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On the Limits of Hedging Inflation Risk in Investment Portfolios

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Abstract

We explore to what extent real returns on investment portfolios can be hedged against inflation risk by using existing financial market instruments. We find that inflation-linked bonds offer only limited protection against inflation risk and that nominal debt and stocks play at least comparable roles in this respect. These findings apply to both a static and a dynamic setting. The demonstrated limits of hedging inflation risk are of particular relevance for long-term investors, such as pension funds with participants concerned about the real value of their pension benefits.

Key words: unhedgeable inflation risk, incomplete markets, welfare loss, mean-variance frontiers, minimum risk portfolio, nominal and index-linked bonds.
JEL Codes: C61, E21, G11, G23.

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1 Introduction

In the aftermath of the Covid-19 crisis, inflation has once again become a major economic concern. With higher average inflation, the uncertainty about future inflation also increases, which in turn raises macroeconomic risks and makes returns on investments more risky. Long-term investors generally aim to protect the real value of their assets. This is especially important for pension funds that try to provide a stream of benefits with stable purchasing power because most pensioners have a much-reduced ability to absorb risk. Chen et al. (2020) have shown that the lack of adequate inflation hedging instruments can lead to welfare losses for retirees ranging from 1% to 8% in terms of certainty equivalent consumption, thus underscoring the importance of limiting inflation risk. However, effectively hedging inflation risk over a long horizon is hampered by market frictions, rollover risks and other complicating factors. Therefore, a key question is to what extent it is possible *in practice* to construct investment portfolios that generate stable real returns and what would be the cost associated with such a strategy.

This paper explores to what extent real returns on investment portfolios composed of (indices of) nominal public debt, index-linked public debt and stocks can be hedged against inflation. The purpose of our analysis is expressly *not* to investigate optimal allocations of a consumption-based capital asset pricing model (CAPM) for real portfolio returns. Instead, it is to explore more narrowly to what extent it is possible *in practice* to hedge inflation risk with instruments that are widely traded on exchanges and therefore are easily available at low trading cost. To our knowledge, there is little to no empirical work that addresses this question. We address the issue by constructing minimum-variance portfolios that include and exclude index-linked bonds (ILBs).

Index-linked bonds (ILBs) are used on a limited scale only to hedge inflation risk, possibly because of their limited availability. Many countries, such as, for example, the Netherlands, do not even issue ILBs linked to their own price index.[1] Although Dutch pension funds can resort to foreign ILBs according to the pension supervisor, the Dutch central bank, these constituted only 1% of their total asset holdings in 2024.[2] Other assets, such as equities and commodities, have historically demonstrated some usefulness in hedging against inflation risk, but those hedges are far from perfect. In addition, financial innovation, derivatives and alternative investment strategies have emerged that do help protect portfolio returns against inflation risk. However, these possibilities come with costs, frictions and roll-over risks that reduce the scope of hedging inflation risks.

We demonstrate for four countries (the Netherlands, Germany, the United

[1] Common arguments against issuing ILBs are high issuance costs because of limited market liquidity, which may well be a circular argument, and the fear that their issuance would signal a reduced commitment to combating inflation. There is little evidence for the latter argument.

[2] https://www.dnb.nl/statistieken/data-zoeken/ belegd vermogen voor risico pensioenfondsen.

Kingdom and the United States) that it is *not* possible to construct a portfolio with ILBs that is *even remotely* able to hedge away inflation risk. We demonstrate the limited effectiveness in hedging inflation risk for both the short- and the long-term. Our findings underscore the critical importance of deeper investigation of inflation hedging possibilities. This is in particular important for long-term investors, such as pension funds, who aim at protecting the purchasing power of their participants' benefits over their lifetime. Our findings should provide investment managers with leads to improve risk versus expected return trade-offs in real terms. From a policy perspective, our findings suggest the need for a better supply of hedging instruments.

Our analysis relates to the literature that analyzes the methodologies and effectiveness of inflation risk hedging. Jeanblanc et al. (2012) present advanced techniques for mean-variance hedging, but do not provide empirical evidence. Kupfer (2019) presents methodologies for estimating the inflation risk premia. Assessments of the effectiveness of hedging inflation risks vary in the literature, of which Arnold and Auer (2015) and Jarrow and Yildirim (2023) provide an overview. Specifically, they explore the relationship between inflation and asset classes like stocks, gold, fixed income and real estate. Our findings are consistent with those of Campbell et al. (2009), who also highlight a strong relationship between nominal bond returns and inflation, while they contrast with those of Spierdijk and Umar (2015), who indicate that nominal bonds have only limited effectiveness in inflation hedging. He et al. (2024) find that 5-year nominal and inflation-protected US public debt securities generate similar returns, while 10-year nominal debt outperforms 10-year inflation-protected debt. Bekaert and Wang (2010) show that the inflation risk is difficult to hedge and that the inflation risk premium is large and volatile. Moreover, Fleckenstein et al. (2014) indicates that inflation-linked bonds are mispriced in the market and that arbitrage opportunities persist. Mispricing limits capital flows into the ILBs, which causes mispricing to persist. This can be an additional source of hedging mismatch. Finally, Neville et al. (2021) explore different passive and active investment strategies for inflationary times. Inspired by Brière and Signori (2012), who show that economic regimes matter for the role of inflation-linked bonds, we also specifically explore inflation risk hedging during the period after the COVID-19 outbreak. Indeed, during this high-inflation period the importance of ILBs in the minimum-variance portfolio increases.

The remainder of this paper is structured as follows. Section 2 describes the methodology and data for our analysis. Section 3 analyses the mean-variance frontiers of portfolio returns, while Section 4 presents the results of an optimal investment strategy for a variable annuity. Section 5 concludes, emphasizing some general lessons from our analysis. The Appendix provides technical details on the analytical solutions and some additional tables and figures with results.

2 Data and methodology

We explore to what extent real returns on investment portfolios can be hedged against inflation. In practice, the mismatch between the inflation rate that needs to be hedged and the protection provided by the instruments (that is, the basis risk) may be driven by several factors. First, not all durations of the relevant instruments are available. Second, the measure of inflation to be hedged may differ from what is available in terms of inflation measures underlying the hedging instrument. This is the case when the domestic government does not issue ILBs. Third, there are rebalancing and transaction costs. Fourth, instruments used for hedging may have credit risks.

2.1 Instruments for inflation hedging

We confine ourselves to nominal bonds, ILBs and stocks as the market instruments to be used for inflation hedging. We could also consider inflation swaps, but this would not add much since their underlying assets are bonds, which we already include. We use the three criteria for the selection of specific instruments suggested by the Solvency II review of EIOPA[3]: market depth, measured by the average daily notional amount traded; market liquidity, measured by the average daily number of trades; and transparency.

The existing mismatch between the inflation that needs to be hedged and the protection provided by the instruments considered (that is, the basis risk) is driven by several factors. First, not all necessary durations of the relevant instruments are available. Second, the measure of inflation to be hedged may actually differ from the measure of inflation underlying the hedging instrument; for example, in the Netherlands, domestic inflation can only be hedged using bonds linked to foreign inflation. Third, there are rebalancing and transaction costs. Fourth, hedging instruments may be exposed to credit risks, like all financial instruments.

2.2 Data

We use ILB price indices from S&P Global, because they are designed to track the performance of local-currency-denominated ILBs. These indices include broad and comprehensive developed market indices and indices specific to individual countries.[4] Each index is based on several instruments.[5]

[3] https://www.eiopa.europa.eu/system/files/2020-12/eiopa-bos-20-750-background-analysis.pdf

[4] https://www.spglobal.com/spdji/en/index-family/fixed-income/inflation-linked/#overview

[5] For example, the S&P Germany Sovereign Bond Index is composed of four instruments with an average maturity of 8.3 years (see https://www.spglobal.com/spdji/en/idsenhancedfactsheet/file.pdf?calcFrequency=M&force_download=true&hostIdentifier=48190c8c-42c4-46af-8d1a-0cd5db894797&indexId=91922574), while the

Table 1: Number of observations and the number of financial instruments

#instruments	**#observations**	$\sqrt{\#\text{instr}} * \sqrt{\#\text{obs}}$
23	1587	191.05
22	2294	224.65
21	2489	228.62
20	2508	223.96
19	4507	292.63
18	4768	292.96
17	**5082**	**293.93**
16	5227	289.19

Note: Trade-off between the number of observations and the number of financial instruments

Table 2: Average and standard deviation of CPI inflation rates in four countries: Germany, the UK, the Netherlands and the US.

inflation rates (in bps)	GE	UK	NL	US
monthly average	17.97	29.13	19.02	20.97
monthly standard deviation	36.96	40.80	45.72	29.58
quarterly average	54.12	89.62	57.42	63.80
quarterly standard deviation	82.02	94.93	80.57	75.50

The complete set of ILB-indices from S&P Global consists of a set of 10 nominal bond indices and 13 ILB indices. All indices are total return indices that reinvest coupon and dividend payments. These 23 indices have 1587 days in common. But Table 1 shows that there is a trade-off between the number of available instruments and the number of trading days in common. To strike a balance between the number of common data days and the number of instruments, we select the set of indices for which the product of the numbers of instruments and observations is maximized. Table 1 shows that this is obtained with 17 bond indices: they have have 5082 trading days in common, from 20 September 2004 to 18 April 2024. Table 3 lists these indices.[6] Using changes in the price levels of the indices, we construct nominal returns.

Figure 1 depicts the monthly Dutch, German, UK and US consumer price

S&P U.K. Gilt Bond Index is based on 32 instruments with an average maturity of 16.7 years (see `https://www.spglobal.com/spdji/en/idsenhancedfactsheet/file.pdf?calcFrequency=M&force_download=true&hostIdentifier=48190c8c-42c4-46af-8d1a-0cd5db894797&indexId=91922572`).

[6]The excluded bond indices are the S&P Global Developed Sovereign Inflation Linked Bond USD Index, S&P U.S. TIPS Index, S&P South Africa Sovereign Inflation-Linked Bond Index, S&P CLX Chile Sovereign Inflation-Linked Bond Index, S&P/B3 Brazil Sovereign Inflation Linked Bond Index and S&P Cyprus Sovereign Bond Index.

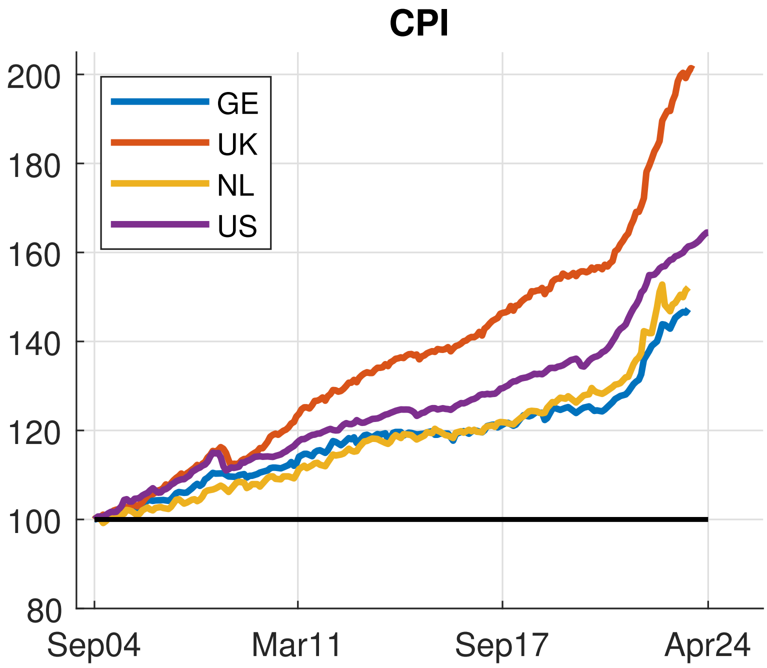


Figure 1: CPI of Germany, the UK, the Netherlands and the US.

indices (CPI). Except for the Netherlands, all of these countries issue ILBs. Table 2 reports the average and standard deviation of the corresponding inflation rates of the CPI per month and quarter. On an annual basis, the average inflation varies from 2.2% (= 12 * 17.97 bps in Germany) to 3.6% (= 4 * 89.62 bps in the U.K). Table 3 reports the correlations (in %) between the returns based on these indices and the CPI inflation rates of each of the corresponding countries. All monthly nominal bond index returns are negatively correlated with Dutch, German, and British inflation, but except for the US Aggregate Bond Index, they are all positively correlated with US inflation. The prevalence of negative correlations between nominal bond index returns and inflation may well have a macroeconomic explanation. Higher inflation is likely followed by monetary tightening, which causes a fall in the price of the outstanding nominal debt. At the quarterly frequency, almost all correlations between nominal bond price indices and inflation are also negative, with two exceptions (see Table 3).

