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How to Pollute a River If You Must

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How to Pollute a River If You Must*

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Abstract

We propose the river pollution claims problem to distribute a pollution budget among agents located along a river. A key distinction from the standard claims problem is that agents are ordered exogenously. For environmental reasons, the location of pollution along the river is an important concern in addition to fairness. We characterize the class of *externality-adjusted proportional rules* and show that they strike a balance between fairness and minimizing environmental damage in the river. We also propose two novel axioms that are motivated by the river pollution context and use them to characterize two priority rules. We illustrate the rules through a case study of the Tuojiang Basin in China.

Keywords: Claims Problem, River Pollution, Pollution Permits, Externality-Adjusted Proportional Rules, Priority Rules

JEL classification: D62, D63, C71, Q25

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1 Introduction

In this paper we propose the river pollution claims problem and offer several solutions to it. Agents are ordered along a river and each of them claims to be allowed the discharge of a certain amount of pollution into the river. For environmental conservation reasons, the budget of total permitted pollution is limited and a solution allocates this limited amount of permits. In this way, our model extends the standard claims problem (O'Neill, 1982) by incorporating an exogenous order of the agents, reflecting their position along the river. This extension is inspired by the problem of river water pollution, such as nutrient pollution originating from agricultural production and chemical pollution originating from industrial processes, which cause more aggregate harm the further upstream they are emitted. Our goal is to allocate pollution permits fairly among the involved parties while also keeping the environmental damage from pollution at a minimum.

Water pollution can cause serious health problems and ecological damage. For example, Ebenstein (2012) estimated the impacts of surface water pollution on human health, showing that a deterioration of water quality by one grade (based on a six-grade scale) could cause a 9.7% increase in the digestive cancer death rate. Besides health, polluted water causes ecological imbalance and eco-remediation costs. For example, Camargo and Alonso (2006) showed that acidification and eutrophication of freshwater ecosystems due to nitrogen pollution may cause severe damage to the survival, growth, and reproduction of aquatic animals. Water pollution has become a severe environmental problem and urgently requires effective control measures. Such measures may be hampered by the mismatch between river basins and the administrative borders of jurisdictions in which they are located. Globally, 286 rivers flow across country borders (UNEP, 2016), and many more rivers cross the borders of lower-level jurisdictions like provinces, regions, and municipalities. Due to this mismatch, the management of river pollution is often shared by multiple jurisdictions. The distribution of water pollution between agents is a challenge for which an analysis of the river pollution claims problem can provide possible directions.

The main difference with the standard claims problem is that, in our model, agents are ordered linearly from upstream to downstream, reflecting the direction of river flow, and this order is exogenously given by the hydrological setting. In addition to concerns over the amount of pollution in the river, a major concern is its distribution over the agents. One reason for this concern is the standard fairness consideration that is inherent to claims problems (see e.g. Thomson, 2003). A second reason, which is novel, is that the location of pollution matters for the re-

sulting damage. A given amount of pollution is likely to cause more damage when it is discharged upstream compared to downstream since upstream pollution will cause damage along a longer section of the river. This environmental externality affects total health- and ecological damage in the river and may also cause tensions along it. After presenting the model in Section 2, in Section 3 we will introduce a class of rules that provides a compromise between these aspects of fairness and environmental externalities. Specifically, we characterize the class of *externality*adjusted proportional rules on the domain of river pollution claims problems. It is inspired by the fixed-fraction rules proposed in Gudmundsson, Hougaard, and Ko (2024). Our characterized class seems appropriate in light of the required balance between fairness and pushing permits downstream. The characterization is based on the axioms independence of upstream null claims, budget additivity, redistribution ad*ditivity*, and *merging/splitting proofness*. A common point to these axioms is that they ensure that the rule is invariant across seemingly arbitrary choices on how to define the problem (e.g., whether to allocate permits on a monthly or yearly basis, or whether to allocate at the country, city, or firm level). The axioms are formally defined in Section 3. In Section 4, we provide a numerical example of pollution in the Tuojiang River Basin.

Our paper relates to four separate strands of the literature. First, a series of recent papers is concerned with the allocation of the global carbon budget in order to assess fairness of countries' efforts to mitigate greenhouse gas emissions (Duro et al., 2020; Giménez-Gómez et al., 2016; Heo & Lee, 2022; Ju et al., 2021). Similar to the current paper, these carbon budget papers model a total budget of pollution that is allowed and they are concerned with the distribution of this budget over all countries. The main difference with the current paper is that agents are not ordered, and the location of pollution is not considered.

Second, there is a small literature that focuses on allocating water quantity in river settings using a cooperative game approach (Ambec & Sprumont, 2002; van den Brink et al., 2012) as well as using the claims problem approach (Ansink & Weikard, 2012, 2015; Estévez-Fernández et al., 2021; van den Brink et al., 2014). Similar to the current paper, these papers use a setting where agents are ordered linearly along the river. A key element of these models are the individual inflows ("endowments") of water that originates on each of the agents' territories in the form of rainfall and tributaries. Applying a claims problem in this setting redistributes the existing water resources under a water balance constraint. In the current paper, as in the standard claims problem, there is a single joint endowment, the pollution budget, and we are concerned with the higher environmental impact of upstream pollution compared to downstream pollution.

Third, a fairly recent literature is concerned with the distribution of welfare due to river cleaning (Gengenbach et al., 2010; Gudmundsson & Hougaard, 2021; Steinmann & Winkler, 2019; van der Laan & Moes, 2016) or the sharing of river water treatment costs (Alcalde-Unzu et al., 2015; Li et al., 2023; Ni & Wang, 2007; van den Brink et al., 2018). Ni and Wang (2007) pioneered this literature with an analysis of how to share the costs of cleaning a river among different agents. While many papers are concerned about the economics of river pollution in terms of the distribution of costs for a given pollution abatement level or the distribution of welfare for the efficient pollution level, we are not aware of papers that focus on the distribution of pollution in the river. Such allocation of a pollution budget is underlying the costs of pollution abatement and the resulting welfare levels.

Fourth, a few papers propose water quality trading as a cost-effective policy instrument to manage externalities due to water pollution (Farrow et al., 2005; Hung & Shaw, 2005; Nguyen et al., 2013). Because of market frictions, such as transaction costs, the initial distribution of pollution permits may impact the effectiveness of this instrument in practice. The class of *externality-adjusted proportional rules* that we propose in this paper offers an attractive starting point for the distribution of permits under a water quality trading system. While this rule potentially causes the most downstream agent to be allocated more permits than his claim, under water quality trading this agent can simply trade any excess permits.

The paper is organized as follows. In the next section, we introduce the river pollution claims problem. In Section 3, we characterize the class of *externality-adjusted proportional rules*. In Section 4, we illustrate a case. In Section 5, we present concluding remarks.

