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Preference estimation from point allocation experiments

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Preference estimation from point allocation experiments

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Abstract

Point allocation experiments are widely used in the social sciences. In these experiments, survey respondents distribute a fixed total number of points across a fixed number of alternatives. This paper reviews the different perspectives in the literature about what respondents do when they distribute points across options. We find three main alternative interpretations in the literature, each having different implications for empirical work. We connect these interpretations to models of utility maximization that account for point and budget constraints and investigate the role of budget constraints in more detail. We show how these constraints impact the regression specifications for point allocation experiments that are commonly used in the literature. We also show how a formulation of a taste for variety as entropy that had been previously used to analyse market shares can fruitfully be applied to choice behaviour in point allocation experiments.

Keywords: constant-sum paired comparison; probabilistic choice; entropy; constrained optimization

Declarations of interest: none.

1. Introduction

Point allocation experiments have been used in various scientific fields for many years. As far as we know, the approach was used first in the field of psychology where subjects rate the intensity of stimuli (Comrey, 1950; Guilford, 1954; Metfessel, 1947). Later, marketing studies have asked respondents to distribute chips or tokens across options to indicate their preferences for these options (e.g. Silk and Urban (1978)). The method has also been used in health economics, asking respondents to distribute donor livers across patients (e.g. Ubel and Lowenstein (1996)), or a fixed budget across patients (e.g. Schwappach (2003)) or across health programmes (e.g. Skedgel et al. (2015)). The point allocation approach is closely related to asking respondents how likely they would be to choose a given brand (e.g. Juster (1966)) or to experiments where respondents assign subjective likelihoods to choice alternatives (e.g. Blass et al. (2010)).

The reason for asking respondents to distribute points, tokens or shares in experiments is that it provides richer and more nuanced information about the respondents' preferences than the widely used discrete choice experiments where respondents indicate their preferred option, and possibly their least preferred option (Marley & Louviere, 2005). But it is not obvious what information exactly can be obtained from point allocation experiments.

In this note, we first provide an overview of the assumptions made in various parts of the social science literature about what respondents do when they distribute points across alternatives. We find three main approaches: interval theory, ratio theory, and a log-ratio model associated with the elicitation of choice probabilities. We link these approaches to existing empirical applications. We then investigate the possibility of making these approaches consistent with utility-maximizing behaviour. The specification of a value or direct utility function and of constraints that impact the decision can be helpful when policies are valued or when researchers want to learn more about the determinants of choice. In particular, we show how to deal with the marginal utility of private income or public budget and to allow for heterogeneity therein. We also show that a model previously used to explain market shares, that models taste for variety as entropy (Anderson et al., 1988) can also be used to analyse the respondents' decisions in a point allocation experiment.

The contribution of this note is very practical and might prove useful for teaching purposes, experimental developers and gives input for the dialogue between economists and choice modelers in the social sciences.

2. Literature review

This section reviews papers where respondent n allocates an amount Q points to rate J options, where each of these options is characterized by a vector of attributes X_{nj}. For simplicity it is assumed that the number of points Q and the number of alternatives J are equal for all respondents. Let us call q_{nj} the number of points allocated to alternative j, under the condition that $\sum_j q_{nj} = Q$. This type of experiment is sometimes called 'constant sum paired comparison' (Moore & Lehmann, 1989; C. D. Skedgel et al., 2015; C. Skedgel & Regier, 2015).[1] Most applications ask respondents to allocate points or the like between two alternatives. Comrey (1950) argues that limiting the task to two alternatives makes answering easier for respondents. Guilford (1954) and Moore & Lehmann (1989) compare asking respondents to rate two or more alternatives at a time, and do not find systematic differences in answers. Some studies in marketing do ask respondents to distribute choice probabilities across a set of brands (e.g. Reibstein (1975)) or to report the likelihood of buying one specific product or type of product (e.g. Juster (1966)).

For the empirical analysis of point allocation experiments, researchers need to make assumptions about the thought process of the respondents when they allocate the points across options. Hauser and Shugan (1980) present an overview of possible assumptions about what respondents are doing when they allocate points. An important question is whether one thinks that points convey meaningful information about the intensity of respondents' preferences, i.e. some cardinal measure of utility, or whether one thinks that points only convey ordinal information. We use the classification of Hauser and Shugan (1980) to structure our literature review, starting with the ordinal interpretation of the point allocation and then discussing the cardinal interpretation. For each possible set of assumptions, we discuss their meaning and which empirical studies are based, explicitly or implicitly, on these assumptions.

2.1. Point allocation as ordinal information

The most limited information one can derive from a point allocation is that more points attributed to an option indicate that it is preferred. This is consistent with an idea of utility that is purely **ordinal** and entirely **deterministic** (Marschak, 1950; Neumann & Morgenstern, 1947). Preferences can then be inferred using conjoint analysis (Krantz & Tversky, 1971).

