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Personalized Pricing, Competition and Welfare

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Abstract

Data-driven AI pricing algorithms in on-line markets collect consumer information and use it in their pricing technologies. In the simplest symmetric Hotelling's model such technologies reduce prices and profits. We extend Hotelling's model with vertically differentiated products, cost asymmetries and arbitrary adjustment costs. We provide a characterization of competition in personalized pricing: Sellers compete in offering consumer surplus, personalized prices are constrained monopoly prices and social welfare is maximal. For linear adjustment costs, adopting personalized pricing technology is a dominant strategy for both sellers. We derive conditions under which the most efficient seller increases her profit through personalized pricing. While aggregate consumer surplus increases, consumers with high switching costs may be hurt. Finally, we discuss several extensions of our approach such as oligopoly.

JEL Classification: L1, D43, L13

Keywords: Industrial Organization, Personalized Prices, Artificial Intelligence, Technology Adaption, Profit Paradox

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1 Introduction

There is an increasing use of data-driven Artificial Intelligence (AI) pricing algorithms in online markets that collect detailed information about consumer characteristics and adopt new pricing technologies incorporating this information. Companies make substantial investments to keep up in this technology race. We analyze the incentives for companies to adopt such pricing technologies and the implications for both individual and aggregate consumer welfare.

A pricing algorithm is a software program for determining the price of a product or service. It takes data on the market environment, such as cost, sales, inventories, rival firms' prices, consumer characteristics, and uses this information to update its own price.[1] The use of AI pricing algorithms can bring economic benefits to all market participants. On the supply side, algorithms are used to optimize prices and quantities. Pricing algorithms can adjust prices to respond to changing market conditions and to monitor rivals. In addition, algorithms are used to improve the quality of search results or to personalize product recommendations, which benefits consumers. Consumers also benefit from the use of algorithms, for example through reduced search and transaction costs as possible information asymmetries between companies and customers can be reduced.[2]

However, recent economic literature also raises concerns about the potential for AI pricing algorithms to lead to consumer harm.[3] Consumer harm can be related to increased ability of suppliers to extract a larger fraction of consumer surplus through excessive pricing or to exploit certain consumer groups through personalized pricing.[4] AI pricing algorithms that are using personal information of consumers can also bring prices close to consumers' maximal willingness to pay effectively approaching the monopoly outcome of perfect price discrimination. Another concern is that algorithmic pricing can be used by dominant incumbent firms to deter new entry or to extend their dominant position into related markets. Furthermore, pricing algorithms can create new opportunities for collusion.

We extend Hotelling's model of spatial competition to incorporate cost asymmetries, both horizontally and vertically differentiated products and arbitrary reservation price functions reflecting consumers' adjustment costs in order to analyze endogenous adoption of data-driven AI pricing technologies. The information about the exact location in Hotelling's model can be seen as comparable to detailed information about consumer characteristics that can be obtained by the sellers in online markets. Data-driven AI pricing technologies collect this information and employ it through the practice of personalized pricing, while uniform pricing strategies ignore it. In this sense, the uniform pricing strategy represents the current pricing technology,

[1] For a more detailed discussion of the definition and effects of pricing algorithms see e.g. Harrington (2019).

[2] See e.g. OECD (2017).

[3] Examples of literature that analyse potential harm for consumers can be found in e.g. Ezrachi and Stucke (2016) and Calvano et al. (2019, 2020).

[4] See for example Competition & Markets Authority (2018).

while the data-driven AI pricing technology has the potential to disrupt the current technology. A related perspective on technology adoption is that within companies already running data-driven AI technologies, companies still need to decide whether to further expand and include additional consumer characteristics. Modifying Hotelling's model is a convenient framework to study endogenous technology adoption of personalized pricing versus uniform pricing.[5]

We start our analysis with personalized pricing and show that the consumer buys at the company that can provide the largest surplus for this consumer (such company can be viewed as more efficient). In equilibrium the less efficient competitor adopts marginal cost pricing. And the more efficient company sets its own price such that the consumer obtains a consumer surplus that is equal to the maximal consumer surplus of the consumer's second-best choice, which is either buying at the competitor at marginal cost pricing or refraining from buying. Social welfare is maximal if both companies adopt personalized pricing. We also provide an extension of this model where we show that these results generalize to oligopoly with more than two companies and general spaces of consumer characteristics.

Comparison of the uniform pricing setting to the setting where only one of the companies adopts personalized pricing technology indicates that there are strong unilateral incentives to invest in the new technology for both, the less and the more efficient company. Also in case one company has already adopted the new technology, the other firm has a strong unilateral incentive to adopt too. This implies that endogenous adoption can be seen as an asymmetric dominant strategy game in which adopting the next-generation personalized-pricing technology dominates the current uniform-pricing strategy. Symmetric settings of Hotelling's model, as first analyzed in Thisse and Vives (1988), can be seen as prisoners' dilemmas in that individual company's profits will be lower by adopting the new technology compared to keeping the current technology. Rhodes and Zhou (2021) extend this result to a class of symmetric distributions of consumer valuations for oligopoly markets with full coverage under uniform pricing. We show that in an asymmetric duopoly setting with full market coverage personalized pricing is still a dominant strategy, however, the more efficient company can benefit even in case of joint adoption of the new technology while the less efficient company never benefits. These findings provide a theoretical underpinning of the empirical findings in de Loecker et al. (2020), who report that the average markup and profits in the US have increased since 1980 and that this is driven mainly by the upper percentiles which have increased sharply.

We also analyze total welfare and consumer welfare implications of switching to the new technology under full marlet coverage. We show that both total welfare and aggregate consumer welfare will increase under personalized pricing because all consumers will buy from the company that provides them their (individualy) largest net consumer surplus and this firm will outcompete all other firms. However, there will be a group of consumers who are worse-off compared to the

[5] In our analysis in section 4.1 we assume that adoption of personalized pricing technology is costless. The model can be easily extended to incorporate costs of technology adoption without affecting the main results.

uniform pricing setting and there is a risk that the information available about characteristics of that particular group will be exploited by the seller in order to extract the full consumer surplus through perfect price discrimination.[6]

Our contribution relates closely to the work of Thisse and Vives (1988).[7] They analyze the choice of a pricing policy in a duopoly model and the incentives that arise in spatial competition for firms to price discriminate or price uniformly. They also analyze the consequences of the price-policy choice for firms and consumers. In a homogeneous products setting they show that price discrimination by all firms in the market emerges as the unique equilibrium outcome even though both firms would make more profit by both following a uniform price policy. We extend their analysis to vertically differentiated products with asymmetric firms and identify a robust subspace of parameter values for which a similar paradox holds, even though both producers make lower profits compared to uniform pricing. However, we also identify another large subspace in which we show that personalized (discriminatory) pricing can be profitable for the more efficient producer. This result holds when there are sizable differences in efficiency among the two producers of differentiated products or when the consumer adjustment costs are relatively low.[8] Furthermore, in that case the more efficient company is able to exploit the information about characteristics of the "more loyal" segment of consumers to extract the full consumer surplus from that segment. And personalized (discriminatory) prices for that segment will be higher than those under uniform pricing policy. This new result may explain why in practice we observe the massive shift to use of data-driven personalized pricing algorithms.

Another related work is by Liu and Serfes (2004). They analyze how quality of information affects the choice of pricing strategies (i.e. uniform vs discriminatory) in a spatial price competition model with identical products. Firms use the available information to classify the consumers into different groups. The number of identifiable consumer segments increases with the information quality. Liu and Serfes (2004) show that when the information quality is low, uniform pricing is more attractive. They also find that for sufficiently high levels of information precision commitment not to price discriminate is a dominated strategy. This corresponds to the

[6] The consumer surplus standard is common in competition policy and competition economics. In practice consumers who feel they have been exploited or overcharged can engage in private law suits. A similar approach might be available for consumers who feel they have been exploited through price discriminatory practies based on information that companies collect.

[7] The Hotelling setup in Thisse and Vives (1988) has been often employed in subsequent research. Shaffer and Zhang (2002) use it to study personalized pricing in case one firm has a brand advantage over the other. Chen and Iyer (2002) use this framework to study personalized pricing when firms first need to advertise to reach consumers. Montes et al. (2019) use it to analyze the behavior of a monopolistic data intermediary that sells data to competing firms, which use this data for personalized pricing. More recently, Chen et al. (2020) use the Hotelling setup in Thisse and Vives (1988) to study consumer identity management.

[8] In a related work, Rhodes and Zhou (2021) also show that the result of Thisse and Vives (1988) can only be reversed if the market is not fully covered and their result extends to symmetric oligopolies. Further, Jullien et al. (2020) study the interaction between personalized pricing and distribution strategies and show that in the case of intra-brand competition between distribution channels, personalized pricing maximizes industry profit in contrast to classical result in Thisse and Vives (1988).

results described in Thisse and Vives (1988). Liu and Serfes (2004) do not observe any consumer exploitation as in their model the prices each firm charges are below the nondiscriminatory (uniform) price for any parameter values (including perfect information quality as assumed in our paper). This is different from our results. So that our extension of spatial competition model with vertically differentiated products seems to alternate and enrich the standard results in the literature.

Work by Armstrong and Vickers (2001) has already attempted to analyze price discrimination strategies in a vertically differentiated spatial competition model. We have a similar setup as we also view producers as competitors that supply different utility levels to consumers. However, Armstrong and Vickers (2001) do not focus on comparison of the profits and consumer welfare implications of various combinations of uniform and personalized pricing strategies.

Our insights are complementary to Rhodes and Zhou (2021). They show that full market coverage is a sufficient condition for extending the prisoners' dilemma result in Thisse and Vives (1988) to a symmetric general setting. Partial market coverage under uniform pricing and a mild technical sufficient condition are needed to increase individual profits and reduce aggregate consumer welfare. They also provide a sufficient condition under which consumers with high enough willingness's to pay suffer from personalized pricing. Our contribution is to stress the role of asymmetries in efficiency as an important and realistic explanation that offsets the prisoners' dilemma result of Thisse and Vives (1988). We also show that a more efficient company may benefit while the less efficient company does not in asymmetric Hotelling's models. It holds the wider implication that asymmetries in efficiency offset "binary" results of the kind that either all companies benefit from personalized pricing or none, such binary results are inevitable under symmetry and symmetry is therefore not without loss of generality.

A similar result also arises in a sequential context where companies decide on the timing of adoption, they want to adopt at the earliest moment in time. Switching to personalized pricing is profitable in the short run because it enlarges a company's market share. Modification of these arguments also explains why companies invest in the latest technology and find it attractive to gather higher dimensions of consumer characteristics.

This paper is organized as follows. In Section 3 we analyze the market configuration in which both companies employ personalized-pricing technologies that are exogenously given and characterize equilibrium pricing strategies and welfare. In Section 4 we focus on an extension of the linear Hotelling's model. In Section 4.1, we analyze the other three market configurations to quantify personalized pricing by both companies compared to uniform pricing by both companies. In Section 4.2, we analyze what endogenous market configuration will arise when AI pricing technologies are endogenously adopted. The simplest setting is when both companies adopt their AI pricing technology simultaneously, but sequential games will be discussed as well. In section 5 we discuss extensions and applications to demonstrate the generalizability of our main results.

2 The Model

In this section we propose a parsimonious framework, which allows to illustrate the personalized pricing paradox and our methodology in a tractable model.[9] Consider two companies that employ online AI pricing technologies to sell differentiated products or services to heterogeneous consumers. Consumer 'big data' characteristics[10] are modelled as the unit interval $[0,1]$ and we refer to a consumer with characteristics $x \in [0,1]$ as consumer x. The distribution of characteristics is described by the cumulative distribution function $F : [0,1] \to [0,1]$. We assume that the corresponding density function f is continuous and $f(\cdot) > 0$.[11]

The companies are called A and B, indexed by $i = A, B$. The constant marginal cost of providing the product is denoted as $c^i \geqslant 0$, $i = A, B$. Their AI pricing technologies are endogenous and either include consumer characteristic x or not.[12] If characteristic x is included, company i's AI technology sets personalized price $p^i(x) \geqslant 0$ to consumer with characteristic x. Otherwise, it sets the uniform price $\bar{p}^i$ to every characteristic $x \in [0,1]$. We refer to these technologies as P (personalized) and U (uniform). For two companies there are four configurations PP (i.e. both companies employ personalized pricing technology), PU (i.e. company A employs personalized pricing, while company B employs uniform pricing technology), UP (i.e. company B employs personalized pricing and company A uniform pricing technology) and UU (i.e. both companies employ uniform pricing). Configuration UU corresponds to Hotelling's model, which can be seen as an important benchmark. The exogenously given PP configuration is analyzed in Section 3, while the other three (exogenously given) configurations are analyzed in Section 4.1. The outcomes of these configurations are needed for the analysis of endogenous adoption of AI pricing technologies in Section 4.2. The most-convenient setting for such an analysis is a 2×2 normal form game in which both companies adopt their AI pricing technology (either U or P) simultaneously, but sequential adoption will also be covered.

Consumer x has a reservation price of $r^i(x)$ for the product of company $i = A, B$, and a surplus of 0 from not buying. Consumer surplus from buying at company i is $r^i(x) - p^i$, where p^i stands for either personalized price $p^i(x)$ or uniform price $\bar{p}^i$. Then, the utility maximization

[9] In Section 5, a number of important extensions are discussed. They are non-monotonicity of reservation-price functions, partial market coverage, discontinuity, oligopoly setting with multiple firms, multidimensional consumer characteristics, or setting where adoption of technology takes place over time. We are able to show that our main results are robust in these more general settings. However, many of these extensions create situations with a multiplicity of possible cases, which makes the model intractable.

[10] This is based on the information collected from previous on-line searches, previous purchases, other consumer characteristics (such as age, occupation, location, links in on-line social networks).

[11] Our main results on personalized pricing in Section 3 are independent of the distribution f.

[12] We assume that if both use AI pricing technology, they possess the same information about consumer x.

problem of consumer x can be stated as

$$\max\left\{0, r^A(x) - p^A, r^B(x) - p^B\right\}.$$

This consumer will buy from company A if $r^A(x) - p^A > \max\left\{0, r^B(x) - p^B\right\}$ and from company B if $r^B(x) - p^B > \max\left\{0, r^A(x) - p^A\right\}$. In case consumer x is indifferent, we will apply the endogenous tie-breaking rule suggested in Simon and Zame (1990) to guarantee existence of equilibrium. For linear reservation prices $r^A(x) = u^A - tx$ and $r^B(x) = u^B - t(1-x)$ as in e.g. Hotellings's model, we may think of company A being located at $x = 0$, company B located at $x = 1$, $u^i > 0$ represents the utility of buying at company $i = A, B$ while tx and $t(1-x)$, $t > 0$, represent the consumer specific transaction or adjustment costs of buying at company A respectively, company B.

To provide structure to the model, we assume that the reservation price for company A is continuous and strictly decreasing in x meaning consumer $x = 1$ has the lowest willingness to pay for company A's product. By symmetry, the reservation price for company B is continuous and strictly increasing in x meaning that the willingness to pay is the lowest for consumer $x = 0$. Furthermore, for each consumer it is assumed that there is at least one company in the market that can provide the product against marginal costs that are below the consumer's reservation price for this company. This excludes market segments where trade would yield negative social welfare and that are trivial to analyze. Formally we have:

Assumption 1 *Reservation price $r^A : [0,1] \to \mathbb{R}_+$ is a continuous and strictly decreasing function in x, reservation price $r^B : [0,1] \to \mathbb{R}_+$ is a continuous and strictly increasing function in $[0,1]$ and $\max\left\{r^A(x) - c^A, r^B(x) - c^B\right\} > 0$ for all $x \in [0,1]$.*

Figure 1 illustrates this assumption for a case with unequal marginal costs. The last inequality of Assumption 1 can also be interpreted as follows: for each consumer there is at least one company that can provide positive social welfare through trade and has an incentive to serve this consumer.

Because reservation prices are monotonic, it can be shown that the market segment of consumers who derive positive social welfare for a particular company is an interval, which is obvious from Figure 1. The bounds on the companies' market segments, where positive social welfare can be offered, play an important role in our results. Consider company A and denote $\overline{x}^A \in [0,1]$ as the upper bound on its interval $\left[0, \overline{x}^A\right)$ where it can offer positive social welfare. There are two mutually exclusive cases. In case the lowest reservation price $r^A(1) \geq c^A$, social welfare of buying at company A is positive for all consumers $x \in [0,1)$ and $\overline{x}^A = 1$, because reservation price $r^A(x)$ is strictly decreasing. In the opposite case, $r^A(1) < c^A$, there exists an $\overline{x}^A < 1$ such that $r^A\left(\overline{x}^A\right) = c^A$ and social welfare $r^A(x) - c^A > 0$ for all $x \in \left[0, \overline{x}^A\right)$ while it is negative for $x \in \left(\overline{x}^A, 1\right]$. The latter case is depicted in Figure 1. For linear reservation prices $r^A(x) = u^A - tx$,

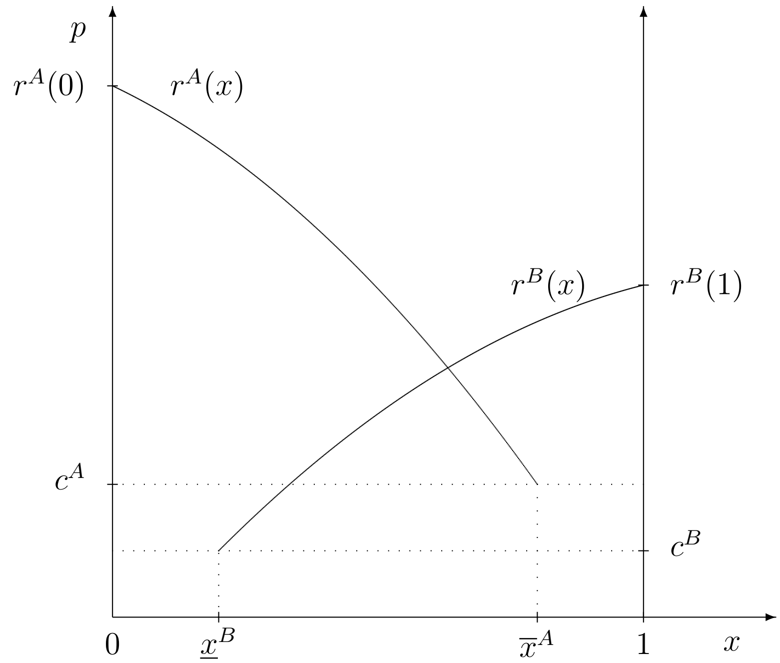

Figure 1: The reservation-price curves (solid) and the marginal cost lines (dotted) in case company A offers higher quality at higher marginal costs than company B.

we have $\overline{x}^A = \frac{u^A - c^A}{t} > 0$ and therefore $\overline{x}^A < 1$ whenever adjustment costs $t > u^A - c^A$. In that case, company A cannot attract consumers in $\left(\overline{x}^A, 1\right)$.

