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Abstract

Politicians respond to incentives when they decide how to allocate their campaigning time and effort. The literature suggests electoral rules impact politicians' incentives. We argue that the candidate selection process is an equally important source of incentives. We develop a two-stage model in which parties select candidates before the election. Elections are under first past the post (FPTP) or closed list proportional representation (PR). Selection is competitive or non-competitive. When selection is not competitive, effort is higher under FPTP. With competitive selection, effort is higher under PR as, under PR, competition motivates candidates to exert effort to be selected (as under FPTP) and to be ranked higher on the list. The results point to a causal relationship between electoral rules and how parties porganize. They suggest empirical studies comparing electoral rules should consider how parties organize.

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1 Introduction

Politicians respond to incentives when they decide how to allocate their electoral and campaigning time and effort. The literature has singled out the electoral rule as the primary source of incentives for politicians.[1] If elections are key, the selection of candidates by parties also plays a major role. Indeed, candidates adapt their behavior to the specificities of the selection process.

In most democracies, political parties control this important stage. To quote Gallagher and Marsh (1988), parties are the "gate keepers of the secret garden of politics". Norris (2006) also points to the importance of parties in candidate selection, when she argues that "the process of recruitment [...] is widely regarded as one of the most important residual functions of parties [...]." The literature (see for instance Poguntke et al. 2016) has documented a lot of variation in how parties select candidates across democracies, with a particular focus on the level of democratization, centralization and inclusiveness.[2] As economists, we believe that the degree of competition of the selection process plays a central role. An open, fair and competitive system provides candidates with strong incentives to perform, while candidates are unlikely to offer their best performance when a handful of elite party members control the process.

In this paper, we contribute to the comparative politics literature by analyzing how electoral rules and candidate selection processes interact to influence the behavior of politicians. We present a two-stage model in which parties first select candidates and then the legislative election takes place. Candidates care not only about being elected to the legislature, but also about their party winning a majority of seats, as the control of the executive allows the party to implement its program. Candidates are thus both opportunistic and ideological. Candidates exert costly effort to improve electoral appeal. Effort represents the time and energy spent by candidates in all activities they undertake to improve their party's electoral

[1]See for example Persson and Tabellini (2000) and Persson, Tabellini and Trebbi (2003). The literature that builds on these papers has identified other important characteristics of the electoral system such as the ballot structure and district magnitude.

[2]See for example the Political Party Database introduced in Poguntke et al. (2016) and Scarrow et al. (2017), or the data sets in Kernell (2015), Lundel (2004) and Shomer (2014).

success. Effort should thus be interpreted broadly as any costly action to mobilize voters who are on the fence about going to the voting booth or to persuade undecided citizens to vote for a specific party. Such campaigning efforts may very well be welfare decreasing for the population at large, but they are instrumental to the electoral strategy and goals of parties. Effort may include, for example, initial fund raising, maintaining or initializing contacts with key pressure groups, helping one's party's craft and develop its electoral platform, participating in debates at different levels of society, recruiting key policy advisors. etc. The effort of a candidate influences both his selection and election prospects.

We focus on two electoral rules: legislative elections under British-style first past the post in single member districts (FPTP in what follows) and Israeli-style proportional representation with closed lists in a single, nationwide district (PR in what follows). These two electoral systems generate different incentives. Under FPTP, voters can compare candidates individually. Elections thus provide strong incentives to candidates. To the contrary, under PR, parties run as a team and voters can't select their favorite candidate. This leads to free-riding and weaker incentives. We aggregate all differences in candidate selection into a single dimension, its degree of competition. We consider two types of selection processes: a competitive and a non-competitive one. Under the competitive process, selection depends on the effort of candidates, while in the non-competitive process, candidates are selected irrespective of their effort choice.

We first compare candidates' efforts when selection is not competitive. We confirm, that, in line with the previous literature, PR provides weaker incentives than FPTP. To quote Persson et al. (2003, p. 961), under PR, "politicians' incentives are [...] diluted by two effects. First, a free-rider problem arises among politicians on the same list. Under proportional representation, the number of seats depends on the votes collected by the whole list, rather than the votes for each individual candidate. Second, [as] the list is closed and voters cannot choose their preferred candidate, an individual's chance of re-election depends on his rank on the list, not his individual performance". Candidates at the top and bottom of the list exert little effort as the marginal benefit of effort on their probability of winning a seat is small. Candidates in the middle of the list face strong incentives. However, their high efforts do not compensate for the low efforts of the other list members. Aggregate effort is higher under FPTP than PR.

We then compare effort when selection is competitive. We find that party's aggregate effort is higher under PR than under FPTP. The key reason for this reversal is that under PR, candidates compete not just to be included on the party list, but to also get the highest possible spot on the list. Accounting for incentives generated by selection at the party level thus overturns the result on the impact of electoral rules on aggregate effort. To wrap up, our results suggest that the selection stage generates stronger incentives than the election stage.

To compare incentives (and effort), we introduce a unified model of elections as imperfectly discriminating contests (see Tullock 1980). Under FPTP, a candidate wins his district with a probability proportional to the ratio of his effort over the sum of the efforts of all candidates running in that district. When selection is non-competitive, candidate effort is irrelevant for selection, for example because it is based purely on seniority. Candidates know whether or not they are selected before they exert effort. When selection is competitive, it takes the form of a simple Tullock contest between potential candidates. Modelling elections under PR is more difficult as parties are teams, and not unitary actors *à la* Downs (1957). We build on Crutzen *et al.* (2020). A party wins a legislative seat according to the standard Tullock contest success function based on the party aggregate effort, which is the sum of the efforts of the candidates on the list. The probability that a party wins x seats then follows a simple binomial distribution with the number of seats in the legislature and the Tullock ratio as parameters. When selection is non-competitive, candidates know their position on the list when they choose their effort. When selection is competitive, a pool of candidates compete for the highest possible ranks on the party list. This stage is modelled as a contest for heterogeneous prizes as in Clark and Riis (1996).

Our findings generate new empirical implications. First, our results hint at a causal relationship between electoral rules and the way political parties organize. The general election provides strong incentives to politicians under FPTP while these incentives are less powerful under PR. As a consequence the need for parties to control the rules of the selection process to generate appropriate incentives is lower under FPTP than under PR. These findings thus predict that party elites want to retain more organizational control under

PR than under FPTP, all else equal.

Second, any empirical comparative politics analysis of candidate choices and actions before and during elections should include variables about the way parties organize their candidate selection. Indeed, our results suggests that different electoral rules are associated with different candidate choices as a function of not only the electoral rule, but also – and perhaps even mainly – of the way parties organize. If electoral rules incentivize parties to prefer some candidate selection processes over others and these in turn affect candidate incentives, not controlling for the candidate selection stage yields biased and inconsistent estimates of the relationship between candidate choices and electoral rules. Such an omitted variable issue could explain for example why the empirical comparative politics literature does not reach any clear-cut conclusion about the impact of electoral rules on perceived political corruption; see for example Persson et al. (2003).

The rest of the paper is structured follows. The next section reviews the related literature. Section 3 presents the basic model and the next Section solves it. Section 5 derives the main comparative politics result. Section 6 discusses the main assumptions of he model ad presents some extensions. The last section concludes. An appendix contains all proofs.

2 Related Literature

In our model, the main function of a party is to select candidates for election. We extend the existing literature by considering candidate selection under closed list proportional representation and comparing outcomes under that electoral rule to those under British-style first past the post. Mattozzi and Merlo (2015), Galasso and Nannicini (2017), Buisseret and Prato (2022) and Crutzen et al. (2020) consider a similar question.

Closest to our work are Mattozzi and Merlo (2015) and Crutzen et al. (2020). Mattozzi and Merlo develop a two stage comparative politics game in which parties first select one candidate who then runs in the election. Their model thus speaks to presidential elections under different electoral rules. Our model, with its explicit focus on the role of party lists under proportional representation, is most relevant for the analysis of legislative elections. Also, in their model, candidate quality – modelled as individually different costs of effort provision – is the key driver of all decisions and both game stages are modelled as all-pay auctions. We rely on the framework of imperfectly discriminating contests as in Tullock (1980) and focus on incentives only, as all candidates are identical. They show that, under the assumption that rents from winning office are more evenly spread across parties under proportional representation than under first past the post, parties are less likely to select their top quality candidate under proportional representation. Their main prediction is therefore that elections under proportional representation should be associated with worse candidates compared to elections run under a winner take all system. We offer predictions about incentives to exert costly effort in legislative elections across different electoral rules.

Crutzen et al. (2020), from whom we borrow the binomial Tullock distribution to map party votes into seats under PR, develop a model of a team contest for multiple, indivisible prizes. They apply the model to elections under proportional representation with both open and closed lists and derive conditions under which closed lists generate stronger incentives than open lists. We extend their analysis to first past the post and to two different candidate selection procedures.

