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Revisiting EWMA in High-Frequency Portfolio Optimization: A Comparative Assessment

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Abstract

This paper compares the statistical and economic performance of state-of-the-art high-frequency based multivariate volatility models with a simpler, widely used alternative—the Exponentially Weighted Moving Average (EWMA) filter. Using over two decades of 100 U.S. stock returns (2002–2023), we assess model performance through a Global Minimum Variance portfolio optimization exercise across various forecast horizons. We find that the EWMA model consistently outperforms more complex HF-based volatility models, delivering significant utility gains when including transaction costs, due in part to its lower turnover. Even in the absence of transaction costs, the EWMA filter cannot be beaten in most cases. Our results are robust to various dimensions, including no-short-selling constraints, varying portfolio sizes, and alternative parameter choices, highlighting the continued relevance of the EWMA model in high-frequency-based portfolio allocation.

Key words: multivariate volatility, high-frequency data, realized (co)variances, GMV portfolios, transaction costs

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1 Introduction

Modeling and forecasting volatility of financial asset returns is a crucial element of quantitative portfolio management. Since the development of the GARCH model (Bollerslev, 1986), a substantial body of literature has emerged on modeling volatility using daily returns (see Bauwens et al. (2006) for a survey). This line of work has been significantly enriched by methods that exploit high-frequency (HF) data to estimate (co)variances more precisely, as demonstrated by Andersen et al. (2003), Barndorff-Nielsen and Shephard (2004), and Barndorff-Nielsen et al. (2011), among others. These HF-based covariance measures have enabled the development of so-called "realized" covariance models, such as the CAW (Conditional Autoregressive Wishart) model of Gourieroux et al. (2009), the HAR-DRD model of Oh and Patton (2016), and the multivariate HAR model of Chiriac and Voev (2011), among others.

Comparative evaluations of high-frequency-based volatility models are often conducted under restrictive conditions that limit their practical relevance for portfolio management. Two common limitations stand out. First, many studies (e.g. Chiriac and Voev, 2011; Golosnoy et al., 2012; Archakov et al., 2025) focus on relatively small cross-sections of assets, which reduces the applicability of their findings to realistically large portfolios. High dimensionality introduces estimation challenges and increases the risk of overfitting, particularly for models with rich parameterizations.

Second, economic performance is often assessed in a frictionless setting that overlooks transaction costs—despite their crucial role in real-world portfolio decisions. Typically, volatility models are compared within a Global Minimum Variance (GMV) framework, without accounting for trading frictions. Examples include Opschoor et al. (2018) and Gribisch and Stollenwerk (2020). A notable exception is the work by Hautsch et al. (2015), which overcomes both limitations. They consider large-scale portfolios of 100 and 350 assets and incorporate transaction costs when evaluating the benefits of high-frequency data. However, their focus is on comparing high-frequency- versus low-frequency-based models within a given modeling framework, rather than comparing the relative merits of different high-frequency-based covariance estimators.

This paper addresses this gap by providing a head-to-head comparison of state-of-the-art high-frequency (HF)- based multivariate volatility models under realistic portfolio constraints and cost considerations. In particular, we investigate whether more sophisticated realized covariance models can consistently outperform a simple yet robust benchmark: the Exponentially Weighted Moving Average (EWMA). The original version of the EWMA model, also known as the RiskMetrics model, was based on daily returns and was popularized by J.P.Morgan (1996), eventually becoming an industry standard. It has been widely adopted by practitioners due to its simplicity, ease of implementation, and transparency. The high-frequency version of the EWMA preserves these advantages thanks to its minimal parameterization and recursive updating, making it appealing for its ability to efficiently capture the autocorrelation in realized (co)variances.

We construct portfolios of U.S. equity returns of various sizes (i.e. 10, 30, 50, and 100) and compare the economic performance of a comprehensive set of state-of-the-art HF-based multivariate volatility models. We evaluate model performance within the GMV framework across multiple investment horizons — daily, weekly, biweekly, and monthly — while explicitly accounting for transaction costs. Economic gains are assessed using the utility-based evaluation framework of Fleming et al. (2001, 2003), as adapted by Hautsch et al. (2015), in which the utility depends solely on HF-based ex-post variances and a risk aversion parameter.[1]

We use subsampled realized (co)variances based on 5-minute returns and overnight returns of 100 highly liquid U.S. stocks spanning the period from 2002 to 2023. The models under evaluation include the HAR-DRD model (Oh and Patton, 2016), the CCHAR model (Chiriac and Voev, 2011; Hautsch et al., 2015), the CAW model of Gourieroux et al. (2009), the DPC-CAW model of Gribisch and Stollenwerk (2020), and the HEAVY GAS model of Opschoor et al. (2018). This selection reflects a diverse set of modeling approaches, including the long-memory behavior of realized (co)variances/correlations, spectral density compositions, parameter reductions, and score-driven dynamics, that have been shown to perform well in the context of forecasting multivariate high-frequency-based volatility. We construct cumulative forecasts of the covariance matrix across each horizon and use them to generate GMV portfolios. To capture time variation in performance, we evaluate results

[1]As noted in Hautsch et al. (2015), evaluating GMV portfolios using ex-post returns may introduce severe biases; see also Voev (2009).

separately for tranquil and crisis periods, including the Global Financial Crisis (2008–2009) and the COVID-19 pandemic (2020–2021).

We find that the EWMA model exhibits strong statistical performance in our GMV portfolio exercise, often delivering ex-post realized portfolio volatilities that are comparable to those of more sophisticated models, particularly during crisis periods. We then go one step further in determining how this statistical performance translates into economic gains for a risk-averse representative investor. Our analysis shows that the EWMA model consistently outperforms alternative models once transaction costs are taken into account, across both tranquil and crisis periods. In most scenarios, switching from other models to EWMA yields substantial improvements, with compensation requirements for moving away from EWMA exceeding 400 basis points per year. Even in a frictionless setting—where transaction costs are set to zero—the EWMA model delivers strong results. In the few cases where an alternative model offers an advantage, the gains are modest—at most 10 basis points per year—and highly context-specific, occurring only during tranquil periods and under strong risk aversion. These results highlight the robustness and practical appeal of the EWMA model in portfolio allocation with high-frequency data, especially when economic considerations are evaluated in addition to statistical performance.

Our main findings are robust across four key dimensions. First, we use nonlinear shrinkage (Ledoit and Wolf, 2020) to estimate a valid and well-conditioned realized covariance matrix of dimension 100, instead of the eigenvalue cleaning method used by Hautsch et al. (2015). Second, we introduce no-short-selling constraints when executing the GMV strategy, bringing the analysis closer to realistic investment conditions and addressing practical limitations faced by many institutional investors. Third, we vary the smoothing parameter used in the EWMA filter. Finally, we consider smaller portfolios of size 10, 30, and 50 to assess whether the results hold across different levels of portfolio dimensionality.

The remainder of the paper is organized as follows. Section 2 briefly describes the dataset. Section 3 presents the set of models included in the comparison. Section 4 outlines the construction and evaluation of the GMV portfolios. Section 5 discusses the main empirical findings. Section 7 concludes.

2 Data

Our data set consists of daily realized (co)variances and daily close-to-close log returns of 100 U.S. equities from various sectors from the S&P 500 index over the period January 2, 2002, until December 31, 2023. These stocks are randomly chosen from a list of the 600 stocks with the highest market capitalization in 2019. This results in a sample of $5,262$ observations, after deleting days without HF trades and half-day observations. For each stock, we observe consolidated trades (transaction prices) extracted from the Trade and Quote (TAQ) database with a time-stamp precision of one second before 2014 and one millisecond after 2014. We first clean the high-frequency data following the guidelines of Brownlees and Gallo (2006) and Barndorff-Nielsen et al. (2009), and construct Realized Covariance matrices using subsampling based on 5-minute return intervals.

Since our goal is to forecast close-to-close volatility, the raw subsampled Realized Covariance matrix only measures the variation in returns during the day (9:30 - 16:00). We follow Bollerslev et al. (2018) and add the outer product of the overnight return vector to incorporate a measure of the overnight variation.

Table 1 provides an overview of 100 Tickers and their GICS sector. As shown in Panel A, the stocks originate from nine different sectors, with the majority belonging to the financial and industrial sectors. Panel B shows abnormal observations in the constructed realized (co)variances. For 15 stocks, the RV can be above 1000. If this is the case, we winsorize the Realized Variance series with a level of 99.9, which corresponds to five values, and then re-calculate the realized covariances. Most of these outliers were observed around the peak of the Global Financial Crisis in 2008 or the COVID-19 pandemic.

Finally, the usage of subsampling using 5-min returns and adding the overnight return will theoretically lead to a positive definite covariance matrix as the number of used observations exceeds K. However, as noted by Hautsch et al. (2015), the resulting realized covariance matrix can still be ill-conditioned. Denote $\boldsymbol{RC}_t$ as the $K \times K$ realized covariance matrix at time t after winsorizing. It holds that

$$\boldsymbol{RC}_t = \operatorname{diag}(\boldsymbol{v}_t)\, \boldsymbol{RL}_t \operatorname{diag}(\boldsymbol{v}_t), \tag{1}$$

where $\boldsymbol{v}_t$ is a $K \times 1$ vector with the square root of the diagonal of the matrix $\boldsymbol{RC}_t$, the

Table 1: Tickers, GICS sectors and large observations

This table provides an overview of the Tickers and corresponding GICS sector of 100 U.S. stocks from the S&P 500 index. Panel A lists the sector, the Ticker, and the number of companies within each sector. Panel B shows abnormal values of the daily (close-to-close) realized variance of our sample. We list the Ticker, the date, the maximum, and the winsorized realized variance at a 99.9% confidence level. The sample goes from 2 January 2002 until 29 December 2023 and contains 5,462 values.

Panel A: Sectors and Tickers

GICS nr	Sector	# Comp	Tickers
10	Energy	8	XOM, WMB, SLB, CVX, HAL, OXY, SU, PXD
15	Materials	3	IP, MLM,NUE
20	Industrials	19	BA, CAT, GE, HON, DOV, NOC, LUV, MMM UPS, NSC, FDX, GD, ROK, ETN, RSG, EFX GWW, CP, URI
25	Consumer Discretionary	6	HD, MCD, F ,TGT, RCL, AAP
30	Consumer Staples	9	KO, PG, WMT, MO, SYY, CL, GIS, CPB, EL
35	Health Care	13	PFE, ABT, BAX, JNJ, LLY, MRK BMY, MDT, A, CI, NVS, LH, EW
40	Financials	21	AXP, JPM, AIG, BAC, C, KEY, MTB, COF USB, WFC, GS, MS, MMC, HIG, NLY, PNC MCO, DB, AJG, FDS, RJF
45	Information Technology	3	IBM, HPQ, TSM
50	Communication services	2	VZ, DIS
55	Utilities	8	AEP, AEE, DUK, SO, AES, EXC, ETR, ATO
60	Real estate	8	BXP, EQR, WY, UDR, VTR, PSA, O, VNO

Panel B: Abnormal values of Realized Variance

Ticker	date	max	ws value
AIG	20080916	15420	1554
C	20081124	2581	666
F	20081013	1846	548
KEY	20230313	1024	548
WMB	20020722	2004	1018
SLB	20200309	1117	148
MS	20081013	2444	1377
HAL	20200309	1228	322
OXY	20200309	3313	358
HIG	20081205	1044	611
CI	20021025	1796	186
PXD	20200309	1507	216
LH	20021004	2890	104
VTR	20070307	3271	332
AAP	20230531	1234	230

realized variances $RV_{i,t}$ $(i = 1, \ldots, K)$, and $\boldsymbol{RL}_t$ the matrix with realized correlations, obtained via $\text{diag}(\boldsymbol{v}_t)^{-1}\boldsymbol{RC}_t\text{diag}(\boldsymbol{v}_t)^{-1}$. We follow Hautsch et al. (2012) and define $\boldsymbol{RL}_t$ to be ill conditioned if $\left|\Lambda_t^1/\Lambda_t^K\right| > 10K$, where Λ_t^1 and Λ_t^K are the largest and smallest eigenvalue of $\boldsymbol{RL}_t$ respectively. In case the matrix is non-positive definite or ill-conditioned, we use the eigenvalue cleaning procedure of Laloux et al. (1999). We refer to (the web appendix of) Hautsch et al. (2015) and references herein for more details. Given the regularized realized correlation matrix $\boldsymbol{RL}_t^E$, we obtain the regularized realized covariance matrix back via

$$\boldsymbol{RC}_t^{\boldsymbol{E}} = \operatorname{diag}(\boldsymbol{v}_t)\, \boldsymbol{RL}_t^E \operatorname{diag}(\boldsymbol{v}_t) \tag{2}$$

3 The modeling framework

Inspired by the work and empirical results of Hautsch et al. (2015), Opschoor et al. (2018) and Gribisch and Stollenwerk (2020), we focus on seven different models to model the full (regularized) realized covariance matrix $\boldsymbol{RC}_t^{\boldsymbol{E}}$. Our selection includes models that have demonstrated strong performance in prior studies using high-frequency data: the CCHAR model (as in Hautsch et al. (2015)), the HAR-DRD model of Oh and Patton (2016), the CAW model of Gourieroux et al. (2009), the DPC-CAW model of Gribisch and Stollenwerk (2020), and the HEAVY GAS model of Opschoor et al. (2018). We describe these models in more detail in this section.

