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Personalized Pricing and Consumer Privacy

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Abstract

Advances in data collection enable firms to use consumer information for personalized pricing. In Clavorà Braulin's (2023) symmetric two-dimensional model, this reduces prices and profits, while partial privacy yields the highest profits. Extending the model to asymmetric firms and vertically differentiated products, we show that these results are not robust under sizable asymmetries. Partial privacy may harm both firm and industry profits, while no privacy can outperform other regimes. Consumer welfare also depends on asymmetry: when large, partial privacy maximizes consumer surplus. These findings challenge prior literature and inform the design of privacy protection regulations.

JEL Classification: L1, D43, L13

Keywords: Personalized Prices, Price Discrimination, Consumer Information, Privacy

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1 Introduction

The advancements in data collection in digital markets may have opposing effects on consumer welfare. The advantage is that more detailed information can intensify competition and benefit consumers (see e.g. Thisse and Vives 1988). More detailed information also helps to improve the quality of recommendations and services offered to consumers (see e.g. Ichihashi 2020). The drawback is that when consumer privacy protection rules are not in place, or enforcement of such rules is weak, firms are free which dimensions of information about consumer characteristics to utilize for pricing. Firms may adopt modes that respect privacy in all dimensions, or respect privacy in a limited number of dimensions, or disrespect privacy in all dimensions. These modes are called full privacy, partial privacy and no privacy, respectively (see Clavorà Braulin 2023).

For the symmetric square city, Clavorà Braulin (2023) shows that partial privacy leads to higher individual profits compared to full or no privacy. This suggests that firms are capable of partial self-constraint in respecting privacy, and this limits the urgency for policy intervention. In this note, we investigate the robustness of the latter findings with respect to the symmetry assumption underlying these theoretical insights.[1] We show that qualitative policy recommendations derived from symmetric models require extra scrutiny.

Extending the model with vertically differentiated products and asymmetric costs, we show that, unlike the symmetric case, the no privacy mode need not be the least profitable. When asymmetries are large, the more efficient firm benefits from personalized pricing, and industry-wide profits under no privacy can exceed those under partial or full privacy. These new results arise due to expanding the dimensions of consumer characteristics, which allows for more sizable asymmetries and enhances the ability of more efficient (or dominant) firms to benefit from personalized pricing.

Turning to consumer surplus effects, our results show that they strongly depend on the degree of asymmetry. While Clavorà Braulin (2023) highlights the superiority of the no privacy mode, we show that this result survives only for sufficiently small asymmetries and is overturned when the asymmetries are sizable. In such environments, partial privacy benefits consumers most, and it may be desirable to impose some restrictions on the use of personalized pricing by limiting the ability of large dominant firms to collect and use consumer information on multiple dimensions.

The paper is organized as follows. Section 2 outlines the model and summarizes the equilibrium prices and profits for the three symmetric configurations, which can be translated into

[1] Our approach is methodologically close to Larralde et al. (2009), Liu and Shuai (2013) and Clavorà Braulin (2023). Our paper is related to the stream of literature focusing on how the availability of consumer data affects the firms' profitability and the competitive environment in which they operate. A common finding in this literature is that firms suffer from the possibility of perfect price discrimination (Thisse and Vives 1988, Corts 1998, Taylor and Wagman 2014). More nuanced effects of information and asymmetries on firms' profits emerge in a series of recent papers that underline how the presence of heterogeneities at the firm level can soften competition (see e.g. Houba et al. 2023, Matsumura and Matsushima 2015, Chen et al. 2020, Shy and Stenbacka 2016, or Colombo et al. 2021, Gehrig et al. 2012).

three modes distinguished by the degree of consumer privacy protection. Section 3 describes the impact of the extent of privacy protection on profitability and consumer welfare. Section 4 discusses the policy implications.

2 The Model

Consider two firms that sell differentiated products or services to heterogeneous consumers. Consumer characteristics[2] are modeled as the square city $X = [0,1]^2$ (or unit cube). We refer to a consumer with characteristics $x = (x_1, x_2) \in [0,1]^2$ as consumer x. The distribution of characteristics is described by the bivariate uniform distribution on X given by the density function $f(x) = 1$.

The firms are called A and B, indexed by $i = A, B$. Firm A is located at $x = (0,0)$ and firm B at $x = (1,1)$. The constant marginal cost of providing the product is denoted as $c^i \geqslant 0$, $i = A, B$. Their pricing technologies are exogenous.[3] Whenever such technology includes both consumer characteristics x_1 and x_2, it sets the fully personalized price $p^i(x) = p^i(x_1, x_2) \geqslant 0$ to consumer x and we refer to it as PP, which is the no privacy regime. In case the technology includes the first characteristic x_1 only, it sets the partly personalized price $p^i(x_1) \geqslant 0$ to consumer x and we refer to it as PU, which is the partial privacy regime.[4] Otherwise, if no characteristics are included in the pricing technology, such technology sets the uniform price $\bar{p}^i$ to consumer x and we refer to it as UU, the full privacy regime.

For two firms, we consider three symmetric technology configurations. Configuration (UU, UU) corresponds to Hotelling's model in the square city, where firms adopt uniform pricing on both dimensions of consumer characteristics. Configuration (PP, PP) corresponds to an extension of Hotelling's model in the square city in which both firms adopt fully personalized pricing. Finally, configuration (PU, PU) corresponds to a square city where both firms adopt the pricing strategy that includes only the first consumer characteristic and is uniform in the second characteristic.

Consumer x has a utility $u^i > c^i$ of buying at firm $i = A, B$, while $tx_1 + tx_2$ and $t(1 - x_1) + t(1 - x_2)$, $t > 0$, represent consumer x's specific transaction or adjustment costs of buying at firm A and B, respectively. The outside option of not buying has a surplus of 0.

We denote the asymmetry in maximal social welfare levels as $\Delta = u^A - c^A - (u^B - c^B)$.[5] We assume that $0 \leq \Delta < 2t$. The first inequality means that firm A offers at least as much consumer

[2] This can be based on information collected from previous on-line searches, previous purchases, other consumer characteristics (such as age, occupation, location, links in on-line social networks).

[3] We assume that if both firms use pricing technologies that incorporate overlapping dimensions of consumer characteristics, they possess the same information about these overlapping dimensions.

[4] Under this regime firms are allowed to use only the least sensitive dimension, such as location, and would not be allowed to use more detailed personalized information such as age, occupation, etc.

[5] Note that in this model we can normalize $c^A = c^B = 0$ without loss of generality due to the isomorphic nature of cost advantages and quality advantages as discussed in Houba et al. (2023). To simplify the derivations, in Appendix A we apply this normalization.

surplus as firm B, while the second inequality can be interpreted (read: rewritten) as firm B offers more consumer surplus to consumer $x = (1, 1)$ than firm A. Our assumption is sufficient to guarantee positive market shares and market coverage in equilibrium of any configuration.