Galasso and Nannicini (2017) have no incentive effects in their analysis, as candidates can be either loyal to the party but relatively incompetent, or competent but relatively independent from the party. They assume that under proportional representation, the share of swing voters in the entire electorate is a key determinant of who gets elected. Under first past the post, the competitiveness of the election, that is, the share of competitive districts is key. They show that there is a U-shaped relationship between the competitiveness of the majoritarian system and its capacty to elect a better pool of politicians than under proportional representation. We focus on the effects of the interaction between electoral rules and candidate selection processses on the campaign effort choices of candidates.

Buisseret and Prato (2022) develop a model under list proportional representation. Parties observe their politicians' choices in the legislature and then have to decide on how to rank these on their party list before the following election. The authors study how the degree of flexibility of the list (they let the list go from fully closed to fully open) and district magnitude impact politicians's incentives to favour the agenda of their party versus that of their local constituency. We consider different electoral rules and focus on campaigning effort as opposed to politicians' choices within the legislature.

Other contributions have focused on intraparty incentive issues, but under one electoral rule only. Galasso and Nanniccini (2015), Besley et al. (2017), Dal Bo et al. (2017), Buisseret et al. (forthcoming), Fiva et al. (2021) Cox et al. (2022) and Crutzen et al. (2022) focus on PR. Galasso and Nanniccini (2015) suggest that parties may exploit party lists to field loyal but possibly less competent candidates in its safe list spots. Besley et al. (2017) analyze the consequences of introducing genrder quotas on the electoral fortunes of male candidates. Dal Bo et al. (2017) analyze how representative are the characteristics of elected representatives relative to the population at large. Buisseret et al. (forthcoming) show that it is especially parties which expect to end up in government that face the strongest incentives to rank candidates in decreasing order of competence. Fiva et al. (2021) show that, in Norway at least, parties balance geographically their electoral lists. Cox et al. (2022) show that, using Norwegian electoral data, the list rank of candidates matters for the geographic level at which they choose to campaign, with only the candidates at the top of the list campaigning nationally. Finally, Crutzen et al. (2022) develop a model to analyze parties' incentives to rank their candidates in decreasing order of competence when parties wish to maximize their electoral success and candidates's efforts are pivotal for their party's electoral fortunes.

Carrillo and Mariotti (2001), Caillaud and Tirole (2002), Hirano, Snyder and Ting (2009), Castanheira et al. (2010) and Snyder and Ting (2011) focus on intraparty incentive issues under first past the post (or with one leading candidate only, as in US-style Presidential regimes). Carrillo and Mariotti (2001) shows that parties have an incentive to replace incumbents less often than what is socially desirable even when they care about their candidate's competence, because of the cost asssociated to possibly losing the incumbecy after an incumbent's replacement. Caillaud and Tirole (2002) is arguably the first explicit model of party organization as a source of incentives for individual politicians.[3] They analyze how different candidate selection processes impact candiates' incentives to invest in campaign effort. Castanheira, Crutzen and Sahuguet (2010) extend the analysis of Caillaud and Tirole (2002) to allow for different political motives and general equilibrium effects. Hirano et al. (2009) analyze how the competitiveness of the candidate selection process impacts

[3]Alesina and Spear (1988) does not focus on how parties organize.

the geographic distribution of spending promises. They find that primaries, a competitive candidate selection procedure, increases the polarization of promises. Their findings about the competitiveness of candidate selection under FPTP thus complements ours.[4] Snyder and Ting (2011) argue that parties will resort to a competitive candidate selection process only in districts in which they enjoy a large electoral avantage, as the competitive selection process may reveal that candidates are incompetent. They offer supporting evidence from US primaries. Relatedly, Galasso and Nanniccini (2011) argue that parties file more competent candidates in more competitive districts and report supporting evidence from Italy.

A growing literature, in political science, analyses different characteristics of candidate selection processes, such as their degree of centralization or inclusiveness. Important contributions include, besides those cited above, Bille (2001), Hazan and Pennings (2001), Hazan and Rahat (2006, 2010), Katz and Mair (1994), Lundell (2004), Norris (1997, 2006), Rahat and Hazan (2001), Shomer (2014, 2017) and Cirone, Cox and Fiva (2020). We contribute to this literature as we suggest that the degree of competition in the candidate selection process matters to understand political outcomes.

We also contribute to the theoretical comparative politics literature that studies candidates' incentives under different electoral rules. The literature is vast but neglects the role of candidate selection. Important contributions on incentive aspects of elections include Bawn and Thies (2003), Lizzeri and Persico (2001, 2005), Milesi-Ferretti, Perotti and Rostagno (2002), Morelli (2004), Myersson (1993a, 1993b and 1999), Persson, Roland and Tabellini (2000) and Persson and Tabellini (1999, 2000). These theories do not all point in the same direction. Myerson (1993a) argues that incentives to offer targeted transfers to small subgroups of the electorate are similar across FPTP and PR if the number of parties is similar across electoral rules, but those incentives get worse under PR if the number of parties is larger under that rule. Lizzeri and Persico (2001) build on Myerson (1993a) to suggest that candidates' incentives are more aligned with voter preferences under PR in large electoral districts: politicians are less tempted to divert resources from the budget of a nationwide public good, to target inefficiently their electoral promises to subgroups of the population.

[4]How the competitiveness of candidate selection impacts incentives to distribute geographically electoral promises under PR is still an open question.

In contrast, Persson and Tabellini (1999) hold the opposite view: incentives are stronger in elections under FPTP because voters can more effectively discipline politicians and make them accountable for their actions. Our paper develops a model in which politicians exert effort and is thus closer in spirit to Persson and Tabellini (1999). Our findings suggest that considering the way parties organize is important to understand the incentives of politicians under various electoral rules.

3 The Model

Consider a society with a mass K of voters, K odd and equal to 3 or more, and an election for K seats in parliament. We analyze two electoral rules:[5] US- or British-style first past the post in K identical single member districts (FPTP hereafter) and Israeli-style proportional representation with closed party lists in a single nationwide district (PR hereafter).

Candidates belong to one of two parties L and R and choose effort e to maximize their expected utility. The cost of effort is quadratic: $c(e) = \frac{1}{2}e^2$. Candidates get a payoff $V \geq 0$ when they win a seat in the legislature. Candidates also get a payoff $M \geq 0$, when their party wins control of government. This second component captures the utility associated with the party of the candidate gaining control of the executive office, which allows it to implement its favored policies. M is thus a proxy for the ideological color of candidates.

Under FPTP, an election takes place in every district between two party candidates. The result of the election depends on their efforts, e^L and e^R. Under PR, parties compete in the election via a list of K candidates. Voters can only cast their ballot for either list, they cannot vote for individual candidates. The aggregate effort of a party is defined as the sum of the efforts of all candidates on its list: $E^P = \sum_{m=1}^{K} e_m^P$. where e_m^P is the effort of the candidate of party P in mth position on the list. Parties' electoral success is a function of these aggregate efforts.

Within parties, we consider two candidate selection processes. When selection is non-competitive, the party chooses the candidates for the election based on candidate character-

[5] Our focus on these two rules is dictated by a desire to follow the extant literature on the incentive effects of different electoral rules.

istics independent of effort.[6] Under FPTP, the party selects one candidate for each district. Under PR, the party chooses K candidates and their order on the list. When selection is competitive, several candidates compete for selection on the basis of their effort. Under FPTP, in each district, $n \geq 1$ candidates compete in a primary election for the right to run in the general election. Under PR, $N \geq K$ candidates compete nationwide for the right to be on the party list and for their position on the list. We now describe each scenario in detail.

3.1 First Past the Post

3.1.1 Non-competitive Selection

In each district, the two party candidates exert effort e^L and e^R. Party $L's$ candidate wins the election with probability:[7]

$$p^L\left(e^L, e^R\right) = \lambda\left(\frac{e^L}{e^L + e^R}\right) + \frac{1-\lambda}{2}. \tag{1}$$

Parameter $\lambda \in [0,1]$ represents the importance of effort in the result of the election. When $\lambda = 1$, we have the lottery contest function. When $\lambda = 0$, the election is random: the result does not depend on candidate effort.

Party $L's$ candidate chooses effort e^L to maximize:

$$p^L\left[V + MP^L\left(\tfrac{K-1}{2}, K-1\right)\right] + M\left[\sum_{k=\frac{K+1}{2}}^{K-1} P^L\left(k, K-1\right)\right] - \tfrac{1}{2}\left(e^L\right)^2, \tag{2}$$

where $P^L\left(k, K-1\right) = C_k^{K-1}\left(p^L\right)^k\left(1-p^L\right)^{K-k-1}$ denotes the probability that party L wins k of the other $K-1$ seats

Candidates choose their effort considering the prospect of both getting elected and their party winning a majority of seats. The candidate's election is pivotal for the party winning a

[6]Remark that selection may not be competitive also because the party leadership, for a variety of reasons, may not be able to commit to a competitive, fair and unbiased selection procedure.

