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How to pollute a river if you must

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Abstract

We propose the river pollution claims problem to distribute a pollution budget among agents located along a river. A key distinction with the standard claims problem is that agents are ordered exogenously. For environmental reasons, the location of pollution along the river is an important concern in addition to fairness. We characterize the class of *externality-adjusted proportional rules* and argue that they strike a balance between fairness and minimizing environmental damage in the river. We also propose two novel axioms that are motivated by the river pollution context and use them to characterize two priority rules.

Keywords: Claims Problem, River Pollution, Pollution Permits, Externality-Adjusted Proportional Rules, Priority Rules

JEL classification: D62, D63, C71, Q25

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1 Introduction

In this paper we propose the river pollution claims problem and offer several solutions to it. Agents are ordered along a river and each of them claims to be allowed the discharge of a certain amount of pollution into the river. For environmental conservation reasons, the budget of total permitted pollution is limited and a solution allocates this limited amount of permits. In this way, our model extends on the standard claims problem (O'Neill, 1982) through an exogenous order of the agents reflecting their position along the river. This extension is inspired by the problem of river water pollution, such as nutrient pollution originating from agricultural production and chemicals pollution originating from industrial processes, causing more aggregate harm the further upstream it is emitted.

Water pollution may cause serious health problems and ecological damage. For example, Ebenstein (2012) estimated the impacts of surface water pollution on human health, showing that a deterioration of water quality by one grade (based on a six-grade scale) could cause a 9.7% increase in the digestive cancer death rate. Besides health, polluted water causes ecological imbalance and eco-remediation costs. For example, Camargo and Alonso (2006) showed that acidification and eutrophication of freshwater ecosystems due to nitrogen pollution may cause severe damage to the survival, growth, and reproduction of aquatic animals. Water pollution has become a severe environmental problem and urgently requires effective control measures. Such measures may be hampered by the mismatch between river basins and the administrative borders of jurisdictions in which they are located. Globally, 286 rivers flow across country borders (UNEP, 2016), and many more rivers cross the borders of lower-level jurisdictions like provinces, regions, and municipalities. As a result of this mismatch, the management of river pollution is often shared by multiple jurisdictions. The distribution of water pollution between agents is a challenge for which an analysis of the river pollution claims problem can provide possible directions.

The main difference with the standard claims problem is that, in our model, agents are ordered linearly from upstream to downstream, reflecting the direction of river flow, and this order is exogenously given by the hydrological setting. In addition to concerns over the amount of pollution in the river, a major concern is its distribution over the agents. One reason for this concern is the standard fairness consideration that is inherent to claims problems (see e.g. Thomson, 2003). A second reason, which is novel, is that the location of pollution matters for the resulting damage. A given amount of pollution is likely to cause more damage when it is emitted upstream compared to downstream since upstream pollution will cause damage along a longer stretch of the river.[1] This environmental externality affects total health- and ecological damage in the river and may also cause tensions along it. After presenting the model in Section 2, in Section 3 we will translate both concerns into axioms and apply them to characterize two solutions. These take the form of so-called priority rules (Moulin, 2000), which however have important drawbacks. These drawbacks will inform the design of the axioms leading to our main result.

In Section 4, we proceed to find a more reasonable solution by weakening *claims boundedness*, which normally would prevent agents from receiving more than their claims. For environmental reasons it may be desirable to push permits downstream, even in settings where downstream agents have low claims. Weakening *claims boundedness* allows exactly this. We characterize the class of *externality-adjusted proportional rules* on the domain of river pollution claims problems. It is inspired by the fixed-fraction rules proposed in Gudmundsson et al. (2023). Our characterized class seems appropriate in light of the required balance between fairness and pushing permits downstream. The characterization is based on the axioms *α-claim excess*, *independence of upstream null claims*, *resource additivity*, *redistribution additivity*, and *merging/splitting proofness*. The axioms are formally defined in Section 4.

Our paper relates to four separate strands of the literature. First, a series of recent papers is concerned with the allocation of the global carbon budget in order to assess fairness of countries' efforts to mitigate greenhouse gas emissions (Duro et al., 2020; Giménez-Gómez et al., 2016; Heo & Lee, 2022; Ju et al., 2021). Similar to the current paper, these carbon budget papers model a total budget of pollution that is allowed and they are concerned with the distribution of this budget over all countries. The main difference with the current paper is that agents are not ordered, and the location of pollution is not considered.

Second, there is a small literature that focuses on allocating water quantity in river settings using a cooperative game approach (Ambec & Sprumont, 2002; van den Brink et al., 2012) as well as using the claims problem approach (Ansink & Weikard, 2012, 2015; Estévez-Fernández et al., 2021; van den Brink et al., 2014). Similar to the current paper, these papers use a setting where agents are ordered linearly along the river. A key element of these models are the individual inflows ("endowments") of water that originates on each of the agents' territories in the form of rainfall and tributaries. Applying a claims problem in this setting is similar to redistributing the existing water resources under a water balance constraint. In

[1] Although damage will be partly mitigated by the absorptive capacity of the river, influenced by ambient pollution concentrations, flow, and temperature, see e.g. Chakraborti (2021). We do not explicitly model this process unlike e.g. Alcalde-Unzu et al. (2015).

the current paper, as in the standard claims problem, there is a single joint endowment, the pollution budget, and we are concerned with the higher environmental impact of upstream pollution compared to downstream pollution.

Third, a fairly recent literature is concerned with the distribution of welfare due to river cleaning (Gengenbach et al., 2010; Gudmundsson & Hougaard, 2021; Steinmann & Winkler, 2019; van der Laan & Moes, 2016) or the sharing of river water treatment costs (Alcalde-Unzu et al., 2015; Ni & Wang, 2007; van den Brink et al., 2018). Ni and Wang (2007) pioneered this literature with an analysis of how to share the costs of cleaning a river among different agents. While many papers are concerned about the economics of river pollution in terms of the distribution of costs for a given pollution abatement level or the distribution of welfare for the efficient pollution level, we are not aware of papers that focus on the distribution of pollution in the river.[2] Such allocation of a pollution budget is underlying the costs of pollution abatement and the resulting welfare levels.