Consumer surplus from buying at firm A is $u^A - tx_1 - tx_2 - p^A$, where p^A stands for either the fully personalized price $p^A(x_1, x_2)$, or the partly personalized price $p^A(x_1)$, or the uniform price $\bar{p}^A$. Similarly, $u^B - t(1 - x_1) - t(1 - x_2) - p^B$ is the consumer surplus offered by firm B. Our analysis will characterize the market segment $\mathcal{A} \subseteq [0, 1]^2$ of consumers who buy at firm A, and the market segment $\mathcal{B} \subseteq [0, 1]^2$, $\mathcal{B} \cap \mathcal{A} = \emptyset$, of consumers who buy at firm B. Then, both market shares are given by

$$\mathcal{B} = \left\{ x \in [0,1]^2 \middle| \, x_1 + x_2 > 1 + \tfrac{1}{2t}\left(u^A - u^B + p^B - p^A\right) \right\} \quad \text{and} \quad \mathcal{A} = [0,1]^2 \setminus \mathcal{B}. \tag{1}$$

The associated aggregate consumer surplus given personalized prices $p_A(x)$ and $p_B(x)$ is

$$CS = \int_{x \in \mathcal{A}} \left(u^A - tx_1 - tx_2 - p^A(x)\right) f(x)\, dx + \int_{x \in \mathcal{B}} \left(u^B - t(1 - x_1) - t(1 - x_2) - p^B(x)\right) f(x)\, dx,$$

aggregate producer surplus is given by

$$PS = \int_{x \in \mathcal{A}} \left(p^A(x) - c^A\right) f(x)\, dx + \int_{x \in \mathcal{B}} \left(p^B(x) - c^B\right) f(x)\, dx,$$

and aggregate social welfare is given by

$$SW = \int_{x \in \mathcal{A}} \left(u^A - tx_1 - tx_2 - c^A\right) f(x)\, dx + \int_{x \in \mathcal{B}} \left(u^B - t(1 - x_1) - t(1 - x_2) - c^B\right) f(x)\, dx.$$

We summarize the equilibrium pricing strategies, market segments and profits for the three symmetric configurations mentioned in Table 1.[6] Under full privacy configuration (UU, UU), we denote π^i_{UU}, $i = A, B$, as the individual profits for firms A and B, respectively, while PS_{UU} denotes the industry level profits. Similarly, individual profits under partial privacy configuration (PU, PU) are π^i_{PU}, $i = A, B$, and industry profit is given by PS_{PU}. Finally, individual profits under personalized pricing (no privacy) are π^i_{PP}, $i = A, B$, and industry profit is PS_{PP}. Note that, at $\Delta = 0$, the asymmetries between firms disappear and the profits and consumer surpluses coincide with those in Clavorà Braulin (2023).

[6]Deriving the equilibrium for each configuration is a routine but cumbersome exercise that we defer to Appendix A. The results for configuration (UP, UP) are equivalent to (PU, PU), and hence omitted.

	No privacy (PP, PP)	Partial privacy (PU, PU)	Full privacy (UU, UU)
p^A	$\max\{\Delta + 2t - 2tx_1 - 2tx_2, 0\}$	$\frac{2}{3}(2 - x_1)t + \frac{1}{3}\Delta$	$-\frac{5}{8}(2t - \Delta) + \frac{3}{8}\sqrt{36t^2 - 4t\Delta + \Delta^2}$
p^B	$\max\{\Delta - 2t + 2tx_1 + 2tx_2, 0\}$	$\frac{2}{3}(1 + x_1)t - \frac{1}{3}\Delta$	$\frac{1}{8}(2t - \Delta) + \frac{1}{8}\sqrt{36t^2 - 4t\Delta + \Delta^2}$
$\mathcal{A}$	$x_1 + x_2 \leq 1 + \frac{\Delta}{2t}$	$\frac{1}{3}x_1 + x_2 \leq \frac{2}{3} + \frac{\Delta}{6t}$	$x_1 + x_2 \leq 1 + \frac{\Delta}{2t} - \frac{1}{2t}(\bar{p}^A - \bar{p}^B)$
π^A	$\frac{1}{24t^2}\left((2t + \Delta)^3 - 2\Delta^3\right)$	$\frac{1}{54t}(28t^2 + 18t\Delta + 3\Delta^2)$	$\frac{\left(128t^2 - \left(2t - \Delta + \sqrt{36t^2 - 4t\Delta + \Delta^2}\right)^2\right)\left(-5(2t - \Delta) + 3\sqrt{36t^2 - 4t\Delta + \Delta^2}\right)}{1024t^2}$
π^B	$\frac{1}{24t^2}(2t - \Delta)^3$	$\frac{1}{54t}(28t^2 - 18t\Delta + 3\Delta^2)$	$\frac{\left(2t - \Delta + \sqrt{36t^2 - 4t\Delta + \Delta^2}\right)^3}{1024t^2}$

Table 1: The equilibrium prices, firm A's equilibrium market segment and the equilibrium profits for the three configurations in which both firms adopt the same pricing technology: Full privacy (UU, UU), partial privacy (PU, PU) and no privacy (PP, PP).

3 Results

3.1 Consumer privacy protection and profitability

In this section, we examine the implications of consumer information availability, which enables firms to employ personalized pricing in any of the two dimensions of consumer characteristics. To establish our results, we conduct pairwise comparisons of profits across all three symmetric configurations. We first present several graphical representations before stating our formal results. Figures 1(a) and 1(b) summarize the individual profit rankings for firms A and B, respectively. Figure 2 does the same at the level of industry profits and identifies four profit rankings that can occur.

Note that the profit rankings corresponding to Region I and I$'$ are in line with the results of the symmetric model in Clavorà Braulin (2023). The rankings in Figures 1(a) and 2 also reflect that the larger the asymmetries, the more likely the efficient firm will benefit from symmetric personalized pricing.

