TI 2022-041/VIII
Tinbergen Institute Discussion Paper

Carbon Capture: Storage vs. Utilization

Michel Moreaux[1]
Jean-Pierre Amigues[1]
Gerard van der Meijden[2]
Cees Withagen[2]

1 Toulouse School of Economics

2 Vrije Universiteit Amsterdam, Tinbergen Institute

Tinbergen Institute is the graduate school and research institute in economics of Erasmus University Rotterdam, the University of Amsterdam and Vrije Universiteit Amsterdam.

Contact: discussionpapers@tinbergen.nl

More TI discussion papers can be downloaded at https://www.tinbergen.nl

Tinbergen Institute has two locations:

Tinbergen Institute Amsterdam
Gustav Mahlerplein 117
1082 MS Amsterdam
The Netherlands
Tel.: +31(0)20 598 4580

Tinbergen Institute Rotterdam
Burg. Oudlaan 50
3062 PA Rotterdam
The Netherlands
Tel.: +31(0)10 408 8900

Carbon Capture: Storage vs. Utilization*

Michel Moreaux†
Toulouse School of Economics

Jean-Pierre Amigues‡
Toulouse School of Economics

Gerard van der Meijden§
Vrije Universiteit Amsterdam
Tinbergen Institute

Cees Withagen¶
Vrije Universiteit Amsterdam
Tinbergen Institute

June 29, 2022

Abstract

Carbon capture and storage (CCS) seems an appealing option to meet the ambitious objectives of the Paris Agreement. Captured CO_2 emissions can also be injected in active fields to enhance recovery: Carbon capture and utilization (CCU). We study a dynamic model of CCS and CCU of an economy subject to a carbon budget. We demonstrate that if the social planner implements CCU, it does so at the beginning of the planning period and stops before the budget has been depleted. On the contrary, if CCS occurs in the social optimum, this happens only once the carbon budget has been depleted. We show that the relationship between the carbon budget and the carbon price can be non-monotonic if CCU occurs. Our model features three state variables: The stock of fossil fuel, the stock of atmospheric CO_2 and the stock of injected CO_2 in active fields. We derive frontiers that separate regions in initial-stock-space with and without CCS and CCU regimes in the social optimum. Finally, we compare the social optimum with the decentralized market outcome.

Keywords: global warming; carbon capture and storage; enhanced recovery; non-renewable resources; renewable resources.

JEL classifications: Q30, Q35, Q42, Q54.

---

*The authors declare that they have no (financial) conflicts of interest. The authors would like to thank participants at the CESifo Area Conference on Energy & Climate Economics (Munich, October 2019), the Natural Resource Management under Catastrophic Threats conference (Rehovot, January 2020), the SURED conference (Ascona, June 2020), the EAERE conference (Berlin, June 2020), the SWEEET conference (Paris, January 2022) and the Eureka seminar (Amsterdam, March 2022) for their valuable comments.

†During the process of writing this paper Michel Moreaux passed away. We miss him.

‡Toulouse School of Economics, INRAE, University of Toulouse Capitole, 1 Esplanade de l'Université, 31080, Cedex 06, Toulouse, France. E-mail: jean-pierre.amigues@inrae.fr.

§Department of Spatial Economics, Vrije Universiteit Amsterdam, De Boelelaan 1105, 1081 HV, Amsterdam, The Netherlands. E-mail: g.c.vander.meijden@vu.nl.

¶Corresponding author. Department of Spatial Economics, Vrije Universiteit Amsterdam, De Boelelaan 1105, 1081 HV, Amsterdam, The Netherlands. E-mail: cwithagen@feweb.vu.nl.

1 Introduction

This paper examines the role of Carbon Capture and Storage (CCS) and Carbon Capture and Utilization (CCU) in the transition from fossil to renewable energy. The remaining carbon budget associated with keeping the increase in the global mean temperature to 1.5 degrees with a probability of 50 percent is equal to 500 gigatonnes of CO_2 (IPCC, 2022), around 14 times the current yearly emissions. One way to stay within this carbon budget is to drastically reduce global fossil fuel consumption. According to Welsby et al. (2021), 89 percent of coal, 56 percent of gas and 58 percent of oil reserves should remain unextracted to secure a '1.5 degree world' with a 50 percent probability.[1] An additional way to reduce emissions is to capture them from large point sources and store them underground in geological formations (CCS). Using this option allows for longer usage of fossil fuels and reduces the value of stranded fossil assets. Moreover, it is a way to lower the emissions from sectors that are hard to decarbonize, such as steel production (cf. Herzog, 2009; IPCC, 2022).

There seems to be no problem in terms of space. The available storage capacity of 1 terratonne CO_2 exceeds the requirements to stay within the 1.5 degrees threshold (IPCC, 2022). But there is a problem in terms of costs. The capture, transport and storage of CO_2 is still quite expensive. Schmelz et al. (2020) report averages for US states varying between 50 and 100 US$ per tonne CO_2. As a result, CCS does not yet occur on a large scale. In 2021, 45 megatonnes of CO_2 were captured, only 0.1 percent of global emissions (IEA, 2021a). However, instead of injecting the captured CO_2 in empty reservoirs, which yields no benefit apart from the prevented climate damage, the captured emissions can also be *utilized* (CCU) to create value (cf. Hepburn et al., 2019). Yearly, around 230 megatonnes of CO_2 are used commercially—mainly to produce fertilizers and for enhanced oil recovery—but also for soft drinks, decaffeination of coffee, chemical solvents, cooling, water treatment and in greenhouses (IEA, 2019, 2021a).

These commercial uses of CO_2 lower the net cost of capturing emissions. Furthermore, there is considerable potential for cost reduction of CCS and CCU (IEA, 2021b).

[1] See McGlade and Ekins (2014, 2015); Rezai and Van der Ploeg (2017); Van der Ploeg and Rezai (2017a, 2020) for more on the subject, and Van der Ploeg and Rezai (2017b) for the financial implications of stranded assets.

The most recent IPCC report explicitly mentions CCS and CCU as viable options to mitigate global warming (IPCC, 2022). Nevertheless also scepticism can be found in the scientific literature. An important exponent of the view is Jacobson (2020). He is not convinced of the positive effect of carbon capture and argues that the expected capturing rate of 85 to 90 percent of emissions is an overestimation. One reason is that a power plant with capture equipment needs 25 to 50 percent more energy to run the equipment, which leads in addition to more emissions at the mining and transportation level. Moreover, leaks of sequestered carbon will occur. Finally, it turns out that actual capturing is not very successful so far (Jacobson, 2020).

In this paper, we examine under which conditions it is optimal to use CCS and/or CCU and in which stage of the energy transition an economy should deploy these technologies, given that cumulative emissions are limited by a carbon budget. Regarding CCU, we focus on the utilization of captured CO_2 for enhanced oil or gas recovery (EOR or EGR). The reason is that, in contrast to most of the other commercial uses of CO_2, EOR and EGR result in permanent storage of CO_2, implying that a genuine emission *reduction* instead of an emission *delay* is realized. The idea behind EOR and EGR is to inject CO_2 into active oil and gas fields to increase the reservoir pressure, which increases the recovery factor, i.e. share of the reservoir that can be exploited.[2]

We derive the following main results from a linear-quadratic specification of our model. First, both CCS and CCU increase cumulative extraction of fossil fuel but lower cumulative CO_2 emissions. Second, if CCS and CCU are both used, the social planner uses CCU initially, then stops with capturing CO_2 and starts with CCS as soon as the carbon budget is depleted. The reason is that CCU is profitable as long as the discounted value of the future extraction cost savings due to increasing the pressure in the well is large enough. This is the case in the beginning, when the remaining extraction horizon is still large. Moreover, the only benefit from CCS is prevention of the climate catastrophe. Due to discounting, it is then beneficial to start as late as possible with CCS, i.e. to capture all emissions only once the carbon budget has been depleted. Third, a *laissez-faire* market economy never uses CCS, but may use CCU initially. Under laissez-faire, CCU stops earlier than in the social optimum. Fourth, we have developed a

[2]EOR has increased recovery factors by as much as 100 percent (IEA, 2018), whereas EGR typically increases the recovery factor by 4 to 15 percentage points. On the different EOR methods, see for example Mischenko (2001) and Núñez-López and Moskal (2019). Details about the process of EGR can be found in Oldenburg et al. (2004).

graphical apparatus to show the conditions under which a CCU and a CCS phase exist in the social optimum. Fifth, we quantify our results with numerical simulations and find that substantial cost reductions in CCS and CCU are required to make a large-scale use of both technologies optimal. Moreover, we numerically show that the relationship between the carbon budget and the carbon price can be non-monotonic if CCU occurs in the social optimum.

Our paper is related to the analytical literature on non-renewable resource use and global warming that allows for CCS as an abatement option. Studies within this field typically impose a carbon budget to prevent a climate catastrophe. An early contribution by Lafforgue et al. (2008) studies optimal carbon sequestration policy with limited storage capacity in carbon sinks. Coulomb and Henriet (2011) and Amigues et al. (2014) examine CCS with different abatement costs between sectors. Jaakkola (2012) deals with CCS infrastructure investment and an imperfectly competitive fossil fuel market. Moreaux and Withagen (2015) study optimal use of CCS with a climate damage function instead of a carbon budget, whereas Amigues et al. (2016) allow for learning-by-doing in CCS. Belfiori (2017) looks at the effect of a difference between the social and private discount rates on the optimal subsidy for carbon sequestration in a market equilibrium. Finally, Durmaz and Schroyen (2019) introduce CCS in the directed technical change model developed by Acemoglu et al. (2012).[3]

A crucial element in our model is the dependence of extraction costs on the pressure within the reservoir. Due to the progressive decline of the pressure in the reservoirs as a result of the extraction process, a substantial part of the exploited oil and gas fields will be left underground. Typically, only around 30 percent of the oil reserves and 65 percent of the gas reserves of the exploited fields can be recovered (cf. Khan et al., 2013; IEA, 2018). The flow of oil and gas through reservoir rock toward the extraction wells depends upon the geological properties of the fields. A first formal characterization of this process has been given by Darcy (1856) in another context.[4] Recently, Mason and Van 't Veld (2013) have proposed a new model of oil extraction based on what is known as the Darcy's Law, which essentially states that the extraction rate is proportional to the pressure differential between the reservoir and the well (cf. Anderson et al., 2018).

Early economic studies of secondary recovery through pressure maintenance can be

[3] A more extensive overview of the literature on the economics of CCS is provided by Durmaz (2018).
[4] Darcy was in charge of the water fountains of the city of Dijon, France.

found in Amit (1986) and Cairns and Davis (2001). More recently, Leach et al. (2011) and Van 't Veld et al. (2013) make the link with carbon sequestration, by developing a model of CCU through EOR. However, these studies abstract from climate damages and focus at the level of an individual firm, or even at the level of a single well. Our aim here is to consider CCU from the perspective of a social social planner facing a cap on cumulative carbon emissions in order to restrict the mean global temperature increase. This implies a shadow cost on carbon emissions that may stimulate CCS and CCU. We present a simplified exploratory model of a fossil fuel industry which can resort to CCS and CCU in a society subject to a carbon budget. We are also interested in the difference between the social optimum and the *laissez-faire* market outcome. We acknowledge the differences between different types of fossil fuels, e.g. in terms of ease of capturing, and between different types of renewable energy sources, but we have chosen to work on a high level of aggregation for expository purposes.

The rest of our paper is organized as follows. Section 2 describes the model. Section 3 characterizes the social optimum. Section 4 derives the social optimum for a linear-quadratic specification of the model. Section 5 provides a numerical illustration. Section 6 concludes.

2 The model

The economy can produce useful energy from two sources. The first one is a polluting fossil non-renewable resource. Let $X(t)$ be the available fossil resource stock at time t and let $x(t)$ denote the extraction rate of fossil fuel, so that:

$$\dot{X}(t) = -x(t), \tag{1}$$

with $X(t) \geq 0$ and $x(t) \geq 0$. Let $X_0 = X(0)$ be the initial endowment of fossil fuel. The other source is renewable and non-polluting (solar, wind). Let $y(t) \geq 0$ be the production rate of renewables. We assume convex production cost, $C_y(y)$. At the end-user stage all energy sources are perfect substitutes. Assuming that energy is not storable we write $q(t) = x(t) + y(t)$, where $q(t)$ denotes the consumption rate of useful energy. The use of energy yields utility $U(x + y)$, U is a concave function.

2.1 CO_2 accumulation

Burning fossil fuel $x(t)$ to produce useful energy generates greenhouse gases, $\zeta x(t)$, in particular CO_2. The fossil energy transformation industry has access to an abatement technology to capture these gases before they are released into the atmosphere. We denote by $a(t)$ the abatement rate of greenhouse gases. Let $Z(t)$ denote the atmospheric carbon stock at time t and $Z_0 = Z(0)$ the initial atmospheric stock inherited from the past. If, as we assume, carbon in the atmosphere does not decay, the motion of the CO_2 stock is given by:

$$\dot{Z}(t) = \zeta x(t) - a(t). \tag{2}$$

We take it that capturing CO_2 directly from the atmosphere is prohibitively costly. Hence, the captured flow is bounded from above by the potential emission flow, that is $x(t)$ and $a(t)$ are subject to the constraint; $\zeta x(t) - a(t) \geq 0$.

Following Chakravorty et al. (2006), we assume that if some critical atmospheric concentration threshold $\bar{Z}$ is crossed by the accumulated CO_2, the climate conditions on earth become catastrophic. Thus, society should stick to the ceiling constraint, $\bar{Z} - Z(t) \geq 0$. Below the ceiling we assume that climate damages are negligible. In order for the model to make sense, we assume that $Z_0 < \bar{Z}$: Initially, the carbon constraint is not binding. Note that in the absence of abatement possibilities, either $\zeta X_0 > \bar{Z} - Z_0$ and burning the entire stock of fossil fuel is incompatible with the carbon ceiling, or $\zeta X_0 < \bar{Z} - Z_0$ and the climate constraint will never become binding. In the first case, some fraction of the resource stock will remain underground and fossil fuel is abundant, yielding no pure scarcity rent. Since with abatement possibilities, the climate constraint cannot bind *a fortiori* if $\zeta X_0 < \bar{Z} - Z_0$, we assume $\zeta X_0 > \bar{Z} - Z_0$ for the climate constraint to be relevant. Depending on the abatement policy, the resource can be either scarce or abundant as we define and show below.

2.2 Abatement and carbon sequestration

Let $C_a(a)$ denote the convex cost of carbon capture. There exist two carbon sinks. Captured gas can be stored at no cost in inert, or inactive, reservoirs that are sufficiently large to never face storage limits. This has been defined as *Carbon Capture and Storage*

(CCS). By $b(t) \geq 0$ we denote the captured gas flow stored in these carbon reservoirs. The other carbon sinks are provided by the fossil fuel extractive industry which injects the captured gas into the wells at a rate denoted by $s(t) \geq 0$. This has been defined as *Carbon Capture and Utilization* (CCU). Full storage of all captured gas implies $a(t) - s(t) - b(t) = 0$ so that (2) can be written as:

$$\dot{Z}(t) = \zeta x(t) - b(t) - s(t). \tag{3}$$

Let $S(t)$ denote the sequestered carbon stock in the fossil fuel wells at time t and let $S_0 = S(0)$ denote the initial carbon stock in the fossil fuel wells. The motion of the gas stock in these wells obeys:

$$\dot{S}(t) = s(t). \tag{4}$$

We assume convex injection cost $C_s(s)$. We define full capture by $\zeta x = a = b + s \geq 0$, full CCS by $b = a > 0$ so that $s = 0$, and full CCU by $s = a > 0$ so that $b = 0$.

