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Non-Transitive Patterns in Long-Term Football Rivalries

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Abstract

The phenomenon of non-transitivity in outcomes, typically observed in non-effort games with predetermined probabilities and immediate clarity, extends to team-based, time-consuming games requiring effort that unfold over a long period of time. This study explores this aspect through an empirical analysis of professional football matches in the Netherlands involving three prominent teams: Feyenoord, Ajax, and PSV. Contrary to conventional expectations, the results reveal a non-transitive pattern over more than three decades, indicating that Feyenoord is more likely to triumph over PSV, PSV over Ajax, and Ajax over Feyenoord than the reverse scenarios.

Keywords: Non-transitive match outcomes, football, bookmakers, Elo-rating

JEL-codes: C25, D01, Z2

Declarations of interest: none

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1 Introduction

Many binary relations demonstrate transitivity, particularly when comparing objective measures. For example, if person A is taller than person B, who is taller than person C, then logically, A is taller than C. Similarly, transitivity often applies to game outcomes. If player A consistently defeats player B in chess matches, and player B consistently defeats player C, it's reasonable to expect that player A would also likely defeat player C. However, this principle isn't universal in the realm of game outcomes. Various factors such as players' form or fitness, environmental conditions (like heat or pollution), luck, and coincidence can influence results. Moreover, players might be experimenting with new strategies, introducing an element of unpredictability. As a result, instances of non-transitivity are regularly observed in sports match outcomes, where in a particular season or tournament Team A beats Team B, Team B beats Team C, and Team C beats Team A.

The current paper is on non-transitivity of outcomes of football matches between three teams that unfolded over several decades. The analysis focuses on three teams that were consistently competing in the highest tier of professional football in the Netherlands: Feyenoord, PSV, and Ajax. Over an extended period of time, the results of matches between these clubs exhibited non-transitivity: Feyenoord defeated PSV, PSV defeated Ajax, and Ajax defeated Feyenoord more frequently than the reverse scenarios. This non-transitive order persists regardless of conditions and becomes even more pronounced when considering differences in the relative strengths of the teams. While factors such as form, luck, strategy, and quality differences may influence individual matches, none of these elements can fully account for the non-transitivity of match outcomes over more than three decades.

The assertion of non-transitivity in certain sports match outcomes is not ground-

breaking, as previous studies have identified evidence of non-transitivity in matches between individual players in sports like tennis and chess, as well as between teams in baseball, basketball, and football. Moreover, demonstrating non-transitivity over a short period, such as a single sports season, is not particularly remarkable. In round-robin setups, where within a season three teams play against each other six times, randomness can significantly influence non-transitive outcomes. The current paper adds to the literature the establishment of non-transitivity over an extended duration. This extended time frame distinguishes it from previous studies, as it underscores the persistence of non-transitivity beyond the confines of a single season or tournament.

The main contribution of this paper to the sports economics literature is threefold. Firstly, it demonstrates that non-transitivity in match outcomes among three teams persisted over an extended period. Secondly, it highlights that the phenomenon of non-transitivity is not reflected in the market's expectations. In fact, when considering match outcome expectations, non-transitivity becomes more pronounced. Thirdly, the paper illustrates that employing a non-transitive betting strategy can be profitable.

The paper is structured as follows. Section 2 discusses previous studies on non-transitivity in games in general and more in particular in sports matches. Section 3 provides an overview of match outcomes between the three teams since the inception of professional football in the Netherlands in 1956. Section 4 introduces a measure of non-transitivity in match outcomes. This measure calculates the sum of the results of three pairs of matches. If the sum equals zero, transitivity is observed; otherwise, if the sum differs from zero, non-transitivity of match outcomes exists. In Section 5, the paper quantifies the relationships between the three clubs estimating ordered logit models of match outcomes in terms of home win, draw and away win. The parameter estimates validate the existence of non-transitivity

in match outcomes. Section 6 expands the analysis of non-transitivity to expected match outcomes and match surprises. Section 7 explores the implications of non-transitivity on potential earnings from the betting market. Section 8 discusses other major European football leagues focusing on non-transitivity in match outcomes in the Portuguese league. Finally, Section 8 provides the main conclusions drawn from the analysis.

2 Non-transitivity in games

There are several simple games illustrating the existence of non-transitivity. A well-known example is the game of rock-paper-scissors (RPS), commonly played by children. In one-on-one matches, players use their hand to signal either rock, scissors, or paper. Choices are revealed simultaneously, and the outcome becomes immediately apparent. Rock defeats scissors, scissors defeat paper, and paper defeats rock. The result is either a win or a loss, or a draw if both players make the same choice. The children's game rock-paper-scissors has garnered significant attention in studies. According to Cason et al. (2014), the game is iconic due to its simple illustration of non-transitive dominance, where outcomes occur with equal probabilities. Simple as it may be, the game is popular. There are RPS championship competitions worldwide and since 2002 there is a World Rock Paper Scissors Society (You (2021)). Outcomes of RPS games are also studied to understand human behavior. Batzilis et al. (2019) analyzed Facebook data concerning the outcomes of hundreds of thousands of players engaging in the RPS game, with varying levels of information about previous player behavior. Their findings indicated that more experienced players utilized this information to their advantage.

Another example of non-transitivity is demonstrated by a two-player game

where players select one of three colored dice and then proceed to roll them. The dice have two identical series of numbers. The red die features the numbers 2-4-9, the green die has 1-6-8, and the blue die contains the numbers 3-5-7. In each roll, the die with the highest number is considered the winner. The outcome of the game depends on the number of wins accumulated over multiple rolls. Over time, the blue die prevails over the red die, the red die prevails over the green die, and the green die prevails over the blue die. If the first player selects a specific die, the second player can then choose the die that ensures victory in the game. Draw outcomes are impossible since no two numbers on the dice are identical.[1] Non-transitive dice sets have been studied from a theoretical point of view. Bozóki (2014) for example presented a more general discussion of non-transitive dice sets extending the number of simultaneous players to more than two and indicating that there are three-player non-transitive dice sets such that if two players pick a die the third player can pick a die that beats both opponents' dice.

Non-transitivity may occur in pairwise comparisons of outcomes of matches between sports teams. Smead (2019) provided a real-life example of non-transitivity in international football during the 1994 FIFA World Cup group E, where Ireland defeated Italy, Italy defeated Norway, Norway defeated Mexico, and Mexico defeated Ireland. Kaiser (2019) investigating the sensitivity of point systems in determining the final ranking in Formula 1 Drivers seasons showed that in the 2007 season there was non-transitivity among the top three drivers: Raikkonen defeated Alonso 11 races to 6, Alonso defeated Hamilton 10 races to 7, and Hamilton defeated Raikkonen 10 races to 7. These examples of non-transitivity were likely caused by the small number of matches, as luck is a determinant of match outcomes.

[1]It's worth noting that the voting paradox, also known as Condorcet paradox, is a prominent non-transitivity paradox suggesting that a perfect democratic voting system is theoretically impossible. If voter preferences for three candidates are non-transitive and voting occurs in pairs of proposals, the order in which the pairs are presented determines the outcome.

