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# Poverty and Uncertainty Attitudes

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# Poverty and Uncertainty Attitudes*

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**Abstract**

This paper demonstrates that well-established biases in decision making under uncertainty can generate poverty traps. A theoretical framework is developed to demonstrate that: i) probability weighting and ambiguity attitude can lead individuals to erroneously undervalue profitable investments, and ii) poverty increases the magnitude of these investment errors. The model predicts that poverty is perpetuated by inducing poor individuals to underinvest in profitable opportunities to a greater extent than rich individuals. We empirically validate these theoretical predictions using data from two experiments conducted on a representative sample of American households.

**JEL Classification**: O12, D81, D09, I3.
**Keywords:** Poverty traps, Probability Weighting, Ambiguity Attitude.

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# 1. Introduction

The poor tend to forgo opportunities that could help them climb out of poverty. They often underinvest in preventive health products (Dupas and Miguel, 2017), they neglect to acquire insurance products designed to reduce their exposure to uncertainty (Giné and Yang, 2009, Ashraf, 2009, Cole et al., 2013), and they undervalue the returns on additional years of schooling (Jensen, 2010, Nguyen, 2008). This paper provides a theoretical framework that can explain these behaviors and formalizes their consequences.

Our claim, in a nutshell, is that behavioral biases in decision making under uncertainty can create poverty traps. These biases impede the accurate evaluation of uncertain but profitable investments, leading individuals to underinvest in these opportunities. Remarkably, poverty exacerbates these mistakes, making underinvestment more severe among the poor. This behavior decreases their expected earnings and thus increases the likelihood that they will be locked in a vicious circle of poverty.

We formalize our proposal using a simple theoretical framework, in which an individual endowed with some level of initial wealth decides how much to invest in a good that generates uncertain returns in the future. While a high level of investment does not guarantee a high future return, it does increase its probability. In other words, we consider a situation in which returns are stochastic with respect to investment. In such a setting, an individual who does not suffer from biases evaluates the costs and benefits from investment using expected utility.

However, there is abundant empirical evidence that individuals deviate from expected utility as a result of probability weighting, i.e. preferences across risky alternatives are not linear in probabilities (Tversky and Kahneman, 1992, Abdellaoui, 2000, Abdellaoui et al., 2011, L'Haridon and Vieider, 2019a), and ambiguity attitude, i.e. a preference or dislike for events with unknown probability relative to equally likely events with known probability (Ellsberg, 1961, Trautmann and van de Kuilen, 2015). We take this empirical evidence at face value and incorporate these biases into our model by characterizing the decision maker's preferences using rank-dependent utility (Quiggin, 1982) and source theory (Abdellaoui et al., 2011, Baillon et al., 2023). Notably, it is assumed that all individuals, regardless of their initial wealth, suffer from probability weighting and ambiguity attitude to the same degree. Thus, our results are not driven by the assumption that the poor suffer more from such biases.

We first focus on probability weighting. The first theoretical result of the model is that probability weighting generates the erroneous perception that investments without extreme returns are not worthwhile. This occurs because an individual suffering from this bias assigns too much probability weight to extreme and unlikely returns, while assigning insufficient weight to non-extreme and likely returns. Such misperception generates an underestimation of the profitability of investments. For example, a pessimistic individual, i.e. someone who underweights all probabilities because she erroneously believes that worse outcomes are more likely, views an investment as profitable only when it yields a sufficiently high minimum return. This causes her to undervalue the higher returns that an investment might yield.

Our framework also predicts that the stronger the bias is, the more likely an individual is to underinvest. This is not a straightforward result. Consider two individuals who exhibit different types of probability weighting: one is pessimistic, i.e. she underweights all probabilities (as explained in the previous paragraph), while the other is optimistic, i.e. she overweights all probabilities because she erroneously believes that best outcomes are more likely. Despite their opposite perceptions of probabilities, they may exhibit the same degree of underinvestment. That is due to the fact that they assign too much probability weight to extreme returns, i.e. the best or the worst returns, generating a corresponding tendency to assign too little weight to other possible returns. Moreover, as their pessimism/optimism becomes more acute, the propensity to underweight non-extreme returns—and consequently underestimate the profitability of the investment—increases.

The second theoretical result is that poverty worsens underinvestment due to probability weighting. It turns out that the poor not only suffer worse consequences by making these erroneous investment choices, due to their disadvantaged position, but they also make more sizeable mistakes. This result emerges when the consumption utility function is sufficiently concave. In that case, the property of diminishing marginal utility implies that the utility gains from investment are the largest among the poor, and in the absence of biases they would invest the most. Therefore, when probability weighting blurs these benefits, the foregone utility gains are the largest among the poor, and they tend to underinvest to a greater extent than wealthier individuals.

We test the theoretical results using two experiments conducted among representative samples of American households. The first was originally conducted by Dimmock et al. (2021) using the American Life Panel, and its most relevant feature is that it elicits the probability weighting function of respondents using a series of binary lotteries. We use these elicitations to determine whether probability weighting has a negative relationship with income and wealth, as predicted by our model. We find that the majority of respondents exhibit probability weighting characterized by pessimism and likelihood insensitivity, i.e. the tendency to assign excessive weight to the probabilities associated with best and worst events while assigning too little weight to other probabilities (Wakker, 2010). More importantly, it turns out that these two components of probability weighting are negatively associated with family income and financial wealth.

While the aforementioned result supports the predictions of our model, it can be explained in other ways. Specifically, it could be argued that causality flows in the opposite direction, i.e. individuals who suffer most from probability weighting become poor as a result of their biases. To dismiss this alternative interpretation and provide causal evidence that supports our theoretical predictions, we use the experiment conducted by Carvalho et al. (2016) in the GfK Knowledge panel. In that experiment, respondents were randomly assigned to fill out a survey before or after payday. We leverage this exogenous variation in financial resources to determine whether poorer respondents, that is those in the before-payday group, are more likely to make investment decisions that deviate from expected utility due to probability weighting. While we find that the deviation from expected utility is on average similar between the experimental groups, it is more pronounced among respondents in the before-payday group who exhibit a sufficiently concave utility function, a condition that also appears in our model to guarantee the existence of a behavioral poverty trap. Thus, the experimental data conclusively validate the predictions of our model.

Finally, in order to incorporate ambiguity attitude, we extend our model by having the individual invest in one of two goods: a risky good, whose distribution of returns is known, and an ambiguous good, whose distribution of returns is unknown. We use Baillon et al. (2023)'s source theory to model the decision-maker's preferences, which are characterized by a different weighting function in the case of the ambiguous good. We refer to this weighting function as the *source function*. The difference in curvature between the probability weighting function, i.e. the weighting function in the case of the risky good, and the source function generates ambiguity attitude. For example, a source function that is more convex (concave) than the probability weighting function generates ambiguity aversion (seeking), that is, an aversion (proneness) to invest in the ambiguous good.

The model predicts that individuals with stronger ambiguity attitude are more likely to underinvest in the ambiguous good relative to the the risky good. Moreover, we find that this tendency is stronger among the poor and therefore they not only heavily underinvest in risky though profitable goods, as shown in our simpler model with only probability weighting, but also underinvest to a greater extent in ambiguous though profitable goods. By not fully taking advantage of profitable opportunities for which probabilistic inference is not available, they perpetuate their poverty. We conclude by discussing relevant empirical evidence that supports this prediction.

### 1.1. Related literature

The paper contributes to several strands of the literature. The first is the emerging literature on behavioral economics and development to which it contributes by formalizing a novel behavioral poverty trap. Previous theoretical research has focused on the relationship between poverty and other behavioral biases, such as time-inconsistent preferences (Bernheim et al., 2015, Banerjee and Mullainathan, 2010) and riskless reference-dependence with aspirations as reference points (Bogliacino and Ortoleva, 2013, Dalton et al., 2016, Genicot and Ray, 2017). To the best of our knowledge, we are the first to demonstrate that probability weighting and ambiguity attitude can generate poverty traps. The core of our contribution is the association between decision-making under uncertainty and economic decision-making by the poor.

Moreover, our framework can be extended to provide novel insights and alternative explanations of existing findings. Appendix B presents an extension of the theoretical framework that includes risky reference dependence. The results of that extension are in line with those in the literature on aspirations and poverty (Dalton et al., 2016, Genicot and Ray, 2017), according to which a low reference point among the poor can worsen their situation. This finding is a consequence of loss aversion, which motivates individuals to overinvest in order to avoid falling short of their reference point. Thus, in the case that the poor have a low reference point, because, for example, it is dictated by their current wealth, they miss out on this effect and lag behind the rich in return on investment. In addition, deviations from expected utility due to probability weighting and ambiguity attitude can provide an alternative explanation for behaviors typically attributed to hyperbolic discounting preferences, such as, such as poor farmers' failure to invest in fertilizer despite prior intentions to purchase it (Duflo et al., 2011).

The model also contributes to the literature on the economic consequences of biases in decision-making under uncertainty. Previous research shows that probability weighting influences behavior in economically relevant situations such as insurance choice (Barseghyan et al., 2013), portfolio choice (Polkovnichenko, 2005, Dimmock et al., 2021), incentive design and contracting (González-Jiménez, 2024), and auctions (Gershkov et al., 2022), among others. Similarly, ambiguity attitude has been shown to also affect choices in these settings (Bose et al., 2006, Amarante et al., 2017, Grant et al., 2018, Dimmock et al., 2016b, Bryan, 2019). Thus, another novelty of this paper is that we are the first to show that these biases can perpetuate poverty.

Finally, our paper reconciles various results from the empirical literature on development economics. It incorporates the finding that individuals in developing countries deviate from expected utility in the presence of risk (Humphrey and Verschoor, 2004, Harrison et al., 2010) and ambiguity (Li, 2017, Bryan, 2019). The model is also in line with the finding that these deviations from expected utility are of similar magnitude to those exhibited by individuals in developed countries (L'Haridon and Vieider, 2019a, l'Haridon et al., 2018). Moreover, by including these patterns of choice under uncertainty in our theoretical framework, we are able to provide an alternative explanation for the regularity that the poor exhibit a low demand for profitable investments.

The remainder of the paper is organized as follows. Section 2 presents a stylized model that serves as an example to establish the intuition of the theoretical framework. Section 3 focuses on risk and shows that the results from Section 2 can be extended to a more general setting. It also shows that the model's predictions are empirically validated by data from two experiments. Section 4 extends the theoretical framework to a setting of unknown probabilities and discusses existing empirical evidence that corroborates the predictions of this more general framework. Finally, Section 5 discusses extensions and robustness.

## 2. A Motivating Example

Consider an individual endowed with a level of initial wealth $x_0 \geq 0$. At period $t = 0$, she decides on a level of investment $e \geq 0$ that will affect her future returns, such as, for instance, the amount of fertilizer to buy in order to enhance the growth of her future crop. For simplicity, we suppose that there are two levels of investment: low, $e_L$, and high, $e_H$, where $e_H > e_L$, which correspond to, say, not buying fertilizer and buying fertilizer, respectively.

Investments carry an immediate disutility denoted by $c(e)$. For simplicity, we consider a setting in which only the higher level of investment is costly: $c(e_H) = c$ where $c > 0$ and $c(e_L) = 0$. At period $t = 1$, the individual learns whether the return on her investment, which we denote by $x$, is high $x = x_H > 0$ or low $x = 0$. Hence, at the moment of decision, the individual does not know with certainty whether the investment will increae her income. It is assumed that the higher level of investment does indeed increase her chances of obtaining the higher level of income. Formally, let $p(e)$ be the probability of obtaining $x_H$ for a given investment level $e$, where it is assumed that $1 > p(e_H) > p(e_L) > 0$.

In our setting, the assumption that the individual perceives probabilities accurately is relaxed, and she may systematically misperceive probabilities. Specifically, $p(e_H)$ and $p(e_L)$ are perceived by the individual as $w(p(e_H))$ and $w(p(e_L))$, respectively. To limit the scope of mistakes that the individual can make, we assume that her misperception of probabilities respects ordering. Thus, $1 > w\big(p(e_H)\big) > w\big(p(e_L)\big) > 0$.

A high level of investment is chosen when the resulting (weighted) utility is larger than that resulting from a low level. That is:

$$\begin{aligned} w\big(p(e_H)\big)u(x_H + x_0) + \big(1 - w\big(p(e_H)\big)\big)u(x_0) - c \geq \\ w\big(p(e_L)\big)u(x_H + x_0) + \big(1 - w\big(p(e_L)\big)\big)u(x_0) \\ \Leftrightarrow \Delta w\big(u(x_H + x_0) - u(x_0)\big) \geq c, \end{aligned} \tag{1}$$

where $u$ is the individual's consumption utility and $\Delta w$ is defined as the difference between the perceived probabilities, i.e. $w\big(p(e_H)\big) - w\big(p(e_L)\big)$. Equation (1) shows that high investment is chosen when the disutility of that action, $c$, is outweighed by the *perceived* utility gain, $\Delta w(u(x_H + x_0) - u(x_0))$. The agent is indifferent between high and low investment for the cost level $\hat{c}_r := \Delta w\big(u(x_H + x_0) - u(x_0)\big)$.

If the individual correctly perceives probabilities, she would choose a high level of investment when:

$$\Delta p\big(u(x_H + x_0) - u(x_0)\big) \geq c, \tag{2}$$

where $\Delta p$ is defined as the difference in probabilities $p(e_H) - p(e_L)$. Let $\hat{c}_e$ be the cost which ensures that (2) binds with equality, i.e. $\hat{c}_e = \Delta p\big(u(x_H + x_0) - u(x_0)\big)$.

We are now in a position to compare the individual's choice of investment with that she would have made in the absence of probability weighting. To that end, subtract the left-hand side of (2) from that of (1) to obtain:

$$(\Delta w - \Delta p)\Big(u(x_H + x_0) - u(x_0)\Big). \tag{3}$$

Notice that (3) is equivalent to computing the difference $\hat{c}_r - \hat{c}_e$.

Suppose that the individual's probability weighting reduces the perceived effectiveness of a high level of investment, that is, let $\Delta p > \Delta w$. The expression in (3) is now negative, which implies that $\hat{c}_r < \hat{c}_e$. Hence, for $c \in (\hat{c}_r, \hat{c}_e)$ the individual does not choose a high level of high investment because she erroneously perceives that the utility gains from making that choice do not offset the costs. In this case, we say that probability weighting prevents the individual from taking advantage of profitable investments.

We now show that the poor make these investment errors to a greater extent than the rich. Assume that the consumption utility function is concave, that is, $u' > 0$, $u'' < 0$, and $u(0) = 0$, which implies that $u'(x_H + x_0) - u'(x_0) < 0$. In other words, marginal increases in initial wealth lead to diminishing utility gains. Hence, the expression in (3) becomes more negative and the set $c \in (\hat{c}_r, \hat{c}_e)$ is extended as initial wealth decreases.

Therefore, the difference between $\hat{c}_e$ and $\hat{c}_r$ is the largest for the poorest individuals. The utility lost by not opting for a high level of investment when $c$ is just above $\hat{c}_r$ is greatest for the poor, leading to higher underinvestment than any other individuals in the society.

This simple example illustrates the underlying mechanism of our poverty trap. The poor, like everyone else in society, misperceive probabilities. However, this has more severe consequences for the poor, since it leads them to underinvest in profitable opportunities to a greater extent. This behavior perpetuates their poverty, since they end up with lower expected income than if they had accurately perceived probabilities.

## 3. Probability Weighting Functions and Poverty Traps

In this section, we focus exclusively on decision making under risk. We first demonstrate that our proposed poverty trap emerges in a general framework. In particular, investment choice in the model is a continuous variable; returns are also continuous; the agent can exhibit different types of probability weighting not just underweighting as in the example presented above; the agent exhibits discounting; and initial wealth is not necessarily independent of income. Subsequently, we provide empirical evidence from two experiments that validate our proposed poverty trap.

### 3.1. The Theoretical Framework

Consider an individual who lives for two periods, $t = 0$ and $t = 1$, and has initial wealth $x_0 \in \chi = [\underline{x}, \bar{x}]$, where $\underline{x} \geq 0$. At period $t = 0$, she chooses the investment level $e \in [0, \bar{e}]$, which affects her future income $x \in \chi$. We assume throughout that a chosen level of investment, $e$, generates disutility $c(e)$ with the following properties:

**Assumption 1.** *The disutility of investment is given by $c(e) : [0, \bar{e}] \to [0, +\infty)$ which is twice continuously differentiable and exhibits $c(0) = 0$, $c'(e) > 0$, and $c''(e) > 0$.*

The variable $x$ captures returns on investment. We assume that the return is realized at $t = 1$, making it uncertain at the time the individual chooses $e$. Thus, we model $x$ as a random variable with distribution $F(x|e)$. We make the following assumption on the function $F(x|e)$:

**Assumption 2.** *The cumulative density function $F(x|e)$ is twice continuously differentiable with respect to $x$ and $e$, and fulfills the following conditions:*

*(i)* $F_x(x|e) = f(x|e) \geq 0$;

*(ii)* $F_{ee}(x|e) > 0$*; and*

*(iii)* $\frac{d}{dx}\left(\frac{f_e(x|e)}{f(x|e)}\right) > 0.$

Assumption 2 includes three properties that are central to our analysis. The first is that $F(x|e)$ admits a probability density function denoted by $f(x|e)$. According to the second, the cumulative density function is convex with respect to $e$, which, in the absence of underweighting or overweighting of probabilities, ensures both an interior solution and the validity of the first-order approach (Laffont and Martimort, 2002). The final property is the monotone likelihood ratio property, which implies that, among other things, a higher level of investment leads to a higher probability of obtaining a high return.

Final wealth is the result of the interplay between initial wealth $x_0$ and the return on investment $x$. We assume that the relationship between these two variables is governed by the function $b(x, x_0)$ about which we make the following assumption:

**Assumption 3.** *The wealth production function* $b : \chi \times \chi \to \mathbb{R}^+$ *is twice continuously differentiable and exhibits* $b_{x_0}(x_0, x) > 0$*,* $b_x(x_0, x) > 0$*, and* $b_{x_0,x}(x_0, x) \geq 0$*.*

Thus, final wealth is monotonically increasing with respect to wealth and income. This assumption entails two possible relationships between initial wealth and income. First, they may be complementary, i.e. $b_{x,x_0}(x_0, x) > 0$, in which case they amplify each other in generating final wealth. Second, they may be independent, i.e. $b_{x,x_0}(x_0, x) = 0$, as in the example in Section 2.

We now describe the individual's risk preferences. Choosing an investment level $e$ not only generates disutility (Assumption 1), but also produces utility from the expected level of final wealth. These utility gains are described by the following expected utility functional:

$$\mathbb{E}\big(u(x, e)\big) = D(1) \int_{\underline{x}}^{\bar{x}} u\Big(b(x_0, x)\Big) \mathrm{d}F(x|e) - c(e), \tag{4}$$

where $D(1) \in (0, 1)$ is the discounting applied to the expected consumption utility that will be derived in period $t = 1$.