2.3 Fossil fuel extraction costs

The gas injected into the fossil fuel wells contributes to an increase of the pressure in the wells, easing the extraction process. To formalize this idea we assume that the unit extraction cost decreases with the matter content of the field. Assume that fossil fuel and CO_2 can mix perfectly in the reservoir.[5] Let $\bar{R}$ denote the size of the reservoir. Then the ratio $(X + \alpha S)/\bar{R}$ is a measure of the pressure in the field, α being a conversion parameter. Normalize the size of the reservoir to unity so that $X + \alpha S$ measures the pressure.[6] Let $G(X + \alpha S)$ denote the unit cost function as a function of the pressure, thus a total extraction cost of $G(X + \alpha S)x$. The unit cost function $G(.)$ is twice continuously differentiable and decreasing.

We want to avoid the possibility that extraction may become cheaper over time by excessively increasing the pressure in the well. The time derivative of G is $G'(X +$

[5] See Mischenko (2001) and Oldenburg et al. (2004) for a description of the CO_2 injection process in petroleum and gas fields, respectively.

[6] An alternative interpretation is that injection of CO_2 increases the amount of fossil fuel that can be extracted at a given cost. The total recoverable stock of fossil fuels then equals $X + \alpha S$, consisting of conventional reserves X and enhanced reserves αS.

$\alpha S)(-x + \alpha s)$. It is negative if $\alpha s > x \geq s/\zeta$, which is ruled out if $\alpha\zeta < 1$. In the sequel this assumption is made.

3 The social optimum

The social planner must determine a fossil fuel extraction policy, a renewable energy production policy, a carbon storage policy in inert reservoirs and a carbon emissions injection policy in the active fossil fuel wells, maximizing social welfare, that is, the social planner solves the following social planning problem:

$$\max_{x,y,b,s} \int_0^{\infty} \left[U(x+y) - G(X+\alpha S)x - C_y(y) - C_a(b+s) - C_s(s)\right] e^{-\rho t} dt \tag{SP}$$
$$s.t.\ (1),\ (3),\ (4),$$
$$\bar{Z} - Z(t) \geq 0,$$
$$\zeta x - b - s \geq 0,$$
$$x \geq 0\ ,\ y \geq 0\ ,\ b \geq 0\ , s \geq 0,$$

where $\rho > 0$ is the constant social discount rate. We omit the time index t when there is no danger of confusion. Let λ_s, λ_x and λ_z denote the co-state variables associated with S, X and Z, respectively. The present-value Hamiltonian associated with optimization problem (SP) reads:

$$\begin{aligned}\mathcal{H} &= \left[U(x+y) - G(X+\alpha S)x - C_y(y) - C_a(b+s) - C_s(s)\right] e^{-\rho t} \\ &\quad +\lambda_s s - \lambda_x x + \lambda_z(\zeta x - b - s).\end{aligned}$$

Denote by properly indexed γs the Lagrange multipliers associated with the positivity constraints on x, y, b and s and by μ the Lagrange multiplier associated with the full abatement constraint, $\zeta x - b - s \geq 0$. Let ν denote the multiplier associated with the carbon budget constraint, $\bar{Z} - Z \geq 0$. The Lagrangian associated with optimization problem (SP) then reads:

$$\mathcal{L} = \mathcal{H} + \gamma_x x + \gamma_y y + \gamma_b b + \gamma_s s + \mu(\zeta x - b - s) + \nu(\bar{Z} - Z).$$

The shadow price λ_z of the atmospheric CO_2 stock is negative. We therefore define $\tau = -\lambda_z$ as the positive social cost of carbon, in utility terms. A first set of necessary conditions is:

$$e^{-\rho t}U'(x+y) = e^{-\rho t}G(X+\alpha S) + \lambda_x + \zeta\tau - \zeta\mu - \gamma_x, \tag{5a}$$

$$e^{-\rho t}U'(x+y) = e^{-\rho t}C_y'(y) - \gamma_y, \tag{5b}$$

$$\tau = e^{-\rho t}C_a'(b+s) + \mu - \gamma_b, \tag{5c}$$

$$\lambda_s + \tau = e^{-\rho t}(C_a'(b+s) + C_s'(s)) + \mu - \gamma_s, \tag{5d}$$

together with the usual complementary slackness conditions. The co-state variables satisfy:

$$\dot{\lambda}_x = e^{-\rho t}G'(X+\alpha S)x, \tag{6a}$$

$$\dot{\lambda}_s = \alpha e^{-\rho t}G'(X+\alpha S)x, \tag{6b}$$

$$\dot{\tau} = -\nu. \tag{6c}$$

Lastly, the following transversality conditions hold:[7]

$$\lim_{t\uparrow\infty} \lambda_x(t)X(t) = 0, \tag{7a}$$

$$\lim_{t\uparrow\infty} \lambda_s(t) = 0, \tag{7b}$$

$$\lim_{t\uparrow\infty} \tau[\bar{Z} - Z(t)] = 0. \tag{7c}$$

The interpretation of the necessary conditions is straightforward. Suppose there is an interval of time, $\mathcal{T}$, with positive extraction, $x(t) > 0$ so that $\gamma_x(t) = 0$, for all $t \in \mathcal{T}$. Then the present value of the marginal benefit associated with fossil fuel extraction equals $e^{-\rho t}U'(q)$. Equation (5a) states that this marginal benefit must equal the full fossil fuel use marginal cost. The marginal cost associated with extraction consists of the sum of the direct extraction cost, $e^{-\rho t}G(X+\alpha S)$, the marginal cost of extraction now rather than in the future, λ_x, and the emission factor, ζ, multiplied by the net

[7] Under the interpretation that injection of CO_2 increases the the amount of fossil fuels that can be extracted at given marginal costs (instead of letting CO_2 injection decrease the marginal extraction costs of the remaining stock), transversality conditions (7a)-(7b) should be replaced by $\lim_{t\uparrow\infty} \lambda_x(t)\,(X(t)+\alpha S(t)) = \lim_{t\uparrow\infty} \lambda_s(t)\,(X(t)+\alpha S(t)) = 0$. Furthermore, we then need $X(t) + \alpha S(t) \geq 0$ instead of $X(t) \geq 0$.

marginal cost of accumulating CO_2, $\tau - \mu$. With less than full carbon capture, $\mu = 0$ and this latter term is equal to the social cost of carbon, τ. With full CCS (full CCU) it equals the marginal capturing costs, $C_a'(a)$ (the sum of the marginal capturing and injection costs, $C_a'(b+s) + C_s'(s)$).

If renewables are used, $y > 0$, equation (5b) states that their marginal benefit equals their marginal cost. Capturing gas for storage in the inert reservoirs, i.e. $b > 0$ (CCS), brings a marginal benefit τ in terms of avoided shadow cost of pollution. Equation (5c) states that this benefit must cover the carbon capture present-value marginal cost, possibly augmented by the scarcity rent on available gas in case of full abatement of emissions. As shown on the left-hand side of (5d), injecting captured emissions in the active fossil fuel wells, $s > 0$ (CCU), brings along an additional benefit λ_s, the carbon rent from CCU, in addition to the avoided carbon pollution shadow cost, τ. This total benefit must balance the sum of the present value capture and injection cost, and, in case of full abatement, the scarcity rent on available gas.

Conditions (6a) and (6b) are the Hotelling rules for the resource stock and for the injected carbon stock, respectively. Equation (6c) states that the shadow cost of pollution, $\tau > 0$, or, equivalently, the optimal carbon tax, is constant in present value terms as long as the carbon cap constraint does not bind, and decreases when it is binding in the end. The interpretation of the transversality conditions is straightforward. If some fossil fuel is left in the ground, its value must be zero; the value of inserted CO_2 must be zero eventually; and the shadow cost of atmospheric CO_2 is zero if the ceiling will not be reached.

4 The linear-quadratic case

With three state variables, X, S and Z, it is generally difficult to obtain clear-cut results. In order to gain more analytical insights, specific functional forms are helpful. They also serve the purpose of attaining numerical results from simulations. Hence, in the sequel we restrict ourselves to the linear-quadratic case.

4.1 Necessary conditions for optimality

We adopt the following functional forms:

$$U(x+y) = \beta(x+y) - \gamma(x+y)^2/2, \tag{8a}$$

$$G(X+\alpha S) = \psi - \delta(X+\alpha S), \tag{8b}$$

$$C_a(b+s) = c_a(b+s), \tag{8c}$$

$$C_s(s) = c_s s, \tag{8d}$$

$$C_y(y) = c_y y, \tag{8e}$$

where β, γ, ψ, δ, c_a, c_s and c_y are positive constants. They must satisfy several conditions to which we will pay attention in due course. Hence, we assume quadratic utility, a per unit fossil fuel extraction cost that is linear in the pressure and linear capture and insertion costs. The necessary conditions now read

$$e^{-\rho t}(\beta - \gamma(x+y)) = e^{-\rho t}(\psi - \delta(X+\alpha S)) + \lambda_x + \zeta\tau - \zeta\mu - \gamma_x, \tag{9a}$$

$$e^{-\rho t}(\beta - \gamma(x+y)) = e^{-\rho t}c_y - \gamma_y, \tag{9b}$$

$$\tau = e^{-\rho t}c_a + \mu - \gamma_b, \tag{9c}$$

$$\lambda_s + \tau = e^{-\rho t}(c_a + c_s) + \mu - \gamma_s, \tag{9d}$$

together with the usual complementary slackness conditions. The co-state variables satisfy:

$$\dot{\lambda}_x = -\delta e^{-\rho t}x, \tag{10a}$$

$$\dot{\lambda}_s = -\alpha\delta e^{-\rho t}x, \tag{10b}$$

$$\dot{\tau} = -\nu. \tag{10c}$$

We assume now, and later prove, that there is a final phase with use of renewables only. This phase starts at a time T_3. For T_3 to be larger than zero it is obviously necessary that the unit cost of renewables, c_y, is larger than $G(X_0 + \alpha S_0)$. Let T_2 be the time at which the carbon cap constraint begins to be active, $Z(T_2) = \bar{Z}$, or, equivalently, the 'carbon budget', $\bar{Z} - Z_0$, to be exhausted. Since after T_3 the stock of atmospheric CO_2 remains constant, emissions being zero, we must have $T_2 \leq T_3$, for the carbon cap constraint to be ever active.

Released carbon emissions will remain in the atmosphere forever. Thus the carbon budget $\bar{Z} - Z$ effectively is an exhaustible resource. Hence, the shadow cost of carbon must fulfill the Hotelling rule, being constant in present value terms and rising at the discount rate in current-value terms before T_2, the depletion time of the carbon budget.

At time T_2 three options arise. First, if the fossil fuel stock is exhausted, then $T_2 = T_3$ and the economy directly enters the 100 percent renewable energy, or carbon-free, regime at the time it exhausts its carbon budget. Second, it may also be that some fossil fuel reserves remain at time T_2, the economy leaves these reserves underground and enters the carbon-free regime, so that $T_2 = T_3$. Third, the economy continues to use fossil fuel and implements full capturing of carbon emissions during a time interval $[T_2, T_3)$ before entering the carbon-free regime. In this last case, it is also possible that the economy ultimately leaves some fraction of the fossil fuel reserves underground at time T_3.

Before T_2 the economy may or may not abate its emissions and positive abatement can be split in several ways between injection in inert reservoirs (CCS) or injection in eventually active fossil fuel wells (CCU). To disentangle all these possibilities we now analyze in detail the implications of the optimality conditions. Under our set of assumptions we show that optimality excludes: (i) the joint production of fossil and renewable energies, (ii) partial positive capturing of emissions (iii) the simultaneous use of CCS and CCU, and (iv) the use of CCU when the carbon budget has been exhausted. More formally, we prove:

Lemma 1

(i) There is no non-degenerate interval of time $\mathcal{T}$ with $x(t) > 0$ and $y(t) > 0$ for all $t \in \mathcal{T}$.

(ii) If there is a non-degenerate interval of time $\mathcal{T}$ with $\zeta x(t) > b(t) + s(t)$ then $s(t) = b(t) = 0$ for all $t \in \mathcal{T}$.

(iii) There is no non-degenerate interval of time $\mathcal{T}$ with $s(t) > 0$ and $b(t) > 0$ for all $t \in \mathcal{T}$.

(iv) Suppose $T_2 < t < T_3$. Then $b(t) > 0$ for all $T_2 < t < T_3$.

Proof. See Appendix A. ■

Since we focus on a scenario where the climate constraint eventually binds so that fossil fuel is exploited, claim (i) of the lemma implies that renewable energy will be produced only when the economy has decided to no longer consume fossil fuel, either because the fossil fuel reserves are exhausted, or because their exploitation cost has become too high compared to the renewable alternative. Claim (ii) of the lemma implies that either the economy does not abate emissions at all, or it implements full abatement. In this last case claim (iii) of the Lemma states that CCU and CCS never occur at the same time: The economy either injects all CO_2 emissions in fossil fuel wells or stores the entire CO_2 flow in inert reservoirs. In case $T_2 < T_3$, claim (iv) states that the CCS option dominates the CCU one throughout the time interval (T_2, T_3). If $X(T_2) > 0$, the economy implements full abatement until the ultimate transition to renewable energy, while storing the CO_2 emissions in inert reservoirs.

Let us now turn to the question whether, when and in which sequence CCU and CCS regimes can occur. Before T_2, a CCS regime can be excluded. Assume, to the contrary, that at some time $t_0 < T_2$ storage in inert reservoirs becomes possible. We know that τ is constant over time from (10c), so that (9c) implies that μ must increase over time. Hence, once a CCS regime occurs, it will not end before the switch to renewables at $t = T_3$. But with full abatement, we have $\zeta x = b$ until the switch to renewables so that $Z(t) = Z(t_0)$ and the carbon budget is never exhausted. This would imply that $\tau = 0$ leading to a contradiction because the first order condition (9c) is not satisfied when $b > 0$. We thus conclude that storage in inert reservoirs can happen only after the depletion of the carbon budget. This is intuitive as well, because the only benefit from CCS is prevention of the climate catastrophe. Due to discounting, it is then beneficial to start as late as possible with CCS, i.e. to capture all emissions only once the ceiling has been reached.

It remains possible that the economy uses the CCU option before T_2. This can be seen from combining (9c) and (9d), which gives

$$\lambda_s = c_s e^{-\rho t} + \gamma_b - \gamma_s. \tag{11}$$

Since $b = 0$ before T_2, and thus $\gamma_b \geq 0$, it follows that $s > 0$ implies $\lambda_s \geq c_s e^{-\rho t}$, because

$\gamma_s = 0$. After T_2, $b > 0$ and $s = 0$ implies that $\gamma_b = 0$ and $\gamma_s \geq 0$ so that $\lambda_s \leq c_s e^{-\rho t}$. Since $\lambda_s(t)$ is non-increasing over time due to (10b), there exists an instant of time $T_1 < T_2$ such that the economy stops using CCU and then waits until the depletion of the carbon budget to perform CCS. The strict inequality $T_1 < T_2$ results from the impossibility to have an injection plan lasting until the depletion of the carbon budget. Full injection of CO_2 emissions prevents the depletion of the carbon budget. Furthermore, if $\lambda_s(0) < c_s$, the injection phase collapses and the economy starts abatement only after exhaustion of its carbon budget.

However, it is not possible to have an initial regime without CCU followed by a CCU regime. To see this, let us assume that this sequence of regimes does exist. Hence, there will first be a switch to a regime with CCU and then to a regime without CCU (as a CCU regime must end before the switch to renewables). Note that at each of these two regime switches, continuity requires $\mu = \gamma_s = 0$. It follows from (9d) and (10b) that the growth rate of the left-hand side of (9d) at these moments is given by $-\alpha\delta x(t)/(c_a + c_s)$. The growth rate of the right-hand side is constant and equal to $-\rho$. In a regime with (without) CCU, the left-hand side of (9d) is higher (lower) than the right-hand side. Hence, the growth rate of the left-hand side decreases between the two switching times. This implies that the extraction rate $x(t)$ increases over this interval. However, the extraction rate is non-increasing over time, as shown in Section 4.2. Hence, we obtain a contradiction.