The monthly nominal returns on the ILBs are all negatively correlated with UK and German inflation and all positively correlated with US inflation. The pattern for Dutch inflation is more mixed: 5 negative correlations and three positive correlation coefficients). For the quarterly returns, the pattern is mixed for all countries, except for the US where the correlations all remain positive. As expected, the correlations of the nominal returns on the ILBs with inflation are generally higher than those on the nominal bonds with inflation. The correlations between the monthly stock index returns and the Dutch and US inflation rates

Table 3: Correlations (in %) between nominal returns and CPI inflation rates

Index name	ticker	monthly correlations GE	UK	NL	US	quarterly correlations GE	UK	NL	US
S&P Eurozone Sov. Bond Index	SPBDEGIT	-31.1	-19.4	-17.4	13.6	-48.3	-31.2	-36.3	1.7
S&P France Sov. Bond Index	SPBDEFRT	-30.1	-19.6	-17.8	11.2	-44.0	-31.4	-36.0	-2.2
S&P Austria Sov. Bond Index	SPBDEATT	-31.4	-20.1	-18.6	10.1	-43.7	-31.0	-34.7	-3.5
S&P Belgium Sov. Bond Index	SPBDEBET	-30.7	-20.9	-17.1	10.3	-43.2	-33.4	-34.2	-2.7
S&P Finland Sov. Bond Index	SPBDEFIT	-29.6	-18.9	-18.6	13.1	-44.5	-30.9	-34.1	0.7
S&P Germany Sov. Bond Index	SPBDEDET	-27.4	-18.1	-17.0	12.6	-43.8	-29.2	-33.2	-0.5
S&P Netherlands Sov. Bond Index	SPBDENLT	-28.8	-18.9	-18.3	11.2	-44.4	-31.5	-35.8	-1.5
S&P U.K. Gilt Bond Index	SPFIGBT	-11.5	-15.5	-17.2	3.6	-20.9	-44.7	-25.5	-0.6
S&P U.S. Aggregate Bond Index	SPUSBMIT	-7.4	-19.6	-3.5	-14.8	-10.4	-22.7	5.5	-37.1
S&P Eurozone Sov. ILB Index	SPFID4IT	-14.4	-12.6	-3.8	28.2	-22.4	-14.2	-9.7	22.6
S&P U.K. Gilt ILB Index	SPFIGBIT	-10.3	-11.6	-17.6	13.4	-20.0	-24.2	-22.6	12.3
S&P 10 Year US TIPS Index	SPBDU1ST	-3.8	-19.3	4.6	7.7	-7.8	-16.5	13.3	4.2
S&P New Zealand ILB Index	SPBNILT	-8.4	-13.4	-6.6	11.6	-5.7	-12.8	-4.6	13.3
S&P Sweden Sov. ILB Index	SPFISEI	-6.1	-8.6	-2.3	29.6	-3.3	-3.6	-1.5	30.5
S&P Andean Sov. ILB Index	SPFIMLUT	-3.1	-17.7	-2.0	17.6	0.3	-1.1	4.8	20.0
S&P Pacific Allce. Sov. ILB Index	SPFIMPUT	-2.4	-14.3	2.5	26.9	4.3	12.7	18.9	32.9
S&P/BMV Mexico Sov. ILB Index	SPVIF0U	-1.8	-9.6	4.8	26.8	4.0	20.0	24.1	35.2
S&P 500	SPXT	-2.7	-7.8	4.5	22.0	2.3	6.9	3.1	34.0
MSCI World EUR	MSDEWIN	-2.5	-7.5	2.5	29.9	4.2	8.0	0.2	37.8
Dow Jones	DJITR	-2.9	-7.7	0.8	18.7	7.7	9.6	-2.3	31.1

Note: "GE" is Germany and "NL" is the Netherlands. The top panel contains nominal bond indices, the middle panel ILB indices and the lower panel stock indices.

are all positive, but all negative for the UK and German inflation rates. The correlations between the quarterly stock market returns and inflation rates are all positive, with one exception: the Dow Jones index-based return is negatively correlated with Dutch inflation. But overall the Table suggests that, at least judging by the quarterly data, stocks can serve as a (partial) hedge against inflation.

Next, we group all the instruments into three broad asset classes: nominal bonds, ILBs, and stocks. Table 4 reports the average correlations between the nominal returns of all pairs of these category-based indices. The average correlations are positive and tend to be high. The lowest average correlations are between stocks and nominal bonds.

Table 5 reports the average monthly return and the quarterly return of each asset class, as well as the volatility of these returns, measured as the average standard deviation of the instruments of the corresponding asset class. The nominal returns are in euros, while the real returns are in the local currency and corrected for the corresponding CPI inflation. The average nominal returns are larger than the average real returns because the average inflation is positive in each country. In general, the returns on stocks have the highest return and the highest volatility, while the returns on nominal bonds have the lowest return and the lowest volatility.

Table 4: Average correlations (in %) between asset classes

	monthly nominal returns		
type of instruments	nominal bonds	ILBs	stocks
nominal bonds	93.4	73.7	46.7
ILBs	73.7	72.1	55.0
stocks	46.7	55.0	97.1
	quarterly nominal returns		
type of instruments	nominal bonds	ILBs	stocks
nominal bonds	93.2	70.5	35.7
ILBs	70.5	69.7	44.4
stocks	35.7	44.4	96.2

Note: Average nominal return correlations between index pairs based on the grouping of all assets into broad instrument classes.

2.3 Constructing mean-variance frontiers

Consider a portfolio of n instruments, with portfolio weights ω:

$$\omega' = \begin{bmatrix} \omega_1 & \omega_2 & \cdots & \omega_n \end{bmatrix}$$

The return of the instrument i at time t is $R_{i,t}$. The vector of (empirical) average returns μ and the variance-covariance matrix Σ are, respectively:

$$\mu' = \begin{bmatrix} \mu_1 & \mu_2 & \cdots & \mu_n \end{bmatrix}$$
$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{1,2} & \cdots & \sigma_{1,n} \\ \sigma_{2,1} & \sigma_2^2 & \cdots & \sigma_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n,1} & \sigma_{n,2} & \cdots & \sigma_n^2 \end{pmatrix}$$

where $\mu_i = \frac{1}{T}\sum_{t=1}^{T} R_{i,t}$ and $\sigma_{i,j} = Cov(R_{i,t}, R_{j,t})$. The global minimum-variance portfolio solves the following minimization problem:

$$\min_{\omega} \sigma_m^2 = \omega'\Sigma\omega \text{ s.t. } \omega'\mathbf{1}_n = 1. \tag{1}$$

where $\mathbf{1}_n$ is a column vector of n 1's. In the Appendix we show that solving Eq.(1) yields the optimal portfolio weights ω_m:

$$\omega_m = (\mathbf{1}_n'\Sigma^{-1}\mathbf{1}_n)^{-1}\Sigma^{-1}\mathbf{1}_n. \tag{2}$$

Table 5: Summary statistics of returns for each asset class

	monthly average					quarterly average				
	nominal	real				nominal	real			
type of instruments		GE	UK	NL	US		GE	UK	NL	US
nominal bonds	20.0	2.3	2.9	13.0	10.1	61.0	8.2	7.9	39.1	28.1
ILBs	43.2	25.4	25.1	35.1	31.4	128.2	74.5	69.8	101.5	87.7
stocks	88.0	70.1	67.9	78.0	72.7	264.1	209.2	196.2	229.1	212.9
	monthly volatility					quarterly volatility				
	nominal	real				nominal	real			
type of instruments		GE	UK	NL	US		GE	UK	NL	US
nominal bonds	153.8	166.7	270.6	268.7	391.4	283.2	320.2	476.7	461.7	642.6
ILBs	307.1	311.2	354.2	351.4	435.8	503.7	513.0	548.3	542.1	671.0
stocks	419.5	421.5	406.0	402.9	439.2	672.4	671.0	571.2	578.2	663.2

Note: Real returns are obtained by subtracting CPI inflation from nominal returns. Average returns and volatility of returns of an asset class are measured by taking the average of mean and standard deviation the returns of the instruments within the corresponding asset class.

and the expected return on the global minimum-variance portfolio is $\mu_m = \omega_m' \mu$.

To construct the mean-variance frontiers we also need to derive the minimum-variance portfolio's subject to given target rates of return μ_p. We obtain the minimum-variance portfolio for a given target expected return μ_p by minimizing the same objective function but subject to an additional constraint pinning down the target return μ_p:

$$\min_{\omega} \sigma_p^2 = \omega' \Sigma \omega \qquad (3)$$
$$\text{s.t. } \omega' \mu = \mu_p \text{ and } \omega' \mathbf{1}_n = 1.$$

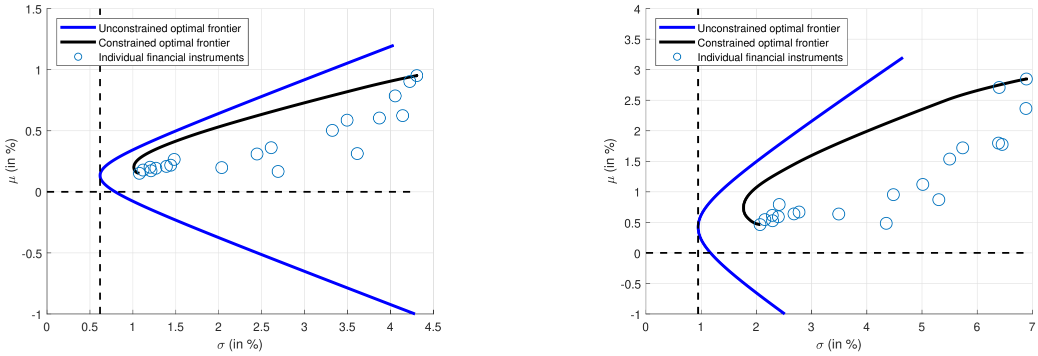
In the Appendix we show that the solution is given by the vector of portfolio weights ω_p:

$$\omega_p = \Sigma^{-1} [\mu \; \mathbf{1}_n] \Theta^{-1} [\mu_p \; 1]' \qquad (4)$$

where Θ is the two-by-two matrix:

$$\Theta = [\mu \; \mathbf{1}_n]' \; \Sigma^{-1} \; [\mu \; \mathbf{1}_n]$$

Both the unconstrained and the constrained minimum-variance portfolio we just derived allow for portfolio weights that are negative or exceed 100%, i.e. they allow in principle for short-selling and borrowing. This is often not allowed in practice; therefore, we also consider minimum-variance portfolios that constrain the



(a) Monthly nominal returns

(b) Quarterly nominal returns

Figure 2: Mean-variance frontiers. *Note: mean-variance frontiers are expressed in percentages and constructed from 17 bond indices and three stock indices based on monthly nominal returns (left) and quarterly nominal returns (right). Returns are derived from prices in EUR.*

portfolio weights not to become negative or exceed 100%. These restrictions add two constraints to the optimization problems in Eq.(1) and Eq.(3), summarized in: $\omega_i \in [0,1], \forall i$. We also solve this constrained optimization problem numerically in the following.

3 Results

We use our $n = 20$ indices (17 bond indices and 3 stock indices) to construct mean-variance portfolios for nominal returns in Section 3.1 and real returns in Section 3.2. Section 3.3 presents a sensitivity analysis in which we explore a number of variations on the preceding setup. Our analysis is carried out for both monthly returns and quarterly returns, in view of the fact that nominal returns may respond with a lag to inflation. Also, we present both portfolios in which short selling and leverage is allowed (labeled "unconstrained") and portfolio's where the portfolio shares ω_i are constrained to lie between zero and one, labeled "constrained".

3.1 Nominal returns

The left panel of Figure 2 shows the mean-variance frontier based on nominal monthly returns in the absence of any restrictions on portfolio weights ω_i, i.e. short selling ($\omega_i < 0$) and leverage ($\omega_i > 1$) are allowed. The minimum-variance portfolio corresponds to the very left point on the frontier. Table 6 shows that the unconstrained global minimum-variance portfolio has a volatility of $\sigma = 61.8$ basis points (bps) and an expected return of $\mu = 13.3$ bps per month. The standard deviation associated with this portfolio is strictly positive, indicating that it is

Table 6: Expected returns (μ_m) and volatility (σ_m) of the minimum-variance portfolios for nominal returns.

	monthly returns			quarterly returns		
minimum-variance portfolio	μ_m	σ_m	$\hat{\mu_m}$	μ_m	σ_m	$\hat{\mu_m}$
unconstrained	13.3	61.8	13.3	42.0	94.4	42.0
constrained	12.1	63.4	17.1	46.5	113.4	80.3

Note: Numbers are in bps. Further, $\hat{\mu_m}$ is the expected return from the unconstrained mean-variance frontier where volatility equals that of the constrained minimum variance portfolio.

Table 7: Portfolio allocations (in %) of the minimum-variance portfolios based on nominal returns.

	monthly		quarterly	
indices	unconstrained	constrained	unconstrained	constrained
nominal bonds	83.5	75.8	73.7	65.1
index-linked bonds	12.5	20.1	17.7	25.3
stocks	4.1	4.2	8.8	9.4

not possible to completely eliminate risk in nominal returns. Moving along the frontier to the right, both the expected return and the standard deviation of the portfolio increase. Table 7 presents the shares of the three asset categories in the minimum-variance portfolio. Note that by far the largest fraction of the portfolio is made up of nominal bond indices.

