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Borrowing Constraints and the Rise of the Private Rental Sector^{*}

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Abstract

Over the last decade, the private rental sector has grown in many developed economies. We construct an assignment model in which investors convert owner-occupied houses to rental houses for credit-constrained households. Our model identifies credit-constrained households as well as households exposed to equilibrium spillovers of investment, allowing us to interpret quasi-experimental evidence as aggregate effects. We apply this method to a series of contractions of the mortgage-payment-to-income constraint in the Netherlands between 2012 and 2016. We find that tighter borrowing constraints make grow the private rental sector, explaining 21 percent of the total increase of the sector over that period.

Keywords: borrowing constraints, housing tenure, arbitrage, buy-to-let investment, assignment models.

JEL classifications: R31, R21, G51.

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1 Introduction

In the wake of the financial crisis, many advanced economies have introduced borrowing constraints (Cerutti et al., 2017). While borrowing constraints as a form of macroprudential policy may contribute to macroeconomic stability, these constraints may also prevent households to become homeowners. Households that are borrowing-constrained would be forced to rent and would thus miss out on the opportunity to acquire wealth via housing capital gains, self-amortizing mort-gages and subsidies to homeownership. The homeownership rate would fall and the private rental sector as a share of the total housing stock would rise. Indeed, private rental sectors have recently been growing in many advanced economies.¹

In this paper, we study whether the tightening of borrowing constraints, and specifically mortgage payment-to-income constraints, causes a rise of the private rental sector. Mortgage payment-to-income constraints have recently been shown crucial to explain debt levels and house prices in the United States (Greenwald, 2018) and Ireland (Higgins, 2024), and are increasingly often imposed in other countries.² Borrowing constraints have long been recognized to induce those whose choices are constrained, to rent rather than to own (Linneman and Wachter, 1989). However, establishing a causal effect of borrowing constraints on the size of the private rental sector suffers from two empirical challenges.

The first challenge is to identify which households are borrowing-constrained. While a researcher may have access to detailed information about the financial situation of a household, identifying whether a constraint is binding generally requires counterfactual information on which house would be bought in the absence of borrowing constraints.

The second challenge is to establish effects on the aggregate housing market from cross-sectional variation. While cross-sectional variation can be leveraged in a natural experiment to provide causal evidence, such evidence often suffers from a missing-intercept problem. For example, one mechanism that prevents a simple aggregation of the effects on constrained households is that borrowing constraints reduce house prices (Acharya et al., 2022; Akinci and Olmstead-Rumsey, 2018; Higgins, 2024; Vigdor, 2006). Reduced house prices potentially alleviate previously binding constraints and allow households again to access the housing market. Consequently, using quasi-experimental evidence to estimate the effect on the aggregate homeownership rate and the relative size of the private rental sector suffers from spillovers.

We develop an assignment model to tackle both challenges. In the model, households that differ in income compete for a given number of houses that dif-

¹See e.g. Gabriel and Rosenthal (2015) and Acolin et al. (2016) for the U.S.; Byrne (2020) for the UK, Ireland and Spain; and Thiel and Zaunbrecher (2023) and Figure 1 for the Netherlands.

²The Online Appendix of Cerutti et al. (2017) documents that in 2013, 26 out of 119 countries imposed a maximum debt service-to-income ratio, up from only 4 countries in 2000.

fer in quality. The richest household lives in the highest-quality house and prices adjust to induce poorer households to buy lower-quality houses. The introduction of mortgage payment-to-income constraints drives a wedge between some households' willingness to pay and their ability to borrow, resulting in segments of the housing market with constrained households and lower prices.

The wedge between households' willingness to pay and their ability to borrow creates an arbitrage opportunity for buy-to-let (or keep-to-let) investors. Because rental payments are not constrained by the ability to borrow, investors can buy houses at depressed prices and rent them out to borrowing-constrained households. Investors thus allow constrained households to spend more on housing than these households would be able to as homeowners. However, competition between investors drives up house prices, resulting in other constrained households and even more investment. This process stops when all arbitrage opportunities have been fully exploited, which occurs when house prices coincide with their levels in the absence of borrowing constraints.³ The assignment model thus identifies the *directly* and *indirectly* treated region of the introduction of mortgage payment-to-income constraints.

We exploit the insights from the assignment model to interpret the causal effects of the introduction and subsequent contraction of a mortgage payment-to-income constraint in the Netherlands between 2011 and 2017. In particular, we exploit that before August 2011 mortgage lenders could easily deviate from mortgagepayment-to-income constraints. From August 2011 onward, these guidelines became binding and such deviations were no longer possible as easily. Moreover, between 2012 and 2016, the constraints were tightened every consecutive year. Figure 1 shows the continued decline of the mortgage payment-to-income constraint between 2012 and 2016. The figure also shows a corresponding rise in the size of the private rental sector, consistent with our model.

This policy reform is the ideal setting to study the effect of borrowing constraints of the size on the private rental sector, because it allows us to determine which constraints apply to which households. Even without macroprudential policy, mortgage lenders will generally tie the size of mortgages to the ability of borrowers to service their mortgage debt. The policy reform replaced such lenderspecific practices with a uniform constraint.⁴ Moreover, in the Netherlands mortgage payment-to-income constraints are the only relevant borrowing constraints. While potentially access to mortgages is limited by three different types of borrowing constraints – loan-to-value (LTV) or down-payment constraints, debt serviceto-income or mortgage payment-to-income constraints, and credit scores (Acolin et al., 2016) – there are neither credit scores nor down-payment requirements in the

³We acknowledge that arbitrage arguments are less forceful on the housing market, which is in many respects imperfect, than on many asset markets, see Glaeser and Gyourko (2007).

⁴Constraints are uniform for households with the same weighted household income and a permanent contract, see Section 3.1.



Figure 1: The change of mortgage payment-to-income constraint and private rental sector

Note: The mortgage payment-to-income differs by income and interest rate. This figure presents the constraint corresponding to an income of 60K euros, with the average mortgage interest rate for the year. We use mortgage interest rates for new loans, with an initial rate fixation of 5 to 10 years, as is most common in the Netherlands, see De Nederlandsche Bank (DNB). The share of private rental houses is calculated based on Statline. We focus exclusively on owner-occupied and private rental housing, excluding social housing. The private rental share is defined as the proportion of private rented houses relative to the combined total of owner-occupied and private rental properties.

Netherlands.⁵ Finally, the extensive Dutch register data provide us with the necessary information for all households, houses, and transactions in the Netherlands.

We consider all transactions by first-time buyers that took place in the two years preceding the year at which the constraint started to be enforced, i.e. the transactions in 2009 and 2010. Using the average interest rate in the month of the transaction and the income and mortgage loan of the first-time buyer, we then determine which of these transactions would not have been possible if the mortgagepayment-to-income constraints present in the years between 2012 and 2017 would have prevailed. Consistent with the assignment model, we find that the transactions that would no longer have been possible are concentrated in one segment of the income distribution.

Exploiting the insights from the assignment model, we then define the trans-

⁵Maximum LTV ratios have always exceeded 100% during our sample period. While LTV constraints affect leverage (Van Bekkum et al., 2024), we argue that they do not affect tenure choice in the Netherlands, see Section 3.1.

acted *houses* as belonging to treatment and control groups based on whether and when constraints became binding. The idea is that the houses bought in 2009 and 2010, would, if the stock of houses and the income distribution did not meaningfully change in between, in the equilibrium of later years be assigned to similar households. However, as the result of borrowing constraints, not all of these households would still be able to buy them, but some would have to rent them instead. The houses that the constrained households would otherwise have bought, provide profitable arbitrage opportunities for buy-to-let or keep-to-let investment. The assignment model also clarifies that such investment does not affect the likelihood of investment in the houses that unconstrained households would buy, despite the prediction that investment would also drive up price for these houses as well. In the terminology of the difference-in-difference literature, the stable unit treatment value assumption (SUTVA) holds, and the aggregate effect on the size of the rental sector can be identified.

Using a staggered difference-in-difference design, we find that owner-occupied houses are significantly more likely to be converted to rental housing from exactly the year onwards at which the corresponding 2009 or 2010 transaction would no longer have been possible, and not before. We find relatively homogeneous effects across event and calendar time groups: an initially owner-occupied house corresponding to a 'constrained transaction' has a 0.6 percentage points higher probability to become a rental unit.⁶ Moreover, we find that the contractions affect buy-to-let investment and keep-to-let investment in equal measures.

Because the effects accumulate over time, the series of contractions that ultimately affected 17 percent of transactions resulted in a cumulative increase of the private rental sector of 1.2 percentage points, which amounts to 21 percent of the overall increase in this sector by the end of 2017. Because the event study coefficients show no sign of flattening off, we conjecture even larger cumulative effects after 2017 resulting from the contractions between 2012 and 2017 alone, but we cannot estimate these effects because the data suffer from a structural break after 2017.

Because houses that are bought in 2009 or 2010 and that are already converted to rental units (although not necessarily transacted) by 2017 may constitute an unrepresentative sample of the housing stock, our most substantive robustness exercise considers a matched sample leveraging the registry of all houses in the Netherlands. For all houses bought by first-time buyers in 2009 and 2010, we search for a house of the same type (apartment or single-family), with the same value (WOZvalue, as assessed for tax purposes), and in the same street (i.e. with the same 6digit postcode). We then define treated and control-group houses in the matched sample based on the original sample, and find that our results continue to hold

⁶The two-way fixed-effect estimator and the heterogeneity-robust estimator of Callaway and Sant'Anna (2021) result in an identical average treatment effect on the treated.

with even somewhat larger effect sizes.⁷

Our paper makes three contributions. Our first contribution is to explain that buy-to-let investment can exist as an equilibrium phenomenon that results from borrowing constraints. Our explanation builds on the assignment model in Braid (1981), who analyzes only a rental market, while in our model the user cost replaces the rent in case housing is owner-occupied. The model assigns heterogeneous households to a fixed housing stock of heterogeneous quality. Such a fixed stock of housing is a realistic feature of housing markets in many urban areas, at least in the short or medium run.

Our second contribution is to show that the causal effects resulting from an appropriately designed difference-in-difference identification strategy can be interpreted as aggregate effects. As such, our hybrid solution to the missing-intercept problem is similar in spirit to Wolf (2023)'s approach to estimating fiscal multipliers. Our empirical strategy focuses on houses rather than people, and allows us to identify the houses that are susceptible to spillovers from borrowing constraints and subsequent buy-to-let investment. Buy-to-let investment drives up house prices to the same level as prices without borrowing constraints. The transactions before the introduction of borrowing constraints are thus informative about which households face binding borrowing constraints, including those that face them as the result of equilibrium spillovers, and allow us to identify the houses that constrained and unconstrained households would want to live in without observing any characteristics of these households. The assignment model shows that spillovers do not contaminate the effect that we can isolate from comparing the houses that constrained and unconstrained households would buy.

Our third contribution concerns the specific empirical effect of the introduction of borrowing constraints on the size of the private rental sector in the Netherlands. We find a statistically significant but only modest effect of the introduction and subsequent tightening of the mortgage payment-to-income constraint. Complementary to Carozzi (2019) and Hanson (2023) who consider only either keep-to-let or buy-to-let investment, respectively, we find that keep-to-let and buy-to-let investment are equally affected by the tightening of mortgage payment-to-income constraints.

⁷In the Appendix, we also consider a version of the model in which households differ in their preference for housing relative to other consumption. We show that such preference heterogeneity results in an attenuation bias. In addition, we consider a version in which borrowing constraints differ among households, for instance because some have wealth that may be invested in owner-occupied housing to circumvent or soften the borrowing constraint. Such heterogeneous borrowing constraints also result in an attenuation bias. Consequently, when households differ in their wealth or taste for housing, we would underestimate the effect of borrowing constraints on the size of the private rental sector.

Related Literature. The literature on the effects of borrowing constraints on the homeownership rate and the relative size of the private rental sector goes back until at least Linneman and Wachter (1989), who study the combined effects of constraints on the loan-to-value ratio (LTV) and the ratio between the mortgage payments and household income (PTI). They set the literature standard by comparing the tenure choice of constrained and unconstrained households, in which they inevitably encounter the challenge of identifying the counterfactual tenure choice of constrained households in the absence of borrowing constraints. While the maximum home purchase price for a household with a certain income and wealth can easily be computed, this counterfactual choice is fundamentally unobserved, and is necessary to categorize constrained and unconstrained households. Linneman and Wachter predict unconstrained choices for all households from a set of observable characteristics for households that are likely to make unconstrained choices. However, this approach is only valid if the price function is linear, while it is generally nonlinear if an assignment model is a good description of the housing market. Moreover, they cannot rule out spillovers due to endogenous home prices, and thus cannot estimate the aggregate effect on the homeownership rate.

Observing counterfactual choices and establishing general equilibrium effects remain the main challenges in the subsequent literature. One solution to the challenge of counterfactual choices, pursued by Fuster and Zafar (2021) in a stated choice experiment, is to ask people for their willingness to pay for housing. Their findings point to the importance of borrowing constraints in tenure choice, but cannot straightforwardly be aggregated to equilibrium results.

The most common solution to the challenge of counterfactual choices, also pursued in this paper, is the use of quasi-experimental methods. However, because prices are likely to respond to borrowing constraints, such quasi-experimental cannot directly be aggregated to predictions on the aggregate homeownership rate. Exploiting a tightening of LTV constraints in Israel, Tzur-Ilan (2023) finds no effect on the homeownership rate. We find that the tightening of PTI constraints did reduce the homeownership rate in the Netherlands. These findings are consistent with Higgins (2024), who finds that the tightening of LTV constraints in Ireland did not reduce house prices, but that the tightening of PTI constraints did so.

