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The river pollution claims problem

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Abstract

We propose the river pollution claims problem to distribute a limited pollution budget among agents located along a river. A key distinction with the standard claims problem is that agents are ordered and they are given priority based on their *location* in this order instead of their *identity*. We propose two new axioms that are relevant in the context of river pollution and use these to characterize two priority rules. Our characterization results show that *Consistency* plays an important role since it makes sure that any asymmetric treatment will be transferred across problems.

Keywords: Claims Problem, River Pollution, Pollution Permits, Priority Rules

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1 Introduction

In this paper we propose the river pollution claims problem. In this problem, agents are ordered linearly along a river and each of them claims a permit to discharge a certain amount of pollution into the river. For environmental conservation reasons, the budget of total permitted pollution is limited and a solution to this problem allocates this limited amount of permits to the agents. The river pollution claims problem is based on the standard claims problem as introduced by (O'Neill 1982), and it is inspired by water pollution in rivers such as nutrients originating from agricultural production and chemicals originating from industrial processes. Water pollution may cause serious health problems. For example, Ebenstein (2012) estimated the impacts of surface water pollution on human health, showing that a deterioration of water quality by one grade (based on a six-grade scale) could cause a 9.7% increase in the digestive cancer death rate. Besides health, polluted water causes ecological imbalance and eco-remediation costs. For example, Camargo and Alonso (2006) used multi-scale data to show that nitrogen pollution could result in acidification and eutrophication of freshwater ecosystems, and cause severe damage to the survival, growth and reproduction of aquatic animals when it reaches toxic levels.

Water pollution has become a severe environmental problem and urgently requires effective control measures. Such measures may be hampered by the mismatch between river basins and the borders of jurisdictions in which they are located. Globally, 286 rivers flow across country borders (UNEP 2016), and many more rivers cross the borders of lower-level jurisdictions like provinces, regions, and municipalities. As a result of this mismatch, the management of river pollution is often shared by multiple jurisdictions. The distribution of water pollution between agents is a challenge for which an analysis of the river pollution claims problem can provide possible directions.

The main difference with the standard claims problem is that in the river pollution claims game agents are ordered linearly from upstream to downstream, reflecting the direction of river flow, and this order is exogenously given by the hydrological setting. In addition, there may be a concern not only about the amount of pollution in the river, but also by its distribution over the agents. One reason is the standard fairness consideration that is inherent to claims problems (see e.g. Thomson 2003). A second reason, which is novel, is that one may be concerned about the location of pollution. A given amount of pollution is likely to cause more damage when it is emitted upstream compared to downstream since upstream pollution will cause damage along a longer stretch of the river, mitigated by the absorptive capacity of the river (joint with ambient pollution concentrations, flow, and temperature, see e.g. Chakraborti 2021). This environmental externality increases total health- and ecological damage in the river and may also cause tensions between regions along a river. In Section 3 we will translate both concerns into axioms that we will then apply to characterize solutions to the river pollution claims problem.

The natural order of the agents in the river pollution claims problem intuitively leads one to consider a solution from the family of priority rules, characterized by Moulin (2000). Priority rules meet agents' claims lexicographically, following some exogenous ordering, until the complete endowment is allocated. If we were to take the ordering of agents along the river as this exogenous ordering, we would end up with a solution that gives priority to upstream agents over downstream agents. In Section 4 we will denote this rule the Upstream Priority Rule and we will provide a characterization result.

We will also consider other priority rules. However, given the natural order of agents in the river pollution claims problem, we arrive at different results compared to Moulin (2000). The reason for this difference is that we use a different criterion to assign priority. Given the river setting, we give priority to agents according to their *location* instead of their *identity*. This alternative interpretation of priority turns out to give particular strength to axioms in which the population of claimants may vary, see e.g. Thomson (2003). *Consistency*, one of the axioms used by Moulin (2000) in his characterization result, is such an axiom. It becomes very strong in the river setting since applying *Consistency* will cause a mismatch between agents' *identity* and their *location*, which determines their position in the priority order. For example, consider a problem with 3 agents located linearly along the river, 1 being upstream of 2, and agent 3 completely downstream. Suppose we apply a priority rule that gives the second agent priority over the upstream agent and the upstream agent priority over the most downstream agent. Note that we define priority by location, not by identity. Now, consider a problem where agent 2 leaves the problem with his claim, as is the case when one applies *Consistency*. In the revised problem, agent 3 will take up the second spot in the river and by his location as second agent it will have strict priority over agent 1, who is still upstream. The only two priority rules that do not violate *Consistency* in the river pollution claims problem seem specifically relevant in the river setting: the Upstream Priority Rule and the Downstream Priority Rule. We will formally define and characterize both solutions in Section 4.

