

TI 2025-064/VII
Tinbergen Institute Discussion Paper

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Equilibrium in the Jungle Edgeworth Box

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November, 2025

Abstract

We introduce the Jungle Edgeworth Box economy as an analytical framework to analyze bilateral conflict and examine the interplay between coercion and voluntary exchange. We characterize the set of equilibria in which no further coercion or voluntary exchange occurs. By assuming that coercion precedes voluntary exchange, we characterize the set of equilibria of a Nash Negotiation Game, where coercion is interpreted as a threat from the stronger to the weaker agent. We conclude that the jungle allocation is rarely the correct snapshot of the economy after coercion is over and exchange, facilitated by effective property rights, is about to begin.

JEL Classification: D610; K110; P480

Keywords: bilateral conflict, coercion, jungle, barter, Edgeworth box

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1 Introduction

The emergence and establishment of property rights in situations characterized by a lack of institutional governance, commonly referred to as the State of Nature, has attracted increasing attention in economic analysis. Traditional economic models typically focus on voluntary exchanges based on mutually beneficial agreements, where agents possess endowments and preferences that determine the distribution of goods through trade. These models assume that individuals respect each other's property rights and engage in peaceful transactions to maximize individual welfare. However, throughout human history, the use of power to seize control of assets has been a common occurrence and, unfortunately, continues to play a significant role in economic interactions, even today.

In many real-world situations, power imbalances, whether stemming from physical strength, social status, or other factors, can result in coercion and the violation of property rights. This phenomenon challenges the conventional view of economic exchange and highlights the need to understand the dynamics of 'jungle' exchange, where stronger agents forcefully appropriate resources from weaker agents. By analyzing the interplay between coercion and voluntary trade, we can gain insights into the mechanisms through which power dynamics shape the initial distribution of resources and influence subsequent economic outcomes, providing a more nuanced understanding of how societies navigate the complexities of property rights, distributional outcomes and the potential for conflict.

In this article, we present a new analytical environment, the Jungle Edgeworth Box (JEB) economy in which we modify the standard Edgeworth box exchange economy developed by Edgeworth (1881) in the following way. We let the Edgeworth box coincide with the agents' consumption sets and assume that a subset of allocations, called the coercion set, are not secured by property rights. The stronger agent can change allocations in the coercion set at will by exercising coercive power. However, in general, such coercion does not result in a bilateral efficient outcome. Hence, after coercion, both agents can still improve their welfare via voluntary trade, resulting in an allocation outside the coercion set, which is secured by

property rights. Our first goal is to characterize equilibria in the JEB economy, that is, identify the set of allocations at which no further coercive nor voluntary exchange is taking place.

However, coercion and voluntary exchange are opposing activities that arguably cannot be performed simultaneously in the JEB economy. Therefore, we extend our analysis by studying the order of events in which the stronger agent first uses coercion against the weaker agent before both engage in voluntary exchange. This specific order helps us better understand how the outcome of jungle exchange explains the endowment division when agents enter the stage of voluntary exchange. We use John Nash’s two-agent negotiation model with threats and a general axiomatic bargaining solution to discuss the strategic motivations of rational agents in this process (Nash, 1953). We interpret the outcome of jungle exchange as the initiation of a threat of the stronger agent against the weaker agent before voluntary exchange through bargaining starts. Our second goal is to characterize which threats are credible in JEB equilibria and how they affect the agents’ final welfare after bargaining. Throughout the analysis, we assume agents to be rational in the standard economic/game theoretic sense, that is, farsighted through subgame perfection.

Literature Overview

Our article adds to the expanding body of research on jungle exchange, a topic that aligns with the broader literature on anarchy models, which in turn serves as a foundation for the study of constitutional political economy. Below, we offer a succinct and non-exhaustive overview of each of these areas of literature.

In their seminal article on jungle exchange, Piccione and Rubinstein (2007) propose a stylized model in which coercion governs the allocation of scarce resources in the absence of markets. Coercion in bilateral encounters is driven by the agents’ preferences over bounded consumption sets and by a power relation that describes an exogenous ranking of agents according to strength. Weaker agents concede their possessions to stronger agents without engaging in costly conflict. In a jungle equilibrium, a stronger agent no longer wants to take

goods from a weaker agent nor from a pile of common goods, that no other agent holds. In the jungle, there is no role for trade, secured property rights and market institutions. By relaxing assumptions on consumption sets and preferences, Houba et al. (2017) demonstrate that allocations other than lexicographic maximizers can be jungle equilibria. They further explore the role of multiple unilateral takings, revealing that jungle equilibria align with lexicographic maximization under certain conditions, while presenting instances of Pareto inefficient equilibria that require voluntary gift-giving and even voluntary trade to achieve Pareto efficiency in the jungle. Crettez (2020) characterizes the subset of jungle equilibria that are also Pareto-minimal.¹ Proposing a model of a stochastic jungle where resource allocation is driven by conflict and bargaining, Schwarz (2019) demonstrates that the stochastic jungle bargaining mechanism, implemented within a finite number of periods, leads to an inefficient Talmud rule allocation that serves as the foundation for a stable exchange economy with Pareto-efficient Walrasian equilibria. Rubinstein and Yildiz (2022) enrich the jungle economy by introducing a language component that specifies legitimate criteria for justifying the assignment of agents to objects. They propose a solution concept called civilized equilibrium, which requires each agent to be justifiable within a group and stronger than any other agent justifiable within the same group and propose conditions under which the equilibrium in a civilized jungle is identical to the jungle equilibrium.

The analysis of jungle economies can be linked to the broader study of anarchy, which examines the functioning of economic systems without formal governance or established property rights, as both explore the implications of power dynamics, conflict, involuntary exchange and property rights in shaping resource distribution and economic outcomes in various societal contexts. Skaperdas (1992) focuses on the trade-off between productive and coercive activities and highlights the possibility of voluntary exchange without enforceable property rights. In his seminal analysis of the emergence and breakdown of anarchy, Hirschleifer (1995) explores the role of self-interest, social norms and the presence of external

¹An allocation is Pareto-minimal if it is impossible to reduce the welfare of one agent without increasing the welfare of another.

threats in shaping the stability or instability of anarchic societies. Grossman and Kim (1995) present a general equilibrium model that examines the allocation of resources between appropriative and productive activities, exploring the role of offensive and defensive technologies in determining the security of property claims. Hafer (2006) provides a dynamic model of conflict and production in a state of nature and demonstrates that, over time, a systematic bias in favour of incumbent landholders develops even when offensive and defensive technologies are the same, no saving or long-term investment in conflict is possible and the initial conditions are symmetric.

Constitutional political economy explores the design, formation and evolution of institutions, particularly constitutions, as a means to facilitate cooperation, protect property rights and promote economic prosperity as economies evolve from anarchy to states. Inspired by Bush (1972), who presents a pessimistic view of anarchy where predation rather than cooperation is omnipresent, Tullock (1972) contends that creating a government, through its power to enforce the law, limits the use of force by others. The ensuing reduction of conflict creates incentives for production rather than predation. Buchanan (1975) emphasizes the importance of studying the rules and constraints that govern political decision-making, highlighting the role of constitutional design in aligning individual and collective interests. Brennan and Buchanan (1985) explore the role of rules as a means to coordinate individuals' actions and promote social cooperation, emphasizing the need for rules to be impartial, predictable and subject to voluntary participation. McGuire and Olson (1996) examine the dynamics of political power and its implications for economic outcomes, explaining how the use of force by those in power can distort resource allocation and hinder economic development. Olson (2000) studies the impact of different political and economic systems on prosperity, demonstrating how inclusive institutions and property rights are crucial for fostering economic growth and innovation.

The article is structured as follows. In Section [2](#), we extend the formal deterministic framework of Piccione and Rubinstein (2007) to include voluntary exchange. The charac-

terization of JEB equilibria is presented in Section 3. Our analysis of the Nash negotiation game in the JEB economy can be found in Section 4. Section 5 illustrates the main insights of our article using a 2-agent 2-commodity example, while Section 6 concludes our article. Appendix A contains the proofs of Propositions 1-3, while Appendix B revisits the analysis of Section 4 under a weaker assumption. Appendix C revisits the characterization of JEB equilibria in a stochastic framework, inspired by Schwarz (2019), that provides a foundation for our specification of the set of contested allocations.

2 Preliminaries

We consider n agents, denoted $i = 1, \dots, n$ and m commodities, denoted $k = 1, \dots, m$. Agent i 's consumption of commodity k is z_k^i . Agent i 's consumption vector is $z^i = (z_1^i, \dots, z_m^i)$. An allocation $z = (z^1, \dots, z^n)$ is an n -tuple of non-negative consumption vectors. Let ω^i denote agent i 's initial endowment. The vector of initial endowments is $\omega = (\omega^1, \dots, \omega^n)$, where $\sum_{i=1}^n \omega^i = \tilde{\omega}$, where $\tilde{\omega}$ denotes the economy's total endowment. The economy's set of feasible allocations is $\mathcal{Z}(\tilde{\omega}) \equiv \{z = (z^1, \dots, z^n) | z^i \in \mathbb{R}_+^m \text{ for all } i \text{ and } \sum_{i=1}^n z^i = \tilde{\omega}\}$.