The closest paper to ours is Thiel and Zaunbrecher (2023), who also study the effect of PTI constraints on the relative size of the private rental sector in the Netherlands in recent years. While we study the tightening of the constraints, they exploit a relative relaxation of the PTI constraints for dual earners compared to single earners. Constrained households are defined as those households that rent before the reform, but are constrained to buy a house that is somewhat similar in location, value and size. In contrast, we study whether the exact same house that was bought in a 'constrained transaction' is more likely to be converted to the rental sector. Defining submarkets based on housing characteristics suffers from the risk that one wrongly classifies the relevant choice set for a household. Only in our robustness exercise, we match houses, and our method allows a majority of these houses to be almost indistinguishable.

The triple-difference design that follows from exploiting the differential treatment of dual and single earners allows Thiel and Zaunbrecher to control for spillovers between treated and untreated households. In contrast, our model allows us to identify the treated houses, including the houses that are susceptible to spillovers. To further understand equilibrium effects, Thiel and Zaunbrecher develop a structural discrete-choice model of the Dutch housing market in which PTI constraints cause a switch of households and houses from the owner-occupied to the private rental sector. They conclude that 21% of the increase in the private rental sector is due to the tightened borrowing constraints. Remarkably, even though our model and identification strategy are very different, and the study period only partly overlaps, our estimate is exactly the same. Kvaerner et al. (2024) also estimate a structural discrete-choice model with borrowing constraints, but focus on neighborhood quality.

A paper that has a similar spirit to ours is Hanson (2023), who uses a portfolio choice model to identify those parts of the market that are most severely exposed to buy-to-let investment in response to the tightening of borrowing constraints. Local income and the interaction between credit supply and local credit scores are used as instruments for investor excess returns, which are then shown to have a significant impact on institutional investor activity. Our analysis differs from Hanson (2023) in the theoretical model, but also by including small buy-to-let investors (e.g. private persons) and keep-to-let investment, and by concentrating the empirical analysis on individual houses rather than aggregate spatial entities. We stress that aggregate spatial entities may not capture the relevant choice set for the house-holds living in them, and that comparing them may thus not adequately deal with spillovers.

Similar to Hanson (2023), other papers exploit variation in the number of constrained households across geographical areas too. Carozzi (2019) studies changes in LTV ratios across UK cities, and finds a fall in transaction volume at the bottom of the market, resulting in keep-to-let investment. We find that keep-to-let investment is only half of the story of the rise of the private rental sector. Mabille (2022) develops a macro-spatial model in which PTI constraints have heterogeneous regional implications due to differences in house prices and incomes. We find that there is only a small correlation between the fraction of constrained transactions and local house prices in the Netherlands, while income is strongly correlated, justifying the use of an assignment model.

Finally, our paper relates to the literature that uses assignment models for the analysis of housing markets. Assignment models have been applied for a long time to allocation of workers over jobs, see Sattinger (1993) for a review. The first

application of such a model to the housing market, as far as we know, is Braid (1981), which is the starting point of the model of the present paper. Braid studies a rental market in which houses differ in one-dimensional quality and households all have the same tastes, but may differ in income. In the assignment model of Määttänen and Terviö (2014) and Määttänen and Terviö (2021), households move from one house to another and use the revenues from selling the initial house to help finance the next one. Braid's model is essentially a version of their model in which the price of the initial house is absent from the wealth constraint, which is the relevant budget constraint of first-time buyers.

The assignment model of Landvoigt et al. (2015) uses a generalized multi-period version in which households face a cash-on-hand constraint rather than a single period budget constraint. They expand their model to a more quantitative version in which heterogeneous households maximize intertemporal utility subject to an intertemporal budget constraint as well as a down-payment borrowing constraint for housing, which they then take to the data. However, they do not formally discuss how the presence of borrowing constraints affects the allocation of households over housing in equilibrium. Epple et al. (2020) present an approach to structurally estimate housing assignment models. Higgins (2023) applies an assignment model to study racial segmentation in the US housing market.

2 Theory

In this section we develop an analysis of tenure choice in a housing market with borrowing constraints. First, we present an assignment model for an owner-occupied market without borrowing constraints. In subsection 2.2, we introduce such constraints. Finally, in subsection 2.3, we allow for buy-to-let investment, so that a private rental sector emerges.

2.1 An assignment model

We consider a market with a population of households who have identical preferences over housing services q and other consumption c.⁸ These preferences can be described by the utility function

$$u = u(q, c), \tag{1}$$

which is increasing in both arguments, strongly quasi-concave and twice differentiable. Housing and other consumption are both normal goods.

Households maximize utility subject to a budget constraint. The budget will be referred to as income, but it should really be interpreted as the amount of money

⁸In Appendix A.5 we discuss an extension of the model to situations with households differing in tastes.

the household is willing to spend on consumption (of housing and other goods) in the period we consider.⁹

Households differ in incomes. The distribution of income is given by the strictly increasing and continuously differentiable function F(y), which has positive support on the interval $[y^{min}, y^{max}]$. We denote the density function by f(y). The total number of households equals B, where $B = F(y^{max})$.

The budget constraint is given by

$$c + p(q) = y, \tag{2}$$

where p(q) denotes the user cost of housing and y is the available budget. The user cost p(q) is a function of the sales price of the house. More specifically, it is the product of the market value and the opportunity cost of the capital invested in the house, adjusted for the costs of maintenance, insurance and taxes, minus the expected increase in the value of the house:

$$p(q) = \gamma P(q) - E(\Delta P), \tag{3}$$

where P(q) denotes the sales price, γ reflects the various cost items (capital, maintenance, insurance, taxes) and ΔP is the (expected) change in the price of the house. In what follows, we focus on the user costs that equilibrate the market in the current period, without paying attention to its composition.¹⁰ Note that it is not assumed that the user cost p(q) (or the transaction price P(q)) is linear in the amount of housing services. That is, the marginal price of housing $\pi(q) = \frac{\partial p}{\partial q}$ may depend on the quantity of housing services consumed.¹¹

Houses are available in a continuum of varieties, characterized by a given number of housing services q. The housing stock is fixed and the distribution of housing services is given by the strictly increasing and continuously differentiable function G(q), which has positive support on the interval $[q^{min}, q^{max}]$. We denote the density function by g(q). The number of houses is S, where $S = G(q^{max})$.

We assume that the number of households is at least equal to the number of houses: $B \ge S$. As a result, some households may not be able to live in any of the available houses. For that reason, we allow for an outside option, which consists of a combination of housing consumption $q^* \le q^{min}$ and user cost p^* that is available

⁹As shown in Appendix A.2, this budget can be derived from an intertemporal utility maximizing framework.

¹⁰Note that the first part of the user cost, $\gamma P(q)$, consists mainly of out-of-pocket expenses like taxes, mortgage interest payments and maintenance, unlike expected price changes. The implication is that monetary outlays on housing can exceed the user cost when house prices are expected to increase. Borrowing restrictions, such as the mortgage qualification constraint discussed later in this paper refer to monetary expenses. Note also that there is in general not a one-to-one correspondence between users cost p(q) and transaction price P(q).

¹¹In later subsections we will encounter situations in which the function p(q) is not differentiable at some points.

to every household. One can interpret this outside option as renting social housing or living in temporary housing.

The outside option results in a reservation utility $u^*(y) = u(q^*, y - p^*)$. Households will thus only participate in the (primary) housing market studied here if this offers them a higher utility than the outside option. In the remainder of the paper, we only consider this non-degenerate case. As a result, there exists some critical income y^c that is needed to participate in the housing market and that is determined by the condition that only *S* households can own a house:

$$B - F(y^c) = S. (4)$$

In equilibrium, user costs will be such that households with incomes below the critical value y^c will choose the outside option. Households with higher incomes compete with each other for the available housing. The following lemma characterizes the resulting assignment of houses to households.

Lemma 1 (Assignment rule). *In equilibrium, the assignment follows the continuous function*

$$y(q) = F^{-1} \left(F(y^c) + G(q) \right).$$
(5)

Proof. First, it is easy to see that p(q) must be increasing and continuous in q. In contradiction, suppose a house with a better quality is less expensive than that of a lower quality. Then there will be no household choosing the lower quality house. Similarly, suppose there is a discontinuity in the house price function. Then the marginal price of housing is infinitely high at the point of the discontinuity, which means that there will be no demand for housing with quality just above the point of discontinuity. Second, since housing is a normal good, a household with a higher income will in equilibrium consume more housing than a household with a lower income, see Lemma 3 in Appendix A.1.1. These two observations imply the assignment rule. This function is continuous because (5) implies that the change in income dy/dq is

$$\frac{dy}{dq} = \frac{g(q)}{f(y)},\tag{6}$$

in which g(q) and f(y) are continuous and positive.

In equilibrium the ranking of households on the basis of housing consumption thus corresponds to the ranking of households on the basis of income. However, there is no reason to suppose that housing expenditure in equilibrium will be proportional to the quality of housing services consumed, q. The main purpose of this section is to find the equilibrium user cost function p(q) on this market.

The user cost function can be derived as follows. The household with the critical income must be indifferent between housing of the lowest quality and the outside option:

$$u(q^{min}, y^{c} - p(q^{min})) = u^{*}(y^{c}).$$
(7)

This equation pins down the value $p(q^{min})$, the user cost of housing of the lowest quality. The following lemma characterizes the equilibrium user cost function.

Lemma 2 (Equilibrium prices). *In equilibrium, the user cost function is given by the first-order differential equation*

$$\pi(q) = \frac{\partial p}{\partial q} = M(q, y(q) - p(q)), \tag{8}$$

with initial condition $p(q^{min})$ from (7), with $M(q,c) = (\partial u/\partial q)/(\partial u/\partial c)$, and in which y(q) follows from the assignment rule in (5).

Proof. The outside option pins down the user cost at q^{min} and y^c . The user cost at higher qualities is determined by the requirement that the slope of the price function, the marginal price of housing $\pi(q)$, must be equal to the marginal rate of substitution M(q,c). p(q) is differentiable because M(q,c), p(q), and y(q) are continuous in their arguments.

We can thus trace out the housing price function by making use of the equilibrium condition and the assignment rule. That is, starting from the critical income y^c , the lowest housing quality q^{min} , and its price $p(q^{min})$, (5) determines the income associated with each housing quality and then (8) determines the equilibrium price for each housing quality.

Finally, our assumptions imply that the second-order condition for utility maximization is satisfied for a linear budget constraint. For a nonlinear user cost function, the budget constraint is also nonlinear. Lemma 4 in Appendix A.1.2 shows that the second-order condition remains satisfied.

2.2 Borrowing constraints

The analysis thus far has assumed that households are not restricted in their choice behavior, except by the budget constraint. Many households need a mortgage loan to finance the purchase of their house and mortgage payments are an important element of their user cost. Lenders usually impose restrictions on the size of these loans. In the Netherlands, the ratio of the mortgage payment to income is the most important indicator used by the lenders, and we will now consider the implications of such a constraint.¹² In particular, we impose that the user cost can at most be equal to a fraction μ of income for all households:

$$p(q) \le \mu y. \tag{9}$$

We refer to this restriction as the mortgage qualification constraint.

¹²In Appendix A.4 we generalize this analysis to an arbitrary distribution of maximum purchase prices or user costs that may depend on household income.



Figure 2: The mortgage qualification constraint and the user cost of housing with high $p(q^{min})$.

We assume that initially the market is in an equilibrium without borrowing constraints, and then consider what changes if such a constraint is imposed. To this end, we consider the user cost as a function of income rather than quality. The assignment rule (5) is a continuous and monotonically increasing relationship between housing quality and income, so that we can write its inverse as q(y). Using this inverse, we derive the user cost of housing as a function of income, p(q(y)). Since the equilibrium housing price is increasing in quality and quality is increasing in income, the user cost p(q) must also be increasing in income, with slope

$$\frac{dp}{dy} = \pi(q)\frac{dq}{dy} = \pi(q)\frac{f(y)}{g(q(y))}.$$
(10)

The equilibrium user cost function p(q(y)) can take various forms, and the interaction with a mortgage qualification constraint thus results in several cases. We first consider a simple and empirically relevant case, and consider other cases later. In this simple case, shown in Figure 2, the outside option and critical income y^c are such that $p(q^{min})$ exceeds the mortgage qualification constraint μy . As can be seen in the figure, borrowing constraints thus bind at the bottom of the income distribution, and there exists some level of income, y', at which the (unconstrained) user function p(q(y)) (in bold) crosses the (dashed) mortgage qualification constraint μy .

One may conjecture that the user cost function in the presence of a mortgage qualification constraint would follow the constraint up to y', and the unconstrained user cost function from that point onward. However, this conjecture is not valid. To see that the mortgage qualification constraint does not describe the constrained user cost function all the way up to y', note that at income y', the marginal will-ingness to pay for housing is smaller than $\mu g(q(y))/f(y)$. Consequently, the con-



Figure 3: The mortgage qualification constraint and the price of housing with low $p(q^{min})$.

straint cannot be binding at y'. Hence there must be a lower income, y'', for which this constraint stops to bind. Moreover, the marginal willingness to pay for housing is a continuous function of income, so the constrained user cost function (in gray) does not have a kink, but slowly bends off at y''.¹³

Figure 3 illustrates another, more complex case that deviates from the previous case in that the user cost of the lowest-quality house is smaller than the mortgage qualification constraint, so that the constraint is not binding at the bottom of the income distribution. This deviation leads to the definition of an extra particular income level: y^* , which denotes the income level at which the constraint starts to bind.