[1] This term is also used in studies that ask survey respondents to distribute points across attributes of a product to measure their relative importance in consumer choice (Zwahlen et al 1996, Netzer & Srinivasan 2011, Zafri et al 2020, Ujjwal & Bandyoppadhyaya 2021). In this paper, we focus on the literature that asks respondents to distribute points across alternatives.

If one is willing to attribute more information value to the points allocated, one can assume that the points allocated convey information about the relative likelihood of choosing the options. This implies introducing randomness in preferences (**stochastic theory**). The stochastic term can reflect mistakes made by the respondents, measurement errors by the researcher, or characteristics of the alternatives that are not observed by the researcher but do matter for the respondents' choice.

Let us denote $P_n(j > k)$ the probability that individual n chooses/prefers option *j* over option *k*. Hauser and Shugan (1980) show for a pairwise comparison of options and a sequential allocation of points, the process is stationary. When preferences are transitive, each individual point allocation is Bernoulli, and the maximum-likelihood estimator for $P_n(j > k)$ is equal to the number of points of an alternative divided by the total:

$$P_n(j > k) = \frac{q_{nj}}{q_{nj} + q_{nk}} = \frac{q_{nj}}{Q}.$$

Another way to interpret the point allocation is to apply random utility maximisation. In the special case of the logit additive random utility model, which assumes logistically distributed error terms which enter the utility function additively, this becomes (McFadden, 1974):

$$P_n\left(U_{nj} > U_{nk}\right) = \frac{e^{V_{nj}}}{e^{V_{nj}} + e^{V_{nk}}}$$

where $V_{nj}(X_{nj},\ \beta_n)$ is the deterministic part of the conditional additive random utility function $U_{nj} = V_{nj} + \varepsilon_{nj}$. This interpretation of point allocation is not applied so much in the discrete choice literature, probably because it in fact reduces information that is collected in a cardinal way to ordinal discrete choice information, which does not seem very efficient. Nevertheless, data from a point allocation experiment can always be transformed to ordinal choice data. An example is Linley and Hughes (2013), who transform budget allocation between two patient groups into an indicator of whether either one of both groups or none is favoured.

2.2. Asking for choice probabilities

Some studies do ask respondents to compare alternatives by indicating the probability that they would choose each alternative. Early applications in the field of marketing include Byrnes (1964), Ferber & Piskie (1965), Juster (1966), Axelrod (1968), Haley (1970), Reibstein (1975), Granbois and Summers (1975). Such probabilities can be interpreted as individual market shares when repeated choices are made. Blass et al. (2010) describe the rationale for asking for choice probabilities. According to them, alternatives presented in choice experiments are incomplete scenarios: not all relevant information is available to the respondent to make his choice, which introduces some uncertainty. By asking respondents to state choice probabilities, they are allowed to express that "resolvable uncertainty": the elicited choice probability q_{nj} is the subjective probability that person n places on the event that the realisations of ε_{nj} will make option j optimal. The choice probability for alternative j is given by:

$$q_{nj} = \frac{e^{V_{nj}}}{\sum_{j=1}^{J} e^{V_{nj}}}$$

which yields the following estimation equation for linear in attributes V_{nj}:

$$ln\left(\frac{q_{nj}}{q_{n1}}\right) = (X_{nj} - X_{n1})\beta_n + u_{nj} \qquad (1)$$

where the alternative *j=1* can be chosen arbitrarily and u_{nj} is an error term that can capture measurement errors or unobserved preferences. Blass et al. (2010) suggest that preference parameters can best be estimated using median regression, because that method is insensitive to the way probabilities equal to 0 or 1 are treated, as at these points log-odds equal to plus or minus infinity.

This method has been used to study preferences for electricity reliability (Blass et al. 2010), for land-use scenarios (Shoyama et al., 2013), for political candidates (Delavande & Manski, 2015), for electric power from different sources (Morita & Managi, 2015), for workplace attributes (Wiswall & Zafar, 2018), for long-term care insurance products (Boyer et al., 2020), and for migration (Koşar et al., 2021).

2.3. Point allocation as cardinal information

The literature in the previous sections does not assume that respondents are able to give meaningful information about the magnitude, or intensity, of their preferences. If one is willing to make this assumption, one can use the point allocation as an indicator of preference in the cardinal sense. Hauser and Shugan (1980) distinguish two possible ways of doing so: interval theory and ratio theory.

