TI 2025-058/IV
Tinbergen Institute Discussion Paper

Carry Trade and Currency Crash Risk

Merve Kutuk[1]
Sweder van Wijnbergen[2]

1 CPB Netherlands Bureau for Economic Policy Analysis

2 DNB, University of Amsterdam, Tinbergen Institute and CEPR

Carry Trade and Currency Crash Risk

Merve Mavuş Kütük*and Sweder van Wijnbergen†

October 1, 2025

Abstract

This paper examines the role of currency crash risk in explaining the persistent profitability of carry trades. Focusing on the US Dollar–Turkish Lira market, we construct three forward-looking measures of crash risk: risk reversals, crash probabilities from option-implied distributions, and jump risk from a jump-diffusion model. Using survey-based exchange rate expectations, we separate ex ante carry premia from ex post surprises. Our results show that higher crash risk significantly increases expected returns, indicating that investors demand compensation for bearing such risk rather than arbitraging away mispricing. Shapley decomposition attributes over 20% of the explained variance in expected carry returns to crash risk, while balance sheet constraints and global risk aversion further reinforce premia. A comparison of hedged and unhedged strategies reveals that 46–77% of carry returns reflect compensation for crash exposure.

JEL codes: F13, G01, G10, G12, G15

Keywords:Exchange rates, currency crash risk, mispricing, dollar exchange rate, bank currency mismatches

*CPB Netherlands Bureau for Economic Policy Analysis

†DNB, University of Amsterdam, Tinbergen Institute and CEPR

1 Introduction

Positive and persistent carry trade returns remain one of the puzzles in the macro-finance literature. A growing body of literature emphasizes the role of risk factors like currency crash risk to explain returns to carry trade. However, there is still little empirical evidence to specifically identify the contribution of currency crash risk to carry trade returns. In this paper we construct a new currency crash risk measure based on asset pricing considerations to test to what extent crash risk contributes to carry trade returns. We show in our analysis of the Turkish Lira that crash risk of the investment currency is one of the main determinants of the carry trade returns. The results are robust to the use of different currency crash risk measures. We find empirical evidence that currency crash risk deters investors from taking sufficiently strong foreign exchange (FX) positions to arbitrage away apparent mispricing, leading carry traders to demand compensation for bearing currency risk in the presence of crash risk in the investment currency.

Understanding the return dynamics of carry trade is important not only for investors engaging in arbitrage but also for policy makers. Carry trade positions are highly volatile by their nature since they are highly leveraged due to the nature of this investment strategy. Any type of shock to the borrowing capacity of carry traders directly affects carry positions in investment currencies. Moreover, carry trade positions are generally short term since investors try to obtain positive returns by exploiting interest rate differentials while minimizing their exposure to crash risk during investment periods. So there always is a risk that arbitrageurs unwind or hesitate to roll over their short term positions just because of their currency crash risk expectations. As a consequence carry trading leads to highly volatile capital flows and accordingly brings high volatility to FX markets.

A substantial literature studies why the carry trade is a profitable investment strategy. Some state that carry traders are compensated for disaster risk (Farhi and Gabaix, 2015) or similarly, currency crash risk (Brunnermeier et al., 2008). Others claim that carry traders are compensated for liquidity spirals arisen from funding constraints (Brunnermeier and Pedersen, 2008), illiquidity in currency markets (Mancini et al., 2013) or global imbalances (Gabaix and Maggiori, 2015 and Corte et al., 2016). So while there are studies linking carry trade returns to currency crash risk [1], empirical literature testing this hypothesis is limited. The studies that do exist use skewness of observed exchange rate changes or risk reversals derived from currency option prices as proxies of currency crash risk. However, measured skewness is a backward looking indicator, it does not show investors' perceived depreciation risk in the investment currency over their investment horizon going forward. And while risk reversals provide information about perceived depreciation risk in the relevant currency, they are not sufficiently informative to make an inference about the whole distribution of a currency's perceived value. We aim to fill this gap in the literature by using new measures of currency crash risk taken from the asset pricing literature in our analysis of carry trade returns.

The empirical studies that do exist use *realized* carry returns to calculate deviations from

[1] We discuss in more detail in Section 2

uncovered interest rate parity (UIP). However, *ex post* realized carry returns will generally not have been equal to ex ante expected carry returns: obviously traders invest in this strategy without knowing exact end-of-period exchange rates. We therefore use investors' exchange rate *expectations* so that we can isolate the carry return from any surprise in the exchange rate and focus solely on the return investors demand to participate in carry trade strategies. For comparison, we replicate our regressions using realized returns and report the results in Appendix B.

Another common assumption in the literature is that covered interest parity (CIP) holds so that researchers can derive interest rate differentials as the residual of the CIP condition using forward and spot exchange rates. However, recent studies (e.g., Du et al. [2018]) show that the CIP condition does not hold in many currency pairs. This means that the carry trade returns using interest rate differentials derived from the CIP condition can be misleading. Therefore, we use interbank cost of borrowing as benchmark interest rates in this study.

We focus on the US Dollar (US$)-Turkish Lira (TL) market, where the TL is the investment currency and the US$ the funding currency. High interest rates in the TL compared to the US$ make the TL an attractive investment currency for carry trades. Moreover, high volatility in the TL likely makes investors' perceived crash risk an important factor in their investment decision. And direct availability of currency option prices and exchange rate expectations in the TL market enable us to conduct this analysis.

In the first part of our paper, we derive three new measures capturing currency crash risk in the TL which make one-by-one increasing use of the information contained in FX derivative prices. First we use skewness of implied volatilities i.e. *risk reversals*. A positive risk reversal suggests that market participants expect the investment currency (the TL) to depreciate against the funding currency. However, the risk reversals reflect only two specific points in volatility smiles, which is not enough to derive the entire distribution of the underlying asset value since an option's price is not linear in the implied volatility of its underlying asset. Therefore, altough we obtain significant and economically meaningful results using the risk reversals, we next derive the entire empirical (risk neutral) probability distribution, again from option prices, following Malz (2014). The risk neutral probability distributions of exchange rates are derived using the pricing formulas of Black and Scholes (1973) as applied to FX option by Garman and Kohlhagen (1983). Then, as a second measure, we calculate the exact perceived *crash probabilities* implied by the currency option prices using those derived distribution functions. Finally, we derive the third measure which is the *jump risk* in the TL derived from a double exponential jump-diffusion option pricing model (Kou, 2002). The third measure provides an opportunity for us to capture actual jump risk in the TL instead of *perceived* crash risk.

In the second part of the paper, we analyze determinants of the carry trade return and investigate whether the currency crash risk is embedded in the returns, using the above-mentioned crash risk measures. We first analyze how crash risk affects the carry trade returns. The results suggest that higher crash risk in the TL discourages investors from arbitraging apparent mispricing away; investors demand compensation for bearing higher crash risk. Secondly, we capture balance sheet constraints of investors by controlling for strength of the US dollar using a broad dollar index. The idea behind using the broad dollar index is that the balance sheets of domestic borrowers with dollar liabilities become weaker during the periods of broad dollar appreciations[2] due to currency mismatch in their positions. The estimations are consistent with, although no direct proof of, the argument that investors can not take large enough positions in the TL due to binding budget constraints during broad dollar appreciation periods. Also, they demand higher premia to invest in the carry strategy when they are constrained in their borrowing capacity. We also investigate the association between global risk aversion and carry trade returns. We show that the carry returns increase during periods of high global risk aversion. Finally, we control for capital flows and liquidity conditions. But we could not find supporting evidence that availability of funds to investors or liquidity conditions contribute to explaining US$-TL carry trade returns.

In the last part of the paper, we derive unhedged and crash-hedged carry trade strategies using daily data for three different maturities. Since the only difference between these two strategies is that one of them is hedged against crash risk in the investment currency against the funding currency and the other is not, the comparison of the returns to these two different strategies indicates how much an investor demands for bearing currency crash risk. Currency crash risk can explain 62 percent of carry returns on average.

The rest of the paper is organized as follows. First, Section 2 reviews the literature on returns to carry trade. Then, in Section 3, we introduce and derive crash risk measures. Section 4 discusses the determinants of carry returns including currency crash risk. Then, we compare the unhedged and the crash-hedged carry returns in Section 5. Finally, Section 6 concludes the paper.

2 Related Literature

UIP implies that if the local currency interest rate in a country is higher than the other country's corresponding local currency rate, one should expect the high-interest currency to depreciate against the low-interest currency. However, following the earlier studies of Tryon [1979], Hansen and Hodrick [1980] and Fama [1984], many studies show that UIP fails to hold: see Froot and Thaler [1990] for a survey. Part of the puzzle can be explained by another anomaly: Eichenbaum and Evans [1995] and Scholl and Uhlig [2008] show empirically that the instantaneous appreciation that should precede this period of gradual depreciation [3] only occurs gradually over a significantly long period. In line with these findings in the literature, one often observes negative rather than a positive correlation between exchange rates and interest rate differentials. During that phase, carry traders not only enjoy high returns because of high interest rates but also gain from any appreciation of currency in which they invest, or, equivalently, from the depreciation of the currency in which they borrow.

In the literature, there are different explanations for this apparent arbitrage failure: apparently there is insufficient arbitrage to enforce UIP. One of them is currency crash risk. In a

[2] Broad dollar appreciation refers to appreciation of the US dollar against a broad range of currencies.
[3] "Dornbusch overshooting", cf Dornbusch [1976]

very early study, Krasker [1980] introduces what has since become known as the *peso problem* in the analysis of the Mark-Pound exchange rate during the German hyperinflation period. He shows that the forward market participants perceive that profit would be unusually large in the runup to a *drastic event* that only occurs with a small probability. He demonstrates that the presence of currency crash risk invalidates the standard test statistics used to empirically test the efficiency of forward markets. Incorporating the possibility of rare disasters, Farhi and Gabaix [2015] propose a richer model for exchange rate determination than the original Dornbusch overshooting model in Dornbusch [1976]. They show that riskier countries have higher interest rates so as to compensate for exchange rate depreciation *risk*, and offer that as an explanation of the well known forward premium puzzle. They also show that the countries with high interest rates also have high crash risk in their currencies since values of those currencies are expected to drop much more during a possible global disaster. As a result, investment in high interest rate currencies brings positive carry returns as long as the crash does not actually occur, and for more than the expected depreciation once the crash takes place, to compensate for bearing disaster risk. Brunnermeier et al. [2008] empirically support this prediction and argue that crash risk discourages speculators from taking sufficiëntly large positions to enforce UIP.

Another set of explanations of UIP failure depends not so much on a risk premium for crash risk but on liquidity shortages in currency markets. Brunnermeier and Pedersen [2008] introduced the concept of *liquidity spirals* to explain positive average return and negative skewness of investments of speculators. Liquidity spirals erupt when speculators, after hitting a funding constraint, close out their positions which then leads to a price drop which in turn further triggers increasing funding constraints of speculators who then close out more positions and so on. In a theoretical framework, Brunnermeier and Pedersen [2008] show that shocks tightening liquidity conditions of speculators are amplified but positive shocks to funding constraints are not. The positive average return on speculators' investments then constitutes a liquidity risk premium and negative skewness of returns is due to the asymmetry in shock amplification.

Brunnermeier et al. [2008] use global risk appetite as an indicator of tightness of global liquidity constraints. They show that when carry traders' risk appetite declines (as indicated by higher levels of VIX[4]), they unwind their positions in high interest rate currencies with, as a result, an increase in the price of insurance against crash risk and lower carry returns. They find that lower risk appetite predicts higher returns for high interest rate currencies. After they control for risk appetite, they obtain a less significant impact of interest rate differentials on carry returns which explains the apparent failure of UIP. Mancini et al. [2013] provide additional empirical support for liquidity spirals. They reveal that high interest rate currencies appreciate in a high liquidity environment while low interest currencies tend to depreciate. They argue that the different liquidity risk exposures of investment and funding currencies contribute to deviations from UIP.

Gabaix and Maggiori [2015] introduce an exchange rate model with imperfect financial

[4]VIX is the implied volatility calculated using S&P500 index options. VIX is generally used as a market based proxy for global risk appetite (Illing and Aaron [2005]).

markets. Carry traders earn positive returns when there are negative shocks to the financial system which lead to a decline in risk bearing capacity of financial intermediaries. In this way, carry traders are exposed to financial conditions and therefore demand a currency risk premium induced by their limited risk bearing capacity. Their model introduces *global imbalances* as a separate risk factor in currency risk premia: net external debtor countries' currencies have positive carry trade return. Corte et al. [2016] also show that net debtor countries offer a currency risk premium to compensate the loss of speculators since their currencies depreciate during global risk aversion periods.

Caballero and Doyle [2012] document that exchange rate options provide a cheap form of systemic insurance which leads to large returns on carry trade strategies with hedging. Although we do not focus on deviations from CIP, the factors which hinder the arbitrage opportunities from being exploited can also affect carry trade returns. Therefore, the literature on the role of limits-to-arbitrage in explaining deviations from CIP is also relevant. A recent study by Du et al. [2018] shows that deviations from CIP can be explained by constraints on financial intermediaries and international imbalances in investment demand and funding supply across currencies. Specifically, they analyze how capital requirements and other banking regulations such as the restrictions on proprietary trading and the introduction of liquidity coverage ratios affect excess returns compared to those implied by CIP. They also discuss limits to arbitrage of hedge funds, money market funds and foreign currency reserve managers and corporate issuers. Cenedese et al. [2019] particularly investigate the role of the maximum leverage ratio introduced by the Basel Committee after the global financial crisis on currency mispricing. They find a significant relation between the leverage ratio of dealer banks and recent violations of CIP. Using contract-specific data, they show that dealers with tighter balance sheets (higher leverage ratio's) demand higher premia from their clients.

Another strand of the literature focuses on slow moving capital into assets whose prices are below their fundamental values. The main argument is that price of an asset can sustainably deviate from its fundamental value because of limited availability of investor capital. Mitchell et al. [2007] discuss the role of slow moving capital in the convertible bond market in 2005 and arbitrage through merger in the stock market crash of 1987 in the US. They show that when external shocks reduce the capital of liquidity providers, turning them into liquidity demanders, it takes more time to restore equilibrium in a dislocated market. In an asset pricing framework, Duffie [2010] shows that asset prices can be distorted due to capital moving slowly to the relevant markets. He shows that the extended period of search for appropriate counterparties in the over-the-counter markets can be a source of delayed trades. Duffie [2010] also emphasizes the role of limits on capital market intermediation due to depleted balance-sheet capacity of the intermediaries: limited capacity of intermediaries leads to asset price distortions.