Having seen these two cases, we are now in the position to show the impact of a mortgage qualification constraint in full generality. However, it is important to keep in mind that we consider a version of the simple case to be the empirically relevant case, and that the we present the following theory only for the sake of transparency. From now on, we will refer to the price function for the situation without borrowing constraints as derived in the previous section by $p^m(q)$, and to the one that with mortgage qualification constraint as $p^{bc}(q)$. We will first consider the case in which the borrowing constraint starts to bind at $y^* \ge y^c$, similar to the situation in Figure 3, and present the case at which the borrowing constraint already binds at y^c afterwards.

¹³If the utility function and housing quality and income distributions are such that user costs increase less with income than borrowing capacity already at y^c , then $y'' = y^c$. In this case, the constrained user cost function equals the constraint only at y^c and lies below the constraint for all higher income levels, as illustrated in Appendix A.3.

Proposition 1. Consider the introduction of a borrowing constraint that starts to bind at $y^* \ge y^c$: $p^m(q^{min}) \le \mu y^c$, and y^* is the smallest $y \ge y^c$ such that in the right-sided neighborhood of y^* , $p^m(q(y)) > \mu y$. Define y'' as the smallest $y > y^*$ for which in the right-sided neighborhood of y'', $M(q(y), (1 - \mu)y)f(y)/g(q(y)) < \mu$ if that occurs, and as y^{max} otherwise. Then,

- *The assignment rule* q(y) *does not change;*
- $p^{bc}(q(y)) = p^m(q(y))$ for $y \in [y^c, y^*]$, and $p^{bc}(q(y)) < p^m(q(y))$ for $y \in (y^*, y^{max}]$, so that utility is the same for all households with $y \leq y^*$ and higher for all households with $y > y^*$:
 - 1. $p^{bc}(q(y)) = \mu y$ for $y \in [y^*, y'']$;
 - 2. $p^{bc}(q(y))$ is described by $\pi^{bc}(q(y)) = M(q(y), y p^{bc}(q(y)))$ with initial condition $p^{bc}(q(y'')) = \mu y''$, for $y \in [y'', y^{**}]$ if y^{**} exists, and for $y \in [y'', y^{max}]$ otherwise, in which y^{**} is the smallest y > y'' such that in the right-sided neighborhood of y^{**} , $p^{bc}(q(y)) > \mu y$.
- If y^{**} exists, the constraint binds the constrained price function again, and 1. and 2. apply recursively with y^{**} replacing y^{*} and y'' redefined accordingly.

Proof. Assume for now that the assignment rule does not change as the result of the mortgage qualification constraint. Then the minimum income of owner-occupiers is the same as in the equilibrium without a borrowing constraint, and because the borrowing constraint becomes binding only later, $p^{bc}(q^{min}) = p^m(q^{min})$. We can now follow the logic of Lemma 2 to show that until income y^* , where the borrowing constraint becomes binding, and the associated housing quality $q(y^*)$, the functions $p^{bc}(q)$ and $p^m(q)$ will coincide.

At income y^* , $p^m(q)$ crosses the borrowing constraint. Using (10), it must thus be that $\pi^m(q(y^*))f(y^*)/g(q(y^*)) > \mu$. A household with an income slightly higher than y^* would thus like to spend a larger income share than μ on housing, but is restricted. The slope of $p^{bc}(q)$ is then thus smaller than $p^m(q)$ and equal to

$$\frac{\partial p^{bc}}{\partial q} = \mu \frac{g(q(y))}{f(y)} < M(q(y), y - p^{bc}(q(y))).$$

$$\tag{11}$$

Now consider two cases. First, the constraint remains binding. Then (11) continues to describe the slope of $p^{bc}(q)$ and thus $p^{bc}(q(y)) = \mu y$ for all $y \in [y^*, y^{max}]$. Then it follows immediately that $p^{bc}(q(y)) < p^m(q(y))$ for $y > y^*$. Second, the constraint stops to bind: there exists some $y'' > y^*$ for which $p^{bc}(q(y'')) = \mu y''$ and $M(q(y''), y'' - p^{bc}(q(y'')))f(y'')/g(q(y'')) = \mu$, after which the left-hand side becomes smaller. Then $p^{bc}(q(y)) = \mu y$ only for $y \in [y^*, y'']$.

For incomes higher than y'', the slope of $p^{bc}(q)$ is thus equal to the marginal willingness to pay for housing, as long as the constraint does not become binding

again, at which point the slope is again given by $\mu g(q(y))/f(y)$. Irrespective of whether that happens, it follows that $p^{bc}(q(y)) < p^m(q(y))$ for y > y''.

The households with $y \ge y''$ pay a lower price for the same housing they would have occupied in the unconstrained market equilibrium and they are satisfied with their situation, as the marginal willingness to pay for housing equals the marginal price they face. The households with $y \in (y^*, y'')$ would like to overbid richer households, but the borrowing constraints prevent them from doing so. The households with $y \in [y^c, y^*]$ still face the marginal price that equals their marginal willingness to pay for housing, so are satisfied too, and will not be overbid by households with $y < y^c$. We conclude that the assignment rule does not change.

Because $p^{bc}(q(y))$ and $p^m(q(y))$ already started diverging for $y > y^*$, prices are lower for $y > y^*$. Because the assignment is the same, it follows that utility is the same for all households with $y \le y^*$ and higher for all households with $y > y^*$.

Summarizing, we have shown that the function $p^{bc}(q(y))$ coincides with $p^m(q(y))$ until this function hits the borrowing constraint. Then $p^{bc}(q(y))$ follows the borrowing constraint until, if that happens, the marginal willingness to pay for housing is so low that households prefer to spend less on housing than is allowed by this constraint. This happens at an income y'' that is lower than the income y' at which $p^m(q(y))$ crosses the borrowing constraint. There is a kink in $p^{bc}(q(y))$ at income y^* but not at y''.

Households with income between y^* and y'' want to consume more housing, but are unable to realize this desire. Due to the binding borrowing constraint they pay less for the same house they would have occupied in the unconstrained market equilibrium, so that their utility will be higher. The utility of households with income larger than y'' will also be higher, but their marginal willingness to pay for housing will equal the marginal price, unless the borrowing constraint binds again. The assignment rule of the unconstrained market equilibrium remains valid, because no household can reach a higher utility by deviating from this rule.

Now consider the introduction of a borrowing constraint that already binds at q^c , as in the simple case of Figure 2. The following corollary shows that this situation closely follows the description above.

Corollary 1. Consider the introduction of a borrowing constraint that already binds at y^c : $p^m(q^{min}) > \mu y^c$. Define y'' as the smallest $y \ge y^c$ for which in the right-sided neighborhood of y'', $M(q(y), (1 - \mu)y)f(y)/g(q(y)) < \mu$ if that occurs, and as y^{max} otherwise. Then,

- *The assignment rule* q(y) *does not change;*
- $p^{bc}(q(y)) < p^m(q(y))$, so that utility is higher for all households:
 - 1. $p^{bc}(q(y)) = \mu y$ for $y \in [y^c, y'']$;

- 2. $p^{bc}(q(y))$ is described by $\pi^{bc}(q(y)) = M(q(y), y p^{bc}(q(y)))$ with initial condition $p^{bc}(q(y'')) = \mu y''$, for $y \in [y'', y^*]$ if y^* exists, and for $y \in [y'', y^{max}]$ otherwise, in which y^* is the smallest y > y'' such that $p^{bc}(q(y)) = \mu y$.
- If *y*^{*} exists, the constraint binds the constrained price function again, and Proposition 1 applies.

In the simple case of Figure 2, the borrowing constraint not only binds already at y^c , but y^c is also the smallest $y \ge y^c$ for which $M(q(y), (1 - \mu)y)f(y)/g(q(y)) < \mu$, so that $y'' = y^c$. As a result, $p^{bc}(q(y)) = \mu y$ only for for y^c , as argued above.

What happens if the mortgage qualification constraint is relaxed? Consider again the situation pictured in Figure 3. Households for whom the constraint is no longer binding will attempt to increase their housing consumption until the constraint binds again, or until they are on their housing demand function. However, if housing supply does not adjust, all households will stay in the same house, which will have become more expensive. If in the new situation the constraint is no longer binding for any household, the market returns to the equilibrium price function $p^m(q(y))$. If some households are still constrained, then in the new equilibrium the interval for which the constraint is binding will be smaller. Prices will increase for all households with an income higher than y^* . The welfare of all these households will decrease, since their housing consumption and income do not change. However, the number of constrained households (for whom the borrowing constraint is binding) will be smaller than with the tighter constraint.

Note that the results of this section are sensitive to the assumption that all households have to borrow all the money needed for purchasing their houses. If some households with a given income experience a binding credit constraint while others own some wealth and are willing to invest it in their houses, the latter group may not experience a binding credit constraint while the former group does. In such a case the allocation of households over the housing stock will be affected by the borrowing constraint, as is shown in Appendix A.4.

2.3 Buy-to-let investors

So far, the market only consisted of owner-occupied housing. Now consider the potential entry of absentee buy-to-let investors. We assume that investors have deep pockets and do not experience borrowing constraints. However, they only include rental housing in their portfolios if the return exceeds their cost of capital adjusted for the various cost items (maintenance, insurance, taxes). We assume that investors face the same cost of capital and other cost items as owner-occupiers, proportional to the house price as in (3). Denoting the rent of a house of quality *q* by R(q), investors thus enter the market if

$$R(q) + E\Delta P(q) > \gamma P(q). \tag{12}$$

When do buy-to-let investors enter the market? Household utility only depends on q and c, not on tenure type. If buy-to-let investors would enter an owner-occupied market without borrowing constraints, they would buy houses at the prevailing market price $P^m(q)$ and would be able to let them at a rent that is at most $p^m(q)$. Substituting $p^m(q)$ in (3) for R(q), the investor earns a return of exactly γ . Because this return is not sufficiently attractive to trigger investments in the housing market, buy-to-let investors will not enter a market like the one described in Section 2.1.

The situation is different when some households experience a binding borrowing constraint. As we have seen above, households restricted by a borrowing constraint have a marginal willingness to pay for housing that exceeds the marginal price. These households are willing to pay more than the user cost $p^{bc}(q)$ as rent if this offers them the possibility to consume more housing than they are able to do in owner-occupied housing with the borrowing constraint present. Buy-to-let investors can thus buy owner-occupied housing at a price $P^{bc}(q)$ but charge a rent that exceeds $p^{bc}(q)$. The return of the buy-to-let investor will then exceed γ .

The model thus predicts that buy-to-let investors will enter the market when borrowing constraints are binding for some households. We argue below that this unleashes an arbitrage process that ends when user costs are equal to the user costs in the equilibrium without borrowing constraints.

Consider the indifference curves and budget constraint depicted in Figure 4, in which the prevailing user costs are affected by the presence of borrowing constraints. The bold line indicates the budget constraint of a household, that is the difference between income and the user cost of housing, $y - p^{bc}(q)$. The household also faces a maximum user cost μy , and the resulting minimum consumption of the other good is indicated by the dashed line. As a result, the highest utility that the household can achieve is u^{bc} and it chooses to consume quality q^{bc} .

However, in the absence of a binding borrowing constraint, but assuming for now that prices remain fixed, the household would choose housing consumption q^{o} . A single buy-to-let investor could then purchase a house of quality q^{0} at the prevailing market price, finance it with user cost $p(q^{o})$ and offer it to the borrowingconstrained household at a rent between $p(q^{o})$ and p^{max} . This investment would offer the investor a return above γ and it would give the household the possibility to increase its utility by consuming the quality it would consume without borrowing constraints. In fact, any offer to rent a house implying that the household reaches a combination of housing and other consumption somewhere in the shaded area means a possibility to improve utility relative to the present state of constrained owner-occupied housing consumption for the household.

However, competition between buy-to-let investors would drive up prices. While price increases only contribute to the incentives to invest in the housing market, they may affect the quality of housing that is available to a household, irrespective



Figure 4: Borrowing constraints and profitable buy-to-let

of the tenure type. The following proposition characterizes the equilibrium with buy-to-let investment.

Proposition 2. The assignment, user cost function and welfare in the equilibrium with borrowing constraints and buy-to-let investment are equal to the assignment, user cost function and welfare of the equilibrium without borrowing constraints.

Proof. The investor charging the highest rent will be able to pay most to obtain the house, so competition ensures that in equilibrium buy-to-let investors charge the highest rent that renters would still choose to pay. Because households do not care about tenure type and investors can arbitrage away the impact of borrowing constraints, the highest rent equals the highest user cost that households would be willing to pay. The highest user cost will be paid by the household with the highest marginal willingness to pay. As a result, the equilibrium assignment follows the ranking of Lemma 1 and the user cost function follows the differential equation of 2. Because the assignment and the user cost function are identical, welfare is also identical to the equilibrium without borrowing constraints. \Box

The activity of buy-to-let investors thus drives up housing prices from a situation of borrowing constraints. The prices of houses for which demand was depressed by borrowing constraints increase until the possibility of profit-making buy-to-let activities has disappeared. In this situation, all households who were initially borrowing-constrained avoid the implied restriction by moving to rental housing. The allocation of housing over households is identical to that in a pure owner-occupied market without borrowing constraints as a result of the arbitrage of buy-to-rent investors.¹⁴

¹⁴In a model in which borrowing constraints have macro-prudential benefits, such constraints

How does this equilibrium with buy-to-let investment play out in Figure 4? We know from Proposition 1 that the assignment is not affected by the presence of borrowing constraints, but that prices are lower for households with income levels higher than the income level at which the borrowing constraints starts to bind, compared to the situation without borrowing constraints. Buy-to-let investment thus does not affect the assignment in the equilibrium with borrowing constraints either, and only drives up prices.