So, if CCU is used, it will be done during a temporary phase in the beginning. The intuition behind this outcome is that CCU is profitable as long as the discounted value of the future extraction cost savings due to increasing the pressure in the well is large enough. This is the case in the beginning, when the remaining extraction horizon is still large.[8]

The following proposition summarizes the discussion.

Proposition 1 *Suppose X_0 is large enough. Then there exist $0 \leq T_1 < T_2 \leq T_3$ such that*

(i) $\zeta x(t) = s(t) > 0$ and $y(t) = 0$ for $0 \leq t \leq T_1$,

[8]If CCU is assumed to increase the physically recoverable stock instead of to lower the extraction costs through an increase in well pressure (see footnote 6), CCU is profitable as long as the profit per unit of the resource stock, i.e. the scarcity rent on fossil fuels, is large enough. This again is the case in the beginning, when unit extraction costs are still relatively small.

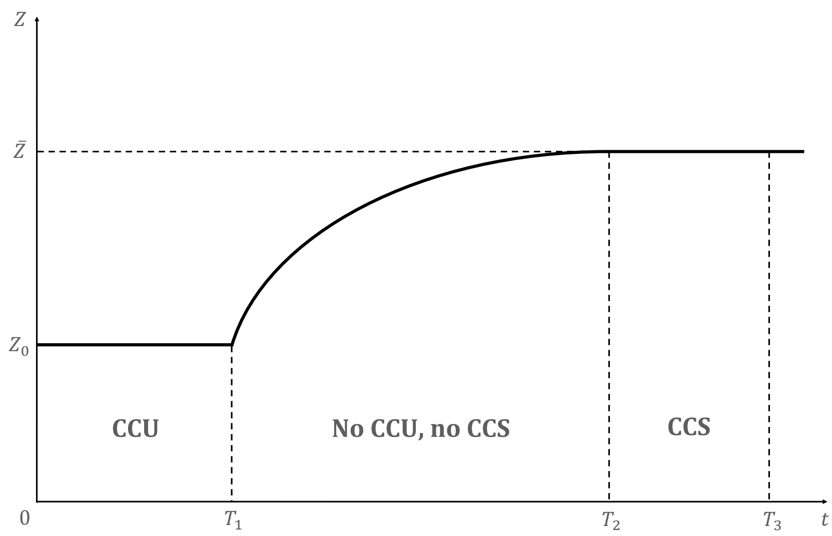
(ii) $\zeta x(t) > s(t) = b(t) = 0$ *and* $y(t) = 0$ *for* $T_1 \leq t \leq T_2$*,*

(iii) $\zeta x(t) = b(t) > 0$ *and* $y(t) = 0$ *for* $T_2 \leq t \leq T_3$*,*

(iv) $x(t) = 0,\ y(t) > 0$ *for* $t > T_3$*.*

In Figure 1 we depict the motion of the atmospheric CO_2 stock over time. It is assumed that there are no degenerate intervals. However, it cannot be excluded that e.g. $T_1 = 0$ or $T_2 = T_3$.

Figure 1: Evolution of the atmospheric CO_2 concentration



4.2 Three regimes

Lemma 1 has shown that an optimal plan is some sequence of at most three transitory regimes before the ultimate transition to renewable energy:

(i) A CCU regime where $b = 0$, $s > 0$ and $x > 0$;

(ii) A regime without CCU or CCS where $s = b = 0$ and $x > 0$;

(iii) A CCS regime where $b > 0$, $s = 0$ and $x > 0$.

We now browse through the different regimes to present their characteristics.

4.2.1 The CCU regime

Along the first regime, $[0, T_1)$, we have CCU. Hence $\zeta x = s > 0$, implying $\gamma_x = \gamma_s = 0$. This implies $\tau - \mu = (c_a + c_s)e^{-\rho t} - \lambda_s$. We use this in (9a) to get

$$\beta - \gamma x - (\psi - \delta(X + \alpha S)) - \zeta(c_a + c_s) = (\lambda_x - \zeta\lambda_s)e^{\rho t}.$$

Taking account of $\zeta x = s$ and (10a) and (10b), after differentiation with respect to time under full injection of CO_2 emissions, we obtain:

$$\begin{aligned} -\gamma\dot{x} &= \rho\left(\lambda_x - \zeta\lambda_s\right)e^{\rho t} \\ &= \rho[\beta - \gamma x - (\psi - \delta(X + \alpha S)) - \zeta(c_a + c_s)]. \end{aligned}$$

Since $\lambda_x - \zeta\lambda_s > 0$ (see the proof of Lemma 1 (i) in Appendix A) we have $\dot{x} < 0$ along the interval. The scarcity rent on the resource being larger than the carbon rent, it never pays to accelerate the extraction of fossil fuel despite the extraction cost reduction effect of CO_2 injection. Observe that it implies also that $s(t)$ decreases throughout the phase: Less and less gas is injected into the fossil fuel wells.

Moreover, after differentiating once more we get the following linear second-order differential equation:

$$\ddot{x} - \rho\dot{x} - \frac{\delta\rho}{\gamma}(1 - \alpha\zeta)x = 0.$$

The general solution is given by:

$$x(t) = R_1 e^{r_1 t} + R_2 e^{r_2 t}, \tag{12}$$

where R_1 and R_2 are constants of integration and the characteristic roots r_1 and r_2 are given by:

$$r_1 = \frac{1}{2}\left(\rho + \sqrt{\rho^2 + 4\frac{\delta\rho}{\gamma}(1 - \alpha\zeta)}\right), \tag{13}$$

$$r_2 = \frac{1}{2}\left(\rho - \sqrt{\rho^2 + 4\frac{\delta\rho}{\gamma}(1 - \alpha\zeta)}\right). \tag{14}$$

4.2.2 The no CCU and no CCS regime

Along the interval $[T_1, T_2)$ we have $x > s = b = 0$ so that $\mu = 0$. Hence, from (9a) we get:

$$\beta - \gamma x = [\psi - \delta(X + \alpha S(T_1)] + \lambda_x e^{\rho t} + \zeta \tau e^{\rho t},$$

with τ constant. Differentiation with respect to time and using (10a) yields:

$$\begin{aligned} -\gamma \dot{x} &= \rho(\lambda_x(t) + \zeta\tau)e^{\rho t} \\ &= \rho[\beta - \gamma x - (\psi - \delta(X + \alpha S(T_1))]. \end{aligned}$$

Observe that $\dot{x} < 0$ during the phase due to the rise of the extraction cost and the exponential rise of the carbon tax in current value terms. After differentiating once more we get the following linear second-order differential equation:

$$\ddot{x}(t) - \rho\dot{x}(t) - \frac{\delta\rho}{\gamma}x(t) = 0.$$

The general solution reads:

$$x(t) = K_1 e^{m_1 t} + K_2 e^{m_2 t}, \tag{15}$$

where K_1 and K_2 are two integration constants and the characteristic roots m_1 and m_2 are given by:

$$m_1 = \frac{1}{2}\left(\rho + \sqrt{\rho^2 + 4\frac{\delta\rho}{\gamma}}\right), \tag{16}$$

$$m_2 = \frac{1}{2}\left(\rho - \sqrt{\rho^2 + 4\frac{\delta\rho}{\gamma}}\right). \tag{17}$$

4.2.3 The CCS regime

Finally, consider the interval $[T_2, T_3)$. Along the interval, $b > 0$. It follows from (9c) that $\tau - \mu = c_a e^{-\rho t}$. Use this in (9a) to get

$$\beta - \gamma x = (\psi - \delta(X + \alpha S)) + \lambda_x e^{\rho t} + \zeta c_a. \tag{18}$$

Once again we find that $\dot{x}(t) < 0$: The fossil extraction rate declines throughout the CCS phase. Since $\zeta x(t) = b(t)$, the economy injects less and less CO_2 emissions into the inert reservoirs. Following the same procedure as in the previous case we arrive at:

$$x(t) = M_1 e^{m_1 t} + M_2 e^{m_2 t}, \tag{19}$$

where M_1 and M_2 are two constants of integration, different from K_1 and K_2.

4.3 Scenarios

Proposition 1 shows that, with an active climate constraint, the different regimes can combine in at most four possible optimal scenarios.

Scenario 1: No CCU and no CCS. The economy does not abate its CO_2 emissions in this scenario, which yields a sequence of two time phases: A first phase, $[0, T_2)$, of consumption of fossil fuel until the carbon budget becomes exhausted at time $T_2 = T_3$ followed by the infinite duration of the carbon-free regime. Discarding the special case, $\zeta X_0 = \bar{Z} - Z$, the economy leaves forever underground the excess amount of fossil fuel reserves, $X_0 - (\bar{Z} - Z_0)/\zeta$.

Scenario 2: CCS only. The economy never uses CCU. The scenario is a sequence of three time phases: A first phase, $[0, T_2)$, without abatement until the carbon budget is depleted followed by a CCS phase, $[T_2, T_3)$, with full abatement of emissions and storage into the inert reservoirs with a transition to the carbon-free regime at T_3. It is possible that the economy fully exhausts the fossil fuel reserves at time T_3. Alternatively, it leaves some remaining amount of the fossil fuel reserves underground.

Scenario 3: CCU and CCS. The economy uses both options to abate CO_2 emissions, first CCU and later CCS. The scenario is a sequence of four phases: A first phase, $[0, T_1)$, of injection of the CO_2 emissions in the fossil fuel wells, followed by a no abatement phase until the carbon budget becomes exhausted, $[T_1, T_2)$, next a CCS abatement phase with storage inside the inert reservoirs, $[T_2, T_3)$, before the carbon-free regime starting at T_3. Both complete and incomplete depletion can occur.

Scenario 4: CCU only. The economy never uses CCS. The scenario consists of three time phases: A first phase, $[0, T_1)$ of CO_2 emission injection into the fossil fuel wells, followed by a no abatement phase, $[T_1, T_2)$ until the depletion of the carbon budget before the transition to the carbon-free regime. It is possible that the economy fully exhausts the fossil fuel reserves at time T_3 or leaves some fraction of the reserves underground.

Including the (in)complete depletion of the fossil fuel reserves variants, there exist potentially 8 different optimal scenarios. Since empirically it seems more likely that fossil fuel exploitation will end by lack of economic profitability rather than for pure physical reasons, we focus on the four scenarios with incomplete depletion of the fossil reserves. Occasionally we point at the other cases.

4.4 Extraction paths

Below, we will give a set of necessary and sufficient conditions for each possible scenario to be optimal. First, we derive a set of conditions for each scenario to solve for the transition times and the constants of integration and hence for the entire fossil extraction path until the carbon-free regime. Observe that once the extraction path is identified, the abatement paths are also identified, since the economy, when it decides to abate, fully abates its carbon emissions. We treat scenarios 1 and 2 in the main text and perform a parallel exercise for scenarios 3 and 4 in Appendix B. We also check the existence and uniqueness of the extraction and abatement paths for the different scenarios in a companion Technical Appendix.

4.4.1 Scenario 1

Suppose that CCU and CCS are so costly that the economy chooses not to abate its CO_2 emissions. Hence, $0 = T_1 < T_2 = T_3$. To avoid triviality we assume $X(T_3) = X_0 - (\bar{Z} - Z_0)/\zeta > 0$. $X(T_3)$ depends only on the carbon budget and the initial availability of fossil fuel. We also have a constant τ. Consider the first phase of only fossil fuel exploitation until depletion of the carbon budget, $[0, T_2]$ where we have (15). We need to determine K_1, K_2 and T_2, the transition time to renewables, which is also the exhaustion time of the carbon budget. This requires three conditions.

First, the energy consumption path has to be continuous, so that $x(T_3^-) = \frac{\beta - c_y}{\gamma}$:

$$x(T_3) = K_1 e^{m_1 T_3} + K_2 e^{m_2 T_3} = \frac{\beta - c_y}{\gamma}. \tag{20}$$

Next, we derive a condition on the time derivative of x at T_3^-. It follows from $X(T_3) > 0$ that $\lambda_x(T_3) = 0$. Therefore,

$$c_y = \psi - \delta(X(T_3) + \alpha S_0) + \zeta \tau e^{\rho T_3}.$$

From

$$\beta - \gamma x(t) = [\psi - \delta(X(t) + \alpha S_0)] + \lambda_x(t) e^{\rho t} + \zeta \tau e^{\rho t}$$

we also have

$$\dot{x}(T_3^-) = -\frac{\rho}{\gamma} \left[c_y - \psi + \delta(X(T_3) + \alpha S_0)\right].$$

This implies

$$\dot{x}(T_3^-) = m_1 K_1 e^{m_1 T_3} + m_2 K_2 e^{m_2 T_3} = -\frac{\rho}{\gamma} \left[c_y - \psi + \delta(X(T_3) + \alpha S_0)\right]. \tag{21}$$

Finally, the carbon budget condition has to hold:

$$\frac{\bar{Z} - Z_0}{\zeta} = \int_0^{T_3} x(t) dt = \frac{K_1}{m_1} \left(e^{m_1 T_3} - 1\right) + \frac{K_2}{m_2} \left(e^{m_2 T_3} - 1\right). \tag{22}$$

Conditions (20)-(22) can be used to solve for (K_1, K_2, T_2).

4.4.2 Scenario 2

Next, we investigate the case where $T_1 = 0$ and $0 = T_1 \leq T_2 \leq T_3$. Hence, there is no CCU at all, so that $S(t) = S_0$ for all $t \geq 0$. We proceed by backward induction. At time T_2, $X(T_2) = X_0 - (\bar{Z} - Z_0)/\zeta$. CCS is implemented for $T_3 > t > T_2$. Assume $\frac{1}{\delta}(\psi + \zeta c_a - c_y) - \alpha S_0 > 0$ implying incomplete depletion of the fossil fuel reserves.

Then

$$X(T_3) = \frac{1}{\delta}(\psi + \zeta c_a - c_y) - \alpha S_0 > 0$$

solves (9a) with $\lambda_x(T_3) = 0$. Taking as given T_2, we need 3 conditions to determine (M_1, M_2, T_3).

First, the energy supply path should be time continuous: $x(T_3^-) = \frac{\beta - c_y}{\gamma} > 0$. Along the final CCS interval we have (19), which yields

$$M_1 e^{m_1 T_3} + M_2 e^{m_2 T_3} = \frac{\beta - c_y}{\gamma}. \tag{23}$$

Second, it follows from (18) with $\lambda_x(T_3) = 0$ that $\dot{x}(T_3) = 0$, thus:

$$\dot{x}(T_3) = m_1 M_1 e^{m_1 T_3} + m_2 M_2 e^{m_2 T_3} = 0. \tag{24}$$

Lastly, the resource balance condition:

$$X(T_2) - X(T_3) = \int_{T_2}^{T_3} x(t)dt = \frac{M_1}{m_1}\left(e^{m_1 T_3} - e^{m_1 T_2}\right) + \frac{M_2}{m_2}\left(e^{m_2 T_3} - e^{m_2 T_2}\right). \tag{25}$$

Along the first interval $[0, T_2)$ we have (15). To determine the vector (K_1, K_2, T_2), we need 3 conditions as well. The first condition is the continuity of the energy consumption path at T_2:

$$M_1 e^{m_1 T_2} + M_2 e^{m_2 T_2} = K_1 e^{m_1 T_2} + K_2 e^{m_2 T_2}. \tag{26}$$

A second condition is a kink condition over the time derivatives of $x(t)$ just before and just after T_2. This can be seen as follows. From

$$\beta - \gamma x = (\psi - \delta(X + \alpha S)) + \lambda_x e^{\rho t} + \zeta c_a,$$

holding in the second interval, we have $-\gamma \dot{x}(T_2^+) = \rho \lambda_x(T_2) e^{\rho t}$, and from

$$\beta - \gamma x = (\psi - \delta(X + \alpha S)) + \lambda_x e^{\rho t} + \zeta \tau e^{\rho t},$$

holding in the first interval, we have $-\gamma \dot{x}(T_2^-) = \rho(\lambda_x(T_2) + \zeta\tau) e^{\rho T_2}$. Moreover, continuity at T_2 requires $\tau e^{\rho T_2} = c_a$, leading to the condition:

$$m_1 K_1 e^{m_1 T_2} + m_2 K_2 e^{m_2 T_2} = m_1 M_1 e^{m_1 T_2} + m_2 e^{m_2 T_2} - \frac{\rho\zeta}{\gamma} c_a. \tag{27}$$

Finally, the resource consumption balance condition needs to be satisfied, stating that

$$\int_0^{T_2} x(t)dt = X_0 - X(T_2) = \frac{K_1}{m_1}\left(e^{m_1 T_2} - 1\right) + \frac{K_2}{m_2}\left(e^{m_2 T_2} - 1\right). \tag{28}$$

The conditions (23)-(25) and (26)-(28) suffice to identify the extraction and abatement paths in Scenario 2, as they can be used to solve for $(K_1, K_2, M_1, M_2, T_2, T_3)$.