2.3.1. Interval theory

According to **interval theory**, respondents allocate points in such a way that the difference between the points allocated reflects the intensity of preference. Therefore:

$$V_{nj} - V_{nk} = q_{nj} - q_{nk} \qquad (2)$$

This relates to Shapley (1975), who derives axioms implying the existence of such a cardinal utility function. Shapley (1975) defines V_{nj} as unique up to an order-preserving linear transformation. In that sense, it is more precise to say that the difference between the points allocated is proportional to the difference in utilities.

Hauser and Shugan (1980) show that interval theory, together with "evaluative independence", implies a representation of utility that is additive in the attributes multiplied by their parameters. The assumption of "evaluative independence" is equal to the well-known assumption of "independence of irrelevant alternatives" for multinomial logit models: respondents' answers only depend upon the attributes varying in the pair considered. Hauser and Shugan (1980) assume a utility function that takes the form: $V_{nj} = \sum_i \beta_{ni} X_{nij}$. One can therefore estimate the utility function using the following equation:

$$q_{nj} - q_{nk} = \beta_n (X_{nj} - X_{nk}) + \varepsilon_{nj} \qquad (3)$$

Positive affine transformations of the utility function will result in a scaling of the parameter vectors β and ε. Here, ε is usually assumed to represent measurement error.

This estimation approach is the one followed in the health economics literature where respondents allocate donor livers across patients with different characteristics (Chan et al., 2006; Ratcliffe, 2000; Ubel & Loewenstein, 1996), or budget across patient groups or health programmes (Schwappach, 2003; Schwappach & Strasmann, 2006; C. D. Skedgel et al., 2015; C. Skedgel & Regier, 2015). Most of these articles are not explicit about the underlying behavioural model. Skedgel and Regier (2015, p.157) state: "*the difference in budget shares can be interpreted as proportional to the difference in latent utility between the underlying alternatives*", thereby explicitly adhering to interval theory. Similarly, Skedgel et al. (2015, p.1232) write: *"coefficients from the (...) CSPC model represent the change in (...) the difference in latent utility (...) associated with a 1-unit change in each attribute"*.

2.3.2. Ratio theory

According to **ratio theory**, respondents allocate points in such a way that the ratio between them indicates the intensity of preferences:

$$\frac{q_{nj}}{q_{nk}} = \frac{V_{nj}}{V_{nk}} \qquad (4)$$

This idea has been developed by Torgerson (1958) in the context of psychological experiments in which subjects were asked to rate the intensity of stimuli. This theory has not been used much in marketing or economics, but there are a few exceptions. In the field of marketing, Silk and Urban (1978) derive preference values for brands from constant-sum paired comparison data using Torgerson's model, and use these preference values to predict purchase behaviour. In health economics, Ubel and Loewenstein (1996) conduct not only an analysis based on differences in allocated points, but also regress the ratio of allocated livers between two groups of patients on the ratio of survival probabilities for these two groups. Hauser and Shugan (1980) show that ratio theory, together with evaluative independence, implies a utility function in which the attributes enter multiplicatively: $V_{nj} = \prod_i \beta_{ni} X_{nij}$. This model implies the following regression equation:

$$\ln\left(\frac{q_{nj}}{q_{nk}}\right) = \ln(V_{nj}) - \ln(V_{nk}) + \varepsilon_{nj} \quad (5)$$

and preference parameters can be estimated by discretization of the attributes. Consistent with this, Moore & Lehmann (1989) estimate preference parameters for brands by assuming that the difference between these preference parameters is equal to the log of the ratio of intentions to purchase measured by a constant-sum comparison task.

3. Utility maximization models for point allocation experiments

In this section, we first model the allocation of points in a constant-sum paired comparison experiment as an optimization process with constraints. This makes the role of prices and budget constraints explicit, and as such, uncovers implicit assumptions made about their role in point allocation experiments. Second, we introduce a model that uses previous insights on representative agent models to analyse product differentiation (Anderson et al., 1988). These models reformulate aggregated discrete choice models as the allocation of consumption shares to different products due to a taste for variety formulated as entropy. We apply the model to individual choices where respondents allocate shares or subjective probabilities to different alternatives in an experiment. This neatly fits the experimental setup of point allocation experiments where a cardinal number is measured instead of a discrete choice.

3.1. Point allocation as an optimization process

The models in section 2 do not see point allocation as the result of an optimisation procedure with constraints. However, the allocation of points itself can be interpreted as the result of an optimization process with a constraint on the total number of points. Indeed, there is typically a unique point allocation that both reflects the respondents' preferences by yielding a specific difference or ratio between the points and satisfies the constraint that all points should sum up to Q. In that process, the respondent can be viewed as optimizing a latent direct utility function that is a function of the number of points attributed to the alternatives. In this setting, the points represent demand rather than approximations of utility.