- RiskMetrics Model

  We begin with the original RiskMetrics model (J.P.Morgan, 1996), which is applied to daily close-to-close returns. Define the $K \times K$ conditional covariance matrix as $\boldsymbol{V}_{t+1}$ and $\boldsymbol{r}_t$ as the $K \times 1$ close-to-close return vector. The RiskMetrics (labeled as RM 94) model reads

$$\boldsymbol{V}_{t+1} = \lambda \boldsymbol{V}_t + (1-\lambda)\boldsymbol{r}_t\boldsymbol{r}_t^\top \tag{3}$$

  with $\lambda = 0.94$. In case of ill-conditioned h-step ahead forecasts of this model, we use the eigenvalue cleaning approach used by Hautsch et al. (2015) to construct a valid matrix.

- EWMA model

  Our benchmark model, the Exponentially Weighted Moving Average Model (EWMA) adapted to high-frequency data, simply replaces the outer product of returns in equation 3 by the regularized realized covariance matrix $\boldsymbol{RC}_t^{\boldsymbol{E}}$:

$$\boldsymbol{V}_{t+1} = \lambda \boldsymbol{V}_t + (1-\lambda)\boldsymbol{RC}_t^{\boldsymbol{E}} \tag{4}$$

where we set $\lambda = 0.96$. We label this model as EWMA to discriminate between (3) and (4).

- CCHAR Model

The CCHAR model (Hautsch et al., 2015; Chiriac and Voev, 2011, see) ensures positive definiteness of the covariance matrix by modeling the columns of the Cholesky decomposition of the realized covariance matrix separately, hence $\boldsymbol{RC}_t^{\boldsymbol{E}} = \boldsymbol{L}_t\boldsymbol{L}_t^\top$. Denote $\boldsymbol{L}_t^{(\cdot g)}$ is the $(K-g+1)\times 1)$ vector of elements from the gth column, with $g = 1, \ldots, K$. The model accounts for slowly declining autocorrelations in realized (co)variances- i.e. the so-called long-memory behavior - by means of HAR dynamics (Corsi, 2009) for each g

$$\boldsymbol{L}_t^{(\cdot g)} = \boldsymbol{c}^{(g\cdot)} + \beta_d^{(\cdot g)}\boldsymbol{L}_{t-1}^{(\cdot g)} + \beta_w^{(\cdot g)}\sum_{s=1}^{5}\boldsymbol{L}_{t-s}^{(\cdot g)}/5 + \beta_m^{(\cdot g)}\sum_{s=1}^{22}\boldsymbol{L}_{t-s}^{(\cdot g)}/22 + \boldsymbol{\epsilon}_t(\cdot g) \tag{5}$$

where $\boldsymbol{c}^{(\cdot g)}$ is an parameter vectors of length $((K-g+1)\times 1)$. The remaining parameters are all scalars. All parameters can be estimated straightforwardly by OLS.

- HAR-DRD Model

Oh and Patton (2016) develop the HAR-DRD model, which assumes a HAR type model for both the realized variances and realized correlations separately. Given the decomposition of (2), the (logarithm of the) individual realized variances are modeled by the HAR model in a first step as

$$\log RV_{i,t+1} = \beta_{0,i} + \beta_{1,i}\log RV_{i,t} + \beta_{2,i}\frac{1}{5}\sum_{k=1}^{5}\log RV_{i,t-k+1} +$$
$$\beta_{3,i}\frac{1}{22}\sum_{k=1}^{22}\log RV_{i,t-k+1} + \eta_{i,t+1}, \tag{6}$$

with coefficients $\beta_{0,i}, \ldots, \beta_{3,i}$ $(i = 1, \ldots, K)$ estimated by OLS. The realized correlations are modeled in a second step by the following HAR model

$$\begin{aligned}\text{vech}(\boldsymbol{RL}_{t+1}^{E}) = (1-a-b-c)\text{vech}(\overline{\boldsymbol{RL}}) + a\,\text{vech}(\boldsymbol{RL}_{t}^{E}) + \\ b\,\frac{1}{5}\sum_{k=1}^{5}\text{vech}(\boldsymbol{RL}_{t-k+1}^{E}) + c\,\frac{1}{22}\sum_{k=1}^{22}\text{vech}(\boldsymbol{RL}_{t-k+1}^{E}) + \boldsymbol{\epsilon}_{t+1},\end{aligned} \tag{7}$$

with $\overline{\boldsymbol{RL}}$ the sample average of $\boldsymbol{RL}_{t}^{E}$. Again, the coefficients (a, b, c) are estimated by OLS. We refer to Oh and Patton (2016) about regular conditions such that $\text{vech}(\boldsymbol{RL}_{t+1}^{E})$ is positive definite.

- CAW Model

The fifth model that fully uses the realized covariance matrix is the CAW model of Gourieroux et al. (2009). Assuming a conditional Wishart distribution for $\boldsymbol{RC}_{t}^{\boldsymbol{E}}$, the filtered covariance matrix $\boldsymbol{V}_t$ is modeled as:

$$\begin{aligned}\boldsymbol{RC}_t|\mathcal{F}_{t-1} &\sim \mathcal{W}(\boldsymbol{V}_t, \nu_W) \\ \boldsymbol{V}_{t+1} &= (1-A-B)\boldsymbol{\Omega} + A\,\boldsymbol{RC}_t + B\,\boldsymbol{V}_t,\end{aligned}$$

where A and B are estimated by maximum likelihood and $\boldsymbol{\Omega}$ is set to the sample mean of $\boldsymbol{RC}_{t}^{\boldsymbol{E}}$.

Gribisch and Stollenwerk (2020) argue that the numerical optimization of the CAW likelihood might be problematic. They propose the Dynamic Principal Component (DPC)-CAW model with the main idea to model the eigenvectors and eigenvalues of the filtered conditional mean of the realized covariance matrix $\boldsymbol{RC}_{t}^{\boldsymbol{E}}$. The full model reads

$$\boldsymbol{RC}_{t}^{\boldsymbol{E}}|\mathcal{F}_{t-1} \sim \mathcal{W}(\boldsymbol{V}_t, \nu_W) \tag{8}$$

$$\boldsymbol{V}_t = \boldsymbol{L}_t\boldsymbol{\Lambda}_t\boldsymbol{L}_t^{\top} \tag{9}$$

with $\boldsymbol{L}_t$ a matrix of eigenvectors and $\boldsymbol{\Lambda}_t$ a diagonal matrix of ascending eigenvalues.

The eigenvectors $\boldsymbol{L}_t$ are modeled by a matrix-variate auxiliary process $\boldsymbol{Q}_t$

$$\boldsymbol{Q}_{t+1} = (1-a-b)\boldsymbol{S} + a\boldsymbol{RC}_t^{\boldsymbol{E}} + b\boldsymbol{Q}_t \tag{10}$$

$$\boldsymbol{Q}_{t+1} = \boldsymbol{L}_{t+1}\boldsymbol{G}_{t+1}\boldsymbol{L}_{t+1}^\top \tag{11}$$

with $\boldsymbol{S} = \underline{\boldsymbol{L}\boldsymbol{\Lambda}\boldsymbol{L}^\top}$ the sample covariance matrix. Note that we are only interested in $\boldsymbol{L}_t$, since the eigenvalues $\gamma_{i,t}$ of the diagonal matrix $\boldsymbol{\Lambda}_t$ are modeled by a GARCH type process

$$\gamma_{i,t+1} = (1-\alpha_i-\beta_i)\omega_i + \alpha_i g_{i,t} + \beta_i\gamma_{i,t} \tag{12}$$

$$g_{i,t} = e_i^\top \boldsymbol{L}_t^\top \boldsymbol{RC}_t^{\boldsymbol{E}} \boldsymbol{L}_t e_i \tag{13}$$

$$\omega_i = \underline{\gamma}_i \tag{14}$$

with e_i a $K \times 1$ vector of zeros with a 1 at position i and $\underline{\gamma}_i$ the i-th diagonal element of $\underline{\boldsymbol{\Lambda}}$. Note that $\mathbb{E}[g_{i,t}|\mathcal{F}_{t-1}] = \gamma_{i,t}$. We follow Gribisch and Stollenwerk (2020) and estimate the parameters $\{\text{vech}(\bar{\boldsymbol{S}}), a, b, \alpha_i, \beta_i\}$ with $i = 1, \ldots, K$ in three steps. First, we estimate the eigenvectors and eigenvalues of the sample covariance matrix $\boldsymbol{S} = \underline{\boldsymbol{L}\boldsymbol{\Lambda}\boldsymbol{L}\prime}$. Then we estimate a and b by assuming a Wishart distribution for $\boldsymbol{Q}_t$, treating ν_W as a nuisance parameter. Finally, we estimate the α_i and β_i using a Gamma distribution.

- HEAVY-GAS Model

The final model we consider is the HEAVY GAS model (Opschoor et al., 2018). This model applies the score-driven framework of Creal et al. (2013) by using the score to update $\boldsymbol{V}_t$. More specifically, the model assumes a conditional fat-tailed Student's t distribution for the close-to-close returns, and a fat-tailed matrix-F distribution for the realized covariance matrix. The innovation for V_t is defined as the sum of (the scaled) score of the two conditional distributions.

$$\begin{aligned}
&\boldsymbol{r}_t|\mathcal{F}_{t-1} \sim t(\boldsymbol{\mu}, \boldsymbol{V}_t, \nu_0) \quad \boldsymbol{RC}_t^{\boldsymbol{E}}|\mathcal{F}_{t-1} \sim F(\boldsymbol{V}_t, \nu_1, \nu_2) \\
&\boldsymbol{V}_{t+1} = (1-B)\boldsymbol{\Omega} + A\,\boldsymbol{S}_t + B\,\boldsymbol{V}_t \qquad (15)
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{S}_t &= \frac{1}{\nu_1+1}\boldsymbol{V}_t\nabla_t\boldsymbol{V}_t \\
&= \frac{w_t(\boldsymbol{r}_t-\boldsymbol{\mu})(\boldsymbol{r}_t-\boldsymbol{\mu})' - \boldsymbol{V}_t}{\nu_1+1} \\
&+ \frac{\nu_1}{\nu_1+1}\left[\frac{\nu_1+\nu_2}{\nu_2-k-1}\,\boldsymbol{RC}_t^{\boldsymbol{E}}\left(\mathbf{I}_k + \frac{\nu_1\,\boldsymbol{V}_t^{-1}\,\boldsymbol{RC}_t^{\boldsymbol{E}}}{\nu_2-k-1}\right)^{-1} - \boldsymbol{V}_t\right], \qquad (16)
\end{aligned}$$

with $\nabla_t = \frac{\partial \log t(\cdot)}{\partial \boldsymbol{V}_t} + \frac{\partial \log F(\cdot)}{\partial \boldsymbol{V}_t}$ and $w_t = (\nu_0+k)/(\nu_0 - 2 + (\boldsymbol{r}_t-\boldsymbol{\mu})^\top \boldsymbol{V}_t^{-1}(\boldsymbol{r}_t-\boldsymbol{\mu}))$. Equation (16) shows that incidental large returns and realized covariance matrices are downweighted via w_t and the inverse of $\left(\mathbf{I}_k + \frac{\nu_1\,\boldsymbol{V}_t^{-1}\,\boldsymbol{RC}_t^{\boldsymbol{E}}}{\nu_2-k-1}\right)$ respectively. The model parameters are estimated by Maximum Likelihood.

4 The Economic Value of Realized Covariance Forecasting

We evaluate the economic value of our covariance forecasts using a Global Minimum Variance Portfolio (GMVP) optimization framework. This approach, originally proposed by Markowitz (1952), is the most common application for assessing the quality of covariance matrix forecasts in asset return modeling. At each point in time, we construct the GMVP by solving an optimization problem in which an investor minimizes the cumulative h-step-ahead portfolio volatility, subject to a fully invested portfolio constraint. The resulting quadratic problem can be written as

$$\min \boldsymbol{w}_{t,t+h}^\top \boldsymbol{V}_{t,t+h}\boldsymbol{w}_{t,t+h}, \qquad \text{s.t.} \quad \boldsymbol{w}_{t,t+h}^\top \boldsymbol{\iota} = 1, \qquad (17)$$

with solution

$$\boldsymbol{w}_{t,t+h}^{opt} = \frac{\boldsymbol{V}_{t,t+h}^{-1}\boldsymbol{\iota}}{\boldsymbol{\iota}^\top \boldsymbol{V}_{t,t+h}^{-1}\boldsymbol{\iota}}. \qquad (18)$$

where $\boldsymbol{V}_{t,t+h} = \sum_{j=1}^{h} \boldsymbol{V}_{t-1+j}$ and $h = (1, 5, 10, 22)$. As a robustness check, we also construct the GMV portfolio under a no-short-selling constraint, i.e., $w^i_{t,t+h} \geq 0 \ \forall i$. In this case, we rely on a nonlinear solver to obtain the optimal weights, as no closed-form solution is available.

We assess the predictive ability of the different models by comparing the implied ex-post realized portfolio volatility, $\sigma^p_{t,t+h} = \sqrt{\boldsymbol{w}^{opt\top}_{t,t+h} \boldsymbol{RC}^{\boldsymbol{E}}_{t,t+h} \boldsymbol{w}^{opt}_{t,t+h}}$, using the Model Confidence Set (MCS) proposed by Hansen et al. (2011) with a significance level of 5%. The MCS tests for the model(s) with the lowest average loss and accounts for the dependence between model outcomes, as all models are estimated from the same data.

In addition to ex-post realized volatility, we include other relevant performance metrics based on the GMV portfolio, such as turnover and total short position. Turnover measures the value of the portfolio bought or sold when rebalancing from time t to $t+h$. Models that produce more stable covariance forecasts typically result in lower turnover, which in turn implies lower transaction costs. This leads to gains in trading strategies. The total turnover at time t is defined as:

$$TO_{t,h} = \sum_{i=1}^{N} \left| w^i_{t,t+h} - w^i_{t-h,t} \frac{1 + r^i_{t-h,t}}{1 + r^p_{t-h,t}} \right|, \tag{19}$$

where $w^i_{t-h,t}$ is the weight of asset i, $r^i_{t-h,t}$ its return over $[t-h, t]$, and $r^p_{t-h,t} = \sum i = 1^K w^i_{t-h,t} r^i_{t-h,t}$ is the return of the portfolio over the same period.