Fourth, a few papers propose water quality trading as a cost-effective policy instrument to manage externalities due to water pollution (Farrow et al., 2005; Hung & Shaw, 2005; Nguyen et al., 2013). Because of market frictions, such as transaction costs, the initial distribution of pollution permits may impact the effectiveness of this instrument in practice. The class of externality-adjusted proportional rules that we propose in this paper offers an attractive starting point for the distribution of permits under a water quality trading system. While this rule potentially causes the most downstream agent to be allocated more permits than his claim, under water quality trading this agent can simply trade any excess permits.

The paper is organized as follows. In the next section, we introduce the river pollution claims problem. In Section 3, we discuss two priority rules. In Section 4, we characterize the class of externality-adjusted proportional rules. In Section 5, we present concluding remarks.

2 The River Pollution Claims Problem

In this section, we introduce the *river pollution claims problem*. This problem adds structure to the well-studied "claims/bankruptcy problems" (O'Neill, 1982); see Thomson (2015, 2019) for surveys of this literature) through the natural order in which agents (countries, regions, cities, ...) are located along the river. The resource to be divided is a budget of pollution permits. As in the bankruptcy problem, agents

[2] There are some exceptions to this claim (see for instance Wu et al. (2019) and references cited there), but these papers tend to focus on specific case studies rather than providing a more general analysis as we do in this paper.

hold uncontested "claims" to this resource through, say, historic pollution levels. We assume that the benefits of an agent's pollution—a byproduct of many industrial processes that create jobs, growth, and welfare—outweigh the costs in terms of environmental and health damages,[3] so agents prefer to be assigned more permits.[4]

2.1 Model

Formally, we consider a variable-population model with an infinite set of "potential" agents indexed by the natural numbers $\mathbb{N}$. To specify a problem, we first draw a finite number of them from this infinite population. Let $\mathcal{N}$ denote the family of nonempty finite subsets of $\mathbb{N}$. Let $N \in \mathcal{N}$ denote a generic **set of agents**, representing the regions located along a river. Throughout, we reserve i, j, and k to denote generic agents in N, where i is upstream of j whenever $i < j$. Let $[i] \in \{1, 2, \dots\}$ denote the ith-most upstream agent. When there is no risk of confusion, we fix the population $N \in \mathcal{N}$. Each agent $i \in N$ has a **claim** $c_i \geq 0$ corresponding to their historical pollution level. The agents' claims are collected in the profile $c \equiv (c_i)_{i \in N} \in \mathbb{R}^N_{\geq 0}$. There is a pollution budget $E \geq 0$ to be distributed among the agents. We are concerned with problems in which the claims add up to at least the budget; that is, a **problem** (c, E) is such that $C \equiv \sum_i c_i \geq E$. Let $\mathcal{D}^N = \{(c, E) \in \mathbb{R}^N_{\geq 0} \times \mathbb{R}_{\geq 0} \mid C \geq E\}$ denote the domain of problems that agents $N \in \mathcal{N}$ may face. A particular subdomain that will be interesting is that of **redistribution problems**, which are such that the claims add up to the pollution budget: $\mathcal{R}^N = \{(c, E) \in \mathcal{D}^N \mid C = E\} \subseteq \mathcal{D}^N$.

Given a problem $(c, E) \in \mathcal{D}^N$, an **allocation** $x \in \mathbb{R}^N_{\geq 0}$ specifies that agent i is awarded $x_i \geq 0$ pollution permits and is such that $\sum_i x_i = E$. For each $E \geq 0$, let $\mathcal{X}^N(E) \equiv \{x \in \mathbb{R}^N_{\geq 0} \mid \sum_i x_i = E\}$ denote the set of allocations. A **rule** φ is a systematic way of selecting allocations; it selects, for each population $N \in \mathcal{N}$ and problem $(c, E) \in \mathcal{D}^N$, an allocation $\varphi(c, E) \in \mathcal{X}^N(E)$.[5] We restrict throughout to continuous rules.

[3] Here, this relation is taken as given and we choose not to model pollution and production separately as is done, for instance, by Gudmundsson and Hougaard (2021).

[4] Implicit throughout is that regions are unrestricted in the amount of permits they can put to use. Although the particular case in which a small region (small c_i) is awarded a large number of permits (x_i close to E, where E far exceeds c_i) is possible, its impact can be limited through our novel *α-claims excess* axiom that we introduce in Section 4. Modeling an uncontested upper bound on the amount a region can be awarded is left for future research.

[5] For allocations and rules, we use subscript "$-i$" to denote the assignment to all but agent i; for instance, $x = (x_i, x_{-i})$.

2.2 Fairness and Environmental Concerns

In what follows, we will take an axiomatic approach to recommend a class of rules. We stress that our objective is not to select *how much* pollution to allow but rather to distribute a fixed pollution budget among the agents. In this way, we only indirectly address the damages to health and the environment caused by pollution through the axioms. A key concern is the simple observation that, due to the flow of the river, pollution is more harmful the further upstream it is emitted. Although this calls for pushing permits downstream, this has to be balanced with the fact that the claims (historic permit allocations) anchor the agents' expectations and act as a reference point from which it may be difficult to make too drastic adjustments.

The two goals suggest two very different rules. For reasons that will become apparent, we denote them φ^0 (*all downstream rule*) and φ^1 (*the proportional rule*):

$$\varphi^0(c,E) = (0,\ldots,0,E)$$
$$\varphi^1(c,E) = (E/C)\cdot c.$$

Before turning to our main characterization result in Section 4, as an intermezzo we first present results based on two new axioms that are inspired by existing axiomatic work on both bankruptcy problems and river pollution. These results will illustrate that imposing conventional axioms from the two literatures may be too demanding and eliminate many interesting rules.

