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Agreeing on public goods or bads

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Abstract

Without regulation or agreement, public goods are underprovided and public bads are overprovided. Both problems are usually seen as flip sides of the same coin. In this paper we examine a situation where a public good is good for some agents but bad for others, and this preference is endogenous to the provisioning level of the good. We allow agents to form a coalition to coordinate provision. Compared with games with only goods (or only bads) we find larger coalitions in equilibrium. Specifically, we analyze a game with quadratic benefit- and cost functions and we find the grand coalition to be stable except for situations where agents have identical or almost identical characteristics. The main driving force of coalition stability is that cooperation avoids a wasteful contest between agents pulling the provision level in opposite directions. We show that, in equilibrium, wasteful contest is confined to a narrow range of the parameter space of our game. This result connects the literatures on public goods and contests.

Keywords: Public goods or bads; Cartel games; Coalition stability; Contests; Geo-engineering

JEL classification: C72; D02; Q20

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1 Introduction

Even a century after Lindahl (1919) published his path-breaking book, private provision of public goods remains a thriving field of research. In this paper we examine the private provision of a public good that is appreciated by some but disliked by others. We do so using a game in which agents can join a binding agreement to coordinate public good provisioning. Our model set-up follows a strand of literature that has emerged under the header of *international environmental agreements* and deals with international cooperation for the provision of global public goods such as mitigating climate change (Carraro and Siniscalco, 1993; Barrett, 1994). The formal analysis of public goods that can be good for some while bad for others has been pioneered in more recent work by Weitzman (2015) who discusses the potential role of geo-engineering in climate policy-making. Geo-engineering measures, like e.g. stratospheric aerosol injection, aim at global cooling. Such measures can be adopted by individual countries at fairly low cost but are potentially undesirable for other countries. Hence, global cooling is perceived as a public good by some while it is perceived as a public bad by others (Heyen et al., 2019; Rickels et al., 2020).

Many examples of a similar structure can be found in the domains of public and environmental policy-making where attitudes with respect to local conditions or environmental standards differ. A road cutting through a forest can be a public good for commuters while being a public bad for conservationists. Nature conservation and agriculture are frequently pulling in opposite directions when it comes to water management or the intensity of farming. The return of wolves to some European countries such as Germany or the Netherlands is appreciated by conservationists but opposed by sheep farmers. Generally, all tax-financed public goods may be seen as overprovided by taxpayers facing a high tax rate or having a low preference for the public good. Conversely, taxpayers with low tax rates or a strong preference for the public good would see them as underprovided. Hence, increased provision would be a public good for the latter group and a public bad for the former. Another example is the level of public security if higher levels come with more extensive policing. Preferred levels may differ between citizens when more police in the streets is appreciated by some but not by others.

We study such situations where a public good can be good for some and bad for others in a setting with private provision of the public good. We borrow terminology from Weitzman (2015) and refer to such goods as *gobs* (goods or bads). Importantly, we consider the case where individual assessments of a gob as good or bad depend on the level of provision. To fix ideas, consider a group of people in a room where each can turn on the heating or open the window for cooling. As the room gets warmer, fewer people prefer additional warming and more people will start to consider the warmth a public bad. Notice that the non-cooperative equilibrium in this situation might be that some people turn on the heating while others open the window, which is grossly inefficient. We refer to such situations as wasteful contest.

In this paper we examine a model of private provision of public gobs where agents can form coalitions. A coalition would internalise the externalities associated with the provision of the public gob and avoid wasteful contest within the coalition. Our interest is to see whether such coalitions can be stable and would help to restrain wasteful contest, also with agents outside the coalition. To obtain sharp results we use a model specification where agents have quadratic benefits with respect to the public gob level and quadratic costs with respect to their own contribution. This specification allows us to derive a closed-form solution for coalition stability. It is also one of the few specifications for which analytical results are available in the literature that considers only public goods (Carraro and Siniscalco, 1993; Barrett, 1994). This allows us to compare public gobs with public goods. Moreover, it is the simplest specification that features non-orthogonal response functions (so that we do not presuppose dominant strategies) while remaining tractable in a setting with heterogeneous agents.

Our results show that the public gobs game has features of a contest. We find larger coalitions compared to public goods games where everyone appreciates the good. Specifically, in Proposition 1 we find the grand coalition to be stable with the exception of situations where agents have identical or almost identical characteristics. This result is driven by our second result that we present in Proposition 2: coalition formation avoids to a large extent wasteful contest between agents pulling in opposite directions.

2 Contribution to the literature

In this paper we connect four strands of literature: (i) The private provision of public goods, (ii) coalition formation for public goods provision, (iii) the provision of public gobs, and (iv) contests.

The analysis of the private provision of public goods is generally framed as a game between agents who derive utility from a public good and a composite numeraire good. Agents are budget constrained and the public good is available at constant prices. Warr (1983) and Bergstrom et al. (1986) are seminal works showing that the unique Nash-equilibrium provision level is not affected by a redistribution of income as long as the set of contributing agents is not affected. Although we study the private provision of public goods, we do not assume constant prices but rather increasing marginal costs of provision. In this setting, a change in the contribution of one agent is usually not completely offset by changes in other agents' contributions.

A second strand of literature is motivated by the general underprovision of transboundary or global public goods. This literature studies the stability of coalitions between countries that are characterised by their costs of provision and their benefits derived from the public good. A key concern of this literature is to spell out determinants of the size of stable coalitions and their effectiveness in terms of the provision of public goods. Coalitions are formed to overcome the inefficiently low provision level in a Nash equilibrium of a game with potentially many players. A coalition is stable when no member has an incentive to leave and no non-member has an incentive to join. In general, larger coalitions would provide more of the public good and a grand coalition would provide the efficient amount. In this literature, a two-stage game is a workhorse model. In stage 1 countries announce whether or not they join the coalition. In stage 2 the coalition members coordinate public good provisioning to maximise their joint net benefits in a game with non-members (Carraro and Siniscalco, 1993; Barrett, 1994). We contribute to this literature (see Benchekroun and Long (2012) for a survey) by generalising the preferences of agents by considering public gobs.

Third, we contribute to a small recent literature that addresses the provision of public gobs. Theoretical work in this domain by Buchholz et al. (2018) extends the private provision of public goods literature, considering utility maximising agents who face a given price of the public gob and thus constant marginal costs of provision. Their model considers two groups of agents; for one group more of the gob is always preferred, while the other always prefers a lower level of provision. Our model differs from this approach in two ways. First, we assume convex costs of provision and, second, our agents are not exogenously grouped into beneficiaries and victims of public gob provision. In our model it depends on the level of provision whether an agent prefers to have more or less of the gob. Close to our paper is some recent work on the implications of geo-engineering options to combat climate change. In particular Weitzman (2015) considers asymmetric damage from a certain gob level where for any agent having too much may be more (or less) costly than having too little. Weitzman does not address the issue of private provision of the gob but suggests a voting mechanism that would implement an efficient gob level and avoid wasteful contest. Barrett (2008), Heyen et al. (2019), Rickels et al. (2020) and Ghidoni et al. (2023) also take issue with geo-engineering, emphasising that measures taken by some countries to stabilise the climate could be opposed by others who differ in their assessment of the benefits and point to the potential dangers of the measures. None of these papers offers a comprehensive analysis of coalition formation in the public gobs game.