We use shorthand notation $V_{nj} = V_{nj}\left(\beta_n, X_{nj}, \varepsilon_{nj}\right)$ for the systematic element of the utility function. The random component in the conditional systematic utility function can be interpreted as perception error, measurement error or random preference. Direct utility for consumption of the bundle of J goods is defined as: $U_n = \sum_{j=0}^{J} H(q_{nj}, V_{nj})$. This utility is maximized subject to the constraint $\sum_{j=0}^{J} q_{nj} = Q$.

3.1.1. A utility maximization model that parallels interval theory

To illustrate the importance of formulating the thought process as utility maximization, let us formulate a model that is consistent with the estimation equation yielded by interval theory. Let us assume that direct utility takes the form:

$$U_n = \sum_{j=0}^{J} \left(q_{nj} V_{nj} - \frac{1}{2} {q_{nj}}^2 \right)$$

Formulating the Lagrangian and solving the first-order conditions leads to optimal demands that are a function of the systematic utilities, the total number of points and the total number of alternatives:

$$ {q_{nk}}^* = V_{nk} - \frac{\sum_{j=0}^{J} V_{nj} - Q}{J}. \tag{6}$$

The first term is equal to the systematic part of the utility function. The second term is equal to the Lagrangian multiplier in the optimum. It shows with how much the optimal or indirect utility increases when the respondent receives an additional point for distribution. Optimal demand is increasing with rate $\frac{1}{J}$ in the total number of points and is decreasing in the number of alternatives. Demand increases with rate $1 - \frac{1}{J}$ in the own conditional systematic utility V_{nk} and decreases with rate $\frac{1}{J}$ in the conditional systematic utilities of the other alternatives V_{nj}. The second term is equal for all alternatives and therefore the difference in points for alternatives k and j is equal to the difference in systematic utility:

$$q_{nj}^* - q_{nk}^* = V_{nj} - V_{nk} \qquad (7)$$

This shows that the interval model can be interpreted as measuring the difference in the marginal utilities for the linear term in the direct utility function which is equal to the difference in the systematic part of the utility function.

3.1.2. The role of prices and budgets

The formulation as a utility maximization problem under constraints is particularly interesting when it comes to studying the roles of prices and budgets.

In many point allocation experiments, prices are not an explicit part of the optimisation problem. For instance, Schwappach (2003) asks respondents to distribute budget between groups, but does not include any attributes that would depend on the budget allocation. This is practically equivalent to a case where the prices do not differ between groups. Linley & Hughes (2003), Skedgel et al. (2013) and Skedgel et al. (2015) who ask respondents to distribute budget between two patient groups, do allow the costs of treatment to differ between both groups. They allow the number of patients treated in two groups to be dependent on the allocation of budget between those two groups, and this is explicitly shown to respondents. However, the total budget to be distributed is fixed in the experiment, so that a preference for a larger number of patients to be treated can either indicate a preference for cost-effectiveness, or a preference for treating more patients regardless of the costs, as recognized explicitly by Skedgel et al. (2013).

In other experiments, however, prices are presented as attributes of the alternatives (Schwappach & Strasmann 2006). This implies that besides the constraint on points, there can be a budget constraint. It is therefore only natural to add a private and/or a public budget constraint, and to let a private and/or a public outside good enter the direct utility function. Let us denote z_{priv} the outside private good with p_{priv} its price, and z_{pub} an outside public good with b_{pub} its price, Y_n the private income of respondent *n*, and B_n the public budget that respondent n can allocate. For simplicity, we assume separable utility. The maximization problem therefore becomes:

$$U_n = \sum_{j=0}^{J} H(q_{nj}, V_{nj}) + \mathrm{f}(z_{priv}) + \mathrm{g}(z_{pub}),$$

subject to:

$$\sum_{j=0}^{J} q_{nj} = Q$$

$$Y_n = \sum_{j=0}^{J} p_{nj} q_{nj} + p_{priv} z_{priv}$$

$$B_n = \sum_{j=0}^{J} b_{nj} q_{nj} + b_{pub} z_{pub}$$

If we assume $H(q_{nj}, V_{nj}) = q_{nj}V_{nj} - \frac{1}{2} q_{nj}{}^2$, then in the optimum, one can substitute the budget constraints into the utility function, and use the first-order conditions for utility maximization to obtain the difference in demand shares:

$$q_{nj}{}^* - q_{nk}{}^* = V_{nj} - V_{nk} - \frac{\partial f / \partial z_{priv}}{p_{priv}} (p_{nj} - p_{nk}) - \frac{\partial g / \partial z_{pub}}{b_{pub}} (b_{nj} - b_{nk}). \quad (8)$$

The first term in this equation is the difference in conditional systematic utility. For experiments with private and governmental expenditures, two additional terms enter the equation: the second and third terms show that the differences in prices of the private and public outside goods are multiplied with a coefficient that is the ratio of the marginal utility of the outside good and the price of the outside good. Without further assumptions, the marginal utilities of the outside goods may endogenously depend on all the conditional systematic utilities and all the prices of the alternatives, so that one cannot estimate the coefficients on the price differences without further assumptions.