The unconstrained minimum-variance portfolio contains portfolio weights on some individual assets that exceed 100% or are negative, implying borrowing and short-selling. For example, the portfolio features an exposure of -214.7% to the Netherlands sovereign bond index and +236.5% to the German sovereign bond index.[7] We consider such weights unrealistic; therefore, we also show the mean-variance frontier when we constrain the weights on all individual assets to lie in the $[0, 1]$ interval, in Figure 2. The constrained minimum-variance portfolio has somewhat higher (volatility $\sigma = 63.4$bps) and marginally lower expected return $\mu = 12.1$bps, as expected given that the constraints bind. Compared to the unconstrained minimum-variance portfolio, the fraction allocated to nominal indices falls but is still high. It is still dominated by the German sovereign bond index, which has a 75.5% allocation in the overall portfolio. Although the constrained minimum-variance portfolio unavoidably features a higher variance and a lower expected return, the differences with the unconstrained minimum-variance portfolio are limited. The expected return from the unconstrained mean-variance frontier at the point where volatility equals that of the constrained minimum-variance portfolio is 17.1 bps, implying a nominal risk premium from the constraints of $17.1 - 12.1 = 5.0$ bps with the returns calculated on a monthly basis.

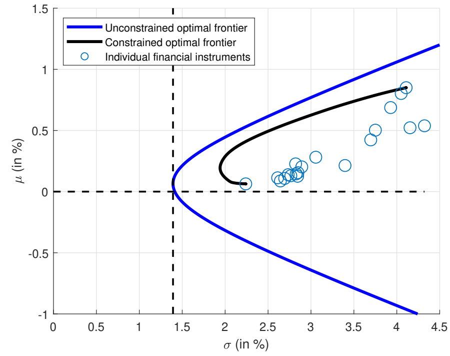
The right panel of Figure 2 shows the mean-variance frontiers based on quarterly nominal returns. These portfolios are also dominated by nominal bond indices, although less so than in portfolios based on monthly returns. Consequently, the weights of both ILBs and stocks are now higher. The implied risk premium of the portfolio weight constraints rises to $80.3 - 46.5 = 33.8$bps per quarter (see Table 6), which when annualized is higher than for portfolios based on monthly nominal returns: 136 bps for the portfolios based on quarterly returns versus 60 basispoints for the portfolios based on monthly returns.

In the remainder of this paper, we confine ourselves to portfolios that exclude borrowing and short-selling, to prevent the unrealistically extreme allocations to individual indices that characterize the unconstrained portfolios.

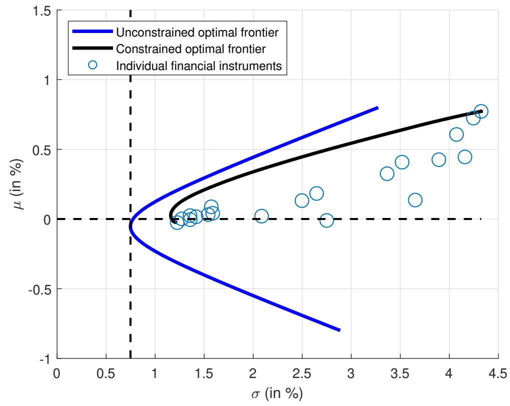
3.2 Real returns

So far, we have focused on mean-variance portfolios based on nominal returns. But we are interested in the hedging of inflation risks because of the riskiness of the purchasing power of an investment portfolio, hence we also construct the minimum-variance portfolio's based on real asset returns. The real returns are

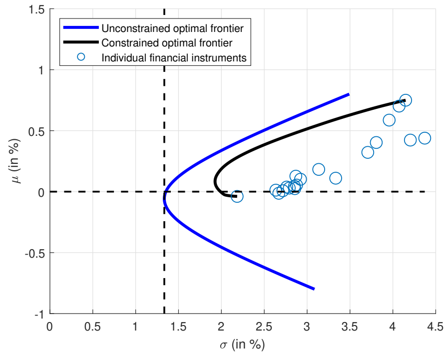
[7] The coexistence of highly-correlated indices in a portfolio frequently leads to extreme weights of opposite sign on these indices.



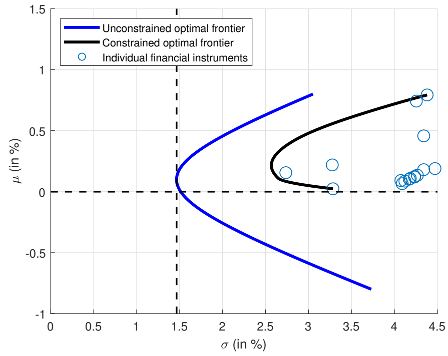
(a) Returns based on Dutch inflation



(b) Returns based on German inflation



(c) Returns based on UK inflation



(d) Returns based on US inflation

Figure 3: Mean-variance frontiers based on monthly real returns. *Notes: allocations are in percent. Real returns are obtained by subtracting CPI inflation from nominal returns.*

adjusted for the corresponding CPI inflation and the relevant currency (i.e. euros for NL and GE, dollars for the US, and pounds for the UK).[8]

Figure 3 shows the mean-variance frontiers of portfolios based on monthly real returns for the aforementioned countries. Panel (a) shows that the unconstrained minimum-variance portfolio based on Dutch inflation has a volatility of $\sigma = 139.3$ bps and an expected return of $\mu = 6.4$ bps, while the corresponding figures for the constrained portfolio are $\sigma = 194.1$ bps and $\mu = 19.3$ bps, respectively. See also Table 8. Panels (b), (c), and (d) depict the corresponding figures for real returns based on German, U.K. and U.S. inflation, respectively. The global minimum-variance portfolio based on German inflation has much lower volatility than the one based on Dutch inflation, while those based on UK and US inflation have volatilities similar to the portfolio based on Dutch inflation. The *constrained* minimum-variance portfolio with the lowest volatility ($\sigma = 115.9$ bps) is again

[8]Appendix B describes the steps that we perform to construct the nominal and real returns from the data.

Table 8: Expected real returns (μ) and volatility (σ) of the minimum-variance portfolios.

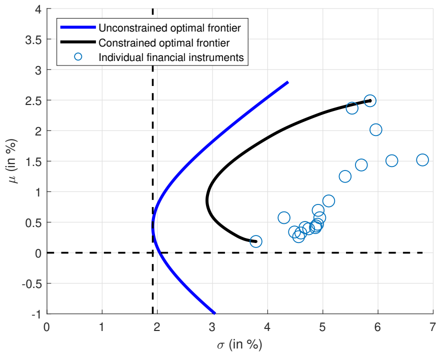
	monthly returns			quarterly returns		
country	μ	σ	$\hat{\mu}$	μ	σ	$\hat{\mu}$
NL unconstrained	6.4	139.3	6.4	43.0	191.7	43.0
NL constrained	19.3	194.1	42.3	85.7	290.0	174.2
GE unconstrained	-5.4	74.8	-5.4	-14.6	129.8	-14.6
GE constrained	2.6	115.9	18.3	26.4	217.1	91.4
UK unconstrained	-5.9	133.4	-5.9	15.3	204.2	15.3
UK constrained	8.4	192.6	31.1	59.5	297.9	146.1
US unconstrained	9.9	146.4	9.9	68.1	210.1	68.1
US constrained	21.6	256.7	65.1	95.3	391.5	266.1

Note: Numbers are in bps. Real returns are constructed by subtracting CPI inflation from the nominal returns. Borrowing and short-selling are excluded. Finally, ($\hat{\mu}$) is the expected return on the unconstrained mean-variance frontier where volatility equals that of the constrained minimum-variance portfolio.

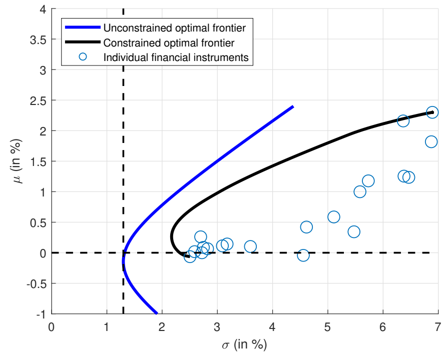
Table 9: Allocations of constrained minimum-variance portfolios based on real returns

	monthly returns				quarterly returns			
indices	NL	GE	UK	US	NL	GE	UK	US
nominal bonds	71.4	73.3	71.8	84.7	49.8	57.2	47.2	67.2
index-linked bonds	14.1	22.4	13.8	0.0	24.8	32.4	24.8	3.6
stocks	14.6	4.1	14.2	15.3	25.3	10.2	27.9	29.1

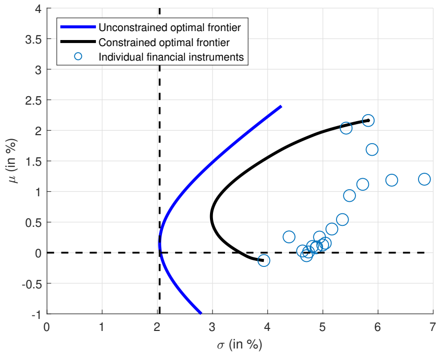
Note: allocations are in percent and exclude borrowing for and short-selling of individual indices. Real returns are obtained by subtracting CPI inflation from the nominal returns.



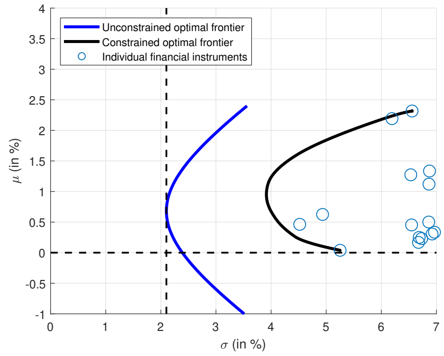
(a) Returns based on Dutch inflation



(b) Returns based on German inflation



(c) Returns based on UK inflation



(d) Returns based on US inflation

Figure 4: Mean-variance frontiers based on quarterly real returns. *Notes: allocations are in percent. Real returns are obtained by subtracting CPI inflation from nominal returns.*

the one based on German inflation. Those based on Dutch and UK inflation have similar volatility, while the portfolio based on US inflation features slightly higher volatility. Overall, the minimum-variance volatilities of the portfolios based on real returns exceed those of the MV portfolios based on nominal returns that we discussed earlier, see Table 6. Most importantly, even though ILBs are part of the portfolios, it is not possible to create portfolios, unconstrained or constrained, that generate returns that are stable in real terms. In other words, inflation risk cannot be hedged away with an appropriate choice of portfolio composition.

Figure 4 shows the mean-variance frontiers of portfolios based on quarterly real returns. As before, the volatility associated with minimum-variance portfolios, unconstrained or constrained, is lowest for Germany; see also Table 8.

Table 9 presents the allocations of the constrained minimum-variance portfolios. Contrary to what one might have expected *a priori*, in all instances nominal bond indices, and not ILBs, again dominate the allocations, with weights varying between 70% and 85% for the monthly real returns and constitute the largest investment class with weights between 50% and slightly less than 70% for the quarterly real returns. Compared to constrained minimum-variance portfolios based on nominal returns, stocks have a higher share in portfolios based on real returns in virtually all instances. ILB's see their role increased for the quarterly real returns, where they reach a maximum allocation of almost one-third for the case of German inflation. Remarkably, however, for real returns based on US inflation the minimum-variance portfolios contain a zero allocation towards ILBs for the portfolio's based on monthly returns and a very low allocation for the portfolio's based on quarterly returns.

Finally, Table 8 also reports $\tilde{\mu}$, the expected return on the unconstrained mean-variance frontier for the point where volatility equals that of the constrained minimum-variance portfolio. For the Netherlands, the implicit risk premium attributable to the constraints is $42.3 - 19.3 = 23.0$ bps per month for the monthly returns and 88.5 bps per quarter for the quarterly returns, almost the same if both are brought on a quarterly basis: 92 bps versus 88.5 bps. For German inflation, the figures are lower, while for UK inflation, they are comparable to those for Dutch inflation. For US inflation, the implicit risk premia are substantially higher than in the other cases, at 43.5 bps and 170.8bps for the monthly, respectively, quarterly case.

3.3 Sensitivity analysis

This subsection explores to what extent our findings so far are affected by variations on the setup used so far. We confine ourselves to the analysis based on real returns.