2 The River Pollution Claims Problem

In this section, we introduce the *river pollution claims problem*. This problem adds structure to the well-studied "claims/bankruptcy problems" (O'Neill (1982); and see Thomson (2015, 2019) for surveys of this literature) through the natural order in which agents (countries, regions, cities, ...) are located along the river. The resource to be divided is a budget of pollution permits. As in the bankruptcy problem, agents hold uncontested "claims" to this resource through, say, historic pollution levels. We assume that the benefits of an agent's pollution—a byproduct of many industrial processes that create jobs, growth, and welfare—outweigh the costs in terms of

¹There are some exceptions to this claim (see for instance Wu et al., 2019, and references cited there), but these papers tend to focus on specific case studies rather than providing a more general analysis as we do in this paper.

environmental and health damages, $\frac{2}{3}$ so agents prefer to be assigned more permits.

2.1 Model

Let $N = \{1, ..., n\}$ denote the **set of agents**, representing the regions (e.g., cities or countries) located along a river [] Throughout, we reserve *i*, *j*, and *k* to denote generic agents in *N*, where *i* is upstream of *j* whenever i < j. Each agent $i \in N$ has a **claim** $c_i \ge 0$ corresponding to their historical pollution level. The agents' claims are collected in the profile $c \equiv (c_i)_{i \in N} \in \mathbb{R}^n_{\ge 0}$. There is a pollution budget $E \ge 0$ to be distributed among the agents. We are concerned with problems in which the claims add up to at least the budget; that is, a **problem** (c, E) is such that $C \equiv \sum_i c_i \ge E$. Let $\mathcal{D}^N = \{(c, E) \in \mathbb{R}^n_{\ge 0} \times \mathbb{R}_{\ge 0} \mid C \ge E\}$ denote the domain of problems that agents *N* may face. A particular subdomain that will be interesting is that of **redistribution problems**, which are such that the claims add up to the pollution budget: $\mathcal{R}^N = \{(c, E) \in \mathcal{D}^N \mid C = E\} \subset \mathcal{D}^N$.

Given a problem $(c, E) \in \mathcal{D}^N$, an **allocation** $x \in \mathbb{R}^n_{\geq 0}$ specifies that agent *i* is awarded $x_i \geq 0$ pollution permits and is such that $\sum_i x_i = E$. For each $E \geq 0$, let $\mathcal{X}^N(E) \equiv \{x \in \mathbb{R}^n_{\geq 0} \mid \sum_i x_i = E\}$ denote the set of allocations. A **rule** φ is a systematic way of selecting allocations; it selects, for each population *N* and problem $(c, E) \in \mathcal{D}^N$, an allocation $\varphi(c, E) \in \mathcal{X}^N(E)$. We restrict throughout to continuous rules. In Remark [1], we illustrate two specific rules.

Remark 1 (Priority rules). An intuitive approach is to prioritize agents based on their location along the river (compare *downstream incremental* in Ambec & Sprumont, 2002). Depending on the "direction", this defines either the *upstream priority* or *downstream priority* rule (details in Appendix A). For *downstream priority*, the most downstream agent is awarded their claim if the budget allows. Then we turn to the second-most downstream agent, and so on. Priority rules satisfy desirable properties such as *claims boundedness* (no one is assigned more than their claim) and *con*-

²Here, this relation is taken as given and we choose not to model pollution and production separately as is done, for instance, by Gudmundsson and Hougaard (2021).

³Implicit throughout is that regions are unrestricted in the amount of permits they can put to use and we do not a priori exclude the possibility that a small region (small c_i) is awarded a large number of permits (x_i close to E, where E far exceeds c_i). This situation is relevant for the class of rules characterized in Section 3 and we offer a pragmatic solution in case it is deemed undesirable.

⁴Formally, we consider a variable-population model (see Axioms 1 and 4). There is an infinite set of "potential" agents indexed by the natural numbers \mathbb{N} . To specify a problem, we first draw a finite number of them from this infinite population. Let \mathcal{N} denote the family of nonempty finite subsets of \mathbb{N} . In this context, $N \in \mathcal{N}$ denotes a generic set of agents. For convenience, we define the rules and axioms for a fixed population N, but they should be understood as applying to every population in \mathcal{N} .

¹⁵For allocations and rules, we use subscript "-i" to denote the assignment to all but agent *i*; for instance, $x = (x_i, x_{-i})$.

sistency (allocations update consistently across problems as participants change; see Moulin, 2000). Indeed, *downstream priority* is the only rule to satisfy *independence of upstream claims* (downstream allocation is independent of upstream claims) together with *claims boundedness* (see van den Brink et al., 2014). However, priority rules also inherently violate the principle of impartiality as they treat agents unequally based on predetermined criteria.

In Appendix A, we provide new characterizations of both *upstream priority* and *downstream priority*. We focus on the intuition here and refer to Appendix A for technical details and proofs. Proposition 1 characterizes *upstream priority* through *claims boundedness, consistency,* and *upstream solidarity*. This axiom states that if some agent has an increased claim, then any change in the allocation of permits to upstream agents should have the same sign. That is, either all receive more, less, or the same. Hence, upstream agents are affected in the same direction if one of the downstream agents increases his claim. A related solidarity condition is *upstream symmetry* (Ni & Wang, 2007) and similar ideas are discussed more broadly in Thomson (2003).

To characterize *downstream priority*, Proposition 2 couples *claims boundedness* and *consistency* with the axiom *don't move up*. This axiom is tailored to the differential environmental impact of upstream versus downstream pollution. The axiom states that any transfer of (part of) a claim from a non-satiated downstream agent to an upstream agent does not decrease the downstream agent's allocation of pollution permits. In other words, it should be challenging to reallocate pollution from downstream to upstream locations in the river. The axiom draws inspiration from the *no transfer paradox* axiom (Chun, 1988).

2.2 Fairness and Environmental Concerns

Building on Remark 1, we will take an axiomatic approach to recommend a class of rules. We stress that our objective is not to select *how much* pollution to allow but rather to distribute a fixed pollution budget among the agents. In this way, we only indirectly address the damages to health and the environment caused by pollution through the axioms. A key concern is the simple observation that, due to the flow of the river, pollution is more harmful the further upstream it is emitted. Although this calls for pushing permits downstream, this has to be balanced with the fact that the claims (historic permit allocations) anchor the agents' expectations and act as a reference point from which it may be difficult to make too drastic adjustments.

The two goals suggest two very different rules. For reasons that will become

apparent, we denote them φ^0 (all downstream rule) and φ^1 (the proportional rule):

$$\varphi^0(c, E) = (0, \dots, 0, E),$$
$$\varphi^1(c, E) = (E/C) \cdot c.$$

In the next section we turn to introduce a class of rules that provides a compromise between these two rules.

Finally, we note that our model and proposed solution have merit even if the permit system only covers a section of the river (see Section 4). In this case, the budget *E* can be interpreted as an environmental standard set to safeguard environmental quality further downstream. Intuitively, a well-designed system for a section of the river may be a key first step to attract more to join and extend the agreement both up- and downstream. Our objective then is, first and foremost, to allocate pollution permits to the involved agents/regions to minimize the environmental damage from pollution. The same intuition applies for sea-bound rivers, where we now may interpret *E* as a standard set to control the river's impact on the sea.