Bozóki et al. (2016) used results from tennis matches to rank male tennis players; Temesi et al. (2024) did the same for female tennis players. Players are sometimes ranked even if they have never played against each other. Both studies find that non-transitivity in match outcomes between three players occurred frequently. Poddiakov (2022) mentioned that non-transitivity not only occurs in stochastic real-world games but also in deterministic positional games in chess and checkers in terms of non-transitive players' positions. Finally, Spearing et al. (2023) studied non-transitivity in baseball using data from 9 seasons of American League Baseball.[2] The main finding was that by allowing for non-transitivity prediction of match outcomes can be improved.

3 Setting the stage: Professional football

Professional football is a game played according to the same rules in competitions all over the world. Two teams of eleven players compete against each other in a match that lasts two halves of 45 minutes each, with a break of 15 minutes in between. Most competitions are organized as round-robin tournaments, where each team faces every other team twice, once at home and once away. The team scoring the most goals in a match wins and earns three points. In the event of a draw, both teams receive one point. The team with the highest number of points at the end of the season wins the championship. Whether a team wins a particular match depends on the relative quality of the teams, the abilities of the coaches, luck, referee decisions, and chance occurrences. In the long run, the quality of the team is the primary determinant of success. However, disparities in quality may not be adequate to ensure a clear hierarchy of teams in every game; thus, match outcomes are uncertain. Indeed, without this uncertainty, football would not be

[2]The model used by Spearing et al. (2023) is discussed in more detail in section 5.

as exciting to watch.

The top tier of professional football in the Netherlands (Eredivisie) started in 1956. In the 67 seasons since then, three clubs have consistently played in the top league: Feyenoord, PSV, and Ajax.[3] Feyenoord and PSV have always played in the same stadiums which were renovated a couple of times but not renewed. Ajax moved to a new stadium in 1996 with a substantial larger capacity.[4]

The 2019/20 season was partly canceled due to Covid-19.[5] When the Eredivisie was put to a standstill in March 2020, Ajax and AZ were on top with the same number of points (56). On the basis of goal difference the Dutch football association declared Ajax to have ended top of the league though not as champion.[6] In the other 66 seasons, Ajax won 28 championships, PSV 21, Feyenoord 11, and other clubs won a total of 6 times. The three clubs are often referred to as the 'big three' of Dutch professional football. Throughout all seasons, Ajax won 67 percent of all its league matches, PSV 61 percent, and Feyenoord 57 percent.

Table 1 displays the match outcomes in terms of the total number of points obtained during all seasons.[7] As shown in the first column, over all matches they

[3]Data on match outcomes are collected from Wikipedia pages.

[4]Van Ours (2024a) analyzing the effect of a new football stadium found that this increased attendance while there were no effects on home advantage.

[5]Because of the Covid-restrictions some football matches were played behind closed doors. Bryson et al. (2021) showed that this removed home advantage. However, Van Ours (2024b) showed that although also in the Dutch top league home advantage disappeared this phenomenon was not present for the top teams including Feyenoord, Ajax, and PSV.

[6]Csató (2021) discussing how to obtain a fair ranking if a competition ends prematurely concluded that Ajax obtained the same number of points as AZ but with a more difficult schedule of matches already played. According to this line of reasoning ranking AZ as second was fair.

[7]Until the 1995/96 season, winning a match generated two points. From then on, three points were awarded for a win. Here, the three points for a win rule was applied throughout the sample period to ensure comparability across seasons. Guedes and Machado (2002) and Hon and Parinduri (2016) are studies on the effects of the introduction of three-points for a win rule. The studies found that the effects on match outcomes or style of play were absent or limited. Two recent studies are Butler and Butler (2017) and Reade and van Ours (2024) investigating the effects of changes in points system for professional football in Ireland and England. The main conclusion was that the change in point system did not affect match outcomes. A potential problem in the analysis is that scheduling effects may have affected match outcomes (Krumer and Lechner (2017)). However, this is not an actual problem since the order in which the three

Table 1: **Match outcomes in points obtained; 1956/57-2022/23**

Feyenoord-PSV		PSV-Ajax		Ajax-Feyenoord		Total
Feyenoord	196	PSV	192	Ajax	228	616
PSV	172	Ajax	183	Feyenoord	132	487
Balance 1956/57-2022/23	24		9		96	129
Balance 1956/57-1989/90	21		-21		6	6
Balance 1990/91-2022/23	3		30		90	123

Note: In Covid season 2019/20 one match (Feyenoord-Ajax) was canceled. 1956/57-2022/23: 401 observations; 1956/57-1989/90: 198 observations; 1990/91-2022/23: 203 observations. In the calculation of the match points three points are allocated to a win even though in reality up to season 1995/96 winning a match generated two points.

played against each other, Feyenoord obtained 24 more points than PSV indicating that overall, Feyenoord was stronger than PSV. The second column shows that PSV acquired 9 more points than Ajax when they played against each other, demonstrating overall slight superiority of PSV over Ajax. The third column demonstrates that Ajax accumulated 96 points more than Feyenoord when they played against each other, showcasing Ajax's superiority over Feyenoord.

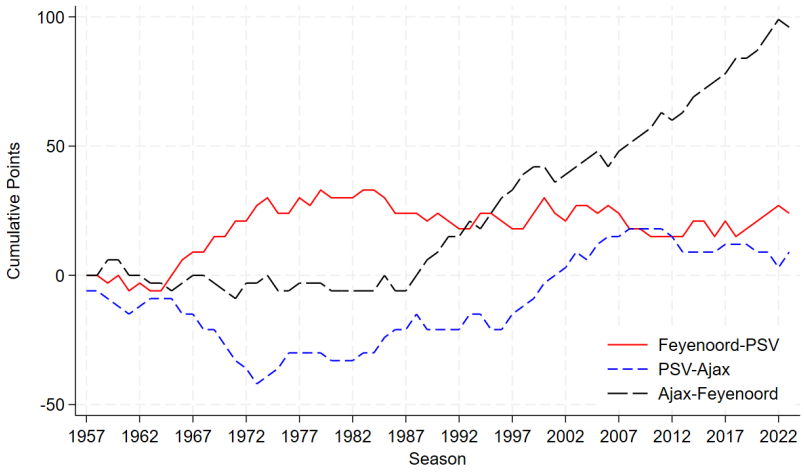
The bottom part of Table 1 shows the points balance is the sample period is roughly split-up in two: from 1956/57 to 1989/90 and from 1990/91 to 2022/23. From this it appears that the dominance of Feyenoord over PSV was present mainly in the first half of the sample period. The dominance of PSV over Ajax and of Ajax over Feyenoord occurred in the second half of the sample period. Using the sum of the number of points obtained in pairwise matches non-transitivity is evident: Feyenoord was stronger than PSV, which was stronger than Ajax, which, in turn, was stronger than Feyenoord. This non-transitivity is clearly related to the second half of the sample period.[8]

Figure 1 illustrates the evolution of the balance of the cumulative points obtained in pairwise matches. The trajectories of the cumulative points are not

teams played against each order changed from season to season.

[8]If the sample is split-up according to the two-points for a win rule (1956/57 to 1994/95) and the three-points for a win rule (1995/96 to 2022/23) the results are very much the same.