4.5 Optimal policy

We now determine necessary and sufficient conditions for all possible scenarios to be the optimal solution to (SP) through a mapping of the scenarios in the initial endowments phase plane (X, S).

In Scenario 2 with only CCS, for a given gas level in the wells, S_0, the economy endowed with sufficiently large initial reserves, X_0, leaves in the ground:

$$\tilde{X}_{CCS} \equiv \frac{1}{\delta}\left[\psi + \zeta c_a - c_y\right] - \alpha S_0.$$

Let $\tilde{X}$ denote the unburned amount of the resource in a no CCS scenario:

$$\tilde{X} \equiv X_0 - \frac{\bar{Z} - Z_0}{\zeta}.$$

If $\tilde{X} < \tilde{X}_{CCS}$ then the amount left into the ground in a no CCS scenario is too small to justify using the CCS option. In the opposite case the economy uses the CCS option and leaves the amount $\tilde{X}_{CCS}$ underground.

We thus obtain a necessary condition for CCS to eventually become an optimal policy. Let $\bar{X}_{CCS}(S_0) \equiv \tilde{X}_{CCS} + (\bar{Z} - Z_0)/\zeta$. Then $X_0 > \bar{X}_{CCS}(S_0)$ is necessary for CCS to become optimal eventually. Conversely, the economy should never perform CCS if $X_0 < \bar{X}_{CCS}(S_0)$. To this condition corresponds a critical frontier in the endowment plane (X, S), which we call the *CCS frontier*. It is a line given by:

$$S = \frac{1}{\alpha\delta}\left[\psi + \zeta c_a - c_y\right] + \frac{\bar{Z} - Z_0}{\alpha\zeta} - \frac{X}{\alpha}.$$

In the optimum, the economy never performs CCS if the initial pair (X_0, S_0) is located below the CCS frontier, whereas it performs CCS eventually if (X_0, S_0) is located above the frontier.

We turn to the CCU option. For CCU to be optimal during an initial interval of time it must be the case that:

$$\lambda_s(0) + \tau \quad \geq \quad c_a + c_s.$$

We show in Appendix C that this condition is associated with another critical frontier, to be called the *CCU frontier*. This is a line with slope $-1/\alpha$, thus parallel to the CCS frontier. To get some intuition, let (X_0, S_0, Z_0) be such that the economy starts with CCU, but for $X(0) < X_0$, $S(0) < S_0$ and $Z(0) = Z_0$ there is no CCU. We can write total welfare as $W(X_0, S_0)$, omitting Z_0 for brevity. Let us now increase $X(0)$ and decrease $S(0)$ such that the social planner starts with CCU and total welfare remains the same, whereas for lower $X(0)$ and lower $S(0)$ there will be no CCU. Formally $dW = W'_X(X_0, S_0)dX(0) + W'_S(X_0, S_0)dS(0) = 0$. We know that $W'_X(X_0, S_0) = \lambda_x$, $W'_S(X_0, S_0) = \lambda_s$ and $\lambda_s = \alpha\lambda_x$. Therefore, we obtain $dX(0) = -\alpha dS(0)$, which defines a line with slope $-1/\alpha$ in the initial endowments phase plane (X, S).

If the initial endowments vector, (X_0, S_0), is located above the CCU frontier, the economy should apply CCU and the contrary holds if (X_0, S_0) is located below the CCU frontier. Depending on the model parameters, it may be the case that the CCS frontier is located above or below the CCU frontier. We now examine these two possibilities.

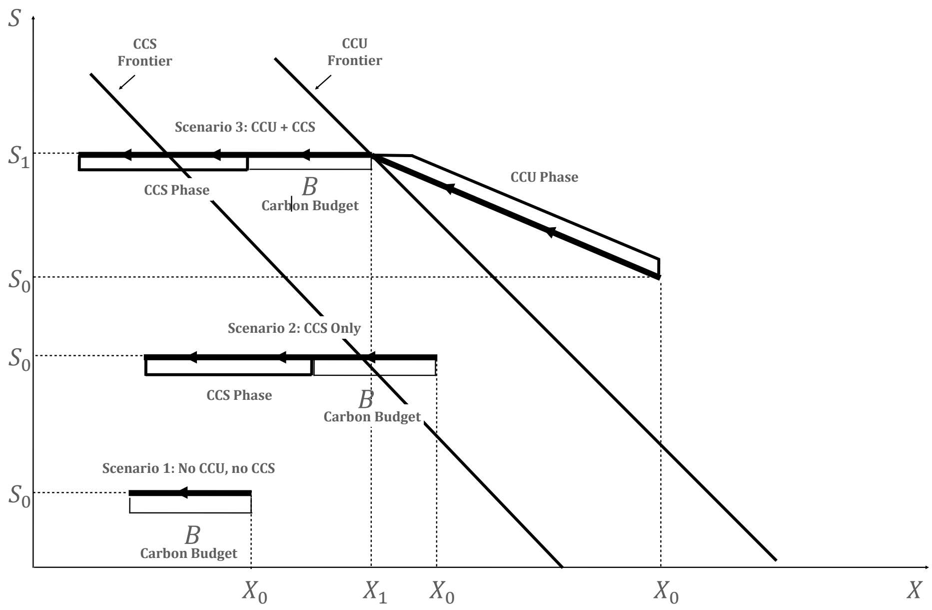
The CCS frontier is located below the CCU frontier. Depending on (X_0, S_0), three optimal scenarios are possible in this configuration of the frontiers.

- **Scenario 1:** If (X_0, S_0) is located below the CCS frontier and thus below the CCU frontier, CCU and CCS both will never be used.

- **Scenario 2:** If (X_0, S_0) is located in between the CCS frontier and the CCU frontier, CCU will never be used but CCS will eventually be used.

- **Scenario 3:** If (X_0, S_0) is located above the CCU frontier and thus above the CCS frontier, the economy fully uses its CO_2 abatement options, with a first CCU

Figure 2: The CCS frontier is located below the CCU frontier



regime before beginning to deplete the carbon budget and next a CCS regime after the exhaustion of the carbon budget.

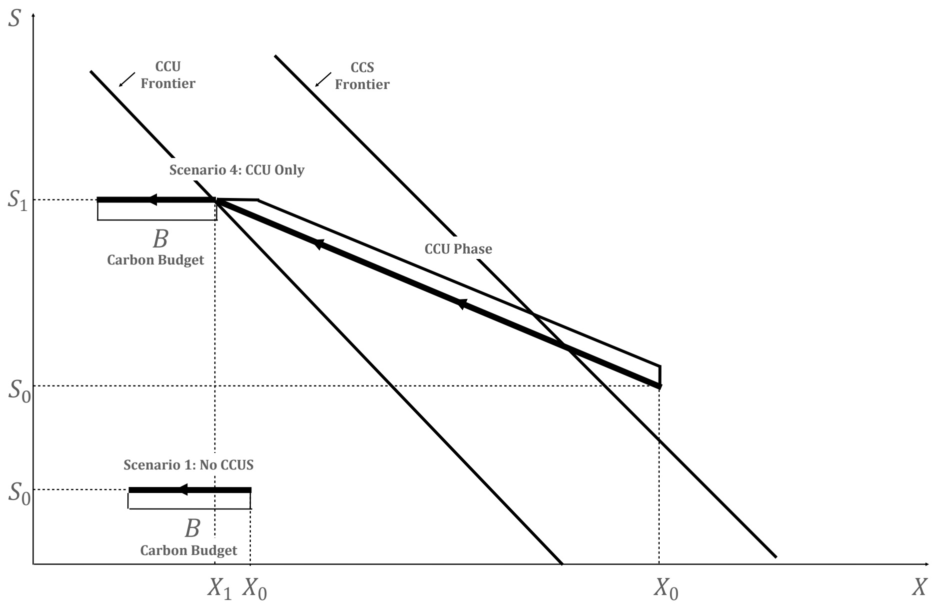
See Figure 2.

The CCS frontier is located above the CCU frontier. In this configuration of the frontiers, only two scenarios are possible.

- **Scenario 1:** If (X_0, S_0) is located below the CCU frontier and thus below the CCS frontier, abating CO_2 emissions is never optimal.
- **Scenario 4:** If (X_0, S_0) is located above the CCU frontier, either in between the frontiers or above the CCS frontier, CCS is never optimal but CCU is optimal during an intitial interval of time.

Figure 3 shows the two possible scenarios. The CCS option is never optimal when the CCS frontier is located above the CCU frontier. If the initial endowment pair is located below the two frontiers this is trivial. For large initial endowments, it should be recalled that the test of the relevance of the CCS option has to be made with respect to the pair (X_1, S_1) and not with respect to the initial endowment vector (X_0, S_0). If the CCU frontier is located below the CCS frontier, no (X_1, S_1) pair is located above the CCS frontier by construction.

Figure 3: The CCS frontier is located above the CCU frontier



Proposition 2 summarizes our findings.

Proposition 2

(i) Scenario 1 without CCU or CCS is an optimal policy if and only if *the initial endowment vector, (X_0, S_0), is located below both the CCS and the CCU frontiers whatever their relative position.*

(ii) Scenario 2 with only CCS is an optimal policy if and only if *the CCS frontier is located below the CCU frontier and the initial endowment vector, (X_0, S_0), is located in between the two frontiers.*

(iii) Scenario 3 with CCS and CCU is an optimal policy if and only if *the CCS frontier is located below the CCU frontier and the initial endowments vector, (X_0, S_0), is located above the CCU frontier.*

(iv) Scenario 4 with only CCU is an optimal policy if and only if *the CCS frontier is located above the CCU frontier and the initial endowments vector, (X_0, S_0), is located above the CCU frontier.*

5 Numerical illustration

In this section, we provide a numerical illustration to quantify our results. Table 1 shows the parameter values that we have used, based as much as possible on the available data.

McGlade and Ekins (2015) report current fossil fuel reserves of 2900 $GtCO_2$. In line with this estimate, we impose an initial carbon stock of 3000 $GtCO_2$. The parameters of the extraction cost function, ψ and δ, are chosen such that extraction cost vary from 25 to 250 US$ per tCO_2. Using the average carbon content of a barrel of oil of 430.80 kg CO_2/bbl, this would correspond to a range of 12 to 120 US$ per barrel, in line with the oil production cost of the ultimately recoverable resources reported by McGlade and Ekins (2015). We choose a choke price, β, equal to 400 US$/$tCO_2$. This corresponds to 192 US$/bbl, roughly twice the current market price of oil. The slope of the demand function, γ is chosen to get an oil price of 200 US$ per $GtCO_2$ if demand equals the yearly global emissions of 35 $GtCO_2$.

Table 1: Parameter values

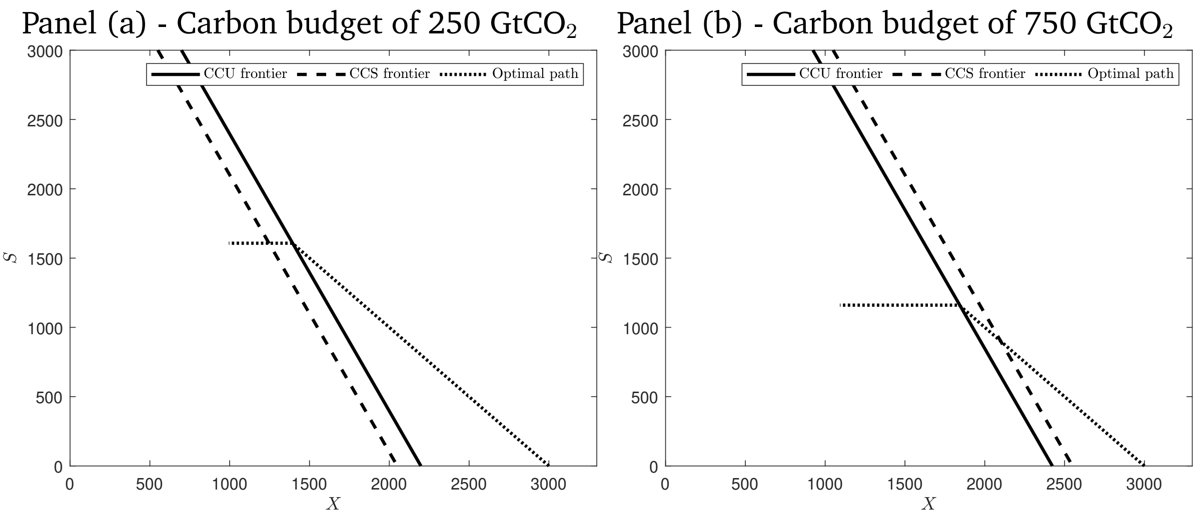
parameters	description	value	unit
α	CCU effectiveness	0.5	-
β	choke price	400	US$
γ	slope demand function	40/7	US$/$tCO_2$
ψ	vertical intercept extraction cost function	250	US$
δ	slope extraction cost function	0.075	US$/$tCO_2$
ρ	interest rate	0.05	-
ζ	emission factor	1	$GtCO_2/GtCO_2$
c_a	marginal capturing cost	10	US$/$tCO_2$
c_s	marginal injection cost	10	US$/$tCO_2$
c_y	marginal cost of renewables	125	US$/$tCO_2$
S_0	initial injected CO_2 stock	0	$GtCO_2$
X_0	initial fossil reserve	3000	$GtCO_2$

The interest rate, ρ, is set to 5 percent. For the price of renewables we pick 125 US$/$tCO_2$, which implies that gross (net) cumulative emissions in the absence of a carbon budget would amount to 1333 (757) $GtCO_2$. These numbers significantly exceed the carbon budget of 500 $GtCO_2$ corresponding to a 50 percent chance of limiting global warming to 1.5 degrees (IPCC, 2021). For the CCU effectiveness parameter, α, we pick

0.5. Based on data from the US, Schmelz et al. (2020) report CCS costs of 52 US$ per ton CO_2 for the cheapest option, with CO_2 sourced from coal-fired power plants and stored onshore in depleted oil and gas fields. Onshore storage of CO_2 sourced from gas-fired power plants costs over 80 US$ per ton. Offshore storage increases these cost by about 10 US$ per ton CO_2 (Schmelz et al., 2020). When using these numbers, the optimum in our benchmark model does not feature CCU nor CCS at all with a carbon budget of 500 $GtCO_2$. In order to still be able to demonstrate how the importance of CCU and CCS in the social optimum depends on the different parameters of interest, we set the capturing and injection costs both to 10 US$ per ton CO_2, which is significantly below the current costs of CCS and CCU.