[7]We use the lottery function for simplicity. All results remain if we were to use a generalized Tullock contest, of the form $\frac{\left(E^L\right)^\gamma}{\left(E^L\right)^\gamma + \left(E^R\right)^\gamma}$ with $\gamma < 1$. Note that small values γ makes the objective function more concave and thus second order conditions are more easily satisfied.

majority of seats when his party wins in exactly $\frac{K-1}{2}$ of the other $K-1$ districts. No matter the result of his election, the party of the candidate also wins a majority of seats when the party wins at least $\frac{K+1}{2}$ districts among the $K-1$ other districts.

We have:[8]

Proposition 1 *Under FPTP, when selection is non-competitive, candidates exert effort equal to* $\sqrt{\frac{\lambda\left(V+\bar{M}\right)}{4}}$, *with* $\bar{M} = MC^{K-1}_{\frac{K-1}{2}}\left(\frac{1}{2}\right)^{K-1}$.

As the maximization problem is symmetric both within and between parties, all candidates exert the same effort.

3.1.2 Competitive Selection

Before the election, parties select candidates on the basis of their efforts, for example in a primary election. We model the primary as a Tullock contest between $n > 1$ party candidates. Candidate i of party L wins the primary and represents his party in his district's general election with probability:

$$Q^{iL} = \frac{e^{iL}}{e^{iL} + \sum_{k \neq i} e^{kL}}, \tag{3}$$

where e^{kL} denotes the effort of candidate k of party L in the primary.

Candidate i chooses effort e^{iL} to maximize:

$$\begin{aligned} &Q^{iL}p^{L}\left[V + MP^{L}\left(\tfrac{K-1}{2}, K-1\right)\right] + \left(1 - Q^{iL}\right)p^{L}_{-i}MP^{L}\left(\tfrac{K-1}{2}, K-1\right) \\ &+M\left[\sum_{j=\frac{K+1}{2}}^{K-1} P^{L}\left(j, K-1\right)\right] - \frac{\left(e^{iL}\right)^{2}}{2}. \end{aligned} \tag{4}$$

The second term represents the payoff of winning a majority when candidate i loses the primary but the winner of that primary still ends up winning the district; we denote the probability of this event with p^{L}_{-i}.

In the symmetric equilibrium, all candidates exert the same effort. We have:

Proposition 2 *Under plurality rule, when candidate selection is competitive among* n *candidates in each district, candidates exert effort equal to:*

$$e^{*} = \sqrt{V\left(\frac{n-1}{2n^{2}} + \frac{\lambda}{4n}\right) + \frac{\lambda}{4n}\bar{M}}.$$

[8] All proofs are in the Appendix.

If $\lambda < \frac{V}{\bar{M}+V}$ *, letting two candidates compete for selection* ($n = 2$) *maximizes effort.*
If $\lambda > \frac{V}{\bar{M}+V}$*, making selection non-competitive* ($n = 1$) *maximizes effort.*

The optimal number of candidates trades off two effects: a dilution and a competition effect. The dilution effect quickly counterbalances the competition effect as the number of candidates increases. Given this, parties severely restrict competition and do not let more than 2 candidates compete in the primary.

The effect of parameters λ, $\bar{M}$ and V on the optimal number of candidates deserves more comments. Parameter λ represents the relative elasticity of the general election result to candidates' effort with that of the primary election. When λ is small, the general election generates weak incentives and competitive selection becomes valuable, it complements the weak incentives generated by the election. The ratio $\frac{V}{\bar{M}+V}$ represents the relative importance of individual and collective objectives for candidates. The more opportunistic are candidates – opportunistic behavior is stronger the larger is V compared to $\bar{M}$ – the more likely it is that λ is smaller than $\frac{V}{\bar{M}+V}$. If $\lambda < \frac{V}{\bar{M}+V}$, having a competitive selection stage provides better incentives to candidates. The more 'ideologically' motivated are candidates – the larger is $\bar{M}$ compared to V – the more likely that λ is larger than $\frac{V}{\bar{M}+V}$. If $\lambda \geq \frac{V}{\bar{M}+V}$ intraparty incentives are not useful; the dilution effect dominates as candidates exert effort to see their party win a majority rather than win a seat themselves.

3.2 Proportional Representation

Under PR, the result of the election depends on the two parties' aggregate efforts. The aggregate effort is the sum of efforts of all candidates on the list.[9] Specifically, the probability that party L wins a seat in the legislature is equal to $p^L = \lambda \frac{E^L}{E^L+E^R} + \frac{1-\lambda}{2}$.[10] We assume that parties' seat shares in the legislature follow a binomial distribution. This Binomial-Tullock distribution is a natural extension of the Tullock contest function to the case of multiple

[9]See Crutzen, Flamand and Sahuguet (2020) for a production function with complementarities between candidates' efforts. Our results remain qualitatively unaltered under this more general setup.

[10]Once again, we use the lottery function for simplicity. All results remain if we were to use a generalized Tullock contest, of the form $\frac{(E^L)^\gamma}{(E^L)^\gamma+(E^R)^\gamma}$ for $\gamma < 1$. Note that small values γ makes the objective function more concave and thus second order conditions are more easily satisfied.

prizes. The probability that party L wins k seats is then $P^L(k) = C_k^K \left(p^L\right)^k \left(1-p^L\right)^K - k$.

3.2.1 Non-Competitive Selection

When selection is non-competitive, parties first select K candidates on the basis of some effort-independent characteristics and order them on a list, and then each selected candidate exerts effort. The candidate in mth position on the list of party L chooses effort e_m^L to maximize:

$$\sum_{s=m}^{K} P^L(s)\, V + \sum_{t=\frac{K+1}{2}}^{K} P^L(t)\, M - \frac{1}{2}\left(e_m^L\right)^2 \tag{5}$$

Incentives to get elected are mediated by the position on the party list. The prospect of helping the party reach a majority always matters for incentives. This is not the case under FPTP. Thus, party-wide performance looms larger under PR. We have:

Proposition 3 *Under PR, when selection is non-competitive, candidates exert effort as a function of their position on the list:*

$$e_m^* = \frac{\sqrt{\lambda}}{2\sqrt{K\left(V+K\bar{M}\right)}}\left(mC_m^K\left(\frac{1}{2}\right)^{K-1}V+\bar{M}\right).$$

A party's aggregate effort is equal to $K\sqrt{\frac{\lambda\left(\frac{V}{K}+\bar{M}\right)}{4}}$.

Thus, under PR, when selection is not competitive, candidates at the top and bottom of a party list exert little effort, as the marginal effect of effort on their election probability is small. Candidates in the middle of the list exert highest effort. The distribution of efforts within parties is bell-shaped and symmetric about the median list position.

3.2.2 Competitive Selection

When selection is competitive, $N \geq K$ candidates compete for the K positions on the party list. As the allocation of seats follows the list, the first position on the list is more valuable than the second position and so on and so forth. We thus model the selection process as a contest between N candidates for K prizes of different values. We use the imperfectly discriminating contest model of Clark and Riis (1996).[11] Denote the effort of candidate i by e_i; the probability that i ends up in position k or higher on the party list is:

$$Q_i(k) = p_1 + \sum_{j=2}^{k} p_j \left[\prod_{s=1}^{j-1} (1 - p_s) \right], \tag{6}$$

where p_j is the probability that i ends up in position $j = 1, ..., k$ on the list and is given by the standard Tullock ratio contest success function among the candidates who have not yet been attributed a slot on the list. Thus, for the jth prize, candidate i competes with $N - j$ other party candidates and wins with probability:

$$p_j = \frac{e_i}{e_i + \sum_{k \neq i} e_k}, \#k = N - j. \tag{7}$$

We can interpret these probabilities as the result of a sequential process. Each candidate exerts effort at the beginning of the contest. Then, a Tullock contest with the contributions of the N contestants determines the winner of the first prize (the top spot the list). The winner and his contribution are then excluded. A Tullock contest with the contributions of the remaining $N - 1$ contestants determines the winner of the second prize. This process continues until all K prizes are awarded.

Candidates choose effort e_i^L to maximize:

$$\sum_{m=1}^{K} P^L(m)\, Q_i(m) V + \sum_{j=\frac{K+1}{2}}^{K} P^L(j)\, M - \frac{\left(e_i^L\right)^2}{2}. \tag{8}$$

Although K may not be the optimal number of candidates , we set $N = K$ for simplicity. We have:

Proposition 4 *Under PR, when K candidates compete within each party to be ranked as high as possible on the list, candidates exert effort equal to:*

$$e^* = \sqrt{\frac{\lambda}{4}\left(\frac{V}{K} + \bar{M}\right) + V \sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K \left[\tfrac{K-m}{K} \sum_{j=1}^{m} \tfrac{1}{K-j+1}\right]}. \tag{9}$$

[11] Clark and Riis (1996) is the standard model of contests with multiple prizes. The model is tractable and allows for closed-form solutions. The model also presents many desirable axiomatic properties (see Fu and Lu, 2012).