Figure 1: **Cumulative balance in points pairwise matches; 1956/57 - 2022/23**



Note: The differences are calculated for the first team for each match pair.

linear. The gap in cumulative points between Feyenoord and PSV increased until the early 1970s, remaining relatively stable thereafter. The cumulative points difference between PSV and Ajax initially decreased, then increased, and has remained relatively constant over the past 20 years. The cumulative points difference between Ajax and Feyenoord was steady until the mid-1980s and then increased steadily.

4 Measuring non-transitivity of match outcomes

The outcome of a match, S_{ij}, between home team i and away team j depends on the home advantage H_i of team i and the difference in quality Q between both teams:[9] $S_{ij} = H_i + Q_i - Q_j$ and $S_{ji} = H_j + Q_j - Q_i$. The assumption is that the

[9] According to Clarke and Norman (1995) match outcomes between two teams can be split-up in two components: Firstly, the difference in quality between the teams (which they refer to as 'ability' rather that 'quality' and secondly home advantage which is supposed to be team specific. Clarke and Norman (1995) used this set-up to calculate seasonal averages in home team

home advantage is club-specific. The net balance of results between club i and club j after playing against each other twice (away and at home), is equal to the difference in home advantages and twice the difference in quality:

$$R_{ij} = S_{ij} - S_{ji} = H_i - H_j + 2(Q_i - Q_j) \tag{1}$$

Similarly,

$$R_{jk} = S_{jk} - S_{kj} = H_j - H_k + 2(Q_j - Q_k) \tag{2}$$

$$R_{ki} = S_{ki} - S_{ik} = H_k - H_i + 2(Q_k - Q_i) \tag{3}$$

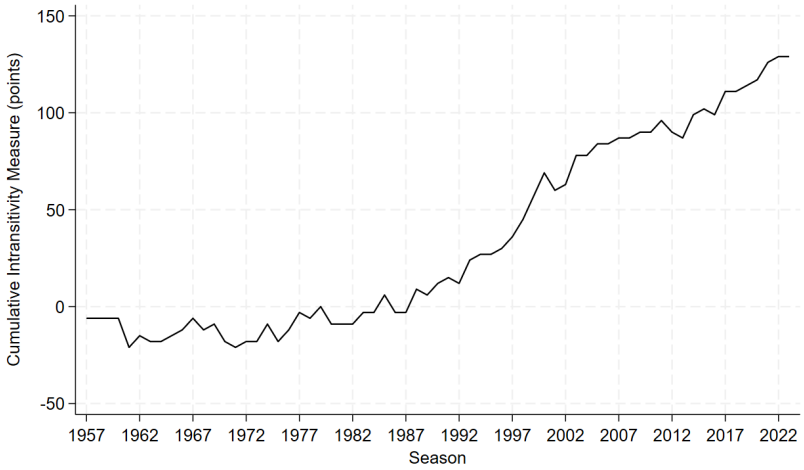
Adding up Eqs (2) to (4), it follows that $R_{ij} + R_{jk} + R_{ki} = 0$. This shows that the results between k and i are fully determined by the results between i and j and j and k: the relationship is transitive. Non-transitivity can be introduced by assuming that there is an interaction-specific effect in one of the pairs, for example between i and j such that $R_{ij} = S_{ij} - S_{ji} = H_i - H_j + 2(Q_k - 2Q_i) + \theta_{ij}$. Then,

$$R_{ij} + R_{jk} + R_{ki} = \theta_{ij} \tag{4}$$

So, if there is transitivity, the three net results add up to zero. If there is non-transitivity, they add up to θ_{ij}. The sum of the net results determines whether or not there is non-transitivity in a particular season. Note that this measure can take any value. If it is not zero, it can be positive as well as negative, depending on the nature of the non-transitive relationship. The cumulative value, aggregated over

advantages and team quality. In a round-robin set-up where all clubs in a league meet each-other twice during a season and imposing the restriction that average team quality in a season is equal to zero, all H_i and Q_i (i=1,..,N) with N = the number of clubs in a league are identified. In this set-up it is not possible to detect non-transitivity. For that, individual match data are required. Whether or not home advantage is team-specific is an empirical issue. See Peeters and van Ours (2021) for an example of a recent empirical analysis of home advantage in English professional football in which team-specific home advantage is picked-up by team fixed effects.

Figure 2: **Cumulative non-transitivity; 1956/57-2023/24**



subsequent seasons, is an indicator of the persistence of a particular non-transitive relationship.

Figure 2 displays the developments of the cumulative non-transitivity indicator with i=Feyenoord, j=PSV, and k=Ajax. Up to the late 1980s, this indicator remained flat but with ups and downs, suggesting that the non-transitivity indicator fluctuated between positive and negative values. This implies that there may have been non-transitivity, but its source varied over time. From the late 1980s onward, the cumulative indicator increased, indicating that the nature of non-transitive match outcomes was consistent over time. Indeed, from the late 1980s onward, it was Feyenoord dominating PSV, PSV dominating Ajax, and Ajax dominating Feyenoord.

5 Parameter estimates pairwise comparisons

The Bradley and Terry (1952) model is the seminal model for ranking on the basis of pairwise comparisons. In terms of match outcomes, the probability that

one team beats another team depends on the difference in skills or quality they possess. The model is transitive, meaning that if team i beats team j and team j beats team k, then team i will beat team k. The model for three teams i, j, and k can be specified as a logit model. The probabilities that team i beats team j, team j beats team k, and team i beats team k are determined by the difference in quality Q between the two teams:

$$p_{ij} = \frac{1}{1 + e^{Q_j - Q_i}}, \quad p_{jk} = \frac{1}{1 + e^{Q_k - Q_j}}, \quad p_{ik} = \frac{1}{1 + e^{Q_k - Q_i}} \tag{5}$$

The Bradley-Terry model is transitive since p_{ik} is fully determined by p_{ij} and p_{jk}:

$$p_{ik} = \frac{p_{ij}p_{jk}}{1 + 2p_{ij}p_{jk} - p_{ij} - p_{jk}} \tag{6}$$

Spearing et al. (2023) introduced non-transitivity by adding a parameter θ_{ik} to the probability that i beats k:

$$p_{ik} = \frac{1}{1 + e^{Q_k - Q_i + \theta_{ik}}} \tag{7}$$

If $\theta_{ik} \neq 0$, p_{ik} is not fully determined by the other two probabilities. If θ_{ik} is negative (positive) this increases (decreases) the probability that team i beats team k relative to the effect of the difference in quality.

As mentioned in section 2, Spearing et al. (2023) applied this model to outcomes of baseball matches. They did not allow for draws to occur and they ignored home advantage, indicating that this can be included by introducing a parameter γ next to θ_{ik}, representing that the win probability, in addition to the difference in quality and non-transitivity, may be influenced by whether or not a team plays at home. Allowing for a draw can be taken into account by estimating an ordered logit model. Hankin (2020), studying the results of chess matches, introduced draws

into pairwise comparisons and first-mover advantage. The white player who moves first has an advantage in chess comparable to the home advantage in football. Draws were modeled by a multinomial logit model in which a hypothetical third party is introduced that wins if the game is drawn. The first-mover advantage was modeled by adding a parameter to the strength of the white player.

