TI 2023-014/IV
Tinbergen Institute Discussion Paper

Flooded House or Underwater Mortgage? The Implications of Climate Change and Adaptation on Housing, Income & Wealth

Revision: October 2023

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The Implications of Climate Change and Adaptation on Housing, Income & Wealth

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September 2, 2023

Abstract

I study the implications of climate change and adaptation on housing and income, and wealth. I embed climate change in a redistributive growth model by introducing exposure of households and firms to extreme weather events, which damage their housing capital and physical capital, respectively. The analysis reveals that climate change is intrinsically redistributive. Low-income workers experience a relatively larger decline in income due to their exposure to climate-related damages, while the rate at which households with positive savings accumulate wealth rises. Importantly, I show that adapting to climate change is more challenging for low-income households who are financially constrained and the failure to reduce vulnerability to climate impacts exacerbates wealth inequality. While houses exposed to climate risk face a price discount in the market, I demonstrate that the materialization of climate change risk puts upward pressure on house prices, as habitat becomes reduced. This general equilibrium effect induces low-income households to spend a larger fraction of their budget on housing as climatic impacts intensify, translating into a widening of the adaptation gap over time.

Keywords— Climate Change, Adaptation, Housing, Financial Constraints, Extreme Weather Events, Income Inequality, Wealth Inequality
JEL classification codes— E44, G51, Q54

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I am grateful to Enrico Perotti for many valuable discussions and suggestions that significantly improved this paper. For helpful comments and feedback I thank Rick van der Ploeg, Björn Brügemann, Eric Bartelsman, Viral Acharya, Franck Moraux, Deniz Aydin, Xingchen Zhu, Fanny Cartellier, Simone Lenzu, Spyros Terovitis, Yenan Wang, Albert Jan Hummel, Elizabeth Proehl, Fabian Greimel and Simon J. Toussaint. I am thankful to Daniel Schmidt for his help with the model simulations, and Andrea Titton and Robert Lee for their advice and suggestions. This paper further benefited from discussions with participants of the 12th Portuguese Financial Network Conference, 1st European Sustainable Finance PhD Workshop, 39th International Conference of the French Finance Association (Affi), 1st VU Finance PhD Workshop, IPWSD 2023 at Columbia University in the City of New York, the Dutch Economist Day 2022, and participants of the Seminars on Environmental Economics and Management of Sustainability as well as the internal seminar of the MInt and Finance group at the University of Amsterdam. All potential errors are my own.

1 Introduction

The significance of *adaptation* as a global response strategy becomes increasingly evident as the impacts of climate change intensify across the globe, and *mitigation* efforts remain inadequate in preventing temperatures from rising above 1.5 degree Celsius (UNFCCC (2022)). Adaptation aims to reduce vulnerability to climatic impacts by adjusting to actual or expected climate change (IPCC (2021)). However, a pressing question remains: can adaptation effectively limit the potentially significant adverse effects of climate change on our economy (Barrage and Nordhaus (2023); Kahn et al. (2021))? In times of rising wealth inequality (Saez and Zucman (2016); Zucman (2019); Blanchet and Martínez-Toledano (2023)), it is essential to understand the differential impacts of climate change and heterogeneity in the ability to respond. How are economic loss and damages distributed? Should we be concerned about feedback effects, given that income and wealth are critical determinants of our ability to adapt? Do financial constraints restrict those most vulnerable to climate impacts when attempting to reduce their vulnerability?

This paper answers the above questions and sheds light on the broader implications of climate change and adaptation on housing, income and wealth. The analysis reveals that climate change is intrinsically redistributive. Wage inequality rises as low-income households experience a relatively larger decline in income due to their exposure to climate-related damages. Additionally, the rate at which households with positive savings accumulate wealth rises, whereas those with debt face a rise in the costs of borrowing. I further show that adapting to climate change is more challenging for financially constrained, low-income households and the failure to reduce vulnerability to climate change results in a disproportionately large reduction of their wealth when hit by an extreme weather event. This exacerbates wealth inequality. As climatic impacts intensify, the relative importance of housing in the consumption bundle of constrained households rises as well as its price. Consequently, constrained households allocate a larger fraction of their budget on housing, translating into a widening of the adaptation gap (defined as the choice of adaptation of constrained households relative to the optimal choice of adaptation) over time. To illustrate the effects, I provide a parameterization based on the Netherlands, a country with a long history in flood risk management, which is likely to face an increase in its exposure to flood risk as sea levels rise. I demonstrate the model outcomes for different scenarios of sea level rise, based on based on a low (RCP 2.6), medium (RCP 4.5), and high (RCP 8.5) greenhouse gas concentration trajectory from the Intergouvernmental Panel for Climate Change (IPCC (2013)).

I embed climate change and adaptation in a redistributive growth model, which is based on Döttling and Perotti (2017). The economy consists of one region, populated by households and firms. Households have preferences over housing and a non-durable consumption good, which is produced by the firm. Households invest in housing when young, and use the revenue from selling the house, as well as the return earned on their financial investments, to the purchase of the non-durable consumption good when old. The investment in housing is risky as households are exposed to *physical* climate risk. Physical risk emerges due to a rise in the probability of being hit by an extreme weather event, such as floods, wildfires, droughts, and hurricanes. Such climate impacts destroy housing capital and the exposure to climate risk lowers the expected revenue from selling the house in the next period - which is consistent with the existence of a "sea level rise discount" for houses exposed to climate risk (see, e.g., Bernstein et al. (2019); Bosker et al. (2019); Baldauf et al. (2020)). However, the destruction of housing capital translates into a decline in the supply of inhabitable houses as climate risk materializes. This leads to a fall in the supply of inhabitable houses (Burzyński et al. (2019)), and increases the marginal benefit of owning a house. This general equilibrium effect puts *upward* pressure on contemporaneous house prices - an effect which is novel to the literature. Moreover, the effect is propagated as the supply of houses shrinks permanently, and becomes amplified as climate risk rises over time.

Households, which are either low-skilled or high-skilled, work in the firm when young, where their labor is complementary to physical and intangible capital, respectively. Firms are also exposed to climate risk, as extreme weather events destroy physical capital. This leads to a capital loss, reducing firm profits and dividends. Additionally, wages fall and firms scale back investments in both types of capital. Due to the greater sensitivity of physical capital to climate risk, firms reduce investments in physical capital relatively more. This produces a stronger decline in the wage of low-skilled households, who work with physical capital and exacerbates income inequality. Climate-related damages also affect the costs of borrowing, as given by the return on physical capital. While the direct loss of capital and the fall in income put downwards pressure on the costs of borrowing, physical capital becomes relatively more scarce, which increases its price. This increases the rate at which households with positive savings accumulate wealth. However, as the rise in the costs of borrowing suppresses the value of the firm's equity, this reduces the financial wealth of shareholders. As a result, price changes become an equalizing force for wealth inequality, while savings form an unequalizing force - a finding which is in contrast with current trends in wealth inequality (see e.g. Bauluz et al. (2022), Fagereng et al. (2022) and Martínez-Toledano (2020) who show that the current rise in wealth inequality is driven by changes in asset prices, rather than savings). The fall in income increases mortgage credit demand. As the cost of borrowing rise at the same time, this raises loan-to-value ratio's and leads to a sharp rise in default rates, as the exposure to climate risk becomes more severe.

The paper further studies the optimal response of households to rising climate risk, and its implications on housing and financial assets, as well as income and wealth inequality. As the analysis is conducted in the context of a closed economy, where climate change is as an exogenous phenomenon, the emphasis is on *adaptation* strategies, rather than *mitigation* measures aimed at reducing climate change itself. Specifically, I study optimal adaptation in response to the growing threat of extreme weather events. By investing in protective measures, such as installing storm-proof windows or building seawalls and stilts (Fried (2022)), households limit losses and damages to their housing capital, an essential component of household wealth. This reduces households' vulnerability to future climatic impacts, leading to a leftward shift in the distribution of idiosyncratic losses. The investment in adaptation is costly, and households trade off the investment costs with the benefits of avoided climate change damage. When climate risk is accurately priced in the market (i.e. when beliefs about climate risk are correct and are accurately reflected in house prices), households internalize the benefits of their adaptive efforts for future generations. This prevents "tragedy of the horizon" effects (Carney (2015)), and the privately optimal investment in climate change adaptation is dynamically efficient. This investment increases in climate risk (as this increases the expected damages to the house) and in the house price (as this increases the value that is potentially lost when an extreme weather event occurs). However, any imperfect pricing of climate risk (e.g., due to incorrect beliefs on climate risk) leads to underinvestment in adaptation. Additionally, the choice of social discount rate is essential to ensure that the privately optimal choice of adaptation coincides with the social optimum.

Finally, when mortgage creditors take into account that part of the housing collateral posted has zero liquidation value, the size of household debt becomes limited to the market value of the *undamaged* housing capital. This introduces financial frictions, which effectively limit access of low-income households to the financial resources required to adapt optimally. As the ability of low-income households to adapt to climate change is critically dependent on their access to finance, the presence of credit constraints prevents these households from reducing their vulnerability to climate impacts. This has significant consequences, as households who fail to reduce vulnerability experience a disproportionately large reduction in their housing wealth when hit by an extreme weather event. This exacerbates wealth inequality. As climate risk rises and habitat becomes reduced, this induces a stronger change in demand for housing of constrained households as the marginal utility from housing of constrained households is higher than the marginal utility of unconstrained households. Housing thus becomes relatively more important in the consumption bundle of constrained households, but also more expensive. Consequently, constrained households allocate a larger fraction of their budget on housing, rather than adaptation, translating into a widening of the adaptation gap over time. As credit constraints on the household level have been shown to be a significant barrier to effective adaptation (IPCC (2023)) and prevent improvements in the sustainability of homes (Havlinova et al. (2022)), the relation with wealth inequality brought forward in this paper underscores the need for targeted policies that address the unequal impacts of climate change on those most vulnerable.

Related literature and contribution This paper contributes to the literature which studies how response strategies to climate change affect inequality. This literature mainly focuses on the implications of the introduction of a carbon tax. Känzig (2021) finds that higher carbon prices come at the cost of a decline in economic activity and that poorer households bear the burden of this decline. As poorer households have a high energy share, they experience a relatively larger reduction in their consumption. This effect is reinforced as these households experience a larger decline in income, as they tend to work in sectors which are more impacted by carbon pricing policies. This paper paper shows that a similar result holds in the presence of physical climate risk. I show that low-skilled workers' income fall relatively more due to the complementarity of these workers with physical capital, which is exposed to climate-related damages. Pedroni et al. (2022) study the effect of inequality on optimal fiscal policy and show that the social costs of carbon are lower when economic inequalities are taken into consideration. [1] To my knowledge, the present paper is the first to study the relation between climate change adaptation and wealth inequality. While this paper highlights that the availability of income and wealth crucially determine our capacity to adapt to climate change, I show that the failure to adapt optimally to climate change leads to a further rise in wealth inequality.

This paper more broadly contributes to the literature on climate change and inequality. Dell et al. (2012) first showed that climate change is a larger problem for countries that are not sufficiently rich, as higher temperatures reduce economic growth rates in poor countries substantially and this effect is persistent. Alvarez and Rossi-Hansberg (2021) formalize this argument and confirm that the economic effects of climate change are heterogeneous across space. The authors show that rising temperatures are associated with a decline in the productivity and demonstrate that the welfare of the developing world is more severely affected as a result of climate change, since developing countries face more extreme changes in temperatures. As migration is an effective, yet costly, adaptation mechanism in this context, restrictions to migration may contribute to the increase in spatial inequities (Desmet and Rossi-Hansberg (2015)). Burzyński et al. (2019) highlight that climate change has heterogeneous effects and that climate-related damages may make certain areas uninhabitable. Moreover, the authors confirm that poorer regions are most prone to the adverse affects of rising temperatures and that climate change reinforce inequality across the globe as a result. This paper contributes to this research area by studying the relationship between climate change and inequality in a within-country setting, and by showing that climate change amplifies income inequality as low-income workers experience are more exposed to climate-related damage.

This paper also contributes to the literature that study climate change adaptation in macro-models as well as efficient adaptation. As far as I am aware, only Fried (2022) studies the macro-economic effects of climate change adaptation. While this study abstracts from the effects of climate change adaptation on house price dynamics and on income and wealth inequality, the author quantifies the interactions between climate change, adaptation and federal disaster policy. Fried (2022) finds that disaster aid policies induce moral hazard in adaptation and this reduces adaptation in the US economy, while federal subsidies for adaptation more than correct for this effect. The present paper complements this result by demonstrating that underinvestment also follows in the presence of credit constraints, and argues that *targeted* policies are required to prevent a further rise in wealth inequality.

This paper also relates to the literature on efficient adaptation. Mendelsohn (2006) emphasizes that markets encourage efficient adaptation in sectors whose goods are traded, such as agriculture and energy[2], and Anderson et al. (2019) highlight that this applies to land markets as well. While the papers argue that the market may facilitate efficient adaptation through price signals, the present paper is the first to formalize the argument, and to highlight the

[1] The authors also find that it is optimal to use half of the carbon tax revenue to increase transfers, with the other half used to reduce distortionary taxes.

[2] The author further highlights that government should focus their attention on markets for public goods, in which the market outcome is not necessarily efficient, and should refrain from intervening in markets from traded goods, as this undermines private incentives.

conditions under which the market outcome fails to be efficient.

Additionally, this paper contributes to the literature studying the pricing of climate risk in housing markets. This line of research predominately focused on the sea level rise (SLR) risk. Harrison et al. (2001). Bin et al. (2008), Keenan et al. (2018), Gibson et al. (2017), Ortega and Tapınar (2018), Bernstein et al. (2019), Hino and Burke (2020), Baldauf et al. (2020) find evidence that SLR risk is - at least, to a certain extent - capitalized into US housing markets in coastal states.[3] However, Murfin and Spiegel (2020) conclude that the price effects of flood risk are limited and argue that this is driven by the fact that climate change believers sort in unexposed neighbourhoods, while deniers sort in exposed neighbourhoods. Indeed, Bakkensen and Barrage (2021) show that heterogeneity in beliefs reconciles the mixed empirical evidence and further argues that flood risk is not fully reflected in house prices due to high degrees of belief heterogeneity in coastal areas. Rather than focusing on heterogeneity in climate-change beliefs, this paper focuses on heterogeneity in households skill levels, which determines their income and wealth and therefore plays a crucial role in determining households' capacity to adapt. Consistent with the aforementioned research, this paper shows that houses exposed to climate risk trade at a discount in the market. However, as climate-related losses reduce the supply of houses over time, the marginal benefit of owning a house rises and this puts upward pressure on contemporaneous house prices. I derive the condition under which this general equilibrium effect dominates.

As the evolution of house prices matter for mortgage market dynamics, this paper also adds to the literature that studies the relationship between the rise in climate risk and mortgage market dynamics. Bakkensen et al. (2022) use heterogeneous beliefs to show that the purchase of a house with a larger exposure to SLR is more likely to be leveraged, despite lower property prices.[4] Issler et al. (2019) show that mortgage delinquency and foreclosure rates significantly increase after a wildfire. [5]. Finally, Ouazad and Kahn (2019) show that mortgage originators are more likely to transfer default risk after natural disasters, using securitization and are more likely to increase the share of mortgages originated below the conforming loan limit after the occurrence of a natural disaster. This paper adds to the existing literature by demonstrating that households become more indebted, as the impacts of climate change intensify. This contributes to an increase in loan-to-value ratio's, and leads to a sharp rise in default rates as the exposure to climate risk becomes more severe. I show that investments in climate change adaptation soften this effect, by weakening the rise of loan-to-value ratio's and by reducing idiosyncratic losses when hit by an extreme weather event.

[3] Bernstein et al. (2019) and Baldauf et al. (2020) aim to identify what drives the existence/level of the discount. Bernstein et al. (2019) find that the sophistication of buyers, as well as climate change beliefs play a key role, and Baldauf et al. (2020) demonstrate that exposed properties in climate change-believer neighborhoods sell at a discount compared to those in denier neighborhoods. Baldauf et al. (2020) conclude that house prices reflect heterogeneity in beliefs about climate risks.

[4] Bakkensen et al. (2022) also find that the underlying mortgage contracts have a longer maturity, and climate change pessimists are more likely to trade their climate risk exposure with banks via long-term debt contracts.

[5] The authors show that the strength of this effect declines in the size of the disaster due to coordination externalities afforded by large fires positively affect the quality of rebuilding projects and thus makes the neighbourhood more valuable than it was before the wildfire occurred

Roadmap The remainder of this paper is structured as follows: Section 2 describes the theoretical framework, and conditions relevant for the definition of an equilibrium are derived in Section 3. In Section 4, climate adaptation is introduced and Section 5 considers credit constraints. Section 6 provides a parameterization to illustrate the effects of the model. Section 7 concludes.

2 Theoretical Framework

Time is discrete and denoted by $t \in \{0,1,...,\infty\}$. The economy is characterized by two overlapping generations, each consisting of a unit mass of households. Households derive utility from consuming housing and a non-durable consumption good, which is produced by firms. Households with some entrepreneurial talent set-up a unit mass of firms in each period. Firms operate for a single period and produce the non-durable consumption good, using physical and intangible capital, as well as labour in its production process. At the start of each period, an extreme weather event occurs, which hits a fraction of households and firms, and damages their housing and physical capital, respectively. All risk is idiosyncratic, and the economy's climate risk exposure rises deterministically over time. Households can invest in protective measures to adapt to climate change, which reduces the expected losses when hit by an extreme weather event. As firms only live for a single period, they do not adapt to climate change.

2.1 Households

Households live for two periods. When young, households purchase housing capital, denoted by L, from the old generation at a relative price p (the price of the consumption good is normalized to 1). Additionally, young households hold financial assets, and can invest in corporate- and mortgage debt, as well as firm equity. Corporate debt and equity are denoted by D_f and e_f, respectively. Once old, households channel their savings, which consist of the proceeds from selling their house, as well as the return earned on their financial assets, to the purchase of the non-durable consumption good, denoted by c. There is an initial generation at $t = 0$, which is endowed with the supply of houses, $\bar{L}_0$.

2.1.1 Preferences

Households have preferences over housing and the non-durable consumption good, which are given by the following quasi-linearutility function

$$U\left(c_{i,t+1}, L_{i,t}\right) = c_{i,t+1} + v\left(L_{i,t}\right)$$

$v\left(L_{i,t}\right)$ captures the utility that household i obtain in period t from owning $L_{i,t}$ housing capital and $v'(\cdot) > 0, v''(\cdot) < 0$. Households maximize expected lifetime utility.

2.1.2 Labour Endowments

Households are heterogeneous in terms of skills, which are exogenously given. A fraction ϕ of households is high-skilled, h, and is endowed with $\bar{h}$ high-skilled labour. The remaining households are low-skilled, l, and are endowed with $\bar{l}$ manual labour. When young, workers supply labour inelastically in a perfectly competitive labour market and earn an income of $y_{i,t} = \{q_t \tilde{h}, w_t \tilde{l}\}$, where q_t respectively w_t denotes the high- respectively low-skilled workers' wage and $\tilde{h}$ respectively $\tilde{l}$ denote the amount of high- respectively low-skilled labour supplied (Döttling and Perotti (2017)).

2.1.3 Innovators

A fraction ε of high-skilled workers has some entrepreneurial talent. When young, these 'innovators' set up a firm, f and create intangible capital, H, by investing $I_{H,t}$, where $I_{H,t} = H_{t+1}$. This is a costly investment, as it requires some effort cost

$$C\left(I_{H,t}\right) = \frac{\beta}{2} I_{H,t}^2$$

where β denotes the ease of innovating.

