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Human-driven vehicles' cruising versus autonomous vehicles' back-and-forth congestion: The effects on traveling, parking and congestion

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Human-driven vehicles' cruising versus autonomous vehicles' back-and-forth congestion: The effects on traveling, parking and congestion

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Abstract

This paper explores how the interaction between human-driven vehicles (HVs) cruising for parking and autonomous vehicles (AVs) traveling back and forth affects travel behavior and congestion. To capture the spatial distribution of parking, we develop a continuous spatial optimization model, with a discrete choice logit model governing the choice between the two modes. Various congestion externalities are considered in the proposed model. Using optimal control method, we derive the social optimum under user-equilibrium constraints and compare it to the unpriced user equilibrium. Without pricing, the introduction of AVs may increase or lower congestion depending on whether cruising or traveling back and forth dominates. Thus, AVs may be underused or overused, as the marginal external benefit of switching to AVs may be positive or negative. In our numerical model, with optimal pricing, the introduction of AVs always reduces travel costs. In terms of parking, the introduction of AVs is efficient in reducing parking demand and results in a smaller and less compact city. The efficiency of pricing is significantly impacted by the congestion interactions. When only one congestion type exists, it increases with the degree of congestion. However, when both congestion types exist, the efficiency of pricing is ambiguous. Specifically, when both HVs' cruising congestion and AVs' back-and-forth congestion are heavy, the efficiency of pricing is lower. Our proposed model reveals that the effects of AVs on travel and urban equilibrium may be more difficult to assess and less beneficial than often thought. Our numerical study provides policy insights into actions that regulators could consider for the operation of AVs.

Keywords: Autonomous vehicles; Cruising; Parking pricing; Congestion

JEL codes: R41, R42, R48

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1. Introduction

Parking is a major challenge for car usage in cities worldwide, particularly in European cities where on-street parking predominates. The potential of autonomous vehicles (AVs) to reduce congestion and parking in city centers has sparked considerable interest among urban planners. AVs could, for example, drop off and pick up travelers in high-cost parking areas and return home or park in less expensive locations. This reduction in parking demand creates opportunities for urban redevelopment, such as reallocating space for cycle paths, green space, or more real estate.

However, the anticipated decrease in parking demand may be offset by increased congestion caused by AVs traveling back and forth without travelers, which warrants serious concern. Previous studies have demonstrated the significant impact of AV parking on congestion, as each traveler generates two AV trips (one to the destination and one to a place to park), thereby increasing travel demand (e.g., Levin and Boyles, 2015; Shafiei et al., 2023). Nonetheless, the interaction between congestion resulting from human-driven vehicles (HVs) parking and AVs traveling back and forth remains largely unexplored.

In the absence of pricing, unassigned parking lots become a common resource that is vulnerable to overuse, so this becomes a modern tragedy of the commons. Implementing optimal parking pricing involves accounting for congestion externalities and equalizing parking demand by setting higher rates at meters closer to the central business district (CBD). Hence, it is essential to explore pricing strategies for both HVs and AVs to decentralize the social optimum and evaluate their impacts on travelers and congestion.

Against this background, we develop a continuous spatial optimization model to explore decisions regarding travel modes and parking location choices in an environment featuring both AVs and HVs, while considering the interaction between HVs cruising and AVs traveling back and forth. Specifically, we aim to address the following three questions: (i) What are the impacts of HVs' cruising for parking and AVs' back-and-forth movements on travelers and congestion? (ii) How can we achieve an equilibrium that maximizes the benefits of AVs while minimizing their negative effects on congestion? (iii) How do travelers respond to such socially optimal pricing?

In the literature, studies have examined the impact of parking on traffic congestion (e.g., Arnott and Inci, 2006; Shoup, 2006; Van Ommeren et al., 2021), modeled the impact of cruising for parking (e.g., Arnott and Inci, 2006; Shoup, 2006; Arnott et al., 2015; Geroliminis, 2015; Arnott and Williams, 2017; Qian and Rajagopal, 2014), and proposed a number of parking management strategies to improve parking and traffic efficiency, such as parking/congestion pricing (Arnott et al., 1991; Zhang et al., 2005;

Zhang et al., 2008; Zheng and Geroliminis, 2016; Liu and Geroliminis, 2017; Lu et al., 2021), parking permit or reservation systems (Liu et al., 2014; Liu et al., 2016; Chen et al., 2019; Wang et al., 2020), and competition among multiple parking facilities (Li et al., 2008; Inci and Lindsey, 2015). For a more comprehensive review, see Inci (2015). However, all these parking-related studies propose approaches for non-AV transportation systems, wherein AVs are not considered.

Recently, parking management with AVs has received increasing attention. For example, Liu (2018), Zhang et al. (2019), and Su and Wang (2020) built on the Vickrey bottleneck model to investigate the departure time and parking location choices of AVs. Tian et al. (2019) extended the bottleneck model by considering both regular and shared AVs with and without parking space constraints and derived optimal solutions for parking capacity and ride-sharing occupancy. Millard-Ball (2019) used a traffic microsimulation model to demonstrate how AVs can implicitly coordinate to reduce the cost of cruising for parking. In their work, Bahrami et al. (2021) used a Wardrop equilibrium to investigate AV parking choice, highlighting the importance of time-based congestion pricing. More recently, Zhang, Liu, and Zhang (2023) investigated the joint network equilibrium of parking and travel route choices with mixed private and shared AVs, in which the private AVs need to make both route and parking choices. Tscharaktschiew and Reimann (2023) developed an economic equilibrium speed and parking choice model to identify the classical (negative) cruising-for-parking externality of HVs, namely a (positive) speeding-when-cruising externality that may occur in the era of AVs. Although existing studies have shed some light on the parking issue with AVs, existing parking models have primarily focused on AVs' parking choices separately, and failed to characterize the congestion interactions.

This study adopts a continuous monocentric spatial optimization approach, whereby HVs have parking location choices at the end of a long, narrow city, and AVs are parked at home. Most of the literature using the linear corridor approach examined congestion tolling (Mun et al., 2003; Verhoef, 2005; Li et al., 2014), curbside parking and park-and-ride facilities (Anderson and de Palma, 2004; Wang et al., 2004; Liu et al., 2009; Lu et al., 2021), joint equilibrium of land use and travel (Li et al., 2012a), and a bi-modal transport system design problem (Li et al, 2012b). Regarding AVs' parking problem in a linear corridor, Liu (2018), Zhang et al. (2019), and Zhang, Liu, Levin, and Waller (2023) analyzed the joint equilibrium of departure time and parking location choices in a linear, continuous, and monocentric city with a bottleneck-constrained highway. However, their focus is on fully AV environments, with constant AV parking densities.

This study is related to Verhoef et al. (1995) and Anderson and de Palma (2004). Verhoef et al. (1995) developed a spatial parking model to investigate the optimal distribution of parking fees, taking residential and business locations and existing infrastructures as given. Anderson and de Palma (2004) considered a long, narrow city with a CBD at the end. They investigated the regulatory parking policies while considering the interaction between cruising congestion and travel congestion. Our study differs from those of Verhoef et al. (1995) and Anderson and de Palma (2004) in two important aspects. First, in addition to HVs' searching and cruising congestion, we also investigate AVs' back-and-forth congestion and the interaction between different congestion types. Second, we examine the effects of optimal pricing on travelers and congestion while considering travelers' stochastic preferences.

The main methodological contributions of the paper are *twofold*. First, we develop an (user) equilibrium model of mode and parking-location choices in a monocentric city system with HVs and AVs. The congestion interaction between HVs' cruising and searching for parking and AVs' back-and-forth movements are explicitly considered. Mode choices are characterized by stochastic user equilibrium. Second, using optimal control method, we derive the social optimum under user-equilibrium constraints and compare it to the unpriced user equilibrium. The optimal parking fee for HVs is space differentiated: it varies with the parking location and equals the marginal external cost (MEC) imposed at a location. The optimal pricing on AVs is constant, as their fee equals their constant MEC.

Our work provides three core policy and societal contributions.

(i) We compare the congestion effects of AVs and HVs to test if introducing AVs lowers congestion in our setting (with and without pricing). In our model, AVs do not automatically lower congestion, which is so in many more optimistic models. Although HVs cruising for parking and AVs traveling back and forth both raise travelers' travel price and total travel cost, their joint effects on travelers depend on which congestion type dominates. The marginal external benefit (MEB) of switching from an HV to an AV may be negative or positive, depending on the specific congestion parameters. If it is positive, a trip in an AV causes less congestion than a trip in an HV, as the extra congestion from traveling back and forth is small compared to the congestion from cruising.

(ii) We examine how AVs and/or (spatially varying) parking pricing affects the parking equilibrium and thus the urban form. Given the number of HV and AV users, HVs' cruising and AVs' back-and-forth congestion both increase HVs' parking span due to increased parking costs. Nonetheless, the introduction of AVs is efficient at reducing parking demand and leads to a smaller and less compact city. In the absence of pricing, HVs' cruising congestion leads to a more compact and larger city, whereas the reverse is true with pricing. In contrast, AVs' back-and-forth congestion results in a less compact and smaller city without pricing, and the opposite with pricing. The joint effects depend on which congestion effect dominates.

(iii) We evaluate the efficiency of (spatially varying) parking pricing in reducing congestion. When only one type of congestion exists, the efficiency of pricing increases with the degree of congestion. When HVs' cruising and AVs' back-and-forth congestion both exist, however, the efficiency of pricing is significantly impacted by their interactions. In particular, when both types of congestion are heavy, the efficiency of pricing is low.

The remainder of this paper is organized as follows. Section 2 presents the proposed (equilibrium) model. Section 3 solves the social optimum model analytically. Section 4 discusses the social optimum. Section 5 uses numerical examples to illustrate the properties of the proposed model. Finally, Section 6 provides conclusions and recommendations for further studies.