Table 2: **Parameter estimates match outcomes ordered logit models**

	1956/57-2022/23		1956/57-1989/90		1990/91-2022/23	
Quality PSV	0.57	(0.23)**	-0.15	(0.32)	1.38	(0.35)***
Quality Ajax	0.54	(0.16)***	0.11	(0.22)	1.05	(0.25)***
Match Feyenoord & PSV	0.67	(0.28)***	0.08	(0.40)	1.36	(0.42)***
Observations	401		204		197	

Note: Match outcomes: loss, draw, win. For identification the effect of quality Feyenoord is normalized to zero. Standard error in parentheses; thresholds not reported; *** (**): significant at a 1% (5%) level

Table 2 presents the relevant parameter estimates when an ordered logit model is used for pairwise ranking, allowing for the probability of a draw and for the potential effect of playing at home.[10] The ordering is threefold: loss, draw, and win. For identification, the effect quality for Feyenoord is normalized to zero so the parameter estimates for quality of PSV and Ajax are quality relative to Feyenoord.

The first column of Table 2 show the parameter estimates over the full sample period, confirming the existence of quality differences between on the one hand Feyenoord and on the other hand Ajax and PSV. There is also evidence of non-transitivity as the match-specific effect of Feyenoord playing against PSV is positive and significantly different from zero. This indicates that the match outcomes between Feyenoord and PSV are different than expected based on the ranking in quality. Conditional on the differences in quality, Feyenoord is more likely to obtain points when playing against PSV, which in itself is a sign of the

[10]The parameter estimates for team-specific home advantage were not significantly different from zero in any of the estimates. Therefore, they are ignored in the estimates presented in Table 2.

existence of non-transitivity.

The second column presents the estimates for the first 34 seasons.[11] The quality indicators are insignificantly different from zero, as is the match-specific effect for Feyenoord playing against PSV. Apparently, there is no systematic difference in outcomes between the various matches, and there is no evidence of non-transitivity occurring.

The third column of Table 2 shows the parameter estimates for the second period of 33 seasons, from 1990 onward. Now the quality indicators are positive and significantly different from zero, although not significantly different from each other. This indicates that both PSV and Ajax were of approximately equal strength, and both were stronger teams than Feyenoord. The match-specific effect of Feyenoord and PSV playing against each other is also significantly different from zero, confirming the existence of non-transitivity in match outcomes.

6 Expected outcomes and match surprises

Match outcomes may be influenced by the relative strengths of the teams. Stronger teams are more likely to win. One way to account for these differences is by using differences in Elo ratings. Originally, Elo ratings were used to indicate the relative strength of chess players (Elo (1978)) but nowadays they are used in many sports to predict match outcomes. Lasek et al. (2013), for example, presented an analysis of national team football match outcomes in the period 2006-2012. They investigated the predictive power of various ranking systems using as a benchmark the official ranking of FIFA, the world football association. The ranking systems are used to generate predictions and by comparing with actual match outcomes to assess the accuracy of the predictions. The main conclusion

[11] If the sample is split-up differently, according to the two-points for a win and three-points for a win the parameter estimates are very much the same.

was that an Elo-rating based system outperformed the then-used FIFA ranking method. In the meantime, since 2018, FIFA started using an Elo-approach for their ranking of countries (FIFA (2018)). Hvattum and Arntzen (2010) studied whether rating systems of football teams based on their past performance are helpful in predicting match outcomes. The authors concluded that using Elo-ratings as a measure of team strength is justified. Arntzen and Hvattum (2021) is a related study showing that Elo-ratings of football teams performed well when used as a basis for predicting match outcomes in professional football in England. The predictions were improved when individual player valuations are also taken into account. Peeters (2018) investigated outcomes of international football matches between national teams comparing various rating methods including FIFA country ranking and Elo-based ratings.

When two teams play against each other, their Elo-ratings are adjusted depending on the match's outcome. Winning increases a team's rating, while losing decreases it. Teams with a history of good results have higher Elo ratings, while those with poor results have lower ones. The differences in Elo ratings before a match are used to predict match outcomes in terms of home win probabilities, draw probabilities and away win probabilities.[12]

Panel a Table 3 indicates that in terms of expected points based on predicted outcomes based on Elo ratings, there was a transitive relationship between the three clubs: Ajax was stronger than PSV, which was stronger than Feyenoord. Ajax was stronger than Feyenoord where the difference in the number of expected points between Ajax and Feyenoord was of the same magnitude as the expected point differences between Feyenoord and PSV and PSV and Ajax.

The expected points serve as an indicator of the team's strength but does not reveal the origin of that strength. Considering the relative strength of the teams,

[12]These probabilities are based on outcomes of past matches with a similar difference in Elo-rating (see Elofootball.com).

Table 3: **Expected points and surprise points; 1956/57-2022/23**

a. Expected points						
Feyenoord-PSV		PSV-Ajax		Ajax-Feyenoord		Total
Feyenoord	170	PSV	167	Ajax	214	551
PSV	198	Ajax	200	Feyenoord	151	549
Balance	-28		-33		63	2
b. Surprise points						
Feyenoord-PSV		PSV-Ajax		Ajax-Feyenoord		Total
Feyenoord	26	PSV	25	Ajax	14	65
PSV	-26	Ajax	-17	Feyenoord	-19	-62
Balance	52		42		33	127
N	134		134		133	401

Note: N = number of matches. In Covid season 2019/20 one match was canceled. The expected match outcomes are provided by elofootball.com in terms of probabilities of home win, draw and away win. These probabilities are based on differences in Elo-ratings at the start of a match. In the calculation of the expected match points three points are allocated to the win probability even though in reality up to season 1995/96 winning a match generated two points.

it's unsurprising that Ajax wins more often against Feyenoord than the other way around. However, it is surprising that Feyenoord wins more often against PSV and PSV wins more often against Ajax. To explore this further, the difference between the expected result and the actual result of a match, termed the match surprise, are also analyzed.

An example of the calculation of a match surprise is as follows: On January 27, 2019, Feyenoord played a home game against Ajax. Based on differences in Elo rating, Feyenoord had a win probability of 24 percent, while Ajax had a win probability of 49 percent.[13] The probability of a draw was 27 percent. The expected number of points for Feyenoord was calculated as 0.24*3 + 0.27*1 = 0.99, and for Ajax, it was 0.27*1 + 0.49*3 = 1.74. The final score was 6-2, resulting in 3 points for Feyenoord and 0 for Ajax. The surprise points for Feyenoord from

[13]The odds at Interwetten – one of the main bookmakers – are not very different. They implied a home win probability of 24%, a draw probability of 23% and an away win probability of 53%.

this match are equal to 3 - 0.99 = 2.01, and for Ajax, it is equal to 0 - 1.74 = -1.74. By summing the surprise points from the start of the Eredivisie in 1956/57 for subsequent seasons, the cumulative number of surprise points is calculated.

Panel b of Table 3 displays the cumulative surprises of the pairs of matches between the three clubs. Feyenoord had a positive surprise of 26 points, PSV had a negative surprise of 26 points, resulting in an overall positive surprise of 52 points for Feyenoord when playing against PSV. PSV had a positive cumulative surprise of 25 points when playing against Ajax, and Ajax had a positive cumulative surprise of 14 points in their matches against Feyenoord. Once again, non-transitivity is observed: Feyenoord dominated PSV, which dominated Ajax, which, in turn, dominated Feyenoord.