In the equilibrium with buy-to-let investment, the household in Figure 4 would thus still consume q^{bc} and would not choose to rent q^0 or any other housing quality. User costs and rents would be higher, so that the budget constraint would lie below the bold line. At q^{bc} , this lower budget constraint would be tangent to an indifference curve corresponding to a lower utility than u^{bc} . Indeed, uniform borrowing constraints increase utility for all constrained households and those with higher incomes, and this utility gain is lost when buy-to-let investment drives up prices.

3 Empirical approach

The assignment model clearly identifies which households are borrowing-constrained, and which houses investors will convert to rental houses. In this section, we explain how we can leverage these insights to get from cross-sectional variation to aggregate effects.

Proposition 2 shows that the assignment and user cost (or rent) function in the equilibrium with borrowing constraints and buy-to-let investment coincides with the assignment and user cost function in the equilibrium without borrowing constraints. This proposition has two crucial implications. First, the result that the assignment does not change implies that the characteristics of the people who live in certain houses before the introduction of borrowing constraints (ex ante) are informative about the characteristics of the residents of those houses after the introduction of borrowing constraints (ex post). Second, the result that the user cost function does not change implies that the mortgage payments of the people who live in certain houses ex ante are informative about the mortgage payments that the residents of those houses ex post pay or *would* pay if they were owneroccupiers.

The combination of these results shows that the mortgage payment-to-income ratios ex ante are informative about the mortgage payment-to-income ratios of residents ex post, including those counterfactual ratios that can no longer be observed because the residents are now renters. As a result, the mortgage payment-

may nevertheless be useful as they protect households (and banks) against the risks associated with mortgage default, at least to the extent that buy-to-let investors are better able to carry these risks.

to-income ratios of the residents of the houses transacted ex ante can be directly compared to the mortgage payment-to-income constraint. Those ratios that violate the constraint identify the houses that in the absence of the constraint would be bought by first-time buyers, but that ex post will be targeted by investors for conversion to the rental sector.

Finally, even though borrowing constraints reduce equilibrium user costs and buy-to-let investors increase these costs, Proposition 2 shows that these effects exactly cancel out. The constrained households, *including* those that are constrained as the result of the spillovers from buy-to-let investment, can thus be identified from the ex ante mortgage payment-to-income ratios. The difference between these directly and indirectly treated regions can be seen most clearly in Figure 2. Whereas the people with incomes between y^c and y'' are the people that are constrained in the absence of buy-to-let investment, the people with incomes between y'' and y' become constrained because of the equilibrium spillovers from buy-to-let investment. The directly and indirectly treated region thus consists of the people with incomes between y^c and y', and the crucial insight is that this entire treated region can be identified from the ex ante mortgage payment-to-income distribution.¹⁵

We apply this insight to study the introduction and tightening of the mortgagepayment-to-income constraint in the Netherlands. In the next section, we provide some institutional background to this policy reform.

3.1 Institutional background

Since 2000, mortgage lenders in the Netherlands have – under pressure from the government and consumers authorities, introduced a Code of Conduct. A main element of this Code of Conduct for mortgage loans is that households are, as a rule, only allowed to have mortgage loans that leave them enough income for other necessary spending categories. A coarse rule of thumb is that the net mortgage payments should not exceed 30% of net income. However, in the Dutch system this rule is formalized and the details are elaborated each year by an independent institute for expenditure research, the National Institute for Budget Information (NIBUD), that annually produces a detailed table indicating the maximum amount that can be borrowed by households without other debt and a specific income.

Before 2011, this 'NIBUD rule' was applied as a formal qualification requirement for the national mortgage guarantee (NHG). On top of that, they were part of the Code of Conduct of Dutch mortgage providers, where they had the status of a guideline. When NHG was not required, borrowing in excess of the NIBUD rule could be justified by general arguments like the expected higher than average

¹⁵We acknowledge that we may misclassify some treated and control-group houses in the case of the more general borrowing constraints as considered in Appendix A.4 or taste differences as considered in Appendix A.5. Such misclassification results in an attenuation bias (Aigner, 1973).

increase in future incomes for the higher educated or plans for investments in the home that would increase its value.

As announced on March 21, the interpretation of the the Code of Conduct changed on August 1, 2011. From now on, the 'comply or explain' principle was applied in a more rigorous way, requiring case-specific reasons for each exception. In the years that followed the formalization of the rules for mortgage lending continued and parts of the Code became elements of the national Dutch law system, applying to everyone.

In the years between 2011 and 2016, the NIBUD rule was tightened every consecutive year. Figure 5 shows the evolution of the maximum mortgage paymentto-income ratio (MPTI) for four income levels from the start of 2008 until the end of 2021. The vertical axis shows the maximum MP2I. The Figure shows a relative large decrease in the maximum MP2I in January 2013, and smaller but still significant changes in first month of the next three years. In 2017 and later years there are modest relaxations of the maximum MP2I for some income levels.



Figure 5: The evolution of the NIBUD rule for a fixed interest rate Note: The graph plot the evolution of the NIBUD rule for a fixed interest rate. We keep the interest rate in the interval of 4.5% to 5% (the interest rate prevailing in 2010 and 2011) to isolate the changes chosen by NIBUD. The data come from NIBUD.

The combination of tightened borrowing restrictions and stricter enforcing makes it likely that some of the housing transactions realized before the tightening of the Code of Conduct in 2011 would have been impossible afterwards. The assignment model developed in the previous sections then suggests that the houses involved in such transactions had a larger probability of being bought by buy-to-let investors than others. We will investigate this in detail in the next subsections.

The tightening of the Code Of Conduct was accompanied by some other changes in Dutch housing policy measures that were all associated with the state of the housing market and the economy in general at the time of the Euro crisis. Until 2011, households could realize an initial LTV that was substantially higher than 1, to allow them to finance additional costs associated with purchasing a home from the mortgage loan. One important item that could be financed with a mortgage was the transaction tax, equal to 6% until 2011, but decreased to 2% in that year. This reduction in the costs was reflected in a lowering of the maximum LTV at the time of purchasing the house to 1.06, a decrease by 4 percentage points. Between 2013 and 2018 this limited was lowered further to 1.00 in annual steps of 1%. It should be noted that these changes in the maximum LTV left the rules with respect to mortgage payment-to-income ratios untouched. They did not change the maximum size of the mortgage loan a household could obtain, but stipulated that this loan could after 2011 only be used to finance the purchase of the house itself, not the associated additional costs.¹⁶

This reduction occurred against the background of increasing concern about the total size of Dutch mortgage debt. The main reason for this situation was the unlimited deduction of mortgage interest paid for the owner-occupied house. Related to this was the popularity of mortgage types that postponed repayment of the loan. Investment mortgages that combined an interest-only mortgage with a life insurance that paid out at the end of the term of the mortgage (commonly referred to as savings mortgages) were popular since the 1980s, but in the 1990s interest-only mortgages (without an accompanying investment obligation) became the most popular type.¹⁷ In 2013 this changed: mortgage interest deductibility was for new cases limited to mortgage loans that are repaid within 30 years as an annuity, or faster. This resulted in a loss of popularity of the savings and interest-only mortgages, while annuity mortgages became again the dominant type. The limitation of the mortgage interest deductibility of 2013 was a major shift in Dutch housing policy. However, its impact on mortgage payment-to-income ratios was limited, since the rules incorporated in the Code of Conduct had always been based on annuity mortgages with a term of 30 years. Households opting for an interest-only mortgage before 2013 realized lower mortgage payment-to-income ratios, but the maximum size of their loan was still determined as if they had chosen an annuity mortgage.

3.2 Data and sample selection

3.2.1 Data

We employ several data sources for our empirical analysis. First of all, we combine the housing transaction datasets of the Land Registry (Kadaster) and NVM (Dutch Association of Real Estate Agents). The properties involved in these transactions

¹⁶See Van Bekkum et al. (2024) for an analysis of this leverage cap.

¹⁷See Bernstein and Koudijs (2024) for the impact of this measure on wealth accumulation by Dutch households.

are our primary focus. We know the properties that were traded and the changes in ownership status of those properties in subsequent years. We also know the individuals and households to which these transactions correspond.

Then, we use rich micro-data from Statistics Netherlands to construct our sample. We have unique address identifiers of houses from GBA-Adresobject and unique person and household identifiers of buyers from GBA-persoon and GBA-huishoudens for each transaction. Through household assets information Vehtab, we match households' total mortgage loans to each buyer and house. Through Inkomen en bestedingen, we obtain gross income of the household involved in the transaction. We obtain houses' ownership status through Eigendomtab.¹⁸

We define the mortgage payment (MP) and mortgage payment to income ratio (MPTI) using, respectively,

$$MP = ML * \frac{r(1+r)^{360}}{(1+r)^{360} - 1'}$$
(13)

and

$$MPTI = \frac{MP}{Income} * 12,$$
(14)

in which, the *ML* refers to the total mortgage loan. The *r* refers to the monthly interest rate, which is equal to the annual interest rate divided by 12. The *Income* refers to yearly household gross income. It is worth noting that we assume the loan repayment period to be 30 years, 360 months. Figure 6 shows the frequency distribution of the MPTI for the transactions we used. It has a relatively fat right tail, suggesting that a non-negligible share of the transactions did not satisfy the NIBUD rules, perhaps associated with their lenient enforcement at the time. However, even in these years the large majority of the transactions (more that 80%) satisfy the NIBUD rule. By comparing the MPTI with the NIBUD rule in subsequent years, we can determine whether the house is subject to borrowing constraints in subsequent years.

3.2.2 Sample selection

We performed the following sample selection procedure.

First, we retained samples of transactions completed between 2009 and 2010 in the Kadaster and NVM datasets. The years 2009 and 2010 are two years when NIBUD was relatively stable, and these were the last two years before the change in the Code of Conduct ws announced. Then, we use transactions related to first-time home buyers. We only retain samples that match the buyer's household income and mortgage loan. Only from these samples can we calculate MPTI to determine whether the property would violate the NIBUD rule in subsequent years. We only

¹⁸This dataset changed definition or statistical methods in 2010 and 2018, so we focus on the comparable period from 2011 to 2017. This was also the time period when NIBUD dropped sharply.





Note: The graph is the frequency histogram of MPTI based on sample summarized in Table A1. We retain all owner-occupied properties transacted in 2009 and 2010. We drop the observations with MPTI larger than 0.5 or equal to 0.

retained those houses that were owner-occupied at the end of 2010. We also removed those samples that had violated the NIBUD rule in 2011 (always treated). At this point, we have a sample of almost 50k unique properties. The descriptive statistics of the house characteristics of this sample are presented in Table A1.

We transformed the baseline housing sample into a panel dataset from 2011 to 2017. Our regression will be conducted based on this panel dataset. The reason for choosing this time period is that the property ownership dataset changed its statistical method in 2011 and 2018, making the data before and after not comparable.

The use of a sample of transactions as the focus of our analysis may be questioned. The transaction sample is peculiar, i.e., they may be a more active sample, causing us to overestimate the effect. We may also underestimate the effect because the sample we use was recently traded in 2009 and 2010, making them less likely to be traded into rental properties in subsequent years. We also matched the houses in the sample to houses with similar characteristics in the baseline transaction sample, to verify whether borrowing restrictions had the same effect on them. We will elaborate this in our empirical results section 4.4.

3.3 A staggered difference-in-differences approach

We would like to introduce a diff-in-diff strategy to test if the houses involved in transactions before the tightening of the Code of Conduct that were no longer feasible in subsequent years were indeed more exposed to buy-to-let activity.

For those transactions for which this rule were satisfied at the time of realization, we could moreover check if they would still have been in accordance with the NIBUD rule in the years that followed (until 2017) if everything else would have remained constant. That is, we ask the question if the same transaction would have been feasible if the income of the purchasing household, the price of the property and the mortgage interest rate would still be the same as in 2009 or 2010, while we use the NIBUD rules of later years. This provides us with a useful granular indicator of a binding borrowing constraint for households that are the likely buyers of such houses, which does not depend on market developments after 2011. According to our theory exactly these houses are exposed to buy-to-let activity.

We use a panel for the years 2011-2017 in our baseline analyses, which ownership status is observed for every house in every year. Ownership status of houses is indicated in our data as a categorical variable with categories: (i) owned by housing association, (ii) owner-occupied, (iii) owned by a private landlord and (iv) unknown. Houses in the third category can be owned by a private person as well as by a firm, for instance an institutional investor. We define a dummy variable to indicate whether the house is a private rent house in that year. Since our sample was all owner-occupied in late 2011, we actually saw a shift from owneroccupied to private rent. In the period considered here there are many switches between private rental and owner-occupied and *vice versa*. As noted above, many of the switches from owner-occupied to private rental resulted from buy-to-let or keep-to-let activities, while many switches in the other direction were from less expensive rent-controlled housing to owner-occupation.

We define whether a house enters the treatment group in a year based on whether the house's fixed MPTI (in 2009 or 2010) violates the NIBUD rule of that year. What we are concerned about is whether buyers with the same income and the same interest rate can still buy a house of the same price when the NIBUD rule changes. When controlling for year fixed effects, it can also be interpreted as, we assume that there is a common trend in the growth of interest rates, housing prices, income, etc over years. All houses were originally in the control group (in 2010) and were owner-occupied. With the gradual decline of the NIBUD rule, many houses began to enter the treatment group, and some houses were converted to private rentals. In other words, we define the year in which a house first violates the NIBUD rule as the treatment year.

3.4 Suggestive evidence

Figure 7 shows the share of the transactions observed in 2009 and 2010 that violate the NIBUD rules in later years. It shows a continuing deepening and staggered timing of treatment, with the exception of 2017. The increase is particularly large in 2013, which is in line with the decrease of NIBUD rule. As NIBUD gradually declined and more households are constrained, the average MPTI of sold homes also gradually declined between 2011 and 2017. The kernel density distribution of MPTI across year in Figure 8 confirms the pattern.