Our paper relates to three separate strands of the literature. First, a series of recent papers is concerned with the allocation of the global carbon budget in order to assess fairness of countries' efforts to mitigate greenhouse gas emissions (Giménez-Gómez, Teixidó-Figueras, and Vilella 2016; Duro, Giménez-Gómez, and Vilella 2020; Ju et al. 2021;

Heo and Lee 2022). Similar to the current paper, these carbon budget papers model a total budget of pollution that is allowed and they are concerned with the distribution of this budget over all countries. The main difference with the current paper is that agents are not ordered. Also, the nature of the climate change problem makes that the proposed axioms are quite different from axioms that are relevant for the setting of river pollution.

Second, there is a small literature that focuses on allocating water quantity in river settings using a cooperative game approach (Ambec and Sprumont 2002; Brink, Laan, and Moes 2012) as well as using the claims problem approach (Ansink and Weikard 2012, 2015; Brink et al. 2014; Estévez-Fernández, Giménez-Gómez, and Solís-Baltodano 2021). Similar to the current paper, these papers use a setting where that agents are ordered linearly along the river. The main difference is that in modeling water quantity, the endowment consists of a vector of endowments of water that originate on each of the agents' territories in the form of rainfall and tributaries. Applying a claims problem in this setting is similar to redistributing the existing water resources under a water balance constraint. In the current paper, as in the standard claims problem, there is a single endowment, the pollution budget and we are concerned with the higher environmental impact of upstream pollution compared to downstream pollution.

Third, a fairly recent literature is concerned with the distribution of welfare due to river cleaning (Gengenbach, Weikard, and Ansink 2010; Laan and Moes 2016; Steinmann and Winkler 2019; Gudmundsson and Hougaard 2021) or the sharing of river water treatment costs (Ni and Wang 2007; Alcalde-Unzu, Gómez-Rúa, and Molis 2015; Brink, He, and Huang 2018). Ni and Wang (2007) pioneered this literature with an analysis of how to share the costs of cleaning a river among different agents. One of their proposed cost sharing rules employs an *Upstream Symmetry* axiom, stating that any given downstream costs should be shared equally by all upstream polluters. We will adapt this axiom to the setting of the river pollution claims problem in Section 3. Surprisingly, while many papers are concerned about the economics of river pollution in terms of the distribution of costs for a given pollution abatement level or the distribution of welfare for the efficient pollution level, we are not aware of similar papers in economics that focus on the distribution of pollution in the river that is underlying the costs of pollution abatement and the resulting level welfare.

We employ three main axioms. We have already discussed *Consistency* and the *Upstream Symmetry* axiom that is inspired by Ni and Wang (2007). The third axiom that we propose is tailored to the environmental impact of upstream versus downstream pollution as discussed above. This *Don't Move Up* axiom comes in different flavors, depending on some parameter $\alpha \in [0, 1]$. In its strictest form, the axiom states that any transfer of (part of) his claim from a downstream to an upstream agent does not affect the allocation of pollution permits. In weaker forms, a limited effect is allowed. In other words, it should be hard to re-allocate pollution from downstream to upstream locations in the river, given that upstream pollution is more damaging to the environment. More details on the axioms are provided in Section 3.

The paper is organized as follows. In the next section, we introduce the river pollution claims problem. In Section 3, we introduce and motivate the relevant axioms. In Section 4, we provide our characterization results. In Section 5, we present concluding remarks.

2 The river pollution claims problem

We analyze the river pollution claims problem as a pollution problem from a conflicting claims point of view. A conflicting claims problem is a particular case of distribution problem, and originates from O'Neill (1982). A prime example that is used to motivate this problem is the problem of how assets should be distributed among its creditors when a firm goes bankrupt. A large number of solutions to solve claims problems have been proposed (see e.g. Thomson 2003, for an overview).