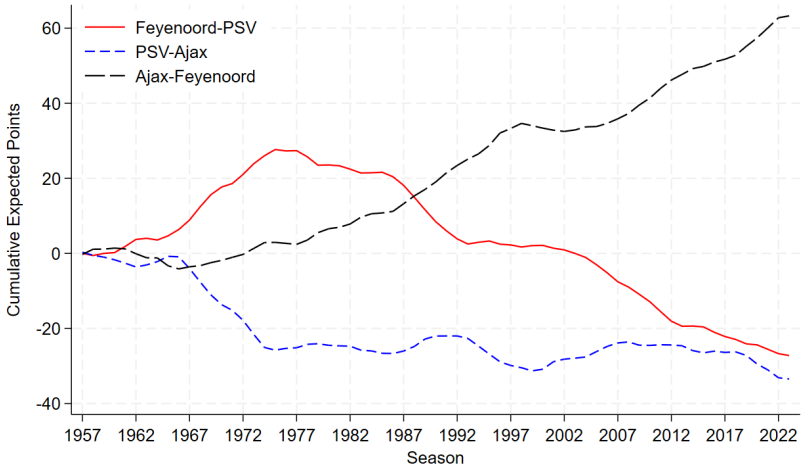
The last column of Table 3 presents the overall balance of the various indicators. For expected points, the sum was close to zero, as was to be expected. However, for points differences, the balance was quite large, reflecting the non-transitivity of the cumulative match surprises.

Panel a of Figure 1 depicts the evolution of cumulative expected points. Until the late 1970s, Feyenoord obtained more expected points than PSV in their matches. However, from then on, the situation reversed, indicating that initially Feyenoord was considered stronger than PSV, while later on PSV was considered stronger than Feyenoord. From the late 1970s onward, PSV was roughly as strong as Ajax.

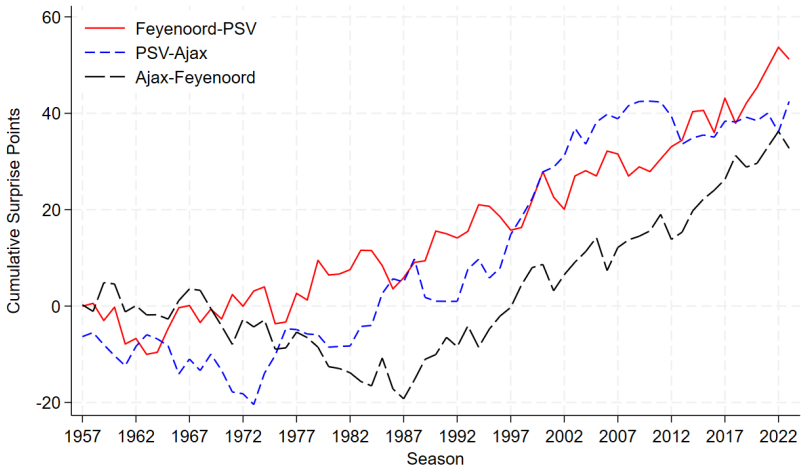
Panel b of Figure 1 presents the development of cumulative surprise points for the three match pairs. It's evident that the upward-sloping trend for each pair of teams was not present from the beginning. However, from the late 1980s onward, all curves began to slope upward.

Figure 3: **Cumulative expected and surprise points; 1956/57 - 2022/23**

a. Cumulative expected points



b. Cumulative surprise points



Note: The differences are calculated for the first team in each match pair.

7 Against the odds: Non-transitive gambling

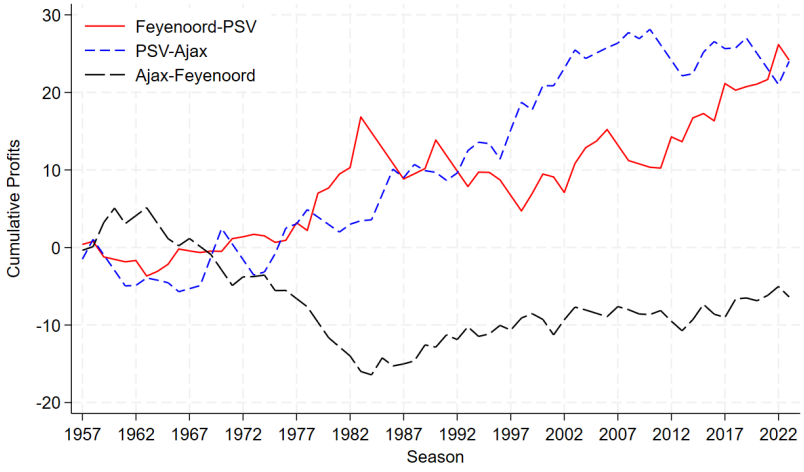
7.1 Odds based on Elo-ratings

The non-transitivity of match outcomes implies that Feyenoord won more often against PSV than one would expect based on the match outcomes between Feyenoord and Ajax, and between PSV and Ajax. To illustrate the consequences of this bias in expected results, the potential profits from a hypothetical non-transitive betting strategy are calculated. The assumption is that the odds derived from differences in Elo ratings are equivalent to betting odds.[14] The calculations of the earnings are based on betting 1 euro for a win for every home match and every away match on the first team in the pairs Feyenoord-PSV, PSV-Ajax, Ajax-Feyenoord. If a win did not occur, the loss was 1 euro; if the win materialized, the pay-out was according to the betting odds. The cumulative outcomes of these hypothetical betting examples are shown in Figure 4a. Betting on a win of Feyenoord against PSV would have been profitable with a cumulative profit of about 24 euros (investing 134 euros). The cumulative profits would have fluctuated over time. For instance, in 2007, the cumulative profits from 50 years of investment would have been approximately 10 euros (investing 100 euros). The increase from 10 to 24 euros would have occurred in the last decade. Betting on a win of PSV against Ajax would have generated a cumulative profit of about 26 euros, but this would have been obtained already at the start of the 21st century. However, betting on a win of Ajax against Feyenoord would not have been profitable. The cumulative losses would have been about 5 euros. In the early 1980s, the cumulative loss would have been around 15 euros, so there would have been an improvement since then.

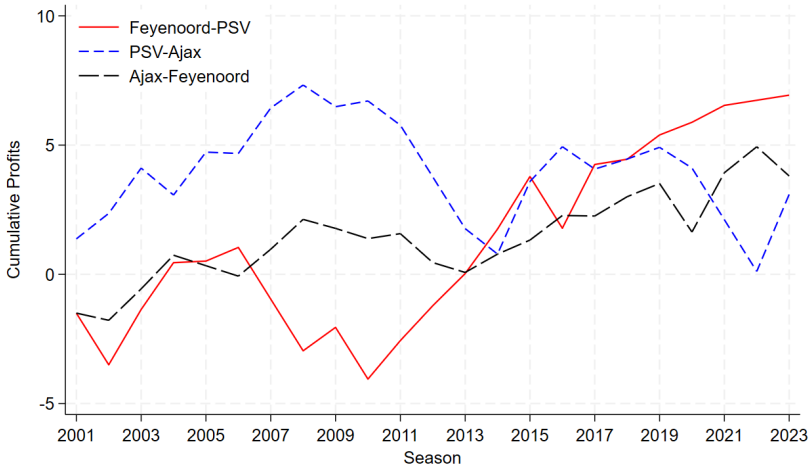
[14] When transformed into probabilities, the betting odds of a home win, a draw, and an away win add up to more than 1 because of the bookmaker's margin (Hvattum and Arntzen (2010)). The assumption here is that the bookmaker's margin is 10 percent.

