CS^{ss} - \hat{CS}^{ss}) > (\hat{CS}^{sf} - CS^{ff})[(1-\beta(x^*)^2 - (1-\beta(x^m))^2]\\ &+ (\hat{CS}^{ss} - CS^{sf})[\beta(x^m)^2 - \beta(x^*)^2],\end{aligned}$$

where we assume $\hat{CS}^{ff} < CS^{ff} < \hat{CS}^{sf} < CS^{sf} < \hat{CS}^{ss} < CS^{ss}$.

Proposition 6(ii) is illustrated in Figure 2. In this figure, we plot the CDF of consumer surplus pre-merger in red, and the CDF of consumer surplus post-merger in blue, assuming that R&D investment increases after the merger. The graph illustrates a situation where the negative price effects dominate the positive innovation effects.

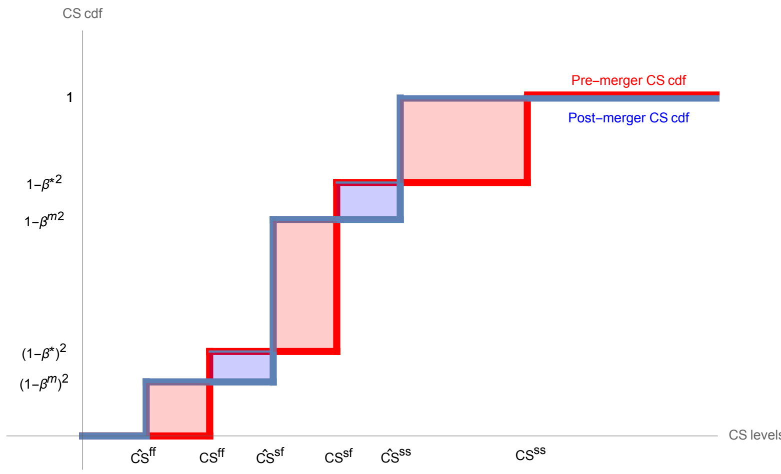


Figure 2: CS CDF pre-merger SOSD CS CDF post-merger

Unfortunately, the notion of SOSD cannot be used to derive conditions under which consumer surplus increases after a merger, because the lower bound of the support of the post-merger CDF of consumer surplus is lower than its pre-merger counterpart, i.e. $\hat{CS}^{ff} < CS^{ff}$. To make further progress, we use the well-known identity that for a random variable $\mathcal{Z}$ with distribution $F(z)$ and support on the interval $[\ell, u]$ it holds that:

$$\mathbb{E}[\mathcal{Z}] = \int_{\ell}^{u} z\, dF(z) = u - \int_{\ell}^{u} F(z)\, dz,$$

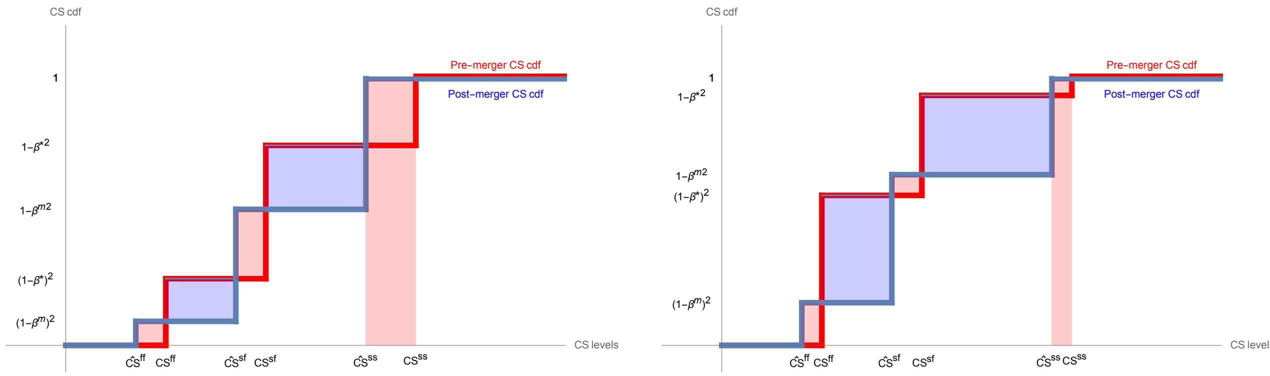
whose proof follows immediately from integration by parts. We then obtain:

Proposition 7. *Assume that $\beta(x^m) > \beta(x^*)$, in which case consumer surplus may either increase or decrease after a merger. Then consumer surplus post-merger is higher than pre-merger if and only if*

$$\begin{aligned}(1-\beta(x^m))^2\left(CS^{ff} - \hat{CS}^{ff}\right) + \left[1-\beta(x^m)^2 - (1-\beta(x^*))^2\right]\left(CS^{sf} - \hat{CS}^{sf}\right) + \left(CS^{ss} - \hat{CS}^{ss}\right) \\ < \left[1-\beta(x^*)^2 - (1-\beta(x^m))^2\right]\left(\hat{CS}^{sf} - CS^{ff}\right) + \left[\beta(x^m)^2 - \beta(x^*)^2\right]\left(\hat{CS}^{ss} - CS^{sf}\right),\end{aligned}$$

where we assume that $\hat{CS}^{ff} < CS^{ff} < \hat{CS}^{sf} < CS^{sf} < \hat{CS}^{ss} < CS^{ss}$.

This proposition is illustrated in Figure 3. The left panel depicts a case in which the pre- and post-merger CDFs cannot be ranked by SOSD, yet expected consumer surplus is higher before the merger. In this configuration, the adverse price effects of the merger dominate the innovation gains. By contrast, the right panel shows a situation where the innovation effect is strong enough to outweigh the price effect, so that expected consumer surplus increases after the merger.



(a) Merger reduces consumer surplus (b) Merger increases consumer surplus

Figure 3: Consumer surplus effects of mergers

We have shown above that R&D output synergies need not increase post-merger incentives to invest. However, when they do, an increase in consumer surplus becomes more likely. To illustrate, consider first the setting in Figure 2, where the pre-merger CDF of consumer surplus second-order stochastically dominates the post-merger CDF, so expected consumer surplus is lower after the merger. Introducing output synergies alters both the distribution of outcomes and their probabilities: the SOSD ranking breaks down and the expected level of consumer surplus becomes higher post-merger. This reversal is illustrated in Figure 4.

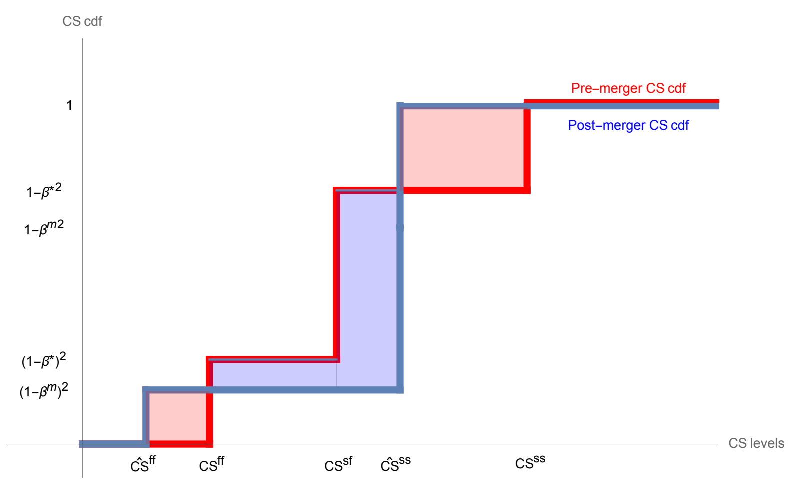


Figure 4: Setting in Figure 1 with R&D output synergies

In the remainder of this section, we return to the examples used to illustrate Proposition 3 and discuss the consumer-surplus effects of mergers in those settings. To assess the impact of a merger on consumer surplus, we must substitute the optimal investment levels into the consumer surplus expressions derived above. This requires solving for the optimal investment choices, which in turn calls for specific functional-form assumptions on the success probability function and the R&D cost function. Except for very simple specifications, these investment levels are not available in closed form, so we compute them numerically and then compare preand post-merger consumer surplus. Further details are provided in Appendix B.

Logit with quality-enhancing and cost-reducing innovations In all the logit examples with single-product firms, mergers do not benefit consumers even when they raise R&D. This occurs because the adverse price-coordination effect dominates the innovation effect. By contrast, when we extend the model so that each firm also supplies an additional, non-overlapping product in a separate market with constant-elasticity demand, the same merger can increase consumer surplus. In that case, higher post-merger R&D improves the quality (or reduces the cost) of both the overlapping and non-overlapping products, while the negative price effect is confined to the overlapping segment, so the innovation gains dominate. This multi-product logit example illustrates one of the advantages of our reduced-form two-stage framework: it can be extended in a tractable way to richer product portfolios.

Hotelling with vertically differentiated products In the Hotelling model, we have seen that, in the absence of synergies, R&D investment increases after a merger. Depending on the parameters and the shape of the success probability and R&D cost functions, consumer surplus may increase or decrease. Specifically, for a linear success probability function and a standard quadratic R&D cost function, consumer surplus decreases despite the increase in R&D. However, if the marginal cost of R&D is very steep, consumer surplus can increase.

Sutton's (2001) model of horizontally and vertically differentiated products and quantity competition As shown above, when the quality difference is large, a merger leads to a reduction in R&D; hence, consumer surplus falls. However, when the quality difference is small, a merger results in higher R&D effort provided the marginal cost of R&D is sufficiently large. Despite this increase in R&D, assuming a linear success probability function and a quadratic cost function, consumer surplus decreases.

Singh and Vives (1984) model of horizontally differentiated products, cost-reducing innovation and Cournot Competition We have seen above that, in the absence of synergies, investment decreases after a merger and so does consumer surplus. However, when synergies are present and the marginal cost of R&D is sufficiently large, post-merger R&D effort is higher than pre-merger. In that case, assuming a linear success probability function, and a quadratic R&D cost function, we observe that consumer surplus post-merger can be higher than pre-merger. (A similar remark is made by Mukherjee (2022).)

Mussa and Rosen (1978) model of vertical product differentiation with price competition. As mentioned before, a merger results in higher R&D effort provided the marginal cost of R&D is sufficiently low. With a linear success probability function and a quadratic R&D cost function, this is not possible though, so consumer surplus decreases.

In conclusion, our micro-founded examples with single-product firms provide very little support for a positive effect of mergers on consumer surplus, even when consolidation raises R&D effort. In the presence of R&D output synergies, we do observe consumer-surplus gains in the Singh and Vives (1984) quantity-competition models, but not in other single-product settings. By contrast, R&D input synergies make it more likely that mergers lead to substantially higher R&D (cf. Proposition 5), and therefore increase the scope for consumer-surplus improvements. Finally, our multi-product logit example with partially overlapping portfolios shows that consumer surplus may also rise when the price effects of the merger are confined to the overlapping segment, while the innovation effects extend to a broader set of non-overlapping products.

6 Concluding remarks

We have studied the implications of mergers for R&D within a broader model of R&D competition. Firms first invest in R&D to reduce costs, improve quality, or enhance demand, and then compete in the product market. In the most general version of our framework, R&D effort affects both the probability of innovation success and the payoff conditional on success, rather than just one of these margins. As a result, our model nests a large class of existing theories of mergers and long-run R&D, including stochastic models (Federico et al., 2017, 2018; Denicolò and Polo, 2018; Jullien and Lefouili, 2020) and deterministic models (Motta and Tarantino, 2021, Section 3.1; Bourreau et al., 2025, online appendix).

A merger modifies the incentives to invest in R&D through three channels: anticipation of post-merger price coordination, internalisation of a success-probability externality, and internalisation of a payoff externality arising from business-stealing in the product market. In the absence of price coordination, for instance because of regulation or a failure to coordinate prices across units, only the latter two forces remain and mergers unambiguously reduce R&D. In general, however, the sign of the merger effect on R&D depends on a large number of payoff levels and derivatives (cf. Proposition 1), so whether a merger ultimately spurs or discourages innovation is inherently model-specific. This complexity motivates our focus on two benchmark classes in which the condition simplifies and the underlying forces can be understood more transparently.

The first class comprises models of stochastic R&D in which investment effort affects the probability of success but does not directly change payoffs conditional on success. In this class, only price coordination and the success-probability externality operate, and Proposition 2 provides a clean characterisation of when mergers raise or lower R&D. We show that the standard assumption that firms earn zero profits when they fail to innovate is restrictive: once firms can obtain positive payoffs even when innovation fails, the pre-merger level of R&D –and thus the shape of the success probability and cost functions– becomes central in determining the effect of a merger on investment. The various cases discussed in Proposition 2 both nest existing results (Federico et al., 2017, 2018; Denicolò and Polo, 2018; Jullien and Lefouili, 2020) and reveal additional configurations in which mergers can unambiguously spur innovation, unambiguously reduce it, or have effects that depend on the initial R&D level.

The second class consists of models in which R&D affects the payoff conditional on success, while the probability of success is exogenous. Here, the relevant forces operate through price coordination and business-stealing, and Proposition 3 characterises the conditions under which a merger results in higher or lower R&D. We show that the usual assumption of deterministic R&D –R&D succeeds with probability one, as in Motta and Tarantino (2021) and Bourreau et al. (2025)– is again restrictive. Once success occurs with probability strictly less than one, both the pre-merger level of R&D and the difficulty of achieving success become crucial for the sign of the merger effect. In this sense, Proposition 2 is informative in environments where conditional payoffs are nearly insensitive to effort, whereas Proposition 3 is the natural benchmark when success probabilities are relatively inelastic. Together, they offer tractable approximations to the general condition in Proposition 1.

We also extend the framework to incorporate R&D synergies, asymmetries, and consumer-surplus effects. On the synergy side, we distinguish between R&D output synergies, whereby a successful innovation by one division can be leveraged across the other division (in the spirit of Farrell and Shapiro, 1990), and R&D input synergies, modelled as intra-firm spillovers in the RJV tradition (d'Aspremont and Jacquemin, 1988; Kamien et al., 1992; Suzumura, 1992). Output synergies can strengthen or weaken post-merger investment incentives: sharing innovation benefits raises the private return from success but also creates scope for free-riding within the merged entity. Input synergies, by effectively reducing the marginal cost of effort, make post-merger increases in R&D more likely in the environments we consider.

On the welfare side, we observe that higher post-merger R&D does not automatically translate into consumer gains. In standard single-product settings without synergies, our micro-founded examples indicate that even when mergers raise R&D effort, innovation effects are often outweighed by adverse price-coordination effects, so expected consumer surplus typically falls. Consumer surplus can increase in some models with R&D output synergies (for instance, in the Singh and Vives (1984) quantity-competition model), and R&D input synergies enlarge the scope for such gains by making R&D increase more likely. Finally, by exploiting the flexibility of our reduced-form approach, we show in a simple multi-product extension of the logit model that consumer surplus may strictly increase when portfolios overlap only partially and innovation applies to the broader product line. In that case, the negative price effects of the merger are confined to the overlapping segment, whereas the benefits of stronger R&D spill over to non-overlapping products, so the innovation effect can dominate despite stronger market power.