In our model, the resource to divide is the pollution budget. This budget is not enough to satisfy all the agents' claims, and the problem is to distribute a limited river pollution budget among agents along a river. Claims may be based, for instance, on particular regional characteristics like population size, local GDP, or on existing or historical pollution levels. The reason that agents have claims to the pollution budget is that, in addition to causing environmental and health damage, pollution is a by-product of many industrial processes that create jobs, growth, and welfare. Our paper takes this relation as given, ignoring the potential de-coupling of pollution from production.

Formally, a river pollution claims problem is defined as follows.

Definition 1 (River Pollution Claims Problem)**.** Let $N = \{1, 2, ..., n\} \subset \mathbb{N}$ be a finite and ordered set such that $n \geq 2$ agents are located along a river, with agent 1 the most upstream and n the most downstream. Agent i is upstream of j whenever $i < j$. Each agent has a claim on the pollution budget $E \in \mathbb{R}^n_+$. Let $c = (c_1, c_2, ..., c_n) \in \mathbb{R}^n_+$ be the claims vector with $\sum_{i=1}^{n} c_i > E$. Then, a river pollution claims problem is a triple (N, E, c).

As discussed in Section 1, we take into account both agents' *identity* as well as their *location* along the river. In general, the two will overlap so that agent 2 is located in the second location starting upstream. However, as the example given in Section 1 demonstrates, whenever we apply e.g. *Consistency*, we may end up with a river pollution claims problem where identity and location do not match. In this particular example, agent 3 is in location 2. We choose to use location as our main descriptor. In order to avoid excessive notation we do not add an index to denote identity but rather we will clearly explain the mismatch in the text, whenever relevant and particularly in the proofs.

Denote by Ω the set of all river pollution claims problems. A river pollution allocation rule allocates permits to discharge a certain amount of pollution to each agent. Doing so, it solves the river pollution claims problem.

Definition 2 (River Pollution Allocation Rule)**.** A river pollution allocation rule is a mapping F that assigns to each river pollution claims problem $\omega \in \Omega$ an allocation vector $x = (x_1, x_2, ..., x_n)$, $x \in \mathbb{R}^n_+$, such that

1. $0 \leq x_i \leq c_i$, $\forall i \in N$ (*Non-Negativity* and *Claims-Boundedness*), and
2. $\sum_{i=1}^{n} x_i = E$, $\forall i \in N$ (*Efficiency*).

We use shorthand notation and write $x_i = F_i(\omega)$. This completes the description of the river pollution claims problem and its solution.

3 Axioms

Claims problems are solved using an axiomatic approach: a set of desirable axioms is applied that jointly arrive at a specific solution – an allocation rule – to the problem. In this section, we present and motivate the axioms that we will use to characterize two allocation rules in the next Section.

We start with *Consistency*, a basic property that is satisfied by some classical solutions to claims problems (Herrero and Villar 2001; Thomson 2012):

Axiom 1 (Consistency)**.** For each river pollution claims problem $\omega = (N, E, c) \in \Omega$ and its related problem $\omega' = (N', E', c') \in \Omega$ such that $N' \subset N,\ E' = \sum_{N'} x_i(\omega),$ and $c' = c_{N'}$, we have $F_i(\omega') = F_i(\omega)$ for all $i \in N'$.

Consistency states that if some agents join or leave the river pollution claims problem, allocations to the remaining agents should not be affected. There are two reasons why this axiom is relevant in the river context. First, as mentioned in Section 1, the natural order of the agents in the river pollution claims problem links to the family of priority rules, which treats agents unequally; it is intuitive that such treatments are supposed to be consistent when the population of agents vary (Thomson 2012). Second, in practice, the allocation of a limited river pollution budget requires cooperation among agents located along the river. If these are under the governance of different jurisdictions, a cooperative agreement is required, which leaves each of the jurisdictions involved the autonomy to decide to leave the agreement.

We will also apply a weaker form of *Consistency*. It requires that adding or deleting agents whose claims are zero should not affect the amount that the other agents receive. Chun (1999) calls this property *Dummy* and Thomson (2003) denotes it *Limited Consistency*. We will use this latter term.