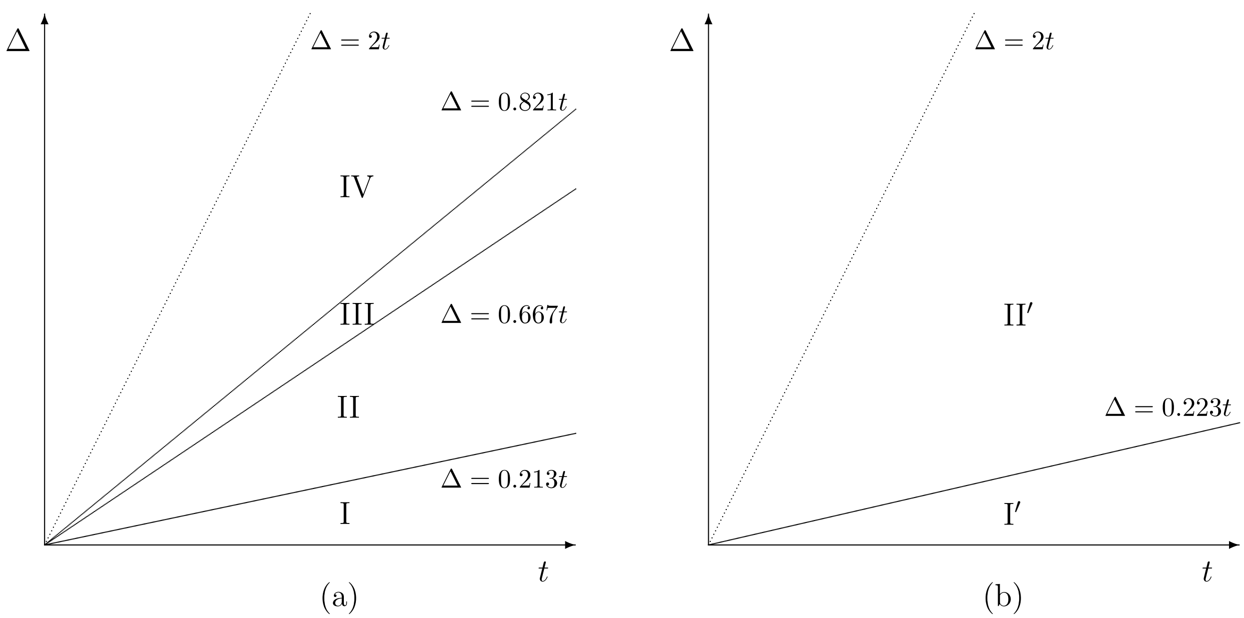

Figure 1: Profit rankings of Firm A (panel a) and Firm B (panel b). The regions in panel (a) refer to I: $\pi^A_{PU} > \pi^A_{UU} > \pi^A_{PP}$, II: $\pi^A_{UU} > \pi^A_{PU} > \pi^A_{PP}$, III: $\pi^A_{UU} > \pi^A_{PP} > \pi^A_{PU}$, and IV: $\pi^A_{PP} > \pi^A_{UU} > \pi^A_{PU}$. The regions in panel (b) refer to I': $\pi^B_{PU} > \pi^B_{UU} > \pi^B_{PP}$ and II': $\pi^B_{UU} > \pi^B_{PU} > \pi^B_{PP}$.

The following three propositions formalize the implications of transitions between the three configurations and the regions depicted in Figures 1 and 2. Each proposition reports on a particular pair of configurations. First, comparing the (PP, PP) and (UU, UU) configurations yields the following result.[7]

[7]The proofs of all propositions can be found in Appendix B.

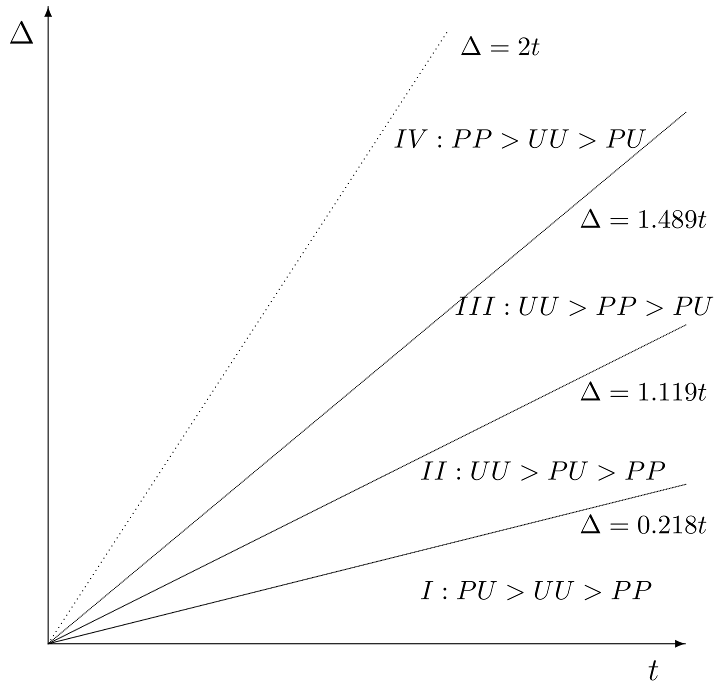

Figure 2: The ranking of industry profits.

Proposition 1 *Firm A benefits from* (PP, PP) *compared to* (UU, UU) *when* $0.821t < \Delta < 2t$*, whereas firm B never benefits. Moreover, the industry profit is higher under* (PP, PP) *for* $1.489t < \Delta < 2t$*.*

We conclude that adopting personalized pricing along both dimensions, corresponding to the no privacy regime, never benefits the less efficient firm B. However, it can benefit the more efficient firm A, when asymmetries are sufficiently large ($\Delta > 0.821t$), as illustrated by Region IV in Figure 1(a). This finding is consistent with Houba et al. (2023). In contrast to previous literature (Thisse and Vives (1988), Clavorà Braulin (2023) or Houba et al. (2023)), our extension demonstrates that the industry profits under personalized pricing can exceed those under uniform pricing when asymmetries are substantial ($\Delta > 1.489t$). These parameter values correspond to Region IV in Figure 2.

Next, comparison of the (UU, UU) and (PU, PU) configurations implies the following result.

Proposition 2 *Firm A benefits from* (PU, PU) *compared to* (UU, UU) *when* $0 < \Delta < 0.213t$*, whereas firm B benefits from* (PU, PU) *compared to* (UU, UU) *when* $0 < \Delta < 0.223t$*. Moreover, the industry profit is higher under* (PU, PU) *for* $0 < \Delta < 0.218t$*.*

We conclude that the transition from full privacy to partial privacy will benefit both firms only for sufficiently low asymmetries, specifically $\Delta < 0.213t$ for A and $\Delta < 0.223t$ for B. These parameter ranges are illustrated by Region I and I$'$ in Figures 1(a) and 1(b), respectively. Consequently, industry profits will show a similar pattern of reversed ranking, where the critical locus ($\Delta = \ 0.218t$) is located between the previous two loci. This aligns with the result of Clavorà Braulin (2023), who concludes that in the symmetric setting ($\Delta = 0$) partial privacy is more profitable compared to full privacy. However, this result can be reversed if the asymmetries are sizable. Then full privacy results in higher individual and industry profits compared to partial privacy, which is illustrated by the Regions II, III, and IV in Figure 1(a) and Figure 2, and Region II′ in Figure 1(b).

Finally, comparing the (PP, PP) and (PU, PU) configurations yields the following result.