Table 10: Expected return and volatility of minimum-variance portfolios excluding one asset class at a time.

	all instruments		excl. nominal bonds		excl. ILBs		excl. stocks	
	expected returns (in bps)							
NL unconstrained	6.4	43.0	25.7	86.7	10.1	47.3	3.5	13.0
NL constrained	19.3	85.7	29.9	111.2	19.8	81.6	14.1	51.4
GE unconstrained	-5.4	-14.6	6.1	22.1	-3.8	-7.0	-5.6	-23.3
GE constrained	2.6	26.4	9.6	40.0	2.2	23.6	1.1	11.3
UK unconstrained	-5.9	15.3	14.2	56.0	-1.6	17.9	-8.0	-18.3
UK constrained	8.4	59.5	19.3	82.5	9.0	55.2	2.8	20.5
US unconstrained	9.9	68.1	34.6	119.5	13.4	63.5	6.8	28.7
US constrained	21.6	95.3	35.8	121.2	21.6	94.4	12.0	46.6
	volatility (in bps)							
NL unconstrained	139.3	191.7	231.3	313.3	149.7	240.1	142.5	216.2
NL constrained	194.1	290.0	233.5	326.5	195.2	297.5	199.9	323.9
GE unconstrained	74.8	129.8	139.1	233.4	82.0	146.6	75.7	134.8
GE constrained	115.9	217.1	144.2	250.2	118.3	225.1	116.7	225.4
UK unconstrained	133.4	204.2	232.4	321.1	146.2	251.0	136.1	228.8
UK constrained	192.6	297.9	235.3	333.2	193.9	306.0	198.4	337.3
US unconstrained	146.4	210.1	279.9	372.0	154.8	260.7	149.8	236.5
US constrained	256.7	391.5	300.0	410.5	256.7	391.5	263.9	429.4

3.3.1 Excluding one of the indices

In order to further assess the contribution of a specific asset class to the possibility of inflation hedging, we check to what extent the properties of the minimum-variance portfolios change when we exclude one specific asset class at a time. Table 10 reports the expected returns and volatilities of the corresponding minimum-variance portfolios. For monthly portfolios, the exclusion of the nominal bond indices has the biggest impact: excluding this asset class leads to significantly higher volatility of the minimum-variance portfolio, but also a higher expected return. This is also the case for all the unconstrained quarterly portfolio's: leaving out nominal bonds leads to the highest increase in the expected return and volatility of the minimum-variance portfolio's. For the constrained quarterly portfolios the exclusion of any of the investment categories has similar, but not quite the same pattern: for the NL, UK and the US leaving out stocks has an impact on the volatility of the MV portfolios that is similar to the impact of leaving out nominal bonds. For Germany, leaving out stocks matters much less.

3.3.2 Aligned volatilities

The relatively low risk of returns on nominal bond indices (see Table 5) may well explain why minimum-variance portfolios are dominated by this asset class. In order to explore this hypothesis, we counterfactually set the volatility of the real return on each index at the average volatility of the real returns across all indices, while rescaling the average real return on each index, so as to keep the Sharpe ratio at its original value.[9] In this way, the individual riskiness of each index becomes the same while maintaining the diversification benefits of the portfolio.

Figure 5 depicts the mean-variance frontiers for the monthly real returns and the first panel of Table 11 shows the asset allocations of the corresponding minimum-variance portfolios.[10] With the above scaling, the only way to reduce the riskiness of the portfolio is through diversification, as each individual index now has the same volatility. The minimum risk portfolio now contains only 30% to one-third nominal bond indices, while the allocation to ILB indices rises to one-third for the quarterly returns and to 40% - 55% for the monthly returns. In addition, the weight of the stock indices increases, especially for the quarterly returns. The large decrease in the portfolio share of nominal bonds, matched by an almost corresponding increase in the share allocated to ILBs confirms that the high nominal bond shares that we found earlier are mostly related to the low volatility in the real returns on nominal bonds. If we eliminate this relative advantage of nominal bonds over ILBs, the dominant allocations to nominal bonds in the minimum-variance portfolio disappear.

3.3.3 Including additional asset classes

The results of Section 3.3.2 strongly indicate that diversification plays an important role in reducing portfolio risk. To see whether diversification benefits can be increased even more, we explore the impact of adding additional asset classes. Specifically, we allow for exposure to global real estate markets and commodities by adding four real estate and three commodity indices to the portfolios.

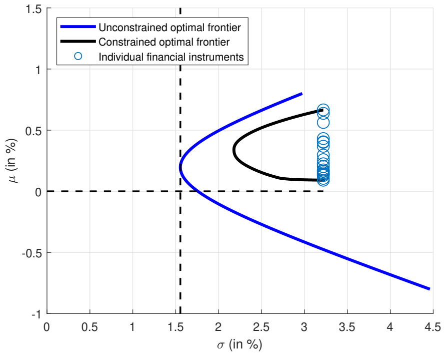
The correlations of the returns of these new indices with the inflation of the CPI are reported in Table 12. The nominal returns on the commodity indices are positively correlated with inflation, while the nominal returns on the real estate indices are positively correlated with US inflation, but mostly negatively correlated with CPI inflation of the other countries, especially in the case of the monthly returns.

Table 13 reports the average monthly return and the quarterly return of these two asset classes, as well as the volatilities of these returns, measured as the average standard deviation of the instruments of the corresponding asset class. Similarly to the asset classes in Table 5, the average nominal returns are higher than the average real returns because the average inflation is positive in each country. Both the average and volatility of the returns for real estate are higher than for commodity.

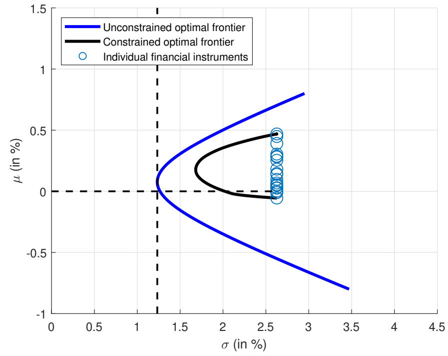
Figure 6 presents the optimal frontiers for monthly real returns when indices

[9] The Sharpe ratio is the ratio of the average return divided by the standard deviation of the return.

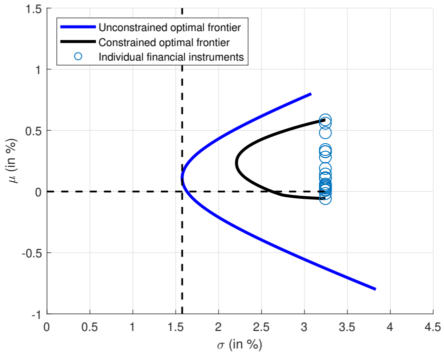
[10] Figure 13 in the Appendix shows the mean-variance frontiers for the quarterly real returns.



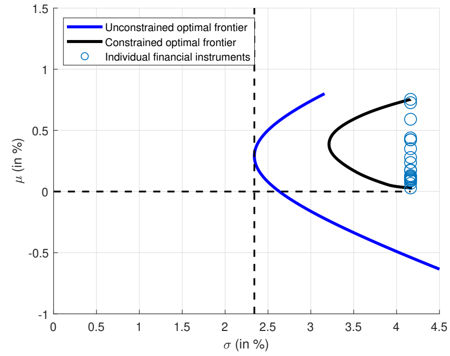
(a) Monthly real returns in terms of Dutch inflation



(b) Monthly real returns in terms of German inflation



(c) Monthly real returns in terms of U.K. inflation



(d) Monthly real returns in terms of U.S. inflation

Figure 5: Mean-variance frontiers based on monthly real returns after scaling the returns to align the volatilities. *Note: Real returns are obtained by subtracting CPI inflation from the nominal returns.*

Table 11: Portfolio allocations of the constrained minimum-variance portfolios.

	monthly returns				quarterly returns			
indices	NL	GE	UK	US	NL	GE	UK	US
Section 3.3.2	scaled returns to align the volatilities							
nominal bonds	31.2	29.1	30.0	32.0	32.3	33.2	33.4	26.4
index-linked bonds	44.4	54.9	45.4	38.9	31.1	36.2	30.0	35.0
stocks	24.3	15.8	24.5	29.0	36.4	30.5	36.3	38.6
Section 3.3.3	including real estate and commodities							
nominal bonds	76.2	78.4	76.0	81.8	64.1	74.3	58.0	71.5
index-linked bonds	0.9	11.2	1.2	0.0	4.2	9.7	8.0	0.0
stocks	5.4	0.2	5.1	7.3	14.1	5.7	17.7	19.0
real estate	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
commodities	17.5	10.1	17.6	10.7	17.2	10.0	16.3	9.4
Section 3.3.4	post COVID-19 (01-03-2020 to 18-04-2024)							
nominal bonds	44.1	62.1	51.6	92.1	9.3	17.1	6.7	32.5
index-linked bonds	55.8	37.7	48.1	0.2	62.2	62.9	70.2	51.3
stocks	0.0	0.0	0.0	7.6	28.3	19.9	22.9	16.2

Note: allocations are in percent and exclude short-selling of and borrowing for individual indices. Real returns are obtained by subtracting CPI from nominal returns.

Table 12: Correlations (in %) between the returns on new indices with CPI inflation rates.

		monthly correlations				quarterly correlations			
Index name	ticker	GE	UK	NL	US	GE	UK	NL	US
FTSE EPRA/NAREIT Developed	RNGL	-6.9	-10.2	-1.2	26.6	-1.1	11.3	-4.3	35.0
RMSG: MSCI US REIT Total Return	RMSG	-5.2	-7.9	1.7	20.4	1.3	16.6	-0.0	33.2
STOXX Europe 600 Real Estate	SX86P	-13.4	-13.0	-10.6	23.6	-11.0	-2.5	-18.5	25.9
Dow Jones U.S. Real Estate	DJUSRE	-5.5	-6.8	0.8	20.4	2.2	18.9	1.7	33.5
S&P GSCI Total return	SPGSCITR	23.7	19.0	19.1	65.7	35.8	37.4	42.0	79.8
Bloomberg Commodity Total return	BCOMTR	20.1	15.0	17.3	58.9	30.1	32.4	39.0	74.9
Dow Jones Commodity	DJCIT	19.2	12.4	15.7	61.7	28.3	29.2	35.7	74.8

Note: The top panel contains the correlations with the real estate indices and the bottom panel the correlations with the commodity indices.

Table 13: Summary statistics of returns for each asset class

	monthly average					quarterly average				
	nominal	real				nominal	real			
type of instruments		GE	UK	NL	US		GE	UK	NL	US
real estate	51.8	34.1	32.1	42.2	38.3	158.3	104.5	86.9	120.4	107.1
commodity	21.5	3.2	0.7	10.8	7.0	59.1	3.2	-7.6	24.1	14.2
	monthly volatility					quarterly volatility				
	nominal	real				nominal	real			
type of instruments		GE	UK	NL	US		GE	UK	NL	US
real estate	583.6	586.4	576.9	575.1	620.8	1029.0	1030.4	932.5	948.6	1029.9
commodity	530.1	522.5	507.9	506.8	563.3	952.2	923.9	890.8	889.9	1004.5

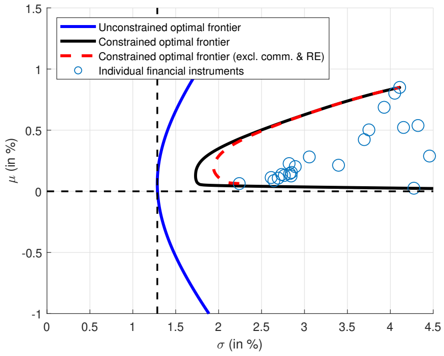
Note: Real returns are obtained by subtracting CPI inflation from nominal returns. Average returns and volatility of returns of an asset class is measured by taking the average of mean and standard deviation the returns of the instruments within the corresponding asset class.

of real estate and commodities are included. The allocations of the corresponding constrained minimum-variance portfolio for each frontier are reported in the second panel of Table 11.

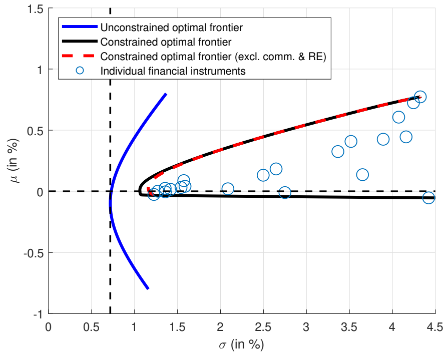
The allocations to the real estate indices are negligible, implying no change in the constrained minimum-variance portfolios. However, the allocations to the commodity indices range from 9.4% (quarterly returns based on US inflation) to 17.6% (quarterly returns based on Dutch inflation) and are accompanied by smaller allocations to the ILB and stock indices. When comparing these frontiers with the original frontiers excluding commodity and real estate indices, indicated by the red dashed lines, we observe a shift to the left, indicating that minimum-variance portfolios with the additional assets have lower volatility. For monthly returns, the reduction in volatility varies from 10 to 22 bps. The Appendix shows the results based on quarterly returns in Figure 14, where the reduction in volatility varies from 14 to 40 bps. A separate analysis with only real estate included as an additional asset class did not result in reductions in the minimum variance. The conclusion is clear: adding real estate exposure does not yield additional diversification benefits, but adding commodity indices does help to further mitigate inflation risk.