3 Externality-Adjusted Proportional Rules

In this section, we will introduce and characterize the class of *externality-adjusted proportional rules*, striking a balance between our concerns for proportionality and pushing permits downstream without necessarily resulting in priority outcomes (see Remark 1). Rules in this class are parameterized by a single parameter, which captures the trade-off between treating claims "fairly" and shifting pollution downstream.

The *externality-adjusted proportional rules* φ^{λ} with parameter $\lambda \in [0, 1]$ allocate the fraction λ of the permits in proportion to the claims, whereas the remaining $1 - \lambda$ are awarded to the most downstream agent to minimize any harmful externalities. With $\lambda = 1$, we obtain the canonical *proportional rule*, giving priority to fairness over minimizing environmental damage. With $\lambda = 0$, permits are exclusively assigned to the most downstream agent, giving priority to minimizing environmental damage over fairness (such a solution would naturally violate *claims boundedness*, an issue that we address at the end of Section 3.2). Any intermediate value of $\lambda \in (0, 1)$ makes for a compromise between these two outcomes.

Definition 1 (The externality-adjusted proportional rule with parameter λ). For

each $(c, E) \in \mathcal{D}^N$,

$$\varphi^{\lambda}(c, E) = \lambda \cdot \varphi^{1}(c, E) + (1 - \lambda) \cdot \varphi^{0}(c, E).$$

That is,

$$\varphi_1^{\lambda}(c, E) = (E/C) \cdot \lambda c_1$$

$$\varphi_2^{\lambda}(c, E) = (E/C) \cdot \lambda c_2$$

$$\vdots$$

$$\varphi_{n-1}^{\lambda}(c, E) = (E/C) \cdot \lambda c_{n-1}$$

$$\varphi_n^{\lambda}(c, E) = (E/C) \cdot (\lambda c_n + (1 - \lambda)C).$$

These rules resemble the *fixed-fraction rules* recently explored by Gudmundsson, Hougaard, and Ko (2024) in the context of assigning liability in the case of sequentially triggered losses. Beyond the setting being completely different, there are two key differences. First, in their paper, liability should be assigned "upstream" (to the initiator of the loss chain); here, pollution permits should ideally be awarded downstream. Second, the *fixed-fraction rules* only apply in the particular case corresponding to C = E (which we will refer to as "redistribution problems"); our solutions extend to $C \ge E$, where the practically most relevant cases will have C > E. In this way, our setting calls for a different set of axioms and results in a novel class of rules.

Next, we introduce a series of desirable axioms and ultimately show that they jointly pin down the *externality-adjusted proportional rules*.

3.1 Axioms

We start from the case in which our two objectives—fairness with respect to claims and avoiding upstream pollution—agree. This is captured in an axiom inspired by the well-known "dummy", independence, and consistency axioms (e.g. Arrow, 1963; Moulin, 2000; Shapley, 1953; Thomson, 2012) as follows:

Consider the case in which the most upstream agent has a claim of zero. We contend then that the agent is irrelevant to the problem and that the rule should be invariant to whether the agent is included or not. That is, for each agent i > 1, $\varphi_i(c, E) = \varphi_{i-1}(c_{-1}, E)$.

Axiom 1 (Independence of upstream null claims). For each $(c, E) \in \mathcal{D}^N$,

$$c_1 = 0 \implies \varphi(c, E) = (0, \varphi(c_{-1}, E)).$$

Next, *budget additivity* is a property closely connected to proportionality (see e.g. Moulin, 1987; Thomson, 2019). It asserts that decomposing the resources (here, permits) in two parts and applying the solution separately should make no difference. This is desirable for instance if there is uncertainty on the total permits that will be made available; the permits can then cautiously be announced in batches without affecting the final distribution. The axiom is introduced by Chun (1988) and explored for instance in Bergantiños and Vidal-Puga (2006).

Axiom 2 (Budget additivity). For each $(c, E) \in \mathcal{D}^N$ and $E', E'' \ge 0$ with E = E' + E'',

$$\varphi(c, E) = \varphi(c, E') + \varphi(c, E'').$$

The remaining axioms apply only to the particular class of *redistribution problems*, $\mathcal{R}^N = \{(c, E) \in \mathcal{D}^N \mid C = E\}$. We require the rule to be additive across redistribution problems. Note here that, if (c, E) and (c', E') are redistribution problems, then $(c + c', E + E') = ((c_1 + c'_1, \dots, c_n + c'_n), E + E')$ is as well. Again, additivity has a long history in the literature on fair allocation (e.g. Shapley, 1953); Thomson (2019) discusses applying it only on a subset of problems.

Axiom 3 (Redistribution additivity). For each (c, E), $(c', E') \in \mathbb{R}^N$,

$$\varphi(c+c', E+E') = \varphi(c, E) + \varphi(c', E').$$

The final axiom applies to a yet smaller, elementary set of problems. The *n*-agent elementary problem $u_n \equiv (c, E) \in \mathbb{R}^N$ is such that only the most upstream agent has a positive claim, which equals one: c = (1, 0, ..., 0) and E = 1. The axiom below pertains to the strategic opportunities for neighboring regions. Imagine two regions requesting a recount because they now want to be treated as one (or one region "splitting" into two): if they benefit from doing so—at the expense of other agents—it would create unnecessary conflict. *Merging/splitting proofness* eliminates this possibility and goes back to the seminal work of O'Neill (1982); see also Chun (1988), de Frutos (1999), and Ju et al. (2007). Intuitively, starting from claims $(1, 0, ..., 0) = (1, 0 + 0, ..., 0) = \cdots = (1, 0, ..., 0, 0 + 0)$. We wish to rule out that they benefit from doing so. Again, this requirement is here only imposed on elementary problems.

Axiom 4 (Merging/splitting proofness). For each i < n,

$$\varphi_i(u_n) + \varphi_{i+1}(u_n) = \varphi_i(u_{n-1}).$$

In summary, in any application one has to decide how to delimit the problem choose where to draw boundaries for who to include (*independence of upstream null claims*), how to treat multiple source of pollution (*additivity*), and what level of aggregation (regions, cities, firms, etc.; *merging/splitting proofness*) to go for. There might not be a clear answer to these questions. Different parties can argue for different options and it could jeopardize the stability of the agreement if these administrative decisions had too much of an impact on the outcome. This is then a common point of our axioms: they make the outcome invariant across seemingly arbitrary choices on how to define the problem.

3.2 Characterization Result

The main result of the paper is the following:

Theorem 1. A rule φ satisfies *independence of upstream null claims, budget additivity, redistribution additivity,* and *merging/splitting proofness* if and only if there is $\lambda \in [0, 1]$ such that $\varphi = \varphi^{\lambda}$.