Proposition 3 *Firm A benefits from (PP, PP) compared to (PU, PU) when $\frac{2}{3}t < \Delta < 2t$, whereas firm B never benefits. Moreover, the industry profit is higher under (PP, PP) for $1.119t < \Delta < 2t$.*

We conclude that adopting personalized pricing along both dimensions instead of partly personalized along a single dimension never benefits the less efficient firm B. However, it can benefit the more efficient firm A when asymmetries are sizable ($\Delta > 0.667t$), as the combined Region III and IV in Figure 1(a) illustrate.[8] The industry profits show a similar pattern, where personalized pricing dominates partial privacy regime for $\Delta > 1.119t$, which is illustrated by the Regions III and IV in Figure 2.

The intuition is that the lack of information about one of the dimensions of consumer characteristics attenuates competition. In the symmetric setting ($\Delta = 0$), both firms will benefit if information on one dimension is unavailable and lose out if it becomes available. However, in asymmetric settings (when Δ is sufficiently large), the more efficient firm can take advantage of an additional dimension of information and increase its own profits. The increase in profit of the more efficient firm is so substantial that, although the weaker competitor loses out, the total industry profit is greater compared to partial privacy regime. We state this result formally in the following corollary.

Corollary 4 *For large enough asymmetries and the adoption of identical pricing technologies, the industry profits under the symmetric (PP, PP) configuration are larger than those under the (UU, UU) configuration, which in turn are larger than the industry profits under the (PU, PU) configuration.*

This result is in sharp contrast to the prior literature. It has not been observed in symmetric models (Thisse and Vives, 1988 or Clavorà Braulin, 2023), nor in a one-dimensional asymmetric linear city setting (e.g., Houba et al., 2023). This novel result arises due to expanding the dimensionality of consumer characteristics, which enlarges the parameter space for which there

[8] Note that this echoes the result in Proposition 2 of Houba et al. (2023), where it is derived that for more efficient firms switching to personalized pricing will be beneficial only when $\Delta > (\frac{6}{7}\sqrt{2} - \frac{3}{7})t = 0.784t$, which is higher than the bound in Proposition 3 indicating that the efficient firm may benefit for a larger range of parameters in this two-dimensional model compared to the Hotelling model.

is a duopoly in equilibrium, and allows for more pronounced asymmetries. Such expansion enhances the ability of the more efficient firm to benefit from personalized pricing. This creates additional advantages for more efficient (dominant) firms, which can increase their profits from personalized pricing to such an extent that even though weaker rivals lose, the industry profit is greater compared to both uniform pricing (full privacy) and partial privacy.

3.2 Privacy protection and consumer welfare

Turning to the impact of privacy protection rules on consumer welfare, we again show that the conclusions derived from the symmetric model can be reversed. When asymmetries are large, partial privacy may outperform other regimes, contrary to the findings of Clavorà Braulin (2023).

To establish these results, we follow a similar approach as in Section 3.1 and summarize our main results in Figure 3 and Proposition 5.[9] In this figure, Region I identifies the parameter space where the aggregate consumer surplus under no privacy exceeds that under full privacy, which in turn exceeds the consumer surplus under partial privacy. This ranking holds for sufficiently low asymmetries ($\Delta < 0.422t$) and confirms the predictions of symmetric models (see, e.g., Thisse and Vives, 1988; Clavorà Braulin, 2023). The no privacy regime remains the best in Region II in which the other two regimes switch their ranking. Finally, Region III, corresponding to large asymmetries ($\Delta > 1.357t$), identifies the set of parameters for which partial privacy generates the highest benefits for consumers. These observations are summarized in Proposition 5.

Proposition 5 *For large enough asymmetries and adoption of identical pricing technologies, the aggregate consumer surplus under symmetric (PU, PU) configuration is larger than the aggregate consumer surplus under (PP, PP) configuration, which in turn is larger than the aggregate consumer surplus under (UU, UU) configuration.*

We conclude that the impact of privacy protection on consumer welfare depends on the degree of asymmetry. When asymmetries are sizable, partial privacy maximizes aggregate consumer surplus and personalized pricing is suboptimal. These findings challenge the previous literature focusing on symmetric models (e.g. Thisse and Vives, 1988 or Clavorà Braulin, 2023) and one-dimensional asymmetric settings (e.g. Houba et al., 2023). These results stem from expanding the dimensionality of consumer characteristics, which accommodates larger asymmetries and enhances the ability of more efficient dominant firms to extract rents by exploiting consumer information through personalized pricing.[10]

[9] Appendix A derives the expressions for aggregate consumer surplus under the three symmetric configurations. Figure 3 follows directly from pairwise comparison of these expressions. The proof of Proposition 5 is similar to Propositions 1-3, and we omit it for brevity.

[10] Note that total welfare is still the highest under the no privacy (or personalized pricing) due to disproportional increase in the profits of the most efficient firm.

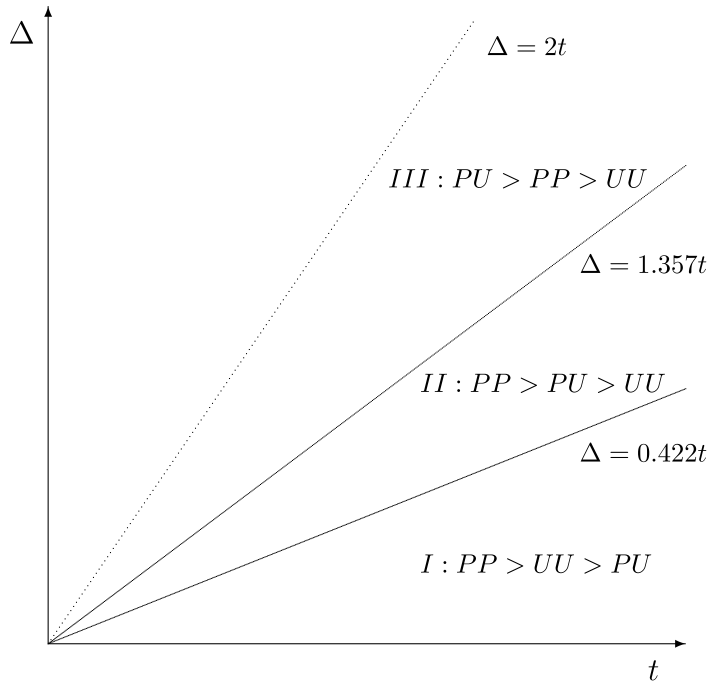

Figure 3: The ranking of aggregate consumer surpluses.

4 Policy implications

Our results provide a warning to consumer protection authorities and regulators as we show that, in markets where asymmetries are sizable (e.g., digital industries dominated by large tech giants), personalized pricing under no privacy protection may result in an outcome that is suboptimal from a consumer welfare perspective. In such a case, stricter protection of consumer privacy by moving to the partial privacy regime is preferable, where only one dimension of consumer characteristics (e.g., location or previous searches) can be used for pricing, while other more sensitive information on age or occupation should not be used for pricing.