A Proofs

Proof of Proposition 2: When the probability of success depends on R&D effort, while the conditional payoffs are independent of it, expression in (5) becomes:

$$FOC_i^m(x^*) = \frac{\partial \beta_i(x^*)}{\partial x_i} \left[\beta_j(x^*)K_1 + (1-\beta_j(x^*))K_2\right].$$

Because $\beta'(x) > 0$, the sign of this expression is equal to the sign of the expression in the squared bracket. Suppose $K_1 > 0$ and $K_2 > 0$. Then, the $FOC_i^m(x^*)$ is clearly positive. The same conclusion holds for $K_1 = 0$, $K_2 > 0$ and $K_1 > 0$ and $K_2 = 0$. This proves the result in (i). Suppose $K_1 < 0$ and $K_2 < 0$, then $FOC_i^m(x^*)$ is negative. The same conclusion holds for $K_1 = 0$, $K_2 < 0$ and $K_1 < 0$ and $K_2 = 0$. This proves the result in (ii). Finally, we note that $x^m = x^*$ can only hold when $K_1 = 0$ and $K_2 = 0$ or when $x^* = \hat{x}$. This completes the proof of parts (i) and (ii).

In situations in which K_1 and K_2 have different signs, the $FOC_i^m(x^*)$ may be positive or negative. Define $\hat{x}$ as the solution to $\beta(x)(K_1 - K_2) + K_2 = 0$, i.e.,

$$\hat{x} = \beta^{-1}\left(\frac{K_2}{K_2 - K_1}\right).$$

Suppose $K_1 < 0$ and $K_2 > 0$, then $FOC_i^m(x^*)$ is positive for all $x^* < \hat{x}$ and negative otherwise. This proves result in (iii). Suppose $K_1 > 0$ and $K_2 < 0$, then $FOC_i^m(x^*)$ is positive for all $x^* > \hat{x}$. This proves result in (iv). ■

Proof of Proposition 3: In this setting, where the conditional payoffs are endogenous and depend on R&D effort while the probability of success is exogenous and is given by $\mu \in (0, 1]$, expression in (5) becomes:

$$FOC_i^m(x^*) = \mu\left[\mu K_3(x^*) + (1-\mu)K_4(x^*)\right].$$

Then it is straightforward to see that

(i) If $K_3(x^*) > 0$ and $K_4(x^*) > 0$, then $FOC_i^m(.)$ evaluated at x^* is positive, in which case a merger results in an increase in R&D. This proves the result in (i).

(ii) Similarly, if $K_3(x^*) < 0$ and $K_4(x^*) < 0$, then $FOC_i^m(.)$ evaluated at x^* is negative. In that case, a merger results in a decrease in R&D. This proves the result in (ii).

(iii) Above we have defined the function $\Phi(x)$. Suppose $\Phi(x)$ is decreasing. If $K_3(x^*) < 0$ and $K_4(x^*) > 0$, then the $FOC_i^m(x^*)$ is positive if $\mu < \frac{K_4(x^*)}{K_4(x)-K_3(x^*)}$. Then, it is easy to see that if the equation $\mu - \Phi(x) = 0$ does not have a solution, because $\Phi(x)$ is positive and decreasing, then $\mu < \frac{K_4(x}{K_4(x)-K_3(x)}$ for all x and so a merger results in an increase in R&D. However, if it has a solution, denoted $\tilde{x}$, the fact that $\Phi(x)$ is decreasing implies that $\mu < \frac{K_4(x)}{K_4(x)-K_3(x)}$ for all $x < \tilde{x}$ and $\mu > \frac{K_4(x)}{K_4(x)-K_3(x)}$ for all $x > \tilde{x}$. As a result, we conclude that a merger results in an increase in R&D if $x^* < \tilde{x}$; otherwise, $x^* > \tilde{x}$, a merger reduces R&D.

If we have the opposite situation in which $K_3(x^*) > 0$ and $K_4(x^*) < 0$, then the $FOC_i^m(x^*)$ is positive if $\mu > \frac{K_4(x^*)}{K_4(x) - K_3(x^*)}$. When $\mu - \Phi(x) = 0$ does not have a solution, because $\Phi(x)$ is positive and decreasing, this condition is never satisfied and so a merger always decreases R&D. When $\mu - \Phi(x) = 0$ has a solution, denoted $\tilde{x}$, we have $\mu > \frac{K_4(x)}{K_4(x) - K_3(x)}$ for all $x > \tilde{x}$. Hence, a merger results in higher R&D, $x^m > x^*$, if and only if $x^* > \tilde{x}$; otherwise, when $x^* < \tilde{x}$, a merger reduces R&D.

(iv) The results when $\Phi(x)$ is increasing are proven analogously. To save on space, we omit the details. ■

B Micro-founded examples

In this appendix, we present a series of micro-founded examples that illustrate Propositions 2 and 4. Each example is developed with enough detail to be read in isolation. This self-contained presentation entails a certain amount of unavoidable repetition across examples.

B.1 Logit model, horizontally differentiated products, quality-improving innovation and Bertrand Competition

This example illustrates the results in Proposition 1(i)-(iii). To characterize the profits in the second stage, we use a logit system of demands and assume Bertrand competition. Firms invest to increase the quality of their products, which we denote s_i, $i = 1, 2$. Initially, firms offer a basic product, whose quality is $s_f > 0$. Upon successful innovation, quality rises to s_s, with $s_s > s_f$. If the innovation effort fails, firms continue to offer the basic quality.

Consumers derive utility

$$v_i = s_i - p_i^\kappa + \epsilon_i,$$

from the inside goods, with $\kappa > 0$ and ϵ_i TIEV distributed. The utility from the outside option, denoted 0, is normalized to $v_0 = \epsilon_0$.

The market shares of the inside goods are then:

$$q_1(p_1, p_2) = \frac{e^{s_1 - p_1^\kappa}}{1 + e^{s_1 - p_1^\kappa} + e^{s_2 - p_2^\kappa}}, \qquad q_2(p_1, p_2) = \frac{e^{s_2 - p_2^\kappa}}{1 + e^{s_1 - p_1^\kappa} + e^{s_2 - p_2^\kappa}}.$$

The market share of the outside good is $q_0 = 1 - q_1 - q_2$. The parameter κ governs the disutility from price and has a bearing on the elasticity of demand, which is given by $\epsilon = \kappa p_i^\kappa (1 - s_i)$. $\kappa = 1$ is the standard multinomial logit; values of κ lower than 1 correspond to less elastic demands, while values of κ greater than 1 to more elastic demands.

In the second stage, firms compete in prices. Given a pair of innovation outcomes (s_i, s_j), and the price of the rival firm, an indididual firm i picks its price to maximize its payoff:

$$\pi_i(p_i; p_j) = (p_i - c) q_i(p_i, p_j), \ \ i, j = 1, 2; \ \ i \neq j.$$

The price equilibrium follows from solving the system of FOCs:

$$1-(p_i-c)(1-q_i(p_i,p_j))\kappa p_i^{\kappa-1}=0,\ i,j=1,2;\ i\neq j.$$

These FOCs define the equilibrium prices for any vector of qualities: $(p_i^*(s_i,s_j),p_j^*(s_i,s_j))$. Unfortunately, they cannot be solved in closed-form so we will solve them numerically to obtain the equilibrium prices corresponding to every innovation subgame. For a given pair of qualities (s_i,s_j), the corresponding second-stage profits are then obtained by plugging the equilibrium prices into the payoff above. As in the main body of the paper, we label the payoffs corresponding to the various innovation subgames as follows:

$$\pi_i^{ss}\equiv\pi_i(s_s;s_s),\quad \pi_i^{sf}\equiv\pi_i(s_s;s_f),\quad \pi_i^{fs}\equiv\pi_i(s_f;s_s),\quad \pi_i^{ff}\equiv\pi_i(s_f;s_f),$$

Post-merger, the merged entity chooses prices (p_1,p_2) to maximize the joint profit:

$$\Pi^m(p_1,p_2)=(p_1-c)q_1(p_1,p_2)+(p_2-c)q_2(p_1,p_2).$$

For each pair (s_i,s_j), the merged entity's optimal prices, denoted $(\hat{p}_i(s_i,s_j),\hat{p}_j(s_i,s_j))$, solve the FOCs:

$$1-(p_i-c)(1-q_i(p_i,p_j))\kappa p_i^{\kappa-1}+(p_j-c)q_j(p_i,p_j)\kappa p_i^{\kappa-1}=0,\ i,j=1,2;\ i\neq j.$$

The profits corresponding to each of the divisions of the merged entity are obtained by plugging the optimal prices corresponding to the various innovation outcomes into the joint payoff:

$$\hat{\pi}_i^{ff}\equiv\hat{\pi}_i(s_f,s_f),\quad \hat{\pi}_i^{ss}\equiv\hat{\pi}_i(s_s,s_s),\quad \hat{\pi}_i^{fs}\equiv\hat{\pi}_i(s_f,s_s),\quad \hat{\pi}_i^{sf}\equiv\hat{\pi}_i(s_s,s_f).$$

Merger increases R&D

We now present examples of parameters for which the results described in Proposition 2(i)-(iii) arise. Assume that $s_f=5$ and $s_s=6$. Moreover, assume $c=1$. When $\kappa=\frac{1}{2}$, we get the result in Proposition 2(ii). The profits levels corresponding to these parameters are:

Pre-merger payoffs:

Firm 1 \ *Firm 2*	s	f
s	7.055, 7.055	9.022, 4.654
f	4.654, 9.022	5.841, 5.841

Table B.1: Firms' payoffs ($\kappa=1/2$)

Post-merger payoffs:

Division 1 \ *Division 2*	s	f
s	11.668, 11.668	14.936, 5.495
f	5.495, 14.936	8.089, 8.089

Table B.2: Merged entity's payoffs ($\kappa=1/2$)

As per Proposition 2, we compute the difference between the incentives of the merged entity to invest in R&D in order to catch up with the other division, and those of a stand-alone firm to catch up with the competitor:

$$K_1 = \hat{\pi}_i^{ss} - \hat{\pi}_i^{fs} - \left(\hat{\pi}_j^{sf} - \hat{\pi}_j^{ss}\right) - \left(\pi_i^{ss} - \pi_i^{fs}\right) \approx 11.668 - 5.495 - (14.936 - 11.668) - (7.055 - 4.654) \approx 0.506.$$

We also compute the corresponding difference in incentives when the merged entity seeks to escape its partner division and the stand-alone firm seeks to escape its competitor:

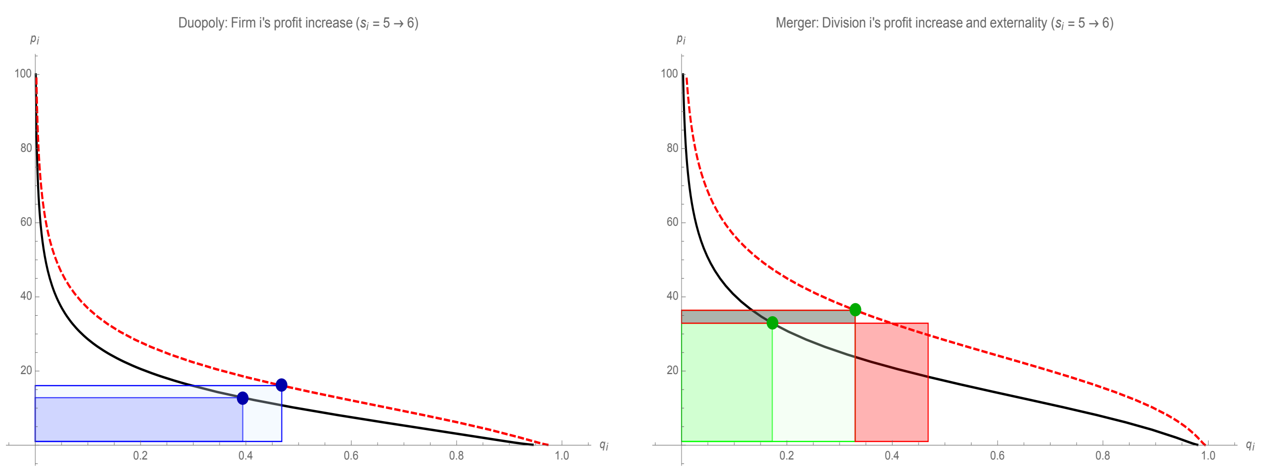
$$K_2 = \hat{\pi}_i^{sf} - \hat{\pi}_i^{ff} - \left(\hat{\pi}_j^{ff} - \hat{\pi}_j^{fs}\right) - \left(\pi_i^{sf} - \pi_i^{ff}\right) \approx 14.936 - 8.089 - (8.089 - 5.495) - (9.022 - 5.841) \approx 1.073.$$

Because the signs of K_1 and K_2 are positive, a merger increases R&D incentives.

We illustrate these differences in incentives for the catch-up and escape situations in Figures 5 and 6. As explained in the main text with reference to Spence (1975), a key reason why a merger has stronger incentives to invest in R&D than a stand-alone firm is that an increase in quality shifts demand more at lower than at higher quantities. Because the merged entity operates at lower quantities, its incentives to raise demand are larger. In addition to this first-order effect, the stand-alone firm is subject to a strategic effect, whereas a division of the merged entity internalizes a cannibalization effect.

Catch-up-case. In the catch-up case we take firm j as already successful ($s_j = s_s$), and consider firm i improving its quality from s_f to s_s.

In Figure 5a we plot the stand-alone gain for firm i when it catches up by increasing its quality from s_f to s_s. The black solid, downward-sloping curve is firm i's pre-innovation demand; the red dashed curve is its post-innovation demand. The left (blue) point marks the pre-innovation price-quantity for firm i, and the dark-blue rectangle is its pre-innovation profit. The right (blue) point is the post-innovation price-quantity, with the light-blue rectangle its post-innovation profit. The stand-alone incentive to catch up is therefore the difference between the two blue rectangles, i.e. $\pi_i^{ss} - \pi_i^{fs}$.



(a) Stand-alone firm (b) Division of merged entity and externality

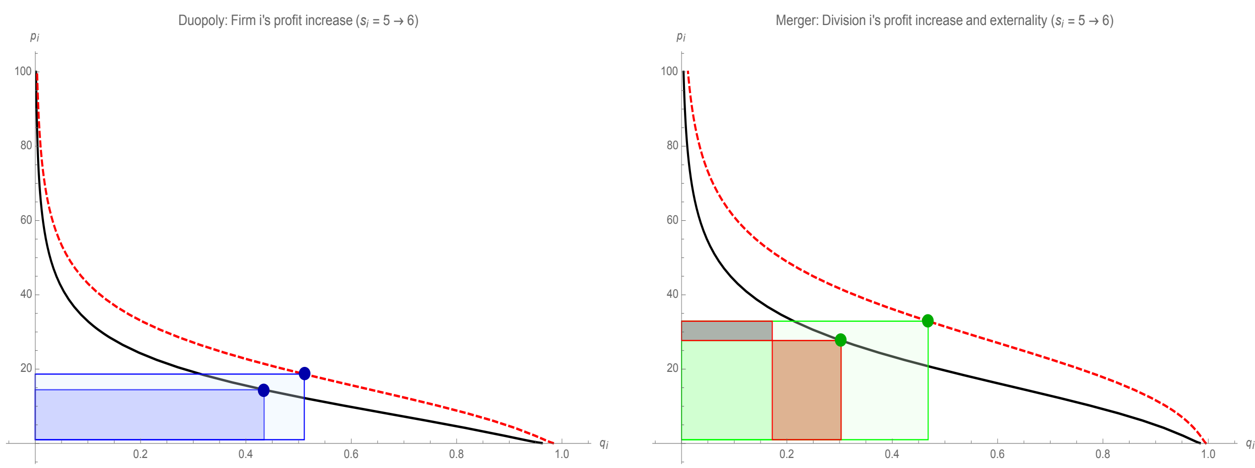
Figure 5: Catching-up ($\kappa = 1/2$)

In Figure 5b we depict the same experiment for a merged entity with two divisions, i and j. The left (green) point marks division i's pre-innovation price-quantity under (s_f, s_s), with the dark-green rectangle its profit share. The right (green) point is the common post-innovation price-quantity under (s_s, s_s), with the light-green area the contribution of division i after innovation (it includes the grey rectangle).