4 Comparative Politics

Putting together Propositions 1-4 above, we derive our central comparative politics prediction:

Theorem 5 *Incentives generated by the candidate selection process dominate those generated by electoral rules. When candidate selection is non-competitive, incentives are stronger under FPTP. When candidate selection is competitive, incentives are stronger under PR.*

The first part of the theorem is in line with previous results in the literature (see for example Persson, Tabellini and Trebbi 2003). Intuitively, under FPTP, the benefit of competitive selection is decreased by the dilution effect, while under PR, the party can reap the benefits of competitive selection without the need to increase the number of candidates and the associated dilution of incentives. Intraparty competition for the best spots on the party list turns out to provide strong incentives to all candidates. The literature seems to have overlooked this effect of party lists. Somewhat ironically, the supposedly unconditional perverse incentives of closed lists are typically at the center of critiques of PR. Our findings suggest that such perverse incentives may not materialize provided that parties allocate the positions on the list in a competitive fashion. Remark finally that, if candidates are purely ideological – if $M > V = 0$ – the two electoral rules generate the same incentives. Thus the above ranking of electoral rules when selection is non-competitive requires candidates to be opportunistic, at least to some extent.

Our findings also complement those of Myerson (1993b) and Buisseret and Prato (2020) on the effects of district magnitude. Both papers conclude that increasing the size of electoral districts leads to better outcomes for voters. Larger districts generate more *interparty* competition, which gives voters larger freedom of choice (Myerson), or allow for a better balancing of the objectives of voters and parties (Buisseret and Prato). Our model adds *intraparty* competition as a source of positive incentives associated with larger districts.

4.1 Implications

Our findings generate some empirical implications. First, our results hint at a causal relationship between electoral rules and the way political parties organize. All else equal, under

FPTP, the general election on its own incentivizes politicians substantially. Electoral incentives are less powerful in elections under PR. As a consequence, parties have less need to generate competition among candidates under FPTP. This may be one of the reasons for the well documented lack of competitiveness of many pre-electoral intraparty primaries in the US; see e.g. Ansolabehere et al. (2007) for an empirical account and Crutzen and Sahuguet (2018) for a model rationalising this lack of competitiveness of primaries. Under PR, because electoral incentives are weaker for many candidates, parties have more to gain to become and remain well-organized machines.

Political parties do take many forms. They go from the "American" model of the party as a decentralized and candidate-centered organization – Katz and Kolodny (1994) refers to the two main American parties as "empty vessels" given the dominant position individual incumbents seem to have in US politics, an aspect of US politics that has, if anything, grown in importance in the last twenty-five years – to European parties which are highly hierarchical organizations that regulate public affairs and in which, quite often, the party elite is in charge of selection; see for example Pogunkte et al. (2016) and Scarrow et al. (2017). Of course, many other forces explain such differences. Our theory predicts that the electoral rule is one of them.

Second and related to the first implication above, any empirical comparative politics analysis which aims to uncover systematic patterns in the choices and actions of individual politicians should include data capturing candidates's selection. This data should focus not only on the formal and informal characteristics of the candidate selection process but also on the party leadership's goals and strengths. Indeed, it is rational for a farsighted party leadership to choose the selection process that maximizes the party's electoral success, as we assume in this paper. Yet, the leadership could use its power to select candidates for other reasons, such as favoring friends or factions or returning favors to other members of the party. Galasso and Nannicini (2011, 2015) report empirical evidence of such issues in Italy. Our results imply that these issues also impact candidate incentives. Then, this omitted variable problem could in itself be sufficient to explain why the existing empirical comparative politics evidence does not deliver clear-cut predictions. If there is enough variation in candidate selection processes across democracies relying on the same electoral system, then, as different selection processes impact candidate incentives differently, the mapping from electoral systems to candidate choices and political outcomes more broadly becomes polluted by this variation at the selection stage of the game. Such an issue may explain why Persson et al. (2003, p.983) conclude their analysis by noting that "a comprehensive electoral reform, going from a Dutch-style electoral system [under proportional representation] with closed party lists in a single national constituency to a UK-style system with first past the post in one-member districts [would produce] a net result close to zero.". Their empirical analysis does not consider how parties organize, and this may be at the root of their lack of significant and clear-cut results.

Existing data sets focus almost exclusively on the level of democratization, centralization and inclusiveness of candidate selection; see for example the Political Party Database introduced in Poguntke et al. (2016) and Scarrow et al. (2017), or the data sets in Kernell (2015), Lundel (2004) and Shomer (2014). Democratization refers to how democratic the selection system is. Centralization refers to the extent to which candidates selection is the sole remit of the party leader (fully centralized process) or, to the contrary, is left exclusively to the party's grass. roots and local branches (full decentralization). Inclusiveness refers to the size and reach of the selectorate choosing party candidates: a fully inclusive system allows any individual, be them registered party members or not, to vote; an exclusive system endows only a few party members "usually the party elite" with this task. Our theory suggests that the degree of competition in selection is also of primary importance. Whereas intuition suggests that competition and decentralization go hand in hand, this need not be the case; see Bille (2001) and Lundel (2004), for arguments along these lines. For example, local party barons may end up controlling selection. Also, a democratic and inclusive system is no guarantee for selection to be competitive; see Hirano and Snyder (2014), Hassell (2016) and (2018), and Crutzen and Sahuguet (2018), for empirical and theoretical accounts of this issue for US primaries.

5 Discussion and Extensions

We first discuss a few key features of our model, then consider two important extensions.

A candidate's effort matters for both his selection (if candidate selection is competitive) and his election. This simplification in our model is in line with previous works – see for example Caillaud and Tirole (2002) and Castanheira et al. (2010) – and is arguably a good first order approximation of reality under PR as a party's manifesto is very much dictated by the candidates who end up on its electoral list. Also, under such an electoral system, the candidates who go through a competitive intraparty selection process are typically screened on their capacity to help their party increase its nationwide electoral appeal. Things are possibly less simple under FPTP, especially when selection is carried out through primaries, as in the US. Some scholars and political commentators argue that candidates' effort choices at the selection stage are quite different from those at the election stage. For example, Jackson et al. (2007) and Hirano et al. (2009) offer theoretical models in which candidates target their efforts to different audiences and thus different policies at the selection and election stages. Yet, the opposite view is put forward by Adams and Merrill (2008, 2014). More importantly for our purposes perhaps, the prediction that candidates' behavior in the selection stage is quite different and possibly contrasts with their behavior in the general election is not borne out empirically. For example, Hirano and Snyder (2014) report that, at least for Congressional elections in the US, most of the issues that are at the forefront of the selection stage are also at the forefront of the election. More recently, Hassell (2016, 2018) and Hirano and Snyder (2019, Ch. 11) report strong evidence that, even though the voters making up the intraparty selectorate and the electorate at large can be quite different, candidates' efforts during the selection contest do not focus on policies and issues that are markedly different from the ones that are put forward during the election. An important reason for this is simply that parties want to make sure that candidates efforts are directed at policies that make them viable and successful *electoral* candidates as opposed to *selectable* candidates only.

Under FPTP, voters do not care about the parties' electoral outputs and only consider the efforts exerted by the two candidates in their district. This assumption follows the previous literature and aims to replicate existing comparative politics results. Our goal is to preserve the prediction that FPTP generates stronger incentives because of the closer link between politicians' decisions and voters' behavior. This advantage of FPTP disappears when voters base their decisions on the same inputs under different electoral systems. Indeed, under the alternative assumption, that voters decide for which party to vote on the basis of the electoral outputs of parties also under FPTP, then two electoral systems lead to the same aggregate efforts when selection is non-competitive.[12] Yet, PR leads to higher aggregate effort than FPTP when selection is competitve.

Under FPTP, we assume that candidate selection takes place in each district independently and not at the national level. We simply do not observe such a grand contest in practice: district-specific party elites want and do influence selection in their district. Furthermore, in many democracies, like the US, candidates must be residents of their district, which hinders the cross-district mobility a national selection would require.

We assume that parameter λ, that drives the noisiness of the election is the same under PR and FPTP. We do not take a stand about which electoral system is more subject to noise. Of course, we can easily relax this assumption. Increasing the noise in the election leads to weaker incentives.

We now turn to two extensions of the model.