Finally, our paper connects the literature on public goods provisioning with the literature on contests. In a standard model of contests, a prize is allocated among agents who can exert costly effort to increase the probability of receiving the prize (Tullock, 1980; Rosen, 1986). Specific contest designs where effort contributes to the value of the prize, but does not affect other contestants' probabilities of winning, resemble the problem of public good provisioning (Konrad, 2009). Such similarities between public goods provisioning and contests have been noticed before (see e.g. Gradstein, 1993; Chung, 1996; Baik, 2016). Furthermore, contest games with alliance formation (Garfinkel, 2004) are close to our game-theoretic setup. When some agents prefer more while others prefer less of the public gob — choosing positive and negative provision levels, respectively — our game can be interpreted as a contest with offensive vs. defensive activities as in Grossman and Kim (1995). Combining this game feature with alliance (or coalition) formation describes a mechanism to prevent dissipation of the prize, which is one of the main concerns in the literature on contests.

3 The gob model

Consider a set $N = \{1, \ldots, n\}$ of agents who have single-peaked preferences with respect to a gob. There is a uniform level of the gob $G \in \mathbb{R}$ to which all agents are exposed. We label as $B_i \in \mathbb{R}$ agent i's preferred level (or bliss level) of the gob. If $B_i > G$, then an increase of G is a public good for agent i. If, however, $B_i < G$, it is a public bad for i. Let $(B_1, \ldots, B_n)$ be the distribution of preferred gob levels and, without loss of generality, we order agents by their bliss levels such that $B_1 \leq \ldots \leq B_n$. Each agent can contribute to the public gob. We denote agent i's contribution by $g_i \in \mathbb{R}$. Note that we allow for negative contributions. Positive (negative) contributions will increase (decrease) the public gob level and contributions are assumed to be additive. Let G_0 be the default level of the gob when no agent takes action, i.e., $g_i = 0$ for all i. The gob level obtained through individual contributions is

$$G = G_0 + \sum_{i \in N} g_i. \tag{1}$$

In what follows we normalise the game such that $G_0 = 0$. We denote aggregate contributions of any subset of agents $S \subseteq N$ as $g_S \equiv \sum_{i \in S} g_i$ and similarly for aggregates of benefits, costs, and payoffs, defined below.

Agents derive benefits from the gob level and incur costs from their own (positive or negative) gob contributions:

$$b_i(G) = -\frac{1}{2}G^2 + \beta_i G + \delta_i, \tag{2}$$

$$c_i(g_i) = \frac{1}{2}g_i^2. \tag{3}$$

As discussed in the introduction, both functions are quadratic. Agents are heterogeneous in terms of their exogenous benefit function parameter β_i. The second benefit function parameter, δ_i, is included to facilitate Examples 1 and 2 below but will drop out in the analysis (and we generally use $\delta_i = 0$ to economize on notation). Benefits peak at $B_i = \beta_i$ while costs have a unique minimum at $g_i = 0$ when contributing nothing. This model specification in which agents only differ in bliss points allows us to obtain sharp analytical results. In Section 6 we will present simulations for a more general model specification that also allows for heterogeneity in costs.

Payoffs are given by

$$\pi_i(g_i, G) = b_i(G) - c_i(g_i). \tag{4}$$

To build intuition, consider the following two-player example.

Example 1. Consider two agents $i = 1, 2$ with bliss points $B_1 = \beta_1 = -1$ and $B_2 = \beta_2 = 1$. Their benefit functions are $b_i(G) = -\frac{1}{2}(G - \beta_i)^2$ and their cost functions are $c_i = \frac{1}{2}g_i^2$. This is a symmetric game in the following sense. Evaluated at the default gob level where $G_0 = 0$, the agents are facing similar marginal costs of contributing to the gob while their marginal benefits are diametrically opposed. Any contribution by agent 1 will be negative while any contribution by agent 2 will be positive, in order to pull the gob level in the direction of their respective bliss levels. In the Nash equilibrium the agents make contributions $g_1 = -1$ and $g_2 = 1$. The resulting gob level is $G = G_0 = 0$ and associated payoffs are $\pi_1 = \pi_2 = -1$. A cooperative agreement in which each agent would reduce her contribution to $g_1 = g_2 = 0$ would yield the same gob level while saving costs. With payoffs $\pi_1 = \pi_2 = -\frac{1}{2}$, it would be advantageous for both.

Example 1 demonstrates that the public gobs game may have features of a contest. Such contest occurs whenever for two players i, j we have $B_i < G_0 + g_{N \setminus \{i,j\}} < B_j$. In such situations, agent i would make a negative contribution while agent j would make a positive one. A coalition formed by agents i and j would avoid a wasteful contest.

4 Coalition formation

We consider the formation of a single coalition $S \subseteq N$. Our game is based on the standard two-stage coalition formation game (see Barrett, 1994; Hagen et al., 2020), often referred to as a cartel game. In the first stage, agents decide whether to join the coalition $S \subseteq N$. We consider an open membership game, that is, all agents who decide to join will be coalition members $i \in S$ and act jointly in the second stage. Agents who do not join are singleton agents $i \in N \setminus S$. Denote this set of singleton agents by $\bar{S}$. In the second stage, the coalition and the singleton agents play a simultaneous-move game of public gobs provision. Since we assume quadratic cost- and benefit functions, the public gobs game has a unique equilibrium.

This equilibrium gob level when coalition S forms is implicitly given by the system of equations

$$\sum_{j \in S} b_j'(G) = c_i'(g_i), \text{ for all } i \in S; \tag{5a}$$

$$b_i'(G) = c_i'(g_i), \text{ for all } i \in \bar{S}. \tag{5b}$$

Solving for contributions, this system gives the following response functions for members and singletons, respectively.[1]

$$g_i = \frac{\sum_{j \in S} \beta_i - s \sum_{j \in N_{-i}} g_j}{s+1} \quad \text{for all} \;\; i \in S, \tag{6a}$$

$$g_i = \frac{\beta_i - \sum_{j \in N_{-i}} g_j}{2} \quad \text{for all} \;\; i \in \bar{S}. \tag{6b}$$

Full cooperation means that the grand coalition $S = N$ is formed, in which case Condition (5a) is the Samuelson condition for the efficient provision of public goods.

Equilibrium uniqueness allows us to define payoffs in terms of the coalition formed. That is, we write the payoff function as a cartel-partition function that uniquely defines a payoff $V_i(S)$ for every singleton agent $i \in \bar{S}$ and a coalition payoff $V_S(S)$ for any coalition S that may form. We allow for transfers between coalition members and assume that transfers are arranged to stabilise a coalition if possible (see e.g. Carraro et al., 2006; Weikard, 2009), such that

$$V_i(S) \geq V_i(S_{-i}) \;\; \text{if and only if} \;\; V_S(S) \geq \sum_{i \in S} V_i(S_{-i}) \text{ for all } i \in S. \tag{7}$$

Condition (7) implies that if coalition S does not earn enough to cover the outside-option payoffs $V_i(S_{-i})$ of its members, no member will receive her outside-option payoff in that coalition. It is thus not advantageous for any agent to join a coalition that would not earn at least the sum of the outside-option payoffs. We can now define the concept of coalition stability.