The firm operates with the intangible capital once the innovator turns old, but intangible capital can only be used in production by high-skilled workers. Due to the inalienability of human capital (Hart and Moore (1994)), high-skilled workers try to capture part of its value. I assume that the innovator grants shares to high-skilled workers, which are backed by the value of the intangible capital that the high-skilled workers appropriate. Denote the bargaining power of the innovator by ω. Then, shareholders capture a fraction $(1-\omega)$ of the value of intangible capital.

2.1.4 Climate Risk and Housing Capital

The economy is exposed to climate risk and an extreme weather event occurs in each period. Let γ_t denote the probability that a given household is hit by an extreme weather event in period, t, which is i.i.d. across households and increases deterministically over time. By the law of large numbers, γ_t corresponds to the fraction of households that suffer climate-related damages in any period t (Fried (2022)). Let μ_L be the fraction of housing capital lost when hit by an extreme weather event and denote by $\xi_{i,t}$ the losses suffered by a given household, i, in period, t. As losses are idiosyncratic, $\xi_{i,t}$ is stochastic and follows some distribution, $F\left(\xi_{i,t}\right)$, which is i.i.d. across households. Then, the expectation of $\xi_{i,t}$ is given by of

$$\begin{aligned}\mathbb{E}\left(\xi_{i,t}\right) &= \mathbb{E}\left(\xi_{i,t} \middle| \text{Hit by Extreme weather event}\right) \cdot \mathbb{P}\left(\text{Hit by Extreme weather event}\right) \\ &= \mu_L \gamma_t\end{aligned}$$

This reduces the housing capital owned by a given household i:

$$L_{i,t+1} = \left(1 - \xi_{i,t}\right) L_{i,t}$$

The probability of being hit, and therefore the expected damages, rise determinisitcally over time. As habitat becomes gradually lost with the occurrence of extreme weather events, the materialization of climate risk translates into a decline in the supply of inhabitable houses over time (Burzyński et al. (2019)). Denote the supply of houses in a given period by $\bar{L}_t$. The supply of houses evolves according to the following law of motion:

$$
\begin{aligned}
\bar{L}_{t+1} &= \int_0^1 \left(1-\xi_{i,t}\right) di \cdot \bar{L}_t \\
&\overset{\text{LLN}}{=} \left(1-\mu_L \gamma_{t+1}\right) \cdot \bar{L}_t
\end{aligned}
$$

This law of motion reflects that habitat is gradually lost due to the materialization of physical climate risk, as certain high-risk areas become uninhabitable (Burzyński et al. (2019)).

2.2 Firms

Innovators set up $\frac{1}{\phi\epsilon}$ identical firms, which implies that there is a unit mass of firms. Firms operate a single period and maximize profits. Firms employ both high- and low-skilled workers as well as physical and intangible capital in the production process.

2.2.1 Production Technology

Each firm produces a non-durable consumption good and use physical and intangible capital in the production process. Intangible capital (H) is complementary to high-skilled labour (h) and physical capital (K) is complementary to manual labour (l). Output, Y_t, is produced according to the following constant elasticity of substitution production technology:

$$
\begin{aligned}
Y_t &= A\mathscr{F}(H_t, h_t, K_t, l_t) \\
&= A\left[\eta\left(H_t^{\alpha} h_t^{1-\alpha}\right)^{\rho} + (1-\eta)\left(K_t^{\alpha} l_t^{1-\alpha}\right)^{\rho}\right]^{\frac{1}{\rho}}
\end{aligned}
$$

where A is a technology parameter, $\rho \in [0,1)$ is the substitution parameter, and η is a distribution parameter reflecting the relative productivity of intangible capital and high-skilled labour.

2.2.2 Scarcity of High-Skilled Labour

To ensure that wages of high-skilled workers are higher than those of low-skilled workers, I assume that high-skilled labour is relatively scarce, i.e. (Döttling and Perotti (2017))

Assumption 1.

$$
\frac{\phi}{1-\phi} \leq \frac{\eta}{1-\eta}
$$

2.2.3 Capital Investments

Intangible capital is created by innovators. Physical capital is invested in upfront, where $I_{f,K,t} = K_{f,t+1}$. Both types of capital depreciate fully (i.e. $\delta_K = \delta_H = 1$) after the production period. Firms operating in $t = 0$ are endowed with an initial stock of physical capital, K_0, and the $t = 0$ old innovators are endowed with an initial stock of intangible capital, H_0.

2.2.4 Firm Financing

To finance the investment in physical capital, each firm f issues corporate debt, $D_{f,t}$, which earns a rate of return equal to r_t and is repaid each period.[6] Innovators also issues equity, which is backed by the value of the share of intangible capital appropriated by equity holders. The price of a share of firm f is denoted by $e_{f,t}$, and the quantity of shares of each firm normalized to 1. Households can trade in shares, which receive a dividend payment, $d_{f,t}$, at the end of the period.

2.2.5 Climate Risk and Firm Capital

Firms are exposed to climate change, as extreme weather events destroy its physical capital.[7] Let γ_t also capture the probability that a given firm is hit by an extreme weather event in period, t. Denote by μ_K the average losses to physical capital conditional on being hit by an extreme weather event and $\xi_{f,t}$ the idiosyncratic losses suffered by given firm, f, in period, t. Then, $\xi_{f,t}$ is stochastic and follows some distribution, $G\left(\xi_{f,t}\right)$, which is i.i.d. across firms. This indicates that idiosyncratic risk is perfectly diversifiable, and what matters for investors, then, is the expectation of $\xi_{f,t}$, which is given by

$$\begin{aligned}\mathbb{E}\left(\xi_{f,t}\right) &= \mathbb{E}\left(\xi_{f,t}\middle|\text{Hit by Extreme weather event}\right)\cdot\mathbb{P}\left(\text{Hit by Extreme weather event}\right)\\ &= \mu_K\gamma_t\end{aligned}$$

and the damages reduce the amount of physical capital owned by firm f which has productive value

$$\tilde{K}_t = \left(1-\xi_{f,t}\right)K_t$$

Then, the firm's production is given by[8]

$$\tilde{Y}_t = A\left[\eta\left(H_t^{\alpha}h_t^{1-\alpha}\right)^{\rho} + (1-\eta)\left(\tilde{K}_t^{\alpha}l_t^{1-\alpha}\right)^{\rho}\right]^{\frac{1}{\rho}}$$

where[9]

[6] I abstract from corporate default in the baseline model. Corporate debt, then, is in essence equivalent to equity, and the rate of return, r_t, can be viewed as the risk-adjusted rate of return.

[7] This follows the empirical evidence. For example, Acharya et al. (2022) show that tangible industries (e.g. construction, mining, oil & gas, utilities, manufactuing and forestry & fishery) are more exposed to physical climate risk than service industries.

[8] The literature following Nordhaus (1992) models climate-related damages by introducing a TFP factor that declines in temperatures. As I focus on physical climate risk in the form of extreme weather events, I model climate-related damages as a capital loss, rather than as a loss in the productivity of physical production factors. Note that the two approaches are isomorphic when $\rho = 0$. In this case, the expression also captures any potential decline in the productivity of manual labor (Acharya et al. (2022)).

[9] Fatica et al. (2022) show that floods have a significant and long-term negative impact on firm performance, as it deteriorates its assets both in the medium- and long-run. Non-exposed firms are positively effected, indicating that there may be relocation of economic activity to less exposed areas - which is in line with the other work arguing that physical climate risk leads to creative destruction (Leiter et al. (2009)). As this paper does not consider firm heterogeneity nor adaptation by firms (as firms only operate in a single period), I follow the macro-literature which shows that climate change has an adverse effects on GDP (Barrage and Nordhaus (2023); Kahn et al. (2021)) and the effect of physical climate risk on the production technology is therefore negative.

$$\tilde{Y}_t = A\mathscr{F}(H_t, h_t, \tilde{K}_t, l_t), \qquad \mathscr{F}'_\gamma(H_t, h_t, \tilde{K}_t, l_t) \leq 0$$

Note on the Climate Risk Exposure of Firms vs Households There is an important distinction between the climate risk exposure of firms as opposed to households. Since firms only operate during one period and its capital stock depreciates fully, climate-related damages only affect the capital stock in a given period (i.e. climate risk has a *flow* effect). In contrast, the supply of housing shrinks permanently over time as a result of climate-related damages to housing capital (i.e. climate risk has a *stock* effect).

2.3 Financial Markets and Credit Risk

Households purchase shares, $s_{i,f,t}$ and lend to the firm, $D_{i,f,t}$ as well as to other households on the financial market. Lending only occurs against collateral and takes the form of mortgage debt backed by housing capital or corporate debt backed by physical capital.

2.3.1 Housing Market Dynamics

There is one housing market on which all home purchases and sales take place. The market opens after the extreme weather takes place, which occurs at the start of the period. Define $S_{i,t}$ as the net savings of a young household, i in period t, after the purchase of housing capital as well as shares of the firm, i.e.

$$S_{i,t} = y_{i,t} - p_t L_{i,t} - s_{i,t} e_t$$

Then, depending on whether $S_{i,t}$ is positive or negative, a given household i is classified as net lender or borrower.

$$S_{i,t} \begin{cases} \geq 0 & \text{net lender} \\ < 0 & \text{net borrower} \end{cases}$$

As housing capital forms the collateral backing the loan, and destroyed housing capital has zero liquidation value, borrowers risk default and pay the risky rate of return, $\hat{r}_t > r_t$.[10]

Default occurs when

$$p_{t+1} L_{i,t+1} \leq -(1 + \hat{r}_{t+1})(S_{i,t})$$

i.e. when the proceeds from selling the undamaged housing capital is smaller than the (absolute) value of the amount borrowed, including interest. Define the loan-to-value ratio as

$$LTV_{i,t+1} = \frac{(1 + \hat{r}_{t+1})(-S_{i,t})}{p_{t+1} L_{i,t}}$$

This implicitly defines the threshold of losses above which a homeowner defaults as

$$\hat{\xi}_{i,t+1} = 1 - LTV_{i,t+1}$$

Then, the probability of default is given by

$$\chi_{i,t} = \left(1 - F\left(\hat{\xi}_{i,t+1}\right)\right)$$

[10] While investors in mortgage debt anticipate the effect of climate-related damages on the value of the pledged housing capital, I postpone the introduction of credit constraints to Section 5.

3 Equilibrium

3.1 Household Optimization Problem

Households maximize utility subject to the budget constraint and limited liability constraint:

$$\max_{c_{i,t+1},L_{i,t},s_{i,t},S_{i,t}} \mathbb{E}\left(U(c_{i,t+1},L_{i,t})\right) = \mathbb{E}_t\left(c_{i,t+1}\right) + v\left(L_{i,t}\right)$$

$$\begin{aligned} s.t. \quad & y_{i,t} \leq p_t L_{i,t} + s_{i,t} e_t + S_{i,t} \\ & c_{i,t+1} \leq \max\{y_{i,t+1} + p_{t+1}(1-\xi_{i,t+1})L_{i,t} + d_{t+1}s_{i,t} + (1+\hat{r}_{t+1})S_{i,t}, 0\} \\ & c_{i,t+1}, L_{i,t} \geq 0, \end{aligned}$$

where $c_{i,t+1}$ is the consumption of household i in period $t+1$, and $\mathbb{E}_t$ denotes expectations formed at date t.

3.1.1 Optimal Demand for Housing Capital

The optimal demand for housing capital in a given period t determines its price :

Lemma 1. *The demand for housing capital of each household i in period t is given by*

$$L_t^* = v'^{-1}\left((1+r_{t+1})p_t - \left(1-\mu_L\gamma_{t+1}\right)p_{t+1}\right)$$

Then, the price of housing capital in a given period, t, becomes

$$p_t = \frac{(1-\mu_L\gamma_{t+1}p_{t+1}) + v'\left(L_t^*\right)}{(1+r_{t+1})}$$

The price of housing capital today is equal to the discounted value of the benefits from owning it, which consists of the marginal benefit of owning housing capital, $v'\left(L_t^*\right)$, as well as the revenue from selling the undamaged housing capital in the next period. This revenue falls in next period's *(i.e., future)* climate risk, γ_{t+1}. This term reflects that houses exposed to climate risk are traded at a discount in the market, which is consistent with the empirically documented sea-level rise discount (e.g. Bernstein et al. (2019); Bosker et al. (2019); Baldauf et al. (2020)).

3.1.2 Optimal Demand for Shares and Corporate Debt

The price of a share is equal to the discounted value of the dividend payment, d_{t+1}

$$e_t = \frac{d_{t+1}}{(1+r_{t+1})}$$

The investment in corporate and household debt follows as residual. Households with net savings lend to others households and firms. Households with negative savings take out a mortgage.

3.2 Firm Optimization problem

Firms maximize the value to its equity holders. Since firms only operate for one period, and pay out all profits, the maximization problem is given by

$$\max_{H_t,h_t,K_t,l_t} \pi_{f,t} = \tilde{Y}_t(A,H_t,h_t,\tilde{K}_t,l_t) - \omega R_t H_t - q_t h_t - (1+r_t)D_t - w_t l_t$$

3.2.1 High- and Low-Skilled Workers' Wage

Labour markets are perfectly competitive, which means that high-skilled and low-skilled workers earn their marginal productivity.

Lemma 2. *Wages of high- and low-skilled workers, q_t respectively w_t are equal to*

$$q_t^* = A^\rho (1-\alpha)\eta \frac{\tilde{Y}_t^{1-\rho}}{h_t^{1-(1-\alpha)\rho}} H_t^{\alpha\rho}$$

$$w_t^* = A^\rho (1-\alpha)(1-\eta) \frac{\tilde{Y}_t^{1-\rho}}{l_t^{1-(1-\alpha)\rho}} \left(1-\mu_K \gamma_t\right)^{\alpha\rho} K_t^{\alpha\rho}$$

and the wage ratio, defined as $\frac{q_t^*}{w_t^*}$, is given by

$$\frac{q_t^*}{w_t^*} = \frac{\eta}{1-\eta} \cdot \left(\frac{H_t}{\left(1-\mu_K\gamma_t\right)K_t}\right)^{\alpha\rho} \cdot \left(\frac{l_t}{h_t}\right)^{1-(1-\alpha)\rho}$$

As income falls in climate-related damages, this reduces the wage of both high- and low-skilled workers. However, the loss of physical capital suppresses wages of low-skilled workers directly relative to wages high-skilled workers. Climate-related damages also contribute indirectly to a rise in wage inequality. While investments in both physical and intangible capital drop as income falls, firms scale back investments in physical capital relatively more due to its exposure to climate-related damages. This suppresses the wages of low-skilled workers further.

Proposition 1. *Wage inequality rises in climate-related damages.*

Proof: See Appendix A.1

3.2.2 Return on Physical Capital

Firms are financially unconstrained and borrow up to the point where the marginal costs of capital are equal to its marginal productivity

Lemma 3. *The return to physical capital is given by*

$$(1+r_t^*) = A^\rho \alpha (1-\eta) \frac{\tilde{Y}_t^{1-\rho}}{\left(\left(1-\mu_K\gamma_t\right)K_t\right)^{1-\alpha\rho}} l_t^{(1-\alpha)\rho}$$

and firms fully finance the investment in physical capital by debt in each period, $I_{K,t}^* = D_t$.

Climate-related damages render physical capital less productive and reduces income, which suppresses its return. On the other hand, extreme weather events lead to increased scarcity of capital and this pushes the return on physical capital up. As the elasticity of physical capital is larger than the elasticity of income with respect to climate related-damages, the latter effect dominates and the costs of borrowing *rises* in climate-related damages.

Proposition 2. *The costs of borrowing rises in climate-related damages*

Proof: See Appendix A.2

3.2.3 Return on Intangible Capital

Competitive firms pay a return on intangible capital equal to its marginal productivity

Lemma 4. *The return on intangible capital is given by:*

$$R_t^* = A^\rho \alpha \eta \frac{\tilde{Y}_t^{1-\rho}}{H_t^{1-\alpha\rho}} h_t^{(1-\alpha)\rho}$$

and the amount of intangible capital created by innovators is

$$I_t^* = \frac{\omega R_{t+1}^*}{\beta}$$

where $I_t^* = H_{t+1}^*$ (Döttling and Perotti (2017)).

3.2.4 Dividends and Share Prices

Share holders capture a fraction $(1-\omega)$ of the return to intangibles capital. Therefore, equilibrium dividends are equal to

$$\begin{aligned} d_t^* &= \tilde{Y}_t(H_t, \tilde{K}_t, h_t, l_t) - \left(\omega R_t^* H_t + q_t^* h_t + (1+r_t^*)K_{t-1} + w_t^* l_t\right) \\ &= (1-\omega) R_t^* H_t \end{aligned}$$

Lemma 5. *Dividends decline in climate-related damages.*

and share prices are given by the discounted value of the dividend payment.

$$e_t^* = \frac{(1-\omega) R_{t+1} H_{t+1}}{1 + r_{t+1}}$$

The decline in dividends is a direct consequence from the reduction in the firm's profitability due to climate-related damages. The downwards revision of dividends, as well as the the rise in the costs of borrowing, triggers a revaluation of firm's equity, which suppresses equity prices.

Proposition 3. *Share prices fall in climate risk*

Proof: See Appendix A.3

As equity values decline, this reduces the financial wealth of shareholders and (financial) wealth inequality falls. Therefore, asset price changes form an *equalizing* force for wealth inequality. On the other hand, the rate at which households with positive savings accumulate wealth increases as financial costs rise. This latter effect contributes to a rise wealth inequality and savings become an *unequalizing* force. This finding is in contrast with current trends in wealth inequality as documented by e.g. Bauluz et al. (2022), Fagereng et al. (2022) and Martínez-Toledano (2020).

3.3 Equilibrium and Market Clearing

A competitive equilibrium is defined as an allocation $\{c_t^l, c_t^h, L_t^l, L_t^h, s_t^l, s_t^h, D_t^l, D_t^h, K_t, H_t, l_t, h_t\}_{t=0}^T$ and prices $\{p_t, e_t, r_t, R_t, w_t, h_t\}_{t=0}^T$ such that in each period, t, given prices

1. Households maximize lifetime utility
2. Firms maximize profits
3. Innovators choose intangible investment

and markets clear, i.e.

1. Total labour demand equals total labour supply:

$$\int_0^1 \left[h_{f,t}^d, l_{f,t}^d\right] df = [h^s, l^s]$$

2. Total housing demand equals total housing supply:

$$\int_0^1 L_{i,t}^* di = \bar{L}_t$$

3. Total share holdings equal total supply of shares:

$$\int_0^1 s_{i,t}^* di = 1$$

4. Total net savings from labour income are equal to the value of the firm's market capitalization and its corporate debt:

$$(1-\alpha)\tilde{Y}_t - p_t\bar{L}_t = e_t + D_t$$

where $(1-\alpha)\tilde{Y}_t = q_t\phi\tilde{h} + w_t(1-\phi)\tilde{l}$.