3.3.4 Post COVID-19 crisis period

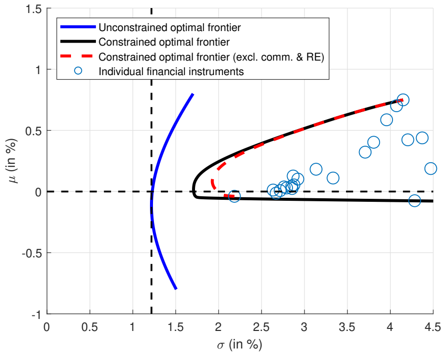
A priori, we would expect ILBs to be a particularly attractive asset class during periods of high inflation, such as those in the aftermath of the COVID-19 crisis. Therefore, we also provide a separate analysis based on the subsample period since



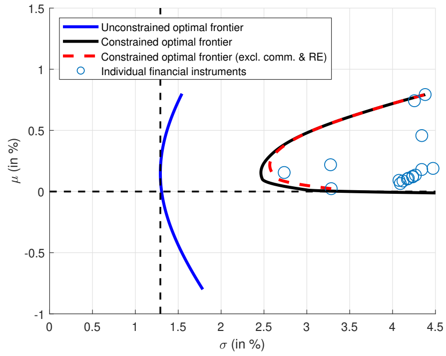
(a) Monthly real returns in terms of Dutch inflation



(b) Monthly real returns in terms of German inflation

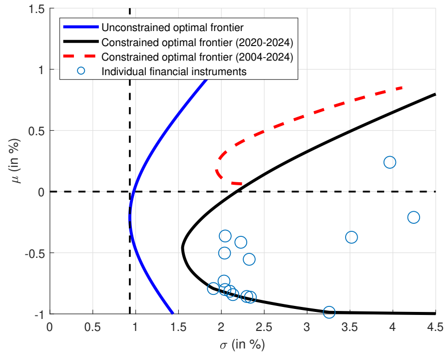


(c) Monthly real returns in terms of U.K. inflation

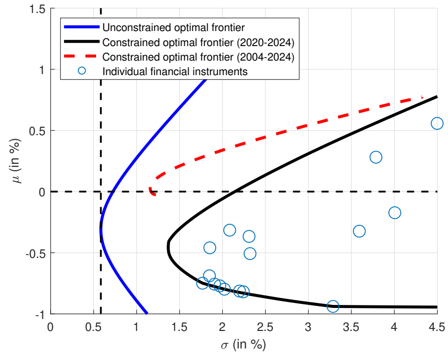


(d) Monthly real returns in terms of U.S. inflation

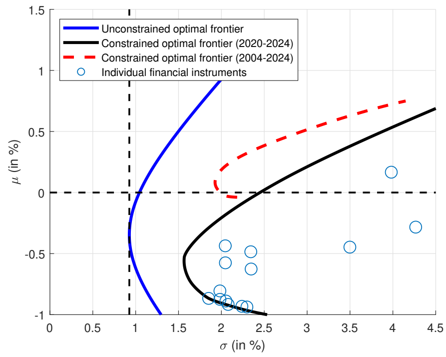
Figure 6: Mean-variance frontiers based on monthly real returns, real estate and commodity indices included. *Note: Real returns are obtained by subtracting CPI inflation from nominal returns.*



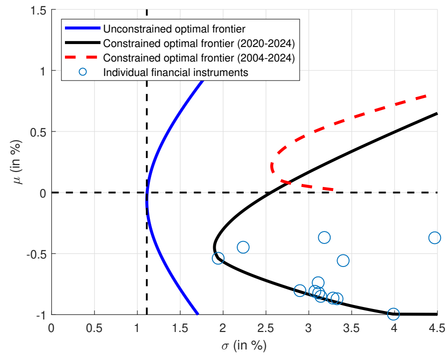
(a) Monthly real returns in terms of Dutch inflation



(b) Monthly real returns in terms of German inflation



(c) Monthly real returns in terms of British inflation



(d) Monthly real returns in terms of American inflation

Figure 7: Mean-variance frontiers based on monthly real returns over the post COVID-19 period. *Notes: Real returns are obtained by subtracting CPI inflation from the nominal returns. Sample period runs from 1 March 2020 to 18 April 2024.*

March 1, 2020. Figure 7 shows the corresponding mean-variance frontiers for the monthly real returns, while the corresponding asset class allocations are reported in the third panel of Table 11. In three of the four cases, i.e. excluding the case when real returns are based on German CPI inflation, the risks associated with the constrained minimum-variance portfolios are lower than for the full sample period. However, the lower minimum risk comes at a substantial cost in terms of expected return, which in all cases turns from positive to negative! Compared to the full sample, the asset allocation of the minimum-variance portfolio exhibits a substantial shift to the ILB indices, except for the case of monthly returns based on US inflation. For quarterly returns, ILBs are now the dominant class in all cases. In general, these findings suggest that ILBs, even during periods of high inflation, provide substantial, but still imperfect, inflation protection.

4 Evaluation of long-term inflation hedging through a variable annuity

So far, we have constructed optimal portfolios based on their performance over a short period of time, that is, a month or a quarter. However, investors tend to stay in the market for substantially longer, which requires evaluating over a longer time horizon. Portfolios that may be optimal in terms of the trade-off between risk and expected return over a short period may not be so when evaluated over a long horizon. Moreover, changes in inflation take time to affect nominal interest rates, while compensation for inflation through ILBs usually occurs with lags. For example, for US TIPs the principal sum is uprated every half year with realized inflation over that half year. Through these lags, optimal portfolio compositions may be affected by the horizon, and we should analyze optimal portfolio compositions over a correspondingly longer evaluation period.

Specifically, we analyze the performance of a variable annuity over a simulation period of 20 years. The length of the simulation period is roughly in line with the time the average worker can be expected to be in retirement. The variable annuity can be considered as a financial product for people who enter retirement and face a trade-off between fluctuations in purchasing power and expected increases in purchasing power. Hence, we no longer confine ourselves to minimum-variance portfolios but consider portfolios that maximize individual investor utility so as to be able to evaluate this trade-off.

4.1 Estimation

We jointly estimate the nominal returns of the $n = 20$ financial indices and the inflation rates. The nominal returns are modeled as:

$$r^n_{i,t} = \mu_i + \sigma_i \varepsilon_{i,t}. \tag{5}$$

where the parameter μ is the same as that used in Section 2.3 to calculate the mean-variance frontiers based on the nominal returns. Inflation rates are assumed to follow the following process:

$$\log(\pi_t + c) = \alpha_0 + \alpha_1 \log(\pi_{t-1} + c) + \sigma_\pi \varepsilon_{\pi,t}. \tag{6}$$

This simple process allows us to capture the possibility that high inflation rates are more likely to occur than low or even negative inflation rates.

We estimate the values for α_0, α_1, and c using nonlinear least squares both in monthly frequency data ($\delta = \frac{1}{12}$) and in quarterly frequency data ($\delta = \frac{1}{4}$), where in the following we use δ to indicate the length of the period between subsequent observations as a fraction of the year. As a final step in the estimation, we construct the $(n + 1) \times (n + 1)$ covariance matrix Σ_ε of the residuals of the estimated processes of nominal returns ($\varepsilon_{i,t}$, $i \in \{1, ..., n\}$) and inflation ($\varepsilon_{\pi,t}$).

From this matrix, we can derive the volatility of each instrument σ_i, the volatility of inflation rates σ_π and the correlations.

Estimates are reported in Table 14. Return numbers apply to the time between observations. Hence, the quarterly parameter estimates are larger than the monthly parameter estimates. Estimates based on the period prior to COVID-19 indicate a lower average inflation rate ($\bar{\pi}$) and lower volatility of inflation rates (σ_π) than we get over the period after COVID-19. Not surprisingly, the estimates based on the full sample period lie in between the sub-sample estimates.

4.2 Simulation

Using the parameter estimates and the variance-covariance matrix, we simulate $Q = 10.000$ scenarios of nominal returns and inflation rates over a period of $T * \delta = 20$ years. That is, for the monthly (quarterly) simulation for each scenario, 240 (80) sets of shocks for the 20 assets and the inflation process are drawn from the matrix of residuals. These are substituted into the estimated return and inflation rate processes to simulate time paths for these variables. To simulate a time series for π_t using Eq.(6) we have to assume an initial value π_0. We assume that the initial value (π_0) in all cases is equal to the steady-state value ($\bar{\pi}$) of the process of Eq.(6):

$$\pi_0 = \bar{\pi} = \exp\left(\frac{\alpha_0}{1-\alpha_1}\right) - c. \tag{7}$$

Finally, we obtain the real returns on the assets as $r_{i,t} = r_{i,t}^n - \pi_t$.

4.3 Evaluation of welfare effects

For each scenario q, we evaluate a time-varying annuity that pays a benefit for each period based on a fixed discount factor β. Assume a fund $W_{q,t}$ from which the annuity is paid each period:

$$b_{q,t} = W_{q,t} / \sum_{s=t}^{T} \beta^{(T-s)\delta}, \text{ for } t \in \{0, 1, 2, ..., T\}. \tag{8}$$

for monthly simulations ($T = 20/\delta = 240$) and for quarterly simulations ($T = 20/\delta = 80$). The fund then evolves in scenario q with the vector of simulated real returns $r_{q,t}$ over the simulation period as:

$$W_{q,0} = 1, \tag{9}$$

$$W_{q,t+1} = (W_{q,t} - b_{q,t}) * (1 + \omega' r_{q,t+1}), \text{ for } t \in \{0, 1, 2, ..., T-1\}. \tag{10}$$

For simplicity, we do not consider dynamic portfolios. This is also in line with the strategy of long-term investors like pension funds that tend to allocate fixed fractions or ranges of their portfolios to the different asset categories. Hence, we

Table 14: Estimates of the parameters of the inflation process.

parameter	monthly ($\delta = \frac{1}{12}$) NL	GE	UK	US	quarterly ($\delta = \frac{1}{4}$) NL	GE	UK	US
	full sample period: 20-09-2004 to 18-04-2024							
α_0	1.02	1.26	1.32	0.80	1.51	1.91	0.94	1.10
α_1	0.50	0.43	0.45	0.60	0.33	0.13	0.49	0.29
c	7.76	9.17	10.97	7.36	9.44	9.03	6.36	4.71
σ_π (in %)	0.05	0.04	0.03	0.03	0.08	0.09	0.13	0.15
$\bar{\pi}$ (in %)	0.18	0.16	0.28	0.19	0.59	0.54	0.87	0.64
	pre COVID-19: 20-09-2004 to 01-03-2020)							
α_0	1.17	1.62	1.86	0.91	2.25	3.50	1.30	1.43
α_1	0.43	0.27	0.22	0.54	-0.00	-0.38	0.30	0.08
c	7.80	9.14	10.92	7.35	9.37	12.53	6.35	4.71
σ_π (in %)	0.04	0.03	0.03	0.03	0.06	0.05	0.09	0.14
$\bar{\pi}$ (in %)	0.11	0.11	0.23	0.14	0.41	0.32	0.71	0.49
	post COVID-19: 01-03-2020 to 18-04-2024							
α_0	0.84	0.82	0.77	0.80	0.99	1.31	0.82	1.12
α_1	0.47	0.58	0.58	0.53	0.43	0.22	0.44	0.29
c	4.82	7.14	6.10	5.49	5.70	5.29	4.37	4.89
σ_π (in %)	0.13	0.05	0.08	0.03	0.18	0.21	0.32	0.14
$\bar{\pi}$ (in %)	0.43	0.42	0.50	0.43	1.34	1.32	1.67	1.35

Note: the parameters are estimated by non-linear least squares to fit the following inflation process: $\log(\pi_t + c) = \alpha_0 + \alpha_1 \log(\pi_{t-1} + c) + \sigma_\pi \varepsilon_{\pi,t}$. *The steady-state inflation rate is given by* $\bar{\pi} = \exp\left(\frac{\alpha_0}{1-\alpha_1}\right) - c$.

evaluate portfolios with fixed allocations ω assuming that an individual investor values annuity benefits according to the utility function $u(.)$ with a constant relative risk aversion parameter γ, i.e., $u(b) = \frac{b^{1-\gamma}}{1-\gamma}$, and with a time discount factor β:

$$U(\omega, \gamma) \equiv \frac{1}{Q} \sum_{q=1}^{Q} \sum_{t=0}^{T} \beta^{t\delta} u(b_{q,t}(\omega)) \tag{11}$$

The implied certainty equivalent benefit (CE) is:

$$CE(\omega, \gamma) = \left(\frac{(1-\gamma)U(\omega, \gamma)}{\sum_{t=0}^{T} \beta^{t\delta}} \right)^{1/(1-\gamma)} \tag{12}$$

With a numerical solver, we can find the portfolio allocation ω^* with the highest certainty equivalent benefit. As a benchmark, we take the constrained minimum risk portfolio based on real returns derived in Section 2.3, which we denote by ω_m, and calculate the welfare gain relative to this benchmark as:

$$Gain(\gamma) = \frac{CE(\omega^*, \gamma)}{CE(\omega_m, \gamma)} - 1. \tag{13}$$

4.4 Results

We set the time discount factor β equal to $\frac{1}{1.01}$ and let the constant relative risk aversion parameter range from 2 to 20. Table 15 reports the constrained portfolio allocations per asset class that provide the largest welfare and Table 16 reports the welfare gains associated with these optimal constrained portfolios.