Definition 1. A coalition is stable if it is internally and externally stable.

1. If transfers are arranged according to Condition (7), a coalition S is internally stable if and only if $V_S(S) \geq \sum_{i \in S} V_i(S_{-i})$.

2. If transfers are arranged according to Condition (7), a coalition S is externally stable if there is no agent $j \notin S$ such that $V_j(S_{+j}) \geq V_j(S)$.

Definition 1 says that a coalition is internally stable if it can guarantee that each member receives at least her outside-option payoff. It is externally stable if no singleton agent has an incentive to join as she would earn less than her outside-option payoff. The following general result follows from Definition 1. It links internal and external stability and we will use it in discussing Example 2 below, as well as in the proof of Proposition 2.

Lemma 1. (*Weikard, 2009*). *Coalition S is externally unstable, i.e. there is an agent $j \notin S$ who prefers to join S over being a singleton agent, if and only if the enlarged coalition S_{+j} is internally stable.*

[1] In the following we use the shorthand notation S_{-i} and S_{+i} for $S \setminus \{i\}$ and $S \cup \{i\}$, respectively.

We can describe the coalitional preferences by the sum of the benefits of the members. A coalition can be characterised by the gob level B_S that maximizes $\sum_{i\in S} b_i(G)$. Because we use quadratic benefit functions, the coalitional benefit function is also quadratic. It follows that B_S is unique and lies strictly between the bliss levels of the members with the lowest and the highest index number in S. In our specification we simply have $B_S = \frac{1}{s}\sum_{i\in S}\beta_i$.

In general, coalition formation will change the gob level. A case like Example 1, where coalition formation prevents wasteful contest but does not change the gob level, is a special case. If the formation of coalition S changes the gob level to the advantage (disadvantage) of agent $k \notin S$, we will say that the formation of S has a positive (negative) spillover effect on agent k. Intuitively, negative spillovers are conducive to the formation of larger coalitions. An agent, by joining the coalition, can impact the coalition's provision level to her advantage or benefit from the transfers provided to stabilise the coalition. For symmetric public goods games with negative spillovers, Yi (2003, Proposition 5.1) finds the grand coalition to be the unique stable coalition. Positive spillovers, by contrast, hamper coalition formation as they generate free-rider incentives. In our game, however, spillovers can be beneficial for some agents and harmful for others. For such public gobs games, Proposition 2 shows that only positive (or only negative) spillovers remain in equilibrium, i.e., when a stable coalition is formed. Exceptions to this result occur only in a small part of the parameter space as we will discuss.

The following example illustrates the above discussion.

Example 2. Consider a game with four players and quadratic benefit- and cost functions. Let their bliss levels be at $B_1 = 1, B_2 = 2, B_3 = 3, B_4 = 4$ and normalise the benefit functions such that $b_i(B_i) = 1$.[2] The example is illustrated in Figure 1. We solve the system of equations (6) and calculate equilibria for all possible coalitions. We report the results in Table 1.

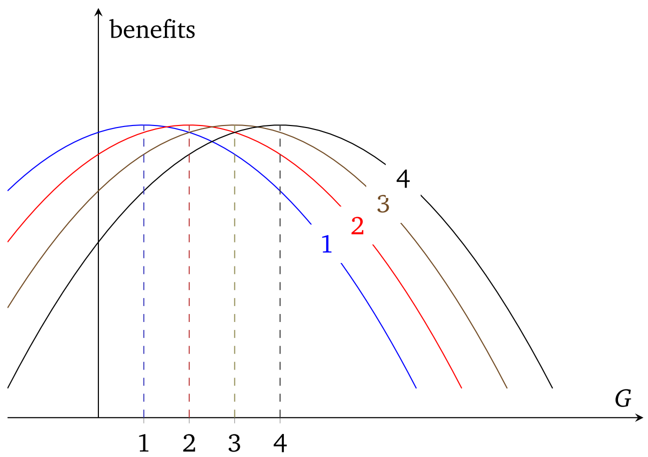

Figure 1: Benefit functions for four agents with bliss points at $G = 1, 2, 3, 4$.

[2] $\beta_1 = 1, \beta_2 = 2, \beta_3 = 3, \beta_4 = 4, \delta_1 = \frac{1}{2}, \delta_2 = -1, \delta_3 = -\frac{7}{2}, \delta_4 = -7$.

Table 1: Provision levels and payoffs for all possible coalitions based on Example 2.

S	g_1	g_2	g_3	g_4	G	V_1	V_2	V_3	V_4	V_S	V_{total}	V_o	IS^*
∅	−1	0	1	2	2	0	1	0	−3	–	−2	–	–
12	−0.71	−0.71	1.14	2.14	1.86	0.38	0.73	−0.31	−3.59	1.11	−2.79	1	0.11
13	0	0	0	2	2	0.5	1	0.5	−3	1	−1	0	1
14	0.71	−0.14	0.86	0.71	2.14	0.09	0.98	0.27	−0.98	−0.89	0.36	−3	2.11
23	−2.14	0.71	0.71	1.86	1.14	−0.31	0.73	0.38	−2.45	1.11	−1.64	1	0.11
24	−1.29	1.43	0.71	1.43	2.29	−0.65	−0.06	0.49	−1.49	−1.55	−1.71	−2	0.45
34	−1.43	−0.43	2.14	2.14	2.43	−1.04	0.82	−1.46	−2.53	−3.99	−4.21	−3	−0.99
123	0	0	0	2	2	0.5	1	0.5	−3	2	−1	0.39	1.61
124	0.45	0.45	0.82	0.45	2.18	0.2	0.88	0.33	−0.76	0.32	0.65	−3.27	3.59
134	0.91	−0.36	0.91	0.91	2.36	−0.34	0.87	0.38	−0.75	−0.71	0.16	−3.78	3.06
234	−1.55	1.36	1.36	1.36	2.55	−1.39	−0.08	−0.03	−0.99	−1.10	−2.47	−1.14	0.04
1234	0.59	0.59	0.59	0.59	2.35	−0.09	0.76	0.62	−0.53	0.76	0.76	−3.19	3.95

* $IS = V_S - V_o$, indicates internal stability if (weakly) positive. V_o refers to the sum of members' outside option payoffs.

Example 2 reveals some interesting differences between public gobs and public goods. A well-known result from the literature on coalitions with public goods is that in a specification with symmetric agents and quadratic benefit- and cost functions (and hence positive spillovers), the equilibrium coalition size will not be larger than 2 (Finus, 2001). Table 1 shows that in the case of asymmetric agents, this result does not hold as the game becomes a gobs game. The last column shows that all coalitions except $S = \{3, 4\}$ are internally stable. By Lemma 1, all coalitions except the grand coalition are externally unstable. As a result, only the grand coalition is stable. Notice that if coalition $S = \{3, 4\}$ is formed, members will increase the gobs level and induce negative spillovers to agents 1 and 2. In response, agents 1 and 2 will, as singletons, increase their countermeasures, thus exacerbating the wasteful contest. Next, we can see that both agents 1 and 2 have an incentive to join $\{3, 4\}$. If agent 2 joins and coalition $\{2, 3, 4\}$ is formed, the gobs provision by these agents increases and so do the negative spillovers to agent 1 who increases, in turn, the countermeasures. Again the wasteful contest can be avoided if agent 1 joins the coalition and the grand coalition is formed.