Axiom 2 (Limited Consistency)**.** For each river pollution claims problem $\omega = (N, E, c) \in \Omega$ and its related problem $\omega' = (N', E, c') \in \Omega$, such that $N' \subset N$, and $c' = c_{N'}$ with $c_i = 0$ for some $i \in N \setminus N'$, we have $F_i(\omega') = F_i(\omega)$ for all $i \in N'$.

Next, we propose our first new axiom, *Upstream Symmetry*, that is relevant in the setting of the river pollution claims problem.

Axiom 3 (Upstream Symmetry)**.** For each river pollution claims problem $\omega = (N, E, c) \in \Omega$, its related problem $\omega' = (N, E, c') \in \Omega$ and each $j > 2$, such that $c'_j > c_j$ and $c'_i = c_i$ for all $i \neq j$, we have $x'_i - x_i = x'_k - x_k$ for any $i, k < j$.

Upstream Symmetry states that, if an increased claim from one of the downstream agents affects the amount of pollution permits available to upstream agents, then all upstream agents should be affected equally. This fairness axiom is inspired by the upstream equal sharing rule proposed by Ni and Wang (2007) and motivated by the fact that it is difficult to distinguish each upstream polluter's contribution to the downstream costs of cleaning pollution. As a result, it seems fair to apply an equal sharing principle. Their *Upstream Symmetry* axiom pertains to the equal sharing of pollution costs by upstream agents. In the river pollution claims game, however, we are not concerned about cost sharing but rather about the distribution of the pollution budget. As a result, we adapt the *Upstream Symmetry* axiom to the setting of a claims game in order to reflect this difference. The motivation is rather similar: when downstream pollution increases, under a given pollution budget it seems fair that pollution permits to all upstream agents are reduced by a similar amount.

In one of our characterization results, we will use the inverse of *Upstream Symmetry*, which we denote *Downstream Symmetry*.

Axiom 4 (Downstream Symmetry)**.** For each river pollution claims problem $\omega = (N, E, c) \in \Omega$, its related problem $\omega' = (N, E, c') \in \Omega$ and each $i < n - 1$, such that $c'_i < c_i$ and $c'_j = c_j$ for all $i \neq j$, we have $x'_j - x_j = x'_k - x_k$ for any $j, k > i$.

Downstream Symmetry states that, if a decreased claim from one of the upstream agents affects the amount of pollution permits available to downstream agents, then all downstream agents should be affected equally. This axiom takes a viewpoint of equality similar to the one used for *Upstream Symmetry*.

Next, we propose a second new axiom named *Don't Move Up*. This axiom employs agents' locations along the river to lessen the environmental harm caused by a given amount of pollution.

Axiom 5 (Don't Move Up)**.** For each river pollution claims problem $\omega = (N, E, c) \in \Omega$ and its related problem $\omega' = (N, E, c') \in \Omega$, such that $0 < c'_i - c_i = c_j - c'_j < c_j - x_j$ for all $i < j$ and $c'_k = c_k$ for all $k \neq i, j$, we have $x_j - x'_j \leq \alpha(c_j - c'_j)$, $\alpha \in [0, 1]$.

The *Don't Move Up* axiom puts a constraint on the allocation to agent j – who is non-satiated in problem ω. While j's claim is lower in problem ω' than in problem ω, the associated change in his allocation may be much smaller, depending on parameter α. Note that agent j's claim cannot decrease too much, since otherwise *Claims-Boundedness* is violated. This constraint is reflected by the inequality $c_j - c'_j < c_j - x_j$ which is equivalent to $x_j < c'_j$. This inequality is also the reason why the axiom does not apply when agent j is satiated.

In its extreme version, when $\alpha = 0$, the last part of the axiom states $x_j - x'_j \leq 0$. This version of the axiom states that agent j's allocation cannot decrease at all despite his lower claim. This strong version of *Don't Move Up* says that an upstream transfer in claims will not result in an upstream transfer of pollution. The motivation is that upstream pollution is likely to cause more damage and hence, we may want to prevent pollution from moving upstream, give a certain pollution budget. *Don't Move Up* with $\alpha = 0$ is similar to the inverse of *No Transfer Paradox* (Chun 1988), which focuses on the case where one agent transfer his claim to another agent, and requires not only that the former should receive at most as much as he did initially, but also that the latter should receive at least as much as he did initially. The *No Transfer Paradox* is satisfied by many classical solutions to claims problems. When we consider such claim transfer situation in a river setting, however, it implies that if a downstream agent move some of the claims towards upstream, the former will get at most as much as he did before. This is not a desirable outcome from an environmental perspective given that pollutants flow from upstream to downstream. Therefore, we propose this inverse version of the axiom in order to prevent undesirable claim transfers and keep pollution downstream as much as possible.