5.1 Social optimum

Figure 4: CCS and CCU frontiers

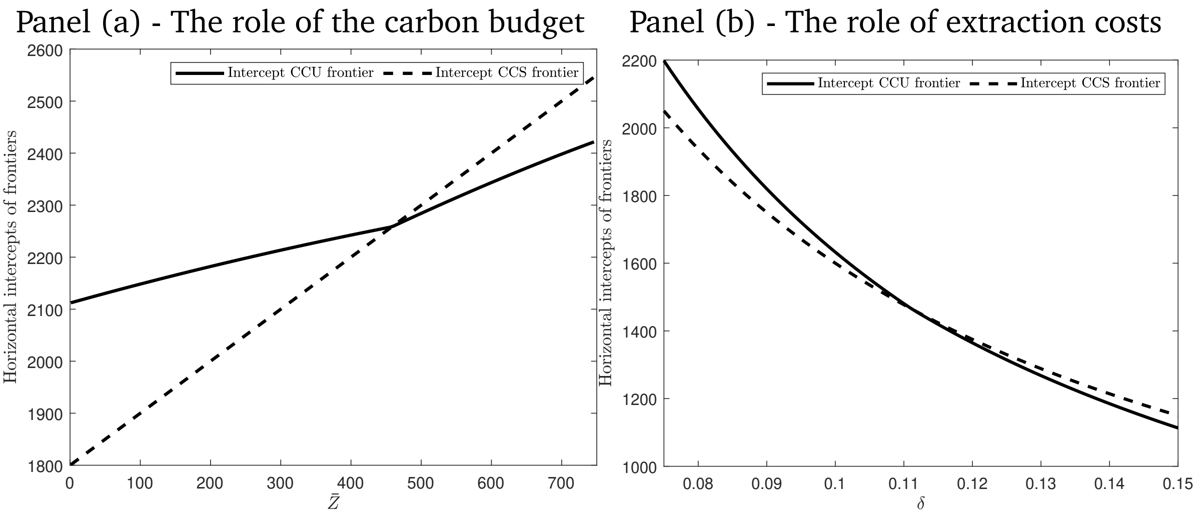


Notes: The solid (dashed) lines show the CCU and CCS frontiers, respectively. The dotted lines represent the poptimal paths. In panel (a), the carbon budget is set to 250 $GtCO_2$. In panel (b), the carbon budget is 750 $GtCO_2$.

Figure 4 depicts the CCU and CCS frontiers and the optimal path in (X, S)-space. Panel (a) shows the case with carbon budget of 250 $GtCO_2$, whereas in panel (b) we have imposed a carbon budget of 750 $GtCO_2$. The low carbon budget in panel (a) implies that the CCU frontier is located above the CCS frontier. Given this constellation and the initial stocks, the optimum is characterized by Scenario 3: first a CCU regime, then a regime without abatement, and finally a CCS regime. The relatively high carbon budget in panel (b) results in a CCU frontier below the CCS frontier. This implies that

CCS does not occur. Still, given the initial stocks, there is an initial regime of CCU, before abatement stops and the carbon budget is depleted.

Figure 5: Horizontal intercepts of CCU and CCS frontiers

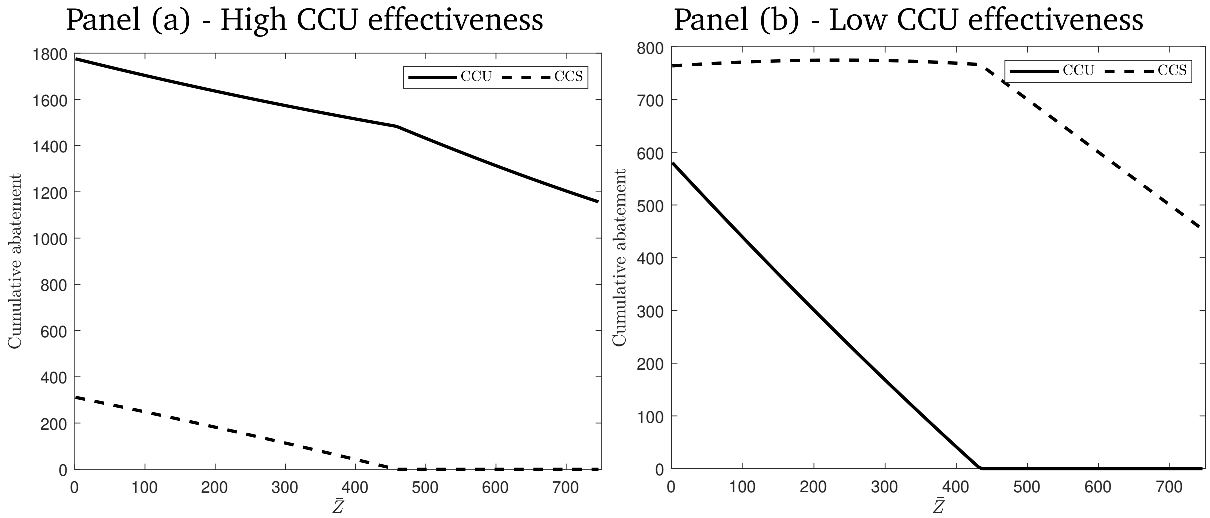


Notes: The solid (dashed) lines show the horizontal intercepts of the CCS and CCU curves for various values of the carbon budget (panel (a)) and the slope of the extraction cost function (panel (b)). The difference between these intercepts is equal to the horizontal distance between the CCS and CCU frontiers.

Figure 5 shows how the horizontal intercepts of the CCS and CCU frontiers depend on the carbon budget $\bar{Z}$ (panel (a)) and on the slope of the extraction cost function, δ, (panel (b)). Note that the difference between the horizontal intercepts measures the horizontal distance between the CCU and CCS frontiers. A CCS regime can only be optimal if the horizontal intercept of the CCU is located to the right that of the CCS frontier. The figure shows that the difference between the intercepts of the CCU and CCS frontiers depends negatively on the carbon budget and the slope of the extraction cost function. Intuitively, if the carbon budget increases, CCS becomes less important. Furthermore, an increase in the slope of the extraction cost function means that CCU becomes more effective in terms of lowering extraction costs.

Figure 6 shows how cumulative abatement due to CCU and CCS depends on the carbon budget. Panel (a) depicts the benchmark case with effective CCU ($\alpha = 0.5$), In panel (b), the effectiveness of CCU is lowered to $\alpha = 0.25$. In panel (a), most abatement is done through CCU, whereas in panel (b) captured emissions are mainly stored into empty reservoirs (CCS). Typically, the two types of abatement are non-increasing in the carbon budget. However, panel (b) shows that CCS abatement is increasing in the budget if the budget is small. The reason is that CCU becomes profitable if the budget

Figure 6: Cumulative abatement



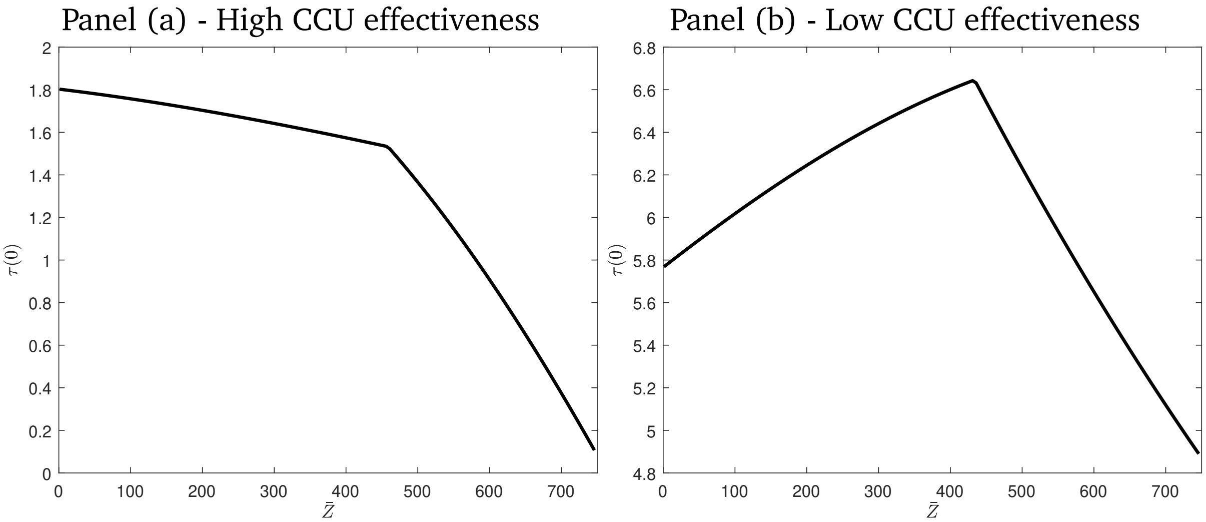
Notes: The solid (dashed) lines show cumulative abatement through CCU (CCS). Panel (a) depicts the benchmark case with high CCU effectiveness ($\alpha = 0.5$), whereas panel (b) shows the case with a relatively lower CCU effectiveness ($\alpha = 0.25$).

is tight enough.

Figure 7 shows the relationship between the carbon budget, $\bar{Z}$, and the initial carbon price, $\tau(0)$. Panel (a) again depicts the benchmark case with effective CCU ($\alpha = 0.5$) and panel (b) the case with less effective CCU ($\alpha = 0.25$). Panel (a) shows an intuitive downward-sloping relationship between the carbon budget and the carbon price, with a lower price if carbon is less scarce. However, panel (b) shows a non-monotonic relationship. The reason is that an increase in the carbon budget now considerably shortens the duration of the initial CCU phase, which eventually even vanishes. The shortening of the CCU phase implies that the depletion of the carbon budget occurs sooner in time, which pushes up the initial carbon price.[9] Moreover, a shorter CCU phase implies lower extraction costs during the no CCU and no CCS phase, which also brings forward the moment of depletion of the carbon budget. These effects together explain the upward-sloping part of the graph in Panel (b). The top of this graph corresponds to the carbon budget that is associated with a CCU phase of zero duration (i.e. with the switch from Scenario 3 to Scenario 2).

[9]This can be seen most clearly if a CCS phase occurs before the switch to renewables. In this case, we have $\tau(0) = c_a e^{-\rho T_2}$, where T_2 denotes the moment at which the carbon budget is depleted.

Figure 7: Carbon price



Notes: Panel (a) depicts the benchmark case with high CCU effectiveness ($\alpha = 0.5$), whereas panel (b) shows the case with a relatively lower CCU effectiveness ($\alpha = 0.25$).

5.2 Laissez-faire

We now briefly compare the social optimum to the market equilibrium under *laissez-faire*. By the definition we employ here, under laissez-faire there is no concern for climate change, implying that there is no carbon budget. But otherwise the market economy functions efficiently and thus satisfies the necessary conditions derived in Section 4. Under laissez-faire, CCU is the only motivation for carbon capture. As a result, $b = 0$ throughout and there are at most three regimes: An initial regime with CCU and a constant CO_2 stock, then a regime without CCU with an increasing CO_2 stock, and a final carbon-free regime. Accordingly, only Scenarios 1 and 4 are possible in equilibrium.[10] The end condition now yields the following amount of untapped fossil fuel:

$$X(T_3) = \frac{\psi - c_y}{\delta} - \alpha S(T_1), \tag{29}$$

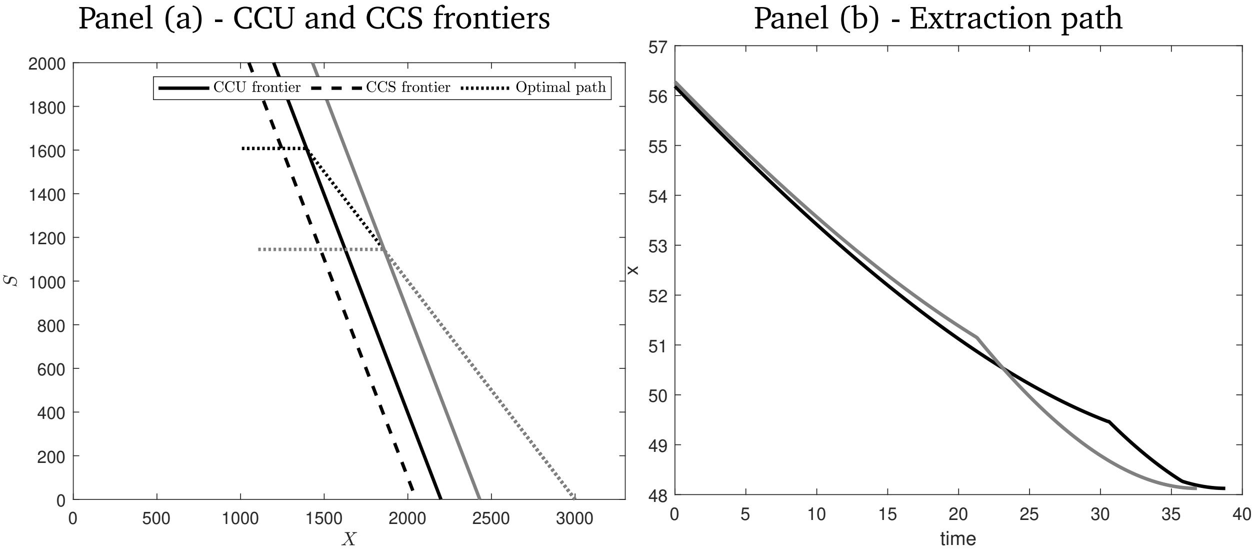
yielding cumulative emissions of

$$\zeta\left(X_0 - X(T_3)\right) - \left(S(T_1) - S_0\right) = S_0 + \zeta\left(X_0 - \frac{\psi - c_y}{\delta}\right) - (1 - \zeta\alpha)S(T_1). \tag{30}$$

[10]In the conditions derived for Scenario 1 and Scenario 4 in Sections 4.4.1 and B.1, the carbon budget $\bar{Z} - Z_0$ needs to be replaced by $\zeta(X(T_3) - X(T_1))$.

Hence, CCS increases cumulative extraction of fossil fuels, but lowers cumulative CO_2 emissions. The CCS frontier does not exist under laissez-faire. However, we can still derive the CCU frontier. The laissez-faire CCU fontier is located above the one corresponding to the social optimum with a binding carbon budget, because the social cost of carbon, τ, is ignored under laissez-faire. This lowers the marginal benefits of CCU, and hence requires a higher threshold composite resource stock $X + \alpha S$ to make CCU profitable.

Figure 8: Optimum vs. laissez-faire



Notes: The black (grey) curves correspond to the social optimum (laissez-faire). The carbon budget is 250 $GtCO_2$. The remaining parameter values are set at their benchmark levels.

Figure 8 graphically compares the social optimum with the outcome under laissez-faire. In the social optimum, we have imposed a carbon budget of 250 $GtCO_2$. The black lines in the figure correspond to the social optimum and the gray lines to the laissez-faire. Panel (a) shows that the laissez-faire CCU frontier is indeed located above that of the social optimum. Furthermore, comparison of the optimal paths in panel (a) and the extraction time paths in panel (b) shows that, although the economy starts with a CCU regime under laissez-faire, it ends sooner (around 20 years instead of 30 years from now) and at a higher remaining composite stock level than in the social optimum. The social optimum leaves less fossil reserves unburned, but emits less carbon because it uses CCS and it uses CCU for a longer period. Panel (a) also shows that initial extraction is higher under laissez-faire, as the social costs of carbon are ignored. Once capturing stops, the extraction rate soon drops below the one of the social optimum, because extraction costs will then increase more rapidly over time.

6 Conclusions and extensions

In the ongoing debates on combating climate change, carbon capture and storage (CCS) as well as carbon capture and utilization (CCU) are important issues. Many countries, including the Netherlands, France, Canada and some states in the US are seriously considering this option. Presently the cost is high but it is expected that technological progress will lead to a significant cost reduction. An additional option is to use the inserted greenhouse gases, CO_2 in particular, to enhance fossil fuel recovery by increasing pressure in the wells. The aim of this paper has been to offer a preliminary investigation of the pros and cons by analyzing a formal model of CCS and CCU. The model is very stylized and we mainly focus on the linear-quadratic case. The main feature of the model is the fact that the marginal extraction cost of fossil fuel is a decreasing function of the pressure in the well. This pressure can be increased by inserting CO_2, recovered from burning fossil fuel. On the one hand, CCS and CCU help to reduce atmospheric CO_2 accumulation, at a cost. On the other hand, CCU leads to more fossil fuel being processed than otherwise, which may lead to less fossil fuel left in situ. In a social optimum the two effects are both taken into account. We also consider a *laissez-faire* market economy which neglects climate change but is efficient otherwise.