Furthermore, consumption utility $u$ is assumed to have the following properties:

**Assumption 4.** *The function* $u : \mathbb{R}^+ \to \mathbb{R}$ *is twice continuously differentiable and exhibits* $u(0) = 0$*,* $u'(x) > 0$*, and* $u''(x) < 0$*.*

An implication of Assumption 4 is that the individual exhibits diminishing returns to final wealth. Under expected utility (henceforth EU), this implies that the individual is risk averse.

*3.1.a. Probability Weighting Functions and their Decomposition.–* We deviate from expected utility by relaxing the assumption that the individual perceives probabilities accurately. It is assumed instead that she can exhibit probability weighting, which will affect her risk attitude. We model this feature by means of a probability weighting function $w(p)$ that transforms objective probabilities. The following assumptions are imposed on $w(p)$:

**Assumption 5.** *Let $p \in [0,1]$. The probability weighting function $w : [0,1] \to [0,1]$ is twice continuously differentiable and exhibits the following properties:*

*(i)* $w(0) = 0$ *and* $w(1) = 1$*;*
*(ii)* $w'(p) > 0$ *for all* $p \in (0,1)$ *;*
*(iii) For some* $\tilde{p} \in [0,1]$, $w''(p) < 0$ *if* $p < \tilde{p}$ *and* $w''(p) > 0$ *if* $p > \tilde{p}$*;*
*(iv)* $\lim_{p\to 0} w'(p) > 1$ *and* $\lim_{p\to 1} w'(p) = 0$ *if* $\tilde{p} = 0$*;*
*(v)* $\lim_{p\to 1} w'(p) > 1$ *and* $\lim_{p\to 0} w'(p) = 0$ *if* $\tilde{p} = 1$*;*
*(vi)* $\lim_{p\to 1} w'(p) > 1$ *and* $\lim_{p\to 0} w'(p) > 1$ *if* $\tilde{p} = 1$*;*
*(vii) If* $\tilde{p} \in (0,1)$*, then there exists a* $\hat{p} \in (0,1)$ *such that* $w(\hat{p}) = \hat{p}$*.*

The probability weighting function is a strictly increasing and continuous function that maps the unitary interval onto itself. It exhibits at least two fixed points: one at impossibility, i.e. $p = 0$, and one at certainty, i.e. $p = 1$. Moreover, $w(p)$ can have three possible shapes: concave, convex, or inverse-S, which are determined by the location of the inflection point $\tilde{p} \in [0,1]$. It is worth emphasizing that when the function has the inverse-S shape (because $\tilde{p} \in (0,1)$) an additional fixed point emerges which we denote by $\hat{p} \in (0,1)$.

The risk preferences of the agent with probability weighting are characterized by rank-dependent utility (henceforth RDU) (Quiggin, 1982):

$$RDU\big(u(x,e)\big) = D(1) \int_{\underline{x}}^{\bar{x}} u\Big(b(x_0,x)\Big) \mathrm{d}\Big(w\big(1 - F(x|e)\big)\Big) - c(e), \tag{5}$$

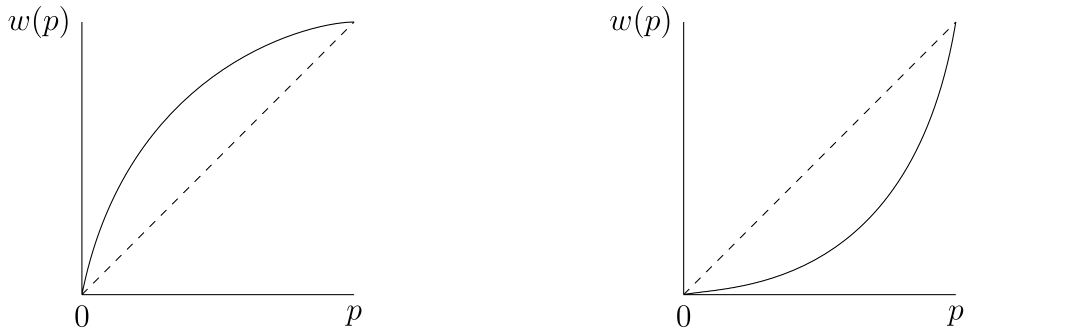
An RDU individual transforms probabilities as follows. For given return and investment levels, i.e. $X \in \chi$ and $e' \in [0,\bar{e}]$, she considers the *rank* or probability of obtaining a higher level of return, where the probability is $1 - F(X|e')$, and is perceived by the decision maker as $w\big(1 - F(x|e')\big)$. In other words, probabilities in our setting are decumulative, such that $p = 1 - F(x|e)$, and are transformed using the function $w$ with properties as described in Assumption 5.

Accordingly, note that a return which is infinitesimally worse than $X$ generates a marginal difference in perceived ranks captured by the expression $\mathrm{d}\Big(w\big(1-F(X|e')\big)\Big)$, which is the differential of the integral in (5). Therefore, the rank-dependent functional given in (5) implies that the utility derived from a return $X$, i.e. $u(X)$, is weighted by its contribution to the perceived rank $\mathrm{d}\Big(w\big(1-F(X|e')\big)\Big)$, and those weighted utilities are summed over all possible returns $x \in \chi$.

Under RDU, the individual's risk attitudes are jointly determined by the curvature of the functions $u$ and $w$. The risk attitude generated by the curvature of $u$ is common to both EU and RDU, while that generated by the curvature of $w$ is exclusive to RDU. This latter influence of probability weighting on risk attitude is known as *probabilistic risk attitude* (Wakker, 1994) and it captures the influence of deviations from expected utility in decision making under risk.

To comprehensively investigate the relationship between poverty and probability weighting, we follow Wakker (2010) by distinguishing between two types of probability weighting. The first is the result of pessimism or optimism. This type of probability weighting captures the idea that the individual irrationally believes that favorable outcomes, in the case of optimism, and unfavorable outcomes, in the case of optimism, realize more often than they actually do.

We incorporate pessimism into the model using convex probability weighting functions, which have the property that larger probability weights are assigned to the probabilities associated with the lowest levels of return. In contrast, optimism is represented by concave probability weighting functions, which assign larger probability weights to the probabilities associated with the highest levels of returns. Figure 1 provides examples of optimism and pessimism.



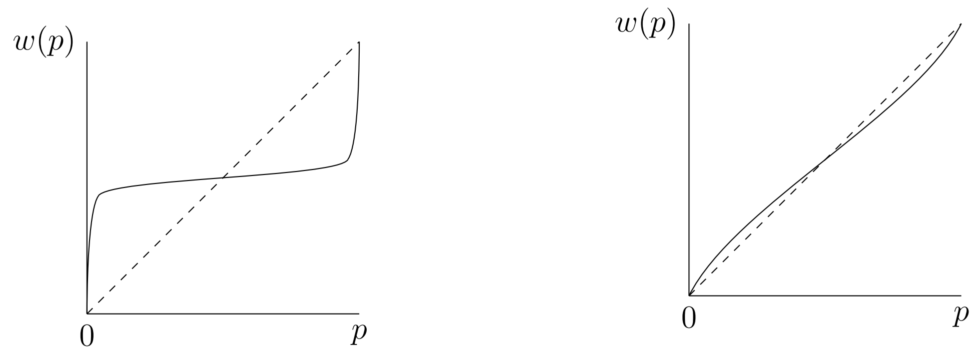

(a) Example of Optimism

(b) Example of Pessimism

Figure 1: Optimism and Pessimism

**Definition 1.** *Optimism (pessimism) is characterized by a $w(p)$ with the properties of Assumption 5 and $\tilde{p} = 1$ ($\tilde{p} = 0$)*

We are interested in investigating the effect of more severe probability weighting due to stronger optimism or pessimism on investment. The following definition, due to Yaari (1987), provides a formal basis for understanding different degrees of optimism and pessimism.




(a) Example of extreme likelihood insensitivity (b) Example of moderate likelihood insensitivity

Figure 2: Examples of likelihood insensitivity

**Definition 2.** *An agent $i$ with weighting function $w_i(p)$ is more optimistic ( pessimistic) than an agent with weighting function $w_j$ if $w_i(p) = \theta\big(w_j(p)\big)$ where the function $\theta : [0, 1] \to [0, 1]$ is twice continuously differentiable and exhibits $\theta' > 0$ and $\theta'' < 0$ ($\theta'' > 0$).*

We are now in a position to establish the influence of stronger pessimism/optimism on risk attitude. The following remark states that they have opposite effects: stronger optimism makes the decision maker more risk seeking, while stronger pessimism makes the decision maker more risk averse.

**Remark 1.** *For a given investment level $e$, stronger optimism (pessimism) leads to more less) risk aversion.*

The second type of probability weighting is generated by likelihood insensitivity (Tversky and Wakker, 1995, Wakker, 2010), which captures the idea that individuals transform probabilities as a result of cognitive and perceptual limitations. More specifically, individuals exhibit extremity-orientedness: they are overly sensitive to changes in extreme probabilities and are not sufficiently sensitive to changes in intermediate probabilities.

We characterize likelihood insensitivity using an inverse-S probability weighting function (see Figure 2) An individual with such a probability weighting function assigns too much probability weight to the probabilities of extreme returns and too little to the probabilities of intermediate returns.

**Definition 3.** *Likelihood insensitivity is characterized by a $w(p)$ with the properties of Assumption 5 and $\tilde{p} = 0.5$ and $\hat{p} = 0.5$.*

Note that our definition of likelihood insensitivity assumes that the probabilities of intermediate outcomes are perceived to be close to $0.5$. This property captures the idea that when assessing the likelihood of intermediate outcomes, the insensitive individual will often have a crude perception of these probabilities as being close to "50-50."

It will later become useful to characterize an individual according to her degree of likelihood insensitivity. The following definition, based on Baillon et al. (2023), states that stronger likelihood insensitivity is reflected in a probability weighting function with a more accentuated inverse-S shape.

**Definition 4.** *An individual $i$ with weighting function $w_i$ is more likelihood insensitive than an individual $j$ with weighting function $w_j$ if $w_i = \phi\big(w_j(p)\big)$ where $\phi : [0,1] \to [0,1]$ is a function with likelihood insensitivity in the sense of Definition 3.*

The following remark states that an individual with a strong degree of likelihood insensitivity assigns more weight to the probabilities of the highest and lowest returns, and less weight to the probabilities of intermediate returns.

**Remark 2.** *If individual $i$ is more likelihood insensitive than individual $j$ in the sense of Definition 4, then her weighting functions exhibits $w_i(p) > w_j(p)$ for all $p \in (0, 0.5)$ and $w_i(p) < w_j(p)$ for all $p \in (0.5, 1)$.*

Whether stronger likelihood insensitivity generates more risk aversion or more risk seeking depends on the distribution of returns $F(x|e)$. When that distribution is right-skewed, stronger likelihood insensitivity generates more risk aversion. This is because the larger weight given to probabilities associated with low returns together with more probability mass being assigned to these outcomes reduces the attractiveness of investing. In this case, the individual overweights the already large probability of obtaining a low return. The opposite is true for a left-skewed distribution, and stronger likelihood insensitivity will generate more risk seeking.

The following lemma presents the crucial result that more extreme probability weighting, regardless of whether it is caused by stronger optimism, stronger pessimism, or stronger likelihood insensitivity, generates a larger segment in which probabilities are given insufficient weight.

**Lemma 1.** *For a given investment level $e$, stronger pessimism, stronger optimism, or stronger likelihood insensitivity generates a larger set of probabilities for which the probability weighting function exhibits $w'(p) < 1$.*

A higher degree of optimism leads to more weight being assigned to the probability of the highest return. Since the probability weighting function must be on average equal to one, i.e. $-\int_{\underline{x}}^{\bar{x}} w'(1 - F(x|e))f(x|e)\mathrm{d}x = 1$, this overweighting of the best outcome implies that lower weights will be assigned to the probabilities of all other possible outcomes. Consequently, the set of probabilities that receive lower weight relative to the expected utility benchmark—that is, a weight less than one—is augmented. The opposite is true for a higher degree of pessimism. In that case, the set of probabilities that receive insufficient weight relative to the EU benchmark is expanded and is located at the high end of the outcome space. Moreover, a higher degree of likelihood insensitivity leads an individual to assign more weight to the probabilities of extreme returns at the expense of assigning less weight to all other returns. Therefore, the set of probabilities that receive insufficient weight is again augmented and is located at the probabilities associated with intermediate returns.

***3.1.b.*** *Behavioral Poverty Traps.–* We are now in a position to investigate the influence of probability weighting on the individual's choices. To do so, we will contrast the optimal investment level of an RDU decision maker to that of an otherwise identical EU decision maker. The following proposition focuses on probability weighting due to pessimism and shows that sufficiently strong pessimism leads the RDU individual to underinvest relative to the hypothetical case in which she is EU.

**Proposition 1.** *Assume that Assumption 1-5 hold and that $w''(p) > 0$ for all $p \in [0,1]$. There exists a threshold level of pessimism at which the RDU individual chooses a lower level of investment than her EU counterpart. If the RDU individual is more pessimistic than that threshold, in the sense of Definition 2, then again there is underinvestment.*

The RDU individual with sufficiently strong pessimism underinvests relative to the EU individual because her probability weighting generates the erroneous perception that investments that do not guarantee a high enough minimal return are not as profitable as they actually are. For example, investments that are on average profitable but have low minimal returns are unappealing to this individual.

In order to elucidate the mathematical rationale underlying Proposition 1, consider two probabilities $p_1, p_2 \in (0,1)$ such that $p_2 = p_1 + \varepsilon$ for small enough $\varepsilon > 0$. In the context of our model, $\varepsilon$ is the improvement in returns from choosing a higher level of investment. Suppose that $w'(p_1) < 1$, which by the definition of a derivative can be rewritten as $\lim_{\varepsilon \to 0} \frac{w(p_1+\varepsilon)-w(p_1)}{\varepsilon} < 1$. This implies that $w(p_2) - w(p_1) < p_2 - p_1$ for close enough $p_2$ and $p_1$. Recall from the example in Section 2 that by evaluating the probabilities $p_1$ and $p_2$ in this way leads individuals to undervalue the benefits of choosing the higher level of investment. Furthermore, Lemma 1 demonstrated that stronger pessimism expands the set of probabilities that yield $w'(p) < 1$, which makes the assumption $w'(p_1) < 0$ less stringent and therefore valid for a larger set of probabilities $p_1$. Therefore, it can be said that stronger pessimism encourages underinvestment.

We can similarly analyze the influence of optimism and likelihood insensitivity on an individual's choices. The following proposition shows that stronger optimism or stronger likelihood insensitivity increases the probability that a decision maker will *forgo* a profitable investment. These types of probability weighting can therefore generate extreme underinvestment since they may cause the individual to avoid investing altogether.

**Proposition 2.** *Assume that Assumption 1-5 hold and that $w''(p) < 0$ for some $p \in [0, 1]$. Under stronger likelihood insensitivity, in the sense of Definition 2, or stronger optimism, in the sense of Definition 4, an RDU individual is more likely to choose an investment level of zero while her EU counterpart chooses a strictly positive level of investment.*

The RDU individual suffering from sufficiently strong optimism or likelihood insensitivity might forgo profitable investments because her probability weighting leads to the incorrect perception that investments which do not generate extreme returns are not worthwhile. In the case of sufficiently strong optimism, the individual erroneously perceives that only investments that guarantee a sufficiently high maximum return are profitable. In the case of sufficiently strong insensitivity, the individual incorrectly perceives that only investments with sufficiently high maximum and minimum returns are profitable. Essentially, these individuals would pass over opportunities that do not exhibit extreme even though they are profitable in expectation.

The mathematical rationale behind this result is similar to that of Proposition 1 given above. In particular, stronger likelihood insensitivity or optimism extends the segment of probabilities in which the probability weighting function exhibits $w'(p) < 1$. This implies that more investments will be undervalued relative to the EU benchmark. Moreover, because of the non-convexities generated by the concavity of the probability weighting function in the case of optimism and likelihood insensitivity, the optimal solution is at the extremes of the set $[0, \bar{e}]$. Thus, the optimal solution is to choose an investment level of zero when an investment is undervalued. This is in contrast to the choices that the individual would have made under EU, in which case the non-convexities due to probability weighting are avoided and as a result the optimal level of investment is an interior solution.

Thus far, Proposition 1 and 2 demonstrate that all individuals, regardless of their initial wealth, may erroneously underinvest or forgo profitable opportunities if they suffer from sufficiently strong probability weighting. The following proposition shows that under reasonable conditions on consumption utility, poor individuals either exhibit the greatest degree of underinvestment or forgo opportunities more often relative to wealthier individuals. Thus, poverty exacerbates the underinvestment caused by probability weighting.

**Proposition 3.** *Assume that Assumption 1-5 hold and the condition $\frac{-u''\left(b(x_0,x)\right)}{u'(x_0,x)} > \frac{b_{x,x_0}(x_0,x)}{b_{x_0}(x_0,x)b_x(x_0,x)}$ hold for all $x$. As $x_0$ decreases, the RDU individual:*

*(i) invests less relative to her EU counterpart when her level of pessimism exceeds the threshold characterized in Proposition 1; or*

*(ii) more frequently chooses an investment level of zero for a given level of likelihood insensitivity or optimism.*

Thus, although the poor may suffer from probability weighting to the same extent as the rich, they experience more acute economic consequences as a result of probability misperception. They either underinvest to a greater extent or completely forgo profitable investments more often. These behaviors reduce their prospects of escaping poverty.

Importantly, Proposition 3 requires that the curvature of the consumption utility function is sufficiently concave (Assumption 4) so as to outweigh the complementarity between initial wealth and investment returns (Assumption 3). This condition guarantees the property of diminishing marginal utility of wealth, which implies that those with the lowest initial wealth experience the largest gains in consumption utility from investment. Consequently, when probability weighting gets in the way of investment, the poor exhibit the largest gap in utility with respect to their EU counterparts.

Our behavioral poverty trap is characterized by Propositions 1-3, which show that probability weighting can lead individuals to underinvest or to forgo profitable opportunities, and that this error is more pronounced among the poor. Consequently, the poor will end up having the lowest final expected wealth even though they had an opportunity to improve their initial condition by exploiting profitable investments. Their poverty is perpetuated by a systematic misperception of probabilities. In the next subsection, this hypothesis will be empirically examined.

### 3.2. Empirical Evidence

In order to determine the empirical validity of our behavioral poverty trap, we use the data of Dimmock et al. (2021) who conducted an incentivized experiment among a representative sample of American households. Their experiment elicited the probability weighting functions of respondents from the American Life Panel (ALP) to analyze the relationship between household portfolio diversification and probability weighting.

The experiment used the method of Abdellaoui (2000), which has the ability to elicit the utility and probability weighting functions of each respondent in a non-parametric way. This is achieved by implementing a set of binary lotteries that keep probabilities fixed, in order to elicit utility function curvature, and a set of binary lotteries that keep outcomes fixed and vary probabilities in order to elicit probability weighting function curvature. Therefore, the data can be used to identify and separate the two components of risk attitude in the case of RDU.