The (negative) externality on division j from i's catch-up is the change in j's profit when the state moves from (s_f, s_s) to (s_s, s_s), i.e. $-(\hat{\pi}_j^{sf} - \hat{\pi}_j^{ss})$. Graphically, the red rectangle captures the loss in quantity at division j due to cannibalization (as products become more similar), while the grey rectangle captures the offsetting gain in price-cost margin. Their net gives $\hat{\pi}_j^{ss} - \hat{\pi}_j^{sf}$.

Hence the merged-entity incentive to invest in division i's catch-up equals the sum of the own gain of i plus the (internalized) externality on j, i.e. $\hat{\pi}_i^{ss} - \hat{\pi}_i^{fs} - (\hat{\pi}_j^{sf} - \hat{\pi}_j^{ss})$. In the graph, this is equal to the difference between the two green rectangles (own gain for i) minus the net red-minus-grey effect on j.

Escape case. Figure 6 mirrors the above construction for an escape move. Here we take firm j as unsuccessful ($s_j = s_f$), and consider firm i improving its quality from s_f to s_s. The colors and geometric elements have the same meaning as above. Now firm i's innovation increases vertical differentiation and typically has a strong strategic effect, that is why we do not appreciate a large profits increase of a stand-alone firm; by contrast, vertical differentiation weakens cannibalization so that the externality on the partner firm is smaller.



(a) Stand-alone firm

(b) Division of merged entity and externality

Figure 6: Escaping ($\kappa = 1/2$)

Consumer surplus. In this single-product logit example with horizontally differentiated products and price competition, mergers do not benefit consumers even when they raise R&D: for $\kappa = 1/2$ the merger increases equilibrium investment, but expected consumer surplus falls because the adverse price-coordination effect dominates the innovation effect. For example, assuming a linear success probability $\beta(x_i) = x_i$ on $[0, 1]$ and a quadratic cost function $C(x_i) = 5x_i^2$, the pre-merger symmetric equilibrium effort is the solution to the $x^*[\pi_i^{ss} - \pi_i^{fs}] + (1 - x^*)[\pi_i^{sf} - \pi_i^{ff}] - 10x^* = 0$ (see FOC (1)), which gives $x^* \approx 0.295$. Likewise, the post-merger optimal equilibrium effort, which solves the FOC (2), is given by $x^m \approx 0.375$. However, expected consumer surplus falls from approximately $\mathbb{E}CS(x^*) \approx 2.23$ to $\mathbb{E}CS^m(x^m) \approx 0.99$. Thus, in this case with single-product firms, higher R&D after a merger does not translate into higher consumer surplus (as per Proposition 6).

We now illustrate how the flexibility of our reduced-form framework can be used to generate new insights in multi-product environments. To this end, we extend the previous logit specification by adding a second, independent market for each firm.[21] Each firm $k \in \{i, j\}$ continues to sell the differentiated good in the market with logit demand, as specified above, and, in addition, supplies an independent product in a separate market with constant-elasticity demand. We denote the demand for firm k's independent product by $q_k^0(p_k^0)$, $k = \{i, j\}$, and assume

$$q_k^0(p_k^0) = A_{z_k} \, (p_k^0)^{-\varepsilon}, \qquad \varepsilon > 1,$$

where $z_k \in \{f, s\}$ indexes the outcome of firm k's R&D project, and A_f and A_s are the corresponding demand levels upon project failure and success, with $A_f < A_s$. Marginal cost in the independent market is c_0. The R&D game is the same as in the single-product benchmark, except that innovation success now upgrades the entire product line: if firm k's project succeeds, the quality parameter of its logit product switches from s_f to s_s and, simultaneously, the demand shifter for its independent product increases from A_f to A_s; if the project fails, both remain at the basic leves s_f and A_f). For a given innovation state, prices in the logit and independent markets are chosen separately, so logit prices and quantities are determined as in the single-product model, while profits and consumer surplus in each state now include an additional contribution from the constant-elasticity markets.

Keeping the logit market specified exactly as above, and assuming that $A_s = 2$, $A_f = 1$ $\varepsilon = 1.1$ and $c_0 = 0.01$, Tables B.3 and B.4 report the pre- and post-merger equilibrium payoffs by innovation state when firms operate both in the logit market and in the independent constant-elasticity markets.

Pre-merger payoffs (multi-product case):

		Firm 2 s	*Firm 2* f
Firm 1	s	12.723, 12.723	14.690, 5.788
	f	5.788, 14.690	6.974, 6.974

Table B.3: Pre-merger profits (logit + additional markets, $\kappa = 1/2$).

Post-merger payoffs (multi-product case):

		Division 2 S	*Division 2* F
Division 1	S	17.337, 17.337	20.604, 6.628
	F	6.628, 20.604	9.222, 9.222

Table B.4: Post-merger profits (logit + additional markets, $\kappa = 1/2$).

Because the additional markets are unaffected by changes in ownership, their profit contributions are identical before and after the merger in every innovation state. They therefore drop out of the replacement expressions and the terms K_1 and K_2 remain exactly identical to those above: $K_1 \approx 0.506$ and $K_2 \approx 1.073$. This means that the merged entity's incentive to invest in R&D continue to be higher than those of a stand-alone firm.

With the linear success probability function and the quadratic R&D cost function the merger increases equilibrium R&D effort from $x^* \approx 0.716$ to $x^m \approx 0.774$. Using these equilibrium investment levels, Table B.5 reports the associated consumer-surplus levels by innovation state:

[21] For a similar market structure with overlapping and independent products, but in the context of start-up acquisitions, see Dijk, Moraga-González and Motchenkova (2024, 2025). In those papers, however, the focus is on the "direction of innovation."

	CS^{ff}	$CS^{sf} = CS^{fs}$	CS^{ss}	$\mathbb{E}[CS]$
Pre-merger	26.973	77.171	127.448	98.865
Post-merger	25.871	75.842	125.775	103.248

Table B.5: Consumer surplus by innovation state: logit with additional independent products

In the multi-product extension, as expected, consumer surplus is lower under the merger than under competition in each innovation state. However, the increase in R&D raises the probability of high-quality outcomes for both the overlapping and non-overlapping products, while the merger-induced price increase is confined to the overlapping logit segment. As shown in Table B.5, the innovation gains in the additional markets are sufficiently strong to dominate the adverse price effect in the main market, so that expected consumer surplus increases from approximately 98.9 to 103.2 after the merger.[22]

Merger decreases R&D

Keeping all the parameters fixed as above, let us suppose now that $\kappa = 2$. In this case, we get the result in Proposition 2(ii). The profits levels corresponding to these parameters are:

Pre-merger payoffs:

	Firm 2		
		s	f
Firm 1	s	0.300, 0.300	0.417, 0.205
	f	0.205, 0.417	0.287, 0.287

Table B.6: Firms' payoffs ($\kappa = 2$)

Post-merger payoffs:

	Division 2		
		s	f
Division 1	s	$0.523, 0.523$	$0.714, 0.263$
	f	$0.263, 0.714$	$0.431, 0.431$

Table B.7: Merged entity's payoffs ($\kappa = 2$)

Again, as per Proposition 2, we compute the differences

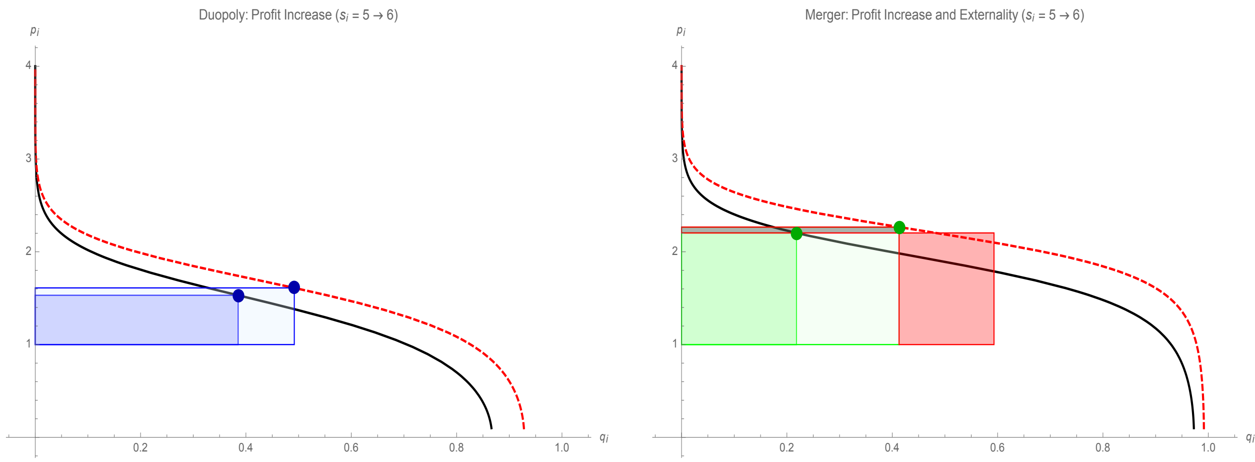
$$K_1 = \hat{\pi}_i^{ss} - \hat{\pi}_i^{fs} - \left(\hat{\pi}_j^{sf} - \hat{\pi}_j^{ss}\right) - (\pi_i^{ss} - \pi_i^{fs}) \approx 0.523 - 0.263 - (0.714 - 0.523) - (0.300 - 0.205) \approx -0.0262.$$

$$K_2 = \hat{\pi}_i^{sf} - \hat{\pi}_i^{ff} - \left(\hat{\pi}_j^{ff} - \hat{\pi}_j^{fs}\right) - (\pi_i^{sf} - \pi_i^{ff}) \approx 0.714 - 0.431 - (0.431 - 0.263) - (0.417 - 0.287) \approx -0.0145.$$

Because both the signs of K_1 and K_2 are negative, a merger decreases R&D incentives.

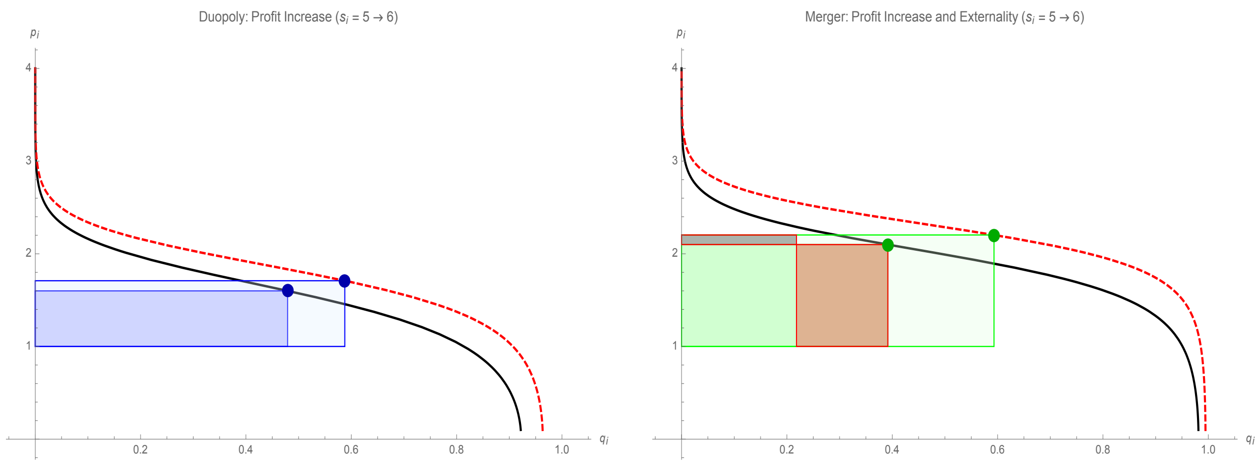
We illustrate these differences in incentives for the catch-up and escape situations in Figures 7 and 8. The key difference compared with the previous illustration is that an increase in quality now shifts demand more at higher than at lower quantities. By the same logic as in Spence (1975), the incentives to raise demand are now weaker for the merged entity than for a stand-alone firm.

[22]In this example, we take the case of a single but relatively large independent market. Similar results obtain if each firm sells many products in smaller independent markets.



(a) Stand-alone firm (b) Division of merged entity and externality

Figure 7: Catching-up ($\kappa = 2$)



(a) Stand-alone firm (b) Division of merged entity and externality

Figure 8: Escaping-up ($\kappa = 2$)

Merger increases R&D if marginal cost is high, decreases otherwise.

To illustrate the result in Proposition 2(iii), assume now the standard setting with $\kappa = 1$ and keep the remaining parameters as above. With $\kappa = 1$, a quality improvement shifts demand by the same amount at all quantities. As a result, the Spence (1975) quantity effect is muted: the gain from a marginal upward shift in demand is roughly independent of whether the firm operates at a high or a low quantity. The difference in R&D incentives between the merged entity and the stand-alone firms is therefore mainly driven by the market power effect, and the strategic and cannibalisation effects. In catch-up situations, the strategic and cannibalisation effects work against the merger, so that the merged entity has weaker incentives to invest than stand-alone firms, whereas in escape situations they work so that the merged entity may have stronger incentives to invest than stand-alone firms.

The profits levels corresponding to these parameters are:

Pre-merger payoffs:

Firm 2

		s	f
Firm 1	s	0.955, 0.955	1.302, 0.657
	f	0.657, 1.302	0.892, 0.892

Table B.8: Firms' payoffs ($\kappa = 1$)

Post-merger payoffs:

Division 2

		s	f
Division 1	s	1.727, 1.727	2.312, 0.850
	f	0.850, 2.312	1.350, 1.350

Table B.9: Merged entity's payoffs ($\kappa = 1$)

Again, as per Proposition 2, we compute the differences:

$$K_1 = \hat{\pi}_i^{ss} - \hat{\pi}_i^{fs} - \left(\hat{\pi}_j^{sf} - \hat{\pi}_j^{ss}\right) - \left(\pi_i^{ss} - \pi_i^{fs}\right) \approx 1.727 - 0.850 - (2.312 - 1.727) - (0.955 - 0.657) \approx -0.00613.$$

$$K_2 = \hat{\pi}_i^{sf} - \hat{\pi}_i^{ff} - \left(\hat{\pi}_j^{ff} - \hat{\pi}_j^{fs}\right) - \left(\pi_i^{sf} - \pi_i^{ff}\right) \approx 2.312 - 1.350 - (1.350 - 0.850) - (1.302 - 0.892) \approx 0.05227.$$

The sign of K_1 is negative while the sign of K_2 is positive. In catch-up situations, the strategic effect is weaker than the cannibalisation effect, so the merged entity has weaker incentives to invest than stand-alone firms (corresponding to $K_1 < 0$), whereas in escape situations the opposite holds and the merged entity has stronger incentives to invest (corresponding to $K_2 > 0$).