3.3.1 Labour Market Clearing

Households supply their entire labour endowment as its marginal product is strictly positive, i.e. $[h^s, l^s] = \{\phi\bar{h}, (1-\phi)\bar{l}\}$ and

$$\int_0^1 \left[h_{f,t}^d, l_{f,t}^d\right] df = \{\phi\bar{h}, (1-\phi)\bar{l}\}$$

3.3.2 Housing Market Clearing

The housing market clearing condition pins down the equilibrium price of housing capital in a given period, t

$$p_t^* = \frac{p_{t+1}\left(1-\mu_L\gamma_{t+1}\right) + v'\left(\bar{L}_t\right)}{1+r_{t+1}}$$

Forward substitution gives

$$p_t^* = \sum_{j=t}^{\infty} \left(\prod_{\tau=t}^{j} \frac{1}{1+r_{\tau+1}} \right) \left[v'\left(\bar{L}_j\right) \right] \prod_{\iota=t}^{j-1} (1-\mu_L \gamma_{\iota+1})$$

While houses exposed to climate risk face a price discount in the market, the materialization of climate risk translates into a decline in the supply of inhabitable houses. This increases the marginal benefit of owning a house and puts *upwards* pressure on the contemporaneous house prices. This general equilibrium effect propagates as the supply of housing capital falls permanently, but is also amplified as climate risk increases further over time.

Proposition 4. *Let climate risk increase in all future periods, i.e.* $\{\gamma_{t+1},...,\gamma_{\infty}\}$ *by factor* $\zeta > 0$ *(i.e., future climate risk is given by* $\{\zeta\gamma_{t+1},...,\zeta\gamma_{\infty}\}$*). At a constant rate of financing, the price of housing capital rises in* ζ *for*

$$\underbrace{-\frac{v''(\bar{L}_j)\cdot \bar{L}_j}{v'(\bar{L}_j)}}_{RRA} \geq 1$$

Proof: See Appendix A.4

i.e. the price of housing capital *rises* in climate risk when households are sufficiently risk-averse with respect to their consumption of housing , which is reflected by a coefficient of relative risk aversion larger than or equal to 1.

3.3.3 Financial Market Clearing

The financial market clearing condition requires the value of assets that carry savings over time (RHS), i.e. housing, shares and corporate debt, to equal the total savings from labour income (LHS). Recall that $D_t = K_t$. Then, the supply of physical capital is given by

$$K_t = (1-\alpha)\tilde{Y}_t - p_t \bar{L}_t - e_t$$

Additionally, the financial market clearing condition highlights that total savings of labour income are large enough to cover the purchase of the stock of houses, the firm's equity as well as its corporate debt. This requires at least one type of workers to have positive savings (Döttling and Perotti (2017)). Assumption 1 ensures that high-skilled workers have higher wages, which implies that high-skilled workers are net lenders and low-skilled are either net lenders or net borrowers. Then, the volume of mortgage credit in the economy, m, is given by

$$m_t = \max\left\{0, (1-\phi)\left(p_t \bar{L}_t - w_t \tilde{l}\right)\right\}$$

Corollary 1. *Let climate risk increase in all future periods, i.e.* $\{\gamma_{t+1},...,\gamma_{\infty}\}$ *by factor* $\zeta > 0$*. Mortgage credit volumes rise in* ζ *when*

$$-\frac{v''(\bar{L}_j)\cdot \bar{L}_j}{v'(\bar{L}_j)} \geq 1$$

Remark that climate-related damages in the current period, t, have an ambiguous effects on mortgage credit volumes. As climate related-damages reduce the supply of housing capital, households are confined to purchase a lower amount of housing capital and this reduces their mortgage credit demand. However, the marginal utility effect induced by the reduction in housing supply increases the price at which housing capital is purchased. Furthermore, the reduction in wages of low-skilled workers increases mortgage credit demand. The total effect, then, also depends on the specification of housing preferences.

4 Adaptation to Climate Change

Households respond to the rise in climate risk exposure by implementing measures that reduce their vulnerability to extreme weather events. By adapting to climate change households reduce their vulnerability to future climatic impacts, leading to a leftward shift in the distribution of the *idiosyncratic* losses. Importantly, climate change adaptation does not affect the probability of being hit by an extreme weather event in a given period (γ_t) nor it's evolution ($\gamma_{t+1},...,\gamma_\infty$), but rather reduces the expected losses of housing capital when hit by an extreme weather event. I assume that households are unable to eliminate all climate-related losses. The primary advantage of adaptation, then, is to reduce the rate at which damages grow, and therefore the rate at which the housing stock falls.

4.1 Adaptation

Households invest in climate change adaptation when they purchase housing capital. This increases their protection against climate-related damages in period $t+1$. Denote by $x_{i,t} \in [0,1)$ the degree of protection invested upon by household i in period t. To achieve a certain degree of protection, $x_{i,t}$, households need to make a costly investment of $\psi(x_{i,t}) = \frac{\theta}{2} L_{i,t}(x_{i,t})^2$, where θ represents the ease of adapting. The investment costs rise in the amount of housing capital, as a larger houses require more significant investments, such as longer seawalls or a larger amount of storm-proof windows, to achieve similar levels of protection (Fried (2022)). However, even the most ambitious investment in adaptation cannot entirely prevent climate-related loss and damages (UNEP (2022)). This is reflected by the convexity of investment costs in the level of adaptation.

Let the degree of protection (hereafter: choice of adaptation), $x_{i,t}$, represent the fraction of expected losses (i.e. $\mu\gamma_{t+1}$) by which idiosyncratic losses are reduced. For a given choice of adaptation, $x_{i,t}$, this leads to a leftward shifts by $x_{i,t}\mu\gamma_{t+1}$ in the distribution of losses, $F(\xi_{i,t})$, of which the mean becomes

$$\mathbb{E}\left(\xi_{i,t+1}\right) = \left(1 - x_{i,t}\right)\mu_L\gamma_{t+1}$$

A given household with $x_{i,t} = 0$ does not undertake any measures to reduce idiosyncratic losses, while $x_{i,t} \to 1$ indicates that a given household has perfectly adapted to climate change and reduced nearly all expected losses. Define X_t as the total private investment in adaptation, i.e.

$$X_t = \int_0^1 x_{i,t} di$$

By its virtue of preserving housing capital when hit by the extreme weather event, climate change adaptation reduces the rate at which the supply of inhabitable houses declines. I assume that adaptive measures are only effective in protecting against next period's extreme weather event (i.e. each $x_{i,t}$ depreciates fully after the extreme weather at the start of period $t+1$). Then, the supply of inhabitable houses evolves according to the following law of motion

$$\begin{aligned} \bar{L}_{t+1} &= \int_0^1 \left(1-\xi_{i,t}\right) di L_t \\ &\overset{\text{LLN}}{=} \left(1-(1-X_t)\mu_L \gamma_{t+1}\right) \bar{L}_t \end{aligned}$$

The law of motion highlights that any investment in adaptation in period t affects the supply of housing in the next period, $\bar{L}_{t+1}$. As adaptation does not change the current supply, $\bar{L}_t$, it does not increase the utility from owning housing capital. Rather, adaptation increases total household utility by raising the amount of undamaged housing capital which can be sold in the next period.

Note on the Insurance I abstract from insurance polices in the present model, as any insurance pay out would be monetary. While this increases the resources available to households to purchase the non-durable consumption good, it does not reduce the fall in the supply of inhabitable houses, which is the dominant force at play. Furthermore, as households are risk-neutral with respect to their consumption of the non-durable consumption good, insurance has no role in the present model.[11]

4.2 Equilibrium with Adaptation

4.2.1 Household Optimization Problem

Since $x_{i,t}$, depreciates fully after the extreme weather at the start of period $t+1$, adaptive measure do not have any resale value (other than increasing the amount of housing capital that has non-zero resale value.) This is reflected in the household maximization problem, which is given by

[11] Beyond any theoretical considerations to abstract from insurance, it deserves mention that insurance is only to a limited extent available in practice. For the parameterization, I focus on the Netherlands, a country which is extremely flood prone. 60% of land area may be affected by overflowing rivers and roughly 70 % of population lives in this densely populated area. Although insurance has added value from a societal point of view, the limited options for diversification make it expensive for any individual insurer to insure flooding at primary flood defences. In particular, the damage burden from any breach of primary flood defenses is potentially unmanageable and may even lead to failure of the insurer if risk materializes (Doll et al. (2022)). Under the 'Wet Tegemoetkoming Schade bij Rampen' (Wts) the government may take up damage that cannot reasonably be insured. As the anticipation of government relief may induce moral hazard, the Minister of Justice and Security only decides whether to declare Wts applicable after the extreme weather event has occured. However, this introduces uncertainty on extent to which the government covers damages, and limits willingness to provide insurance (Doll et al. (2022)).

$$\max_{c_{i,t+1},x_{i,t},L_{i,t},S_{i,t},s_{i,t}} \mathbb{E}\left(U(c_{i,t+1},L_{i,t})\right) = \mathbb{E}_t\left(c_{i,t+1}\right) + v\left(L_{i,t}\right)$$

$$s.t. \quad y_{i,t} \leq \left(p_t + \frac{\theta}{2}x_{i,t}^2\right)L_{i,t} + s_{i,t}e_t + S_{i,t}$$

$$c_{i,t+1} \leq \max\left\{y_{i,t+1} + p_{t+1}\left(1-\xi_{i,t+1}\right)L_{i,t} + d_{t+1}s_{i,t} + (1+\hat{r}_{t+1})\,S_{i,t}, 0\right\}$$

$$c_{i,t+1}, x_{i,t}, L_{i,t} \geq 0$$

4.2.2 Optimal Demand for Housing and Adaptation

Climate change adaptation has an ambiguous effect on the demand for housing capital. As investments in adaptation absorb part of households' savings, this reduces demand. On the other hand, the reduction in idiosyncratic losses increases the fraction of undamaged housing capital that has a non-zero resale value, which raises demand.

Lemma 6. *When households adapt to climate change, the demand for housing capital of a given household, i, in a given period, t, is given by*

$$L_t^* = v'^{-1}\left((1+r_{t+1})\left(p_t + \frac{\theta}{2}x_{i,t}^{*2}\right) - \left(1-(1-x_{i,t}^*)\mu_L\gamma_{t+1}\right)p_{t+1}\right)$$

and the price of housing capital in a given period, t, is given by

$$p_t = \frac{\left(1-(1-x_{i,t}^*)\mu_L\gamma_{t+1}\right)p_{t+1} + v'(L_t^*)}{(1+r_{t+1})} - \frac{\theta}{2}x_{i,t}^{*2}$$

The trade-off between the present costs and future benefits of adaptation is also reflected in the price equation. The discounted value of the benefits from owning a house now equals the total per unit amount spent on housing capital, $p_t + \frac{\theta}{2}x_{i,t}^{*2}$. This implies that investment costs directly reduce the price of housing capital, which is a consequence of the associated fall in demand.

Households invest in adaptation as long as its marginal benefits outweigh its marginal cost.

Lemma 7. *The privately optimal choice of adaptation for each household i is given by*

$$x_{i,t}^* = \frac{\mu_L\gamma_{t+1}\cdot p_{t+1}}{\theta\cdot(1+r_{t+1})}$$

The optimal choice of adaptation increases in households' climate risk exposure $\left(\gamma_{t+1}\right)$ as well as the fraction of housing capital lost when hit by an extreme weather event, μ_L, as both increase the expected damages to the house. Therefore, the rise in climate exposure rises creates stronger incentives to invest in adaptation over time. Additionally, the optimal choice of adaptation increases increases in the house price, as this increases the value that is potentially lost when an extreme weather event occurs. Importantly, the privately optimal choice of adaptation is equal to the social optimum. In particular, when an unconstrained social planner maximizes utilitarian welfare, i.e.

$$\max_{x_{S,t}} \sum_{t=0}^{\infty}\left(\prod_{\tau=1}^{t}\frac{1}{1+r_{\tau+1}}\right)\left[-(1+r_{t+1})\frac{\theta}{2}x_{S,t}^2\bar{L}_t + v(\bar{L}_t)\right]$$

subject to

$$\bar{L}_j = \bar{L}_t \prod_{\iota=t}^{j-1} \left(1 - (1 - x_{S,\iota})\mu_L \gamma_{\iota+1}\right)$$

Then

$$x^*_{i,t} = x^*_{S,t}$$

This occurs as climate change adaptation increases the amount of undamaged housing capital that can be sold to the next generation. Because climate risk (and, consequently, adaptation) is accurately priced in the market, households internalize the benefits of their adaptive efforts for future generations, which prevents ”tragedy of the horizon” effects (Carney (2015)).

Proposition 5. *The privately optimal investment of climate change adaptation is dynamically efficient.*

Proof: See Appendix A.5

When climate risk is not properly reflected in market pricing (e.g. due to heterogeneous beliefs about climate change), households may not be aware of the risks faced by their housing capital and prices fail to signal this. Consequently, households would underinvest in adaptation measures. This underscores the importance of the accurate pricing of climate risk in housing markets, as this encourage households to adapt optimally.

Additionally, this result is based on the assumption that the social planner maximizes utilitarian welfare and weights generations based on market discount rates. If the welfare of future generations is valued at a rate lower than the market discount rate (i.e. a higher discount factor), underinvestment in climate change adaptation follows. This highlights the relevance of the discounting debate in climate change (Stern (2007); Nordhaus (2008)) for the determination of optimal climate change adaptation policies.

Corollary 2. *When the social planner discounts the welfare of future generations at rate* $r^{SP} \in [0,1]$ *and* $\left(\frac{1}{1+r^{SP}}\right)^{j-t} \geq \prod_{\tau=t+1}^{j}\left(\frac{1}{1+r_{\tau+1}}\right)$, *households* underinvest *in adaptation. The difference between the social and privately optimal choice of adaptation is given by*

$$\frac{\mu_L \gamma_{t+1}}{(1+r_{t+1})\theta} \cdot \sum_{j=t+1}^{\infty} \left(\left(\frac{1}{1+r^{SP}}\right)^{j-t} - \prod_{\tau=t+1}^{j} \left(\frac{1}{1+r_{\tau+1}}\right) \right) \left[-(1+r_{j+1})\frac{\theta}{2}x_j^2 + v'(L_j) \right] \prod_{\iota=t+1}^{j-1} \left(1-(1-x_\iota)\mu_L \gamma_{\iota+1}\right)$$

Proof: See Appendix C.1

4.2.3 Optimal Demand for Shares and Corporate Debt

The price of shares is unchanged (see Section 3.1.2). As the investment in adaptation is costly, it increases the total costs associated with the purchase of housing capital. This reduces households' savings, and investments in corporate and household debt follow as residual.

4.2.4 Equilibrium and Market Clearing

A competitive equilibrium is defined as an allocation $\{c_t^l, c_t^h, L_t^l, L_t^h, x_t^l, x_t^h, s_t^l, s_t^h, D_t^l, D_t^h, K_t, H_t, l_t, h_t\}_{t=0}^T$ and prices $\{p_t, e_t, r_t, R_t, w_t, q_t\}_{t=0}^T$ such that given prices

1. Households maximize lifetime utility
2. Firms maximize profits
3. Innovators choose intangible investment

and markets clear. The labour market clearing condition and stock market clearing condition remain unchanged (see Section 3.3). Climate change adaptation does affect the housing market clearing, which becomes

$$\int_0^1 L_{i^*,t} di = \bar{L}_t$$

where $\bar{L}_t = (1-(1-X_t)\mu\gamma_t)\bar{L}_{t-1}$.

The clearing condition pins down the equilibrium price of a house in period t:

$$\begin{aligned} p_t^* &= \frac{\left(1-(1-X_t^*)\mu_L\gamma_{t+1}\right)p_{t+1} + v'(\bar{L}_t)}{(1+r_{t+1})} - \frac{\theta}{2}X_t^{*,2} \\ &= \sum_{j=t}^{\infty}\left(\prod_{\tau=t}^{j}\frac{1}{1+r_{\tau+1}}\right)\left[-(1+r_{j+1})\frac{\theta}{2}X_j^{*,2} + v'(\bar{L}_j)\right]\prod_{\iota=t}^{j-1}\left(1-(1-X_\iota)\mu_L\gamma_{\iota+1}\right) \end{aligned}$$

which reveals that climate change adaptation further softens the upwards pressure on the price of housing capital by reducing the speed at which supply of inhabitable houses falls.

The financial market clearing condition must now account for the investments in adaptation, as it absorbs part of workers' savings:

$$(1-\alpha)Y_t - \left(p_t(X_t) + \frac{\theta}{2}X_t^{2*}\right)\bar{L}_t = e_t + D_t$$

Moreover, climate change adaptation increase the funding needs of those who borrow:

$$m_t = \max\left\{0, (1-\phi)\left(p_t(X_t) + \frac{\theta}{2}X_t^2\right)\bar{L}_t - w_t\tilde{l}\right\}$$

5 Endogenous Credit Constraints

Housing capital is collateralized to prevent households with large amounts of debt from repudiating on their debt contract. Until now, investors did not take into consideration the effect of climate-related damages on pledged housing capital's value. However, investors in mortgage debt have common knowledge on the rise in the probability of a household being hit by an extreme weather event, and therefore anticipate the associated losses. Since destroyed housing capital has zero liquidation value, mortgage creditors would never allow the size of household debt (gross of interest) to exceed next period's (expected) liquidation value of the housing capital (Kiyotaki and Moore (1997)):

$$-(1+\hat{r}_{t+1})S_t \leq \left(1-(1-\mathbb{E}(\bar{x}_t))\,\mu_L\gamma_{t+1}\right)p_{t+1}L_{l,t}$$

Climate risk affects the credit constraint along various dimensions. Demand for mortgage credit (LHS)increases as interest rates rise and wages fall, which both tighten the constraint. A rise in *current* house prices further increases mortgage credit demand and therefore tightens the constraint as well, while a change in *future* house prices increase borrowing capacity (RHS) and thus loosens the constraint. The borrowing capacity fall in climate risk exposure, as this reduces the amount of undamaged housing capital which can be sold in the next period. Adaptation offers a countervailing force against the tightening of the constraint by reducing expected damages. However, as households cannot commit in advance to a specific choice of investment (i.e. the level of adaptation is non-contractible), creditors form expectations on the expected average choice of adaptation, denoted by $\bar{x}_t$.

5.1 Equilibrium with Credit Constraints

High-skilled households are net lenders in equilibrium and, consequently, do not face any credit constraints. When low-skilled workers are net borrowers, they maximize expected utility subject to the credit constraint, as well as the budget constraint

$$\begin{aligned}
\max_{c_{l,t+1},x_{l,t},L_{l,t},S_{l,t}} \mathbb{E}\left(U(c_{l,t+1},L_{l,t})\right) &= \mathbb{E}_t\left(c_{l,t+1}\right)+v\left(L_{l,t}\right)\\
s.t.\quad w_t &\leq \left(p_t+\frac{\theta}{2}x_{l,t}^2\right)L_{l,t}+S_{l,t}\\
c_{l,t+1} &\leq \max\left\{p_{t+1}\left(1-\xi_{l,t+1}\right)L_{l,t}+(1+\hat{r}_{t+1})\,S_{l,t},0\right\}\\
-(1+\hat{r}_{t+1})S_{l,t} &\leq \left(1-(1-\mathbb{E}(\bar{x}_t))\mu_L\gamma_{t+1}\right)p_{t+1}L_{l,t}\\
c_{l,t+1},x_{l,t},L_{l,t} &\geq 0
\end{aligned}$$

5.1.1 Optimal Demand for Housing and Adaptation of Constrained, Low-Income Households

Credit constrained, low-income households have limited resources available to purchase housing capital and invest in adaptation. In equilibrium, low-income households borrow up to the point where the constraint binds

Lemma 8. *The demand for housing capital of constrained, low-income households is given as function of $x_{l,t}^*$:*

$$L_{l,t}^* = \frac{(1+r_{t+1})\,w_t}{(1+r_{t+1})\left(p_t+\frac{\theta}{2}x_{l,t}^2\right)-\left(1-\left(1-\mathbb{E}(\bar{x}_{l,t})\right)\mu_L\gamma_{t+1}\right)p_{t+1}}$$

The choice of adaptation of constrained, low-skilled households, $x_{l,t}^*$, is given by

$$x_{l,t}^* = \frac{\mu_L\gamma_{t+1}\cdot p_{t+1}}{\theta\cdot(1+r_{t+1})(1+\lambda_t)}$$

where $\lambda_t \geq 0$ denotes the shadow price of the constraint and $\mathbb{E}(\bar{x}_{l,t}) = x_{l,t}^*$ in a symmetric equilibrium.