A disadvantage of Dimmock et al. (2021)'s elicitation is that it confounds probability weighting due to likelihood insensitivity with probability weighting due to pessimism/optimism. To deal with this, we fit each respondent's choices to parametric forms of probability weighting that identify these factors of probability weighting. Specifically, we first fit the data to Prelec (1998)'s probability weighting function. This is an empirically desirable function since it accounts for changes at both small and large probabilities (Wakker, 2010). Formally, for each respondent $i$, we estimate the functional:

$$w(p_{ij}) = \exp\Big( -\beta_i\big( -\ln(p_{ij})\big)^{\alpha_i}\Big). \tag{6}$$

In the estimation, we only used the questions designed to elicit probability weighting functions, which are indexed by $j$. The parameters $\alpha_i$ and $\beta_i$ are estimated using non-linear least squares, a method that has been widely used to estimate parameters of the probability weighting function (Abdellaoui et al., 2011, Dimmock et al., 2021, Li et al., 2018)

Importantly, the estimate $\hat{\alpha}_i$ captures the respondent $i$'s likelihood insensitivity (Wakker, 2010). The closer $\hat{\alpha}_i$ is to zero, the more insensitive the respondent is, and conversely a value of $\hat{\alpha}_i$ closer to one implies a perception of probabilities closer to EU. Therefore, we use $-\hat{\alpha}_i$ (if $\hat{\alpha}_i < 1$) as a continuous index of insensitivity that we refer to as"Inverse-S." Conversely, we use $\hat{\alpha}_i$ (if $\hat{\alpha}_i > 1$) as a continuous index of *oversensitivity* to probabilities that we refer to as"S-shape." The larger is the latter index, the more pronounced will be the S-shape of the respondent's probability weighting function. In this case, the decision maker also deviates from EU by assigning too much weight to intermediate outcomes.

Furthermore, the estimate $\hat{\beta}_i$ indicates whether participant $i$ exhibits pessimism or optimism (Wakker, 2010). If $\hat{\beta}_i < 1$, the respondent exhibits pessimism, while if $\hat{\beta}_i > 1$, she exhibits optimism. That estimate also captures the degree of optimism or pessimism. Accordingly, lower values of $\hat{\beta}_i$ denote stronger pessimism when $\hat{\beta}_i < 1$, while higher values of $\hat{\beta}_i$ denote stronger optimism when $\hat{\beta}_i > 1$.

We also estimate the respondent's consumption utility function. The questions in the survey that were designed to elicit utility curvature were used to estimate the following CRRA utility for each respondent $i$:

$$u(x_{ij}) = x_{ij}^{1-\gamma_i}. \tag{7}$$

We used non-linear least squares to estimate the parameter $\gamma$. Importantly, the estimation of the utility curvature parameter was performed simultaneously with the estimation of probability weighting parameters.

Table 1 presents descriptive statistics of $\hat{\alpha}_i$ and $\hat{\beta}_i$. We applied a 95% winzorization to the estimates of $\beta_i$ in order to reduce the effect of outliers.[1] On average, respondents exhibited likelihood insensitivity and pessimism, given that the average value of $\hat{\alpha}_i$ is less than one and that of $\hat{\beta}_i$ is greater than one. Figure 3a illustrates the median probability weighting function, which is also characterized by pessimism and likelihood insensitivity.

These findings are further corroborated by analyzing the estimates at the individual level. We find that a majority of respondents, 2012 out of 2640, exhibit $\hat{\alpha}_i < 1$, which indicates likelihood insensitivity. Furthermore, a majority of subjects, 1872 out of 2640, exhibit $\hat{\beta}_i > 1$, which indicates pessimism. These results are in line with experimental findings (Abdellaoui, 2000, Abdellaoui et al., 2011, Bruhin et al., 2010, L'Haridon and Vieider, 2019b).

Table 1: Estimates of Probability Weighting

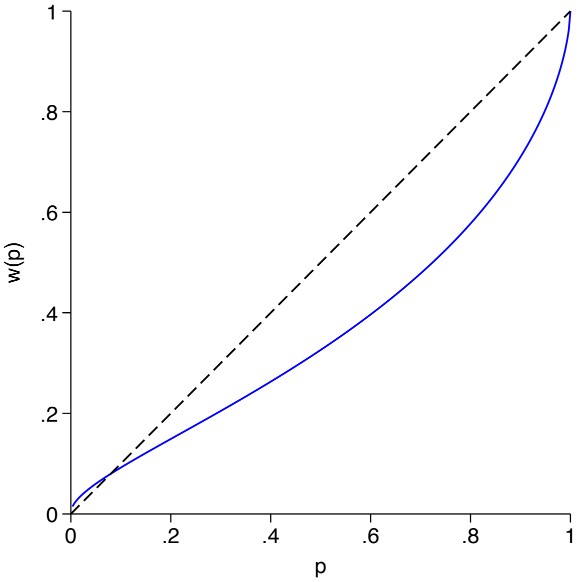
| | **Prelec (1998)** | | **Chateauneuf et al. (2007)** | | **Goldstein and Einhorn (1987)** | |
|---|---|---|---|---|---|---|
| | $\hat{\alpha}_i$ | $\hat{\beta}_i$ | $\hat{s}_i$ | $\hat{c}_i$ | $\hat{\phi}_i$ | $\hat{\delta}_i$ |
| **Mean** | 0.815 | 1.855 | 0.594 | 0.028 | 0.885 | 0.934 |
| **25th perc.** | 0.361 | 0.932 | 0.257 | -0.118 | 0.540 | 0.187 |
| **50th perc.** | 0.630 | 1.411 | 0.611 | 0.001 | 0.743 | 0.491 |
| **75th perc.** | 0.972 | 2.329 | 0.891 | 0.056 | 1.024 | 1.066 |
| **St. Dev.** | 1.211 | 1.550 | 0.358 | 0.067 | 0.615 | 1.268 |

This table presents the descriptive statistics for estimates of probability weighting obtained at the respondent level. The first two columns present the descriptive statistics of the parameters when the form $w(p_{ij}) = \exp\Big(-\beta_i\big(-\ln(p_{ij})\big)^{\alpha_i}\Big)$, due to Prelec (1998), is assumed. Columns 3 and 4 present the descriptive statistics of the parameters when the form $w(p_{ij}) = \begin{cases} 0 & \text{if } p = 0, \\ c_i + s_i \cdot p_{ij} & \text{if } p \in (0,1), \\ 1 & \text{if } p = 1. \end{cases}$, due to Chateauneuf et al. (2007), is assumed. Columns 5 and 6 present the descriptive statistics of the parameters when the form $w(p_{ij}) = \frac{\delta_i p_{ij}^{\phi_i}}{\delta_i p_{ij}^{\phi_i} + (1-p_{ij})^{\phi_i}}$, due to Goldstein and Einhorn (1987), is assumed. All estimates were obtained using non-linear least squares.

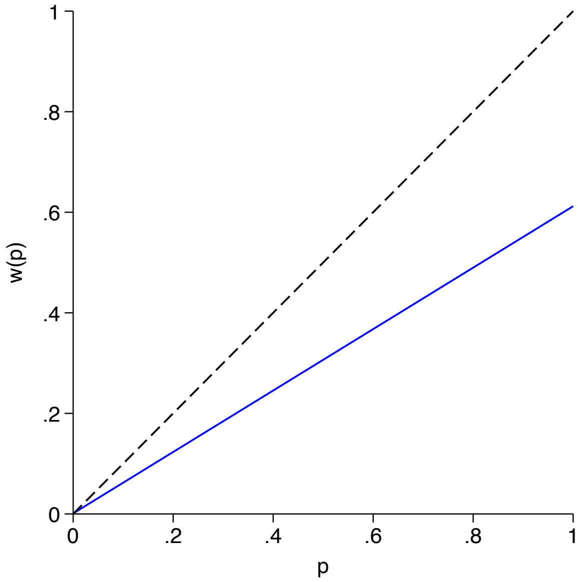
Another advantage of these data is that in previous waves of the ALP, the same respondents were also asked to report their levels of income and wealth, which makes it

[1] Prior to transforming the data, the mean of $\hat{\beta}_i$ was equal $6.148$, which is considerably higher than estimates reported in previous studies. Moreover, the standard deviation of $\hat{\beta}_i$ was $27.92$, which indicates an exceptionally high variance.

Figure 3: Median probability weighting functions




(a) using Prelec (1998)




(b) using Chateauneuf et al. (2007)

Note: The blue lines represent the median probability weighting function in the sample while the dashed lines represent the accurate perception of probabilities benchmark.

possible to analyze the relationship between the aforementioned indexes of insensitivity and pessimism, and those responses. In particular, we included in our analyses the variables "Financial Wealth", which is the household's self-reported financial wealth; "Return Stock", which is the households' self-reported return on individual stocks and stock mutual funds in retirement accounts, "Family Income", which is the household's self-reported income; and "Housing Wealth", which is the household's self-reported housing wealth. Table 2 presents the descriptive statistics of these variables, which shows these variables to be continuous and expressed either in dollars or thousands of dollars. Furthermore, the standard deviation of each variable is larger than its mean, which indicates substantial variance. To stabilize this variability, we work with natural logarithms of these variables, as in Dimmock et al. (2021) and Dimmock et al. (2016b).

To examine the influence of probability weighting on self-reported income and wealth, we regress each of the transformed income and wealth variables on our indexes of probability weighting. The advantage of running regressions in which each of these variables serves as the dependent variable in turn is that they capture different dimensions of income and wealth, which can inform us about the specific context in which our poverty trap operates. For example, Return Stock captures income with lower liquidity relative to Family Income, and might therefore be less relevant in the case of the poorest households. Similarly, Housing Wealth represents low-liquidity asset relative to Financial Wealth, which, again, might be less important in the case of

Table 2: Descriptive Statistics of Income and Wealth

| Variable | Unit | Mean | 25th Perc. | 50th Perc. | 75th Perc. | St. Dev. | Obs. |
|---|---|---|---|---|---|---|---|
| Financial Wealth | 1000s USD | 139.673 | 0 | 2.500 | 38 | 1897.933 | 1954 |
| Return Stock | 1 USD | 260275.800 | 30000 | 114000 | 350000 | 439687.8 | 951 |
| Family Income | 1000s USD | 76.516 | 32.5 | 67.500 | 112.5 | 53.350 | 2659 |
| Housing Wealth | 1000s USD | 483.909 | 0 | 100 | 250 | 13054.750 | 1943 |

This table presents descriptive statistics for the variables that capture the respondents' self-reported income and wealth. The variables are "Financial Wealth" which captures the household's self-reported financial wealth in thousands of US dollars, "Return Stock" which captures the households self-reported return on individual stocks and stock mutual funds in retirement accounts in US dollars, "Family Income" which captures the household's self-reported income in thousands of US dollars, and "Housing Wealth" which captures the household's self-reported housing wealth in thousands of US dollars.

the poorest households.

Moreover, we control for the respondent's utility curvature in all the regressions, with the goal of focusing on the effect of probabilistic risk attitudes above and beyond the risk attitude generated by utility curvature. In some specifications, we also include other control variables that might moderate the relationship between probability weighting and income (or wealth), such as the respondent's age, gender, ethnicity, level of education, state of residence, spoken language, and employment status.

The OLS estimates are presented in Table 3. Our main finding is that higher likelihood insensitivity, as captured by the index Inverse-S, is associated with lower financial wealth, lower return on stocks, and lower family income. Furthermore, these relationships remain highly significant even after the introduction of controls. These results are in line with the prediction of our model that sufficiently strong insensitivity generates underinvestment, which in turn lowers expected income and wealth, and this underinvestment is more pronounced among the poor (Propositions 2 and 3). Furthermore, the coefficient Inverse-S is not significant when Housing Wealth is the dependent variable, suggesting that for the measures of low wealth liquidity, which as mentioned above are less relevant in the case of poorest households, our hypothesis cannot be empirically confirmed. The same qualitative results are obtained without winsorization (see Table 7 in Appendix C).

Notably, the estimates in Table 3 also show that neither pessimism nor optimism affects the respondent's income and wealth. This result contradicts the prediction of Propositions 1 and 3 that sufficiently strong pessimism or optimism generates underinvestment and that poverty amplifies this tendency. This lack of support for our hypothesis can be explained by the fact that $\hat{\beta}_i$ can be confounded by factors other than pessimism or optimism, such as insensitivity and utility curvature (Gonzalez and

Wu, 1999, Abdellaoui et al., 2011, Li et al., 2018). To overcome this potential problem, we perform the same empirical exercise using other parametric forms of probability weighting which are better able to separate optimism and pessimism from other determinants of risk attitude.

Table 3: The Relationship between Prelec (1998)'s Probability Weighting Function and Income or Wealth

| | (1) Ln Financial Wealth | (2) Ln Return Stock | (3) Ln Family Income | (4) Ln Housing Wealth | (5) Ln Financial Wealth | (6) Ln Return Stock | (7) Ln Family Income | (8) Ln Housing Wealth |
|---|---|---|---|---|---|---|---|---|
| Inverse-S | -1.301*** | -1.520*** | -0.184*** | -0.138 | -1.127*** | -1.236*** | -0.152** | -0.099 |
| | (0.400) | (0.434) | (0.067) | (0.212) | (0.347) | (0.406) | (0.060) | (0.184) |
| S-shaped | -0.137 | -0.087 | -0.041** | -0.079 | -0.101 | -0.027 | -0.020 | -0.054 |
| | (0.095) | (0.098) | (0.018) | (0.049) | (0.067) | (0.090) | (0.015) | (0.041) |
| Opt./Pess. | -0.037 | -0.052 | -0.011 | -0.008 | -0.084 | -0.104 | -0.014 | -0.041 |
| | (0.096) | (0.105) | (0.017) | (0.050) | (0.087) | (0.101) | (0.016) | (0.045) |
| U. Curv | 0.013** | 0.014** | 0.001 | 0.004 | 0.012* | 0.013** | 0.001 | 0.004 |
| | (0.006) | (0.006) | (0.001) | (0.003) | (0.006) | (0.006) | (0.001) | (0.003) |
| Constant | 6.110*** | 4.373*** | 10.891*** | 3.409*** | 2.293* | -3.305*** | 9.859*** | -3.126*** |
| | (0.281) | (0.300) | (0.049) | (0.147) | (1.320) | (1.266) | (0.248) | (0.742) |
| $R^2$ | 0.015 | 0.012 | 0.010 | 0.004 | 0.217 | 0.129 | 0.153 | 0.234 |
| N | 1902 | 2245 | 2629 | 1921 | 1901 | 2244 | 2628 | 1920 |

This table presents OLS estimates of the model $y_i = b_0 + b_1\text{Inverse-S}_i + b_2\text{S-shaped}_i + b_3$ Opt./Pess.$_i$ + $b_3$U.curv$_i$.+Controls$_i'\Gamma + \varepsilon_i$ . The dependent variable $y_i$ are the respondent's self-reported measures of income and wealth. "Inverse-S" is an index of likelihood insensitivity, "S-shaped" is an index of oversensitivity to probabilities, "Opt./pess." is an index of optimism and pessimism, and "U.curv" captures the respondent's utility function curvature. The estimates presented in Columns 1-4 do not include additional control variables, and the estimates presented in Columns 5-8 do include them. Robust standard errors are presented in parentheses. *** denotes significance at the 0.01 level, ** denotes significance at the 0.05 level, * denotes significance at the 0.1 level.

In particular, we assume the parametric form of Chateauneuf et al. (2007):

$$w(p_{ij}) = \begin{cases} 0 & \text{if } p = 0, \\ c_i + s_i \cdot p_{ij} & \text{if } p \in (0,1), \\ 1 & \text{if } p = 1. \end{cases} \tag{8}$$

This probability weighting function is recommended when constructing indexes of pessimism and insensitivity because its parameters have a clean and simple interpretation (Wakker, 2010). We estimate the parameters $c$ and $s$ for each respondent $i$, and simultaneously estimate the power utility function given in (7). We again use non-linear least squares to perform the estimations.

Columns 3 and 4 in Table 1 present the descriptive statistics of the estimated parameters $\hat{c}_i$ and $\hat{s}_i$. We did not apply winsorization to these data.[2] Our previous finding that most respondents exhibit insensitivity and pessimism again emerges when using this alternative parametric form of probability weighting. Specifically, respondents exhibit insensitivity on average given that the average estimated value of $\hat{s}_i$ is less than one. They also exhibit pessimism on average given that, on average, $1 - \hat{c}_i - \hat{s}_i < 1$. Figure 3b shows that these conclusions also hold for the median probability weighting function.

Following Wakker (2010), if $\hat{s}_i < 1$, then we use the estimate $-\hat{s}_i$ as a continuous index of likelihood insensitivity which we refer to as"Inverse-S." If $\hat{s}_i > 1$, we use $\hat{s}_i$ as an index of oversensitivity to probabilities that we refer to as "S-shape." Finally, we use the fraction $\frac{2\hat{s}_i + \hat{c}_i}{2}$ as an index of optimism and pessimism. That measure compares the extent to which a respondent overweights the smallest probabilities, i.e. those of best outcomes, to the extent to which that same respondent underweights the largest probabilities, i.e. those of worst outcomes.[3]

The estimates using these alternative indexes of probability weighting are presented in Table 4. In line with our previous estimations, we find that higher likelihood insensitivity is associated with lower financial wealth, lower return on stocks, and lower family income. Notably, we also find that stronger pessimism is related to lower family income and financial wealth, which are the relevant measures of income and wealth for the poorest households. Hence, using this alternative parametric form of probability weighting function, we find support for Propositions 1-3. Table 8 in Appendix C shows that the results are similar when 95% winsorization is applied to the estimate $\hat{c}_i$, which is the analog of $\hat{\beta}_i$ in Prelec (1998)'s weighting function. Thus, whether or not the data is transformed is not crucial to the findings.

We repeat the same exercise but this time using the parametric form proposed by Goldstein and Einhorn (1987). It has been shown to fit data well because it correctly accounts for heterogeneity (Wu et al., 2004) and fully separates likelihood insensitivity from pessimism (Li et al., 2018). Formally, that functional is given by:

$$w(p_{ij}) = \frac{\delta_i p_{ij}^{\phi_i}}{\delta_i p_{ij}^{\phi_i} + (1 - p_{ij})^{\phi_i}}. \tag{9}$$

We estimate the coefficients $\phi_i$ and $\delta_i$ for each respondent, and simultaneously es-

[2]When we apply a 95% winzorization to the estimated values of $c_i$, which is the analog of the parameter $\beta_i$ in Prelec (1998)'s weighting function, its mean is 0.0261 and its standard deviation is 0.067, which are very close to the mean and standard deviation in Table 1. Thus, these data are potentially less prone to generate incorrect conclusions as a result of outliers.

[3]As $p$ approaches zero, the weighting function becomes $w(p) = c$. In contrast, as $p$ approaches one, it becomes $w(p) \approx 1 = c + s$. Thus, comparing a respondent's level of optimism to her level of pessimism is equivalent to computing the difference $c - (1 - c - s)$. The index $\frac{2\hat{s} + \hat{c}}{2}$ is a linear transformation of that difference (Wakker, 2010).