When a laggard firm or division catches up, the strategic effect is relatively weak, because closing the quality gap only modestly fosters price competition with the rival. By contrast, the cannibalisation effect within the merged entity is strong: much of the additional demand attracted by the improving division comes at the expense of the leading partner, so the gain in joint profit is limited. When a firm escapes, the ranking reverses. The strategic effect becomes very strong: by moving further ahead in quality, the leading firm induces the rival to cut its price substantially in order to protect sales of its lower-quality product. At the same time, the cannibalisation effect within the merged entity is relatively weak, because a large part of the additional sales generated by the escaping division comes at the expense of the outside alternative rather than from the partner division.

Because the sign of K_1 is negative while the sign of K_2 is positive, a merger increases R&D incentives when the pre-merger level of investment is sufficiently low, and decreases them otherwise. For example, assuming a linear probability of success function $\beta(x) = x$ and a cost function $C(x) = \frac{1}{4}x^2$, we obtain $x^* > x^m$, while for $C(x) = \frac{1}{2}x^2$ we get $x^* < x^m$. ■

B.2 Logit model, horizontally differentiated products, cost-reducing innovation and Bertrand competition

This example illustrates how Proposition 2(iii) can arise in a logit model with cost-reducing innovation and Bertrand competition. Firms invest to decrease the marginal of production, which we denote c_i, $i = 1, 2$. Initially, firms produce at a cost equal to $c_f > 0$. Upon successful innovation, marginal cost goes down to c_s, with $c_f > c_s$. If the innovation effort fails, firms continue to produce at cost c_f. In state ss both firms have low cost; in state sf only firm i has low cost; in fs only firm j has low cost; and in ff both have high cost.

Consumers derive utility

$$v_i = s - p_i^{\kappa} + \epsilon_i,$$

from the inside goods, with $\kappa > 0$ and ϵ_i TIEV distributed. The utility from the outside option, denoted 0, is normalized to $v_0 = \epsilon_0$. The corresponding logit market shares are

$$q_1(p_1, p_2) = \frac{e^{s_1 - p_1^\kappa}}{1 + e^{s_1 - p_1^\kappa} + e^{s_2 - p_2^\kappa}}, \qquad q_2(p_1, p_2) = \frac{e^{s_2 - p_2^\kappa}}{1 + e^{s_1 - p_1^\kappa} + e^{s_2 - p_2^\kappa}}.$$

The market share of the outside good is $q_0 = 1 - q_1 - q_2$. The parameter κ governs the disutility from price and has a bearing on the elasticity of demand, which is given by $\epsilon = \kappa p_i^\kappa (1 - s_i)$. $\kappa = 1$ is the standard multinomial logit; values of κ lower than 1 correspond to less elastic demands, while values of κ greater than 1 to more elastic demands.

In the second stage, firms compete in prices. Given an innovation outcome (c_i, c_j), and the price of the rival firm, an individual firm i picks its price to maximize its payoff:

$$\pi_i(p_i, p_j) = (p_i - c_i)\, q_i(p_i, p_j), \ i, j = 1, 2, \ i \neq j.$$

The FOC for firm i is:

$$1 - \kappa p_i^{\kappa - 1}(p_i - c_i)\big(1 - q_i(p_i, p_j)\big) = 0, \ i, j = 1, 2, \ i \neq j.$$

For each cost state (c_i, c_j), the unique pre-merger Bertrand equilibrium (p_i^*, p_j^*) is obtained by solving this system of equations. As mentioned before, these FOCs cannot be solved in closed-form so we will solve them numerically to obtain the equilibrium prices for every innovation subgame: $(p_i^*(c_i, c_j), p_j^*(c_i, c_j))$. The corresponding second-stage profits are then obtained by plugging the equilibrium prices into the payoff above. As in the main body of the paper, we label the payoffs corresponding to the various innovation subgames as follows:

$$\pi_i^{ss} \equiv \pi_i(c_s; c_s), \quad \pi_i^{sf} \equiv \pi_i(c_s; c_f), \quad \pi_i^{fs} \equiv \pi_i(c_f; c_s), \quad \pi_i^{ff} \equiv \pi_i(c_f; c_f),$$

After the merger, a single owner controls both products and chooses (p_i, p_j) to maximize the joint profit:

$$\Pi^m(p_i, p_j; c_i, c_j) = \pi_i(p_i, p_j; c_i) + \pi_j(p_i, p_j; c_j).$$

The FOCs for the merged entity are:

$$1 - \kappa p_i^{\kappa - 1}(p_i - c_i)(1 - q_i(p_i, p_j)) + \kappa p_j^{\kappa - 1}(p_j - c_j) q_j(p_i, p_j) = 0, \ i, j = 1, 2, \ i \neq j.$$

For each cost state (c_i, c_j), we solve these two equations numerically to obtain post-merger equilibrium prices $(\hat{p}_i(c_i, c_j), \hat{p}_j(c_i, c_j))$. The profits corresponding to each of the divisions of the merged entity are obtained by plugging the optimal prices corresponding to the various innovation outcomes:

$$\hat{\pi}_i^{ff} \equiv \hat{\pi}_i(c_f, c_f), \quad \hat{\pi}_i^{ss} \equiv \hat{\pi}_i(c_s, c_s), \quad \hat{\pi}_i^{fs} \equiv \hat{\pi}_i(c_f, c_s), \quad \hat{\pi}_i^{sf} \equiv \hat{\pi}_i(c_s, c_f).$$

Merger increases investment if marginal cost of R&D is high, decreases otherwise.

To illustrate the result in Proposition 2(iii), assume that $\kappa = 2$. Moreover, let $s = 2$ and the cost reduction from $c_f = 1$ to $c_s = 0$. The pre- and post-merger profits levels corresponding to these parameters are:

Pre-merger payoffs:

Firm 2

		s	f
Firm 1	s	0.403, 0.403	0.588, 0.094
	f	0.094, 0.588	0.151, 0.151

Table B.10: Firms' payoffs (logit, cost-reducing, $\kappa = 2$)

Post-merger payoffs:

Division 2

		s	f
Division 1	s	0.476, 0.476	0.731, 0.044
	f	0.044, 0.731	0.160, 0.160

Table B.11: Merged entity's payoffs (logit, cost-reducing, $\kappa = 2$)

Using the definitions in Proposition 2, the relevant replacement terms are:

$$K_1 = \hat{\pi}_i^{ss} - \hat{\pi}_i^{fs} - (\hat{\pi}_j^{sf} - \hat{\pi}_j^{ss}) - (\pi_i^{ss} - \pi_i^{fs}) \approx 0.4761 - 0.0436 - (0.7313 - 0.4761) - (0.4028 - 0.0944) \approx -0.131 < 0,$$

$$K_2 = \hat{\pi}_i^{sf} - \hat{\pi}_i^{ff} - (\hat{\pi}_j^{ff} - \hat{\pi}_j^{fs}) - (\pi_i^{sf} - \pi_i^{ff}) \approx 0.7313 - 0.1604 - (0.1604 - 0.0436) - (0.5884 - 0.1505) \approx 0.016 > 0,$$

where K_1 measures the change in the net replacement effect conditional on partner success (catch-up) and K_2 conditional on partner failure (escape).

The signs $K_1 < 0$ and $K_2 > 0$ match the qualitative discussion in the main text. In catch-up situations (partner success), a cost-reducing innovation by firm i mainly serves to match the rival's marginal cost. The stand-alone firm produces a larger quantity at a given cost, so the direct cost-saving effect is stronger pre-merger than for a division of the merged entity, while the strategic effect (the rival's price response) is relatively weak. As a consequence, the net gain from cost reduction is smaller post-merger, which is reflected in $K_1 < 0$.

In escape situations (partner failure), a successful innovation pushes firm i far ahead of a high-cost rival. The stand-alone firm then faces a strong strategic effect: the laggard rival reacts by cutting its price aggressively to defend its sales, sharply reducing the stand-alone firm's gain from becoming low cost. Within the merged entity, this large negative strategic effect is internalised, while the direct cost-saving effect remains. This is why K_2 is positive: conditional on partner failure, the merged entity has stronger incentives to reduce costs than a stand-alone firm, even though each division sells less output.

To close the example, suppose that the probability of project success is given by $\beta(x) = x$ on $[0, 1]$ and that R&D costs are $C(x) = 5x^2$. In the pre-merger duopoly, the symmetric equilibrium R&D effort is $x^* \approx 0.043$, while after the merger it becomes $x^m \approx 0.044$.

Merger decreases investment.

The result in Proposition 2(ii) also arises in this model if we instead assume $\kappa = 1$ and keep the rest of the parameters as above. The pre- and post-merger profits levels corresponding to $\kappa = 1$ are:

Pre-merger payoffs:

Firm 2

		s	f
Firm 1	s	0.599, 0.599	0.746 , 0.317
	f	0.317, 0.746	0.401, 0.401

Table B.12: Firms' payoffs (logit, cost-reducing, $\kappa = 1$)

Post-merger payoffs:

Division 2

		s	f
Division 1	s	0.687, 0.687	0.850, 0.313
	f	0.313, 0.850	0.426, 0.426

Table B.13: Merged entity's payoffs (logit, cost-reducing, $\kappa = 1$)

The relevant differences in replacement effects are:

$$K_1 = \hat{\pi}_i^{ss} - \hat{\pi}_i^{fs} - \left(\hat{\pi}_j^{sf} - \hat{\pi}_j^{ss}\right) - \left(\pi_i^{ss} - \pi_i^{fs}\right) \approx 0.687 - 0.313 - (0.850 - 0.687) - (0.599 - 0.317) \approx -0.069 < 0$$

$$K_2 \hat{\pi}_i^{sf} - \hat{\pi}_i^{ff} - \left(\hat{\pi}_j^{ff} - \hat{\pi}_j^{fs}\right) - \left(\pi_i^{sf} - \pi_i^{ff}\right) \approx 0.850 - 0.426 - (0.426 - 0.313) - (0.746 - 0.401) \approx -0.035 < 0.$$

Because both K_1 and K_2 are negative, a merger reduces investment.

B.3 Hotelling with vertically differentiated products

This example illustrates the result in Proposition 2*(i)*. To characterize the profits in the second stage we use a Hotelling model similar to that in Gilbert and Katz (2022) and Houba, Motchenkova and Wang (2023). Two firms locate at the two ends of a linear city that extends over the unit interval. If a firm successfully innovates, it provides a high-quality product, denoted s_s; otherwise, it provides a low-quality product, s_f, and stays in the market. Consumers are distributed uniformly along the linear city. The utility of a consumer located at $x \in [0,1]$ is $u_i(x) = s_i - tx - p_i$ when buying from i, and $u_j(x) = s_j - t(1-x) - p_j$ when buying from j. We assume that $s_s - s_f < t$, which ensures that both firms have positive market shares.[23] Demands are given by

$$D_i = \frac{s_i - s_j - (p_i - p_j)}{2t} + \frac{1}{2} \text{ and } D_j = \frac{1}{2} - \frac{s_i - s_j - (p_i - p_j)}{2t}$$

We normalize the marginal cost of production to zero.[24] Under these assumptions, the profit levels corresponding to the different subgames in Table 1 are given by:[25]

$$\pi_i^{ss} = \frac{t}{2},\ \pi_i^{sf} = \frac{(3t + (s_s - s_f))^2}{18t},\ \pi_i^{fs} = \frac{(3t - (s_s - s_f))^2}{18t},\ \pi_i^{ff} = \frac{t}{2} \qquad (15)$$

We now derive the payoffs post-merger that appear in Table 2. When the two divisions of the merged entity sell the same quality each division serves half of the market, since we assume the market is covered. Consequently, the furthest away consumer each division serves incurs a transportation cost equal to $\frac{t}{2}$. Hence, if, say, the two divisions sell high quality, the merged entity's optimal prices are: $p_i = p_j = s_s - \frac{t}{2}$. As a result, the profits of the two divisions are $\hat{\pi}_i^{ss} = \hat{\pi}_j^{ss} = \frac{s_s}{2} - \frac{t}{4}$. The merged entity's total profits are then $\hat{\pi}^{ss} = s_s - \frac{t}{2}$. Similarly, if the two divisions sell low quality, the prices

[23] Condition $s_s - s_f < t$ ensures that quality differences are not too large so that, both pre- and post-merger, consumers buy from both firms. Further, we assume that $s_s + s_f > 3t$, which ensures that the duopolists and the merged firm will cover the market. Also it is assumed that $\min\{s_s, s_f\} > 3t/2$.

[24] This is without loss of generality, since cost and quality advantages are isomorphic in the Hotelling model.

[25] The derivations of pre-merger profits are straightforward and can be found in Table 1 of Houba et al. (2023).

are $p_i = p_j = s_f - \frac{t}{2}$ and each division earns $\hat{\pi}_i^{ff} = \hat{\pi}_j^{ff} = \dfrac{s_f}{2} - \dfrac{t}{4}$, with the merged entity earning $\hat{\pi}^{ff} = s_f - \dfrac{t}{2}$.

When, instead, one of the divisions sells high quality and the other low quality, as in Gilbert and Katz (2021), we need to derive the optimal split of the consumers across the two divisions of the merged entity. Let y be the merged entity's profits maximizing location of the consumer indifferent between the offerings of the two divisions (to be found). Pricing optimality implies that such a consumer must obtain zero utility; otherwise, the monopolist could raise the prices of both divisions without losing any sales. This implies that $p_s = s_s - ty$ and $p_f = s_f - (1-y)t$. As a result, the merged entity's profit is given by $\hat{\pi}^{sf} = yp_s + (1-y)p_f$. Optimization with respect to y gives $\dfrac{\partial \hat{\pi}^{sf}}{\partial y} = s_s - s_f - 4ty + 2t = 0$. Solving for y, we obtain

$$y^* = \frac{1}{2} + \frac{s_s - s_f}{4t}.$$

As a result, the prices at which the merged entity sells high- and low-quality products are:

$$p_s = \frac{3s_s + s_f - 2t}{4} \text{ and } p_f = \frac{s_s + 3s_f - 2t}{4},$$

with corresponding profits:

$$\hat{\pi}_i^{sf} = \frac{(3s_s + s_f - 2t)(s_s - s_f + 2t)}{16t}, \hat{\pi}_j^{fs} = \frac{(3s_f + s_s - 2t)(s_f - s_s + 2t)}{16t}, \text{ and}$$

$$\hat{\pi}^{sf} = \frac{s_s + s_f}{2} - \frac{4t^2 - (s_s - s_f)^2}{8t}.$$

Summarizing, the merged entity's payoffs corresponding to the different innovation outcomes are:

$$\hat{\pi}^{ss} = s_s - \frac{t}{2},\ \hat{\pi}^{sf} = \hat{\pi}^{fs} = \frac{s_s + s_f}{2} - \frac{4t^2 - (s_s - s_f)^2}{8t},\ \hat{\pi}^{ff} = s_f - \frac{t}{2}. \tag{16}$$

Using the profits expressions in (15) and (16), we can compute the expressions for K_1 and K_2 in Proposition 1:

$$K_1 = \hat{\pi}_i^{ss} - \hat{\pi}_i^{fs} - \left(\hat{\pi}_j^{sf} - \hat{\pi}_j^{ss}\right) - \left(\pi_i^{ss} - \pi_i^{fs}\right) = \frac{(s_s - s_f)(12t - 5(s_s - s_f))}{72t}$$
$$K_2 = \hat{\pi}_i^{sf} - \hat{\pi}_i^{ff} - \left(\hat{\pi}_j^{ff} - \hat{\pi}_j^{fs}\right) - \left(\pi_i^{sf} - \pi_i^{ff}\right) = \frac{(s_s - s_f)(12t + 5(s_s - s_f))}{72t}.$$

It is straightforward to see that $K_1 > 0$ because $0 < s_s - s_f < t$, and $K_2 > 0$ because $s_s > s_f$. As a result, in this model a merger will always result in an increase in R&D.