With low risk aversion ($\gamma \leq 5$), the optimal portfolio mainly relies on investing in the stock index. The corresponding welfare gains can be substantial: in all cases, more than 29% for the monthly simulations and more than 20% for the quarterly simulations. Higher risk aversion ($\gamma \geq 10$) implies more diversification, resulting in about a one-third weight invested in nominal bond indices, ILB indices and stock indices each. Hence, the weight of ILB indices is substantially larger than in the one-period minimum variance portfolios (cf Table 9). The corresponding welfare gains vary from 16% to 36%.

5 Conclusion

This paper has investigated the effectiveness of existing financial instruments in hedging real portfolio returns against inflation, a question of growing relevance for long-term investors such as pension funds. Using mean-variance frontiers, we have assessed the hedging effectiveness of nominal bonds, inflation-linked bonds (ILBs), and equities during the period 2004-2024. We found that it is not possible

Table 15: Optimal portfolio allocations (in %) for the variable annuity including constraints that prevent borrowing and short-selling.

asset class	monthly returns					quarterly returns				
NL	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	$\gamma = 20$	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	$\gamma = 20$
nominal bonds	0.0	0.1	1.4	2.8	3.2	0.0	0.0	0.1	0.1	0.4
index-linked bonds	0.0	2.3	4.6	4.1	4.2	0.0	1.4	3.9	4.7	4.6
stocks	24.9	20.9	13.6	11.3	10.3	25.0	22.2	17.1	15.4	14.9
GE	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	$\gamma = 20$	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	$\gamma = 20$
nominal bonds	0.0	0.2	0.7	1.6	1.9	0.0	0.0	1.8	2.6	2.7
index-linked bonds	0.0	3.1	6.8	7.0	7.3	0.0	2.3	3.7	4.3	4.7
stocks	24.9	18.9	10.1	7.5	6.6	25.0	20.4	13.6	10.6	9.5
UK	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	$\gamma = 20$	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	$\gamma = 20$
nominal bonds	0.0	0.1	3.7	5.1	5.7	0.0	0.0	0.0	0.1	0.5
index-linked bonds	0.0	1.7	1.8	1.4	1.3	0.0	0.9	3.0	3.9	3.8
stocks	24.9	22.0	14.2	11.2	9.8	25.0	23.2	18.8	17.0	16.2
US	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	$\gamma = 20$	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	$\gamma = 20$
nominal bonds	0.0	0.2	4.4	4.8	4.9	0.0	0.0	0.0	1.0	2.3
index-linked bonds	0.0	1.0	0.2	0.7	1.0	0.0	1.0	3.7	2.7	1.2
stocks	24.9	22.6	14.8	12.8	12.1	25.0	23.0	17.6	17.5	17.6

Note: Simulations are based on real returns. Allocations are in percent. Real returns are obtained by subtracting CPI from nominal returns.

Table 16: Welfare gain compared to the minimum-variance portfolios including constraints that prevent borrowing and short-selling.

	monthly returns				quarterly returns			
γ	NL	GE	UK	US	NL	GE	UK	US
2	68.6	82.4	73.1	57.9	55.7	74.4	56.5	43.3
5	32.6	40.2	39.9	29.0	26.2	39.1	31.9	20.6
10	23.9	26.6	30.0	25.4	15.9	25.2	25.1	16.2
15	29.2	24.9	28.8	29.0	18.7	25.0	29.4	20.5
20	35.8	25.2	28.0	30.8	26.6	27.4	34.0	22.2

Note: numbers are expressed in percent. Welfare gains compare the certainty equivalent benefit from the optimal portfolio (ω^*) with the certainty equivalent benefit from the minimum risk portfolio (ω_m).

to construct portfolios of these asset categories that completely eliminate the effect of inflation risk on real returns. Introducing realistic market constraints, such as no borrowing or short-selling, further limits the scope for protecting portfolio returns against inflation risk. Although ILBs are commonly believed to offer the best protection against inflation risk, our findings actually show that nominal bond indices have been a more effective hedge against inflation risk during this period.

The benefits of diversification are confirmed by several sensitivity analyses. Once we (counterfactually) eliminate the comparative advantage of nominal bonds in terms of the volatility in their real returns, the minimum-variance portfolio becomes more balanced across asset classes. Moreover, focusing on the recent high-inflation period in the aftermath of the Covid-19 crisis creates a more substantial role for ILBs in hedging inflation risks.

To connect the short-term nature of the mean-variance analysis with the long-term investment horizon often relevant to investors, we also constructed optimal investment portfolios for a long-term investor receiving a variable annuity over a 20-year period, roughly the amount of time individuals can be expected to spend in retirement. These results underscore the importance of diversification: for investors with a substantial aversion to fluctuations in real returns, optimal portfolios appear much more balanced across ILBs, nominal bonds and equities than is the case for the minimum-variance portfolio over a short period of time. Moreover, the welfare gains relative to the minimum-variance portfolios are substantial. However, while ILBs do play an important role for these long-term investors, they do not dominate their optimal portfolio allocation, highlighting the limits to inflation hedging and the high cost, in terms of forgone average return, of protection against inflation risk.

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A Solving for the optimal portfolio analytically

This section closely follows the discussion of mean variance frontiers in Gale (2005).

A.1 Global minimum portfolio

The global minimum-variance portfolio is the solution of the following minimization problem:

$$\begin{aligned} \min_{\omega} \sigma_m^2 &= \omega' \Sigma \omega \qquad (14) \\ \text{s.t. } \omega' \mathbf{1}_n &= 1. \end{aligned}$$

In order to solve this, we compute the Langrange function:

$$L(\omega, \lambda,) = \omega' \Sigma \omega + \lambda(\omega' \mathbf{1}_n - 1). \qquad (15)$$

The corresponding first-order conditions are

$$\frac{\partial L(\omega, \lambda)}{\partial \omega} = 2\Sigma\omega + \lambda \mathbf{1}_n = \mathbf{0} \qquad (16)$$

$$\frac{\partial L(\omega, \lambda)}{\partial \lambda} = \omega' \mathbf{1}_n - 1 = 0. \qquad (17)$$

Then we can derive the global minimum portfolio ω_m as follows:

$$\mathbf{1}_n' \omega_m = -\frac{1}{2} \lambda \mathbf{1}_n' \Sigma^{-1} \mathbf{1}_n = 1 \qquad (18)$$

$$\Rightarrow \lambda = -2(\mathbf{1}_n' \Sigma^{-1} \mathbf{1}_n)^{-1} \qquad (19)$$

$$\Rightarrow \omega_m = (\mathbf{1}_n' \Sigma^{-1} \mathbf{1}_n)^{-1} \Sigma^{-1} \mathbf{1}_n. \qquad (20)$$

A.2 Mean variance portfolio

The mean-variance frontier is built up by solving the optimal portfolio problem for given target expected returns μ_p. Such a mean variance frontier portfolio is the solution of the following minimization problem:

$$\begin{aligned} \min_{\omega} \; \sigma_p^2 &= \omega' \Sigma \omega \qquad (21) \\ \text{s.t. } \mu_p &= \omega' \mu \text{ and } \omega' \mathbf{1}_n = 1. \end{aligned}$$

In order to solve this, we compute the Langrange function:

$$L(\omega, \lambda_1, \lambda_2) = \omega' \Sigma \omega + \lambda_1(\omega' \mu - \mu_p) + \lambda_2(\omega' \mathbf{1}_n - 1). \qquad (22)$$

The corresponding first-order conditions are

$$\frac{\partial L(\omega,\lambda_1,\lambda_2)}{\partial \omega} = 2\Sigma\omega + \lambda_1\mu + \lambda_2\mathbf{1}_n = \mathbf{0}_n \tag{23}$$

$$\frac{\partial L(\omega,\lambda_1,\lambda_2)}{\partial \lambda_1} = \omega'\mu - \mu_p = 0 \tag{24}$$

$$\frac{\partial L(\omega,\lambda_1,\lambda_2)}{\partial \lambda_2} = \omega'\mathbf{1}_n - 1 = 0. \tag{25}$$

Then we can derive the mean variance portfolio ω_p as follows:

$$\omega_p = -\frac{1}{2}\lambda_1\Sigma^{-1}\mu - \frac{1}{2}\lambda_2\Sigma^{-1}\mathbf{1}_n = -\frac{\lambda'}{2}\Sigma^{-1}\left[\mu\ \mathbf{1}_n\right] \tag{26}$$

with $\lambda = \begin{bmatrix}\lambda_1 \\ \lambda_2\end{bmatrix}$. Then we can use Eq.(26) to obtain:

$$\mu_p = \mu'\omega_p = -\mu'\frac{\lambda'}{2}\Sigma^{-1}\left[\mu\ \mathbf{1}_n\right] \tag{27}$$

$$1 = \mathbf{1}'\omega_p = -\mathbf{1}_n'\frac{\lambda'}{2}\Sigma^{-1}\left[\mu\ \mathbf{1}_n\right]. \tag{28}$$

We can solve for λ:

$$\lambda = -2\Theta^{-1}\begin{bmatrix}\mu_p \\ 1\end{bmatrix} \tag{29}$$

With Θ defined as the two-by-two matrix:

$$\Theta = \begin{pmatrix}\mu'\Sigma^{-1}\mu & \mu'\Sigma^{-1}\mathbf{1}_n \\ \mathbf{1}_n'\Sigma^{-1}\mu & \mathbf{1}_n'\Sigma^{-1}\mathbf{1}_n\end{pmatrix} \tag{30}$$

$$= \left[\mu\ \mathbf{1}_n\right]'\Sigma^{-1}\left[\mu\ \mathbf{1}_n\right] \tag{31}$$

which we can then use in Eq.(26) to obtain the following expression for the optimal portfolio weights ω_p:

$$\omega_p = \Sigma^{-1}[\mu\ \mathbf{1}_n]\Theta^{-1}\begin{bmatrix}\mu_p \\ 1\end{bmatrix} \tag{32}$$

B Constructing nominal and real returns

The nominal returns are in euros, while the real returns are adjusted for the corresponding CPI inflation and the relevant currency (i.e. euros for NL and GE, dollars for the US, and pounds for the UK). We perform the following steps to construct these nominal and real returns:

- For each index we take the prices in euros. The indices are total return indices that reinvest coupon and dividend payments.

- The nominal returns in euros are derived by taking the percentage changes in these euro prices.
- The real returns for Germany and for the Netherlands are derived (i) by dividing the euro prices by, respectively, the German CPI and the Dutch CPI, and (ii) by taking the percentage changes in these CPI adjusted price indices.
- The real returns for the UK and for the US are derived (i) by converting the euro prices to pounds and dollars, respectively, (ii) by dividing the resulting converted prices by, respectively, the UK CPI and the US CPI, and (iii) by taking the percentage changes in these CPI adjusted price indices.

C Additional results: for online publication

C.1 Additional figures

Figure 8 shows the prices of the 17 bond indices plus the 3 stock indices during the sample period.

Figure 9 and Figure 10 show the mean-variance frontiers of each subset based on monthly and quarterly returns, respectively, where portfolios are restricted by preventing borrowing and short-selling. The nominal bond indices are generally less risky, but also provide a lower return. Stock indices provide the highest return, but are more risky. This explains why the minimum risk portfolios mostly contain the nominal bond indices, as it appears that nominal bond indices are more helpful in hedging against inflation risk, even better than ILB indices.

Similarly to the graphs presented in Figure 9 and Figure 10, the graphs in Figure 11 and Figure 12 show the mean-variance frontiers of each subset, but without any restrictions on the portfolio weights to prevent borrowing and short-selling.

C.2 Results for quarterly returns of the sensitivity analysis

Figure 13 and Figure 14 show the mean-variance frontiers of the sensitivity analysis in Section 3.3 based on quarterly returns. The mean-variance frontiers based on quarterly returns during the period after the COVID-19 outbreak are not shown, as our sample data have too few observations. to construct those frontiers.