In addition, we document heterogeneous effects of borrowing constraints across income groups. Figure 9 describes the distribution of household income of baseline sample and treated sample. This suggests that households subject to borrowing constraints are concentrated among low-income households, which has a perfect match with our assignment model. Correspondingly, growth in the private rental sector between 2011 and 2017 was also concentrated among low-income groups. Figure 10 documents that the probability of entering the private rental sector increases by 4 to 6 percentage points for low-income people below the 20th percentile. The probability of entering the private rental sector decreases with increasing income and disappears at the 50th percentile. These figures provide suggestive evidence that the growth of the private rental sector is likely to be due to borrowing constraints.



Figure 7: Share of properties violating NIBUD rule over year Note: The graph shows the share of houses violating NIBUD rule over years. This shows both how the treatment group grew, and the treatment distribution over year.



Figure 8: The kernel density distribution of MPTI across year Note: This is the kernel density distribution of MPTI of all transactions across year.



Figure 9: Income distribution of whole sample and treated sample

Note: The graph is the histogram of household income. The lighter bar is the income distribution of households in the baseline sample, and the darker bar is the income distribution of households subject to financing constraints from 2011 to 2017.



Figure 10: Change in the probability of entering the private rental sector Note: The graph shows the change in the probability of entering the private rental sector between 2011 and 2017 for households in each income quantile.

4 Empirical results

4.1 **Baseline estimation**

We employed the DID strategy at property-year level. With the panel dataset constructed in the previous section, we start our estimation with a canonical two-way fixed effect specification:

$$Private_rent_{it} = \alpha + \beta Violate_{it} + \gamma_i + \phi_t + \epsilon_{it}.$$
(15)

In this equation, $Private_rent_{it}$ is a dummy variable indicating whether house *i* is a private rental house in year *t*. We define $Private_rent_{it}$ as 100 if the house *i* is a privately rented house in year *t*, and 0 otherwise. $Violate_{it}$ is a dummy variable to indicate whether the $MPTI_i$ of house *i* is larger than $NIBUD_t$ in year *t*. If $MPTI_i$ is larger than $NIBUD_t$, we define $Violate_{it} = 1$, otherwise, 0. γ_i indicates property fixed effects and ϕ_t indicates year fixed effects, which capture the time-invariant property characteristics and common trend of properties over year, respectively. ϵ_{it} indicates the error term. We have no time-variant covariates at the property level, but in some specifications we control for property type (apartment or not) multiplied by a linear year trend.¹⁹ Because Glaeser and Gyourko (2007) find that rental homes in the US are more likely to be apartments, while owner-occupied homes are more likely to be single-family houses, we thus allow for different time

¹⁹We also experimented with controlling for nonlinear time trends, but the estimates are almost the same as those under linear trends. To maintain brevity, we do not present those.

trends in ownership status for apartments and single-family houses.

In all estimations we use property and year fixed effects, as indicated in the equation. However, to take into account possible local trends in buy-to-let activity, we also estimate specifications with postcode*year fixed effects.²⁰ Controlling for local trends at a small geographical scale may be useful because it is known that buy-to-let investment activities vary substantially within cities.

Table 1 presents the results of our baseline estimation. These results capture the average treatment effects on the treated (ATT) of the violation of NIBUD rule. We find the expected positive and significant coefficients in all specifications. The estimators show that houses subject to borrowing constraints have a 0.43% to 0.84% higher probability of becoming a private rental property than houses that do not violate NIBUD. We conclude that our empirical analysis confirms a key result of our assignment model: tighter borrowing constraints open up profitable arbitrage possibilities for buy-to-let investors in those parts of the market where demand is depressed.

We then discuss the magnitude of the treatment effects starting with sample mean. In the treatment group, the average probability of a house being privately rented over the years was 3.3%, initially 0, and reaching 6.8% by the end of 2017. Our estimates show that the probability of a house entering the private rental sector increases by 0.4% to 0.8%, which is a large figure relative to the sample mean. In addition, in the following section 4.3, we used the new CSDID estimator and obtained a cumulative treatment effect of 1% to 1.4% by the end of 2017. In other words, by the end of 2017, the private rental sector share in the treatment group sample was 6.8%, of which 1% to 1.4% was contributed by borrowing constraints, accounting for approximately 15% to 21%. This suggests that borrowing constraints have strong explanatory power for the growth of the private rented sector. In addition, with the linear year trend, it is intuitive that apartments have a greater probability of becoming private rental housing.

4.2 Event study

The key assumption for identifying ATT is that the parallel trend assumption holds, which allows us to interpret the control group as a counterfactual of the treatment group. The parallel trend assumption is untestable, but we can build some confidence that it holds if we observe a parallel pre-trend of treatment and control group. We conducted an event study to check the parallel trend and to investigate the dynamic effects of borrowing constraint. We carried out event studies by estimating:

²⁰We use the 4 position postcode areas defined by the 4 first digits of the Dutch postcode. The 4,770 areas defined in this way cover the whole Netherlands and consist of a small number of contiguous streets.

| | (1) | (2) | (3) | (4) |
|---------------|--------------|-----------------|--------------|----------------|
| | TWFE | +Pc4*year F.E. | TWFE | +Pc4*year F.E. |
| | | Dependent Varia | able: Priva | te rent |
| | | | | |
| violate | 0.840*** | 0.561*** | 0.574*** | 0.433*** |
| | (0.155) | (0.159) | (0.154) | (0.159) |
| type*year | | | 1.656*** | 1.394*** |
| | | | (0.043) | (0.054) |
| Observations | 336,521 | 333,378 | 336,521 | 333,378 |
| R-squared | 0.449 | 0.481 | 0.456 | 0.483 |
| Property F.E. | \checkmark | \checkmark | \checkmark | \checkmark |
| Year F.E. | \checkmark | \checkmark | \checkmark | \checkmark |
| Pc4*Year F.E. | | \checkmark | | \checkmark |

Table 1: Baseline dif-in dif estimates

Notes: *** p < 0.01, ** p < 0.05, * p < 0.1. Standard errors are clustered at Pc4*year level. Column (1) shows the estimation results specified in equation (15), column (2) includes Pc4*year fixed effects. Columns (3) and (4) include the interaction of whether the property is an apartment and the linear year trend.

$$Private_rent_{it} = \alpha + \sum_{\tau = -5, \tau \neq -1}^{\tau = 4} \beta_{\tau} \mathbb{I}(T_i - t = \tau) + \gamma_i + \rho_p \phi_t + \epsilon_{it}.$$
(16)

 T_i denotes the year in which house *i* first violated the NIBUD rule. $\rho_p \phi_t$ denotes the 4 digit postcode times year fixed effect. The results are presented in Figure 11. The plots show that there are no significant differences between the treatment and control groups in the private rented sector before violating the NIBUD rule. The parallel pre-trend makes us trust that the parallel trend assumption is likely to hold. After violating the NIBUD rule, however, properties subject to borrowing restrictions start to have a significant greater probability of entering the private rental sector. We also saw a cumulative treatment effects over period. After a house violates NIBUD rules for the first time, the probability of it becoming a private rent becomes greater and slowly increases.

4.3 Robustness checks

We conduct several checks to show the robustness of our baseline estimates.

Treatment effects heterogeneity. We adopt prevailing "heterogeneity-robust" diagnostics for staggered treatment timing (In our case, the house could start to violate NIBUD rule in any year between 2011 and 2017). A recent econometric literature pointed out that the two-way fixed effect estimator can be interpreted as



Figure 11: Event study

Note: This figure is a coefficient plot and 95% confidence interval of the estimation results in equation (16). Standard errors are clustered at Pc4*year level. The graph above is the coefficient plot with apartment times year trend and without apartment times year trend, which is corresponding to column (2) and (4) in Table 1.

a weighted average estimator of each staggered treatment effect (de Chaisemartin and D'Haultfœuille, 2020; Goodman-Bacon, 2021). If there were heterogeneity of treatment effects over time and units, we may get negative weights which make the TWFE estimator hard to interpret.

We calculate and plot the weights of each treatment following de Chaisemartin and D'Haultfœuille (2020). Figure A4 shows that almost all the weights associated with each treatment are non-negative and there is little variation. Moreover, we adopt the heterogeneity-robust estimator of Callaway and Sant'Anna (2021). The estimator is obtained by estimating all 2x2 DID estimators and aggregation. The CSDID estimators are presented in Table A5 and corresponding event-study plots are presented in Figure A5. The average treatment effects on the treated with the new estimator is between 0.53 and 0.76. Comparing with column (1) and (2) in Table 1, these estimators are slightly smaller but no substantial differences.

We further decompose the treatment effects by group of year entering treatment and by calendar years, presented in Figure A6. The results show that the treatment effects are almost positive for all groups, however, only significant in group 2013. It suggests that estimated treatment effects are dominated by the sample entering the treatment group in 2013. The results also show that there is a cumulative effects, by the end of 2017, the cumulative treatment effects are from 0.979 to 1.366. **Placebo test.** One concern regarding the empirical results is that the observed treatment effects may be driven by some unknown random factors. To address this, we conduct a placebo test. In this placebo test, we randomly generated a fictitious treatment group within the original sample for 500 times. The size of the fictitious group matches that of the actual treatment group, and the distribution of their treatment timing is identical to that of the real treatment group. This approach allows us to randomly assign each sample to the treatment group, enabling us to examine whether such a random distribution would produce similar treatment effects.

We plot the estimators and p-value of these 500 estimations in the Figure A7. These estimators exhibit a distribution resembling a normal distribution. The actual treatment effect of 0.43 lies on the right tail of this distribution. Statistically, only 2.2% of the fictitious treatment effects exceed the true value of 0.43. Thus, we can reject the hypothesis that the real treatment effect is equal to the fictitious treatment effect at the 95% confidence level. Therefore, we rule out the possibility that the treatment effect is driven by randomness.

4.4 Matching sample

In the previous subsection we always restricted the analysis to houses transacted just before the change in the Code of Conduct of 2011. However, it may be argued that is an idiosyncratic set of houses, for example, may because dwelling that are frequently traded are over-represented. This may cause these houses to be more prone to trades or change ownership status, thus biasing our estimates. To address this issue we reconstruct the sample by using for properties similar to these transaction samples. That is, we assume that houses with characteristics that are similar to those in the sample used thus far, also have a similar probability of becoming treated in the period 2012-2017. We use location, type, price and floor area to construct this matched sample (e.g. other apartments in the same condominium) and continue to use the MPTI of the property in the original sample to determine whether the new property violates NIBUD. Our hypothesis is that the ownership status of the matched property is as likely to switch from owner-occupied to private rental at that of the original property.

We conducted a following matching procedure. First of all, we make sure the new property has same 6-digit post code and type with baseline property. For the many-to-many matching of properties with the same PC6 and type, we first identified a group of properties that exactly matched the benchmark property. Next, we progressively reduced the disparity in assessed home values between the baseline and matched properties, resulting in a more comparable set of matched properties. We obtained three additional matched samples with assessed value differences of less than 10k, less than 1k, and exactly equal. The matching accuracy of these four matched samples improves sequentially, while the sample sizes gradually decrease.

The estimates using 4 matching samples are presented in Table 2. The column (1) shows that When we ensure that the matched sample has the same house value, postcode 6, and type as the baseline sample, we obtain the largest estimate of 0.64, which is also larger than the baseline estimator. This is what we expect, the effect of borrowing constraints should be higher for the matched sample. Since the sample that traded in 2009-2010 has a lower probability of transaction in subsequent years, the matched sample has a relatively higher probability of transaction in 2012-17. The results also indicate that as we progressively relax the matching criteria for property values, the matches become less precise, and the estimated treatment effects gradually decline. When we match only on house type and postcode 6, the estimated coefficient in column (4) is already close to 0. This suggests that house value is important for finding similar matched samples and similar matched samples also lead to larger estimates.

We also present the event study plot of results in column (1) in Figure 12, showing that the parallel trend is still there. All the results suggest that the higher probability of being converted to the private rental sector was not limited to houses transacted in 2010 or the first half of 2011 but occurred for all houses with similar characteristic in the entire housing stock.

| | (1) | (2) | (3) | (4) | | |
|-----------------|--------------|----------------------------------|--------------|--------------|--|--|
| | Same value | ≤ 1000 | ≤ 10000 | All | | |
| | Depen | Dependent Variable: Private rent | | | | |
| | | | | | | |
| violate | 0.635* | 0.350* | 0.094 | 0.050 | | |
| | (0.339) | (0.202) | (0.083) | (0.051) | | |
| type*year trend | 0.992*** | 1.117*** | 1.003*** | 0.776*** | | |
| | (0.112) | (0.077) | (0.036) | (0.030) | | |
| Observations | 140 116 | 340 851 | 1 01/ 831 | 5 795 001 | | |
| Observations | 140,110 | 540,051 | 1,914,031 | 5,795,001 | | |
| R-squared | 0.827 | 0.826 | 0.827 | 0.833 | | |
| Property F.E. | \checkmark | \checkmark | \checkmark | \checkmark | | |
| Year F.E. | \checkmark | \checkmark | \checkmark | \checkmark | | |
| Pc4*Year F.E. | \checkmark | \checkmark | \checkmark | \checkmark | | |

Notes: *** p < 0.01, ** p < 0.05, * p < 0.1. Standard errors are clustered at Pc4*year level. The table shows the estimation results specified in equation (15). This is the estimation based on 4 different matching samples.