The total short position measures the extent of negative portfolio weights. Portfolios with fewer short positions are generally easier and less costly to implement, as rebalancing requires fewer adjustments. Again, more stable forecasts of $\boldsymbol{V}_{t+1}$ should lead to less extreme portfolio weights. The total short position $SP_{t,h}$ is given by:

$$SP_{t,h} = \sum_{i=1}^{K} w^i_{t+h,t} \cdot I[w^i_{t,t+h} < 0], \tag{20}$$

with $I[\cdot]$ an indicator function that takes the value one if the i-the element of the weight vector is lower than zero.

Beyond statistically testing on differences in the ex-post realized portfolio volatility, we also use the adapted utility-based framework of Fleming et al. (2001, 2003); Hautsch et al. (2015) to assess the relative economic advantages of using different forecasting models. This approach is based on the assumption that an investor has quadratic utility with a risk aversion parameter γ. The realized utility of the portfolio return based on the forecasted covariances equals

$$U(r^p_{t,t+h}) = (1 + r^p_{t,t+h}) - \frac{\gamma}{2(1+\gamma)}(1 + r^p_{t,t+h})^2. \tag{21}$$

with γ typically set to 1 and 10, respectively. Given two different models, the return Δ_γ that the investor with risk aversion γ would like to pay in order to switch from model I to II can be obtained by solving

$$\sum_{t=1}^{T-h} \mathbb{E}[U(r^{p,I}_{t,t+h})|\mathcal{F}_t] = \sum_{t=1}^{T-h} \mathbb{E}[U(r^{p,II}_{t,t+h} - \Delta_\gamma)|\mathcal{F}_t]. \tag{22}$$

We follow Hautsch et al. (2015) in estimating Δ_γ, assuming constant expected returns across time and assets and using high-frequency-based conditional variances only.[2] We set the annual expected return to 5%. The null hypothesis $\Delta_\gamma = 0$ is tested using the Reality Check procedure of White (2000), with p-values obtained via the stationary bootstrap of Politis and Romano (1994) (999 bootstrap samples, average block length of 22 days).

Although the above analysis extends the traditional comparison of ex-post realized portfolio volatility, it remains within a stylized setting. Our main measure of economic significance is therefore the annualized performance fee, Δ_γ, adjusted for transaction costs. Following Hautsch et al. (2015), we extend (22) by incorporating a cost-adjustment term:

$$\Delta^c_\gamma = \Delta_\gamma - c(\overline{TO}^{II} - \overline{TO}^{I}), \tag{23}$$

where c denotes proportional transaction costs per dollar traded (set to 0%, 1%, and 2%), and $\overline{TO}^{i}$ is the average turnover implied by the GMV strategy using model i's forecasts $(i = I, II)$. We again test the null hypothesis $\Delta^c_\gamma = 0$.

According to (23), the adjusted performance fee Δ^c_γ reflects the economic value of switching from model I to model II after accounting for differences in transaction costs. A

[2] See Hautsch et al. (2015)'s online appendix for a detailed solution.

positive value of Δ_γ means that an investor would be willing to pay this amount to switch to model II based on expected utility gains. However, if model II has higher average transaction costs than model I, the adjusted fee Δ_γ^c becomes smaller. Conversely, if model II involves lower transaction costs, the adjusted fee increases, making the switch even more attractive. This adjustment ensures that any economic gains are evaluated in light of the practical costs of implementing the strategy.

5 Results

We use a moving estimation window of 1,000 observations (corresponding to roughly four calendar years), leaving P = 4,441 observations for the out-of-sample period, starting on January 11th, 2006. We re-estimate the models' parameters after 22 days (one month) and construct h-step ahead forecasts at each day t. The cumulative h-step ahead forecasts are constructed using the recursive forecasting method, except for the HAR-type models. Following Bollerslev et al. (2018), we use a direct forecasting approach for the CCHAR and HAR-DRD models.

Tables 2 and 3 present our baseline results for a 100-dimensional portfolio. Each table reports statistics for the Global Minimum Variance (GMV) portfolio—ex-post minimum variance (σ_ϵ^{HF}), turnover (TO), and short positions (SP)—based on covariance matrix forecasts at various horizons: daily ($h = 1$), weekly ($h = 5$), biweekly ($h = 10$), and monthly ($h = 22$). We compare the performance of the following models: EWMA, RiskMetrics (RM 94), HAR-DRD, CCHAR, DPC-CAW, CAW, and HEAVY GAS, during crisis and non-crisis periods, as defined in Section 2. Our baseline GMV portfolio analysis allows for short-selling.

Following the approach of Fleming et al. (2003), we also report the economic value of switching from the EWMA model to each of the alternative models. These gains, expressed in annual basis points, represent the compensation a mean-variance investor with quadratic utility would require to prefer the specified alternative over the EWMA model. Positive (negative) bold values indicate that the investor would be significantly inclined (disinclined) to use the EWMA model instead of the particular model specified in each row.

We begin by evaluating the ex-post minimum realized portfolio volatility, denoted by σ_ϵ^{HF}. Tables 2 and 3 show that across all forecast horizons, the EWMA model is consistently included in the 95% Model Confidence Set (MCS) during crisis periods (the rows where σ_{ϵ}^{HF} appears in bold indicate models that are included in the 95% confidence set). In contrast, during tranquil periods, the HEAVY GAS model is the only model retained in the MCS, implying that it achieves significantly lower ex-post realized portfolio volatility relative to its competitors.

This statistical advantage of the HEAVY GAS model over the EWMA benchmark during non-crisis periods translates into only modest utility losses when transaction costs are excluded. For an investor with moderate risk aversion ($\gamma = 1$), the loss is approximately one basis point per year. For an investor with high risk aversion ($\gamma = 10$), the loss increases to around 10 basis points per year—still a modest figure. In contrast, for the remaining models, the estimated relative gains are mostly statistically insignificant; when significant, they tend to favor the EWMA model. For example, in the case of biweekly forecasts during non-crisis periods (Table 3), the performance fee for switching from the DPC-CAW (or CAW) model to the EWMA filter reaches up to 20 basis points annually for a highly risk-averse investor. These results underscore that, even under a frictionless setting, the EWMA filter is economically difficult to beat.

Turning to a more realistic setting that incorporates transaction costs, the findings are more pronounced. When proportional transaction costs of 1% and 2% are applied (see Tables 2 and 3), the EWMA model is never significantly outperformed by any alternative. In fact, for every forecast horizon, investors would require substantial compensation to justify switching away from the EWMA benchmark. These utility gains are not only statistically significant but often economically meaningful, and they increase with higher transaction cost levels.

For instance, with 1% transaction costs (and excluding the RM 94 model), the fees range from 4.5 (CAW, monthly forecasts, off-crises) to 118.6 (HAR-DRD, daily forecasts, off-crisis) basis points for an investor with $\gamma = 1$. In case of a strongly risk-averse investor, these values range from 10 (HEAVY GAS, $h = 10$, off-crisis) to 437 (HAR-DRD, monthly forecasts, crisis) annual bp. The performance differentials become even larger when c is set to 2%.

These substantial utility gains can be attributed to two key factors. First, the EWMA model consistently delivers the lowest portfolio turnover among the evaluated high-frequency models. This result follows naturally from its parsimonious structure, with only one smoothing parameter close to unity. This pattern holds true across both crisis and tranquil periods, as well as across all forecast horizons. Second, despite its simplicity, the EWMA model captures the key characteristic of realized volatility: its high degree of persistence.

Finally, our findings support those of Hautsch et al. (2015), demonstrating that incorporating high-frequency data into multivariate volatility models significantly improves GMV performance compared to using only close-to-close returns. The RiskMetrics (RM 94) model, which relies on daily data, produces markedly higher ex-post realized volatility than any of the high-frequency models considered. Combined with its relatively high turnover, the utility cost of using RM 94 instead of EWMA ranges from 16 to over 600 basis points per year.

In conclusion, for large portfolios and across forecast horizons, the EWMA model is not economically outperformed by any high-frequency-based volatility model once transaction costs are taken into account. In fact, significant utility gains are observed when switching from competing models to the EWMA benchmark. Even in the absence of transaction costs, the EWMA model remains difficult to beat. The only exception is the HEAVY GAS model, which marginally outperforms EWMA during non-crisis periods for daily and (bi)weekly forecasts, yielding at most a gain of 10 annual basis points for highly risk-averse investors.

Table 2: GMV Portfolio Performance Measures and Economic Gains from Switching to the EWMA Model -Short-Selling allowed (Daily and Weekly Forecasts)

This table shows portfolio statistics of the Global Minimum Variance portfolio, based on daily ($h = 1$), weekly ($h = 5$) predictions of the 100×100 covariance matrix, according to the following models: EWMA (with $\lambda = 0.96$), RM 94, HAR-DRD, CCHAR, DPC-CAW, CAW, HEAVY GAS. Parameters are estimated with a moving window of 1,000 observations and re-estimated after 22 observations. In the GMV exercise we assume an annual expected return of 5% to be fixed and identical across all stocks. We report the ex-post minimum realized portfolio volatility σ_ϵ^{HF}, as well as the turnover (TO) and short position (SP). The lowest portfolio volatilities obtained using the true realized covariance matrix are marked in bold if they belong to the model confidence set (MCS) based on a 5% significance level. The economic gains Δ_1 and Δ_{10} represent the annualized gain in basis points that a risk averse investor with risk aversion parameter $\gamma \in 1, 10$ and transaction costs $c \in 0\%, 1\%, 2\%$ would receive when switching from the model described in the row to the EWMA model. The gains that are significant at a 5% level are also marked bold. The out-of-sample period spans from January 2006 to December 2023, comprising 4,441 observations.

				$c = 0\%$		$c = 1\%$		$c = 2\%$					$c = 0\%$		$c = 1\%$		$c = 2\%$	
	σ_ϵ^{HF}	TO	SP	Δ_1	Δ_{10}	Δ_1	Δ_{10}	Δ_1	Δ_{10}	σ_ϵ^{HF}	TO	SP	Δ_1	Δ_{10}	Δ_1	Δ_{10}	Δ_1	Δ_{10}
	$h = 1$									$h = 5$								
Crisis																		
EWMA	**13.22**	0.217	-0.760	-	-	-	-	-	-	**13.74**	0.248	-0.760	-	-	-	-	-	-
RM 94	15.91	0.677	-1.052	**58.30**	**583.4**	**104.3**	**629.4**	**150.2**	**675.3**	16.45	0.697	-1.052	**59.98**	**601.6**	**104.9**	**646.5**	**149.7**	**691.4**
HAR-DRD	**13.21**	1.346	-0.763	3.830	38.37	**116.7**	**151.3**	**229.6**	**264.2**	13.93	1.037	-0.705	16.30	164.2	**95.16**	243.1	**174.0**	**322.0**
CCHAR	**13.13**	0.872	-0.736	-4.247	-42.55	**61.25**	22.95	**126.7**	**88.44**	**13.65**	0.798	-0.687	-3.020	-30.49	**51.93**	24.47	**106.9**	**79.42**
DPC-CAW	**13.22**	0.845	-0.580	2.738	27.43	**65.49**	90.19	**128.2**	**152.9**	**13.82**	0.739	-0.596	2.376	23.97	**51.46**	**73.05**	**100.5**	**122.1**
CAW	**13.13**	0.816	-0.776	-4.954	-49.63	**54.89**	10.21	**114.7**	**70.05**	**13.81**	0.706	-0.772	-0.928	-9.360	**44.86**	36.43	**90.66**	**82.22**
HEAVY GAS	**12.88**	0.463	-0.448	6.376	63.87	**30.99**	88.48	**55.60**	113.1	**13.49**	0.460	-0.451	5.327	53.72	**26.52**	74.91	**47.71**	96.10
Non-Crisis																		
EWMA	7.935	0.209	-0.680	-	-	-	-	-	-	8.306	0.219	-0.680	-	-	-	-	-	
RM 94	9.486	0.652	-0.980	**15.04**	**150.6**	**59.32**	**194.9**	**103.6**	**239.2**	9.875	0.658	-0.980	**15.84**	**159.6**	**59.68**	**203.4**	**103.5**	**247.3**
HAR-DRD	7.972	1.387	-0.766	0.791	7.925	**118.6**	**125.7**	**236.3**	**243.5**	8.336	1.020	-0.709	0.864	8.714	**80.95**	**88.80**	**161.0**	**168.9**
CCHAR	8.148	0.876	-0.742	0.960	9.617	**67.61**	**76.27**	**134.3**	**142.9**	8.317	0.672	-0.671	-0.552	-5.570	**44.71**	**39.70**	**89.98**	**84.96**
DPC-CAW	8.037	0.705	-0.512	0.011	0.114	**49.58**	**49.68**	**99.14**	**99.25**	8.474	0.510	-0.528	1.084	10.94	**30.21**	**40.06**	**59.33**	**69.18**
CAW	8.082	0.644	-0.693	-0.066	-0.660	**43.41**	**42.81**	**86.88**	**86.29**	8.547	0.456	-0.673	1.116	11.26	**24.83**	**34.97**	**48.55**	**58.69**
HEAVY GAS	**7.707**	0.428	-0.448	**-1.070**	**-10.72**	**20.79**	**11.15**	**42.66**	**33.01**	**8.148**	0.411	-0.455	**-0.901**	**-9.093**	**18.32**	**10.13**	**37.55**	**29.36**

Table 3: GMV Portfolio Performance Measures and Economic Gains from Switching to the EWMA Model -Short-Selling allowed (Biweekly and Monthly Forecasts)

This table shows portfolio statistics of the Global Minimum Variance portfolio, based on biweekly ($h = 10$), monthly ($h = 22$) predictions of the 100×100 covariance matrix, according to the following models: EWMA (with $\lambda = 0.96$), RM 94, HAR-DRD, CCHAR, DPC-CAW, CAW, HEAVY GAS. Parameters are estimated with a moving window of 1,000 observations and re-estimated after 22 observations. In the GMV exercise we assume an annual expected return of 5% to be fixed and identical across all stocks. We report the ex-post minimum realized portfolio volatility σ_{ϵ}^{HF}, as well as the turnover (TO) and short position (SP). The lowest portfolio volatilities obtained using the true realized covariance matrix are marked in bold if they belong to the model confidence set (MCS) based on a 5% significance level. The economic gains Δ_1 and Δ_{10} represent the annualized gain in basis points that a risk averse investor with risk aversion parameter $\gamma \in 1, 10$ and transaction costs $c \in 0\%, 1\%, 2\%$ would receive when switching from the model described in the row to the EWMA model. The gains that are significant at a 5% level are also marked bold. The out-of-sample period spans from January 2006 to December 2023, comprising 4,441 observations.