For company B we also have two cases. In case the lowest reservation price $r^B(0) \geq c^B$, social welfare of buying at company B is positive for all consumers $x \in (0, 1]$ and $\underline{x}^B = 0$. Otherwise, there exists a $\underline{x}^B > 0$ such that $r^B\left(\underline{x}^B\right) = c^B$ and social welfare $r^B(x) - c^B > 0$ for all $x \in \left(\underline{x}^B, 1\right]$ while it is negative for $x \in \left(0, \underline{x}^B\right]$, which is depicted in Figure 1. For linear reservation prices $r^B(x) = u^B - t(1 - x)$, we have $\underline{x}^B = 1 - \frac{u^B - c^B}{t} < 1$ and therefore $\underline{x}^B > 0$ whenever adjustment costs $t > u^B - c^B$. In that case, company B cannot attract any consumer $x \in \left(0, \underline{x}^B\right)$.

Assumption 1 guarantees that market segments $\left[0, \overline{x}^A\right)$ and $\left(\underline{x}^B, 1\right]$ overlap, i.e. $\underline{x}^B < \overline{x}^A$. Consequently, both companies may provide positive social welfare to consumers in the nonempty 'middle' market segment $\left(\underline{x}^B, \overline{x}^A\right)$ and correspond to local monopolies outside. Because both companies have an incentive to serve consumers in the middle market segment, both companies have to compete for these consumers. For linear reservation prices, $\underline{x}^B < \overline{x}^A$ translates into $t < u^A - c^A + u^B - c^B$. To summarize, in what follows we will distinguish the (possibly empty) subintervals $\left[0, \underline{x}^B\right]$, $\left(\underline{x}^B, \overline{x}^A\right)$ and $\left[\overline{x}^A, 1\right]$. Figure 1 illustrates these three intervals.

Our analysis will characterize a market segment $\mathcal{A} \subseteq [0, 1]$ of consumers who buy at company A, and market segment $\mathcal{B} \subseteq [0, 1]$, $\mathcal{B} \cap \mathcal{A} = \emptyset$, of consumers that buy at company B.[13] The

[13]Consumers outside segment $\mathcal{B} \cup \mathcal{A}$ do not buy, have consumer surplus of 0, generate social welfare of 0 and may be neglected.

associated aggregate consumer surplus of personalized prices $p^A(x)$ and $p^B(x)$ is given by

$$CS = \int_{x\in\mathcal{A}} \left(r^A(x) - p^A(x)\right) f(x)\,dx + \int_{x\in\mathcal{B}} \left(r^B(x) - p^B(x)\right) f(x)\,dx,$$

aggregate producer surplus is given by[14]

$$PS = \int_{x\in\mathcal{A}} \left(p^A(x) - c^A\right) f(x)\,dx + \int_{x\in\mathcal{B}} \left(p^B(x) - c^B\right) f(x)\,dx.$$

Aggregate social welfare is given by

$$SW = \int_{x\in\mathcal{A}} \left(r^A(x) - c^A\right) f(x)\,dx + \int_{x\in\mathcal{B}} \left(r^B(x) - c^B\right) f(x)\,dx.$$

3 Competition in Personalized Pricing

In this section, we analyze the exogenously given configuration of AI pricing technologies in which both companies compete in personalized prices. The analysis is broken down in a number of cases depending upon the equilibrium market shares of the companies, where each case is discussed in a separate subsection. We first focus on a duopoly in which both firms have positive market shares in equilibrium. After that we consider the case in which one company is a monopolist who is contested by its competitor. A final subsection discusses the welfare implications of our equilibrium.

3.1 Duopoly with positive market shares

In this subsection, we characterize the unique equilibrium for pricing technologies in which market shares of both companies are positive and the market is covered. Moreover, this equilibrium is independent of the distribution of consumers.

We first state the necessary and sufficient condition for duopoly competition with positive market shares to arise. To ensure this we analyze the inequality $r^A(0) - c^A > r^B(0) - c^B$ which states that the consumer at location 0 generates more social welfare from trading with company A than from trading with company B, which is positive by Assumption 1. It also means that company A can offer, through setting its price, more consumer surplus (up to social welfare) to the consumer at location 0 compared to both his competitor and to the option not to trade.[15] In equilibrium, company A will indeed outcompete its competitor at location 0. Similarly, inequality $r^B(1) - c^B > r^A(1) - c^A$ states that company B can offer more consumer

[14] Similar expressions for consumer and producer surplus can be stated if one or both of the companies would set a uniform price, i.e. $\bar{p}^A$ or $\bar{p}^B$.

[15] To see this, define $\bar{p} \in \left[c^A, r^A(0)\right]$ as the price for which $r^A(0) - \bar{p} = \max\left\{0, r^B(0) - c^B\right\}$. For $r^A(0) - c^A > r^B(0) - c^B > 0$, we have that $c^A < \bar{p} < r^A(0)$. Then, for each $p \in \left(c^A, \bar{p}\right)$ company A offers more consumer surplus $r^A(0) - p$ than the maximal consumer surplus $r^B(0) - c^B$ company B is able to offer while company A generates a positive profit.

surplus to the consumer at location 1 than company A and attract this consumer. Combining the above conditions implies

$$r^A(1) - r^B(1) < c^A - c^B < r^A(0) - r^B(0), \tag{D}$$

where condition (D) stands for duopoly. In other words, this condition describes the relative efficiency of the companies at the endpoint locations of the consumer characteristics.

We state our first main result. All mathematical proofs can be found in Appendix A.

Proposition 2 *If condition (D) holds, then there exist a unique* $x^* \in (\underline{x}^B, \overline{x}^A)$ *such that*

1. $x \in [0, x^*) : p^B(x) = c^B$, $p^A(x) = r^A(x) - \max\{0, r^B(x) - c^B\} > c^A$ *and* x *buys at* A,
2. $x \in (x^*, 1] : p^A(x) = c^A$, $p^B(x) = r^B(x) - \max\{0, r^A(x) - c^A\} > c^B$ *and* x *buys at* B,
3. $p^A(x^*) = c^A$, $p^B(x^*) = c^B$, *and* x *is indifferent where to buy.*

Moreover, x^* *solves* $r^A(x) - c^A = r^B(x) - c^B$.

This proposition implies that given Assumption 1, both personalized-pricing functions are continuous in consumer characteristic x on $[0, 1]$. Figure 2 illustrates the curve of personalized prices of Proposition 2 as set by each company, where the dashed curves correspond to the curves for reservation prices and marginal costs as depicted in Figure 1. Company A's pricing schedule is strictly decreasing on its own market segment $[0, x^*)$ and constant at marginal costs on its competitor's market segment $(x^*, 1]$. By symmetry, company B's pricing schedule is constant at marginal costs on $[0, x^*)$ and strictly increasing on $(x^*, 1]$. As Figure 2 illustrates, $x^* \in (\underline{x}^B, \overline{x}^A)$. Note that $r^i(x) - c^i$ for $i = A, B$ can be seen as the 'nonnegative profit' upper bound on consumer surplus that consumer x can obtain from buying at company i. The market share x^* of company A is determined by the consumer for whom upper bound $r^A(x) - c^A$ from buying at company A equals the upper bound $r^B(x) - c^B$ from buying at company B, as illustrated by the equally-long double arrows in Figure 2.

The economic intuition underlying the proof is based upon the insight that both companies compete for every individual consumer. In comparing differentiated prices of differentiated products, only the net consumer surplus matters to consumers as does the consumer surplus of not buying. That is, consumer x compares $r^A(x) - p^A(x)$ to $r^B(x) - p^B(x)$, provided these are nonnegative. Therefore, one may say that the companies are involved in an auction with money being replaced by consumer surplus and the company that is able to set a price that offers the largest nonnegative consumer surplus will attract this consumer. In this analogy, "not buying" can be seen as a third bidder who bids zero consumer surplus. In equilibrium, the company that attracts the consumer will bid a consumer surplus that is equal to the maximum of consumer surpluses offered by the competitor and not buying, f.e. $\max\{0, r^B(x) - c^B\}$ for $x < x^*$.

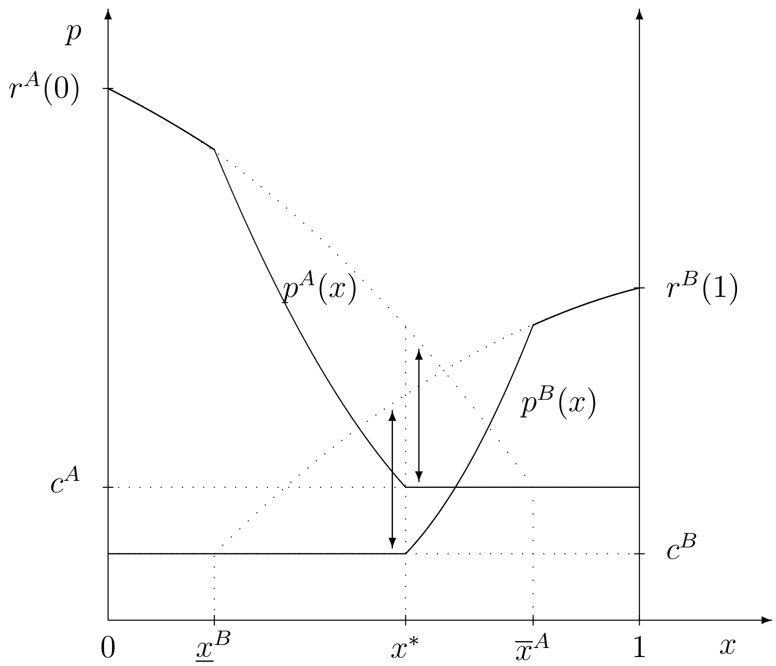

Figure 2: The personalized-pricing strategies of Proposition 2 (solid) for reservation-price curves and the marginal cost lines of Figure 1 (all dotted).

This raises the question what ensures that the consumer chooses for the "right" company if he is indifferent between buying at this company and the second-best alternative of either going to the competitor or not buying at all. In the proof, the endogenous tie-breaking rule proposed in Simon and Zame (1990) is invoked. This rule treats consumers as additional players who choose where to buy (or not) and this simple modification restores existence of equilibrium in games with an exogenously given tie-breaking rule such as for example equal market sharing whenever consumers are indifferent in the standard Bertrand price competition with differentiated products. Having consumers as additional players in such standard Bertrand price competition, one of the companies will equate its price to marginal costs; the other company will match that company's consumer surplus and is able to do so by setting its price above marginal costs; and the consumer will buy at the company that could lower its price slightly to become the strictly preferred company.

These insights are clarifying in discussing the qualitatively different market segments. Recall market segments $\left[0, \underline{x}^B\right]$, $\left(\underline{x}^B, \overline{x}^A\right)$ and $\left[\overline{x}^A, 1\right]$ that have been introduced in Section 2. Our first result indicates that we need to split the middle segment into sub-segments $\left(\underline{x}^B, x^*\right)$ and $\left(x^*, \overline{x}^A\right)$. We discuss these four cases in detail.

Consider market segment $\left[0, \underline{x}^B\right] \subset [0, x^*)$ first. It is common knowledge that company B can only offer positive consumer surplus to consumer x in this market segment by setting a price below marginal costs and incurring a loss. Company B avoids this by adopting marginal cost pricing and offering negative consumer surplus and not buying is the consumer's second-best option, i.e., $\max\left\{0, r^B(x) - c^B\right\} = 0$. As a consequence, company A does not face any competition, sets its price equal to the reservation price of consumer x who then buys at this company, and company A extracts all consumer surplus from consumer x. In the absence of competition there is perfect price discrimination on this market segment. The profit per consumer is given by $r^A(x) - c^A > 0$.

On the nonempty market segment $\left(\underline{x}^B, x^*\right)$, company B can offer positive consumer surplus by setting a price at or above its marginal costs and this company can compete with company A. However, the maximal consumer surplus company A can offer to consumer x on this market segment is larger than the maximal surplus that company B is able to offer, i.e., $r^A(x) - c^A > r^B(x) - c^B > 0$. In this situation, both firms are involved in a race to the bottom in competing for consumer x. The company that is able to set a price that offers the largest nonnegative consumer surplus will attract this consumer. Company B bids up to its maximal consumer surplus $r^B(x) - c^B$ by setting $p^B(x) = c^B$. Company A matches this consumer surplus by setting $p^A(x) = r^A(x) - \left(r^B(x) - c^B\right) > c^A$, wins the auction by attracting consumer x and attains a positive profit.[16]

There is an alternative interpretation in terms of constrained monopoly in which company B's presence and bidding for consumers on market segment $[0, x^*)$ forces company A to refrain from perfect price discrimination and to lower its price by $\max\left\{0, r^B(x) - c^B\right\}$.[17] This implies that company A's price is capped by perfect-price discrimination minus the consumer surplus of the second-best alternative of either buying at company B or not buying at all.

The discussion of the other two cases, i.e., $\left(x^*, \overline{x}^A\right)$ and $\left[\overline{x}^A, 1\right]$, is similar to the previous two cases after reversing the roles of the companies. For consumers in market segment $\left(x^*, \overline{x}^A\right)$, both companies can provide positive consumer surplus and it is company B that can offer the most and attracts these consumers by matching company A's consumer surplus. On market segment $\left[\overline{x}^A, 1\right]$ company B does not face competition from company A and it can extract full consumer surplus through prefect price discrimination.

Note also that equilibrium bidding for consumers in the auction is independent of the concentration of consumers at location x and therefore the equilibrium in personalized-pricing is independent of the distribution of consumers characteristics.

The next example illustrates how to apply Proposition 2 in Hotelling's model with linear costs.

Example 3 *Consider the reservation prices $r^A(x) = u^A - tx$ and $r^B(x) = u^B - t(1 - x)$ in Hotelling's model with linear adjustment costs. We consider the case $u^i > c^i$, $i = A, B$, which ensures that the middle market segment is $[0, 1]$. Then, market share x^* solves $r^A(x) - r^B(x) =$*

[16] Company A's profit from attracting consumer $x \in \left(\underline{x}^B, x^*\right)$ is equal to $p^A(x) - c^A = r^A(x) - c^A - \left(r^B(x) - c^B\right) > 0$.

[17] Constrained monopoly is studied in e.g. Funaki et al. (2019).

$u^A - u^B - t(2x-1)$ and this yields

$$x^* = \tfrac{1}{2} + \frac{(u^A - c^A) - (u^B - c^B)}{2t}.$$

Note that $u^i - c^i > 0$ is the maximal consumer surplus company i can offer consumer x if company i would apply marginal cost pricing and there would be no costs of adjustment. The company with the larger maximal surplus has a competitive advantage. Next, $x^* \in (0,1)$ if and only if $\frac{(u^A-c^A)-(u^B-c^B)}{2t} \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ from which we obtain

$$t > \left|(u^A - c^A) - (u^B - c^B)\right|.$$

The personalized prices are given by

$$\begin{aligned} x \in [0, x^*):&\quad p^A(x) = c^B + t + u^A - u^B - 2tx, \quad p^B(x) = c^B, \\ x \in (x^*, 1]:&\quad p^A(x) = c^A, \qquad p^B(x) = c^A + t + u^B - u^A - 2t(1-x). \end{aligned}$$

It is a routine exercise to verify that for every $x \in [0,1]$ it holds that

$$u^A - tx - p^A(x) = u^B - t(1-x) - p^B(x)$$

The endogenous tie-breaking rule implies that consumer x will buy from company A if $x < x^*$ and from company B if $x > x^*$. We briefly discuss several subcases:
***Full symmetry**: $u^A = u^B$ and $c^A = c^B$. Then, $x^* = \frac{1}{2}$ for all $t > 0$ and both companies equally share the market. For all $t > 0$, the personalized prices simplify to*

$$\begin{aligned} x \in \left[0, \tfrac{1}{2}\right):&\quad p^A(x) = c + t - 2tx, \quad p^B(x) = c, \\ x \in \left(\tfrac{1}{2}, 1\right]:&\quad p^A(x) = c, \qquad p^B(x) = c + t - 2t(1-x). \end{aligned}$$

Note that both $p^A(x)$ and $p^B(x)$ are below $c+t$, which represents the uniform prices in Hotelling's model with a uniform distribution of consumers, see e.g. Tirole (1988). Taking the limit of t goes to 0, implies the classic case of Bertrand competition in which $p^A(x) = p^B(x) = c$ for all $x \in [0,1]$ and $\lim_{t \to 0} x^* = \frac{1}{2}$.
***Homogeneous products** produced at asymmetric costs: $u^A = u^B$ and $c^A < c^B$.[18] Then, an equilibrium with positive market shares exists when $t > c^A - c^B$. Furthermore, $x^* > \frac{1}{2}$ implies the most efficient company, i.e. A, attracts the largest market segment of consumers. The personalized prices are given by*

$$\begin{aligned} x \in \left[0, \tfrac{1}{2} + \tfrac{c^B - c^A}{2t}\right):&\quad p^A(x) = c^B + t - 2tx, \quad p^B(x) = c^B, \\ x \in \left(\tfrac{1}{2} + \tfrac{c^B - c^A}{2t}, 1\right]:&\quad p^A(x) = c^A, \qquad p^B(x) = c^A + t - 2t(1-x). \end{aligned}$$

[18] This case corresponds to case (D, D) of Thisse and Vives (1988, p. 130), who assume $c^A = 0 \leq c^B$ and locations at the endpoints.