Finally, our paper relates to the literature on measuring currency crash risk. In the literature, only mesasured skewness and risk reversals are used to capture currency crash risk. Although skewness of realized exchange rate changes can be used to measure currency crash risk, it is a backward looking indicator. There are other methods to measure perceived currency crash risk and exact jump risk in currencies. For example, Olijslagers et al. [2019] use a variety of approaches to extract crash risk measures for the Euro from FX option prices to analyze the impact of unconventional monetary policy measures of the European Central Bank (ECB) on crash risk of the Euro. Specifically, they use three main measures: risk reversals, the crash probability implied by the option-price-derived risk neutral probability distribution and a measure derived from an explicit jump-diffusion option pricing model. Although risk reversals are suggestive, they are sometimes hard to interpret because of the non-linearity of vega, the sensitivity of option prices with respect to volatility. Therefore we use two alternative measures to measure crash risk in the US$-TL market: a crash risk measure calculated as the integral over the left tail of the risk neutral probability distribution derived from option prices at various degrees of moneyness and a jump risk measure derived from an explicit mixed jump-diffusion risk model.

3 Crash Risk Measures

In this section, we introduce the three different measures to capture currency crash risk: the risk reversal, the crash probability derived from the entire risk neutral probability distribution and the jump risk implied by an explicit jump risk model with exponentially distributed jump sizes and Poisson arrival rates, like in (Kou [2002] and Olijslagers et al. [2019]).

3.1 Risk Reversal

The risk reversal is the difference between two implied volatilities derived respectively from an out-of-the-money (OTM) call and from an OTM put option at the same delta (δ):

$$RR_\delta = \sigma_\delta^C - \sigma_\delta^P \tag{1}$$

We use a standard foreign currency option pricing model (see Garman and Kohlhagen [1983]) to derive the implied volatilities from observed option prices. The technical details are in Appendix A.1. The risk reversal measures the skewness of the implied distribution and can therefore be seen as an indicator of currency crash risk. When the risk reversal is positive, the implied volatility of an OTM call option is larger than for an OTM put option with the same δ (Figure 1). If the underlying distribution of exchange rate movements is symmetric, the implied volatilities are the same when evaluated for the same δ and the risk reversal then is zero. When the underlying asset is a foreign currency, a positive risk reversal can be interpreted as more market participants betting on appreciation of the foreign currency (equivalently, depreciation of the domestic currency) than depreciation (equivalently, appreciation of the domestic currency). Hence, the risk reversal can be used as an indicator for the crash risk of the domestic currency.

The data for risk reversals for the US$-TL options with different maturities are available in Bloomberg for four different delta values (10, 15, 25 and 35). Figure 2 displays the 10-delta risk reversal derived from 2-week US$-TL currency options between 2006 and 2019. The positive risk reversal throughout this period shows that the market participants expect depreciation more than appreciation in the TL against the US$. The rise in the risk reversal corresponds to

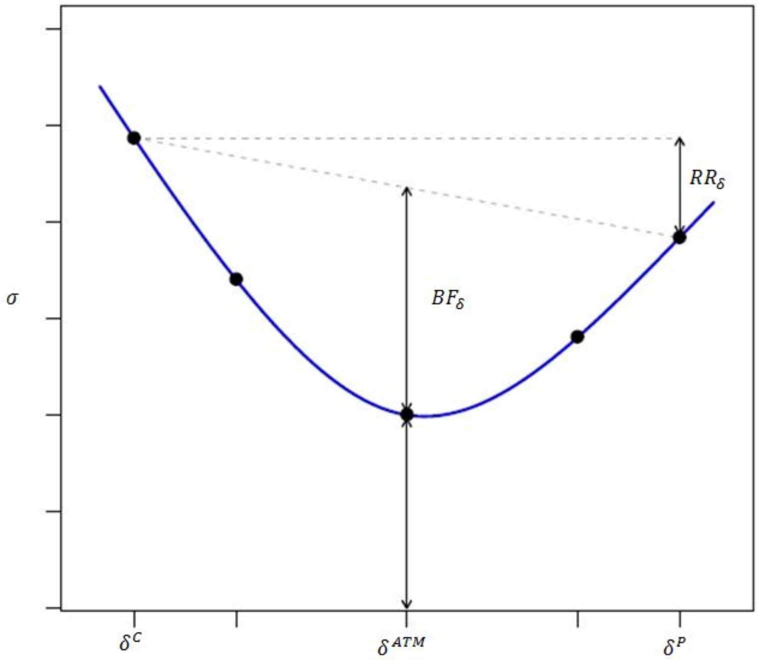


Figure (1) Graphical Illustration of Risk Reversal and Butterfly

higher expected crash risk within two weeks for the TL against the US$.

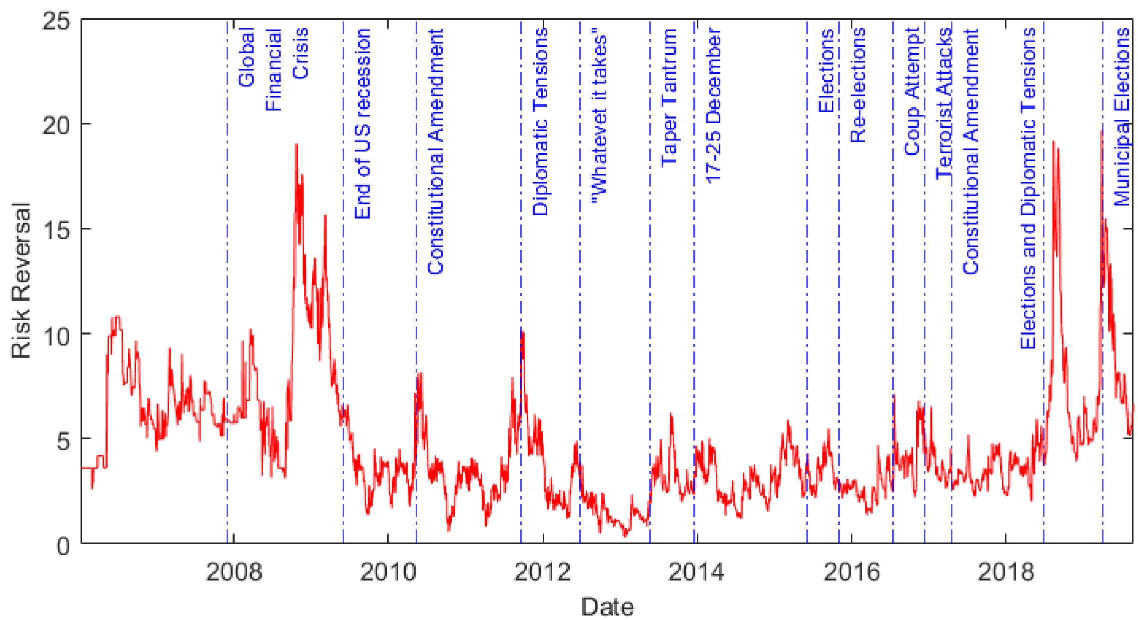


Figure (2) Risk Reversal, 2-Week 10-Delta US$-TL currency options
Source: Bloomberg

3.2 Currency Crash Risk derived from the Risk Neutral Probability Distribution (RNPD)

The risk reversal represents two specific points in the volatility smile. We can get a sharper focus on the currency crash risk by deriving the entire RNPD and using it to assess tail risk specifically. Although the data for implied volatilities for FX options are available for a long period of time, deriving RNPDs for underlying asset values using available data requires a lot of processing and different steps specific to different types of options and data structures. We explain how we derive RNPDs for the TL in Appendix A.2. After the derivation of daily RNPDs, we use them to compute currency crash probabilities by integrating the RNPD over the left tail.

We downloaded the daily US$-TL 1-month currency option data from Bloomberg for the period 2006 January - 2019 August. Although there are data available for 2004 and 2005, the quality and frequency of the data are not enough to include those years into the analysis. The data includes implied volatility of the at-the-money (ATM) option (σ_{ATM}), the risk reversal and the butterfly [5] at deltas 10, 15, 25 and 35 (RR_{10}, RR_{15}, RR_{25}, RR_{35}, BF_{10}, BF_{15}, BF_{25}, BF_{35}). Also, we downloaded spot TL/US$ exchange rates and 1-month interbank interest rates for Turkey and the US from FactSet.

Figure 3 shows the derived RNPDs for TL against US$ between 2006 and 2019 August using 1-month US$-TL currency options as shown in Equation 38 in Appendix A.2. Using the derived RNPDs, we can obtain the cumulative RNPDs (CRNPDs) as shown in Equation 39 in Appendix A.2. The CRNPDs allow us to compute tail risk for different crash probabilities. We define an x% crash probability in the TL at date t as the probability that the TL depreciates x% or more against the US$ in the following month[6] (Equation 3). Figures 4 to 6 shows the calculated crash probabilities in the TL against US$ during the sample period.

$$CrashProb_t^{x\%} = 1 - CRNPD_t\Big(S_t\big(1 + \frac{x}{100}\big)\Big) \tag{3}$$

3.3 Currency Options Pricing Incorporating Jump Risk

For many FX return distributions some of the assumptions made sofar do not hold. First, the Black-Scholes Model assumes that returns are normally distributed but empirically returns often have an asymmetric leptokurtic distribution (Kou, 2002), i.e., i.e. the return distribution is left-skewed with a higher peak and fatter tails than the normal distribution. Second, the Black-Scholes Model assumes that the volatility is constant across strike prices; obviously non-zero risk reversals are at variance with that assumption. In fact the option data show that implied volatility is a convex function of strike prices.

[5] Butterfly:

$$\text{BF}_\delta = \frac{\sigma_\delta^C + \sigma_\delta^P}{2} - \sigma_{\text{ATM}} \tag{2}$$

[6] Since we use 1-month currency option prices, the implied probability by the RNPD shows the perceived exchange rate change for the next month.

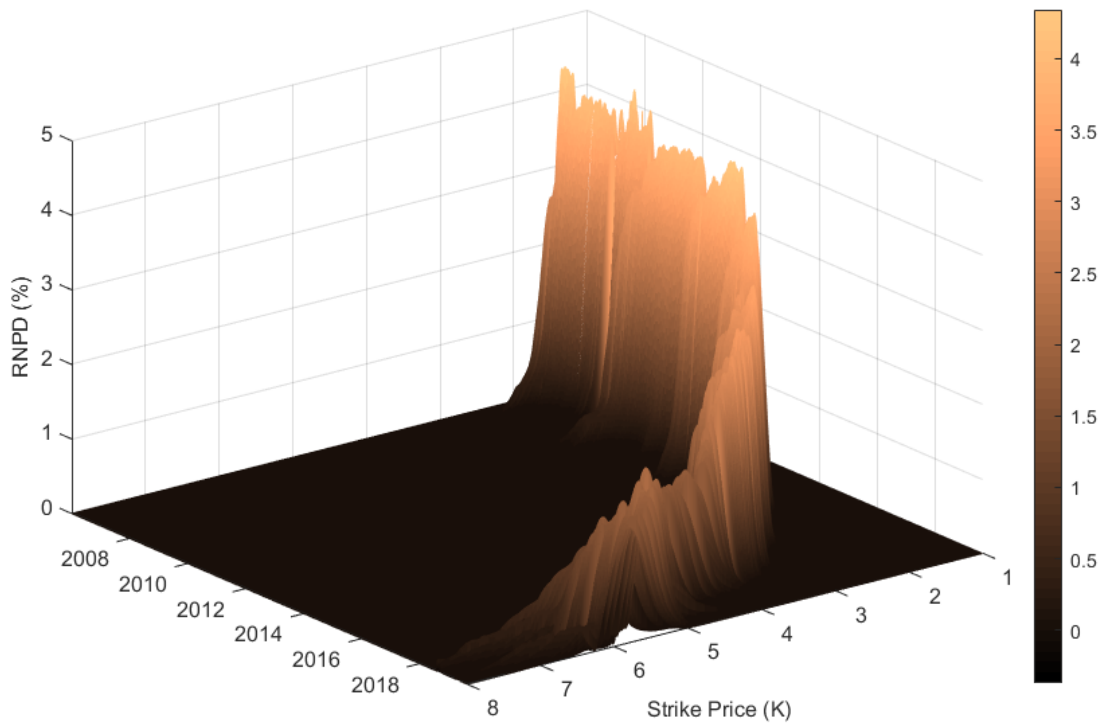


Figure (3) Risk Neutral Probability Distribution for TL Against US$
Source: Bloomberg and Authors' calculation

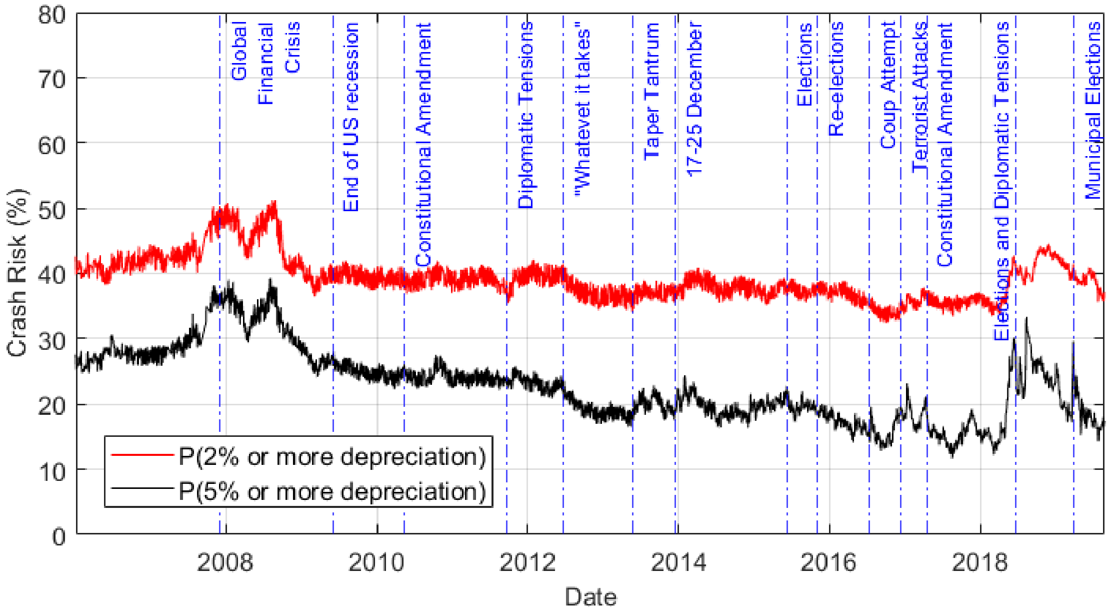


Figure (4) Crash Risk in the TL (2 and 5 percent)
Source: Bloomberg and Authors' calculations

In order to address these problems, Kou [2002] proposes a double exponential jump-diffusion model. The introduction of jump risk fits the presumption of a potential crash much better. And the model allows for analytical expressions of the option prices. Moreover, the distributional assumptions of the model fit the asymmetric leptokurtic features of empirical returns. Therefore