3 Intermezzo: Priority Rules

For the bankruptcy problem, an almost universally assumed requirement is that agents receive at most their claim.[6] On top of this, inspired by Ni and Wang (2007)'s seminal paper, we may impose that downstream permit allocation be independent of upstream claims.[7] Unfortunately, these two modest conditions alone eliminate all but one rule, namely *downstream priority*.[8] This rule only awards permits to agent i if every downstream agent $j > i$ is awarded permits equal to their claim—that is, downstream agents get to pollute at their historic pollution levels while upstream agents may leave with nothing.

[6] In the present context, *claims boundedness* would assert that, for each problem $(c,E) \in \mathcal{D}^N$ and agent $i \in N$, $\varphi_i(c,E) \leq c_i$.

[7] Formally, *independence of upstream claims* would assert that, for all problems $\{(c,E),(c',E)\} \subseteq \mathbb{D}^N$, if there is $i \in N$ such that, for each $j > i$, $c_j = c'_j$, then also $\varphi_j(c,E) = \varphi_j(c',E)$ for each $j > i$.

[8] See van den Brink et al. (2014, Theorem 6.6(ii)). Formally, the *downstream priority rule* φ^d is such that, for each $(c,E) \in \mathcal{D}^N$, $\varphi^d_n(c,E) = \min\{c_n, E\}$ and otherwise $\varphi^d_i(c,E) = \min\{c_i, E - \sum_{j>i}\varphi^d_j(c,E)\}$.

This priority result is not a coincidence. The order of agents in the river pollution claims problem implies that axioms that use this order will often end up prioritizing either upstream or downstream agents. We illustrate this feature with two characterization results based on two new axioms. Both characterizations employ *consistency*[9] (see e.g. Moulin, 2000), which ensures that any asymmetric treatment of agents will be transferred across problems when the population varies. The two new axioms exploit the exogenous order of agents along the river and both express concern about the location of pollution. The first is *upstream solidarity*; it states that if some agent has an increased claim, then any change in the allocation of permits to upstream agents should have the same sign. That is, either all receive more, less, or the same. The second new axiom, *don't move up*, is tailored to the environmental impact of upstream versus downstream pollution as discussed above. It states that any transfer of (part of) their claim from a non-satiated downstream agent to an upstream agent does not decrease the former agent's allocation of pollution permits. In other words, it should be challenging to reallocate pollution from downstream to upstream locations in the river, given that upstream pollution is more damaging to the environment. More details on these axioms are provided in Appendix A where we also present the formal characterization results for both *upstream priority* and *downstream priority* (Propositions 1 and 2 in Appendix A).

We argue that neither the *upstream priority* nor the *downstream priority* rule provides a satisfying outcome in the river pollution claims problem. We distinguish four impediments. First, *upstream priority* goes against our key concern of pushing permits downstream. Second, the priority rules may cause drastic adjustments in the distribution of pollution permits, which goes against our concern for proportionality. Third, the priority rules may lead to a highly concentrated distribution of permits.[10] Finally, *downstream priority* might be too extreme since some pollution could be mitigated by the absorptive capacity of the river and this capacity may now remain unused.

In the next section we turn to introduce a class of rules that allows for pushing permits downstream when downstream agents have low claims but without resulting in the impediments discussed above.

[9] In the present context, *consistency* would assert that, for each problem $(c, E) \in \mathcal{D}^N$, and agents $\{i, j\} \subseteq N$ such that $i \neq j$, $\varphi_i(c, E) = \varphi_i(c_{-j}, E - \varphi_j(c, E))$.

[10] That is to say, there is an agent $i \in N$ such that, for each problem $(c, E) \in \mathcal{D}^N$ with $c_i \geq E$, $\varphi_i(c, E) = E$.

4 Externality-Adjusted Proportional Rules

In this section, we will introduce and characterize the class of *externality-adjusted proportional rules*, striking a balance between our concerns for proportionality and pushing permits downstream without necessarily resulting in priority outcomes. Rules in this class are parameterized by a single parameter, which captures the trade-off between treating claims "fairly" and shifting pollution downstream.

The *externality-adjusted proportional rule* φ^λ with parameter $\lambda \in [0,1]$ allocates the fraction λ of the permits in proportion to the claims, whereas the remaining $1-\lambda$ is awarded to the most downstream agent to minimize any harmful externalities. With $\lambda = 1$, we obtain the canonical *proportional rule*, giving priority to fairness over minimizing environmental damage. With $\lambda = 0$, permits are exclusively assigned the most downstream agent, giving priority to minimizing environmental damage over fairness. Any intermediate value of $\lambda \in (0,1)$ makes for a compromise between these two outcomes. For convenience, we define the rules and axioms for a fixed population $N \equiv \{1,\ldots,n\} \in \mathcal{N}$.

Definition 1 (The externality-adjusted proportional rule with parameter λ)**.** For each $(c,E) \in \mathcal{D}^N$,

$$\varphi^\lambda(c,E) = \lambda \cdot \varphi^1(c,E) + (1-\lambda)\cdot \varphi^0(c,E).$$

That is,

$$\begin{aligned}
\varphi_1^\lambda(c,E) &= (E/C)\cdot \lambda c_1\\
\varphi_2^\lambda(c,E) &= (E/C)\cdot \lambda c_2\\
&\;\;\vdots\\
\varphi_n^\lambda(c,E) &= (E/C)\cdot (\lambda c_n + (1-\lambda)C)\,.
\end{aligned}$$

These rules resemble the *fixed-fraction rules* recently explored by Gudmundsson et al. (2023) in the context of assigning liability in the case of sequentially triggered losses. Beyond the setting being completely different, there are two key differences. First, in their paper, liability should be assigned "upstream" (to the initiator of the loss chain); here, pollution permits should ideally be awarded downstream. Second, the *fixed-fraction rules* only apply in the particular case corresponding to $C = E$ (which we will refer to as "redistribution problems"); our solutions extend to $C \geq E$, where the practically most relevant cases will have $C > E$. In this way, our setting calls for a different set of axioms and results in a novel class of rules.