Differentiated products *produced at symmetric costs:* $u^A > u^B$ *and* $c^A = c^B = c$*. Then, an equilibrium with positive market shares exists exists when* $t > u^A - u^B$*. Furthermore,* $x^* > \frac{1}{2}$ *implies that the company with the most-preferred product, i.e. company A, attracts the largest market segment of consumers. The personalized prices become*

$$x \in \left[0, \tfrac{1}{2} + \tfrac{u^A - u^B}{2t}\right) : \quad p^A(x) = c + t + u^A - u^B - 2tx, \quad p^B(x) = c,$$
$$x \in \left(\tfrac{1}{2} + \tfrac{u^A - u^B}{2t}, 1\right] : \quad p^A(x) = c, \qquad p^B(x) = c + t + u^B - u^A - 2t(1-x).$$

3.2 Constrained Monopoly

In this subsection, we characterize the unique equilibrium for pricing technologies in which one company captures the entire market while the other company contests this market. Moreover, this equilibrium is independent of the distribution of consumers.

We first state the necessary and sufficient conditions for companies A or B to be constrained monopolists. Note that consumer $x = 1$ generates more social welfare from buying at company A than from buying at company B when $r^A(1) - c^A > r^B(1) - c^B$. To consumer $x < 1$, company A generates even more social welfare than company B. Therefore, company A can offer at least as much consumer surplus to each consumer $x \in [0,1)$ than its competitor. In equilibrium, company A will indeed outcompete its competitor and become a constrained monopolist. Similar under condition $r^A(0) - c^A < r^B(0) - c^B$, company B can offer more consumer surplus to each consumer $x \in (0,1]$ and attract all consumers. The two conditions described above can be rewritten as

$$r^A(1) - r^B(1) > c^A - c^B, \tag{A}$$
$$r^A(0) - r^B(0) > c^A - c^B, \tag{B}$$

where (A) stands for company A being the constrained monopolist and (B) for company B being constrained monopolist. Finally, note that condition (A), (B) and (D) are mutually exclusive and cover all possible cases under Assumption 1.

We state our second main result.

Proposition 4 *For all* $x \in (0,1)$ *:*

1. *under (A):* $p^B(x) = c^B$, $p^A(x) = r^A(x) - \max\{0, r^B(x) - c^B\} > c^A$ *and* x *buys at* A*,*
2. *under (B):* $p^A(x) = c^A$, $p^B(x) = r^B(x) - \max\{0, r^A(x) - c^A\} > c^B$ *and* x *buys at* B*.*

The personalized pricing strategy of the company that attracts consumer x is similar to that in Proposition 2 and illustrated in Figure 3 for condition (A). Under this condition, company A captures the entire market as if $x^* = 1$ in terminology of the previous subsection. Company A offers surplus $\max\{0, r^B(x) - c^B\}$ to consumer $x \in [0,1]$ because its competitor, company B, is

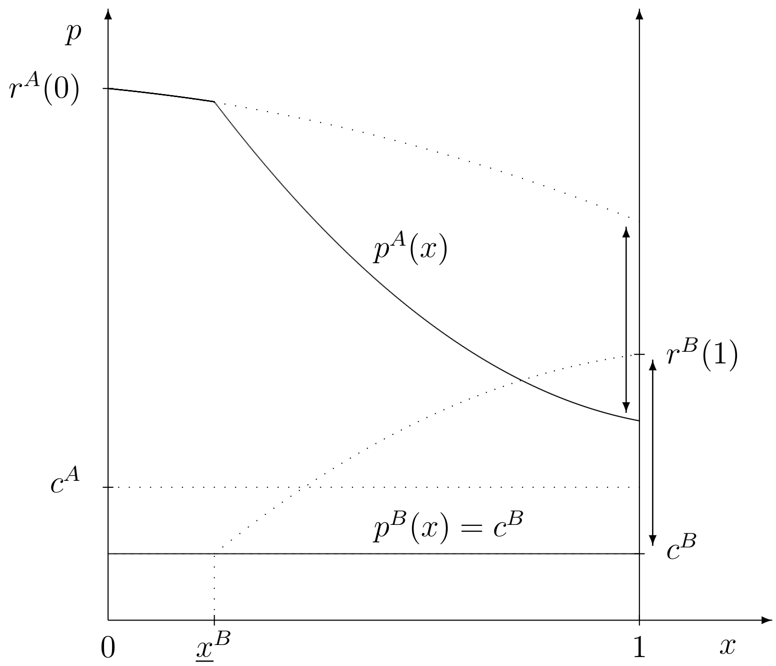

Figure 3: The personalized-pricing strategies of Proposition 4 (solid) under condition (A) given the reservation-price curves and company A's marginal cost line (dotted).

driven to its marginal cost price and offers little to no consumer surplus to this consumer. The intuition for the only two cases, being $x \in \left[0, \underline{x}^B\right]$ and $x \in \left(\underline{x}^B, 1\right)$, immediately applies. By symmetry, under condition (B) company B captures the entire market (as if $x^* = 0$) and the intuition for the cases $x \in \left(0, \overline{x}^A\right)$ and $x \in \left[\overline{x}^A, 1\right]$ of the previous subsection applies.

The following example illustrates the constrained monopoly situation.

Example 5 *Consider the linear reservation prices $r^A(x) = u^A - tx$ and $r^B(x) = u^B - t(1 - x)$ of Example 3. Recall that*

$$x^* = \tfrac{1}{2} + \frac{(u^A - c^A) - (u^B - c^B)}{2t}.$$

Then, $x^* \notin (0, 1)$ if and only if $t \leqslant \left|(u^A - c^A) - (u^B - c^B)\right|$, i.e. the consumers' costs of adjustment are relatively small. If additionally $u^A - c^A > u^B - c^B$, then company A covers the entire market. By symmetry, if additionally $u^A - c^A < u^B - c^B$, company B covers the entire market.

3.3 Welfare

Next we analyze the welfare implications of the personalized pricing and show that it maximizes aggregate social welfare.

Consider the duopoly case with positive market shares first. This means that condition (D) holds, that the market shares are characterized by Proposition 2 and that both companies cover the entire market. For arbitrary market segment $[0,x)$, $x \in (0,1)$, for company A, aggregate social welfare $SW(x)$ consists of the sum of social welfare generated on this and the complementary market segment. Formally,

$$SW(x) = \int_0^x \left(r^A(\hat{x}) - c^A\right) f(\hat{x})\, d\hat{x} + \int_x^1 \left(r^B(\hat{x}) - c^B\right) f(\hat{x})\, d\hat{x},$$

which is differentiable in x. The first-order condition for a maximum is given by[19]

$$\frac{\partial}{\partial x} SW(x) = \left[\left(r^A(x) - c^A\right) - \left(r^B(x) - c^B\right)\right] f(x) = 0$$

and the socially optimal market share x of company A solves $r^A(x) - c^A = r^B(x) - c^B$.[20] Hence, the socially optimal market share of company A coincides with this company's market share x^* characterized by Proposition 2.[21]

Proposition 6 *Social welfare in the personalized pricing equilibrium is maximal.*

Graphically speaking, maximal social welfare is reached at the upper envelope of the graphs of $r^A(x) - c^A$ and $r^B(x) - c^B$, which intersect once at x^*. By Assumption 1, this upper envelope lies above the horizontal axis. The previous arguments establish that the upper envelope is attained under personalized pricing. This establishes that personalized pricing achieves the social optimum.[22]

3.4 Discussion

Proposition 2 and 4 are derived under Assumption 1. Based upon the economic intuition obtained from these two results, we can also characterize equilibrium if we would drop this assumption. Then obviously, $\overline{x}^A \leq \underline{x}^B$ and the middle segment where the companies compete has vanished and the different middle market segment $\left[\overline{x}^A, \underline{x}^B\right]$ has appeared in which both companies are unable to provide positive social welfare and will not serve these consumers. The other two market segments, i.e., $\left[0, \overline{x}^A\right)$ and $\left(\underline{x}^B, 1\right]$, have become isolated local monopolies in which perfect price discrimination prevails. The social welfare consequences are evident, social welfare is maximal but consumers have zero consumer surplus.

Competition in personalized pricing resembles insights from constrained monopoly and is also equivalent to an auction with bidding in consumer surplus instead of money. Because of this equivalence, such competition maximizes social welfare and is efficient. The company that

[19] Recall $\frac{\partial}{\partial x} \int_0^x g(\hat{x})\, d\hat{x} = \frac{\partial}{\partial x} \left[G(\hat{x})\right]_0^x = g(x)$.

[20] We assume $f(x) > 0$ for $x \in [0,1]$.

[21] Similar arguments apply for the constrained monopoly case, in which company A serves the entire market and condition (A) holds. To see this, condition (A) implies that $r^A(x) - c^A > r^B(x) - c^B$ for all $x \in [0,1]$. Hence, $SW(x)$ is increasing in x and the socially optimal market share of company A is the boundary solution at $x = 1$ as if $x^* = 1$.

[22] More detailed discussion of social welfare, consumer and producer surplus can be found in Appendix A.3.

wins the consumer at a particular location sets its price above marginal costs and equal to the perfectly price discriminating one, because in general this company has to lower its bid by the consumer's second-best alternative, being either buying at the competitor who sets its price at marginal costs in order to generate maximal consumer surplus or refrain from buying. Only on market segments where the competitor cannot provide a positive consumer surplus do we expect to see perfect price discrimination.

4 Endogenous adoption of pricing technologies

Personalized pricing in the Hotelling's model with linear costs of adjustment, constant marginal costs and fully symmetric companies has been discussed in Example 3. The personalized prices are lower than the uniform equilibrium prices in Hotelling's model with uniformly distributed consumers while both companies share the market equally in both equilibria. This obviously implies that both companies make less profit under personalized pricing than under uniform pricing and that they would be better off by refraining from adopting personalized pricing technologies. This seems paradoxical to what we observe in reality where companies are eager to adopt personalized pricing. What may explain this paradox?

We answer this question by investigating endogenous adoption of pricing technologies in which both companies simultaneously choose their pricing technologies: either uniform pricing (U) or personalized pricing (P). For the fully symmetric case, as also reported in e.g. Thisse and Vives (1988) and Liu and Serfes (2004), the technology adoption game is given by

	U	P
U	$\frac{1}{2}t, \frac{1}{2}t$	$\frac{1}{8}t, \frac{9}{16}t$
P	$\frac{9}{16}t, \frac{1}{8}t$	$\frac{1}{4}t, \frac{1}{4}t$

where the numbers represent the companies' profits in each configuration under the assumptions that the costs of technology are negligible. Inspection of this game reveals that personalized pricing is a dominant strategy for each company. Consequently, the Nash equilibrium in dominant strategies is that both companies adopt personalized pricing. However, in equilibrium adoption by both companies reduces the prices and profits due to intensified competition for each consumer. This raises the question how robust the results of this example are.

To investigate this, we first analyze closed-form solutions for equilibrium pricing, market shares and profits per market configuration in asymmetric Hotelling's model with linear reservation prices and uniformly distributed consumers. Then, we analyze endogenous adoption of pricing technologies and investigate how robust the paradox is.

4.1 Equilibrium in each configuration

The analysis of endogenous adoption of technologies requires comparing equilibrium profits across market configurations and closed-form solutions for profits would facilitate such analysis. Solutions for equilibrium pricing, market shares and profits for all four configurations can be obtained for asymmetric Hotelling's model with linear reservation prices and uniformly distributed consumers. These solutions are reported in Table 1.[23] Inspection of this table shows that it is convenient to think about the five model parameters as social welfare levels $u^A - c^A$ and $u^B - c^B$ that are provided by the two companies and the cost of adjustment t.

In investigating the paradox, we focus on two competing companies that have positive market shares in all four configurations. To guarantee this, and also that all prices are above marginal costs and that consumers are willing to buy in all four configurations, we derive the corresponding restrictions on the parameter values in Appendix B and state these here as an assumption.[24]

Assumption 7 *Consider the pair of conditions*[25]

$$\begin{cases} 0 < \frac{1}{2}\left(u^A - c^A\right) < u^B - c^B < u^A - c^A, \\ u^A - c^A - \left(u^B - c^B\right) < t < \frac{1}{3}\left(u^A - c^A + u^B - c^B\right). \end{cases}$$

The first condition states that, neglecting costs of adjustment, company A is more efficient and generates more social welfare than less-efficient company B and that the asymmetry between companies should not be too sizeable such that company B is driven out of the market. The second condition restricts the range of adjustment costs, which is a nonempty range if (and only if) the first condition holds. Therefore, both conditions are needed and specify a nonempty set of the parameter space. Note that two companies that provide the same welfare, such as in the fully symmetric case of Example 3, can be seen as a boundary case in which $t \in \left(0, \frac{2}{3}(u - c)\right)$ is the only condition. Figure 4 illustrates the parameter values that satisfy Assumption 7. As we make clear in the appendix the upward sloping line is derived from full market coverage under the UU configuration, while the decreasing 45-degree line indicates the boundary between the personalized-pricing duopoly with positive market shares and the constrained monopoly of the more efficient company A.

Analysis of the profits and market shares in Table 1 shows that compared to the UU configuration, in both the PU and UP configurations market shares and profits of the company

[23] The first column of Table 1 reports the asymmetric Hotelling's game with uniform prices. The second column of Table 1 repeats results from Example 3. The last two columns are combinations of uniform and personalized pricing strategies. The derivations are straightforward and are available upon request from the authors.

[24] In Appendix B, we derive all conditions including Assumption 7, its symmetric counterpart and the general necessary and sufficient condition.

[25] Another range where interior solutions are possible is given by $u^B - c^B > u^A - c^A > \frac{1}{2}\left(u^B - c^B\right) > 0$ and $u^B - c^B - \left(u^A - c^A\right) < t < \frac{1}{3}\left(u^B - c^B + u^A - c^A\right)$. The analysis for this range is identical reversing the roles of the players.

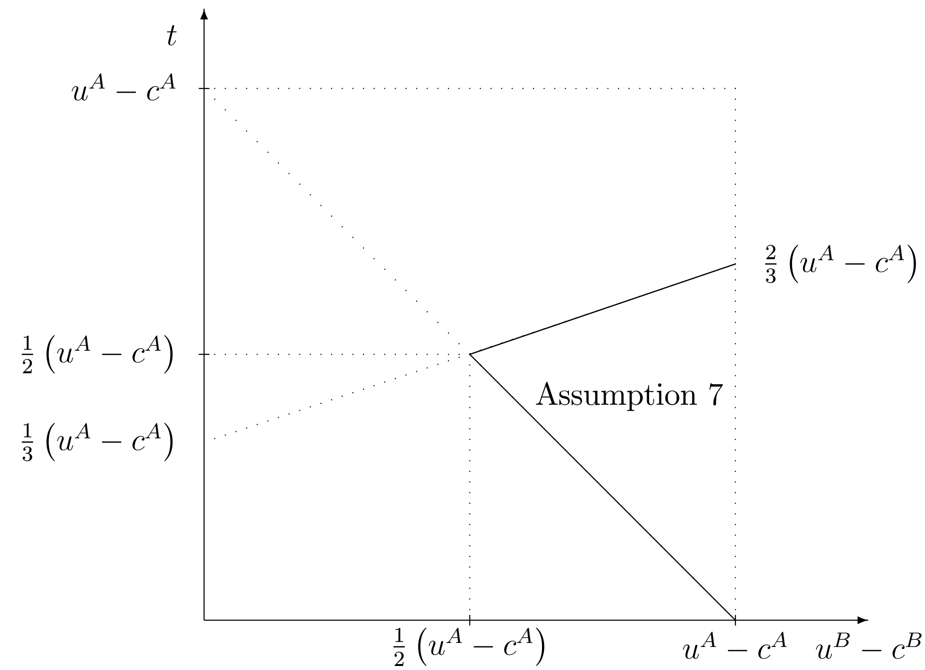

Figure 4: Parameter values of satisfying Assumption 7 for fixed social welfare $u^A - c^A$ in the $\left(u^B - c^B, t\right)$ −space.

adopting the personalized pricing technology increase at the expense of the other company that does not adopted the new technology. So, competing with uniform pricing against a rival that possesses the personalized pricing technology reduces the profits compared to the UU configuration. This indicates that there are strong unilateral incentives to invest in the new technology for both companies, whether it is the less or more efficient one.

Further analysis of the profits and market shares in Table 1 shows that compared to either the UP or PU configuration, the company that employs the uniform pricing technology can increase both its market share and its profit by also adopting the personalized pricing technology and this is at the expense of the other company that did already adopted the new technology. Or, competing with uniform pricing against a rival that possesses the personalized pricing technology reduces the profits compared to the PP configuration. This indicates that there are also strong unilateral incentives to invest in the new technology for companies that are behind in the technology race, whether it is the less or more efficient one.

The comparison of configurations UU and PP is less clear. It indicates that, whether the company will benefit or loose from joint adoption of personalized pricing technology will depend on the relative efficiency of the companies. We analyze this issue in details in the next subsection.

	UU	PP	PU	UP
p^A	$c^A+t+\frac{u^A-c^A-\left(u^B-c^B\right)}{3}$	$c^A+t+u^A-c^A-\left(u^B-c^B\right)-2tx$	$c^A+\frac{3t}{2}+\frac{u^A-c^A-\left(u^B-c^B\right)}{2}-2tx$	$c^A+\frac{t}{2}+\frac{u^A-c^A-\left(u^B-c^B\right)}{2}$
p^B	$c^B+t-\frac{u^A-c^A-\left(u^B-c^B\right)}{3}$	$c^B+t-\left(u^A-c^A-\left(u^B-c^B\right)\right)-2t(1-x)$	$c^B+\frac{t}{2}-\frac{u^A-c^A-\left(u^B-c^B\right)}{2}$	$c^B-\frac{t}{2}-\frac{u^A-c^A-\left(u^B-c^B\right)}{2}+2tx$
x^A	$\frac{1}{2}+\frac{u^A-c^A-\left(u^B-c^B\right)}{6t}$	$\frac{1}{2}+\frac{u^A-c^A-\left(u^B-c^B\right)}{2t}$	$\frac{3}{4}+\frac{u^A-c^A-\left(u^B-c^B\right)}{4t}$	$\frac{1}{4}+\frac{u^A-c^A-\left(u^B-c^B\right)}{4t}$
x^B	$\frac{1}{2}-\frac{u^A-c^A-\left(u^B-c^B\right)}{6t}$	$\frac{1}{2}-\frac{\left(u^A-c^A\right)-\left(u^B-c^B\right)}{2t}$	$\frac{1}{4}-\frac{u^A-c^A-\left(u^B-c^B\right)}{4t}$	$\frac{3}{4}-\frac{u^A-c^A-\left(u^B-c^B\right)}{4t}$
π^A	$\frac{\left[3t+u^A-c^A-\left(u^B-c^B\right)\right]^2}{18t}$	$\frac{\left[t+\left(u^A-c^A\right)-\left(u^B-c^B\right)\right]^2}{4t}$	$\frac{\left[3t+u^A-c^A-\left(u^B-c^B\right)\right]^2}{16t}$	$\frac{\left[t+u^A-c^A-\left(u^B-c^B\right)\right]^2}{8t}$
π^B	$\frac{\left[3t-\left(u^A-c^A\right)+u^B-c^B\right]^2}{18t}$	$\frac{\left[t-\left(u^A-c^A\right)+\left(u^B-c^B\right)\right]^2}{4t}$	$\frac{\left[t-\left(u^A-c^A\right)+u^B-c^B\right]^2}{8t}$	$\frac{\left[3t-\left(u^A-c^A\right)+u^B-c^B\right]^2}{16t}$

Table 1: Assumption 7 implies the following prices (p), market shares (x) and profits (π) per company for each market configuration of AI pricing technologies, where U means uniform pricing and P personalized pricing.