We assume that every agent i has a standard weak preference ordering $\succeq^i$ over agent i 's consumption vectors in $\mathcal{Z}(\tilde{\omega})$.² Its symmetric and asymmetric parts are denoted $\sim^i$ and $\succ^i$ respectively.

All exchange, whether coercive or voluntary, is a bilateral activity undertaken within pairs ij of individual agents i and j , where $i < j$. As in Piccione and Rubinstein (2007), $i < j$ means that agent i is stronger than agent j and that i can take from j what j is forced to concede to i , but not vice versa. Hence, we can rank agents with respect to strength, where agent 1 is the strongest agent and agent n the weakest. Every pair of agents and their consumption bundles in a feasible allocation form a bilateral Edgeworth box of feasible bilateral allocations that may be reached through exchange. Given the pair ij and their consumption bundles in feasible allocation z , a bilateral allocation is $\hat{z}^{ij} = (\hat{z}^i, \hat{z}^j)$ and the

²The standard assumptions are completeness, transitivity, continuity, monotonicity and strict convexity.

bilateral Edgeworth box is $\mathcal{E}^{ij}(z) \equiv \{\hat{z}^{ij} = (\hat{z}^i, \hat{z}^j) | \hat{z}^i, \hat{z}^j \in \mathbb{R}_+^m \text{ for all } i, j \text{ and } \hat{z}^i + \hat{z}^j = z^i + z^j\}$, which is a non-empty and compact set.³ In case $z = \omega$, we write $\mathcal{E}^{ij}(\omega)$. We denote agent i 's origin as $O^i(z) = (0, z^i + z^j)$ and agent j 's origin as $O^j(z) = (z^i + z^j, 0)$.

Each bilateral Edgeworth box is partitioned in a subset of contested and uncontested bilateral allocations, denoted $\mathcal{C}^{ij}(z) \subseteq \mathcal{E}^{ij}(z)$ and $\mathcal{U}^{ij}(z) = \mathcal{E}^{ij}(z) \setminus \mathcal{C}^{ij}(z)$, respectively. For every pair ij and $z \in \mathcal{Z}(\tilde{\omega})$, we assume that $\mathcal{C}^{ij}(z)$ is a compact set that is strict comprehensive with respect to agent i 's origin $O^i(z)$, that is, for all $\hat{z}^{ij} \in \mathcal{C}^{ij}(z)$ and $\tilde{z}^{ij} \in \mathcal{E}^{ij}(z)$, such that $\tilde{z}^i \leq \hat{z}^i$, it holds that $\tilde{z}^{ij} \in \text{Int}(\mathcal{C}^{ij}(z))$.⁴ For all initial endowments and allocations in $\mathcal{C}^{ij}(z)$, the stronger agent is physically capable of changing z^{ij} (or ω) into $\hat{z}^{ij}$, provided that $\hat{z}^{ij} \in \mathcal{C}^{ij}(z)$. Coercion can take the form of theft when $\hat{z}^i \geq z^i$ or the form of coercive trade when commodities are exchanged, but the stronger agent i unilaterally dictates the terms of the exchange in $\mathcal{C}^{ij}(z)$. Denote the set of maximal elements of the stronger agent's consumption bundles associated with allocations in the coercion set as $\mathcal{FC}^{ij}(z)$.⁵ This frontier is a subset of the boundary of the coercion set and the uncontested set.

For a given pair ij , we denote the subset $\mathcal{PU}^{ij}(z) \subseteq \mathcal{U}^{ij}(z)$ the set of Pareto efficient allocations in the set of uncontested allocations. An allocation z is Pareto efficient if there does not exist another allocation $\hat{z}$ such that $\hat{z}^i \succeq^i z^i$ for all $i = 1, \dots, n$ and $\hat{z}^i \succ^i z^i$ for at least one agent i . We refer to $\mathcal{PU}^{ij}(z)$ as the contract curve of $\mathcal{U}^{ij}(z)$. When $\mathcal{C}^{ij}(z) = \emptyset$, $\mathcal{PU}^{ij}(z) \equiv \mathcal{PE}^{ij}(z)$, that is, the classic contract curve in $\mathcal{E}^{ij}(z)$. Since $\mathcal{U}^{ij}(z)$ and $\mathcal{PU}^{ij}(z)$ are open sets, we denote and $\bar{\mathcal{U}}^{ij}(z)$, respectively, $\bar{\mathcal{PU}}^{ij}(z)$ as their closure.

We also define the set of the stronger agent's best allocations in the contested set as well as the set of the weaker agent's worst allocations in the contested set. Denote the set of agent i 's best allocations in the contested set as $\mathcal{J}^{ij}(z) \subset \mathcal{C}^{ij}(z)$. An element of $\mathcal{J}^{ij}(z)$ is denoted $J^{ij}(z) = (J^i(z), J^j(z))$, which is a bilateral jungle allocation in the JEB economy.

³Under monotonic preference relations, it is without loss of generality to write $\hat{z}^i + \hat{z}^j = z^i + z^j$ instead of $\hat{z}^i + \hat{z}^j \leq z^i + z^j$, because there will be no spillage in equilibrium.

⁴Vector inequalities: we write $a \leq b$ if for all vector components k we have $a_k \leq b_k$; $a \leq b$ if for all vector components k we have $a_k \leq b_k$ and there exist at least one component for which the inequality is strict; and $a < b$ if for all vector components k we have $a_k < b_k$.

⁵Formally, $\bar{z}^{ij} = (\bar{z}^i, \bar{z}^j) \in \mathcal{FC}^{ij}(z)$ if there does not exist a $\hat{z}^{ij} = (\hat{z}^i, \hat{z}^j) \in \mathcal{C}^{ij}(z)$ such that $\hat{z}^i \geq \bar{z}^i$.

Next, denote the set of agent j 's worst allocations in the contested set as $\mathcal{W}^{ij}(z) \subset \mathcal{C}^{ij}(z)$. An element of $\mathcal{W}^{ij}(z)$ is denoted $W^{ij}(z) = (W^i(z), W^j(z))$.

Finally, we define three specific allocations in $\overline{\mathcal{PU}}^{ij}(z)$. First, denote as $G^{ij}(z)$ the intersection of $\overline{\mathcal{PU}}^{ij}(z)$ and, since the stronger agent is indifferent between all jungle allocations, the indifference curve through all $J^{ij}(z) \in \mathcal{J}^{ij}(z)$. Second, denote as $H^{ij}(z)$ the intersection of $\overline{\mathcal{PU}}^{ij}(z)$ and, since the weaker agent is indifferent between all her worst contested allocations, the indifference curve through all $W^{ij}(z) \in \mathcal{W}^{ij}(z)$. Third, denote as $S^{ij}(z) = (S^i(z), S^j(z))$ the intersection given by $\overline{\mathcal{PU}}^{ij}(z) \cap \mathcal{C}^{ij}(z)$.

3 Equilibrium in the Jungle Edgeworth Box Economy

First, we introduce the equilibrium concept of our JEB economy. Such an economy is in equilibrium if, in any pairwise encounter, neither the stronger agent unilaterally coerces further, nor any pair of agents bilaterally Pareto improves further through voluntary exchange. We denote this equilibrium concept a *JEB equilibrium*.

Definition 1 *A JEB equilibrium is a feasible allocation z such that, for any pair ij ,*

- $\nexists \hat{z}^{ij} \in \mathcal{C}^{ij}(z)$ such that $\hat{z}^i \succ^i z^i$ whenever $\hat{z}^{ij} \in \mathcal{C}^{ij}(z)$,
- $\nexists \hat{z}^{ij} \in \mathcal{U}^{ij}(z)$ such that $\hat{z}^i \succeq^i z^i$, $\hat{z}^j \succeq^j z^j$ (with at least one $\succeq$ being $\succ$) whenever $\hat{z}^{ij} \in \mathcal{E}^{ij}(z)$.

Note that the concept of JEB equilibrium assumes that weaker agents have the wit to avoid contested allocations in voluntary exchange.

The following proposition provides a characterization of the set of JEB equilibria in the JEB economy.⁶ It distinguishes between equilibria located in the contested and uncontested sets of the JEB economy. The former equilibria are subtle and degenerate: either the weaker agent does not possess any endowments, or the stronger agent ends up with her

⁶All proofs are relegated to Appendix [A](#).

worst allocation in the closure of the set of Pareto efficient allocations belonging to the uncontested set. Simultaneously, this allocation has to be a jungle allocation. The latter equilibria coincide with the intersection of the standard contract curve and the uncontested set.

Proposition 1 *Allocation $z \in \mathcal{Z}(\tilde{\omega})$ is a JEB equilibrium if and only if, for any pair of ij ,*

$$\begin{aligned} z^{ij} &\in \mathcal{C}^{ij}(z) : \text{either } z^j = 0 \text{ or } z^{ij} = S^{ij}(z) \in \mathcal{J}^{ij}(z), \\ z^{ij} &\in \mathcal{U}^{ij}(z) : z^{ij} \in \mathcal{PU}^{ij}(z). \end{aligned}$$

Several remarks are worth making.