This offers a novel policy insight that the regulation of privacy protection should depend on the market structure and the degree of asymmetries in the market. In markets dominated by large firms, e.g. online markets with dominant gate-keepers and large asymmetries, regulators may need to adopt stricter privacy protection rules that restrict the scope and use of consumer data. By contrast, in more competitive environments, where asymmetries are modest, consumers may benefit from personalized pricing across multiple dimensions of consumer characteristics.

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A Appendix with Derivations

A.1 No Privacy *(PP,PP)*

In this section, we analyze the exogenously given configuration of data-driven no-privacy pricing technologies in which both firms compete in personalized prices $p^A(x_1, x_2)$ and $p^B(x_1, x_2)$.

Price competition in personalized pricing is strategically equivalent to a procurement auction in which A can offer at most $u^A - c^A - tx_1 - tx_2$ to consumer x at marginal cost pricing and, similarly, B can offer at most $u^B - c^B - t(1 - x_1) - t(1 - x_2)$. The firm with the higher maximal value outbids the competitor by matching. In equilibrium the competitor is driven down to marginal cost pricing. The auction outcome is efficient and maximizes social welfare for all $x \in [0, 1]^2$. Normalizing $c^A = c^B = 0$, the equilibrium pricing strategies are given by

$$p^A(x) = \max\{\Delta + 2t - 2tx_1 - 2tx_2, 0\}, \tag{2}$$

$$p^B(x) = \max\{\Delta - 2t + 2tx_1 + 2tx_2, 0\}. \tag{3}$$

The boundary of A's market segment (1) is where the positive profit margin $p^A(x)$ becomes zero. A's market segment in (1) is given by

$$x_2 \le 1 + \tfrac{\Delta}{2t} - x_1,$$

where the right-hand side is positive, decreasing in x_1 and intersects the upper boundary $x_2 = 1$ at $x_1 = \frac{\Delta}{2t} < 1$. Hence, the profit of firm A must be computed in two parts:

$$\begin{aligned}\pi^A_{PP,PP} &= \int_0^{\frac{\Delta}{2t}} \int_0^1 (\Delta + 2t - 2tx_1 - 2tx_2)\, dx_2 dx_1 + \int_{\frac{\Delta}{2t}}^1 \int_0^{1+\frac{\Delta}{2t}-x_1} (\Delta + 2t - 2tx_1 - 2tx_2)\, dx_2 dx_1 \\ &= \frac{1}{4t}\Delta(2t + \Delta) + \frac{1}{24t^2}\left(8t^3 - \Delta^3\right) = \frac{(2t + \Delta)^3 - 2\Delta^3}{24t^2} > 0. \end{aligned} \tag{4}$$

Similarly, the profit of B is equal to

$$\pi^B_{PP,PP} = \int_{\frac{\Delta}{2t}}^1 \int_{\frac{\Delta}{2t}+1-x_1}^1 (2tx_1 + 2tx_2 - \Delta - 2t)\, dx_2 dx_1 = \frac{(2t - \Delta)^3}{24t^2} > 0. \tag{5}$$

Then, the industry profit is given by

$$PS_{PP,PP} = \pi^A_{PP,PP} + \pi^B_{PP,PP} = \frac{8t^3 + 6t\Delta^2 - \Delta^3}{12t^2} > 0.$$

Social welfare generated by consumers buying from A must be computed in two parts similar to A's profit:

$$\begin{aligned}SW^A_{PP,PP} &= \int_0^{\frac{\Delta}{2t}} \int_0^1 \left(u^A - tx_1 - tx_2\right) dx_2 dx_1 + \int_{\frac{\Delta}{2t}}^1 \int_0^{\frac{\Delta}{2t}+1-x_1} \left(u^A - tx_1 - tx_2\right) dx_2 dx_1 \\ &= \frac{\Delta\left(4u^A - (\Delta + 2t)\right)}{8t} + \frac{(\Delta - 2t)\left((\Delta + 2t)^2 + \Delta t - 3(\Delta + 2t)u^A\right)}{24t^2} \\ &= \frac{12t^2\left(u^A - \Delta\right) + 3\Delta(4t - \Delta)u^A + \Delta^3 - 8t^3}{24t^2}. \end{aligned} \tag{6}$$

Similarly, social welfare generated by B is equal to

$$SW^B_{PP,PP} = \int_{\frac{\Delta}{2t}}^{1} \int_{\frac{\Delta}{2t}+1-x_1}^{1} \left(u^B - t\left(1-x_1\right) - t\left(1-x_2\right)\right) dx_2 dx_1 = \frac{\left(2t-\Delta\right)^2 \left(3u^B + \Delta - 2t\right)}{24t^2}.$$

Then, aggregate social welfare is given by

$$SW_{PP,PP} = \frac{12t^2\left(u^A + u^B\right) + 6t\Delta^2 - 16t^3 - \Delta^3}{24t^2}.$$

Aggregate consumer welfare can be computed by subtracting aggregate producer surplus from aggregate social welfare:

$$\begin{aligned} CS_{PP,PP} &= SW_{PP,PP} - SP_{PP,PP} \\ &= \frac{12t^2\left(u^A + u^B\right) + 6t\Delta^2 - 16t^3 - \Delta^3}{24t^2} - \frac{8t^3 + 6t\Delta^2 - \Delta^3}{12t^2} \\ &= \frac{12t^2\left(u^A + u^B\right) - 6t\Delta^2 - 32t^3 + \Delta^3}{24t^2}. \end{aligned} \tag{7}$$

A.2 Partial Privacy *(PU,PU)*

In this section, we analyze the exogenously given configuration of data-driven pricing technologies that partially respect privacy. As in Clavorà Braulin (2023), we consider symmetry in the firms' pricing technologies and that firms use information x_1 and ignore information x_2. That is, they compete in setting prices $p^A(x_1)$ and $p^B(x_1)$.