Figure 1: Illustration of Theorem 1 for c = (2, 1, 0) and E = 1. With parameter $\lambda = 1/2$, the allocation is $\varphi^{1/2}(c, E) = (1/3, 1/6, 1/2)$. The allocations selected by the *externality-adjusted proportional rules* with parameters $\lambda > 1/2$ are located on the solid line within the simplex between $\varphi^{1/2}$ and φ^1 . The dashed line covers other *externality-adjusted proportional rules* with parameters $0 < \lambda < 1/2$.

The proof is deferred to Appendix **B**. From the class of *externality-adjusted proportional rule*, decision-makers or practitioners can select one rule based on their preferred value of parameter $\lambda \in [0, 1]$ that controls the trade-off between proportionality and pushing permits downstream. Figure **1** illustrates how the value of λ affects the permit allocation in a simple numerical example.

Claims can be such that a low value of λ causes the externality-adjusted proportional rule to violate *claims boundedness* (specifically, the most downstream agent may receive more than their claim when $c_n < E$). To prevent such potentially undesirable outcomes, a decision-maker may select λ on a case-by-case basis. For instance, selecting the parameter based on historical data or specific problem characteristics could ensure that the rule remains within acceptable bounds. A pragmatic procedure is to restrict to values of λ which allocate the most downstream agent (at most) his claim. In the relevant case $c_n < E \leq C$, solving $\varphi_n^{\lambda}(c, E) = c_n$ for $\lambda \in [0, 1]$ yields

$$\frac{E}{C} \cdot (\lambda c_n + (1 - \lambda)C) = c_n \iff \lambda = \frac{C}{E} \cdot \frac{E - c_n}{C - c_n} \le 1.$$

The final inequality follows as $E \leq C$, so $E \cdot c_n \leq C \cdot c_n$, and therefore $C(E - c_n) = EC - C \cdot c_n \leq EC - E \cdot c_n = E(C - c_n)$. This shows that, in each specific problem, there is an externality-adjusted proportional rule for which no agent receives more than their claim. In particular, it identifies the smallest λ that accomplishes this (that is, the one that also best addresses the concern of pushing pollution downstream).

4 Case Study

In this section, we provide a numerical example using data from the Tuojiang River Basin (China) to illustrate how the choice for some parameter λ affects the resulting permit allocation results when using a *externality-adjusted proportional rule*.

The Tuojiang Basin is a primary tributary to the Yangtze, originating in Mianzhu City, Sichuan Province, it flows from north to south through the cities of Deyang, Chengdu, Ziyang, Neijiang, Zigong, and Luzhou within the Sichuan Province, ultimately joining the Yangtze River. It spans a total length of 638 km and holds prominence as one of the economically important rivers in the Sichuan Province. While pollution from the Tuojiang Basin ends up in the Yangtze River, for this case we only consider cities within the Tuojiang basin. Clearly, one can incorporate additional cities along the Yangtze River, say all the way down to Shanghai. A potential benefit of using an *externality-adjusted proportional* rule is that the inclusion of these downstream cities would have a limited effect on the permit allocation to most of the upstream cities.

We focus on the volume of untreated sewage discharge in this case. The data were obtained from the 2017 and 2021 Sichuan Provincial Statistical Yearbook. The total volume of untreated sewage discharged was approximately reduced from 81.24 million cubic meters (MCM) in 2016 to 64.30 MCM in 2020. We use these data

⁶This approach defines a rule φ that for each problem (c, E) selects the same allocation as some φ^{λ} but where the parameter λ depends on the problem. We leave a characterization of this rule for future research, but note that φ satisfies *claims boundedness* and all axioms in Theorem 1 except *budget additivity*.

⁷We label the cities Deyang, Chengdu, Ziyang, Neijiang, Zigong, Luzhou by 1 through 6.

to construct the claims vector and endowment parameter. In the left section of Table 1, we take the 2016 historical discharge levels as the claims vector, and we use the 2020 total discharge of 64.30 MCM as the budget to be distributed. Column 3 shows the result of the downstream priority rule φ^d (see Appendix A for its definition) and columns 4-5 show the results of the *externality-adjusted proportional rules* for two levels of parameter λ . A case-by-case approach to selecting parameter λ such that pollution is pushed downstream without violating *claims boundedness* (footnote 6) results in $\varphi^{0.94}$. Under this specific rule the most downstream city, Luzhou, receives its full claim, whereas the other cities receive proportional shares of their claims. The rules allocate significantly less to regions 2 and 5 compared to the actual 2020 emissions (column 6).

In the right section of Table 1, we instead take 2020 discharge levels as claims. We examine three more stringent policy targets aimed at reducing pollution. Intuitively, a lower budget *E* provides more flexibility in distributing permits without exceeding claims and lowers the threshold for the parameter λ . Put differently, by reducing total emissions *E*, we can push a larger share of permits downstream.

	Claims are 2016 discharge levels, budget is total 2020 discharge				Claims are 2020 discharge levels, variable budgets (emission targets)			
i	Claim	$arphi_i^d$	$arphi_i^{0.94}$	φ_i^1	Claim	$arphi_i^{0.97}$	$arphi_i^{0.95}$	$\varphi_i^{0.90}$
1	4.17	0.00	3.10	3.30	2.78	2.11	1.63	1.16
2	53.98	41.21	40.14	42.72	50.26	38.08	29.56	21.04
3	2.13	2.13	1.58	1.69	0.83	0.63	0.49	0.35
4	3.30	3.30	2.45	2.61	1.59	1.20	0.94	0.67
5	2.48	2.48	1.84	1.96	3.53	2.67	2.08	1.48
6	15.18	15.18	15.18	12.01	5.31	5.31	5.31	5.31
	81.24	64.30			64.30	50.00	40.00	30.00

Table 1: Discharge Permits Allocation in Tuojiang River Basin (units: MCM).

5 Conclusions

We propose the river pollution claims problem to distribute a pollution budget among agents located along a river. A key distinction from standard claims problems is that agents are ordered and the location of pollution is an important concern in addition to fairness. We characterized the class of *externality-adjusted proportional rules* in Theorem []. We apply the solution in a case study of the Tuojiang River Basin to show that it is well-suited to balance fairness considerations with concern for minimizing environmental damage due to river pollution. In addition to our main result, we propose two new axioms in the context of Remark 1. *upstream solidarity* and *don't move up*, to characterize the *upstream priority rule* and the *downstream priority rule*.

We end on three suggestions for future work. The first is to consider another class of rules that could potentially be used to balance fairness and minimizing environmental damage. This is the class of *bubbling-up rules* proposed by Hougaard et al. (2017). In this class of rules, agents 'bubble down' a share of their claim to their immediate downstream neighbor. The second is to explore an extension of the model that replaces the river-wide budget *E* with agent-specific bounds $E_i \ge 0$. That is, there may be environmental concerns that make it desirable to emit at most E_i units at or upstream of agent *i*. The present model is a special case in which $E_1 = \cdots = E_n = E$. Third, our study implicitly assumes the involved participants will keep emissions to the levels selected by the rule. It would be interesting to dig deeper into this and search for game-theoretic, decentralized solutions (compare Gudmundsson, Hougaard, & Ansink, 2024; Gudmundsson et al., 2019). We leave these possibilities for future research.