Proposition 6. *Credit constrained, low-income households adapt relatively less to climate change than high-income households:*

$$x_l^* < x_h^*$$

Proof: See Appendix A.6

Corollary 3. *Credit constrained, low-income households invest less in adaptation than unconstrained, high-income households:*

$$\psi_l^* < \psi_h^*$$

Credit constraints limit access of low-income households to the required financial resources, they prevent low-income households from investing optimally in adaptation to climate change. As a result, constrained households fail to reduce vulnerability to climate impacts and remain more exposed to extreme weather events than unconstrained households. When hit by the extreme weather event, this results in a disproportionately large reduction in the housing wealth of low-income households, exacerbating wealth inequality. Additionally, the underinvestment by constrained households produces an excess reduction in the supply of housing capital, which has negative spillover effects to future generations as this leaves them with less housing capital to derive utility from. To capture the degree of underinvestment, define the adaptation gap as the adaptation of constrained households relative to the optimal level of adaptation:

$$\begin{aligned}\Lambda &= \frac{x_{h,t}^*}{x_{l,t}^*} \\ &= (1 + \lambda_t)\end{aligned}$$

Proposition 7. *At a constant rate of borrowing, the adaptation gap rises in climate risk when the utility function for housing is characterized by constant relative risk aversion.*

Proof: See Appendix A.7

The rise in climate risk induces a stronger change in demand for housing of constrained households (i.e. the elasticity of the demand for housing of low-income households' with respect to climate risk is larger than the elasticity of housing demand of high-income households when credit constraints bind). This occurs as the marginal utility from housing of constrained households is higher than the marginal utility of unconstrained households. Housing thus becomes relatively more important in the consumption bundle of constrained households, which is reflected by an increase in the shadow value of the constraint. Consequently, constrained households allocate a larger fraction of their budget on housing, rather than adaptation, which translates into a widening of the adaptation gap over time.

6 Parameterization and Counterfactual Analysis

To illustrate the equilibrium effects of rising climate risk on house prices, mortgage credit volumes and default rates, I provide a parameterization based on data of the Netherlands as the example, a country with a long history in flood risk management, which is likely to face an increase in its exposure to flood risk as sea levels rise. I set one time period equal to 30 years and run the model forward from 2010 to 2100. Additionally, I conduct counterfactual analysis to demonstrate the model outcomes for different scenarios of sea level rise, based on based on a low (RCP 2.6), medium (RCP 4.5), and high (RCP 8.5) greenhouse gas concentration trajectories from the IPCC (IPCC (2013)).

6.1 Parameterization and Functional Form Specification

I use externally calibrated parameters, as well as a number of normalizations. Externally calibrated parameters are based on data of the Netherlands, and are reported as of 2010. Other parameter values are taken from Döttling and Perotti (2017).

Parameter	Description	Value	Target/Source
A	TFP in final-good production	1	Normalization
$\tilde{h}$	Inelastic supply of high-skilled labour	40	Döttling and Perotti (2017)
$\tilde{l}$	Inelastic supply of low-skilled labour	12.5	Döttling and Perotti (2017)
$\bar{L}$	Initial stock of houses	1	Normalization
α	Capital share in final-good production	0.33	Bengtsson and Waldenström (2018)
β	Ease of innovating	1	Normalization
ϵ	Fraction of high-skilled innovators	0.05	% wealthy households with substantial interest (CBS (2010))
η	Relative productivity of intangible inputs	0.6	Döttling and Perotti (2017)
θ	Ease of adapting	1	Normalization
μ_L	Fraction of damages to housing capital	1	Normalization
μ_K	Fraction of damages to tangible capital	0.7	Target $\mu_L/\mu_K = 0.7$ (Fried (2022))
ρ	Substitution parameter	0	Cobb-Douglas Production
ϕ	Fraction of high skilled labour	0.20	Van der Mooren and De Vries (2022)
ω	Bargaining power of innovators	0.9	

Table 1: *Parameter Values*

I further assume that $v(L) = ln(L)$.

6.2 The Evolution of Climate Risk

For the purpose of illustrating the effects of the model, I let γ_t represent the fraction of the currently flood safe houses that a future rise in sea levels would put at risk of flooding. To establish a relationship between this fraction and various levels of sea level rise, I use estimates of Bosker et al. (2019).[12] To approximate the evolution of γ_t under different scenarios of sea level rise, I use the Climate Scenario Tables from IPCC (2013). These tables provide projections of global mean sea level rise for every decade from 2000 to 2100, with the period 1986-2005 as the reference period.[13] Additionally, projections are reported under a low (RCP 2.6),

[12] Bosker et al. (2019) provides estimates of the number of the currently flood safe houses that a future rise in sea levels would put at risk of flooding in a best-, medium-, and worst-case scenario with sea levels rising by 24, 100 or 150 cm respectively, based on Dutch elevation data.

[13] While the sea level in the North Sea may be more relevant for the Netherlands than the global mean, the sea level in the North Sea is more erratic. At time scales of several decades, sea level at the coast of the Netherlands

medium (RCP 4.5, RCP 6.0) and high (RCP 8.5) greenhouse gas concentration trajectory.[14] The evolution of γ under the different RCP trajectories is depicted in Figure 1.

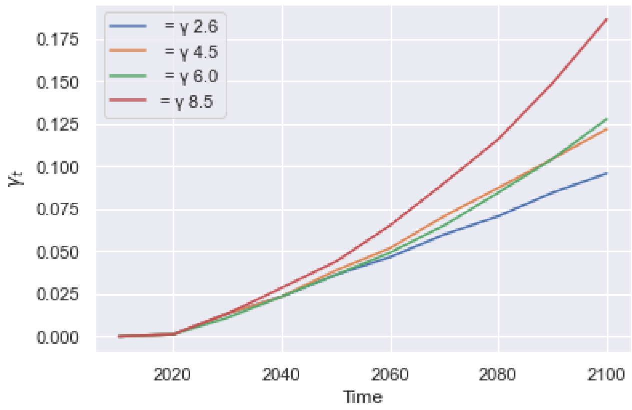

Figure 1: *The evolution of γ_t under the RCP 2.6, RCP 4.5, RCP 6.0 and RCP 8.5 trajectory.*

6.3 Results

The model is simulated in the absence of climate risk and I conduct counterfactual analysis for different Representative Concentration Pathway (RCP) trajectories. As projections of global mean sea level rise are only available up to 2100, a steady-state assumption is made for the parameter γ_t beyond this point. The steady-state equations are determined and the model is solved backwards. The simulation without climate risk reveals the steady-state, while the simulations with climate risk show deviations from it. The results are compared to those obtained when households adapt climate change. The privately optimal choice of adaptation, x_t, is determined endogenously and the evolution of x_t is depicted in Figure 2 under the different RCP trajectories.

rises at a similar rate as global sea level, indicating that the latter serves as a valid approximation (Van den Hurk et al. (2006)).

[14] The Representative Concentration Pathways trajectories describe different climate futures depending on the volume of future greenhouse gas emissions (IPCC (2014)). Under the RCP 2.6 (RCP 4.5 respectively RCP 6.0) trajectory, emissions peak in 2020 (2040 respectively 2080) and the rise in global mean temperatures is likely to stay between 0.3 to 1.7 (1.1 to 2.6 respectively 1.4 to 3.1) degrees Celsius, relative to the reference period. This translates into a rise in global mean sea levels as of 2100 of 0.26 to 0.55 (0.32 to 0.63 respectively 0.33 to 0.63) meters relative to the reference period (IPCC (2014)). Under RCP 8.5, emissions continue to rise throughout the 21st century and global mean temperatures are likely to rise by approximately 2.6 to 4.8 degrees Celsius. This translates into a rise in global mean sea levels of 0.45 to 0.82 meters (IPCC (2014)).

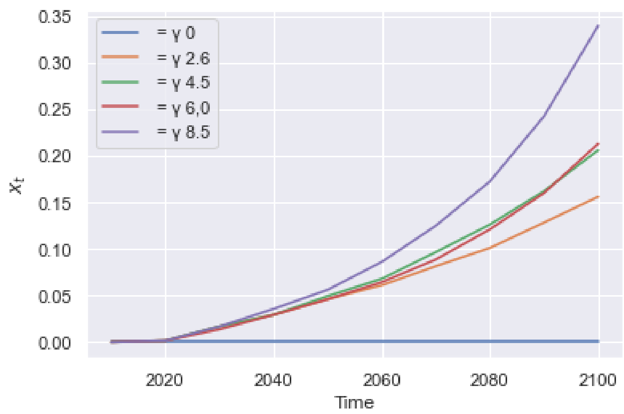

Figure 2: *The evolution of the degree of adaptation, x (right), under the RCP 2.6, 4.5, 6.0 and 8.5 trajectory.*

6.3.1 Climate Change and Adaptation

The supply of inhabitable houses falls exogenously in the model with climate risk, and the housing stock is reduced by approximately half at the end of the century in the RCP 8.5 trajectory (see left panel of Figure 3). The right panel of Figure 4 illustrates the impact of households' endogenous adaptation to climate change. By investing in adaptation, households reduce the rate at which damages grow, and therefore the rate at which the supply of inhabitable houses falls. While adaptation softens the blow, the effect remains rather small (see right panel of Figure 3).

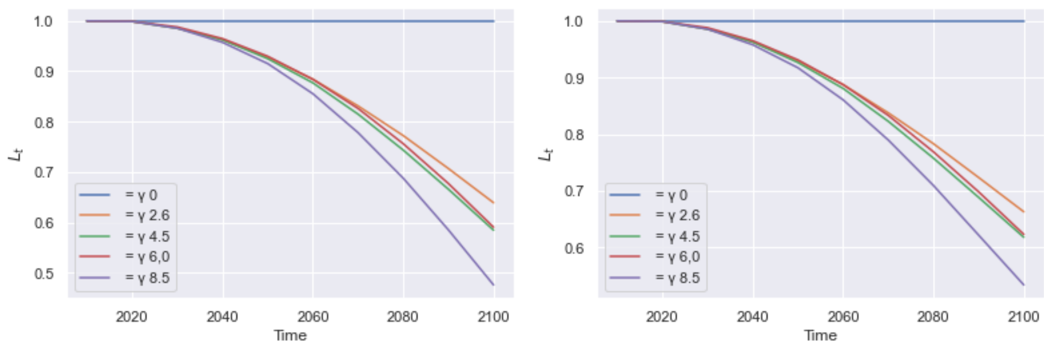

Figure 3: *The steady-state of the supply of inhabitable houses in the absence of climate change and deviations under the different RCP trajectories, for the model with climate change (left) and the model with adaptation to climate change (right).*

The strong fall in the supply of inhabitable over time (see Figure 3), which puts upwards pressure on house prices and dominates any discount of the price due to climate-related damages. This leads to a sharp growth of house prices relative to income over time, under the present functional form specification (see Figure 4).

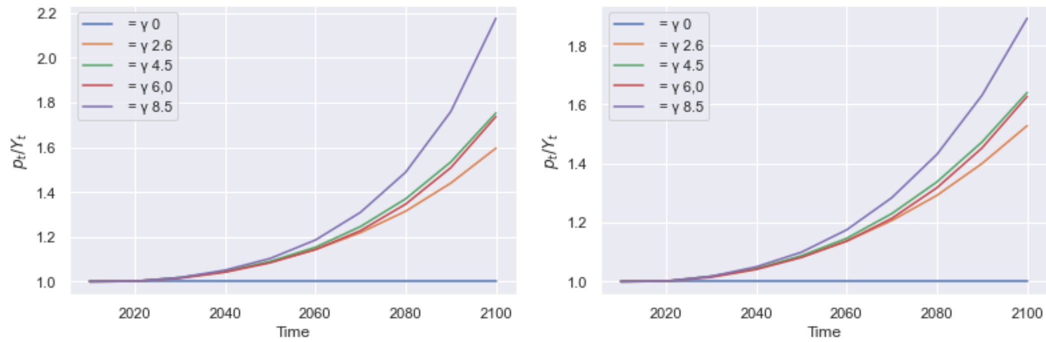

Figure 4: *The steady-state of house prices to income in the absence of climate change and deviations under the different RCP trajectories, for the model with climate change (left) and the model with adaptation to climate change (right).*

As the rate at which the supply of inhabitable houses falls when households endogenously adapt to climate change, this weakens the marginal utility effect. As a consequence, the rise in house prices relative income becomes less pronounced.

While fall in the supply of houses suppresses mortgage credit demand, the supply of houses is purchased at a higher price. The rise in prices relative to income and the fall in housing supply rather appear to reach a balance. Nevertheless, mortgage credit relative income rises in the exposure to climate risk.

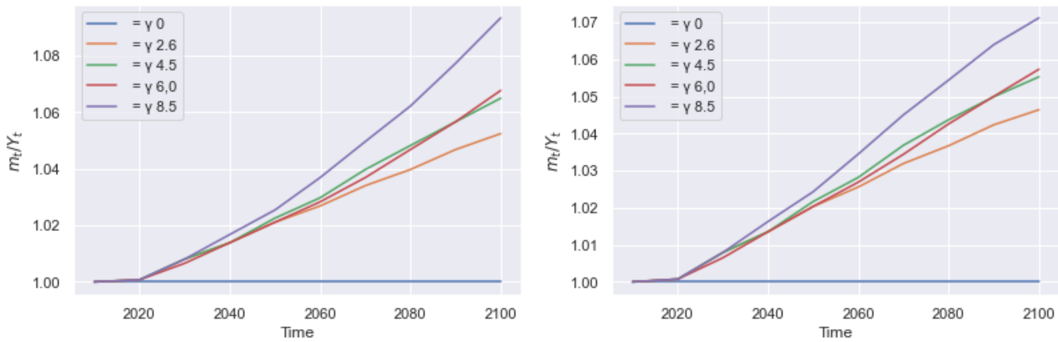

Figure 5: *The steady-state of mortgage credit to income in the absence of climate change and deviations under the different RCP trajectories, for the model with climate change (left) and the model with adaptation to climate change (right).*

This is a result of the reduction in income (as illustrated in Figure 6), which reduces wages of borrowers and leads to a higher demand for mortgage credit.

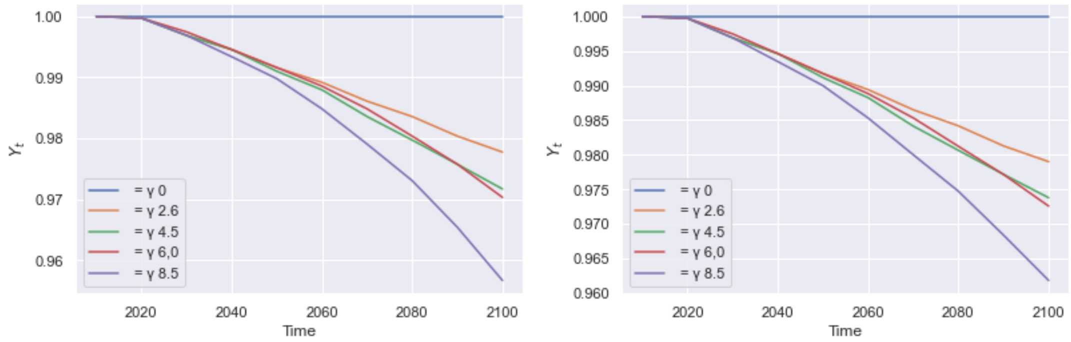

Figure 6: *The steady-state of income in the absence of climate change and deviations under the different RCP trajectories, for the model with climate change (left) and the model with adaptation to climate change (right).*

Figure 7 shows that the costs of borrowing rises in the exposure to climate risk (see Figure 7). As highlighted in Proposition 2, this occurs as physical capital becomes more scarce due climate-related damages, which raises the costs of borrowing.

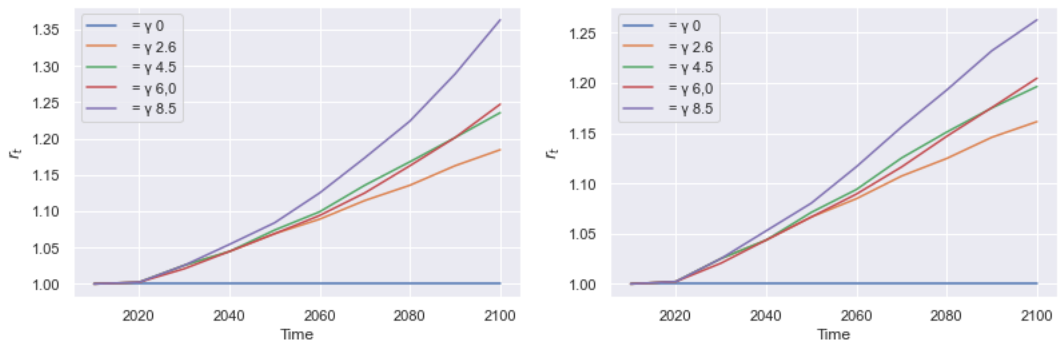

Figure 7: *The steady-state of the costs of borrowing in the absence of climate change and deviations under the different RCP trajectories, for the model with climate change (left) and the model with adaptation to climate change (right).*

The effect is again softened when households adapt endogenously to climate change, as depicted in right panel of Figure 7 Adaptation limits the rise in prices relative to income, and this channels households' (excess) savings to the market for corporate debt, rather than the housing market. This makes physical capital relatively less scarce, and reduces the increase in the costs of borrowing. Moreover, it softens the fall in income (see Figure 6, right panel).

As the costs of borrowing rises along with mortgage credit relative to income, households' financial position worsens. This is reflected by a sharp rise in loan-to-value ratios.

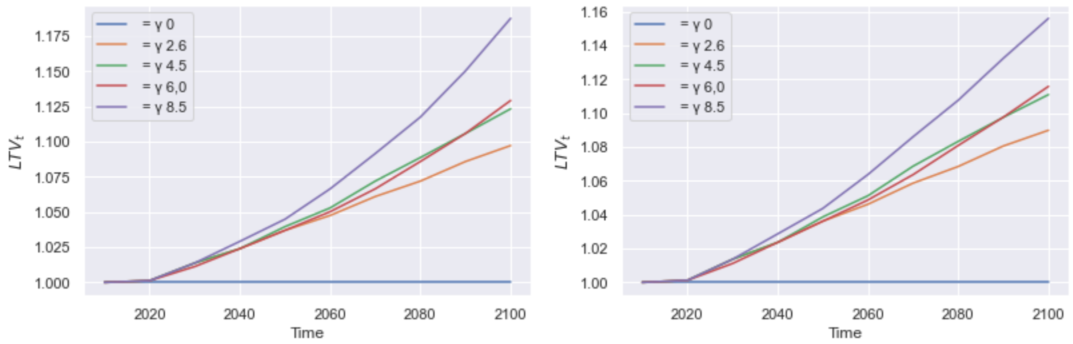

Figure 8: *The steady-state of loan-to-value ratio's in the absence of climate change and deviations under the different RCP trajectories, for the model with climate change (left) and the model with adaptation to climate change (right).*

Finally, share prices relative to income fall in the exposure to climate risk (see Figure 9), which illustrates the finding of Proposition 3.