2. Model formulation

2.1. Problem description

We investigate travelers' traveling and parking issues when there is a mix of HVs and AVs in a monocentric urban system. We consider that there is a common destination located at x=0, and there are N travelers located far away. The CBD is at the end of a long, narrow city and is served by parallel access roads. Perpendicular to these access roads are side streets that are used for on-street parking. Such a city system has been adopted in some relevant studies, such as Anderson and de Palma (2004), reflecting the main characteristics of traveling, parking, and congestion. Travelers travel from home to the CBD using either HVs or AVs. Other transport modes such as public transport and taxis are ignored in this paper. An illustration of the monocentric bi-modal urban system is shown in Fig. 1.

Suppose that parking is not assigned to individuals in advance. HVs can park on the street at any vacant location but, before parking, they need to cruise and search for a vacant parking space. Suppose that if drivers stop somewhere, they will search at this location until they find a vacant parking spot. Referring to Fig. 1, with an HV, travelers first drive toward downtown, then start to cruise to find a parking location (side street), then look for an empty parking space on this side street. Once an available parking spot is found, they walk to the CBD.

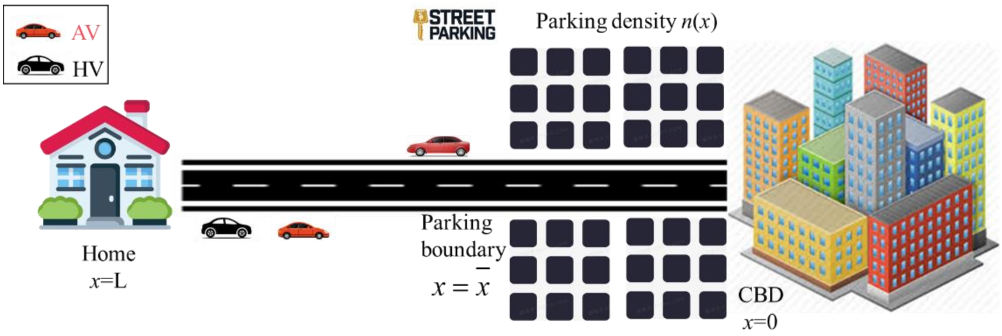

Fig. 1. Monocentric bi-modal urban system established.

For AV users, it is assumed that, due to the limited space in the city center, there are no parking spaces for them. In this sense, AVs must park outside the city. To simplify, we consider AVs must park at home. Other parking places for AVs are not considered in this paper but will be explored in a subsequent study. As AVs can drive themselves, users with an AV will first reach the destination, and then the AVs turn back home to park themselves.

The exclusion of normal traffic congestion from the present study stems from a methodological imperative that focuses on the interaction between HVs' parking and AVs' back-and-forth congestion. A flow congestion function could be adopted to characterize the normal traffic congestion. However, this would only complicate the technical analysis without providing significantly different insights for our model.

2.2. Congestion externalities

In the bi-modal urban transport system investigated, there are three types of congestion externalities: HVs searching for parking, HVs cruising for parking, and AVs traveling back and forth. We do not consider normal traffic congestion, which has already been studied extensively in the literature.

The searching externality refers to the effect of one parker increasing the search time for subsequent parkers. Essentially, finding a vacant spot takes longer when more parkers are searching. The cruising externality arises from HVs cruising for parking, slowing down other vehicles. This directly impacts traffic flow on the main arteries and indirectly affects side streets if cruisers search there. Increased cruising traffic leads to amplified delays at traffic lights and disrupts traffic flow, reducing the speed of through traffic—a secondary effect of the cruising issue. AVs' back-and-forth congestion involves the externality of deadheading—empty return trips to remote parking after completing trips in the city. This adds to traffic and may offset the anticipated congestion reduction from AV technology. Specifically, congestion from AVs' return trips could exacerbate the cruising externality, further slowing traffic on both main arteries and side streets, intensifying delays and flow interference.

2.3. HVs' generalized travel cost

As discussed in section 2.1, an HV trip from home to the CBD includes four phases: regular driving, cruising for a parking street in the parking area, searching for a parking space in a side street, and walking to the CBD.

Let x denote the distance of a location from the CBD, and $\bar{x}$ denote the farthest distance parked, or the parking span, which is determined endogenously. Let $n(x)$ denote the number of cars parking at x; we also call it the parking density at x. At the parking boundary $\bar{x}$, $n(\bar{x}) = 0$ holds, meaning that the car parked the farthest away incurs the minimum searching cost.

Let $C_n(x)$ denote the generalized travel cost by HVs parking at x. This cost thus includes the regular travel cost from home to the parking span, the cruising cost from the parking span to the parking street x, the searching cost at x, the walking cost from x to the CBD, and the monetary automobile cost:

$$C_n(x) = \alpha_n \cdot (T_1(\bar{x}) + T_2(x)) + C_s(x) + C_w(x) + \psi_n(x), \tag{1}$$

where α_n is the value of time for HV users. $T_1(\bar{x})$ is the driving time from home to the parking boundary $\bar{x}$, which can be expressed as $T_1(\bar{x}) = (L - \bar{x}) / V_{\max}$, where $V_{\max}$ represents the free-flow speed assumed to be identical for HVs and AVs (see, e.g., Van den Berg and Verhoef, 2016; Yu et al., 2022a, b). $T_2(x)$ represents the driving time from $\bar{x}$ to the parking location x. $C_s(x)$ denotes the searching cost to find a vacant space at location x, $C_w(x)$ represents the walking time cost, and $\psi_n(x)$ is the monetary automobile cost.

We now formulate the driving time from the parking boundary $\bar{x}$ to parking location x, $T_2(x)$. A cruising HV at location x introduces additional delay for all vehicles passing x, whereas an AV passing through x during its return trip adds delay for all vehicles crossing x. We assume the travel delay induced by the cruising HVs is proportional to the number of drivers cruising for parking at x, and for AVs it is proportional to the number of AVs. Accordingly, we model the travel speed at x, $V(x)$, through the following speed-flow function:

$$\frac{1}{V(x)}=\frac{1}{V_{\max}}+\frac{w_n n(x)+w_a N_a}{s}, \tag{2}$$

where w_n measures the cruising congestion caused by HVs and w_a measures the extra congestion caused by AVs traveling back and forth, with $0 \le w_a \le 1$ and $0 \le w_n \le 1$. A larger w_i means a greater degree of congestion externality. Specifically, when $w_n = w_a = 0$, the travel time reduces to the free-flow travel time $(\bar{x}-x)/V_{\max}$. Therefore, the travel time from $\bar{x}$ to x is:

$$T_2(x)=\int_x^{\bar{x}}\frac{1}{V(x)}dx=\int_x^{\bar{x}}\left(\frac{1}{V_{\max}}+\left(\frac{w_a N_a + w_n \cdot n(x)}{s}\right)\right)dx, \tag{3}$$

This analysis emphasizes the impact of HVs' parking and AVs' back-and-forth on traffic flow and travel delay.

Next, we address HVs' searching process for finding a parking space at location *x*. Existing literature models the searching time as a convex function of the parking occupancy rate (e.g., Anderson and de Palma, 2004; Qian and Rajagopal, 2015; Arnott and Williams, 2017; Leclercq et al., 2017).[1] We adopt the searching cost function proposed by Anderson and de Palma (2004), assuming that each parking spot at location *x* is equally likely to be vacant. The vacancy probability is determined by the ratio of HVs choosing *x* to the total parking spots at *x*, denoted as $1-n(x)/K(x)$, where $n(x)$ is the number of HV drivers choosing to park at *x*, and $K(x)$ is the number of parking spots at *x*. For simplicity, we set $K(x)=K$. Therefore, the expected spaces searched for before finding a vacant spot is $1/(1-n(x)/K)$. Let γ denote the search cost incurred by a driver checking whether or not a spot is occupied. The expected cost to HV drivers who choose to search for parking at location *x* is:

$$C_s(x)=\frac{\gamma \cdot K}{K-n(x)}. \tag{4}$$

The expected searching cost increases with the number of cars parked, $n(x)$, and approaches infinity as the number of parkers approaches the parking capacity, *K*.

Let V_w denote the walking speed, which is assumed to be constant. The walking cost for HV users parking at *x* is thus:

[1] Specifically, search time remains steady at low or medium occupancy levels but sharply increases at high occupancy, especially for the last few available spots. If travelers are aware of high occupancy (e.g., 99%) and lack a designated spot, they are unlikely to choose that lot due to the expected high cruising time.

$$C_w(x) = \frac{\alpha_w \cdot x}{V_w}, \tag{5}$$

where α_w is the value of time for walking.

The monetary cost of traveling by HVs, $\psi_n(x)$, is assumed to be a linear function of the distance traveled, as in Wang et al. (2004), Liu et al. (2009), and Li et al. (2012a), expressed as:

$$\psi_n(x) = mc_n \cdot (L - x) + mc_{0n}, \tag{6}$$

where mc_n is the variable cost (e.g., fuel cost per unit of distance) and mc_{0n} is the fixed cost of traveling by HVs (e.g., insurance or road taxes).

Consequently, the travel cost of choosing a parking space at location *x* is:

$$C_n(x) = \left(\frac{\alpha_n}{V_{\max}} + mc_n\right) \cdot (L - x) + \alpha_n \cdot \int_x^{\bar{x}} \left(\frac{w_a N_a + w_n n(x)}{s}\right) dx + \frac{\gamma K}{K - n(x)} + \frac{\alpha_w \cdot x}{V_w} + mc_{0n}. \tag{7}$$

It should be noted that the externality caused by HVs cruising is a non-localized externality, since drivers at *x* impact all drivers parking closer to the CBD.

2.4. AVs' generalized travel cost

As AVs do not require parking spaces, AV travelers will first travel to the CBD, then the AVs return home to park themselves. The generalized travel cost by AVs comprises the travel time cost to the CBD and the return journey cost. Although AVs do not need to cruise and search for a parking spot, they are still subject to congestion caused by cruising HVs. The travel time per unit of distance by AVs can thus be separated into two parts: before entering the parking area and traveling within the parking area. Let C_a be the generalized travel cost of AVs, expressed as:

$$C_a = \alpha_a \cdot \frac{L - \bar{x}}{V_{\max}} + \alpha_a \cdot \int_0^{\bar{x}} \frac{1}{V(x)} dx + \psi_a, \tag{8}$$

where the first term gives the cost of traveling from home to the parking area, and α_a is the value of time for AV users. Because travel time in an AV can be used for other purposes, such as relaxing or handling email, the value of travel time is lower than the cost of travel time in an HV, implying that $\alpha_a \le \alpha_n$. ψ_a gives the monetary automobile cost of the full journey. Note that when AVs drive back from the CBD, the vehicles are empty, thus the congestion delay cost is zero, and only the monetary cost matters.