C.3 Optimal portfolio allocations for the minimum risk portfolios

The tables Section 3 and Section 4 provide the minimum risk portfolios per asset class. The corresponding portfolio allocation per index is presented in Table 17 to

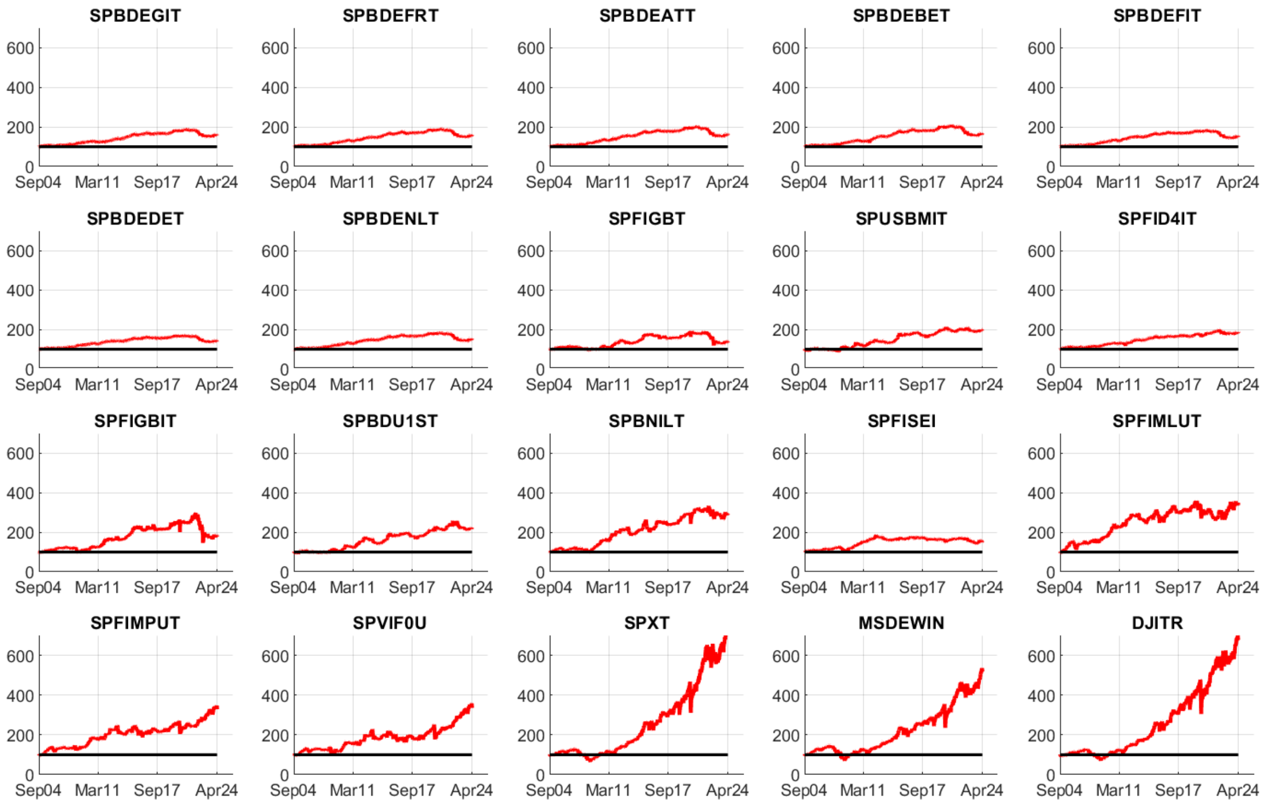
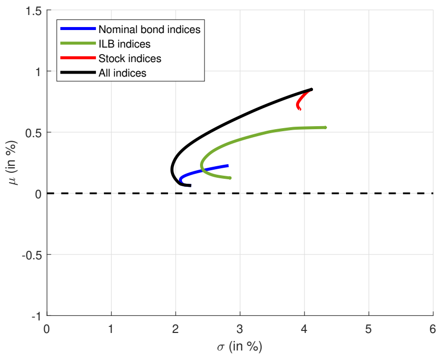
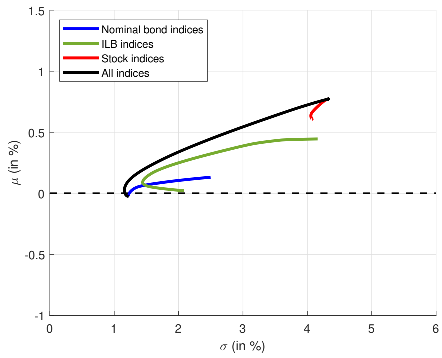


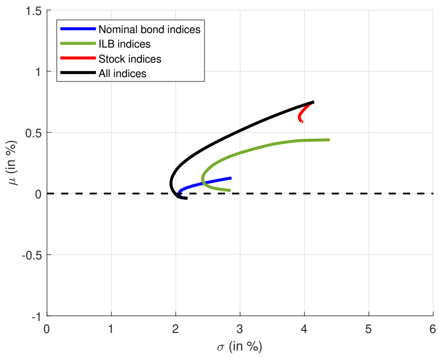
Figure 8: Daily prices of bond and stock indices. *Note: The first nine graphs are nominal bond indices, the following eight graphs are ILB indices and the last three graphs are stock prices.*



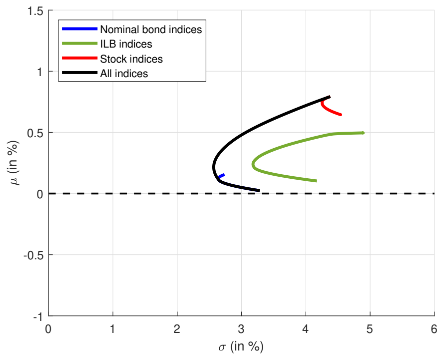
(a) Monthly real returns in terms of Dutch inflation



(b) Monthly real returns in terms of German inflation

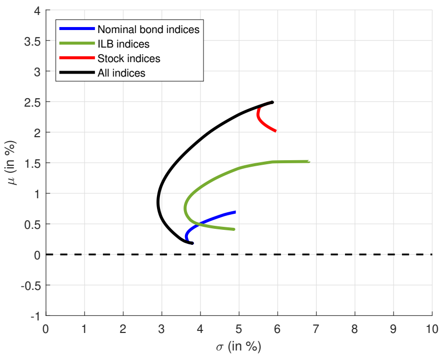


(c) Monthly real returns in terms of U.K. inflation

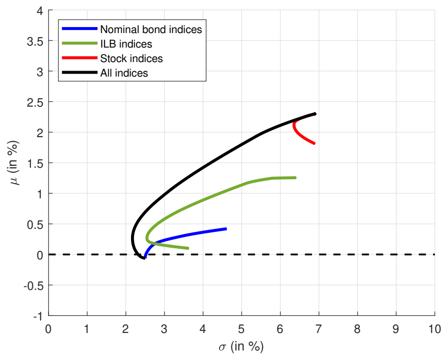


(d) Monthly real returns in terms of U.S. inflation

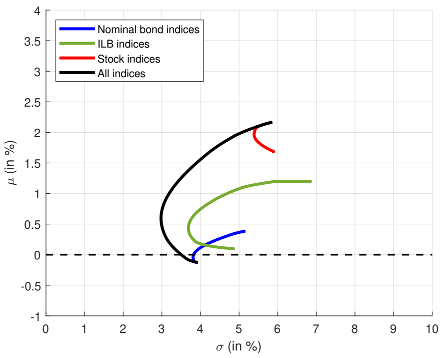
Figure 9: Mean-variance frontiers for constrained portfolios based on monthly real returns. *Note: allocations are in percent. Real returns are obtained by subtracting CPI from nominal returns.*



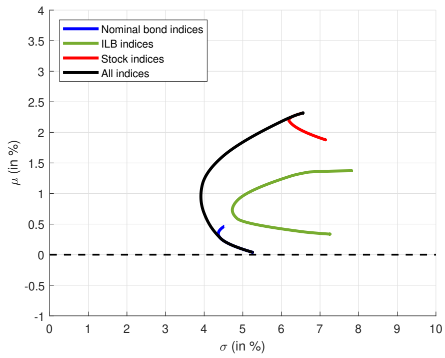
(a) Quarterly real returns in terms of Dutch inflation



(b) Quarterly real returns in terms of German inflation

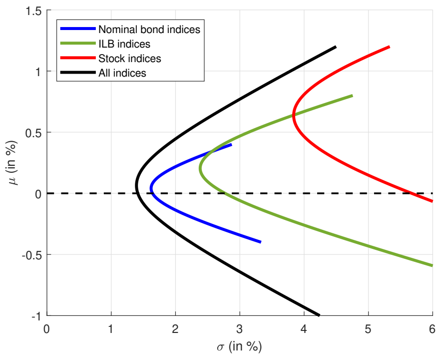


(c) Quarterly real returns in terms of U.K. inflation

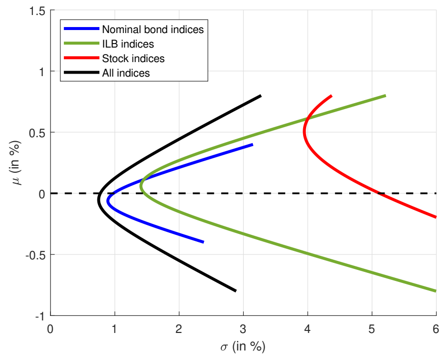


(d) Quarterly real returns in terms of U.S. inflation

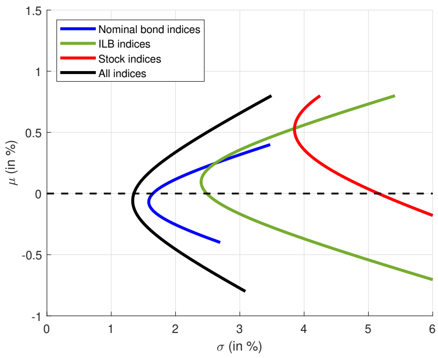
Figure 10: Mean-variance frontiers for constrained portfolios based on quarterly real returns. *Note: allocations are in percent. Real returns are obtained by subtracting CPI from nominal returns.*



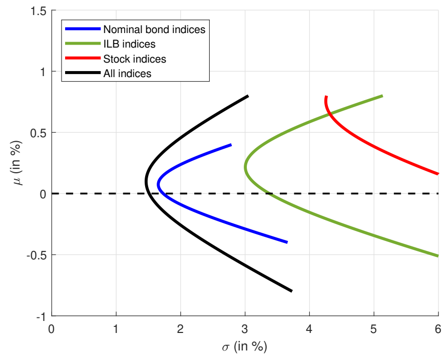
(a) Monthly real returns in terms of Dutch inflation



(b) Monthly real returns in terms of German inflation

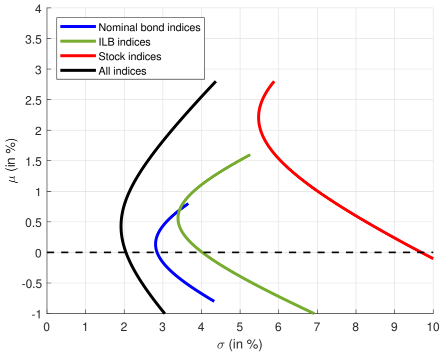


(c) Monthly real returns in terms of U.K. inflation

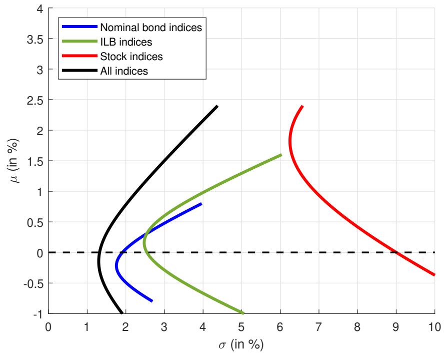


(d) Monthly real returns in terms of U.S. inflation

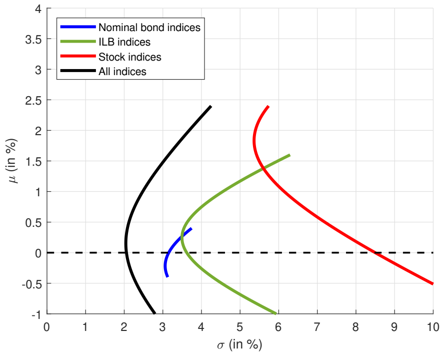
Figure 11: Mean-variance frontiers for unconstrained portfolios based on monthly real returns. *Note: allocations are in percent. Real returns are obtained by subtracting CPI from nominal returns.*



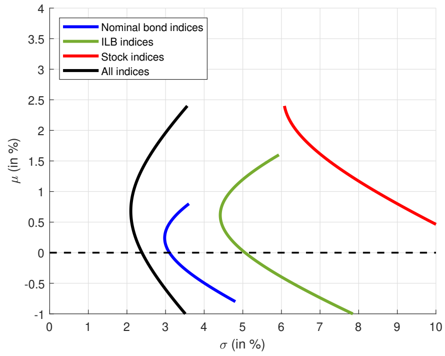
(a) Quarterly real returns in terms of Dutch inflation



(b) Quarterly real returns in terms of German inflation

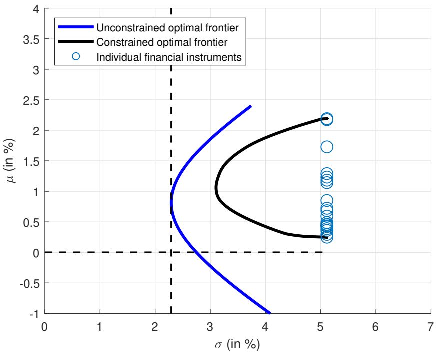


(c) Quarterly real returns in terms of U.K. inflation

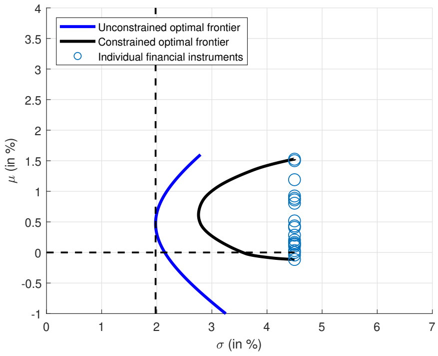


(d) Quarterly real returns in terms of U.S. inflation

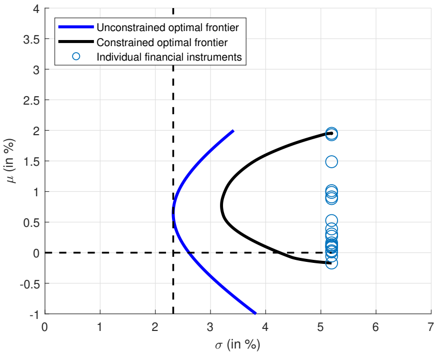
Figure 12: Mean-variance frontiers for unconstrained portfolios based on quarterly real returns. *Note: allocations are in percent. Real returns are obtained by subtracting CPI from nominal returns.*