To evaluate how a merger impacts consumer surplus, it is inevitable to compute the equilibrium investment levels. This means that we have to make assumptions about the success probability function and R&D cost function. Assume the success probability function is linear, i.e. $\beta(x_i) = x_i$; moreover, assume quadratic investment costs $C(x_i) = \frac{\gamma x_i^2}{2}$, where γ is a parameter reflecting the steepness of the cost function. For these functional forms, we can compute the pre- and post-merger investment levels explicitly:

$$x^* = \frac{\left(\pi_i^{sf} - \pi_i^{ff}\right)}{\gamma + \left(\pi_i^{sf} - \pi_i^{ff}\right) - \left(\pi_i^{ss} - \pi_i^{fs}\right)} = \frac{6t(s_s - s_f) + (s_s - s_f)^2}{18t\gamma + 2(s_s - s_f)^2}$$

$$x^m = \frac{\left(\hat{\pi}^{sf} - \hat{\pi}^{ff}\right)}{\gamma + (\hat{\pi}^{sf} - \hat{\pi}^{ff}) - (\hat{\pi}^{ss} - \hat{\pi}^{fs})} = \frac{4t(s_s - s_f) + (s_s - s_f)^2}{8t\gamma + 2(s_s - s_f)^2}$$

It is straightforward to show that $x^m > x^*$, which confirms the result mentioned above.

R&D output synergies We now assume that innovation developed by one division of the merged entity can be leveraged across the other division (cf. Proposition 3). While the pre-merger profits continue to be the same as before, based on the arguments mentioned above, it is straightforward to see that the post-merger profits change to:

$$\hat{\pi}^{ss} = \hat{\pi}^{sf} = \hat{\pi}^{fs} = s_s - \frac{t}{2}, \; \hat{\pi}^{ff} = s_f - \frac{t}{2}.$$

We are now ready to compute the expressions for D_1 and D_2 in Proposition 3:

$$D_1 = -\left(\pi_i^{ss} - \pi_i^{fs}\right) = \frac{(s_s - s_f)(s_s - s_f - 6t)}{18t} < 0,$$

where the sign follows from $s_s - s_f < t$.

$$D_2 = \hat{\pi}_i^{ss} - \hat{\pi}_i^{ff} - \left(\hat{\pi}_j^{ff} - \hat{\pi}_j^{ss}\right) - \left(\pi_i^{sf} - \pi_i^{ff}\right) = \frac{(s_s - s_f)(12t - (s_s - s_f))}{18t} > 0,$$

where, again, the sign follows from $s_s - s_f < t$.

Because $D_1 < 0$ and $D_2 > 0$, whether the merger with R&D output synergies increases or decreases investment depends on the magnitude of x^* (which, in turn, depends on the shape of the investment cost function). This illustrates Proposition 4(ii). When $x^* < \hat{x}$, we get $x^m > x^*$, where $\hat{x}$ is the solution to

$$\beta(x) = \frac{K_2}{K_2 - K_1} = \frac{12t + s_f - s_s}{2(9t + s_f - s_s)};$$

otherwise, the merger leads to less investment.

To derive the consumer surplus effects of mergers, we need explicit values for the pre- and post-merger optimal investments. Suppose, again, the success probability function is linear and the investment costs are quadratic. Then, the pre- and post-merger investment levels are equal to:

$$x^* = \frac{\left(\pi_i^{sf} - \pi_i^{ff}\right)}{\gamma + \left(\pi_i^{sf} - \pi_i^{ff}\right) - \left(\pi_i^{ss} - \pi_i^{fs}\right)} = \frac{6t(s_s - s_f) + (s_s - s_f)^2}{18t\gamma + 2(s_s - s_f)^2},$$

$$x^m_{syn} = \frac{\left(\hat{\pi}^{ss} - \hat{\pi}^{ff}\right)}{\gamma + (\hat{\pi}^{ss} - \hat{\pi}^{ff})} = \frac{s_s - s_f}{\gamma + s_s - s_f}$$

Comparing these two investment levels gives a lower bound on the cost parameter γ for which a merger can result in higher R&D investments:

$$\overline{\gamma} = \frac{6t(s_s - s_f) - (s_s - s_f)^2}{12t + s_f - s_s}.$$

Hence, compared to the pre-merger market equilibrium, a merger will result in an increase in R&D for all $\gamma > \overline{\gamma}$; otherwise, R&D investments will decrease. As mentioned above, this illustrates the scenario described in Proposition 3(ii).

Further, we note that, relative to the situation without R&D output synergies –where we obtained an unambiguous increase in R&D post-merger– synergies paradoxically restrict the set of parameters under which mergers can spur R&D. However, the level of innovation post-merger in the presence of output synergies need not be lower than in the model without synergies. In fact, in the case of a quadratic investment cost function and a linear success probability, we find that $x^* < x^m < x^m_{syn}$.

Consumer surplus The consumer surplus levels corresponding to the different subgames in are given by (see e.g. Gilbert and Katz (2022)):

$$CS^{ss} = s_s - \frac{5t}{4},\ CS^{sf} = \frac{2s_f - t}{2} + \frac{(s_s - s_f)^2 + 18t\,(s_s - s_f) - 27t^2}{36t},\ CS^{ff} = s_f - \frac{5t}{4}.$$

As a result, the expected consumer surplus in the pre-merger market is

$$\begin{aligned}\mathbb{E}CS(x^*) = (\beta^*)^2 \left(s_s - \frac{5t}{4}\right) + 2\beta^*\,(1-\beta^*) \left(\frac{2s_f - t}{2} + \frac{(s_s - s_f)^2 + 18t\,(s_s - s_f) - 27t^2}{36t}\right) \\ + (1-\beta^*)^2 \left(s_f - \frac{5t}{4}\right),\end{aligned}$$

where, to shorten the expression, we have written β^* to refer to $\beta(x^*)$.

After merger, the levels attained under the different innovation outcomes are:

$$\hat{CS}^{ss} = \frac{t}{4},\ \hat{CS}^{sf} = \hat{CS}^{fs} = \frac{t}{4} + \frac{(s_s - s_f)^2}{16t},\ \hat{CS}^{ff} = \frac{t}{4}.$$

The corresponding expected consumer surplus is then:

$$\mathbb{E}CS^m(x^m) = (\beta^m)^2 \frac{t}{4} + 2\beta^m\,(1-\beta^m) \left(\frac{t}{4} + \frac{(s_s - s_f)^2}{16t}\right) + (1-\beta^m)^2\,\frac{t}{4},$$

where, similarly, we have written β^m instead of $\beta(x^m)$.

As mentioned above, it is quite difficult to compare the consumer surplus levels pre- and post-merger because their distributions cannot be ranked according to the FOSD criterion. Inevitably, we have to factor the optimal investment levels and proceed numerically. This means that this comparison depends on the choice of functional forms. For the case of linear success probability and quadratic investment cost, we can plug the investment levels derived above and compare numerically the consumer surplus levels. Our numerical results show that consumer surplus decreases. However, if we use a much steeper marginal cost function, specifically, $C'(x) = 0.7x^{1/9}$, consumer surplus may increase (depending on the other parameters).[26]

[26] If we set $s_s = 2.5$, $s_f = 1.5$ and $t = 1$, consumer surplus increases after a merger by more than 5%.

With R&D output synergies, the expression for the expected consumer surplus is given by

$$\mathbb{E}CS^m_{syn}(\beta(x^m_{syn})) = \frac{t}{4},$$

which is independent of the assumed success probability and investment cost functions.

We now show that with R&D output synergies, consumer surplus always goes down after a merger. For this we compute the difference:

$$\mathbb{E}CS(x^*) - \mathbb{E}CS^m_{syn}(\beta^*(x^m_{syn})) = \frac{9t(2s_f - 3t) + \beta^*(s_s - s_f)(s_s - s_f + 18t) - (\beta^*)^2(s_s - s_f)^2}{18t}. \quad (17)$$

The derivative of this difference with respect to β^* is

$$\frac{(s_s - s_f)((2\beta^* - 1)s_f - 2\beta^* s_s + s_s + 18t)}{18t}$$

This expression is decreasing in β^*, which means that (17) is a strictly concave function of β^*. Setting $\beta^* = 0$ in (17) and simplifying gives $s_f - 3\frac{t}{2} > 0$. Setting $\beta^* = 1$ in (17) and simplifying gives $s_s - 3\frac{t}{2} > 0$. As a result, (17) is always positive; hence, a merger always reduces consumer surplus in the presence of R&D output synergies.

B.4 Singh and Vives (1984) model of horizontally differentiated products, cost-reducing innovation and Cournot Competition[27]

This example illustrates the result in Proposition 2(ii). At the same time, this example shows that the possibility Mukherjee (2022) mentions in his paper that R&D may increase after a merger in this model is not possible in the absence of synergies, irrespective of the success probability and R&D cost functions (see also Valletti (2025)). Notice that our approach allows us to show this for any formulation of the success probability and R&D cost functions.

To characterize the profits in the second stage, we use Singh and Vives' (1984) system of demands and assume Cournot competition. The demand functions are given by:

$$p_1(q_1, q_2) = a - bq_1 - dq_2,\ p_2(q_1, q_2) = a - bq_2 - dq_1,$$

and the Cournot equilibrium in the second-stage of the pre-merger market for arbitrary marginal costs c_1 and c_2 is:

$$q_1 = \frac{2b(a - c_1) - d(a - c_2)}{4b^2 - d^2},\ q_2 = \frac{2b(a - c_2) - d(a - c_1)}{4b^2 - d^2},$$

The corresponding expressions for the reduced-form profits are:

$$\pi_1(c_1; c_2) = \frac{b(2b(a - c_1) - d(a - c_2))^2}{(4b^2 - d^2)^2},\ \pi_2(c_2; c_1) = \frac{b(2b(a - c_2) - d(a - c_1))^2}{(4b^2 - d^2)^2}$$

In the first stage, firms invest to lower their marginal costs of production. Initially, firms marginal costs are equal to c. Upon innovation failure, they stay equal to c, so $c_f = c$. Upon successful innovation,

[27] We have also analysed the cases of cost-reducing innovations under Bertrand competition, and demand-enhancing innovations under Cournot competition. We obtain similar results. The detailed derivations are omitted to save on space, but can be obtained from the authors upon request.

firms lower their marginal costs by $\Delta > 0$, i.e. $c_s = c - \Delta$. Like in Mukherjee (2022), we normalize the cost-reduction so that $c_s = 0$ to shorten the expressions.

The conditional reduced-form profits corresponding to the various innovation subgames are:

$$\pi_i^{ss} = \frac{a^2 b}{(2b+d)^2},\ \pi_i^{sf} = \frac{b(a(2b-d)+cd)^2}{\left(d^2-4b^2\right)^2},\ \pi_i^{fs} = \frac{b(a(2b-d)-2bc)^2}{\left(d^2-4b^2\right)^2},\ \pi_i^{ff} = \frac{b(a-c)^2}{(2b+d)^2}$$

and symmetrically for firm j.

Post-merger, the merged entity chooses quantities to maximize the joint profit:

$$\pi^m = q_1(p_1(q_1,q_2)-c_1) + q_2(p_2(q_1,q_2)-c_2)$$

The merged entity's profit-maximizing quantities are:

$$q_1^m = \frac{a(b-d)-bc_1+c_2 d}{2b^2-2d^2},\ q_2^m = \frac{a(b-d)-bc_2+c_1 d}{2b^2-2d^2}$$

Observe that for these quantities to be strictly positive, it must be the case that $a(b-d) > bc$. We maintain this assumption in what follows.

The profits corresponding to each division of the merged entity are:

$$\hat{\pi}_1(c_1,c_2) = \frac{(a-c_1)(a(b-d)-bc_1+c_2 d)}{4\left(b^2-d^2\right)},\ \hat{\pi}_2(c_1,c_2) = \frac{(a-c_2)(a(b-d)-bc_2+c_1 d)}{4\left(b^2-d^2\right)}$$

Using the expressions above, and setting $c_s = 0$ and $c_f = c$ as appropriate, we obtain the conditional reduced-form profits for division i of the merged entity corresponding to the various innovation outcomes:

$$\hat{\pi}_i^{ss} = \frac{a^2}{4(b+d)},\ \hat{\pi}_i^{sf} = \frac{a(a(b-d)+cd)}{4\left(b^2-d^2\right)},\ \hat{\pi}_i^{fs} = \frac{(a-c)(a(b-d)-bc)}{4\left(b^2-d^2\right)},\ \hat{\pi}_i^{ff} = \frac{(a-c)^2}{4(b+d)},$$

and symmetrically for division j.

We are now ready to compute the expressions for K_1 and K_2 in Proposition 1. For K_1, we have:

$$K_1 = \hat{\pi}_i^{ss} - \hat{\pi}_i^{fs} - \left(\hat{\pi}_j^{sf} - \hat{\pi}_j^{ss}\right) - \left(\pi_i^{ss} - \pi_i^{fs}\right) = -\frac{cd\left[2a(8b^3-d^3)(b-d)+cdb(8b^2+d^2)\right]}{4(b-d)(b+d)\left(4b^2-d^2\right)^2} < 0,$$

where the sign follows from $c > 0$ and $b > d$.

After simplifying, we can write K_2 as follows:

$$K_2 = \hat{\pi}_i^{sf} - \hat{\pi}_i^{ff} - \left(\hat{\pi}_j^{ff} - \hat{\pi}_j^{fs}\right) - \left(\pi_i^{sf} - \pi_i^{ff}\right) = \frac{cd\left[(2a-c)(8b^3 d+bd^3)-(a-c)(16b^4+2d^4)\right]}{4(b-d)(b+d)\left(4b^2-d^2\right)^2}.$$

We now show that K_2 is also negative for all possible parameters. First, notice that the sign of K_2 is equal to the sign of the expression

$$(2a-c)(8b^3 d+bd^3)-(a-c)(16b^4+2d^4) \tag{18}$$

This expression is decreasing in a because its derivative with respect to a is:

$$-16b^4 - 2d^4 + 2(8b^3 d + bd^3) = -16b^4 - 2d^4 + 16b^3 d + 2bd^3 = -(16b^3 - 2d^3)(b-d) < 0,$$

where the last sign follows from $b > d$. Hence, if (18) is negative for the lowest admissible a, it is always negative.