There are two ways to come back to a regression equation that is the same as the one following from interval theory. The first is an experimental design where prices of pairs of alternatives are assumed to be equal (a bit like in Schwappach (2003)). One can then interpret the difference in allocated points as the difference in the systematic part of the utility function. Second, one can assume that the marginal utility of income is close to 0. The model then gives a reasonable approximation of the interval model.

However, the studies that introduce price as an attribute in the experiments do estimate a price coefficient (e.g. Schwappach & Strasmann 2006). It is therefore interesting to uncover the assumptions that underlie such estimations. It turns out that estimating a price coefficient in a model consistent with interval theory amounts to assuming that *f(.)* and *g(.)* take a linear form:

$\mathrm{f}(z_{priv}) = \beta_{priv} z_{priv}$ and $\mathrm{f}(z_{pub}) = \beta_{pub} z_{pub}$. If we make this assumption on top of assuming that $H(q_{nj}, V_{nj}) = q_{nj}V_{nj} - \frac{1}{2}{q_{nj}}^2$, then substituting the budget constraints into the utility function and maximizing with respect to q_{nj} yields:

$$ {q_{nj}}^* - {q_{nk}}^* = V_{nj} - V_{nk} - \frac{\beta_{priv}}{p_{priv}}(p_{nj} - p_{nk}) - \frac{\beta_{pub}}{b_{pub}}(b_{nj} - b_{nk}) \qquad (9) $$

The estimated marginal utilities of private income and public budget are the ratios of the marginal utilities of the outside private and private goods to the prices of the outside private and private goods, respectively. It is possible to imagine that these marginal utilities are heterogeneous in the sample studied and therefore one might employ methods to allow for heterogeneity in these coefficients in the estimations.

If one wants to relax the assumption that the utility function is linear in the outside goods, one can for instance assume that *f(.)* and *g(.)* take the *ln(.)* form, for instance $\mathrm{f}(z_{priv}) = \beta_{priv}\ln(z_{priv})$ and $\mathrm{f}(z_{pub}) = \beta_{pub}\ln(z_{pub})$. In this case, we obtain:

$$ {q_{nj}}^* - {q_{nk}}^* = V_{nj} - V_{nk} - \beta_{priv}\frac{p_{nj} - p_{nk}}{Y_n - \sum_{j=0}^{J} p_{nj}{q_{nj}}^*} - \beta_{pub}\frac{b_{nj} - b_{nk}}{B_n - \sum_{j=0}^{J} b_{nj}{q_{nj}}^*} \qquad (10) $$

The marginal utilities of private income and public budget are then proportional to the reciprocals of the remaining income (or public budget) after consumption of the chosen bundle. When income (or public budget) is higher, the marginal utility of income (or public budget) is lower, and price differences are less relevant for the respondent. In the presence of information on private income of the respondents and on the public budget available, one can rescale the price differences as in the equation above to obtain unbiased estimates of all parameters.

The question remains whether the ln(.) assumption for f(.) and g(.) is valid. One can test this by assuming Box-Cox transformations with homogeneous curvature parameters ρ and κ respectively in the population. The specification of the dependent variable then becomes:

$$ {q_{nk}}^* - {q_{nj}}^* = V_{nk} - V_{nj} - \left(\frac{Y_n - \sum_{j=0}^{J} p_{nj}{q_{nj}}^*}{p_{priv}}\right)^{\rho} \beta_{priv}\frac{p_{nk} - p_{nj}}{Y_n - \sum_{j=0}^{J} p_{nj}{q_{nj}}^*} - \left(\frac{Y_n - \sum_{j=0}^{J} b_{nj}{q_{nj}}^*}{b_{pub}}\right)^{\kappa} \beta_{pub}\frac{b_{nk} - b_{nj}}{Y_n - \sum_{j=0}^{J} b_{nj}{q_{nj}}^*}. \qquad (11) $$

One can use a grid search on (ρ, κ) to see which assumptions are most appropriate. Models of choice with budget transfers in experiments are analysed by Dekker et al. (2019), and have been applied applied in various contexts (Mouter et al., 2017, 2021). These budget transfers can help to identify the relationship between the marginal utility of income and the marginal utility of public budget.