4.2 Equilibrium of endogenous pricing technologies

Similar to the simple example that illustrates the personalized-pricing paradox, we model and analyze endogenous adoption of pricing technologies as a simultaneous choice to adopt either uniform or personalized pricing technologies. A dynamic technology race will be discussed in the section on extensions.

The closed-form solutions for profits in Table 1 provide payoffs of the bi-matrix game that represents endogenous adoption of pricing technologies: either uniform pricing (U) or personalized pricing (P). We obtain the following game.[26]

	U	P
U	$\frac{\left(3t+u^A-c^A-\left(u^B-c^B\right)\right)^2}{18t}, \frac{\left(3t-\left(u^A-c^A\right)+u^B-c^B\right)^2}{18t}$	$\frac{\left(t+u^A-c^A-\left(u^B-c^B\right)\right)^2}{8t}, \frac{\left(3t-\left(u^A-c^A\right)+u^B-c^B\right)^2}{16t}$
P	$\frac{\left(3t+u^A-c^A-\left(u^B-c^B\right)\right)^2}{16t}, \frac{\left(t-\left(u^A-c^A\right)+u^B-c^B\right)^2}{8t}$	$\frac{\left(t+u^A-c^A-\left(u^B-c^B\right)\right)^2}{4t}, \frac{\left(t-\left(u^A-c^A\right)+u^B-c^B\right)^2}{4t}$

Inspection of this game reveals that adopting the personalized pricing technology dominates adopting a uniform technology.[27] Therefore, joint adoption of the personalized pricing technology is the unique equilibrium in dominant strategies. We state this result without further proof.

Proposition 8 *For each company, personalized pricing dominates uniform pricing. The unique equilibrium is where both companies adopt the personalized pricing technology.*

The fact that adoption of personalized pricing technology is a dominant strategy equilibrium generalizes a result in Thisse and Vives (1988), who observe that their Table 1 is a symmetric Prisoners' Dilemma if costs are assumed equal also. They did not investigate whether their low cost firm could make more profit under discriminatroy pricing. We therefore investigate when companies are worse-off under competition in personalized pricing compared to competition in uniform pricing. By comparing the corresponding profits, we can show the following result, its proof can be found in Appendix C.[28]

Proposition 9 *Let Assumption 7 hold. Less-efficient company B never benefits from competition in personalized pricing compared to competition in uniform pricing, whereas more-efficient company A does not benefit if and only if*

$$\begin{cases} \frac{\sqrt{2}}{1+\sqrt{2}}\left(u^A-c^A\right) < u^B-c^B < u^A-c^A, \\ \frac{1+2\sqrt{2}}{3}\left(u^A-c^A-\left(u^B-c^B\right)\right) < t < \frac{1}{3}\left(u^A-c^A+u^B-c^B\right), \end{cases}$$

[26] Table 1 in Thisse and Vives (1988, p. 131) is a special case if we additionally assume $u^A = u^B$ (homogeneous products) and $c^A = 0 \leq c^B$.

[27] Consider company A. The numerator expressing A's profit is the same for the top and bottom row in each column, while the denominator of the bottom row is smaller than the denominator of the top row.

[28] From Table 1 in Thisse and Vives (1988) the condition $\frac{1+2\sqrt{2}}{3}c^B < t$ can be derived, which is the lower bound on t of our second condition.

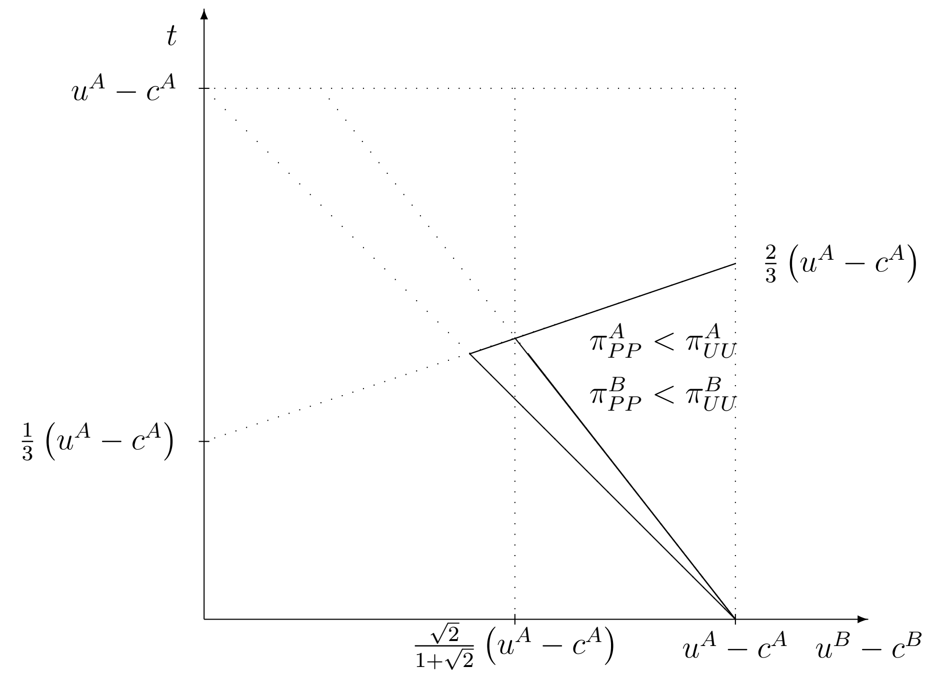

Figure 5: The parameter values for which uniform pricing is more profitable than personalized pricing for fixed social welfare $u^A - c^A$ in the $\left(u^B - c^B, t\right)$ −space.

where $\frac{1}{2} < \frac{\sqrt{2}}{1+\sqrt{2}} < 1$ *and* $\frac{1+2\sqrt{2}}{3} > 1$.

This result implies that less-efficient company B can never benefit from competition in personalized pricing compared to competition in uniform pricing. Also, we obtain that the pair of conditions under which more-efficient company A does not benefit from competition in personalized pricing is a subset of the parameter space specified by Assumption 7 and each condition (of this pair) is more restrictive than the corresponding condition of the pair stated in Assumption 7. The first condition states that, neglecting costs of adjustment, company A is more efficient and generates more social welfare than less-efficient company B and that the asymmetry between companies should be less sizeable than under Assumption 7, because $\frac{\sqrt{2}}{1+\sqrt{2}} > \frac{1}{2}$. The second condition restricts the range of cost of adjustment, which is a nonempty range if (and only if) the first condition holds. This range is smaller than the range under Assumption 7 by excluding low values of cost t, since $\frac{1+2\sqrt{2}}{3} > 1$. As before, both conditions of the pair are needed and specify a nonempty set of the parameter space. Figure 5 illustrates both sub-regions of parameter values that satisfy Assumption 7 where the large triangle indicates where the paradox holds and the small triangle where the efficient company gains from competition in personalized prices.

This proposition shows that the paradox, where competition in personalized pricing deteriorates individual profits, reported in Thisse and Vives (1988), Liu and Serfes (2004) and Rhodes and Zhou (2021) for the fully symmetric case is robust and can also arise in an asymmetric setting with full market coverage.

More interesting, our asymmetric setting enriches the literature and points out that under full market coverage it is also possible that the more efficient company can benefit from simultaneous switching to personalized pricing technology and this case is also robust. As Proposition 9 states this happens for the situations where there are sizeable difference in providing social welfare/efficiency among two companies and low cost of adjustment for consumers.[29] Our findings provides a theoretical underpinning of the empirical findings in de Loecker et al. (2020), who reports that the most succesfull companies in each sector had increased profits over the past decades, while the least succesfull companies had declining or at best stagnating profits.

Closer inspection of several special cases is revealing. For the boundary case of full symmetry in Example 3 (i.e. a vertical line at $u^B - c^B = u^A - c^A$ in Figure 5), both conditions of Proposition 9 are trivially met for $0 < t < \frac{2}{3}\left(u^A - c^A\right)$. For this boundary case, the paradox arises. And it continues to hold until asymmetries (or differences in efficiency) become large (i.e. $u^B - c^B < \frac{\sqrt{2}}{1+\sqrt{2}}\left(u^A - c^A\right)$) and each company is able to exploit certain segments of consumers through perfect price discrimination. Furthermore, when asymmetries are too large, company A monopolizes the market and extracts full consumer surplus.

To summarize, we have shown that the less efficient company will never benefit if both companies move to personalized pricing, while the more efficient company may benefit if certain conditions are met. In that case exploiting information about consumer preferences helps to extend its own market share. However it is only possible when the differences in the efficiency levels are substantial, i.e. $\frac{u^A - c^A}{2} < u^B - c^B < \frac{\sqrt{2}}{1+\sqrt{2}}\left(u^A - c^A\right)$, and the adjustment costs are relatively low, i.e. $u^A - c^A - \left(u^B - c^B\right) < t < \frac{1+2\sqrt{2}}{3}\left(u^A - c^A - \left(u^B - c^B\right)\right)$.

4.3 Welfare Analysis

In this section, we analyze the welfare effects of the technology race for linear reservation prices by comparing the initial configuration of uniform pricing and the final configuration of personalized pricing. As in the previous section, the analysis is done under Assumption 7, meaning company A is more efficient and generates more social welfare than less-efficient company B.

4.3.1 Social Welfare

For linear reservation prices, more can be said than that personalized pricing maximizes social welfare, a result already established in Proposition 6. Comparing the expressions of market shares in Table 1 implies the following welfare ranking of the four configurations.[30]

[29] In Appendix E we extend this result to a wider range of parameter values including the setting of constrained monopoly. We relax Assumption 7 and obtain that the result that more efficient company benefits from switching to personalized pricing technology extends to the entire area of low adjustment costs given by $t < \min\left\{\frac{1}{3}\left(u^A - c^A + \left(u^B - c^B\right)\right), u^A - c^A - \left(u^B - c^B\right)\right\}$ for $0 \leq u^B - c^B < u^A - c^A$.

[30] For completeness, $x^A_{UP} < x^A_{UU} \iff t > \frac{1}{3}\left(u^A - c^A - \left(u^B - c^B\right)\right)$. The latter is satisfied under Assumption 7.

Proposition 10 *(i) The market segments* $\left[0, x_{PP}^{A}\right)$ *of company* A *and* $\left(x_{PP}^{A}, 1\right]$ *of company* B *under personalized pricing* (PP) *coincide with the socially optimal market segments.*

(ii) Moreover, company A*'s market share* $x_{UP}^{A} < x_{UU}^{A} < x_{PP}^{A} < x_{PU}^{A}$.

The first part of Proposition 10(i) is easy to verify by noting that in the Hotelling's model the general expression for total welfare is independent of the type of pricing technology. It is determined by the difference in the consumers' willingness to pay and marginal costs minus the adjustment cost. In particular, for a given market share x_{PP}^{A}, company A generates welfare given by

$$SW^{A}\left(x_{PP}^{A}\right) = \left(u^{A} - c^{A}\right) x_{PP}^{A} - \tfrac{1}{2}t\left(x_{PP}^{A}\right)^{2} = \left(\left(u^{A} - c^{A}\right) - \tfrac{1}{2}t x_{PP}^{A}\right) x_{PP}^{A}.$$

For a given market share $1 - x_{PP}^{A}$, company B generates welfare

$$SW^{B}\left(x_{PP}^{A}\right) = \left(u^{B} - c^{B}\right)(1-x_{PP}^{A}) - \tfrac{1}{2}(1-x_{PP}^{A})t(1-x_{PP}^{A}) = \left(\left(u^{B} - c^{B}\right) - \tfrac{1}{2}t(1 - x_{PP}^{A})\right)(1-x_{PP}^{A}).$$

The aggregate social welfare is given by

$$SW\left(x_{PP}^{A}\right) = SW^{A}\left(x_{PP}^{A}\right) + SW^{B}\left(x_{PP}^{A}\right) = \left(u^{A} - c^{A} - \left(u^{B} - c^{B}\right) + t\right) x_{PP}^{A} + \left(u^{B} - c^{B}\right) - \tfrac{1}{2}t - t\left(x_{PP}^{A}\right)^{2}.$$

Taking the FOC with respect to x_{PP}^{A}, we get

$$\frac{\partial SW}{\partial x_{PP}^{A}} = u^{A} - c^{A} - \left(u^{B} - c^{B}\right) + t - 2t x_{PP}^{A} = 0.$$

Hence, the socially optimal level of company A market share is $\frac{u^{A} - c^{A} - \left(u^{B} - c^{B}\right) + t}{2t}$, which obviously coincides with that under personalized pricing as specified in Table 1.

Part (ii) of Proposition 10 states that the equilibrium market share of company A under uniform pricing (UU configuration) is smaller than the socially optimal level, while the equilibrium market share of company A under PU regime is bigger than the socially optimal level. Moreover, market share of company A increases as companies move from UU configuration to PU, i.e. when A is the leader in the technology adoption raise. However, when less efficient player B also adopts personalized pricing, i.e. market moves from PU to PP, more efficient company A loses some of its customers. Furthermore, comparison of expressions for market shares in Table 1 reveals that market share of more efficient company A is higher than that of less efficient company B under all market configurations except of UP.

4.3.2 Consumer Surplus

In this section, we first establish that aggregate consumer surplus under personalized pricing is higher than that under uniform pricing. Then, we show that not all consumers benefit and we identify the market segment of individual consumers that are worse off under personalized pricing.

The first result compares aggregate consumer surplus. Detailed proof can be found in Appendix D.

Proposition 11 *Under Assumption 7, aggregate consumer surplus under personalized pricing* (PP) *is larger than aggregate consumer surplus under uniform pricing* (UU).

This proposition implies that under Assumption 7 in an asymmetric duopoly market with linear reservation prices personalized pricing on aggregate benefits consumers. However, dispite of the fact that on average consumer surplus is improved under personalized pricing, still there are some groups of consumers that are exploited by the more efficient company. More precisely, consumers located to the right of $\bar{x} = \frac{1}{3t}\left(u^A - c^A - \left(u^B - c^B\right)\right)$ will benefit due to prices that are lower than uniform prices that arise in the Hotelling's model. At the same time, consumers located in $[0, \bar{x})$ will lose, since they will be paying prices above the level implied by the uniform pricing in the Hotelling's model and will also be exploited by more efficient company A through perfect price discrimination.

The following proposition formalizes this result. The proof can be found in Appendix D.

Proposition 12 *(i) Individual consumer surplus of consumers* $x \in \left[0, \frac{1}{3t}\left(u^A - c^A - \left(u^B - c^B\right)\right)\right)$ *is lower under personalized pricing compared to uniform pricing.*

(ii) Individual consumers in the interval $x \in \left(\frac{1}{3t}\left(u^A - c^A - \left(u^B - c^B\right)\right), 1\right]$ *benefit from switching from uniform to personalized pricing.*

We observe that an increase in aggregate consumer surplus is insufficient to conclude that all consumers benefit from personalized pricing. We show that there is a market segment of consumers that are worse off because these consumers will be charged more. These are consumers that are most loyal to the more efficient company and for whom it is costly to switch. Hence, from the perspective of individual consumers, personalized prices fail as a Pareto improvement. Nevertheless, social welfare is maximal under personalized pricing and there is sufficient increase in welfare to compensate consumers who are worse off.

In practice it may be challenging to develop new yardsticks that combine insights from individual and aggregate consumer surplus. Price caps set at the level of uniform prices could protect those consumers, for whom it is costly to switch, from exploitation through personalized pricing. However, such protection of individual consumers that are vulnerable to exploitation reduces profits and, therefore, reduces incentives to adopt new technologies. It is still an open question what policy should antitrust agencies pursue and how to balance individual versus aggregate consumer surplus considerations.

4.3.3 Producer Surplus revisited

The discussion of welfare implications of moving from uniform pricing to personalized pricing identified several facts that also explain the impact of personalized pricing on producer surplus as reported in Proposition 9. We briefly discuss producer surplus once more.

The lower producer surplus for less-efficient company B under personalized pricing arises from a combined effect of a smaller market share and charging consumers in its remaining market segment lower personalized prices compared to this company's uniform price. Similar effects on producer surplus for efficient company A are ambiguous and depend upon the parameter values, as Figure 5 illustrates. From the discussion on welfare effects, we can identify two effects of adopting personalized pricing. A positive effect that company A enjoys from the expansion of its market segment. And an ambiguous effect with respect to the personalized prices company A can charge to its 'old' market segment under uniform pricing. This latter market segment consists of two subsegments, one where higher prices can be charged and one where lower prices can be charged. For a net gain in profits, the loss of profit on consumers that are charged less but that buy at company A under both personalized and uniform prices must be compensated by the higher personalized prices on the market segment under uniform pricing plus the profit obtained on company A's newly concurred market expansion. The net effect on the producer surplus of company A is ambiguous and Figure 5 illustrates the parameter values for which producer surplus of company A is either larger or smaller.

As a final remark, we report that aggregate producer surplus will always be lower. This is trivial for parameter values in which both companies have lower producer surplus, but it also holds when company A enjoys an increase. Then the increase in company A's producer surplus does not offset company B's loss of producer surplus. This also means that the total increase in social welfare from adopting personalized pricing plus the aggregate loss of producer surplus accrues to consumers at the aggregate level. Adopting personalized pricing does have an impact on the balance of aggregate market power. We summarize this discussion in the following result, and its proof can be found in Appendix D.