Next, we introduce a series of desirable axioms and ultimately show that they jointly pin down the externality-adjusted proportional rules.

4.1 Axioms

First, we revisit *claims boundedness*, which prevents any redistribution of permits when $C = E$. This leaves no room to take into account the downstream externality of pollution—if permits historically have been awarded upstream, then they will continue to be so. Taken to its extreme, $\varphi_2(c, E) = 0$ for any two-agent problem with $c = (E, 0)$. We believe that a desirable rule should take the environmental externality into consideration and therefore allow also $\varphi_i(c, E) > c_i$, at least to some degree. To address this, we suggest a weaker, parameterized version of the axiom, giving the practitioner a choice in how strictly to impose such bounds. We recover *claims boundedness* for parameter $\alpha = 1$ and the axiom has no bite for $\alpha = 0$. Intermediate values of $\alpha \in (0, 1)$ allow agents to receive more than their claim but not all of the permits.

Axiom 1 (α-Claim excess)**.** For each $(c, E) \in \mathcal{D}^N$ and $i \in N$,

$$\varphi_i(c, E) \leq \alpha c_i + (1 - \alpha)E.$$

There is one case in which our two objectives—fairness with respect to claims and avoiding upstream pollution—agree. This is captured next in an axiom inspired by the well-known "dummy", independence, and consistency axioms (e.g. Arrow, 1963; Moulin, 2000; Shapley, 1953; Thomson, 2012).

Consider the case in which the most upstream agent has a claim of zero. We contend then that the agent is irrelevant to the problem and that the rule should be invariant to whether the agent is included or not.[11] That is, for each agent $i > 1$, $\varphi_i(c, E) = \varphi_{i-1}(c_{-1}, E)$.

Axiom 2 (Independence of upstream null claims)**.** For each $(c, E) \in \mathcal{D}^N$,

$$c_1 = 0 \implies \varphi(c, E) = (0, \varphi(c_{-1}, E)).$$

Next, *resource additivity* is a property closely connected to proportionality (see e.g. Moulin, 1987; Thomson, 2019). It asserts that decomposing the resources (here, permits) in two parts and applying the solution separately should make no difference. This is desirable for instance if there is uncertainty on the total permits that will be made available; the permits can then cautiously be announced in batches

[11] This is related to *independence of upstream claims* but strictly speaking not a weakening of it.

without affecting the final distribution. The axiom is introduced by Chun (1988) and explored for instance in Bergantiños and Vidal-Puga (2006).

Axiom 3 (Resource additivity). For each $(c, E) \in \mathcal{D}^N$ and $E', E'' \geq 0$ with $E = E' + E''$,

$$\varphi(c, E) = \varphi(c, E') + \varphi(c, E'').$$

The remaining axioms apply only to the particular class of *redistribution problems*, $\mathcal{R}^N = \{(c, E) \in \mathcal{D}^N \mid C = E\}$. These are trivially solved under *claims boundedness* but generally not under *α-claim excess*. We require the rule to be additive across redistribution problems. Note here that, if (c, E) and (c', E') are redistribution problems, then $(c + c', E + E') = ((c_1 + c'_1, \ldots, c_n + c'_n), E + E')$ is as well. Again, additivity has a long history in the literature on fair allocation (e.g. Shapley, 1953); Thomson (2019) discusses applying it only on a subset of problems.

Axiom 4 (Redistribution additivity). For each $(c, E), (c', E') \in \mathcal{R}^N$,

$$\varphi(c + c', E + E') = \varphi(c, E) + \varphi(c', E').$$

The final axiom applies to a yet smaller, elementary set of problems. The n-agent *elementary problem* $u_n \equiv (c, E) \in \mathcal{R}^N$ is such that only the most upstream agent has a positive claim, which equals one: $c = (1, 0, \ldots, 0)$ and $E = 1$. The axiom below pertains to the strategic opportunities for neighboring regions. Imagine two regions requesting a recount because they now want to be treated as one (or one region "splitting" into two): if they benefit from doing so—at the expense of other agents—it would create unnecessary conflict. *Merging/splitting proofness* eliminates this possibility and goes back to the seminal work of O'Neill (1982); see also Chun (1988), de Frutos (1999), and Ju et al. (2007).

Intuitively, starting from claims $(1, 0, \ldots, 0)$, any pair of adjacent agents i and $i + 1$ can coordinate to claims $(1 + 0, 0, \ldots, 0) = (1, 0 + 0, \ldots, 0) = \cdots = (1, 0, \ldots, 0, 0 + 0)$. We wish to rule out that they benefit from doing so.

Axiom 5 (Merging/splitting proofness). For each $i < n$,

$$\varphi_i(u_n) + \varphi_{i+1}(u_n) = \varphi_i(u_{n-1}).$$

4.2 Characterization Result

The main result of the paper is the following:

Theorem 1. For each $\alpha \in [0,1]$, a rule φ satisfies *α-claim excess*, *independence of upstream null claims*, *resource additivity*, *redistribution additivity*, and *merging/splitting proofness* if and only if there is $\lambda \geq \alpha$ such that $\varphi = \varphi^\lambda$.