Table 4: The Relationship between Chateauneuf et al. (2007)'s Probability Weighting Function and Income or Wealth

| | (1) Ln Financial Wealth | (2) Ln Return Stock | (3) Ln Family Income | (4) Ln Housing Wealth | (5) Ln Financial Wealth | (6) Ln Return Stock | (7) Ln Family Income | (8) Ln Housing Wealth |
|---|---|---|---|---|---|---|---|---|
| Inverse-S | -1.943** | -4.004*** | -0.453*** | -1.370*** | -0.421*** | -3.365*** | -0.421*** | -0.845* |
| | (0.959) | (1.022) | (0.153) | (0.506) | (0.146) | (0.971) | (0.146) | (0.450) |
| S-shaped | 0.070 | 1.781* | 0.148 | 0.879* | 0.202 | 1.629* | 0.202 | 0.551 |
| | (0.879) | (0.941) | (0.142) | (0.463) | (0.135) | (0.897) | (0.135) | (0.418) |
| Opt./Pess. | -2.386 | -4.358** | -0.665** | -2.280*** | -0.560** | -2.942* | -0.560** | -1.143 |
| | (1.646) | (1.735) | (0.271) | (0.881) | (0.260) | (1.665) | (0.260) | (0.796) |
| U.curv. | 0.037 | -0.052 | -0.006 | 0.003 | -0.005 | -0.056 | -0.005 | -0.010 |
| | (0.040) | (0.042) | (0.006) | (0.022) | (0.005) | (0.041) | (0.005) | (0.019) |
| Constant | 6.412*** | 4.134*** | 10.913*** | 3.409*** | 9.887*** | -3.590*** | 9.887*** | -3.184*** |
| | (0.290) | (0.307) | (0.046) | (0.155) | (0.256) | (1.295) | (0.256) | (0.756) |
| Controls | NO | NO | NO | NO | YES | YES | YES | YES |
| $R^2$ | 0.017 | 0.016 | 0.013 | 0.006 | 0.154 | 0.130 | 0.154 | 0.232 |
| N | 1902 | 2245 | 2629 | 1921 | 2628 | 2244 | 2628 | 1920 |

This table presents OLS estimates of the model $y_i = b_0 + b_1\text{Inverse-S}_i + b_2\text{S-shaped}_i + b_3$ Opt./Pess.$_i$ + $b_3$U.curv$_i$.+Controls$'_i\Gamma + \varepsilon_i$ . The dependent variable $y_i$ are the respondent's self-reported measures of income and wealth. "Inverse-S" is an index of likelihood insensitivity, "S-shaped" is an index of oversensitivity to probabilities, "Opt./pess." is an index of optimism and pessimism, and "U.curv" captures the respondent's utility function curvature. The estimates presented in Columns 1-4 do not include additional control variables, and the estimates presented in Columns 5-8 do include them. Robust standard errors are presented in parentheses. *** denotes significance at the 0.01 level, ** denotes significance at the 0.05 level, * denotes significance at the 0.1 level.

timate the utility function in (7). As before, we use non-linear least squares to perform the estimations. Table 1 presents the descriptive statistics of the estimates. As in the previous analyses, we use the estimated $\hat{\phi}_i$ to construct an index of insensitivity and the estimated $\hat{\delta}_i$ to construct an index of optimism and pessimism.[4] The estimates presented in Table 9 in Appendix C show that our results continue to hold when using these alternative indexes of insensitivity and pessimism. In particular, higher pessimism and likelihood insensitivity are associated with lower family income and financial wealth.

All in all, the empirical evidence supports the predictions of our model. However, the empirical analysis does not establish causality. Thus, these results are also in line with the alternative explanation that individuals who suffer the most from probability weighting end up in poverty as a consequence of their probabilistic misperception. Notice that this conclusion is different from our hypothesis that poverty worsens the consequences of probability weighting. In the next section, we provide experimental evidence that rules out this alternative explanation and conclusively confirms our predictions.

### 3.3. Experimental Evidence

The data of Carvalho et al. (2016) will be used to conclusively demonstrate that poverty worsens the consequences of probability weighting. They conducted experiments using two panels of representative American households in order to investigate the influence of financial resources on economic decision-making. In both experiments respondents were randomly assigned to one of two groups, and each group completed a survey either before or after payday. The survey included a battery of questions that elicited risk preferences and time preferences, and measured decision-making quality.

Unfortunately, the elicitation of risk preferences included in their experiment does not make it possible to cleanly elicit probability weighting and utility functions. However, we are able to obtain those from the questions originally designed to measure decision-making quality. Those questions were included in the survey administered to the participants of the GfK Knowledge panel and therefore, our empirical analysis focuses on that sample.

Decision-making quality is measured using the method of Choi et al. (2007), which essentially requires respondents to invest an endowment in two securities whose pay-

[4] Specifically, if $\hat{\phi}_i < 1$, the estimate $-\hat{\phi}_i$ is used as an index of likelihood insensitivity that we refer to as"Inverse-S." If $\hat{\phi}_i > 1$, the estimated $\hat{\phi}_i$ is an index of probability oversensitivity that we refer to as "S-shaped." $\hat{\delta}_i$ is an index of optimism and pessimism that we refer to as "Opt./Pess." If $\hat{\delta}_i > 1$, the respondent is optimistic, and if $\hat{\delta}_i < 1$, she is pessimistic.

outs are risky. More specifically, in each question respondents were asked to choose the fraction of their endowment to be invested in good $x_1$ and the rest of their endowment was automatically invested in good $x_2$. When making the decision, respondents knew that with probability $1/2$ only their investment in good $x_1$ or $x_2$ would generate a return. In total, the survey included 25 such questions, which differed in the relative prices of $x_1$ and $x_2$. That is, the amount of investment that could be afforded in one good relative to the other was varied in each question.

We recovered the participant's risk preferences using the method of Halevy et al. (2018). The most important property of this method for our analysis is its ability to recover their probability weighting functions and utility functions for a *given level of decision-making quality*. That is because it is based on a theoretical result that separates the participant's consistency of choices with respect to the maximization of a non-satiated utility function, which is Carvalho et al. (2016)'s criterion for decision-making quality, from misspecification, which refers to the fit of the parametric forms that are assumed in order to recover risk preferences. This separation is advantageous since it allows us to ignore decision-making quality and focus on risk preferences.

In line with our theoretical model, we assume that participant $i$ makes choices in each question using the following preferences:

$$RDU_i = \omega_i \cdot u_i\big(\max\{x_1, x_2\}\big) + (1 - \omega_i) \cdot u_i\big(\min\{x_1, x_2\}\big). \tag{10}$$

The variable $\omega_i$ captures her probability weighting, which is assumed to have the parametric form:

$$\omega_i = \frac{1}{2 + \beta_i} \text{ with } \beta_i > -1. \tag{11}$$

Notice that when $\beta_i > 0$, the probability associated with the better outcome, namely $\max\{x_1, x_2\}$, is overweighted, i.e. perceived to be larger than $1/2$. This overweighting of probabilities can be due to either optimism or likelihood insensitivity combined with optimism. In contrast, when $\beta_i < 0$, the probability associated with the better outcome is underweighted. This can be due to either pessimism or insensitivity combined with pessimism. Finally, when $\beta_i = 0$ the decision maker is an expected utility maximizer.

We also assume that the consumption utility function belongs to the CRRA family:

$$u_i(z) = \begin{cases} \frac{z^{1-\rho_i}}{1-\rho_i} & \text{if } \rho_i \geq 0 \text{ and } \rho_i \neq 1, \\ \ln(z) & \text{if } \rho_i \geq 0 \text{ and } \rho_i = 1. \end{cases} \tag{12}$$

The parameter $\rho$ captures risk aversion due to utility curvature. For the sake of robustness, and to avoid potential misspecification due to the assumption that the utility function belongs to the CRRA family, we also estimate an alternative model in which the utility functional is assumed to belong to the CARA family. In that specification, we use the parametric form $u(z) = -\exp\big(-Az\big)$ where $A \geq 0$.

We use the Money Metric Index (henceforth MMI) method of Halevy et al. (2018) to estimate, for each respondent, the parameters $\beta$ and $\rho$ in (11), as well as to estimate the parameters $\beta$ and $A$ in the model with CARA utility. We winsorized the variable that includes all individual estimates of $\hat{\rho}_i$ since it included a maximum value of 331 and had a variance of 528, which have no empirical interpretation. Table 5 presents the descriptive statistics for the resulting estimates of $\hat{\rho}_i$ and $\hat{\beta}_i$. The table shows that the average estimate $\hat{\beta}_i$ is equal to $0.393$, which implies that the probability $0.5$ is perceived by participants to be, on average, equal to $0.417$. This underweighting of probabilities is also obtained when the consumption utility function is assumed to belong to the CARA family. In that case, the probability $0.5$ is perceived to be $0.390$.

Table 5: Risk preference estimates obtained from the MMI method

| | **CRRA utility** | | **CARA utility** | |
|---|---|---|---|---|
| | $\hat{\beta}_i$ | $\hat{\rho}_i$ | $\hat{\beta}_i$ | $\hat{A}_i$ |
| **Mean** | 0.393 | 0.624 | 0.566 | 0.501 |
| **25th perc.** | 0.102 | 0.265 | 0.164 | 0.023 |
| **50th. perc.** | 0.238 | 0.399 | 0.341 | 0.037 |
| **75th. perc.** | 0.526 | 0.817 | 0.703 | 0.067 |
| **St. Dev.** | 0.608 | 0.524 | 0.709 | 2.37 |

This table presents the descriptive statistics for estimates of probability weighting and utility curvature obtained for each participant using the MMI method (Halevy et al., 2018). The first two columns present estimates obtained when utility is assumed to belong to the CRRA family, i.e. $u_i(z) = \begin{cases} \frac{z^{1-\rho_i}}{1-\rho_i} & \text{if } \rho_i \geq 0 \text{ and } \rho_i \neq 1, \\ \ln(z) & \text{if } \rho_i \geq 0 \text{ and } \rho_i = 1. \end{cases}$ Columns 3 and 4 present estimates obtained when utility is assumed to belong to the CARA family, i.e. $u(z) = -\exp\big(-Az\big)$, where $A \geq 0$. Probability weighting is assumed to follow the parametric form $\omega_i = \frac{1}{2+\beta_i}$ with $\beta_i > -1$.

Our theoretical framework predicts that being poor generates greater underinvestment relative to the EU benchmark. Therefore, we examine whether respondents assigned to the treatment group, i.e. the group that was financially constrained, are more likely to make investment decision that deviate from EU than the control group, i.e. the group that was not financially constrained. To do so, we first classify each respondent as EU or RDU based on the value of their estimated parameter $\hat{\beta}_i$. Specifically, a respondent $i$ is classified as EU if we cannot reject the null hypothesis $\hat{\beta}_i = 0$, implying that she does not exhibit probability weighting. In contrast, we classify the respondent as RDU if the null hypothesis is rejected. The test is performed by computing 95% confidence intervals of the estimated parameter $\hat{\beta}_i$ using resampling.

According to our classification, the majority of subjects are RDU. In particular, we find that 509 respondents out of 1131 (45% of the sample) can be classified as EU, while 622 exhibit significant probability weighting. When the CARA utility function is used, we find that 449 respondents can be classified as EU (39% of the sample) while 682 exhibit significant probability weighting.

In order to investigate the relationship between our classification and the treatment assignment, we regress a binary variable referred to as " EU" (which takes a value of one if a respondent is classified as EU and zero otherwise) on a treatment indicator referred to as "Before Payday". In all the regression specifications, we control for utility curvature, which in the case of CRRA utility is given by the estimated $\hat{\rho}_i$. Furthermore, in some of the regression specifications we control for decision-making quality using Varian's Index (Varian, 1982), and for cognitive abilities using the time spent on the Stroop test, which was included in the survey administered to the GfK Knowledge panel sample.[5]

Table 6 presents the probit estimates of the regressions. The estimates in columns (1), (3) and (5) show that the treatment, on its own, does not lead to a lower probability of being classified as EU, and this result is robust to the inclusion of control variables. Moreover, this statistically insignificant finding is in line with the results of Carvalho et al. (2016). It is also in line with model's assumption that the poor and non-poor exhibit similar probability weighting.

Recall that a condition for the existence of our proposed poverty trap is that utility curvature is sufficiently concave (see Proposition 3). If that condition holds, then our result that poverty exacerbates the underinvestment caused by probability weighting follows. We account for this interplay between utility curvature and poverty by including an interaction term between the variable Before Payday and the coefficient $\hat{\rho}_i$. The results of regressions that include that interaction are presented in columns (2), (4), and (6) of Table 6, while the average treatment effect for different average levels of $\hat{\rho}_i$ is depicted in Figure 4. These results imply that being financially constrained and having a more concave utility curvature increases the likelihood that investment choices deviate from the EU benchmark.

We interpret this as evidence of underinvestment relative to the EU benchmark

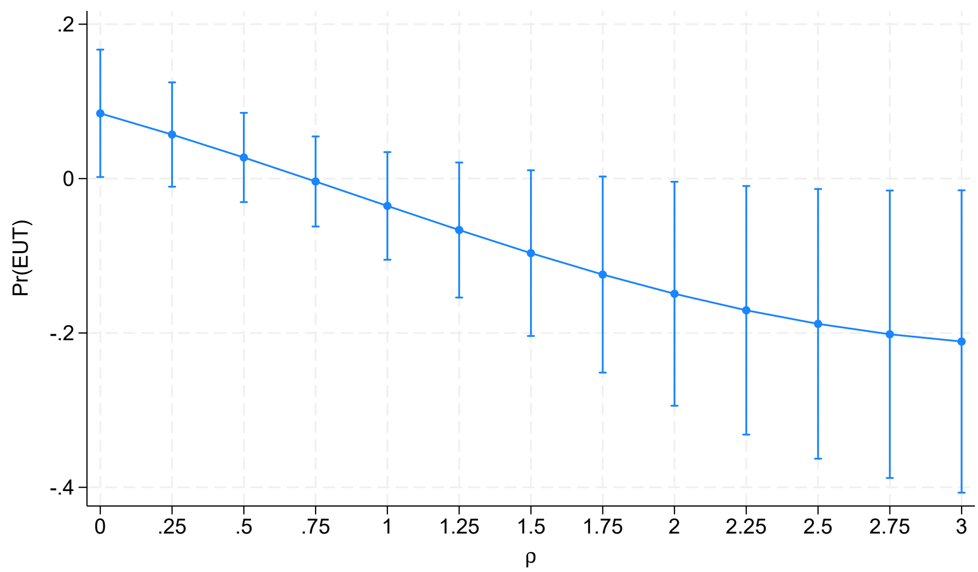
[5]We also control for demographic variables such as ethnicity, age, education, gender, employment status, and type of occupation.

Table 6: The Effects of Payday on the Probability of Expected Utility

| | (1) EU | (2) EU | (3) EU | (4) EU | (5) EU | (6) EU |
|---|---|---|---|---|---|---|
| Before Payday | 0.024 | 0.204* | 0.023 | 0.210* | 0.028 | 0.240** |
| | (0.075) | (0.116) | (0.076) | (0.119) | (0.077) | (0.120) |
| $\hat{\rho}_i$ | 0.162** | 0.311*** | 0.378*** | 0.534*** | 0.395*** | 0.572*** |
| | (0.071) | (0.103) | (0.082) | (0.113) | (0.082) | (0.113) |
| Before Payday$\times\hat{\rho}_i$ | | -0.286** | | -0.295** | | -0.333** |
| | | (0.143) | | (0.146) | | (0.146) |
| Varian Index | | | 10.115*** | 10.171*** | 10.451*** | 10.550*** |
| | | | (1.950) | (1.959) | (1.996) | (2.007) |
| Time Stroop test | | | -0.000 | -0.001 | 0.000 | -0.000 |
| | | | (0.003) | (0.003) | (0.004) | (0.004) |
| Constant | -0.240*** | -0.334*** | -0.451 | -0.397 | -0.681 | -0.635 |
| | (0.070) | (0.084) | (1.207) | (1.208) | (1.326) | (1.325) |
| Controls | NO | NO | NO | NO | YES | YES |
| Log-likelihood | -775.677 | -773.686 | -748.394 | -746.331 | -733.984 | -731.407 |
| N | 1131 | 1131 | 1116 | 1116 | 1116 | 1116 |

This table presents probit estimates of the model $\text{EU}_i = b_0 + b_1\text{Before Payday}_i + b_2\hat{\rho}_i + b_3\text{Before Payday}\times\hat{\rho}_i + \text{Controls}_i'\Gamma + \varepsilon_i$. The dependent variable $\text{EU}_i$ is a binary variable that takes a value of one if respondent $i$ is classified as an expected utility maximizer and zero otherwise. "Before Payday" is a binary variable that takes a value of one if respondent $i$ is assigned to the group that completed the survey before payday. The variable $\hat{\rho}_i$ captures subject's $i$ utility curvature. "Varian Index" captures the extent to which participant's $i$ responsens are consistent with the maximization of a non-satiated utility function. Time Stroop Test captures the time in seconds that respondent $i$ spent on answering the Stroop test questions. *** denotes significance at the 0.01 level, ** denotes significance at the 0.05 level, * denotes significance at the 0.1 level.

Figure 4: Marginal effects of Payday for different levels of $\rho$




Note: 95% confidence intervals.

since, on average, subjects underweight the probability associated with the highest outcome. Thus, their probability perception will lead them to invest more in the lowest outcome, which lowers their average earnings. Our findings indicate that this effect is more pronounced among the poor who have a sufficiently concave consumption utility function.

Table 10 and Figure 5 in Appendix C show that these results also emerge in the case of a CARA consumption utility. The findings are therefore not an artifact of assuming a specific functional of utility. In that appendix, we also show that our results are robust to excluding respondents who exhibit probability overweighting. Those subjects constitute only 5.92% of the sample when it is assumed to be CRRA and 3.27% when utility is assumed to be CARA. Thus, our conclusions are not driven by excluding or including a few overoptimistic respondents.

## 4. Ambiguity Attitudes and Behavioral Poverty Traps

### 4.1. Extension of the Theoretical Framework

In this section, we incorporate ambiguity attitude within the model. To that end, we slightly modify our framework by considering a setting in which the individual chooses to invest in one of two goods: one that is risky, which means that the objective probabilities of obtaining a particular level of return are objectively known, and another that is ambiguous, which means that those probabilities are not known. Intuitively, the ambiguous situation arises when an individual has limited experience with the investment good, or when returns are determined by multiple factors external to the control of the individual.

Notice that when the individual chooses to invest in the risky good, the results of the previous section follow immediately. That case will serve as the benchmark of this analysis. Specifically, our analysis involves comparing investment choices between the ambiguous situation and the risky situation.

There is abundant empirical research on ambiguity attitude. In the context of our model, it is commonly found that most individuals prefer betting on the event $x > \hat{x}$, where $\hat{x}$ is a return threshold, in the case of a risky good rather than in the case of an ambiguous good. In addition, the same individuals typically prefer betting on the event $x \leq \hat{x}$ in the case of a risky good than in the case of an ambiguous good. These preferences imply an aversion to betting on events in the ambiguous good that violates subjective expected utility (Savage, 1954).[6] This type of behavior has been documented in prominent laboratory experiments (Ellsberg, 1961, Halevy, 2007, Abdellaoui et al., 2011, Baillon et al., 2018a). Moreover, recent research shows that when the events under consideration are extreme, individuals typically exhibit ambiguity seeking (Abdellaoui et al., 2011, Baillon et al., 2018b, Baillon and Emirmahmutoglu, 2018). We refer to "ambiguity attitude" as the conjunction of ambiguity seeking in the case of unlikely events and ambiguity aversion in the case of likely ones.