5 Analysis

In this section, we derive analytical results on coalition stability for the public gobs game as well as associated levels of wasteful contest. We do so employing replacement functions that have been introduced in the literature on aggregative games (Cornes and Hartley, 2007; Cornes, 2016). The key idea of this approach is to write agent i's contribution to the public good not as a response function, i.e., a function of other agents' contributions, but as a function of the total contribution of all agents including i. For our game such replacement functions for singletons and members can be obtained from the FOCs when maximising (4).

We obtain

$$g_i = \sum_{j \in S} \beta_j - sG \quad \text{for all} \quad i \in S, \tag{8a}$$

$$g_i = \beta_i - G \quad \text{for all} \quad i \in \bar{S}. \tag{8b}$$

Aggregating all contributions given by (8) and solving for G we obtain the equilibrium provision level

$$G(S) = \frac{s \sum_{i \in S} \beta_i + \sum_{i \in \bar{S}} \beta_i}{s^2 - s + n + 1}. \tag{9}$$

Because of the aggregative structure of the game, the provision level, the coalition payoff, and the sum of the outside-option payoffs only depend on aggregates of the benefit parameters of the agents. We will exploit this feature of the game in the analysis of coalition stability below.

To identify internally stable coalitions we need to check whether the coalition payoff $V_S(S)$ is sufficient to cover the sum of the outside-option payoffs $V_o(S) \equiv \sum_{i \in S} V_i(S_{-i})$. We call $\Phi(S) \equiv V_S(S) - V_o(S)$ the stability function which, if weakly positive, indicates the internal stability of S; see Definition 1. To construct the stability function, notice that a deviation of agent $i \in S$ such that coalition S_{-i} is formed will change the equilibrium provision level from $G(S)$ to $G(S_{-i})$, which we construct using (9):

$$G(S_{-i}) = \frac{(s-1) \sum_{j \in S_{-i}} \beta_j + \sum_{j \in \bar{S}_{+i}} \beta_j}{s^2 - 3s + n + 3}. \tag{10}$$

Using the equilibrium provision levels (9) and (10), we can derive both terms of the stability function. Skipping the summation index for notational ease we obtain

$$\begin{aligned} V_S(S) = & \left(\sum_S \beta \right)^2 \left(\frac{2ns^2 - sn^2 + s}{2(s^2 - s + n + 1)^2} \right) \\ & + \sum_S \beta \sum_{\bar{S}} \beta \left(\frac{(s^2 + 1)(n - s + 1)}{(s^2 - s + n + 1)^2} \right) \\ & - \left(\sum_{\bar{S}} \beta \right)^2 \left(\frac{(s^3 + s)}{2(s^2 - s + n + 1)^2} \right) \end{aligned} \tag{11}$$

and

$$
\begin{aligned}
V_o(S) = &\left(\sum_S \beta\right)^2 \left(\frac{(s-1)(s^2-3s+2n+2}{(s^2-3s+n+3)^2}\right) \\
&+ \sum_S \beta \sum_{\bar{S}} \beta \left(\frac{2(n-s+1)}{(s^2-3s+n+3)^2}\right) \\
&- \left(\sum_{\bar{S}} \beta\right)^2 \left(\frac{s}{(s^2-3s+n+3)^2}\right) \\
&- \sum_S \beta^2 \left(\frac{-7+n^2+10s-3s^2-2s^3+s^4+2n(-1-s+s^2)}{2(s^2-3s+n+3)^2}\right).
\end{aligned}
\tag{12}
$$

Our first main result, stated in Proposition 1, assesses the internal stability of the grand coalition (GC). Since the GC cannot be enlarged, external stability does not apply and internal stability implies stability. The proposition demonstrates that the grand coalition is stable except for situations where agents have identical or almost identical characteristics. After proving and illustrating the proposition we derive a number of additional insights.

Proposition 1 (Stability of the Grand Coalition)**.** *In a quadratic public gobs coalition formation game the stability condition for the grand coalition is*

$$
\Phi(N) \geq 0 \iff \sum_N \beta^2 \geq \frac{1}{n}\left(\sum_N \beta\right)^2 \left(\frac{n(n^5-2n^3+4n^2-3n-4)}{(1+n^2)(n^4-4n^2+8n-7)}\right).
$$

The grand coalition is stable for a large range of distributions of bliss points and is only unstable when agents are (almost) symmetric.

Proof. For the grand coalition we have $s = n$ and, since there are no remaining singletons, $\sum_{\bar{S}} \beta = 0$. Therefore the second and third terms of both (11) and (12) cancel. This simplifies the stability condition to

$$
\begin{aligned}
\Phi(N) = &\left(\sum_N \beta\right)^2 \left(\frac{n}{2} - \frac{2(n-1)}{3-2n+n^2} + \frac{-4+7n-4n^2+n^3}{(3-2n+n^2)^2} - \frac{n^3+n^5}{2(1+n^2)^2}\right) \\
&+ \sum_N \beta^2 \left(\frac{1}{2} - \frac{2(2-n)}{3-2n+n^2} + \frac{(n-2)^2}{(3-2n+n^2)^2}\right) \geq 0 \\
\iff &\sum_N \beta^2 \geq \frac{1}{n}\left(\sum_N \beta\right)^2 \left(\frac{n(n^5-2n^3+4n^2-3n-4)}{(1+n^2)(n^4-4n^2+8n-7)}\right).
\end{aligned}
\tag{13}
$$

The term on the LHS and the first two terms on the RHS give Chebyshev's inequality, which always holds. The remaining term on the RHS depends only on n and is always positive; it is larger than 1 for all $n \geq 3$, has a maximum value for $n = 3$ at $\frac{159}{155} \approx 1.026$ and is approaching 1 for increasing n. Therefore the stability condition is violated only if Chebyshev's inequality holds (approximately) with equality, i.e. when agents are (almost) symmetric. □

Proposition 1 is illustrated in Figure 2. Any possible distribution of agents' bliss points is characterised by $\sum_N \beta^2$ and $\left(\sum_N \beta\right)^2$. The area below the 45° line is not feasible because Chebyshev's inequality must hold. On the 45° line Chebyshev's inequality holds with equality. This represents the case of symmetric agents. The area of instability is a narrow wedge between the 45° line and the stability line for some given n, here depicted for $n = 3$ and $n = 10$. Notice that the wedge gets narrower for larger n.