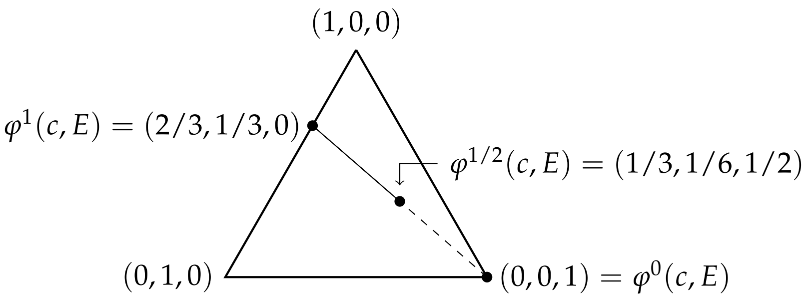

Figure 1: Illustration of Theorem 1 for $c = (2,1,0)$ and $E = 1$ with parameter $\alpha = 1/2$. The allocations selected by the externality-adjusted proportional rules with parameters $\lambda \geq \alpha$ are located on the solid line within the simplex between $\varphi^{1/2}$ and φ^1. The dashed line covers other externality-adjusted proportional rules with parameters $\lambda < \alpha$.

The proof is deferred to Appendix B. To interpret the result, it leaves the practitioner to make two decisions. The first is to set the parameter $\alpha \in [0,1]$ that governs the extent to which an agent's assignment may exceed the agent's claim. The second is to set the parameter $\lambda \geq \alpha$ that controls the trade-off between proportionality and pushing permits downstream. Figure 1 illustrates how these parameter values affect the permit allocation in a simple numerical example.

5 Conclusions

We propose the river pollution claims problem to distribute a pollution budget among agents located along a river. A key distinction from the earlier literature is that agents are ordered and the location of pollution is an important concern in addition to fairness. An unorthodox choice in our analysis was to weaken *claims boundedness* in Section 4 since this axiom is constraining solutions that seek to give some weight to minimizing environmental damage relative to fairness considerations. We argue that this balance is a core aspect of solutions to the river pollution claims problem and weakening *claims boundedness* allows a range of non-conventional solutions. From these, we have characterized the class of *externality-adjusted proportional rules* in Theorem 1. They seem particularly well-suited to balance fairness considerations with concern for minimizing environmental damage due to river pollution.

In addition to our main result, we propose two new axioms, *upstream solidarity* and *don't move up*, to characterize the *upstream priority rule* and the *downstream* *priority rule*. Although the two new axioms are specifically relevant in the river context, we argue how the resulting priority rules are undesirable solutions in the river pollution claims problem.

Our contribution extends beyond introducing the river pollution claims problem. We contribute to the literature by proposing a type of claims problem in which equal treatment of equals is not desirable because the agents are ordered. There are many other problems where agents have exogenous priority orders (Moulin, 2000). Our approach may shed new light on such problems but perhaps also on other problems where there exist externalities between agents.

Finally, another class of rules that violates *claims boundedness* and could also be used to balance fairness and minimizing environmental damage is the class of *bubbling-up rules* proposed by Hougaard et al. (2017). In this class of rules, agents 'bubble down' a share of their claim to their immediate downstream neighbor. We leave the analysis of this class of rules on the domain of river pollution claims problems for future research.

A Priority Rules

In this appendix, we characterize the *upstream priority rule* and the *downstream priority rule*. In addition to *claims boundedness* (see footnote 6) and *consistency* (see footnote 9), introduced in the main text, we now provide formal definitions of the two new axioms that are tailored to the river pollution claims problem. The first one is *upstream solidarity*.

Axiom 6 (Upstream solidarity)**.** For each problem $(c, E) \in \mathcal{D}^N$, amount $\Delta > 0$, and agents $\{i, j, k\} \subseteq N$ such that $i < j < k$,

$$\varphi_i(c, E) < \varphi_i((c_k + \Delta, c_{-k}), E) \iff \varphi_j(c, E) < \varphi_j((c_k + \Delta, c_{-k}), E)$$

and

$$\varphi_i(c, E) > \varphi_i((c_k + \Delta, c_{-k}), E) \iff \varphi_j(c, E) > \varphi_j((c_k + \Delta, c_{-k}), E).$$

Upstream solidarity asserts that, if agent k's claim increases while the pollution budget E is unchanged, then the number of permits assigned the agents upstream of k should be affected in the same direction, either increase, decrease, or remain the same. This fairness axiom is inspired by the *upstream symmetry* as proposed by Ni and Wang (2007). *Upstream symmetry* pertains to the equal sharing of pollution costs by upstream agents because it is difficult to distinguish each upstream polluter's contribution to the downstream costs of cleaning pollution. In the river pollution claims problem, however, we are not concerned about cost sharing but rather about the distribution of the pollution budget. Therefore, we adapt the *upstream solidarity* axiom to reflect this difference.

Proposition 1 characterizes the *upstream priority rule*. Formally, the *upstream priority* rule φ^u is such that, for each $(c, E) \in \mathcal{D}^N$, $\varphi^u_1(c, E) = \min\{c_1, E\}$ and otherwise $\varphi^u_j(c, E) = \min\{c_j, E - \sum_{i<j} \varphi^u_i(c, E)\}$.

Proposition 1. A rule φ satisfies *claims boundedness*, *consistency*, and *upstream solidarity* if and only if $\varphi = \varphi^u$.

Proof. It is straightforward to show that the *upstream priority* rule satisfies *claims boundedness*, *consistency* and *upstream solidarity*. We proof the inverse statement as follows.

Consider the two related problems $d = (c, E) \in \mathcal{D}^N$ and $d' = (c', E) \in \mathcal{D}^N$, where d' differs from d by adding a dummy agent completely upstream, i.e. an agent with a zero claim ordered before agent 1 that we, with slight abuse of notation, refer to as agent 0. Hence, $N \equiv N' \setminus 0$. By *claims boundedness*, the dummy agent will receive a zero allocation. By *consistency*[12], allocations to all the other agents remain the same, so that $\varphi_i(c, E) = \varphi_i(c', E)$ $\forall i \in N$.