For given $x_1 \in [0,1]$, competition in prices $p^A(x_1)$ and $p^B(x_1)$ is strategically equivalent to uniform price competition on the line segment between $(x_1, 0)$ and $(x_1, 1)$. Given the uniform distribution of (x_1, x_2) on $[0,1]^2$, the conditional distribution $f(x_2|x_1)$ is the one-dimensional uniform distribution on the interval $[0,1]$. Furthermore, conditional on x_1, the value to consumer x of buying from A is $u^A - p^A(x_1) - tx_1 - tx_2$ and $u^B - p^B(x_1) - t(1-x_1) - t(1-x_2)$ from B. This is equivalent to the standard Hotelling model in which A offers value $v^A(x_1) = u^A - tx_1$, B offers value $v^B(x_1) = u^A - t(1-x_1)$ and a uniform pair of prices $\left(\bar{p}^A, \bar{p}^B\right)$ such that $p^A(x_1) = \bar{p}^A$ and $p^B(x_1) = \bar{p}^B$. For this case, expressions are readily available in e.g. Houba et al. (2023). Straightforward application yields

$$\begin{aligned} \Delta(x_1) = v^A(x_1) - v^B(x_1) &= \Delta + t - 2tx_1 = \Delta + t(1-2x_1), \\ \bar{p}^A = t + \tfrac{1}{3}\Delta(x_1) &= t + \tfrac{1}{3}(\Delta + t - 2tx_1) = \tfrac{2}{3}(2-x_1)t + \tfrac{1}{3}\Delta, \\ \bar{p}^B = t - \tfrac{1}{3}\Delta(x_1) &= t - \tfrac{1}{3}(\Delta + t - 2tx_1) = \tfrac{2}{3}(1+x_1)t - \tfrac{1}{3}\Delta. \end{aligned}$$

And the market share of A is given by

$$MS^A(x_1) = \tfrac{1}{2} + \tfrac{\Delta(x_1)}{6t} = \tfrac{1}{2} + \tfrac{\Delta + t - 2tx_1}{6t} = \tfrac{2}{3} + \tfrac{\Delta}{6t} - \tfrac{1}{3}x_1 = \tfrac{1}{6t}(4t + \Delta - 2tx_1).$$

The boundary of A's market segment (1) is given by

$$x_2 \leq \tfrac{2}{3} + \tfrac{\Delta}{6t} - \tfrac{1}{3}x_1,$$

where the upper bound is positive and smaller than 1 for all $x_1 \in [0,1]$. Hence, the profit of A is a single double integral and given by[11] [12]

$$\pi^A_{PU,PU} = \int_0^1 \int_0^{\frac{1}{6t}(4t+\Delta-2tx_1)} \left(\tfrac{2}{3}(2-x_1)t + \tfrac{1}{3}\Delta\right) dx_2 dx_1 = \frac{28t^2 + 18t\Delta + 3\Delta^2}{54t} \tag{8}$$

and the profit of B is given by

$$\pi^B_{PU,PU} = \int_0^1 \int_{\frac{1}{6t}(4t+\Delta-2tx_1)}^1 \left(\tfrac{2}{3}(1+x_1)t - \tfrac{1}{3}\Delta\right) dx_2 dx_1 = \frac{28t^2 - 18t\Delta + 3\Delta^2}{54t}. \tag{9}$$

Then, the industry profit is given by

$$PS_{PU,PU} = \pi^A_{PU,PU} + \pi^B_{PU,PU} = \frac{28t^2 + 3\Delta^2}{27t}. \tag{10}$$

Social welfare generated by consumers buying from A must be computed in two parts similar to A's profit:

$$\begin{aligned} SW^A_{PU,PU} &= \int_0^1 \int_0^{\frac{1}{6t}(4t+\Delta-2tx_1)} \left(u^A - tx_1 - tx_2\right) dx_2 dx_1 \\ &= \frac{36(3t+\Delta)u^A - 36t\Delta - 3\Delta^2 - 76t^2}{216t}. \end{aligned} \tag{11}$$

Similarly, social welfare of B is equal to

$$\begin{aligned} SW^B_{PU,PU} &= \int_0^1 \int_{\frac{1}{6t}(4t+\Delta-2tx_1)}^1 \left(u^B - t(1-x_1) - t(1-x_2)\right) dx_2 dx_1 \\ &= \frac{36(3t-\Delta)u^B + 36t\Delta - 3\Delta^2 - 76t^2}{216t}. \end{aligned} \tag{12}$$

Then, aggregate social welfare is given by

$$SW_{PU,PU} = SW^A_{PU,PU} + SW^B_{PU,PU} = \frac{15\Delta^2 - 76t^2 + 54t\left(u^A + u^B\right)}{108t}.$$

[11] Profits in the configuration (UP, UP) are identical due to symmetry.

[12] An alternative way to derive or verify π^A is to apply the expressions in Houba et al. (2023) for the conditional profits $\pi^A(x_1)$ and then integrate over x_1. Doing so, yields

$$\int_0^1 \pi^A(x_1) \cdot 1 dx_1 = \int_0^1 \frac{(2t + 2t(1-x_1) + \Delta)^2}{18t} dx_1 = \frac{28t^2 + 18t\Delta + 3\Delta^2}{54t}.$$

Similarly,

$$\int_0^1 \pi^B(x_1) \cdot 1 dx_1 = \int_0^1 \frac{(2t + 2tx_1 - \Delta)^2}{18t} dx_1 = \frac{28t^2 - 18t\Delta + 3\Delta^2}{54t}.$$

This confirms the stated expressions.

Aggregate consumer welfare can be computed by subtracting aggregate producer surplus from aggregate social welfare:

$$\begin{aligned}CS_{PU,PU} = SW_{PU,PU} - PS_{PU,PU} &= \frac{15\Delta^2 - 76t^2 + 54t\left(u^A + u^B\right)}{108t} - \frac{28t^2 + 3\Delta^2}{27t}\\ &= \frac{54t\left(u^A + u^B\right) + 3\Delta^2 - 188t^2}{108t}.\end{aligned}$$

A.3 Full Privacy *(UU,UU)*

In this section, we analyze the exogenously given configuration of data-driven pricing technologies that respect full privacy in which both firms compete in setting uniform prices $\bar{p}_A$ and $\bar{p}_B$. One might say personalized prices are given by $p^A(x_1, x_2) = p^A$ and $p^B(x_1, x_2) = p^B$, where we supress the upper bar in our derivations.

The boundary of A's market share is derived from

$$u^A - p^A - tx_1 - tx_2 \geq u^B - p^B - 2t + tx_1 + tx_2 \quad \Longrightarrow \quad x_2 \leq 1 + \frac{\Delta - \left(p^A - p^B\right)}{2t} - x_1.$$

B's market share $MS^B\left(p^A, p^B\right)$ is the triangle with corners $\left(1, \frac{1}{2t}\left(\Delta - \left(p^A - p^B\right)\right)\right)$, $(1,1)$ and $\left(\frac{1}{2t}\left(\Delta - \left(p^A - p^B\right), 1\right)\right)$, which gives $MS^B\left(p^A, p^B\right) = \frac{1}{8t^2}\left(2t - \Delta + p^A - p^B\right)^2$.