Figure 4: **Cumulative profits from betting on the first team in the pair; 1956/57-2022/23 and 2000/01-2022/23 (Euro)**

a. 1956/57-2022/23



b. 2000/01-2022/23



Note: The line 'Feyenoord-PSV' shows the cumulative profits of betting 1 euro on a Feyenoord win every match they played against PSV. Same for betting on a PSV win for PSV-Ajax and betting on an Ajax win for Ajax-Feyenoord. The betting odds in panel a are calculated using estimated win-draw-loss probabilities according to differences in Elo-ratings (Elofootball.com). The betting odds in panel b are from bookmakers Interwetten (Football-data.co.uk).

7.2 Bookmaker data

Instead of expected match outcomes as related to Elo-ratings actual bookmaker data can be used.[15] Figure 4b shows the hypothetical cumulative profits from betting one Euro every match on the first team mentioned in each pair. If one would have done this over a period of 22 seasons profits would have been made. The total profit of betting 6 Euro every year would have been 15 Euro based on bookmaker odds.

8 What about other European leagues?

8.1 Top seven European football leagues

According to the UEFA-ranking the current big seven European leagues are England, Spain, Italy, Germany, France, Netherlands and Portugal. To explore whether the main findings of non-transitivity in match outcomes between the top three teams is a peculiarity of the Netherlands this section presents an exploratory analysis of the other six leagues in the big seven European leagues. In the past 25 seasons (1998/99-2022/23) the situation has been as follows.

In the English Premier League, Manchester United won 9 championships, Manchester City 7, Chelsea 5, and other teams won twice (Arsenal) or once (Liverpool, Leicester City). Although this is suggestive of a top three dominating the league this is actually not the case over the full time period of 25 seasons. The 9 championships of Manchester United were prior to 2014, the 7 championships of Manchester City are from 2012 onward. In fact, in the first four of the 25 seasons Manchester City was not in the Premier League. In the Spanish La Liga,

[15] Unlike other bookmakers, Interwetten reported odds for all matches in the sample. Therefore, Interwetten odds are used. Appendix B compares these bookmaker data with expected points based on Elo-ratings.

Barcelona won 12 championships, Real Madrid 8, Valencia and Atletico Madrid 2 and Deportivo one. So, Spain has more of a top two than a top three. In the Italian Serie A, Juventus won 11 championships, Inter Milan 6, AC Milan 4, AS Roma, Lazio Roma and Napoli one while in season (2004/05) no championship was awarded; original winner Juventus was stripped from the title due to the so-called 'Calciopoli' sports scandal (see Buraimo et al. (2016) for details). This scandal also implied that for the season 2005/06 the championship title was transferred from Juventus to Inter Milan. And, in 2006 Juventus was forced to relegate and played in Serie B in season 2006/07.

In the German Bundesliga, there is at most a top two but since Bayern München won 19 championships this could also be referred to as a top one. Borussia Dortmund won 3 championships and other teams, Werder Bremen, Stuttgart and Wolfsburg one. The French Ligue 1 had 9 championships of Paris Saint Germain (PSG), 7 of Olympique Lyon (OL) while other teams won twice (Bordeaux, Monaco, Lille) or once (Nantes, Marseille, Montpellier). Similar to England, the top two were not competing throughout the full time period. The nine championships of PSG are from 2013 onward, the seven championships of OL are before 2009. In the Portuguese Primeira Liga, Porto won the championship 13 times, Benfica 8, and Sporting CP 3 while Boavista won once.

The Italian league and the Portuguese league are comparable to the Dutch league in terms of big 3 teams competing over a long period of time. However, the Italian league has been in somewhat of a distress in recent decades. Since the size and ranking of the league is comparable to the Dutch league, the analysis in the next section is on the Portuguese league.

8.2 The case of Portugal

Table 4 shows the outcomes of the three pairs of matches over the past 25 years in the Dutch Eredivisie and the Portuguese Primeira Liga. There is a clear difference between the two leagues in terms of matches between the big three. In the Eredivisie, PSV outperformed Ajax but while Ajax outperformed Feyenoord, PSV and Feyenoord were approximately balanced. A clear example of non-transitivity.

Table 4: **Match results Dutch Eredivisie and Portuguese Primeira Liga: 25 seasons; 1998/99-2022/23**

	Eredivisie		Primeira Liga	
a.	Feyenoord-PSV		Benfica-Sporting CP	
	Feyenoord	73	Benfica	83
	PSV	67	Sporting CP	50
	Balance	6		33
b.	PSV-Ajax		Sporting CP-Porto	
	PSV	79	Sporting CP	47
	Ajax	58	Porto	86
	Balance	21		-39
c.	Ajax-Feyenoord		Porto-Benfica	
	Ajax	94	Porto	90
	Feyenoord	37	Benfica	45
	Balance	57		45
d.	Total balance	84		39

In the Primeira Liga, Porto was stronger than both Benfica and Sporting CP while the point difference between Porto and Sporting CP was about the same as the point difference between Benfica and Sporting CP. Nevertheless, Benfica outperformed Sporting CP when playing against them. This is also suggestive of non-transitivity but less clear as in the Dutch top league. Porto dominated Benfica that dominated Sporting CP. However, Sporting CP did not dominate Porto. Therefore, a non-transitive betting strategy is unlikely to be profitable.

Table 5: **Parameter estimates match outcomes ordered logit models 25 seasons Dutch Eredivise and Portuguese Primeira Liga; 1998/99 - 2022/23**

a. Eredivisie				
Quality PSV	1.16	(0.39)***		
Quality Ajax	0.86	(0.27)***		
Match Feyenoord & PSV	1.25	(0.47)***		
b. Primeira Liga				
Quality Porto	0.63	(0.27)**	0.69	(0.19)***
Quality Benfica	-0.12	(0.38)	–	
Match Benfica & Sporting CP	0.64	(0.47)	0.52	(0.27)*

Note: Eredivisie 149 observations; Primeira Liga 150 observations; For identification the effects of quality Feyenoord and quality Sporting CP are normalized to zero. Standard errors in parentheses; thresholds not reported; *** (**): significant at a 1% (5%) level

Table 5 shows the parameter estimates of ordered logit models over the past 25 seasons for both leagues. For the Dutch leagues the results shown in panel a are very similar to those presented in Table 2. The qualities of PSV and Ajax are significantly higher than the quality of Feyenoord while there is evidence of non-transitivity since the variable match Feyenoord – PSV has a significant positive effect. Panel b of Table 5 shows that the quality of Porto is significantly larger than the quality of Sporting CP but the quality of Benfica is not significantly different. The variable match Benfica – Sporting CP has a large effect but this is not significantly different from zero. If the quality of Benfica is imposed to be zero – equal to the quality of Sporting CP – the match effect becomes different from zero at a 10%-level of significance. The conclusion is that also in Portugal there is non-transitivity of match outcomes for the three main clubs but not as strong as in the Netherlands.