Figure 12: Event study of the matching sample

Note: This figure is a coefficient plot and 95% confidence interval of the estimation results of equation (16) using matched sample. Standard errors are clustered at Pc4*year level. The graph is the coefficient plot without apartment*year.

4.5 Buy-to-let or keep-to-let

So far, we have demonstrated how borrowing constraints influence the likelihood of properties transitioning into private rental housing. Our next question is whether the growth of the private rental housing market is driven by buy-to-let (BTL) investments or let-to-buy (KTL) strategies. To distinguish them, we match ownership status transitions of the properties to transaction dataset. When a property is private rent in a year and has transaction records from July 2011 onwards to the end of that year, we define it as private rent with BTL. In contrast, we define it as private rent with KTL if the property is private rental property but there is not transaction record.

The estimators using BTL and KTL respectively as the explained variables are shown in the Table 3. The results indicate that violating the NIBUD rule resulted in significant and similar positive effects for BTL and KTL. The effect of borrowing constraints on KTL is larger overall. This may also be due to the lower transaction probability in the baseline sample during 2012-2017, resulting in a lower BTL probability.

5 Conclusion

This paper provides an analysis of the effect of borrowing constraints on buy-tolet investment where the housing stock can be considered as given, at least in the

| | (1) | (2) | (3) | (4) | | |
|----------------|----------------------------------|--------------|------------------|--------------|--|--|
| Panel A: | Private rent with transaction | | | | | |
| | TWFE | Pc4 trend | TWFE | Pc4 trend | | |
| | | | | | | |
| violate | 0.350*** | 0.184* | 0.186* | 0.094 | | |
| | (0.097) | (0.101) | (0.096) | (0.100) | | |
| apartment*year | | | 1.020*** | 0.981*** | | |
| | | | (0.031) | (0.039) | | |
| | | | | | | |
| Observations | 336,521 | 333,378 | 336,521 | 333,378 | | |
| R-squared | 0.374 | 0.409 | 0.382 | 0.412 | | |
| Panel B: | Private rent without transaction | | | | | |
| | TWFE | Pc4 trend | TWFE | Pc4 trend | | |
| | | | | | | |
| violate | 0.490*** | 0.377*** | 0.388*** | 0.339** | | |
| | (0.129) | (0.132) | (0.128) | (0.133) | | |
| apartment*year | | | 0.636*** | 0.412*** | | |
| | | | (0.029) | (0.037) | | |
| | | | | | | |
| Observations | 336 <i>,</i> 521 | 333,378 | 336 <i>,</i> 521 | 333,378 | | |
| R-squared | 0.458 | 0.485 | 0.460 | 0.485 | | |
| | | | | | | |
| Property F.E. | \checkmark | \checkmark | \checkmark | \checkmark | | |
| Year F.E. | \checkmark | \checkmark | \checkmark | \checkmark | | |
| Pc4*Year E.E. | | \checkmark | | \checkmark | | |

Table 3: Borrowing constraint and buy-to-let activity

Notes: *** p<0.01, ** p<0.05, * p<0.1. Standard errors are clustered at Pc4*year level. Column (1) shows the estimation results specified in equation (15), column (2) includes Pc4*year fixed effects. Columns (3) and (4) include the interaction of whether the property is an apartment and the linear year trend. The explained variable in Panel A is that private rentals with transaction records; the explained variable in Panel B is that private rentals without transaction records.

short run. Binding borrowing restrictions open up possibilities for profitable arbitrage by buy-to-let investors. By offering the houses preferred by the restricted households as rental housing, they allow them to reach the same level of housing consumption as in the case without borrowing constraints, albeit at a higher price than is relevant in the situation when borrowing constraints are binding. In the equilibrium with free entry of buy-to-let investors the allocation of housing over households is the same as in the situation without borrowing constraints.

We use this assignment model to predict which owner-occupied houses will be converted to rental houses, and test these predictions exploiting a series of contractions of the debt-service-to-income constraint in the Netherlands between 2012 and 2017. The empirical analysis showed that in the Netherlands switches from the owner-occupied to private rental sector were significantly more prevalent among houses that had been involved in transactions that became infeasible after the tightening of the the borrowing constraints. The series of contractions that ultimately affected 17 percent of households resulted in a cumulative increase of the private rental sector of 1.2 percentage points, accounting for 14 to 21 percent of the overall increase in this sector by the end of 2017.

These findings show that the borrowing constraints that have been introduced in many countries in the wake of the financial crisis, have likely significantly contributed to the observed rise of the private rental sectors in many developed economies. However, our findings suggest that that there is still substantial explanatory power left for other factors.

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Appendix

A Additional theoretical results

A.1 Supporting derivations

A.1.1 High-income households consume more housing

In an assignment model, the marginal price of housing is not necessarily constant. The following lemma shows that housing consumption is still increasing in income.

Lemma 3. Households with a higher income y consume more housing services q.

Proof. Consider two households, 1 and 2, with incomes y_1 and y_2 with $y_1 > y_2$, without loss of generality. Household 1 thus reaches a higher utility, u_1 , than household 2, u_2 . Now suppose that the households have housing consumption q_1 and q_2 with $q_1 < q_2$. Then both households can benefit from switching houses. Define $M(q,c) = (\partial u/\partial q)/(\partial u/\partial c)$ as the marginal rate of substitution between housing and other consumption. Housing being a normal good implies that the marginal willingness to pay for housing M(q,c) is increasing in *c* for given *q*. The willingness to pay of household 1 for the larger house can be written as

$$WTP = \int_{q_1}^{q_2} M(q, c(q, u_1)) dq,$$
 (A1)

where $c(q, u_1)$ denotes the value of other consumption that keeps the household on its initial indifference curve when housing consumption is *q*. Similarly, we can write the minimum required compensation (willingness to accept) of household 2 for the smaller house as

$$WTA = \int_{q_1}^{q_2} M(q, c(q, u_2)) dq,$$
 (A2)

where the interpretation of $c(q, u_2)$ is analogous. Since $M(q, c(q, u_1)) > M(q, c(q, u_2))$ for all q, we must have that WTP > WTA, which implies that both households can reach a higher utility level if they switch houses.

This lemma shows that the high-income household is able to compensate the low-income household for moving to the lower quality house and still reach a higher utility. Hence they can engage in a transaction that is beneficial to both, which shows that that initial situation is incompatible with equilibrium. Figure A1 shows that WTP > WTA if housing is a normal good, i.e. if indifference curves become steeper for the same *q* but higher *c*.



Figure A1: Illustration of switching houses

A.1.2 The second-order condition

The first-order condition (8) requires that the slope of the budget line is equal to that of the indifference curve. The second-order condition requires that a move along the budget line starting from the point where the first-order condition is satisfied results in a lower utility. This is the case if the budget line is locally less convex than the indifference curve. The following lemma shows that this is always the case when the other equilibrium conditions hold.

Lemma 4. In the optimum, the budget line is less convex than the indifference curve:

$$-\frac{d\pi}{dq} < -\frac{dM}{dq}\Big|_{u \text{ constant}}$$
(A3)

Proof. The slope of the user cost function is equal to the marginal willingness to pay of the households that have been assigned to the houses of the quality considered. Taking the total derivative of the marginal rate of substitution in (8),

$$d\pi = \frac{\partial M}{\partial q} dq + \frac{\partial M}{\partial (y-p)} d(y-p)$$

= $\frac{\partial M}{\partial q} dq + \frac{\partial M}{\partial (y-p)} \left[\frac{g(q)}{f(y)} dq - \pi dq \right],$

which follows from the definition of the marginal price of housing and the assignment rule in (6). Consequently,

$$\frac{d\pi}{dq} = \frac{\partial M}{\partial q} + \frac{\partial M}{\partial (y-p)} \left[\frac{g(q)}{f(y)} - \pi \right].$$
 (A4)

By the definition of the marginal willingness to pay, on an indifference curve it must be the case that dc = -Mdq. The total derivative of *M* along an indifference curve is thus

$$dM = \frac{\partial M}{\partial q} dq + \frac{\partial M}{\partial (y-p)} dc$$

= $\frac{\partial M}{\partial q} dq - \frac{\partial M}{\partial (y-p)} M dq$
= $\left[\frac{\partial M}{\partial q} - \pi \frac{\partial M}{\partial (y-p)}\right] dq,$

because in the optimum c = y - p(q) and $\pi(q) = M(q, c)$. The expression in square brackets is the second derivative of the indifference curve with reversed sign, while (A4) gives the second derivative of the equilibrium housing price function, $d\pi/dq$, also with reversed sign. Comparison of these second derivatives shows that (A15) holds if $\partial M/\partial cg(q)/f(y) > 0$, which is true because housing is a normal good and because g(q) and f(y) are positive.

A.2 Intertemporal utility maximization

The model discussed in the main text refers to a single period, but in this section of the Appendix we show that the static model can be embedded in a life-cycle utility maximization framework. We first present preferences, endowments and decisions that are common across environments. Then we show how the static model results from the environments without and with borrowing constraints.

A.2.1 Preferences, endowments and decisions

We consider an economy that consists of overlapping generations of households that live a known, finite number of periods 1, ..., T. Households have preferences that are additively separable across periods and that can be represented by the utility function

$$U = \Sigma_{t=1}^T \beta^{t-1} u(q, c), \tag{A5}$$

where u(q, c) is the same periodical utility function as in the static model. $\beta \in (0, 1)$ denotes the discount factor. Households have no bequest motive.

Households are endowed with a deterministic stream of earnings $\{e_1, ..., e_t, ..., e_T\}$ that is non-decreasing across periods: $e_t \ge e_{t-1}$. Households start their life without any wealth. When a household dies, any wealth remaining after repayment of debts is lost. Deceased households are replaced by newborns, so that the environment is stationary. In particular, the aggregate earnings distribution is constant, just as the house quality distribution.

Each period, households choose their house quality and other consumption. The consumption of owner-occupied housing requires buying a house, and changing housing consumption across periods requires both buying and selling. However, because there are no transaction costs, households can re-optimize every period. Moreover, because the earnings and house quality distributions are constant, house prices are constant over time.

Households have the possibility to save, but can only borrow for the purchase of a house. To finance the purchase of a house, the market offers interest-only mortgage loans with a loan-to-value ratio of 1. The mortgage interest rate is denoted by *i* and there are no other costs associated with housing. The mortgage interest rate *i* equals the interest rate on savings and households' rate of time preference, $\beta = 1/(1+i)$.

A.2.2 No borrowing constraints

Consider the environment of Section 2.1 in which households have access to mortgages with an unconstrained amount M, as long as M equals the price P(q) of the house that they buy. Of course, when households sell their house, they also need to repay their mortgage. Denoting accumulated savings at the beginning of period t by b_{t-1} , the budget constraint of a household in period t thus equals

$$c_t + P(q_t) - M_t + iM_t + b_t \le e_t + b_{t-1} + P(q_{t-1}) - M_{t-1},$$

$$c_t + p(q_t) + b_t \le e_t + b_{t-1},$$
(A6)

because $P(q_t) = M_t$ and $iP(q_t) = p(q_t)$, and where $b_t \ge 0$ and $b_0 = 0$. Because we ignore other costs associated with housing, housing expenditure is thus equal to the amount of interest paid on a loan that equals the price of the house. Effectively, durable housing thus turns into a nondurable good.

Because $\beta = 1/(1+i)$, each period households would like to consume the permanent component of their lifetime earnings. However, because their stream of earnings is non-decreasing and households cannot borrow for other consumption, $b_t = 0$ in every period. The budget in each period thus simply consists of earnings e_t , ans we can think of the income y in the static model as e_t . If earnings are increasing over the life-cycle, consumption of housing and other consumption are thus increasing as well.

The intratemporal first-order conditions of the maximization of (A5) subject to (A6) equal the first-order condition of the static problem in (8) for every period. We can thus conclude that the static model in the body of the paper can be embedded in a life-cycle utility maximization framework.²¹

A.2.3 Uniform borrowing constraints

Now consider an environment with a maximum mortgage payment-to-income constraint. Households can still save and borrow for housing, but now the maxi-

²¹Some of the assumptions above can be relaxed. For instance, the conclusion remains valid if the rate of time preference is higher than the interest rate or if the periodical utility functions change over time.

mum mortgage loan available to a household in period *t* is a function of its earnings e_t . In particular, households have access to interest-only mortgage loans with interest rate *i* up to size $\mu e_t/i$.

Households are thus able to finance housing quality up to a limit q^{bc} that is determined by the equality $P(q^{bc}) = \mu e_t/i$ through a mortgage loan. We assume that the borrowing constraint prevents households access to a mortgage if they use their earnings to buy a house with a price that exceeds $\mu e_t/i$. Consequently, if households would like to consume housing quality that exceeds q^{bc} , they need to pay the entire house price in cash.

The relevant case is the case in which the borrowing constraint binds. The household can then accept the constraint, as we have assumed in the main text, or it can avoid it by paying the entire house price without a mortgage. However, in this case, the budget constraint tilts. It still cannot spend more than its current earnings e_t , but the price of housing is no longer equal to its user cost p(q) = iP(q) but equals the full price P(q). Although the household can sell the house after the period and get its value back, it remains true that it has to give up consumption of other goods to an amount equal to the full price of housing. It is easy to come up with a utility function in which the household finds it optimal to accept the situation with uniform borrowing constraints as discussed in the main text, where y replaces e_t .

A.3 Additional case uniform borrowing constraint

This section presents a third case for the impact of a linear mortgage qualification constraint. In this case, the constrained user cost function equals the constraint only at y^c and lies below the constraint for all higher income levels, as illustrated by the gray curve in Figure A2. This case holds when the marginal willingness to pay for housing is smaller than $\mu g(q(y))/f(y)$ for all income levels, and thus the user cost never equals the constraint except for the lowest-quality house.