The best response of B can be determined from the standard profit maximization under uniform pricing and is given by

$$p^B = \tfrac{1}{3}\left(2t - \Delta + p^A\right).$$

From our analysis of market shares we have:

$$\left(2t - \Delta + p^A - p^B\right)\left(p^A + p^B\right) = 4t^2.$$

Substitution of B's best response into last equation yields

$$\begin{aligned}\left(2t - \Delta + p^A - \tfrac{1}{3}\left(2t - \Delta + p^A\right)\right)\left(p^A + \tfrac{1}{3}\left(2t - \Delta + p^A\right)\right) &= 4t^2,\\ \left(2t - \Delta + p^A\right)\left(2t - \Delta + 4p^A\right) &= 18t^2,\\ \left(2t - \Delta + p^A\right)\left(2t - \Delta + 4p^A\right) - 18t^2 &= 0,\\ 4p_A^2 + 5\left(2t - \Delta\right)p^A + \left(2t - \Delta\right)^2 - 18t^2 &= 0.\end{aligned}$$

Solving this quadratic equation we obtain expressions for equilibrium prices

$$\begin{aligned}p^A &= \frac{5}{8}\Delta - \frac{5}{4}t + \frac{3}{8}\sqrt{36t^2 - 4t\Delta + \Delta^2} = -\tfrac{5}{8}\left(2t - \Delta\right) + \tfrac{3}{8}\sqrt{36t^2 - 4t\Delta + \Delta^2},\\ p^B &= \tfrac{1}{3}\left(2t - \Delta - \tfrac{5}{8}\left(2t - \Delta\right) + \tfrac{3}{8}\sqrt{36t^2 - 4t\Delta + \Delta^2}\right) = \tfrac{1}{8}\left(2t - \Delta\right) + \tfrac{1}{8}\sqrt{36t^2 - 4t\Delta + \Delta^2}.\end{aligned}$$

The market share of B :

$$\begin{aligned} MS^B\left(p^A, p^B\right) &= \frac{1}{8t^2}\left(2t-\Delta+p^A-p^B\right)^2 \\ &= \frac{1}{8t^2}\left(2t-\Delta-\tfrac{3}{4}\left(2t-\Delta\right)+\frac{1}{4}\sqrt{36t^2-4t\Delta+\Delta^2}\right)^2 \\ &= \frac{1}{128t^2}\left(2t-\Delta+\sqrt{36t^2-4t\Delta+\Delta^2}\right)^2. \end{aligned}$$

The market share of A :

$$\begin{aligned} MS^A\left(p^A, p^B\right) &= 1-MS^B\left(p^A, p^B\right) = 1-\frac{1}{128t^2}\left(2t-\Delta+\sqrt{36t^2-4t\Delta+\Delta^2}\right)^2 \\ &= \frac{128t^2-\left(2t-\Delta+\sqrt{36t^2-4t\Delta+\Delta^2}\right)^2}{128t^2}. \end{aligned}$$

Then,

$$\begin{aligned} x_2 &\le 1+\frac{\Delta-\left(p^A-p^B\right)}{2t}-x_1 = 1+\frac{\Delta-\left(-\frac{3}{4}\left(2t-\Delta\right)+\frac{1}{4}\sqrt{36t^2-4t\Delta+\Delta^2}\right)}{2t}-x_1 \\ &= 1+\frac{6t+\Delta-\sqrt{36t^2-4t\Delta+\Delta^2}}{8t}-x_1. \end{aligned}$$

Furthermore, the profit of firm B is

$$\pi^B_{UU,UU} = \frac{1}{1024t^2}\left(2t-\Delta+\sqrt{36t^2-4t\Delta+\Delta^2}\right)^3. \tag{13}$$

Profit of A is

$$\pi^A_{UU,UU} = \frac{\left(128t^2-\left(2t-\Delta+\sqrt{36t^2-4t\Delta+\Delta^2}\right)^2\right)\left(3\sqrt{36t^2-4t\Delta+\Delta^2}-5\left(2t-\Delta\right)\right)}{1024t^2}. \tag{14}$$

The industry profit is given by $PS_{UU,UU} = \pi^A_{UU}+\pi^B_{UU} =$

$$\frac{\left(128t^2-\left(2t-\Delta+\sqrt{36t^2-4t\Delta+\Delta^2}\right)^2\right)\left(3\sqrt{36t^2-4t\Delta+\Delta^2}-5\left(2t-\Delta\right)\right)+\left(2t-\Delta+\sqrt{36t^2-4t\Delta+\Delta^2}\right)^3}{1024t^2}.$$

Social welfare of firm A is given by

$$\begin{aligned} &SW^A_{UU,UU} = \\ &= \int_0^{\frac{6t+\Delta-\sqrt{36t^2-4t\Delta+\Delta^2}}{8t}}\int_0^1\left(u^A-tx_1-tx_2\right)dx_2dx_1 \\ &+ \int_{\frac{6t+\Delta-\sqrt{36t^2-4t\Delta+\Delta^2}}{8t}}^1\int_0^{1+\frac{6t+\Delta-\sqrt{36t^2-4t\Delta+\Delta^2}}{8t}-x_1}\left(u^A-tx_1-tx_2\right)dx_2dx_1. \end{aligned}$$

We set $Z=\frac{6t+\Delta-\sqrt{36t^2-4t\Delta+\Delta^2}}{8t}$ to simplify the derivations. From which we get

$$\begin{aligned}
&SW^{A}_{UU,UU}\\
&=\int_0^Z\int_0^1\left(u^A-tx_1-tx_2\right)dx_2dx_1+\int_Z^1\int_0^{1+Z-x_1}\left(u^A-tx_1-tx_2\right)dx_2dx_1\\
&=\frac{1}{2}u^A-\frac{1}{3}t+Zu^A+\frac{1}{3}Z^3t-\frac{1}{2}Z^2u^A-Zt\\
&=\frac{t}{3}\left(Z^3-3Z-1\right)+\frac{1}{2}\left(1+2Z-Z^2\right)u^A.
\end{aligned}$$

Social welfare of firm B is given by

$$\begin{aligned}
SW^{B}_{UU,UU}&=\int_Z^1\int_{1+Z-x_1}^1\left(u^B-t\left(1-x_1\right)-t\left(1-x_2\right)\right)dx_2dx_1\\
&=-\frac{1}{6}\left(Z-1\right)^2\left(2t-2Zt-3u^B\right).
\end{aligned}$$

Aggregate welfare under uniform pricing is given by

$$\begin{aligned}
&SW_{UU,UU}\\
&=\frac{t}{3}\left(Z^3-3Z-1\right)+\frac{1}{2}\left(1+2Z-Z^2\right)u^A-\frac{1}{6}\left(Z-1\right)^2\left(2t-2Zt-3u^B\right)\\
&=\frac{t}{3}\left(2Z^3-3Z^2-2\right)+\left(Z-\frac{Z^2}{2}\right)\left(u^A-u^B\right)+\frac{1}{2}\left(u^A+u^B\right)\\
&=\frac{t}{3}\left(2Z^3-3Z^2-2\right)+\left(Z-\frac{Z^2}{2}\right)\Delta+\frac{1}{2}\left(u^A+u^B\right).
\end{aligned}$$