5.1 More Than Two Parties Under PR

PR typically displays a larger number of parties than FPTP. As candidates' incentives to exert effort are decreasing in the number of parties, FPTP can be associated to higher aggregate effort than PR irrespective of the degree of competition in the selection of candidates:

Proposition 6 *Suppose that 2 parties compete under FPTP and $Z > 2$ parties compete under PR. If Z is large enough, a party's aggregate effort is lower under PR than that under FPTP irrespective of the type of candidate selection process.*

Proposition 6 suggests a novel trade-off in PR systems between the desire to represent citizen preferences and the need for incentives. Our theory also offers a rationale for the existence of broad, mainstream parties, as their coverage of many societal issues reduces the

[12]This result is true even though individual efforts differ across the two systems, because of party lists under PR.

need for a large number of parties. Lizzeri and Persico (2005) also highlights the cost of an 'excessive' number of parties under PR.

5.2 Ideological Voters

In the model, parties do not take any ideological position. Candidates only differ in their choice of effort, and candidates' ideology appears only in the payoff parameter M. Allowing for more general voter ideological preferences would make the model intractable. Still, to understand the interaction between ideology and electoral incentives, we carry out the following thought experiment. We assume symmetry at the national level, so that electoral competition does not change under PR. Under FPTP, some districts are party strongholds: one party wins the election for sure. In other districts, competition is as in the basic model. When selection is not competitive, the candidate running in a stronghold district doesn't exert effort, as their election is certain. And the other party does not bother filing in a candidate. This implies that, when selection is not competitive, introducing ideology reduces incentives under FPTP but not under PR. Still, as long as there are not too many party strongholds, FPTP would still lead to more effort than PR.

When selection is competitive, candidates will exert effort even in party strongholds. Suppose there are s party strongholds, $s/2$ per party. Each candidate in a party stronghold knows that his party wins for sure in his party's $s/2$ strongholds (and loses for sure in the strongholds of the other party) and that his party probability of winning a majority of seats is $P^L\left(\frac{K-s+1}{2}, K-s\right)$ irrespective of whether or not the candidate is selected by the party. Thus, the problem each candidate faces in a party stronghold is to maximize:[13]

$$Q^{iL}V + MP^L\left(\tfrac{K-s+1}{2}, K-s\right) - \tfrac{\left(e^{iL}\right)^2}{2}. \tag{10}$$

The FOC to a candidate's problem yields:

$$e^* = \sqrt{\frac{n-1}{n^2}V}, \tag{11}$$

[13] As long as the number of strongholds is small, we can consider that incentives in the other districts are unaffected by the presence of strongholds, that is, the effect of the presence of these strongholds on the value of $\bar{M}$ is negligible.

which is maximized for $n = 2$, yielding $e^* = \sqrt{V/4}$.

Thus, in each party's stronghold, each candidate *of the advantaged party* exerts effort equal to e^* whereas no effort takes place in the other party. From above we know that, when ideology plays no role in the electorate, the optimal number of candidates in the intraparty selection process is either one or two. For both cases, as long as V is not too large compared to M, our comparative politics prediction still goes through.[14]

6 Conclusion

We develop a model of legislative elections preceded by an intraparty selection stage. Selection can be either competitive or not. When selection is non-competitive, FPTP generates stronger incentives than PR. To the contrary, when selection is competitive, PR generates stronger incentives than FPTP.

Finally, the incentives generated by the candidate selection process appear to be at least as important as those generated by electoral rules. Our results also point to a causal relationship of electoral rules on the way parties organize. Thus, any empirical analysis of the impact of electoral rules or any other institution affecting elections should include data about the organizational choices of parties, to avoid an important omitted variable problem. Existing data sets focus almost exclusively on the level of democratization, centralization and inclusiveness of candidate selection. Our theory suggests that the degree of competition in selection is also of primary importance.

[14]The condition for our comparative politics result to go through is the same for both cases and is given by: $V < \frac{\lambda}{1-\lambda}\bar{M}$.

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Appendix : Proofs

Proof of Proposition 1

Party L's candidate chooses effort e^L to maximize:

$$\left(\tfrac{e^L}{e^L+e^R}+\tfrac{1-\lambda}{2}\right)\left[V+MP^L\left(\tfrac{K-1}{2},K-1\right)\right]+M\left(\sum_{k=\frac{K+1}{2}}^{K-1}P^L\left(k,K-1\right)\right)-\tfrac{1}{2}\left(e^L\right)^2$$

The first-order condition yields:

$$\tfrac{e^R}{\left(e^L+e^R\right)^2}\lambda\left[V+MP^L\left(\tfrac{K-1}{2},K-1\right)\right]-e^L=0.$$

Second order conditions are satisfied.

In the symmetric equilibrium, $P^L\left(\frac{K-1}{2},K-1\right)=C^{K-1}_{\frac{K-1}{2}}\left(\frac{1}{2}\right)^{K-1}$ and equilibrium effort is given by:

$$e^*=\sqrt{\frac{\lambda}{4}\left(V+MC^{K-1}_{\frac{K-1}{2}}\left(\frac{1}{2}\right)^{K-1}\right)}=\sqrt{\frac{\lambda}{4}\left(V+\bar{M}\right)}.$$

Each party's aggregate effort is then equal to:

$$E^*=Ke^*=K\sqrt{\frac{\lambda}{4}\left(V+\bar{M}\right)}.$$

Proof of Proposition 2

Candidate i chooses effort e^{iL} to maximize:

$$\begin{gathered}Q^{iL}p^L\left(V+MP^L\left(\tfrac{K-1}{2},K-1\right)\right)+\left(1-Q^{iL}\right)\left(p^L_{-i}MP^L\left(\tfrac{K-1}{2},K-1\right)\right)\\+M\left(\textstyle\sum_{j=\frac{K+1}{2}}^{K-1}P^L\left(j,K-1\right)\right)-\tfrac{\left(e^{iL}\right)^2}{2}.\end{gathered}.$$

The first order condition yields:

$$\left[\frac{dQ^{iL}}{de^{iL*}}p_L+\frac{dp_L}{de^{iL*}}Q^{iL}\right]\left(V+MP^L\left(\tfrac{K-1}{2},K-1\right)\right)-\frac{dQ^{iL}}{de^{iL*}}\frac{e^{kL}}{e^{kL}+e^R}MP^L\left(\tfrac{K-1}{2},K-1\right)-e^{iL*}=0$$

We have:

$$\frac{dQ^{iL}}{de^{iL*}} = \frac{\sum_{j=1}^{n} e^{jL} - e^{iL}}{\left(\sum_{j=1}^{n} e^{jL}\right)^2}$$
$$\frac{dp_L}{de^{iL*}} = \frac{\lambda e^R}{\left(e^{iL} + e^R\right)^2}.$$

In the symmetric equilibrium, we get:

$$e^* = \sqrt{\left(\frac{n-1}{2n^2} + \frac{\lambda}{4n}\right) V + \frac{\lambda}{4n}\bar{M}}.$$

Straightforward manipulation of e^* implies that effort is maximized for $n = 1$ or $n = 2$ depending on whether $\lambda \gtrless \frac{V}{V+\bar{M}}$.

The second order condition is $(Q''p + 2Q'p' + Qp'')\left(V + \bar{M}\right) - Q''p^L_{-i} - 1$.

We also have that:

$$p' = \lambda \tfrac{p(1-p)}{e^L},\ Q' = \tfrac{Q(1-Q)}{e^L},\ p'' = \tfrac{\lambda p(1-p)}{\left(e^L\right)^2}\left(\lambda\left(1-2p\right) - 1\right) =,\ Q'' = \tfrac{-2}{\left(e^L\right)^2} Q^2\left(1-Q\right).$$

Both p'' and Q'' are negative but both Q' and p' are positive. Using our last derivations, the SOC can be rewritten as:

$$\begin{aligned}&\left(V + \bar{M}\right)\left(\tfrac{-2}{\left(e^L\right)^2} Q^2\left(1-Q\right)p + 2\tfrac{\lambda(1-p)(1-Q)pQ}{\left(e^L\right)^2} + \tfrac{\lambda p(1-p)}{\left(e^L\right)^2}\left(\lambda\left(1-2p\right)-1\right)Q\right) + \tfrac{2}{\left(e^L\right)^2} p^L_{-i} Q^2\left(1-Q\right)\bar{M} - 1 \\ &= \tfrac{\left(V+\bar{M}\right)Qp}{\left(e^L\right)^2}\left(-2Q\left(1-Q\right) + 2\lambda(1-p)(1-Q) + \lambda\left(1-p\right)\left(\lambda\left(1-2p\right)-1\right)\right) + \tfrac{2}{\left(e^L\right)^2} p^L_{-i} Q^2\left(1-Q\right)\bar{M} - 1\end{aligned}$$

The FOC implies that

$$\left(e^L\right)^2 = \left(Q(1-Q)p + Q\lambda p(1-p)\right)\left(V + \bar{M}\right)$$

Thus the SOC simplifies further to

$$\frac{-2Q\left(1-Q\right) + 2\lambda(1-p)(1-Q) + \lambda\left(1-p\right)\left(\lambda\left(1-2p\right)-1\right)}{(1-Q) + \lambda(1-p)} + \frac{2p^L_{-i}/pQ\left(1-Q\right)}{\left((1-Q) + \lambda(1-p)\right)} \frac{\bar{M}}{\left(V + \bar{M}\right)} - 1$$

and to

$$- 2Q\left(1-Q\right)+2\lambda(1-p)(1-Q)+\lambda\left(1-p\right)\left(\lambda\left(1-2p\right)-1\right)-\left(1-Q\right)-\lambda(1-p)+2p_{-i}^{L}/pQ\left(1-Q\right)\frac{\bar{M}}{\left(V+\bar{M}\right)}$$

$$= -2Q\left(1-Q\right)+2\lambda(1-p)(1-Q)+\lambda\left(1-p\right)\left(\lambda\left(1-2p\right)\right)-2\lambda\left(1-p\right)-\left(1-Q\right)+2p_{-i}^{L}/pQ\left(1-Q\right)\frac{\bar{M}}{\left(V+\bar{M}\right)},$$

which is negative.