A.4 Extension: General borrowing constraints

In this subsection we consider a more general situation in which households can experience borrowing constraints of a general nature. The borrowing constraint is a household-specific maximum imposed on the purchase price of a house.²² Since there is a one-to-one relationship between purchase price and user cost, we include this in our model as a maximum user cost ρ . We assume that ρ has positive support on $[\rho^{min}, \rho^{max}]$ for some $0 \le \rho^{min} < \rho^{max} \le \infty$.

Households are characterized by their income and maximum user cost, an ordered pair (y, ρ) . We denote the simultaneous distribution of income and maxi-

²²This household-specific maximum can be conditional on the mortgage underwriting rules and the mortgage interest rate.



Figure A2: The mortgage qualification constraint and the price of housing with a small marginal willingness to pay for housing.

mum user cost as $F(y, \rho)$. $F(y, \rho)$ is thus the number of households with income at most equal to y who can bid at most ρ for a house. The corresponding density is $f(y, \rho)$. In the previous subsection we discussed a case in which ρ is a function of income, $\rho = \mu y$. This is a special case of the situation considered here, in which the density $f(y, \rho)$ is only positive for $\rho = \mu y$. The discussion that follows considers a different and much more general case in which $f(y, \rho)$ is a continuous function of its two arguments. Such a situation is compatible with the mortgage constraint discussed in the previous section if households also have wealth that can be used, in addition to a mortgage loan to finance a house. The wealth distribution is continuous and may be income-specific.

The ρ that is relevant for a particular household should be interpreted as the maximum user cost that the household can afford. For instance, if the mortgage qualification constraint of the previous section holds, this allows a user cost $\rho^m(y)$ = μy , If the household also has wealth that can be used to help finance the house, there is a second part of the user cost, denoted ρ^w which is a function of the household's wealth w.²³ The constraint is now that the actual user cost p is at most equal to the sum $\rho = \rho^m + \rho^w$.

Note that the analysis is also relevant for other cases. For instance, if there is a down-payment constraint instead of a mortgage qualification constraint, the

²³For concreteness, one may assume that user cost is proportional to the purchase price of a house, with the constant of proportionality equal to the mortgage interest rate for mortgage financing. If it is further assumed that the opportunity cost of wealth is equal to this interest rate there is a conveniently simple relationship between the purchase price of a house and its user cost. The discussion below covers to this case.

household must have enough wealth to pay a share σ that equals one minus the maximum loan-to-value ratio. Using the assumptions of the previous paragraph, the maximum user cost may be determined as follows. The down-payment constraint is: $\sigma P < w$, where *P* denotes the purchase price. Multiplication of both sides of the inequality by the mortgage interest rate and dividing by σ gives: $rP < rw/\sigma$. The left-hand side of this inequality is the user cost, while the right-hand side gives the maximum of the user cost, ρ , in this situation. It does not depend on income, only on wealth. The model discussed here is therefore also relevant for situations with a down-payment constraint.

The supply side of the market is unchanged. The number of households assigned to a house must be equal to the number of houses that is available. These households must have an income that is at least as high as the critical level at which housing of the lowest quality is consumed and a maximum user cost that is larger than that of the lowest quality housing. There are thus potentially two groups of households demanding housing of minimum quality: (i) those with a maximum user cost $\rho = p(q^{min})$ and income $y \ge y^c$, and (ii) those with income $y = y^c$ and maximum user cost $\rho \ge p(q^{min})$:

$$F(y^{max}, \rho^{max}) - F(y^{c}, \rho^{max}) - F(y^{max}, p(q^{min})) + F(y^{c}, p(q^{min})) = S,$$
(A7)

in which the last term shows up to avoid double-counting.

The value of $p(q^{min})$ is determined in the same way as before, namely by the condition in (7) that a household with the critical income must be indifferent between the housing of minimum quality and the outside option. Because housing is a normal good, $p(q^{min})$ is an increasing function of the critical income y^c , and the number of households with a maximum user cost above $p(q^{min})$ must thus be a decreasing function of y^c . It follows that the left-hand side of (A7) is a decreasing function of y^c and that y^c is uniquely determined.

To trace out the user cost function we consider what happens at a combination of income y, quality q and an associated user cost p(q). The idea is that all households with an income lower than y or a maximum user cost lower than p(q) either have been assigned a house, or will not participate in the housing market. The supply of housing of quality q is g(q)dq and this must be equal to the demand. Demand originates both from households experiencing borrowing constraints and from those who do not, so that

$$g(q)dq = f^{bc}(y, p(q))dp + f^{uc}(y, p(q))dy,$$
 (A8)

in which $f^{bc}(y, p(q))$ is the density of households who have not been assigned a house but are constrained at user cost p(q),

$$f^{bc}(y, p(q)) = \int_{y}^{y^{max}} f(y, p(q)) dy,$$
 (A9)



Figure A3: Constrained and unconstrained housing demand

while $f^{uc}(y, p(q))$ is the density of unconstrained households choosing a house with quality *q*,

$$f^{uc}(y,p(q)) = \int_{p(q)}^{\rho^{max}} f(y,\rho)d\rho.$$
(A10)

Figure A3 illustrates. The box indicates the combinations of income $y \in [y^{min}, y^{max}]$ and maximum user costs $\rho \in [\rho^{min}, \rho^{max}]$ for which the distribution $F(y, \rho)$ has positive support. The housing price is given as a function of income. It starts at the critical income and is shown until some higher *y* corresponding to housing demand *q* that commands price p(q). The two narrow (blue) rectangles indicate the demand for housing at this point. The vertical one refers to unconstrained households, that is households with income *y* who are able to bid at least p(q). The horizontal box refers to constrained households, who can just afford to bid p(q)but cannot afford more expensive housing because of a borrowing constraint. Total demand for housing of quality *q* is equal to the number of households whose combinations of income and maximum loan belong to these two boxes.

Observe that unconstrained households will only choose the combination (q, p(q)) of housing quality and user cost if the first-order condition (8) holds. Using the definition of the marginal price $\pi(q) = dp/dq$, participation of unconstrained households at (q, p(q)) thus requires

$$dp = M(q, y - p(q))dq.$$
(A11)

Substituting (A11), we can rewrite (A8) as

$$[g(q) - f^{bc}(y, p(q))M(q, y - p(q))]dq = f^{uc}(y, p(q))dy.$$
 (A12)

Since the right-hand side is non-negative, the expression in square brackets on the left-hand side must also be non-negative. If this is the case, there is a 'mixed' local equilibrium in which a given type of housing is inhabited by both types of households, a situation that did not occur with the uniform mortgage qualification constraint studied in the previous subsection.

However, if the expression in square brackets on the left-hand side of (A12) is negative, (A11) does not hold at (q, p(q)) and a mixed equilibrium is not feasible. In this situation, there are so many households with a binding borrowing constraint at p(q) that they consume all housing with quality q and nothing is left for unconstrained households. The second term on the right-hand side of (A8) thus disappears ($f^{uc} = 0$), and instead we have

$$g(q)dq = f^{bc}(y, p(q))dp.$$
(A13)

The slope of the user cost function will now be determined by the densities of housing and of borrowing-constrained households in such a way that all constrained households are exactly on their constraint:

$$\pi(q)(=\frac{dp}{dq}) = \frac{g(q)}{f^{bc}(y, p(q))}.$$
(A14)

Since all households are constrained, the marginal price of housing is lower than the marginal willingness to pay.²⁴

Note that (A13) allows the density of borrowing-constrained households to be larger than the density of housing. If many borrowing-constrained households are clustered in a particular price-quality range, they may occupy all housing for a range of qualities, as happened in the previous subsection. Unconstrained households will be put 'on hold' until all the constrained households have been served. This implies that these unconstrained households are directed to higher quality housing. Because constrained households are forced to accept lower housing consumption, unconstrained households are enabled to consume housing of higher quality relative to the unconstrained equilibrium.

As will be clear by now, the equilibrium with borrowing constraints in the model of the present subsection differs substantially from that in the previous subsection with a uniform mortgage qualification constraint. In particular, the assignment rule in the constrained equilibrium now differs from that in the unconstrained one. Households experiencing a binding borrowing constraint will in general consume less housing than unconstrained households with the same income level. Some of them may even be pushed out of the housing market, while

²⁴To verify this, note that the expression in square brackets on the left-hand side of (A12) is negative implying that $\frac{g(q)}{f^{bc}(y,p(q))} < M(q, y - p(q))$ and use (A14).

other households with lower incomes but less tight borrowing constraints will be able to enter. However, with the generalized borrowing constraints of the present subsection it is still true that - relative to the corresponding equilibrium without constraints, house prices will be lower.

Lemma 5. In the equilibrium with general borrowing constraints (i) housing consumption of an unconstrained household consuming quality q^* in the corresponding unconstrained equilibrium will never be lower in the constrained equilibrium and strictly higher if households with a higher income have been allocated to housing quality that is lower than q^* , (ii) house prices p(q) will never be higher than in the corresponding unconstrained equilibrium and strictly lower if some households consumer lower housing quality that q^* are constrained.

$$-\frac{d\pi}{dq} < -\frac{dM}{dq}\Big|_{u \text{ constant}}$$
(A15)

Proof. (i) Since all houses are occupied relegating constrained households to lower quality means that unconstrained ones must move to higher quality. Note that for unconstrained households a higher income still implies higher housing consumption. (ii) Suppose the price function would remain unchanged. Then all unconstrained households that have moved to higher quality have lower incomes than the households living in the same houses in the situation without borrowing constraints. Their marginal willingness to pay for housing is lower. The marginal price for houses only occupied by constrained households is also lower than that on the unconstrained households that were put on hold for that quality. Constrained households thus imply lower housing prices, also for the households that they push to higher quality. That leaves us with the possibility that for higher qualities some households live in the same houses as in the situation without constraints. The housing price of the house of the lowest quality occupied by these households is lower than in the situation without borrowing constraints. The marginal willingness to pay for housing is therefore higher, which moves the housing price towards the one in the situation without borrowing constraints for higher qualities, but it will never exceed this price. The situation is similar to that with a uniform borrowing constraint.

A.5 Extension: Taste differences

A.5.1 Introduction

The assumption that all households have the same tastes appears unrealistic and we will therefore in this section consider what changes in the model if we allow heterogeneity in tastes. More specifically, we assume that there are $n \ge 2$ groups of households. Within each group all households have the same tastes, but their incomes differ. We use a super-fix i = 1...n to refer to groups. Incomes belong to group-specific intervals $[y^{(i,min)}, y^{(i,max)}]$ and the income distributions $F^i(y^i)$ are

differentiable and increasing on that interval. The total number of households in all groups exceeds the number of houses. Moreover, the housing distribution G(q) is differentiable and increasing on that same interval. For each group the same regularity assumptions hold as were assumed in the main text for the situation with a single group. Hence there would exist a market equilibrium for each of the groups separately if the housing stock were smaller than the number of households in this group. We refer to the situation just defined as the assignment model with multiple groups.

To provide some intuition for the results that follow, it is helpful to recall the familiar graphical analysis associated with Rosen (1974) in which households that differ in tastes or incomes maximize utility while taking a hedonic price function, referring to a differentiated good with a single characteristic, as given. Different households in general choose different positions, but it is possible that households with different tastes and incomes choose the same position on the hedonic price function. The necessary condition for optimal choice is that the marginal will-ingness to pay for the characteristic equals its marginal price, while the sufficient condition is that the household's indifference curve is more convex than the budget constraint. Market equilibrium requires that all varieties present in the market will be chosen by some households and that for each variety there is equilibrium between supply and demand.

A.5.2 When multiple groups demand the same houses

In this subsection we describe how we still trace out the housing price function when households from two or more groups may express demand for the same houses. That is, for a given housing quality and at a given price, households belonging to two or more groups have the same marginal willingness to pay for housing. The assignment of households to housing is now described by a generalization of (6):

$$g(q)dq = \sum_{i} \delta^{i}(q)f^{i}(y)dy^{i}$$
(A16)

In this equation $\delta^i(q)$ is a 0-1 variable indicating that households of type *i* chose houses with quality *q*. For $\delta^i(q)$ to be equal to 1, the following conditions have to be satisfied (i) the house offers a utility that exceeds that of the outside option, (ii) the first-order condition is satisfied for an income $y \in [y^{i,min}, y^{i,max}]$ and (iii) the second-order condition is also satisfied. The equation states that all available houses of quality *q* will be occupied by households that reach their optimal housing demand there.

This equation does not yet make clear how the houses are distributed over the households of the various groups. To address this issue we consider the change in the first-order condition (8) that occurs if we move to a slightly higher housing quality. If households of group *i* continue to express demand at this higher quality,

the equality between marginal willingness to pay for housing and the marginal price of housing must be maintained. This requires: $d\pi/dq = dM^i/dq$ for all groups that continue to express demand. Elaboration of this condition gives:

$$\frac{d\pi}{dq} = \left[\frac{\partial M^{i}}{\partial q} - \pi \frac{\partial M^{i}}{\partial (y-p)}\right] + \frac{\partial M^{i}}{\partial (y-p)} \frac{dy^{i}}{dq}$$
(A17)

The expression in square brackets on the right-hand side is closely related to the second-order condition as discussed in A.1.2, which requires:

$$\frac{d\pi}{dq} \ge \left[\frac{\partial M^{i}}{\partial q} - \pi \frac{\partial M^{i}}{\partial (y-p)}\right]$$
(A18)

If the first-order condition is satisfied, but the second-order condition fails, households express no demand. If the first as well as the second order condition is satisfied, equation (A17) implies that $dy^i/dq > 0$. Since the change in the marginal price of housing must be the same for all households the changes in income associated with the movement to a higher housing quality for the different groups must be aligned to each other. To see how this works, we solve (A17) for dy^i/dq and substitute the result into (A16). This gives us an expression for the change in the marginal price of housing that is compatible with equilibrium:

$$\frac{d\pi}{dq} = \frac{\sum_{i} \delta^{i}(q) f^{i}(y^{i}) (\frac{\partial M^{i}}{\partial (y^{i}-p)})^{-1} [\frac{\partial M^{i}}{\partial q} - \pi \frac{\partial M^{i}}{\partial (y^{i}-p)}] + g(q)}{\sum_{i} \delta^{i}(q) f^{i}(y^{i}) (\frac{\partial M^{i}}{\partial (y^{i}-p)})^{-1}}$$
(A19)

The change in the marginal price implied by (A19) can now be used to find the change in income for each group of households that expresses demand.