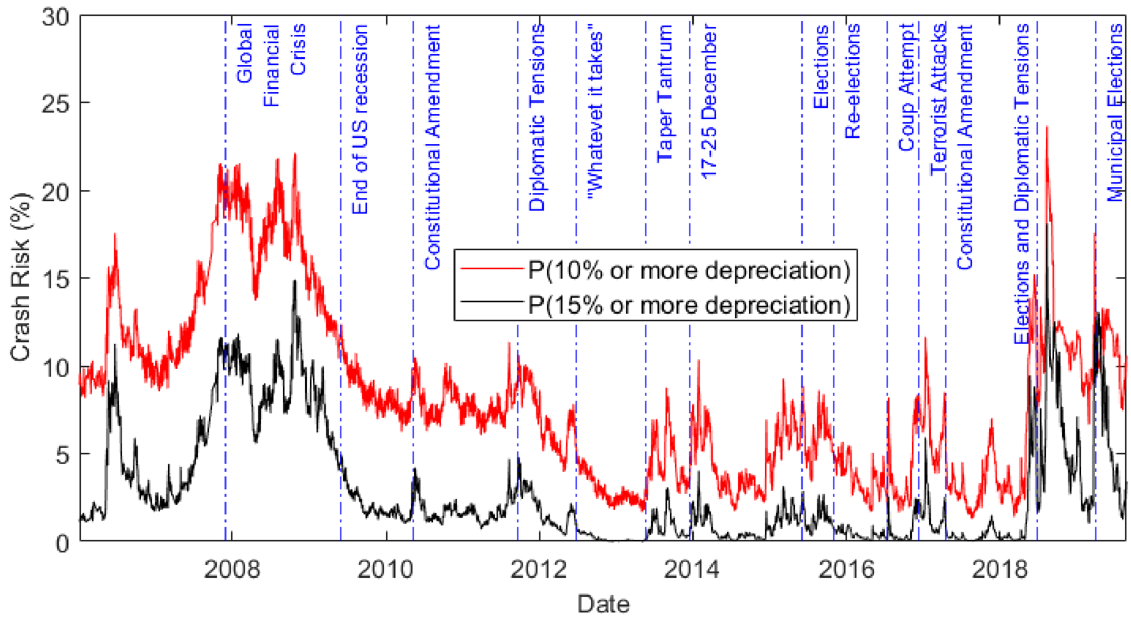


Figure (5) Crash Risk in the TL (10 and 15 percent)
Source: Bloomberg and Authors' calculations

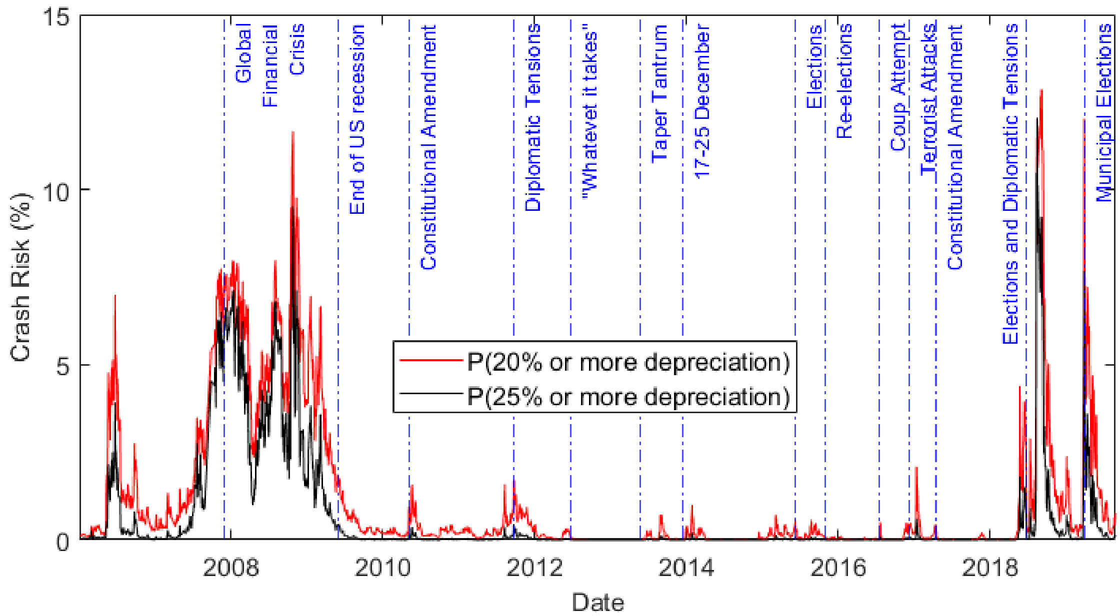


Figure (6) Crash Risk in the TL (20 and 25 percent)
Source: Bloomberg and Authors' calculations

we use the Kou Model to create our third currency crash risk measure to calculate daily the exact jump risk in the currency.

Under the Kou Model, the asset price at time t has the dynamics given in Equation 4. W(t) is a standard Brownian motion and $N(t)$ is a Poisson process with arrival rate λ. $\{V_i\}$ is a sequence of independent identically distributed non-negative random variables such that $Y = log(V)$ has an asymmetric double exponential distribution with the following density:

$$\frac{dS(t)}{S(t-)} = \mu dt + \sigma dW(t) + d\Bigg(\sum_{i=1}^{N(t)}(V_i - 1)\Bigg) \tag{4}$$

$$f_Y(y) = p\eta_1 e^{-\eta_1 y}\mathbb{1}_{y\geq 0} + q\eta_2 e^{\eta_2 y}\mathbb{1}_{y<0} \quad \eta_1 > 1, \eta_2 > 0 \tag{5}$$

where $p \geq 0$ represents the probability of an upward jump and $q \geq 0$ represents the probability of downward jump, while $p + q = 1$. $\mathbb{1}_{y\geq 0}$ is a zero-one index equal to one if $y \geq 0$ (an upward jump); similarly, $\mathbb{1}_{y<0}$ equals 1 if $y < 0$ (a downward jump). η_1 and η_2 are parameters for upward and downward jumps respectively. Since $Y = log(V)$, we can re-write the distribution of Y as follows:

$$log(V) = Y \stackrel{d}{=} \begin{cases} \xi^+, & \text{with probability } p \\ \xi^-, & \text{with probability } q \end{cases} \tag{6}$$

where ξ^+ and ξ^- are exponential random variables with mean $\frac{1}{\eta_1}$ and $\frac{1}{\eta_2}$ respectively.

Solving the stochastic differential equation in 4 leads to the following asset price formula:

$$S(t) = S(0)exp\Bigg\{(\mu - \frac{1}{2}\sigma^2)t + \sigma W(t)\Bigg\}\prod_{i=1}^{N(t)} V_i \tag{7}$$

We assume that $N(t)$, $W(t)$ and Y are independent, and that the drift μ and the volatility σ are constant. In this set-up, $(V - 1)$ is the percentage change of the asset price after a jump and $\mathbb{E}(V)$, the expected value of V, equals:

$$\mathbb{E}(V) = \mathbb{E}(e^Y) = q\frac{\eta_2}{\eta_2 + 1} + p\frac{\eta_1}{\eta_1 - 1}, \quad \eta_1 > 1 \text{ and } \eta_2 > 0 \tag{8}$$

To obtain the daily jump risk in domestic currency we need both the probability of a jump and the expected jump size. We make some simplifying assumptions. First, we assume that only one jump ($n = 1$) can take place during the duration of a currency option. Since we use 1-month, 3-month and 6-month US$/TL currency option data, this assumption implies that only one jump can occur within a given six-month period. Second, we assume that only an upward jump can take place, i.e. by assumption the TL can only jump in a depreciation direction against the US$. This implies that $p = 1$ and $q = 0$.

Under these assumptions the expected jump size ($\mathbb{E}(V - 1)$) and the probability of having no-jumps ($n = 0$) during the duration of an option (π_0) are given in Equations 9 and 10, respectively:

$$\mathbb{E}(V - 1)|_{q=0,p=1} = \frac{\eta_1}{\eta_1 - 1} - 1 = \frac{1}{\eta_1 - 1} \tag{9}$$

$$\pi_0 = P^*\big(N(\tau) = 0\big) = e^{-\lambda\tau} \tag{10}$$

Therefore, the probability of having one jump ($n = 1$) during the duration of an option becomes:

$$\pi_1 = 1 - \pi_0 = 1 - P^*\big(N(\tau) = 0\big) = 1 - e^{-\lambda\tau} \tag{11}$$

We define $JumpRisk_t$, the jump risk at time t in a currency, as the product of the probability of a jump (π_1) and the expected jump size ($\mathbb{E}(V-1)$) given that one occurs in equation 12 below:

$$JumpRisk_t = \mathbb{E}_t(V-1) \cdot \pi_{1,t} = \frac{1}{\eta_{1,t} - 1}\big(1 - e^{-\lambda_t \tau}\big) \tag{12}$$

Finally we need to estimate the parameters σ, λ and η_1. Olijslagers et al. [2019] show that distinguishing between diffusion risk and jump risk is difficult: several combinations of σ and λ give a comparably good fit so we have an identification problem. Therefore we use historical time series data of the pre-crisis period to fix the volatility of the diffusion process and find σ equal to 0.141171; this is the average annualized volatility of the pre-crisis period between 2002 and 2007. In Appendix A.3, the call price equation adapted to currency options is derived explicitly we we explain how λ and η_1 are estimated. This leads to the series for $JumpRisk_t$ shown below in Figure 7:

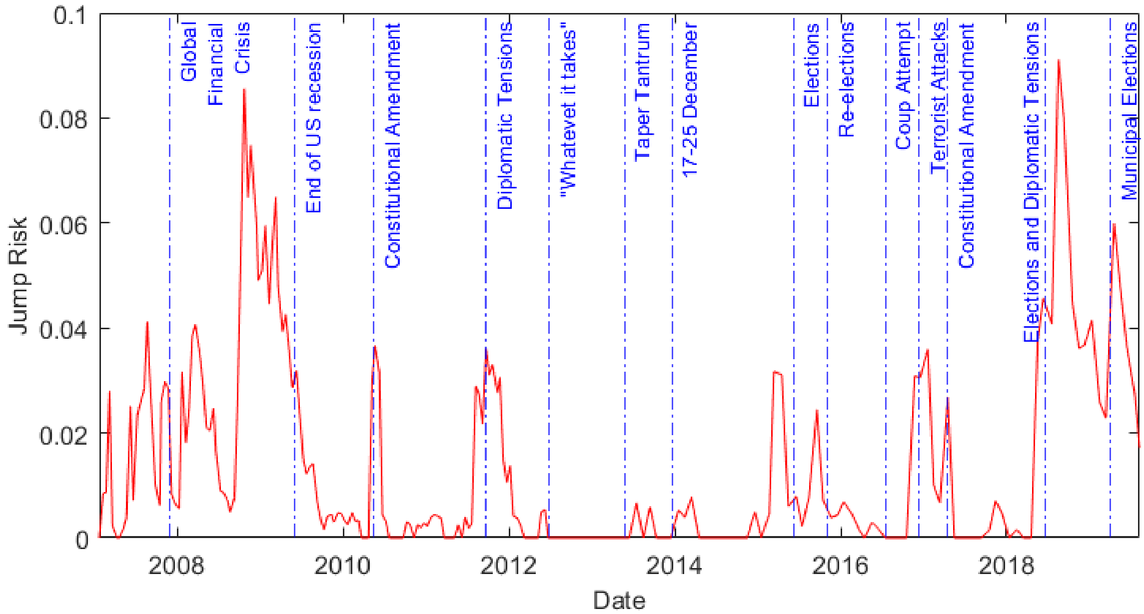


Figure (7) Jump Risk in the TL against the US$ Implied by the Kou Jump-Diffusion Option Pricing Model
Source: Bloomberg, FactSet and Authors' calculations

3.4 Major Events and Crash Risk Measures

Figure 2 to 7 show that all three crash risk measures react to both global and local events which might have an effect on the TL. We see peaks in the crash risk measures during the global financial crisis from December 2007 until the end of US recession, June 2009. And another crash risk jump in May 2010 after the approval of series of constitutional amendments by the President of Turkey. Later on, we observe high levels of crash risk around September

2011 due to diplomatic tensions between Turkey and Israel.

After the ECB's then president Mario Draghi's famous "whatever it takes" speech in June 2012, the crash risk measures decline and stay at very low levels, presumably because of the easing of monetary policy in the Euro Area that followed. This low-crash-risk period lasts until the speech of Federal Reserve Chairman Ben Bernanke in May 2013 in which he announced that the Federal Reserve System (Fed) is likely to start slowing the monthly pace of bond purchases later in 2013. We observe some jumps in the crash risk measures after the monetary policy tightening signals in the US. In the remainder of that year, some increases in the crash risk measures are observed in response to domestic political developments.

The rest of the period includes several elections and re-runs. It is clear that investors see the elections as a risk for the TL: the crash risk increases during the period of political uncertainty. In the summer of 2018, the crash risk measures reach their peak after the presidential elections and the emergence of diplomatic tension between the US and Turkey. It is apparent that the three crash risk measures respond in intuitive ways to both domestic and foreign events between 2006 and 2019 that are plausibly of relevance for crash risk in the TL. We conclude that they apparently are good proxies for the currency crash risk in the TL.

4 Carry Trade Return and Crash Risk

4.1 Carry Trade Returns

A carry trade strategy involves borrowing in low interest currencies to fund investments in high interest currencies. We look at the US\$-TL strategy and analyze the role crash risks play in such strategies. There are several attractive features of such a US\$-TL carry trade strategy. The first is the consistently positive interest differential: TL always offers higher interest rates than US\$ returns on comparable instruments (Figure 8). The persistently high interest rate differential between the TL and the US\$ makes the TL an attractive candidate investment currency for a carry trade strategy. Second, Turkey follows a floating exchange rate regime since the 2001 economic crisis in Turkey. Due to high volatility in the TL, carry traders' perceived crash risk is likely to play an important role in their investment decision. If investors actually require a currency crash risk premium, it will reveal itself in the US\$-TL investment strategy. Hence, measuring currency crash risk in the TL allows us to analyze the role of crash risk in carry trade returns. Also, the availability of detailed and relatively high frequency currency options data since 2006 allows us to derive all the different crash risk measures we have discussed in the previous subsections. Finally, direct measures of investors' expectations about TL exchange rate developments exist which enables us to calculate expected carry returns.

Let $r^{TL}_{t-1,t}$ and $r^{US\$}_{t-1,t}$ denote the risk-free interest rate between $t-1$ and t in the TL and US\$, respectively. The spot exchange rate S_t is expressed in units of TL per US\$ at time t, so an increase in S_t indicates a depreciation of the TL against the US\$. $\mathbb{E}_{t-1}(S_t)$ represents the exchange rate expected at $t-1$ to prevail at t. Equation 13 shows ECR_t, the expected carry trade return at time t, as deviation from the UIP condition.