In order to incorporate ambiguity attitude in our model, let $\chi$ be the set of possible returns on the ambiguous good. Note that it coincides with the set of possible returns on the risky good. An event in the context of the ambiguous good is any subset $E \subset \chi$, while the set of all such events is denoted by $\Sigma$, which we endow with the Borel $\sigma$-algebra.

The most commonly used model for decision making under ambiguity is Choquet Expected Utility (Schmeidler, 1989). In our context, it is described by the functional:

$$RDU\big(u(x,e)\big) = D(1)\int_{\chi} u\Big(b(x_0,x)\Big)\mathrm{d}W - c(e), \tag{13}$$

where $W$ is a weighting function that exhibits $W(\varnothing|e) = 0$ and $W(\chi|e) = 1$, and for which $E_1 \subset E_2$ implies $W(E_1|e) < W(E_2|e)$. This model generalizes subjective expected utility by allowing $W$ to be non-additive, a feature that accounts for ambiguity attitudes by giving up probabilistic beliefs. For instance, the aforementioned aversion to invest in the ambiguous good is incorporated in this model by assuming that $W$ is subadditive.[7] The main problem in modeling ambiguity attitudes using (13) is that there might be potentially many weighting functions $W$ that can account for an individual's ambiguity attitude (Abdellaoui et al., 2011). This makes the identification of ambiguity attitude indeterminate and renders a comparison between choices under risk and choices under ambiguity imprecise.

We address this problem by adopting an alternative approach to model ambiguity attitude known as *source theory* (Abdellaoui et al., 2011, Baillon et al., 2023). Accordingly, we assume that each type of good generates an algebra of events, which we call a *source*. Intuitively, each good is a distinct random mechanism generating a group of events (Tversky and Fox, 1995). A crucial assumption of this theory is that probabilistic beliefs hold *within* sources of uncertainty but not *between* them (Chew and Sagi,

[6]Formally, assume without loss of generality that in the case of the risky good it is known that $prob(x > \hat{x}) = 0.6$ and that $u(0) = 0$. The individual's aversion to betting an amount $M$ on the ambiguous good implies that the subjective probability of the event $x > \hat{x}$ is $P(x > \hat{x}) < 0.6$. Similarly, the individual's aversion to betting on the event $x \leq \hat{x}$ implies $P(x \leq \hat{x}) < 0.4$. Note that the inequalities $P(x > \hat{x}) < 0.6$ and $P(x \leq \hat{x}) < 0.4$ violate probability laws.

[7]Using the example in footnote 6, subadditivity implies that $W(x \leq \hat{x}) < 0.4$ and $W(x > \hat{x}) < 0.6$.

2008). Accordingly, denote by $P_u$ the probability measure generated by $\Sigma$, i.e. the algebra of events generated by the ambiguous good, and, as before, let $F(x|e)$ be the probability measure when probabilities are known.

This approach makes it possible to define attitudes toward probabilities of different sources. In the case of the ambiguous investment, there exists a function $w_u$ with the properties of Assumption 5, such that, for any $e$:

$$W(E|e) = w_u\big(1 - P_u(E|e)\big) \text{for any } E \in \chi. \tag{14}$$

The function $w_u$ carries subjective probabilities to decision weights and is referred to as the *source function*. It can exhibit a different shape than $w$, the probability weighting function which carries objective probabilities to decision weights. The difference in shape between $w_u$ and $w$ fully identifies ambiguity attitude. For instance, when $w_u$ is more convex (concave) than $w$, the individual exhibits ambiguity aversion (seeking), i.e. she irrationally believes that unfavorable (favorable) events are more likely in the case of the ambiguous good than in the case of the risky good. Moreover, if $w_u$ exhibits a more pronounced inverse-S shape than $w$, the decision maker exhibits a-insensitivity, i.e. she erroneously assigns more probability weight to extreme events in the case of the ambiguous good than to equally unlikely events in the case of the risky good (Baillon et al., 2018b).

Substituting (14) in (13) gives the following evaluation of returns in the case of the ambiguous good:

$$RDU\big(u(x,e)\big) = -D(1)\int_\chi u\Big(b(x_0,x)\Big)\mathrm{d}w_u\big(1 - p_u(x|e)\big) - c(e). \tag{15}$$

Equation (15) is analogous to (5) for unknown probabilities. Since it features the source function $w_u$, which is endowed with the properties of Assumption 5, and given the regularity conditions imposed on the set $\Sigma$, the results presented in Propositions 1-3 immediately follow for the ambiguous good. Therefore, a poor individual with sufficiently strong ambiguity attitude underinvests in the ambiguous good to a greater extent than richer individuals with the same ambiguity attitude.

Nevertheless, the most relevant result of this analysis emerges when the optimal investment of an individual with preferences characterized by (15) is contrasted with that of the same individual when choosing the risky good. The following proposition states that ambiguity attitude, regardless of its type, worsens the poverty trap described in Propositions 1-3.

**Proposition 4.** *Assume that Assumption 1-5, exchangeability (Chew and Sagi, 2008), and the condition* $\frac{b_{x,x_0}(x_0,x)}{b_{x_0}(x_0,x)b_x(x_0,x)} < \frac{u''\big(b(x_0,x)\big)}{u'(x_0,x)}$ *hold for all* $x$. *Then:*

*(i) An ambiguity-averse individual chooses a lower level of investment in the ambiguous good than in the risky good if her probability weighting function exhibits a level of pessimism exceeding the threshold characterized in Proposition 1.*

*(ii) An ambiguity-seeking or a-insensitive individual is more likely to forgo investing in the ambiguous good relative to the risky good.*

*These differences in investment between the ambiguous and risky good become more pronounced as $x_0$ decreases.*

Ambiguity attitude worsens the evaluation of investments because individuals assign larger weights to the probabilities of extreme returns in the case of the ambiguous good than to those of equally unlikely returns in the case of the risky good. This tendency to focus on extreme events means that individuals assign smaller probability weights to less extreme outcomes of investment in the ambiguous good, relative to equally likely outcomes of investment in the risky good. This larger bias when evaluating the probabilities of the ambiguous good causes individuals to mistakenly perceive the ambiguous asset as less profitable than the risky asset when its returns are not sufficiently extreme.

Proposition 4 also demonstrates that poverty exacerbates underinvestment in the ambiguous good relative to the risky good. As a result, the poor suffer more severe economic consequences from having ambiguity attitudeas the result of acute underinvestment.

### 4.2. Empirical Evidence

To conclude this section, we discuss empirical evidence that corroborates the prediction of our model that poverty worsens the underinvestment generated by ambiguity attitude. Li (2017) showed that poor rural adolescents in China exhibit more ambiguity aversion and a-insensitivity than their poor urban counterparts. Since the former group is poorer than the latter, her result suggests that ambiguity attitude increases as poverty worsens.

Dimmock et al. (2016a) designed an experiment to elicit ambiguity attitude in a representative sample of Dutch households. One of their main empirical results is that strong a-insensitivity is related to low stock market participation and a lower level of private business ownership. These results are in line with our theoretical prediction that stronger a-insensitivity leads individuals to forgo investments.

Bryan (2019) found that ambiguity attitude causes poor individuals to forgo profitable investments based on two randomized control trials. The first experiment showed that ambiguity-averse farmers in Malawi are less inclined to adopt new crop types when doing so requires the purchasing of rainfall insurance. Notice that this requirement makes the adoption of new crop types more ambiguous, which disincentivizes ambiguity-averse farmers from investing even though the complementarity between rainfall insurance and the new seed type generates higher average returns. In the second experiment, ambiguity-averse farmers in Kenya were less inclined to adopt new crop types even when credit was made available. Hence, the farmer's tendency to underinvest in this new technology is indeed due to ambiguity attitude rather than credit constraints.

# 5. Further extensions

## 5.1. Reference dependence

There is abundant empirical evidence that individuals tend to evaluate risky alternatives relative to a reference point (Kahneman and Tversky, 1979, Tversky and Kahneman, 1992, Von Gaudecker et al., 2011, Baillon et al., 2020). This way of evaluating risky alternatives represents a deviation from expected utility because a decision maker may exhibit a considerably different risk attitude toward outcomes that are evaluated as *gains*, i.e.outcomes surpassing the reference point, than outcomes that are evaluated as *losses*, i.e. outcomes that fall short of the reference point. One of the factors responsible for this difference is loss aversion, which is the notion that losses result in a greater reduction in utility than the increase in utility from commensurate gains.

We incorporate reference dependence within our model by characterizing the individual's risk preferences using Cumulative Prospect Theory (Tversky and Kahneman, 1992). In the interest of brevity, the complete analysis is presented in Appendix C. Interestingly, we find that loss aversion exacerbates the poverty trap presented in Propositions 1-3.

Unlike probability weighting, loss aversion leads to overinvestment because a high level of investment increases the likelihood that the reference point will be surpassed and losses avoided. In our model, the individual's reference point is assumed be the *status quo* or initial wealth, which has empirical support (Terzi et al., 2016, Baillon et al., 2020). Thus, the poor have the lowest reference points and, as a result, are less prone to overinvest as a result of loss aversion. This generates a disparity between the rich who overinvest, thus taking advantage of profitable opportunities even though they suffer from probability weighting, and the poor who forgo such opportunities due to the combination of probability weighting and modest reference points. This result echoes the findings of Dalton et al. (2016) and Genicot and Ray (2017) who show that low reference points among the poor can perpetuate their condition.

### 5.2. Dynamic inconsistency

Our theoretical framework considers an intertemporal setting in which the level of investment is chosen in the present and returns are realized in the future. A possible extension of the model would be to incorporate time-inconsistent preferences, according to which individuals will tend to exhibit preference reversals when they face intertemporal tradeoffs (Thaler, 1981, Frederick et al., 2002, Halevy, 2015). The conventional approach to capture these preferences is to use quasi-hyperbolic discounting (Laibson, 1997), i.e. to assume that $D(t) = \beta\delta^t$.

However, this approach ignores the fact that non-expected utility models, such as RDU, imply dynamic inconsistency (Machina, 1989, Karni and Schmeidler, 1991). Hence, RDU, on its own, can explain behaviors that are typically attributed to quasi-hyperbolic discounting such as, for example, the low usage of fertilizer by poor farmers (Duflo et al., 2011).

This alternative interpretation of the findings can be incorporated into our model using a simple extension, which can be described using the simplified version of the model presented in Section 2. Suppose that in period $t = 0$ the RDU individual evaluates the probability of obtaining a high level of return on an investment in fertilizer, i.e. $w(p(e_H))$. In period $t = 1$, she receives an independent signal about the expected quality of the crop, such as a favorable weather forecast. Let $r$ be the probability of a high return when favorable weather is predicted. At $t = 1$, the decision-maker also makes an investment choice, denoted by $e$. Finally, the return on the investment is realized in period $t = 2$.

If the return on investment is sufficiently overweighted in $t = 0$, i.e. $w(p(e_H)) - w(p(e_L)) > p(e_H) - p(e_L)$, then the individual would be willing to buy fertilizer for more values of $c$ than her EU counterpart. However, if the weather forecast is not sufficiently favorable, which makes the probability $r$ small, then the optimistic or likelihood-insensitive individual might overweight the gains from good weather without investment, i.e. $w(r(1 - p(e_H)))$, which motivates her not to invest in fertilizer. Thus, the individual exhibits a gap between her intention to buy fertilizer in $t = 0$ and her action in $t = 1$.

This brief explanation illustrates how our model can be conveniently extended to incorporate the dynamic inconsistency generated by probability weighting in order to explain results that were hitherto attributed to time-inconsistent preferences.

### 5.3. Overlapping generations model

Our theoretical framework considered stylized models characterized by short time horizons and the absence of markets. To address the possibility that our results do not hold in more complex economic settings, we investigate the effect of probability weighting on investment decisions in an overlapping generations model. That framework includes markets for credit and allows for long-run equilibria. Thus, it can be used to establish whether our proposed poverty trap emerges as a stationary equilibrium, or whether the presence of a perfect credit market corrects the adverse consequences of behavioral biases.

The detailed description of the model and its results can be found in Appendix D. Our theoretical analysis reveals that probability weighting leads individuals to underinvest in capital. This investment error can be costly in the long run because it potentially traps individuals in a low steady-state equilibrium which they would have avoided, and instead achieve a high steady state equilibrium, if they perceived probabilities accurately. These results generalize Proposition 1 and 2.

## 6. Conclusion

We have introduced a poverty trap generated by an individuals tendency to misperceive objective and subjective probabilities. Due to these biases, profitable opportunities are not evaluated accurately, which explain why poor individuals often fail to exploit investments that would improve their condition. We also showed that the consequences of this misperception are stronger among the poor. Not only are they in a more vulnerable position, and suffer more from their mistakes, their mistakes are also the largest in scope. As a consequence, their chances of escaping poverty are slimmer.

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# Appendix A. Proofs

## For Online Publication Only

The following preliminary result is used to proof Remark 1 and Proposition 1.

**Lemma 2.** *If agent $i$ is more optimistic than agent $j$, then $w_i(p) > w_j(p) \forall p \in (0,1)$. In contrast, if agent $i$ is more pessimistic than agent $j$, then $w_i(p) > w_j(p) \forall p \in (0,1)$.*

*Proof.* According to Definition 1, $w_i(p) = \theta\big(w_j(p)\big)$. It is straightforward to show that, under that equality

$$\frac{w_i''(p)}{w_i'(p)} = \frac{\theta_i''(p)}{\theta_i'(p)} w_j'(p) + \frac{w_j''(p)}{w_j'(p)}. \tag{A.1}$$

Since $\theta''(p) < 0$, the previous equation implies that

$$\frac{w_i''(p)}{w_i'(p)} < \frac{w_j''(p)}{w_j'(p)}. \tag{A.2}$$

Let $p_0, p_1 \in [0,1]$ such that $p_1 > p_0$. Integrate (A.2) over $[p_0, p_1]$ to obtain

$$\int_{p_0}^{p_1} \frac{w_i''(s)}{w_i'(s)} \mathrm{d}s < \int_{p_0}^{p_1} \frac{w_j''(s)}{w_j'(s)} \mathrm{d}s \Leftrightarrow \frac{w_j'(p_1)}{w_j'(p_0)} > \frac{w_i'(p_1)}{w_i'(p_0)}. \tag{A.3}$$

Integrating the resulting inequality with respect to $p_0$ gives

$$\int_{0}^{p_1} w_i'(p_1) w_j'(s) \mathrm{d}s < \int_{0}^{p_1} w_j'(p_1) w_i'(s) \mathrm{d}s \Leftrightarrow \frac{w_j'(p_1)}{w_j(p_1)} > \frac{w_i'(p_1)}{w_i(p_1)}. \tag{A.4}$$

Integrating again, but this time with respect to $p_1$ gives

$$\int_{p_0}^{1} \frac{w_j'(s)}{w_j(s)} \mathrm{d}s > \int_{p_0}^{1} \frac{w_i'(s)}{w_i(s)} < \mathrm{d}s \Leftrightarrow w_i(p_0) > w_j(p_0) \text{ for any } p_0 \in (0,1). \tag{A.5}$$

Similar steps lead to the conclusion that when $i$ is more pessimistic than $j$, then $w_j(p_0) > w_i(p_0)$ for any $p \in (0,1)$.

■

**Remark 1.**

*Proof.* Fix $e$. Denote by $w_j$ the probability weighting function of an individual $j$. Using integration by parts, rewrite (5) as:

$$RDU\big(u(x,e)\big) = u\big(b(x_0,\underline{x})\big) + \int_{\underline{x}}^{\bar{x}} u'\big(b(x_0,x)\big) b_x(x_0,x) w_j\big(1 - F(x|e)\big)\mathrm{d}x - c(e). \quad \text{(A.6)}$$

Denote by $w_i$ the probability weighting function of an individual $i$ who is more optimistic than $j$. Using (A.6) it can be established that

$$RDU_i\big(u(x,e)\big) - RDU_j\big(u(x,e)\big) \Leftrightarrow \int_{\underline{x}}^{\bar{x}} u'\big(b(x_0,x)\big) b_x(x_0,x)\Big(w_i\big(1-F(x|e)\big) - w_j\big(1-F(x|e)\big)\Big)\mathrm{d}x. \quad \text{(A.7)}$$

Since, $w_i(p) > w_j(p)$ for any $p \in (0,1)$ (Remark 2), it can be established using (A.7) that $RDU_i\big(u(x,e)\big) - RDU_j\big(u(x,e)\big) > 0$.

We next define a certainty equivalent for each individual. Denote by $F_i$ the certain and fixed amount that makes the individual indifferent between investing $e$ and obtaining $F_i$, and by $F_j$ the certain and fixed amount that makes the individual indifferent between investing $e$ and obtaining $F_j$. Since, $RDU_i\big(u(x,e)\big) - RDU_j\big(u(x,e)\big) > 0$ it must be that $F_i > F_j$, individual $i$ tolerates more risk, as she strictly prefers to invest $e$ and obtain the utility $RDU_i\big(u(x,e)\big)$ over obtaining $F_j$ whereas individual $j$ is indifferent between these two choices.

■

**Remark 2.**

*Proof.* According to Definition 3, the probability weighting function of $w_i$ can be written as $w_i(p) = \phi\big(w_j(p)\big)$. It is straightforward to show that the following equality holds.

$$\frac{w_i''(p)}{w_i'(p)} = \frac{\phi_i''(p)}{\phi_i'(p)} w_j'(p) + \frac{w_j''(p)}{w_j'(p)}. \quad \text{(A.8)}$$

Because $\phi''(p) < 0$ in $p \in (0, 0.5)$, it must be that in that segment

$$\frac{w_i''(p)}{w_i'(p)} < \frac{w_j''(p)}{w_j'(p)}. \quad \text{(A.9)}$$

Instead, if $p \in (0.5, 1)$, then, using similar steps, we obtain $\frac{w_i''(p)}{w_i'(p)} > \frac{w_j''(p)}{w_j'(p)}$.

Let $p_0, p_1 \in [0, 0.5]$ such that $p_1 > p_0$. Integrate (A.9) over $[p_0, p_1]$ to obtain

$$\int_{p_0}^{p_1} \frac{w_i''(s)}{w_i'(s)}\mathrm{d}s < \int_{p_0}^{p_1} \frac{w_j''(s)}{w_j'(s)}\mathrm{d}s \Leftrightarrow \frac{w_j'(p_1)}{w_j'(p_0)} > \frac{w_i'(p_1)}{w_i'(p_0)}. \quad \text{(A.10)}$$

Integrating (A.10) with respect to $p_0$ gives

$$\int_0^{p_1} w_i'(p_1)w_j'(s)\mathrm{d}s < \int_0^{p_1} w_j'(p_1)w_i'(s)\mathrm{d}s \Leftrightarrow \frac{w_j'(p_1)}{w_j(p_1)} > \frac{w_i'(p_1)}{w_i(p_1)}. \tag{A.11}$$

Integrating again (A.11) but this time with respect to $p_0$ gives

$$\int_{p_1}^{1} \frac{w_j'(s)}{w_j(s)}\mathrm{d}s > \int_{p_1}^{1} \frac{w_i'(s)}{w_i'(s)}\mathrm{d}s \Leftrightarrow w_i(p_0) > w_j(p_0) \text{ for any } p_0 \in (0, 0.5) \tag{A.12}$$

Following similar steps it is possible to arrive to $w_i(p_0) < w_j(p_0)$ for any $p_0 \in (0.5, 1)$.