Our analysis has shown that if CCS occurs in the social optimum, it will occur only at the end of the fossil era, just before the switch to renewable energy. On the contrary, if CCU occurs in the social optimum or under laissez-faire, it will take place in the *beginning*. The reason is that cumulative future extraction cost savings from CCU are largest at the outset, when the remaining extraction horizon is still relatively long. CCS, however, does not yield benefits on top of emission reduction. For discounting reasons, it is then better to postpone using it as long as possible. Under laissez-faire, CCS does not occur at all. CCU lowers cumulative emissions under laissez-faire. We also have shown that the relationship between the carbon budget and the carbon price can be non-monotonic if CCU occurs in the social optimum. Finally, we have developed a graphical apparatus to show under which conditions a CCU and a CCS regime exist in the social optimum, and we have performed a numerical illustration to quantify our results.

For reasons of analytical tractability, we have mainly restricted attention to a linear-quadratic specification of our model. When it comes to the cost of CCS and CCU this is not an innocuous assumption. Linearity of the capturing and injection costs is responsible for the fact that CCS and CCU cannot occur simultaneously and for the outcome that they either are operated at full scale or are not used at all. Linearity of production cost of renewables seems less harmful, although technological progress in renewable energy is pertinent but has not been taken into account. The linear specification of the extraction cost function with additivity of existing reserves and inserted CO_2 may pose a problem as well. Additional insights from the field of engineering might be helpful here.

Another important extension would be to make a distinction between different sources of fossil fuels and CO_2 emissions. For example, capturing CO_2 from oil used in transportation is much more costly that capturing it from gas turbines. Furthermore, we have left out coal, although it is a main contributor to global CO_2 emissions. The reason is that inserting CO_2 into coal mines is not productive. Nevertheless, it would be interesting to include emissions from coal-fired power plants as an exogenous source of CO_2 emissions that can be injected into oil and gas wells. When it comes to storage and use, a distinction should be made between inserting in oil wells, gas wells and onshore and offshore inert wells. Moreover, concerns related to environmental and health damages from capturing and storing, including potential leakage of CO_2 from storage locations, should be taken into account. Finally, we have abstracted from directly capturing CO_2 from the air. It would be interesting to allow for this, especially if costs of direct air capture keep on decreasing over time.

References

ACEMOGLU, D., P. AGHION, L. BURSZTYN AND D. HEMOUS, "The Environment and Directed Technical Change," *American Economic Review* 102 (February 2012), 131–166.

AMIGUES, J.-P., G. LAFFORGUE AND M. MOREAUX, "Optimal Timing of CCS Policies with Heterogeneous Energy Consumption Sectors," *Environmental and Resource Economics* 57 (March 2014), 345–366.

———, "Optimal timing of carbon capture policies under learning-by-doing," *Journal of Environmental Economics and Management* 78 (July 2016), 20–37.

AMIT, R., "Petroleum reservoirs exploitation: Switching from primary to secondary recovery," *Operations Research* 34 (August 1986), 534–549.

ANDERSON, S. T., R. KELLOGG AND S. W. SALANT, "Hotelling under Pressure," *Journal of Political Economy* (June 2018).

BELFIORI, M. E., "Carbon pricing, carbon sequestration and social discounting," *European Economic Review* 96 (July 2017), 1–17.

CAIRNS, R. D. AND G. A. DAVIS, "Adelman's Rule and the Petroleum Firm," *The Energy Journal* 22 (2001), 31–54.

CHAKRAVORTY, U., B. MAGNÉ AND M. MOREAUX, "A Hotelling model with a ceiling on the stock of pollution," *Journal of Economic Dynamics and Control* 30 (December 2006), 2875–2904.

COULOMB, R. AND F. HENRIET, "Carbon price and optimal extraction of a polluting fossil fuel with restricted carbon capture," Technical Report 322, Banque de France, 2011, publication Title: Working papers.

DARCY, H., *Les fontaines publiques de la ville de Dijon* (V. Dalmont, 1856).

DURMAZ, T., "The economics of CCS: Why have CCS technologies not had an international breakthrough?," *Renewable and Sustainable Energy Reviews* 95 (November 2018), 328–340.

Durmaz, T. and F. Schroyen, “Evaluating carbon caputre and storage in a climte model with endogenous technical change,” *Climate Change Economics* (December 2019), publisher: World Scientific Publishing Company.

Hepburn, C., E. Adlen, J. Beddington, E. A. Carter, S. Fuss, N. Mac Dowell, J. C. Minx, P. Smith and C. K. Williams, “The technological and economic prospects for CO2 utilization and removal,” *Nature* 575 (November 2019), 87–97, number: 7781 Publisher: Nature Publishing Group.

Herzog, H., “Carbon dioxide capture and storage,” in D. Helm and C. Hepburn, eds., *The Economics and Politics of Climate Change* (OUP Oxford, 2009).

IEA, “Whatever happened to enhanced oil recovery?,” Technical Report, Paris, 2018.

———, “Putting CO2 to Use,” Technical Report, Paris, 2019.

———, “About CCUS,” Technical Report, Paris, 2021a.

———, “Is carbon capture too expensive?,” Technical Report, Paris, 2021b.

IPCC, *Climate Change 2021: The Physical Science Basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change* (Cambridge University Press, 2021).

———, *Climate Change 2022: Mitigation of Climate Change. Contribution of Working Group III to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change* (Cambridge University Press, 2022).

Jaakkola, N., “Monopolistic sequestration of European carbon emissions,” Technical Report 098, Oxford Centre for the Analysis of Resource Rich Economies, University of Oxford, October 2012, publication Title: OxCarre Working Papers.

Jacobson, M., “Evaluation of coal and natural gas with carbon capture as proposed solutions to global warming, air pollution, and energy security,” in *100% Clean, Renewable Energy and Storage for Everything* (New York: Cambridge University Press, 2020).

KHAN, C., R. AMIN AND G. MADDEN, “Carbon dioxide injection for enhanced gas recovery and storage (reservoir simulation),” *Egyptian Journal of Petroleum* 22 (December 2013), 225–240.

LAFFORGUE, G., B. MAGNÉ AND M. MOREAUX, “Optimal Sequestration Policy with a Ceiling on the Stock of Carbon in the Atmosphere,” in R. Guesnerie and H. Tulkens, eds., *The Design of Climate Policy* (MIT Press, 2008).

LEACH, A., C. F. MASON AND K. VAN ’T VELD, “Co-optimization of enhanced oil recovery and carbon sequestration,” *Resource and Energy Economics* 33 (November 2011), 893–912.

MASON, C. F. AND K. V. , “Hotelling Meets Darcy: A New Model of Oil Extraction,” mimeo (2013).

MCGLADE, C. AND P. EKINS, “Un-burnable oil: An examination of oil resource utilisation in a decarbonised energy system,” *Energy Policy* 64 (January 2014), 102–112.

———, “The geographical distribution of fossil fuels unused when limiting global warming to 2°C,” *Nature* 517 (January 2015), 187–190.

MISCHENKO, I. T., *Enhanced Oil Recovery Methods* (Gubkin Russian State University of Oil and Gas, Moscow, Russia, 2001).

MOREAUX, M. AND C. WITHAGEN, “Optimal abatement of carbon emission flows,” *Journal of Environmental Economics and Management* 74 (November 2015), 55–70.

NÚÑEZ-LÓPEZ, V. AND E. MOSKAL, “Potential of CO2-EOR for Near-Term Decarbonization,” *Frontiers in Climate* 1 (2019).

OLDENBURG, C. M., S. H. STEVENS AND S. M. BENSON, “Economic feasibility of carbon sequestration with enhanced gas recovery (CSEGR),” *Energy* 29 (July 2004), 1413–1422.

REZAI, A. AND F. VAN DER PLOEG, “Abandoning Fossil Fuel: How Fast and How Much,” *The Manchester School* 85 (2017), e16–e44.

SCHMELZ, W., G. HOCHMAN AND K. MILLER, “Total Cost of Carbon Capture and Storage Implemented at a Regional Scale: Northeastern and Midwestern United States,” *Interface focus: a theme supplement of Journal of the Royal Society interface* 10 (June 2020).

Van der Ploeg, F. and A. Rezai, "Cumulative emissions, unburnable fossil fuel, and the optimal carbon tax," *Technological Forecasting and Social Change* 116 (March 2017a), 216–222.

———, "The simple arithmetic of carbon pricing and stranded assets," *Energy Efficiency* 11 (2017b).

———, "The risk of policy tipping and stranded carbon assets," *Journal of Environmental Economics and Management* 100 (March 2020), 102258.

Van 't Veld, K., C. F. Mason and A. Leach, "The Economics of CO2 Sequestration Through Enhanced Oil Recovery," *Energy Procedia* 37 (January 2013), 6909–6919.

Welsby, D., J. Price, S. Pye and P. Ekins, "Unextractable fossil fuels in a 1.5 °C world," *Nature* 597 (September 2021), 230–234, number: 7875 Publisher: Nature Publishing Group.

Appendix

A Proof of Lemma 1

Proof. (i) Suppose, to the contrary, that there exists an interval of time with $x(t) > 0$ and $y(t) > 0$. Then $\gamma_x = \gamma_y = 0$.

We first consider the case where along the interval $\zeta x > b + s$. Then $\mu = 0$ because $\zeta x > b+s$. Partial abatement is only possible when $Z < \bar{Z}$, so that $\dot{\tau}_z = 0$ because $\nu = 0$. If $\mu = 0$ and $\dot{\lambda}_z = 0$ then it follows from (9c) that $\gamma_b \neq 0$ so that $\gamma_b > 0$. Therefore $b = 0$. If $s > 0$, then, from (9d),

$$\lambda_s = (c_a + c_s)e^{-\rho t} - \tau. \tag{A.1}$$

Use this together with (9b) in (9a) to get

$$e^{-\rho t}\left(c_y - (\psi - \delta(X + \alpha S)) - \zeta(c_a + c_s)\right) = \lambda_x - \zeta\lambda_s. \tag{A.2}$$

If $\lambda_x \leq \zeta\lambda_s$ along a (sub)interval then we get from (10a) and (10b) that $\dot{\lambda}_x - \zeta\dot{\lambda}_s = e^{-\rho t}\delta x(\alpha\zeta - 1) < 0$ because it has been assumed that $x > 0$ and $\alpha\zeta < 1$. This implies that $\lambda_s(t) > 0$ forever during and after the interval under consideration, contradicting the transversality condition $\lim_{t\uparrow\infty}\lambda_s(t)S(t) = 0$ with $S > 0$ since $s > 0$ in the interval. Therefore $\lambda_x - \zeta\lambda_s > 0$. Take the derivative of (A.2) with respect to time to get

$$-\rho\left(c_y - (\psi - \delta(X + \alpha S)) - \zeta(c_a + c_s)\right) = \delta\alpha(\zeta x - s). \tag{A.3}$$

The left-hand side is negative, due to $\lambda_x - \zeta\lambda_s > 0$. The right-hand side is positive because $\zeta x > s$ by assumption. We have reached a contradiction. Therefore, $s = 0$. We have from (9b), (9a) and $\mu = 0$ that

$$e^{-\rho t}(c_y - (\psi - \delta(X + \alpha S)) = \lambda_x - \zeta\lambda_z. \tag{A.4}$$

Take the time derivative and use (10a) as well as $\dot{\tau} = s = 0$ to obtain

$$-\rho(c_y - (\psi - \delta(X + \alpha S))) = 0. \tag{A.5}$$

Therefore $X + \alpha S$ is constant, contradicting $x > 0$ and $s = 0$. We conclude that $\zeta x > b + s$ implies $y = 0$.

It remains to be shown that $\zeta x = b + s > 0$ and $y > 0$ is non-optimal either. If $b > 0$ then $\gamma_b = 0$. Use (9a), (9b) and (9c) to get

$$\beta - \gamma(x(0) - (\psi - \delta(X_0 + \alpha S_0)) = \lambda_x + \zeta\tau - \zeta\mu - \gamma_x, \tag{A.6}$$

$$e^{-\rho t}(c_y - (\psi - \delta(X + \alpha S)) - \zeta c_a) = \lambda_x. \tag{A.7}$$

Take the derivative with respect to time and use (10a). Then:

$$-\rho\left(c_y - (\psi - \delta(X + \alpha S)) - \zeta c_a\right) + \delta\alpha s = 0. \tag{A.8}$$

If $s = 0$ we get a contradiction, because X is not constant. Hence, $s > 0$. But then

$$-\tau + \mu = \lambda_s - (c_a + c_s)e^{-\rho t}, \tag{A.9}$$

and we get (A.2). Differentiation with respect to time yields, as before,

$$-\rho\left(c_y - (\psi - \delta(X + \alpha S)) - \zeta(c_a + c_s)\right) = \delta\alpha(\zeta x - s). \tag{A.10}$$

Accordingly, we obtain a contradiction as before. We conclude that $b = 0$ and $s = \zeta x > 0$. But, then still $X + \alpha S$ is constant which has been ruled out by the assumption that $\alpha\zeta < 1$. Therefore, we cannot have $x > 0$ and $y > 0$ at the same time.

(ii) Suppose $\zeta x(t) > b(t) + s(t)$ along some interval of time. Then along the interval $\dot{\tau} = \mu = 0$. Therefore, if $b > 0$, so that $\gamma_b = 0$, we obtain a contradiction to (9c). If $s > 0$ then $\gamma_s = 0$. Hence, $\dot{\lambda}_s = -\rho(c_a + c_s)e^{-\rho t}$ and therefore, from (10b), x is constant. But using $\lambda_s = (c_a + c_s)e^{-\rho t} - \tau$ in (9a) with $y = 0$ and differentiating with respect to time, taking into account that x and τ are constant, yields

$$-\rho\left(\beta - \gamma x - (\psi - \delta(X + \alpha S)) - \zeta(c_a + c_s)\right) = \delta\alpha(\zeta x - s). \tag{A.11}$$

We find a contradiction, because $\zeta x > b + s$ so that the right hand side is positive, whereas the left hand side is negative because $\lambda_x - \zeta\lambda_s > 0$, as shown above.

(iii) In view of part (ii) of the lemma, we only have to consider the case where

$\zeta x(t) = b(t) + s(t) > s(t) > 0$. Then it follows from (9c), (9d) and (10b) that x is constant. In view of part (i) of the lemma we also have

$$e^{-\rho t}(\beta - \gamma x - (\psi - \delta(X + \alpha S)) - \zeta(c_a + c_s)) = \lambda_x - \zeta\lambda_s. \tag{A.12}$$

Hence, we get a contradiction as in part (ii) of the lemma.

(iv) It follows from (9c) and (9d) that

$$\lambda_s = c_s e^{-\rho t} + \gamma_b - \gamma_s.$$

Suppose that $b = 0$ along some subinterval of (T_2, T_3). Along the interval $\zeta x = s$. If $s = 0$ at some instant of time in the interval then also $x = 0$ at that instant of time. The state of the system does not change and for the rest of the program it holds that $x = b = s = 0$. Hence, we are not in the interior of $\{T_2, T_3\}$. Therefore $s > 0$ and the value of insertion of CO_2, λ_s, remains positive, which violates the transversality condition $\lim_{t\to\infty} \lambda_s(t)S(t) = 0$. ■

B Scenarios 3 and 4

Since the most complex Scenario 3 combines the characteristics of Scenarios 2 and 4 we start with the simpler Scenario 4.

B.1 Scenario 4: The economy performs only CCU

In this scenario, the economy applies CCU over a first time interval $[0, T_1]$ and next stops abating its CO_2 emissions during a second time phase $[T_1, T_3]$, until the depletion of the carbon budget and the transition to the 100 % renewable energy regime. We proceed through backward induction.