The monetary cost of traveling by AVs, ψ_a, is assumed to be a linear function of the distance traveled, like the HV mode:

$$\psi_a = mc_a \cdot 2L + mc_{0a}, \tag{9}$$

where mc_a represents the variable cost of traveling by AVs (e.g., fuel cost per unit of distance) and mc_{0a} represents the fixed cost (e.g., the cost of buying the autonomous car). $\bar{x}$ is the travel distance from the parking boundary to the CBD, and the factor "2" represents the round trip.

Consequently, the generalized travel cost for an AV can be rewritten as:

$$C_a = \frac{\alpha_a \cdot L}{V_{\max}} + \alpha_a \cdot \int_0^{\bar{x}} \left(\frac{w_n n(x) + w_a N_a}{s} \right) dx + mc_a \cdot (2L) + mc_{0a}, \tag{10}$$

where "2" denotes the full trip of AVs traveling to CBD, amd parking themselves at home.

2.5. Joint travel mode and parking location choice

Travelers' decisions regarding parking location and travel mode choices follow a hierarchical choice structure. This can be studied without loss of generality as a two-step maximization.

First, for any N_a and N_n, HV drivers minimize their travel costs by choosing the parking location. The associated individual cost-minimization problem is:

$$\min_x \quad C_n(x) = \left(\frac{\alpha_n}{V_{\max}} + mc_n \right) \cdot (L - x) + \alpha_n \cdot \int_x^{\bar{x}} \left(\frac{w_a N_a + w_n n(x)}{s} \right) dx + \frac{\gamma K}{K - n(x)} + \frac{\alpha_w \cdot x}{V_w} + mc_{0n}.$$
$$\text{s.t.} \quad x \geq 0, \ x \leq \bar{x} \tag{11}$$

Solving Eq. (11) yields the user-equilibrium parking density $n(x)$ and parking boundary $\bar{x}$, for a given N_a and N_n. The generalized cost is constant in the equilibrium regardless of the parking location x. We can thus omit x in the generalized cost function, and use $C_n(N_a, N_n)$ and $C_a(N_a, N_n)$ to denote the equilibrium generalized cost of HV users and of AV users, respectively.

Second, travelers determine the travel mode based on random utility maximization. This random utility comprises a deterministic component, $C_i(N_a, N_n)$, and a stochastic idiosyncratic mode preference. When these idiosyncratic preferences follow an independently and identically distributed (i.d.d.) extreme distribution, the number of AV and HV users can be determined using the following logit formula:

$$N_a = \frac{e^{-\theta C_a}}{e^{-\theta C_a} + e^{-\theta C_n}} \cdot (N_a + N_n);\ N_n = \frac{e^{-\theta C_n}}{e^{-\theta C_a} + e^{-\theta C_n}} \cdot (N_a + N_n), \tag{12}$$

where the scale parameter θ governs the relative importance of the unobserved idiosyncratic component in the travel cost. Specifically, when θ tends toward infinity, it reduces to the deterministic mode choice model.

2.6. Total travel cost and welfare

The total travel cost, *TC*, is the sum of the total travel cost of HV and AV users:

$$TC = \int_0^{\bar{x}} C_n(x) \cdot n(x) dx + C_a \cdot N_a. \tag{13}$$

Given that the unobserved component of mode utility reflects individual preferences, there are benefits of variety. Introducing an additional alternative typically results in higher expected utility (Koster et al., 2018). In the absence of income effects, the welfare, *W*, is calculated as:

$$W = -TC - \frac{1}{\theta} \cdot \left(N_a \ln\left[\frac{N_a}{N_a + N_n}\right] + N_n \ln\left[\frac{N_n}{N_a + N_n}\right]\right), \tag{14}$$

where the first part represents the deterministic total travel cost, and the second term is consistently non-negative and encapsulates the total benefits of variety.

3. Analytical analysis on the unpriced user equilibrium

3.1. Step one: HVs' parking location choice equilibrium

Given the number of HV and AV users, we can characterize HV users' optimization problem as the minimization of their generalized travel cost, as presented in Eq. (11). To provide a clearer picture of the individual optimization process, we solve the Lagrangian implied by Eq. (11), $L = C_n(x) - \mu \cdot (x - \bar{x})$, with μ representing the Lagrangian multiplier associated with the constraint $x \le \bar{x}$.

The Kuhn-Tucker conditions for this Lagrangian are:

$$\begin{aligned} &\frac{\partial L}{\partial x} = -\frac{\alpha_n}{V_{\max}} - mc_n - \alpha_n \cdot \left(\frac{w_a N_a + w_n n(x)}{s}\right) - \frac{\gamma K}{(K - n(x))^2} \cdot \frac{\partial n(x)}{\partial x} + \frac{\alpha_w}{V_w} - \mu = 0 \\ &\frac{\partial L}{\partial \lambda} = \bar{x} - x \ge 0;\quad \mu \ge 0 \text{ and } \mu \cdot (\bar{x} - x) = 0 \end{aligned}. \tag{15}$$

Hence, if $x<\bar{x}$, the equilibrium parking pattern is determined by the following differential equation:

$$\frac{\gamma K}{(K-n(x))^2}\cdot\frac{\partial n(x)}{\partial x}=\frac{\alpha_n}{V_{\max}}+mc_n+\alpha_n\cdot\frac{w_a N_a+w_n n(x)}{s}-\frac{\alpha_w}{V_w}. \tag{16}$$

Proposition 1. Given N_a and N_n, in the absence of pricing, the equilibrium parking span is smaller when: (i) HVs' parking search cost, γ, is lower, i.e., $\partial\bar{x}/\partial\gamma>0$; (ii) HVs' cruising congestion cost, w_n, is lower, i.e., $\partial\bar{x}/\partial w_n>0$; and (iii) AVs' extra congestion cost of traveling back and forth, w_a, is lower, i.e., $\partial\bar{x}/\partial w_a>0$.

Proof. See Appendix A.

Proposition 1 indicates that the parking span $\bar{x}$ increases with all types of congestion. The intuition behind Proposition 1 is as follows. A lower parking searching cost means that drivers are less sensitive to parking congestion. The new equilibrium therefore involves more congested parking at each location close to the CBD and the total area devoted to parking falls. A decrease in w_n and w_a lessens the congestion annoyance from HVs cruising for parking and AVs traveling back and forth, causing HV drivers to drive further inward. Again, this creates more intense usage of parking spaces further in.

By combining Eq. (16) with the boundary condition $n(\bar{x})=0$, we can derive the expression for the parking density at *x*, $n(x)$. Substituting this into the condition that all HVs park between $[0,\bar{x}]$, i.e., $\int_0^{\bar{x}} n(x)dx=N_n$, enables us to obtain the expression for $\bar{x}$, which is a function of N_n.

3.2. Step two: travelers' mode choice equilibrium

Considering that the furthest parking location involves $n(\bar{x})=0$, the relationship between HVs users' equilibrium cost C_n and $\bar{x}$ is:

$$C_n=C_n(\bar{x})=\left(\frac{\alpha_n}{V_{\max}}+mc_n\right)\cdot(L-\bar{x})+\gamma+\frac{\alpha_w\cdot\bar{x}}{V_w}+mc_{0n}. \tag{17}$$

Let $\Lambda=\left(\frac{\alpha_a}{V_{\max}}-\frac{\alpha_n}{V_{\max}}-mc_n\right)\cdot L+mc_a\cdot(2L)+mc_{0a}-\gamma-mc_{0n}$. Calculating the difference between the equilibrium travel cost of AVs and HVs yields:

$$\Delta c = C_a - C_n = \alpha_a \cdot \frac{w_n N_n + w_a \bar{x} N_a}{s} - \left(\frac{\alpha_w}{V_w} - \frac{\alpha_n}{V_{\max}} - mc_n \right) \cdot \bar{x} + \Lambda \,. \tag{18}$$

Eq. (18) shows that the difference in the generalized travel cost can be positive or negative, depending on the parameters. Specifically, taking the derivative of Eq. (18) with respect to w_n and w_a and combining Proposition 1 yields $\partial \Delta c / \partial w_n > 0$ and $\partial \Delta c / \partial w_a > 0$, meaning that both the HVs' cruising congestion parameter and the AVs' back-and-forth congestion parameter tend to amplify the travel cost difference between HV and AV users.

As we use stochastic preferences, in the equilibrium, the number of HVs and AVs is set by Eq. (12).