■

**Lemma 1.**

*Proof.* Consider an individual $j$ with probability weighting function $w_j$ and who exhibits optimism in the sense of Definition 1. Due to the the continuity of $w_j(p)$ for all $p$, and that at extreme probabilities that probability weighting function exhibits $\lim_{p\to 0} w_j'(p) > 1$ and $\lim_{p\to 1} w_j'(p) < 1$ (Assumption 5), there must exist a probability $p_k \in (0, 1)$ such that $w_j'(p_k) = 1$. Accordingly, for all $p < p_k$ then $w_j'(p) > 1$ whereas for all $p > p_k$ then $w_j'(p) < 1$.

Consider now an individual $i$ with probability weighting function $w_i$ who exhibits stronger optimism than individual $j$ (Definition 2). Using the same reasoning given above, there exists a $p_l \in (0, 1)$ that satisfies $w_i'(p_l) = 1$, and for all $p < p_l$ then $w_i'(p) > 1$ whereas for all $p > p_l$ then $w_i'(p) < 1$.

According to Lemma 2, $w_i(p) > w_j(p)$. Therefore, the second equivalence in eq. (A.4) when evaluated at $p_1 = p_k$ implies that

$$w_i'(p_k) < 1 = w_j'(p_k). \tag{A.13}$$

We now proceed by contradiction. Assume that $p_l \geq p_k$, then $w_i'(p_l) = 1 \leq w_i'(p_k)$, which contradicts (A.13). Hence, it must be that $p_k > p_l$. Consequently, the set $p > p_l$, which generates $w_i'(p) < 1$, is larger than the set $p > p_k$. Under pessimism, i.e. when $i$ is more pessimistic than $j$, the arguments of the proof can be mirrored to obtain the result that the set $p < p_l$ is larger than the set $p < p_k$.

■

The following lemma is relevant to characterize a solution to the maximization problem of the individual. It shows the conditions under which an interior solution is guaranteed.

**Lemma 3.** *An interior solution to the individual's problem of maximizing investment is guaranteed if the individual exhibits pessimism.*

*Proof.* The second derivative of (A.6) with respect to $e$ gives

$$\int_{\underline{x}}^{\bar{x}} u'\big(b(x_0,x)\big)b_x(x_0,x)\Big(w_j'\big(1-F(x|e)\big)F_{ee}(x|e) - w_j''\big(1-F(x|e)\big)(F_e(x|e))^2\Big)\mathrm{d}x - c_{ee}(e). \tag{A.14}$$

A sufficient and necessary condition for an interior solution is that (A.14) is negative. Since $c_{ee}(e) > 0$ (Assumption 1), $b_x(x_0,x) \geq 0$ (Assumption 3), and $u' \geq 0$ (Assumption 4), it suffices that

$$\int_{\underline{x}}^{\bar{x}} w_j'\big(1-F(x|e)\big)F_{ee}(x|e)\mathrm{d}x < \int_{\underline{x}}^{\bar{x}} w_j''\big(1-F(x|e)\big)(F_e(x|e))^2\mathrm{d}x \tag{A.15}$$

Since $w_j'\big(1-F(x|e)\big) > 0$ for all $x$ and $e$ (Assumption 5), and $F_e(x|e) > 0$ and $F_{ee}(x|e) > 0$ for all $x$ and $e$ (Assumption 2), the inequality in (A.15) requires pessimism, which implies $w''\big(1-F(x|e)\big) > 0$ for all $x$ and $e$. ■

The following remark will also be useful to prove Proposition 1.

**Remark 3.** *Assumption 2 implies the existence of a return level $\hat{x} \in [\underline{x}, \bar{x}]$ such that $\frac{f_e(x|\tilde{e})}{f(x|\tilde{e})} \leq 0$ if and only if $x < \hat{x}$.*

*Proof.* The probability that a realized investment $X$ is greater than a given $x$ when effort $\tilde{e} \in [0, \bar{e}]$ is exerted is $1 - F(X|\tilde{e})$. Let us check that increasing $\tilde{e}$ raises this probability when Assumption 2 holds. We have

$$F_e(x|\tilde{e}) = \int_{\underline{x}}^{x} \frac{f_e(s|\tilde{e})}{f(s|\tilde{e})} f(s|\tilde{e})\mathrm{d}s = \int_{\underline{x}}^{x} \eta(s,\tilde{e})f(s|\tilde{e})\mathrm{d}s \tag{A.16}$$

where $\eta(x,\tilde{e}) = \frac{f_e(x|\tilde{e})}{f(x|\tilde{e})}$ a given $x$. By Assumption 2, $\eta(x,\tilde{e})$ increases in $x$. Notice that $\eta(x,\tilde{e})$ cannot be everywhere negative, because, by definition, $F_e(\bar{x}|\tilde{e}) = 0 = \int_{\underline{x}}^{\bar{x}} \eta(x,\tilde{e})f(x|\tilde{e})\mathrm{d}x$. Hence, there exists $\hat{x}$ such that $\eta(x,\tilde{e}) \leq 0$ if and only if $x \leq \hat{x}$. $F_e(x|\tilde{e})$ is decreasing in $x$ (resp. decreasing) on $[\underline{x}, \hat{x}]$ (resp. $[\hat{x}, \bar{x}]$). Since $F_e(\underline{x}|\tilde{e}) = F_e(\bar{x}|\tilde{e}) = 0$, we necessarily have $F_e(x|\tilde{e}) \leq 0$ for any $x$.

■

**Proposition 1**

*Proof.* According to Lemma 3, a probability weighting function with pessimism guarantees an interior solution. Hence, the optimal investment level, which we denote by $e_r^*$, satisfies the following first-order condition obtained from deriving (A.6) with respect to $e$:

$$-D(1)\int_{\underline{x}}^{\bar{x}} u'\big(b(x_0,x)\big)b_x(x_0,x)w_j'\big(1-F(x|e_r^*)\big)F_e(x|e_r^*)\mathrm{d}x - c'(e_r^*) = 0. \tag{A.17}$$

Expected utility holds when $w_j(p) = p$. In the case of those preferences, equation (A.17) becomes:

$$-D(1)\int_{\underline{x}}^{\bar{x}} u'\big(b(x_0,x)\big)b_x(x_0,x)F_e(x|e_u^*)\mathrm{d}x - c'(e_u^*) = 0. \tag{A.18}$$

where $e_u^*$ is the investment level that satisfies the first order condition given in (A.18).

We proceed by contradiction. Suppose that $e_r^* \geq e_u^*$ for all $x$. This assumption can be expressed, using (A.17) and (A.18), as:

$$-D(1)\int_{\underline{x}}^{\bar{x}} w_j'\big(1-F(x|e_r^*)\big)F_e(x|e_r^*)\Big)\mathrm{d}x \geq -D(1)\int_{\underline{x}}^{\bar{x}} u'\big(b(x_0,x)\big)b_x(x_0,x)\Big(F_e(x|e_u^*)\Big)\mathrm{d}x, \tag{A.19}$$

which holds if:

$$\int_{\underline{x}}^{\bar{x}} w_j'\big(1-F(x|e_r^*)\big)F_e(x|e_r^*)\mathrm{d}x \geq -\int_{\underline{x}}^{\bar{x}} F_e(x|e_u^*)\mathrm{d}x. \tag{A.20}$$

Denote by $\Omega = \{w^1, w^2, ...\}$ the set of all probability weighting functions with pessimism in the sense of Definition 1. Assume that $\Omega$ is strict partial order with respect to pessimism. Accordingly, $w^1 \in \Omega$ is the function with the least possible pessimism and $w^{\aleph} \in \Omega$ the function with most severe pessimism.

Suppose that $w_j = w^{\aleph}$. Lemma 1 states that such degree of pessimism is characterized by $\lim_{x\to\underline{x}} w_j'(1-F(x|\tilde{e})) = +\infty$ for given $\tilde{e} \in [0,\bar{e}]$ and by $w_j'(1-F(x|e)) < 1$ for all $\chi\backslash\{\underline{x}\}$. Furthermore, Remark 3 shows that $F_e(\underline{x}|e) = 0$. Therefore, when $w_j = w^{\aleph}$ the inequality in (A.20) cannot hold and it must be that $e_u^* > e_r^*$.

Lemma 1 also implies that when $w_j \in \Omega\backslash\{w^n\}$, that is when pessimism is less severe, less weight is given to $\underline{x}$ and more weight is given to all other possible returns. The continuity of $w''(p)$ (Assumption 5) implies the existence of a $w^k \in \Omega$ such that $e_r^* = e_u^*$. Therefore, for the partition $\Omega\backslash\{w^1, ..., w^k\}$ it holds that $e_r^* < e_u^*$.

■

**Proposition 2**

*Proof.* Lemma 3 shows that an interior solution might not be guaranteed when $w''(p) < 0$ for some $p \in [0,1]$. Suppose that an interior solution is indeed disregarded. In that case, optimal investment is chosen from the set $e = \{0, \bar{e}\}$. Using Eq. (A.6), it can be established that $\bar{e}$ is chosen when:

$$\begin{gathered} RDU_j(u(x,\bar{e})) > RDU_j(u(x,0)) \Leftrightarrow \\ D(1)\int_{\underline{x}}^{\bar{x}} u'\big(b(x_0,x)\big)b_x(x_0,x)\Big(w_j\big(1-F(x|\bar{e})\big) - w_j\big(1-F(x|0)\big)\Big)\mathrm{d}x > c(\bar{e}). \end{gathered} \tag{A.21}$$

Assumptions 2 and 5 imply $w_j\big(1-F(x|\bar{e})\big) - w_j\big(1-F(x|0)\big) \geq 0$ and Assumption 3 states that $u' > 0$. Therefore, the left-hand side of the second equivalence in (A.21) is weakly positive.

Under EU, $e = \bar{e}$ is chosen over $e = 0$ when:

$$D(1)\int_{\underline{x}}^{\bar{x}} u'\big(b(x_0,x)\big)b_x(x_0,x)\Big(\big(1-F(x|\bar{e})\big) - \big(1-F(x|0)\big)\Big)\mathrm{d}x > c(\bar{e}). \tag{A.22}$$

Using eqs. (A.21) and (A.22), it can be established that probability weighting steers the individual into choosing $\bar{e}$ less often if

$$D(1)\int_{\underline{x}}^{\bar{x}}\int_{1-F(x|0)}^{1-F(x|\bar{e})} u'\big(b(x_0,x)\big)b_x(x_0,x)\big(w_j'(s)-1\big)\mathrm{d}s\mathrm{d}x < 0, \tag{A.23}$$

which holds if

$$\int_{\underline{x}}^{\bar{x}}\int_{1-F(x|0)}^{1-F(x|\bar{e})} w_j'(s) - 1 \;\mathrm{d}s\mathrm{d}x < 0. \tag{A.24}$$

Equation (A.24) shows that $w_j'(p) < 1$ over a large segment of $p$ enhances the likelihood that the RDU individual chooses $e = 0$ while her EU counterpart, if confronted with the choice set $e = \{0, \bar{e}\}$, would choose $e = \bar{e}$. According to Lemma 1, $w_j'(p) < 1$ occurs for larger segments of $p$ for stronger optimism or likelihood insensitivity.

Next, we show that an interior solution can be disregarded. The inequality in (**??**) can hold under optimism or likelihood insensitivity if $w''(p) < 0$ holds for a small segment of $p$ and/or that concavity is moderate. In that case, the solution $e_r^* \in ]0, \bar{e})$ is given by (A.17). Following the rationale of the proof given for Proposition 1, it can be stated that $e_r^* < e_u^*$ holds for sufficiently strong likelihood insensitivity or optimism in the sense of Definition 2 and Definition 4, respectively. However, this implies that the concavity of $w(p)$ is enhanced, making the inequality in (A.15) more stringent. Thus, it must be that the optimal investment under strong levels of optimism and insensitivity must belong in the set $e_r^* \in \{0, \underline{e}\}$.

Finally, we characterize underinvestment. The properties $c_{ee}(e) > 0$ and $F_{ee})(x|e) <$

0 from Assumption 1 and 2, respectively, imply that the solution is interior if $w(p) = p$. That solution is given by (A.18) and is denoted by $e_u^* > 0$. As established above, as optimism or insensitivity become stronger, the RDU individual chooses more often $e_r^* = 0$. Hence, underinvestment is given by $e_r^* = 0 < e_u^*$.

■

**Proposition 3**

*Proof.* Consider pessimism and assume that it is sufficiently strong so as to ensure $e_u^* > e_r^*$ (Proposition 1). Derive (A.20) with respect to $x_0$ to obtain:

$$-D(1)\int_{\underline{x}}^{\bar{x}} \Big(u''\big(b(x_0,x)\big)b_x(x_0,x)b_{x_0}(x_0,x) + u'\big(b(x_0,x)\big)b_{x,x_0}(x_0,x)\Big) \cdot \Big(F_e(x|e_u^*) - w_j'\big(1-F(x|e_r^*)\big)F_e(x|e_r^*)\Big)\mathrm{d}x. \tag{A.25}$$

The expression in (A.25) shows that if

$$\int_{\underline{x}}^{\bar{x}} u''\big(b(x_0,x)\big)b_x(x_0,x)b_{x_0}(x_0,x) + u'\big(b(x_0,x)\big)b_{x,x_0}(x_0,x)\mathrm{d}x < 0, \tag{A.26}$$

the investment difference given by the left-hand side of equation (A.20) becomes smaller as $x_0$ becomes higher. Equation (A.26) is implied by $-\frac{u''\left(b(x_0,x)\right)}{u'\left(b(x_0,x)\right)} > \frac{b_{x,x_0}(x_0,x)}{b_x(x_0,x)b_{x_0}(x_0,x)}$ for all $x$.

Consider now optimism or likelihood insensitivity and assume they are sufficiently strong so as to ensure $e_u^* > e_r^*$ (Proposition 2). Derive (A.23) with respect to $x_0$ to obtain

$$D(1)\int_{\underline{x}}^{\bar{x}} \Big(u''\big(b(x_0,x)\big)b_x(x_0,x)b_{x_0}(x_0,x)+u'\big(b(x_0,x)\big)b_{x,x_0}(x_0,x)\Big)\int_{1-F(x|0)}^{1-F(x|\bar{e})}(w'(s)-1)\mathrm{d}s\mathrm{d}x. \tag{A.27}$$

Equation (A.27) shows that if the inequality in (A.26) holds, then the left-hand side of (A.23) becomes less negative as $x_0$ increases, which implies that $e_r^* = 0$ is more likely with lower $x_0$. Therefore, $e_u^* - e_r^* > 0$ is the largest when $x_0$ is the lowest if $-\frac{u''\left(b(x_0,x)\right)}{u'\left(b(x_0,x)\right)} > \frac{b_{x,x_0}(x_0,x)}{b_x(x_0,x)b_{x_0}(x_0,x)}$ for all $x$.

■

The following corollary is useful for proving Proposition 4.

**Corollary 1.** *A lower investment due to probability weighting among the poor (Propositions 1-3), is enhanced by stronger optimism, pessimism, or likelihood insensitivity in the sense of Definitions 2 and 4.*

*Proof.* Consider an individual $j$ with probability weighting function $w_j(p)$ exhibiting pessimism (Definition 1). Assume that for this individual $e_r^* < e_u^*$ (Proposition 1). Also, consider an individual $i$ who is more pessimistic than $j$ in the sense of Definition 2 and denote her optimal investment level by $e_r^{**}$. According to Lemma 1, the set $p \in (p_l, 1)$ such that $w_i'(p) < 1$ is larger than the set $p \in (p_k, 1)$ such that $w_j'(p) < 1$. Hence, using (A.20), we obtain:

$$-D(1)\int_{\underline{x}}^{\bar{x}} u'\big(b(x_0,x)\big)b_x(x_0,x)\Big(F_e(x|e_u^*) - w_i'\big(1-F(x|e_r^{**})\big)F_e(x|e_r^{**})\Big)\mathrm{d}x > \\ -D(1)\int_{\underline{x}}^{\bar{x}} u'\big(b(x_0,x)\big)b_x(x_0,x)\Big(F_e(x|e_u^*) - w_j'\big(1-F(x|e_r^*)\big)F_e(x|e_r^{**})\Big)\mathrm{d}x > 0. \tag{A.28}$$

Equation (A.28) implies that stronger pessimism enhances the likelihood that $e_r^{**} < e_r^*$ and thus that underinvestment is exacerbated.

Assume now that individual $j$ exhibits optimism or likelihood insensitivity and denote her optimal level of investment by $e_r^*$. Assume that the inequality in equation (A.23) holds, which implies that $e_u^* > e_r^* = 0$ (Proposition 2). Moreover, let $i$ be more optimistic or insensitive than $j$, and denote her optimal investment level by $e_r^{**}$. Due to Lemma 1, the set $p \in (p_l, 1)$ such that $w_i'(p) < 1$ is larger than the set $p \in (p_k, 1)$ such that $w_j'(p) < 1$. Using (A.23), it can be established that

$$-\int_{\underline{x}}^{\bar{x}} u'\big(b(x_0,x)\big)b_x(x_0,x)\int_{1-F(x|\underline{e})}^{1-F(x|\bar{e})}(w_i'(s)-1)\mathrm{d}s\mathrm{d}x \leq \\ -\int_{\underline{x}}^{\bar{x}} u'\big(b(x_0,x)\big)b_x(x_0,x)\int_{1-F(x|\underline{e})}^{1-F(x|\bar{e})}(w_j'(s)-1)\mathrm{d}s\mathrm{d}x < 0. \tag{A.29}$$

Therefore, $e_r^{**} - e_u^* \leq e_r^* - e_u^*$.

■

**Proposition 4**

*Proof.* **Part i)** Let $w_j$ and $w_{uj}$ be the *probability weighting function* and *source function*, respectively, of an individual $j$ who suffers from ambiguity aversion. Chew and Sagi (2008)'s exchangeability implies that for a given event $E_k \in \Sigma$, there exists a probability $p_k$ such that $1 - P(E_K) = p_k$.

This property enables us to define ambiguity aversion as $w_{uj}(1-P(E_K))$ exhibiting more pessimism than $w_j(p_k)$ for all $E_k \in \Sigma$. Accordingly, let $w_{uj} = w_i$ and let $i$ be more pessimistic than $j$. Denote by $e_r^*$ the optimal investment level of $j$ and by $e_r^{**}$ that of $i$.

Lemma 1 shows that ambiguity aversion implies that the set of subjective probabilities $p \in (0, p_l)$ such that $w_i'(p) < 1$ is larger than the set $p \in (0, p_k)$ such that $w_j'(p) < 1$. Hence, if $e_r^* < e_u^*$, then using Eq. (A.28) we obtain,

$$-D(1) \int_{\underline{x}}^{\bar{x}} u'\big(b(x_0, x)\big) b_x(x_0, x) \Big( w_j' \big(1 - F(x|e_r^*)\big) F_e(x|e_r^*) - w_i' \big(1 - P(x|e_r^{**})\big) P_e(x|e_r^{**}) \Big) \mathrm{d}x > 0. \tag{A.30}$$

Equation (A.30) shows that ambiguity aversion, in the sense of Definition 1, generates a larger difference between $e_r^{**}$, the optimal investment level in the ambiguous good, and $e_r^*$, the optimal investment level in the risky good.