An investor borrowing 1 US\$ at time $t-1$ ows $(1+r^{US}_{t-1,t})$ US\$ at time t. But the investor

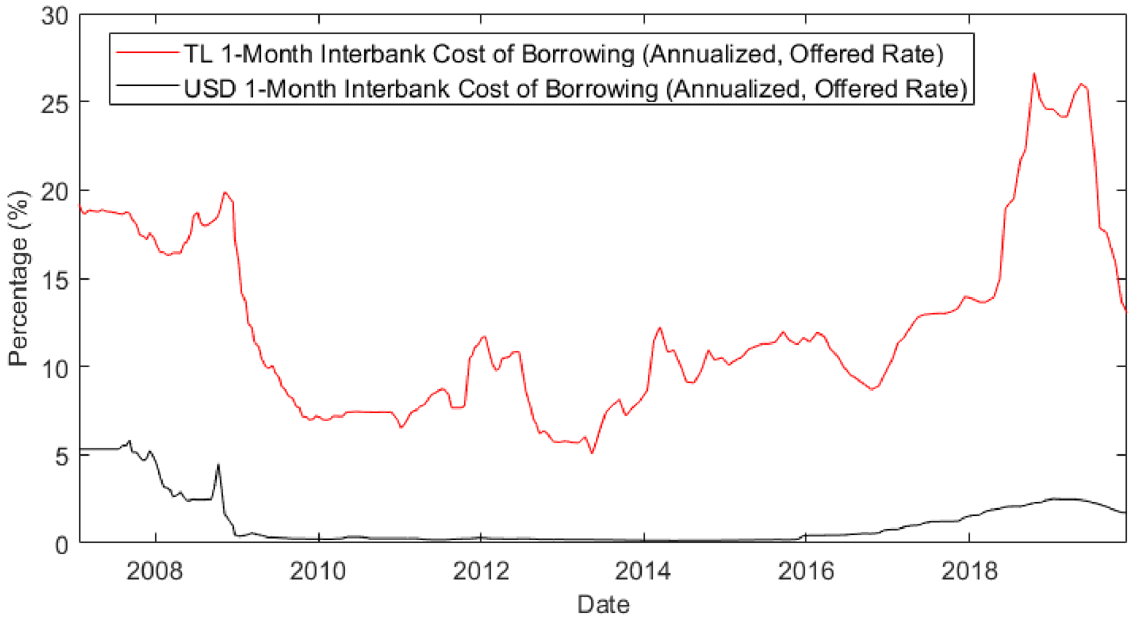


Figure (8) **1-Month Interbank Cost of Borrowing in the TL and the US$**
Source: FactSet

can alternatively convert his/her 1 US$ into S_{t-1} TL at time$t-1$, own $\left(1+r_{t-1,t}^{TL}\right)S_{t-1}$ TL at time t and convert that amount back into dollars at the exchange rate at time t for an overall *ex post actual* return $\left(1+r_{t-1,t}^{TL}\right)\frac{S_{t-1}}{S_t}$ US$. But the investor at time $t-1$ does not know the value of the exchange rate at time t, so s/he has to take the investment decision based on the expected value of the exchange rate at $t-1$, which gives the following expression for the *expected* carry trade return:

$$ECR_t = \left(1+r_{t-1,t}^{TL}\right)\frac{S_{t-1}}{\mathbb{E}_{t-1}(S_t)} - (1+r_{t-1,t}^{US\$}) \tag{13}$$

The general approach in empirical carry trade return analysis is to assume that speculators have rational expectations and substitute the actual exchange rate (S_t) instead of the usually unobservable *expected* exchange rate ($\mathbb{E}_{t-1}(S_t)$) when calculating the carry returns used in empirical analysis. Call the carry return using actual exchange rates RCR, for Realized Carry Return:

$$RCR_t = \left(1+r_{t-1,t}^{TL}\right)\frac{S_{t-1}}{S_t} - (1+r_{t-1,t}^{US\$}) \tag{14}$$

Using actual rates to proxy for expected rates implies that the Realized Carry Return RCR is used as a proxy for ECR. But since the actual exchange rate is not known at time $t-1$, this assumption introduces a measurement error in Equation 13.

However in this study, we do not need to rely on this simplifying assumption and so can avoid this measurement error: we use the Central Bank of Turkey's (CBT) Survey of Expectations as a direct measure of investors' exchange rate expectations. The CBT conducts the survey monthly and monitors expectations of market participants and experts about financial markets and real sectors [7]. In questionnaires, the participants are asked about their expectations of the US$ rate at the end of the current month, at the end of the year and at the end of the next 12 months in the interbank foreign exchange market. Since carry trades are assumed to have a short-term investment horizon, we use the measure of end-of-month exchange rate expectations, assuming that the carry investment takes place at the survey day and is held until the end of corresponding month (Figure 9). The average investment horizon between 2007 January and 2019 August is 15 days.

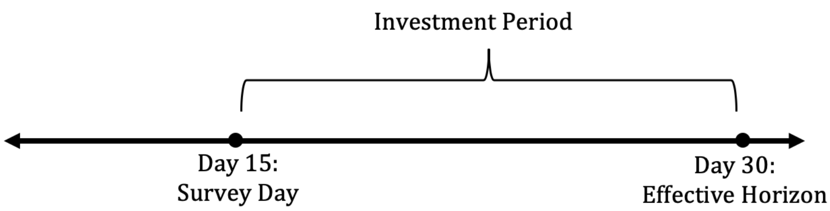


Figure (9) Investment Timeline: For each survey date, the expected carry return is calculated assuming that carry investment takes place at the survey day and held until the end of corresponding month. In this strategy, TL/US$ expectation of participants at the end of the month is taken into account. The survey dates are not exactly at the 15^{th} day of each of month but the investment horizon between 2007 January and 2019 August is 15 days on average.

Figure 10 shows the annualized expected carry return of this 15-day investment strategy over the period from January 2007 until August 2019. The average expected carry return is 8.37 percent over that period and is statistically different from zero and positive[8].

Figure 11 shows histogram and kernel density representations of both the expected and realized carry returns over the sample, period. The histograms and Kernels show that both the expected and realized carry trade return distributions have positive skew: the skewness of the expected and realized carry trade return distributions are 3.1 and 1.05 respectively: the expected carry return distribution has a higher skew. The only difference between the expected and realized carry returns is the difference between exchange rate expectations and the actual exchange rate realizations, we can interpret the higher positive skewness in the expected carry return distribution to imply that carry trade investors are likely to demand risk premia for the crash risk in the TL.

The expected carry return has higher skewness and a lower mean and median compared to the realized carry return. This suggests that investors' exchange rate expectations about the TL are higher (less valuable TL) on average compared to the realized *ex post* exchange rate over the sample period; so investors are pessimistic about the value of TL. The right skew is in the same direction: apparently investors lean more towards higher than to lower than expected depreciation in the $TL - US$ rate. Therefore, carry trade investors are likely to demand risk premia for the crash risk in the TL.

[7]The survey frequency before 2013 was twice monthly. Since 2013, the survey has been released once in a month.

[8]The standard deviation is 0.519 for 224 observations so the t-value is 2.415.

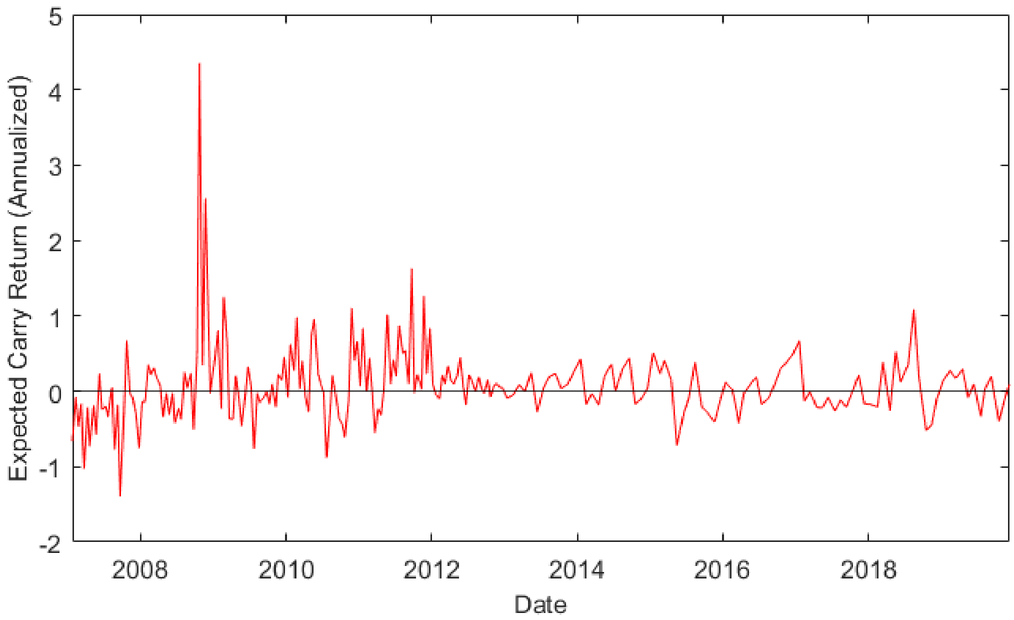
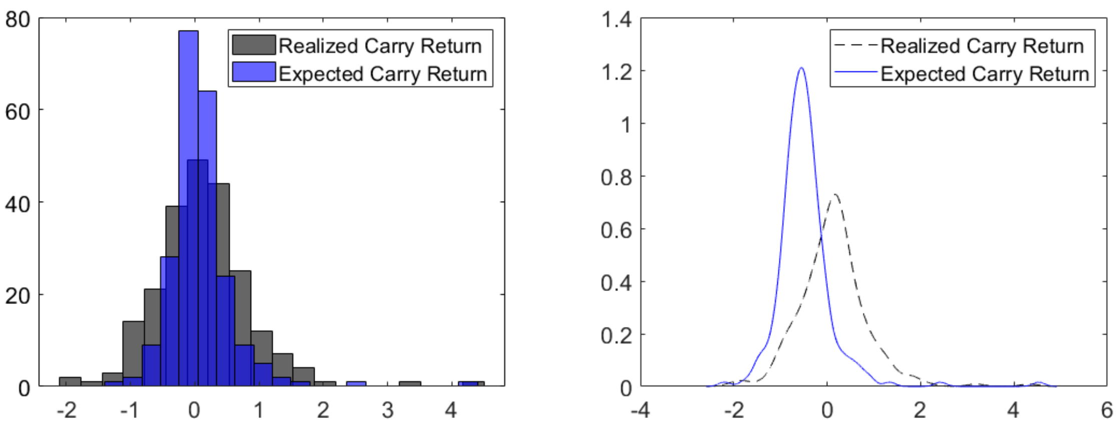


Figure (10) Expected Carry Return: This figure shows annualized expected carry return defined in Equation 13 using 1-month interbank offered cost of borrowing in the TL and the US$ for an 15-day investment strategy. The carry return for each period is calculated assuming the investment occurs at the survey date and held until the end of the corresponding month. The average investment horizon is 15 days. The returns are expressed as fractions. Source: CBT, FactSet and Authors' calculations



((a)) Histogram

((b)) Kernel Density Estimate

Figure (11) Histogram and Kernel Density Estimate of Realized and Expected Carry Returns: This figure shows histogram and kernel density estimates of realized and expected carry returns of the US$-TL carry strategy. Realized carry return is calculated as shown in Equation 14 using the realized exchange rate S_t instead of expected exchange rate. Source: CBT, FactSet and Authors' calculations

4.2 Expected Carry Returns and Currency Crash Risk

We next consider the relation between the expected carry return and the currency crash risk measures using the time series data discussed in the earlier sections. We expect to see that higher crash risk in the TL against the US$ is positively correlated with the expected carry return since investors can be expected to demand compensation via a risk premium for the currency crash risk they are exposed to.

We regress expected carry returns on changes in the three different crash risk measures one by one: the risk reversals (Section 3.1), the currency crash probability implied by the integral of the left tail of the RNDP (Section 3.2) and the explicit jump risk measure derived from the Kou Model (Section 3.3). The regression specification is given by Equation 15 below. ECR_t shows the expected carry return between *time t-1* and *t*. The variable ΔCCR_t indicates changes in currency crash measures: change in 10-delta 2-week risk reversal in the US$-TL options ($\Delta RR_t$), the change in the risk of a TL crash of 20% or more (ΔCP_t^{20}) and jump risk in the TL ($\Delta JumpRisk_t$).

$$\begin{aligned} ECR_t =& \alpha_1 + \alpha_2 \Delta CCR_{t-1} + \alpha_3 \Delta BDI_{t-1} + \alpha_4 Debt\&EquityFlow_{t-1} \\ &+ \alpha_5 VIX_{t-1} + \Delta BidAskSpread_{t-1} + u_t \end{aligned} \tag{15}$$

We also include potential drivers of the carry returns as control variables to avoid omitted variable bias. Avdjiev et al. [2019] and Du et al. [2018] show that limits on bank leverage play an important role in the failure of CIP. During binding balance sheet constraints, investors are not able to borrow at the lower interest rate and lend out at the higher interest rate to exploit any arbitrage opportunities. Although our focus is not on the violations of the no-arbitrage conditions implied by violations of CIP, binding balance sheet constraints of banks are likely to play a role for UIP violations too. Bruno and Shin [2015] show that after an appreciation of the US dollar, domestic borrowers' balance sheets become weaker because of currency mismatch. This leads to an increase in the global bank's perception of riskiness of domestic borrowers and a decrease in domestic bank lending capacity. Therefore, the supply of available US dollar credit to domestic borrowers declines after appreciation of dollar[9]. We expect to observe that a stronger US dollar leads speculators to demand higher risk premia when investing in carry strategies with higher carry trade returns as a consequence. In order to capture the strength of dollar, we use a US trade weighted nominal effective exchange rate broad dollar index as a proxy of limits on bank leverage due to exchange rate related debt overhang. The data are from the Federal Reserve Board Statistics. ΔBDI_t denotes changes in the broad dollar index (BDI): an increase in the BDI indicates appreciation in the US$ against all trading partners' currencies in weighted-average terms.

Several authors have identified slow-moving capital as a cause of temporary mispricing in financial markets (Fleckenstein et al. [2014], Brunnermeier and Pedersen [2008] and Duffie [2010]). According to this hypothesis we expect to see that net larger inflows into Turkey before the investment date have a negative impact on expected carry returns and vice versa. We use weekly net debt and equity flows into Turkey provided by the Institute of International Finance.[10] We do not take capital flows in the same week as the investment but look at the prior week's net inflow, which should also avoid endogeneity bias problems. $Debt\&EquityFlow_t$ equals the net debt and equity inflows into Turkey in million US$ during one week prior to time

[9] For detailed discussion of the mechanism between strength of the dollar and CIP deviations see Avdjiev et al. [2019] and Bruno and Shin [2015].