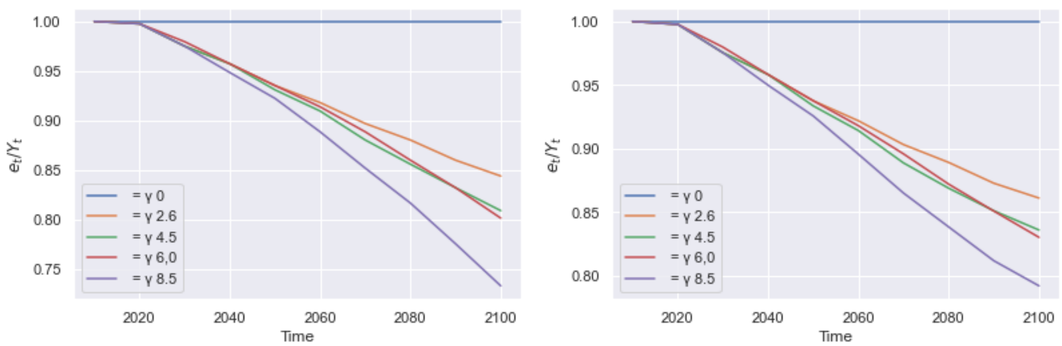

Figure 9: *The steady-state of share prices to income in the absence of climate change and deviations under the different RCP trajectories, for the model with climate change (left) and the model with adaptation to climate change (right).*

6.3.2 Default Rates

While the model outcomes thus far relied on the value of expectation of ξ, the evaluation of default rates requires the specification of its distribution. I assume that $\xi_{i,t}$ follows a Beta distribution with shape parameters $\upsilon \geq 0, \nu \geq 0$: $\xi_{i,t} \sim Beta(\upsilon, \nu)$. The Beta distribution is defined on the interval $[0,1]$, and its shape depends on the values of its parameters. As large idiosyncratic losses are relatively rare, while some losses are rather common when hit by an extreme weather event, I choose the values of the shape parameters such to ensure that probability mass is concentrated among low values of $\xi_{i,t}$, and large values are in the tail of the distribution. Specifically, I set $\upsilon = \mu_L$ (which is equal to one in the parameterization). Under the $Beta(\upsilon, \nu)$ distribution, the expectation is defined as

$$\mathbb{E}\left(\xi_{i,t}\right) = \frac{\upsilon}{\upsilon + \nu}$$

which implies that ν is given by the following expression in the model with climate change

$$\nu_t^c = \frac{1}{\mu_L \gamma_t} - 1$$

Note that this parameter is time-varying due to its dependence on γ_t.[15] The cumulative density function associated with each RCP trajectory are displayed in Figure 10.

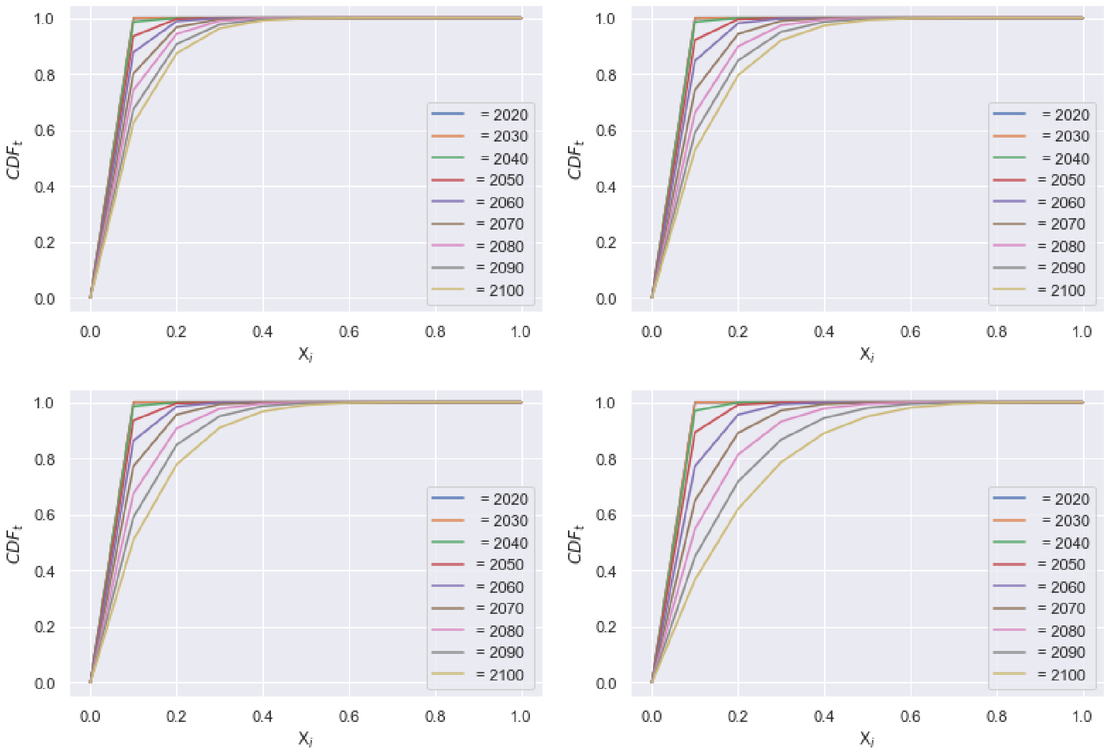

Figure 10: *The cumulative density function of $\xi_{i,t}$ for different periods in time in the model with climate change. The left upper panel plots the cumulative density functions over time under the RCP 2.6 scenario, and the right upper panel plots it under the RCP 4.5 scenario. The left lower panel plots the cumulative density functions over time under the RCP 6.0 scenario, and the right lower panel plots it under the RCP 8.5 scenario.*

The figure highlights that small losses (i.e., ξ_t) are relatively more common across all scenarios. As flood risk exposure (i.e., γ_t) rises over time, small losses become relatively less common while larger losses are suffered more frequently. This becomes clear when comparing the probability density functions across different time periods within each graph, and when comparing different RCP trajectories between graphs.

The cumulative density functions for the model with climate change adaptation are shown in Figure 11.

[15]Since γ_t remains relatively small under each RCP trajectory for the majority of the time period covered, ν_t becomes very large. This creates computational difficulties, as the estimation of the probability density function involves the Gamma function $\Gamma(\nu_t)$, which is computationally not defined for most of the values ν_t. To overcome this, I calculate the probability density function without applying any normalization (using the Gamma function) first. Then, to ensure that the area under the curve becomes equal to 1, I divide each value along the curve by the integral of the "non-normalized" probability density function.

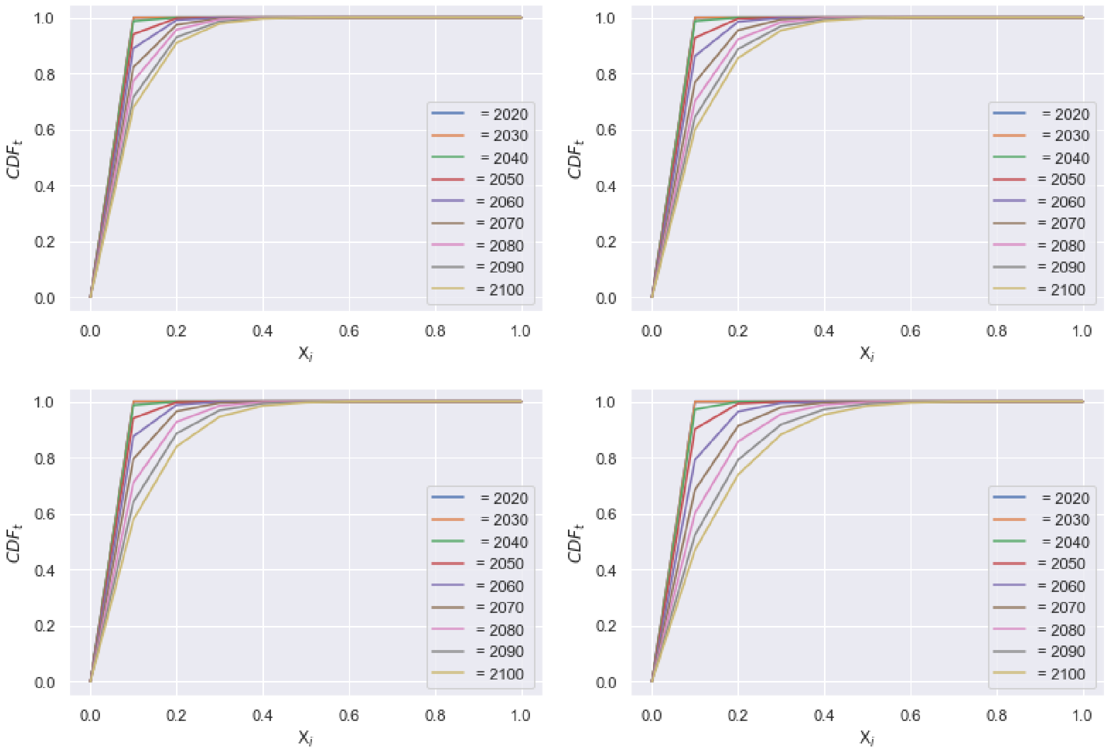

Figure 11: *The cumulative density function of $\xi_{i,t}$ for different periods in time in the model with climate change adaptation. The left upper panel plots the cumulative density functions over time under the RCP 2.6 scenario, and the right upper panel plots it under the RCP 4.5 scenario. The left lower panel plots the cumulative density functions over time under the RCP 6.0 scenario, and the right lower panel plots it under the RCP 8.5 scenario.*

The distribution of idiosyncratic losses becomes shifted leftward when households endogenously adapt to climate change. To see this, note that the shape parameter ν is given by the following expression in the model with climate change adaptation

$$\nu_t^a = \frac{1}{(1-x_{t-1})\mu_L \gamma_t} - 1$$

The cumulative density functions are used to determine default rates, which are given by

$$\chi_{i,t} = \left(1 - F\left(\hat{\xi}_{i,t+1}\right)\right)$$

where

$$\hat{\xi}_{i,t+1} = 1 - LTV_{i,t+1}$$

The results are displayed in Figure 12. As loan-to-value ratio's become higher as climate risk exposure rises, and larger losses become relatively more common, default rates rise over time. Moreover, the effect on default rates becomes stronger under more severe scenario's of climate change, and default rates reach roughly 10% at the end of the century under the RCP 8.5 trajectory.

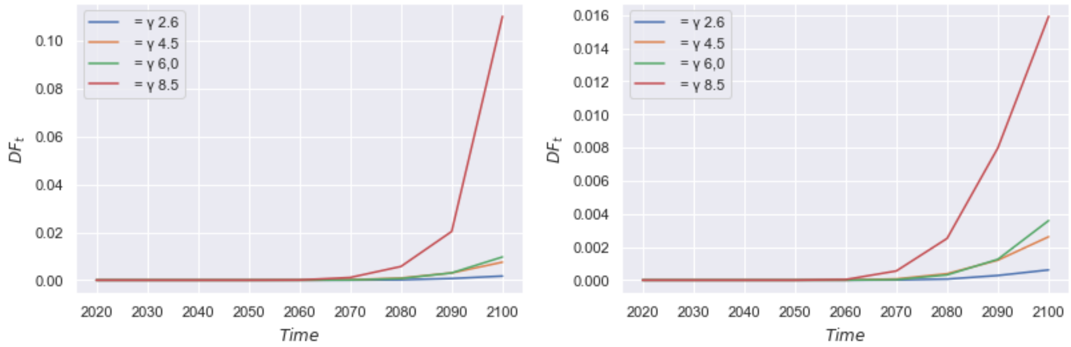

Figure 12: *The evolution of default under the different RCP trajectories, for the model with climate change (left) and the model with adaptation to climate change (right).*

As climate change adaptation reduces idiosyncratic losses (see Figure 11), and softens the rise in loan-to-value ratio's (see Figure 8), default rates become remarkably lower. Default rates remain well below 2% throughout the century, even under the most severe climate change scenario. This highlights the (social) benefits generated by climate change adaptation, and underlines the crucial importance of investing adequately in measures that reduce our vulnerability to the impacts of climate change.

7 Conclusion

This paper explores the broader implications of climate change and adaptation, through the lens of a redistributive growth model. The analysis reveals that climate change is intrinstically redistributive. Low-income workers experience a relatively larger decline in income due to their exposure to climate-related damages, while households with positive savings accumulate wealth at a higher rate. While several existing papers study the effects of climate change mitigation policies on wealth inequality, this paper is the first to focus on the relation between climate adaptation and wealth inequality. Binding financial constraints prevent those most exposed to the consequences of climate risk from investing optimally in climate change adaptation and thus from reducing their vulnerability to climatic impacts. This leads to feedback effects on wealth inequality, which become stronger over time as the relative importance of housing rises in the consumption bundle of constrained households as climatic impacts intensify. As credit constraints on the household level are a significant barrier to effective adaptation (IPCC (2023)) and prevent investments that improve sustainability of houses (Havlinova et al. (2022)), this finding underscores the need for targeted policies that address the differential impacts of climate change on those most vulnerable.

The interaction between climate change and skill-biased technological change potentially increases the severity of this problem. Both climate-change and skill-biased technological change reduce relative wages of low-skilled households and drive house prices up, causing households to become more indebted over time. At the same time, the costs of borrowing rise, and the worsening of the financial position of households is reflected by a sharp rise in default rates. This occurs even in the absence of any wealth effects driven by a failure to adapt to climate change. However, a potential limitation is that migration is not considered in the analysis. Migration is an important, yet costly, adaptation mechanism (Desmet and Rossi-Hansberg (2015); Alvarez and Rossi-Hansberg (2021)) that has a potentially strong influence on house price dynamics. Moreover, Varela (2019) shows that a rise in post-flood neighborhood segregation may increase preexisting spatial inequities. In this context, the location of firms crucially determines segregation patterns. Due to their complementariy to physical capital, only low-skilled workers are confined to live in the same area as the firm. Therefore, when firms are located in an area with higher climate risk exposure, low-skilled workers again remain more exposed to climatic impacts. Therefore, migration only offer an alternative to investments in protective measures for those that are able to move more exposed to climate risk.

This paper has shown that climate risk affects the costs of borrowing. Climate-related damages render physical capital more scarce and this increases its price. Moreover, the reduction in the firm's profits and downwards revision of dividends may triggers a revaluation of firm's equity, which suppresses equity prices. Importantly, this finding does not take into account any effects driven by firm's exposure to climate *transition* risk (as the analysis focuses exclusively on *physical* climate risk). As the transition to a green economy requires large investments, this puts additional pressure on the costs of borrowing (Bolton et al. (2021); Mongelli et al. (2022)). Accordingly, the empirical evidence shows that investors demand compensation for exposure to climate policy risk in the form of higher returns (Bolton and Kacperczyk (2021); Hsu et al. (2022)). Therefore, as a "carbon premium" further depresses share prices, exposure to climate *transition* risk would reinforce the effects of physical climate risk. However, a proper investigation of the interaction of both types of climate risk is left for further research.

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Appendix A: Proof of Propositions

A.1 Proof of Proposition 1

Wage inequality increases with climate-related damages when

$$\frac{\partial q_t / w_t}{\partial \gamma_t} = \frac{\eta}{(1-\eta)} \cdot \left(\frac{(1-\phi)\tilde{l}}{\phi \tilde{h}} \right)^{1-(1-\alpha)\rho} \cdot \frac{\partial}{\partial \gamma_t} \left(\frac{H_t}{\left(1-\mu_K \gamma_t\right) K_t} \right)^{\alpha\rho} \geq 0$$

For this to hold, it must be that

$$\frac{\mu_L}{\left(1-\mu_K \gamma_t\right)} + \frac{\partial H_t / \partial \gamma_t}{H_t} - \frac{\partial K_t / \partial \gamma_t}{K_t} \geq 0$$

or equivalently

$$\frac{\mu_L}{\left(1-\mu_K \gamma_t\right)} \geq \frac{\partial}{\partial \gamma_t} ln \left(\frac{K_t}{H_t} \right)$$

which implies that the losses of tangible capital (*i.e. the direct effect*) should be larger than the change in investment in tangible capital relative to that of intangible capital *(i.e. the indirect effect)*.To proof this, it suffices to show that

$$\frac{\partial}{\partial \gamma_t} ln \left(\frac{K_t}{H_t} \right) \leq 0$$

Note first that

$$H_t = I^*_{t-1} = \frac{\omega}{\beta} \cdot A^{\rho} \alpha \eta \frac{{Y_t}^{(1-\rho)}}{H_t^{1-\alpha\rho}} h_t^{(1-\alpha)\rho}$$

Using logarithmic differentiation, the derivative of H_t to γ_t becomes

$$(2-\alpha\rho) \frac{\partial}{\partial \gamma_t} ln(H_t) = (1-\rho) \frac{\partial}{\partial \gamma_t} ln(Y_t)$$

The derivative of Y_t to γ_t is given by

$$\begin{aligned} \frac{\partial}{\partial \gamma_t} Y_t &= \frac{\partial Y_t}{\partial H_t} \cdot \frac{\partial H_t}{\partial \gamma_t} + \frac{\partial Y_t}{\partial K_t} \cdot \frac{\partial K_t}{\partial \gamma_t} \\ &= R_t \cdot \frac{\partial H_t}{\partial \gamma_t} + (1+r_t) \cdot \frac{\partial K_t}{\partial \gamma_t} \end{aligned}$$

This equation is used to find a relation between the partial derivatives of capital to climate-related damages

$$\frac{\partial K_t}{\partial \gamma_t} = \frac{\frac{(2-\alpha\rho)\cdot Y_t}{H_t \cdot (1-\rho)} - R_t}{(1+r_t)} \cdot \frac{\partial H_t}{\partial \gamma_t}$$

Now, it must be shown that

$$\frac{\partial}{\partial \gamma_t} ln \left(\frac{K_t}{H_t} \right) \leq 0 \Leftrightarrow \frac{1}{H_t} \cdot \frac{\partial H_t}{\partial \gamma_t} \geq \frac{1}{K_t} \cdot \frac{\partial K_t}{\partial \gamma_t}$$

There are two cases;

1. $\partial H_t / \partial \gamma_t \leq 0$. In this case, the following condition is obtained by combining equation (81) and inequality (82)

$$R_t H_t + (1 + r_t) K_t \leq \frac{(2 - \alpha\rho) Y_t}{(1 - \rho)}$$

Using that

$$\alpha Y_t = R_t H_t + (1 + r_t) K_t$$

gives

$$\alpha \leq 2$$

which is always satisfied. Therefore,

Lemma 9. *The elasticity of tangible capital to climate-related damages is higher than the elasticity of intangible capital, i.e.*

$$\frac{1}{K_t} \cdot \left| \frac{\partial K_t}{\partial \gamma_t} \right| \geq \frac{1}{H_t} \cdot \left| \frac{\partial H_t}{\partial \gamma_t} \right|$$

and wage inequality increases in climate-related damages when $\partial H_t / \partial \gamma_t \leq 0$.