When $\alpha = 1$, the last part of the axiom states $x_j - x'_j \leq c_j - c'_j$. The upper bound becomes larger than 0, which weakens the axiom considerably, requiring only that if one (non-satiated) downstream agent moves some of his claim upstream, the decrease in his allocation should not exceed the decrease in his claim. The axiom still aims to keep pollution downstream but becomes less strict than the case where $\alpha = 0$. Many rules satisfy this weaker version of the axiom including the Proportional Rule, Constrained Equal Awards Rule and Constrained Equal Loss Rule.

4 Characterization Results

Equal Treatment of Equals is a compelling property in many claims problems where discrimination among agents is undesirable; for instance, when dividing the global carbon budget (see Giménez-Gómez, Teixidó-Figueras, and Vilella (2016) or designing a tax schedule (see Chambers and Moreno-Ternero (2017). However, in many other cases, one may want to treat agents unequally with regards to their claims. A typical example is that creditors of a bankruptcy firm generally belong to different priority classes so that agents with the same claims may get different shares because of their identities. In the setting of the river pollution claims problem, agents are ordered linearly so that downstream agents could suffer some damage caused by upstream polluters. We therefore consider an agent's location as the relevant characteristic to determine its priority class. In this section, we characterize two priority rules that follow this logic and base priority on location. One gives priority to upstream agents over downstream agents, another gives priority to downstream agents over upstream agents.

Definition 3 (Upstream Priority Rule)**.** The Upstream Priority Rule for a river pollution claims problem $\omega = (N, E, c) \in \Omega$ allocates discharge rights such that agent $j \in N$ receives $x_j = min\{c_j, E - \sum_{i<j} x_i\}$.

Definition 4 (Downstream Priority Rule)**.** The Downstream Priority Rule for a river pollution claims problem $\omega = (N, E, c) \in \Omega$ allocates discharge rights such that agent $i \in N$ receives $x_i = min\{c_i, E - \sum_{j>i} x_j\}$.

We apply the relevant axioms introduced in Section 3 and characterize these two solutions to the river pollution claims problem, starting with the Upstream Priority Rule.

Theorem 1. On the class of river pollution claims problems, the Upstream Priority Rule is the only rule satisfying *Limited Consistency* and *Upstream Symmetry*.

Proof. It is straightforward to show that the Upstream Priority Rule satisfies *Limited Consistency* and *Upstream Symmetry*. We proof the inverse statement as follows.

Consider the two related problems $\omega = (N, E, c) \in \Omega$ and $\omega' = (N', E, c') \in \Omega$, where ω' differs from ω by adding a dummy agent completely upstream, i.e. an agent with a zero claim ordered before agent 1 that we, with slight abuse of notation, refer to as agent 0. Hence, $N' \equiv N \setminus 0$. By *Claims-Boundedness*, the dummy agent will receive a zero allocation. By *Limited Consistency*, allocations to all the other agents remain the same, so that $x_i = x'_i \; \forall i \in N$.

Next, consider the related problem $\omega'' = (N', E, c'') \in \Omega$ where c'' differs from c' only by $c''_j > c'_j$. By *Upstream Symmetry*, we have

$$x''_i - x'_i = x''_k - x'_k \text{ for any } i, k < j. \tag{1}$$

Each agent upstream of agent j has an equal change in his allocation. By *Non-Negativity* we have

$$x''_i - x'_i \geq -min\{x'_i | i \in N'\} = 0 \text{ for any } i < j. \tag{2}$$

The RHS is equal to zero because of the presence of the dummy agent. Next, by *Claims-Boundedness* we have

$$x''_i - x'_i \leq min\{c'_i - x'_i\} = 0 \text{ for any } i < j. \tag{3}$$

Again, the RHS is equal to zero because of the presence of the dummy agent. Combining (2) and (3), we now have a lower and upper bound that coincide such that $x''_i - x'_i = 0$ for any $i < j$. That is, allocations to agents upstream of agent j will not be affected by agent j's increased claim.