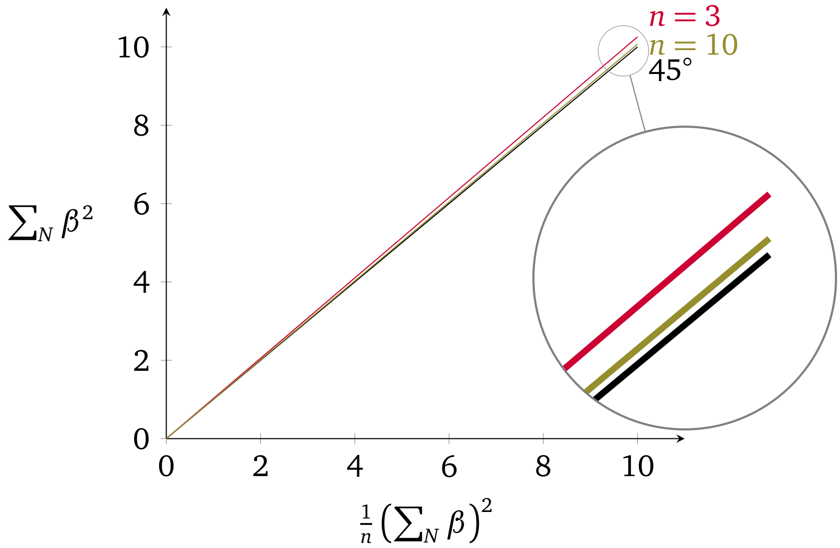

Figure 2: The area of instability is a wedge between the 45° line and the stability line for some given n, here depicted for $n = 3$ and $n = 10$.

In public gobs games, like in public goods games, player heterogeneity helps stabilise larger coalitions. For public goods games this feature has been discussed by Weikard (2009), Pavlova and de Zeeuw (2013) and Finus and McGinty (2019). For public gobs games, however, Proposition 1 shows that the stabilising effect of agent heterogeneity is much stronger, leading to stable grand coalitions in a large part of the parameter space. Only when agents are (almost) symmetric, which makes the public gobs game a public goods game, the GC is unstable. Indeed, it is well-known that such games have no stable grand coalition whenever $n > 2$ (see Section 6).

To see Proposition 1 at work consider the following example.

Example 3. Extending Example 2, consider a game with n players having their bliss points at $B_1 = k + 1$, $B_2 = k + 2$, …, $B_n = k + n$, where $k \geq 0$ is a positive constant. Example 2 is obtained for $n = 4$ and $k = 0$. This uniform distribution allows us to obtain closed-form expressions for $\sum_N \beta^2$ and $\left(\sum_N \beta\right)^2$:

$$\sum_N \beta^2 = \frac{1}{6}\left(n + 6kn + 6k^2 n + 3n^2 + 6kn^2 + 2n^3\right), \tag{14}$$

$$\left(\sum_N \beta\right)^2 = \frac{1}{2}\left(n + 2kn + n^2\right). \tag{15}$$

An increase in k shifts all bliss levels away from the default gob level $G = 0$. As a result, coalition members become 'more similar'. The gob becomes a conventional public good whenever the smallest bliss level becomes sufficiently large: $B_1 = k+1 > G(N)$. Substituting (14) and (15) into stability condition (13), we find a stable GC for any n if $k = 0$. For larger k, however, GC stability breaks down for sufficiently small n. For example, when $n = 3$ ($n = 5$) the threshold value for k where the GC is still stable is 3.08 (7.33). If $k > 6.49$ ($k > 11.64$) only a trivial coalition $s = 1$ is stable and the game is a conventional public goods game.[3]

Corollaries of Proposition 1 follow for two special cases: two-player games (Corollary 1) and games where agents have opposed preferences (Corollary 2).

Corollary 1. *If and only if $n = 2$, the GC is stable regardless of agents' bliss points.*

Proof. Evaluating the stability function (13) for $n = 2$ we obtain the stability condition $\sum_N \beta^2 \geq \frac{1}{2}\big(\sum_N \beta\big)^2 \frac{44}{45}$. This always holds since Chebyshev's inequality requires $\sum_N \beta^2 \geq \frac{1}{2}\big(\sum_N \beta\big)^2$ and the factor $\frac{44}{45} < 1$ makes the RHS even smaller. In the proof of Proposition 1 we have established that there is a 'region' of instability if $n \geq 3$. This region includes the case of symmetric agents located on the 45° line of Figure 2. □

Corollary 2. *If agents' bliss levels are distributed such that $\sum_N \beta = 0$, then the GC is stable for all n.*

Proof. The LHS of the stability function (13) is always (weakly) positive because the bliss level parameters are squared, while the RHS is equal to zero. □

We now turn to examining the internal stability of partial coalitions, i.e., we zoom in on cases where the GC is unstable. The analysis leads to the remarkable result that, in equilibrium, wasteful contest is absent in the largest part of the parameter space. We formally define 'no wasteful effort' and then characterise the parameter space where such wasteful effort is avoided both within the coalition and between the coalition and singletons.

Definition 2. There is no wasteful effort if and only if

1. $g_S \geq 0$ and $g_j \geq 0$ for all $j \notin S$, or
2. $g_S \leq 0$ and $g_j \leq 0$ for all $j \notin S$.

The definition says that all agents are exerting effort in the *same* direction and contest is avoided.

Proposition 2 (No Wasteful Effort)**.** *In a quadratic public gobs coalition formation game wasteful effort does not occur in equilibrium. There are two exceptions. Wasteful effort can occur in equilibrium only if:*

[3] Mathematica code for this example is available upon request.

(i) $n = 3$, $s = 2$, $G(S) < B_S$ ($G(S) > B_S$), and the remaining singleton agent's bliss level is weakly smaller (larger) than and close to $G(S)$.
(ii) $n > 3$, $G(S) < B_S$ ($G(S) > B_S$), at least one singleton agent's bliss level is weakly smaller (larger) than and close to $G(S)$ and the coalition is sufficiently small, i.e., a size-s coalition in a game with n agents lies below the contour in the (n, s)-space that solves the polynomial $\Omega(n, s) = 0$ (given in the Appendix and illustrated by Figure 4).

Proof. The result is proven in the Appendix. □

Proposition 2 says that, generally, in equilibrium all agents exert effort in the same direction. Exceptions can occur for small coalitions (compared to the number of agents) consisting of fairly homogeneous agents while any counteracting singleton agent's bliss level would be close to the equilibrium provision level and thus this agent spends little counteracting effort. Hence, even in cases where wasteful effort occurs, the wasted effort is limited, as can be seen from our proof in the Appendix. For example, for $n = 4$ and $n = 5$, no wasteful effort can occur in equilibrium. For $6 \leq n \leq 14$ wasteful effort can only occur when $s = 1$, the case of a trivial coalition.

The intuition for this result is as follows. Suppose a coalition exerts positive effort, pulling up the level of G, then a singleton agent pulling down could be integrated into the coalition which increases overall payoffs (since wasteful effort is avoided) and stabilises the enlarged coalition. By Lemma 1, a stable enlargement implies that the initial coalition is externally unstable and, therefore, not an equilibrium. This intuition only fails in cases where agents are sufficiently similar such that the cost savings of integrating a counteracting agent are insufficient to stabilise the enlarged coalition.

In the next section we will extend the analysis by introducing a cost parameter to examine how cost heterogeneity helps stabilising larger coalitions. To set a benchmark for this analysis we close this section by showing that a coalition of symmetric agents cannot be stable regardless of the bliss levels of singleton agents. In the next section, however, we will see that this result does not generalise to a model where we introduce a cost parameter.