				$c=0\%$		$c=1\%$		$c=2\%$					$c=0\%$		$c=1\%$		$c=2\%$	
	σ_{ϵ}^{HF}	TO	SP	Δ_1	Δ_{10}	Δ_1	Δ_{10}	Δ_1	Δ_{10}	σ_{ϵ}^{HF}	TO	SP	Δ_1	Δ_{10}	Δ_1	Δ_{10}	Δ_1	Δ_{10}
	$h=10$									$h=22$								
Crisis																		
EWMA	**13.98**	0.266	-0.760	-	-	-	-	-	-	**14.06**	0.305	-0.760	-	-	-	-	-	-
RM 94	16.59	0.712	-1.052	**53.13**	**535.2**	**97.73**	**579.8**	**142.3**	**624.4**	16.59	0.745	-1.052	**48.38**	**492.7**	**92.37**	**536.7**	**136.4**	**580.7**
HAR-DRD	14.40	0.930	-0.682	27.27	276.1	**93.69**	342.6	**160.1**	**409.0**	14.94	0.799	-0.639	37.89	387.6	**87.22**	**437.0**	**136.5**	**486.3**
CCHAR	**13.91**	0.753	-0.668	-2.635	-26.85	**46.05**	21.84	**94.74**	**70.53**	**14.16**	0.681	-0.655	3.423	35.58	**41.00**	73.16	**78.58**	**110.7**
DPC-CAW	14.18	0.663	-0.620	3.384	34.43	**43.13**	**74.17**	**82.87**	**113.9**	14.40	0.570	-0.664	3.185	33.11	**29.63**	**59.55**	**56.07**	**85.99**
CAW	14.15	0.617	-0.770	1.878	19.12	**36.96**	54.20	**72.04**	**89.28**	14.32	0.506	-0.771	2.710	28.18	**22.79**	48.26	**42.88**	68.35
HEAVY GAS	**13.75**	0.454	-0.456	3.843	39.10	**22.69**	57.95	**41.54**	76.80	**13.80**	0.446	-0.471	-1.537	-16.01	**12.50**	-1.979	**26.53**	12.06
Non-Crisis																		
EWMA	8.514	0.229	-0.680	-	-	-	-	-	-	8.910	0.251	-0.680	-	-	-	-	-	-
RM 94	10.09	0.665	-0.980	**16.05**	**162.9**	**59.59**	**206.5**	**103.1**	**250.0**	10.45	0.681	-0.980	**15.95**	**164.8**	**58.97**	**207.8**	**102.0**	**250.9**
HAR-DRD	8.568	0.891	-0.697	0.690	7.029	**66.89**	**73.23**	**133.1**	**139.4**	8.975	0.748	-0.674	1.182	12.30	**50.85**	**61.96**	**100.5**	**111.6**
CCHAR	8.490	0.588	-0.648	-0.679	-6.919	**35.19**	**28.95**	**71.07**	**64.83**	8.881	0.487	-0.628	-0.516	-5.376	**23.10**	**18.24**	**46.71**	**41.85**
DPC-CAW	8.757	0.409	-0.546	**1.999**	**20.35**	**19.93**	**38.28**	**37.85**	**56.20**	9.226	0.311	-0.572	**3.072**	**31.94**	**9.012**	**37.88**	**14.95**	**43.82**
CAW	8.812	0.355	-0.664	**1.969**	**20.04**	**14.50**	**32.57**	**27.02**	**45.10**	9.250	0.267	-0.658	**2.934**	**30.50**	**4.531**	**32.10**	**6.127**	**33.69**
HEAVY GAS	**8.380**	0.394	-0.464	**-0.972**	**-9.895**	**15.53**	6.610	**32.04**	**23.11**	**8.800**	0.365	-0.482	-0.814	-8.478	**10.55**	2.887	**21.92**	**14.25**

6 Robustness checks

To assess the reliability and consistency of our findings, we conduct a series of robustness checks that examine the sensitivity of our results to alternative assumptions. Specifically, we focus on four relevant dimensions: (i) the construction of a 100 by 100 well-conditioned positive definite realized covariance matrix, (ii) the presence of short-selling restrictions, (iii) the choice of smoothing parameter λ in the EWMA model and (iv) portfolio dimensionality. These dimensions are motivated by practical considerations faced by investors, such as regulatory constraints, computational feasibility, and model calibration, as well as by the broader literature's focus on how model performance varies with portfolio size and parameter tuning.

6.1 Using non-linear shrinkage

Our first robustness check addresses the construction of a well-conditioned, positive-definite 100-by-100 realized covariance matrix. As an alternative to the eigenvalue cleaning approach of Hautsch et al. (2015), we employ the non-linear shrinkage method proposed by Ledoit and Wolf (2020). Using these regularized covariance matrices, we re-estimate the parameters of all volatility models within the same framework—namely, a moving window of 1,000 observations with parameter updates every 22 days. The corresponding results are reported in Tables A.1 and A.2. Overall, our main findings remain robust to the method used for constructing the realized covariance matrix. In terms of ex-post realized portfolio volatility, our benchmark model consistently belongs to the Model Confidence Set during crisis periods. It also remains within the set during tranquil periods, with the exception of daily forecast horizons. Notably, our benchmark model does not exhibit statistically significant underperformance relative to competing models across any of the fee specifications, with or without transaction costs. On the contrary, the EWMA model continues to deliver significant economic gains relative to most alternatives, particularly when accounting for transaction costs.

6.2 Imposing no short-selling constraints

Second, we revisit our primary analysis under the constraint of no short-selling, reflecting real-world limitations often imposed by regulation or risk management policies. Tables A.3 and A.4 in the Appendix show us the new results. As expected, imposing this constraint increases the realized ex-post portfolio volatility due to the restricted optimization space. We also see that the realized turnover decreases, indicating less aggressive portfolio rebalancing. Despite these changes, the EWMA model continues to deliver positive and significant economic gains in scenarios with transaction costs. In scenarios without transaction costs, no alternative model consistently outperforms EWMA, and the model remains in the 95% confidence set during crisis periods across all forecast horizons. Only the HEAVY GAS model performs slightly better in crises, for both daily and weekly forecasts; however, the economic significance is rather small, with a maximum of 8.3 annual basis points.

6.3 The smoothing parameter

We use $\lambda = 0.96$ when applying the EWMA filter. Since this value is imposed rather than estimated, understanding how sensitive our results are to its calibration is essential. Figures 1 and 2 present the minimum gain from switching to EWMA across different values of $\lambda \in 0.94, 0.95, 0.96, 0.97, 0.98$. For each setting, we report the smallest statistically significant gain an investor would achieve by switching to the EWMA model. If none of these gains are significant, we instead report the smallest observed gain (regardless of significance) among all alternative models.

During non-crisis periods, we find that higher values of λ (i.e., greater persistence in the covariance estimates) are associated with larger economic gains (or smaller losses when excluding transaction costs) from using EWMA. Lower values ($\lambda = 0.94, 0.95$) can result in modest losses relative to other models, though these are generally statistically significant only in the absence of transaction costs or for monthly horizons. The patterns are magnified for $\gamma = 10$ with relatively less gains due to the incremental weight of the ex-post portfolio volatility versus the impact of transaction costs on the estimated fee. Still, the losses remain economically small.

In crisis periods, the relationship reverses when there are no transaction costs: losses will increase and even become statistically significant when $\lambda = 0.98$, especially when risk aversion is high. This likely reflects the fact that excessive smoothing during volatile periods dampens important short-term signals in the covariance dynamics, which could otherwise enhance portfolio performance. Taking into account transaction costs, there is always a (significant) gain when using the EWMA filter when $\gamma = 1$. For a strongly risk-averse investor, our benchmark model is on par with all competitors.

We complement the third robustness check by re-running our exercise with respect to the smoothing parameter λ when short selling is not allowed. Figures A.1 and A.2 in the Appendix. The results are similar to those described above.

6.4 The portfolio size

We conclude by examining how the performance of the EWMA model changes with portfolio size, ranging from 10 to 100 assets (our baseline). Since model rankings may shift depending on portfolio dimensionality, this analysis helps assess the robustness of our findings. More sophisticated realized covariance models might outperform EWMA in smaller portfolios. While previous studies (Chiriac and Voev, 2011; Golosnoy et al., 2012, among others) have primarily evaluated the statistical performance of these models in low-dimensional settings, less is known about their relative economic performance in such contexts. To summarize our results, we report the minimum economic gain an investor would achieve by switching from any model in our set to the EWMA model across different portfolio sizes.

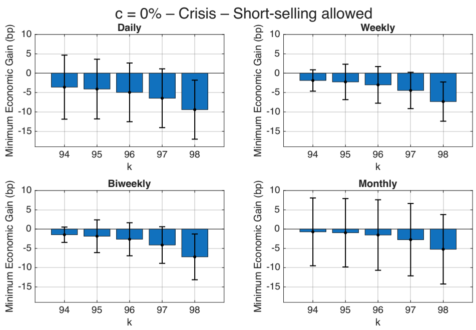
Figures 3 and 4 present these results under different levels of risk aversion ($\gamma = 1$ and $\gamma = 10$, respectively). Each bar represents the minimum gain, with 95% confidence intervals, and we distinguish between crisis and non-crisis periods, as well as transaction cost levels ($c \in 0\%, 1\%, 2\%$). Again, each bar shows the smallest statistically significant gain an investor would achieve by switching to the EWMA model. If no gains are significant, we instead report the smallest observed gain (regardless of significance) among all alternative models.

When investors exhibit lower risk aversion (Figure 3), no model significantly outperforms EWMA during crisis periods. Consistent with our baseline results, when transaction costs are present, switching to EWMA always yields positive and significant gains across all

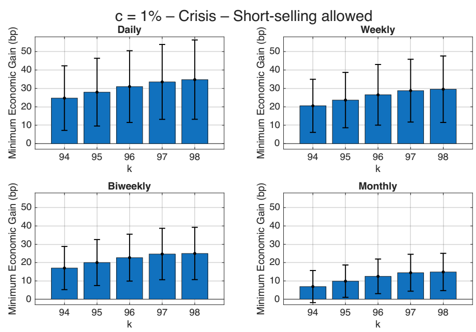
Figure 1: Minimum Economic Gain when Switching to the EWMA Model for Different Values of λ. Low Risk Aversion Scenario ($\gamma = 1$)

This figure shows the minimum economic gain, expressed in annual basis points (bp), obtained from switching from the models included in our analysis (RM 94, HAR-DRD, CCHAR, DPC-CAW, CAW, HEAVY GAS) to the EWMA model, based on the utility framework developed by Fleming et al. (2003), adopted by Hautsch et al. (2015), with a risk aversion parameter of $\gamma = 1$. Each bar shows the smallest statistically significant gain an investor would achieve by switching to the EWMA model. If none of these gains are significant, we instead report the smallest observed gain (regardless of significance) among all alternative models. Estimates are based on a 100-dimensional portfolio. Short-selling is allowed. Each row corresponds to a different transaction cost parameter (c), and each column represents a regime (Crisis vs. Non-Crisis). Each plot displays bars and 95% confidence intervals for different values of the smoothing parameter λ in the EWMA model, across various forecast horizons (daily, weekly, biweekly, and monthly).