Next, we use this result to derive the Upstream Priority Rule. Consider problem $\omega''' = (N, E, c''')$ where the claims vector $c''' = (c_1, \ldots, c_{j-1}, c'''_j, 0, \ldots, 0)$ is such that the sum of claims is exactly equal to the pollution budget, i.e. $\sum_{i \leq j} c'''_i = E$. Obviously, we have $x_i = c'''_i$ for all $i \in N$. Now, create a sequence of $n + 1 - j$ problems $\omega_{i \geq j}$ to transform problem ω''' back into problem ω by lexicographically increasing agents' claims back to their original level, i.e. $c'''_i = c_i$. We do so starting with the claim by agent j and subsequently going downstream with claims by agent $j + 1$, $j + 2$, etc. In each of these games, we can apply the above result. Since we do this sequentially, we end up with $x_j = min\{c_j, E - \sum_{i<j} x_i\}$ for all $j \in N$. This defines the Upstream Priority Rule.

Finally, we show logical independence of the axioms.

1. The Proportional Rule satisfies *Limited Consistency* (Thomson 2003) but violates *Upstream Symmetry*.

2. The Second-Agent-First Rule satisfies *Upstream Symmetry* but violates *Limited Consistency*. This rule assigns the resource according to the order of agents, but unlike the Upstream Priority Rule, it swaps the priority of the first and second agent. Formally, the second-agent-first rule for a river pollution claims problem $\omega = (N, E, c) \in \Omega$

allocates discharge rights such that

$$x_j = \begin{cases} min\{c_2, E\} \text{ if } j = 2 \\ min\{c_1, E - x_2\} \text{ if } j = 1 \\ min\{c_j, E - \sum_{i<j} x_i\} \text{ if } j \geq 3. \end{cases}$$

□

In Theorem 1, *Limited Consistency* rules out all the priority orders that could lead to a mismatch between agents' identity and their location, leaving us with the Upstream Priority Rule, the Downstream Priority Rule and a class of symmetric rules that treats agents equally independent of their location. *Upstream Symmetry* excludes all the symmetric rules and most priority orders. Combining these two axioms, we get a unique solution to the river pollution claims problem. We proposed the *Upstream Symmetry* axiom in order to make sure that upstream agents are affected equally if one of the downstream agents increases his claim. It turns out that the resulting solution is not desirable from an environmental perspective since a given amount of pollution is likely to cause more negative impacts when it is discharged upstream compared to downstream. Clearly, there is a trade-off where imposing *Upstream Symmetry* – an axiom inspired by fairness considerations – comes at the cost of environmental damage.

The following corollary is closely linked to Theorem 1, replacing upstream priority with downstream priority and using the associated symmetry axiom. The proof and interpretation of this result is otherwise similar.

Corollary 1. On the class of river pollution claims problems, the Downstream Priority Rule is the only rule satisfying *Limited Consistency* and *Downstream Symmetry*.

We continue with characterizing the Downstream Priority Rule.

Theorem 2. On the class of river pollution claims problems, the Downstream Priority Rule is the only rule satisfying *Consistency* and *Don't Move Up* with $\alpha = 0$.

Proof. It is straightforward to show that the Downstream Priority Rule satisfies *Consistency* and *Don't Move Up* with $\alpha = 0$. We proof the inverse statement as follows.

Consider the two related problems $\omega = (N, E, c) \in \Omega$ and $\omega' = (N', E', c') \in \Omega$, where ω' differs from ω by removing a subset of agents. Specifically, remove all but two agents, such that only agents $i =$ and j remain with $i < j$. By *Consistency*, the remaining endowment is $E' \equiv E - \sum_{k \neq i,j} x_k$, and the corresponding agents and claims vectors are $N' = \{i, j\}$ and $c' = \{c_i, c_j\}$.