Proposition 13 *Under Assumption 7, aggregate producer surplus under personalized pricing* (PP) *is smaller than aggregate producer surplus under uniform pricing* (UU)*.*

5 Extensions

In this section we discuss a number of important extensions. In particular: non-monotonicity of reservation-price functions, partial market coverage, discontinuity, oligopoly setting with multiple firms, different consumer groups, multidimensional consumer characteristics and settings with timing of technological adoption over time. We show that our main results are robust in these more general settings. Technical details are deferred to Appendix F.

We first discuss the implications of relaxing Assumption 1 and personalized pricing. These are extensions looking at nonmonotonicity, discontinuity and partial market coverage. Assumption 1 can be motivated as capturing the essence of Hotelling's model in which the boundaries of the interval $[0, 1]$ coincide with the locations of the companies and the market is fully covered because the the negative effect of distance on consumer surplus are relatively modest. The second set of extensions is motivated by the practical applications of our model, where more firms can be present, consumer 'big data' that has various dimensions (such as age, occupation, location), or data that may not be as fine grained as it often may seem.

All these extensions investigate the more general oligopoly setting with multiple firms, multidimensional consumer characteristics and classification into different consumer groups. The main insight will be that our framework and results can be accommodated easily to include these features.

5.1 Discontinuity

Discontinuity is of no concern as we argue in Appendix F and either has no effect or would exclude the case $x = x^*$ in Proposition 2 in case a discontinuous drop fails to equate $r^A(x) - c^A = r^B(x) - c^B$. This insight extends to other relaxations of Assumption 1 that we discuss next.

5.2 Non-monotonicity of Reservation-price Functions

Dropping monotonicity allows for interesting cases and novel results. The most well-known case is Hotelling's model with arbitrary locations, denoted as $\hat{x}^A$ and $\hat{x}^B$, and linear adjustment costs $t\,|\hat{x}^i - x|$ for $i = A, B$, see e.g. Hotelling (1929). The reservation price functions are piecewise-linear and single-peaked at the companies' locations. Proposition 2 immediately generalizes and requires only a minor modification of condition (D) for duopoly, namely location $\hat{x}^A$ replaces 0 and location $\hat{x}^B$ replaces 1. Moreover, $x^* \in \left(\hat{x}^A, \hat{x}^B\right)$ in case of market coverage because the line pieces describing r^A and r^B on the subdomains $\left[0, \hat{x}^A\right]$ and $\left[\hat{x}^B, 1\right]$ are paralel and do not intersect in the generic case. Proposition 4 also generalizes after a similar minor modification and strict inequalities in conditions (A) and (B) for constrained monopoly.[31] Consequently, the equilibrium in personalized prices also maximizes social welfare, as in Proposition 6.

Hotelling's model is build upon two implicit assumptions, namely that the company-specific adjustment costs to buy at company i, denoted as $d^i(x)$, are *symmetric* around company i's most-valued consumer $\hat{x}^i$ and that these costs are also *homogeneous* across companies. Whether homogeneous and symmetric adjustment costs are realistic and too restrictive for econometric applications is outside the scope of this theoretical study.

[31] Under the boundary case of equality at least one pair of parallel line pieces coincides, which is a nongeneric case of little interest.

We extend the above analysis to asymmetric company-specific adjustment costs to the left and right of each peak, and heterogeneity between companies with respect to these asymmetric (non)linear adjustment costs in Appendix F. Without going into details, Pandora's box is opened because even for heterogeneous and asymmetric *linear* adjustment costs an explosion of cases emerges. Although the companies' locations and endpoints of the interval $[0, 1]$ pin down the six endpoints of the four line pieces that characterize the piecewise-linear single peaked reservation price functions, we obtain in total five cases some involving e.g. $x^* \leq \hat{x}^A$ or $x^* \geq \hat{x}^B$ for duopoly. The conditions for constrained monopoly become more demanding. For a constrained monopoly of company A, these require a combination consisting of condition (A) at the upper endpoint of $[0, 1]$ and similar conditions at the lower endpoint and at company B's location.

Novel cases for duopoly arise in case condition (A) holds but the two additional modified conditions just mentioned do not hold. In these cases one of the companies serves two disconnected market segments, each containing either consumer 0 or 1, separated by the other company's market segment, i.e., segments are ordered either A, B, A or B, A, B under market coverage. Characterizing whether the boundaries of the market segments lie to the left or right of the companies' locations gives rise to a further plurality of cases.

For models with single-peaked reservation price functions corresponding to nonlinear adjustment costs even more cases can be expected because then the curvatures of the reservation price functions will play a role too. Our approach and way of analyzing can be extended further to multi-peak reservation price functions that capture situations with multiple targeted consumer groups and nonmonotonic adjustment costs for consumers, which is left for future research.

5.3 Partial Market Coverage

Relaxing the assumption of full market coverage would imply a nonempty segment of consumers who do not buy in equilibrium. These are characterized by negative maximal consumer surpluses that both companies are able to offer at loss-avoiding personalized prices and these are less than the consumer surplus of not buying. Hence, social welfare is maximal, as before. Maintaining monotonicity yields market segments and a segment of consumers who do not buy and these correspond to three possibly empty subintervals of $[0, 1]$ that are disjoint and cover $[0, 1]$. Moreover, in case all three subintervals are nonempty, these are ordered $A, 0, B$, where A's market segment lies to the left of the segment 0 of consumers that do not buy, which in turn lies to the left of B's market segment. In case of empty subintervals some of these subintervals will drop out of the order $A, 0, B$, such as emptiness of segment 0 under Assumption 1 or constrained monopoly. As a final remark, we may also assume that the consumer surplus of not buying is consumer specific by introducing a third function $r^0 : [0, 1] \rightarrow \mathbb{R}$, which we do not investigate further.

5.4 Oligopoly

Many interesting online markets, such as airlines, hotels and car rentals, are oligopoly markets with more than two competing companies and possibly partial market coverage. Such markets also feature multidimensional consumer characteristics, the topic of the next subsection. The techniques developped for our parsimonious framework in Appendix A and the insights obtained in Section 3 can be easily extended to deal with such oligopoly markets and companies that produce single-products, as we demonstrate in Appendix F. In this section, we summarize this general framework and the main generalized results, where we mantain the unit interval of Hotelling's model.

Central to our approach are excess surplus functions for individual consumers that are company specific. Refraining from buying is redefined as an additional company, denoted company 0.[32] Formally, the excess surplus that the most efficient firm can offer consumer x compared to the second-best option for this consumer can be defined by the function $g^i : [0,1] \to \mathbb{R}$, $i = 0, \ldots, n$, as

$$g^i(x) = r^i(x) - c^i - \max_{j=0,\ldots,n; j \neq i} \left\{ r^j(x) - c^j \right\}, \tag{1}$$

where $i, j = 0$ represents not buying with convention $r^0(x) - c^0 = 0$. Obviously, company i is able to offer consumer x the highest maximal surplus $r^i(x) - c^i$ if its excess surplus is nonnegative, i.e, $g^i(x) \geq 0$. It is the only company that offers the maximal surplus if its excess surplus is positive, i.e., $g^i(x) > 0$. If positive, company i can outcompete all other companies and attract consumer x. The last term of (1) equals consumer x's second-best maximal surplus and its maximizers identify company i's closest competitors. Under standard assumptions, like those of Assumption 1 for duopoly, there will be a unique closest competitor for most consumer characteristics x. In case this unique closest competitor is refraining from buying $(j = 0)$, company i is the local monopolist at consumer x. The possibly multiple market segments where company i provides maximal consumer surplus consist of all $x \in [0,1]$ where the excess surplus function is nonnegative and its boundaries coincide where this function is zero. Without further structure, the boundaries of company i's market segments are intractable.

The excess surplus functions of (1) hold the key to characterizing the equilibrium in personalized pricing. To see this, let us apply these functions to our parsimonious duopoly framework of Section 3 first. Then, under condition (D) for a duopoly $g^A(x) > 0$ is equivalent to both $x < x^*$ and $x^* \in (0,1)$, while the condition for a constrained monopoly for company A, namely $g^A(x) > 0$ for all $x \in [0,1]$, can be rewritten as condition (A). Furthermore, except for a single term, the function $g^A(x)$ resembles the equilibrium personalized pricing schedule for company

[32] Company 0 has reservation price function $r^0(x) = 0$, constant marginal costs $c^0 = 0$ and personalized price function $p^0(x) = 0$. It can be extended to any $r^0 : [0,1] \to \infty$ capturing outside options that yield positive or negative consumer surplus, which is left for future research.

A in Proposition 2 and 4 on this company's market share. Formally, $p^A(x) = c^A + g^A(x)$ and $g^A(c) > 0$ immediately implies $p^A(x) > c^A$ and that the equilibrium profit company A makes by selling to consumer x is $g^A(x)$. Company A's total profit can then be expressed as

$$\int_{x\in[0,1]:g^A(x)>0} g^A(\hat{x}) f(\hat{x}) d\hat{x}.$$

The intuition derived from our parsimonious duopoly framework extends to the oligopoly case. In case $g^i(x) > 0$ company i is able to and will outcompete all other companies in terms of providing most consumer surplus by setting the equilibrium personalized price $p^i(x) = c^i + g^i(x) > c^i$ and driving its closest competitor j to marginal cost pricing $p^j(x) = c^j$.[33] The equilibrium profit company i makes by selling to consumer x is $g^i(x) > 0$ and this company's equilibrium market segments is characterized by the area where $g^i(x) > 0$. Because the condition for the equilibrium market segments coincides with the condition for maximal social welfare, social welfare will be maximal in equilibrium too. In case refraining from buying is the second-best alternative, company i is a local monopolist. Finally, in case $g^0(x) > 0$ for some area of x, there will be partial market coverage. The boundaries of company i's market segments are found at those $x \in [0,1]$ where $g^i(x) = 0$. As mentioned before, these boundaries are intractable without further structure and we refer to Appendix F for tractable examples with additional structure.

To summarize, the excess surplus functions of (1) fully characterize the equilibrium in personalized pricing and all results obtained for the parsimonious duopoly model generalize. The characterization of the boundaries of each company's market segments are intractable.

5.5 Multidimensional Consumer Characteristics

Consumer 'big data' characteristics have multiple dimensions such as occupation, location, links in on-line social networks. Also Thisse and Vives (1988) and empirical Industrial Organization take into account many attributes, see e.g. Berry et al. (1995), Nevo (2001) and Goldberg and Verboven (2001). Each of such characteristics or attributes can be modeled as either a continuous or discrete variable that are stacked on each other to form a multidimensional vector. For convenience of exposition, we restrict attention to continuous variables. Formally, multidimensional characteristics are defined as vectors in the nonempty and compact set $X \subseteq \mathbb{R}^m_+$, where $m \geq 1$. We refer to a consumer with vector of characteristics $x \in X$ as consumer x. The reservation price functions on the extended domain are defined as $r^A, r^B : X \longrightarrow \mathbb{R}$ and assumed to be continuous for convenience. Such functions admit a global maximum, and if unique, these can be interpreted as locations. For example, in case $\arg\max_{x\in X} r^i(x) = \{\hat{x}^i\}$, $i = A, B$, maximizer $\hat{x}^i$ can be said to represent the location of company i. Note that this framework can also accommodate multiple global maxima, local maxima and economic situations lacking the

[33] Formally, there is an indeterminacy in equilibrium determing the other $n-2$ company's prices and also among multiple closest competitors. For details, we refer to Appendix F.

notion of location. Finally, the distribution of characteristics is described by some cumulative multivariate distribution function $F : X \to [0, 1]$, but as before this distribution does not play any part in the results.

The qualitative economic logic underlying Proposition 2, 4 and 6 and oligopoly as discussed in the previous subsection remains valid in case of multidimensional characteristics. As before, companies compete in offering consumer surplus, each company can offer a company-specific maximal consumer surplus against marginal cost pricing and the company who is able to offer the highest maximal consumer surplus to consumer x attracts this consumer by charging consumer's reservation price minus either the maximal consumer surplus offered by its competitor or the consumer surplus of not buying. In this equilibrium, social welfare is maximal. Without further structure this equilibrium is intractable.

Formally, redefine the domain of the excess surplus function (1) as X instead of $[0, 1]$, i.e. $g^i : X \to \mathbb{R}$. Then applications of the same arguments as in the previous subsection implies that company i, $i = 0, 1, \ldots, n$, attracts consumer $x \in X$ if $g^i(x) > 0$ with equilibrium personalized prices given by $p^i(x) = c^i + g^i(x) > c^i$ and $p^j(x) = c^j$ for the closest competitor. The equilibrium profit made by selling to consumer x is $g^i(x) > 0$. The equilibrium in personalized pricing maximizes social welfare.

As before, $g^i(x) \geq 0$ defines equilibrium market segments and identifying the boundaries of market segments in the multi-dimensional space X is intractable. In Appendix F we discuss a tractable example of a two-dimensional square city with heterogeneous adjustment costs that are quadratic, see e.g. Example 32. This example extends Larralde et al. (2009) to a personalized pricing setting and derives the boundary of market segments corresponding to the equilibrium in personalized prices.

Finally, previous insights obtained can also be applied to explain why firms seem to prefer expanding the dimension of characteristics in data gathering. Doing so is always a weakly dominant strategy. Suppose a company contemplates expanding its current dimension of n chararcteristics to $n + m$, where $y \in Y \subseteq \mathbb{R}^m$ denotes m augmented dimensions. If $p^i(x)$ is the current optimal personalized pricing strategy on X against either a uniform or personalized strategy of the competitor, then the optimal personalized pricing strategy $p^i(x, y)$ on the expanded set $X \cup Y$ generates at least the same profit as $p^i(x)$ does, because it could ignore those extra m dimensions. Carefull selection of augmented characteristics can be expected to generate a higher profit. Therefore, companies have an ongoing incentive to further expand the dimension of their space of consumer characteristics up to the point where all additional returns are exhausted.

5.6 Timing of Technology

In reality, adoption of technologies takes place over time. If employing pricing technologies is reversible and does not require 'time-to-build', which in some sense is realistic because it only requires either contracting on-the-shelf personalized pricing services of specialized IT companies or renting cloud computing facilities and (re)programming of the code how to use available information about consumer characteristics. One may say that company A and B are involved in an infinitely repeated game with the bi-matrix game as stated in the previous subsection as the stage game. Given that personalized pricing dominates uniform pricing for each company in the stage game, there is a trivial stationary (subgame perfect) equilibrium in which each company employs personalized pricing in each period.

For pricing technologies that are irreversible, companies compete according to some current technology while an innovative technology becomes available. The issue then becomes whether and when to adopt this innovative technology. Then uniform pricing can be seen as the current technology employed by both companies and each company strategically times when to implement new personalized pricing technology. Formally, one would have a modified repeated game with employed technologies last time as the state variables and some state transition function that obeys irreversibility of each company's switch to the personalized pricing technology. The initial state is the configuration in which both companies employ uniform pricing, while future states can be any of the four configurations considered thus far. Given that personalized pricing dominates uniform pricing for each company, there is a unique Markov perfect equilibrium in which each company switches to personalized pricing in every state where it did not yet employ such pricing and irreversibility of technology implies it continues employing personalized pricing.[34] So, after one round of competition the initial state has switched to both companies employing personalized pricing technologies and the state stays that way. To put it differently, both companies are involved in a technology race in which each company adopts the innovative personalized pricing technology as soon as possible.

Information about rivals may be imperfect in reality. Technological innovation makes personalized pricing available to the companies and it may take some time before companies realize the potential of the new technology. It may be even the case that, say, company A learns about the potential first. Given uniform pricing by its competitor, adopting the technology increases its market share and profits. Company B may experience that its market share shrinks, and also learns that company A adopted personalized pricing. Given personalized pricing by company A, it is profitable for company B to adopt personalized pricing too resulting in the joint adoption of personalized pricing technology. The underlying model of information transmission and exact timing of adoption is not that important, each company has a strong incentive to adopt personalized pricing as early as possible independent of the rival's pricing technology. This story holds independent whether competitor's learn about the other company adoption through public

[34] In terms of repeated games, always playing the Nash equilibrium PP of the stage game is a Markov perfect equilibrium. Obviously, for sufficiently large discount factors, there is an alternative pure subgame perfect equilibrium in trigger strategies that supports playing UU and in which playing the Nash equilibrium PP of the stage game forever is the punishment strategy. We leave the welfare implications of such strategy to future research.

channels or through a decrease of their market demand.

6 Concluding Remarks

Competition in personalized pricing resembles insights from constrained monopoly and is also equivalent to an auction with bidding in consumer surplus instead of money. Because of this equivalence, such competition maximizes social welfare and is efficient. The company who wins the consumer at a particular location sets its price above marginal costs and equal to the perfectly price discriminating one, because in general this company has to lower its bid by the consumer's second-best alternative, being either buy at the competitor who sets its price at marginal costs in order to generate maximal consumer surplus or refrain from buying. Only on market segments where the competitor cannot provide a positive consumer surplus do we expect to see perfect price discrimination.

Comparison of the uniform pricing setting to the setting where only one of the companies adopts personalized pricing technology indicates that there are strong unilateral incentives to invest in the new technology for both less efficient and more efficient companies.

Endogenous adoption can be seen as a game with dominant strategies in which adopting the next-generation personalized-pricing technology dominates the current uniform-pricing strategy. In a symmetric setting this game is similar to the prisoners' dilemma and the companies' profits will be lower by adopting the new technology compared to keeping the current technology, which induces the personalized-pricing paradox. A similar result also arises in a sequential context where companies decide on the timing of adoption, they want to adopt at the earliest moment in time.

In an asymmetric setting personalized pricing is still a dominant strategy, however, the more efficient company can now benefit even in case of joint adoption of the new technology. The less efficient company never benefits. The analogy to the prisoners' dilemma explains why companies invest in the latest technology, to keep up in the technology race. It also explains why companies have a nonsatiated appetite for higer dimensions of consumer characteristics.

We also analyze total welfare and consumer welfare implications of switching to the new technology. We show that both total welfare and aggregate consumer welfare will increase under personalized pricing. However, there will be a group of consumers who are worse-off compared to the uniform pricing setting and there is a risk that the information available about characteristics of that particular group will be exploited by the seller in order to extract the full consumer surplus through perfect price discrimination.

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A Appendix: Derivations Proposition 2, 4 and 6

Before stating the proofs of our main results, we first introduce some preliminary results that return in several of these proofs.

A.1 Preliminary results

Define the function $g : [0, 1] \to \mathbb{R}$ as

$$g(x) = r^A(x) - r^B(x) - c^A + c^B \tag{2}$$

Straightforward rewriting yields the following equivalence, which is stated without proof.