Deriving (A.30) with respect to $x_0$ gives:

$$-\delta \int_{\underline{x}}^{\bar{x}} \Big( u''\big(b(x_0, x)\big) b_x(x_0, x) b_{x_0}(x_0, x) + u'\big(b(x_0, x)\big) b_{x, x_0}(x_0, x) \Big) \Big( w_j' \big(1 - F(x|e_r^*)\big) F_e(x|e_r^*) - w_i' \big(1 - P(x|e_r^{**})\big) P_e(x|e_r^{**}) \Big) \mathrm{d}x > 0 \tag{A.31}$$

Equation (A.31) shows that if the condition in (A.26) hold, then the investment difference captured by equation (A.30) becomes smaller the higher $x_0$ is. Notice that that condition is implied by $-\frac{u''\big(b(x_0,x)\big)}{u'\big(b(x_0,x)\big)} > \frac{b_{x,x_0}(x_0,x)}{b_x(x_0,x) b_{x_0}(x_0,x)}$ for all $x$ Thus, the difference between $e_r^*$ and $e_r^{**}$ is the largest when $x_0$ is the lowest.

**Part ii)** similar steps can be used to show that ambiguity seeking and ambiguity aversion lead to a higher likelihood of low investment.

■

## Appendix B. Reference Dependence

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To incorporate reference dependence, we characterize risk preferences with Cumulative Prospect Theory (Tversky and Kahneman, 1992). Accordingly, the individual contrast final wealth to her reference point, $r > 0$. Final wealth levels that fall below a reference point are classified as losses while final wealth levels above that point are evaluated as gains. The main departure of CPT with respect to EUT and RDU is that the individual can exhibit different risk preferences for gains and losses. This is captured with two ingredients. First, wealth levels enter the agent's utility differently depending on whether they are classified as gains or losses, property that is captured by the following assumption on the agent's utility.

**Assumption 6.** *The agent's value function is the piece-wise function*

$$V(w,r) = \begin{cases} v(b(x,x_0) - r) & \textit{if } (b(x,x_0) \geq r, \\ -\lambda v(r - b(x,x_0)) & \textit{if } (b(x,x_0) < r. \end{cases}$$

*where* $\lambda > 1$, $r > 0$, *and* $v : \mathbb{R}_+ \to \mathbb{R}_+$ *is a twice-continuously differentiable function that exhibits* $u(0) = 0$, $u' \geq 0$ *and* $u'' < 0$.

Utility is assumed to be convex for losses, which generates risk seeking attitudes, and concave for gains, which generates risk aversion. Furthermore, Assumption 6 introduces loss aversion which means that losses loom larger than commensurate gains. This property is captured by the parameter $\lambda > 1$.

The second ingredient is that the probability weighting function is defined separately over gains and losses. Probabilities associated with gains are transformed by the probability weighting function $w$, introduced in Assumption 5. On the other hand, probabilities associated with losses are transformed with a probability weighting function which we denote by $z$ that applies transformations to cumulative probabilities, $F(x|e)$ rather than to decumulative probabilities.

We simplify the problem by assuming that $z$ adopts the properties of $w$.

**Assumption 7.** *A probability weighting function for losses is a function* $z : [0,1] \to [0,1]$ *that satisfies the duality condition* $z(F(x|e)) = 1 - w(1 - F(x|e))$ *for any* $x$ *and* $e$.

Throughout, we assume that $r$ is exogenous to the alternatives faced by the decision-maker. Specifically, we assume that the reference point is the status quo or the individuals' initial wealth $b(x_0, x_0)$, which has been received strong empirical support (Baillon et al., 2020).

**Assumption 8.** *The reference point is the individual's initial wealth $r = x_0$.*

All in all, the utility of an agent with CPT preferences is

$$CPT\Big(u(x,e)\Big) = D(1) \int_{b(x_0,x)\geq x_0} v\Big(b(x_0,x) - x_0\Big) \mathrm{d}\Big(w\big(1 - F(x|e)\big)\Big) - D(1) \int_{x_0 < b(x_0,x)} v\Big(x_0 - b(x_0,x)\Big) \mathrm{d}\Big(z\big(F(x|e)\big)\Big) - c(e), \tag{A.32}$$

We present the solution to the investment problem when the individual exhibits reference-dependent preferences. It turns out that the behavioral poverty trap defined in Propositions 1-3 emerges under more stringent conditions as compared to the setting in which the agent exhibits RDU preferences. This is a consequence of loss aversion, which incentivizes individuals to exert high effort to avoid the potential losses from failing short of their initial wealth. However, this effect vanishes as initial wealth becomes lower.

**Proposition 5.** *Suppose assumptions 1-8 hold. The CPT decision-maker is more likely to choose lower investment as compared to the expected utility individual under more stringent conditions than those in Propositions 1-3.*

*Proof.* Using Assumption 7 and Assumption 8 , rewrite (A.32) as:

$$CPT\Big(u(x,e)\Big) = D(1) \int_{\bar{x}}^{x_0} u\Big(b(x_0,x) - x_0\Big) \mathrm{d}w\big(1 - F(x|e)\big) - D(1) \int_{x_0}^{\underline{x}} \lambda u\Big(x_0 - b(x_0,x)\Big) \mathrm{d}\Big(1 - w\big(1 - F(x|e)\big)\Big) - c(e), \tag{A.33}$$

Using integration by parts, rewrite (A.33) as

$$CPT\Big(u(x,e)\Big) = D(1) \int_{x_0}^{\bar{x}} u'\Big(b(x_0,x) - x_0\Big) b_x(x_0,x) w\big(1 - F(x|e)\big) \mathrm{d}x - D(1) \int_{\underline{x}}^{x_0} \lambda u'\Big(x_0 - b(x_0,x)\Big) b_x(x_0,x) \Big(1 - w\big(1 - F(x|e)\big)\Big) \mathrm{d}x - c(e), \tag{A.34}$$

Optimal effort, $e_c^*$ , satisfies the following first-order condition obtained from deriving (A.34) with respect to $e$:

$$
\begin{aligned}
&- D(1)\int_{x_0}^{\bar{x}} u'\Big(b(x_0,x)-x_0\Big)b_x(x_0,x)w'\big(1-F(x|e_c^*)\big)F_e(x|e_c^*)\mathrm{d}x \\
&- D(1)\int_{\underline{x}}^{x_0} \lambda u'\Big(x_0-b(x_0,x)\Big)b_x(x_0,x)w'\big(1-F(x|e_c^*)\big)F_e(x|e_c^*)\mathrm{d}x - c'(e_c^*) = 0,
\end{aligned}
\tag{A.35}
$$

Recall that the optimal effort level, $e_u^*$, chosen by the expected utility decision-maker satisfies the first-order condition given in (A.18). Suppose that $e_c^* > e_u^*$. From (A.34) and (A.19), the assumed inequality can be rewritten as

$$
\begin{aligned}
&- D(1)\int_{x_0}^{\bar{x}} b_x(x_0,x)\Big[u'\Big(b(x_0,x)-x_0\Big)w'\big(1-F(x|e_c^*)\big)F_e(x|e_c^*) - u'\big(b(x_0,x)\big)F_e(x|e_u^*)\Big]\mathrm{d}x \\
&- D(1)\int_{\underline{x}}^{x_0} b_x(x_0,x)\Big[\lambda u'\Big(x_0-b(x_0,x)\Big)w'\big(1-F(x|e_c^*)\big)F_e(x|e_c^*) - u'\big(b(x_0,x)\big)F_e(x|e_u^*)\Big]\mathrm{d}x \geq 0.
\end{aligned}
\tag{A.36}
$$

Consider the first integral in (A.36). That expression is positive as long as

$$
\int_{x_0}^{\bar{x}} u'\Big(b(x_0,x)-x_0\Big)w'\big(1-F(x|e_c^*)\big)F_e(x|e_c^*)\mathrm{d}x < \int_{x_0}^{\bar{x}} u'\big(b(x_0,x)\big)F_e(x|e_u^*)\mathrm{d}x. \tag{A.37}
$$

In turn, equation (A.37) holds if

$$
w'\big(1-F(x|e_c^*)\big) > \frac{u'\big(b(x_0,x)\big)}{u'\Big(b(x_0,x)-x_0\Big)}\cdot\frac{F_e(x|e_u^*)}{F_e(x|e_c^*)}, \tag{A.38}
$$

for all $x \in [x_0, \bar{x}]$. Assumption 2 and Assumption 4 imply $F_e(x|e_c^*) > F_e(x|e_u^*)$ and $u'\Big(b(x_0,x)-x_0\Big) > u'\big(b(x_0,x)\big)$, respectively. Thus, the condition in (A.40) contradicts the property of probability weighting function that $\lim_{x\to\bar{x}} w'(1-F(x|e)) = 0$ from Assumption 5.

Similarly, the second integral in (A.36) is positive when

$$
\int_{\underline{x}}^{x_0} \lambda u'\Big(x_0-b(x_0,x)\Big)w'\big(1-F(x|e_c^*)\big)F_e(x|e_c^*)\mathrm{d}x < \int_{\underline{x}}^{x_0} u'\big(b(x_0,x)\big)F_e(x|e_u^*)\mathrm{d}x, \tag{A.39}
$$

which holds if

$$\lambda w'\big(1 - F(x|e_c^*)\big) > \frac{u'\big(b(x_0, x)\big)}{u'\Big(x_0 - b(x_0, x)\Big)} \cdot \frac{F_e(x|e_u^*)}{F_e(x|e_c^*)} \tag{A.40}$$

for all $x \in [\underline{x}, x_0]$. The inequality in (A.40) holds if $w'\big(1 - F(x|e_c^*)\big) \geq \frac{1}{\lambda}$ for all $x \in [x_0, \bar{x}]$. Such condition is weaker than that included in (A.40) but nonetheless contradicts $\lim_{x \to \bar{x}} w'(1 - F(x|e)) = 0$ from Assumption 5.

Suppose now that $e_u^* > e_c^*$. The first integral in (A.36) is negative if

$$w'\big(1 - F(x|e_c^*)\big) < \frac{u'\big(b(x_0, x)\big)}{u'\Big(b(x_0, x) - x_0\Big)} \cdot \frac{F_e(x|e_u^*)}{F_e(x|e_c^*)} \tag{A.41}$$

for all $x \in [x_0, \bar{x}]$. Assumption 2 and Assumption 4, imply $\frac{F_e(x|e_u^*)}{F_e(x|e_c^*)} > 1$, and $\frac{u'\big(b(x_0,x)\big)}{u'\Big(b(x_0,x)-x_0\Big)} < 1$, respectively. Notice that as $x \to x_0$, the expression $\frac{u'\big(b(x_0,x)\big)}{u'\Big(b(x_0,x)-x_0\Big)}$ approaches one and the right-hand side of (A.41) becomes the largest. Instead, if $x \to \bar{x}$ the right-hand side of (A.41) becomes smaller. Thus, the condition in (A.41) precludes probability weighting functions defined by Assumption 5, such as any $w$ such that $\lim_{x_0 \to \bar{x}} w'(1 - F(x|e)) > \frac{F_e(x|e_u^*)}{F_e(x|e_c^*)}$. Hence, It must be that $e_c^* < e_u^*$ holds for some $x \in [x_0, \bar{x}]$

Moreover, the second integral in (A.36) is negative if

$$\lambda w'\big(1 - F(x|e_c^*)\big) < \frac{u'\big(b(x_0, x)\big)}{u'\Big(x_0 - b(x_0, x)\Big)} \frac{F_e(x|e_u^*)}{F_e(x|e_c^*)} \tag{A.42}$$

for all $x \in [\underline{x}, x_0]$. Eq. (A.42) shows that, relative to the condition in (A.41), more probability weighting functions defined by Assumption 5 are precluded, such as any $w$ such that $\lim_{x_0 \to \bar{x}} w'(1 - F(x|e)) > \frac{F_e(x|e_u^*)}{\lambda F_e(x|e_c^*)}$. Consequently, It must be that $e_c^* < e_u^*$ holds for some $x \in [\underline{x}, x_0)$

Next, we characterize the values $x$ for which $e_c^* < e_u^*$ holds. Equations (A.37) and (A.39) show that $e_c^* < e_u^*$ is less stringent when $w'\big(1 - F(x|e_c^*)\big) < \varepsilon$ for arbitrary $\varepsilon$. According to Lemma 1 this property is obtained as pessimism, optimism, or insensitivity become stronger. Moreover, the left-hand side of (A.39) is more stringent than that of (A.37) since $\lambda > 1$; loss aversion makes $e_c^* < e_u^*$ more stringent.

We conclude by showing that underinvestment is stronger among the poor. Differentiation of (A.35) with respect to $x_0$ gives:

$$\begin{aligned}
&-D(1)\int_{x_0}^{\bar{x}}\Big[u''\Big(b(x_0,x)-x_0\Big)\big(b_{x_0}(x_0,x)-1\big)b_x(x_0,x)\Big]w'\big(1-F(x|e_c^*)\big)F_e(x|e_c^*)\mathrm{d}x\\
&-D(1)\int_{x_0}^{\bar{x}}\Big[u'\Big(b(x_0,x)-x_0\Big)b_{x,x_0}(x_0,x)\Big]w'\big(1-F(x|e_c^*)\big)F_e(x|e_c^*)\mathrm{d}x\\
&-D(1)\lambda\int_{\underline{x}}^{x_0}\Big[u''\Big(x_0-b(x_0,x)\Big)\big(1-b_{x_0}(x_0,x)\big)b_x(x_0,x)\Big]w'\big(1-F(x|e_c^*)\big)F_e(x|e_c^*)\mathrm{d}x\\
&-D(1)\lambda\int_{\underline{x}}^{x_0}\Big[u'\Big(x_0-b(x_0,x)\Big)b_{x,x_0}(x_0,x)\Big]w'\big(1-F(x|e_c^*)\big)F_e(x|e_c^*)\mathrm{d}x.
\end{aligned} \tag{A.43}$$

Equation (A.43) is negative if

$$u''\Big(b(x_0,x)-x_0\Big)\big(b_{x_0}(x_0,x)-1\big)b_x(x_0,x)+u'\Big(b(x_0,x)-x_0\Big)b_{x,x_0}(x_0,x)\Big]<0, \tag{A.44}$$

for all $[x_0,\bar{x}]$, and

$$u''\Big(b(x_0,x)-x_0\Big)\big(b_{x_0}(x_0,x)-1\big)b_x(x_0,x)+u'\Big(b(x_0,x)-x_0\Big)b_{x,x_0}(x_0,x)\Big]<0, \tag{A.45}$$

for all $[\underline{x},x_0]$. These conditions are implied by

$$-\frac{u''\Big(b(x_0,x)-x_0\Big)}{u'\Big(b(x_0,x)-x_0\Big)}>\frac{b_{x,x_0}(x_0,x)}{\big(b_{x_0}(x_0,x)-1\big)b_x(x_0,x)}, \tag{A.46}$$

and

$$-\frac{u''\Big(x_0-b(x_0,x)\Big)}{u'\Big(x_0-b(x_0,x)\Big)}>\frac{b_{x,x_0}(x_0,x)}{\big(1-b_{x_0}(x_0,x)\big)b_x(x_0,x)}, \tag{A.47}$$

respectively. The inequalities in (A.46) and (A.47) are analogous to that of Proposition 3 and do not depend on $\lambda$. ■

Loss aversion can counteract the behavioral poverty trap characterized in Propositions 1-3. The possibility of falling below the reference point, and experiencing disutility from losses, motivates individuals to exert higher effort. This effect, however, is nonexistent for the poorest individuals. Since the reference point is assumed to be initial wealth, the poor feel less of a motivation from losing what they have. Instead, for the rich, this motivational effect is strong and might overcome the irrational decision of investing little due to probability weighting . Therefore, Proposition 5 implies that the inequality between rich and poor can be exacerbated due to loss aversion.

Consistent with the result in Proposition C.1. is the notion that stronger loss aversion makes the existence of a behavioral poverty trap from Proposition 1 and 2 more stringent. This rationale is formalized in the next result.

**Corollary 2.** *The stronger underinvestment among the poor is worsened as $\lambda$ becomes larger.*

*Proof.* Equation (A.43) becomes more negative if the inequality in (A.47) holds and $\lambda$ is increased. The difference between $e_u^*$ and $e_c^*$ is largest for lowest values $x_0$ and is enhances with higher values $\lambda$.

■

## Appendix C. Additional Empirical Analyses.

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Table 7: The Relationship between Prelec (1998)'s Probability Weighting Function and Income or Wealth

| | (1) Ln Financial Wealth | (2) Ln Return Stock | (3) Ln Family Income | (4) Ln Housing Wealth | (5) Ln Financial Wealth | (6) Ln Return Stock | (7) Ln Family Income | (8) Ln Housing Wealth |
|---|---|---|---|---|---|---|---|---|
| Inverse-S | -1.292*** | -1.506*** | -0.185*** | -0.140 | -1.131*** | -1.243*** | -0.155*** | -0.108 |
| | (0.400) | (0.435) | (0.067) | (0.212) | (0.347) | (0.407) | (0.060) | (0.185) |
| S-shape | -0.132 | -0.080 | -0.039** | -0.077 | -0.084 | -0.007 | -0.017 | -0.044 |
| | (0.092) | (0.095) | (0.018) | (0.047) | (0.063) | (0.086) | (0.014) | (0.040) |
| Optimism | -0.006 | -0.009 | -0.001 | -0.000 | -0.006 | -0.008 | -0.000 | -0.001 |
| | (0.011) | (0.010) | (0.002) | (0.006) | (0.012) | (0.011) | (0.002) | (0.006) |
| U. Curv. | 0.008 | 0.006 | 0.000 | 0.004 | 0.009 | 0.009 | 0.001 | 0.005 |
| | (0.014) | (0.014) | (0.002) | (0.007) | (0.014) | (0.014) | (0.002) | (0.008) |
| Constant | 6.056*** | 4.302*** | 10.872*** | 3.394*** | 2.204* | -3.409*** | 9.844*** | -3.168*** |
| | (0.214) | (0.230) | (0.036) | (0.113) | (1.313) | (1.261) | (0.248) | (0.743) |
| Controls | NO | NO | NO | NO | YES | YES | YES | YES |
| $R^2$ | 0.015 | 0.012 | 0.010 | 0.004 | 0.217 | 0.129 | 0.153 | 0.233 |
| N | 1902 | 2245 | 2629 | 1921 | 1901 | 2244 | 2628 | 1920 |

This table presents OLS estimates of the model $y_i = b_0 + b_1\text{Inverse-S}_i + b_2\text{S-shaped}_i + b_3$ Opt./Pess.$_i$ + $b_3$U.curv$_i$.+Controls$_i'\Gamma + \varepsilon_i$ . The dependent variable $y_i$ captures the respondent's self-reported measures of income and wealth. "Inverse-S" is an index of likelihood insensitivity, "S-shaped" is an index of oversensitivity to probabilities, "Opt./pess." is an index of optimism and pessimism, and "U.curv" captures the respondent's utility curvature. The estimates presented in Columns 1-4 do not include additional control variables. The estimates presented in Columns 5-8 include additional control variables. Robust standard errors are presented in parentheses. *** denotes significance at the 0.01 level, ** denotes significance at the 0.05 level, * denotes significance at the 0.1 level.