Proof of Proposition 3

The candidate in mth position on the electoral list of party L chooses effort e_m^L to maximize:

$$\sum_{k=m}^{K} P^{L}\left(k\right)V+\sum_{j=\frac{K+1}{2}}^{K} P^{L}\left(j\right)M-\frac{1}{2}\left(e_m^L\right)^2.$$

The first order condition to the problem of that candidate is given by:

$$\begin{aligned}
e_m^L &= \lambda V\sum_{k=m}^{K} C_k^K P_L^{k-1}\left(1-P_L\right)^{K-k}k\tfrac{E^R}{\left(E^L+E^R\right)^2}\\
&-\lambda V\sum_{k=m}^{K} C_k^K\left(P_L\right)^k\left(1-P_L\right)^{K-k-1}\left(K-k\right)\tfrac{E^R}{\left(E^L+E^R\right)^2}\\
&+\lambda M\sum_{j=\frac{K+1}{2}}^{K} C_j^K P_L^{j-1}\left(1-P_L\right)^{K-j}j\tfrac{E^R}{\left(E^L+E^R\right)^2}\\
&-\lambda M\sum_{j=\frac{K+1}{2}}^{K} C_j^K\left(P_L\right)^j\left(1-P_L\right)^{K-j-1}\left(K-j\right)\tfrac{E^R}{\left(E^L+E^R\right)^2}
\end{aligned}$$

The first order condition evaluated in the symmetric equilibrium yields:

$$\begin{aligned}
e_m^* &= \frac{\lambda V}{4E^*}\left(\frac{1}{2}\right)^{K-1}\sum_{l=m}^{K}\left(2l-K\right)C_l^K+\frac{\lambda M}{4E^*}\left(\frac{1}{2}\right)^{K-1}\sum_{j=\frac{K+1}{2}}^{K}\left(2j-K\right)C_j^K\\
&= \frac{\lambda}{4E^*}\left(mC_m^K\left(\frac{1}{2}\right)^{K-1}V+\left(\frac{1}{2}\right)^{K-1}\left(\frac{K+1}{2}\right)C_{\frac{K+1}{2}}^K M\right).
\end{aligned}$$

where the second line exploits the fact that $\sum_{l=j}^{K}\left(2l-K\right)C_l^K=jC_j^K$.

Then, sum the effort over all candidates on the list to derive $E^* = \sum_{m=1}^{K} e_m$:

$$E^* = \frac{\lambda V}{4E^*}\sum_{m=1}^{K}\left(\frac{1}{2}\right)^{K-1} mC_m^K + \frac{\lambda M}{4E^*}\left(\left(\frac{K+1}{2}\right)\left(\frac{1}{2}\right)^{K-1} C^K_{\frac{K+1}{2}} M\right)$$
$$= \frac{\lambda}{4E^*}\left(VK + MK\left(\frac{K+1}{2}\right)C^K_{\frac{K+1}{2}}\left(\frac{1}{2}\right)^{K-1}\right)$$

Exploiting the fact that $C_k^n = \frac{n}{k}C_{k-1}^{n-1}$, we have that $\frac{K+1}{2}C^K_{\frac{K+1}{2}} = \frac{K+1}{2}\frac{2K}{K+1}C^{K-1}_{\frac{K-1}{2}} = KC^{K-1}_{\frac{K-1}{2}}$. We can then rewrite E^* as:

$$E^* = \frac{\sqrt{\lambda\left(VK + K^2\bar{M}\right)}}{2} = K\sqrt{\frac{\lambda}{4}\left(V/K + \bar{M}\right)}.$$

The second-order conditions are satisfied adapting the argument in Crutzen, Flamand and Sahuguet (2020) by adding the additional payoff $\bar{M}$.

Proof of Proposition 4

Candidate i in party L chooses effort e_i^L to maximize:

$$\sum_{m=1}^{K} Q_i(m)\left(P^L(m)\,V\right) + \sum_{j=\frac{K+1}{2}}^{K} P^L(j)\,M - \frac{\left(e_i^L\right)^2}{2}.$$

The first order condition is given by:

$$\sum_{m=1}^{K} Q_i(m)\frac{\partial P^L(m)}{\partial e_i^L}V + \sum_{j=\frac{K+1}{2}}^{K}\frac{\partial P^L(j)}{\partial e_i^L}M + \sum_{m=1}^{K}\frac{\partial Q_i(m)}{\partial e_i^L}P^L(m)V - e_i^L = 0.$$

In the symmetric equilibrium, we have:

$$Q_i^*(m) = \frac{m}{K};$$

$$\frac{\partial P^L(m)}{\partial e} = \lambda\frac{C_m^K}{4E^*}(2m-K)\left(\frac{1}{2}\right)^{K-1} = \lambda\frac{C_m^K}{Ke^*}(2m-K)\left(\frac{1}{2}\right)^{K+1};$$

Also:

$$p_j = \frac{1}{K-j+1}, 1-p_j = \frac{K-j}{K-j+1}, \frac{\partial p_j}{\partial e} = \frac{K-j}{(K-j+1)^2 e}$$

and thus we have:

$$\begin{aligned}
\frac{\partial Q_i(m)}{\partial e} &= \frac{\partial p_1}{\partial e} + \frac{\partial p_2}{\partial e}(1-p_1) + \frac{\partial p_3}{\partial e}(1-p_1)(1-p_2) + ... \\
&= \sum_{j=1}^{m} \frac{\partial p_j}{\partial e} \frac{1}{1-p_j} \left[\prod_{s=1}^{m} (1-p_s)\right] \\
&= \sum_{j=1}^{m} \frac{1}{e} \frac{K-j}{(K-j+1)^2} \frac{K-j+1}{K-j} \left[\prod_{s=1}^{m} \frac{K-s}{K-s+1}\right] \\
&= \frac{1}{e}\left[1-\frac{m}{K}\right] \sum_{j=1}^{m} \frac{1}{K-j+1}.
\end{aligned}$$

as

$$\prod_{s=1}^{m} \frac{K-s}{K-s+1} = 1 - \frac{m}{K}.$$

The FOC in the symmetric equilibrium can thus be rewritten as:.

$$\begin{aligned}
&\frac{\lambda V}{Ke^*} \sum_{j=1}^{K} \frac{j}{K} \left(C_j^K (2j-K) \left(\frac{1}{2}\right)^{K+1} \right) + \frac{\lambda M}{Ke^*} \sum_{j=\frac{K+1}{2}}^{K} (2j-K) C_j^K \left(\frac{1}{2}\right)^{K+1} \\
&+ \frac{V}{Ke^*} \sum_{m=1}^{K} \left(C_m^K \left(\frac{1}{2}\right)^K \right) \left[\left(1-\frac{m}{K}\right) \sum_{j=1}^{m} \frac{1}{K-j+1} \right] - e^* = 0,
\end{aligned}$$

We know that $\sum_{j=x}^{K} (2j-K) C_l^K = x C_x^K$ and that $\frac{K+1}{2} C_{\frac{K+1}{2}}^K = \frac{K+1}{2} \frac{2K}{K+1} C_{\frac{K-1}{2}}^{K-1} = K C_{\frac{K-1}{2}}^{K-1}$. Thus $\frac{\lambda M}{Ke^*} \sum_{j=\frac{K+1}{2}}^{K} (2j-K) C_j^K \left(\frac{1}{2}\right)^{K+1} = \frac{\lambda M}{4Ke^*} \sum_{j=\frac{K+1}{2}}^{K} (2j-K) C_j^K \left(\frac{1}{2}\right)^{K-1} = \frac{\lambda \bar{M}}{4e^*}$.