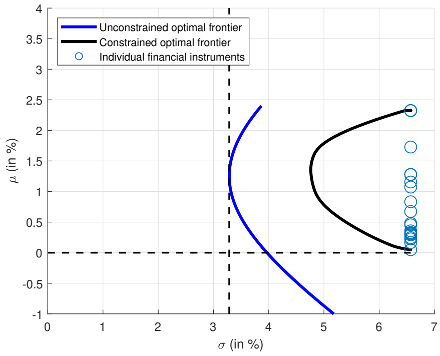
(a) Quarterly real returns in terms of Dutch inflation



(b) Quarterly real returns in terms of German inflation

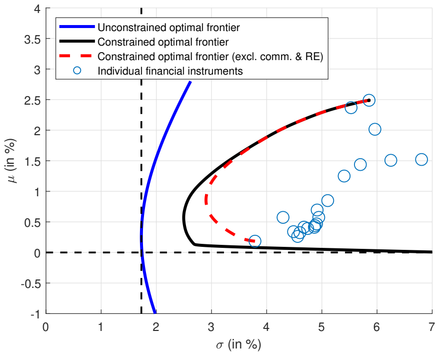


(c) Quarterly real returns in terms of U.K. inflation

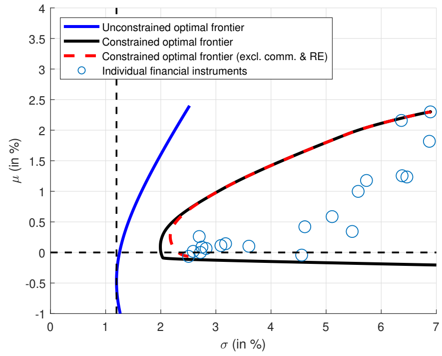


(d) Quarterly real returns in terms of U.S. inflation

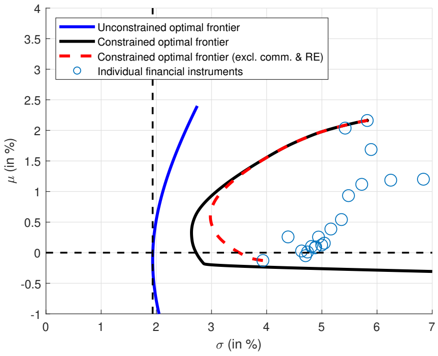
Figure 13: Mean-variance frontiers based on quarterly real returns after scaling the returns to align the volatilities. *Note: Real returns are obtained by subtracting CPI from nominal returns.*



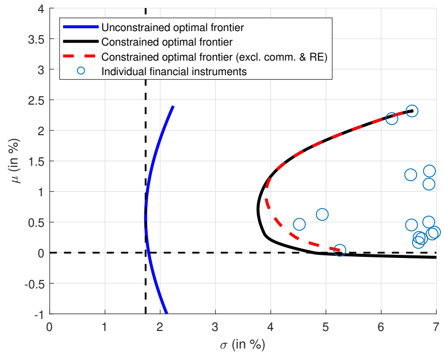
(a) Quarterly real returns in terms of Dutch inflation



(b) Quarterly real returns in terms of German inflation



(c) Quarterly real returns in terms of U.K. inflation



(d) Quarterly real returns in terms of U.S. inflation

Figure 14: Mean-variance frontiers based on quarterly real returns, also including real estate (RE) and commodity (comm) indices. *Note: allocations are in percent. Real returns are obtained by subtracting CPI from nominal returns.*

Table 17: Portfolio allocations (in %) of the minimum-variance portfolios based on nominal returns.

	monthly		quarterly	
index ticker	unconstrained	constrained	unconstrained	constrained
SPBDEGIT	40.2	0.1	36.9	0.0
SPBDEFRT	32.9	0.0	37.3	0.0
SPBDEATT	-41.2	0.0	-95.1	0.0
SPBDEBET	-61.5	0.0	-51.7	0.0
SPBDEFIT	80.0	0.1	87.8	0.0
SPBDEDET	236.5	75.5	194.8	65.1
SPBDENLT	-214.7	0.0	-136.3	0.0
SPFIGBT	-2.8	0.0	-1.8	0.0
SPUSBMIT	14.1	0.1	1.8	0.0
SPFID4IT	19.1	14.5	23.2	20.1
SPFIGBIT	-1.7	0.0	-1.4	0.0
SPBDU1ST	-15.9	0.0	-11.7	0.0
SPBNILT	-2.1	0.0	-3.4	0.0
SPFISEI	10.7	5.2	6.8	5.2
SPFIMLUT	1.9	0.3	-25.4	0.0
SPFIMPUT	-6.2	0.1	61.2	0.0
SPVIF0U	6.7	0.0	-31.6	0.0
SPXT	-7.4	0.0	-7.8	0.0
MSDEWIN	8.0	2.5	10.4	3.6
DJITR	3.5	1.7	6.2	5.8

Note: The upper part contains nominal bond indices, the middle part index-linked bond indices, and the lower part stock indices.

Table 21, based on real returns with respect to the inflation of the Netherlands, Germany, the U.K., and the U.S, respectively.

Table 18: Portfolio allocations of the minimum-variance portfolios including constraints that prevent borrowing and short-selling based on real returns

	monthly returns				quarterly returns			
index ticker	NL	GE	UK	US	NL	GE	UK	US
SPBDEGIT	0.1	0.0	0.1	0.0	0.0	0.0	0.0	0.0
SPBDEFRT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEATT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEBET	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEFIT	0.1	0.1	0.1	0.0	0.0	0.0	0.0	0.0
SPBDEDET	0.3	73.1	0.2	0.0	0.0	57.2	0.0	0.0
SPBDENLT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPFIGBT	56.2	0.0	60.5	22.3	49.0	0.0	47.2	9.5
SPUSBMIT	14.7	0.1	10.9	62.4	0.8	0.0	0.0	57.7
SPFID4IT	3.6	15.4	2.7	0.0	18.1	22.1	17.7	0.0
SPFIGBIT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDU1ST	0.0	0.0	0.0	0.0	0.1	0.0	0.0	3.6
SPBNILT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPFISEI	9.9	6.4	10.9	0.0	0.1	9.2	0.8	0.0
SPFIMLUT	0.0	0.3	0.0	0.0	0.0	0.1	0.0	0.0
SPFIMPUT	0.2	0.2	0.1	0.0	6.5	1.0	6.2	0.0
SPVIF0U	0.4	0.1	0.1	0.0	0.0	0.0	0.1	0.0
SPXT	0.1	0.0	0.0	0.1	0.0	0.0	0.0	16.1
MSDEWIN	13.9	2.4	11.2	0.0	10.6	0.2	8.7	0.0
DJITR	0.6	1.7	3.0	15.2	14.7	10.0	19.2	13.0

Note: allocations are in percent. Real returns are obtained by subtracting CPI from nominal returns. The upper part contains nominal bond indices, the middle part index-linked bond indices, and the lower part stock indices.

Table 19: Portfolio allocations of the minimum-variance portfolios including constraints that prevent borrowing and short-selling, based on real returns after scaling the returns to align the volatilities.

	monthly returns				quarterly returns			
index ticker	NL	GE	UK	US	NL	GE	UK	US
SPBDEGIT	0.2	0.1	0.1	0.0	0.0	0.1	0.0	0.0
SPBDEFRT	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0
SPBDEATT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEBET	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEFIT	0.1	0.0	0.1	0.0	0.0	0.0	0.0	0.0
SPBDEDET	1.0	10.9	0.7	0.0	9.7	27.5	9.4	0.0
SPBDENLT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPFIGBT	18.6	0.9	17.7	11.4	22.3	0.0	23.9	12.1
SPUSBMIT	11.3	17.2	11.4	20.6	0.3	5.5	0.1	14.3
SPFID4IT	5.2	15.1	5.0	0.0	0.1	0.1	0.0	0.0
SPFIGBIT	12.4	9.2	14.1	17.5	0.0	0.1	0.0	0.0
SPBDU1ST	0.0	0.0	0.0	0.1	10.7	0.0	6.1	16.8
SPBNILT	1.4	1.8	1.6	0.0	0.0	0.0	0.0	0.0
SPFISEI	4.0	9.5	4.6	0.0	3.2	14.7	4.6	0.0
SPFIMLUT	7.4	9.4	6.5	7.7	11.8	15.6	12.6	13.6
SPFIMPUT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPVIF0U	14.0	9.9	13.6	13.6	5.3	5.7	6.7	4.6
SPXT	13.8	0.1	5.9	10.2	0.1	0.0	0.0	27.8
MSDEWIN	0.8	8.2	3.5	0.0	28.0	13.7	23.8	0.1
DJITR	9.7	7.5	15.1	18.8	8.3	16.8	12.5	10.7

Note: Allocations are in percent. Real returns are obtained by subtracting CPI from nominal returns. The upper part contains nominal bond indices, the middle part index-linked bond indices, and the lower part stock indices.

Table 20: Portfolio allocations of the minimum-variance portfolios including constraints that prevent borrowing and short-selling, based on real returns also including real estate and commodity indices.

	monthly returns				quarterly returns			
index ticker	NL	GE	UK	US	NL	GE	UK	US
SPBDEGIT	0.3	0.1	0.2	0.0	0.0	0.0	0.0	0.0
SPBDEFRT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEATT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEBET	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEFIT	0.2	0.1	0.2	0.0	0.0	0.0	0.0	0.0
SPBDEDET	4.9	78.1	3.5	0.0	0.0	74.3	0.1	0.0
SPBDENLT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPFIGBT	57.7	0.0	62.0	19.2	55.2	0.0	57.0	9.6
SPUSBMIT	13.1	0.1	10.1	62.6	8.9	0.0	0.9	61.9
SPFID4IT	0.2	9.9	0.1	0.0	4.2	8.0	6.8	0.0
SPFIGBIT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDU1ST	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBNILT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPFISEI	0.7	1.3	1.1	0.0	0.0	1.7	0.0	0.0
SPFIMLUT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPFIMPUT	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0
SPVIF0U	0.0	0.0	0.0	0.0	0.0	0.0	1.1	0.0
SPXT	0.1	0.0	0.0	0.0	0.0	0.0	0.0	11.2
MSDEWIN	0.4	0.0	0.1	0.0	5.4	0.0	0.5	0.0
DJITR	4.9	0.2	5.0	7.3	8.7	5.7	17.2	7.8
RNGL	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
RMSG	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SX86P	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
DJUSRE	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPGSCITR	8.5	2.1	8.4	10.7	8.6	6.3	7.9	9.4
BCOMTR	5.9	0.2	6.6	0.0	0.0	1.6	0.3	0.0
DJCIT	3.1	7.8	2.6	0.0	8.6	2.1	8.1	0.0

Note: allocations are in percent. Real returns are obtained by subtracting CPI from nominal returns. The first part contains nominal bond indices, the second part index-linked bond indices, the third part stock indices, the fourth part real estate indices and the fifth part commodity indices.

Table 21: Portfolio allocations of the minimum-variance portfolios including constraints that prevent borrowing and short-selling, based on real returns over the post COVID-19 period.

	monthly returns				quarterly returns			
index ticker	NL	GE	UK	US	NL	GE	UK	US
SPBDEGIT	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEFRT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEATT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEBET	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEFIT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEDET	0.1	11.3	8.1	0.0	0.0	0.0	0.0	0.0
SPBDENLT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPFIGBT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPUSBMIT	44.0	50.7	43.5	92.1	9.3	17.1	6.7	32.5
SPFID4IT	23.4	5.7	22.4	0.0	28.8	11.4	42.2	0.0
SPFIGBIT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDU1ST	3.7	0.0	0.0	0.2	8.1	0.0	0.0	26.7
SPBNILT	0.0	0.0	0.0	0.0	1.6	8.7	0.0	0.0
SPFISEI	22.1	18.4	19.6	0.0	0.0	15.1	0.0	0.0
SPFIMLUT	6.6	13.6	6.1	0.0	23.7	27.7	28.0	0.1
SPFIMPUT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	17.7
SPVIF0U	0.0	0.0	0.0	0.0	0.0	0.0	0.0	6.8
SPXT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
MSDEWIN	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
DJITR	0.0	0.0	0.0	7.6	28.3	19.9	22.9	16.2

Notes: allocations are in percent. The real returns are obtained by subtracting CPI from the nominal returns. The sample period is 01-03-2020 to 18-04-2024. The upper part contains nominal bond indices, the middle part index-linked bond indices, and the lower part stock indices.