B.1.1 The last phase of fossil fuel exploitation $[T_1, T_3]$

Let $S_1 \equiv S(T_1)$, be the inherited gas inside the fossil fuel wells at the beginning of the phase and denote by $X_1 \equiv X(T_1)$, the remaining fossil fuel reserves at time T_1. For a given triplet (X_1, S_1, T_1), the optimal scenario after T_1 can be characterized as follows.

Since $b(t) = s(t) = 0$ for $t \geq T_1$, $\mu(t) = 0$ and the optimality condition reads:

$$\beta - \gamma x(t) = (\psi - \delta(X(t) + \alpha S_1)) + (\lambda_x(t) + \zeta\tau)\, e^{\rho t} \;\; \text{for} \;\; t \in [T_1, T_3].$$

Time differentiation yields:

$$-\gamma \dot{x}(t) = \delta x(t) - \delta x(t) + \rho\,(\lambda_x(t) + \zeta\tau)\, e^{\rho t} = \rho\,(\lambda_x(t) + \zeta\tau)\, e^{\rho t}.$$

Hence, an equivalent expression of the optimality condition reads:

$$\beta - \gamma x(t) = (\psi - \delta(X(t) + \alpha S_1)) - \frac{\gamma}{\rho}\dot{x}(t).$$

This implies:

$$\dot{x}(t) - \rho x(t) = \frac{\rho}{\gamma}\,(\psi - \beta - \delta(X(t) + \alpha S_1)\,. \tag{B.1}$$

Since we assume incomplete extraction of the fossil fuel reserves, then $\lambda_x(T_3) = 0$ and the amount of the resource left unburnt is determined by the carbon budget:

$$X(T_3) = X_1 - \frac{\bar{Z} - Z_0}{\zeta}.$$

Through the optimality condition

$$c_y = \psi - \delta\,(X(T_3) + \alpha S_1) + \zeta\tau e^{\rho T_3}$$

we get the corresponding level of the shadow cost of carbon evaluated at time T_3 as a function of $X(T_3)$ and thus of X_1, the remaining amount of the fossil fuel reserves at the beginning of the phase, S_1 the gas stored in the wells at the end of the CCU phase, and the size of the carbon budget. This gives a condition on the time derivative of $x(t)$ at time T_3:

$$\dot{x}(T_3^-) = -\frac{\rho}{\gamma}\zeta\tau e^{\rho T_3} = -\frac{\rho}{\gamma}\left[c_y - \psi + \delta(X_1 - \frac{\bar{Z} - Z_0}{\zeta} + \alpha S_1)\right].$$

We now prove that $\lambda_s(t) = \alpha\lambda_x(t)$ over the phase. Since $x(t) = 0$, $t > T_3$, (10b) implies that $\lambda_s(t) = \bar{\lambda}_s$, some constant after T_3. The transversality condition, $\lim_{t\uparrow\infty} \bar{\lambda}_s S_1 =$

0 then implies that $\bar{\lambda}_s = 0$. Thus $\lambda_x(T_3) = \lambda_s(T_3) = 0$. Integrating (10a) and (10b) over $[t, T_3)$ with these terminal conditions, we get:

$$\lambda_s(t) = \alpha \int_t^{T_3} x(\tau) e^{-\rho\tau} d\tau = \alpha \lambda_x(t).$$

Thus $\dot{x}(T_1^+) = -(\rho/\gamma)(\lambda_x(T_1) + \zeta\tau)e^{\rho T_1}$ gives an expression of $\lambda_s(T_1)$:

$$\lambda_s(T_1) = \frac{\alpha\gamma}{\rho} \left(\dot{x}(T_3^-) e^{-\rho T_3} - \dot{x}(T_1^+) e^{-\rho T_1} \right).$$

Differentiating twice with respect to time yields:

$$-\gamma \ddot{x}(t) = -\rho\delta x(t) - \rho\gamma\dot{x}(t),$$

showing that the motion of $x(t)$ during the phase is governed by the second order differential equation:

$$\ddot{x}(t) - \rho\dot{x}(t) - \frac{\delta\rho}{\gamma} x(t). \tag{B.2}$$

With K_1 and K_2 the two integration constants, the general solution of the above equation reads:

$$x(t) = K_1 e^{m_1 t} + K_2 e^{m_2 t},$$

where the roots m_1 and m_2 have been already computed.

For a given vector (X_1, Z_1, T_1) inherited from the previous phase, the vector (K_1, K_2, T_3) is a solution of the following system of three conditions.

(i) The continuity condition over the energy consumption path at time T_3:

$$x(T_3^-) = \frac{\beta - c_y}{\gamma}$$

Substitution of general solution (B.2) yields

$$K_1 e^{m_1 T_3} + K_2 e^{m_2 T_3} = \frac{\beta - c_y}{\gamma}. \tag{B.3}$$

(ii) The condition over the time derivative of $x(t)$ at time T_3^-:

$$m_1K_1e^{m_1T_3} + m_2K_2e^{m_2T_3} = -\frac{\rho}{\gamma}\left[c_y - \psi + \delta(X_1 - \frac{\bar{Z} - Z_0}{\zeta} + \alpha S_1)\right].$$

(iii) The depletion condition of the carbon budget during the time interval $[T_1, T_3]$:

$$\begin{aligned}\frac{\bar{Z} - Z_0}{\zeta} &= \int_{T_1}^{T_3} x(t)dt \\ &= \frac{K_1}{m_1}\left(e^{m_1T_3} - e^{m_1T_1}\right) + \frac{K_2}{m_2}\left(e^{m_2T_3} - e^{m_2T_1}\right). \qquad \text{(B.4)}\end{aligned}$$

We express by $K_1(X_1, S_1, T_1)$, $K_2(X_1, S_1, T_1)$ and $T_3(K_1, S_1, T_1)$ the functional dependency of K_1, K_2 and T_3 on the vector (X_1, S_1, T_1).

B.1.2 The CCU phase $[0, T_1]$

During this phase, the economy implements full injection of the CO_2 emissions inside the oil wells and:

$$\tau - \mu = (c_a + c_s)e^{-\rho t} - \lambda_s.$$

Hence, the optimality condition reads:

$$\beta - \gamma x(t) = \psi - \delta(X(t) + \alpha S(t)) + \lambda_x(t)e^{\rho t} + \zeta(c_a + c_s) - \zeta\lambda_s(t)e^{\rho t}.$$

Time differentiating while remembering that $\zeta x(t) = s(t)$ under full injection of CO_2 emissions, yields:

$$\begin{aligned}-\gamma\dot{x}(t) &= \delta x(t) - \delta\alpha s(t) - \delta x(t) + \rho\lambda_x(t)e^{\rho t} + \zeta\alpha\delta x(t) - \zeta\rho\lambda_s(t)e^{\rho t} \\ &= \rho\left(\lambda_x(t) - \zeta\lambda_s(t)\right)e^{\rho t}.\end{aligned}$$

Lemma 1 shows that $\lambda_x(t) - \zeta\lambda_s(t) > 0$. We conclude that $\dot{x}(t) < 0$ and thus that $\dot{s}(t) = \zeta\dot{x}(t) < 0$ during the injection phase. Despite the positive effect on the extraction cost of gas injection, the extraction rate declines and so the injection rate also declines. At time T_1 the injected flow rate jumps down to zero and remains nil thereafter.

Evaluating the optimality condition before and after T_1 yields:

$$\begin{aligned}\beta - \gamma x(T_1^-) &= \psi - \delta(X_1 + \alpha S_1) + \lambda_x(T_1)e^{\rho T_1^-} + \zeta(c_a + c_s) - \zeta\lambda_s(T_1^-)e^{\rho T_1^-},\\ \beta - \gamma x(T_1^+) &= \psi - \delta(X_1 + \alpha S_1) + \Big(\lambda_x(T_1^+) + \zeta\tau\Big)\, e^{\rho T_1^+}.\end{aligned}$$

Since the extraction path must be time continuous at time T_1, $x(T_1^-) = x(T_1^+)$ and $\lambda_x(t)$ is time continuous, we conclude that $\zeta(c_a + c_s) - \zeta\lambda_s(T_1^-)e^{\rho T_1} = \zeta\tau e^{\rho T_1}$. This implies:

$$\lambda_s(T_1^-) = (c_a + c_s)e^{-\rho T_1} - \tau.$$

Taking account of our previous computations of $\lambda_s(T_1)$ and τ, an equivalent expression of this condition reads:

$$\alpha\zeta\dot{x}(T_1^+) + (1 - \alpha\zeta)\dot{x}(T_3^-)e^{-\rho(T_3 - T_1)} = -\frac{\zeta\rho}{\gamma}(c_a + c_s).$$

Through the optimality condition, an equivalent expression of $\dot{x}(t)$ reads:

$$\dot{x}(t) \;=\; -\frac{\rho}{\gamma}\left[\beta - \gamma x(t) - \psi + \delta(X(t) + \alpha S(t)) - \zeta(c_a + c_s)\right].$$

Evaluating before and after the time T_1:

$$\begin{aligned}\dot{x}(T_1^-) &= -\frac{\rho}{\gamma}\Big[\beta - \gamma x(T_1^-) - \psi + \delta(X_1 + \alpha S_1) - \zeta(c_a + c_s)\Big],\\ \dot{x}(T_1^+) &= -\frac{\rho}{\gamma}\Big[\beta - \gamma x(T_1^+) - \psi + \delta(X_1 + \alpha S_1)\Big].\end{aligned}$$

Since $x(T_1^-) = x(T_1^+)$, we conclude that the extraction rate trajectory has a kink at time T_1 and $\dot{x}(T_1^-) = \dot{x}(T_1^+) + \frac{\rho\zeta}{\gamma}(c_a + c_s)$, which implies $|\dot{x}(T_1^-)| < |\dot{x}(T_1^+)|$. Differentiating twice the optimality condition w.r.t. time we obtain:

$$\begin{aligned}-\gamma\ddot{x}(t) &= \rho\left(-\delta x(t) + \zeta\alpha\delta x(t)\right) - \rho\gamma\dot{x}(t)\\ &= -\rho\delta(1 - \alpha\zeta)x(t) - \rho\gamma\dot{x}(t).\end{aligned}$$

The motion of $x(t)$ obeys the second order differential equation:

$$\ddot{x}(t) - \rho\dot{x}(t) - \frac{\delta\rho}{\gamma}(1 - \alpha\zeta)x(t) \;=\; 0.$$

Let R_1 and R_2 be two integration constants, then a general solution is given by:

$$x(t) \quad = \quad R_1 e^{r_1 t} + R_2 e^{r_2 t},$$

with:

$$r_1 = \frac{1}{2}\left[\rho + \sqrt{\rho^2 + 4\frac{\delta\rho}{\gamma}(1-\alpha\zeta)}\right],$$
$$r_2 = \frac{1}{2}\left[\rho - \sqrt{\rho^2 + 4\frac{\delta\rho}{\gamma}(1-\alpha\zeta)}\right].$$

The vector $(R_1, R_2, X_1, S_1, T_1)$ is the solution of the following system of five conditions:

(i) The continuity condition over $x(t)$ at time T_1, $x(T_1^-) = x(T_1^+)$, which implies:

$$R_1 e^{r_1 T_1} + R_2 e^{r_2 T_1} = K_1(X_1, S_1, T_1)e^{m_1 T_1} + K_2(X_1, S_1, T_1)e^{m_2 T_1}. \tag{B.5}$$

(ii) The kink condition over the time derivative of $x(t)$ at time T_1^+, $\dot{x}(T_1^-) = \dot{x}(T_1^+) + \frac{\rho\zeta}{\gamma}(c_a + c_s)$, which implies:

$$r_1 R_1 e^{r_1 T_1} + r_2 R_2 e^{r_2 T_1} = m_1 K_1(X_1, S_1, T_1)e^{m_1 T_1} + m_2 K_2(X_1, S_1, T_1)e^{m_2 T_1} + \frac{\rho\zeta}{\gamma}(c_a + c_s). \tag{B.6}$$

(iii) The limit condition over λ_s at time T_1, $\alpha\zeta\dot{x}(T_1^+) + (1-\alpha\zeta)\dot{x}(T_3^-)e^{-\rho(T_3 - T_1)} = -\frac{\zeta\rho}{\gamma}(c_a + c_s)$, which implies:

$$m_1 K_1(X_1, S_1, T_1)e^{m_1 T_1}\left[\alpha\zeta + (1-\alpha\zeta)e^{-m_2(T_3 - T_1)}\right]$$
$$+ m_2 K_2(X_1, S_1, T_1)e^{m_2 T_1}\left[\alpha\zeta + (1-\alpha\zeta)e^{-m_1(T_3 - T_1)}\right] = -\frac{\zeta\rho}{\gamma}(c_a + c_s). \tag{B.7}$$

(iv) The fuel resource stock balance condition over the time interval $[0, T_1]$, $X_0 = \int_0^{T_1} x(t)dt + X_1$, which yields:

$$X_0 = \frac{R_1}{r_1}\left(e^{r_1 T_1} - 1\right) + \frac{R_2}{r_2}\left(e^{r_2 T_1} - 1\right) + X_1. \tag{B.8}$$

(v) The injected gas balance condition over the interval $[0, T_1]$, $S_1 = S_0 + \zeta\int_0^{T_1} x(t)dt$, which gives:

$$S_1 = S_0 + \zeta(X_0 - X_1). \tag{B.9}$$

B.2 The economy performs CCU and CCS in sequence

In this scenario, CCU and CCS happen on two disjoint time intervals with CCU being first used and next CCS is introduced only when the carbon budget becomes depleted. Hence $0 < T_1 < T_2 < T_3$. We proceed by backward induction starting with the last CCS phase, next move to the no abatement phase and last to the initial CCU phase.

B.2.1 The CCS phase $[T_2, T_3]$

Let $X_2 = X(T_2)$ and $S_1 = S(T_1) = S(T_2)$. For a given vector (X_2, S_1, T_2) inherited from the previous CCU and no abatement phases, the vector (M_1, M_2, T_3) is the solution of the following system of three conditions.

(i) The supply continuity condition at T_3:

$$M_1 e^{m_1 T_3} + M_2 e^{m_2 T_3} = \frac{\beta - c_y}{\gamma}. \tag{B.10}$$

(ii) The condition over the time derivative of $x(t)$ at time T_3:

$$\begin{aligned} \dot{x}(T_3) &= m_1 M_1 e^{m_1 T_3} + m_2 M_2 e^{m_2 T_3} \\ &= -\frac{\rho}{\gamma}\left[c_y - \psi - \zeta c_a + \delta(X(T_3) + \alpha S_1)\right] = 0. \end{aligned} \tag{B.11}$$

(iii) The fossil resource stock balance condition over the CCS phase:

$$\begin{aligned} X_Z - X(T_3) &= \int_{T_2}^{T_3} x(t)dt \\ &= \frac{M_1}{m_1}\left(e^{m_1 T_3} - e^{m_1 T_2}\right) + \frac{M_2}{m_2}\left(e^{m_2 T_3} - e^{m_2 T_2}\right). \end{aligned} \tag{B.12}$$

B.2.2 The no abatement phase $[T_1, T_2]$

During this phase the economy does not abate its CO_2 emissions until the depletion of its carbon budget. The remaining fossil fuel stock at time T_2, X_Z, is determined through the carbon budget as a function of X_1:

$$X_Z = X_1 - \frac{\bar{Z} - Z_0}{\zeta}.$$

Hence (M_1, M_2, T_3) are functions of (X_1, S_1, T_2). Let $M_1(X_1, S_1, T_2)$ and $M_2(X_1, S_1, T_2)$ express this functional dependency by a slight abuse of notation.