References

- Alcalde-Unzu, J., Gómez-Rúa, M., & Molis, E. (2015). Sharing the costs of cleaning a river: The upstream responsibility rule. *Games and Economic Behavior*, 90, 134–150.
- Ambec, S., & Sprumont, Y. (2002). Sharing a river. *Journal of Economic Theory*, 107(2), 453–462.
- Ansink, E., & Weikard, H.-P. (2012). Sequential sharing rules for river sharing problems. *Social Choice and Welfare*, *38*(2), 187–210.
- Ansink, E., & Weikard, H.-P. (2015). Composition properties in the river claims problem. *Social Choice and Welfare*, 44(4), 807–831.
- Arrow, K. J. (1963). Social Choice and Individual Values. Yale University Press.
- Bergantiños, G., & Vidal-Puga, J. J. (2006). Additive rules in discrete allocation problems. *European Journal of Operational Research*, 172(3), 971–978.
- Camargo, J. A., & Alonso, A. (2006). Ecological and toxicological effects of inorganic nitrogen pollution in aquatic ecosystems: A global assessment. *Environment International*, 32(6), 831–849.
- Chun, Y. (1988). The proportional solution for rights problems. *Mathematical Social Sciences*, *15*(3), 231–246.
- de Frutos, M. Á. (1999). Coalitional manipulations in a bankruptcy problem. *Review* of Economic Design, 4(3), 255–272.

- Duro, J. A., Giménez-Gómez, J.-M., & Vilella, C. (2020). The allocation of CO2 emissions as a claims problem. *Energy Economics*, *86*, 104652.
- Ebenstein, A. (2012). The consequences of industrialization: Evidence from water pollution and digestive cancers in China. *Review of Economics and Statistics*, 94(1), 186–201.
- Estévez-Fernández, A., Giménez-Gómez, J.-M., & Solís-Baltodano, M. J. (2021). Sequential bankruptcy problems. *European Journal of Operational Research*, 292(1), 388–395.
- Farrow, R. S., Schultz, M. T., Celikkol, P., & Van Houtven, G. L. (2005). Pollution trading in water quality limited areas: Use of benefits assessment and costeffective trading ratios. *Land Economics*, 81(2), 191–205.
- Gengenbach, M. F., Weikard, H.-P., & Ansink, E. (2010). Cleaning a river: An analysis of voluntary joint action. *Natural Resource Modeling*, 23(4), 565–590.
- Giménez-Gómez, J.-M., Teixidó-Figueras, J., & Vilella, C. (2016). The global carbon budget: A conflicting claims problem. *Climatic Change*, *136*(3), 693–703.
- Gudmundsson, J., & Hougaard, J. L. (2021). *River pollution abatement: Decentralized solutions and smart contracts*, IFRO Working Paper No. 2021/07, University of Copenhagen.
- Gudmundsson, J., Hougaard, J. L., & Ansink, E. (2024). *Towards fully decentralized environmental regulation* (No. 2024-035/VIII), Tinbergen Institute Discussion Paper.
- Gudmundsson, J., Hougaard, J. L., & Ko, C. Y. (2019). Decentralized Mechanisms for River Sharing. *Journal of Environmental Economics and Management*, 94, 67– 81.
- Gudmundsson, J., Hougaard, J. L., & Ko, C. Y. (2024). Sharing sequentially triggered losses: Automated conflict resolution through smart contracts. *Management Science*, 70(3), 1773–1786.
- Heo, E. J., & Lee, J. (2022). Allocating CO2 emissions: A dynamic claims problem. *Review of Economic Design*, 1–24.
- Hougaard, J. L., Moreno-Ternero, J. D., Tvede, M., & Østerdal, L. P. (2017). Sharing the proceeds from a hierarchical venture. *Games and Economic Behavior*, 102, 98–110.
- Hung, M.-F., & Shaw, D. (2005). A trading-ratio system for trading water pollution discharge permits. *Journal of Environmental Economics and Management*, 49(1), 83–102.
- Ju, B.-G., Kim, M., Kim, S., & Moreno-Ternero, J. D. (2021). Fair international protocols for the abatement of GHG emissions. *Energy Economics*, *94*, 105091.

- Ju, B.-G., Miyagawa, E., & Sakai, T. (2007). Non-manipulable division rules in claim problems and generalizations. *Journal of Economic Theory*, 132(1), 1–26.
- Li, W., Xu, G., & van den Brink, R. (2023). Two new classes of methods to share the cost of cleaning up a polluted river. *Social Choice and Welfare*, *61*(1), 35–59.
- Moulin, H. (1987). Equal or proportional division of a surplus, and other methods. *International Journal of Game Theory*, *16*, 161–186.
- Moulin, H. (2000). Priority rules and other asymmetric rationing methods. *Econometrica*, *68*(3), 643–684.
- Nguyen, N. P., Shortle, J. S., Reed, P. M., & Nguyen, T. T. (2013). Water quality trading with asymmetric information, uncertainty and transaction costs: A stochastic agent-based simulation. *Resource and Energy Economics*, 35(1), 60–90.
- Ni, D., & Wang, Y. (2007). Sharing a polluted river. *Games and Economic Behavior*, 60(1), 176–186.
- O'Neill, B. (1982). A problem of rights arbitration from the Talmud. *Mathematical Social Sciences*, 2(4), 345–371.
- Shapley, L. S. (1953). A Value for n-Person Games. In H. Kuhn & A. Tucker (Eds.), *Contributions to the theory of games ii* (pp. 307–317). Princeton University Press.
- Steinmann, S., & Winkler, R. (2019). Sharing a river with downstream externalities. *Games*, *10*(2), 23.
- Thomson, W. (2003). Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: A survey. *Mathematical Social Sciences*, 45(3), 249–297.
- Thomson, W. (2012). On the axiomatics of resource allocation: Interpreting the consistency principle. *Economics & Philosophy*, 28(3), 385–421.
- Thomson, W. (2015). Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: An update. *Mathematical Social Sciences*, 74, 41–59.
- Thomson, W. (2019). *How to divide when there isn't enough: from Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation.* Cambridge University Press.
- UNEP. (2016). Transboundary River Basins: Status and Trends.
- van den Brink, R., Estévez-Fernández, A., van der Laan, G., & Moes, N. (2014). Independence of downstream and upstream benefits in river water allocation problems. *Social Choice and Welfare*, 43(1), 173–194.
- van den Brink, R., He, S., & Huang, J.-P. (2018). Polluted river problems and games with a permission structure. *Games and Economic Behavior*, *108*, 182–205.
- van den Brink, R., van der Laan, G., & Moes, N. (2012). Fair agreements for sharing international rivers with multiple springs and externalities. *Journal of Environmental Economics and Management*, 63(3), 388–403.