Proposition 3. *(No Stable Coalition of Symmetric Agents). In a quadratic public gobs coalition formation game no coalition of agents with the same bliss levels can be stable, except for the specific case where members do not make a contribution in equilibrium.*

Proof. We construct the stability function $\Phi(S)$ from (11) and (12). Denote every member's bliss level by β_m, that is, $\beta_i = \beta_m \; \forall i \in S$. The stability function can be rewritten as

$$\Phi(S) = \left(\sum_{\bar{S}} \beta + \beta_m(-1 - n + s) \right)^2 \left(\frac{(s - s^2)\big(-7 + 7s - 11s^2 + 9s^3 - 5s^4 + s^5 + n^2(1 + s) + 2n(-1 - 2s^2 + s^3)\big)}{2(3 + n - 3s + s^2)^2(1 + n - s + s^2)^2} \right).$$

The first factor contains the bliss level parameters. Because it is squared, it is always (weakly) positive. It can be zero and hence a symmetric coalition is just stable in the special case that $\beta_m = \frac{1}{n-s+1}\sum_{\bar{S}}\beta$. This is the case when the equilibrium provision level $G(S) = \beta_m$ and members are indifferent between membership and being a singleton. If the first factor is (strictly) positive, the second factor determines the sign of the stability function. We find that it is negative for all $n \geq 3$ and all $s \geq 2$. That is, the stability condition is violated for all non-trivial coalitions, except when members' contribution is zero. □

In general, no non-trivial ($s \geq 2$) coalition consisting of symmetric agents can be stable regardless of the bliss levels of members and singletons, and regardless of the number of agents and the size of the coalition.

6 Cost heterogeneity

In this section we introduce a more general model specification that allows for heterogeneity not only in benefits but also in costs. Specifically, we add an agent-specific cost parameter $\gamma_i > 0$ and update the cost function (3) to:

$$c_i(g_i) = \frac{\gamma_i}{2} g_i^2. \tag{16}$$

As discussed in Section 5, agent heterogeneity helps stabilising larger coalitions and we expect that adding cost heterogeneity will boost the stability of gobs coalitions even beyond what was shown in Proposition 1. Using cost function (16), we derive generalised versions of the replacement functions (8):

$$g_i = \frac{\sum_{j\in S}\beta_j - sG}{\gamma_i} \quad \text{for all} \quad i \in S, \tag{17a}$$

$$g_i = \frac{\beta_i - G}{\gamma_i} \quad \text{for all} \quad i \notin S. \tag{17b}$$

Using these replacement functions, we repeat the analysis of Section 5 and ultimately obtain generalised terms of the stability function (11) and (12):

$$V_S(S) = \sum_{i\in S}\left(-\frac{1}{2}\big(G(S)\big)^2 + \beta_i G(S) - \frac{1}{2\gamma_i}\Big(\sum_{j\in S}\beta_j - sG(S)\Big)^2\right), \tag{18}$$

$$V_o(S) = \sum_{i\in S}\left(-\frac{1}{2}\big(G(S_{-i})\big)^2 + \beta_i G(S_{-i}) - \frac{1}{2\gamma_i}\big(\beta_i - G(S_{-i})\big)^2\right), \tag{19}$$

with

$$G(S) = \frac{\sum_{i\in S}\frac{1}{\gamma_i}\sum_{i\in S}\beta_i + \sum_{i\in\bar{S}}\frac{\beta_i}{\gamma_i}}{1+\sum_{i\in S}\frac{s}{\gamma_i}+\sum_{i\in\bar{S}}\frac{1}{\gamma_i}}, \tag{20}$$

$$G(S_{-i}) = \frac{\sum_{j\in S_{-i}}\frac{1}{\gamma_j}\sum_{j\in S_{-i}}\beta_j + \sum_{j\in\bar{S}_{+i}}\frac{\beta_j}{\gamma_j}}{1+\sum_{j\in S_{-i}}\frac{(s-1)}{\gamma_j}+\sum_{j\in\bar{S}_{+i}}\frac{1}{\gamma_j}}. \tag{21}$$

Before analyzing the impact of heterogeneity in costs and benefits, we first examine a benchmark case with full symmetry. This gives us a public goods game as a special case.

Taking $\beta_i = \beta$ and $\gamma_i = \gamma$ for all $i \in N$ simplifies (18) and (19) considerably. From (18)–(21) we obtain for a two-player coalition $s = 2$ the internal stability condition $\Phi \geq 0 \Leftrightarrow \frac{2\gamma n+(2+n)^2}{(2+\gamma+n)^2} \geq \frac{n^2+\gamma(-1+2n)}{(\gamma+n)^2}$. Note that it is independent of β. Solving for γ gives $\gamma \geq -4+n+2\sqrt{3-3n+n^2}$ as the condition for a stable two-player coalition, given $n \geq 3$. Further, we examine the stability condition for $s = 3$ and find that it is always negative. Taking derivatives we find $\frac{\partial\Phi}{\partial s} < 0$. Thus, no coalition $s \geq 3$ can be stable. These results have first been reported by De Cara and Rotillon (2003). Thus the finding of Proposition 3 does not generalise to a model with a general cost parameter. But the scope for stable coalitions in a symmetric game is limited to $s = 2$.

Having established this benchmark case, we now proceed to assess how stability changes if we introduce asymmetry in both β and γ parameters. Because of the complexity of (18) and (19), we use simulations to illustrate such changes. Since we employ replacement functions, we can introduce asymmetry simply by using a setting where all agents but one have similar parameters $\beta_i = \beta$ and $\gamma_i = \gamma$ for all $i \in N \setminus j$. This 'odd' agent $j \in S$ is a coalition member with parameters β_j and γ_j. A variation of this parameter suffices to assess the impact of benefit- and cost asymmetry on the internal stability of any coalition S.

In Figure 3 we plot stability of the grand coalition as a function of $s = n \in \{2,\ldots,20\}$ for parameters $\beta_j \in \left\{\frac{1}{2},1,\frac{3}{2}\right\}$ and $\gamma_j \in \left\{\frac{1}{2},1,\frac{3}{2}\right\}$, while normalizing $\beta = \gamma = 1$. The central subplot features the symmetric case where $\beta_j = \beta = \gamma_j = \gamma = 1$ as discussed above. We observe that stability holds for $n = 2$ only. That is, given symmetric parameter values the only stable grand coalition in this subplot occurs for $n = 2$. All three subplots in the middle column of Figure 3 feature $\gamma = \gamma_j = 1$, so that Proposition 1 applies. That is, for β_i parameters that are sufficiently close to symmetry, we find that the grand coalition is unstable. In Figure 3 this occurs in the central subplot where $\beta = \beta_j = 1$. The grand coalition is unstable (i.e., it is in the 'wedge' of Figure 2), except when $n = 2$, in line with Corollary 1. In its two neighbouring subplots where $\beta_j = \frac{1}{2}$, respectively $\beta_j = \frac{3}{2}$, the asymmetry in bliss points leads to stable grand coalitions for all levels of n.