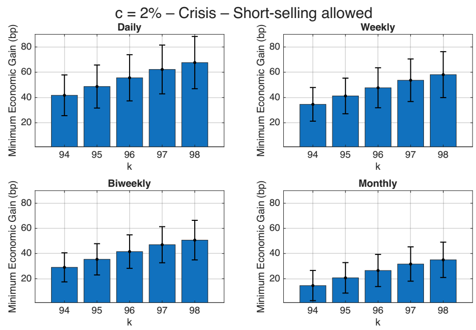
Crisis period



(a) $c = 0\%$

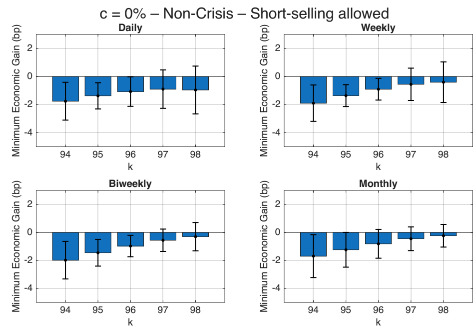


(b) $c = 1\%$

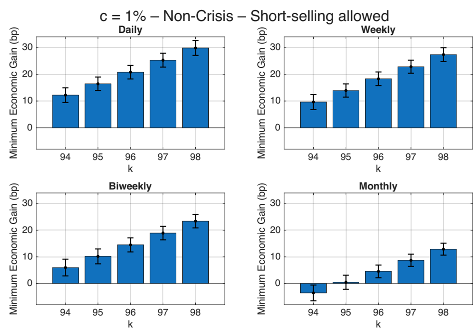


(c) $c = 2\%$

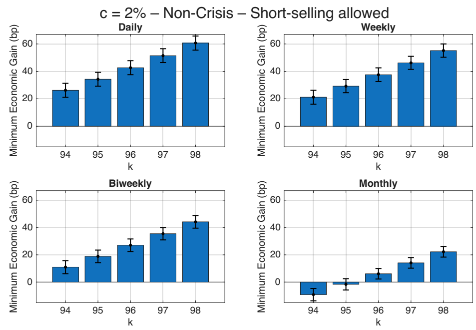
Non-Crisis period



(d) $c = 0\%$



(e) $c = 1\%$



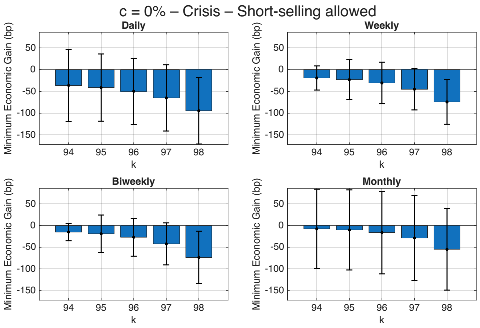
(f) $c = 2\%$

Figure 2: Minimum Economic Gain when Switching to the EWMA Model for Different Values of λ. High Risk Aversion Scenario ($\gamma = 10$)

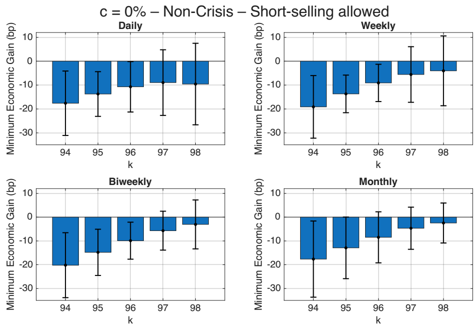
This figure shows the minimum economic gain, expressed in annual basis points (bp), obtained from switching from the models included in our analysis (RM 94, HAR-DRD, CCHAR, DPC-CAW, CAW, HEAVY GAS) to the EWMA model, based on the utility framework developed by Fleming et al. (2003), adopted by Hautsch et al. (2015), with a risk aversion parameter of $\gamma = 10$. Each bar shows the smallest statistically significant gain an investor would achieve by switching to the EWMA model. If none of these gains are significant, we instead report the smallest observed gain (regardless of significance) among all alternative models. Estimates are based on a 100-dimensional portfolio. Short-selling is allowed. Each row corresponds to a different transaction cost parameter (c), and each column represents a regime (Crisis vs. Non-Crisis). Each plot displays bars and 95% confidence intervals for different values of the smoothing parameter λ in the EWMA model across various forecast horizons (daily, weekly, biweekly, and monthly).

Crisis period

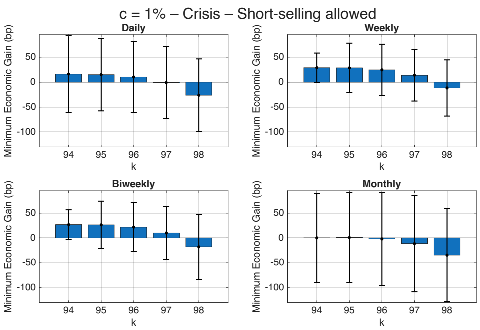
Non-Crisis period



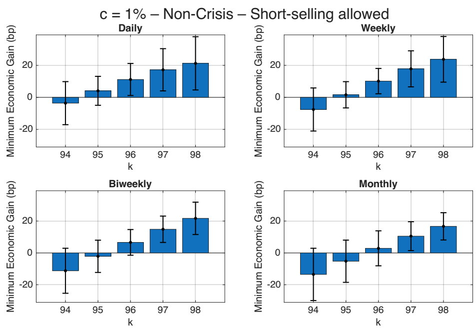
(a) $c = 0\%$



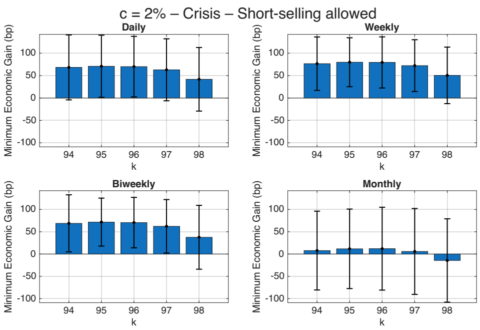
(d) $c = 0\%$



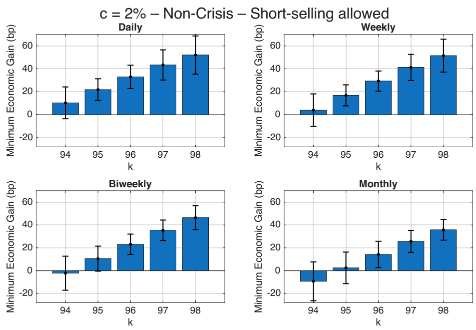
(b) $c = 1\%$



(e) $c = 1\%$



(c) $c = 2\%$



(f) $c = 2\%$

forecast horizons. These gains are smaller for small portfolios (10 assets) or longer horizons (monthly), typically around 15–25 basis points (bp), but may increase to over 60 bp for portfolios with 30–50 assets and shorter forecast horizons. This highlights the practical value of the EWMA model in turbulent periods, especially when transaction costs are a concern.

In non-crisis periods, the economic advantage of EWMA is more modest, particularly for small portfolios. In fact, in the absence of transaction costs, the EWMA is marginally outperformed (by the HEAVY GAS and the HAR-DRD models) in one case—the 10-asset portfolio—though the economic loss is negligible (maximum of 4 bp). For portfolios with 30 to 100 assets, EWMA again yields positive and statistically significant gains, especially for daily horizons (up to 40 bp at $c = 2\%$), with smaller gains at monthly horizons.

With higher risk aversion ($\gamma = 10$), the magnitudes and standard errors of the gains increase, as expected. Even under these conditions, the EWMA model is only significantly outperformed in one case: the 10-asset portfolio, in non-crisis periods with no transaction costs, where the loss ranges between 20 and 30 bp. In all other settings, the minimum gain from switching to EWMA remains positive and becomes statistically significant in larger portfolios during non-crisis periods.

The results of this robustness case in a scenario without short-selling are presented in Figures A.3 and A.4 in the Appendix. Although the EWMA gains are relatively smaller in this scenario, we find again that the EWMA model is not significantly outperformed in the presence of transaction costs, regardless of the forecast horizon or the market volatility conditions (crisis or non-crisis). Consistent with our previous findings, we find that the model is only outperformed in the absence of transaction costs and only for the smaller portfolios (10 assets). Again, the gains of switching to the best model are negligible for low-aversion investors.

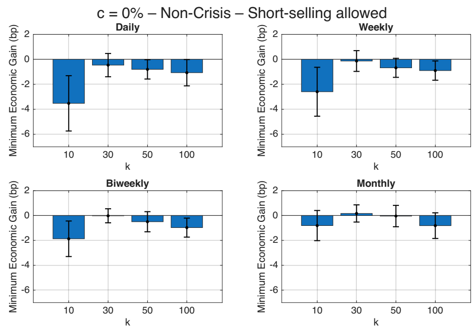
In sum, our main finding is robust against the construction of large realized covariance matrices and no-short-selling conditions. Furthermore, the EWMA model still behaves well when $\lambda \in 0.94, 0.95, 0.97, 0.98$, though too much (less) smoothing is not favorable in times of (non) crisis periods. Finally, our results are robust against portfolio size, particularly for 30- and 50-dimensional portfolios. For small portfolios ($k = 10$), the EWMA could economically be outperformed by competitors, but this only occurs in the stylized setting of no transaction costs. Overall, these robustness checks confirm the stability of our main findings. The EWMA model proves resilient across a wide range of settings and assumptions, and continues to offer competitive—often superior—economic performance compared to models that incorporate high-frequency information.

Figure 3: Minimum Economic Gain when Switching to the EWMA Model Across Portfolio Dimensions. Low Risk Aversion Scenario ($\gamma = 1$)

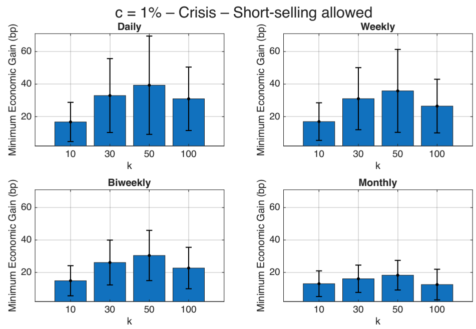
This figure shows the minimum economic gain, expressed in annual basis points (bp), obtained from switching from the models included in our analysis (RM 94, HAR-DRD, CCHAR, DPC-CAW, CAW, HEAVY GAS) to the EWMA model, based on the utility framework developed by Fleming et al. (2003), adopted by Hautsch et al. (2015), with a risk aversion parameter of $\gamma = 1$. Each bar shows the smallest statistically significant gain an investor would achieve by switching to the EWMA model. If none of these gains are significant, we instead report the smallest observed gain (regardless of significance) among all alternative models. Short-selling is allowed. Each row corresponds to a different transaction cost parameter (c), and each column represents a regime (Crisis vs. Non-Crisis). Each plot displays bars and 95% confidence intervals for different portfolio sizes (k assets) across various forecast horizons (daily, weekly, biweekly, and monthly).

Crisis period | **Non-Crisis period**

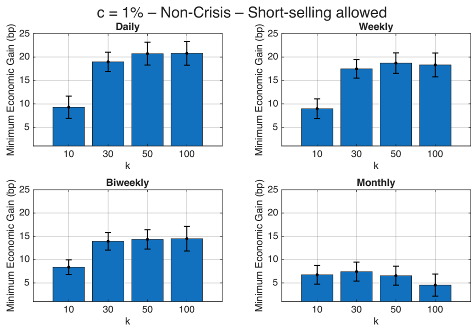
(a) $c = 0\%$



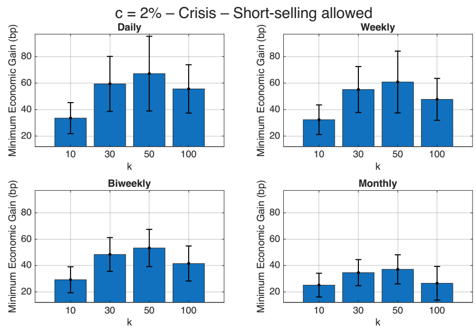
(d) $c = 0\%$



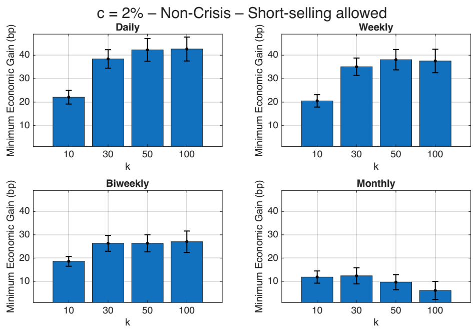
(b) $c = 1\%$



(e) $c = 1\%$



(c) $c = 2\%$

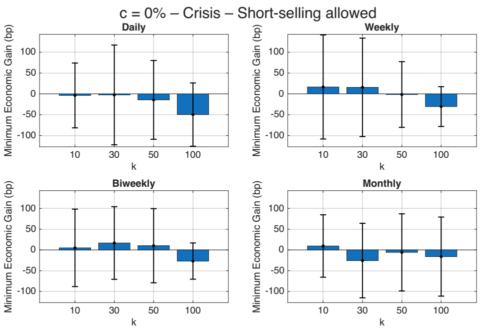


(f) $c = 2\%$

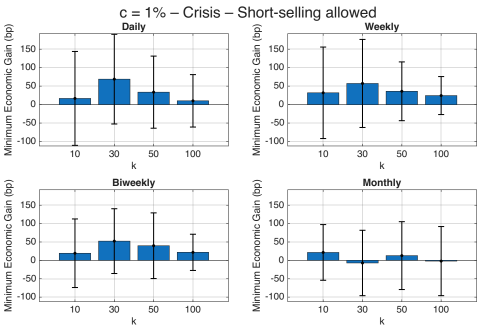
Figure 4: Minimum Economic Gain when Switching to the EWMA Model Across Portfolio Dimensions. High Risk Aversion Scenario ($\gamma = 10$)

This figure shows the minimum economic gain, expressed in annual basis points (bp), obtained from switching from the models included in our analysis (RM 94, HAR-DRD, CCHAR, DPC-CAW, CAW, HEAVY GAS) to the EWMA model, based on the utility framework developed by Fleming et al. (2003), adopted by Hautsch et al. (2015), with a risk aversion parameter of $\gamma = 10$. Each bar shows the smallest statistically significant gain an investor would achieve by switching to the EWMA model. If none of these gains are significant, we instead report the smallest observed gain (regardless of significance) among all alternative models. Short-selling is allowed. Each row corresponds to a different transaction cost parameter (c), and each column represents a regime (Crisis vs. Non-Crisis). Each plot displays bars and 95% confidence intervals for different portfolio sizes (k assets) across various forecast horizons (daily, weekly, biweekly, and monthly).