First, a JEB equilibrium exists, because the allocation where the strongest agent in the JEB economy holds all resources is a JEB equilibrium.

Second, the necessary and sufficient conditions for a jungle allocation to be a JEB equilibrium are very restrictive. Such a jungle allocation also has to be Pareto efficient in the bilateral Edgeworth box. As already mentioned, these conditions are degenerate. Rather, the generic and interesting case is that a jungle allocation fails to be a JEB equilibrium. Therefore, the snapshot of a jungle allocation can better be interpreted as a starting allocation for further negotiations. Often, the JEB economy will not even reach this specific starting allocation, as the analysis in the next section will demonstrate.

Third, can a First Fundamental Theorem of Welfare Economics be formulated for the JEB economy? While our JEB equilibrium, as characterized in Proposition [1](#), is restricted to bilateral efficiency, it is possible to obtain overall Pareto efficiency. Rader (1968) shows that if there is a trader who can deal in all commodities under a given bilateral efficient allocation, that allocation is Pareto efficient. Essentially, Rader's work shows how important a broker might be in an exchange economy where multilateral trades are impossible. Given our assumption of the strict comprehensiveness of the contested set, the strongest agent will take on the role of a broker in our framework, as she can and will possess positive amounts of all commodities obtainable through coercion.

4 A Nash Negotiation Game in the JEB Economy

The two economic activities that describe the JEB economy, on the one hand coercion and on the other hand voluntary exchange, are pursuits that contrast sharply with each other. Furthermore, arguably, both cannot be performed simultaneously. However, our analysis in the previous section neglected any ‘order of events’ in which coercion and trade take place. In this section, we explicitly assume that the stronger agent uses coercion against the weaker agent before, in what is possibly a new encounter, both agents engage in voluntary exchange. As explained in the Introduction, this specific chronology of events is motivated by the attempt to analyze the transition from jungle to barter economy, that is, to obtain a better understanding of how the outcome of jungle exchange helps explain the division of initial endowments with which economic agents enter the stage of voluntary exchange.

Of course, the introduction of a specific timing of events opens up a discussion on the strategic motivations that the agents may have, assuming both stronger and weaker agents are rational in the standard game theoretic sense, that is, farsighted through subgame perfection, and the rules and outcomes of the game are common knowledge while information is complete and perfect. In the following, we take inspiration from John Nash’s classic two-agent negotiation model where negotiators choose actions, called threats, that determine the disagreement point in an axiomatic bargaining solution (Nash, 1953).⁷ We interpret the outcome of coercive exchange as the initiation of a threat of the stronger agent against the weaker agent, a threat the stronger agent makes prior to the negotiations with the aim of maneuvering the disagreement outcome of the bargaining process in order to achieve a more favorable voluntary exchange. In the following, we are interested in characterizing which threats are credible in a JEB equilibrium and how they affect the agents’ welfare.

As in Nash (1953), our analysis is performed backward in two stages. The second stage of bargaining is analyzed prior to the first stage in which the stronger agent chooses her

⁷The axiomatic approach is taken for convenience, but can be replaced by a strategic approach as in e.g. Binmore et al. (1986) or Bolt and Houba (1998). Also, dictator bargaining power can be linked to being the proposer in an ultimatum game.

threats.

Bargaining Stage

In order to analyze the bargaining stage, we need to introduce additional notation. For $z \in \mathcal{Z}(\tilde{\omega})$ and any pair ij , denote an arbitrary threat allocation as $\hat{T}^{ij} = (\hat{T}^i, \hat{T}^j) \in \mathcal{E}^{ij}(z)$. The equilibrium threat allocation for a given z is denoted by the function $T^{ij} : \mathcal{E}^{ij}(z) \rightarrow \mathcal{E}^{ij}(z) : T^{ij}(z) = (T^i(z), T^j(z))$ and expresses agent i 's choice of threat (as a bilateral allocation) in equilibrium. Of course, $T^{ij}(z) = z^{ij}$ if $z^{ij} \in \mathcal{U}^{ij}(z)$. In addition, we introduce an axiomatic bargaining solution as a function of arbitrary threats. Let the function $B^{ij} : \mathcal{E}^{ij}(z) \rightarrow \overline{\mathcal{U}}^{ij}(z) : B^{ij}(\hat{T}^{ij}) = (B^i(\hat{T}^{ij}), B^j(\hat{T}^{ij}))$ represent the axiomatic bargaining solution for a given pair of agents and a given threat allocation. Bargaining solutions map to the set of uncontested allocations, because both agents are rational and foresee that any contested allocation will invoke another cycle of future coercion followed by voluntary exchange.⁸ In the following, we admit any axiomatic bargaining solution that satisfies the standard axioms of Pareto Efficiency and Individual Rationality with respect to the threat allocation.⁹

Our next proposition considers an arbitrary allocation in the JEB economy as the threat point in the bargaining stage. It establishes a pair of lower bounds on welfare that bargaining can achieve in each bilateral encounter, one for the stronger and another for the weaker agent. Obviously, the case of interest is when the allocation is contested.

⁸To avoid technical details, we take the closure of the set of uncontested allocations as the image of bargaining solutions with the understanding that whenever the weaker agent would be able to negotiate an allocation belonging to the boundary of the contested set, this agent settles for an allocation inside the uncontested set arbitrarily close to this boundary allocation. A technical discussion is deferred to Appendix [B](#).

⁹ These axioms hold for e.g. the symmetric and asymmetric Nash bargaining solution (Nash (1950) and Kalai (1977a), respectively), the Kalai-Smorodinsky solution (Kalai and Smorodinsky, 1975), the egalitarian solution (Kalai, 1977b) and the unified bargaining solution (Haake and Qin, 2018).

Proposition 2 *For all $z \in \mathcal{Z}(\tilde{\omega})$ and all pairs ij , it holds that*

$$\begin{aligned} z^{ij} \in \mathcal{C}^{ij}(z) : H^i(z) \succeq^i B^i(T^{ij}(z)) \succeq^i J^i(z) \text{ for all } J^{ij}(z) \in \mathcal{J}^{ij}(z) \text{ and} \\ G^j(z) \succeq^j B^j(T^{ij}(z)) \succeq^j W^j(z) \text{ for all } W^{ij}(z) \in \mathcal{W}^{ij}(z), \\ z^{ij} \in \mathcal{U}^{ij}(z) : B^i(T^{ij}(z)) \succeq^i S^i(z), B^i(T^{ij}(z)) \succeq^i z^i \text{ and } B^j(T^{ij}(z)) \succeq^j z^j. \end{aligned}$$

Moreover, $J^i(z) \succ^i S^i(z)$ is the non-degenerate case.

First, consider an initial allocation in the contested set and any axiomatic bargaining equilibrium arising from this allocation. Such an equilibrium belongs to the open curve consisting of the uncontested set's contract curve and the endpoint allocations $G^{ij}(z)$ and $H^{ij}(z)$. Recall that this former endpoint allocation can be identified as the intersection of the contract curve and the stronger agent's indifference curve through all jungle allocations. In general, it lies within the uncontested set and every axiomatic bargaining allocation is weakly preferred by the stronger agent to any jungle allocation. In the richer JEB economy setting, allowing for the combination of coercion and voluntary exchange, the stronger agent's security level is obtained via coercion. Benefiting from the option to trade, most axiomatic bargaining solutions will assign the stronger agent even more welfare. Similarly, the other endpoint allocation is the intersection of the contract curve and the weaker agent's indifference curve through all allocations in $\mathcal{W}^{ij}(z)$. This allocation demonstrates the limit of what the stronger agent can ultimately obtain, being advantaged by the ability to coerce the weaker agent before engaging in voluntary exchange. Our motivating example shows that, when one of the agent's possesses dictator power, it is impossible to derive tighter bounds than these endpoint allocations.

Second, consider an initial allocation in the uncontested set and any axiomatic bargaining equilibrium arising from this allocation. Even in such an equilibrium, coercion casts its shadow. It limits the set of uncontested allocations the agents can agree on without provoking coercion that would make the weaker agent worse off. The weaker agent wants to avoid this scenario out of self-protection. In particular, when the stronger agent prefers allocation

$S^{ij}(z)$ to z^{ij} , the set of possible Pareto efficient allocations is reduced. Self-protection adds an additional constraint to the standard lens of individual rational allocations starting from z^{ij} , favoring the stronger agent.

Threat Stage

We turn to the first stage of the negotiation model in which the stronger agent selects her threat. This agent only chooses a threat if the initial allocation is contested. Thereby, the stronger agent correctly predicts how this threat will lead to a future agreement as characterized in Proposition 2. In other words, the stronger agent's best threat in the first stage is the one that gives this agent the best consumption bundle among all allocations belonging to the contract curve of the uncontested set that can be reached by threats from the coercion set. Formally, the condition for stronger agent's equilibrium threat $T^{ij}(z) \in \mathcal{C}^{ij}(z)$ states that $B^i(T^{ij}(z)) \succeq^i B^i(\bar{T}^{ij})$ for all $\bar{T}^{ij} \in \mathcal{C}^{ij}(z)$.