3.2. A behavioural model with a taste for variety

3.2.1. Modelling a taste for variety

The literature review in section 2 shows that the points allocated by respondents to alternatives are either assumed to reflect cardinal utility, or interpreted as choice probabilities that incorporate "resolvable uncertainty". Another plausible reason for spreading points or probabilities across different options is a taste for variety when repeated choices are made. Reibstein (1975) and Silk and Urban (1978) already mention variety-seeking as a possible reason to ask respondents what percent of the time they would choose a given brand, or how likely they would be to choose a given brand, respectively. They do not, however, formalize this idea in more detail. We do so here using earlier results in the literature.

If we want to model taste for variety in a behavioural model, direct utility for consumption of the bundle of J goods can then be written as (see Anderson et al. 1988 for a model with point and budget constraints):

$$U_n = \sum_{j=0}^{J} q_{nj} V_{nj} - \sum_{j=0}^{J} q_{nj} \cdot \ln[q_{nj}]. \tag{12}$$

The first part of this utility function is related to the systematic utility of the alternatives. This systematic utility V_{nj} is multiplied with the allocated share. The second term is the Shannon entropy and captures a taste for variety. Point allocations closer to equal shares receive a higher entropy bonus than allocations where all points are allocated to one alternative. The size of the systematic part of the utility determines the relative size of the entropy in direct utility. Again, it is assumed that the individual allocates points optimally subject to the point constraint. Formulating the Lagrangian and solving the first-order conditions leads to logit expressions for optimal demands that are a function of the systematic utilities only (Anderson et al. 1988):

$$q_{nk}^* = \frac{e^{V_{nk}}}{\sum_{j=1}^{J} e^{V_{nj}}}.$$

The log-ratio of the points can be used in a linear regression model as it gives the difference between the systematic utility of the alternatives:

$$\ln\left[\frac{q_{nj}{}^{*}}{q_{nk}{}^{*}}\right] = V_{nj} - V_{nk}. \tag{13}$$

The scale of the systematic utility functions is incorporated in the size of the estimated coefficients for the different attributes and the scale of the error term. If the attributes enter V_{nj} linearly, this yields the following regression equation:

$$ln\left(\frac{q_{nj}}{q_{nk}}\right) = (X_{nj} - X_{nk})\beta_n + u_{nj} \tag{14}$$

where u_{nj} is an error term that captures measurement error and unobserved preferences. It is interesting to note that this regression equation is the same as the one derived by Blass et al. (2010). While they derive this regression equation using "resolvable uncertainty" and i.i.d. extreme-value distributed unknowns, our model derives it from the maximization of a direct utility function with a taste for variety formulated as entropy. Just like the model of Blass et al. (2010), our model is consistent with an ordinal interpretation of utility and still allows to use the cardinal information given by the respondents to measure the intensity of preference for attributes. More complicated extensions of direct utility functions with nested structures are given by Verboven (1996) and Fosgerau and De Palma (2016). Swait & Marley (2013) discuss other uses of entropy to model variety-seeking and conceptualize probabilistic choice.

3.2.2. Link with ratio theory

The model we use to formalize a taste for variety can also lead to the same regression estimation as ratio theory. If we assume $V_{nj} = \ln[W_{nj}]$, this results in:

$$\ln\left[\frac{q_{nk}{}^{*}}{q_{nj}{}^{*}}\right] = \ln[W_{nk}] - \ln[W_{nj}] \tag{15}$$

This is the estimation equation that Hauser & Shugan (1980) show to be consistent with ratio theory. Note that utility, in this setting, is no longer separable in the attributes of an alternative.

3.2.3. The role of prices and budgets

The role of prices and budgets is similar to the role demonstrated above in the model consistent with interval theory. When private prices or governmental prices are part of the experiment, these enter the optimality conditions resulting in:

$$\ln\left[\frac{q_{nk}{}^{*}}{q_{nj}{}^{*}}\right] = V_{nk} - V_{nj} - \frac{\partial f/\partial z_{priv}}{p_{priv}}\left(p_{nj} - p_{nk}\right) - \frac{\partial g/\partial z_{pub}}{b_{pub}}\left(b_{nj} - b_{nk}\right). \qquad (16)$$

The marginal utilities of the private and public outside goods may depend on all systematic utilities and all prices leading to endogeneity issues.