Aggregate consumer surplus is given by

$$\begin{aligned}
&CS_{UU,UU}=SW_{UU,UU}-PS_{UU,UU}=SW^{A}_{UU}+SW^{B}_{UU}-\left(\pi^{A}_{UU}+\pi^{B}_{UU}\right)\\
&=\frac{t}{3}\left(2Z^3-3Z^2-2\right)+\left(Z-\frac{Z^2}{2}\right)\Delta+\frac{1}{2}\left(u^A+u^B\right)\\
&-\frac{\left(-4\Delta^3+24t\Delta^2+240t^2\Delta-544t^3-\left(16t\Delta-176t^2-4\Delta^2\right)\sqrt{36t^2-4t\Delta+\Delta^2}\right)}{512t^2}=\\
&=\frac{1}{2}\left(u^A+u^B\right)-\frac{\left(2\Delta^3-12t\Delta^2+168t^2\Delta-304t^3+\left(264t^2-2\Delta^2+8t\Delta\right)\sqrt{36t^2-4t\Delta+\Delta^2}\right)}{768t^2}.
\end{aligned}$$

It is easy to verify that all above expressions converge to those in Clavorà Braulin (2023), when $\Delta=0$.

B Appendix with Proofs

Proof of Proposition 1

We compare each firm's profits under configuration (PP,PP) in (4)-(5) with those under configuration (UU,UU) in (14)-(13). For firm B, inequality

$$\pi^{B}_{UU,UU}-\pi^{B}_{PP,PP}=\frac{\left(2t-\Delta+\sqrt{36t^2-4t\Delta+\Delta^2}\right)^3}{1024t^2}-\frac{(2t-\Delta)^3}{24t^2}>0$$

holds under our assumption $0 \leq \Delta < 2t$. This implies that for the less efficient firm, adopting personalized pricing is never profitable compared to uniform pricing irrespective of whether the model is one dimensional or two-dimensional.

For firm A, inequality

$$\pi^A_{UU,UU} - \pi^A_{PP,PP} = \frac{\left(128t^2 - \left(2t-\Delta+\sqrt{36t^2-4t\Delta+\Delta^2}\right)^2\right)\left(-5(2t-\Delta)+3\sqrt{36t^2-4t\Delta+\Delta^2}\right)}{1024t^2} - \frac{(2t+\Delta)^3-2\Delta^3}{24t^2} > 0$$

holds for $0 \leq \Delta < 0.821t$.[13] The left-hand side is negative when $0.821t < \Delta < 2t$. This implies that the expression can be both positive and negative depending on the degree of asymmetry.

Finally, summing both fractions yields

$\pi^A_{PP,PP} + \pi^B_{PP,PP} < \pi^A_{UU,UU} + \pi^B_{UU,UU}$ holds when $0 \leq \Delta < 1.489t$.[14] For $1.489t < \Delta < 2t$ the sign is reversed. QED

Proof of Proposition 2

We compare each firm's profits under UU, UU in (14)-(13) to profits under configuration PU, PU in (8)-(9). For firm B,

$$\pi^B_{UU,UU} - \pi^B_{PU,PU} = \frac{\left(2t-\Delta+\sqrt{36t^2-4t\Delta+\Delta^2}\right)^3}{1024t^2} - \frac{\left(28t^2-18t\Delta+3\Delta^2\right)}{54t} < 0$$

holds for $0 \leq \Delta < 0.223t$.[15] Similarly for firm A,

$$\pi^A_{UU,UU} - \pi^A_{PU,PU} = \frac{\left(128t^2 - \left(2t-\Delta+\sqrt{36t^2-4t\Delta+\Delta^2}\right)^2\right)\left(-5(2t-\Delta)+3\sqrt{36t^2-4t\Delta+\Delta^2}\right)}{1024t^2} - \frac{\left(28t^2+18t\Delta+3\Delta^2\right)}{54t} < 0$$

holds for $0 \leq \Delta < 0.213$.[16]

Finally, we obtain $\pi^A_{UU,UU} + \pi^B_{UU,UU} < \pi^A_{PU,PU} + \pi^B_{PU,PU}$ for $0 \leq \Delta < 0.218t$.[17] QED

Proof of Proposition 3

We compare each firm's profits under configuration (PP, PP) in (4)-(5) to those under configuration (PU, PU) in (8)-(9). For firm B, inequality

$$\pi^B_{PP,PP} - \pi^B_{PU,PU} = \frac{(2t-\Delta)^3}{24t^2} - \frac{28t^2-18t\Delta+3\Delta^2}{54t} = \frac{t\left[42\left(\frac{\Delta}{t}\right)^2 - 40 - 36\left(\frac{\Delta}{t}\right) - 9\left(\frac{\Delta}{t}\right)^3\right]}{216} < 0$$

[13] Computed with assistance of software package Maple. First, equating both denominators, Then, simplifying and division by t^3 of the numerator followed by the change of variable $\frac{\Delta}{t} = x$. Finally, we obtained $x = 0.82084$ as the unique numerical root on $[0, 2)$. The latter was verified by plotting the function of x.

[14] Similar to previous footnote, $\frac{\Delta}{t} = x = 1.48908$ is computed with assistance of software package Maple as the unique numerical root on $[0, 2)$.

[15] Solution $\frac{\Delta}{t} = 0.22304$ is obtained in Maple as the unique numerical root on $[0, 2)$.

[16] Solution $\frac{\Delta}{t} = 0.21275$ is obtained in Maple as the unique numerical root on $[0, 2)$.

[17] Solution $\frac{\Delta}{t} = 0.21765$ is obtained in Maple as the unique numerical root on $[0, 2)$.

holds under our assumption $0 \leq \Delta < 2t$. This implies that the less efficient firm never benefits from extending personalized pricing to both dimensions. For firm A, inequality

$$\pi^A_{PP,PP} - \pi^A_{PU,PU} = \frac{(2t+\Delta)^3 - 2\Delta^3}{24t^2} - \frac{28t^2+18t\Delta+3\Delta^2}{54t} = \frac{t\left[-40+36\left(\frac{\Delta}{t}\right)+42\left(\frac{\Delta}{t}\right)^2-9\left(\frac{\Delta}{t}\right)^3\right]}{216} < 0$$

holds for $0 \leq \Delta < \frac{2}{3}t$. The inequality flips for $\frac{2}{3}t < \Delta < 2t$. This shows that the fraction can be either negative or positive depending on the degree of asymmetry.

Summing both equation lines implies that aggregate profits $\pi^A_{PP,PP} + \pi^B_{PP,PP} < \pi^A_{PU,PU} + \pi^B_{PU,PU}$ for $0 \leq \Delta < 1.119t$.[18] QED

[18]Similar to above, solution $\frac{\Delta}{t} = 1.11933$ is obtained in Maple as the unique numerical root on $[0, 2)$.