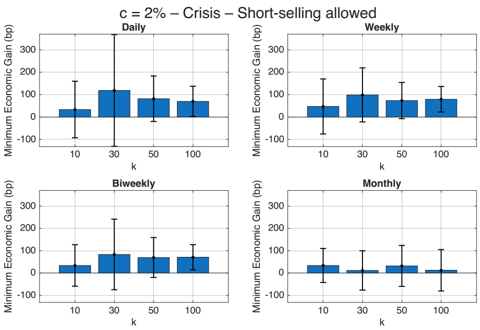
Crisis period



(a) $c = 0\%$

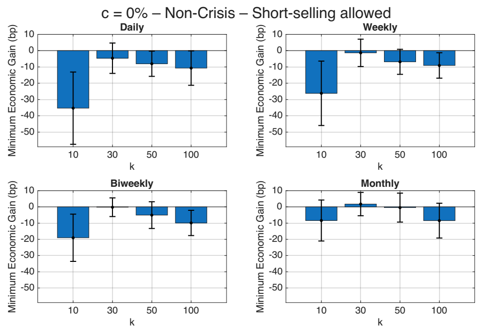


(b) $c = 1\%$

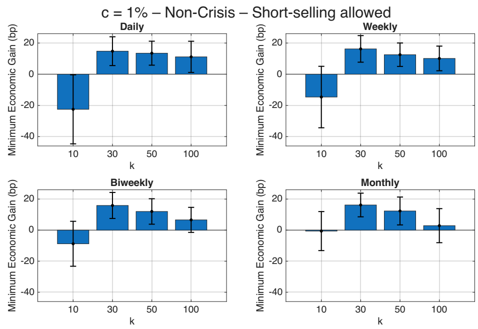


(c) $c = 2\%$

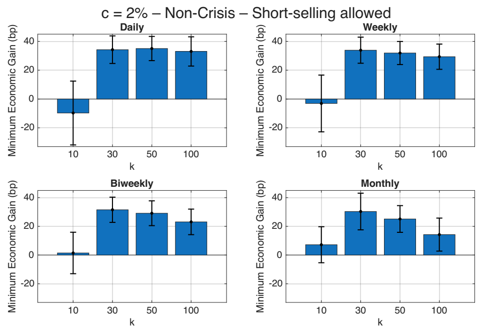
Non-Crisis period



(d) $c = 0\%$



(e) $c = 1\%$



(f) $c = 2\%$

7 Conclusions

This paper contributes to the growing literature on portfolio optimization with high-frequency data by providing a comprehensive, out-of-sample comparison of alternative covariance forecasting models within the Global Minimum Variance (GMV) portfolio framework. While prior work often focuses on statistical accuracy, or assesses the economic value of high-frequency data within a single modeling framework, we offer a systematic comparison across a broad set of models, including a variety of approaches, such as multivariate GARCH-type models, score-driven multivariate models (HEAVY GAS), and longer-term memory structures (HAR-DRD), that have been recognized in the literature for their superior statistical performance. We shift the focus toward economic gains from an investor's perspective, explicitly accounting for differences in risk aversion, transaction costs, and market volatility regimes.

Our central finding is that the EWMA model, a simple yet powerful framework that leverages high-frequency returns for covariance estimation, consistently delivers strong performance relative to more sophisticated alternatives, including models with complex dynamics and structural components. This result holds across a wide range of portfolio sizes—from 30 to 100 assets—and is particularly robust during crisis periods, when volatility is elevated and model performance is most critical.

We find that even in the absence of transaction costs, no alternative model offers a substantial economic advantage over the EWMA. This robustness speaks to the model's intrinsic performance qualities. When transaction costs are included in the analysis, the EWMA model consistently delivers significant economic gains under a variety of scenarios. While our analysis employs a static, proportional cost structure, which is widely used in the literature, we acknowledge that in practice, transaction costs are stochastic and endogenous, linked to portfolio turnover, market conditions, and liquidity. Accounting for these features would transform the optimization problem and represents a promising direction for future research, offering a richer understanding of real-world investment performance. Recent work by Hautsch and Voigt (2019) provides important steps in this direction by modeling transaction costs as part of the optimization problem, demonstrating their link with turnover penalization and covariance shrinkage.

We also contribute to the practitioner-oriented literature by incorporating realistic portfolio constraints, such as restrictions on short-selling, which are often imposed by regulation or institutional mandates. Our results show that even under these constraints, the EWMA model maintains its relative advantage, especially in the presence of transaction costs. The framework we propose thus offers both theoretical insight and practical value, highlighting that relatively simple high-frequency models can be preferable to more complex alternatives under realistic conditions.

In summary, our results suggest that a simple, high-frequency-based EWMA approach remains a highly effective tool for covariance forecasting in portfolio management, offering a valuable balance between empirical robustness, economic performance, and practical implementation.

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A Additional results

Table A.1: GMV Portfolio Performance Measures and Economic Gains from Switching to the EWMA Model - Non-Linear Shrinkage (Daily and Weekly Forecasts)

This table shows portfolio statistics of the Global Minimum Variance portfolio, based on daily ($h = 1$), weekly ($h = 5$) predictions of the 100×100 covariance matrix, according to the following models: EWMA (with $\lambda = 0.96$), RM 94, HAR-DRD, CCHAR, DPC-CAW, CAW, HEAVY GAS. Parameters are estimated with a moving window of 1,000 observations and re-estimated after 22 observations. In the GMV exercise we assume an annual expected return of 5% to be fixed and identical across all stocks. We report the ex-post minimum realized portfolio volatility σ_ϵ^{HF}, as well as the turnover (TO) and short position (SP). The lowest portfolio volatilities obtained using the true realized covariance matrix are marked in bold if they belong to the model confidence set (MCS) based on a 5% significance level. The economic gains Δ_1 and Δ_{10} represent the annualized gain in basis points that a risk averse investor with risk aversion parameter $\gamma \in 1, 10$ and transaction costs $c \in 0\%, 1\%, 2\%$ would receive when switching from the model described in the row to the EWMA model. The gains that are significant at a 5% level are also marked bold. The out-of-sample period spans from January 2006 to December 2023 and comprises 4,441 observations.

				$c = 0\%$		$c = 1\%$		$c = 2\%$					$c = 0\%$		$c = 1\%$		$c = 2\%$	
	σ_ϵ^{HF}	TO	SP	Δ_1	Δ_{10}	Δ_1	Δ_{10}	Δ_1	Δ_{10}	σ_ϵ^{HF}	TO	SP	Δ_1	Δ_{10}	Δ_1	Δ_{10}	Δ_1	Δ_{10}
	$h = 1$									$h = 5$								
Crisis																		
EWMA	**13.24**	0.201	-0.699	-	-	-	-	-	-	**13.73**	0.232	-0.699	-	-	-	-	-	-
RM 94	16.46	0.611	-1.139	**66.53**	**665.6**	**107.5**	**706.6**	**148.6**	**747.7**	16.96	0.635	-1.139	**67.53**	**676.8**	**107.8**	**717.2**	**148.1**	**757.5**
HAR-DRD	**13.13**	1.126	-0.638	1.660	16.63	**94.11**	**109.1**	**186.6**	**201.5**	**13.80**	0.883	-0.591	12.47	125.6	**77.53**	190.7	**142.6**	**255.8**
CCHAR	**13.14**	0.844	-0.669	-2.904	-29.09	**61.43**	35.24	**125.8**	**99.58**	**13.65**	0.773	-0.634	-2.267	-22.88	**51.83**	31.22	**105.9**	**85.31**
DPC-CAW	**13.20**	0.744	-0.549	3.246	32.52	**57.54**	86.81	**111.8**	**141.1**	**13.81**	0.666	-0.571	1.930	19.47	**45.29**	**62.84**	**88.66**	**106.2**
CAW	**13.19**	0.714	-0.728	0.074	0.740	**51.38**	52.05	**102.7**	**103.4**	**13.79**	0.628	-0.734	-0.343	-3.456	**39.19**	36.08	**78.72**	**75.61**
HEAVY GAS	**13.43**	0.425	-0.350	26.43	264.7	**48.82**	287.0	**71.20**	309.4	**14.04**	0.427	-0.353	24.97	251.3	**44.39**	270.8	**63.81**	290.2
Non-Crisis																		
EWMA	7.817	0.192	-0.639	-	-	-	-	-	-	**8.176**	0.203	-0.639	-	-	-	-	-	-
RM 94	9.575	0.583	-1.046	**16.92**	**169.5**	**55.98**	**208.5**	**95.04**	**247.6**	9.922	0.590	-1.046	**17.22**	**173.5**	**55.94**	**212.2**	**94.65**	**250.9**
HAR-DRD	7.771	1.121	-0.640	-0.675	-6.764	**92.20**	**86.11**	**185.1**	**179.0**	**8.115**	0.847	-0.605	-0.594	-5.996	**63.85**	**58.45**	**128.3**	**122.9**
CCHAR	8.010	0.768	-0.657	**1.084**	**10.86**	**58.63**	**68.41**	**116.2**	**126.0**	8.225	0.614	-0.619	-0.060	-0.608	**41.08**	**40.53**	**82.21**	**81.67**
DPC-CAW	7.910	0.611	-0.494	0.155	1.554	**42.06**	**43.46**	**83.97**	**85.37**	8.359	0.459	-0.511	**1.307**	**13.18**	**26.96**	**38.84**	**52.62**	**64.50**
CAW	7.960	0.506	-0.645	0.079	0.792	**31.47**	**32.19**	**62.87**	**63.58**	8.421	0.378	-0.634	**1.293**	**13.05**	**18.79**	**30.54**	**36.28**	**48.03**
HEAVY GAS	**7.706**	0.332	-0.339	0.761	7.620	**14.77**	**21.63**	**28.77**	**35.63**	**8.139**	0.326	-0.345	0.814	8.210	**13.18**	**20.58**	**25.55**	**32.94**

Table A.2: GMV Portfolio Performance Measures and Economic Gains from Switching to the EWMA Model -Non-Linear Shrinkage (Biweekly and Monthly Forecasts)

This table shows portfolio statistics of the Global Minimum Variance portfolio, based on biweekly ($h = 10$), monthly ($h = 22$) predictions of the 100×100 covariance matrix, according to the following models: EWMA (with $\lambda = 0.96$), RM 94, HAR-DRD, CCHAR, DPC-CAW, CAW, HEAVY GAS. Parameters are estimated with a moving window of 1,000 observations and re-estimated after 22 observations. In the GMV exercise we assume an annual expected return of 5% to be fixed and identical across all stocks. We report the ex-post minimum realized portfolio volatility σ_{ϵ}^{HF}, as well as the turnover (TO) and short position (SP). The lowest portfolio volatilities obtained using the true realized covariance matrix are marked in bold if they belong to the model confidence set (MCS) based on a 5% significance level. The economic gains Δ_1 and Δ_{10} represent the annualized gain in basis points that a risk averse investor with risk aversion parameter $\gamma \in 1, 10$ and transaction costs $c \in 0\%, 1\%, 2\%$ would receive when switching from the model described in the row to the EWMA model. The gains that are significant at a 5% level are also marked bold. The out-of-sample period spans from January 2006 to December 2023 and comprises 4,441 observations.

				$c = 0\%$		$c = 1\%$		$c = 2\%$					$c = 0\%$		$c = 1\%$		$c = 2\%$	
	σ_{ϵ}^{HF}	TO	SP	Δ_1	Δ_{10}	Δ_1	Δ_{10}	Δ_1	Δ_{10}	σ_{ϵ}^{HF}	TO	SP	Δ_1	Δ_{10}	Δ_1	Δ_{10}	Δ_1	Δ_{10}
	$h = 10$									$h = 22$								
Crisis																		
EWMA	**13.96**	0.250	-0.699	-	-	-	-	-	-	**14.04**	0.288	-0.699	-	-	-	-	-	-
RM 94	17.06	0.652	-1.139	**60.33**	**606.9**	**100.5**	**647.1**	**140.7**	**687.3**	17.03	0.691	-1.139	**55.26**	**561.1**	**95.56**	**601.4**	**135.9**	**641.7**
HAR-DRD	**14.21**	0.804	-0.573	20.95	212.4	**76.27**	267.7	**131.6**	**323.0**	**14.70**	0.701	-0.541	31.69	325.2	**72.97**	366.5	**114.3**	**407.7**
CCHAR	**13.87**	0.733	-0.621	-3.463	-35.29	**44.78**	12.96	**93.03**	**61.20**	**14.06**	0.663	-0.614	0.601	6.259	**38.09**	43.75	**75.58**	**81.23**
DPC-CAW	**14.13**	0.603	-0.596	1.404	14.29	**36.64**	**49.53**	**71.87**	**84.76**	**14.32**	0.524	-0.644	1.031	10.73	**24.61**	34.31	**48.19**	57.89
CAW	**14.11**	0.555	-0.738	1.483	15.09	**31.91**	45.52	**62.34**	75.95	**14.26**	0.464	-0.744	2.012	20.92	**19.63**	38.54	**37.25**	56.16
HEAVY GAS	**14.25**	0.427	-0.358	22.24	225.5	**39.90**	243.1	**57.56**	260.8	**14.25**	0.427	-0.371	13.65	141.2	**27.58**	155.2	**41.51**	169.1
Non-Crisis																		
EWMA	**8.381**	0.213	-0.639	-	-	-	-	-	-	**8.781**	0.236	-0.639	-	-	-	-	-	-
RM 94	10.12	0.598	-1.046	**17.25**	**175.0**	**55.74**	**213.5**	**94.24**	**252.0**	10.47	0.618	-1.046	**17.10**	**176.6**	**55.28**	**214.8**	**93.45**	**252.9**
HAR-DRD	**8.350**	0.753	-0.598	-0.225	-2.292	**53.74**	**51.67**	**107.7**	**105.6**	**8.780**	0.646	-0.586	0.511	5.320	**41.45**	**46.26**	**82.38**	**87.19**
CCHAR	**8.392**	0.542	-0.605	-0.269	-2.738	**32.58**	**30.11**	**65.43**	**62.96**	**8.769**	0.455	-0.591	-0.425	-4.428	**21.41**	**17.41**	**43.25**	**39.24**
DPC-CAW	8.641	0.373	-0.528	**2.192**	**22.31**	**18.13**	**38.25**	**34.07**	**54.19**	9.106	0.290	-0.551	**3.019**	**31.39**	**8.363**	**36.73**	**13.71**	**42.07**
CAW	8.685	0.303	-0.628	**2.061**	**20.98**	**10.98**	**29.90**	**19.90**	**38.82**	9.121	0.239	-0.624	**2.763**	**28.73**	**3.011**	**28.98**	3.259	**29.23**
HEAVY GAS	**8.368**	0.320	-0.352	0.850	8.650	**11.56**	**19.36**	**22.26**	**30.06**	**8.775**	0.311	-0.369	0.916	9.536	**8.395**	**17.01**	**15.87**	**24.49**