- van der Laan, G., & Moes, N. (2016). Collective decision making in an international river pollution model. *Natural Resource Modeling*, 29(3), 374–399.
- Wu, W., Gao, P., Xu, Q., Zheng, T., Zhang, J., Wang, J., Liu, N., Bi, J., Zhou, Y., & Jiang, H. (2019). How to allocate discharge permits more fairly in China? A new perspective from watershed and regional allocation comparison on socio-natural equality. *Science of The Total Environment*, 684, 390–401.

A Priority Rules

In this appendix, we characterize the *upstream priority rule* and the *downstream priority rule*.

Definition 2 (Upstream priority rule). For each $(c, E) \in \mathcal{D}^N$ and j > 1,

$$\varphi_1^u(c, E) = \min\{c_1, E\}$$

$$\varphi_j^u(c, E) = \min\{c_j, E - \sum_{i < j} \varphi_i^u(c, E)\}.$$

Definition 3 (Downstream priority rule). For each $(c, E) \in \mathcal{D}^N$ and i < n,

$$\varphi_i^d(c, E) = \min\{c_i, E - \sum_{j>i} \varphi_j^d(c, E)\}$$
$$\varphi_n^d(c, E) = \min\{c_n, E\}.$$

We now introduce a series of axioms, taking inspiration from the literature on fair allocation. We start with *claims boundedness*, which asserts that no agent should receive more permits than their claim.

Axiom 5 (Claims boundedness). For each problem $(c, E) \in \mathcal{D}^N$ and agent $i \in N$,

 $\varphi_i(c, E) \leq c_i.$

We continue with *consistency*, which addresses the situation in which agent *j* "leaves" with their assigned permits. When we reevaluate the reduced problem, the assignment to the remaining agents should be unchanged.

Axiom 6 (Consistency). For each problem $(c, E) \in D^N$, and agents $\{i, j\} \subseteq N$ such that $i \neq j$,

$$\varphi_i(c, E) = \varphi_i(c_{-j}, E - \varphi_j(c, E)).$$

We now provide formal definitions of the two new axioms that are tailored to the river pollution claims problem. The first one is *upstream solidarity*.

Axiom 7 (Upstream solidarity). For each problem $(c, E) \in D^N$, amount $\Delta > 0$, and agents $\{i, j, k\} \subseteq N$ such that i < j < k,

$$\begin{split} \varphi_i(c,E) &< \varphi_i((c_k + \Delta, c_{-k}), E) \iff \varphi_j(c,E) < \varphi_j((c_k + \Delta, c_{-k}), E) \\ \text{and} \\ \varphi_i(c,E) &> \varphi_i((c_k + \Delta, c_{-k}), E) \iff \varphi_j(c,E) > \varphi_j((c_k + \Delta, c_{-k}), E). \end{split}$$

Upstream solidarity asserts that, if agent *k*'s claim increases while the pollution budget *E* is unchanged, then the number of permits assigned to the agents upstream of *k* should be affected in the same direction, either increase, decrease, or remain the same. This fairness axiom is inspired by the *upstream symmetry* as proposed by Ni and Wang (2007). *Upstream symmetry* pertains to the equal sharing of pollution costs by upstream agents because it is difficult to distinguish each upstream polluter's contribution to the downstream costs of cleaning pollution. In the river pollution claims problem, however, we are not concerned about cost sharing but rather about the distribution of the pollution budget. Therefore, we adapt the *upstream solidarity* axiom to reflect this difference.

Proposition 1 characterizes the *upstream priority rule*. Formally, the *upstream priority* rule φ^{u} is such that, for each $(c, E) \in \mathcal{D}^{N}$, $\varphi_{1}^{u}(c, E) = \min\{c_{1}, E\}$ and otherwise $\varphi_{i}^{u}(c, E) = \min\{c_{j}, E - \sum_{i < j} \varphi_{i}^{u}(c, E)\}.$

Proposition 1. A rule φ satisfies *claims boundedness, consistency,* and *upstream solidarity* if and only if $\varphi = \varphi^u$.

Proof. It is straightforward to show that the *upstream priority* rule satisfies *claims boundedness, consistency* and *upstream solidarity*. We proof the converse statement as follows.

Consider the two related problems $(c, E) \in \mathcal{D}^N$ and $(c', E) \in \mathcal{D}^{N'}$, where (c', E) differs from (c, E) by adding a dummy agent completely upstream, i.e. an agent with a zero claim ordered before agent 1 that we, with slight abuse of notation, refer to as agent 0. Hence, $N \equiv N' \setminus 0$. By *claims boundedness*, the dummy agent will receive a zero allocation. By *consistency*, allocations to all the other agents remain the same, so that $\varphi_i(c, E) = \varphi_i(c', E) \forall i \in N$.

Next, consider the related problem $(c'', E) \in \mathcal{D}^{N''}$ where c'' differs from c' only

⁸Note that we can use *independence of upstream null claims* instead of *consistency*, see Axiom 1

by $c_k'' > c_k'$, where i, j < k, by *upstream solidarity*, we have,

$$\varphi_i(c'', E) - \varphi_i(c', E) = \varphi_j(c'', E) - \varphi_j(c', E) = \varphi_0(c'', E) - \varphi_0(c', E) = 0.$$
(1)

So allocations to agents upstream of agent k will not be affected by agent k's increased claim.

Next, we use this result to derive the *upstream priority* rule. Consider problem $(c''', E) \in \mathcal{D}^{N'''}$ where profile $c''' = (c_1, \ldots, c_{j-1}, c''_j, 0, \ldots, 0) \in \mathbb{R}^N_{\geq 0}$ is such that the sum of claims is exactly equal to the pollution budget, i.e. $\sum_{i \leq j} c''_i = E$. By *claims boundedness*, we have $\varphi_i(c''', E) = c''_i$ for all $i \in N$. Now, create a sequence of n + 1 - j problems $(c, E)^{k \leq j}$ to transform problem (c''', E) back into problem (c, E) by lexicographically increasing agents' claims back to their original level, we do so starting with the claim by agent j and subsequently going downstream with claims by agent j + 1, j + 2, etc. In each of these problems, we can apply the above result. Since we do this sequentially, we end up with $\varphi_j(c, E) = \min\{c_j, E - \sum_{i < j} \varphi_i(c, E)\}$ for all $j \in N$. This defines the *upstream priority* rule.

With concerns for limiting environmental damage in mind, we introduce our next axiom: *don't move up*.