2. $\partial H_t / \partial \gamma_t \geq 0$. From Lemma 9, it follows that

Corollary 4. *The partial derivatives of H_t, K_t and Y_t to γ_t have the same sign, i.e.*

$$\frac{\partial H_t}{\partial \gamma_t} \geq 0 \implies \frac{\partial K_t}{\partial \gamma_t} \geq 0 \implies \frac{\partial Y_t}{\partial \gamma_t} \geq 0$$

Therefore, $\partial H_t / \partial \gamma_t \geq 0 \implies \partial Y_t / \partial \gamma_t \geq 0$, which is ruled out as $\mathscr{F}_\gamma \leq 0$.

Concluding, wage inequality increases in climate-related damages.

A.2 Proof of Proposition 2

The return to tangible capital is given by

$$(1+r_t^*) = A^\rho \alpha(1-\eta) \frac{\tilde{Y}_t^{1-\rho}}{\left(\left(1-\mu_K\gamma_t\right)K_t\right)^{1-\alpha\rho}} l_t^{(1-\alpha)\rho}$$

Using logarithmic differentiation, the derivative of r_t^* to γ_t becomes

$$\frac{\partial r_t^*}{\partial \gamma_t} = \frac{(1-\rho)}{Y_t} \cdot \frac{\partial Y_t^{net}}{\partial \gamma_t} - (1-\alpha\rho)\left[\frac{1}{K_t} \cdot \frac{\partial K_t}{\partial \gamma_t} - \frac{\mu_K}{\left(1-\mu_K\gamma_{t+1}\right)}\right]$$

For $\rho = 0$, this derivative becomes

$$\left.\frac{\partial r_t^*}{\partial \gamma_t}\right|_{\rho=0} = \frac{1}{Y_t^{net}} \cdot \frac{\partial Y_t}{\partial \gamma_t} - \frac{1}{K_t} \cdot \frac{\partial K_t}{\partial \gamma_t} + \frac{\mu_K}{\left(1-\mu_K\gamma_{t+1}\right)}$$

Recall that

$$\begin{aligned}\frac{\partial}{\partial \gamma_t}\tilde{Y}_t &= \frac{\partial \tilde{Y}_t}{\partial H_t} \cdot \frac{\partial H_t}{\partial \gamma_t} + \frac{\partial \tilde{Y}_t}{\partial K_t} \cdot \frac{\partial K_t}{\partial \gamma_t} \\ &= R_t \cdot \frac{\partial H_t}{\partial \gamma_t} + (1+r_t) \cdot \frac{\partial K_t}{\partial \gamma_t}\end{aligned}$$

and that

$$\left.\frac{\partial K_t}{\partial \gamma_t}\right|_{\rho=0} = \frac{\frac{2\cdot\tilde{Y}_t}{H_t} - R_t}{(1+r_t)} \cdot \frac{\partial H_t}{\partial \gamma_t} \Leftrightarrow \left.\frac{\partial H_t}{\partial \gamma_t}\right|_{\rho=0} = (1+r_t) \cdot \frac{\partial K_t}{\partial \gamma_t} \cdot \frac{H_t}{2\tilde{Y}_t - R_t H_t}$$

then

$$\begin{aligned}\left.\frac{\partial r_t^*}{\partial \gamma_t}\right|_{\rho=0} &= \frac{\partial K_t}{\partial \gamma_t} \cdot \left[(1+r_t) \cdot \left(\frac{2}{2\tilde{Y}_t - R_t H_t}\right) - \frac{1}{K_t}\right] + \frac{\mu_K}{\left(1-\mu_K\gamma_{t+1}\right)} \\ &= \frac{\partial K_t}{\partial \gamma_t} \cdot \frac{2}{K_t \cdot (2\tilde{Y}_t - R_t H_t)} \cdot \left((1+r_t)K_t + R_t H_t - \tilde{Y}_t - 1/2 R_t H_t\right) + \frac{\mu_K}{\left(1-\mu_K\gamma_{t+1}\right)}\end{aligned}$$

Recall that

$$\alpha \tilde{Y}_t = R_t H_t + (1+r_t)K_t$$

Then

$$\left.\frac{\partial r_t^*}{\partial \gamma_t}\right|_{\rho=0} = \underbrace{\frac{\partial K_t}{\partial \gamma_t}}_{\leq 0} \cdot \underbrace{\frac{2}{K_t \cdot (2\tilde{Y}_t - R_t H_t)}}_{\geq 0} \cdot \underbrace{\left((\alpha-1)\tilde{Y}_t - 1/2 R_t H_t\right)}_{\leq 0} + \underbrace{\frac{\mu_K}{\left(1-\mu_K\gamma_{t+1}\right)}}_{\geq 0} \geq 0$$

Now, $\frac{\partial r_t^*}{\partial \gamma_t}$ falls in ρ, as

$$\begin{aligned}
\frac{\partial r_t^*/\partial \gamma_t}{\partial \rho} &= -\frac{1}{Y_t}\cdot\frac{\partial Y_t}{\partial \gamma_t} + \frac{\alpha}{K_t}\cdot\frac{\partial K_t}{\partial \gamma_t} - \frac{\alpha\mu_K}{\left(1-\mu_K\gamma_t\right)} \\
&= \frac{\partial K_t}{\partial \gamma_t}\cdot\left[\frac{\alpha}{K_t} - \frac{2(1+r_t)}{2Y_t - R_t H_t}\right] - \frac{\alpha\mu_K}{\left(1-\mu_K\gamma_t\right)} \\
&= \underbrace{\frac{\partial K_t}{\partial \gamma_t}}_{\leq 0}\cdot\underbrace{\left[\frac{2}{K_t(2Y_t - R_t H_t)}\right]}_{\geq 0}\cdot\underbrace{[\alpha Y_t - (1+r_t)K_t - \alpha R_t H_t + \alpha/2\cdot R_t H_t)]}_{\geq 0} - \underbrace{\frac{\alpha\mu_K}{\left(1-\mu_K\gamma_t\right)}}_{\geq 0} \leq 0
\end{aligned}$$

However, for $\rho = 1$, the derivative remains negative, i.e.

$$\frac{\partial r_t^*}{\partial \gamma_t} = (1-\alpha)\Big[-\frac{1}{K_t}\cdot\underbrace{\frac{\partial K_t}{\partial \gamma_t}}_{\leq 0} + \underbrace{\frac{\mu_K}{(1-\mu_K\gamma_{t+1})}}_{\geq 0}\Big] \geq 0$$

As $\rho \in [0,1)$, it holds that the costs of borrowing *rise* is climate-related damages.

A.3. Proof of Proposition 3

Share prices are given by

$$e_t^* = \frac{(1-\omega)R_{t+1}H_{t+1}}{1+r_{t+1}}$$

i.e.

$$e_t^* = (1-\omega)\cdot\frac{\eta}{(1-\eta)}\cdot\left(\frac{\tilde{h}}{\tilde{l}}\right)^{(1-\alpha)\rho}\cdot H_{t+1}^{\alpha\rho}\cdot\left(\left(1-\mu_K\gamma_{t+1}\right)K_{t+1}\right)^{(1-\alpha\rho)}$$

Using logarithmic differentiation, the derivative of e_t^* to γ_{t+1} becomes

$$\frac{\partial e_t^*}{\partial\gamma_{t+1}} = \frac{\alpha\rho}{H_{t+1}}\cdot\frac{\partial H_{t+1}}{\partial\gamma_{t+1}} + (1-\alpha\rho)\left[\frac{1}{K_{t+1}}\cdot\frac{\partial K_{t+1}}{\partial\gamma_{t+1}} - \frac{\mu_K}{\left(1-\mu_K\gamma_{t+1}\right)}\right]$$

For $\rho = 0$, this derivative becomes

$$\left.\frac{\partial e_t^*}{\partial\gamma_{t+1}}\right|_{\rho=0} = \frac{1}{K_{t+1}}\cdot\underbrace{\frac{\partial K_{t+1}}{\partial\gamma_{t+1}}}_{\leq 0} - \underbrace{\frac{\mu_K}{\left(1-\mu_K\gamma_{t+1}\right)}}_{\geq 0} \leq 0$$

Now, $\frac{\partial e_t^*}{\partial\gamma_{t+1}}$ increases in ρ, as

$$\frac{\partial e_t^*/\partial\gamma_{t+1}}{\partial\rho} = \alpha\left[\frac{\mu_K}{\left(1-\mu_K\gamma_{t+1}\right)} + \frac{1}{H_{t+1}}\cdot\frac{\partial H_{t+1}}{\partial\gamma_{t+1}} - \frac{1}{K_{t+1}}\cdot\frac{\partial K_{t+1}}{\partial\gamma_{t+1}}\right]$$

which, following the proof in Appendix A.1, is positive, since $\mu_K, \mu_L \geq 0$.

However, for $\rho = 1$, the derivative remains negative, i.e.

$$\frac{\partial e_t^*}{\partial\gamma_{t+1}} = \frac{\alpha}{H_{t+1}}\cdot\underbrace{\frac{\partial H_{t+1}}{\partial\gamma_{t+1}}}_{\leq 0} + (1-\alpha)\left[\frac{1}{K_{t+1}}\cdot\underbrace{\frac{\partial K_{t+1}}{\partial\gamma_{t+1}}}_{\leq 0} - \underbrace{\frac{\mu_K}{\left(1-\mu_K\gamma_{t+1}\right)}}_{\geq 0}\right] \leq 0$$

As $\rho \in [0,1)$, it holds that share prices *rise* is climate risk.

A.4. Proof of Proposition 4

Suppose climate risk rises in all future periods, $\{\gamma_{t+1},...,\gamma_\infty\}$ by some factor $\zeta > 0$, i.e. $\{\zeta\gamma_{t+1},...,\zeta\gamma_\infty\}$. Then, the price of house capital is given by

$$p_t^* = \sum_{j=t}^{\infty}\left(\prod_{\tau=t}^{j}\frac{1}{1+r_{\tau+1}}\right)\left[v'\left(\bar{L}_j\right)\right]\prod_{\iota=t}^{j-1}\left(1-\mu_L\zeta\gamma_{\iota+1}\right)$$

which, for a constant rate of financing, r, becomes

$$p_t^* = \sum_{j=t}^{\infty}\left(\frac{1}{1+r}\right)^{j-t+1}\left[v'\left(\bar{L}_j\right)\right]\prod_{\iota=t}^{j-1}\left(1-\mu_L\zeta\gamma_{\iota+1}\right)$$

Then, the FOC of p_t with respect to ζ is given by

$$\frac{\partial p_t}{\partial\zeta} = \sum_{j=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{j-t+1}\left[\frac{\partial v'(\bar{L}_j)}{\partial\bar{L}_j}\cdot\frac{\partial\bar{L}_j}{\partial\zeta}\cdot\prod_{\iota=t}^{j-1}(1-\mu_L\zeta\gamma_{\iota+1})+v'\left(\bar{L}_j\right)\cdot\frac{\partial}{\partial\zeta}\left(\prod_{\iota'=t}^{j-1}\left(1-\mu_L\zeta\gamma_{\iota'+1}\right)\right)\right]$$

Remark that

$$\bar{L}_j = \bar{L}_t\prod_{\iota=t}^{j-1}\left(1-\mu_L\zeta\gamma_{\iota+1}\right)$$

and

$$\frac{\partial\bar{L}_j}{\partial\zeta} = -\mu_L\bar{L}_t\sum_{\iota=t}^{j-1}\gamma_{\iota+1}\prod_{\iota'=t,\iota'\neq\iota}^{j-1}\left(1-\mu_L\zeta\gamma_{\iota'+1}\right)$$

then

$$\frac{\partial p_t}{\partial\zeta} = \sum_{j=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{j-t+1}\left[-\frac{\partial v'(\bar{L}_j)}{\partial\bar{L}_j}\cdot\bar{L}_j - v'\left(\bar{L}_j\right)\right]\cdot\mu_L\sum_{\iota=t}^{j-1}\gamma_{\iota+1}\prod_{\iota'=t,\iota'\neq\iota}^{j-1}\left(1-\mu_L\zeta\gamma_{\iota'+1}\right)$$

This is positive when

$$-\frac{\partial v'(\bar{L}_j)}{\partial\bar{L}_j}\cdot\bar{L}_j - v'\left(\bar{L}_j\right)\geq 0$$

or, equivalently, when

$$-\frac{\frac{\partial v'(\bar{L}_j)}{\partial\bar{L}_j}\cdot\bar{L}_j}{v'\left(\bar{L}_j\right)}\geq 1$$

A.5 Proof of Proposition 5

A

The unconstrained social planner maximizes utilitarian welfare, i.e.

$$\sum_{t=0}^{\infty}\left(\prod_{\tau=1}^{t}\frac{1}{1+r_{\tau+1}}\right)\left[-(1+r_{t+1})\frac{\theta}{2}x_t^2\bar{L}_t+v(\bar{L}_t)\right]$$

subject to

$$\bar{L}_j=\bar{L}_t\prod_{\iota=t}^{j-1}\left(1-(1-x_\iota)\mu_L\gamma_{\iota+1}\right)$$

The first order condition for x_t is

$$\left(\prod_{\tau=1}^{t}\frac{1}{1+r_{\tau+1}}\right)(1+r_{t+1})\theta x_t L_t=\sum_{j=t+1}^{\infty}\left(\prod_{\tau=1}^{j}\frac{1}{1+r_{\tau+1}}\right)\left[-(1+r_{j+1})\frac{\theta}{2}x_j^2+v'(\bar{L}_j)\right]\frac{\partial\bar{L}_j}{\partial x_t}$$

Using that

$$\frac{\partial\bar{L}_j}{\partial x_t}=\mu_L\gamma_{t+1}\bar{L}_t\prod_{\iota=t+1}^{j-1}\left(1-(1-x_\iota)\mu_L\gamma_{\iota+1}\right)$$

this becomes

$$(1+r_{t+1})\theta x_t=\mu_L\gamma_{t+1}\sum_{j=t+1}^{\infty}\left(\prod_{\tau=t+1}^{j}\frac{1}{1+r_{\tau+1}}\right)\left[-(1+r_{j+1})\frac{\theta}{2}x_j^2+v'(\bar{L}_j)\right]\prod_{\iota=t+1}^{j-1}\left(1-(1-x_\iota)\mu_L\gamma_{\iota+1}\right)$$

The first-order condition of the unconstrained household is

$$(1+r_{t+1})\theta x_t=\mu_L\gamma_{t+1}\cdot p_{t+1}$$

and the first-order condition of the unconstrained social planner is

$$(1+r_{t+1})\theta x_t=\mu_L\gamma_{t+1}\sum_{j=t+1}^{\infty}\left(\prod_{\tau=t+1}^{j}\frac{1}{1+r_{\tau+1}}\right)\left[-(1+r_{j+1})\frac{\theta}{2}x_j^2+v'(\bar{L}_j)\right]\prod_{\iota=t+1}^{j-1}\left(1-(1-x_\iota)\mu_L\gamma_{\iota+1}\right)$$

A necessary and sufficient condition for the privately optimal level of investment to be efficient is

$$p_{t+1}=\sum_{j=t+1}^{\infty}\left(\prod_{\tau=t+1}^{j}\frac{1}{1+r_{\tau+1}}\right)\left[-(1+r_{j+1})\frac{\theta}{2}x_j^2+v'(\bar{L}_j)\right]\prod_{\iota=t+1}^{j-1}\left(1-(1-x_\iota)\mu_L\gamma_{\iota+1}\right)$$

Remark that the demand for housing capital of unconstrained households is

$$L^*_{t+1}=v'^{-1}\left((1+r_{t+2})\left(p_{t+1}+\frac{\theta}{2}x_{t+1}^2\right)-\left(1-(1-x_{t+1})\mu_L\gamma_{t+2}\right)p_{t+2}\right)$$

Together with the housing market clearing condition, this becomes

$$p_{t+1}=\left(\frac{1}{1+r_{t+2}}\right)\left[-(1+r_{t+2})\frac{\theta}{2}x_{t+1}^2+v'(\bar{L}_{t+1})+\left(1-(1-x_{t+1})\mu_L\gamma_{t+2}\right)\cdot p_{t+2}\right]$$

and forward substitution of this expression gives

$$p_{t+1}=\sum_{j=t+1}^{\infty}\left(\prod_{\tau=t+1}^{j}\frac{1}{1+r_{\tau+1}}\right)\left[-(1+r_{j+1})\frac{\theta}{2}x_j^2+v'(\bar{L}_j)\right]\prod_{\iota=t+1}^{j-1}\left(1-(1-x_\iota)\mu_L\gamma_{\iota+1}\right)$$

Therefore, the first-order condition of the unconstrained social planner is equivalent to the first-order condition of the unconstrained household, which implies that the market outcome is efficient.

A.6 Proof of Proposition 6

The first-order condition for L_l^* is derived from the constrained household problem as

$$-(1+r_{t+1})(1+\lambda)\left(\frac{\theta}{2}x_{l,t}^{*,2}(\lambda)+p_t\right)+(1+\lambda)\left(1-\left(1-x_{l,t}^*\right)\mu_L\gamma_{t+1}\right)p_{t+1}+v'\left(L_{l,t}^*\right)=0$$

This condition defines an implicit expression for λ_t, i.e.

$$\lambda_t=\frac{\left(1-\left(1-x_{l,t}^*(\lambda_t)\right)\mu_L\gamma_{t+1}\right)p_{t+1}+v'\left(L_{l,t}^*\right)-(1+r_{t+1})\left(\frac{\theta}{2}x_{l,t}^{*,2}(\lambda_t)+p_t\right)}{(1+r_{t+1})\left(\frac{\theta}{2}x_{l,t}^{*,2}(\lambda_t)+p_t\right)-p_{t+1}\left(1-\left(1-x_{l,t}^*(\lambda_t)\right)\mu\gamma_{t+1}\right)}$$

Since λ_t denotes the change in the optimal level of utility for loosening the constraint (and the marginal utility of owning housing is strictly positive), it holds by construction that $\lambda_t \geq 0$. What remains to be determined, is under which condition $\lambda_t = 0$.

As the denominator is strictly positive (see Proof of Lemma 8), the following condition must hold for λ_t to be zero

$$p_t=\frac{\left(1-\left(1-x_{l,t}^*(0)\right)\mu_L\gamma_{t+1}\right)p_{t+1}+v'\left(L_{l,t}^*\right)}{1+r_{t+1}}-\frac{\theta}{2}x_{l,t}^{*,2}(0)$$

Now, $\lambda_t = 0 \implies x_{l,t}^* = x_{h,t}^*$. Recall that the price of housing capital is defined as

$$p_t=\frac{\left(1-(1-x_{h,t}^*)\mu_L\gamma_{t+1}\right)p_{t+1}+v'\left(L_{h,t}^*\right)}{1+r_{t+1}}-\frac{\theta}{2}x_{h,t}^{*,2}$$

Then, a necessary and sufficient condition for the above condition to hold is that $v'\left(L_{l,t}^*\right) = v'\left(L_{h,t}^*\right)$. This implies that $L_{l,t}^* = L_{h,t}^*$, which means that the constraint doesn't bind. However, when the credit constraint binds, $L_{l,t}^* < L_{h,t}^* \implies v'\left(L_{l,t}^*\right) > v'\left(L_{h,t}^*\right)$. By contradiction, it must then be the case that $\lambda_t > 0$.