Next, consider the related problem $d'' = (c'', E) \in \mathcal{D}^N$ where c'' differs from c' only by $c''_k > c'_k$. By *upstream solidarity*, we have

$$\varphi_i(c'', E) - \varphi_i(c', E) = \varphi_j(c'', E) - \varphi_j(c', E) = 0 \text{ for any } i, j < k. \tag{1}$$

So allocations to agents upstream of agent k will not be affected by agent k's increased claim.

Next, we use this result to derive the *upstream priority* rule. Consider problem $d''' = (c''', E) \in \mathcal{D}^N$ where profile $c''' = (c_1, \ldots, c_{j-1}, c'''_j, 0, \ldots, 0) \in \mathbb{R}^N_{\geq 0}$ is such that the sum of claims is exactly equal to the pollution budget, i.e. $\sum_{i \leq j} c'''_i = E$. Hence, we have $\varphi_i(c''', E) = c'''_i$ for all $i \in N$. Now, create a sequence of $n + 1 - j$ problems $d_{i \geq j}$ to transform problem d''' back into problem d by lexicographically increasing agents' claims back to their original level, i.e. $c'''_i = c_i$. We do so starting with the claim by agent j and subsequently going downstream with claims by agent $j + 1$, $j + 2$, etc. In each of these games, we can apply the above result. Since we do this sequentially, we end up with $\varphi_j(c, E) = \min\{c_j, E - \sum_{i<j} \varphi_i(c, E)\}$ for all $j \in N$. This defines the *upstream priority* rule. $\square$

With concerns for limiting environmental damage in mind, we introduce our next axiom: *don't move up*.

Axiom 7 (Don't move up)**.** For each problem $(c, E) \in \mathcal{D}^N$, agents $\{i, j\} \subseteq N$ such that $i < j$, and amount $0 < \Delta < c_j - \varphi_j(c, E)$,

$$\varphi_j(c, E) \leq \varphi_j((c_i + \Delta, c_j - \Delta, c_{-i,j}), E).$$

Don't move up says that an upstream transfer in claims will not result in an upstream transfer of pollution. Note that the axiom applies only when agent j is non-satiated in the original problem since otherwise *claims boundedness* is violated; this constraint is reflected by the inequality $\Delta < c_j - \varphi_j(c, E)$. The motivation for *don't move up* is that upstream pollution is likely to cause more damage and we may want to prevent pollution from moving upstream given a certain pollution budget. *Don't move up* is similar to the inverse of *no transfer paradox* (Chun, 1988). The *no transfer paradox* axiom focuses on the case where one agent transfers his claim to another agent, and requires not only that the former should receive at most as much as he did initially, but also that the latter should receive at least as much as

[12] Note that we can use *independence of upstream null claims* instead of *consistency*, see Axiom 2.

he did initially. This axiom is satisfied by many classical solutions to claims problems. When we consider such a claim transfer situation in a river setting, however, *no transfer paradox* implies that if a downstream agent transfers part of its claim towards upstream, the former will get at most as much as he did before. This is not a desirable outcome from an environmental perspective given that pollutants flow from upstream to downstream. Therefore, we propose this inverse version of *no transfer paradox* in order to prevent undesirable claim transfers and keep pollution downstream as much as possible.

Proposition 2 characterizes the *downstream priority rule*.

Proposition 2. A rule φ satisfies *claims boundedness*, *consistency*, and *don't move up* if and only if $\varphi = \varphi^d$.

Proof. It is straightforward to show that the *downstream priority* rule satisfies *claims boundedness*, *consistency* and *don't move up*. We proof the inverse statement as follows.

Consider the two related problems $d = (c, E) \in \mathcal{D}^N$ and $d' = (c', E) \in \mathcal{D}^N$, where d' differs from d by removing a subset of agents. Specifically, remove all but two agents, such that only agents i and j remain with $i < j$. By *consistency*, the remaining endowment is $E' \equiv E - \sum_{k \neq i,j} \varphi_k(c, E)$, and the corresponding claims vector is $c' = \{c_i, c_j\}$.

Next, consider the related problem $d'' = (c'', E) \in \mathcal{D}^N$, where the claims vector $c'' = (0, c_i + c_j)$ is such that the claim by agent i is transferred and added to agent j's claim. This transfer implies $0 < c'_i - c''_i = c''_j - c'_j$. Whenever we also have $\varphi_j(c'', E) < c'_j$, by *don't move up* applied to problems d'' and d', we have $\varphi_j(c'', E) \leq \varphi_j(c', E)$, and given that there are only two agents, this implies $\varphi_i(c'', E) \geq \varphi_i(c', E)$. By *claims boundedness*, $c''_i = 0$ implies $\varphi_i(c'', E) = 0$. By *Non-Negativity*, $\varphi_i(c'', E) = 0 \geq \varphi_i(c', E)$ implies $\varphi_i(c', E) = 0$. Agent i will always get a zero allocation under problem d' even though his claim is not zero, implying that agent j has priority over agent i: $\varphi_i(c, E) = \min\{c_i, E - \sum_{j>i} \varphi_j(c, E)\}$. This defines the *downstream priority* rule. □

B Proof of Theorem 1

It is straightforward to show that the *externality-adjusted proportional rules* satisfies *α-claim excess*, *independence of upstream null claims*, *resource additivity*, *redistribution additivity*, and *merging/splitting proofness*. We prove the other direction as follows. Fix $\alpha \in [0, 1]$ and take as given a rule φ that satisfies the axioms of the statement.