Axiom 8 (Don't move up). For each problem $(c, E) \in D^N$, agents $\{i, j\} \subseteq N$ such that i < j, and amount $0 < \Delta < c_j - \varphi_j(c, E)$,

$$\varphi_j(c, E) \leq \varphi_j((c_i + \Delta, c_j - \Delta, c_{-i,j}), E).$$

Don't move up says that an upstream transfer in claims will not result in an upstream transfer of pollution. Note that the axiom applies only when agent *j* is nonsatiated in the original problem since otherwise *claims boundedness* is violated; this constraint is reflected by the inequality $\Delta < c_j - \varphi_j(c, E)$. The motivation for *don't move up* is that upstream pollution is likely to cause more damage and we may want to prevent pollution from moving upstream given a certain pollution budget. *Don't move up* is similar to the inverse of *no transfer paradox* (Chun, 1988). The *no transfer paradox* axiom focuses on the case where one agent transfers his claim to another agent, and requires not only that the former should receive at most as much as he did initially, but also that the latter should receive at least as much as he did initially. This axiom is satisfied by many classical solutions to claims problems. When we consider such a claim transfer situation in a river setting, however, *no transfer paradox* implies that if a downstream agent transfers part of its claim towards upstream, the former will get at most as much as he did before. This is not a desirable outcome from an environmental perspective given that pollutants flow from upstream to downstream. Therefore, we propose this inverse version of *no transfer paradox* in order to prevent undesirable claim transfers and keep pollution downstream as much as possible.

Proposition 2 characterizes the *downstream priority rule*.

Proposition 2. A rule φ satisfies *claims boundedness, consistency,* and *don't move up* if and only if $\varphi = \varphi^d$.

Proof. It is straightforward to show that the *downstream priority* rule satisfies *claims boundedness, consistency* and *don't move up*. We proof the converse statement as follows.

Consider the two related problems $(c, E) \in \mathcal{D}^N$ and $(c', E') \in \mathcal{D}^N$, where (c', E') differs from (c, E) by removing a subset of agents. Specifically, remove all but two agents, such that only agents *i* and *j* remain with i < j. By *consistency*, the remaining endowment is $E' \equiv E - \sum_{k \neq i,j} \varphi_k(c, E)$, and the corresponding claims vector is $c' = \{c_i, c_j\}$.

Next, consider the related problem $(c'', E) \in \mathcal{D}^N$, where the claims vector $c'' = (0, c_i + c_j)$ is such that the claim by agent *i* is transferred and added to agent *j*'s claim. In other words, $c''_i = 0$ and $c''_j = c_i + c_j$. This transfer implies $0 < c'_i - c''_i = c''_j - c'_j$. Whenever we also have $\varphi_j(c'', E) < c'_j$, by *don't move up* applied to problems (c'', E) and (c', E'), we have $\varphi_j(c'', E) \leq \varphi_j(c', E)$, and given that there are only two agents, this implies $\varphi_i(c'', E) \geq \varphi_i(c', E)$. By *claims boundedness*, $c''_i = 0$ implies $\varphi_i(c'', E) = 0$. As allocations are non-negative, $\varphi_i(c'', E) = 0 \geq \varphi_i(c', E)$ implies $\varphi_i(c', E) = 0$. Agent *i* will always get a zero allocation under problem *d'* even though his claim is not zero, implying that agent *j* has priority over agent *i*: $\varphi_i(c, E) = \min\{c_i, E - \sum_{j>i} \varphi_j(c, E)\}$. This defines the *downstream priority* rule.

B Proof of Theorem **1**

It is straightforward to show that the *externality-adjusted proportional rules* satisfies *independence of upstream null claims, budget additivity, redistribution additivity,* and *merging/splitting proofness.* We prove the other direction as follows.

PART 1: Elementary problems: merging/splitting proofness

We start by pinning down the selection of the rule for elementary problems. Recall that, for $n \in \mathbb{N}$, we have $u_n = (c, E) \in \mathcal{R}^N$ with c = (1, 0, ..., 0) and E = 1. By *merging/splitting proofness*, for each i < n, $\varphi_i(u_{n-1}) = \varphi_i(u_n) + \varphi_{i+1}(u_n)$. We use this first for each i < n and then rearrange the right-hand side:

$$\varphi_1(u_{n-1}) + \dots + \varphi_{n-1}(u_{n-1}) = (\varphi_1(u_n) + \varphi_2(u_n)) + \dots + (\varphi_{n-1}(u_n) + \varphi_n(u_n)) = \varphi_1(u_n) + \dots + \varphi_n(u_n) + (\varphi_2(u_n) + \dots + \varphi_{n-1}(u_n))$$

By balance, $\varphi_1(u_{n-1}) + \cdots + \varphi_{n-1}(u_{n-1}) = 1$ and $\varphi_1(u_n) + \cdots + \varphi_n(u_n) = 1$. Hence, the equation simplifies to $1 = 1 + \varphi_2(u_n) + \cdots + \varphi_{n-1}(u_n)$. As each $\varphi_i(u_n) \ge 0$, we have $\varphi_2(u_n) = \cdots = \varphi_{n-1}(u_n) = 0$. Therefore, by balance, there exists $\lambda \in [0, 1]$ such that, for each $n \in \mathbb{N}$, $\varphi(u_n) = (\lambda, 0, \dots, 0, 1 - \lambda) = \varphi^{\lambda}(u_n)$.

PART 2: Redistribution problems: redistribution additivity and independence of upstream null claims

Next, we extend to generic redistribution problems $(c, E) \in \mathbb{R}^N$. Recall that the sum of redistribution problems is also a redistribution problem. Moreover, the budget *E* can be inferred from the claims *c* through E = C. Hence, to simplify notation, we refer to the problems in this part only through the claims vector.

First, we show that *redistribution additivity* implies that, for $\beta \ge 0$,

$$\varphi(\beta \cdot c) = \varphi(\beta c_1, \dots, \beta c_n) = \beta \cdot \varphi(c).$$

We consider three cases as follows:

1. (Integer) If $\beta \in \mathbb{N}$, then by repeatedly applying *redistribution additivity*, we have

$$\varphi(\beta \cdot c) = \varphi(c) + \cdots + \varphi(c) = \beta \cdot \varphi(c).$$

2. (Rational) If $\beta = (p/q) \in \mathbb{Q} \setminus \mathbb{N}$ for some $p, q \in \mathbb{N}$, then by *redistribution additivity*, we have

$$q \cdot \varphi(\beta \cdot c) = \varphi(\beta q \cdot c) = \varphi(p \cdot c) = p \cdot \varphi(c).$$

Divide by *q* on both sides to obtain the desired conclusion.