A.7 Proof of Proposition 7

To evaluate the effect of a rise in γ_{t+1} on λ_t, note first that the expression for λ_t can be rewritten as (see Proof of Lemma 8)

$$(1+\lambda_t) = \frac{v'\left(L^*_{l,t}\right)}{v'\left(L^*_{h,t}\right) + \frac{\theta}{2}(1+r_{t+1})x^{*2}_{l,t}(\lambda_t)\lambda_t^2}$$

then, the FOC becomes

$$\frac{\partial\lambda_t}{\partial\gamma_{t+1}} = \frac{1}{\left(v'\left(L^*_{h,t}\right) + \frac{\theta}{2}(1+r_{t+1})x^{*2}_{l,t}(\lambda_t)\lambda_t^2\right)^2} \cdot \left(\frac{\partial v'\left(L^*_{l,t}\right)}{\partial L^*_{l,t}} \cdot \frac{\partial L^*_{l,t}}{\partial\gamma_{t+1}} \cdot \left[v'\left(L^*_{h,t}\right) + \frac{\theta}{2}(1+r_{t+1})x^{*2}_{l,t}\lambda_t^2\right]\right.$$

$$\left. - v'\left(L^*_{l,t}\right) \cdot \left[\frac{\partial v'\left(L^*_{h,t}\right)}{\partial L^*_{h,t}} \cdot \frac{\partial L^*_{h,t}}{\partial\gamma_{t+1}} + \frac{\theta}{2} \cdot \frac{\partial(1+r_{t+1})}{\partial\gamma_{t+1}} \cdot x_l^{*2}\lambda_t^2 + \theta(1+r_{t+1}) \cdot \left(\frac{\partial x^*_{l,t}}{\partial\lambda_t} \cdot \frac{\partial\lambda_t}{\partial\gamma_{t+1}} \cdot x^*_{l,t}\lambda_t^2 + x^{*2}_{l,t}\lambda_t \cdot \frac{\partial\lambda_t}{\partial\gamma_{t+1}}\right)\right]\right)$$

For this to be positive, it must hold that

$$\frac{\partial v'\left(L^*_{l,t}\right)}{\partial L^*_{l,t}} \cdot \frac{\partial L^*_{l,t}}{\partial\gamma_{t+1}} \cdot \left[v'\left(L^*_{h,t}\right) + \frac{\theta}{2}(1+r_{t+1})x^{*2}_{l,t}\lambda_t^2\right]$$

$$- v'\left(L^*_{l,t}\right) \cdot \left[\frac{\partial v'\left(L^*_{h,t}\right)}{\partial L^*_{h,t}} \cdot \frac{\partial L^*_{h,t}}{\partial\gamma_{t+1}} + \frac{\theta}{2} \cdot \frac{\partial(1+r_{t+1})}{\partial\gamma_{t+1}} \cdot x_l^{*2}\lambda_t^2 + \theta(1+r_{t+1}) \cdot \left(\frac{\partial x^*_{l,t}}{\partial\lambda_t} \cdot \frac{\partial\lambda_t}{\partial\gamma_{t+1}} \cdot x^*_{l,t}\lambda_t^2 + x^{*2}_{l,t}\lambda_t \cdot \frac{\partial\lambda_t}{\partial\gamma_{t+1}}\right)\right] \geq 0$$

This expression can be rewritten as

$$\frac{\partial\lambda_t}{\partial\gamma_{t+1}}\left[1 + v'\left(L^*_{l,t}\right) \cdot \theta(1+r_{t+1}) \cdot \left(\frac{\partial x^*_{l,t}}{\partial\lambda_t} \cdot x^*_l\lambda_t^2 + x^{*2}_{l,t}\lambda_t\right)\right]$$

$$= \frac{\partial v'\left(L^*_{l,t}\right)}{\partial L^*_{l,t}} \cdot \frac{\partial L^*_{l,t}}{\partial\gamma_{t+1}} \cdot \left[v'\left(L^*_{h,t}\right) + \frac{\theta}{2}(1+r_{t+1})x^{*2}_{l,t}\lambda_t^2\right] - v'\left(L^*_{l,t}\right) \cdot \left[\frac{\partial v'\left(L^*_{h,t}\right)}{\partial L^*_{h,t}} \cdot \frac{\partial L^*_{h,t}}{\partial\gamma_{t+1}} + \frac{\theta}{2} \cdot \frac{\partial(1+r_{t+1})}{\partial\gamma_{t+1}} \cdot x_l^{*2}\lambda_t^2\right]$$

Using that

$$\frac{\partial x^*_{l,t}}{\partial\lambda_t} = -\frac{x^*_{l,t}}{(1+\lambda_t)}$$

the LHS is rewritten

$$\frac{\partial \lambda_t}{\partial \gamma_{t+1}}\left[1+v'\left(L^*_{l,t}\right)\cdot\theta(1+r_{t+1})\cdot\left(\frac{\partial x^*_{l,t}}{\partial \lambda_t}\cdot x^*_l\lambda_t^2+x^{*2}_{l,t}\lambda_t\right)\right]$$

$$=\frac{\partial \lambda_t}{\partial \gamma_{t+1}}\left[1+\lambda_t v'\left(L^*_{l,t}\right)\cdot\frac{\left(\mu\gamma_{t+1}p_{t+1}\right)^2}{\theta(1+r_{t+1})(1+\lambda_t)^2}-\lambda_t^2 v'\left(L^*_{l,t}\right)\cdot\frac{\left(\mu\gamma_{t+1}p_{t+1}\right)^2}{\theta(1+r_{t+1})(1+\lambda_t)^3}\right]$$

$$=\frac{\partial \lambda_t}{\partial \gamma_{t+1}}\left[1+\lambda_t v'\left(L^*_{l,t}\right)\cdot\frac{\left(\mu\gamma_{t+1}p_{t+1}\right)^2}{\theta(1+r_{t+1})(1+\lambda_t)^2}\cdot\underbrace{\left(1-\underbrace{\frac{\lambda_t}{(1+\lambda_t)}}_{\leq 1}\right)}_{\geq 0}\right]$$

$$=\frac{\partial \lambda_t}{\partial \gamma_{t+1}}\underbrace{\left[1+\lambda_t v'\left(L^*_{l,t}\right)\cdot\theta(1+r_{t+1})x^2_{l,t}\cdot\left(1-\frac{\lambda_t}{(1+\lambda_t)}\right)\right]}_{\geq 0}$$

Therefore, the LHS of the equation is positive and it remains to be evaluated whether the RHS is positive as well. In order to do so, the RHS is first rewritten:

$$\frac{\partial v'\left(L^*_{l,t}\right)}{\partial L^*_{l,t}}\cdot\frac{\partial L^*_{l,t}}{\partial \gamma_{t+1}}\cdot\left[v'\left(L^*_{h,t}\right)+\frac{\theta}{2}(1+r_{t+1})x^{*2}_{l,t}\lambda_t^2\right]-v'\left(L^*_{l,t}\right)\cdot\left[\frac{\partial v'\left(L^*_{h,t}\right)}{\partial L^*_{h,t}}\cdot\frac{\partial L^*_{h,t}}{\partial \gamma_{t+1}}+\frac{\theta}{2}\cdot\frac{\partial(1+r_{t+1})}{\partial \gamma_{t+1}}\cdot x^{*2}_l\lambda_t^2\right]$$

$$=\frac{\partial v'\left(L^*_{l,t}\right)}{\partial L^*_{l,t}}\cdot\frac{\partial L^*_{l,t}}{\partial \gamma_{t+1}}\cdot\frac{1}{v'\left(L^*_{l,t}\right)}-\frac{\partial v'\left(L^*_{h,t}\right)}{\partial L^*_{h,t}}\cdot\frac{\partial L^*_{l,t}}{\partial \gamma_{t+1}}\cdot\frac{1}{v'\left(L^*_{h,t}\right)}$$

$$+\frac{\left(\frac{\theta}{2}(1+r_{t+1})x^{*2}_{l,t}\lambda_t^2\right)}{v'\left(L^*_{h,t}\right)}\cdot\left[\frac{\partial v'\left(L^*_{l,t}\right)}{\partial L^*_{l,t}}\cdot\frac{\partial L_{l,t}}{\partial \gamma_{t+1}}\cdot\frac{1}{v'\left(L^*_{l,t}\right)}-\frac{\partial(1+r_{t+1})}{\partial \gamma_{t+1}}\cdot\frac{1}{(1+r_{t+1})}\right]$$

Now, for a constant cost of borrowing, this becomes

$$\underbrace{\frac{\partial v'\left(L^*_{l,t}\right)}{\partial L^*_{l,t}}}_{\leq 0}\cdot\underbrace{\frac{\partial L^*_{l,t}}{\partial \gamma_{t+1}}}_{\leq 0}\cdot\underbrace{\frac{1}{v'\left(L^*_{l,t}\right)}}_{\geq 0}-\underbrace{\frac{\partial v'\left(L^*_{h,t}\right)}{\partial L^*_{h,t}}}_{\leq 0}\cdot\underbrace{\frac{\partial L^*_{l,t}}{\partial \gamma_{t+1}}}_{\leq 0}\cdot\underbrace{\frac{1}{v'\left(L^*_{h,t}\right)}}_{\geq 0}+\frac{\left(\frac{\theta}{2}(1+r_{t+1})x^{*2}_{l,t}\lambda_t^2\right)}{v'\left(L^*_{h,t}\right)}\cdot\underbrace{\frac{\partial v'\left(L^*_{l,t}\right)}{\partial L^*_{l,t}}}_{\leq 0}\cdot\underbrace{\frac{\partial L^*_{l,t}}{\partial \gamma_{t+1}}}_{\leq 0}\cdot\underbrace{\frac{1}{v'\left(L^*_{l,t}\right)}}_{\geq 0}$$

Then, for $\partial\lambda_t/\partial\gamma_{t+1}$ to be positive, it must hold that

$$\frac{\partial v'\left(L^*_{l,t}\right)}{\partial L^*_{l,t}}\cdot\frac{\partial L^*_{l,t}}{\partial \gamma_{t+1}}\cdot\frac{1}{v'\left(L^*_{l,t}\right)}-\frac{\partial v'\left(L^*_{h,t}\right)}{\partial L^*_{h,t}}\cdot\frac{\partial L^*_{l,t}}{\partial \gamma_{t+1}}\cdot\frac{1}{v'\left(L^*_{h,t}\right)}\geq 0$$

When the utility function is characterized by CRRA with relative risk aversion coefficient ς, this becomes

$$-\varsigma\left(\underbrace{\frac{\partial L^*_{l,t}}{\partial \gamma_{t+1}}}_{\leq 0}\cdot\frac{1}{L^*_{l,t}}-\underbrace{\frac{\partial L^*_{h,t}}{\partial \gamma_{t+1}}}_{\leq 0}\cdot\frac{1}{L^*_{h,t}}\right)\geq 0$$

which holds as the elasticity of the demand for housing of constrained households is larger than the elasticity for housing of unconstrained households in the presence of binding financial constraints.

Thus, for a constant cost of borrowing, λ_t rises in γ_{t+1} when the utility function for housing is characterized by CRRA with RRA coefficient ς.

Note that for $\partial\lambda_t/\partial\gamma_{t+1}$ to be positive in the general equilibrium, it must additionally hold that

$$\left[\frac{\partial v'\left(L^*_{l,t}\right)}{\partial\gamma_{t+1}}\cdot\frac{1}{v'\left(L^*_{l,t}\right)}-\frac{\partial(1+r_{t+1})}{\partial\gamma_{t+1}}\cdot\frac{1}{(1+r_{t+1})}\right]\geq 0$$

while this cannot be verified analytically, it is likely that this condition holds due to the concavity of the utility function. However, this result remains to be established by means of model simulations.

Appendix B: Proof of Lemmas

B.1 Proof of Lemma 1

Households maximize utility subject to the budget constraint and limited liability constraint:

$$\max_{c_{i,t+1},L_{i,t},s_{i,t},S_{i,t}} \mathbb{E}\left(U(c_{i,t+1},L_{i,t})\right) = \mathbb{E}_t\left(c_{i,t+1}\right) + v\left(L_{i,t}\right)$$

$$s.t. \quad y_{i,t} \leq p_t L_{i,t} + s_{i,t}e_t + S_{i,t}$$

$$c_{i,t+1} \leq \max\left\{y_{i,t+1} + p_{t+1}\left(1-\xi_{i,t+1}\right)L_{i,t} + d_{t+1}s_{i,t} + (1+\hat{r}_{t+1})S_{i,t}, 0\right\}$$

$$c_{i,t+1}, L_{i,t} \geq 0,$$

where $c_{i,t+1}$ is the consumption of household i in period $t+1$ and $\mathbb{E}_t$ denotes expectations formed at date t.

Given the probability of default, the expectation of household i's consumption in period $t+1$, $c_{i,t+1}$, as formed at date t, becomes:

$$\mathbb{E}_t\left(c_{i,t+1}\right) = G\left(\hat{\xi}_{i,t+1}\right)\left(p_{t+1}\left(1-\mathbb{E}\left(\xi_{i,t+1}|\xi_{t+1} \leq \hat{\xi}_{i,t+1}\right)\right)L_{i,t} + (1+\hat{r}_{t+1})S_{i,t}\right) + d_{t+1}s_{i,t}$$

No arbitrage requires that the expected payoff of investing in corporate debt is equal to the expected payoff of holding household debt:

$$(1+r_{t+1})(-S_{i,t}) = G\left(\hat{\xi}_{i,t+1}\right)(1+\hat{r}_{t+1})(-S_{i,t}) + \left(1-G\left(\hat{\xi}_{i,t+1}\right)\right)p_{t+1}\left(1-\mathbb{E}\left(\xi_{i,t+1}|\xi_{i,t+1} > \hat{\xi}_{i,t+1}\right)\right)L_{i,t}$$

where the expected payoff of holding household debt is equal to the repayment of the loan with interest in case the household does not default and the revenue from selling the collateral in case of default.

The no-arbitrage condition can be rewritten as

$$G(\hat{\xi}_{i,t+1})\left(p_{t+1}\left(1-\mathbb{E}\left(\xi_{i,t+1}|\xi_{i,t+1} \leq \hat{\xi}_{i,t+1}\right)\right)L_{i,t} + (1+\hat{r}_{t+1})(S_{i,t})\right) = (1+r_{t+1})(S_{i,t}) + p_{t+1}\left(1-\mathbb{E}\left(\xi_{i,t+1}\right)\right)L_{i,t+1}$$

and the expectation of household i's consumption in period $t+1$, $c_{i,t+1}$, as formed at date t, becomes

$$\mathbb{E}_t\left(c_{i,t+1}\right) = (1+r_{t+1})(S_{i,t}) + p_{t+1}\left(1-\mathbb{E}\left(\xi_{i,t+1}\right)\right)L_{i,t} + d_{t+1}s_{i,t}$$

Using that $\mathbb{E}\left(\xi_{i,t+1}\right) = \mu\gamma_{t+1}$, the household optimization problem can be written as

$$\max_{c_{i,t+1},L_{i,t},s_{i,t}} \mathbb{E}\left(U(c_{i,t+1},L_{i,t})\right) = (1+r_{t+1})(S_{i,t}) + p_{t+1}\left(1-\mu_L\gamma_{t+1}\right)L_{i,t} + d_{t+1}s_{i,t} + v\left(L_{i,t}\right)$$

$$s.t. \quad y_{i,t} \leq p_t L_{i,t} + s_{i,t}e_t + S_{i,t}$$

$$c_{i,t+1}, L_{i,t} \geq 0,$$

and the budget constraint is substituted to obtain

$$\max_{c_{i,t+1},L_{i,t},s_{i,t}} \mathbb{E}\left(U(c_{i,t+1},L_{i,t})\right) = (1+r_{t+1})(y_{i,t} - p_t L_{i,t} - s_{i,t}e_t) + p_{t+1}\left(1-\mu_L\gamma_{t+1}\right)L_{i,t} + d_{t+1}s_{i,t} + v\left(L_{i,t}\right)$$

$$s.t. c_{i,t+1}, L_{i,t} \geq 0,$$

The FOC for $L_{i,t}$ is given by

$$-(1+r_{t+1})p_t + p_{t+1}\left(1-\mu_L\gamma_{t+1}\right) + v'\left(L_{i,t}\right) = 0$$

and the demand for housing capital of each household i in period t is given by

$$L_t^* = v'^{-1}\left((1+r_{t+1})p_t - p_{t+1}\left(1-\mu_L\gamma_{t+1}\right)\right)$$

and the price of housing capital in a given period, t, becomes

$$p_t = \frac{\left(1-\mu_L\gamma_{t+1}\right)p_{t+1} + v'\left(L_t^*\right)}{(1+r_{t+1})}$$

B.2 Proof of Lemma 2

The wages of high-skilled workers, q_t, follows from the FOC for h_t from the firm optimization problem:

$$\begin{aligned} q_t^* &= \frac{\partial \tilde{Y}_t}{\partial h_t} \\ &= A^\rho(1-\alpha)\eta\frac{\tilde{Y}_t^{1-\rho}}{h_t^{1-(1-\alpha)\rho}}H^{\alpha\rho} \end{aligned}$$

and the wages low-skilled workers, l_t, follow from the FOC for l_t from the firm optimization problem:

$$\begin{aligned} w_t^* &= \frac{\partial \tilde{Y}_t}{\partial l_t} \\ &= A^\rho(1-\alpha)(1-\eta)\frac{\tilde{Y}_t^{1-\rho}}{l_t^{1-(1-\alpha)\rho}}K^{\alpha\rho} \end{aligned}$$

Then, the wage ratio becomes

$$\frac{q_t^*}{w_t^*} = \frac{\eta}{(1-\eta)}\cdot\left(\frac{H_t}{\left(1-\mu_K\gamma_t\right)K_t}\right)^{\alpha\rho}\cdot\left(\frac{l_t}{h_t}\right)^{1-(1-\alpha)\rho}$$

B.3 Proof of Lemma 3

The return to tangible capital follow from the FOC for K_t from the firm optimization problem:

$$\begin{aligned} (1+r_t^*) &= \frac{\partial \tilde{Y}_t}{\partial K_t} \\ &= A^\rho\alpha(1-\eta)\frac{\tilde{Y}_t^{1-\rho}}{\left(\left(1-\mu_K\gamma_t\right)K_t\right)^{1-\alpha\rho}}l^{(1-\alpha)\rho} \end{aligned}$$

B.4 Proof of Lemma 4

The return to intangible capital follow from the FOC for H_t from the firm optimization problem:

$$\begin{aligned} R_t^* &= \frac{\partial \tilde{Y}_t}{\partial H_t} \\ &= A^{\rho}\alpha\eta\frac{\tilde{Y}_t^{1-\rho}}{H_t^{1-\alpha\rho}}h^{(1-\alpha)\rho} \end{aligned}$$

The productive use of intangible capital requires the commitment of innovators, who capture a fraction ω of its value. This means that the return earned by innovators on the intangible capital they create, H_{t+1}, is $\omega R_{t+1}H_{t+1}$. The effort cost associated with creating H_{t+1} units of intangible capital is $C(I_{H,t+1}) = \frac{1}{2}I_{H,t+1}^2$. Innovators, then, create intangible capital until its marginal benefits are equal to its marginal costs and invest

$$I_t^* = \frac{\omega R_{t+1}^*}{\beta}$$

where $I_t^* = H_{t+1}^*$.

B.5 Proof of Lemma 5

Dividends of each firm f in period t are given by

$$\begin{aligned} d_t^* &= (1-\omega)R_t^* H_t^* \\ &= A^{\rho}\alpha\eta\tilde{Y}_t^{1-\rho}H_t^{\alpha\rho}h^{(1-\alpha)\rho} \end{aligned}$$

Using logarithmic differentiation, the derivative of d_t^* to γ_t becomes

$$\frac{\partial d_t^*}{\partial \gamma_t} = \frac{1-\rho}{\tilde{Y}_t}\frac{\partial \tilde{Y}_t}{\partial \gamma_t} + \frac{\alpha\rho}{H_t}\frac{\partial H_t}{\partial \gamma_t}$$

which is smaller than 0.