Next, consider the related problem $\omega'' = (N', E', c'') \in \Omega$, where the claims vector $c'' = (0, c_i + c_j)$ is such that the claim by agent i is transferred and added to agent j's claim. This transfer implies $0 < c'_i - c''_i = c''_j - c'_j$. Whenever we also have $x''_j < c'_j$, by *Don't Move Up* with $\alpha = 0$ applied to problems ω'' and ω', we have $x''_j \leq x'_j$, and given that there are only two agents, this implies $x''_i \geq x'_i$.

By *Claims-Boundedness*, $c''_i = 0$ implies $x''_i = 0$. By *Non-Negativity*, $x''_i = 0 \geq x'_i$ implies $x'_i = 0$. Hence, agent i will always get a zero allocation under problem ω' even though his claim is not zero, implying that agent j has priority over agent i: $x_i = min\{c_i, E - \sum_{j>i} x_j\}$. This defines the Downstream Priority Rule.

Finally, we show logical independence of the axioms.

1. The Constrained Equal Loss Rule satisfies *Consistency* (Thomson 2003) but violates *Don't Move Up* with $\alpha = 0$.

2. The following rule satisfies the *Don't Move Up* with $\alpha = 0$ but violates *Consistency*: First assign the river pollution budget by the Downstream Priority Rule, then take ϵ away from the most downstream agent and assign ϵ to agent 1. Note that ϵ should be small enough such that *Claims-Boundedness* or *Efficiency* is not violated. It is obvious that when agent 1 leaves the claims problem with his share, the former agent 2 will become the new agent 1 and receive a different allocation, which violates *Consistency*.

□

In Theorem 2, *Consistency* rules out all the priority orders that could lead to a mismatch between agents' identity and their location. The *Don't Move Up* axiom with $\alpha = 0$ excludes all the equal treatment rules and most priority orders. Combining these two axioms, we get a unique solution to the river pollution claims problem. The Downstream Priority Rule could ideally keep the pollution downstream and this solution is as expected given that we propose the *Don't Move Up* axiom to prevent undesirable claim transfers from downstream to upstream, but there are two reasons why this rule may not be realistic. First, although pollutants discharged from upstream will cause damage along a longer stretch of the river, some pollution could be mitigated by the absorptive capacity of the river. The Downstream Priority Rule may lead to situations where upstream agents get a zero allocation and downstream agents get a share that is far beyond the river absorptive capacity, resulting in a situation where this capacity is left unused. Such a solution is not efficient. Second, if the jurisdictions involved have the autonomy to decide whether to join or leave the claims problem, upstream agents lack incentives to participate in such an agreement. While this criticism applies to all solutions to the river pollution claims problem, it is stronger in case of solutions with clear winners and losers like the Upstream and Downstream Priority Rules.

5 Conclusion

We propose the river pollution claims problem to distribute a limited pollution budget among agents located along a river. A key distinction with the earlier literature is that agents are ordered and they are given priority based on their *location* in this order instead of their *identity*. The two characterization results show that *Consistency* plays an important role since it makes sure that any asymmetric treatment will be transferred across problems. As a result, agents that are treated particularly (un)favorable in the initial problem because of their location, will not be in the same location in some reduced problem. When we apply *Consistency*, which says that agents should be treated 'as asymmetrically as' they were initially (Thomson 2012), only two priority rules survive. We propose two new axioms, *Upstream Symmetry* and *Don't Move Up*, to characterize each of these rules. Both are specifically relevant in the river context. *Upstream Symmetry* is inspired by fairness considerations but it ignores the transfer of pollution along the river, which is covered by *Don't Move Up*. We only discuss the two extreme cases of this latter axiom, where $\alpha = 0$ or $\alpha = 1$, leaving other cases for future research.

Our contribution extends beyond introducing the river pollution claims problem and proposing two solutions to this problem. We contribute to the literature by proposing a type of claims problem in which *Equal Treatment of Equals* is not desirable because the agents are ordered, and in which it is desirable that the agents' position in this order leads to asymmetric treatment. There are many other problems where agents have exogenous priority orders. For instance, in the classic bankruptcy problem, creditors may belong to different priority classes because there are secured debts and unsecured debts. In the school choice problem proposed in Abdulkadiroğlu and Sönmez (2003), priority orders of students are determined by exogenous factors such as the number of siblings or the distance from home to school and these orders are imposed by state or local laws. In the estate distribution problem, priority orders are determined by exogenous factors such as age or income level. Our approach may shed new lights on such related problems.

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