3.3. Congestion effects

We investigate the congestion effects by examining the MEB of switching from an HV to an AV. By combining Eqs (10), (17), and (13), the total travel cost can be expressed as:

$$\begin{aligned} TC = &\left(\frac{\alpha_n}{V_{\max}} + mc_n \right) \cdot (L - \bar{x}) N_n + \gamma N_n + \frac{\alpha_w \cdot \bar{x}}{V_w} N_n + mc_{0n} N_n \\ &+ \frac{\alpha_a \cdot L}{V_{\max}} N_a + \alpha_a \cdot \int_0^{\bar{x}} \left(\frac{w_n n(x) + w_a N_a}{s} \right) dx \cdot N_a + mc_a \cdot (2L) \cdot N_a + mc_{0a} \cdot N_a \end{aligned} , \tag{19}$$

which can be further rewritten as:

$$\begin{aligned} TC = &\left(\frac{\alpha_n}{V_{\max}} + mc_n \right) \cdot (L - \bar{x}) N_n + \gamma N_n + \frac{\alpha_w \cdot \bar{x}}{V_w} N_n + mc_{0n} N_n \\ &+ \frac{\alpha_a \cdot L}{V_{\max}} N_a + \alpha_a \cdot \frac{w_n N_n + w_a \bar{x} N_a}{s} \cdot N_a + mc_a \cdot (2L) \cdot N_a + mc_{0a} \cdot N_a \end{aligned} . \tag{20}$$

It should be noted that the total cruising congestion cost imposed on HV users is $\frac{\alpha_n w_1}{s} \cdot \int_0^{\bar{x}} \left(\int_x^{\bar{x}} (n[x]) dx \right) n(x) dx$, which can be expressed as $\frac{\alpha_n w_n}{s} \cdot \frac{N_n^2}{2}$, and the total cruising congestion cost imposed on AV users is $\alpha_a \cdot w_n N_n \cdot N_a / s$. Accordingly, the cruising congestion cost imposed by HVs is independent of the specific parking distributions, and increases with the cruising congestion parameter w_n.[2]

Let F denote the proportion of AV users, i.e., $F = N_a / (N_a + N_n)$. The MEB of

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[2] This stems from considering that the congestion resulting from cruising increases linearly with the number of cruisers.

switching from an HV to an AV can be calculated as: $MEB = C_a - C_n - \dfrac{\partial TC/\partial F}{N}$. Combining Eqs (20) and (18), the MEB is:

$$MEB = \left(\frac{\alpha_n}{V_{\max}} + mc_n - \frac{\alpha_w}{V_w}\right)\cdot\frac{\partial\bar{x}}{\partial F}(1-F) - \alpha_a\cdot\frac{(w_a\bar{x} - w_n)N + w_a\dfrac{\partial\bar{x}}{\partial F}FN}{s}\cdot F\,. \tag{21}$$

As $\partial\bar{x}/\partial F < 0$ and $\alpha_n/V_{\max} + mc_n < \alpha_w/V_w$, the first term in Eq. (21) is always positive. The sign of the second term depends on the values of w_a , w_n , and the corresponding parking distribution, making the MEB positive or negative. It is positive when the reduction in HVs' cruising and searching congestion outweighs the increase in AVs' back-and-forth congestion, and vice versa. Eq. (21) demonstrates how these different forces jointly determine the MEB. A positive MEB means the overuse of AVs without tolling, whereas a negative MEB means their underuse. Specifically, when $w_a = w_n = 0$, the MEB is always positive, since AVs do not need to search for a parking spot.

4. Social optimum

Given various types of congestion externalities, the total welfare is not, in general, minimized in the unregulated equilibrium. This section discusses the social optimum, whereby the total welfare should be maximized.

4.1. Formulation of the social optimum problem

The welfare-maximizing problem can be mathematically formulated as:

$$\begin{aligned}
&\max_{n(x),\bar{x},\,N_a,N_n} \quad W\\
&s.t. \quad \int_0^{\bar{x}} n(x)dx = N_n\,. \\
&\qquad\quad n(x) \ge 0
\end{aligned} \tag{22}$$

The above welfare maximization problem can be studied without loss of generality as a two-step maximization.

First, for any N_a and N_n, the regulator minimizes the total travel cost, $TC(N_a, N_n)$:

$$\min_{n(x),\ \bar{x}} \quad TC(N_a, N_n) = \int_0^{\bar{x}} C_n(x) \cdot n(x)dx + C_a \cdot N_a$$

$$s.t. \quad \begin{cases} \int_0^{\bar{x}} n(x)dx = N_n \\ n(x) \geq 0 \end{cases}. \qquad . \tag{23}$$

Solving Eq. (23) yields the optimal parking distribution under total cost minimization for a given number of HV and AV users.

Next, the regulator maximizes welfare by deciding N_a and N_n:

$$\max_{N_a, N_n} \quad W = -\frac{1}{\theta} \cdot \left(N_a \ln\left[\frac{N_a}{N_a + N_n} \right] + N_n \ln\left[\frac{N_n}{N_a + N_n} \right] \right) - TC^*(N_a, N_n), \tag{24}$$

where $TC^*(N_a, N_n)$ is the optimal value of the objective function as found in Eq. (23). In the following analytical analysis, we first focus on the first step, how to regulate the congestion and parking distribution for a given number of AV and HV users, and second on how the interaction of different types of congestion affects travelers. In the numerical examples, we use simulation to further examine the effects of w_a and w_n.

4.2. Step one: optimal distribution of parking fee

This section uses the Hamiltonian-based dynamic optimization to derive the optimal toll, given the number of HVs and AVs. Substituting Eqs (7) and (10) into Eq. (23), the total cost minimization problem in the first step can be reformulated as[3]:

$$\begin{aligned} \min_{n(x),\bar{x}} \quad TC\left(N_a, N_n\right) &= \int_0^{\bar{x}} \left[\left(\frac{\alpha_n}{V_{\max}} + mc_n \right) \cdot (L - x) + \frac{\gamma K}{K - n(x)} + \frac{\alpha_w \cdot x}{V_w} + mc_{0n} \right] \cdot n(x)dx \\ &- \frac{\alpha_n w_a N_a}{s} \cdot \int_0^{\bar{x}} x \cdot n(x)dx + \alpha_a N_a \cdot \int_0^{\bar{x}} \left(\frac{w_n n(x) + w_a N_a}{s} \right) dx - \frac{\alpha_n w_n}{s} \int_0^{\bar{x}} A_n(x) \cdot n(x)dx, \\ &+ \frac{\alpha_a \cdot L}{V_{\max}} N_a + \frac{\alpha_n w_a N_a N_n \cdot \bar{x}}{s} + \frac{\alpha_n w_n}{s} \cdot N_n^2 + mc_a \cdot (2L) \cdot N_a + mc_{0a} \cdot N_a \end{aligned} \tag{25}$$

subject to

$$\begin{cases} \dfrac{dA_n(x)}{dx} = n(x) \\ A_n(\bar{x}) = N_n,\ A_n(0) = 0,\ \bar{x} \text{ is choosen free} \end{cases}, \tag{26}$$

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[3] It should be noted that $\int_0^{\bar{x}} \alpha_n \cdot \int_x^{\bar{x}} \left(\frac{w_n n(x)}{s} \right) dx \cdot n(x)dx = \frac{w_n}{s} \alpha_n \cdot N_n^2 - \frac{\alpha_n w_n}{s} \int_0^{\bar{x}} A_n(x) \cdot n(x)dx$.

where $A_n(x)$ is the number of HVs parking between 0 and *x*, i.e., $A_n(x)=\int_0^x n(x)dx$.

We use optimal control method to solve the above total cost minimization problem. The associated Hamiltonian for (25) and (26) is:

$$\begin{aligned} H=&-\left[\left(\frac{\alpha_n}{V_{\max}}+mc_n\right)\cdot(L-x)+\frac{\gamma K}{K-n(x)}+\frac{\alpha_w\cdot x}{V_w}+mc_{0n}\right]\cdot n(x) \\ &+\frac{\alpha_n w_a N_a}{s}\cdot x\cdot n(x)-\frac{\alpha_a N_a\cdot w_n n(x)}{s}+\frac{\alpha_n w_n A_n(x)}{s}\cdot n(x)-\frac{\alpha_a N_a w_a N_a}{s}+\lambda\cdot n(x)\end{aligned}, \quad (27)$$

with $A_n(x)$ being the state variable, $n(x)$ being the control variable, and λ the adjoint (or costate) variable.

The necessary first-order conditions of Eq. (27) are:

$$\begin{aligned}\frac{\partial H}{\partial n(x)}=&-\left(\frac{\alpha_n}{V_{\max}}+mc_n\right)\cdot(L-x)+\frac{\alpha_n w_a N_a x}{s}-\frac{\alpha_a w_n N_a}{s}+\frac{\alpha_n w_n A_n(x)}{s} \\ &-\frac{\gamma K^2}{\left(K-n(x)\right)^2}-\frac{\alpha_w x}{V_w}-mc_{0n}+\lambda=0\end{aligned}, \quad (28)$$

$$\frac{\partial\lambda}{\partial x}=-\frac{\partial H}{\partial A_n(x)}=-\frac{\alpha_n w_n}{s}\cdot n(x). \quad (29)$$

Because λ gives the shadow price for the state variable $A_n(x)$, it signifies the marginal value of having an HV traveler park at an additional unit of distance. Consequently, this value is closely linked to the spatial distribution of the optimal parking fees, influencing individual behavior and the spatial parking pattern.

Taking the spatial derivative of Eq. (28) and combining with Eq. (29), we obtain:

$$\frac{\alpha_n}{V_{\max}}+mc_n+\frac{\alpha_n w_a N_a}{s}+\frac{\alpha_n w_n n(x)}{s}-\frac{\partial\left[\frac{\gamma K^2}{\left(K-n(x)\right)^2}\right]}{\partial x}-\frac{\alpha_w}{V_w}-\frac{\alpha_n w_n n(x)}{s}=0 \quad (30)$$

as a necessary condition for the optimum. Comparing Eq. (30) with individual optimizing behavior in Eq. (16) reveals that optimality requires the following parking pricing strategies:

$$\frac{d\tau_n(x)}{dx}=\frac{\partial\left[\frac{\gamma K^2}{\left(K-n(x)\right)^2}\right]}{\partial x}-\frac{\partial\left[\frac{\gamma K}{K-n(x)}\right]}{\partial x}+\frac{\alpha_n w_n\cdot n(x)}{s}. \quad (31)$$

Eq. (31) implies that the slope of the parking fee gradient should exactly reflect the increase in the externality of HV drivers' searching and cruising. Here, the first two terms represent the increase in the searching externality caused by parking at *x*, and the last term represents the increase in the cruising externality caused by cruising at *x*. Eq. (31) indicates that when regulating HV parking, only spatial parking fee differentials matter, and not the absolute values of parking fees. According to Eq. (31), the optimal parking fee for HVs can be expressed as the sum of a space-varying parking fee and a constant $\tau_n^{\#}$:

$$\tau_n(x) = \frac{\gamma K^2}{\left(K - n(x)\right)^2} - \frac{\gamma K}{K - n(x)} + \frac{\alpha_n w_n}{s} \cdot \int_0^x n(x)dx + \tau_n^{\#}. \tag{32}$$

Comparing Eqs (28) and (11), we note that a choice of $\tau_n^{\#} = \alpha_a w_n N_a / s$ implies that travelers at *x* are also charged for their externality on AV drivers, caused by cruising at *x*. Therefore, the spatial distribution of parking fees for HVs parking at location *x* is given by:

$$\begin{aligned} \tau_n(x) &= \frac{\gamma K^2}{\left(K - n(x)\right)^2} - \frac{\gamma K}{K - n(x)} + \frac{\alpha_n w_n}{s} \cdot \int_0^x n(x)dx + \frac{\alpha_a w_n N_a}{s} \\ &= \frac{\gamma K \cdot n(x)}{\left(K - n(x)\right)^2} + \frac{\alpha_n w_n}{s} \cdot \int_0^x n(x)dx + \frac{\alpha_a w_n N_a}{s} \end{aligned}. \tag{33}$$

In Eq. (33), the first term means the searching externality caused by HVs parking at *x*, the second term means the cruising externality on HV users parking between [0, *x*], and the third term means the congestion externality imposed on AV users. Note that the last term does not affect HVs' parking distribution; it only mitigates congestion costs by reducing HV demand. Since parking closer to the center is more desirable, the optimal parking fee increases with closeness to the CBD to counteract this effect and reduce the over-congestion, which is most pronounced closest to the CBD.