We further verify how asymmetry affects the stability of grand coalitions by comparing the central subplot with its eight neighbours. Moving to the left or right we keep $\beta_j = 1$ but we vary γ_j. This results in only minor changes and our main result holds. For other

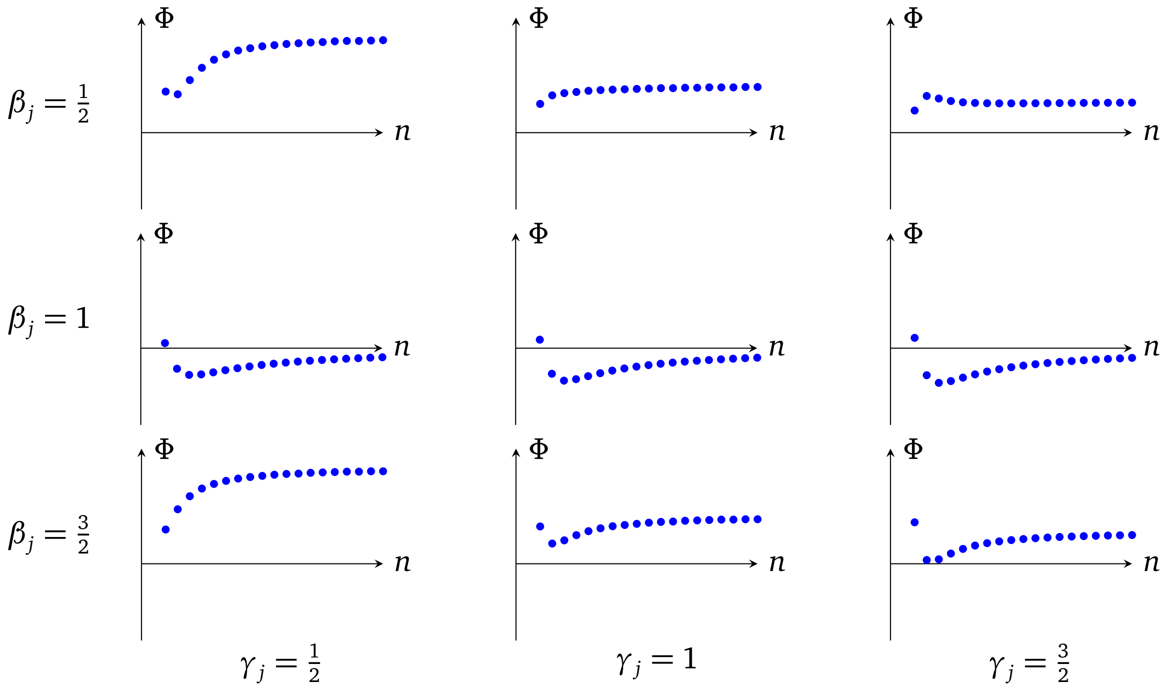

Figure 3: Stability $\Phi(N)$ of the grand coalition as a function of $s = n \in \{2, \dots, 20\}$, using $\beta = \gamma = 1$ and 3×3 combinations of β_j and γ_j for the 'odd' agent.

levels of β_j, such changes in γ_j have more substantial effects, generally leading to increases in stability when γ_j decreases. Moving up and down we keep $\gamma_j = 1$ but we vary β_j. This results in substantial improvements in stability. For other levels of γ_j, such changes in β_j have a similar stabilising effect. We conclude that asymmetry in costs, in particular the presence of a low-cost agent, has the potential to increase coalition stability. Asymmetry in benefits seems to be more powerful in the sense that it does not only affect the absolute value but also the sign of the stability function.

7 Conclusion

In this paper, we offer a comprehensive analysis of the private provision of a public good when additional provision is good for some agents and bad for others. Such goods have been called *gobs* in the recent literature. We study coalition formation in the gobs provision game in a setting with quadratic benefits from public gobs and quadratic costs of provision. We establish two main findings. First, we characterise the class of games where the grand coalition is stable (i.e., an equilibrium outcome) and the provision level is efficient. We find a stable and efficient grand coalition if agents are sufficiently heterogeneous (Proposition 1). Second, we find the remarkable result that, even in cases where the grand coalition is not an equilibrium, a potential contest between agents exerting effort in different directions is generally avoided in equilibrium. In short, if agents are (sufficiently) heterogeneous, they can form a stable coalition which avoids a wasteful contest while, if agents are (sufficiently) homogeneous, coalition stability breaks down, but the similarity of agents implies that all exert effort in the same direction. The cases where moderate wasteful effort may occur lie between these polar cases and are characterised in Proposition 2.

Appendix: Proof of Proposition 2

Proof. To prove Proposition 2 we distinguish two main cases. (i) When the grand coalition is stable, all agents coordinate their actions, wasteful contest is avoided, and the proposition holds. (ii) When the grand coalition is unstable, we need to show whether in equilibrium members and singletons exert effort in the same direction. In this case agents are (almost) symmetric as we know from Proposition 1. We distinguish two sub-cases. (iia) First consider that all agents are fully symmetric. Then, by Proposition 3, we have no stable coalition. However, since agents have the same bliss level, they would exert the same effort and the proposition holds. (iib) The remaining case is when agents have different bliss levels, but not so different that the GC would be stable, i.e., we are inside the wedge depicted in Figure 2. The remainder of the proof deals with this case.

Since it is not generally true that wasteful effort cannot occur in equilibrium, we determine a no-wasteful-effort condition. We do so for the first part of Definition 2 where members exert positive effort, i.e., we assume $g_S \geq 0$ and therefore $B_S \geq G(S)$. Wasteful effort implies that there exists some singleton $j \notin S$ with $g_j < 0$ and therefore $B_j = \beta_j < G(S)$. The proof for the case $g_S \leq 0$ works in the same way and can be skipped.

The strategy to characterise the no-wasteful-effort condition is as follows. We first obtain the internal stability function from equations (11) and (12). Next, we consider constraints on the distribution of bliss levels (β) that need to be satisfied for a coalition $S \subset N$ to be stable. Finally, we distinguish the classes of equilibria (i.e., stable coalitions) in the (n,s)-space for which wasteful effort can and cannot occur. We employ this strategy first for part (i) of the proposition where $n = 3$ and subsequently for part (ii) where $n > 3$.

Combining (11) and (12), the stability function is given by

$$\begin{aligned}\Phi(S) = &\left(\sum_S \beta\right)^2 \left(\frac{(1-s)(2+2n-3s+s^2)}{(3-3s+s^2+n)^2} + \frac{s(1-n^2+2ns)}{2(1+n-s+s^2)^2}\right) \\ &+ \sum_S \beta \sum_{\bar{S}} \beta \left(-\frac{2(1+n-s)}{(3-3s+s^2+n)^2} + \frac{(1+n-s)(1+s^2)}{(1+n-s+s^2)^2}\right) \\ &+ \left(\sum_{\bar{S}} \beta\right)^2 \left(\frac{s}{(3-3s+s^2+n)^2} - \frac{s(1+s^2)}{2(1+n-s+s^2)^2}\right) \\ &+ \sum_S \beta^2 \left(\frac{-7+n^2+2ns^2-2ns-2n+s^4-2s^3-3s^2+10s}{2(3-3s+s^2+n)^2}\right). \end{aligned} \tag{22}$$

Part (i) (n = 3) First, consider $s = 1$ such that the coalition is trivially internally stable. Without loss of generality, we normalise $\sum_S \beta = s$ such that $B_S = 1$. By (9) this implies that $G(S) = \frac{1+\sum_{\bar{S}} \beta}{4}$. Since we assume $B_S \geq G(S)$, we obtain $\sum_{\bar{S}} \beta \leq 3$. Next, assume we have wasteful effort. Then there is one singleton j, with $\beta_j < \frac{1+\sum_{\bar{S}} \beta}{4}$. Denote the other singleton agent by k and combine $\sum_{\bar{S}} \beta = \beta_j + \beta_k$ with the previous inequality to obtain

$\beta_k > 3\beta_j - 1$. We use (22) to calculate stability of the enlarged coalition $S \cup \{j\}$ and write it in the (β_k, β_j)-space: $\beta_k \geq 1 + \beta_j - 3\sqrt{2}\sqrt{1 - 2\beta_j + \beta_j^2}$. This last condition implies $\beta_k > 3\beta_j - 1$ so that if there is wasteful effort for $s = 1$, then there is also an internally stable enlargement of S. By Lemma 1, this implies that we do not find an equilibrium with wasteful effort when $s = 1$.