When one assumes linear outside private goods and linear outside governmental goods with unit marginal utilities and unit prices, these multipliers are equal to 1 (Anderson et al., 1988). For a linear value of outside good consumption, one can normalize these regression coefficients to -1, or rewrite the dependent variable to include the difference in prices and budgets:

$$\ln\left[\frac{q_{nk}{}^{*}}{q_{nj}{}^{*}}\right] + \left(p_{nj} - p_{nk}\right) + \left(b_{nj} - b_{nk}\right) = V_{nk} - V_{nj}.$$

If we assume linear outside goods, but neither unit marginal utilities nor unit prices, then price coefficients can again be estimated that are the ratios of the marginal utilities and the prices of the outside goods:

$$\ln\left[\frac{q_{nk}{}^{*}}{q_{nj}{}^{*}}\right] = V_{nk} - V_{nj} - \frac{\beta_{priv}}{p_{priv}}\left(p_{nk} - p_{nj}\right) - \frac{\beta_{pub}}{b_{pub}}\left(b_{nk} - b_{nj}\right) \qquad (17)$$

For a specification with logged utilities of the outside goods, this can be rewritten as:

$$\ln\left[\frac{q_{nk}{}^{*}}{q_{nj}{}^{*}}\right] = V_{nk} - V_{nj} - \beta_{priv}\frac{p_{njk} - p_{nkj}}{Y_n - \sum_{j=0}^{J} p_{nj} q_{nj}{}^{*}} - \beta_{pub}\frac{b_{njk} - b_{nkj}}{B_n - \sum_{j=0}^{J} b_{nj} q_{nj}{}^{*}}. \qquad (18)$$

One can decide on the basis of model fit which specification of the direct utility function is most appropriate. Again Box-Cox parameters can be added to test the assumptions on the marginal utility of income and public budget.

3.2.4. Link with asking for choice probabilities

It is interesting to discuss how this relates to the work of Blass et al. (2010) and their followers, since they derive the estimation equation (1), which is the same as (14), starting from a discrete choice framework and asking respondents to report choice probabilities.

Let us therefore start from a discrete choice framework, with the following direct utility function:

$$U_n = \sum_{j=1}^{J} y_{nj} V_{nj} + f(z_{priv})$$

For the sake of brevity, we only discuss the case with a private outside good here, but the argument is the same with a public outside good. Where V_{nj} is defined as above, but y_{nj} is an indicator variable taking value 1 if alternative *j* is chosen and 0 if not. We introduce the outside good z_{n0} because we are interested in the role of prices and budgets. Let us define p_{nj} as the price of alternative *j* for individual *n*, p_{n0} as the price of the outside good for individual *n*, and Y_n as the disposable income of the same individual. The following budget constraint has to be respected:

$$Y_n = \sum_{j=1}^{J} y_{nj} p_{nj} + p_{priv} z_{priv}$$

For simplicity, let us start by assuming that the outside good enters the utility function linearly:

$$U_n = \sum_{j=1}^{J} y_{nj} V_{nj} + \beta_{priv} z_{priv}$$

Here β_{priv} is the preference parameter for the outside good. When a single alternative i is chosen, substituting the budget constraint into conditional direct utility yields:

$$V_{ni} + \beta_{priv} \frac{Y_n - p_{ni}}{p_{priv}}.$$

Alternative *i* is thus chosen over alternative *j* if:

$$V_{ni} + \beta_{priv} \frac{Y_n - p_{ni}}{p_{priv}} > V_{nj} + \beta_{priv} \frac{Y_n - p_{nj}}{p_{priv}}, \forall i \neq j,$$

$$V_{ni} - \frac{\beta_{priv}}{p_{priv}} p_{ni} > V_{nj} - \frac{\beta_{priv}}{p_{priv}} j, \forall i \neq j. \quad (19)$$

Assuming a random linear conditional utility $V_{ni} = \beta_n X_{ni} + \varepsilon_{ni}$, results in

$$\beta_n X_{ni} - \frac{\beta_{priv}}{p_{priv}} p_{ni} + \varepsilon_{ni} > \beta_n X_{nj} - \frac{\beta_{priv}}{p_{priv}} p_{nj} + \varepsilon_{nj}, \forall i \neq j,$$

If one assumes that the random terms are extreme value Type I distributed, the difference in the random terms has a logistic distribution and the probability takes the conditional logit form.

The price coefficient can be estimated, and interpreted as the ratio of the direct utility of the outside good and the price of the outside good. Note that the disposable income of the individual does not play a role here. This is consistent with the approach followed by Blass et al. (2010) when they estimate a coefficient for the price of electricity. They do allow for heterogeneity in this price coefficient by income groups.

If we assume that the external good enters the utility function in a log-linear way, we arrive at the result that, to estimate the marginal utility of income, one should include the term $\ln[Y_n - p_{ni}]$ in the regression equation. Due to discrete choice, it is no longer necessary to include the consumption and prices of the other goods, as in the $\frac{p_{njk}-p_{nkj}}{Y_n-\sum_{j=0}^{J} p_{nj}q_{nj}^*}$ term following form the "taste for variety" model above.

3.2. Estimation

Standard OLS or median regression can be used where linear and non-linear impacts of the attributes can be included using polynomial terms (Blass et al. 2010). The error term in this regression can be interpreted as the difference in random systematic utility between the status quo and the selected alternative or as misperception or measurement error.