The congestion toll on AVs equals their MEC; that is, the difference between the marginal social cost and the private cost:

$$\tau_a = \frac{\partial TC}{\partial N_a} - C_a = \frac{\alpha_a w_a \bar{x}}{s} \cdot N_a + \frac{\alpha_n w_a}{s} \cdot \int_0^{\bar{x}} (\bar{x} - x) n(x)dx. \tag{34}$$

Unlike the spatially varying pricing for HVs, the pricing for AVs remains constant across space. In Eq. (34), the first term means the congestion externality on AV users, and the second term is the congestion externality on HV users.

4.3. Step one: discussion of the optimal parking distribution

After discussing the distribution of the parking fees and congestion pricing, we now investigate HVs' parking patterns. According to Eq. (7) and (33), the travel price of HV users parking at *x* is:

$$P_n(x) = C_n(x) + \tau_n(x)$$
$$= \left(\frac{\alpha_n}{V_{\max}} + mc_n\right)\cdot(L-x) + \alpha_n \cdot \int_x^{\bar{x}} \left(\frac{w_a N_a + w_n n(x)}{s}\right) dx + \frac{\gamma K}{K-n(x)} + \frac{\alpha_w \cdot x}{V_w} + mc_{0n} \quad (35)$$
$$+ \frac{\gamma K \cdot n(x)}{\left(K-n(x)\right)^2} + \frac{\alpha_n w_n}{s}\cdot\int_0^x n(x)dx + \frac{\alpha_a w_n N_a}{s}$$

In the equilibrium, the travel price remains constant over *x*. Solving $dP_n(x)/dx = 0$ yields:

$$\frac{2K^2\gamma}{\left(K-n(x)\right)^3}\cdot\frac{\partial n(x)}{\partial x} = \frac{\alpha_n}{V_{\max}} + \frac{w_a \cdot \alpha_n N_a}{s} - \frac{\alpha_w}{V_w} + mc_n. \quad (36)$$

Similar to the unpriced equilibrium, we only consider the case of $\partial n(x)/\partial x < 0$; therefore, $\frac{\alpha_w}{V_w} > \frac{\alpha_n}{V_{\max}} + \frac{\alpha_n \cdot w_a N_a}{s} + mc_n$. Eq. (36) implies that the value of w_n has no impact on HV drivers' parking distribution. The reason is that the cruising congestion cost is linear in our modeling approach.

Proposition 2. Given N_a and N_n, under the social optimum, HVs' parking span: (i) is independent of HVs' cruising congestion parameter, w_n; (ii) increases with AVs' back-and-forth congestion parameter, w_a; and (iii) increases with HVs' parking searching congestion parameter, γ.

Proof. See Appendix B.

4.4. Step two: optimal number of HVs and AVs

We now proceed to the second step of determining the optimal proportion of AVs, *F*, by maximizing the following welfare:

$$\max_F \quad \mathrm{W} = -\frac{1}{\theta}\cdot\left((1-F)N\ln\left[\frac{(1-F)N}{N}\right] + FN\ln\left[\frac{FN}{N}\right]\right) - TC^*(F). \quad (37)$$

Taking the derivative of *W* with respect to F yields:

$$\frac{\partial TC^*(F)}{\partial F}+\frac{N}{\theta}\cdot\ln\frac{F}{1-F}=0. \quad (38)$$

which can be simplified as $\frac{\partial TC^*(F)}{\partial F}=\frac{N}{\theta}\cdot\ln\frac{1-F}{F}$.

In comparison to the deterministic user equilibrium, in the stochastic user equilibrium, the regulator equals the marginal benefit to the marginal cost, where $\frac{1}{\theta}\cdot\ln\frac{1-F}{F}$ is the benefit of variety per traveler, which is then multiplied by the total number of travelers to obtain the total benefits of variety. Specifically, as θ tends to infinity, the stochastic equilibrium approaches the deterministic equilibrium.

5. Numerical studies

This section presents numerical studies that examine the model proposed above. The analytics cannot decisively compare HVs' cruising and AVs' back-and-forth congestion, as the net effects depend on the parameters. This comparison of course lends itself to numerical analysis.

5.1. Numerical model

The parking model presented is a continuous space optimization, involving the determination of the number of AV and HV users, the parking distribution, and tolls. To address this, we first discretize the feasible parking region and transform the associated continuous functions into discrete counterparts. We then employ an iterative backward solution method. The underlying principle is to employ a global search method to identify the number of HV and AV users and use the method of successive averages to determine the parking density and parking span.

Base case parameters are listed in Table 1; they are chosen to reflect realistic travel costs. We consider a city 20 km in length, with 40,000 parking spots per kilometer (Anderson and de Palma, 2004). Following Van den Berg and Verhoef (2016), the value of time in a normal car is €10/h, and the value of time in an autonomous car is €8/h. The free-flow travel speeds of AVs and HVs are 60 km/h. The value of walking time is 1.8 times the value of travel time, i.e., €18/h. The cost of searching for one spot is €0.5. The parameter in the congestion function, δ , is 0.15, consistent with the BPR-type congestion function. The road capacity is 4,000 vehicles/h. The scale of utility, θ , is assumed to be 1. The fixed and variable components of the monetary cost of travel by HV, mc_{0n} and mc_n are set as €1.51 and €0.05/km, respectively. For AV users, the fixed and variable components of the monetary travel cost, mc_{0a} and mc_a, are set as €2.51 and €0.09/km, respectively. Under this calibration, when all cars are HVs without cruising congestion, in the absence of pricing, the equilibrium travel cost is €12.78, and the total travel cost is €115,020. In the social optimum, the equilibrium travel price is €15.96, and the total travel cost is €98,352.

Table 1. Input parameters for the numerical illustration

Parameter	Definition	Baseline value
α_n	The value of time in an HV	€10/h
α_a	The value of time in an AV	€8/h
α_w	The value of walking time	€18/h
γ	The cost of searching for one spot	€0.5
δ	Parameter in the congestion function	0.15
V_n	Free-flow travel speed of HVs	60 km/h
V_a	Free-flow travel speed of AVs	60 km/h
V_w	Walking speed	5 km/h
L	Length of the city	20 km
K	Parking spots per kilometer	40,000
s	Capacity of the road	4,000
θ	Scale of utility	1
N	Total number of users	9,000
mc_n	Variable cost of traveling by HVs	€0.05/km
mc_{0n}	Fixed cost of traveling by HVs per trip	€1.51
mc_a	Variable cost of traveling by AVs	€0.09/km
mc_{0a}	Fixed cost of AVs per trip	€2.51

5.2. Numerical results

Table 2 presents the outcomes under unpriced equilibrium. We compare the results under various scenarios, including only searching congestion (w_a=w_n=0), HVs' searching and cruising congestion (w_a=0, w_n=0.4), HVs' searching and AVs' back-and-forth congestion (w_a=0.4, w_n=0), and all types of congestion (w_a=w_n=0.4).

Table 2. Outcomes under unpriced equilibrium

Outcome	No AVs	No HV cruising (w_a=w_n=0)	Only HV cruising (w_a=0, w_n=0.4)	Only AV back and forth (w_a=0.4, w_n=0)	Both HV cruising and AV back and forth (w_a=w_n=0.4)
Number of AVs	0	5370	5141	6123	5842
Number of HVs	9,000	3630	3859	2877	3158
Proportion of AVs	0	0.60	0.57	0.68	0.65
Price with AVs	-	8.78	9.24	10.21	10.52
Price with HVs	12.78	9.20	9.54	10.99	11.15
HVs' parking span	2.05	0.63	0.63	0.53	0.63
TC of AVs	-	47127	47504	62507	61468
TC of HVs	115,020	33404	36806	31632	35213
Total travel cost	115,020	80531	84310	94140	96681
Welfare*	-115,020	-74461	-78163	-88500	-90848

Note: Welfare is negative as we only consider downsides of travel and thus costs. We could add a constant of integration measuring travel benefits—e.g., getting to your job—or include price-sensitive demand. This would make welfare positive.

Table 2 indicates that introducing HVs' cruising congestion increases the number of HVs while decreasing the number of AVs. This occurs because HVs' cruising congestion affects AV drivers more than it does on HVs.[4] In contrast, introducing AVs' back-and-forth congestion raises the number of AVs and lowers the number of HVs. Although AVs affect all HV parkers and all other AVs, HVs are impacted more due to their higher value of time. When all congestion types are taken into account, the number of AVs is reduced and the number of HVs is raised, meaning that HVs' cruising congestion effects dominate AVs' back-and-forth congestion.

Table 3 presents the outcomes for the social optimum, showing a different pattern to the unpriced case. Introducing cruising congestion reduces the number of HVs, as charging for a cruising externality prompts users to switch from HVs to AVs. Similarly, introducing AVs' back-and-forth congestion decreases the number of AVs. In the full case of w_a=w_n=0.4, the number of AVs decreases and the number of HVs increases, indicating that AVs' back-and-forth congestion effects dominate HVs' cruising congestion effects.