[10] The debt and equity flows to Turkey in daily frequency are not available. Therefore, we use the data in weekly frequency.

t.

Brunnermeier et al. [2008] and Brunnermeier and Pedersen [2008] also suggest a different reason for the apparent persistence of unexploited investment opportunities. Since speculators make investments on behalf of their clients, there might be a principal agent problem leading to limited arbitrage. Speculators are more likely to unwind their carry positions during periods with losses, higher margins, higher risk aversion and capital redemption. These funding constraints are likely to be more binding during periods of higher risk aversion or less risk appetite. Thus we expect expected carry returns to be higher in periods of lower global risk appetite since it then becomes harder to borrow in foreign markets (in the US$) to invest in domestic markets (in the TL). In order to capture these risk appetite related funding constraints of investors on the expected carry returns, we include the Chicago Board Options Exchange (CBOE) equity volatility index VIX_t which is the implied volatility calculated from S&P500 index options. The VIX is widely believed to capture risk appetite as it consistently exceeds the actual implied volatilities derived from the same basket (cf Bekaert and Hoerova [2014]). We obtain the data using FactSet. Higher VIX values imply lower global risk appetite.

Finally any excess return in the US$-TL market can also be due to a liquidity premium. Mancini et al. [2013] show that high interest currencies have a positive exposure to liquidity risk and those currencies tend to appreciate during better liquidity periods. On the other hand, low interest rate currencies offer insurance against liquidity risk so that those tend to depreciate during higher liquidity periods. In order to control for the effect of liquidity conditions, we include the bid-ask spread ratio ($BidAskSpread_t$) in the spot exchange rate (the difference between ask and bid price in the spot exchange rate divided by the mid-exchange rate (Equation 16)).

$$BidAskSpread_t = \frac{S_t^{Ask} - S_t^{Bid}}{S_t^{Mid}} \tag{16}$$

Higher bid-ask spreads in the TL/US$ rate imply higher trading costs so higher illiquidity in the market. We expect investors to be compensated by higher returns if there is higher illiquidity.

Table (1) shows our first set of regression results, with the risk reversals as a measure of crash risk (ΔRR_{t-1}). We regress the expected carry return (ECR_t) on ΔRR_{t-1} and other control variables. The coefficients of ΔRR_{t-1} are positive and significant across all specifications. The estimated coefficient of ΔRR_{t-1} in column (1) suggests that a 1-unit increase in the change in the risk reversals leads to a 13.1-percentage-point increase in the expected carry return. When we include all other control variables in column (5), we find that a 1-unit increase in change in the risk reversals is associated with a smaller but still sizable and significant 8.12-percentage-point increase in the expected carry return. Also, the change in the broad dollar index has a significant and positive coefficient. It suggests that increase in the strength of the US dollar leads to higher expected carry returns, which is consistent with our prediction that an appreciating US$ will trigger increased risk premia on carry trades. The debt and equity flows and the bid-ask spread do not enter the regression significantly but the estimated coefficient for the VIX index is positive and significant. This suggests that an increase in VIX, which indicates lower risk appetite globally, is associated with higher expected carry returns: apparently investors demand a higher premium during periods of high risk aversion. Equivalently, it suggests that lower global risk aversion makes it easier to borrow from foreign markets and invest in domestic markets so it is associated with lower expected carry returns. But this regression does not find evidence for the slow moving capital hypothesis and there is no significant impact of our measure of liquidity: neither $Debt\&EquityFlow_t$ nor $BidAskSpread_t$ enter significantly .

Table (1) **Regression of Expected Carry Returns on the Risk Reversal and Other Control Variables:**

	(1)	(2)	(3)	(4)	(5)
ΔRR_{t-1}	0.131***	0.0840***	0.0829***	0.0798***	0.0812***
	(3.70)	(3.50)	(3.54)	(4.24)	(4.39)
ΔBDI_{t-1}		0.132***	0.127***	0.106***	0.105***
		(4.34)	(4.31)	(4.58)	(4.36)
$Debt\&EquityFlow_{t-1}$			-0.0000472	-0.0000321	-0.0000347
			(-1.06)	(-0.81)	(-0.92)
VIX_{t-1}				0.0172***	0.0174***
				(3.38)	(3.49)
$BidAskSpread_{t-1}$					-27.71
					(-0.62)
Constant	0.0842***	0.0731***	0.0765***	-0.286***	-0.267**
	(2.72)	(2.64)	(2.66)	(-2.91)	(-2.25)
N	223	223	223	223	223
adj. R^2	0.213	0.325	0.325	0.446	0.446

This table reports summary statistics for the regression of the expected carry return (ECR_t) on the change in the risk reversal (ΔRR_{t-1}). ECR_t shows the expected carry return defined in Equation 13. ΔRR_{t-1} denotes change in the risk reversal for 2-Week, 10-delta US$-TL options. An increase in the risk reversal implies higher hedging cost and larger crash risk in the TL. ΔBDI_{t-1} denotes changes in the US trade weighted nominal effective exchange rate broad dollar index and an increase in the index shows appreciation in the US$ against all trading partners' currencies. $Debt\&EquityFlow_{t-1}$ shows the net debt and equity inflows to Turkey in million US$ during one week prior to time t. VIX_{t-1} is the implied volatility calculated using S&P500 index options. Higher VIX values imply lower global risk appetite. $BidAskSpread_{t-1}$ is defined as a ratio of bid-ask spread in spot exchange rate divided by the mid-exchange rate (Equation 16). *t-statistics* are in parentheses and are calculated using heteroscedasticity consistent standard errors. * ($p < 0.10$), ** ($p < 0.05$), *** ($p < 0.01$).

We repeat the same regression but now using the currency crash risk measure implied by the integral of the left tail of the implied RNPDs (ΔCR^{20}_{t-1}). The regression results are given in Table 2. The estimated coefficients for the change in the crash probability ΔCR^{20}_{t-1} are again positive and significant across all specifications. The estimated coefficient of ΔCR^{20}_{t-1} in column (1) indicates that a 1-percentage-point increase in the change in the crash probability integral leads to 15.2-percentage-point increase in the expected carry return. Including the other control variables reduces that coefficient to a still significant 11.9-percentage-point increase in the expected carry return. Like in Table 1, both ΔBDI_{t-1} and VIX_{t-1} enter the regression significantly but neither $Debt\&EquityFlow_t$ nor $BidAskSpread_t$ enter significantly.

Finally, Table 3 shows the results of the regression in which the change in the jump risk implied by the Kou Model ($\Delta JumpRisk_{t-1}$) is used as a currency crash risk measure. The estimated coefficients for $\Delta JumpRisk_{t-1}$ are again positive and significant across all specifications. The estimated coefficient of $\Delta JumpRisk_{t-1}$ in column (1) suggests that a 1 point increase in the change in jump risk leads to a 19.68-percentage-point increase in the expected carry return.

Table (2) **Regression of the Expected Carry Return on Crash Probability and Other Controls**

	(1)	(2)	(3)	(4)	(5)
ΔCP^{20}_{t-1}	0.152**	0.109**	0.109**	0.115***	0.119***
	(2.45)	(2.41)	(2.44)	(3.29)	(3.42)
ΔBDI_{t-1}		0.157***	0.150***	0.126***	0.124***
		(4.94)	(4.95)	(5.37)	(5.14)
$Debt\&EquityFlow_{t-1}$			-0.0000595	-0.0000431	-0.0000469
			(-1.37)	(-1.14)	(-1.30)
VIX_{t-1}				0.0180***	0.0183***
				(3.76)	(3.86)
$BidAskSpread_{t-1}$					-37.76
					(-0.97)
Constant	0.0839**	0.0708**	0.0751***	-0.303***	-0.279**
	(2.57)	(2.54)	(2.60)	(-3.25)	(-2.53)
N	223	223	223	223	223
adj. R^2	0.127	0.317	0.319	0.452	0.453

This table reports summary statistics for the regression of the expected carry return (ECR_t) on the change in the crash probability (ΔCP^{20}_{t-1}). ΔCP^{20}_{t-1} is the change in currency crash probability implied by the RNPD defined in Equation 3 in percentage terms. ΔBDI_{t-1} denotes changes in the US trade weighted nominal effective exchange rate broad dollar index; an increase in the index shows appreciation in the US$ against all trading partners' currencies. $Debt\&EquityFlow_{t-1}$ shows the net debt and equity inflows to Turkey in million US$ during one week prior to time t. VIX_{t-1} is the implied volatility calculated using S&P500 index options. Higher VIX values imply lower global risk appetite. $BidAskSpread_{t-1}$ is defined as the ratio of the bid-ask spread in spot exchange rate divided by the mid-range exchange rate (cf Equation 16). *t-statistics* are in parentheses and are calculated using heteroscedasticity consistent standard errors. * ($p < 0.10$), ** ($p < 0.05$), *** ($p < 0.01$).

When we include all other control variables in column (5), the impact is still significant but much lower: 10.61- percentage-point increase in the expected carry return after a 1 point increase in the change in jump risk. Similar to the previous two sets of regression results (Table 1 and 2), the change in the broad dollar index has a positive and significant coefficient; the broad dollar index -the proxy of limits on bank leverage- is apparently robustly and positively associated with expected carry returns. Moreover, the significant coefficients estimated for VIX_{t-1} imply that the positive relation between the VIX and the carry trade return is also robust to different specifications. On the other hand, we again could not find evidence supporting a liquidity premium effect nor support for the slow-moving capital argument.

In order to understand the importance of the crash risk measures in the explanation of (changes in) the expected carry return, we complement the regression results by a Shapley Decomposition Analysis (Huettner et al., 2012). This allows us to calculate the individual contribution of each explanatory variable to the R^2, the goodness of fit. This analysis shows the relative importance of one explanatory variable compared to others in explaining Expected Carry Returns. Table 4 shows the Shapley R^2 values for each variable in the regressions that include all control variables (i.e. corresponding to the column (5)'s in Table 1 to 3). According to these Shapley R^2 values, 28.78 percent, 21.27 percent and 21.59 percent of the explained variance in the expected carry trade return can respectively be attributed to the risk reversals, the crash risk probability or the jump risk (column (2), (4) and (6)). These results indicate that currency crash risk is an important driver of the expected carry return. Furthermore the broad dollar index's contribution to goodness of fit is more than one-third in all specifications, Table (3) **Regression of the Expected Carry Return on Jump Risk and Other Control Variables**

	(1)	(2)	(3)	(4)	(5)
$\Delta JumpRisk_{t-1}$	19.68***	11.33***	11.06***	10.44***	10.61***
	(4.01)	(3.26)	(3.27)	(3.25)	(3.38)
ΔBDI_{t-1}		0.144***	0.139***	0.119***	0.118***
		(4.04)	(4.07)	(4.65)	(4.43)
$Debt\&EquityFlow_{t-1}$			-0.0000452	-0.0000304	-0.0000325
			(-1.00)	(-0.76)	(-0.84)
VIX_{t-1}				0.0172***	0.0174***
				(3.20)	(3.32)
$BidAskSpread_{t-1}$					-22.85
					(-0.47)
Constant	0.0826**	0.0712**	0.0744**	-0.288***	-0.273**
	(2.60)	(2.57)	(2.58)	(-2.77)	(-2.15)
N	223	223	223	223	223
adj. R^2	0.161	0.298	0.298	0.419	0.417

This table reports summary statistics for the regression of the expected carry return (ECR_t) on change in the jump risk ($\Delta JumpRisk_{t-1}$). ECR_t shows the expected carry return defined in Equation 13. $\Delta JumpRisk_{t-1}$ denotes change in the jump risk in Kou Model defined in Equation 12. ΔBDI_{t-1} denotes changes in the US trade weighted nominal effective exchange rate broad dollar index and an increase in the index shows appreciation in the US$ against all trading partners' currencies. $Debt\&EquityFlow_{t-1}$ shows the net debt and equity inflows to Turkey in million US$ during one week prior to time t. VIX_{t-1} is the implied volatility calculated using S&P500 index options. Higher VIX values imply lower global risk appetite. $BidAskSpread_{t-1}$ is defined as a ratio of bid-ask spread in spot exchange rate divided by the mid-exchange rate (Equation 16). *t-statistics* are in parentheses and they are calculated using heteroscedasticity consistent standard errors. * ($p < 0.10$), ** ($p < 0.05$), *** ($p < 0.01$).

suggesting the importance of currency mismatch in Turkish bank balance sheets. The high contribution of the broad dollar index is indicative of the high dependence of Turkey's financial system on cross border lending. Due to currency mismatch in the balance sheets of financial intermediaries in Turkey, changes in the broad dollar index capture changes in leverage and their ability to engage in arbitrage. For an in depth analysis of the impact of constraints on financial intermediaries on carry trade returns in the US-$-TL market see Kütük [2024].

Taken together the regression results show that currency crash risk plays a significant role in expected carry returns; we find robust evidence that investors ask for higher compensation when currency crash risk is higher, irrespective of which one of our three crash Risk measures is used. Moreover, the broad dollar index and the VIX are also robust accross all regression model specifications and with signs in line with expectations: during stricter limits on balance sheets of financial intermediaries and lower risk appetite in global markets, investors demand a higher premium to invest carry trades: we observe higher expected carry returns when balance sheets of financial intermediaries tighten and when global risk appetite goes down (and VIX goes up).

Of course our regression analysis does have its limitations. We use measures of the interbank cost of borrowing in the calculation of mispricing since it represents the effective cost of funding for speculators and arbitrageurs. However, the banks in those countries are not risk-free, most probably they have time-varying bank risks while riskiness of the banks in the US and Turkey obviously are not the same. Therefore, part of the carry trade return can possibly be attributed to the differential riskiness of US and Turkish banks. Moreover, we use monthly data because our measure of exchange rate expectations is only available at that frequency. Also, we do not

Table (4) **Shapley Decomposition Analysis**

	(1) ECR_t	(2) Shapley R^2 (%)	(3) ECR_t	(4) Shapley R^2 (%)	(5) ECR_t	(6) Shapley R^2 (%)
ΔRR_{t-1}	0.0812***	28.78				
	(4.39)					
ΔCR^{20}_{t-1}			0.119***	21.27		
			(3.42)			
$\Delta JumpRisk_{t-1}$					10.61***	21.59
					(3.38)	
ΔBDI_{t-1}	0.105***	33.47	0.124***	38.59	0.118***	38.13
	(4.36)		(5.14)		(4.43)	
$Debt\&EquityFlow_{t-1}$	-0.0000347	3.63	-0.0000469	4.21	-0.0000325	3.87
	(-0.92)		(-1.30)		(-0.84)	
VIX_{t-1}	0.0174***	33.87	0.0183***	35.54	0.0174***	36.21
	(3.49)		(3.86)		(3.32)	
$BidAskSpread_{t-1}$	-27.71	0.24	-37.76	0.39	-22.85	0.20
	(-0.62)		(-0.97)		(-0.47)	
Constant	-0.267**		-0.279**		-0.273**	
	(-2.25)		(-2.53)		(-2.15)	
N	223		223		223	
adj. R^2	0.446		0.453		0.417	

Shapley R^2 (%) values shows individual contribution of explanatory variables into the goodness of fit (R^2). *t-statistics* are in parentheses and calculated using heteroscedasticity consistent standard errors. * ($p < 0.10$), ** ($p < 0.05$), *** ($p < 0.01$).

have dealers' data at transaction level similar to Cenedese et al. (2019). High frequency data supported by detailed capital flow information and bank specific risk factors can enrich this regression analysis.