We introduce additional structure on the axiomatic bargaining solution in order to specify how the solution responds to a specific change in threats. Consider two contested threat allocations, denoted by $\bar{T}^{ij}$ and $\hat{T}^{ij}$, such that the stronger agent weakly prefers the former over the latter, while the weaker agent has the opposite preference. Then, we assume that the stronger agent weakly prefers the axiomatic bargaining solution corresponding to $\bar{T}^{ij}$ to the solution corresponding to $\hat{T}^{ij}$.¹⁰ By Pareto Efficiency, the weaker agent has the opposite preference. Formally, consider the threat allocations $\bar{T}^{ij}, \hat{T}^{ij} \in \mathcal{C}^{ij}(z)$ such that $\bar{T}^i \succeq^i \hat{T}^i$ and $\hat{T}^j \succeq^j \bar{T}^j$, with at least one strict preference. Then, we require that $B^i(\bar{T}^{ij}) \succ^i B^i(\hat{T}^{ij})$.

Our next proposition considers an arbitrary allocation in the JEB economy as the starting point in the threat stage. Obviously, the case of interest is when the allocation is contested. Our proposition then establishes that equilibrium threat allocations lie on the boundary of the coercion and uncontested set.

¹⁰In terms of utility, we assume that the stronger agent's utility of the axiomatic bargaining solution is increasing in the stronger agent's disagreement utility while decreasing in the weaker agent's disagreement utility. Many axiomatic bargaining solutions have this property, e.g. the Nash bargaining solution and the Kalai-Smorodinsky bargaining solution.

Proposition 3 For all $z \in \mathcal{Z}(\tilde{\omega})$ and all pairs ij , it holds that

$$z^{ij} \in \mathcal{C}^{ij}(z) : T^{ij}(z) \in \left\{ \hat{T}^{ij} \in \mathcal{FC}^{ij}(z) \left| \begin{array}{l} \hat{T}^i \succeq^i W^i(z), \quad \forall W^{ij}(z) \in \mathcal{W}^{ij}(z) \quad \text{and} \\ J^j(z) \succeq^j \hat{T}^j, \quad \forall J^{ij}(z) \in \mathcal{J}^{ij}(z) \end{array} \right. \right\},$$

$$z^{ij} \in \mathcal{U}^{ij}(z) : T^{ij}(z) = z.$$

Moreover, if agent i has dictator bargaining power, then $T^{ij}(z) \in \mathcal{W}^{ij}(z)$. And if agent j has dictator bargaining power, then $T^{ij}(z) \in \mathcal{J}^{ij}(z)$.

Any equilibrium threat allocation belongs to a subset of the uncontested set's frontier. This set is bounded by the weaker agent's worst allocation in the contested set and the stronger agent's Jungle allocation, respectively.^[11] As our illustrating example in the next section will show, if one agents has dictator power, then these boundary threat allocations become equilibrium threats. Consequently, it is impossible to derive tighter bounds.

5 Illustrating Example

In this section, we present a 2-agent 2-commodity example illustrating the main insights of our analysis. Consider an economy with two agents $i = 1, 2$ and two (infinitely divisible) commodities $k = 1, 2$. Agent i 's consumption of commodity k is denoted z_k^i . Assume agent 1 has symmetric Cobb-Douglas preferences $z_1^1 z_2^1$, agent 2 has symmetric constant-elasticity-of-substitution preferences $\sqrt{z_1^2} + \sqrt{z_2^2}$ and assume that total resources equal $(4, 4)$.^[12] The Edgeworth box is a square with corners $(0, 0)$, $(0, 4)$, $(4, 0)$ and $(4, 4)$. Without coercive power, the contract curve, which is the set of tangency points between the indifference curves of the two agents and, hence, the locus of Pareto-efficient allocations, is given by the 45°-line connecting the corners $(0, 0)$ and $(4, 4)$.

However, assume that agent 1 is stronger than agent 2. Assume that the set of contested allocations (the coercion set) seen from agent 1's origin, is given by

$$\{z^1 \in \mathbb{R}_+^2 \mid 4z_1^1 + 3z_2^1 \leq 16, \quad z^1 \leq (4, 4)\}.$$

¹¹More precisely, tightest bounds $W^{ij}(z)$ are also best elements with respect to $\succeq^i$ in $\mathcal{W}^{ij}(z)$. And, tightest bounds $J^{ij}(z)$ are worst elements with respect to $\succeq^j$ in $\mathcal{J}^{ij}(z)$.

¹²We motivate the asymmetry in preferences later on in footnote ^[15]

Note that the coercion set is a subset of the Edgeworth box.¹³

The set of JEB equilibria is the intersection of the contract curve and the set of uncontested allocations, given by that part of the 45⁰-line connecting (but not including) allocation $((\frac{2}{7}, \frac{2}{7}), (1\frac{5}{7}, 1\frac{5}{7}))$ and the corner $(4, 4)$.

While the identification of JEB equilibria is rather simple, it is instructive to introduce the specific timing of events where the stronger agent first uses coercion against the weaker agent (stage 1) to manoeuvre herself in a better position before both agents negotiate a voluntary exchange (stage 2).

What activities take place when we start from an initial endowment outside equilibrium? For those initial endowments located in the uncontested set, there is no stage 1 and the standard textbook explanation of agents engaging in voluntary trade towards the contract curve, while avoiding the coercion set, applies.

However, for those initial endowments located in the coercion set, the description of consecutive actions taken by the agents is intriguing.

First consider a jungle economy with only coercion and no trade. Suppose the stronger agent 1's origin forms the initial endowment allocation, denoted ω_1 in Figure 1. The stronger agent can improve her utility by coercion. We obtain the jungle allocation of Piccione and Rubinstein (2007) when agent 1 selects her best allocation $((2, \frac{8}{3}), (2, \frac{4}{3}))$ in the coercion set, denoted z_1 in Figure 1. Note that this jungle allocation is Pareto inefficient as opportunities for Pareto improving trade inside the uncontested set are present. Whatever the outcome of individually rational trade that follows, it is clear that agent 1 can secure herself a minimal utility $u_1(z_1)$ when starting from ω_1 .

In stage 2, barter requires wit and bargaining skill which is modelled as possible asymmetries in bargaining power between the agents. Coercive power and bargaining skill are different traits and agents need not be well equipped in both. Assume that the outcome of bargaining between the stronger and the weaker agent corresponds to the asymmetric Nash

¹³It is easy to check that, seen from agent 2's origin, the complement set of the coercion set (the uncontested set) is given by $\{z^2 \in \mathbb{R}_+^2 | 4z_1^2 + 3z_2^2 < 12, z^2 \leq (4, 4)\}$ in our example.

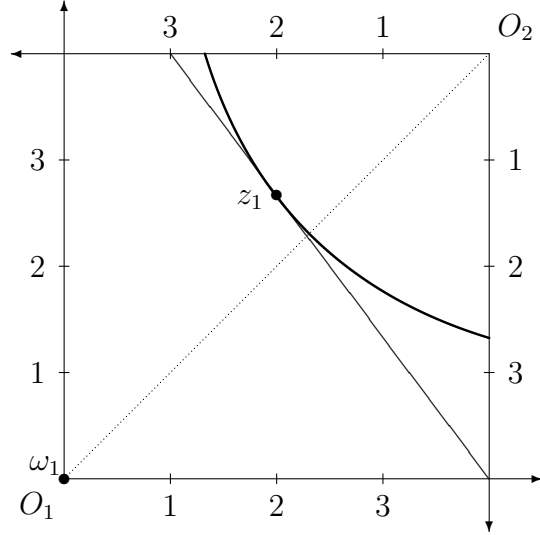


Figure 1: Jungle allocation. Agent 1 uses coercion to change the allocation from ω_1 to Pareto inefficient z_1 .

bargaining solution, with a and $1 - a$ denoting the respective bargaining power of the former and the latter. For the ease of exposition, we consider first the two extreme cases in which either the stronger agent has dictator power ($a = 1$) or the weaker agent has dictator power ($a = 0$).¹⁴ One crucial question stands out: how does the possession of dictator power, or the lack thereof, in stage 2 influence agent 1's behavior in stage 1, if we assume both agents to be farsighted rational?

Let us begin by assuming that agent 1 lacks bargaining power. In this scenario, she anticipates being constrained to the utility level corresponding to the coerced allocation selected in stage 1 during the subsequent stage. Consequently, her optimal strategy in stage 1 is to modify the initial endowment allocation ω_1 to the allocation that maximizes her utility over the coercion set, which in this case is the jungle allocation z_1 (as shown in Figure 1).

Since z_1 is not Pareto efficient, both agents engage in bargaining during stage 2 and ultimately settle on z_2 , which lies at the intersection of the contract curve and the lower

¹⁴Each case can be interpreted as if one agent is in a position to make a take-it-or-leave-it offer in the uncontested set to the other agent.