Comparing results under unpriced parking and the social optimum reveals that when

[4] AVs are subject to cruising congestion between 0 and $\bar{x}$, whereas HVs parking between [0, x] are only affected by the cruising congestion at x.

ignoring HVs' cruising congestion and AVs' back-and-forth congestion, no tolling leads to an underuse of AVs. Including HVs' cruising congestion still results in underusing AVs under unpriced parking, whereas considering AVs' back-and-forth congestion leads to overuse of AVs. In the full model with w_a=w_n=0.4, unpriced parking results in an overuse of AVs, meaning that the effects of AVs' back-and-forth congestion dominate.

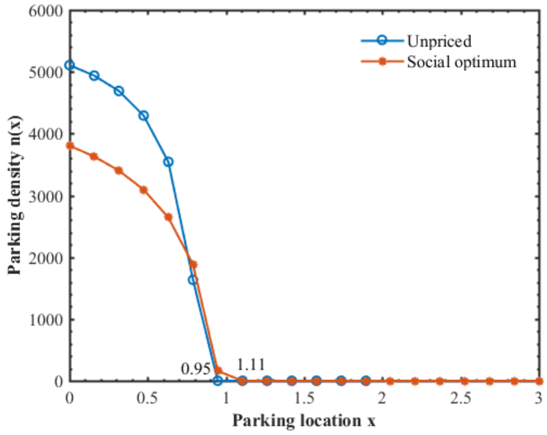
In terms of total travel cost, both HVs' cruising congestion and AVs' back-and-forth congestion increase total travel cost. Specifically, under the social optimum, HVs' cruising congestion raises the total travel cost of all AV users while reducing the total travel cost of all HV users due to the lower proportion of HVs.

Table 3. Outcomes under the social optimum

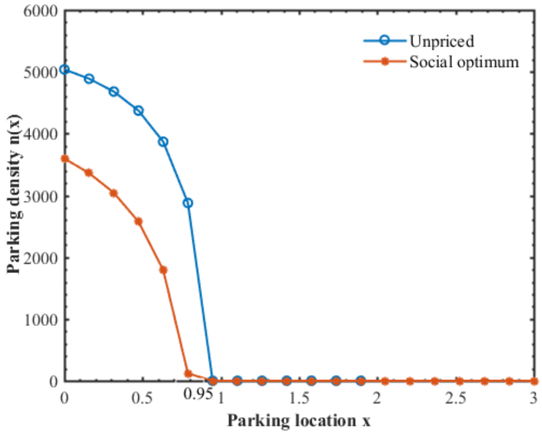
Outcome	No AVs	No HV cruising (w_a=w_n=0)	Only HV cruising (w_a=0, w_n=0.4)	Only AV back and forth (w_a=0.4, w_n=0)	Both HV cruising and AV back and forth (w_a=w_n=0.4)
Number of AVs	0	6209	6828	4385	4046
Number of HVs	9,000	2791	2172	4615	4954
Proportion of AVs	0	0.69	0.76	0.49	0.45
Price with AVs	-	9.28	10.54	12.37	15.18
Price with HVs	15.96	10.08	11.69	12.32	14.98
HVs' parking span	2.53	1.11	0.95	1.58	1.74
TC of AVs	-	54492	61703	42642	42928
TC of HVs	98,352	22674	17366	45397	53372
Total travel cost	98,352	77166	79069	88039	96300
Welfare	-98,352	-71593	-74096	-81804	-9010

5.3. Numerical results: Distribution of HVs parking

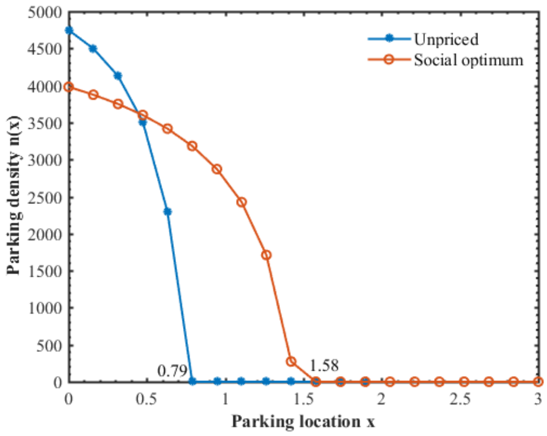
Fig. 2 depicts HVs' parking distribution under different congestion degrees. HVs' parking distribution is closely related to the number of HVs. Without pricing, excessive parking near the CBD occurs because HV drivers fail to consider that selecting a spot close to the CBD increases the search cost for numerous other drivers attempting to park there, and increases the crowding cost for all users passing by.



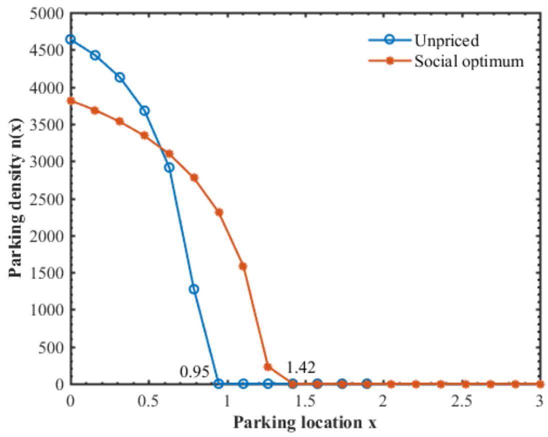
(a) $w_n = w_a = 0$



(b) $w_a = 0,\ w_n = 0.4$



(c) $w_a = 0.4,\ w_n = 0$

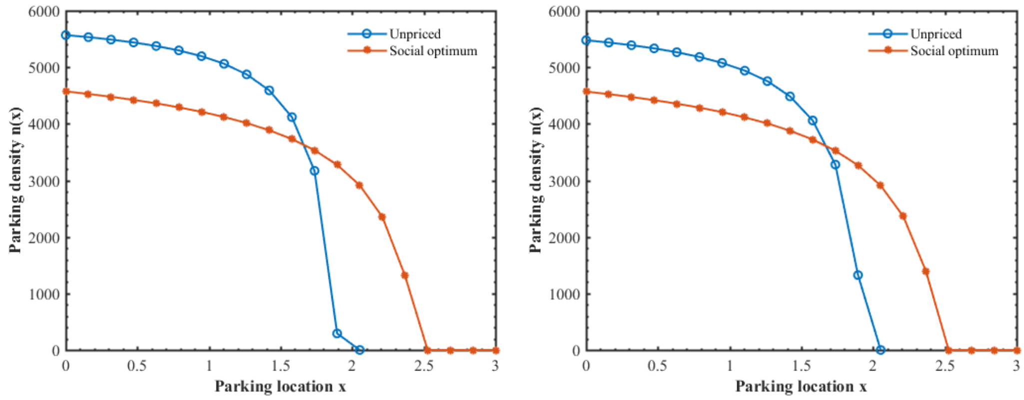


(d) $w_a = 0.4,\ w_n = 0.4$

Fig. 2. Effects of congestion degree on HVs' parking distribution.

Comparing Fig. 2a and Fig. 2b, we observe that in the absence of pricing, the introduction of HVs' cruising congestion leads to a more compact city, whereas the social optimum results in a less compact and smaller city. This is mainly because without pricing, the congestion associated with HVs searching and cruising raises AV users' travel price more, inducing some users to switch from AVs to HVs. Comparing Fig. 2a and Fig. 2c, we observe that AVs' back-and-forth congestion results in a larger and more compact city under the social optimum, and a smaller and less compact city in the absence of pricing. This is because the social optimum uses space-differential pricing to optimize HVs' parking distribution and increases the number of HV users. In the full model incorporating all our congestion types, the outcome is a combination of the separate effects. Fig. 2d shows that the parking resembles that in Fig. 2c, indicating that the effects of AVs' back-and-forth congestion dominates HVs' cruising congestion.

Fig. 3 depicts HVs' parking distribution before introducing AVs. Comparing Fig. 2 and Fig. 3, we can find the introduction of AVs is efficient at reducing parking demand in the city center, and results in a smaller and less compact city.



(a) $w_n = 0$ (b) $w_n = 0.4$

Fig. 3. HVs' parking distribution before introducing AVs.