5 Crash-Hedged Carry Returns

In the previous section, we analyzed the links between crash risk on expected carry returns focusing on unhedged carry trade strategies. In this section, we calculate both crash-hedged and unhedged carry returns on daily frequency for different investment horizons and then compare these two return structures. The difference between these two carry strategies gives sharper focused information about the additional risk premium investors demand for bearing currency risk.

A currency call option provides an opportunity for carry traders to hedge themselves against currency crash risk in an investment currency. A US$-TL call option gives the right to buy US$ at a strike price (K) in the TL. C shows the price of the call option, which is also denominated in TL. This works as follows. At time t, a carry trader borrows 1 US$ and converts this into TL using the current exchange rate S_t. If the carry trader with the S_t TL buys a call option which gives the right to buy 1 US$ at K_t and pays a call price (C_t), then the carry trader is hedging his investment position against any crash in the value of TL. He can invest his remaining funds $S_t - C_t$ in TL with return $r^{TL}_{t,t+1}$.

At time $t+1$, the carry trade return realizes. If the exchange rate at time $t+1$ (S_{t+1}) is less than K_t, the carry trader does not exercise the call option and the crash-hedged carry return becomes:

$$CR_t^{CH} = (1 + r_{t,t+1}^{TL})\frac{S_t - C_t}{S_{t+1}} - (1 + r_{t,t+1}^{US\$}) \quad \text{if} \quad S_{t+1} \leq K_t \tag{17}$$

On the other hand, if the exchange rate depreciates more, to the extent that $S_{t+1} > K_t$, the carry trader will exercise the call option and convert his TL investment back into US$ using the more favorable rate represented by K_t. So then the return to the carry trader becomes:

$$CR_t^{CH} = (1 + r_{t,t+1}^{TL})\frac{S_t - C_t}{K_t} - (1 + r_{t,t+1}^{US\$}) \quad \text{if} \quad S_{t+1} > K_t \tag{18}$$

Taking the two cases together the over-all crash-hedged carry return CR_t^{CH} equals:

$$CR_t^{CH} = (1 + r_{t,t+1}^{TL})\frac{S_t - C_t}{min\{S_{t+1}, K_t\}} - (1 + r_{t,t+1}^{US\$}) \quad \forall S_{t+1}, K_t \tag{19}$$

This compares to the unhedged carry return:

$$CR_t^{UH} = (1 + r_{t,t+1}^{TL})\frac{S_t}{S_{t+1}} - (1 + r_{t,t+1}^{US\$}) \tag{20}$$

Next we calculate crash-hedged (CR_t^{CH}) and unhedged (CR_t^{UH}) carry returns in the US$-TL investment strategy on a daily frequency between 2006 and 2019. We downloaded spot exchange rates and and the 1-Month, 3-Month and 6-Month interbank costs of borrowing both for Turkey and the US from the FactSet. We use daily US$-TL currency option data from Bloomberg and obtain call prices and strike prices for the at-the-money call options. We assume that the carry traders buy at-the-money currency call options to hedge themselves against the crash risk in the TL. Table 5 and Figure 12 to 14 give the summary statistics and time series of unhedged and crash-hedged annualized carry returns for 1-Month, 3-Month and 6-Month US$-TL investment strategies.

	1-Month		**3-Month**		**6-Month**	
	Unhedged Carry Return (%)	Crash Hedged Carry Return (%)	Unhedged Carry Return (%)	Crash Hedged Carry Return (%)	Unhedged Carry Return (%)	Crash Hedged Carry Return (%)
Mean	1.78	0.67	1.68	0.91	1.51	0.34
Standard Deviation	52.81	25.82	31.77	15.20	22.13	10.97
Minimum	-320.52	-66.39	-134.07	-24.45	-84.52	-14.35
Maximum	197.36	154.10	140.30	97.42	91.54	52.86
Skewness	-0.99	1.53	-0.39	1.70	-0.22	1.54

Table (5) **Summary Statistics of Unhedged and Crash-Hedged Annualized Carry Return**

Not surprizingly returns to the crash-hedged carry strategies are lower than the unhedged carry returns on average: insurance does not come for free. Investors demand higher returns for their unhedged carry strategies in return for bearing the currency crash risk. Standard deviations are higher for the unhedged carry returns since these strategies can experience both unconstrained losses but also higher positive returns. In line with this observation we see that the time series for unhedged returns shows both a higher maximum value and a lower minimum value than the time series for hedged carry trade returns. Finally, the skewness measure shows that we have left-skewed unhedged carry returns and right-skewed crash-hedged carry returns which is what one should expect from the distribution of the $S(t)$ and the limits on losses implied by the option hedge. The difference between unhedged and crash-hedged carry returns represents the premium investors demand for taking on crash risk. The numbers in Table 5 show that on average between 46 to 77 percent of total unhedged carry returns is compensation for bearing the crash risk in this US$-TL carry trade strategy[11]. Of course these calculations rely on the assumption that carry traders hedge using at-the-money currency call options but the carry traders might follow other hedging strategies. Under different hedging schemes the contribution of crash risk on carry return might be different. For instance, Jurek (2014) follows a delta-hedging scheme and he finds that crash risk premia are at most one-third of total currency carry trade returns among G10 currencies. Of course that difference may be due to different hedging strategies or to the use of different currency pairs.

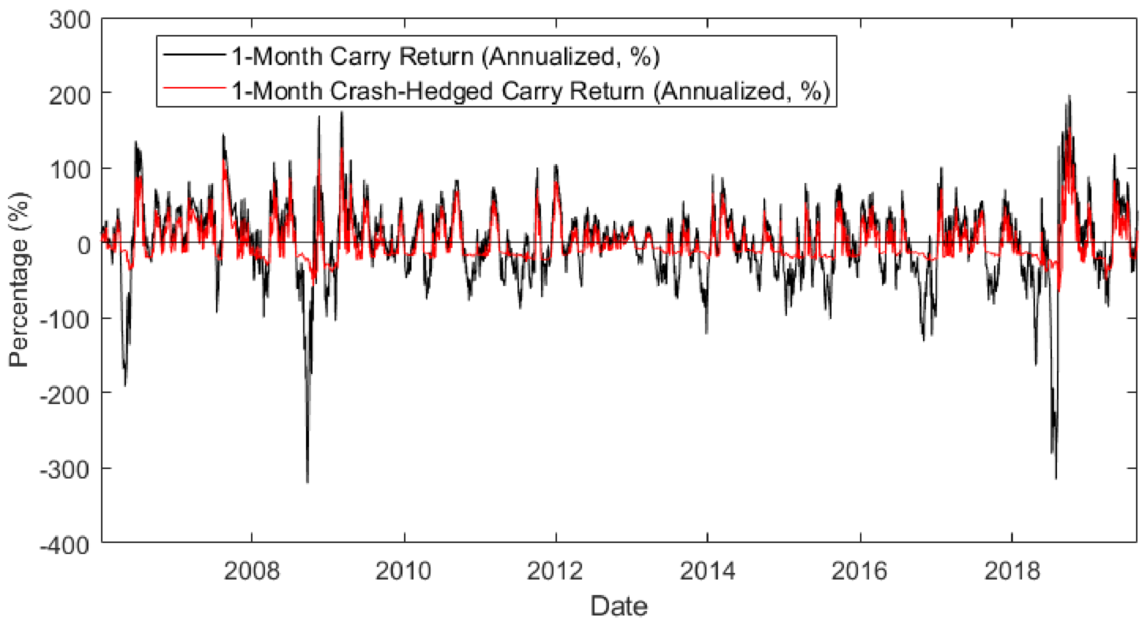


Figure (12) **Unhedged and Crash-Hedged Annualized Carry Return of 1-Month Investment Strategy**
Source: Authors' calculations

[11] 62% for 1 month , 46% for 3 months and 77% for 6 months strategies respectively

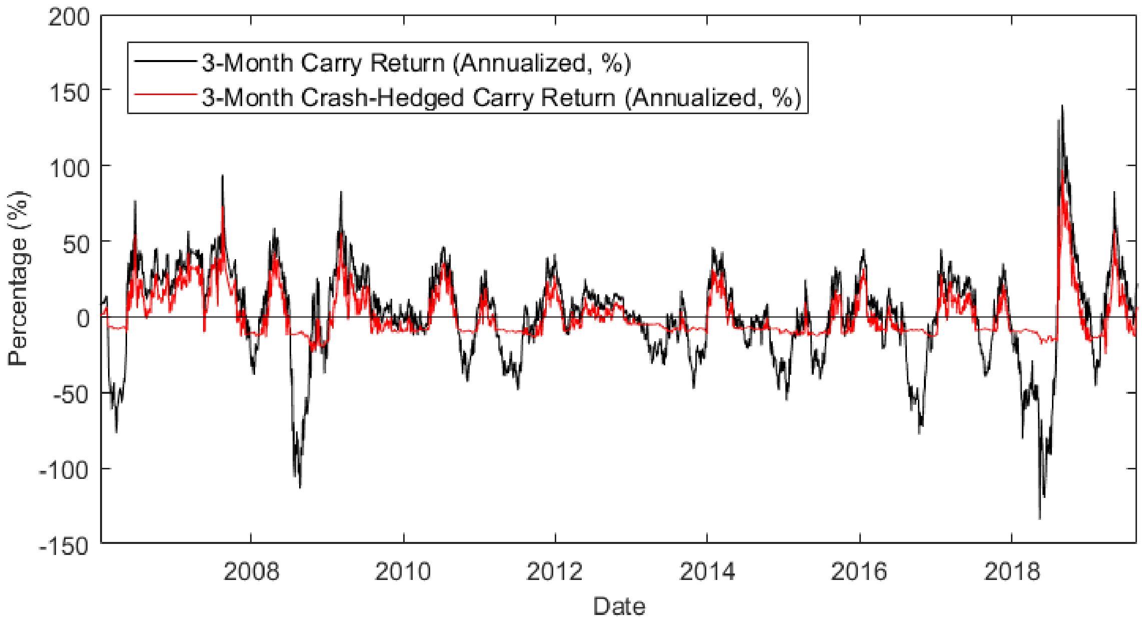


Figure (13) **Unhedged and Crash-Hedged Annualized Carry Return of 3-Month Investment Strategy**
Source: Authors' calculations

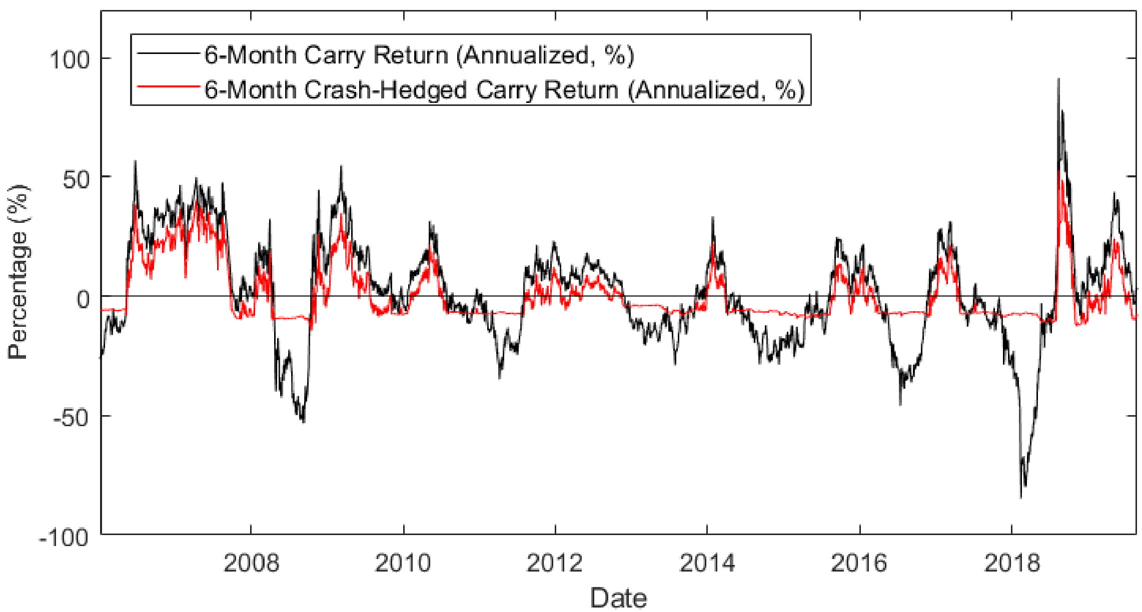


Figure (14) **Unhedged and Crash-Hedged Annualized Carry Return of 6-Month Investment Strategy**
Source: Authors' calculations

6 Conclusion

There is an extensive empirical literature on the impact of currency crash risk on carry trade returns, but it does not use forward looking direct measures of crash risk. This study fills this gap in the literature by introducing several different crash risk measures derived from currency option prices. Moreover, we incorporate direct measures of investors' expected exchange rate changes over the Carry Trade horizon collected by the Central Bank of Turkey. By not relying on *ex post* exchange rates as a proxy for the expected rate at the end of the carry trade horizon we can separate out *ex ante* carry trade returns from *ex post* expectational errors: through our use of a direct measure of exchange rate expectations, this paper is the first study that focues directly on carry trade returns unpolluted by *ex post* surprizes in the exchange rate. We can in this way accurately measure the *ex-ante* currency risk premia demanded by investors' for their participation in carry trades.

We analyze US$-TL carry trade strategies because of the persistently high interest rate differential between TL and US$, high volatility in the TL. Also the availability of direct measures of exchange rate expectations allows us to measure *ex ante* carry trade returns directly, thereby avoiding the pollution of *ex ante* measures of carry trade returns by *ex post* expectational errors. We use three different crash risk measaures: risk reversals, crash probabilities derived from the empirical probability distribution of future TL-rates derived from option prices, and an explicit measure of jump risk derived from a mixed distribution asset pricing model including jump risk explicitly. All these three crash risk measures are shown to be very sensitive to both global and local relevant events in plausible ways.