9 Conclusions

The current paper shows that non-transitive outcomes may occur in real-life games that require time, skills, coordination, and effort. The paper presents an empirical analysis of professional football matches between the top-three clubs in the Netherlands: Feyenoord, PSV, and Ajax. The surprising finding is that for these three clubs, non-transitivity of outcomes occurred over more than three decades. Despite frequent changes in team composition, coaches, and possibly the average ability of the teams, Feyenoord has beaten PSV, PSV has beaten Ajax, and Ajax has beaten Feyenoord more often than the other way around. When considering matches between Feyenoord and PSV, and PSV and Ajax, the prediction would be that Feyenoord is likely to win matches against Ajax, but in reality, it is the opposite. This non-transitive ordering persists over time and is even stronger when the relative strengths of the teams are taken into account. An exploratory analysis of the Portuguese Primeira Liga shows that also here between the three top teams there is evidence of non-transitivity of match outcomes though not as strong as in the Netherlands.

The origin of the non-transitivity of outcomes between the three long-term football rivals is not clear. Since the non-transitive relations are present over several decades, it cannot be solely attributed to team composition or the preference of a particular coach for a certain style of play. It likely has something to do with the playing style preferred by the management of the club, coaches, players, and supporters. Developing a playing style requires time and is difficult to adjust at will over a short period. While having an identity in terms of playing style is advantageous for players to anticipate each others' behavior, it also carries the risk that opposing teams will anticipate and adjust their own style accordingly. Perhaps, this is a risk that the three teams are willing to take, considering they play only a few matches against each other. Alternatively, it could be that there

is a non-transitive psychological explanation. Consciously or unconsciously, players are influenced by past results even though they were not responsible for these results.

Previous studies have shown that non-transitivity may occur in sports outcomes, but the current paper adds to this evidence by demonstrating that non-transitivity may persist over a long time. This finding may be surprising to many and highlights the importance of considering non-transitive outcomes in forecasting economic events and outcomes. Firms, for example, may encounter unexpected non-transitivity in competition with their main rivals, affecting their strategic decisions. If firm A dominates firm B, which in turn dominates firm C, and then firm C dominates firm A, it creates a complex competitive landscape. In such cases, firm A might choose not to directly engage in competition with firm C, recognizing the likelihood of losing. Similarly, direct confrontation with firm B may not be desirable for firm A's strategic interests. Instead, firm A may strategically delay its competition with firm B until after firm B has weakened firm C. This strategic approach allows firm A to enter the competition with firm B at a more opportune moment, increasing its chances of success. Even if the origin is unclear, understanding that non-transitivity may exist can be crucial in making informed decisions in competitive industries.

This paper shows that non-transitivity in match outcomes is not taken into account when calculating expected outcomes. Betting against the odds based on the non-transitivity of match outcomes would have been profitable. Bookmakers are supposed to use all available information when setting odds but the non-transitivity of match outcomes is a phenomenon that is ignored.

References

Arntzen, H. and L. M. Hvattum (2021). Predicting match outcomes in association football using team ratings and player ratings. *Statistical Modelling 21*(5), 449–470.

Batzilis, D., S. Jaffe, S. Levitt, J. A. List, and J. Picel (2019). Behavior in strategic settings: Evidence from a million Rock-Paper-Scissors games. *Games 10*(2), 18.

Bozóki, S. (2014). Nontransitive dice sets realizing the Paley tournaments for solving schütte's tournament problem. *Miskolc Mathematical Notes 15*(1), 39–50.

Bozóki, S., L. Csató, and J. Temesi (2016). An application of incomplete pairwise comparison matrices for ranking top tennis players. *European Journal of Operational Research 248*(1), 211–218.

Bradley, R. A. and M. E. Terry (1952). Rank analysis of incomplete block design: I. The method of paired comparisons. *Biometrika 29*(3/4), 324–345.

Bryson, A., P. Dolton, J. J. Reade, D. Schreyer, and C. Singleton (2021). Causal effects of an absent crowd on performances and refereeing decisions during Covid-19. *Economics Letters 198*, 109664.

Buraimo, B., G. Migali, and R. Simmons (2016). An analysis of consumer response to corruption: Italy's Calciopoli scandal. *Oxford Bulletin of Economics and Statistics 78*(1), 22–41.

Butler, D. and R. Butler (2017). Rule changes and incentives in the League of Ireland from 1970 to 2014. *Soccer & Society 18*(5-6), 785–799.

Cason, T. N., D. Friedman, and E. D. Hopkins (2014). Cycles and instability in a rock–paper–scissors population game: A continuous time experiment. *The Review of Economic Studies 81*(1), 112–136.

Clarke, S. R. and J. M. Norman (1995). Home ground advantage of individual clubs in English soccer. *The Statistician 44*(4), 509–521.

Csató, L. (2021). Coronavirus and sports leagues: Obtaining a fair ranking when the sason cannot resume. *IMA Journal of Management Mathematics 32*(4), 547–560.

Elo, A. E. (1978). *The Rating of Chessplayers, Past and Present*. London Batsford.

FIFA (2018). Revision of the FIFA/Coca-Cola World Ranking. *https://img.fifa.com/image/upload/edbm045h0udbwkqew35a.pdf*.

Guedes, J. C. and F. S. Machado (2002). Changing rewards in contests: Has the three-point rule brought more offense to soccer? *Empirical Economics 27*(4), 607–630.

Hankin, R. K. (2020). A generalization of the Bradley–Terry model for draws in chess with an application to collusion. *Journal of Economic Behavior & Organization 180*, 325–333.

Hon, L. Y. and R. A. Parinduri (2016). Does the three-point rule make soccer more exciting? Evidence from a Regression Discontinuity Design. *Journal of Sports Economics 17*(4), 377–395.

Hvattum, L. and H. Arntzen (2010). Using ELO ratings for match result prediction in association football. *International Journal of Forecasting 26*(3), 460–470.

Kaiser, B. (2019). Strategy and pardoxes of Borda count in Formula 1 racing. *Decyzje* (31), 115–132.

Krumer, A. and M. Lechner (2017). First in first win: Evidence on scheudle effects in round-robin tournaments in mega-events. *European Economic Review 100*, 412–427.

Lasek, J., zoltan Szlávik, and S. Bhulai (2013). The predictive power of ranking systems in association football. *International Journal of Applied Pattern Recognition 1*(1), 27–46.

Peeters, T. (2018). Testing the wisdom of crowds in the field: Transfermarkt valuations and international soccer results. *International Journal of Forecasting 34*, 17–29.

Peeters, T. and J. C. van Ours (2021). Seasonal home advantage in English professional football; 1974-2018. *De Economist 169*, 107–126.

Poddiakov, A. (2022). *Intransitively winning chess players positions.* Manuscript. DOI:10.48550/arXiv.2212.11069.

Reade, J. J. and J. C. van Ours (2024). Consumer perceptions matter: A case study of an anomaly in English football. *Tinbergen Instituut Discussion Paper 24-23*.