5.4. Numerical results: Distribution of optimal parking pricing

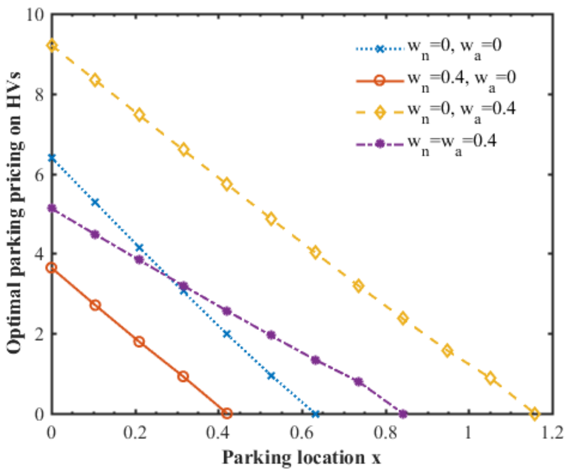


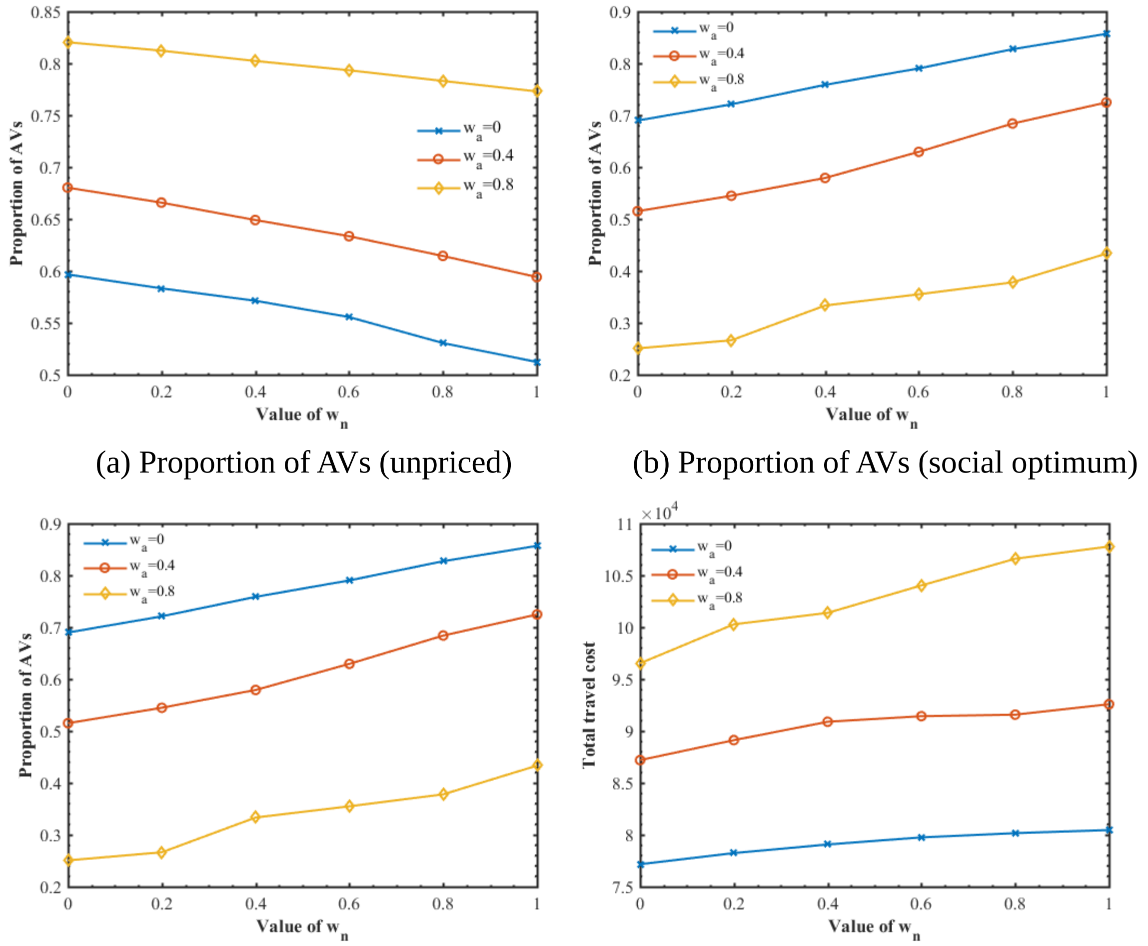
Fig. 4. Distribution of space-varying parking fees for HVs.

Fig. 4 presents the optimal parking fee distribution. As vehicles parked nearer to the CBD contribute more to congestion through cruising and searching, parking fees increase as the parking location x approaches the CBD. The parking fees are lowest under w_a=w_n=0 and highest under w_a=w_n=0.4. This is intuitively because more congestion is induced by introducing HVs cruising and AVs traveling back and forth. When comparing scenarios under w_n=0.4, w_a=0 and w_a=0.4, w_n=0, we can observe an intersection occurs at x=0.3. This indicates that the congestion closer to the CBD is induced more by HVs cruising for parking, and the congestion further from the CBD is induced more by AVs traveling back and forth.

5.5. Effects of w_n and w_a on travelers' mode choices and total travel cost

It is interesting to see what happens when the degree of different types of congestion varies, by varying the values of w_n and w_a.

5.5.1. Varying w_n



(a) Proportion of AVs (unpriced) (b) Proportion of AVs (social optimum)

(c) Total travel cost (unpriced) (d) Total travel cost (social optimum)

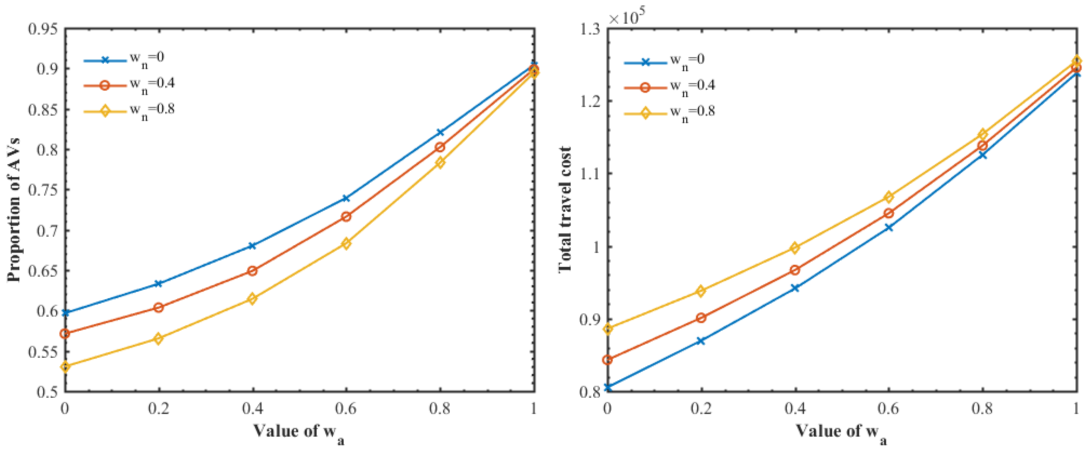
Fig. 5. Effects of w_n on travelers and total travel cost.

We vary HVs' cruising congestion parameter w_n from 0 to 1. In the absence of pricing, Fig. 5a shows that as HVs' cruising congestion parameter w_n increases, the number of AVs decreases, implying a greater impact on AVs compared to HVs. Fig. 5b shows that in the social optimum, travelers' mode choices present contrasting patterns. Specifically, as w_n increases, the number of AVs increases. The space-varying toll raises the travel price for HV users, particularly with a larger w_n. As for the resulting total travel cost, the total travel cost increases with HVs' cruising congestion parameter w_n, both in the absence of pricing and in the social optimum, as shown in Fig. 5c and Fig. 5d. This is clearly due to the increasing congestion for HV and AV users.

5.5.2. Varying w_a

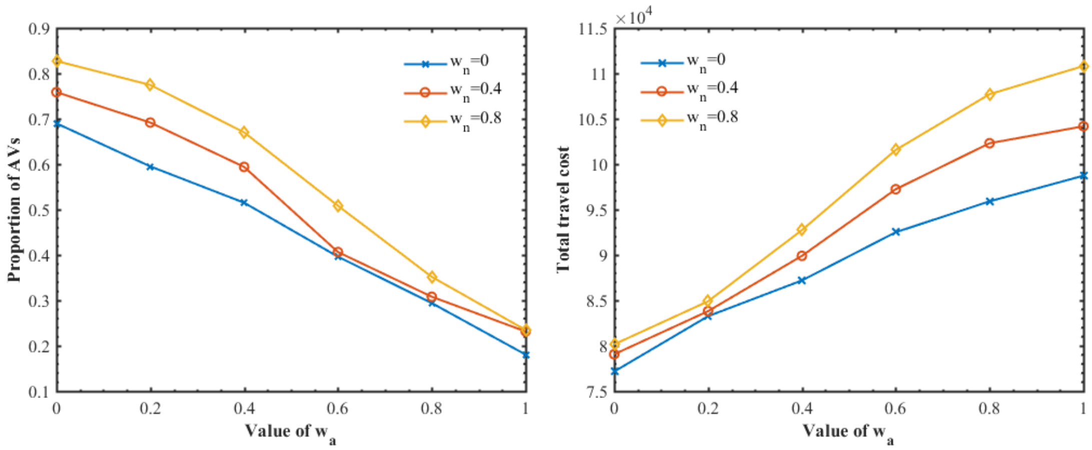
We now vary AVs' back-and-forth congestion parameter w_a from 0 to 1. In the absence of pricing, Fig. 6a shows that as AVs' congestion parameter w_a increases, the number of AVs increases: the AVs' back-and-forth congestion affects HV users more than AV users, as AVs drive through the entire city twice. As depicted in Fig. 6b, the resulting total travel cost rises with w_a, due to the increased congestion caused by AVs moving back and forth.

Fig. 7 enables the same analysis for the social optimum. It can be seen that the proportion of AV users decreases with w_a, and the total travel cost increases with w_a.



(a) Proportion of AVs (b) Total travel cost

Fig. 6. Unpriced equilibrium (varying w_a).



(a) Proportion of AVs (b) Total travel cost

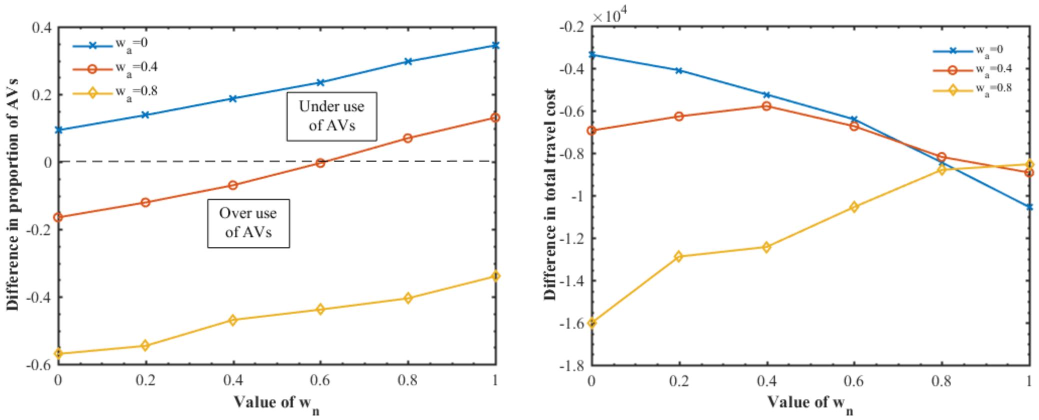
Fig. 7. Social optimum (varying w_a).

5.5.3. Comparison: Unpriced equilibrium versus the social optimum

In this subsection, we compare the outcomes under no tolling and the social optimum. As we will see, the interaction between HVs' cruising congestion and AVs' back-and-forth congestion significantly impacts AVs' use and pricing efficiency. It should be noted that in Fig. 8b and Fig. 9b, a negative value is good and represents a higher pricing efficiency.