Next we use these data in a direct analysis of the interest differential leading to US$-TL carry trades. We show first of all that crash risk affects speculators' premia demand significantly. Higher crash risk in the investment currency of the carry trades discourages investors from taking long positions so they need to be convinced by substantial compensation for bearing higher risk: we show that the component of total carry trade returns purely to compensate for crash risk is between 46% and 77%, depending on the maturity of the strategy followed. Furthermore we show that during periods of a generally stronger US dollar, investors' balance sheet constraints are more binding and speculators demand higher premia to invest in carry trades. Also we find, in line with the existing literature, that higher global risk aversion discourages carry trades. Finally, by comparing hedged and unhedged carry strategies we show that currency crash risk is substantial: as already said we find that pure currency crash risk accounts for between 46 and 77 percent of the total unhedged carry trade returns.

References

S. Avdjiev, W. Du, C. Koch, and H. S. Shin. The Dollar, Bank Leverage, and Deviations from Covered Interest Parity. *American Economic Review: Insights*, 1(2):193–208, 2019.

G. Bekaert and M. Hoerova. The vix, the variance premium and stock market volatility. *Journal of Econometrics*, 183(2):181–192, 2014.

F. Black and M. Scholes. The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3):637–654, 1973.

R. R. Bliss and N. Panigirtzoglou. Option-implied risk aversion estimates. *Journal of Finance*, 59(1):407–446, 2004.

R. P. Brent. *Algorithms for minimization without derivatives.* Courier Corporation, 2013.

M. K. Brunnermeier and L. H. Pedersen. Market liquidity and funding liquidity. *Review of Financial Studies*, 22(6):2201–2238, 2008.

M. K. Brunnermeier, S. Nagel, and L. H. Pedersen. Carry trades and currency crashes. *NBER Macroeconomics Annual*, 23(1):313–348, 2008.

V. Bruno and H. S. Shin. Cross-Border Banking and Global Liquidity. *The Review of Economic Studies*, 82(2):535–564, 2015.

R. Bu and K. Hadri. Estimating option implied risk-neutral densities using spline and hypergeometric functions. *The Econometrics Journal*, 10(2):216–244, 2007.

R. J. Caballero and J. B. Doyle. Carry Trade and Systemic Risk: Why are FX Options so Cheap? Technical Report w18644, National Bureau of Economic Research, 2012.

G. Cenedese, P. Della Corte, and T. Wang. Currency mispricing and dealer balance sheets. *Available at SSRN 3327088*, 2019.

P. D. Corte, S. J. Riddiough, and L. Sarno. Currency premia and global imbalances. *The Review of Financial Studies*, 29(8):2161–2193, 2016.

R. Dornbusch. Expectations and exchange rate dynamics. *Journal of Political Economy*, 84(6): 1161–1176, 1976.

W. Du, A. Tepper, and A. Verdelhan. Deviations from covered interest rate parity. *Journal of Finance*, 73(3):915–957, 2018.

D. Duffie. Presidential address: Asset price dynamics with slow-moving capital. *Journal of Finance*, 65(4):1237–1267, 2010.

M. Eichenbaum and C. L. Evans. Some empirical evidence on the effects of shocks to monetary policy on exchange rates. *Quarterly Journal of Economics*, 110(4):975–1009, 1995.

E. F. Fama. Forward and spot exchange rates. *Journal of Monetary Economics*, 14(3):319–338, 1984.

E. Farhi and X. Gabaix. Rare disasters and exchange rates. *Quarterly Journal of Economics*, 131(1):1–52, 2015.

M. Fleckenstein, F. A. Longstaff, and H. Lustig. The TIPS-treasury bond puzzle. *Journal of Finance*, 69(5):2151–2197, 2014.

K. A. Froot and R. H. Thaler. Anomalies: foreign exchange. *Journal of Economic Perspectives*, 4(3):179–192, 1990.

X. Gabaix and M. Maggiori. International liquidity and exchange rate dynamics. *The Quarterly Journal of Economics*, 130(3):1369–1420, 2015.

M. B. Garman and S. W. Kohlhagen. Foreign currency option values. *Journal of International Money and Finance*, 2(3):231–237, 1983.

L. P. Hansen and R. J. Hodrick. Forward exchange rates as optimal predictors of future spot rates: An econometric analysis. *Journal of Political Economy*, 88(5):829–853, 1980.

F. Huettner, M. Sunder, et al. Axiomatic arguments for decomposing goodness of fit according to Shapley and Owen values. *Electronic Journal of Statistics*, 6:1239–1250, 2012.

M. Illing and M. Aaron. A brief survey of risk-appetite indexes. In *Bank of Canada Financial System Review*. Citeseer, 2005.

J. W. Jurek. Crash-neutral currency carry trades. *Journal of Financial Economics*, 113(3): 325–347, 2014.

H. I. Korkmaz, D. Kucuksarac, Y. Onay, A. Senol, et al. Estimation of FX Option Implied Density Functions: Nonparametric-Malz Approach. *Central Bank of the Republic of Turkey Working Paper Series*, 19(03), 2019.

S. G. Kou. A jump-diffusion model for option pricing. *Management science*, 48(8):1086–1101, 2002.

W. S. Krasker. The 'peso problem'in testing the efficiency of forward exchange markets. *Journal of Monetary Economics*, 6(2):269–276, 1980.

M. M. Kütük. Financiary intermediary constraints and carry trade returns. *mimeo, Available at SSRN 4947364*, 2024.

A. M. Malz. A simple and reliable way to compute option-based risk-neutral distributions. *FRB of New York Staff Report*, (677), 2014.

L. Mancini, A. Ranaldo, and J. Wrampelmeyer. Liquidity in the foreign exchange market: Measurement, commonality, and risk premiums. *Journal of Finance*, 68(5):1805–1841, 2013.

M. Mitchell, L. H. Pedersen, and T. Pulvino. Slow moving capital. *American Economic Review*, 97(2):215–220, 2007.

S. Olijslagers, A. Petersen, N. de Vette, and S. van Wijnbergen. What Option Prices tell us about the ECB's Unconventional Monetary Policies. *De Nederlandsche Bank Working Paper*, 2019.

D. Reiswich and U. Wystup. A guide to FX options quoting conventions. *Journal of Derivatives*, 18(2):58, 2010.

A. Scholl and H. Uhlig. New evidence on the puzzles: Results from agnostic identification on monetary policy and exchange rates. *Journal of International Economics*, 76(1):1–13, 2008.

R. W. Tryon. Testing for rational expectations in foreign exchange markets. *International Finance Discussion Papers*, (139), 1979.

APPENDIX

A Appendix-A

A.1 Risk Reversal

We use the standard foreign currency option pricing model of Garman and Kohlhagen (1983) which relies on Black-Scholes' (1973) option pricing model to derive the implied volatility surface. The following notation is used:

S: spot exchange rate (domestic currency (TL) units per unit of foreign currency (US\$))
K: Strike price of the option
τ: Time remaining until maturity of the option
r_d: Domestic interest rate (TL)
r_f: Foreign interest rate (US\$)
μ: Drift of the exchange rate
σ: Volatility of the spot exchange rate
$\Phi(.)$: Standard normal cumulative probability distribution
$C^{BS}(.)$: The price of a European foreign exchange call option
$P^{BS}(.)$: The price of a European foreign exchange put option

In the Garman and Kohlhagen (1983) option pricing model, four assumptions are usually and so do we: first, the currency spot price follows a geometric Brownianprocess.Second, option prices are a function of only one stochastic variable S. Third, markets are frictionless and fourth, foreign and domestic interest rates are constant over th interval considered.

Equation (21) and (22) show the expressions for the price of European FX calls and puts using the Garman and Kohlhagen model (1983) applying Black and Scholes (1973) to currency options:

$$\begin{gathered} C^{BS}(S,\tau,K,r_d,r_f,\sigma^2) = e^{-r_f\tau}S\Phi(d_1) - e^{-r_d\tau}K\Phi(d_2) \\ d_1 = \frac{ln\big(\frac{S}{K}\big) + (r_d - r_f + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}} \\ d_2 = d_1 - \sigma\sqrt{\tau} \end{gathered} \tag{21}$$

$$P^{BS}(S,\tau,K,r_d,r_f,\sigma^2) = e^{-r_f\tau}S\big(\Phi(d_1) - 1\big) - e^{-r_d\tau}K\big(\Phi(d_2) - 1\big) \tag{22}$$

The delta, the sensitivity of the value of the option to the price of the underlying asset, of a call (δ^C) and a put option (δ^P) respectively equals:

$$\delta^C = \frac{\partial C^{BS}(S, \tau, K, r_d, r_f, \sigma^2)}{\partial S} = e^{-r_f \tau}\Phi(d_1) \tag{23}$$

$$\delta^P = \frac{\partial P^{BS}(S, \tau, K, r_d, r_f, \sigma^2)}{\partial S} = -e^{-r_f \tau}\Phi(-d_1) \tag{24}$$

The strike price of an option (K) can be then written as a function of delta (δ) from the call option using Equation (23) as follows:

$$K = S\ exp\big((r_d - r_f + \frac{\sigma^2}{2})\tau - \sigma\sqrt{\tau}\Phi^{-1}(e^{r_f \tau}\delta)\big) \tag{25}$$

We need one adjustment: the standard option pricing formulas from (23) to (25) assume that payment of the premium is in domestic currency. However, the premium of derivatives written in emerging markets can be in terms of foreign currency. The data for US$-TL options provided by Bloomberg is reported by taking premium payment in foreign currency into account[12] (Korkmaz et al., 2019). Therefore, we need to take the premium adjustment into account to obtain a more precise derivation of the risk neutral probability distribution. The premium-adjusted spot delta can be found using Equation (26):

$$\begin{aligned}\delta^C_{PA} &= \frac{\partial C^{BS}(S, \tau, K, r_d, r_f, \sigma^2)}{\partial S} - \frac{C^{BS}(S, \tau, K, r_d, r_f, \sigma^2)}{S} \\ &= e^{-r_f \tau}\Phi(d_1) - \frac{C^{BS}(S, \tau, K, r_d, r_f, \sigma^2)}{S} = e^{-r_d \tau}\frac{K}{S}\Phi(d_2)\end{aligned} \tag{26}$$

We can calculate the premium-adjusted spot delta of a put option (Equation (28)) using the put-call premium-adjusted spot delta parity (Equation (27)).

$$\delta^C_{PA} - \delta^P_{PA} = \frac{K}{S}e^{-r_d \tau} \tag{27}$$

$$\delta^P_{PA} = -e^{-r_d \tau}\frac{K}{S}\Phi(-d_2) \tag{28}$$

In order to find the strike price of an at-the-money (ATM) option, we use the ATM definition that an option with strike such that a straddle[13] has zero net delta in line with the data provided by Bloomberg. Using $\delta^C_{PA} = -\delta^P_{PA}$ for an ATM option, we drive $d_2 = 0$ to get the corresponding strike price as shown in Equation (29):

$$K^{ATM} = Sexp\big((r_d - r_f - \frac{\sigma^2_{ATM}}{2})\tau\big) \tag{29}$$

The risk reversal is the difference between the out-of-the-money (OTM) call and OTM put option implied volatility at the same delta (δ):

[12] See Reiswich and Wystup (2010) for the details related to the spot delta, forward delta, premium adjusted spot delta and premium-adjusted forward delta.

[13] A straddle is a combination of a long call and a long put with the same strike and maturity.

$$RR_\delta = \sigma_\delta^C - \sigma_\delta^P \tag{30}$$

Furtheremore we use the so called *butterfly statistic* to describe the convexity of the volatility smile. The butterfly (BF_δ) is the average of the volatility of an OTM call option and put option for a given delta minus the volatility of an ATM option (Equation (31)):

$$BF_\delta = \frac{\sigma_\delta^C + \sigma_\delta^P}{2} - \sigma_{ATM} \tag{31}$$

. We use the data for butterflies in our calculation of the implied volatility of call and put options to derive the underlying risk neutral probability distributions.

A.2 Currency Crash Risk Implied by a Risk Neutral Probability Distribution (RNPD)

Calculating implied volatility for call and put options: Using the definitions of risk reversal (Equation (30)) and butterfly (Equation (31)), we can calculate implied volatilities of call σ_δ^C and put options σ_δ^P for four delta values. We repeat these calculation for each specific date.

$$\sigma_\delta^C = \sigma_{ATM} + BF_\delta + \frac{RR_\delta}{2} \tag{32}$$

$$\sigma_\delta^P = \sigma_{ATM} + BF_\delta - \frac{RR_\delta}{2} \tag{33}$$

Calculating the delta of the ATM option: In order to have a complete volatility smile in σ-δ space, we need to know the delta value of the ATM options. Assuming that ATM options have 50 delta can be misleading for premium-adjusted deltas[14]. Therefore, we calculate the delta value for the ATM options as follows. First, the strike price (K) of the ATM option is calculated using σ_{ATM} values using Equation (29). Then, using the domestic interest rate (r_d), the foreign interest rate (r_f), time to maturity (τ), the calculated K, the spot exchange rate (S) and σ_{ATM}, we calculate the corresponding delta for the ATM option using Equation (26).

Transformation of out-of-money (OTM) put options to in-the-money (ITM) call options: We have nine datapoints after completing the previous steps: the implied volatility for OTM calls (σ_δ^C) for 4 delta values, the implied volatility for OTM puts (σ_δ^P) for 4 delta values and the implied volatility for the ATM option for its own delta (σ_{ATM}). We need to transform deltas of OTM put options to ITM call options to complete the volatility smile in $\sigma - \delta$ space. Using the Put-Call premium-adjusted spot delta parity (Equation (27)), we can calculate premium-adjusted spot deltas that correspond to the in-the-money call options. However, to

[14] As a robustness check, we did our whole analysis assuming that ATM options have 50 delta. We obtained very similar result both in crash risk calculation and regression analyses throughout the paper

use Equation (27) we need the strike price K to calculate corresponding deltas for ITM call options, we first need strike prices for OTM put options. We can re-write Equation (28) as follows:

$$\delta_{PA}^{P} = -e^{-r_d \tau}\frac{K}{S}\Phi(-d_2) = -e^{-r_d \tau}\frac{K}{S}\Phi(-d_1 + \sigma\sqrt{\tau}) \tag{34}$$

$$= -e^{-r_d \tau}\frac{K}{S}\Phi\left(\frac{-ln(\frac{S}{K}) - (r_d - r_f + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}} + \sigma\sqrt{\tau}\right) \tag{35}$$

$$= -e^{-r_d \tau}\frac{K}{S}\Phi\left(\frac{-ln(\frac{S}{K}) - (r_d - r_f - \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}\right) \tag{36}$$

$$\tag{37}$$

The strike price (K) appears both in the cumulative distribution function and as a coefficient so we cannot solve for K aanalytically Therefore we use the Brent (2013) root search methodology combined with the approach suggested by Reiswich and Wystup (2010) to obtain strike prices K.