Lemma 14 *For all $x \in [0, 1]$ the following equivalence holds:*

$$g(x) > 0 \quad \Longleftrightarrow \quad \max\left\{r^A(x) - c^A, r^B(x) - c^B\right\} = r^A(x) - c^A > r^B(x) - c^B.$$

Moreover by Assumption 1, it also holds that $g(x) > 0 \implies r^A(x) - c^A > 0$. Similar equivalences hold for $g(x) < 0$ and $g(x) = 0$.[35]

The economic interpretation of $g(x) > 0$ is that the social welfare generated by consumer x from buying at company A is larger than the social welfare of buying at company B. And under Assumption 1, the social welfare of buying at A is larger than that of not buying at all. The sign of the function g embodies these simple facts that become useful later on.

The next result specifies the necessary and sufficient conditions under which function g is positive (or negative) on the entire domain $[0, 1]$, and a condition for when there is a unique separator in the 'middle', denoted $x^* \in (0, 1)$, such that the function g is positive to the right of this separator and negative to the left. These are the three mutually exclusive conditions from the main text that we repeat here.

$$r^A(1) - r^B(1) \geq c^A - c^B, \tag{A}$$
$$r^A(0) - r^B(0) \leq c^A - c^B, \tag{B}$$
$$r^A(1) - r^B(1) < c^A - c^B < r^A(0) - r^B(0). \tag{D}$$

These conditions imply the following result.

Lemma 15 *The following equivalences hold*

$$\begin{aligned} (A) \quad &\Longleftrightarrow \quad g(x) > 0 \quad \textit{for all } x \in [0, 1), \\ (B) \quad &\Longleftrightarrow \quad g(x) < 0 \quad \textit{for all } x \in (0, 1], \\ (D) \quad &\Longleftrightarrow \quad \exists\, x^* \in (0, 1) : g(x^*) = 0. \end{aligned}$$

Moreover, for (D) it holds that x^* is unique; $g(x) > 0$ if $x \in [0, x^*)$; and $g(x) > 0$ if $x \in (x^*, 1]$.

[35] For completeness, $g(x) < 0$ implies $r^B(x) - c^B > 0$. Also, $g(x) = 0$ implies $r^A(x) - c^A = r^B(x) - c^B > 0$.

Proof of Lemma 15

By Assumption 1, $g(x)$ is continuous and strictly decreasing in x.

Consider (A). It can be rewritten as $g(1) = r^A(1) - r^B(1) - c^A + c^B \geq 0$. Because g is continuous and strictly decreasing, $g(x) > 0$ for all $x \in [0,1)$ immediately follows.

Consider (B). It can be rewritten as $g(0) = r^A(0) - r^B(0) - c^A + c^B \leq 0$. Because g is continuous and strictly decreasing, $g(x) < 0$ for all $x \in (0,1]$ immediately follows.

Consider (D). Similar as the previous two cases, it can be rewritten as $g(0) > 0$ and $g(1) < 0$. Because $g(x)$ is also continuous, the Intermediate Value Theorem implies the existence of an $x^* \in (0,1)$ such that $g(x^*) = 0$. Next, $g(x)$ is strictly decreasing implies uniqueness of x^*. Moreover, for all $x < x^*$ we have that $g(x) > 0$ and $g(x) < 0$ for all $x > x^*$. QED

These conditions can be rewritten. For example, condition (A) is equivalent to $r^A(1) - c^A \geq r^B(1) - c^B$, which means that buying at company A by the consumer located at $x = 1$ has a social welfare that is at least equal to the social welfare of buying by that consumer at company B. Because Lemma 15 implies $g(x) > 0$ for all $x \in [0,1)$, the interpretation of Lemma 14 states that social welfare of buying at company A is larger than social welfare from buying at company B or not buying for all consumers $x \in [0,1)$. Hence, company A is superior or dominant in the sense that it has the most consumer surplus (up to social welfare) to offer to every consumer in the market. Similar, company B is superior under condition (B). Finally, there is only local superiority under condition (D). We summarize these preliminary results in the following corollary.

Corollary 16 *The following equivalences hold*

$$
\begin{aligned}
(A): &\quad \max\left\{r^A(x) - c^A, r^B(x) - c^B\right\} = r^A(x) - c^A > 0 \quad \text{for all } x \in [0,1), \\
(B): &\quad \max\left\{r^A(x) - c^A, r^B(x) - c^B\right\} = r^B(x) - c^B > 0 \quad \text{for all } x \in (0,1],
\end{aligned}
$$

$$
(D): \quad
\begin{array}{l}
\max\left\{r^A(x) - c^A, r^B(x) - c^B\right\} = r^A(x) - c^A > 0 \quad \text{for all } x \in [0,x^*), \\
\max\left\{r^A(x) - c^A, r^B(x) - c^B\right\} = r^B(x) - c^B > 0 \quad \text{for all } x \in (x^*,1].
\end{array}
$$

A.2 Proofs of main results

We continue with the proofs of our main results as stated in the main text. Theses proofs refer to the preliminary results of A.1.

Proof of Proposition 2

Nonnegative profit per consumer imposes $p^i(x) \geq c^i$, $i = A, B$. The option not to buy caps the price from any trade by the reservation price, i.e. $p^i(x) \leq r^i(x)$, $i = A, B$. By Assumption 1, at least one company has an incentive to trade with consumer x and integrable areas of consumers that do not trade cannot be an equilibrium. Hence, it is without loss of generality to consider $p^i(x) \in [c^i, r^i(x)]$, $i = A, B$.

We first show two claims that do not require condition (D) to hold.

Claim 17 *Any pure Nash equilibrium excludes*

$$p^A(x) > p^B(x) + r^A(x) - r^B(x) > \max\{c^A, c^B\},$$
$$p^B(x) > p^A(x) - r^A(x) + r^B(x) > \max\{c^A, c^B\}.$$

Proof of Claim 17

Suppose not and that the first inequality holds in some Nash equilibrium. Then, $r^A(x) - p^A(x) < r^B(x) - p^B(x)$. Consumer x will get more surplus from company B than from company A and will buy at company B. Firm A makes zero profit. However, company A could make a positive profit from consumer x by slightly lowering its price to increase its surplus above the surplus offered by company B. This requires setting the price

$$\widetilde{p}^A(x) = p^B(x) + r^A(x) - r^B(x) - \varepsilon > c^A,$$

where $\varepsilon > 0$ is sufficient small. Then, consumer x prefers $\widetilde{p}^A(x)$ over $p^B(x)$ and switches to buy from company A. This contradicts that the first inequality of Claim 17 can occur in a Nash equilibrium. By symmetry, there does not exist a Nash equilibrium when the second inequality of Claim 17 holds. QED

Claim 18 *Any pure Nash equilibrium excludes*

$$p^A(x) = p^B(x) + r^A(x) - r^B(x) > \max\{c^A, c^B\},$$
$$p^B(x) = p^A(x) - r^A(x) + r^B(x) > \max\{c^A, c^B\}.$$

Proof of Claim 18

Suppose not and that the first inequality holds in some Nash equilibrium. Then, consumer x is indifferent between both companies and whatever the probability $p \in [0, 1)$ is that this consumer buys at A, company A can do better by slightly undercutting the price of the other company and attract consumer x for sure by setting

$$\widetilde{p}^A(x) = p^B(x) + r^A(x) - r^B(x) - \varepsilon > \max\{c^A, c^B\}.$$

Formally, there exists a sufficient small $\varepsilon > 0$ such that $\widetilde{p}^A(x)$ is still above c^A and the unilateral deviation $\widetilde{p}^A(x)$ secures company A a larger profit than $p^A(x)$. Hence $p^A(x)$ does not maximize company A's profit against $p^B(x)$ and this contradicts such $p^A(x)$ can occur in any Nash equilibrium. For completeness consider $p = 1$. Then company B makes zero profit and is better off to slightly undercut company A's price. We conclude that no Nash equilibrium exists when the first inequality of Claim 18 holds. By symmetry, there does not exist a Nash equilibrium when the second inequality of Claim 18 holds. QED

Both claims exclude that both $p^A(x), p^B(x) > \max\{c^A, c^B\}$ in any pure Nash equilibrium. This means that at least one of two prices belongs to the interval $\left[\min\left\{c^A, c^B\right\}, \max\left\{c^A, c^B\right\}\right]$. For convenience and without loss of generality, suppose $c^B \geqslant c^A$ such that the interval becomes $\left[c^A, c^B\right]$. Then, there are two cases for Nash equilibrium:

$$p^B(x) \in \left[c^A, c^B\right], \; p^A(x) = p^B(x) + r^A(x) - r^B(x) \text{ and } p^A(x) \in \left[c^A, r^A(x)\right], \tag{3}$$

$$p^A(x) \in \left[c^A, c^B\right], \; p^B(x) = p^A(x) - r^A(x) + r^B(x) \text{ and } p^B(x) \in \left[c^B, r^B(x)\right]. \tag{4}$$

Consider $p^B(x) \in [c^A, c^B]$ first. Nonnegative profit for company B requires $p^B(x) \geqslant c^B$ and we obtain $p^B(x) = c^B$. Substituted into (3), we obtain

$$p^A(x) = r^A(x) - \left(r^B(x) - c^B\right) \quad \text{and} \quad r^A(x) - r^B(x) + c^B \in \left[c^A, r^A(x)\right],$$

from which we derive the equilibrium conditions

$$r^A(x) - c^A \geqslant r^B(x) - c^B \quad \text{and} \quad r^B(x) - c^B \geq 0,$$

where the first inequality can be rewritten as $g(x) \geq 0$ and the latter holds if and only if $x \geq \underline{x}^B$.

Consider $g(x) \geq 0$ first. Recall we impose condition (D) in Proposition 2. Then by Lemma 14 and 15 and Corollary 16, there exists a unique $x^* \in (0,1)$ such that $g(x^*) = 0$, $x^* > \underline{x}^B$ and $p^A(x) = r^A(x) - \left(r^B(x) - c^B\right) = c^A + g(x) > c^A$ for all $x \in [0, x^*)$. Hence, the profit per consumer $p^A(x) - c^A = g(x) > 0$ for all $x \in [0, x^*)$. Furthermore, the price $p^A(x) = c^A + g(x)$ makes consumer x indifferent between buying at company A and buying at company B, i.e. $r^A(x) - p^A(x) = r^B(x) - c^B$. For $x \in \left(\underline{x}^B, x^*\right)$, the surplus $r^B(x) - c^B$ from buying at company B is positive and the two companies compete for consumer x. However, for $x \in \left[0, \underline{x}^B\right]$ (which may be empty), consumer x derives negative surplus from buying at company B (at marginal cost pricing), and an alternative option for this consumer is not to buy, which generates 0 consumer surplus. Then company A competes against competitor "not buy". For $x \in \left[0, \underline{x}^B\right] \cup \left(\underline{x}^B, x^*\right)$, the consumer surplus of both alternative options is $\max\left\{0, r^B(x) - c^B\right\}$.

Incorporating both cases ($x < \underline{x}^B$ and $x \geq \underline{x}^B$) is captured by company A setting the price $p^A(x) = r^A(x) - \max\left\{0, r^B(x) - c^B\right\}$, which makes consumer x indifferent between buying at company B if that generates a positive surplus, or refrain from buying otherwise. In order to have existence of equilibrium, we apply the endogenous tie-breaking rule proposed in Simon and Zame (1990). It treats every consumer x as an additional second-mover in an extended game who also chooses an equilibrium action. Here, it requires that consumer x will buy at company A. The reason is that company B is not willing (or able) to decrease its price below its marginal costs and only company A has slack to lower its price to attract consumer $x \in [0, x^*)$ while still making a positive profit on this consumer. A situation in which $p^B = c^B$, $p^A(x) = r^A(x) - \max\{0, r^B(x) - c^B\}$ and the consumer would either buy at company B or not buy cannot be equilibrium, because company A has a profitable deviation to slightly lower its price. If consumer x buys at company A, neither company B nor consumer x have an incentive to change their strategy. So, the endogenous tie-breaking rule lets the consumer decide in favor of the company who is able to slightly decrease its price, even though the consumer is indifferent to the best alternative option.

Next, consider the case $p^A(x) \in [c^A, c^B]$. If $c^A = c^B$, the previous arguments immediately apply. Without loss of generality consider $c^A < c^B$. We show the following claim.

Claim 19 *If in any pure Nash equilibrium it holds that*

$$p^A(x) \in \left(c^A, c^B\right], \; p^B(x) = p^A(x) - r^A(x) + r^B(x), \; \textit{and } p^B(x) \in \left[c^B, r^B(x)\right],$$

then consumer x buys from company A and $p^B(x) = c^B$.

Proof of Claim 19

Similar as in the proof of Claim 18, consumer x is indifferent between both companies. Suppose the condition holds and that $p \in [0, 1)$ is the probability that this consumer buys at A, company A can do better by slightly undercutting the price of the other company and attract consumer x for sure by setting

$$\widetilde{p}^A(x) = p^B(x) + r^A(x) - r^B(x) - \varepsilon > c^A.$$

Formally, there exists a sufficient small $\varepsilon > 0$ such that $\widetilde{p}^A(x)$ is still above c^A and the unilateral deviation $\widetilde{p}^A(x)$ secures company A a larger profit than $p^A(x)$. Hence, if $p \in [0, 1)$ price $p^A(x)$ does not maximize company A's profit against $p^B(x)$ and in Nash equilibrium we must have that $p = 1$. Then, if $p^B(x) > c^B$, company B makes zero profit and is better off to slightly undercut company A's price. This also contradicts Nash equilibrium. We conclude that if the condition holds, then both $p = 1$ and $p^B(x) = c^B$ must hold. QED

Claim 19 implies that $p^A(x) \in \left(c^A, c^B\right]$ is already covered under case (3) with $p^B(x) = c^B$. This leaves $p^A(x) = c^A$ as the remaining case for (4). Substitution of $p^A(x) = c^A$ into (4) implies

$$p^B(x) = c^A - r^A(x) + r^B(x).$$

Then, the nonnegative profit condition $p^B(x) \geqslant c^B$ imposes $g(x) \leq 0$. Similar arguments as for case (3) apply and together with the endogenous tie-breaking rule proposed in Simon and Zame (1990), there exists a unique $x^* \in (0, 1)$ such that consumer $x \in (x^*, 1]$ buys at company B and pays $p^B(x) = r^B(x) - \max\{0, r^A(x) - c^A\}$. QED

Proof of Proposition 4
Consider condition (A). Then $g(1) \geq 0$ and, by Lemma 15, $g(x) > 0$ for all $x \in [0,1)$. All arguments of the proof of Proposition 2 apply as if $x^* = 1$ for the interval $[0, x^*)$. Hence the stated results follow immediately.
Consider condition (B). Then $g(0) \leq 0$ and, by Lemma 15, $g(x) < 0$ for all $x \in (0,1]$. All arguments of the proof of Proposition 2 apply, as if $x^* = 0$ for the interval $(x^*, 0]$. Hence, the stated results follow immediately. QED

Proof of Proposition 6
By Lemma 14,

$$
\begin{aligned}
g(x) > 0 &\Longleftrightarrow r^A(x) - c^A = \max\left\{r^A(x) - c^A, r^B(x) - c^B\right\}, \\
g(x) < 0 &\Longleftrightarrow r^B(x) - c^B = \max\left\{r^A(x) - c^A, r^B(x) - c^B\right\}.
\end{aligned}
$$

Under condition (D), we have market share $x^* \in (0,1)$ for company A, market share $1-x^* \in (0,1)$ for company B and social welfare given by

$$
\begin{aligned}
&\int_0^{x^*} \left[r^A(x) - p^A(x) + p^A(x) - c^A\right] f(x)\, dx + \int_{x^*}^1 \left[r^B(x) - p^B(x) + p^B(x) - c^B\right] f(x)\, dx \\
&= \int_0^{x^*} \max\left\{r^A(x) - c^A, r^B(x) - c^B\right\} f(x)\, dx + \int_{x^*}^1 \max\left\{r^A(x) - c^A, r^B(x) - c^B\right\} f(x) \\
&= \int_0^1 \max\left\{r^A(x) - c^A, r^B(x) - c^B\right\} f(x)\, dx.
\end{aligned}
$$

The latter expression is maximal social welfare.

Under condition (A), company A captures the entire market (as if $x^* = 1$ above) and only the first integral of each line applies leading to the same result.
Similar for condition (B), company B captures the entire market (as if $x^* = 0$) for which only the second integral of each line applies leading to the same result. QED

A.3 Consumer and Producer Surplus in the Model of Section 3

Let us first consider the constrained monopoly case in which company A serves the entire market. Recall that company A competes in utilities and offers consumers their opportunity cost of foregone consumer surplus, which is $\max\left\{0, r^B(x) - c^B\right\}$. On the (possibly empty) market segment $\left[0, \underline{x}^B\right]$ individual consumer surplus is constant and equals 0, while individual consumer surplus is $r^B(x) - c^B$ on market segment $\left(\underline{x}^B, 1\right)$. Then, aggregate consumer surplus is given by

$$
\int_{\underline{x}^B}^1 \left(r^B(x) - c^B\right) f(x)\, dx.
$$

Obviously, individual consumers on market segment $\left[0,\underline{x}^{B}\right]$ will never profit from personalized pricing, because company A is able to capture the entire consumer surplus of buying from this company. For consumers on market segment $\left(\underline{x}^{B},1\right)$, it depends upon the uniform price (in configuration UU) whether individual consumers benefit from competition in personalized pricing. Without knowing the uniform pricing equilibrium, intuition hints at that market segment $\left(\underline{x}^{B},1\right)$ is partitioned into a segment close to $\underline{x}^{B}$ that may not benefit and a segment close to 1 of individual consumers who may benefit.

The producer surplus for this constrained monopoly case also considers these two market segments. On market segment $\left[0,\underline{x}^{B}\right]$ company A can perfectly price discriminate and profit per consumer is $r^{A}(x)-c^{A}$, while this company offers consumer surplus $r^{B}(x)-c^{B}$ to individual consumers on market segment $\left(\underline{x}^{B},1\right)$ with profit $r^{A}(x)-c^{A}-\left(r^{B}(x)-c^{B}\right)$ per consumer. Then, aggregate producer surplus is given by

$$\begin{aligned}&\int_{0}^{1}\left(r^{A}(x)-c^{A}-\min\left\{0,r^{B}(x)-c^{B}\right\}\right)f(x)\,dx\\&=\int_{0}^{1}\max\left\{r^{A}(x)-c^{A},r^{A}(x)-c^{A}-\left(r^{B}(x)-c^{B}\right)\right\}f(x).\end{aligned}$$

Condition (B) reverses the roles and the order of the intervals, condition (D) combines all cases of both condition (A) and (B).