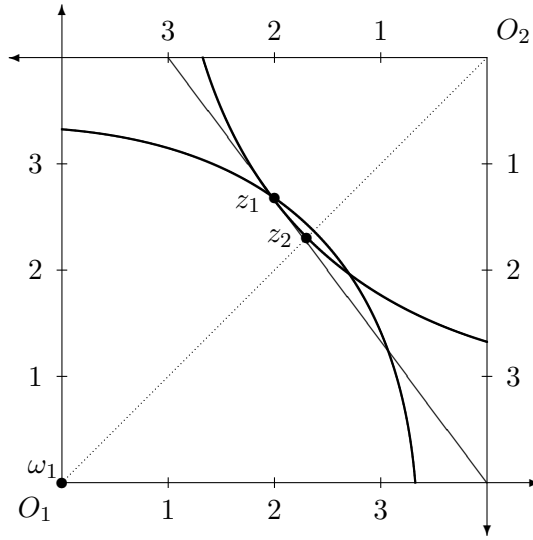


Figure 2: Agent 1's security level. If $a = 0$, coercion by agent 1 moves ω_1 to z_1 in stage 1 and agent 2's dictator power moves z_1 further to the uncontested Pareto efficient allocation z_2 .

section of the Pareto improvement lens emanating from z_1 (as illustrated in Figure 2). It is worth noting that this intersection lies entirely within the uncontested set.

Next, let us consider the scenario where agent 1 holds all the bargaining power. In this case, she anticipates being able to limit the other agent's utility in stage 2 to the level of the coerced allocation chosen in stage 1. Therefore, in stage 1, agent 1's optimal strategy is to modify the initial endowment allocation ω_1 to the allocation that minimizes the other agent's utility over the coercion set, which is $z_3 = ((1, 4), (3, 0))$ (as depicted in Figure 3).

Since z_3 is not Pareto efficient, both agents engage in bargaining during stage 2 and ultimately settle on $z_4 = ((3\frac{1}{4}, 3\frac{1}{4}), (\frac{3}{4}, \frac{3}{4}))$, which lies at the intersection of the contract curve and the upper part of the lens of Pareto improvements emanating from z_3 (as shown in Figure 3).¹⁵

¹⁵ Note that if we assume symmetric Leontief or symmetric Cobb-Douglas preferences for agent 2, the final allocation z_4 would lie on agent 2's origin. However, given the complementary nature of both commodities in the preferences of the agents, the fact that agent 2 is entirely deprived of commodity 2 in z_3 also renders her possession of commodity 1 useless.

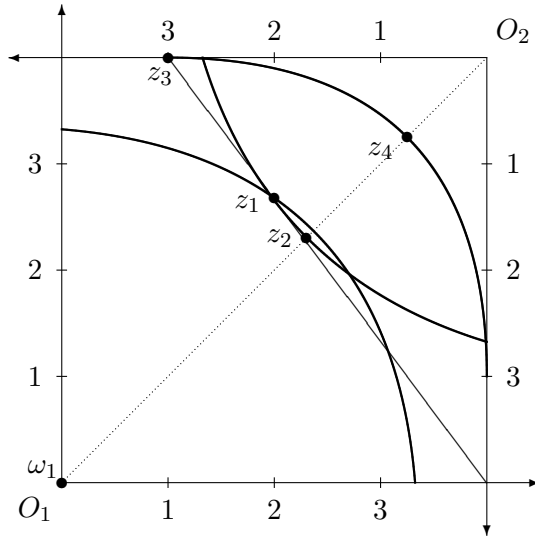


Figure 3: Agent 1's bliss point. If $a = 1$, coercion by agent 1 moves ω_1 to z_3 in stage 1 and agent 1's dictator power moves z_3 further to the uncontested Pareto efficient allocation z_4 .

Our example highlights the conditions under which the stronger agent will use the Jungle allocation z_1 as a threat against the weaker agent in stage 1. Specifically, this will only occur when agent 1 lacks sufficient bargaining power to obtain a utility level in stage 2 that exceeds her security level $u_1(z_1)$. As bargaining power gradually shifts from $a = 0$ to $a = 1$, the stronger agent's threat continuously transitions from the Jungle allocation z_1 to the weaker agent's worst allocation in the coercion set, z_3 . As a result, the JEB equilibrium continuously moves from allocation z_2 to allocation z_4 . Hence, a crucial insight from our analysis is that the jungle allocation fails as a predictor of the initial endowments with which agents engage in voluntary exchange as soon as the stronger agent possesses bargaining power.

6 Conclusion

In this article, we presented the Jungle Edgeworth Box economy as a new analytical framework for bilateral conflict. We examined the interaction between coercive power and voluntary exchange and characterized the set of equilibria in which neither coercion nor voluntary trade occurs. Furthermore, we investigated a general Nash Negotiation Game in the JEB

economy, assuming that coercion occurs prior to voluntary exchange. We identified which threats are credible in JEB equilibria and discussed how these threats affect the agents' welfare. Our results indicate that, on the continuation path from the inefficient State of Nature to the efficient barter equilibrium, only in degenerate cases the traditional jungle equilibrium of Piccione and Rubinstein (2007) is the correct snapshot of the economy after coercion and before voluntary exchange.

What can a benevolent social planner, only able to determine the initial endowments in uncontested sets of the JEB economy, learn from our analysis? Is it possible to formulate a result similar to the classic Second Fundamental Theorem of Welfare Economics in our setting? Without coercion, the planner can achieve any desired bilateral efficient allocation. However, in the presence of coercion, the stronger agent can use her power to obtain a more favorable allocation, making it impossible for the planner to achieve any bilateral efficient allocation within the coercion set. Our characterization result of Proposition [1](#) explicitly demonstrates how the instrument of redistribution of the planner is weakened.

A Proofs

Proof of Proposition [1](#)

We only prove the necessity part as the sufficiency part is straightforward. Consider a JEB equilibrium z , a pair ij and corresponding allocation $z^{ij} \in \mathcal{E}^{ij}(z)$. There are two mutually exclusive cases:

1. $z^{ij} \in \mathcal{C}^{ij}(z)$: The absence of further unilateral coercion can only hold if z^i corresponds to agent i 's best allocation in $\mathcal{C}^{ij}(z)$. There are two sub-cases.
 - a. $O^j(z) \in \mathcal{C}^{ij}(z)$: By comprehensiveness, $\mathcal{C}^{ij}(z) = \mathcal{E}^{ij}(z)$. Agent i 's best allocation coincides with $O^j(z)$ in which agent i obtains $z^i + z^j$. Since z is a JEB equilibrium, it must hold that $z^i = z^i + z^j$, implying $z^j = 0$.
 - b. $O^j(z) \notin \mathcal{C}^{ij}(z)$: A JEB equilibrium imposes two conditions on z . First, there

should be no coercive exchange possible: $z^i \succeq^i \hat{z}^i$ for all $\hat{z}^{ij} \in \mathcal{C}^{ij}(z)$. Hence, $z^{ij} \in \mathcal{J}^{ij}(z)$. Second, there should be no Pareto improving voluntary exchange possible: $z^{ij} \in \mathcal{PE}^{ij}(z)$, which is a curve due to strict convexity. By monotonicity of preferences and comprehensiveness of the contested set, there is a unique intersection point of the contract curve and the boundary. To see why, suppose to the contrary that there exist two intersection points, denoted z^{ij} and $\hat{z}^{ij}$. Because both lie on the contract curve, agent i can rank these allocations, say $z^i \succ^i \hat{z}^i$. By monotonicity and strict convexity, $z^i > \hat{z}^i$, contradicting that $\hat{z}$ lies on the boundary of the comprehensive set $\mathcal{C}^{ij}(z)$. This intersection point coincides with the unique JEB equilibrium allocation in the contested set. Formally, the intersection between $\mathcal{PE}^{ij}(z)$ and the boundary between $\mathcal{C}^{ij}(z)$ and $\mathcal{U}^{ij}(z)$ is the singleton $\{S^{ij}(z)\}$. Since $\mathcal{PU}^{ij}(z)$ is open at this boundary only, the closure $\overline{\mathcal{PU}}^{ij}(z)$ is equal to the union of this singleton and $\mathcal{PU}^{ij}(z)$. Since the JEB equilibrium is also contested, we must have that $z^{ij} \in \overline{\mathcal{PU}}^{ij}(z) \cap \mathcal{C}^{ij}(z)$. Hence, $z^{ij} = S^{ij}(z)$. Combining both conditions implies $z^{ij} = S^{ij}(z) \in \mathcal{J}^{ij}(z)$.

2. $z^{ij} \in \mathcal{U}^{ij}(z)$: The absence of further Pareto improving voluntary exchange implies $z^{ij} \in \mathcal{PU}^{ij}(z)$. ■

Proof of Proposition 2

First, consider $z^{ij} \in \mathcal{C}^{ij}(z)$.