3. (Real) If $\beta \in \mathbb{R} \setminus \mathbb{Q}$, let $(a_1, a_2, ...) \in \mathbb{Q}^{\infty}$ be a rational sequence that converges to β . By case 2 above, $\varphi(a_k \cdot c) = a_k \cdot \varphi(c)$. As φ is continuous,

$$\varphi(\beta \cdot c) = \lim_{k \to \infty} \varphi(a_k \cdot c) = \lim_{k \to \infty} a_k \cdot \varphi(c) = \beta \cdot \varphi(c).$$

By applying redistribution additivity repeatedly,

$$\varphi(c) = \sum_{j} \varphi(0, \dots, 0, c_{j}, 0, \dots, 0) = \sum_{j} c_{j} \cdot \varphi(0, \dots, 0, 1, 0, \dots, 0).$$

Independence of upstream null claims allows us to further decompose the problem, eventually reaching an elementary problem. These were solved in PART 1. That is, for each $j \in N$ and claims vector (0, ..., 0, 1, 0, ..., 0) with a 1 in the *j*th position,

$$\varphi(0,\ldots,0,1,0,\ldots,0) = (\underbrace{0,\ldots,0}_{j-1 \text{ terms}},\underbrace{\varphi(1,0,\ldots,0)}_{n-(j-1) \text{ terms}}) = (0,\ldots,0,\lambda,0,\ldots,0,1-\lambda).$$

Hence, for each agent i < n, $\varphi_i(c) = \lambda c_i = \varphi^{\lambda}(c)$. By balance, $\varphi_n(c) = \varphi_n^{\lambda}(c)$.

PART 3: Full domain: budget additivity

Finally, we generalize the results to the full domain, \mathcal{D}^N . Fix $(c, E) \in \mathcal{R}^N$. First, we show that *budget additivity* implies that, for $\gamma \ge 1$,

$$\frac{1}{\gamma} \cdot \varphi(c, E) = \varphi(c, E/\gamma).$$

We again consider three cases as follows:

1. (Integer) If $(1/\gamma) \in \mathbb{N}$, then by *budget additivity*, we have

$$\varphi(c, E) = \varphi(c, E/\gamma) + \dots + \varphi(c, E/\gamma) = \gamma \cdot \varphi(c, E/\gamma).$$

Divide by γ on both sides to obtain the desired conclusion.

2. (Rational) If $\gamma = (p/q) \in \mathbb{Q} \setminus \mathbb{N}$ for some $p, q \in \mathbb{N}$, then by *budget additivity*, we have

$$p \cdot \varphi(c, E/\gamma) = \varphi(c, pE/\gamma) = \varphi(c, qE) = q \cdot \varphi(c, E).$$

Divide by *p* on both sides to obtain the desired conclusion.

3. (Real) If $(1/\gamma) \in \mathbb{R} \setminus \mathbb{Q}$, let $(a_1, a_2, ...) \in \mathbb{Q}^{\infty}$ be a rational sequence that converges to $1/\gamma$. By case 2 above, $\varphi(c, a_k \cdot E) = a_k \cdot \varphi(c, E)$. As φ is continuous,

$$\varphi(c,\frac{1}{\gamma}\cdot E) = \lim_{k\to\infty} \varphi(c,a_k\cdot E) = \lim_{k\to\infty} a_k\cdot \varphi(c,E) = \frac{1}{\gamma}\cdot \varphi(c,E).$$

This finally allows us to relate the solution of any problem $(c, E/\gamma) \in \mathcal{D}^N$ to that

of the redistribution problem $(c, E) \in \mathbb{R}^N$. These were solved in PART 2:

$$\varphi(c, E/\gamma) = \frac{1}{\gamma} \cdot \varphi(c, E) = \frac{1}{\gamma} \cdot \varphi^{\lambda}(c, E) = \varphi^{\lambda}(c, E/\gamma).$$

This completes the proof.

C Independence of Axioms

We show independence of the axioms in Theorem **1** *independence of upstream null claims, budget additivity, redistribution additivity,* and *merging/splitting proofness*. For each axiom, we identify a rule that is not an externality-adjusted proportional rule yet satisfies the other axioms.

Without *budget additivity* The *downstream priority* rule satisfies *claims boundedness* and *consistency*, so it also satisfies *independence of upstream null claims*. For redistribution problems $(c, E) \in \mathbb{R}^N$, it coincides with the proportional rule: $\varphi(c, E) = c = \varphi^1(c, E)$. Hence, it satisfies *redistribution additivity* and *merging/splitting proofness* as well. To see that it fails *budget additivity*, let c = (1, 3), E = 4, and E' = E'' = 2. Then

$$\varphi_1(c, E) = 1 \neq 1 + 1 = \varphi_1(c, E') + \varphi_1(c, E'').$$

Without *redistribution additivity* Define a rule similar to the *externality-adjusted proportional rules* but let the parameter λ vary with *c*. Specifically, let $\lambda = (C - c_n)/C$ and

$$\varphi(c, E) = \lambda \cdot \varphi^{1}(c, E) + (1 - \lambda) \cdot \varphi^{0}(c, E).$$

It is immediate that the rule satisfies *merging/splitting proofness*. As λ is independent of *E*, it is also *budget additive*. To see that it fails *redistribution additivity*, let $(c, E) = u_n$ and (c', E') be such that c' = (0, E') and $E' \ge 0$. Then $\varphi(c, E) = (\lambda, 1 - \lambda)$ and $\varphi(c', E') = (0, E')$. For the "joint" problem (c + c', E + E') = ((1, E'), 1 + E'), we then have $\lambda = (C - c_n)/C = 1/(1 + E')$. Hence,

$$\varphi_1(c+c',E+E') = \frac{1}{1+E'} \cdot 1 + \frac{E'}{1+E'} \cdot 0 \neq \varphi_1(c,E) + \varphi_1(c',E').$$

Without *merging/splitting proofness* Define a rule as follows:

$$\varphi_i(c,E) = \frac{E}{C} \cdot \left(\frac{c_i}{n-i+1} + \frac{c_{i-1}}{n-i+2} + \cdots + \frac{c_1}{n} \right).$$

To see that it fails *merging/splitting proofness*, let c = (1, 0, 0), E = 1, and c' = (1, 0), E' = 1. Then $\varphi(c, E) = (1/3, 1/3, 1/3)$ and $\varphi(c', E') = (1/2, 1/2)$. Hence,

$$\varphi_2(c, E) + \varphi_3(c, E) = \frac{2}{3} \neq \varphi_2(c', E') = \frac{1}{2}$$

Without *independence of upstream null claims* In terms of the order along the river, define the "opposite" of the externality-adjusted proportional rule as follows: still allocate the fraction λ of the permits in proportion to the claims but now the remaining $1 - \lambda$ are awarded the most upstream agent. That is, say now instead

$$\varphi_1^{\lambda}(c, E) = (E/C) \cdot (\lambda c_1 + (1 - \lambda)C)$$
$$\varphi_2^{\lambda}(c, E) = (E/C) \cdot \lambda c_2$$
$$\vdots$$
$$\varphi_n^{\lambda}(c, E) = (E/C) \cdot \lambda c_n.$$

It is immediate that it satisfies *merging/splitting proofness* as, for each n, $\varphi^{\lambda}(u_n) = (1, 0, ..., 0)$. Furthermore, we have $\varphi^{\lambda}((0, 1, 0, ..., 0), 1) = (1 - \lambda, \lambda, 0, ..., 0)$, which shows that the rule fails *independence of upstream null claims*. Finally, it satisfies the two additivity axioms.