As mentioned above, it must be the case that $a(b-d) > bc$, or $a > \frac{bc}{b-d}$. Hence, the lowest admissible a is $\frac{bc}{b-d}$. Plugging this lowest possible value of a in the expression (18) gives:

$$
\begin{aligned}
&(2\frac{bc}{b-d} - c)(8b^3d + bd^3) - (\frac{bc}{b-d} - c)(16b^4 + 2d^4) \\
&= \frac{c(b+d)}{b-d}(8b^3d + bd^3) - \frac{cd}{b-d}(16b^4 + 2d^4) \\
&= \frac{1}{b-d}\left[cd\left(b(b+d)\left(8b^2 + d^2\right) - 2\left(8b^4 + d^4\right)\right)\right] \\
&= cd\left(-8b^3 + bd^2 + 2d^3\right) < 0,
\end{aligned}
$$

where, again, the last sign follows from $b > d$.

Because both K_1 and K_2 are negative, we conclude that in this model a merger always results in a decrease in R&D effort. As a result, consumer surplus decreases after a merger. This is in contrast to Mukherjee (2022). The reason is that he allows for synergies: if one division of the merged entity succeeds in lowering marginal costs, the other division also benefits and produces at the same lower marginal cost. We now explore the role of synergies in this model.

R&D output synergies Assume now that innovations developed by one division of the merged entity are leveraged across the other division (cf. Proposition 3). In such a case, the pre-merger profits continue to be the same as above but the post-merger profits change to:

$$\hat{\pi}^{ss} = \hat{\pi}^{sf} = \hat{\pi}^{fs} = \frac{a^2}{2(b+d)}, \ \hat{\pi}^{ff} = \frac{(a-c)^2}{2(b+d)}.$$

We are now ready to compute the expressions for D_1 and D_2 in Proposition 3:

$$D_1 = -\left(\pi_i^{ss} - \pi_i^{fs}\right) = -\frac{4b^2c(a(2b-d) - bc)}{(d^2 - 4b^2)^2} < 0.$$

(The negative sign is as expected.) For D_2, we have:

$$D_2 = \hat{\pi}_i^{ss} - \hat{\pi}_i^{ff} - \left(\hat{\pi}_j^{ff} - \hat{\pi}_j^{ss}\right) - \left(\pi_i^{sf} - \pi_i^{ff}\right) = -\frac{c\left(8ab^2d(b+d) - (2a-c)(8b^4 + d^4)\right)}{2(b+d)\left(4b^2 - d^2\right)^2}.$$

The sign of D_2 depends of the sign of the numerator. It is easy to veryfy that $D_2 > 0$ if and only if $a > \frac{c(8b^4+d^4)}{2(8b^4-4b^3d-4b^2d^2+d^4)}$. In that case, because $D_1 < 0$ and $D_2 > 0$ and, as per Proposition 3(ii), if investment cost is sufficiently large, the merger leads to more investment. Otherwise, there is less investment after merger. This shows that the result in Mukherjee (2022) is due to R&D output synergies.

To derive the consumer surplus effects of mergers, we will need explicit values for the pre- and post-merger optimal investments. Assuming that the success probability function is linear and the investment costs quadratic, the pre- and post-merger investment levels are equal to:

$$x^* = \frac{\left(\pi_i^{sf} - \pi_i^{ff}\right)}{\gamma + \left(\pi_i^{sf} - \pi_i^{ff}\right) - \left(\pi_i^{ss} - \pi_i^{fs}\right)} = \frac{4b^2c(a(2b-d) - c(b-d))}{4b^2c^2d + \gamma\left(4b^2 - d^2\right)^2},$$

$$x_{syn}^m = \frac{\left(\hat{\pi}^{ss} - \hat{\pi}^{ff}\right)}{\gamma + \left(\hat{\pi}^{ss} - \hat{\pi}^{ff}\right)} = \frac{c(2a-c)}{c(2a-c) + 2\gamma(b+d)}$$

Comparing these two investment levels gives a lower bound on the cost parameter γ for which a merger can result in higher R&D investments:

$$\overline{\gamma} = \frac{4b^2c(2a-c)(a(2b-d)-bc)}{2a\left(8b^4 - 4b^3d - 4b^2d^2 + d^4\right) - c\left(8b^4 + d^4\right)}.$$

Hence, compared to the pre-merger market equilibrium, a merger will result in an increase in R&D for all $\gamma > \overline{\gamma}$; otherwise, R&D investments will decrease. This illustrates the result in Proposition 3(ii).

Consumer surplus As mentioned above, in the absence of R&D output synergies, investment goes down after a merger so consumer surplus cannot increase. Hence, in what follows, we focus on the case in which R&D output synergies are present.

The consumer surplus levels corresponding to the different subgames are given by (detailed derivations are omitted to save on space):

$$CS^{ss} = \frac{a^2\,(b+d)}{(2b+d)^2},\ CS^{sf} = CS^{fs} = \frac{2a\,(a-c)\,d^3 + \left(4b^2 - 3d^2\right)\left((a-c)\,2ab + bc^2\right)}{2(2b-d)^2(2b+d)^2},\ CS^{ff} = \frac{(a-c)^2\,(b+d)}{(2b+d)^2}.$$

As a result, the expected consumer surplus in the pre-merger market is

$$\mathbb{E}CS(x^*) = (\beta^*)^2\frac{a^2\,(b+d)}{(2b+d)^2} + 2\beta^*\,(1-\beta^*)\left(\frac{2a\,(a-c)\,d^3 + \left(4b^2-3d^2\right)\left((a-c)\,2ab + bc^2\right)}{2(2b-d)^2(2b+d)^2}\right)$$
$$+ (1-\beta^*)^2\left(\frac{(a-c)^2\,(b+d)}{(2b+d)^2}\right),$$

where, to shorten the expression, we have written β^* to refer to $\beta(x^*)$.

With R&D output synergies, after merger, the levels attained under the different innovation outcomes are:

$$\hat{CS}^{ss} = \hat{CS}^{sf} = \hat{CS}^{fs} = \frac{a^2}{4(b+d)},\ \hat{CS}^{ff} = \frac{(a-c)^2}{4(b+d)}.$$

The corresponding expected consumer surplus is then:

$$\mathbb{E}CS^m(x^m) = \left[(\beta^m)^2 + 2\beta^m\,(1-\beta^m)\right]\frac{a^2}{4(b+d)} + (1-\beta^m)^2\,\frac{(a-c)^2}{4(b+d)},$$

where, similarly, we have written β^m instead of $\beta(x^m)$.

To compare the consumer surplus levels pre- and post-merger we have to factor the optimal investment levels. This signifies that such a comparison depends on the choice of functional forms. For the case of linear success probability and quadratic investment cost, we can plug the investment levels derived above and compare numerically the consumer surplus levels. Our numerical results show that, depending on parameters, consumer surplus can increase or decrease.[28]

[28] For example, fix the parameters to $a = 10$, $b = 1$, $d = 0.1$ and $c = 2$. Then, for $\gamma = 10$, investment decreases after merger and hence consumer surplus too. However, for $\gamma = 20$, both investment and consumer surplus increase.

B.5 Sutton's (2001) model of horizontally and vertically differentiated products and quantity competition

This example illustrates the results in Proposition 2*(ii)* and *(iii)*. To characterize the profits in the second stage, we use Sutton's (2001) system of demands for horizontally and vertically differentiated products and Cournot competition. For the sake of exposition, we assume away horizontal product differentiation by setting $\sigma = 2$ and set marginal cost of production to zero. These normalizations are not crucial (see footnote 31). Initially, firms sell a basic product of low quality, denoted $s_f > 0$. If a firm's investment turns out successful, we assume that the firm offers a product of higher quality s_s, with $s_f < s_s < 2s_f$.[29] Otherwise, the firm continues offering the low-quality product. Under these assumptions, utility maximization yields the following system of demands for the (possibly) vertically differentiated products of the two players i and j:[30]

$$p_i(q_i, q_j) = \alpha - \frac{2\beta^2 q_i}{s_i^2} - \frac{2\beta^2}{s_i} \sum_{i \neq j} \frac{q_j}{s_j}, \; p_j(q_i, q_j) = \alpha - \frac{2\beta^2 q_j}{s_j^2} - \frac{2\beta^2}{s_j} \sum_{j \neq i} \frac{q_i}{s_i}.$$

For the Cournot equilibrium in the different innovation subgames, we refer to Dijk et al. (2024). The equilibrium profits corresponding to the subgames are:

$$\pi_i^{ss} = \frac{\alpha^2 s_s^2}{18\beta^2}, \; \pi_i^{sf} = \frac{\alpha^2 (2s_s - s_f)^2}{18\beta^2}, \; \pi_i^{fs} = \frac{\alpha^2 (2s_f - s_s)^2}{18\beta^2}, \; \pi_i^{ff} = \frac{\alpha^2 s_f^2}{18\beta^2} \tag{19}$$

Post-merger, the merged entity chooses quantities to maximize the joint profit:

$$\pi^m = q_1(p_1(q_1, q_2)) + q_2(p_2(q_1, q_2))$$

The optimal quantities can be found in Dijk et al. (2024) and the expressions for the profits corresponding to the different innovation outcomes are:

$$\hat{\pi}^{ss} = \hat{\pi}^{sf} = \hat{\pi}^{fs} = \frac{\alpha^2 s_s^2}{8\beta^2}, \; \hat{\pi}^{ff} = \frac{\alpha^2 s_f^2}{8\beta^2}.$$

We are now ready to compute the expressions for K_1 and K_2 in Proposition 1. For K_1, we have:

$$K_1 = \hat{\pi}^{ss} - \hat{\pi}^{fs} - (\pi_i^{ss} - \pi_i^{fs}) = 0 - \left(\frac{\alpha^2 s_s^2}{18\beta^2} - \frac{\alpha^2 (2s_f - s_s)^2}{18\beta^2} \right) < 0.$$

Note now that

$$K_2 = \frac{\alpha^2 s_s^2}{8\beta^2} - \frac{\alpha^2 s_f^2}{8\beta^2} - \left(\frac{\alpha^2 (2s_s - s_f)^2}{18\beta^2} - \frac{\alpha^2 s_f^2}{18\beta^2} \right),$$

which is negative for $\frac{9}{7} < \frac{s_s}{s_f} < 2$, and positive for the rest of parameters, $1 < \frac{s_s}{s_f} < \frac{9}{7}$. Hence, in the first scenario we always have $x^m < x^*$, while in the second scenario the outcome depends on the magnitude of x^* (which in turn depends on the shape of the investment cost function). Specifically, we obtain $x^m > x^*$ for $x^* < \hat{x}$, where $\hat{x}$ is the solution to $\beta(x) = \frac{K_2}{K_2 - K_1}$.

[29] The restriction $s_s < 2s_f$ rules out drastic innovations.

[30] The utility function underlying this system of demands is: $U = \sum_{i=1}^{2} \left[\alpha q_i - \left(\frac{\beta q_i}{s_i} \right)^2 \right] - \sigma \sum_{i=1}^{2} \sum_{i<j} \frac{\beta q_i}{s_i} \frac{\beta q_j}{s_j} - \sum_{i=1}^{2} p_i q_i$ (see Sutton, 1997; 2001).

Further, if we use a linear specification for the probability of success, $\beta(x) = x$, we can compute $\hat{x}$ explicitly:

$$\hat{x} = \frac{7s_s - 9s_f}{7s_s - 25s_f},$$

which is strictly positive for the parameter range corresponding to the second scenario $1 < \frac{s_s}{s_f} < \frac{9}{7}$. These two scenarios illustrate the results in Proposition 1(ii) and Proposition 1(iii).

If we further assume investment costs are quadratic, $c(x_i) = \frac{\gamma x_i^2}{2}$, where γ is a parameter reflecting the steepness of the cost function, we can compute closed-form solutions for both the pre- and post-merger investment levels. They are given by:

$$x^* = \frac{\left(\pi_i^{sf} - \pi_i^{ff}\right)}{\gamma + \left(\pi_i^{sf} - \pi_i^{ff}\right) - \left(\pi_i^{ss} - \pi_i^{fs}\right)} = \frac{2\alpha^2 s_s\left(s_s - s_f\right)}{9\gamma\beta^2 + 2\alpha^2\left(s_s - s_f\right)^2}$$

$$x^m = \frac{\left(\hat{\pi}^{sf} - \hat{\pi}^{ff}\right)}{\gamma + \left(\hat{\pi}^{sf} - \hat{\pi}^{ff}\right) - \left(\hat{\pi}^{ss} - \hat{\pi}^{fs}\right)} = \frac{\alpha^2(s_s^2 - s_f^2)}{8\gamma\beta^2 + \alpha^2\left(s_s^2 - s_f^2\right)}$$

Comparing these two expressions gives a lower bound on the parameter γ, for which a merger can result in higher R&D investments:

$$\overline{\gamma} = \frac{\left(\hat{\pi}^{sf} - \hat{\pi}^{ff}\right)\left(\pi_i^{ss} - \pi_i^{fs}\right) - \left(\hat{\pi}^{ss} - \hat{\pi}^{fs}\right)\left(\pi_i^{sf} - \pi_i^{ff}\right)}{\left(\hat{\pi}^{sf} - \hat{\pi}^{ff}\right) - \left(\pi_i^{sf} - \pi_i^{ff}\right)} = \frac{2\alpha^2 s_f(s_f^2 - s_s^2)}{\beta^2(7s_s - 9s_f)}.$$

Hence, for $1 < \frac{s_s}{s_f} < \frac{9}{7}$ and $\gamma > \overline{\gamma}$, a merger will result in an increase in R&D investment compared to pre-merger equilibrium.

This example illustrates parts (ii) and (iii) of Proposition 1. The result in part (ii) also arises in the numerical simulations reported in Federico et al. (2018), who model Bertrand competition instead. To the best of our knowledge, the result in part (iii) showing the importance of the shape of the R&D cost function has not been identified in the literature so far.[31]

R&D output synergies A key aspect of this model (with the Sutton's (2001) system of demands and vertically differentiated products) is that in the post-merger market, once one of the divisions' R&D projects succeeds, the merged entity chooses to offer only the high-quality product. This product repositioning implies that mergers involving R&D output synergies result in outcomes identical to those described above.