Equilibrium threat $T^{ij}(z) \in \mathcal{C}^{ij}(z)$ must be agent i 's best consumption bundle $B^i(\hat{T}^{ij})$ among all possible threats $\hat{T}^{ij} \in \mathcal{C}^{ij}(z)$. Because $J^{ij}(z) \in \mathcal{C}^{ij}(z)$ for any $J^{ij}(z) \in \mathcal{J}^{ij}(z)$, agent i 's equilibrium consumption bundle $B^i(T^{ij}(z))$ is as least as good as any consumption bundle $B^i(J^{ij}(z))$. By Individual Rationality, agent i weakly prefers the latter consumption bundles to her consumption bundles in the set of jungle allocations $\mathcal{J}^{ij}(z)$. Hence,

$$B^i(T^{ij}(z)) \succeq^i B^i(J^{ij}(z)) \succeq^i J^i(z) \text{ for all } J^{ij}(z) \in \mathcal{J}^{ij}(z).$$

By definition of $G^{ij}(z) \in \overline{\mathcal{PU}}(z)$, $G^i(z) \sim^i J^i(z)$ for all $J^{ij} \in \mathcal{J}^{ij}$ and, combined with $B^i(T^{ij}(z)) \succeq^i J^i(z)$, it holds that $B^i(T^{ij}(z)) \succeq^i G^i(z)$. By Pareto efficiency, $G^j(z) \succeq^j B^j(T^{ij}(z))$ follows. The lens of individual rational allocations starting in $J^{ij}(z)$ is a subset of $\overline{\mathcal{U}}^{ij}(z)$. If not, this lens would intersect with the contested set such that some elements of the former lie in the interior of the latter, implying there would exist a bilateral allocation in the contested set that is better for agent i than the Jungle allocation, a contradiction. Combining the definitions of $G^{ij}(z)$ and $S^{ij}(z)$, we obtain $G^i(z) \sim^i J^i(z) \succeq^i S^i(z)$ for all $J^{ij} \in \mathcal{J}^{ij}$. The non-degenerate case is that all $J^{ij}(z)$ are Pareto inefficient and do not coincide with $O^j(z)$. This implies $G^{ij}(z) \neq S^{ij}(z)$. Since both $G^{ij}(z)$ and $S^{ij}(z)$ are Pareto efficient, $G^i(z) \sim^i J^i(z) \succ^i S^i(z)$ for all $J^{ij}(z) \in \mathcal{J}^{ij}(z)$ follows.

For agent j , by Individual Rationality and the definition of this agent's worst allocation in the contested set, we obtain

$$B^j(T^{ij}(z)) \succeq^j T^j(z) \succeq^j W^j(z) \text{ for all } W^{ij}(z) \in \mathcal{W}^{ij}(z).$$

By definition of $H^{ij}(z) \in \overline{\mathcal{PU}}(z)$, $H^j(z) \sim^j W^j(z)$ for all $W^{ij} \in \mathcal{W}^{ij}$ and, combined with $B^j(T^{ij}(z)) \succeq^j W^j(z)$, it holds that $B^j(T^{ij}(z)) \succeq^j H^j(z)$. By Pareto efficiency, $H^i(z) \succeq^i B^i(T^{ij}(z))$ follows.

Second, consider $z^{ij} \in \mathcal{U}^{ij}(z)$.

Combining Pareto efficiency of $B^{ij}(T^{ij}(z))$ and the definition of $S^{ij} \in \overline{\mathcal{PU}}^{ij}(z)$ implies that $B^i(T^{ij}(z)) \succeq^i S^i(z)$. The two other preference inequalities follow directly from Individual Rationality with respect to z^{ij} . ■

Proof of Proposition 3

Suppose $T^{ij}(z) \notin \mathcal{FC}^{ij}(z)$. Then, there exists a bilateral allocation $\hat{T}^{ij} \in \mathcal{C}^{ij}(z)$ such that $\hat{T}^i \geq T^i(z)$ and, by definition of the bilateral Edgeworth box, $\hat{T}^j \leq T^j(z)$. Because $\succeq^i$ and $\succeq^j$ are monotone, $\hat{T}^i \succ^i T^i(z)$ and $T^j(z) \succ^j \hat{T}^j$. By the additional assumption imposed on the axiomatic bargaining solution, $B^i(\hat{T}^{ij}) \succ^i B^i(T^{ij}(z))$ follows. But the latter contradicts that $T^{ij}(z)$ is agent i 's best threat in $\mathcal{C}^{ij}(z)$. Hence, $T^{ij}(z) \in \mathcal{FC}^{ij}(z)$.

Recall our assumption that (i) for threat allocations $\bar{T}^{ij}, \hat{T}^{ij} \in \mathcal{C}^{ij}(z)$ such that $\bar{T}^i \succeq^i \hat{T}^i$ and $\hat{T}^j \succeq^j \bar{T}^j$, with at least one strict preference, we require that $B^i(\bar{T}^{ij}) \succ^i B^i(\hat{T}^{ij})$. For $T^{ij}(z) \in \mathcal{FC}^{ij}(z)$, suppose that (ii) $T^j(z) \succ^j J^j(z)$ for a $J^{ij}(z) \in \mathcal{J}^{ij}(z)$. Also, by definition of the jungle allocation, (iii) $J^i(z) \succeq^i T^i(z)$ holds. Combining (i), (ii) and (iii), we obtain $B^i(J^{ij}(z)) \succ^i B^i(T^{ij}(z))$, contradicting that agent i 's equilibrium threat is $T^{ij}(z)$. Hence, $J^j(z) \succeq^j T^j(z)$ for all $J^{ij}(z) \in \mathcal{J}^{ij}(z)$.

For $T^{ij}(z) \in \mathcal{FC}^{ij}(z)$ suppose that (iv) $W^i(z) \succ^i T^i(z)$ for a $W^{ij}(z) \in \mathcal{W}^{ij}(z)$. Also, by definition of $W^{ij}(z)$, (v) $T^j(z) \succeq^j W^j(z)$. Combining (i), (iv) and (v), we obtain $B^i(W^{ij}(z)) \succ^i B^i(T^{ij}(z))$, contradicting that agent i 's equilibrium threat is $T^{ij}(z)$. Hence, $T^i(z) \succeq^i W^i(z)$ for all $W^{ij}(z) \in \mathcal{W}^{ij}(z)$.

If agent i has dictator power, then $B^j(\hat{T}^{ij}) \sim^j \hat{T}^j$ for all $\hat{T}^{ij} \in \mathcal{C}^{ij}(z)$, that is, the bargaining solution $B^{ij}(\hat{T}^{ij})$ is the intersection of agent j 's indifference curve through $\hat{T}^{ij}$ and the contract curve $\overline{\mathcal{PU}}^{ij}(z)$. By choosing $\hat{T}^{ij} \in \mathcal{W}^{ij}(z)$, agent i can obtain allocation $H^{ij}(z)$ of Proposition 2. Similarly, if agent j has dictator power, then $B^i(\hat{T}^{ij}) \sim^i \hat{T}^i$ for all $\hat{T}^{ij} \in \mathcal{C}^{ij}(z)$ and, consequently, $\hat{T}^{ij} \in \mathcal{J}^{ij}(z)$. ■

B Equilibrium Existence in Nash's Negotiation Game

In Section 5, we took the stance that non-existence of equilibrium in the Nash negotiation model is not a problem (see Footnote 8). We assumed it away by taking the closure of the set of uncontested allocations as the image of the bargaining solution, ensuring compactness and hence existence. In this Appendix, we drop this assumption and assume that the bargaining solution is now $B^{ij} : \mathcal{E}^{ij}(z) \rightarrow \mathcal{U}^{ij}(z)$ rather than $B^{ij} : \mathcal{E}^{ij}(z) \rightarrow \overline{\mathcal{U}}^{ij}(z)$. We identify sufficient conditions for equilibrium existence and derive a necessary condition when equilibrium existence cannot be guaranteed. We argue that the latter does not pose a problem.

Bargaining Stage

As in the main text, we distinguish two cases. We assume that equilibrium threat $T^{ij}(z) \in \mathcal{FC}^{ij}(z)$ exists and is fixed.

The first case is $z^{ij} \in \mathcal{C}^{ij}(z)$. Recall that the characterization of Proposition 2 states that $B^i(\hat{T}^{ij}(z)) \succeq^i J^i(z)$ for all $J^{ij}(z) \in \mathcal{J}^{ij}(z)$. We denote by $\mathcal{X}^{ij}(z)$ the subset of all allocations in the classic Edgeworth box that (i) the stronger agent weakly prefers to any of the jungle allocation in $\mathcal{J}^{ij}(z)$ and (ii) the weaker agent weakly prefers to any of her worst contested allocations in $\mathcal{W}^{ij}(z)$. Formally,

$$\mathcal{X}^{ij}(z) = \left\{ \hat{z}^{ij} \in \mathcal{E}^{ij}(z) \left| \begin{array}{l} \hat{z}^i \succeq^i W^i(z), \quad \forall W^{ij}(z) \in \mathcal{W}^{ij}(z) \quad \text{and} \\ J^j(z) \succeq^j \hat{z}^j, \quad \forall J^{ij}(z) \in \mathcal{J}^{ij}(z) \end{array} \right. \right\}.$$

By the agents' convex preferences, the set $\mathcal{X}^{ij}(z)$ is a compact, convex and nonempty subset of $\mathcal{E}^{ij}(z)$. From the proof of Proposition 2 in Appendix A, we know that $\mathcal{X}^{ij}(z) \setminus \mathcal{J}^{ij}(z) \subseteq \mathcal{U}^{ij}(z)$. Either degenerate $S^{ij}(z) \in \mathcal{J}^{ij}(z)$, which is a JEB equilibrium by Proposition 1, implies existence. Or $S^{ij}(z) \notin \mathcal{J}^{ij}(z) \subseteq \mathcal{FC}^{ij}(z)$ implies $J^{ij}(z) \in \mathcal{J}^{ij}(z)$ is not Pareto efficient. Then, by the axiom of Pareto Efficiency, $B^{ij}(T^{ij}(z)) \notin \mathcal{J}^{ij}(z)$. Consequently, $B^{ij}(T^{ij}(z)) \in \mathcal{U}^{ij}(z)$. Hence, it is without loss of generality to replace the image $\mathcal{U}^{ij}(z)$ of axiomatic bargaining solution B^{ij} by $\mathcal{X}^{ij}(z)$, which is compact and nonempty. Hence, $B^{ij}(T^{ij}(z))$ exists.