Table A.3: GMV Portfolio Performance Measures and Economic Gains from Switching to the EWMA Model -Short-Selling allowed (Daily and Weekly Forecasts)

This table shows portfolio statistics of the Global Minimum Variance portfolio, based on daily ($h = 1$), weekly ($h = 5$) predictions of the 100×100 covariance matrix, according to the following models: EWMA (with $\lambda = 0.96$), RM 94, HAR-DRD, CCHAR, DPC-CAW, CAW, HEAVY GAS. Parameters are estimated with a moving window of 1,000 observations and re-estimated after 22 observations. In the GMV exercise we assume an annual expected return of 5% to be fixed and identical across all stocks. We report the ex-post minimum realized portfolio volatility σ_ϵ^{HF}, as well as the turnover (TO) and short position (SP). The lowest portfolio volatilities obtained using the true realized covariance matrix are marked in bold if they belong to the model confidence set (MCS) based on a 5% significance level. The economic gains Δ_1 and Δ_{10} represent the annualized gain in basis points that a risk averse investor with risk aversion parameter $\gamma \in 1, 10$ and transaction costs $c \in 0\%, 1\%, 2\%$ would receive when switching from the model described in the row to the EWMA model. The gains that are significant at a 5% level are also marked bold. The out-of-sample period spans from January 2006 to December 2023 and comprises 4,441 observations.

			$c = 0\%$		$c = 1\%$		$c = 2\%$				$c = 0\%$		$c = 1\%$		$c = 2\%$	
	σ_ϵ^{HF}	TO	Δ_1	Δ_{10}	Δ_1	Δ_{10}	Δ_1	Δ_{10}	σ_ϵ^{HF}	TO	Δ_1	Δ_{10}	Δ_1	Δ_{10}	Δ_1	Δ_{10}
	$h = 1$								$h = 5$							
Crisis																
EWMA	**15.08**	0.075	-	-	-	-	-	-	**15.59**	0.078	-	-	-	-	-	-
RM 94	16.70	0.281	**49.28**	**493.2**	**69.88**	**513.8**	**90.48**	**534.4**	17.22	0.282	**47.41**	**476.2**	**67.76**	**496.5**	**88.10**	**516.9**
HAR-DRD	**15.09**	0.592	3.559	35.65	**55.28**	87.37	**107.0**	**139.1**	**15.65**	0.465	2.453	24.74	**41.07**	63.36	**79.69**	**102.0**
CCHAR	15.32	0.483	-0.367	-3.673	**40.39**	37.09	**81.15**	77.84	**15.74**	0.410	0.238	2.403	**33.43**	35.59	**66.62**	68.79
DPC-CAW	**15.08**	0.383	-4.188	-41.96	**26.60**	-11.17	**57.39**	19.62	**15.58**	0.333	-3.612	-36.45	**21.88**	-10.96	**47.37**	14.52
CAW	**15.05**	0.342	-4.356	-43.64	**22.38**	-16.90	**49.13**	9.838	**15.61**	0.288	-2.224	-22.45	**18.70**	-1.525	**39.62**	19.40
HEAVY GAS	**14.96**	0.227	6.276	62.86	**21.47**	78.05	**36.66**	93.25	**15.55**	0.220	3.835	38.68	**17.95**	52.80	**32.07**	66.91
Non-Crisis																
EWMA	8.637	0.088	-	-	-	-	-	-	9.054	0.091	-	-	-	-	-	-
RM 94	9.691	0.305	**10.99**	**110.0**	**32.61**	**131.7**	**54.24**	**153.3**	10.13	0.306	**11.27**	**113.6**	**32.76**	**135.1**	**54.24**	**156.5**
HAR-DRD	8.587	0.617	0.460	4.613	**53.31**	**57.46**	**106.2**	**110.3**	**9.017**	0.464	0.163	1.642	**37.49**	**38.97**	**74.82**	**76.30**
CCHAR	8.855	0.446	**1.454**	**14.57**	**37.19**	**50.30**	**72.92**	**86.03**	9.090	0.337	-0.451	-4.553	**24.20**	**20.10**	**48.85**	**44.75**
DPC-CAW	8.740	0.358	-0.403	-4.036	**26.51**	**22.87**	**53.42**	**49.78**	9.194	0.254	0.519	5.234	**16.82**	**21.53**	**33.11**	**37.83**
CAW	8.763	0.303	-0.506	-5.073	**20.92**	16.35	**42.35**	**37.78**	9.258	0.205	0.779	7.858	**12.22**	19.29	**23.65**	**30.73**
HEAVY GAS	**8.512**	0.213	**-0.830**	**-8.316**	**11.65**	4.166	**24.13**	**16.65**	**8.981**	0.201	**-0.624**	**-6.301**	**10.41**	4.733	**21.44**	**15.77**

Table A.4: GMV Portfolio Performance Measures and Economic Gains from Switching to the EWMA Model -Short-Selling allowed (Biweekly and Monthly Forecasts)

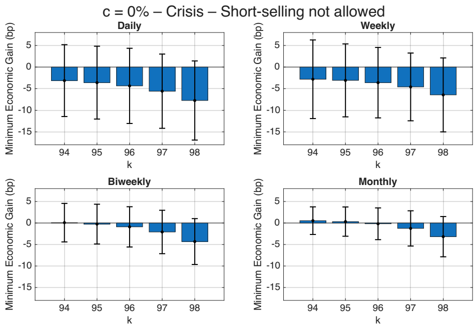
This table shows portfolio statistics of the Global Minimum Variance portfolio, based on biweekly ($h = 10$), monthly ($h = 22$) predictions of the 100×100 covariance matrix, according to the following models:EWMA (with $\lambda = 0.96$), RM 94, HAR-DRD, CCHAR, DPC-CAW, CAW, HEAVY GAS. Parameters are estimated with a moving window of 1,000 observations and re-estimated after 22 observations. In the GMV exercise we assume an annual expected return of 5% to be fixed and identical across all stocks. We report the ex-post minimum realized portfolio volatility σ_{ϵ}^{HF}, as well as the turnover (TO) and short position (SP). The lowest portfolio volatilities obtained using the true realized covariance matrix are marked in bold if they belong to the model confidence set (MCS) based on a 5% significance level. The economic gains Δ_1 and Δ_{10} represent the annualized gain in basis points that a risk averse investor with risk aversion parameter $\gamma \in 1, 10$ and transaction costs $c \in 0\%, 1\%, 2\%$ would receive when switching from the model described in the row to the EWMA model. The gains that are significant at a 5% level are also marked bold. The out-of-sample period spans from January 2006 to December 2023 and comprises 4,441 observations.

			$c = 0\%$		$c = 1\%$		$c = 2\%$				$c = 0\%$		$c = 1\%$		$c = 2\%$	
	σ_{ϵ}^{HF}	TO	Δ_1	Δ_{10}	Δ_1	Δ_{10}	Δ_1	Δ_{10}	σ_{ϵ}^{HF}	TO	Δ_1	Δ_{10}	Δ_1	Δ_{10}	Δ_1	Δ_{10}
	$h = 10$								$h = 22$							
Crisis																
EWMA	**15.76**	0.083	-	-	-	-	-	-	**15.72**	0.090	-	-	-	-	-	-
RM 94	17.39	0.285	**43.43**	**438.4**	**63.63**	**458.6**	**83.83**	**478.8**	17.37	0.288	**37.14**	**380.2**	**56.99**	**400.0**	**76.83**	**419.8**
HAR-DRD	16.04	0.418	**12.95**	**131.5**	**46.54**	**165.1**	**80.13**	**198.7**	16.36	0.356	**22.97**	**236.6**	**49.59**	**263.2**	**76.22**	**289.8**
CCHAR	15.93	0.379	2.907	29.58	**32.60**	**59.27**	**62.29**	**88.96**	**15.82**	0.322	1.287	13.39	**24.54**	**36.65**	**47.80**	**59.90**
DPC-CAW	**15.88**	0.294	4.454	45.31	**25.59**	66.44	**46.73**	**87.58**	15.98	0.241	9.855	102.1	**24.99**	117.3	**40.12**	132.4
CAW	**15.81**	0.244	-0.896	-9.126	**15.21**	6.982	**31.32**	23.09	**15.82**	0.187	0.539	5.611	**10.26**	15.33	**19.97**	25.05
HEAVY GAS	**15.74**	0.212	2.907	29.58	**15.82**	42.50	**28.73**	55.41	**15.69**	0.196	-0.179	-1.863	**10.46**	8.773	**21.09**	19.41
Non-Crisis																
EWMA	**9.267**	0.093	-	-	-	-	-	-	**9.659**	0.099	-	-	-	-	-	-
RM 94	10.36	0.307	**11.74**	**119.2**	**33.12**	**140.6**	**54.50**	**162.0**	10.76	0.310	**12.01**	**124.3**	**33.10**	**145.4**	**54.18**	**166.5**
HAR-DRD	**9.246**	0.405	-0.363	-3.694	**30.76**	**27.43**	**61.89**	**58.56**	**9.649**	0.333	0.092	0.954	**23.48**	**24.34**	**46.87**	**47.73**
CCHAR	**9.262**	0.292	-0.697	-7.099	**19.20**	12.80	**39.10**	**32.69**	**9.641**	0.234	-0.442	-4.602	**13.07**	8.913	**26.59**	**22.43**
DPC-CAW	9.450	0.198	1.199	12.20	**11.66**	**22.66**	**22.12**	**33.12**	9.887	0.142	**2.343**	**24.37**	**6.648**	**28.67**	**10.95**	**32.98**
CAW	9.518	0.153	1.566	15.95	**7.497**	**21.88**	**13.43**	**27.81**	9.952	0.106	**2.791**	**29.01**	**3.508**	**29.73**	**4.226**	**30.45**
HEAVY GAS	**9.205**	0.188	-0.761	-7.749	**8.752**	1.765	**18.27**	**11.28**	**9.612**	0.166	-0.594	-6.188	**6.065**	0.471	**12.72**	7.131

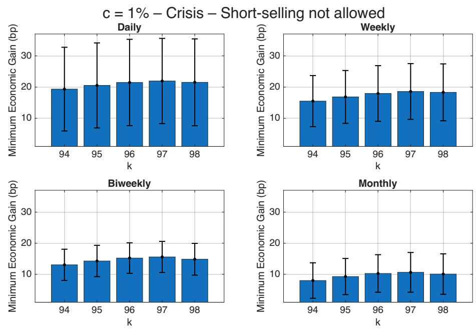
Figure A.1: Minimum Economic Gain when Switching to the EWMA Model for Different Values of λ. Low Risk Aversion Scenario ($\gamma = 1$). No Short-Selling.

This figure shows the minimum economic gain, expressed in annual basis points (bp), obtained from switching from the models included in our analysis (RM 94, HAR-DRD, CCHAR, DPC-CAW, CAW, HEAVY GAS) to the EWMA model, based on the utility framework developed by Fleming et al. (2003), adopted by Hautsch et al. (2015), with a risk aversion parameter of $\gamma = 1$. Each bar shows the smallest statistically significant gain an investor would achieve by switching to the EWMA model. If none of these gains are significant, we instead report the smallest observed gain (regardless of significance) among all alternative models. Estimates are based on a 100-dimensional portfolio. Short-selling is not allowed. Each row corresponds to a different transaction cost parameter (c), and each column represents a regime (Crisis vs. Non-Crisis). Each plot displays bars and 95% confidence intervals for different values of the smoothing parameter λ in the EWMA model across various forecast horizons (daily, weekly, biweekly, and monthly).