PART 1: *Elementary problems: merging/splitting proofness and α-claim excess*

We start by pinning down the selection of the rule for elementary problems. Recall that, for $n \in \mathbb{N}$, we have $u_n = (c, E) \in \mathcal{R}^N$ with $c = (1, 0, \ldots, 0)$ and $E = 1$. By *merging/splitting proofness*,

$$\begin{aligned}
&\varphi_1(u_{n-1}) + \cdots + \varphi_{n-1}(u_{n-1}) \\
&\quad = (\varphi_1(u_n) + \varphi_2(u_n)) + \cdots + (\varphi_{n-1}(u_n) + \varphi_n(u_n)) \\
&\quad = \varphi_1(u_n) + \cdots + \varphi_n(u_n) + (\varphi_2(u_n) + \cdots + \varphi_{n-1}(u_n)).
\end{aligned}$$

As each $\varphi_i(u_n) \geq 0$ and $\varphi_1(u_{n-1}) + \cdots + \varphi_{n-1}(u_{n-1}) = \varphi_1(u_n) + \cdots + \varphi_n(u_n) = 1$, we have $\varphi_2(u_n) = \cdots = \varphi_{n-1}(u_n) = 0$. Therefore, there exists $\lambda \in [0, 1]$ such that, for each $n \in \mathbb{N}$, $\varphi(u_n) = (\lambda, 0, \ldots, 0, 1-\lambda) = \varphi^\lambda(u_n)$. By *$\alpha$-claim excess*, $\varphi_n(u_n) = 1 - \lambda \leq \alpha c_n + (1-\alpha)E = 1 - \alpha$. Hence, $\lambda \geq \alpha$.

PART 2: *Redistribution problems: redistribution additivity and independence of upstream null claims*

Next, we extend to generic redistribution problems $(c, E) \in \mathcal{R}^N$. Recall that the sum of redistribution problems is also a redistribution problem. Moreover, the budget E can be inferred from the claims c through $E = C$. Hence, to simplify notation, we refer to the problems in this part only through the claims vector.

First, we show that *redistribution additivity* implies that, for $\beta \geq 0$,

$$\varphi(\beta \cdot c) = \varphi(\beta c_1, \ldots, \beta c_n) = \beta \cdot \varphi(c).$$

We consider three cases as follows:

1. (Integer) If $\beta \in \mathbb{N}$, then by repeatedly applying *redistribution additivity*, we have

   $$\varphi(\beta \cdot c) = \varphi(c) + \cdots + \varphi(c) = \beta \cdot \varphi(c).$$

2. (Rational) If $\beta = (p/q) \in \mathbb{Q} \setminus \mathbb{N}$ for some $p, q \in \mathbb{N}$, then by *redistribution additivity*, we have

   $$q \cdot \varphi(\beta \cdot c) = \varphi(\beta q \cdot c) = \varphi(p \cdot c) = p \cdot \varphi(c).$$

   Divide by q on both sides to obtain the desired conclusion.

3. (Real) If $\beta \in \mathbb{R} \setminus \mathbb{Q}$, let $(a_1, a_2, \ldots) \in \mathbb{Q}^\infty$ be a rational sequence that converges to β. By case 2 above, $\varphi(a_k \cdot c) = a_k \cdot \varphi(c)$. As φ is continuous,

   $$\varphi(\beta \cdot c) = \lim_{k \to \infty} \varphi(a_k \cdot c) = \lim_{k \to \infty} a_k \cdot \varphi(c) = \beta \cdot \varphi(c).$$

By applying *redistribution additivity* repeatedly,

$$\varphi(c) = \sum_j \varphi(0,\dots,0,c_j,0,\dots,0) = \sum_j c_j \cdot \varphi(0,\dots,0,1,0,\dots,0).$$

Independence of upstream null claims allows us to further decompose the problem, eventually reaching an elementary problem. These were solved in PART 1. That is, for each $j \in N$ and claims vector $(0,\dots,0,1,0,\dots,0)$ with a 1 in the jth position,

$$\varphi(0,\dots,0,1,0,\dots,0) = (\underbrace{0,\dots,0}_{j-1 \text{ terms}}, \underbrace{\varphi(1,0,\dots,0)}_{n-(j-1) \text{ terms}}) = (0,\dots,0,\lambda,0,\dots,0,1-\lambda).$$

Hence, for each agent $i < n$, $\varphi_i(c) = \lambda c_i = \varphi^\lambda(c)$. By balance, $\varphi_n(c) = \varphi^\lambda_n(c)$.

PART 3: *Full domain: resource additivity*

Finally, we generalize the results to the full domain, $\mathcal{D}^N$. Fix $(c,E) \in \mathcal{R}^N$. First, we show that *resource additivity* implies that, for $\gamma \geq 1$,

$$\frac{1}{\gamma} \cdot \varphi(c,E) = \varphi(c,E/\gamma).$$

We again consider three cases as follows:

1. (Integer) If $(1/\gamma) \in \mathbb{N}$, then by *resource additivity*, we have

   $$\varphi(c,E) = \varphi(c,E/\gamma) + \cdots + \varphi(c,E/\gamma) = \gamma \cdot \varphi(c,E/\gamma).$$

   Divide by γ on both sides to obtain the desired conclusion.

2. (Rational) If $\gamma = (p/q) \in \mathbb{Q} \setminus \mathbb{N}$ for some $p,q \in \mathbb{N}$, then by *resource additivity*, we have

   $$p \cdot \varphi(c,E/\gamma) = \varphi(c,pE/\gamma) = \varphi(c,qE) = q \cdot \varphi(c,E).$$

   Divide by p on both sides to obtain the desired conclusion.

3. (Real) If $(1/\gamma) \in \mathbb{R} \setminus \mathbb{Q}$, let $(a_1,a_2,\dots) \in \mathbb{Q}^\infty$ be a rational sequence that converges to $1/\gamma$. By case 2 above, $\varphi(c,a_k \cdot E) = a_k \cdot \varphi(c,E)$. As φ is continuous,

   $$\varphi(c,\frac{1}{\gamma} \cdot E) = \lim_{k\to\infty} \varphi(c,a_k \cdot E) = \lim_{k\to\infty} a_k \cdot \varphi(c,E) = \frac{1}{\gamma} \cdot \varphi(c,E).$$

This finally allows us to relate the solution of any problem $(c,E/\gamma) \in \mathcal{D}^N$ to that of the redistribution problem $(c, E) \in \mathcal{R}^N$. These were solved in PART 2:

$$\varphi(c, E/\gamma) = \frac{1}{\gamma} \cdot \varphi(c, E) = \frac{1}{\gamma} \cdot \varphi^\lambda(c, E) = \varphi^\lambda(c, E/\gamma).$$

This completes the proof.