The second case is $z^{ij} \in \mathcal{U}^{ij}(z)$. Because $T^{ij}(z) = z^{ij}$, we write $B^i(T^{ij}(z))$ as $B^i(z^{ij})$. By Proposition 2, both $B^i(z^{ij}) \succeq^i S^i(z)$ and $B^i(z^{ij}) \succeq^i z^i$ have to hold. We denote by $\mathcal{L}^{ij}(z)$ the subset of all allocations in the classic Edgeworth box that (i) the stronger agent weakly prefers to both z^{ij} and S^{ij} and (ii) the weaker agent weakly prefers to z^{ij} . Formally,

$$\mathcal{L}^{ij}(z) = \{ \hat{z}^{ij} \in \mathcal{E}^{ij}(z) \mid \hat{z}^i \succeq^i z^i, \hat{z}^i \succeq^i S^i(z), \hat{z}^j \succeq^j z^j \}.$$

By the agents' convex preferences, the set $\mathcal{L}^{ij}(z)$ is a compact, convex and nonempty subset of $\mathcal{E}^{ij}(z)$.

When $z^i \succ^i S^i(z)$, the intersection of $\mathcal{PE}^{ij}(z)$ and $\mathcal{L}^{ij}(z)$ belongs to $\mathcal{PU}^{ij}(z)$. Then, by the axiom of Individual Rationality, it is without loss of generality to replace the image

$\mathcal{U}^{ij}(z)$ of B^{ij} by $\mathcal{L}^{ij}(z)$. Hence, $B^{ij}(z^{ij}) \in \mathcal{PU}^{ij}(z)$ exists. When $S^i(z) \succeq^i z^i$, the set $\mathcal{L}^{ij}(z)$ contains the contested allocation $S^{ij}(z)$ and thus always intersects the coercion set $\mathcal{C}^{ij}(z)$. Moreover, since $S^{ij}(z)$ is also Pareto efficient, the intersection of $\mathcal{L}^{ij}(z)$ and $\mathcal{PE}^{ij}(z)$ also contains $S^{ij}(z)$. Then, existence cannot be guaranteed. To see why, consider a weaker agent with dictator power. This weaker agent is able to negotiate $S^{ij}(z)$, an allocation outside the uncontested set. This is in contradiction with our assumption that the axiomatic bargaining solution maps into the uncontested set. More generally, the necessary condition for non-existence is the following: $S^{ij}(z)$ is the axiomatic bargaining solution when negotiations are restricted to the set $\mathcal{L}^{ij}(z)$.

We do not consider the nonexistence of an axiomatic bargaining solution in the bargaining stage a serious issue. This is because the standard assumption of perfectly divisible goods in the Edgeworth box, although a convenient technical assumption, is not a realistic representation of the real world. In reality, goods are discrete, with atoms and molecules or units like milliliter and milligram being the smallest possible measures. Therefore, when weaker agents negotiate an allocation that belongs to the boundary of the contested set, we should interpret the weaker agent settling for an allocation inside the uncontested set that is arbitrarily close to this boundary allocation as if the agents belong to a discrete world with sufficiently tiny ‘smallest units’ per good. In such a discrete world, existence is guaranteed. It is the standard modeling assumption of perfectly divisible goods that becomes problematic and creates an artificial nonexistence problem.

Threat Stage

Recall that the condition for stronger agent’s equilibrium threat $T^{ij}(z) \in \mathcal{C}^{ij}(z)$ states that $B^i(T^{ij}(z)) \succeq^i B^i(\bar{T}^{ij})$ for all $\bar{T}^{ij} \in \mathcal{C}^{ij}(z)$. As above, it is without loss of generality to assume that B^{ij} maps into the compact, convex and nonempty set $\mathcal{X}^{ij}(z)$. To obtain existence, it is sufficient to impose that the bargaining solution B^{ij} is continuous in threat allocation $\bar{T}^{ij}$ belonging to the coercion set.¹⁶ Under this mild condition existence of the stronger agent’s

¹⁶Continuity is ensured for all well-known axiomatic bargaining solutions mentioned in Footnote [9](#)

best threat allocation is guaranteed.

C A Stochastic Extension of the Jungle Edgeworth Box Economy

This appendix extends the deterministic jungle economy developed in the main text to a stochastic setting in which the outcomes of coercive confrontations are no longer predetermined by a fixed hierarchy of strength, but instead governed by probabilistic success in contests. This probabilistic extension builds in particular on the class of Tullock-style contest success functions (CSFs) and connects to recent work by Schwarz (2019), where a similar model is used to study the transition from jungle to market.

From Deterministic to Stochastic JEBs

Recall that in the deterministic model, coercion between agents is dictated by a fixed exogenous strength ranking: whenever agent $i < j$, agent i is strictly stronger than agent j and can unilaterally dictate outcomes within the set $\mathcal{C}^{ij}(z)$. In the stochastic setting, we generalize this by allowing agents to exert effort in pairwise contests, with the probability of winning depending on the relative effort levels. As in the jungle and deterministic JEB, power is one-sided with coercive threats from the strong to the weak. The stronger agent initiates the contest and unilaterally determines the intensity of the confrontation. The weaker agent does not respond strategically or symmetrically in contests initiated by a stronger agent. In this sense, our stochastic extension preserves the asymmetric structure of coercion while introducing uncertainty about outcomes through effort-based probabilistic conflict resolution.

A CSF assigns to each effort profile the probability that agent j concedes to agent i . In such a case, agent i successfully takes from j . Otherwise, agent j does not concede and maintains the contested allocation, but this agent lacks the strength to take from agent i . If the CSF is everywhere equal to 1 for $i < j$, then we obtain the deterministic jungle as

analyzed in the main text.

Formally, let $N = 1, \dots, n$ denote a finite set of agents and $\tilde{\omega} \in \mathbb{R}_+^m$ the aggregate endowment of available goods. The economy is extended with effort that can be exerted in a possible contest. Formally, $e_i \in \mathbb{R}_+$ denotes agent i 's (intangible) individual level of effort and the tuple $e = (e_1, \dots, e_n) \in \mathbb{R}_+^n$ captures all individual effort levels. A feasible allocation is a tuple $(z, e) \equiv ((z^1, e_1), \dots, (z^n, e_n))$ such that $z^i \in \mathbb{R}_+^m$, $\sum_{i \in N} z^i = \tilde{\omega}$ and $e_i \in [0, 1]$. The set of all feasible allocations is defined as $\hat{\mathcal{Z}}(\omega) = \mathcal{Z}(\omega) \times [0, 1]^n$. Given the pair ij , their consumption bundles and their effort levels in feasible allocation (z, e) , a bilateral allocation is $(\hat{z}^{ij}, \hat{e}^{ij}) \equiv ((\hat{z}^i, \hat{e}_i), (\hat{z}^j, \hat{e}_j))$ and the bilateral Edgeworth box is

$$\hat{\mathcal{E}}^{ij}(z, e) \equiv \{(\hat{z}^{ij}, \hat{e}^{ij}) \mid \text{for all } i, j : \hat{z}^i, \hat{z}^j \in \mathbb{R}_+^m \text{ such that } \hat{z}^i + \hat{z}^j = z^i + z^j; \hat{e}_i, \hat{e}_j \in [0, 1]\},$$

which is a non-empty and compact set.

Let $p^{ij}(e_i, e_j)$ denote the CSF giving the probability that agent j concedes to agent i when their effort levels are e_i and e_j , respectively.¹⁷ In our understanding, the CSF depends upon the distance between $i < j$ and j , because even for the strongest agent $i = 1$ it is likely to be more difficult to achieve success against agent $j = 2$ than against, say, the weakest agent $j = n$.¹⁸ We assume that $p^{ij}(e_i, e_j)$ is non-decreasing in e_i and non-increasing in e_j .