To simplify $\sum_{m=1}^{K} \frac{m}{K} C_m^K (2m-K) \left(\frac{1}{2}\right)^{K+1}$, use the moment generating function for the binomial distribution to find:

$$\sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K m = \frac{K}{2}, \text{ and } \sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K m^2 = \left(K+K^2\right)/4.$$

Thus

$$\frac{\lambda V}{2K^2 e^*} \sum_{j=1}^{K} j C_j^K (2j-K) \left(\frac{1}{2}\right)^K = \frac{\lambda V}{4Ke^*}.$$

Finally:

$$\sum_{j=\frac{K+1}{2}}^{K} (2j - K)\, C_j^K = \frac{K+1}{2} C_{\frac{K+1}{2}}^K.$$

Therefore, the FOC simplifies to:

$$e^* = \sqrt{\tfrac{\lambda V}{4K} + \frac{\lambda \bar{M}}{4} + V \sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K \left[\tfrac{K-m}{K} \sum_{j=1}^{m} \tfrac{1}{K-j+1}\right]}.$$

As in Fu and Lu (2009), for the above candidate equilibrium to be an equilibrium, we need to show that the most profitable deviation for a candidate, namely exerting zero effort, is not profitable. This is verified if M is not too large compared to V and λ is not too small. As we mention in the main text, using a generalized Tullock contest function makes the problem more concave, and the second order conditions are immediately satisfied for small values of γ.

The condition that needs to be satisfied for the deviation to zero effort to not be profitable is:

$$\begin{aligned}
&V/4 + M/2 - (e^*)^2/2 \\
&\geq \\
&\left(\tfrac{\lambda(K-1)}{2K-1} + \tfrac{(1-\lambda)}{2}\right)^K V + \sum_{j=\frac{K+1}{2}}^{K} C_j^K \left(\tfrac{\lambda(K-1)}{2K-1} + \tfrac{(1-\lambda)}{2}\right)^j \left(1 - \tfrac{\lambda(K-1)}{2K-1} - \tfrac{(1-\lambda)}{2}\right)^{K-j} M,
\end{aligned}$$

where the RHS of the inequality is a candidate's payoff when he exerts zero effort and thus ends up last on the party list.

Case 1: $M = 0$

When $M = 0$, the condition is satisfied if the term in V on the LHS is larger than the one on the RHS. To prove this, we proceed in three steps. First, remark that, for any value of λ, the RHS is never larger than $\left(\frac{1}{2}\right)^K V$ and is also strictly decreasing in K.

Second, For K equal to 3 and 5, it is easy to check by direct computation that the above condition is satisfied.

Last, e^* is increasing in K but it is never larger that $\sqrt{\frac{\ln 2}{2}V}$.[15] Thus, for any $K > 5$, the LHS is never smaller than $\left(\frac{1}{4} - \frac{\lambda}{8K} - \frac{\ln 2}{4}\right) V > 0.05V$ and is thus strictly larger than the RHS, which is strictly smaller than $\frac{V}{2^5} \simeq 0.03V$ for any $K > 5$. This concludes the proof that the deviation to zero effort is not profitable when $M = 0$.

Case 2: $M > 0$

The term in M on the RHS may be large enough to violate the no deviation condition when λ is small. In that case, a sufficient condition for the no deviation condition to be satisfied is simply that M is not too large.

Proof of Theorem 5

We start by comparing efforts when selection is non-competitive. Comparing party aggregate efforts in propositions 1 and 3 yields the first part of the Theorem.

We are left with the comparison when selection is competitive. As all candidates exert the same effort within their party across the two electoral rules, we can compare individual efforts in this case. We thus need to compare effort under competitive FPTP,

$$e^* = \sqrt{V\left(\frac{n-1}{2n^2} + \frac{\lambda}{4n}\right) + \frac{\lambda}{4n}\tilde{M}},$$

and effort under competitive PR,

$$e^* = \sqrt{\tfrac{\lambda}{4}\left(\frac{V}{K} + \tilde{M}\right) + V\sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K \left[\tfrac{K-m}{K}\sum_{j=1}^{m}\tfrac{1}{K-j+1}\right]}.$$

Remember from Proposition 2 that effort under competitive FPTP is maximized at $n = 1$ or $n = 2$ depending on whether $\lambda \gtrless \frac{V}{V+\bar{M}}$. We deal with each case in turn.

For the first case, for which $\lambda \geq \frac{V}{V+\bar{M}}$ ($n = 1$), we need to compare $\sqrt{\frac{\lambda}{4}\left(V + \bar{M}\right)}$ to $\sqrt{\frac{\lambda}{4}\left(\frac{V}{K} + \bar{M}\right) + V\sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K \left[\frac{K-m}{K}\sum_{j=1}^{m}\frac{1}{K-j+1}\right]}$. For the second case, for which $\lambda \leq \frac{V}{V+\bar{M}}$ ($n = 2$), we need to compare $\sqrt{\left(\frac{1+\lambda}{8}\right)V + \frac{\lambda}{8}\bar{M}}$ to $\sqrt{\frac{\lambda}{4}\left(\frac{V}{K} + \bar{M}\right) + V\sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K \left[\frac{K-m}{K}\sum_{j=1}^{m}\frac{1}{K-j+1}\right]}$. In both cases, the term in $\bar{M}$ is weakly

[15] We provide the detailed derivation of these results in the proof of Theorem 5, where these characteristics of e^* are needed for our comparative politics result.

larger in the expression for effort under competitive PR. We can thus focus on the term in V in what follows.

Consider $\left(\frac{1}{2}\right)^K \sum_{m=1}^{K} C_m^K \left[\frac{K-m}{K} \sum_{j=1}^{m} \frac{1}{K-j+1}\right] \equiv \Lambda(K)$ first. We use the following result about combinatorial sums of finite differences. Identity 14 in Spivey (2007) states that $\sum_{m=1}^{K} 2^{-K} C_m^K \left(H_K - H_{K-m}\right) = \sum_{m=1}^{K} \frac{1}{m2^m}$ with $H_K = \sum_{j=1}^{K} \frac{1}{j}$ being the K^{th} harmonic number. We then have:

$$
\begin{aligned}
\Lambda(K) &= \sum_{m=1}^{K} \left(\frac{1}{2}\right)^K C_m^K \left(1 - m/K\right) \left(\sum_{j=1}^{m} \frac{1}{K+1-j}\right) \\
&= \sum_{m=1}^{K-1} \left(\frac{1}{2}\right)^K C_m^{K-1} \left(H_K - H_{K-m}\right) \\
&= \sum_{m=1}^{K-1} \left(\frac{1}{2}\right)^K C_m^{K-1} \left(H_{K-1} - H_{K-1-m} + \frac{1}{K} - \frac{1}{K-m}\right) \\
&= \frac{1}{2} \sum_{m=1}^{K-1} \frac{1}{m2^m} + \frac{1}{2} \sum_{m=1}^{K-1} \left(\frac{1}{2}\right)^{K-1} C_m^{K-1} \left(\frac{1}{K} - \frac{1}{K-m}\right)
\end{aligned}
$$

Now:

$$
\begin{aligned}
&\frac{1}{2} \sum_{m=1}^{K-1} \left(\frac{1}{2}\right)^{K-1} C_m^{K-1} \left(\frac{1}{K} - \frac{1}{K-m}\right) = \frac{1}{K} \sum_{m=1}^{K-1} \left(\frac{1}{2}\right)^{K-1} C_m^{K-1} \left(\frac{-m}{K-m}\right) \\
&= -\frac{1}{K^2} \sum_{m=1}^{K-1} \left(\frac{1}{2}\right)^{K-1} C_m^K m = -\frac{1}{K} \left(1 - \left(\frac{1}{2}\right)^{K-1}\right)
\end{aligned}
$$

Thus

$$
\left(\frac{1}{2}\right)^K \sum_{m=1}^{K} C_m^K \left[\frac{K-m}{K} \sum_{j=1}^{m} \frac{1}{K-j+1}\right] = \frac{1}{2} \left(\sum_{m=1}^{K-1} \left(\frac{1}{m2^m}\right) - \frac{1}{K} + \frac{1}{K} \left(\frac{1}{2}\right)^{K-1}\right).
$$

This implies in turn that $\Lambda(K)$ is indeed increasing in K as we have:

$$
\begin{aligned}
\Lambda(K+1) - \Lambda(K) &= \left(\frac{1}{(K)\, 2^K} - \frac{1}{K+1} + \frac{1}{K+1} \left(\frac{1}{2}\right)^K + \frac{1}{K} - \frac{1}{K} \left(\frac{1}{2}\right)^{K-1}\right) \\
&= \frac{1}{2^K K} \frac{2^K - 1}{K+1} > 0.
\end{aligned}
$$

To see that $\lambda\frac{V}{4K}+\left(\frac{1}{2}\right)^K\sum_{m=1}^{K}VC_m^K\left[\frac{K-m}{K}\sum_{j=1}^{m}\frac{1}{K-j+1}\right]$ is also increasing in K, simple algebra implies that:

$$\begin{aligned}\frac{\lambda}{4(K+1)}+\Lambda\left(K+1\right)-(\frac{\lambda}{4(K}+\Lambda\left(K\right)) &= \lambda\left(\frac{1}{4(K+1)}-\frac{1}{4K}\right)+\frac{1}{2^K K}\frac{2^K-1}{K+1}\\ &= \frac{1}{K(K+1)}\left(\frac{2^K-1}{2^K}-\frac{\lambda}{4}\right);\end{aligned}$$

For $K\geq 3$ and $\lambda\in[0,1]$, $\frac{2^K-1}{2^K}\geq 7/8>\lambda$.