Santulli, T. V. (2006). Using simulations in the mathematics class. *The Mathematics Teacher 100*(4), 258–263.

Smead, R. (2019). Sports tournaments and social choice theory. *Philosophies 28*(4), 1–10.

Spearing, H., J. Tawn, D. Irons, and T. Paulden (2023). Modeling intransitivity in pairwise comparisons with application to baseball data. *Journal of Computational and Graphical Statistics 32*(4), 1383–1392.

Temesi, J., Z. Szádoczki, and S. Bozóki (2024). Incomplete pairwise comparison matrices: Ranking top women tennis players. *Journal of the Operational Research Society 75*(1), 145–157.

Van Ours, J. C. (2024a). No novelty effect but a honeymoon that lasts: On the attendance effects of new football stadiums. *Sports Economics Review 5*, 100029.

Van Ours, J. C. (2024b). They didn't know what they got till the crowd was gone. *Economics Letters 236*, 11615.

You, J. S. (2021). Random actions in experimental zero-sum games. *Journal of Economics and Behavioral Studies 13*(1), 69–81.

Appendix A: Non-transitivity in match outcomes

To illustrate how non-transitive outcomes may occur this appendix provides two types of example. The first one is with a fixed distribution of goal scoring; the second one is based on differences in quality of defense, midfield and attack.

A1. Fixed distribution of goal scoring

Santulli (2006) provided a numerical example of non-transitivity in pairwise comparison of pitchers in baseball showing that average performance is not always a good indicator of winning probability. This example is adjusted to professional football with the situation presented in panel a of Table A.1. The assumption is that each of the three teams has a discrete and fixed distribution of goal scoring, independent of their opponent. Over three games all three teams score a total of five goals but the distribution of these goals is different for each team. For Team A this is 2-3-0, for team B 1-2-2, and for team C 0-4-1.

Table A.1: **Distribution of goal scoring and match outcomes**

a. Goal scoring 3 matches	1	2	3	Total
Team A	2	3	0	5
Team B	1	2	2	5
Team C	0	4	1	5
b. Results 9 matches	Wins	Draws	Losses	Outcomes
A plays B	4	2	3	A defeats B
B plays C	5	1	3	B defeats C
C plays A	4	1	4	Draw

Note: The distribution of goal scoring is $\frac{1}{3} - \frac{1}{3} - \frac{1}{3}$

Since goal scoring is assumed to be independent of the opponent the results are similar to two opponents rolling dice. With three possible outcomes of goal scoring there are nine possible match outcomes. Panel b of Table A.1 summarizes the outcomes over nine matches for every pair of matches. If A plays B two matches will end in a draw, A wins four matches and loses three. Overall A is more likely to beat B than the other way around. Similarly B is more likely to beat C than the other way around. Thus, if the results were transitive A would be more likely to beat C. However, this is not the case. Both A and C are expected to win four matches and draw one. Clearly, the match outcomes are non-transitive.

A2. Differences in defense – midfield – attack

Smead (2019) presented a numerical example showing that a non-transitive relationship in outcomes of matches between three tennis players is possible. This

relationship is based on differences between two opponents in terms of strength of various dimensions of tennis play, such as serving, returning, and volleying. Unlike tennis, football is a sport in which scoring is not very frequent. And, unlike tennis, in football, a match may end up in a draw when both teams scored an equal number of goals or no goals at all. The numerical example of Smead (2019) on tennis is adjusted to an example that is more fitting for football. In football, a match between two teams is won by the team that scores the most goals. If there is an equal score, the match ends in a draw.

The results of three football teams playing against each other are compared. Teams are assumed to differ in terms of the quality of attack, defense, and midfield, but the average quality of 6 is assumed to be the same for the three teams. Panel a of Table A.2 gives an overview of these assumptions.

Table A.2: **Three dimensions of quality of teams and match outcomes**

a. Quality	Attack	Defense	Midfield	Average
Team A	6	7	5	6
Team B	7	5	6	6
Team C	5	6	7	6
b. Match	Expected goals		Outcome	
A plays B	A: 2	B: 0.5	A defeats B	
B plays C	B: 2	C: 0.5	B defeats C	
C plays A	C: 1	A: 0	C defeats A	

Team A has its best players in attack, team B in defense, and team C in midfield. The assumption is that the score of team i against team j depends on the quality of the attack of team i and the defense of team j, as well as on the relative quality of the midfield players of the two teams. The model specification is chosen to generate non-transitive match outcomes. The number of goals scored by team i against team j is given by:

$$S_{ij} = \text{Max}(0, 2 * (A_i - D_j)) + \text{Max}(0, (0.5 * (M_i - M_j))) \tag{8}$$

There are three parts in Eq (8). The first part shows how the differences in attack of team i and defense of team j are transferred into goal scoring. This only occurs if the attack is stronger than the defense. In such a case, the number of goals is equal to twice the difference in quality. The second term specifies goal scoring from midfield. If the difference in the quality of the midfield is equal to 1, goal scoring equals 0.5. Panel b of Table A.2 shows how many goals are scored in the match between the two teams. Clearly, the match results are non-

transitive: team A defeats B, B defeats C, and C defeats A. If rounded to discrete numbers above, A beats B with 2-1, B beats C with 2-1, and C beats A with 1-0. Obviously, the example presented is limited as there is more to football than the difference in the quality of defense, midfield, and attack. Factors such as style of play, luck, referee decisions, and more also play significant roles. How well a team performs is a combination of physical strength, technical skills, and tactics. Perhaps Feyenoord is a bit more physical, focusing on a strong defense; PSV more technical, focusing on a strong attack; and Ajax more tactical, aiming for a strong midfield. However, this interpretation is not set in stone. The main issue is that if quality is multidimensional, a non-transitive outcome may be a real phenomenon occurring over a long period of time.

Appendix B. Bookmaker data and Elo-based match outcome expectations

Bookmaker data on football matches are available from season 2000/01 onward. Table B.1 provides descriptive statistics. The first column gives the actual results in terms of cumulative points. Columns (2) and (3) summarize the match predictions based on Elo-ratings and bookmaker odds. The last two columns present the surprise points, i.e., the difference between actual points and expected points.

Table B.1: **Actual and expected cumulative match points;2000/01-2022/23**

	Actual	Expected		Surprise points		
	points	Elo	BM	Δ	Δ	
	(1)	(2)	(3)	(1)-(2)	(1)-(3)	N
a. Feyenoord-PSV						46
Feyenoord	61	48	52	13	9	
PSV	67	78	73	-11	-6	
Balance	-6	-30	-21	24	15	
b. PSV- Ajax						46
PSV	69	62	61	7	8	
Ajax	57	64	65	-7	-8	
Balance	12	-2	-4	14	16	
c. Ajax-Feyenoord						45
Ajax	88	77	75	11	13	
Feyenoord	34	47	49	-13	-15	
Balance	54	30	26	24	28	
d. Total balance	60	-2	1	62	59	137

Note: In the calculation of the match points three points are allocated to a win and one point to a draw. Expected points are based on probabilities based on differences in Elo-ratings (Elofootball.com) and bookmaker odds (BM) from Interwetten (Football-data.co.uk).