[31] In a more general setting with both vertical and horizontal product differentiation, we get more complicated expressions for the profits corresponding to the different innovation outcomes:

$$\pi_i^{ss} = \frac{2\alpha^2 s_s^2}{(4+\sigma)^2\beta^2},\ \pi_i^{sf} = \frac{2\alpha^2(4s_s - \sigma s_f)^2}{(16-\sigma^2)^2\beta^2},\ \pi_i^{fs} = \frac{2\alpha^2(4s_f - \sigma s_s)^2}{(16-\sigma^2)^2\beta^2},\ \pi_i^{ff} = \frac{2\alpha^2 s_f^2}{(4+\sigma)^2\beta^2} \tag{20}$$

$$\hat{\pi}^{ss} = \frac{\alpha^2 s_s^2}{2(2+\sigma)\beta^2},\ \hat{\pi}^{sf} = \hat{\pi}^{fs} = \frac{\alpha^2(s_s^2 + s_f^2 - \sigma s_s s_f)}{2(4-\sigma^2)\beta^2},\ \hat{\pi}^{ff} = \frac{\alpha^2 s_f^2}{2(2+\sigma)\beta^2}.$$

However, this does not affect the main insights presented in this example.

Consumer surplus For this model, the consumer surplus levels corresponding to the different subgames are given by (see e.g. Dijk et al. (2024)):

$$CS^{ss} = \frac{\alpha^2 s_s^2}{9\beta^2},\ CS^{sf} = CS^{fs} = \frac{\alpha^2 \left(s_s + s_f\right)^2}{36\beta^2},\ CS^{ff} = \frac{\alpha^2 s_f^2}{9\beta^2}.$$

Hence, the expected consumer surplus in the pre-merger market is:

$$\mathbb{E}CS(x^*) = (\beta^*)^2 \frac{\alpha^2 s_s^2}{9\beta^2} + 2\beta^*\left(1-\beta^*\right)\frac{\alpha^2\left(s_s+s_f\right)^2}{36\beta^2} + \left(1-\beta^*\right)^2 \frac{\alpha^2 s_f^2}{9\beta^2},$$

where we have written β^* to refer to $\beta(x^*)$.

After merger, irrespective whether there are R&D synergies or not, the levels attained under the different innovation outcomes are:

$$\hat{CS}^{ss} = \hat{CS}^{sf} = \hat{CS}^{fs} = \frac{\alpha^2 s_s^2}{16\beta^2},\ \hat{CS}^{ff} = \frac{\alpha^2 s_f^2}{16\beta^2}.$$

The corresponding expected consumer surplus is then:

$$\mathbb{E}CS^m(x^m) = \left[(\beta^m)^2 + 2\beta^m\left(1-\beta^m\right)\right]\frac{\alpha^2 s_s^2}{16\beta^2} + \left(1-\beta^m\right)^2 \frac{\alpha^2 s_f^2}{16\beta^2},$$

where, similarly, we have written β^m instead of $\beta(x^m)$.

To compare the consumer surplus levels pre- and post-merger we have to factor the optimal investment levels, which have been derived above for the case of linear success probability and quadratic investment cost. After plugging them, numerical calculations reveal that consumer welfare decreases after the merger, despite of the possible increase in R&D effort observed for some parameter ranges.

B.6 Mussa and Rosen (1978) model of vertical product differentiation with price competition.

This example illustrates the results in Proposition 2*(iv)*. To characterize the profits in the second stage we adopt the Mussa and Rosen's (1978) model of vertical product differentiation, further studied by Motta (1993). Two firms offer vertically differentiated products. Initially, they sell low-quality products, denoted s_f. If a firm successfully innovates, it sells high-quality products, denoted s_s, with $s_s > s_f$. Consumers' utility is given by $u_i = \theta s_i - p_i$. θ is the quality taste, which follows the uniform distribution on $[0,1]$. The market is always uncovered. We assume zero marginal cost.[32] Under these assumptions, the system of demands for high- and low-quality products is given by

$$q_s(p_s,p_f) = 1 - \frac{p_s - p_f}{s_s - s_f},\ q_f(p_s,p_f) = \frac{p_s - p_f}{s_s - s_f} - \frac{p_f}{s_f}$$

The profits corresponding to the different innovation subgames are given by (see Motta (1993)):[33]

$$\pi_i^{ss} = 0,\ \pi_i^{sf} = \frac{4s_s^2(s_s - s_f)}{(4s_s - s_f)^2},\ \pi_i^{fs} = \frac{s_s s_f(s_s - s_f)}{(4s_s - s_f)^2},\ \pi_i^{ff} = 0 \tag{21}$$

[32] With positive marginal cost, results are qualitatively similar.

[33] Because firms compete in prices, they each earn zero profit in case both have the same quality level. Hence, $\pi_i^{ss} = \pi_j^{ss} = \pi_i^{ff} = \pi_j^{ff} = 0$. Asymmetric pre-merger profits, π_i^{sf} and π_i^{fs}, are derived following Motta (1993).

Post-merger, the merged entity chooses quantities to maximize the joint profit. The merged entity's profits corresponding to the different innovation outcomes are:[34]

$$\hat{\pi}^{ss} = \hat{\pi}^{sf} = \hat{\pi}^{fs} = \frac{s_s}{4},\ \hat{\pi}^{ff} = \frac{s_f}{4}.$$

Using these payoffs, we can now compute the expressions for K_1 and K_2 in Proposition 1:

$$K_1 = \hat{\pi}^{ss} - \hat{\pi}^{fs} - (\pi_i^{ss} - \pi_i^{fs}) = \frac{s_s s_f (s_s - s_f)}{(4s_s - s_f)^2} > 0,$$

and

$$K_2 = \hat{\pi}^{sf} - \hat{\pi}^{ff} - (\pi_i^{sf} - \pi_i^{ff}) = \frac{s_f(s_s - s_f)(s_f - 8s_s)}{4(4s_s - s_f)^2} < 0,$$

for all $s_s - s_f > 0$. As a result, this example illustrates the scenario identified in part (iv) of Proposition 1.

To the best of our knowledge, this case has not been identified in the previous literature on the effects of mergers. In this scenario, the impact of a merger depends on the level of investment pre-merger, x^*, which in turn depends on the shape of the investment cost function. When $x^* > \hat{x}$, where $\hat{x}$ is the solution to $\beta(x) = \frac{K_2}{K_2 - K_1}$, we obtain $x^m > x^*$.

If we assume a linear success probability function, we can compute $\hat{x}$ explicitly:

$$\hat{x} = \frac{8s_s - s_f}{12s_s - s_f}.$$

Further, if we assume investment costs are quadratic, $c(x_i) = \frac{\gamma x_i^2}{2}$, we can compute closed form solutions for both pre-merger and post-merger investment levels. They are given by

$$x^* = \frac{\left(\pi_i^{sf} - \pi_i^{ff}\right)}{\gamma + \left(\pi_i^{sf} - \pi_i^{ff}\right) - \left(\pi_i^{ss} - \pi_i^{fs}\right)} = \frac{4s_s^2\,(s_s - s_f)}{\gamma(4s_s - s_f)^2 + s_s(4s_s^2 - 3s_s s_f - s_f^2)}$$

$$x^m = \frac{\left(\hat{\pi}^{sf} - \hat{\pi}^{ff}\right)}{\gamma + (\hat{\pi}^{sf} - \hat{\pi}^{ff}) - (\hat{\pi}^{ss} - \hat{\pi}^{fs})} = \frac{s_s - s_f}{4\gamma + s_s - s_f}$$

Comparing these two expressions gives an upper bound on parameter γ, for which a merger results in higher R&D investments:

$$\overline{\gamma} = \frac{\left(\hat{\pi}^{ss} - \hat{\pi}^{fs}\right)\left(\pi_i^{sf} - \pi_i^{ff}\right) - \left(\hat{\pi}^{sf} - \hat{\pi}^{ff}\right)\left(\pi_i^{ss} - \pi_i^{fs}\right)}{\left(\pi_i^{sf} - \pi_i^{ff}\right) - \left(\hat{\pi}^{sf} - \hat{\pi}^{ff}\right)} = \frac{\left(s_s^2 - s_f s_s\right)}{8s_s - s_f}$$

Otherwise, for $\gamma > \overline{\gamma}$, a merger results in a reduction in R&D investment compared to pre-merger equilibrium. Because the case $\gamma < \overline{\gamma}$ fails to satisfy the SOCs (see Appendix C), we conclude that, for quadratic R&D costs and linear success probabilities, in this model a merger can only result in lower

[34]When the two divisions of the merged entity sell the same quality, the monopolist's profits are given by $\hat{\pi}^{ss} = \frac{s_s}{4}$ $(\hat{\pi}_i^{ss} = \hat{\pi}_j^{ss} = \frac{s_s}{8})$ and $\hat{\pi}^{ff} = \frac{s_f}{4}$ $(\hat{\pi}_i^{ff} = \hat{\pi}_j^{ff} = \frac{s_f}{8})$. When the R&D project of one of the divisions is successful and the other not, the monopolist only sells the high-quality product. Hence, $\hat{\pi}^{sf} = \hat{\pi}^{fs} = \frac{s_s}{4}$, $(\hat{\pi}_i^{sf} = \frac{s_s}{4}$ and $\hat{\pi}_j^{fs} = 0)$.

R&D and, hence, lower consumer surplus.

R&D output synergies This model also has the feature that in the post-merger market, once the R&D project of one of the divisions succeeds, the merged entity chooses to offer only the high-quality product. This product repositioning implies that mergers involving R&D output synergies result in outcomes identical to those described above.

C Micro-founded example illustrating Proposition 3

In this appendix, we provide a micro-founded example to illustrate the results in Proposition 3.

Singh and Vives's (1984) model of price competition, horizontally differentiated products and cost-reducing innovation

We consider the Sighn and Vives's (1984) system of demands, investments that decrease the marginal cost of the products and price competition.

The demands are given by

$$q_1(p_1, p_2) = a - bp_1 + dp_2,\ q_2(p_1, p_2) = a - bp_2 + dp_1,$$

where, as usual, $b > d$.

In the pre-merger market, the second-stage equilibrium is:

$$p_1 = \frac{2b(a + bc_1) + d(a + bc_2)}{4b^2 - d^2},\ p_2 = \frac{d(a + bc_1) + 2b(a + bc_2)}{4b^2 - d^2}.$$

The reduced-form profits of firms 1 and 2 are:

$$\pi_1 = \frac{b\left(2b(a - bc_1) + d(a + bc_2) + c_1 d^2\right)^2}{(4b^2 - d^2)^2},\ \pi_2 = \frac{b\left(2b(a - bc_2) + d(a + bc_1) + c_2 d^2\right)^2}{(4b^2 - d^2)^2}.$$

We use these expressions to build our payoffs for stage 1, in which the firms decide on R&D efforts to reduce their marginal costs of production. We assume that initially firms have a marginal cost equal to c, with $a > (b - d)c$. If a firm invests an amount $C(x_i) = \gamma\frac{x_i^2}{2}$, then it gets a cost decrease of x_i. Hence, the payoffs of firm i corresponding to the different innovation subgames are:

$$\pi_i^{ss} = \frac{b\left(2b(a - b(c - x_i)) + d(a + b(c - x_j)) + (c - x_i)d^2\right)^2}{(4b^2 - d^2)^2},\ \pi_i^{ff} = \frac{b\left(2b(a - bc) + d(a + bc) + (cd^2\right)^2}{(4b^2 - d^2)^2},$$

$$\pi_i^{sf} = \frac{b\left(2b(a - b(c - x_i)) + d(a + bc) + (c - x_i)d^2\right)^2}{(4b^2 - d^2)^2},\ \pi_i^{fs} = \frac{b\left(2b(a - bc) + d(a + b(c - x_j)) + (cd^2\right)^2}{(4b^2 - d^2)^2},$$

and symmetrically for firm j.

In the pre-merger market, the problem of a firm i is to maximize:

$$\mathbb{E}\pi_i = (1 - \mu)^2\pi_i^{ff} + (1 - \mu)\mu\pi_i^{fs} + \mu(1 - \mu)\pi_i^{sf} + \mu^2\pi_i^{ss} - \gamma\frac{x_i^2}{2}. \tag{22}$$

Taking the FOC with respect to x_i and applying symmetry we get the equilibrium investment in the pre-merger market:

$$x^* = \frac{2b\mu(2b+d)\left(2b^2-d^2\right)(a-(b-d)c)}{\gamma\left(4b^2-d^2\right)^2 - 2b\mu\left(2b^2-d^2\right)\left(2b^2-d^2-bd\mu\right)}. \tag{23}$$

x^* is positive when γ is not too low, which is ensured by the SOC (see Appendix D).

In the second stage of the post-merger market, the merged entity chooses prices to maximize the joint profit:

$$\pi^m(p_1, p_2) = q_1(p_1, p_2)(p_1 - c_1) + q_2(p_1, p_2)(p_2 - c_2).$$

The merged entity's second-stage optimal prices are:

$$p_1 = \frac{1}{2}\left(\frac{a}{b-d} + c_1\right),\ p_2 = \frac{1}{2}\left(\frac{a}{b-d} + c_2\right)$$

Given these prices, the reduced-form profits of the two division of the merged entity are:

$$\pi_1^m = \frac{(a + c_1(d-b))(a - bc_1 + c_2 d)}{4(b-d)},\ \pi_2^m = \frac{(a + c_2(d-b))(a - bc_2 + c_1 d)}{4(b-d)}.$$

Hence, the conditional reduced-form profits for division i of the merged entity corresponding to the various innovation outcomes are:

$$\hat{\pi}_i^{ss} = \frac{(a + (c-x_i)(d-b))(a - b(c-x_i) + (c-x_j)d)}{4(b-d)},\ \hat{\pi}_i^{sf} = \frac{(a + (c-x_i)(d-b))(a - b(c-x_i) + cd)}{4(b-d)},$$

$$\hat{\pi}_i^{fs} = \frac{(a + c(d-b))(a - bc + (c-x_j)d)}{4(b-d)},\ \hat{\pi}_i^{ff} = \frac{(a + c(d-b))(a - bc + cd)}{4(b-d)},$$

and symmetrically for division j.

We are now ready to compute the relevant expressions $K_3(x^*)$ and $K_4(x^*)$ in Proposition 3. For $K_3(x^*)$ we get:

$$\begin{aligned} K_3(x^*) = &-\frac{1}{4}\left(a\left(\frac{b}{d-b} - 1\right) + 2b(c-x^*) - (c-x^*)d\right) - \frac{d(a + (c-x^*)(d-b))}{4(b-d)} + \\ &\frac{2b\left(d^2-2b^2\right)\left(2b(a - b(c-x^*)) + d(a + b(c-x^*)) + (c-x^*)d^2\right)}{\left(d^2-4b^2\right)^2} \\ = &\frac{d\left(-4b^2+2bd+d^2\right)(a-(b-d)(c-x^*))}{2(2b-d)^2(2b+d)} < 0,\ \text{because } b > d \text{ and } a-(b-d)(c-x^*) > 0. \end{aligned}$$

So $K_3(x^*)$ is always negative. This demonstrates the result in Motta and Tarantino (2021, section 3.1) that, if innovation is surely successful (deterministic innovation), a merger always reduces investment in this model.