Agent i may bilaterally contest the current allocation in the pair ij . This means that this agent initiates the contest and unilaterally determines the intensity of the confrontation, where $\hat{e}_i > 0$ and $\hat{e}_j = 0$. In case agent j concedes, the allocation becomes $(\hat{z}^{ij}, \hat{e}^{ij})$. Should coercion be unsuccessful, the contested allocation of available goods remains as before. It is important to stress that we assume that the weaker agent lacks the strength to take from the stronger agent.¹⁹

Each agent i has a preference relation $\succeq^i$ over (z^i, e_i) that satisfies completeness, tran-

¹⁷The special case $p^{ij}(e_i, e_j) = 1$ for $i < j$ and 0 otherwise reproduces the deterministic JEB exactly.

¹⁸For Tullock specifications of the CSF, the distance between i and j may be reflected by coefficients $\alpha_{ij} \in [0, 1]$, $i < j$, such that $\alpha_{1j} \geq \dots \geq \alpha_{i-1j} \geq \alpha_{ij} \geq \alpha_{ij+1} \geq \dots \geq \alpha_{in}$ and $p^{ij}(e_i, e_j) = \frac{e_i}{e_i + \alpha_{ij} e_j}$. Obviously, $\alpha_{ij} = 0$ is needed to obtain $p^{ij}(e_i, e_j) = 1$ in the deterministic JEB.

¹⁹Alternatively, Schwarz (2019) assumes that, in each contest between i and j , either i imposes her most preferred reallocation on j or vice versa.

sitivity, continuity, strict convexity and monotonicity in the sense that more is better with respect to the consumption package z^i and more is worse with respect to effort e_i . In evaluating uncertain outcomes, these preferences additionally satisfy the Independence Axiom. Hence, we assume the existence of a utility function $u_i(z^i, e_i)$ and the applicability of expected utility theory.

Coercive Accessibility and Voluntary Exchange

Given a bilateral allocation $(z^{ij}, e^{ij}) = ((z^i, e_i), (z^j, e_j))$ between agents $i < j$, the set of effort-contested (EC) allocations that are coercively accessible (CA) for agent i is:

$$\mathcal{EC}^{ij}(z, e) := \{(\hat{z}^{ij}, \hat{e}^{ij}) \mid \hat{e}^{ij} = (\hat{e}_i, 0), \mathbb{E}\{u_i(\hat{z}^i, \hat{e}_i)\} \geq u_i(z^i, 0)\},$$

where $\mathbb{E}\{u_i(\hat{z}^i, \hat{e}_i)\} = p^{ij}(\hat{e}_i, 0)u_i(\hat{z}^i, \hat{e}_i) + (1 - p^{ij}(\hat{e}_i, 0))u_i(z^i, \hat{e}_i)$.²⁰ Note that the defender j is assumed to exert zero effort. This set provides a foundation for the coercion set in the deterministic case. The frontier of coercive accessible allocations in which agent i is exactly indifferent between remaining at z^i and contesting is denoted:

$$\hat{\mathcal{F}}^{ij}(z, e) := \{(\hat{z}^{ij}, \hat{e}^{ij}) \mid \hat{e}^{ij} = (\hat{e}_i, 0), \mathbb{E}\{u_i(\hat{z}^i, \hat{e}_i)\} = u_i(z^i, 0)\}.$$

This frontier plays the same boundary role as the frontier of the coercion set in the deterministic case.

Define $\mathcal{EU}^{ij}(z, e) \equiv \hat{\mathcal{E}}^{ij}(z, e) \setminus \mathcal{EC}^{ij}(z, e)$ as the set of allocations that cannot be effort-contested. The set of bilateral voluntary and uncontested Pareto improvements is then defined as:

$$\hat{\mathcal{P}}^{ij}(z, e) := \left\{ (\hat{z}^{ij}, \hat{e}^{ij}) \in \mathcal{EU}^{ij}(z, e) \mid \begin{array}{l} (\hat{z}^i, \hat{e}_i) \succeq^i (z^i, e_i), \quad (\hat{z}^j, \hat{e}_j) \succeq^j (z^j, e_j), \\ \text{with at least one strict preference} \end{array} \right\}.$$

Note that Pareto efficiency excludes positive effort levels, i.e. $\hat{e}_i = \hat{e}_j = 0$.²¹

Equilibrium Concept and Characterization

We now introduce the stochastic analog of the JEB equilibrium.

²⁰This set is non-empty because it contains $(\hat{z}^{ij}, \hat{e}^{ij}) = (z^{ij}, 0)$. It is also compact by the continuity of the expected utility function on a compact domain in combination with the weak inequality.

²¹To see this, for every (z^{ij}, e^{ij}) such that $e^{ij} \neq 0$, $(z^{ij}, 0)$ is a Pareto improvement.

Definition 2 (SJEB Equilibrium) An allocation (z, e) is a Stochastic Jungle Edgeworth Box (SJEB) equilibrium if for all $i < j$:

- (i) There is no effort-contested allocation $(\hat{z}^{ij}, \hat{e}^{ij}) \in \mathcal{EC}^{ij}(z, e)$ such that $(\hat{z}^i, \hat{e}_i) \succ^i (z^i, 0)$;
- (ii) There is no voluntary Pareto improvement $(\hat{z}^{ij}, \hat{e}^{ij}) \in \hat{\mathcal{P}}^{ij}(z, e)$.

We now characterize the set of SJEB equilibria.

Proposition 4 (Characterization of SJEB Equilibrium) An allocation $(z, e) \in \hat{\mathcal{Z}}(\omega)$ is a SJEB equilibrium if and only if for all $i < j$:

- (a) If $\mathcal{EC}^{ij}(z, e) \neq \emptyset$ and $(z^{ij}, e^{ij}) \in \mathcal{EC}^{ij}(z, e)$, then $(z^{ij}, 0) \in \hat{\mathcal{F}}^{ij}(z, e)$.
- (b) If $\mathcal{EU}^{ij}(z, e) \neq \emptyset$ and $(z^{ij}, e^{ij}) \in \mathcal{EU}^{ij}(z, e)$, then $(z^{ij}, 0) \in \hat{\mathcal{P}}^{ij}(z, 0)$.

Proof of Proposition 4

We only prove the necessity part as the sufficiency part is straightforward. Let (z, e) be a SJEB equilibrium, and let $z^{ij} = (z^i, z^j)$ be the bilateral allocation between agents $i < j$. We distinguish two mutually exclusive cases:

1. Case 1: $(z^{ij}, e^{ij}) \in \mathcal{EC}^{ij}(z, e)$

By definition of $\mathcal{EC}^{ij}(z, e)$, this means that there exists some allocation $(\hat{z}^{ij}, \hat{e}^{ij}) \in \mathcal{EC}^{ij}(z, e)$ and effort levels $\hat{e}^{ij} = (\hat{e}_i, 0)$ such that agent i would strictly prefer the expected utility from contesting to her current utility, i.e. $\mathbb{E}\{u_i(\hat{z}^i, \hat{e}_i)\} > u_i(z^i, 0)$. However, since (z, e) is a SJEB equilibrium, agent i does not in fact initiate coercion. This is only possible if (z^{ij}, e^{ij}) lies on the boundary of $\mathcal{EC}^{ij}(z, e)$, i.e. in $\hat{\mathcal{F}}^{ij}(z, e)$. Consequently, $\mathbb{E}\{u_i(\hat{z}^i, \hat{e}_i)\} \leq u_i(z^i, 0)$ for all $(\hat{z}^{ij}, \hat{e}^{ij}) \in \mathcal{EC}^{ij}(z, e)$. Because $(\hat{z}^{ij}, \hat{e}^{ij}) = (z^{ij}, 0)$ is feasible and induces $\mathbb{E}\{u_i(\hat{z}^i, \hat{e}_i)\} = u_i(z^i, 0)$, equality has to hold. That is, agent i is *indifferent* between coercing and not coercing $(z^{ij}, 0)$. This is precisely the definition of $(z^{ij}, 0) \in \hat{\mathcal{F}}^{ij}(z, e)$.

2. Case 2: $(z^{ij}, e^{ij}) \in \mathcal{EU}^{ij}(z, e)$

In this case, agent i has no incentive to coerce, meaning that for all $(\hat{z}^{ij}, \hat{e}^{ij}) \in \hat{\mathcal{E}}^{ij}(z, e)$ it holds that $\mathbb{E}\{u_i(\hat{z}^i, \hat{e}_i)\} \leq u_i(z^i, 0)$. In particular, for all $(\hat{z}^{ij}, \hat{e}^{ij}) \in \mathcal{EU}^{ij}(z, e)$. The only remaining possibility of improvement is through voluntary bilateral exchange in $\mathcal{EU}^{ij}(z, e)$, which requires Pareto improvement in $\mathcal{EU}^{ij}(z, e)$. However, since $(z^{ij}, e^{ij}) \in \mathcal{EU}^{ij}(z, e)$ is an SJEB equilibrium, Pareto improvement is impossible. Hence, (z^{ij}, e^{ij}) must be Pareto efficient in $\mathcal{EU}^{ij}(z, e)$, and thus $(z^{ij}, e^{ij}) \in \hat{\mathcal{P}}^{ij}(z, e)$. As mentioned, Pareto efficiency requires $e^{ij} = 0$. ■

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