One can check by direct computation that, for $K=3$, equilibrium effort under competitive PR is greater than that under competitive FPTP. The previous result shows that it is still the case for higher values of K.

Finally, for completeness and still ignoring $\bar{M}$, $\sqrt{\frac{\lambda}{4}\frac{V}{K}+V\sum_{m=1}^{K}C_m^K\left(\frac{1}{2}\right)^K\left[\frac{K-m}{K}\sum_{j=1}^{m}\frac{1}{K-j+1}\right]}$ approaches $\sqrt{\frac{\ln 2}{2}V}\simeq 0.59\sqrt{V}$ as K goes to infinity, which is higher than maximal equilibrium effort under competitive FPTP, $\frac{\sqrt{V}}{2}$.

Proof of Proposition 6

For the sake of simplicity and wlog, we set M equal to 0. Equilibrium effort under FPTP when selection is non-competitive is $\frac{\sqrt{\lambda V}}{2}$. Let there be T identical parties under PR. The probability that party p wins l seats is given by:

$$P_p(l)=\frac{K!}{(K-l)!l!}\left(P_p\right)^l\left(1-P_p\right)^{K-l},$$

with $P_p=\frac{1-\lambda}{T}+\lambda\frac{E_p}{\sum_{j=1}^{T}E_j}$.

The problem for the candidate in position m on the list of party p is to maximize with respect to their own effort e_m^p:

$$\sum_{k=m}^{K}P_p(k)V-\frac{1}{2}\left(e_m^p\right)^2.$$

The first order condition to the problem of the candidate in position m on the list of party p is given by:

$$e_m^p = \lambda V \sum_{k=m}^{K} C_k^K (P_p)^{k-1} (1-P_p)^{K-k} k \frac{\sum_{j=1}^{T} E_j - E_p}{\left(\sum_{j=1}^{T} E_j\right)^2}$$
$$- \lambda V \sum_{k=m}^{K} C_k^K (P_p)^{k} (1-P_p)^{K-k-1} (K-k) \frac{\sum_{j=1}^{T} E_j - E_p}{\left(\sum_{j=1}^{T} E_j\right)^2}$$

In the symmetric equilibrium, effort choices of candidates in the same position on the list are equal across parties and thus $E^{1*} = ... = E^{P*} = E^*$ and $P_p = 1/T$. We can simplify the above first order condition to find:

$$\begin{aligned} e_m^L &= \frac{(T-1)\lambda V}{T^2 E^*} \sum_{k=m}^{K} C_k^K \left[k \left(\frac{1}{T}\right)^{k-1} \left(\frac{T-1}{T}\right)^{K-k} - (K-k) \left(\frac{1}{T}\right)^{k} \left(\frac{T-1}{T}\right)^{K-k-1} \right] \\ &= \frac{(T-1)\lambda V}{T^2 E^*} \sum_{k=m}^{K} C_k^K \left[\left(Tk - \frac{T(K-k)}{T-1}\right) \left(\frac{1}{T}\right)^{k} \left(\frac{T-1}{T}\right)^{K-k} \right] \\ &= \frac{(T-1)\lambda V}{T^2 E^*} \sum_{k=m}^{K} C_k^K \left[\frac{T}{T-1} (Tk-K) \left(\frac{1}{T}\right)^{k} \left(\frac{T-1}{T}\right)^{K-k} \right] \\ &= \frac{\lambda V}{T E^*} \sum_{k=m}^{K} C_k^K \left[(Tk-K) \left(\frac{1}{T}\right)^{k} \left(\frac{T-1}{T}\right)^{K-k} \right] \end{aligned}$$

Exploiting the fact that $\sum_{k=m}^{K} C_k^K \left[(Tk-K)\left(\frac{1}{T}\right)^k \left(\frac{T-1}{T}\right)^{K-k}\right] = \left(\frac{1}{T}\right)^K (T-1)^{K-m+1} m C_m^K$, this simplifies further to:

$$e_m^L = \frac{\lambda V}{T E^*} \left(\frac{1}{T}\right)^K (T-1)^{K-m+1} m C_m^K.$$

Summing these optimal effort decisions over all party list members and exploiting the fact that $\sum_{m=1}^{K} \left(\frac{1}{T}\right)^K (T-1)^{K-m+1} m C_m^K = K \frac{T-1}{T}$, we get:

$$E^* = \frac{\lambda V}{T E^*} K \frac{T-1}{T} \iff E^* = \frac{\sqrt{K(T-1)\lambda V}}{T}$$

Comparing (21) to the party output under FPTP, $K\frac{\sqrt{\lambda V}}{2}$, we see that the result of the first part of Theorem 5 is reinforced.

We now turn to competitive selection. Effort under FPTP is equal to $\sqrt{\frac{(1+\lambda)V}{8}}$. Under PR, the first order condition to the problem faced by any politician i (in party p, say) is:

$$\left[\sum_{m=1}^{K} \frac{\partial P_p(l)}{\partial e_i} Q_i(m) + \sum_{m=1}^{K} P_p(m) \frac{\partial Q_i(m)}{\partial e_i}\right] V = e_i^*$$

As there are K candidates competing for one of the K list slots, the equilibrium probability of being offered slot m on the list is $Q_i^*(m) = \frac{m}{K}$;

Given that $P_p(m) = C_m^K \left(\frac{1-\lambda}{T} + \lambda \frac{E_i}{\sum_{j=1}^T E_j}\right)^m \left(1 - \frac{1-\lambda}{T} - \lambda \frac{E_i}{\sum_{j=1}^T E_j}\right)^{K-m}$, we have that, exploiting some of the algebra above:

$$\frac{\partial P_p(m)}{\partial e_i} = \frac{\lambda V}{T K e^*} C_m^K \left[(Tm - K)\left(\frac{1}{T}\right)^m \left(\frac{T-1}{T}\right)^{K-m}\right]$$

Also:

$$\frac{\partial Q_i(m)}{\partial e_{iL}} = \frac{1}{e_i^*}\left(1 - \frac{m}{K}\right) \sum_{j=1}^{m} \frac{1}{K-j+1}$$

Finally:

$$P_p^*(m) = C_m^K \left(\frac{E_i^*}{\sum_{j=1}^T E_j^*}\right)^m \left(1 - \frac{E_i^*}{\sum_{j=1}^T E_j^*}\right)^{K-m} = C_m^K \left(\frac{1}{T}\right)^m \left(\frac{T-1}{T}\right)^{K-m} \tag{12}$$

Thus, in the symmetric equilibrium, effort is equal to:

$$\sqrt{\sum_{m=1}^{K} \frac{\lambda V}{TK} C_m^K \left[(Tm-K)\left(\frac{1}{T}\right)^m \left(\frac{T-1}{T}\right)^{K-m}\right] \frac{m}{K} + V \sum_{m=1}^{K} C_m^K \left(\frac{1}{T}\right)^m \left(\frac{T-1}{T}\right)^{K-m} \left(1 - \frac{m}{K}\right) \sum_{j=1}^{m} \frac{1}{K-j+1}}$$

We know that $\sum_{m=1}^{K} C_m^K m^2 \left(\frac{1}{T}\right)^m \left(\frac{T-1}{T}\right)^{K-m} = K(K-1)\left(\frac{1}{T}\right)^2 + \frac{K}{T}$. Thus the first term under the square root simplifies to $\frac{\lambda V(T-1)}{T^2 K}$ and equilibrium effort boils down to:

$$\sqrt{\frac{\lambda V (T-1)}{T^2 K} + V \sum_{m=1}^{K} C_m^K \left(\frac{1}{T}\right)^m \left(\frac{T-1}{T}\right)^{K-m} \left(1 - \frac{m}{K}\right) \sum_{j=1}^{m} \frac{1}{K-j+1}}$$

We need to compare the above to $\sqrt{\frac{(1+\lambda)V}{8}}$. Given that both the first and second term in the square root making up equilibrium effort under PR are decreasing in T, there must be a value of T beyond which equilibrium effort under PR is smaller than that under FPTP.