We can thus still trace out the demand function when households with different tastes express demand for housing of the same quality. Households of a particular group that express demand for housing of a particular quality will only stop doing so for higher qualities if the maximum income is reached or if the second-order condition is no longer satisfied. In such a case the households can always reach a higher utility by putting their demand on hold. Their utility will increase if the housing price function is traced out further.

A.5.3 An allocation procedure

In this subsection we describe an allocation procedure that starts from an arbitrary price $p(q^{min}) = p^s$ for the housing of minimum quality. This procedure does in general not correspond to an equilibrium, but it will nevertheless be useful for demonstrating the existence of such an equilibrium in the subsections that follow.

We start by determining for each group of households the lowest income at which households are willing to occupy the housing of minimum quality at the given price p^s . This can be a household that is indifferent between the outside option and this housing quality, but it can also be the lowest income level y^{min} of a

group, that strictly prefer the housing of minimum quality to the outside option. It is also possible that even for the maximum income of a group, households prefer the outside option to the housing of minimum quality.

In what follows we discuss the situation in which there is at least one group for which a household indifferent between the outside option and the housing of minimum quality. If there is only one such group, we start the allocation procedure as in the case with homogeneous households. If the are two or more groups for which some households are indifferent between housing of minimum quality and the outside alternative, we select the one with the lowest value of the marginal willingness to pay M at the housing of minimum quality and start the allocation process with this group. ²⁵ Note that the households in other groups that are indifferent between the housing of minimum quality and the outside alternative have a willingness to pay for housing of the minimum quality that exceeds the marginal price. Their utility will increase if they put their demand on hold and follow the tracing out of the housing price function. ²⁶

If we trace out the house price function in this way, it may happen that the marginal price becomes equal to the marginal willingness to pay of a household belonging to a group that did not yet express demand for housing. Then for this group the indicator variable $delta^i(q)$ switches from 0 to 1 and that group will take part in the further tracing out of the housing price function. It may also happen, if at least two groups are involved in that process, that one of them stops expressing demand because the second-order condition is no longer fulfilled. It may be again taken on board for higher housing qualities.²⁷

The allocation procedure continues until either all houses are occupied or all households are allocated. In the former case there is excess demand equal to the number of unallocated households, in the former there is excess supply, equal to the number of vacant houses.

²⁷It is possible that all groups involved in the allocation process (that all groups for which $delta^i(q) = 1$) stop expressing demand because their maximum income is reached. Demand for the housing that is still vacant can then only result from households that were not involved in the last phase of the allocation procedure as realized thus far. The allocation procedure should then be re-started by selecting the group with the lowest marginal willingness to pay for housing at the price-quality combination at which the process stopped, and continued in the same way. In this exceptional situation their will be a kink in the price function.

²⁵If there are two or more groups with the lowest marginal willingness to pay, start tracing out the price function with all of them simultaneously as discussed above.

²⁶Note that if we would start tracing out the price function with a group that had a higher willingness to pay for the housing of minimum quality, the group with the lowest marginal willingness to pay would be excluded from housing. However, this is incompatible with equilibrium, because there are households of this group with higher incomes who strictly prefer housing of minimum quality to the outside alternative. They will drive up the price of the housing of minimum quality.

A.5.4 Existence of a market equilibrium

We start with two lemmas.

Lemma 6. Lemma. If we start the allocation process from a higher value of p^s , the price housing will be higher for all qualities. Moreover, the price of housing of any quality is a continuous function of the starting price p^s .

Proof. Suppose that the first statement is false. Then there must be a housing quality for which the price is lower. The two price functions must have crossed at some housing quality, say $q^{@}$. There the slope of the price function starting from the higher price must have been lower than that of the price function starting from the lower price. That slope is equal to the marginal willingness to pay of the households allocated to that quality. We know that for at least one group the income of the households allocated to quality $q^{@}$ must be higher if the price function starting from the higher p^s is relevant. However, for the same price $p(q^{@})$ the marginal willingness to pay for housing must be higher for the higher income households. This implies a contradiction with the flatter slope of the price function starting from the higher p^s . Suppose that the second statement is false. Then there must be at least one discontinuity in the price function: for a certain quality, say $q^{@}$ and a starting price $p^{s@}$ the price function jumps upwards by a strictly positive amount, no matter how small the increase in the starting price is. However, this implies an immediate contradiction with the fact that the increase $dp(q^{@})$ is equal to the marginal willingness to pay for housing, which is a continuous function of the income of the households occupying the houses of that quality, while this income is a continuous function of p^s . \square

Now define the critical income level $y^{(i,c)}(p^s)$ of group i, i = 1...n as the lowest income for which this group expresses demand for housing. All households with income higher than the critical level will consume housing. Existence of a market equilibrium thus requires that the sum of all households with at least the critical income is equal to the number of houses.

Lemma 7. Lemma. The critical incomes $y^{(i,c)}(p^s)$ are continuous increasing functions of the price p^s of minimum quality housing.

Proof. At the critical income, a household is indifferent between the outside option and the preferred housing quality. For a higher price of that housing the household prefers the outside option. All lower incomes already did so, and will not change. Hence the critical income must increase. Continuity of the critical income in the starting price p^s is obvious for the groups that consume housing of minimum quality. So consider a group for which the critical income chooses a housing quality larger than q^{min} , say $q^{@}$. Because of the higher price $p(q^{min})$ the housing price function shifts upwards. The initial critical income will no longer result in indifference between housing of quality $q^{@}$ and the outside alternative. However, higher income will do and since the price function is continuous in the starting price, the same will be true for this higher income. The indifference curve corresponding to this higher income may not be tangent to the new price function, at the same quality. However, the quality at which the tangency will hold must be close to $q^{@}$ and approach that value if the change in the starting price gets arbitrarily small.

To establish existence of a market equilibrium, we must demonstrate the existence of a price p^s for housing of minimum quality that implies values of the critical incomes of all groups for which the total number of households with incomes at least equal to the critical income of their group is equal to the housing stock.

Assumption A1. If the price of the outside option and the price of minimum quality housing are equal, the sum over all groups of the number of households preferring housing of minimum quality exceeds the housing stock.

Assumption A2. If the price of housing of minimum quality p^s is sufficiently high, all households prefer the outside option.

Proposition 3. *If Assumptions A1 and A2 hold, there exist a unique market equilibrium in the assignment model with multiple groups.*

Proof. The critical incomes are continuously increasing functions of the price p^s that are equal to $y^{i,min}$ when $p^s = p^*$ and to $y^{i,max}$ if that price is sufficiently high. Let y^h be the total number of households (in all groups) with income at least equal to the critical value of their group. It follows that y^h is a continuously decreasing function of p^s that is equal to the total number of households in all groups for $p^s = p^*$ and 0 for p^s sufficiently high. Somewhere between these two prices must be a value $p^s = p^{eq}$ for which y^h equals the total number of houses.

A.5.5 A special case

It may be noted that two different aspects of heterogeneity are important in the analysis offered above. One is that households of different groups may realize the same marginal utility of housing at different income levels, the other is that the convexity of the indifference curves (the first derivative of the marginal willingness to pay for housing) may be different even though households belonging to different groups have the same marginal willingness to pay for housing. It is possible that the first aspect is relevant, while the second is not. That situation occurs if the indifference curves of households belonging to different groups are parallel to each other, that is if $u^i(c,q) = u(\theta^i + c,q)$. Two households belonging to different groups choose exactly the same location on the hedonic price function if the difference between their incomes is equal to minus the difference between their θ s.²⁸

²⁸To see this, consider two groups of households i = 1. Their budget constraints are $y^i = c + p(q)$. Substitution in the utility functions gives: $u^1 = u(\theta^1 + y^1 - p(q))$ and $u^2 = u(\theta^2 + y^2 - p(q))$.

When a particular value of housing consumption q is chosen by households of group i with income y, that same value of housing consumption will also be chosen by households of group j with income $y - \theta^i + \theta^j$. Note that the share of households of the two groups may still vary over housing qualities, depending on their income distributions.

To illustrate consider a population where all households have a linear demand function for housing, with a group-specific intercept:

$$q = \alpha^i + \beta \pi + \gamma y, \tag{A20}$$

differences in the constant term α^i have a similar impact on the demand for housing as differences in income. Hence in equilibrium households with different income levels will occupy the same houses, as we see in reality. The demand function refers to a linear budget constraint, whereas in the assignment model the budget constraint may be nonlinear. However, with a nonlinear budget constraint it is still the case that housing consumption is always identical for households belonging to different groups, say 1 and 2, if $y^1 - y^2 = -(\alpha^1 - \alpha^2)/\gamma$.²⁹

Other consumption is equal for both households if $\theta^1 - \theta^2 = (\alpha^1 - \alpha^2)/\gamma$. Although the direct utility function that generates the linear demand curve is unknown, we can therefore be sure that we are in the situation described above, where a group-specific constant is added to other consumption in a utility function that is otherwise identical for all groups.

A.5.6 Borrowing constraints

What happens if we have multiple groups of households and borrowing constraints? Even if the borrowing constraints are homogeneous in the sense that they are always given by the same function of income, the taste differences imply that their impact will differ over the groups. If houses of all qualities are occupied by households from different groups and some of them are constrained and other not, the constrained households have to reduce their housing consumption, which implies that the unconstrained households may shift to higher quality levels.

Lemma 8. Lemma 5 continues to hold with heterogeneous tastes.

The utility functions are therefore identical when $y^1 - y^2 = -(\theta^1 - \theta^2)$.

²⁹To see this, note that with a nonlinear budget constraint y = c + p(q) demand can be written as $q = \alpha^1 + \beta \pi + \gamma (y - p(q) + piq)$ where the expression in brackets denotes the 'virtual income.' This equation is based on a linearization of the nonlinear budget constraint at the optimal consumption of housing and other goods. It is easy to verify that for two groups i = 1, 2 housing consumption is still equal if $y^1 - y^2 = -(\alpha^1 - \alpha^2)/\gamma$.

B Empirical Tables and Figures

B.1 Tables

| Variable | Obs | Mean | SD | p5 | p95 |
|--------------|--------|-----------|-----------|---------|---------|
| MPTI | 49,166 | 0.2173 | 0.0545 | 0.121 | 0.297 |
| Sale price | 49,166 | 207,453.7 | 91,566.52 | 112,000 | 358,000 |
| Loan | 49,166 | 213,635.3 | 91,823.87 | 105,796 | 358,000 |
| Income | 49,166 | 66,915.02 | 33,083.02 | 33,725 | 117,345 |
| Constr. year | 49,166 | 1961.744 | 56.54873 | 1,906 | 2,003 |
| Floor area | 49,093 | 108.8317 | 455.5371 | 56 | 163 |
| Туре | 49,166 | 0.7040 | 0.4565 | 0 | 1 |

Table A1: Descriptive of the baseline properties

Notes: This is the house characteristic description of the baseline sample after dropping always treated sample and non-owner-occupied properties.

| | (1) | (2) | (3) | (4) | |
|-----------------|----------------------------------|--------------|--------------|--------------|--|
| | (1) | (2) | (3) | (+) | |
| | All co | ntrol . | Not yet | treated | |
| | Dependent Variable: Private rent | | | | |
| T 7' 1 . | 0 26 4444 | 0 = 41 *** | 0 - | | |
| Violate | 0.764^{***} | 0.541*** | 0.747*** | 0.528** | |
| | (0.209) | (0.209) | (0.210) | (0.210) | |
| Observations | 336,025 | 336,025 | 336,065 | 336,065 | |
| type*year trend | | \checkmark | | \checkmark | |
| Property F.E. | \checkmark | \checkmark | \checkmark | \checkmark | |
| Year F.E. | \checkmark | \checkmark | \checkmark | \checkmark | |

Table A2: Robustness check: alternative estimators

Notes: *** p < 0.01, ** p < 0.05, * p < 0.1. Standard errors are clustered at Pc4*year level. The results are got from the estimation following Callaway and Sant'Anna (2021).



Figure A4: Weight distribution of each treatment Notes: This is the weight associated with each treatment. The vertical line is the average weight. This is calculated through the command *twowayfeweights* in STATA 16.0.



Figure A5: Event study using CSDID

Note: This figure is a coefficient plot and 95% confidence interval of the estimation results following Callaway and Sant'Anna (2021). Standard errors are clustered at house level.



Figure A6: Decomposition of treatment effects by group and year Note: This is decomposition of treatment effects by group and year following Callaway and Sant'Anna (2021). The graph above is the coefficient plot of treatment effects by treatment group, and the graph below is the coefficient plot of treatment effects by calendar year.



Figure A7: Treatment effects by group

Note: We randomly generated a fictitious treatment group within the original sample for 500 times. This is a distribution of treatment effects (and corresponding p-value) with 500 fictitious treatment groups. The vertical red line indicates the actual treatment effects:0.43.