Fig. 8a shows that without tolling, AVs can either be underused or overused, depending on the relative intensities of AVs' back-and-forth congestion and HVs' cruising congestion. Specifically, when AVs' back-and-forth congestion is low, the absence of tolling results in AVs being underused. Conversely, when AVs' back-and-forth congestion is heavy, the lack of tolling leads to overuse. Moreover, with moderate AVs' back-and-forth congestion, an increase in the HVs' cruising congestion parameter shifts the scenario from overuse to underuse.

In Fig. 8b, we observe that tolling is efficient in reducing total travel cost. When AVs' back-and-forth congestion is low, the efficiency of pricing increases with HVs' cruising congestion. Conversely, when AVs' back-and-forth congestion is heavy, the efficiency of pricing decreases with HVs' cruising congestion. For moderate AVs' back-and-forth congestion, our numerical result shows that the efficiency of pricing first decreases and then increases, indicating that AVs' back-and-forth congestion dominates first, and then HVs' cruising congestion dominates.

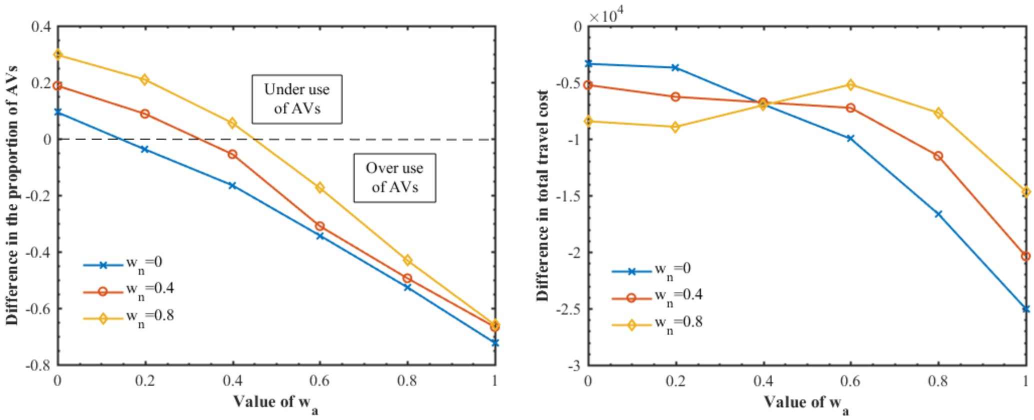


(a) Difference in the proportion of AVs (b) Difference in total travel cost

Fig. 8. Varying HVs' cruising congestion parameter w_n.

Note: The difference refers to the comparison between the results with tolling and without tolling.

Fig. 9 depicts the impact of AVs' cruising congestion parameter w_a. The patterns observed differ from those in Fig. 8. In Fig. 9a, an increase in the AVs' back-and-forth congestion parameter shifts AVs from underuse to overuse, regardless of the value of w_n. In Fig. 9b, when w_n=0, the efficiency of pricing increases with w_a. When w_n=0.8, the efficiency of pricing initially increases with w_a, and then falls. With moderate w_n, the efficiency of pricing depends on which effect dominates, and lies between w_n=0 and w_n=0.8.



(a) Difference in the proportion of AVs (b) Difference in total travel cost

Fig. 9. Varying AVs' cruising congestion parameter w_a.

Note: The difference refers to the comparison between the results with tolling and without tolling.

6. Conclusion

We aimed to compare the cruising congestion of HVs with the back-and-forth congestion of AVs, caused by AVs dropping off their passengers at the destination and then driving back out of the city to park where this is cheap. To do so, we derived the joint equilibrium of travel mode and parking location choices by developing a spatial optimization model that incorporates stochastic user preferences. Our model explicitly considers the interaction between HVs and AVs, addressing various congestion externalities, such as HVs' searching congestion and cruising congestion, and AVs' back-and-forth congestion. The model properties were analytically and numerically explored under both the unpriced equilibrium and the social optimum.

Some insightful findings were obtained. First, without pricing, the introduction of AVs may raise or lower congestion depending on whether the cruising or back-and-forth form of congestion dominates. Specifically, if HVs' cruising congestion dominates, the introduction of AVs tends to lower overall congestion; if AVs' back-and-forth congestion dominates, the introduction of AVs tends to raise overall congestion. Accordingly, AVs may be underused or overused, as the MEB of switching to AVs may be positive or negative.

For HVs' parking distribution, given the number of HV and AV users, HVs' cruising and AVs' back-and-forth congestion both increase HVs' parking span due to increased parking costs. So, in this setting, AVs may or may not help with congestion. Still, the introduction of AVs is efficient at reducing parking demand in the city center, resulting in a smaller and less compact city.

Space-varying tolling can change travelers' travel choice behavior and urban spatial parking structure. The space-varying parking fee on HVs comprises the searching externality caused by HVs parking at location x, the cruising externality on HV drivers parking between specified locations, and the congestion externality imposed on AV users. The congestion tolling on AVs equals the MEC imposed by AV users. Without pricing, parking close to the CBD will be excessive. In the social optimum, the parking fee for HVs decreases as the parking location approaches the CBD.

The congestion interactions between HVs and AVs significantly impact the efficiency of pricing. When only one congestion type exists, the efficiency of pricing increases with the degree of congestion. However, when both congestion types exist, the efficiency of pricing is ambiguous. Specifically, when both HVs' cruising congestion and AVs' back-and-forth congestion are heavy, the efficiency of pricing is low.

Our model shows that the effects of AVs on travel and urban equilibrium may be more difficult to assess and less beneficial than often thought. It also provides a new avenue for investigating the traveling and parking equilibrium when there are both human-driven and autonomous vehicles. This leads to interesting follow-up directions. First, it is important to extend this study to cover price-sensitive demand and compare the efficiency of space-varying pricing with more realistic policies, such as uniform or step pricing. Previous studies, such as Millard-Ball (2019), have shown that more realistic policies can lead to different outcomes. Second, it was assumed that AVs would be parked at home. Other parking location choices could be studied, such as parking near the workplace, a dedicated parking belt, and cruising on the road. Third, we only considered the trip from home to the CBD. The return trip from CBD to home seems reasonably symmetrical, other than that there may be less searching and cruising time at the destination. But not necessarily so as in many cities most of the searching and cruising occurs in the evening at home. It would be interesting to extend our model to examine the integrated daily travel and parking patterns, such as Zhang et al. (2008), Lu et al. (2021), and Zhang et al. (2019). Fourth, it was assumed that all travelers live away from the CBD. However, in reality, people differ in their preferences for residence, and some want to live in the city center. Therefore, it would be worthwhile to determine the location endogenously. Fifth, we could consider AVs raising road capacity as they can drive closer together in platoons than can HVs and use intersections more efficiently (e.g., Chang and Lai, 1997; Van den Berg and Verhoef, 2016). Sixth, we used a monocentric city. However, many cities have multiple centers. Therefore, a multicentric urban model should be developed for further study.

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Appendix

Appendix A. Proof of Proposition 1.

Proof. We illustrate with Proposition 1 (i). Suppose $0<\gamma_1<\gamma_2$, and the corresponding parking density is $n_1(x)$ and $n_2(x)$, respectively. According to Eq. (16), we therefore have:

$$\frac{\gamma_1 K}{(K-n_1(x))^2}\cdot\frac{\partial n_1(x)}{\partial x}=\frac{\alpha_n}{V_{\max}}+mc_n+\alpha_n\cdot\frac{w_a N_a+w_n n_1(x)}{s}-\frac{\alpha_w}{V_w}<0 \text{ and}$$

$$\frac{\gamma_2 K}{(K-n_2(x))^2}\cdot\frac{\partial n_2(x)}{\partial x}=\frac{\alpha_n}{V_{\max}}+mc_n+\alpha_n\cdot\frac{w_a N_a+w_n n_2(x)}{s}-\frac{\alpha_w}{V_w}<0.$$

Because $n_1(x)$ and $n_2(x)$ are both continuous, with positive density at x=0 and the integral of each of them over its support is N_n, there must be at least one crossing of these parking densities. Due to $\partial n_1(x)/\partial x < 0$, we consider that these parking densities cross once, at x. Then at any such crossing, x, $n_1(x) = n_2(x)$ holds. Denote derivatives with primes. From $0 < \gamma_1 < \gamma_2$, we can obtain that $n_1'(x) < n_2'(x)$ holds. That $n'(x) < n_o'(x)$ is true at any crossing implies that there can be only one crossing. That the equilibrium density slopes down more steeply at the crossing means that the equilibrium density is $n_1(x) > n_2(x)$ for $x < x$, and the converse is true for $x > x$. As a result, $\overline{x_1} < \overline{x_2}$ holds. This completes the proof of Proposition 1(i). Using the same logic, we can prove Proposition 1(ii) and (iii). □

Appendix B. Proof of Proposition 2.

Proof. The logic is the same as that in Proposition 1. Suppose $w_a^1 < w_a^2$, and the corresponding parking density is $n_1(x)$ and $n_2(x)$, respectively. According to Eq. (36), we therefore have:

$$\frac{2K^2\gamma}{\left(K - n_1(x)\right)^3} \cdot \frac{\partial n_1(x)}{\partial x} = \frac{\alpha_n}{V_{\max}} + \frac{w_a^1 \cdot \alpha_n N_a}{s} - \frac{\alpha_w}{V_w} + mc_n < 0 \text{ and}$$

$$\frac{2K^2\gamma}{\left(K - n_2(x)\right)^3} \cdot \frac{\partial n_2(x)}{\partial x} = \frac{\alpha_n}{V_{\max}} + \frac{w_a^2 \cdot \alpha_n N_a}{s} - \frac{\alpha_w}{V_w} + mc_n < 0 .$$

Similar to Proposition 1, we can derive that the two parking densities cross once. Hence, at any such crossing, x, $n_1(x) = n_2(x)$ holds. Due to $w_a^1 < w_a^2$, we have that $n_1'(x) < n_2'(x)$ holds. This means the equilibrium density is $n_1(x) > n_2(x)$ for $x < x$, and the converse is true for $x > x$. As a result, $\overline{x_1} < \overline{x_2}$ holds. This completes the proof of Proposition 2(i). Using the same logic, we can prove Proposition 2(ii) and (iii). □