Next, consider $s = 2$ with only one singleton agent k. Again, we normalise $\sum_S \beta = s = 2$. By (9) this implies that $G(S) = \frac{4+\beta_k}{6}$. For wasteful effort to occur we require $\beta_k < G(S)$, which implies $\beta_k < \frac{4}{5}$. Assuming internal stability of S we use (22) to calculate a minimum value for $\sum_S \beta^2$. When the bliss level of agent k gets closer to $\frac{4}{5}$, we find that $\sum_S \beta^2$ tends to $\frac{51}{25}$ when S is just minimally stable. We find a small range of β parameter values for the two coalition members around $\left(1 - \frac{\sqrt{2}}{10}, 1 + \frac{\sqrt{2}}{10}\right)$ where a two-player coalition is stable and exerts positive effort, while the remaining singleton exerts negative effort and so we have wasteful contest. Integrating the contesting singleton leads to an unstable GC, and hence, by Lemma 1, we find an equilibrium with wasteful effort when $s = 2$.

Part (ii) (n > 3) As before, without loss of generality we normalise $\sum_S \beta = s$ so that the average β of members is $B_S = 1$. Next, we fix $\sum_S \beta^2$ (which captures the degree of heterogeneity of members) at a value such that S is just internally stable, i.e. $\Phi(S) = 0$. This leaves us with an internal stability condition $\Phi(S)$ that depends only on n, s, and $\sum_{\bar{S}} \beta$, i.e., the sum of singleton agents' bliss levels.[4]

Now, we need to determine for which combination of parameters n, s, and β, if any, wasteful effort can occur in equilibrium. Note that stability of S is equivalent to the internal stability of S and, by Lemma 1, the external instability of the enlarged coalition $S \cup \{j\}$ for any singleton $j \notin S$. In light of the latter condition, notice that S is most likely to be stable if S is internally stable and any enlarged coalition $S \cup \{j\}$ is likely to be unstable. The latter is true if agents in $S \cup \{j\}$ and therefore in S are more homogeneous in terms of bliss levels. This is the reason why we can fix $\Phi(S) = 0$ as we did before.[5] For any wasteful effort to occur in equilibrium there must exist a singleton j such that $\beta_j < G(S)$ and at the same time β_j must be sufficiently close to B_S since, when a relatively 'similar' agent joins the coalition, the enlarged coalition is the least likely to be internally stable and thus S is most likely to be stable.

Formally, then, for any internally stable coalition S with $B_S \geq G(S)$ and $j \notin S$ with $B_j = \beta_j < G(S)$ we assess the internal stability of $S \cup \{j\}$ (which is equivalent to the external instability of S). We find

$$\Phi(S \cup \{j\}) = \frac{\text{NUM}}{\text{DEN}} s^2 \left(\sum_{\bar{S}} \beta - n + s - 1 \right)^2, \tag{23}$$

[4]For details and subsequent calculations in the proof, our Mathematica script is available upon request.

[5]If S were more than minimally internally stable, then an enlarged coalition $S \cup \{j\}$ is more likely to be stable which implies that S is more likely to be externally unstable and, thus, not an equilibrium coalition.

where numerator NUM and denominator DEN are higher-degree polynomials containing only parameters n and s. Since the second and third terms of the stability function are always positive, we are left with examining the fraction $\frac{\text{NUM}}{\text{DEN}}$. It can be proven that $\text{DEN} > 0$ for all $n \geq 3$ and $s \geq 1$. For NUM we have

$$\begin{aligned}\text{NUM} = &-3 - n^6 + 2s - 4s^2 + 7s^3 - 10s^4 - 6s^5 + 48s^6 - 49s^7 + 22s^8 - 22s^9 + 8s^{10} \\ &- 5s^{11} + s^{12} - 2n^5(1 - 5s + 2s^2) + n^4(7 - 10s - 12s^2 + 35s^3 - 5s^4) \\ &+ 4n^3(3 - 17s + 27s^2 - 22s^3 - 4s^4 + 10s^5) + n^2(-3 - 48s + 136s^2 - 206s^3 \\ &+ 211s^4 - 168s^5 + 4s^6 + 10s^7 + 5s^8) + 2n(-5 + s + 4s^2 - 28s^3 + 77s^4 - 89s^5 \\ &+ 66s^6 - 56s^7 + 9s^8 - 5s^9 + 2s^{10}). \end{aligned} \tag{24}$$

The pairs (n, s) such that $\text{NUM} \geq 0$ indicate internal stability of the enlarged coalition $S \cup \{j\}$ and thus external instability of coalition S. Such coalitions are not equilibria. Hence, equilibria with wasteful effort can only occur when $\text{NUM} < 0$. As a result, $\text{NUM} \geq 0$ constitutes our no-wasteful-effort condition. Figure 4 gives the contour for $\text{NUM} = 0$ in the (n, s)-space. Points below the contour satisfy $\text{NUM} < 0$, that is we find no stable enlargement of S, thus S is stable while agent $j \setminus S$ exerts negative effort. Conversely, at points above the contour wasteful effort cannot occur. □

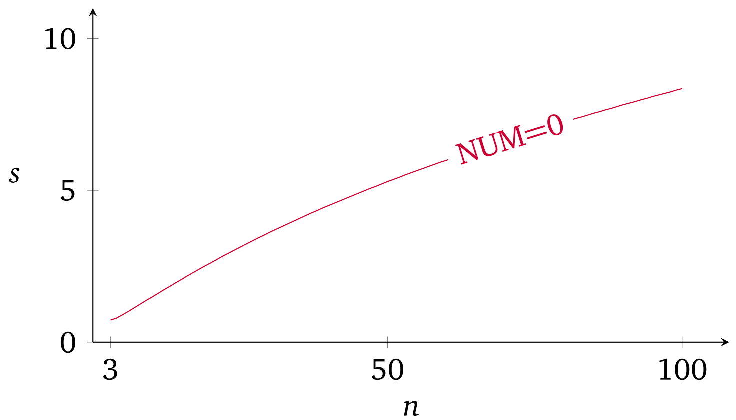

Figure 4: Contour for $\text{NUM} = 0$ in the (n, s)-space for $n > 3$. Points below the contour satisfy $\text{NUM} < 0$ and vice versa.

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