As the mean of this difference might differ over alternatives, it is useful to include alternative-specific fixed effects. These fixed effects capture the intrinsic preferences for each of the alternatives relative to the status quo alternative. To allow for preference heterogeneity in the attributes one can employ random effects regression with a stochastic distribution on β_n or individual fixed effects.

Iterative procedures can be used for determining the curvature of the outside good consumption.

4. Conclusion

Various strands of the literature in psychology, marketing and economics let respondents allocate points across alternatives, with very different motivations for doing so. We have briefly reviewed these motivations for three cases: interval theory, ratio theory, and a log-ratio model derived from the elicitation of choice probabilities, each suggesting a different specification of the dependent variable in point allocation regressions (interval, ratio, log-ratio of points).

Some authors assume that the allocation of points directly reflects cardinal utility derived from the alternatives. Others suggest that choice probabilities reported by respondents are driven by "resolvable uncertainty", i.e. the beliefs of the respondents about the chances that a given option will turn out to be optimal once the factors unknown at the time of the experiment become known.

We have introduced behavioural models for point allocation experiments and have shown potential behavioural underpinnings for the regression equations that are estimated in the literature, thereby making explicit which assumptions are made in the different parts of the literature, mainly about the way income enters the utility function. In particular, we have shown that a utility function with a taste for variety formulated as entropy that had been used previously to model market shares (Anderson et al. 1988) can also be used to model point allocation by one individual. These results might be useful for the conversation between economists and choice modellers in the social sciences. Table 1 offers an overview of the models we discussed.

One of the advantages of point allocation experiments is that for the analysis of choices one can employ linear regression techniques without further assumptions on aggregation. We provided extensions to the literature to provide more detail on the marginal utilities of public budget and private income. We showed how to allow for heterogeneity in the marginal utility of income while keeping tractable specifications of the regression equation. The curvature can be tested by employing iterative regression procedures with fixed Box-Cox parameters.

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Table 1 Overview of models discussed

	Theory of point allocation	Regression equation	Direct utility function leading to the same regression equation when optimized
Interval theory	Difference between the points allocated reflects the intensity of preference: $$U_{nj} - U_{nk} = q_{nj} - q_{nk}$$	$q_{nj} - q_{nk} = \beta_n(X_{nj} - X_{nk}) + \varepsilon_{nj}$ (e.g. Ratcliffe 2000, Schwappach 2003, Skedgel et al. 2013)	$U_n = \sum_{j=0}^{J}\left(q_{nj}V_{nj} - \frac{1}{2}{q_{nj}}^2\right)$
Ratio theory	Ratio between the points allocated reflects the intensity of preference. It implies that attributes enter utility multiplicatively (Hauser & Shugan 1980): $$\frac{q_{nj}}{q_{nk}} = \frac{V_{nj}}{V_{nk}} = \frac{\prod_i x_{inj}\beta_{ni}}{\prod_i x_{ink}\beta_{ni}}$$	$\ln\left(\frac{q_{nj}}{q_{nk}}\right) = \ln(V_{nj}) - \ln(V_{nk}) + \varepsilon_{nj}$ which can only be estimated with discrete attributes, or by estimating only a constant for each alternative (as in Moore & Lehmann 1989).	$U_n = \sum_{j=0}^{J} q_{nj}V_{nj} - \sum_{j=0}^{J} q_{nj} \cdot \ln[q_{nj}]$.
Eliciting choice probabilities	The elicited choice probability q_{nj} is the subjective probability that person n places on the event that the realisations of the unknowns will make option j optimal (Blass et al. 2010): $$q_{nj} = \frac{e^{V_{nj}}}{\sum_{j=1}^{J} e^{V_{nj}}}$$	$ln\left(\frac{q_{nj}}{q_{nk}}\right) = (X_{nj} - X_{nk})\beta_n + \varepsilon_{nj}$ (e.g. Blass et al. 2010)	V_{nj} can take any form (as long as it is additive in the attributes).
Taste for diversity	The points are allocated so as to maximize a utility function with a taste for diversity modelled as entropy: $$U_n = \sum_{j=0}^{J} q_{nj}V_{nj} - \sum_{j=0}^{J} q_{nj} \cdot \ln[q_{nj}]$$ (this paper, inspired by Anderson et al. 1988)	$ln\left(\frac{q_{nj}}{q_{nk}}\right) = (X_{nj} - X_{nk})\beta_n + \varepsilon_{nj}$	$U_n = \sum_{j=0}^{J} q_{nj}V_{nj} - \sum_{j=0}^{J} q_{nj} \cdot \ln[q_{nj}]$.

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