Table 8: The Relationship between Chateauneuf et al. (2007)'s Probability Weighting Function and Income or Wealth

| | (1) Ln Financial Wealth | (2) Ln Return Stock | (3) Ln Family Income | (4) Ln Housing Wealth | (5) Ln Financial Wealth | (6) Ln Return Stock | (7) Ln Family Income | (8) Ln Housing Wealth |
|---|---|---|---|---|---|---|---|---|
| Inverse-S | -1.985** | -4.020*** | -0.449*** | -1.406*** | -0.418*** | -3.356*** | -0.418*** | -0.851* |
| | (0.979) | (1.046) | (0.157) | (0.516) | (0.150) | (0.994) | (0.150) | (0.460) |
| S-shaped | 0.118 | 1.810* | 0.146 | 0.920* | 0.201 | 1.631* | 0.201 | 0.560 |
| | (0.903) | (0.971) | (0.147) | (0.476) | (0.140) | (0.925) | (0.140) | (0.429) |
| Opt./Pess. | -2.479 | -4.406** | -0.658** | -2.358*** | -0.555** | -2.937* | -0.555** | -1.158 |
| | (1.696) | (1.797) | (0.282) | (0.906) | (0.270) | (1.724) | (0.270) | (0.819) |
| U.curv. | 0.037 | -0.052 | -0.006 | 0.003 | -0.005 | -0.056 | -0.005 | -0.010 |
| Constant | 6.415*** | 4.137*** | 10.913*** | 3.412*** | 9.885*** | -3.601*** | 9.885*** | -3.186*** |
| | (0.291) | (0.307) | (0.046) | (0.155) | (0.256) | (1.295) | (0.256) | (0.756) |
| Controls | NO | NO | NO | NO | YES | YES | YES | YES |
| $R^2$ | 0.017 | 0.016 | 0.013 | 0.006 | 0.154 | 0.130 | 0.154 | 0.232 |
| N | 1902 | 2245 | 2629 | 1921 | 2628 | 2244 | 2628 | 1920 |

This table presents OLS estimates of the model $y_i = b_0 + b_1\text{Inverse-S}_i + b_2\text{S-shaped}_i + b_3$ Opt./Pess.$_i$ + $b_3$U.curv$_i$.+Controls$_i'\Gamma + \varepsilon_i$ . The dependent variable $y_i$ captures the respondent's self-reported measures of income and wealth. "Inverse-S" is an index of likelihood insensitivity, "S-shaped" is an index of oversensitivity to probabilities, "Opt./pess." is an index of optimism and pessimism, and "U.curv" captures the respondent's utility curvature. The estimates presented in Columns 1-4 do not include additional control variables. The estimates presented in Columns 5-8 include additional control variables. Robust standard errors are presented in parentheses. *** denotes significance at the 0.01 level, ** denotes significance at the 0.05 level, * denotes significance at the 0.1 level.

Table 9: The Relationship between Goldstein and Einhorn (1987)'s Probability Weighting Function and Income or Wealth

| | (1) Ln Financial Wealth | (2) Ln Return Stock | (3) Ln Family Income | (4) Ln Housing Wealth | (5) Ln Financial Wealth | (6) Ln Return Stock | (7) Ln Family Income | (8) Ln Housing Wealth |
|---|---|---|---|---|---|---|---|---|
| Inverse-S | -1.218** | -0.853 | -0.253*** | -0.110 | -0.993** | -0.591 | -0.151** | -0.009 |
| | (0.487) | (0.526) | (0.080) | (0.257) | (0.438) | (0.500) | (0.074) | (0.224) |
| S-shaped | -0.874*** | -0.696*** | -0.147*** | -0.126 | -0.825*** | -0.525** | -0.091*** | -0.105 |
| | (0.204) | (0.223) | (0.036) | (0.108) | (0.194) | (0.215) | (0.034) | (0.097) |
| Optimism | -0.000 | -0.000 | 0.000 | -0.000** | -0.000 | -0.000 | 0.000 | -0.000 |
| | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| U.curv. | 0.154 | -0.054 | -0.026 | -0.012 | 0.154 | 0.041 | -0.012 | 0.017 |
| | (0.172) | (0.192) | (0.032) | (0.096) | (0.162) | (0.183) | (0.029) | (0.084) |
| Constant | 7.424*** | 5.467*** | 11.095*** | 3.502*** | 3.314** | -2.439* | 9.999*** | -3.066*** |
| | (0.310) | (0.335) | (0.050) | (0.164) | (1.318) | (1.286) | (0.251) | (0.752) |
| Controls | NO | NO | NO | NO | YES | YES | YES | YES |
| $R^2$ | 0.014 | 0.005 | 0.007 | 0.002 | 0.217 | 0.123 | 0.151 | 0.232 |
| N | 1902 | 2245 | 2629 | 1921 | 2628 | 2244 | 2628 | 1920 |

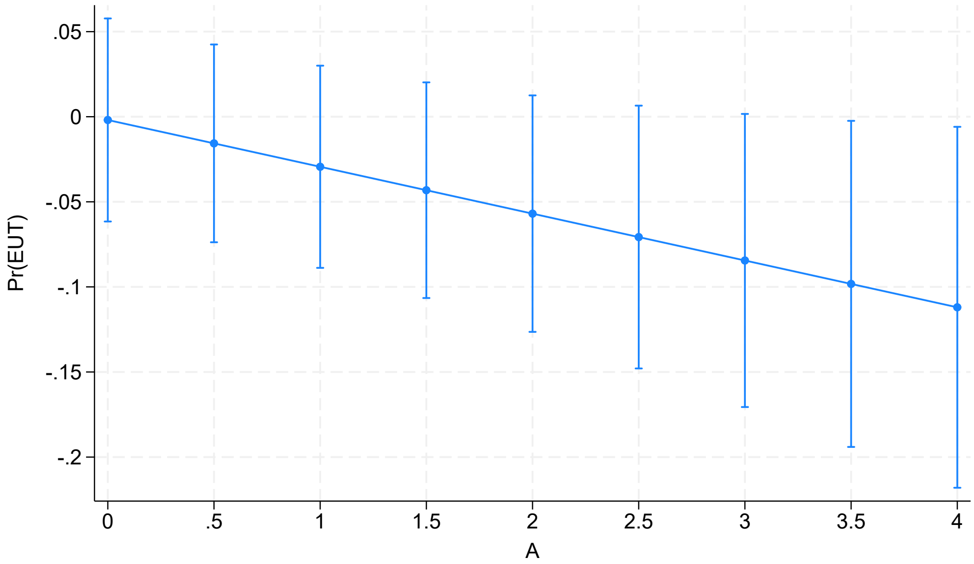
This table presents OLS estimates of the model $y_i = b_0 + b_1\text{Inverse-S}_i + b_2\text{S-shaped}_i + b_3$ Opt./Pess.$_i$ + $b_3\text{U.curv}_i.+\text{Controls}_i'\Gamma + \varepsilon_i$ . The dependent variable $y_i$ captures the respondent's self-reported measures of income and wealth. "Inverse-S" is an index of likelihood insensitivity, "S-shaped" is an index of oversensitivity to probabilities, "Opt./pess." is an index of optimism and pessimism, and "U.curv" captures the respondent's utility curvature. The estimates presented in Columns 1-4 do not include additional control variables. The estimates presented in Columns 5-8 include additional control variables. Robust standard errors are presented in parentheses. *** denotes significance at the 0.01 level, ** denotes significance at the 0.05 level, * denotes significance at the 0.1 level.

Table 10: The effects of Payday and Utility curvature on being unbiased

| | (1) EUT | (2) EUT | (3) EUT | (4) EUT | (5) EUT | (6) EUT |
|---|---|---|---|---|---|---|
| Before Payday | -0.014 | 0.020 | -0.015 | 0.018 | -0.039 | -0.005 |
| | (0.075) | (0.077) | (0.075) | (0.077) | (0.076) | (0.078) |
| $\hat{A}_i$ | 0.013 | 0.040*** | 0.016 | 0.043*** | 0.017 | 0.044*** |
| | (0.015) | (0.015) | (0.015) | (0.015) | (0.015) | (0.015) |
| Before Payday$\times\hat{A}_i$ | | -0.071** | | -0.069** | | -0.071** |
| | | (0.035) | | (0.035) | | (0.034) |
| Varian Index | | | 2.077 | 1.916 | 1.774 | 1.599 |
| | | | (1.540) | (1.546) | (1.626) | (1.630) |
| Time Stroop test | | | | | 0.006* | 0.006* |
| | | | | | (0.004) | (0.004) |
| Constant | -0.176*** | -0.188*** | -0.217*** | -0.226*** | -2.613** | -2.603** |
| | (0.054) | (0.055) | (0.062) | (0.063) | (1.304) | (1.299) |
| Controls | NO | NO | NO | NO | YES | YES |
| Log-likelihood | -772.400 | -770.465 | -771.496 | -769.700 | -754.071 | -752.202 |
| N | 1131 | 1131 | 1131 | 1131 | 1116 | 1116 |

This table presents probit estimates of the model $\text{EU}_i = b_0 + b_1\text{Before Payday}_i + b_2\hat{\rho}_i + b_3\text{Before Payday}\times\hat{\rho}_i + \text{Controls}_i'\Gamma + \varepsilon_i$. The dependent variable $\text{EU}_i$ is a binary variable that takes a value of one if respondent $i$ is classified as expected utility maximizer and zero otherwise. "Before Payday" is a binary variable that takes a value of one if responent $i$ is assigned to the group that completed the survey before payday. The variable $\hat{A}_i$ is captures subject's $i$ utility curvature. " Varian Index" captures participant's $i$ consistency with the maximization of a non-satiated utility function. Time Stroop Test captures the time in seconds that respondent $i$ spent on answering the questions of the Stroop test. *** denotes significance at the 0.01 level, ** denotes significance at the 0.05 level, * denotes significance at the 0.1 level.

Figure 5: Marginal Effects of treatment by different levels of $A$




Note: 95% confidence intervals

## Appendix D. Overlapping Generations Model

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Consider a small open economy in which there is a constant population of agents with unit mass. Each agent lives for two periods and belongs to a dynasty of overlapping generations connected through capital transfers. Each parent has one parent and one child, inheriting capital from the former and bequesting capital to the later. Each agent is a potential capital investor when young, and a producer and consumer when old.

Agents exhibit preferences defined over old-age consumption, $x_{t+1}$, from which they derive a lifetime utility $u_t = u(x_{t+1})$. Under EUT the agent's objective is to maximize $\mathbb{E}\big(u(x_{t+1})\big)$. We consider instead a setting in which the agent distorts probabilities through the probability weighting function from Assumption 5.

In the first period of life, the agent makes a decision about a level of investment $e$. As in our motivating example, we consider two levels of investment $e_H$ and $e_L$ such that $e_H > e_L$. We assume that there is a fixed cost of investment $c(e)$. For simplicity, we assume that only the high investment level generates a cost, so $c(e_H) = c$ and $c(e_L) = 0$ where $c > 0$.

Since all agents are endowed with zero resources, choosing to invest has to be financed with borrowing. Whatever decision is made about investment, we assume that an agent accumulates capital $k_{t+1}$ according to

$$k_{t+1} = \beta + p(e)Hk_t + \big(1 - p(e)\big)Lk_t, \tag{A.48}$$

where $p(e)$ is a probability that exhibits $p(e_H) > p(e_L)$, $\beta > 0$, and $1 > H > L > 0$. Equation (A.48) shows that the agent is more likely to accumulate higher capital when an investment is made. Note, however, that a high investment does not guarantee higher capital accumulation. Our model incorporates uncertainty about the agents' future income through uncertainty about productive efficiency.

In the second period, the agent produces output $y_{t+1}$ using capital according to:

$$y_{t+1} = Ak_{t+1}. \tag{A.49}$$

where $A > 0$. The agent realizes a final income of $x_{t+1}$ which determines final consumption and utility. This level of consumption depends on the agent's past actions. If she abstained from capital investment by choosing $e = e_L$, then she consumes all realized output. However, if investment was performed, $e = e_H$, she needs to pay back lenders their return on the loan $c$. Throughout, we assume that agents have access to competitive financial intermediaries which have access to a perfectly elastic supply of funds at the world interest rate of $r$. Since competition between intermediaries drives their profits to zero, the rate of interest is equal to the intermediaries own cost of borrowing. All in all, the consumption profile of agents is:

$$x_{t+1} = \begin{cases} A\big[\beta + k_t(H-L)p(e_L) + Lk_t\big] & \text{if } e = e_L \\ A\big[\beta + k_t(H-L)p(e_L) + Lk_t\big] - c(1+r) & \text{if } e = e_L \end{cases} \tag{A.50}$$

Finally, we discuss the agent's utility. When there is no investment, the agent's utility is given by

$$RDU(e_L) = A\beta + A(H-L)k_t w(p(e_L)) + Ak_t L. \tag{A.51}$$

Under investment, the agent's utility is given by

$$RDU(e_H) = A\beta + Ak_t(H-L)w(p(e_H)) + A[k_t L] - (1+r)c \tag{A.52}$$

Thus, the RDU agent will decide to invest as long as

$$RDU(e_H) \geq RDU(e_L) \Leftrightarrow A(H-L)k_t\big(w(p(e_H)) - w(p(e_L))\big) \geq (1+r)c. \tag{A.53}$$

The following Proposition characterizes a threshold capital level $\hat{k}$such that the agent invests whenever her inherited capital surpasses is larger. We provide such capital level for the RDU agent and also for her EU counterpart.

**Proposition 6.** *There exist unique capital levels $\hat{k}_r > 0$ and $\hat{k}_e > 0$ such that the RDU agent invests if $k_t > \hat{k}_r$ and the EUT agent invests if $k_t > \hat{k}_e$. These capital levels are such that $k_e < \hat{k}_r$ whenever $w(p(e_H)) - w(p(e_L)) < p(e_H) - p(e_L)$.*

*Proof.* Fix $c$. Note that the expression $A(H-L)k_t\big(w(p(e_H)) - w(p(e_L))\big)$ smoothly increases in $k_t$ over the domain $[0, +\infty)$. Moreover, the expression $(1+r)c$ is constant in capital. Therefore, there exists a unique capital level such that (A.53) holds with equality. Denote by $\hat{k}_r$ the capital level that satisfies the following equality:

$$A(H-L)\hat{k}_r\big(w(p(e_H)) - w(p(e_L))\big) = (1+r)c. \tag{A.54}$$

Given that the $A(H-L)k_t\big(w(p(e_H)) - w(p(e_L))\big)$ is increasing in $k_t$, any capital level such that $k_t > \hat{k}_r$ implies $RDU(e_H) \geq RDU(e_L)$; the individual engages in investment.

Under expected utility, the benefit from capital investment becomes $A(H-L)k_t\big((p(e_H))-(p(e_L))\big)$, which also smoothly increases in $k_t$ over $[0, +\infty)$. Therefore, there also exists a unique capital level $\hat{k}_e$ such that

$$A(H-L)\hat{k}_r\big(p(e_H - p(e_L)\big) = (1+r)c. \tag{A.55}$$

Suppose that $p(e_H) - p(e_L) > w(p(e_H)) - w(p(e_L))$. Then, using (A.55) it must be that

$$(1+r)c = A(H-L)\hat{k}_e\big(p(e_H) - p(e_L)\big) > A(H-L)\hat{k}_e\big(w(p(e_H)) - w(p(e_L))\big). \tag{A.56}$$

Hence, the capital level $\hat{k}_e$ that guarantees (A.54) must exhibit $\hat{k}_r > \hat{k}_e$.

■

The decision to invest in capital is affected by the agent's probability weighting. When probabilities are underweighted, the decision to invest is made when capital is sufficiently high. This behavior generates a behavioral poverty trap: at levels $k \in (\hat{k}_e, \hat{k}_r$ the decision maker erroneously believes that returns to investment are lower than they actually are and refrains from investing even though she would choose to invest if she did not suffer from probability weighting.

Next, we show that stronger deviations from expected utility due to optimism, pessimism, or insensitivity decrease the threshold level $\hat{k}_r > 0$. Therefore, the segment under which the agent does not invest due to irrationalities, $k \in (\hat{k}_e, \hat{k}_r$, becomes larger and the behavioral poverty trap happens for a wider range of capital levels.

**Corollary 3.** *Stronger pessimism, optimism, and likelihood insensitivity leads to a lower $\hat{k}_r$. It also enlarges the segment in which $w(p(e_H)) - w(p(e_L)) < p(e_H) - p(e_L)$ holds.*

*Proof.* Lemma 1 shows that stronger optimism, likelihood insensitivity, or pessimism lead to a larger segment in which $w'(p) < 1$. In this outcome environment, that condition implies that $\int_{p(e_L)}^{p(e_H)} w'(s)\mathrm{d}s < \int_{p(e_L)}^{p(e_H)} \mathrm{d}s \Leftrightarrow w(p(e_H)) - w(p(e_L)) < p(e_H) - p(e_L)$ for a wider range of values of $p(e_H)$ and $p(e_L)$.

Since $A(H-L)k_t\big(w(p(e_H)) - w(p(e_L))\big)$ is increasing in $k_t$, then it must be that stronger optimism, likelihood insensitivity, and pessimism, through their influence on reducing the difference $w(p(e_H)) - w(p(e_L))$, lead to a lower value $k_r$ such that (A.54) holds.

■

Given the above, the intergenerational evolution of capital for an individual dynasty satisfies.

$$k_{t+1} = \begin{cases} \beta + p(e_H)Hk_t + \big(1 - p(e_H)\big)Lk_t & \text{if } k \geq \hat{k}_r \\ \beta + p(e_L)Hk_t + \big(1 - p(e_L)\big)Lk_t & \text{if } k < \hat{k}_r. \end{cases} \tag{A.57}$$

Each of these lineage transition equations correspond to a stable stochastic difference equation. The intersections with the 45 degree line are given by the stationary points:

$$k^{**} = \frac{\beta}{(1 - p(e_H)(H - L) - L)}, \tag{A.58}$$

$$k^{*} = \frac{\beta}{(1 - p(e_L)(H - L) - L)}. \tag{A.59}$$

The transition equations are drawn under the restrictions $\beta < 1 - p(e_H)(H - L) - L$, which makes our analysis non-trivial.

The long-run distribution of capital in our economy is such that only investors with capital accumulation are those agents who are endowed with capital levels $k_0 > \hat{k}_r$. These agents converge to the high steady-state equilibrium. All other agents who start off with $k_0 < \hat{k}_r$ remain forever as non-investors. Note that agents with $\hat{k}_e > k_0 > \hat{k}_r$ will not engage in investment even though they would end-up in the high steady state if they had an accurate perception of probabilities.