For a given vector (X_1, S_1, T_1), the vector (K_1, K_2, T_2) is a solution of the system of three conditions:

(i) The continuity condition over the extraction trajectory at time T_2, $x(T_2^-) = x(T_2^+)$, which implies:

$$K_1 e^{m_1 T_2} + K_2 e^{m_2 T_2} = M_1(X_1, S_1, T_2) e^{m_1 T_2} + M_2(X_1, S_1, T_2) e^{m_2 T_2}. \tag{B.13}$$

(ii) The kink condition over the time derivative of $x(t)$ at the time T_2, $\dot{x}(T_2^-) = \dot{x}(T_2^+) - \frac{\rho\zeta}{\gamma} c_a$, which yields:

$$m_1 K_1 e^{m_1 T_2} + m_2 K_2 e^{m_2 T_2} = m_1 M_1(X_1, S_1, T_2) e^{m_1 T_2} + m_2 M_2(X_1, S_1, T_2) e^{m_2 T_2} - \frac{\rho\zeta}{\gamma} c_a. \tag{B.14}$$

(iii) The carbon budget depletion condition, $\frac{\bar{Z} - Z_0}{\zeta} = \int_{T_1}^{T_2} x(t) dt$, which implies:

$$\frac{\bar{Z} - Z_0}{\zeta} = \frac{K_1}{m_1} \left(e^{m_1 T_2} - e^{m_1 T_1} \right) + \frac{K_2}{m_2} \left(e^{m_2 T_2} - e^{m_2 T_1} \right). \tag{B.15}$$

To express the functional dependency of K_1, K_2 and T_2 with respect to the vector (X_1, S_1, T_1) we introduce the notations $K_1(X_1, S_1, T_1)$, $K_Z(X_1, S_1, T_1)$ and $T_2(X_1, S_1, T_1)$. Note that when the vector (X_1, S_1, T_1) has been determined the whole characteristics of the extraction path, and thus of the abatement path during the CCS phase, are determined.

B.2.3 The CCU phase $[0, T_1]$

During this phase, the economy injects the whole flow of CO_2 emissions in the oil wells. The motion of the extraction rate, $x(t)$, obeys the differential equation:

$$\ddot{x}(t) - \rho\dot{x}(t) - \frac{\delta\rho}{\gamma}(1-\alpha\zeta)x(t) = 0.$$

Let R_1 and R_2 be two integration constants, then a general solution is given by:

$$x(t) = R_1 e^{r_1 t} + R_2 e^{r_2 t},$$

where r_1 and r_2 have been previously defined.

Since the economy performs CCS between times T_2 and T_3, $\tau = c_a e^{-\rho T_2}$. $\lambda_x(T_3) = \lambda_s(T_3) = 0$ still holds, implying that $\lambda_s(t) = \alpha\lambda_x(t)$ during the CCS phase $[T_2, T_3]$. Thus $\lambda_s(T_2) = \alpha\lambda_x(T_2)$ implies that $\lambda_s(t) = \alpha\lambda_x(t)$ also during the CCU phase $[0, T_1]$. Hence, $\lambda_s(T_1) = \alpha\lambda_x(T_1)$.

The condition $\lambda_s(T_1) + \tau = (c_a + c_s)e^{-\rho T_1}$ must hold at time T_1. Using $\dot{x}(T_1^+) = -(\rho/\gamma)(\lambda_x(T_1) + \zeta\tau)e^{\rho T_1}$ and the expression of τ, an equivalent expression of the condition over $\lambda_s(T_1)$ reads

$$-\frac{\alpha\gamma}{\rho}\dot{x}(T_1^+) + (1-\alpha\zeta)c_a e^{-\rho(T_2-T_1)} = (c_a + c_s).$$

The vector $(R_1, R_2, X_1, S_1, T_1)$ is a solution of the system of five equations:

(i) The continuity condition over $x(t)$ at time T_1:, $x(T_1^-) = x(T_1^+)$, which implies:

$$R_1 e^{r_1 T_1} + R_2 e^{r_2 T_1} = K_1(X_1, S_1, T_1)e^{m_1 T_1} + K_2(X_1, S_1, T_1)e^{m_2 T_1} \quad \text{(B.16)}$$

(ii) The kink condition over the time derivative of $x(t)$ at time T_1, $\dot{x}(T_1^-) = \dot{x}(T_1^+) + \frac{\rho\zeta}{\gamma}(c_a + c_s)$, which yields:

$$r_1 R_1 e^{r_1 T_1} + r_2 R_2 e^{r_2 T_1} = m_1 K_1(X_1, S_1, T_1)e^{m_1 T_1} + m_2 K_2(X_1, S_1, T_1)e^{m_2 T_1} + \frac{\rho\zeta}{\gamma}(c_a + c_s) \quad \text{(B.17)}$$

(iii) The limit condition over λ_s at time T_1, $-\frac{\alpha\gamma}{\rho}\dot{x}(T_1^+) + (1-\alpha\zeta)c_a e^{-\rho(T_2-T_1)} = (c_a + c_s)$, which implies

$$-\frac{\alpha\gamma}{\rho}\left[m_1K_1(X_1,S_1,T_1)e^{m_1T_1}+m_2K_2(X_1,S_1,T_1)e^{m_2T_1}\right]$$
$$+(1-\alpha\zeta)c_ae^{-\rho(T_2(X_1,S_1,T_1)-T_1)}=(c_a+c_s). \tag{B.18}$$

(iv) The fuel resource stock balance condition over the time interval $[0,T_1]$, $X_0-X_1=\int_0^{T_1}x(t)dt$, which yields:

$$X_0=\frac{R_1}{r_1}\left(e^{r_1T_1}-1\right)+\frac{R_2}{r_2}\left(e^{r_2T_1}-1\right). \tag{B.19}$$

(v) The injected gas balance condition over the interval $[0,T_1]$, $S_1=S_0+\zeta\int_0^{T_1}x(t)dt$, which implies:

$$S_1=S_0+\zeta(X_0-X_1). \tag{B.20}$$

C The CCU frontier

We proceed in two steps. First we describe the CCU frontier in Scenario 1 without CCS. Second we study Scenario 2 with CCS.

C.1 Scenario 1: No CCS

For CCU to be economically irrelevant in Scenario 1, it must be the case that:

$$\lambda_s(0)+\tau \quad\leq\quad c_a+c_s.$$

In Appendix A we have shown that $\lambda_s(t)=\alpha\lambda_x(t)$ in Scenario 1. Next, since $\lambda_x(T_3^-)=0$, we get $\dot{x}(T_3^-)=-\frac{\rho}{\gamma}\zeta\tau e^{\rho T_3}$, which yields:

$$\tau=-\frac{\gamma}{\zeta\rho}\dot{x}(T_3^-)e^{-\rho T_3}. \tag{C.1}$$

Hence, evaluating $\dot{x}(0^+)$:

$$\begin{aligned}\dot{x}(0^+) &= -\frac{\rho}{\gamma}\left(\lambda_x(0) + \zeta\tau\right) \\ &= -\frac{\rho}{\gamma}\left(\lambda_x(0) - \frac{\gamma}{\rho}\dot{x}(T_3^-)e^{-\rho T_3}\right) \\ &= -\frac{\rho}{\gamma}\lambda_x(0) + \dot{x}(T_3^-)e^{-\rho T_3},\end{aligned}$$

which yields the following expression of $\lambda_x(0)$:

$$\lambda_x(0) = \frac{\gamma}{\rho}\left(\dot{x}(T_3^-)e^{-\rho T_3} - \dot{x}(0^+)\right).$$

Using the computed expressions of $\lambda_x(0)$ and τ, we obtain:

$$\begin{aligned}\alpha\lambda_x(0) + \tau &= \frac{\alpha\gamma}{\rho}\left(\dot{x}(T_3^-)e^{-\rho T_3} - \dot{x}(0^+)\right) - \frac{\gamma}{\zeta\rho}\dot{x}(T_3^-)e^{-\rho T_3} \\ &= \frac{\gamma}{\zeta\rho}\left[\alpha\zeta\left(\dot{x}(T_3^-)e^{-\rho T_3} - \dot{x}(0^+)\right) - \dot{x}(T_3^-)e^{-\rho T_3}\right] \\ &= -\frac{\gamma}{\zeta\rho}\left[(1-\alpha\zeta)\dot{x}(T_3^-)e^{-\rho T_3} + \alpha\zeta\dot{x}(0^+)\right].\end{aligned}$$

Taking into account the expressions of $\dot{x}(T_3^-)$, $\dot{x}(0^+)$ and remembering that $\rho = m_1 + m_2$, the term between brackets can be written as:

$$\begin{aligned}[.] &= (1-\alpha\zeta)\left[m_1K_1e^{m_1T_3} + m_2K_2e^{m_2T_3}\right]e^{-\rho T_3} + \alpha\zeta\left[m_1K_1 + m_2K_2\right] \\ &= (1-\alpha\zeta)\left[m_1K_1e^{-m_2T_3} + m_2K_2e^{-m_1T_3}\right] + \alpha\zeta\left[m_1K_1 + m_2K_2\right] \\ &= m_1K_1\left[\alpha\zeta + (1-\alpha\zeta)e^{-m_2T_3}\right] + m_2K_2\left[\alpha\zeta + (1-\alpha\zeta)e^{-m_1T_3}\right].\end{aligned}$$

The integration constants K_1, K_2 and T_3 are functions of (X_0, S_0). Let $K_1(X_0, S_0)$, $K_2(X_0, S_0)$, $T_3(X_0, S_0)$ denote the resulting functions by a slight abuse of notations. Let

$$\begin{aligned}\Phi(X_0, S_0) \equiv &-\frac{\gamma}{\zeta\rho}\left\{m_1K_1(X_0, S_0)\left[\alpha\zeta + (1-\alpha\zeta)e^{-m_2T_3(X_0,S_0)}\right]\right. \\ &\left.+m_2K_2(X_0, S_0)\left[\alpha\zeta + (1-\alpha\zeta)e^{-m_1T_3(X_0,S_0)}\right]\right\}.\end{aligned}$$

Upon differentiation we find:

$$\begin{aligned}\frac{\partial\Phi}{\partial X_0} &= -\frac{\gamma}{\zeta\rho}\left\{m_1\frac{\partial K_1}{\partial X_0}\left[\alpha\zeta+(1-\alpha\zeta)e^{-m_2T_3(X_0,S_0)}\right]\right.\\ &\quad +m_2\frac{\partial K_2}{\partial X_0}\left[\alpha\zeta+(1-\alpha\zeta)e^{-m_1T_3(X_0,S_0)}\right]\\ &\quad \left.+(-m_1m_2)(1-\alpha\zeta)\left(K_1e^{-m_2T_3}+K_2e^{-m_1T_3}\right)\frac{\partial T_3}{\partial X_0}\right\}.\end{aligned}$$

Since $m_1+m_2=\rho$, it is immediate that:

$$\begin{aligned}K_1e^{-m_2T_3}+K_2e^{-m_1T_3} &= K_1e^{m_1T_3}e^{-\rho T_3}+K_2e^{m_2T_3}e^{-\rho T_3}\\ &= \left[K_1e^{m_1T_3}+K_2e^{m_2T_3}\right]e^{-\rho T_3}=\bar{y}e^{-\rho T_3}>0.\end{aligned}$$

In the companion technical appendix we show that $\partial K_1/\partial S=\alpha\partial K_1/\partial X$, $\partial K_2/\partial S=\alpha\partial K_2/\partial X$, and $\partial T_3/\partial S=\alpha\partial T_3/\partial X$, hence $\partial\Phi/\partial S_0=\alpha\partial\Phi/\partial X_0$. We show also that $\partial K_1/\partial X_0<0$, $\partial K_2/\partial X_0>0$ and $\partial T_3/\partial X_0<0$, hence:

$$\begin{aligned}\frac{\partial\Phi}{\partial X_0}=-\frac{\gamma}{\zeta\rho}&\left\{m_1\underbrace{\frac{\partial K_1}{\partial X_0}}_{(<0)}\left[\alpha\zeta+(1-\alpha\zeta)e^{-m_2T_3(X_0,S_0)}\right]\right.\\ &+\underbrace{m_2}_{(<0)}\underbrace{\frac{\partial K_2}{\partial X_0}}_{(>0)}\left[\alpha\zeta+(1-\alpha\zeta)e^{-m_1T_3(X_0,S_0)}\right]\\ &\left.+\underbrace{(-m_1m_2)}_{(>0)}(1-\alpha\zeta)\bar{y}e^{-\rho T_3}\underbrace{\frac{\partial T_3}{\partial X_0}}_{(<0)}\right\}>0,\\ \frac{\partial\Phi}{\partial S_0}&=\alpha\frac{\partial\Phi}{\partial X_0}>0.\end{aligned}$$

Now consider the boundary condition $\Phi(X_0,S_0)=c_a+c_s$. In the plane (X,S) this condition defines an implicit relationship between X and S such that:

$$\frac{dS}{dX} = -\frac{\partial\Phi/\partial X}{\partial\Phi/\partial S}=-\frac{1}{\alpha},$$

which denotes a line of slope $-1/\alpha$. Fixing S and increasing X slightly, Φ is increased, so that $\Phi>c_a+c_s$ to the right of the boundary and $\Phi<c_a+c_s$ to the left of it. Thus the line of equation $\Phi(X_0,S_0)=c_a+c_s$ defines the CCU abatement frontier. CCU is

economically relevant if and only if (X, S) is located above the CCU frontier and is irrelevant if it is located below the frontier.

C.2 Scenario 2: Only CCS

Now assume that the economy uses the CCS option. We still have $\lambda_s(t) = \alpha\lambda_x(t)$ in Scenario 2, so that $\lambda_s(0) = \alpha\lambda_x(0)$. Since $\tau = c_a e^{\rho T_2}$:

$$\dot{x}(0^+) \;=\; -\frac{\rho}{\gamma}\left[\lambda_x(0) + \zeta\tau\right] = -\frac{\rho}{\gamma}\left[\lambda_x(0) + \zeta c_a e^{-\rho T_2}\right].$$

Hence:

$$\lambda_x(0) \;=\; -\frac{\gamma}{\rho}\dot{x}(0^+) - \zeta c_a e^{-\rho T_2}.$$

Thus:

$$\begin{aligned}\lambda_s(0) + \tau &= \alpha\lambda_x(0) + c_a e^{-\rho T_2}\\ &= -\frac{\alpha\gamma}{\rho}\dot{x}(0^+) - \alpha\zeta c_a e^{-\rho T_2} + c_a e^{-\rho T - Z}\\ &= -\frac{\alpha\gamma}{\rho}\dot{x}(0^+) + (1-\alpha\zeta) c_a e^{-\rho T_2}.\end{aligned}$$

Using the expression for $\dot{x}(0^+)$, we define

$$\Phi(X_0, S_0) \;\equiv\; -\frac{\alpha\gamma}{\rho}\left[m_1 K_1(X_0, S_0) + m_2 K_2(X_0, S_0)\right] + (1-\alpha\zeta) c_a e^{-\rho T_2(X_0, S_0)},$$

where we express by $K_i(X_0, S_0)$, $i = 1, 2$, and by $T_2(X_0, S_0)$ the dependency of (K_1, K_2, T_2) on (X_0, S_0). For a scenario with CCS and no CCU to be optimal it must be the case that $\Phi(X_0, S_0) \leq c_a + c_s$.

In the companion technical appendix, we show that $\partial K_1/\partial X_0 < 0$ and $\partial K_2/\partial X_0 > 0$, thus $m_2 < 0$ implies that:

$$m_1\frac{\partial K_1}{\partial X_0} + m_2\frac{\partial K_2}{\partial X_0} < 0.$$

We show also in the technical appendix that $\partial T_2/\partial X_0 < 0$, hence:

$$\frac{\partial \Phi}{\partial X_0} = -\frac{\alpha\gamma}{\rho}\underbrace{\left[m_1\frac{\partial K_1}{\partial X_0} + m_2\frac{\partial K_2}{\partial X_0}\right]}_{(<0)} - \rho\underbrace{\frac{\partial T_2}{\partial X_0}}_{(<0)}(1-\alpha\zeta)c_a e^{-\rho T_2} > 0,$$

leading to the same type of frontier condition as in the no CCS case for CCU to be economically relevant.