C Independence of Axioms

We show independence of the axioms in Theorem 1: *α-claim excess*, *independence of upstream null claims*, *resource additivity*, *redistribution additivity*, and *merging/splitting proofness*. Fix $\alpha \in [0, 1]$. For each axiom, we identify a rule that is not an externality-adjusted proportional rule with parameter $\lambda \geq \alpha$ yet satisfies the other axioms.

Without *α-claim excess* We distinguish here between $\alpha = 0$ and $\alpha > 0$. As noted in the text, *α-claim excess* is vacuous for $\alpha = 0$. Hence, the rules that satisfy the other axioms are the externality-adjusted proportional rules with parameter $\lambda \geq 0 = \alpha$. That is, for $\alpha = 0$, there is no other rule that satisfies the remaining axioms. For $\alpha > 0$, on the other hand, the *all downstream* rule (parameter $\lambda = 0 < \alpha$) satisfies all axioms except *α-claim excess*.

Without *resource additivity* The *downstream priority* rule satisfies *claims boundedness* and *consistency*, so it also satisfies the weaker *α-claim excess* and *independence of upstream null claims*. For redistribution problems $(c, E) \in \mathcal{R}^N$, it coincides with the proportional rule: $\varphi(c, E) = c = \varphi^1(c, E)$. Hence, it satisfies *redistribution additivity* and *merging/splitting proofness* as well. To see that it fails *resource additivity*, let $c = (1, 3)$, $E = 4$, and $E' = E'' = 2$. Then

$$\varphi_1(c, E) = 1 \neq 1 + 1 = \varphi_1(c, E') + \varphi_1(c, E'').$$

Without *redistribution additivity* For $\alpha = 1$, *α-claim excess* already implies that each redistribution problem $(c, E) \in \mathcal{R}^N$ is solved through $\varphi(c, E) = c = \varphi^1(c, E)$. Hence, *$\alpha$-claim excess* then implies *redistribution additivity* (and *merging/splitting proofness*), so we are only be able to show independence for $\alpha < 1$.

Define a rule similar to the externality-adjusted proportional rules but let the parameter λ vary with c. Specifically, let $\lambda = \min\{\alpha, (C - c_n)/C\}$ and

$$\varphi(c, E) = \lambda \cdot \varphi^1(c, E) + (1 - \lambda) \cdot \varphi^0(c, E).$$

To see that it satisfies *α-claim excess*, the most testing case is agent n in the elementary problem u_n. For $c = (1, 0, \dots, 0)$, we have $\lambda = \min\{\alpha, 1\} = \alpha$ and thus $\varphi_n(u_n) = 1 - \alpha = \alpha c_n + (1-\alpha)E$, as desired. It is immediate that the rule satisfies *merging/splitting proofness*. As λ still is independent of E, it is also *resource additive*. To see that it fails *redistribution additivity*, let $(c, E) = u_n$ and (c', E') be such that $c' = (0, E')$ and $E' \geq 0$. Then $\varphi(c, E) = (\alpha, 1-\alpha)$ and $\varphi(c', E') = (0, E')$. Let E' be such that $1/(1+E') < \alpha$, that is, $E' > 1/\alpha - 1$. For the "joint" problem $(c + c', E + E') = ((1, E'), 1 + E')$, we then have $\lambda = \min\{\alpha, (C - c_n)/C\} = 1/(1+E')$. Hence,

$$\varphi_1(c + c', E + E') = \frac{1}{1+E'} \cdot 1 + \frac{E'}{1+E'} \cdot 0 \neq \alpha = \varphi_1(c, E) + \varphi_1(c', E').$$

Without *merging/splitting proofness* As just noted, it is necessary to restrict to $\alpha < 1$. For elementary problems, let $\gamma = \min\{1/n, 1-\alpha\}$ and

$$\varphi(u_n) = (1 - (n-1)\gamma, \gamma, \dots, \gamma).$$

For instance, if $1/n \leq 1 - \alpha$, then $\varphi(u_n) = (\gamma, \dots, \gamma)$. As in the previous case, this satisfies *α-claim excess*: for each agent $i > 1$, $\varphi_i(u_n) = \gamma \leq 1 - \alpha = \alpha c_i + (1-\alpha)E$. Following the construction in the proof of Theorem 1, we can use *redistribution additivity* to extend the solution to any redistribution problem and then *resource additivity* to extend further to the full domain. To see that it fails *merging/splitting proofness*, it suffices to note that $\varphi_2(u_3) = \gamma > 0$.

Without *independence of upstream null claims* Define the "opposite" of the externality-adjusted proportional rule as follows: still allocate the fraction λ of the permits in proportion to the claims but now the remaining $1 - \lambda$ are awarded the most upstream agent. That is, say now instead

$$\begin{aligned}
\varphi_1^{\lambda}(c, E) &= (E/C) \cdot (\lambda c_1 + (1-\lambda)C) \\
\varphi_2^{\lambda}(c, E) &= (E/C) \cdot \lambda c_2 \\
&\vdots \\
\varphi_n^{\lambda}(c, E) &= (E/C) \cdot \lambda c_n.
\end{aligned}$$

It is immediate that it satisfies *merging/splitting proofness* as, for each n, $\varphi^{\lambda}(u_n) = (1, 0, \dots, 0)$. Furthermore, we have $\varphi^{\lambda}((0, 1, 0, \dots, 0), 1) = (1 - \lambda, \lambda, 0, \dots, 0)$, which shows that the rule fails *independence of upstream null claims*. With $\lambda \geq \alpha$, the rule satisfies *α-claim excess*. Finally, it satisfies the two additivity axioms.

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