Proof of Lemma 6

With adaptation, households' optimization problem becomes

$$\begin{aligned} \max_{c_{i,t+1},x_{i,t},L_{i,t},S_{i,t},s_{i,t}} \quad & \mathbb{E}\left(U(c_{i,t+1},L_{i,t})\right) = \mathbb{E}_t\left(c_{i,t+1}\right) + v\left(L_{i,t}\right) \\ s.t. \quad & y_{i,t} \le \left(p_t + \frac{\theta}{2}x_{i,t}^2\right)L_{i,t} + s_{i,t}e_t + S_{i,t} \\ & c_{i,t+1} \le \max\left\{y_{i,t+1} + p_{t+1}\left(1-\xi_{i,t+1}\right)L_{i,t} + d_{t+1}s_{i,t} + (1+\hat{r}_{t+1})S_{i,t}, 0\right\} \\ & c_{i,t+1}, x_{i,t}, L_{i,t} \ge 0 \end{aligned}$$

where $c_{i,t+1}$ is the consumption of household i in period $t+1$ and $\mathbb{E}_t$ denotes expectations formed at date t.

The same no-arbitrage condition as in Appendix B.1 can be used to find the expectation, formed at date t, of household i's consumption in period $t+1$, $c_{i,t+1}$, becomes

$$\mathbb{E}_t\left(c_{i,t+1}\right) = (1+r_{t+1})(S_{i,t}) + p_{t+1}\left(1-\mathbb{E}\left(\xi_{i,t+1}\right)\right)L_{i,t} + d_{t+1}s_{i,t}$$

With adaptation $\mathbb{E}\left(\xi_{i,t+1}\right) = \left(1 - x_{i,t}\right)\mu\gamma_{t+1}$ and the household optimization problem can be written as

$$\max_{c_{i,t+1},x_{i,t},L_{i,t},S_{i,t},s_{i,t}} \mathbb{E}\left(U(c_{i,t+1},x_{i,t},L_{i,t})\right) = (1+r_{t+1})\left(S_{i,t}\right) + p_{t+1}\left(1-(1-x_{i,t})\mu_L\gamma_{t+1}\right)L_{i,t} + d_{t+1}s_{i,t} + v(L_{i,t})$$
$$s.t. \quad y_{i,t} \leq \left(p_t + \frac{\theta}{2}x_{i,t}^2\right)L_{i,t} + S_{i,t} + s_{i,t}e_t$$
$$c_{i,t+1}, x_{i,t}, L_{i,t} \geq 0$$

and the budget constraint is substituted to obtain

$$\max_{c_{i,t+1},x_{i,t},L_{i,t},S_{i,t},s_{i,t}} \mathbb{E}\left(U(c_{i,t+1},x_{i,t},L_{i,t})\right) = (1+r_{t+1})\left(y_{i,t} - \left(p_t + \frac{\theta}{2}x_{i,t}^2\right)L_{i,t} - s_{i,t}e_t\right)$$
$$+ p_{t+1}\left(1-(1-x_{i,t})\mu_L\gamma_{t+1}\right)L_{i,t} + d_{t+1}s_{i,t} + v(L_{i,t})$$
$$s.t. \quad c_{i,t+1}, x_{i,t}, L_{i,t} \geq 0$$

The FOC for $L_{i,t}$ is given by

$$-(1+r_{t+1})\left(p_t + \frac{\theta}{2}x_{i,t}^2\right) + p_{t+1}\left(1-(1-x_{i,t})\mu_L\gamma_{t+1}\right) + v'\left(L_{i,t}\right) = 0$$

and the demand for housing capital of each household i in period t is given by

$$L_t^* = v'^{-1}\left((1+r_{t+1})\left(p_t + \frac{\theta}{2}x_{i,t}^2\right) - p_{t+1}\left(1-(1-x_{i,t})\mu_L\gamma_{t+1}\right)\right)$$

and the price of housing capital in a given period, t, becomes

$$p_t = \frac{\left(1-(1-x_{i,t})\mu_L\gamma_{t+1}\right)p_{t+1} + v'\left(L_t^*\right)}{(1+r_{t+1})} - \frac{\theta}{2}x_{i,t}^2$$

B.7 Proof of Lemma 7

The optimal choice of adaptation of household i in period t follow from the FOC from the household problem in Section B.6 to $x_{i,t}$:

$$x_t^* = \frac{\mu_L\gamma_{t+1}p_{t+1}}{\theta(1+r_{t+1})}$$

B.8 Proof of Lemma 8

Low-skilled households maximize expected utility subject to their budget constraint and the credit constraint

$$\max_{c_{l,t+1},x_{l,t},L_{l,t},S_{l,t}} \mathbb{E}\left(U(c_{l,t+1},L_{l,t})\right) = \mathbb{E}_t\left(c_{l,t+1}\right) + v\left(L_{l,t}\right)$$
$$s.t. \quad w_t \leq \left(p_t + \frac{\theta}{2}x_{l,t}^2\right)L_{l,t} + S_{l,t}$$
$$c_{l,t+1} \leq \max\left\{y_{l,t+1} + p_{t+1}\left(1-\xi_{l,t+1}\right)L_{l,t} + (1+\hat{r}_{t+1})S_{l,t}, 0\right\}$$
$$-(1+\hat{r}_{t+1})S_{l,t} \leq \left(1-(1-\mathbb{E}(\bar{x}_t))\mu_L\gamma_{t+1}\right)p_{t+1}L_{l,t}$$
$$c_{l,t+1}, x_{l,t}, L_{l,t} \geq 0$$

where $c_{l,t+1}$ is the consumption of household i in period $t+1$ and $\mathbb{E}_t$ denotes expectations formed at date t.

The same no-arbitrage condition as in Appendix B.6 can be used to find the expectation, formed at date t, of low-skilled household l's consumption in period $t+1$, $c_{l,t+1}$, becomes

$$\mathbb{E}_t\left(c_{l,t+1}\right) = (1+r_{t+1})(S_{l,t}) + p_{t+1}\left(1-\mathbb{E}\left(\xi_{l,t+1}\right)\right)L_{l,t} + d_{t+1}s_{l,t}$$

Using that $\mathbb{E}\left(\xi_{l,t+1}\right) = \left(1-x_{l,t}\right)\mu\gamma_{t+1}$, the optimization problem of low-skilled households can be written as

$$\begin{aligned}
\max_{c_{l,t+1},x_{l,t},L_{l,t},S_{l,t}} \mathbb{E}\left(U(c_{l,t+1},x_{l,t},L_{l,t})\right) &= (1+r_{t+1})\left(S_{l,t}\right) + p_{t+1}\left(1-(1-x_{l,t})\mu_L\gamma_{t+1}\right)L_{l,t} + v\left(L_{l,t}\right)\\
s.t. \quad w_t &\leq \left(p_t + \frac{\theta}{2}x_{l,t}^2\right)L_{l,t} + S_{l,t}\\
-(1+r_{t+1})S_{l,t} &\leq \left(1-(1-\mathbb{E}(\bar{x}_t))\mu_L\gamma_{t+1}\right)p_{t+1}L_{l,t}\\
c_{l,t+1}, x_{l,t}, L_{l,t} &\geq 0
\end{aligned}$$

and the budget constraint is substituted to obtain

$$\begin{aligned}
\max_{c_{l,t+1},x_{l,t},L_{l,t},S_{l,t}} \mathbb{E}\left(U(c_{l,t+1},x_{l,t},L_{l,t})\right) &= (1+r_{t+1})\left(w_t - \left(p_t + \frac{\theta}{2}x_{l,t}^2\right)L_{l,t}\right) + p_{t+1}\left(1-(1-x_{l,t})\mu_L\gamma_{t+1}\right)L_{l,t} + v\left(L_{l,t}\right)\\
s.t. \quad -(1+r_{t+1})S_{l,t} &\leq \left(1-(1-\mathbb{E}(\bar{x}_t))\mu_L\gamma_{t+1}\right)p_{t+1}L_{l,t}\\
c_{l,t+1}, x_{l,t}, L_{l,t} &\geq 0
\end{aligned}$$

Define the Lagrangian for this constrained maximization problem as

$$\begin{aligned}
\mathscr{L} = &(1+r_{t+1})\left(w_t - \left(p_t + \frac{\theta}{2}x_{l,t}^2\right)L_{l,t}\right) + p_{t+1}\left(1-(1-x_{l,t})\mu_L\gamma_{t+1}\right)L_{l,t} + v\left(L_{l,t}\right)\\
&+\lambda\left(\left(1-(1-\mathbb{E}(\bar{x}_t))\mu_L\gamma_{t+1}\right)p_{t+1}L_{l,t} - \left((1+r_{t+1})\left[\left(p_t + \frac{\theta}{2}x_{l,t}^{*2}\right)L_{l,t} - w_t\right]\right)\right)
\end{aligned}$$

where $\lambda \geq 0$ denotes the Lagrangian multiplier.

The FOC for the demand for housing by low-skilled household l, $L_{l,t}$ is given by

$$-(1+r_{t+1})(1+\lambda)\left(p_t + \frac{\theta}{2}x_{l,t}^{*2}\right) + p_{t+1}\left(1-(1-x_{l,t}^*)\mu_L\gamma_{t+1}\right) + v'\left(L_{l,t}^*\right) + \lambda\left(1-(1-\mathbb{E}(\bar{x}_t))\mu_L\gamma_{t+1}\right)p_{t+1} = 0$$

where $\mathbb{E}(\bar{x}_{l,t}) = x_{l,t}^*$ in a symmetric equilibrium. The optimal demand for housing by low-skilled household l, $L_{l,t}^*$ is defined in terms of the optimal choice of adaptation by low-skilled household, $x_{l,t}^*$

$$L_{l,t}^* = v'^{-1}\left((1+r_{t+1})(1+\lambda)\left(p_t + \frac{\theta}{2}x_{l,t}^{*2}\right) - (1+\lambda)\left(1-\left(1-x_{l,t}^*\right)\mu_L\gamma_{t+1}\right)p_{t+1}\right)$$

As the degree to which households invest in adaptation is not contractible, the FOC for the choice of adaptation of low-skilled household, $x_{l,t}$, is given by

$$-\theta(1+r_{t+1})(1+\lambda)x_{l,t}^*L_{l,t}^* + \mu\gamma_{t+1}p_{t+1}L_{l,t}^* = 0$$

and the optimal choice of adaptation of low-skilled household, $x^*_{l,t}$, is given by

$$x^*_{l,t} = \frac{\mu\gamma_{t+1}p_{t+1}}{\theta(1+r_{t+1})(1+\lambda)}$$

The FOC for the Lagrangian multiplier, λ is given by

$$\left(1-(1-\mathbb{E}(\bar{x}_t))\mu_L\gamma_{t+1}\right)p_{t+1}L_{l,t} - \left((1+r_{t+1})\left[\left(p_t + \frac{\theta}{2}x^2_{l,t}\right)L_{l,t} - w_t\right]\right) = 0$$

where $\mathbb{E}(x_{l,t}) = x^*_{l,t}$ in a symmetric equilibrium. This equation provides the corner-solution for the optimal demand for housing by low-skilled household, $L^*_{l,t}$ in terms of their optimal choice of adaptation, $x^*_{l,t}$

$$L^*_{l,t} = \frac{(1+r_{t+1})w_t}{(1+r_{t+1})\left(p_t + \frac{\theta}{2}x^2_{l,t}\right) - \left(1-\left(1-x^*_{l,t}\right)\mu_L\gamma_{t+1}\right)p_{t+1}}$$

As $L_{l,t} \geq 0$, it must hold that

$$(1+r_{t+1})\left(p_t + \frac{\theta}{2}x^2_{l,t}\right) - \left(1-\left(1-x^*_{l,t}\right)\mu_L\gamma_{t+1}\right)p_{t+1} > 0$$

Substituting the price of housing capital gives

$$(1+r_{t+1})\left(\frac{\left(1-(1-x^*_{h,t})\mu_L\gamma_{t+1}\right)p_{t+1} + v'\left(L^*_{h,t}\right)}{1+r_{t+1}} - \frac{\theta}{2}\left(x^{*,2}_{h,t} - x^2_{l,t}\right)\right) - \left(1-\left(1-x^*_{l,t}\right)\mu_L\gamma_{t+1}\right)p_{t+1} > 0$$

which can be rewritten to obtain the following condition

$$v'\left(L^*_{h,t}\right) > \left(x^*_{l,t} - x^*_{h,t}\right)\mu_L\gamma_{t+1}\cdot p_{t+1} - (1+r_{t+1})\frac{\theta}{2}\left(x^{*,2}_{l,t} - x^{*,2}_{h,t}\right)$$

Now, there are two cases

1. When the credit constraint doesn't bind, $x_{l,t} = x_{h,t}$. Then, the RHS becomes 0 and the condition is satisfied for any strictly positive $L_{h,t}$.

2. When the credit constraint binds (i.e. $\lambda > 0$), the expression for $x^*_{l,t}$ and $x^*_{h,t}$ can be used to rewrite the RHS to

$$\begin{aligned}
&-\frac{(\mu\gamma_{t+1}p_{t+1})^2}{(1+r)\theta}\left[\frac{1}{2} - \frac{1}{(1+\lambda_t)} + \frac{1}{2(1+\lambda_t)^2}\right] \\
&= -\frac{(\mu\gamma_{t+1}p_{t+1})^2\cdot\lambda_t^2}{2(1+r)\theta(1+\lambda_t)^2} \\
&= -\frac{\theta}{2}(1+r_{t+1})x^{*2}_{l,t}(\lambda_t)\lambda_t^2
\end{aligned}$$

which is negative. Therefore, the condition is again satisfied for any positive $L_{h,t}$.

Appendix C: The Discounting Debate in Climate Change

The privately optimal choice of adaptation is dynamically efficient as long as the social planner maximizes utilitarian welfare, and weights generations based on the market discount rate. This brings forward what has been regarded "the thorniest issues in all climate-change economics: how should we compare present and future costs and benefits?" (Nordhaus (2013)). While this question has received considerable attention within the environmental economic literature, disagreements about the value of the appropriate social discount rate continue to be at the heart of the climate policy debate (Heal and Millner (2014)). In particular, there is no consensus about the choice of r in the dynamic social welfare function of utilitarian form, i.e.

$$\int_{t=0}^{\infty} e^{-rt} u(c(t))dt$$

which, in discrete time, is equivalent to

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t u(c(t))$$

Stern (2007) argues that it is immoral to use a social discount rate based on market discount rates when evaluating the welfare of future generations and favours a more a priori approach, where $r = 0.014$.[16] On the other hand, Nordhaus (2008) argues that economists have no particular expertise in what is morally right and should ensure that models replicate reality. Therefore, Nordhaus (2008) advocates a more market based approach where $r = 0.055$.[17] While the differences appear small, such small differences lead to large disparities between the estimated social cost of carbon and consequently the recommended intensity of climate mitigation policies (Heal and Millner (2014)). This paper shows that the choice of social discount rates also matters crucially for the determination of optimal adaptation policies. If the social planner were to evaluate the welfare of future generations using a social discount rate smaller than the market discount rate (i.e. a larger discount factor), the market outcome is no longer be efficient and households would underinvest in adaptation.

D.1 Proof of Corollary 2

Suppose the social planner discounts the welfare of future generations at rate $r^{SP} \in [0, 1]$. Then, the unconstrained social planner maximizes

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r^{SP}}\right)^t \left[-(1+r_{t+1})\frac{\theta}{2}x_t^2 L_t + v(L_t)\right]$$

subject to

$$L_j = L_t \prod_{\iota=t}^{j-1} \left(1-(1-x_i)\mu_L \gamma_{\iota+1}\right)$$

[16]Following the Ramsey rule, the relationship between the equilibrium real return on capital, r^*, and the growth rate of the economy, g^* is given by $r^* = \varsigma + \sigma \cdot g^*$, where σ denotes the elasticity of consumption and ς denotes the generational rate of time preference (Nordhaus (2007)). Stern (2007) assumes that $\sigma = 1$, $\varsigma = 0.001$ and $g^* = 0.013$. This gives a real return of capital equal to $r^* = 0.014$..

[17]Nordhaus (2008) assumes that $\sigma = 2$, $\varsigma = 0.015$ and $g^* = 0.02$. This gives a real return on capital equal to $r^* = 0.055$.

The first order condition for x_t is

$$(1+r_{t+1})\theta x_t = \mu_L \gamma_{t+1} \sum_{j=t+1}^{\infty} \left(\frac{1}{1+r^{SP}}\right)^{j-t} \left[-(1+r_{j+1})\frac{\theta}{2}x_j^2 + v'(L_j)\right] \prod_{\iota=t+1}^{j-1} \big(1-(1-x_i)\mu_L \gamma_{\iota+1}\big)$$

As the first order condition of the unconstrained household is given by

$$\begin{aligned}(1+r_{t+1})\theta x_t &= \mu_L \gamma_{t+1} \cdot p_{t+1} \\ &= \mu_L \gamma_{t+1} \sum_{j=t+1}^{\infty} \left(\prod_{\tau=t+1}^{j} \frac{1}{1+r_{\tau+1}}\right) \left[-(1+r_{j+1})\frac{\theta}{2}x_j^2 + v'(L_j)\right] \prod_{\iota=t+1}^{j-1} \big(1-(1-x_i)\mu_L \gamma_{\iota+1}\big)\end{aligned}$$

the private solution is not efficient for $\left(\frac{1}{1+r^{SP}}\right)^{j-t} \neq \prod_{\tau=t+1}^{j} \left(\frac{1}{1+r_{\tau+1}}\right)$.

In particular, when

1. $\left(\frac{1}{1+r^{SP}}\right)^{j-t} \geq \prod_{\tau=t+1}^{j} \left(\frac{1}{1+r_{\tau+1}}\right)$, households *underinvest* in adaptation. The difference between the social and privately optimal choice of adaptation is given by

$$\frac{\mu_L \gamma_{t+1}}{\theta(1+r_{t+1})} \cdot \sum_{j=t+1}^{\infty} \left(\left(\frac{1}{1+r^{SP}}\right)^{j-t} - \prod_{\tau=t+1}^{j} \left(\frac{1}{1+r_{\tau+1}}\right)\right) \left[-(1+r_{j+1})\frac{\theta}{2}x_j^2 + v'(L_j)\right] \prod_{\iota=t+1}^{j-1} \big(1-(1-x_i)\mu_L \gamma_{\iota+1}\big)$$

2. $\left(\frac{1}{1+r^{SP}}\right)^{j-t} \leq \prod_{\tau=t+1}^{j} \left(\frac{1}{1+r_{\tau+1}}\right)$, households *overinvest* in adaptation. The difference between the social and the private optimal choice of adaptation is given by

$$\frac{\mu_L \gamma_{t+1}}{\theta(1+r_{t+1})} \cdot \sum_{j=t+1}^{\infty} \left(\prod_{\tau=t+1}^{j} \left(\frac{1}{1+r_{\tau+1}}\right) - \left(\frac{1}{1+r^{SP}}\right)^{j-t}\right) \left[-(1+r_{j+1})\frac{\theta}{2}x_j^2 + v'(L_j)\right] \prod_{\iota=t+1}^{j-1} \big(1-(1-x_i)\mu_L \gamma_{\iota+1}\big)$$