We use these strike prices (K) for each premium-adjusted delta value for the OTM put option in (Equation (27)) to derive the corresponding delta values for the ITM call option. We then have nine data points for the volatility smile in δ-σ space (see Figure (1).

Interpolation by the clamped cubic spline method: Once we obtain nine data points in the volatility smile for each day in the observation period, we carry out the clamped cubic spline interpolation in this space. This cubic spline methodology allows us to have continuous first and second derivatives at all points through the volatility smile. In the interpolation with a clamped cubic spline, the slope takes on specific values at the boundary knot points. Following Malz (2014), we construct a clamped cubic spline with a slope of zero at the boundary knot points. Specifically, the extrapolated spline values beyond those points are assumed equal to the observed implied volatility for the highest and lowest delta values respectively.

Transformation of the Strike Price (K) into the Corresponding Delta (δ_K): To calculate the underlying RNPD for the TL we need to transform the interpolated data into strike-volatility space (K-σ space). Bliss and Panigirtzoglou (2004) show that it is not possible to map directly between the option delta and the strike price due to the fact that the implied volatility varies both with strike price and the delta. Instead, we take strike price intervals of 0.01 between 1 and 15. Then we transform all strike prices into a corresponding delta (δ_K) using Equation (26) with the implied volatility of the ATM option (σ_{ATM}). Bu and Hadri (2007) explain why the ATM volatility should be used instead of implied volatility at the different strike prices. Using the ATM volatility keeps the ordering of δ_K the same as that of K, so unintended kinks in volatility smiles are prevented.

Evaluation of the fitted implied volatility (σ_K): Using the estimated spline function, we evaluate the fitted implied volatility (σ_K) at each delta corresponding to strike price (δ_K) for the intervals $(i, i+1)$ in which theδ_K falls. In this way we get the fitted implied volatility (σ_K) corresponding to strike prices (K) from 1 to 15.

Evaluation of call option price and taking the second derivative to obtain the RNPD: Using strike price (K) and the corresponding implied volatility (σ_K), the call price $C^{BS}(S, \tau, K, r_d, r_f, \sigma^2)$ in Equation (21) is calculated. The second derivative of the call price $C^{BS}(S, \tau, K, r_d, r_f, \sigma^2)$ with respect to the strike price gives the Risk Neutral Probability Density (RNPD) function. We can calculate the $RNPD(K)$ as shown in Equation (38) where $K_i \in [1, 15]$ and $\Delta K_i = 0.01\ \forall i$.

$$RNPD_K(K_i) = e^{r_d \tau} \frac{\partial^2 C^{BS}}{\partial K^2} \approx e^{r_d \tau} \frac{C^{BS}(K_i + \Delta K_i) - 2C^{BS}(K_i) + C^{BS}(K_i - \Delta K_i)}{(\Delta K_i)^2} \tag{38}$$

Using the $RNPD(K)$, we can calculate the cumulative RNPD (CRNPD) for each specific date using the expression below:

$$CRNPD(K_i) = \mathbb{P}\big(K \leq K_i\big) = \sum_{K \leq K_i} RNPD_K(K_i) \tag{39}$$

A.3 Currency Options Pricing Incorporating Jump Risk

Equation (40) gives the call price function for a European call option (Theorem 2. in Kou (2002)).

$$\begin{aligned}
C^{Kou} &= S(0)\Upsilon\left(r + \frac{1}{2}\sigma^2 - \lambda\zeta, \sigma, \tilde{\lambda}, \tilde{p}, \tilde{\eta_1}, \tilde{\eta_2}; log(K/S(0)), \tau\right) \\
&\quad - Ke^{-r\tau}\Upsilon\left(r - \frac{1}{2}\sigma^2 - \lambda\zeta, \sigma, \lambda, p, \eta_1, \eta_2; log(K/S(0)), \tau\right) \\
&\text{where} \\
&\tilde{p} = \frac{p}{1+\zeta} \cdot \frac{\eta_1}{\eta_1 - 1} \\
&\tilde{\eta_1} = \eta_1 - 1 \\
&\tilde{\eta_2} = \eta_2 + 1 \\
&\tilde{\zeta} = p\frac{\eta_1}{\eta_1 - 1} + q\frac{\eta_2}{\eta_2 + 1} - 1
\end{aligned} \tag{40}$$

We first need to adapt this option price to a currency option formula. To do that we derive the following currency call price formula under the assumptions explained in section (3.3) using Proposition B.1 in Appendix B.2 and Theorem B.1 in Appendix B3 in Kou [2002]:

$$
\begin{aligned}
C^{Kou} &= S(0)e^{-r_f\tau}a_1\pi_1\sigma\sqrt{\tau}\eta_1 I_0(h; 1-\eta_1, -\frac{1}{\sigma\sqrt{\tau}}, -\sigma\eta_1\sqrt{\tau}) \\
&- Ke^{-r_d\tau}d_1\pi_1\sigma\sqrt{\tau}\eta_1 I_0(h; -\eta_1, -\frac{1}{\sigma\sqrt{\tau}}, -\sigma\eta_1\sqrt{\tau}) \\
&+ \pi_0\Big(e^{-r_f\tau}S(0)e^{-\lambda\zeta\tau}\Phi(b_+) - Ke^{-r_d\tau}\Phi(b_-)\Big)
\end{aligned}
$$

$$
\begin{aligned}
&\text{where} \\
&b_{\pm} = \frac{log(\frac{S(0)}{K}) + \big(r_d - r_f \pm \frac{\sigma^2}{2} - \lambda\zeta\big)\tau}{\sigma\sqrt{\tau}} \\
&\zeta = \frac{\eta_1}{\eta_1 - 1} - 1; \\
&a_1 = e^{-(\lambda\zeta + \frac{\sigma^2}{2})\tau}d_1; \\
&d_1 = \frac{e^{(\sigma\eta_1)^2\tau/2}}{\sigma\sqrt{2\pi\tau}}; \\
&h = log\bigg(\frac{K}{S(0)}\bigg) + \lambda\zeta\tau - \bigg(r_d - r_f - \frac{\sigma^2}{2}\bigg)\tau; \\
&\pi_0 = e^{-\lambda\tau}; \\
&\pi_1 = 1 - e^{-\lambda\tau}; \\
&\text{If } \beta < 0 \text{ and } \alpha < 0, \text{ then } \forall n \geq -1, \\
&I_0(c; \alpha, \beta, \sigma) = -\frac{e^{\alpha c}}{\alpha}Hh_0(\beta c - \delta) - \frac{1}{\alpha}\sqrt{2\pi}e^{\frac{\alpha\delta}{\beta} + \frac{\alpha^2}{2\beta^2}}\Phi\big(\beta c - \delta - \frac{\alpha}{\beta}\big); \\
&Hh_0(x) = \sqrt{2\pi}\Phi(-x).
\end{aligned}
\tag{41}
$$

In order to calibrate the λ and η_1 parameters so we can calculate $JumpRisk$ as time series, we minimize the weighted square of the distance between the call price implied by Kou Model and the calculated market call prices. We solve the minimization problem shown in (43) using N days data, I different maturities and J different strike prices which corresponds to different δ values. We set $N = 2$ so we use the current (time t) and previous day's (time $t-1$) data to estimate $\lambda_t, \eta_{1,t}$. We use 1-month, 3-month and 6-month US$/TL option pricing data so we have three different maturities ($I = 3$). Finally, the US$/TL option prices include 9 different deltas so we have 9 different strike prices ($J = 9$). Since more liquid option prices are more reliable and indicative, we assign higher weights to more liquid options by introducing weights in the minimization problem. We use weights $\omega_{n,i,j} = \frac{1}{Vega_{n,i,j}} = \frac{1}{S_n\phi(d_{1,n,i,j})\sqrt{\pi}}$. After calculating all weights in this way, we normalize them such that the sum of all weights equals to 1:

$$
\omega_{n,i,j}^{normalized} = \frac{\omega_{n,i,j}}{\sum_n^N \sum_i^I \sum_j^J \omega_{n,i,j}} \tag{42}
$$

..

We can now solve:

$$\min_{\lambda_t,\eta_{1,t}} \quad \sum_{n}^{N}\sum_{i}^{I}\sum_{j}^{J}\omega_{n,i,j}^{normalized}\left(C_{n,i,j}^{Kou} - C_{n,i,j}^{market}\right)^2 \tag{43}$$

B Appendix-B

	(1)	(2)	(3)	(4)	(5)
ΔRR_{t-1}	-0.0268	-0.0305	-0.0330	-0.0367	-0.0421
	(-0.59)	(-0.72)	(-0.79)	(-0.98)	(-1.08)
ΔBDI_{t-1}		0.0104	-0.00102	-0.0261	-0.0206
		(0.21)	(-0.02)	(-0.62)	(-0.49)
$Debt\&EquityFlow_{t-1}$			-0.000106*	-0.0000873	-0.0000772
			(-1.71)	(-1.42)	(-1.27)
VIX_{t-1}				0.0213**	0.0203**
				(2.22)	(2.18)
$BidAskSpread_{t-1}$					109.5
					(1.62)
Constant	0.132***	0.131***	0.138***	-0.309*	-0.381**
	(2.65)	(2.68)	(2.81)	(-1.66)	(-1.99)
N	223	223	223	223	223
adj. R^2	-0.000	-0.004	0.000	0.089	0.100

Table (6) **Results from Regression of the Realized Carry Return on the Risk Reversal and Other Control Variables:** This table reports summary statistics for the regression of realized carry return (RCR_t) on the change in the risk reversal (ΔRR_{t-1}). RCR_t shows the realized carry return defined in Equation 14 between time t-1 and t. ΔRR_{t-1} denotes change in the risk reversal for 2-Week, 10-delta US$-TL options. An increase in the risk reversal implies higher hedging cost and larger crash risk in the TL. ΔBDI_{t-1} denotes changes in the US trade weighted nominal effective exchange rate broad dollar index and an increase in the index shows appreciation in the US$ against all trading partners' currencies. $Debt\&EquityFlow_{t-1}$ shows the net debt and equity inflows to Turkey in million US$ during one week prior to time t. VIX_{t-1} is the implied volatility calculated using S&P500 index options. Higher VIX values imply lower global risk appetite. $BidAskSpread_{t-1}$ is defined as a ratio of bid-ask spread in spot exchange rate divided by the mid-exchange rate (Equation 16). *t-statistics* are in parentheses and they are calculated using heteroscedasticity consistent standard errors. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	(1)	(2)	(3)	(4)	(5)
ΔCP^{20}_{t-1}	0.00633	0.00849	0.00839	0.0147	0.00416
	(0.08)	(0.11)	(0.11)	(0.20)	(0.06)
ΔBDI_{t-1}		-0.00792	-0.0201	-0.0483	-0.0437
		(-0.16)	(-0.41)	(-1.15)	(-1.06)
$Debt\&EquityFlow_{t-1}$			-0.000101	-0.0000818	-0.0000718
			(-1.60)	(-1.32)	(-1.18)
VIX_{t-1}				0.0212^{**}	0.0203^{**}
				(2.26)	(2.20)
$BidAskSpread_{t-1}$					100.4
					(1.49)
Constant	0.132^{***}	0.133^{***}	0.140^{***}	-0.305^{*}	-0.370^{*}
	(2.65)	(2.73)	(2.83)	(-1.66)	(-1.97)
N	223	223	223	223	223
adj. R^2	-0.004	-0.009	-0.005	0.082	0.091

Table (7) **Results from Regression of the Realized Carry Return on the Risk Reversal and Other Control Variables:** This table reports summary statistics for the regression of realized carry return (RCR_t) on the change in the crash probability (ΔCP^{20}_{t-1}). RCR_t shows the realized carry return defined in Equation 14 between time t-1 and t. $\Delta CP20_{t-1}$ denotes change in currency crash probability implied by the RNPD defined in Equation 3 in percentage term. ΔBDI_{t-1} denotes changes in the US trade weighted nominal effective exchange rate broad dollar index and an increase in the index shows appreciation in the US$ against all trading partners' currencies. $Debt\&EquityFlow_{t-1}$ shows the net debt and equity inflows to Turkey in million US$ during one week prior to time t. VIX_{t-1} is the implied volatility calculated using S&P500 index options. Higher VIX values imply lower global risk appetite. $BidAskSpread_{t-1}$ is defined as a ratio of bid-ask spread in spot exchange rate divided by the mid-exchange rate (Equation 16). *t-statistics* are in parentheses and they are calculated using heteroscedasticity consistent standard errors. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	(1)	(2)	(3)	(4)	(5)
$\Delta JumpRisk_{t-1}$	-1.623	-1.522	-2.152	-2.912	-3.712
	(-0.24)	(-0.22)	(-0.32)	(-0.45)	(-0.56)
ΔBDI_{t-1}		-0.00174	-0.0124	-0.0372	-0.0324
		(-0.03)	(-0.23)	(-0.82)	(-0.72)
$Debt\&EquityFlow_{t-1}$			-0.000104^{*}	-0.0000856	-0.0000762
			(-1.67)	(-1.39)	(-1.25)
VIX_{t-1}				0.0212^{**}	0.0203^{**}
				(2.25)	(2.21)
$BidAskSpread_{t-1}$					105.1
					(1.49)
Constant	0.132^{***}	0.132^{***}	0.140^{***}	-0.306^{*}	-0.376^{**}
	(2.66)	(2.73)	(2.85)	(-1.66)	(-1.98)
N	223	223	223	223	223
adj. R^2	-0.004	-0.009	-0.005	0.083	0.093

Table (8) **Results from Regression of the Realized Carry Return on the Risk Reversal and Other Control Variables:** This table reports summary statistics for the regression of realized carry return (RCR_t) on change in the jump risk ($\Delta JumpRisk_{t-1}$). RCR_t shows the realized carry return defined in Equation 14 between time t-1 and t. $\Delta JumpRisk_{t-1}$ denotes change in the jump risk in Kou Model defined in Equation 12. ΔBDI_{t-1} denotes changes in the US trade weighted nominal effective exchange rate broad dollar index and an increase in the index shows appreciation in the US$ against all trading partners' currencies. $Debt\&EquityFlow_{t-1}$ shows the net debt and equity inflows to Turkey in million US$ during one week prior to time t. VIX_{t-1} is the implied volatility calculated using S&P500 index options. Higher VIX values imply lower global risk appetite. $BidAskSpread_{t-1}$ is defined as a ratio of bid-ask spread in spot exchange rate divided by the mid-exchange rate (Equation 16). *t-statistics* are in parentheses and they are calculated using heteroscedasticity consistent standard errors. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.