For $K_4(x^*)$, we obtain:

$$\begin{aligned} K_4(x^*) = &-\frac{1}{4}\left(a\left(\frac{b}{d-b} - 1\right) + 2b(c-x^*) - cd\right) - \frac{d(a + c(d-b))}{4(b-d)} \\ &+\frac{2b\left(d^2-2b^2\right)\left(d(a+bc) + 2b(a - b(c-x^*)) + d^2(c-x^*)\right)}{\left(4b^2-d^2\right)^2} \end{aligned}$$

$$= \frac{d\left[bdx^*\left(8b^2-3d^2\right)-\left(8b^3-4bd^2-d^3\right)(a-c(b-d))\right]}{2\left(4b^2-d^2\right)^2},$$

whose sign is in principle ambiguous. Note, however, that if investment costs are very steep and correspondingly the pre-merger equilibrium investment level is very low ($x^* \to 0$), the sign of $K_4(x^*)$ is negative. In that case, because $K_3(x^*)$ is also negative, investment post-merger is definitely lower than pre-merger. However, if investment costs are not that large, $K_4(x^*)$ may be positive. So, we conclude that there are two relevant cases here: $K_3(x^*) < 0$ and $K_4(x^*) < 0$, and $K_3(x^*) < 0$ and $K_4(x^*) > 0$. The first case illustrates part (ii) of Proposition 3 and implies an unambiguous reduction in R&D post-merger. The second case illustrates part (iv)(a) of Proposition 3, in which case the impact of a merger on R&D depends on the magnitude of the success probability and the shape of the R&D cost function. We now examine further details on this second case.

Note that $K_3(x)$ is decreasing in x, while $K_4(x)$ is increasing in x. Further, it is easy to see that $\Phi(x)$ is increasing in x, where, recall, $\Phi(x) = \frac{K_4(x)}{K_4(x)-K_3(x)}$. Solving the equation $\mu - \Phi(x) = 0$ in x gives:

$$\widetilde{x} = \frac{(a-(b-d)c)(2b+d)\left(4b^2-2bd-d^2\right)}{d(8b^3-3bd^2)-\mu\left(8b^4+d^4-4b^2d^2\right)}.$$

Inspection of this expression reveals that $\tilde{x}$ is strictly positive for any $\mu < \hat{\mu} \equiv \frac{d(8b^3-3bd^2)}{8b^4+d^4-4b^2d^2} < 1$. We then conclude that for $\mu < \hat{\mu}$, the post-merger investment level is higher than pre-merger, i.e. $x^m > x^*$, if and only if $x^* > \tilde{x}$; this requires the marginal cost of R&D not to be too steep. Otherwise, if $x^* < \tilde{x}$, we get a decrease in investment after a merger, i.e., $x^m < x^*$.

To identify a condition on the steepness of the R&D cost function that results in lower R&D after a merger, we compare expressions for pre-merger and post-merger investment levels. The equilibrium investment in the pre-merger market is given above in equation (23). To obtain the merged entity's optimal investment, we maximize the joint profits:

$$\mathbb{E}\pi^m = (1-\mu)^2(\hat{\pi}_i^{ff}+\hat{\pi}_j^{ff})+(1-\mu)\mu(\hat{\pi}_i^{fs}+\hat{\pi}_j^{sf})+\mu(1-\mu)(\hat{\pi}_i^{sf}+\hat{\pi}_f^{fs})+\mu^2(\hat{\pi}_i^{ss}+\hat{\pi}_j^{ss})-\gamma\frac{x_i^2}{2}-\gamma\frac{x_j^2}{2}. \quad (24)$$

Taking the FOC with respect to x_i, applying symmetry, and solving for the optimal investment post-merger gives:

$$x^m = \frac{\mu(a-c(b-d))}{2\gamma-\mu(b-d\mu)}.$$

Comparing x^* and x^m gives an upper bound on the parameter γ, denoted $\overline{\gamma}$, for which a merger can result in a higher level of R&D investment:

$$\overline{\gamma} = \frac{\mu(1-\mu)\left(4b^4-2bd^3+4b^3d-2b^2d^2\right)}{8b^3-4bd^2-d^3},$$

which is strictly positive for all $\mu < 1$. Hence, we conclude that with $\mu < \hat{\mu}$ and $\gamma < \overline{\gamma}$, and provided that the second-order conditions hold which require μ to be small relative to γ (in particular $\gamma > \frac{1}{4}\mu(b+d\mu)$, see Appendix D), a merger will result in an increase in R&D compared to pre-merger. Otherwise, for $\gamma > \overline{\gamma}$ (and fixing the other parameters), R&D will decrease after a merger.[35]

[35] For example, if the parameters are $a = 3, b = 1.7, d = 0.7, c = 2.3$ and $\mu = 0.19$ then this threshold is approximately $\overline{\gamma} = 0.1865$. So for $0.0871 < \gamma < 0.1865$ (low investment costs for which the SOC in Appendix D holds) post-merger investment is greater than pre-merger. However, when $\gamma > 0.1865$ (high investment costs) we get the opposite.

D Concavity and Sufficient Conditions for a Maximum

In the main Propositions of our paper we have assume the strict concavity of the payoffs. However, in the examples we have used to illustrate them (see Appendix B), we have employed specific functional forms for the success probability function and the cost of R&D function. In this appendix, we verify that the second-order conditions (SOCs) are satisfied for the examples we have presented, or imposed conditions on the magnitude of the marginal cost of R&D to ensure our analysis pertains to a true maximum.

Concavity in the examples illustrating Proposition 2

Recall that the pre- and post-merger expected profits in Proposition 2 are, respectively, given by:

$$\begin{aligned}\mathbb{E}\pi_i(x_i;x_j) = \beta_i(x_i)\left[\beta_j(x_j)\pi_i^{ss} + (1-\beta_j(x_j))\pi_i^{sf}\right] \\ +(1-\beta_i(x_i))\left[\beta_j(x_j)\pi_i^{fs} + (1-\beta_j(x_j))\pi_i^{ff}\right] - C(x_i),\end{aligned}$$

and

$$\begin{aligned}\mathbb{E}\pi^m(x_i,x_j) = \beta_i(x_i)\left[\beta_j(x_j)\hat{\pi}_i^{ss} + (1-\beta_j(x_j))\hat{\pi}_i^{sf}\right] \\ +(1-\beta_i(x_i))\left[\beta_j(x_j)\hat{\pi}_i^{fs} + (1-\beta_j(x_j))\hat{\pi}_i^{ff}\right] - C(x_i) \\ +\beta_j(x_j)\left[\beta_i(x_i)\hat{\pi}_j^{ss} + (1-\beta_i(x_i))\hat{\pi}_j^{sf}\right] \\ +(1-\beta_j(x_j))\left[\beta_i(x_i)\hat{\pi}_j^{fs} + (1-\beta_i(x_i))\hat{\pi}_j^{ff}\right] - C(x_j).\end{aligned}$$

To ensure strict concavity of these payoffs, we evaluate the principal leading minors of the Hessian matrix for the functional forms we have used in the examples in Appendix B, i.e. linear success probabilities, $\beta_i(x_i) = x_i$, and quadratic cost functions, $C(x_i) = \gamma x_i^2/2$.

The Hessian matrix pre-merger is given by:

$$H = \begin{bmatrix} -\gamma & \pi_i^{ss} - \pi_i^{fs} - (\pi_i^{sf} - \pi_i^{ff}) \\ \pi_i^{ss} - \pi_i^{fs} - (\pi_i^{sf} - \pi_i^{ff}) & -\gamma \end{bmatrix}.$$

The conditions on the leading principal minors for the strict concavity of the pre-merger payoff are:

$$\begin{aligned} L_1 &= -\gamma < 0 \\ L_2 &= \gamma^2 - \left(\pi_i^{ss} - \pi_i^{fs} - (\pi_i^{sf} - \pi_i^{ff})\right)^2 > 0. \end{aligned} \tag{25}$$

It is clear that these inequalities hold for sufficiently large γ.

Similarly, the Hessian matrix post-merger is given by:

$$H^m = \begin{bmatrix} -\gamma & \hat{\pi}^{ss} - \hat{\pi}^{fs} - (\hat{\pi}^{sf} - \hat{\pi}^{ff}) \\ \hat{\pi}^{ss} - \hat{\pi}^{fs} - (\hat{\pi}^{sf} - \hat{\pi}^{ff}) & -\gamma \end{bmatrix},$$

and the conditions on the leading principal minors ensuring the strict concavity of the payoff are:

$$L_1^m = -\gamma < 0 \tag{26}$$

$$L_2^m = \gamma^2 - \left(\hat{\pi}^{ss} - \hat{\pi}^{fs} - (\hat{\pi}^{sf} - \hat{\pi}^{ff})\right)^2 > 0.$$

Again, it is clear that the post-merger SOCs are satisfied for sufficiently large γ.

Note that, for the micro-founded examples based on the Hotelling model and the Singh and Vives (1984) model (under both price and quantity competition) where the results regarding the positive or negative impact on R&D hold across all parameter values, both conditions (25) and (26) can be satisfied, as the choice of the parameter γ is unrestricted (γ large suffices). Regarding the example based on the Sutton's (2001) system of demands, we have seen that R&D will increase after a merger if γ is large enough and the quality difference sufficiently small. Then, because γ large suffices for (25) and (26), the SOCs hold. Finally, we observe that for the example based on the Mussa and Rosen's (1978) demand model, the range of parameters leading to an increase in R&D post-merger identified in the example based on Mussa and Rosen (1978) does not satisfy the SOCs. This implies that, despite $K_1 > 0$ and $K_2 < 0$ in this model, we always get $x^m < x^*$.[36]

Concavity in the example illustrating Proposition 3

Here we consider the Sighn and Vives's (1984) system of demands, investments that decrease the marginal cost of the products and price competition.

In the pre-merger market the problem of a firm i is to maximize (23):

$$\begin{aligned}\mathbb{E}\pi_1 &= (1-\mu)^2 \frac{b\left(2b(a-bc)+d(a+bc)+cd^2\right)^2}{\left(4b^2-d^2\right)^2} + (1-\mu)\mu \frac{b\left(2b(a-bc)+d(a+b(c-x_2))+cd^2\right)^2}{\left(4b^2-d^2\right)^2} \\ &\quad +\mu(1-\mu)\frac{b\left(2b(a-b(c-x_1))+d(a+bc)+(c-x_1)d^2\right)^2}{\left(4b^2-d^2\right)^2} \\ &\quad +\mu^2 \frac{b\left(2b(a-b(c-x_1))+d(a+b(c-x_2))+(c-x_1)d^2\right)^2}{\left(4b^2-d^2\right)^2} - \gamma\frac{x_1^2}{2}.\end{aligned}$$

To ensure the strict concavity of this payoff, we analyze the Hessian matrix, which in this example has the following form:

$$H = \begin{bmatrix} 2b\mu\frac{\left(2b^2-d^2\right)^2}{\left(4b^2-d^2\right)^2} - \gamma & -2b^2 d\mu^2 \frac{2b^2-d^2}{\left(4b^2-d^2\right)^2} \\ -2b^2 d\mu^2 \frac{2b^2-d^2}{\left(4b^2-d^2\right)^2} & 2b\mu\frac{\left(2b^2-d^2\right)^2}{\left(4b^2-d^2\right)^2} - \gamma \end{bmatrix}$$

As before, for strict concavity, the leading principal minors of the Hessian matrix have to satisfy $L_1 < 0$, $L_2 > 0$ where

$$L_1 = 2b\mu\frac{\left(2b^2-d^2\right)^2}{\left(4b^2-d^2\right)^2} - \gamma,$$

[36] Indeed, substituting the upper bound on the parameter γ derived in Appendix B, $\overline{\gamma} = \frac{s_s^2 - s_f s_s}{8s_s - s_f}$, into the expressions in (25) and (26), we obtain:

$$L_2 = \left(\frac{s_s^2 - s_f s_s}{8s_s - s_f}\right)^2 - \left(\frac{s_s(4s_s^2 - s_f^2 - 3s_f s_s)}{(4s_s - s_f)^2}\right)^2 = -\frac{8s_s^3(s_f - s_s)^2}{(s_f - 4s_s)^4 (s_f - 8s_s)^2}\left(s_f^3 - 18s_f^2 s_s + 64 s_f s_s^2 + 96 s_s^3\right) < 0.$$

Similarly, post-merger $L_2^m = \left(\frac{s_s^2 - s_f s_s}{8s_s - s_f}\right)^2 - \left(\frac{s_s}{4} - \frac{s_f}{4}\right)^2 = -\frac{1}{16}\frac{(s_f - s_s)^2}{(s_f - 8s_s)^2}\left(s_f^2 - 16 s_f s_s + 48 s_s^2\right) < 0.$

$$L_2 = \left(2b\mu\frac{\left(2b^2-d^2\right)^2}{\left(4b^2-d^2\right)^2} - \gamma\right)^2 - \left(-2b^2 d\mu^2 \frac{2b^2-d^2}{\left(4b^2-d^2\right)^2}\right)^2.$$

Note that L_2 is strictly convex in γ. This implies that the SOC is satisfied for all

$$\gamma > 2b\mu\frac{\left(2b^2-d^2\right)^2}{\left(4b^2-d^2\right)^2} + 2b^2 d\mu^2 \frac{2b^2-d^2}{\left(4b^2-d^2\right)^2} = \frac{2b\mu(2b^2-d^2)}{(4b^2-d^2)^2}(2b^2-d^2+bd\mu).$$

In the post-merger market the joint entity maximizes the joint profits in (24):

$$\begin{aligned}
\mathbb{E}\pi^m \quad = \quad & (1-\mu)^2\left(\frac{(a+c(d-b))(a-bc+cd)}{4(b-d)} + \frac{(a+c(d-b))(a-bc+cd)}{4(b-d)}\right) \\
& +(1-\mu)\mu\left(\frac{(a+c(d-b))(a-bc+(c-x_2)d)}{4(b-d)} + \frac{(a+(c-x_2)(d-b))(a-b(c-x_2)+cd)}{4(b-d)}\right) \\
& +\mu(1-\mu)\left(\frac{(a+(c-x_1)(d-b))(a-b(c-x_1)+cd)}{4(b-d)} + \frac{(a+c(d-b))(a-bc+(c-x_1)d)}{4(b-d)}\right) \\
& +\mu^2\left(\frac{(a+(c-x_1)(d-b))(a-b(c-x_1)+(c-x_2)d)}{4(b-d)} + \frac{(a+(c-x_2)(d-b))(a-b(c-x_2)+(c-x_1)d)}{4(b-d)}\right) \\
& -\gamma\frac{x_1^2}{2} - \gamma\frac{x_2^2}{2}.
\end{aligned}$$

The Hessian matrix corresponding to this payoff is:

$$H^m = \begin{bmatrix} \frac{1}{2}b\mu - 2\gamma & -\frac{1}{2}d\mu^2 \\ -\frac{1}{2}d\mu^2 & \frac{1}{2}b\mu - 2\gamma \end{bmatrix}.$$

The conditions for the strict concavity of the payoff are $L_1^m < 0$, $L_2^m > 0$, where

$$\begin{aligned}
L_1^m &= \frac{1}{2}b\mu - 2\gamma \\
L_2^m &= \left(\frac{1}{2}b\mu - 2\gamma\right)^2 - \left(-\frac{1}{2}d\mu^2\right)^2.
\end{aligned}$$

The SOC is satisfied for all $\gamma > \frac{1}{4}b\mu + \frac{1}{4}d\mu^2 = \frac{\mu}{4}(b+d\mu)$.

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