The constrained monopoly case in which company B serves the entire market is similar with market segment $\left(0,\overline{x}^{A}\right)$ on which consumer surplus $r^{A}(x)-c^{A}$ decreases and market segment $\left[\overline{x}^{A},1\right]$ on which individual consumer surplus equals 0. Aggregate consumer surplus is given by

$$\int_{0}^{\overline{x}^{A}}\left(r^{A}(x)-c^{A}\right)f(x)\,dx.$$

Finally, the duopoly case with positive market shares has individual consumers in one of the four above market segments and aggregate consumer surplus given by

$$\begin{aligned}&\int_{\underline{x}^{B}}^{x^{*}}\left(r^{B}(x)-c^{B}\right)f(x)\,dx+\int_{x^{*}}^{\overline{x}^{A}}\left(r^{A}(x)-c^{A}\right)f(x)\,dx\\&=\int_{\underline{x}^{B}}^{\overline{x}^{A}}\min\left\{r^{A}(x)-c^{A},r^{B}(x)-c^{B}\right\}f(x)\,dx.\end{aligned}$$

For producer surplus, we also consider the constrained monopoly case in which, say, company A serves the entire market. On market segment $\left[0,\underline{x}^{B}\right]$ company A can perfectly price discriminate and profit per consumer is $r^{A}(x)-c^{A}$, while this company offers consumer surplus $r^{B}(x)-c^{B}$ to individual consumers on market segment $\left(\underline{x}^{B},1\right)$ with profit $r^{A}(x)-c^{A}-\left(r^{B}(x)-c^{B}\right)$ per consumer. Then, aggregate producer surplus is given by

$$\int_{0}^{1}\left(r^{A}(x)-c^{A}-\min\left\{0,r^{B}(x)-c^{B}\right\}\right)f(x)\,dx.$$

The constrained monopoly case in which company B serves the entire market is similar with market segment $\left(0, \overline{x}^A\right)$ with profit $r^B(x) - c^B - \left(r^A(x) - c^A\right)$ per consumer and market segment $\left[\overline{x}^A, 1\right]$ with profit $r^B(x) - c^B$ per consumer. Aggregate consumer surplus is given by

$$\int_0^1 \left(r^B(x) - c^B - \min\left\{0, r^A(x) - c^A\right\}\right) f(x)\, dx.$$

Finally, the duopoly case with positive market shares has individual consumers in one of the four above market segments and aggregate producer surplus given by

$$\int_0^{x^*} \left(r^A(x) - c^A - \min\left\{0, r^B(x) - c^B\right\}\right) f(x)\, dx + \int_{x^*}^1 \left(r^B(x) - c^B - \min\left\{0, r^A(x) - c^A\right\}\right) f(x)\, dx.$$

B Appendix: Conditions for an interior equilibrium

In this appendix, we consider the case of linear reservation prices and derive the necessary and sufficient conditions that ensure that both companies have positive profits and positive market shares that cover the whole market *per* configuration before we combine these conditions to arrive at the necessary and sufficient conditions for *all four* configurations. These conditions are stated in the main text as Assumption 7.

Linear reservation prices are given by $r^A(x) = u^A - tx$ and $r^B(x) = u^B - t(1-x)$ and we also assume a uniform distribution of consumers on the interval $[0,1]$. The derivation of equilibrium prices, equilibrium market shares, profits and consumer surplus of Table 1 on which the derivations in this appendix are based are routine exercises that are available upon request from the authors.

B.1 Configuration UU

Table 1 states that

$$\overline{p}^A = c^A + t + \tfrac{\left(u^A - c^A\right) - \left(u^B - c^B\right)}{3}, \qquad \overline{p}^B = c^B + t + \tfrac{-\left(u^A - c^A\right) + \left(u^B - c^B\right)}{3},$$

$$x^A = \tfrac{1}{2} + \tfrac{u^A - c^A - \left(u^B - c^B\right)}{6t}, \qquad x^B = \tfrac{1}{2} + \tfrac{-\left(u^A - c^A\right) + \left(u^B - c^B\right)}{6t},$$

$$\pi^A = \tfrac{\left(3t + \left(u^A - c^A\right) - \left(u^B - c^B\right)\right)^2}{18t} > 0, \qquad \pi^B = \tfrac{\left(3t - \left(u^A - c^A\right) + \left(u^B - c^B\right)\right)^2}{18t} > 0,$$

where x^i, $i = A, B$, denotes company i's market share.

Consumer $x \leq x^A$ is willing to buy from A if

$$u^A - \left(c^A + t + \frac{\left(u^A - c^A\right) - \left(u^B - c^B\right)}{3}\right) - tx \geq 0.$$

Substituting x^A and rewriting yields $3t \leq u^A - c^A + \left(u^B - c^B\right)$.

Similarly, consumer $x \geq x^A$ is willing to buy from B if

$$u^B - \left(c^B + t + \frac{-\left(u^A - c^A\right) + \left(u^B - c^B\right)}{3}\right) - t(1-x) \geq 0.$$

Substituting x^B and rewriting yields $3t \leq u^A - c^A + \left(u^B - c^B\right)$,which is the same condition as the previous one.

Both companies set prices at or above marginal costs if

$$t + \tfrac{\left(u^A - c^A\right) - \left(u^B - c^B\right)}{3} \geq 0 \iff u^B - c^B - \left(u^A - c^A\right) \leq 3t,$$

$$t + \tfrac{-\left(u^A - c^A\right) + \left(u^B - c^B\right)}{3} \geq 0 \iff u^A - c^A - \left(u^B - c^B\right) \leq 3t.$$

Which is equivalent to

$$\left|u^A - c^A - \left(u^B - c^B\right)\right| \leq 3t.$$

Finally, both market shares are positive if $0 < x^A < 1$, which can be rewritten as

$$-\tfrac{1}{2} < \frac{u^A - c^A - \left(u^B - c^B\right)}{6t} < \tfrac{1}{2} \iff \left|u^A - c^A - \left(u^B - c^B\right)\right| < 3t.$$

Hence, for the case $u^A - c^A > u^B - c^B$, we obtain the nonempty interval

$$\tfrac{1}{3}\left(u^A - c^A - \left(u^B - c^B\right)\right) < t < \tfrac{1}{3}\left(u^A - c^A + u^B - c^B\right). \tag{5}$$

B.2 Configuration PP

Table 1 states that

$$p^A(x) = c^A + t + u^A - c^A - \left(u^B - c^B\right) - 2tx, \qquad p^B(x) = c^B + t + u^B - c^B - \left(u^A - c^A\right) - 2t(1-x),$$

$$x^A = \tfrac{1}{2} + \tfrac{u^A - c^A - \left(u^B - c^B\right)}{2t}, \qquad x^B = \tfrac{1}{2} + \tfrac{-\left(u^A - c^A\right) + \left(u^B - c^B\right)}{2t},$$

$$\pi^A = \tfrac{\left(t + \left(u^A - c^A\right) - \left(u^B - c^B\right)\right)^2}{4t} > 0, \qquad \pi^B = \tfrac{\left(t - \left(u^A - c^A\right) + \left(u^B - c^B\right)\right)^2}{4t} > 0,$$

Consumer $x \leq x^A$ is willing to buy from A if

$$u^A - \left(u^A - \left(u^B - c^B\right) + t(1-2x)\right) - tx \geq 0.$$

Substituting x^A and rewriting yields $t \leq u^A - c^A + \left(u^B - c^B\right)$.

Similarly, consumer $x \geq x^A$ is willing to buy from B if

$$u^B - \left(u^B - \left(u^A - c^A\right) + t(2x-1)\right) - t(1-x) \geq 0,$$

which can be rewritten as $t \leq u^A - c^A + \left(u^B - c^B\right)$.

Both companies set prices at or above marginal costs if

$$u^A - \left(u^B - c^B\right) + t(1-2x) \geq c^A \iff u^A - c^A - \left(u^B - c^B\right) \geq t(2x-1) \iff x \leq x^A,$$

$$u^B - \left(u^A - c^A\right) + t(2x-1) \geq c^B \iff u^B - c^B - \left(u^A - c^A\right) \geq t(1-2x) \iff x \geq x^A.$$

Hence, both conditions hold for all $t \geq 0$.

Finally, both market shares are positive if $0 < x^A < 1$, which can be rewritten as

$$-\tfrac{1}{2} < \frac{u^A - c^A - \left(u^B - c^B\right)}{2t} < \tfrac{1}{2} \iff \left|u^A - c^A - \left(u^B - c^B\right)\right| < t.$$

Hence, for the case $u^A - c^A > u^B - c^B$, we obtain the nonempty interval

$$u^A - c^A - \left(u^B - c^B\right) < t < u^A - c^A + \left(u^B - c^B\right). \tag{6}$$

B.3 Configuration PU

Table 1 states that

$$p^A(x) = c^A + \tfrac{3}{2}t - 2tx + \tfrac{u^A - c^A - \left(u^B - c^B\right)}{2}, \qquad \overline{p}^B = c^B + \tfrac{1}{2}t - \tfrac{u^A - c^A - \left(u^B - c^B\right)}{2},$$

$$x^A = \tfrac{3}{4} + \tfrac{u^A - c^A - \left(u^B - c^B\right)}{4t}, \qquad x^B = \tfrac{1}{4} - \tfrac{u^A - c^A - \left(u^B - c^B\right)}{4t},$$

$$\pi^A = \tfrac{\left(3t + \left(u^A - c^A\right) - \left(u^B - c^B\right)\right)^2}{16t} > 0, \qquad \pi^B = \tfrac{\left(t - \left(u^A - c^A\right) + \left(u^B - c^B\right)\right)^2}{8t} > 0,$$

Consumer $x \leq x^A$ is willing to buy from A if

$$u^A - \left(c^A + \tfrac{3}{2}t - 2tx + \frac{u^A - c^A - \left(u^B - c^B\right)}{2}\right) - tx \geq 0.$$

Substitution and rewriting yields

$$t \leq u^A - c^A + \frac{u^B - c^B}{3}.$$

Similarly, consumer $x \geq x^A$ is willing to buy if from B if

$$u^B - \left(c^B + \frac{t}{2} + \frac{-\left(u^A - c^A\right) + \left(u^B - c^B\right)}{2}\right) - t(1-x) \geq 0.$$

Substitution and rewriting yields

$$t \leq u^A - c^A + \frac{u^B - c^B}{3}.$$

Both companies set prices at or above marginal costs if

$$c^A + \tfrac{3}{2}t - 2tx + \tfrac{u^A - c^A - \left(u^B - c^B\right)}{2} \geq c^A \iff x \leq \tfrac{3t + u^A - c^A - \left(u^B - c^B\right)}{4t} = x^A,$$

$$c^B + \tfrac{1}{2}t - \tfrac{u^A - c^A - \left(u^B - c^B\right)}{2} \geq c^B \iff t \geq u^A - c^A - \left(u^B - c^B\right).$$

The first condition holds by construction for all $t \geq 0$.

Finally, both market shares are positive if $0 < x^A < 1$, which can be rewritten as

$$t > \max\left\{u^A - c^A - \left(u^B - c^B\right), \tfrac{1}{3}\left(u^B - c^B - \left(u^A - c^A\right)\right)\right\}.$$

Hence, for the case $u^A - c^A > u^B - c^B$, we obtain the nonempty interval

$$u^A - c^A - \left(u^B - c^B\right) < t < u^A - c^A + \tfrac{1}{3}\left(u^B - c^B\right). \tag{7}$$

B.4 Configuration UP

Table 1 states that

$$\overline{p}^A = c^A + \tfrac{t}{2} + \frac{u^A - c^A - \left(u^B - c^B\right)}{2}, \qquad p^B(x) = c^B - \tfrac{t}{2} + 2tx - \frac{u^A - c^A - \left(u^B - c^B\right)}{2},$$

$$x^A = \tfrac{1}{4} + \frac{u^A - c^A - \left(u^B - c^B\right)}{4t}, \qquad x^B = \tfrac{3}{4} - \frac{u^A - c^A - \left(u^B - c^B\right)}{4t},$$

$$\pi^A = \frac{\left(t + u^A - c^A - \left(u^B - c^B\right)\right)^2}{8t} > 0, \qquad \pi^B = \frac{\left(3t - \left(u^A - c^A\right) + u^B - c^B\right)^2}{16t} > 0,$$

Consumer $x \leq x^A$ is willing to buy from A if

$$u^A - \left(c^A + \tfrac{1}{2}t + \frac{u^A - c^A - \left(u^B - c^B\right)}{2}\right) - tx \geq 0.$$

Substitution and rewriting yields

$$t \leq \frac{u^A - c^A}{3} + u^B - c^B.$$

Similarly, consumer $x \geq x^A$ is willing to buy from B if

$$u^B - \left(c^B - \tfrac{1}{2}t + 2tx - \frac{u^A - c^A - \left(u^B - c^B\right)}{2}\right) - t(1-x) \geq 0.$$

Which can be rewritten as

$$t \leq \frac{u^A - c^A}{3} + u^B - c^B.$$

Both companies set prices at or above marginal costs if

$$c^A + \tfrac{1}{2}t + \frac{u^A - c^A - \left(u^B - c^B\right)}{2} \geq c^A \iff t \geq u^B - c^B - \left(u^A - c^A\right),$$

$$c^B - \tfrac{1}{2}t + 2tx - \frac{\left(u^A - c^A\right) - \left(u^B - c^B\right)}{2} \geq c^B \iff x \geq \frac{t + u^A - c^A - \left(u^B - c^B\right)}{4t} = x^A.$$

The last condition holds by construction for all $t \geq 0$.

Finally, both market shares are positive if $0 < x^A < 1$, which can be rewritten as

$$t > \max\left\{u^B - c^B - \left(u^A - c^A\right), \tfrac{1}{3}\left(u^A - c^A - \left(u^B - c^B\right)\right)\right\}.$$

Hence, for the case $u^A - c^A > u^B - c^B$, we obtain the nonempty interval

$$\tfrac{1}{3}\left(u^A - c^A - \left(u^B - c^B\right)\right) < t < \tfrac{1}{3}\left(u^A - c^A\right) + u^B - c^B. \tag{8}$$

B.5 Assumption 7

The necessary and sufficient conditions of Assumption 7 are derived from combining conditions (5), (6), (7) and (8) for the case $u^A - c^A > u^B - c^B > 0$. This yields that t should be larger than the maximum over all lower bounds

$$\max\left\{\tfrac{1}{3}\left(u^A - c^A - (u^B - c^B)\right), u^A - c^A - \left(u^B - c^B\right)\right\} = u^A - c^A - \left(u^B - c^B\right),$$

and smaller than the minimum over all upper bounds

$$\min\left\{\tfrac{1}{3}\left(u^A - c^A + u^B - c^B\right), u^A - c^A + u^B - c^B, u^A - c^A + \tfrac{1}{3}\left(u^B - c^B\right), \tfrac{1}{3}\left(u^A - c^A\right) + u^B - c^B\right\}$$
$$= \tfrac{1}{3}\left(u^A - c^A + u^B - c^B\right).$$

Hence, combining all conditions reduces to

$$u^A - c^A - \left(u^B - c^B\right) < t < \tfrac{1}{3}\left(u^A - c^A + u^B - c^B\right).$$

In order to have a nonempty interval, the upper bound on t should be larger than the lower bound on t. Formally,

$$u^A - c^A - \left(u^B - c^B\right) < \tfrac{1}{3}\left(u^A - c^A + u^B - c^B\right) \Longleftrightarrow \tfrac{1}{2}\left(u^A - c^A\right) < u^B - c^B < u^A - c^A.$$

To summarize, we have derived the necessary and sufficient conditions on all parameters, i.e. u^A, c^A, u^B, c^B and t, that ensure that both companies have positive profits and positive market shares that cover the whole market in all four configurations. These conditions are stated in the main text as Assumption 7 and are given by

$$\begin{cases} \tfrac{1}{2}\left(u^A - c^A\right) < u^B - c^B < u^A - c^A, \\ u^A - c^A - \left(u^B - c^B\right) < t < \tfrac{1}{3}\left(u^A - c^A + u^B - c^B\right). \end{cases}$$

It is also of interest to keep track which of the many constraints analyzed in this appendix are binding and that were the bottleneck in obtaining this pair of conditions. This allows us to better understand the model and to gain economic intuition. The upper bound on t is derived from

- the lower bound of (6) that arises from imposing full market coverage in configuration UU,

while the lower bound on t is derived from *two* cases, being

- the lower bound of (6) that arises from imposing positive market shares in configuration PP, which marks the common boundary between condition (D) and condition (A) and
- the lower bound of (7) that arises from equilibrium condition $\bar{p}^B \geq c^B$ in configuration PU.

For completeness, we also discuss the other case and the general condition. By symmetry, $u^B - c^B > u^A - c^A > 0$ requires

$$\begin{cases} \frac{1}{2}\left(u^B - c^B\right) < u^A - c^A < u^B - c^B, \\ u^B - c^B - \left(u^A - c^A\right) < t < \frac{1}{3}\left(u^A - c^A + u^B - c^B\right). \end{cases}$$

With respect to the restriction on social welfare we derive the following general necessary and sufficient condition for a nonempty range of parameters t:

$$\frac{1}{2}\max\left\{u^A - c^A, u^B - c^B\right\} < \min\left\{u^A - c^A, u^B - c^B\right\} < \max\left\{u^A - c^A, u^B - c^B\right\}.$$

This condition implies that the asymmetry between companies should be sizable, but not too large such that one of the companies is driven out of the market.

C Appendix: Proof of Proposition 9

Denote π^i_{UU} and π^i_{PP}, $i = A, B$, as the profit of company i for the configurations UU, respectively, PP. Recall that for the fully symmetric case the paradox arises with both $\pi^A_{PP} < \pi^A_{UU}$ and $\pi^B_{PP} < \pi^B_{UU}$. Assumption 7 imposes $u^A - c^A > u^B - c^B > 0$, i.e. company A provides more social welfare than company B if cost of adjustment are ignored. The proof consists of several claims that we prove separately.