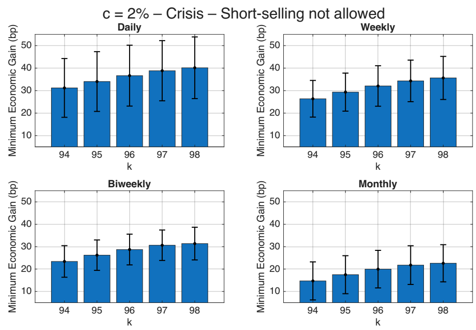
Crisis period



(a) $c = 0\%$

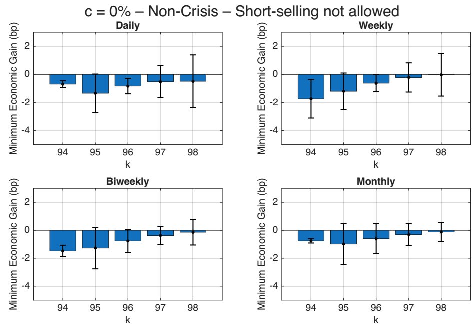


(b) $c = 1\%$

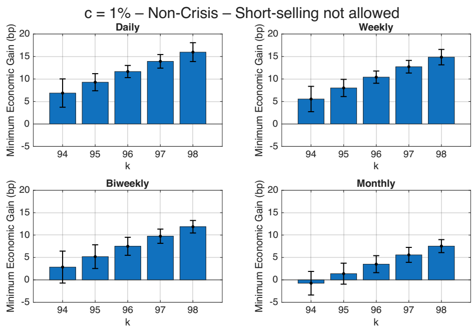


(c) $c = 2\%$

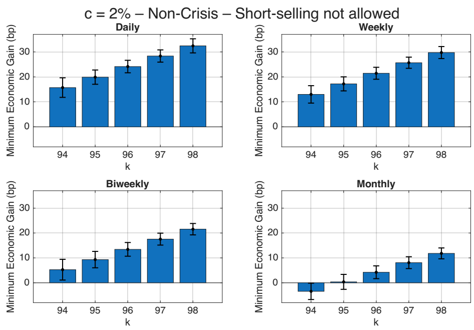
Non-Crisis period



(d) $c = 0\%$



(e) $c = 1\%$

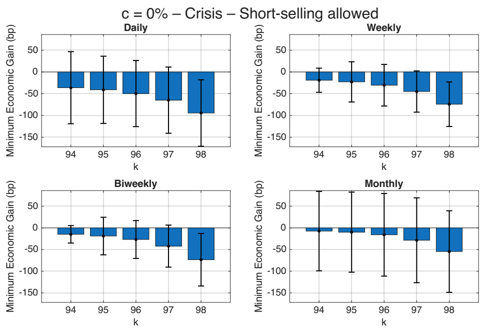


(f) $c = 2\%$

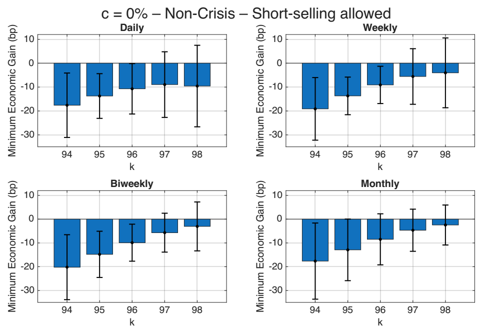
Figure A.2: Minimum Economic Gain when Switching to the EWMA Model for Different Values of λ. High Risk Aversion Scenario ($\gamma = 10$). No Short-Selling.

This figure shows the minimum economic gain, expressed in annual basis points (bp), obtained from switching from the models included in our analysis (RM 94, HAR-DRD, CCHAR, DPC-CAW, CAW, HEAVY GAS) to the EWMA model, based on the utility framework developed by Fleming et al. (2003), adopted by Hautsch et al. (2015), with a risk aversion parameter of $\gamma = 10$. Each bar shows the smallest statistically significant gain an investor would achieve by switching to the EWMA model. If none of these gains are significant, we instead report the smallest observed gain (regardless of significance) among all alternative models. Estimates are based on a 100-dimensional portfolio. Short-selling is not allowed. Each row corresponds to a different transaction cost parameter (c), and each column represents a regime (Crisis vs. Non-Crisis). Each plot displays bars and 95% confidence intervals for different values of the smoothing parameter λ in the EWMA model across various forecast horizons (daily, weekly, biweekly, and monthly).

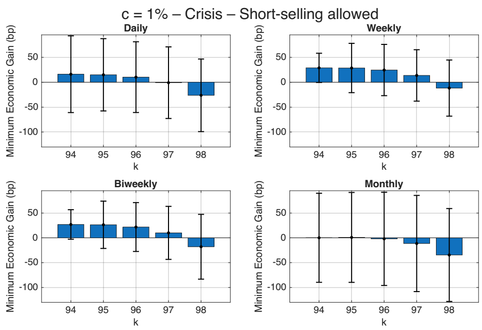
Crisis period | **Non-Crisis period**



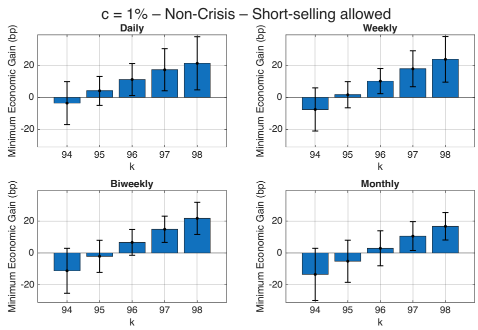
(a) $c = 0\%$



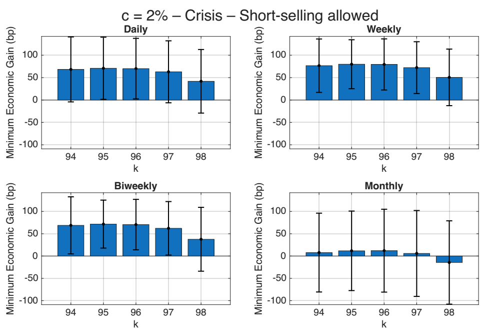
(d) $c = 0\%$



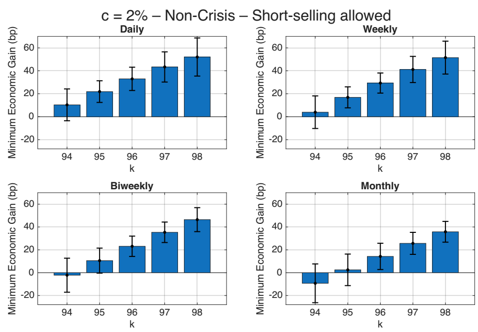
(b) $c = 1\%$



(e) $c = 1\%$



(c) $c = 2\%$



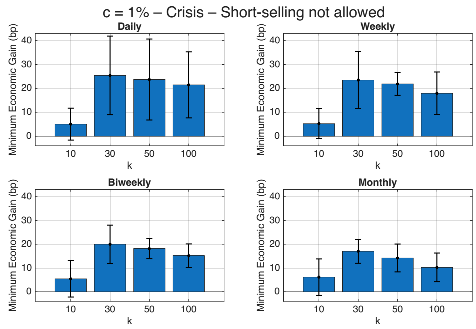
(f) $c = 2\%$

Figure A.3: Minimum Economic Gain when Switching to the EWMA Model Across Portfolio Dimensions. Low Risk Aversion Scenario ($\gamma = 1$). No Short-Selling.

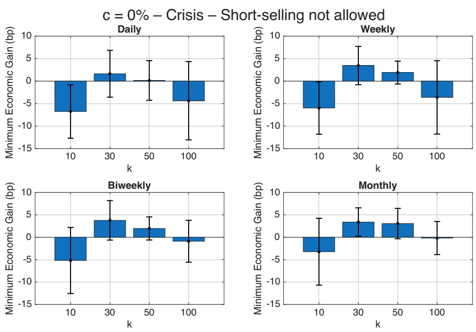
This figure shows the minimum economic gain, expressed in annual basis points (bp), obtained from switching from the models included in our analysis (RM 94, HAR-DRD, CCHAR, DPC-CAW, CAW, HEAVY GAS) to the EWMA model, based on the utility framework developed by Fleming et al. (2003), adopted by Hautsch et al. (2015), with a risk aversion parameter of $\gamma = 1$. Each bar shows the smallest statistically significant gain an investor would achieve by switching to the EWMA model. If none of these gains are significant, we instead report the smallest observed gain (regardless of significance) among all alternative models. Short-selling is not allowed. Each row corresponds to a different transaction cost parameter (c), and each column represents a regime (Crisis vs. Non-Crisis). Each plot displays bars and 95% confidence intervals for different portfolio sizes (k assets) across various forecast horizons (daily, weekly, biweekly, and monthly).

Crisis period

(a) $c = 0\%$

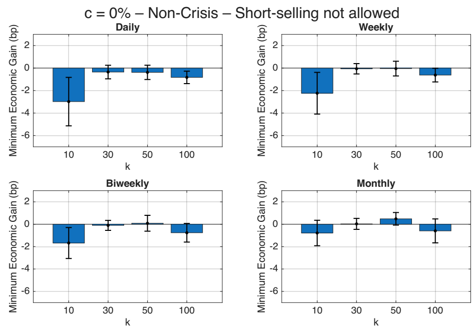


(b) $c = 1\%$

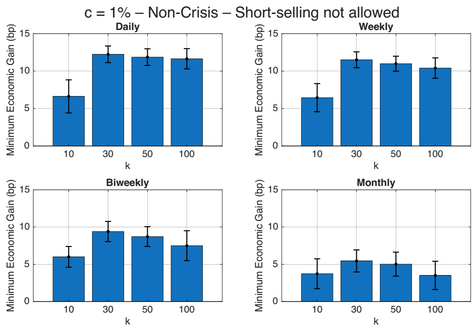


(c) $c = 2\%$

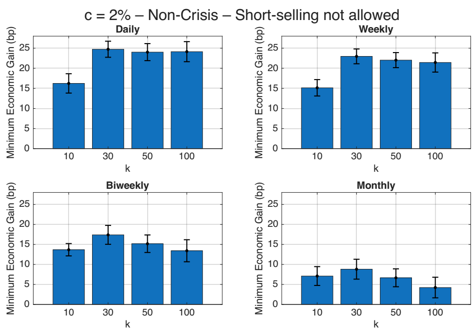
Non-Crisis period



(d) $c = 0\%$



(e) $c = 1\%$

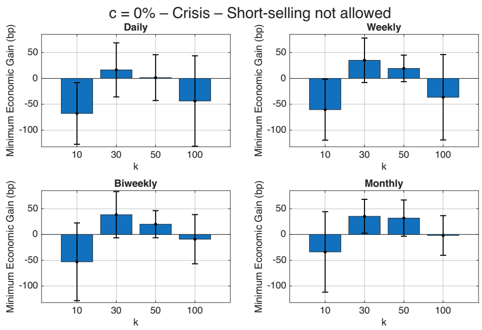


(f) $c = 2\%$

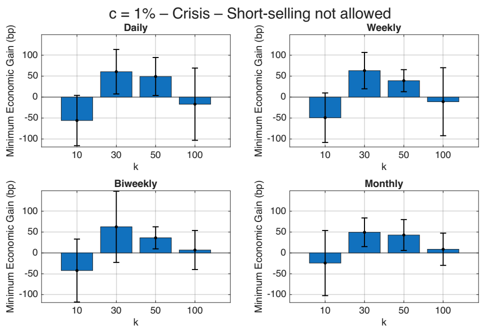
Figure A.4: Minimum Economic Gain when Switching to the EWMA Model Across Portfolio Dimensions. High Risk Aversion Scenario ($\gamma = 10$). No Short-Selling

This figure shows the minimum economic gain, expressed on an annual basis points (bp), obtained from switching from the models included in our analysis (RM 94, HAR-DRD, CCHAR, DPC-CAW, CAW, HEAVY GAS) to the EWMA model. These economic gains are based on the utility framework developed by Fleming et al. (2003), adopted by Hautsch et al. (2015), with a risk aversion parameter of $\gamma = 10$. Each bar shows the smallest statistically significant gain an investor would achieve by switching to the EWMA model. If none of these gains are significant, we instead report the smallest observed gain (regardless of significance) among all alternative models. Short-selling is not allowed. Each row corresponds to a different transaction cost parameter (c), and each column represents a regime (Crisis vs. Non-Crisis). Each plot displays bars and 95% confidence intervals for different portfolio sizes (k assets) across various forecast horizons (daily, weekly, biweekly, and monthly).

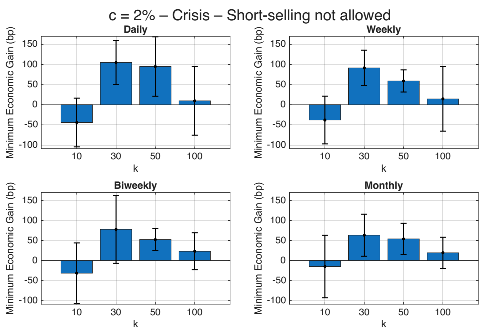
Crisis period



(a) $c = 0\%$

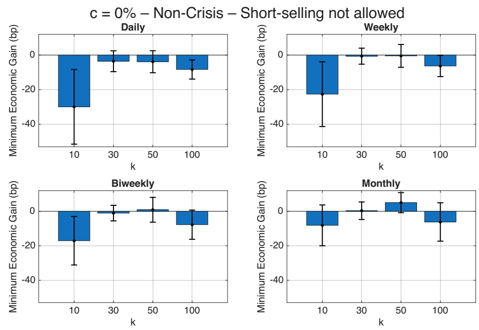


(b) $c = 1\%$

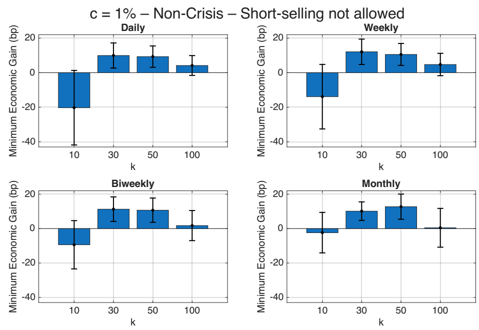


(c) $c = 2\%$

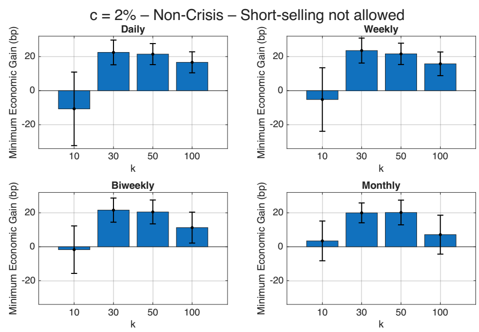
Non-Crisis period



(d) $c = 0\%$



(e) $c = 1\%$



(f) $c = 2\%$