Claim 20 *Under Assumption 7, we have*

$$\pi^A_{UU} > \pi^A_{PP} \iff \begin{cases} \frac{\sqrt{2}}{1+\sqrt{2}}\left(u^A - c^A\right) < u^B - c^B < u^A - c^A, \\ \frac{1+2\sqrt{2}}{3}\left(u^A - c^A - \left(u^B - c^B\right)\right) < t < \frac{1}{3}\left(u^A - c^A + u^B - c^B\right). \end{cases}$$

$$\pi^A_{UU} < \pi^A_{PP} \iff \textit{either} \begin{cases} \frac{\sqrt{2}}{\sqrt{2}+1}\left(u^A - c^A\right) < u^B - c^B < u^A - c^A, \\ u^A - c^A - \left(u^B - c^B\right) < t < \frac{1+2\sqrt{2}}{3}\left(u^A - c^A - \left(u^B - c^B\right)\right). \end{cases}$$

$$\textit{or} \begin{cases} \frac{1}{2}\left(u^A - c^A\right) < u^B - c^B \leq \frac{\sqrt{2}}{\sqrt{2}+1}\left(u^A - c^A\right), \\ u^A - c^A - \left(u^B - c^B\right) < t < \frac{1}{3}\left(u^A - c^A + u^B - c^B\right), \end{cases}$$

All three pairs of conditions partition the nonempty parameter space specified by Assumption 7 into three nonempty subsets.

Proof of Claim 20
Substitution of the closed-form solutions of Table 1 in $\pi^A_{UU} > \pi^A_{PP}$ and rewriting yields

$$\frac{\left(3t + \left(u^A - c^A\right) - \left(u^B - c^B\right)\right)^2}{18t} > \frac{\left(t + \left(u^A - c^A\right) - \left(u^B - c^B\right)\right)^2}{4t} \iff$$

$$18t^2 - 12\left(u^A - c^A - \left(u^B - c^B\right)\right)t - 14\left(u^A - c^A - \left(u^B - c^B\right)\right)^2 > 0. \quad \text{(C.1)}$$

This polynomial in t has as its roots $\frac{1-2\sqrt{2}}{3}\left(u^A - c^A - \left(u^B - c^B\right)\right)$ and $\frac{1+2\sqrt{2}}{3}\left(u^A - c^A - \left(u^B - c^B\right)\right)$, where $\frac{1-2\sqrt{2}}{3} < 1$ and $\frac{1+2\sqrt{2}}{3} > 1$ (it is roughly 1.276). Combined with the bounds on t as stated in Assumption 7, we obtain

$$\tfrac{1+2\sqrt{2}}{3}\left(u^A - c^A - \left(u^B - c^B\right)\right) < t < \tfrac{1}{3}\left(u^A - c^A + u^B - c^B\right).$$

This interval is nonempty if and only if $\frac{\sqrt{2}}{1+\sqrt{2}}\left(u^A - c^A\right) < u^B - c^B < u^A - c^A$, where $\frac{1}{2} < \frac{\sqrt{2}}{1+\sqrt{2}} < 1$ (it is roughly 0.586). Note that both conditions are more strict than their corresponding condition specified under Assumption 7 and both conditions have to hold simultaneously to obtain $\pi^A_{UU} > \pi^A_{PP}$ and cannot be weakened. Hence, under Assumption 7 this pair of conditions is both necessary and sufficient.

Similarly, $\pi^A_{UU} < \pi^A_{PP}$ and rewriting yields that t should lie between the roots of the polynomial in t given by (C.1). Combined with the bounds on t as stated in Assumption 7, we obtain the nonempty interval given by

$$u^A - c^A - \left(u^B - c^B\right) < t < \min\left\{\tfrac{1+2\sqrt{2}}{3}\left(u^A - c^A - \left(u^B - c^B\right)\right), \tfrac{1}{3}\left(u^A - c^A + u^B - c^B\right)\right\}.$$

The minimum is attained by the second term if and only if $u^B - c^B \leq \frac{\sqrt{2}}{\sqrt{2}+1}\left(u^A - c^A\right)$, where $\frac{1}{2} < \frac{\sqrt{2}}{\sqrt{2}+1} < 1$. Combined with Assumption 7, we have the following two cases:

$$\begin{cases} \tfrac{1}{2}\left(u^A - c^A\right) < u^B - c^B \leq \tfrac{\sqrt{2}}{\sqrt{2}+1}\left(u^A - c^A\right), \\ u^A - c^A - \left(u^B - c^B\right) < t < \tfrac{1}{3}\left(u^A - c^A + u^B - c^B\right), \end{cases}$$

and

$$\begin{cases} \tfrac{\sqrt{2}}{\sqrt{2}+1}\left(u^A - c^A\right) < u^B - c^B < u^A - c^A, \\ u^A - c^A - \left(u^B - c^B\right) < t < \tfrac{1+2\sqrt{2}}{3}\left(u^A - c^A - \left(u^B - c^B\right)\right). \end{cases}$$

Each pair of conditions corresponds to a nonempty subset of parameter values under Assumption 7 and each pair is more strict than their corresponding condition specified under Assumption 7. Furthermore, both conditions in each pair have to hold simultaneously to obtain $\pi^A_{UU} < \pi^A_{PP}$.

To summarize, we derived three mutually exclusive pairs of conditions that cover the conditions of Assumption 7. Hence, these form a partition of the set of parameter values for which this assumption holds.

QED

Claim 21 *Under Assumption 7, we have* $\pi^B_{UU} > \pi^B_{PP}$.

Proof of Claim 21

Substitution of the closed-form solutions of Table 1 in $\pi_{UU}^B > \pi_{PP}^B$ and rewriting yields

$$\frac{\left(3t - \left(u^A - c^A\right) + u^B - c^B\right)^2}{18t} > \frac{\left(t - \left(u^A - c^A\right) + u^B - c^B\right)^2}{4t} \iff$$

$$18t^2 + 12\left(u^A - c^A - \left(u^B - c^B\right)\right)t - 14\left(u^A - c^A - \left(u^B - c^B\right)\right)^2 > 0. \quad \text{(C.2)}$$

This polynomial in t has as its roots $-\frac{1+2\sqrt{2}}{3}\left(u^A - c^A - \left(u^B - c^B\right)\right) < 0$ and $\frac{2\sqrt{2}-1}{3}\left(u^A - c^A - \left(u^B - c^B\right)\right)$, where $\frac{1}{2} < \frac{2\sqrt{2}-1}{3} < 1$ (it is roughly 0.609). Combined with bounds on t as stated in Assumption 7, we obtain

$$\tfrac{2\sqrt{2}-1}{3}\left(u^A - c^A - \left(u^B - c^B\right)\right) < t < \tfrac{1}{3}\left(u^A - c^A + u^B - c^B\right),$$

where $\frac{2\sqrt{2}-1}{3} < 1$. Hence, this interval for t is larger than the nonempty interval for t stated in Assumption 7. Furthermore, this interval is nonempty under Assumption 7. In other words, for all parameter values satisfying Assumption 7 it holds that $\pi_{UU}^B > \pi_{PP}^B$ and there are no additional restrictions on t or the other parameters. QED

To summarize both claims, less-efficient company B can never gain from competition in personalized pricing compared to competition in uniform pricing. For more-efficient company A it depends. To avoid repeating these two pairs of conditions in the main text, we refer to these two pairs of conditions as follows:

- one pair of conditions for the same range of social welfare $u^B - c^B$ as stated in Claim 21 for $\pi_{UU}^A > \pi_{PP}^A$ but for low levels of t such that $\frac{t}{u^A - c^A - (u^B - c^B)} \in \left(1, \frac{1+2\sqrt{2}}{3}\right)$;
- and another pair of conditions for the range of low levels of social welfare $u^B - c^B$ such that $\frac{u^B - c^B}{u^A - c^A} \in \left(\frac{1}{2}, \frac{\sqrt{2}}{1+\sqrt{2}}\right)$ and the same range for t as in Assumption 7.

D Appendix: Proofs of Propositions 11, 12 and 13

Proof of Proposition 11

The expressions for Total Welfare in each of the cases can be derived by substituting the relevant level of market share into the total welfare expression

$$SW(x) = SW^A(x) + SW^B(x) = \left(u^A - c^A - \left(u^B - c^B\right) + t\right)x + \left(u^B - c^B\right) - \tfrac{1}{2}t - tx^2.$$

Recall, $x_{PP}^A = \frac{u^A - c^A - \left(u^B - c^B\right) + t}{2t}$. Substituting this into expression above we obtain

$$SW(x_{PP}^A) = \frac{\left(u^A - c^A - \left(u^B - c^B\right) + t\right)^2}{4t} + \left(u^B - c^B\right) - \frac{1}{2}t.$$

Next, substituting $x_{UU}^A = \frac{u^A - c^A - \left(u^B - c^B\right) + 3t}{6t}$ into total welfare expression we obtain

$$SW(x_{UU}^A) = \left(\frac{u^A - c^A - \left(u^B - c^B\right) + 3t}{6t}\right)\left(\frac{5\left(u^A - c^A\right) - 5\left(u^B - c^B\right) + 3t}{6}\right) + \left(u^B - c^B\right) - \frac{1}{2}t.$$

The expressions for consumer surpluses under the configurations PP and UU can now be derived by subtracting profits of both companies (A and B) from the total welfare. This gives CS^{PP} and CS^{UU} specified below:

$$CS^{PP} = \left(u^B - c^B\right) - \frac{1}{2}t - \frac{\left(t - \left(u^A - c^A - \left(u^B - c^B\right)\right)\right)^2}{4t},$$

$$CS^{UU} = \left(u^B - c^B\right) - \tfrac{1}{2}t + \frac{4\left(u^A - c^A - \left(u^B - c^B\right)\right)\left(u^A - c^A - \left(u^B - c^B\right) + 3t\right)}{36t} - \frac{\left(u^A - c^A - \left(u^B - c^B\right) + 3t\right)^2 + 2(3t - \left(u^A - c^A - \left(u^B - c^B\right)\right))^2}{36t}.$$

To simplify the exposition we will denote $u^A - c^A - \left(u^B - c^B\right) = A - B = C$. Then,

$$CS^{PP} - CS^{UU} = \frac{12C(t - C) + 2(3t - C)^2}{36t} > 0 \quad for\ all\ t > C.$$

The latter holds under Assumption 7. QED

Proof of Proposition 12

From Table 1 and Proposition 10.(ii) it follows that the market share of company A under uniform pricing is smaller than this company's market segment under personalized pricing. This means that consumers that buy at company A under uniform pricing, i.e. $x \in \left[0, x_{UU}^A\right)$ where $x_{UU}^A = \frac{1}{2} + \frac{1}{6t}\left(u^A - c^A - \left(u^B - c^B\right)\right)$, will also do so under personalized pricing. Similarly, consumers that buy at company B under personalized pricing, i.e. $x \in \left(x_{PP}^A, 1\right]$ where $x_{PP}^A = \frac{1}{2} + \frac{1}{2t}\left(u^A - c^A - \left(u^B - c^B\right)\right)$, will also do so under uniform pricing. Furthermore, consumers in between $x \in \left(x_{UU}^A, x_{PP}^A\right)$ switch companies. In particular, they buy at company B under uniform pricing and switch to company A under personalized pricing.

Proof of part (i):

By comparing $p^A(x)$ and $\bar{p}^A$ as stated in Table 1, we obtain that

$$p^A(x) > \bar{p}^A \text{ when } c^A + t + u^A - c^A - \left(u^B - c^B\right) - 2tx > c^A + t + \tfrac{1}{3}\left(u^A - c^A - \left(u^B - c^B\right)\right).$$

This implies that

$$x < \frac{u^A - c^A - \left(u^B - c^B\right)}{3t} < x_{UU}^A < x_{PP}^A.$$

Moreover,

$$\frac{u^A - c^A - \left(u^B - c^B\right)}{3t} > 1 - \frac{u^B - c^B}{t} = \underline{x}^B \text{ implies that } t < \frac{u^A - c^A + 2\left(u^B - c^B\right)}{3},$$

where the latter inequality holds under Assumption 7, i.e. $t < \frac{u^A - c^A + u^B - c^B}{3}$. This partitions market segment $\left[0, x_{PP}^A\right)$ of company A into $\left[0, \frac{1}{3t}\left(u^A - c^A - \left(u^B - c^B\right)\right)\right)$, which includes $\left[0, \underline{x}^B\right)$, and $\left[\frac{1}{3t}\left(u^A - c^A - \left(u^B - c^B\right)\right), x_{PP}^A\right)$.

Since $p^A(x) > \bar{p}^A$ is equivalent to $x < \frac{u^A - c^A - \left(u^B - c^B\right)}{3t}$, consumers in $\left[0, \frac{1}{3t}\left(u^A - c^A - \left(u^B - c^B\right)\right)\right)$ segment are worse off under personalized pricing compared to uniform pricing.

Proof of part (ii):
On market segment $\left[\frac{1}{3t}\left(u^A - c^A - \left(u^B - c^B\right)\right), 1\right]$ the result will be different. This segment can be partitioned into three parts: $\left[\frac{1}{3t}\left(u^A - c^A - \left(u^B - c^B\right)\right), x_{UU}^A\right)$, $\left[x_{UU}^A, x_{PP}^A\right)$, and $\left(x_{PP}^A, 1\right]$.

On market segment $\left[\frac{1}{3t}\left(u^A - c^A - \left(u^B - c^B\right)\right), x_{UU}^A\right)$ we have $p^A(x) < \bar{p}^A$. Hence, consumers that stay with A will benefit from lower personalized prices.

Next, for segment $\left(x_{PP}^A, 1\right]$, by comparing $p^B(x)$ and $\bar{p}^B$ as stated in Table 1, we obtain

$$p^B(x) < \bar{p}^B \text{ when } c^B + t - \left(u^A - c^A - \left(u^B - c^B\right)\right) - 2t(1-x) < c^B + t - \frac{u^A - c^A - \left(u^B - c^B\right)}{3}.$$

This implies that

$$x < 1 + \frac{u^A - c^A - \left(u^B - c^B\right)}{3t},$$

which holds for all $x \in [0, 1]$. Hence, every consumer $x \in \left(x_{PP}^A, 1\right]$ is better off under personalized prices set by company B than uniform prices set by company B.

The latter result also holds on the market segment $\left[x_{UU}^A, x_{PP}^A\right)$,where consumers switch from company B to company A. There by Proposition 2 we have

$$r^A(x) - p^A(x) = r^B(x) - p^B(x) > r^B(x) - \bar{p}^B.$$

Hence, $p^A(x) < \bar{p}^B$ for consumers that switch on the market segment $\left[x_{UU}^A, x_{PP}^A\right)$. QED

Proof of Proposition 13
The result is trivial for parameter values in which both companies have lower producer surplus. For the nontrivial case, we have

$$\pi_{UU}^A + \pi_{UU}^B = \frac{18t^2 + 2\left(u^A - c^A - \left(u^B - c^B\right)\right)^2}{18t},$$
$$\pi_{PP}^A + \pi_{PP}^B = \frac{9t^2 + 9\left(u^A - c^A - \left(u^B - c^B\right)\right)^2}{18t}$$

from which we obtain that

$$\pi_{UU}^{A}+\pi_{UU}^{B}-\left(\pi_{PP}^{A}+\pi_{PP}^{B}\right)=\frac{9t^{2}-7\left(u^{A}-c^{A}-\left(u^{B}-c^{B}\right)\right)^{2}}{18t}.$$

The right-hand side is negative whenever $t<\frac{\sqrt{7}}{3}\left(u^{A}-c^{A}-\left(u^{B}-c^{B}\right)\right)$, where $\frac{\sqrt{7}}{3}<1$. Recall that both the "either" and "or" condition for $\pi_{UU}^{A}>\pi_{PP}^{A}$ derived in Appendix C require $t>u^{A}-c^{A}-\left(u^{B}-c^{B}\right)$. Hence, these bounds on t are incompatible and $\pi_{UU}^{A}+\pi_{UU}^{B}>\pi_{PP}^{A}+\pi_{PP}^{B}$. QED

E Appendix: Low Adjustment Costs

In this appendix we investigate whether the result that more efficient company benefits from switching to personalized pricing technology extends to the entire area of low adjustment costs. The many cases that needed to be checked for interior equilibria in Appendix B indicate that relaxing Assumption 7 gives rise to many other situations in Figure 4. We do not intend to provide a full analysis of all cases, but the area of parameter values with low values of parameter t are of interest. First of all, low distance or adjustment costs intensify competition according to the literature, in particular the boundary case of zero costs in the symmetric Hotelling's model is the classic Bertrand cut-throat competition. Furthermore, we aim to investigate whether the result that company A benefits from PP instead of UU extends to this area of intensified competition.

Relaxing Assumption 7

The area of interest is given by $t<\min\left\{\frac{1}{3}\left(u^{A}-c^{A}+\left(u^{B}-c^{B}\right)\right),u^{A}-c^{A}-\left(u^{B}-c^{B}\right)\right\}$ for $0\leq u^{B}-c^{B}<u^{A}-c^{A}$, which is the area below the two lines that define the triangle in Figure 4. According to Appendix B.1, the area below the increasing line is the area with positive market shares for both companies in UU, while the area below the decreasing 45-degree line corresponds to the situation in which company B drops out of the market under PP and company B's uniform price drops below marginal cost pricing under PU. Furthermore, Appendix B.1 also indicates that uniform prices above marginal costs under UU require $t>\frac{1}{3}\left(u^{A}-c^{A}-\left(u^{B}-c^{B}\right)\right)$, while Appendix B.4 indicates that the same condition is needed for a positive market share of company B under UP. This latter condition partitions the area of low t in two subcases that are captured by the following two assumptions.

Assumption 22 *Consider the pair of conditions*

$$\begin{cases} 0\leq u^{B}-c^{B}<u^{A}-c^{A}, \\ \frac{1}{3}(u^{A}-c^{A}-\left(u^{B}-c^{B}\right))<t<\min\left\{\frac{1}{3}\left(u^{A}-c^{A}+\left(u^{B}-c^{B}\right)\right),u^{A}-c^{A}-\left(u^{B}-c^{B}\right)\right\}. \end{cases}$$

Assumption 23 *Consider the pair of conditions*

$$\begin{cases} 0 \leq u^B - c^B < u^A - c^A, \\ t < \frac{1}{3}(u^A - c^A - \left(u^B - c^B\right)). \end{cases}$$

The line piece $t = \frac{1}{3}\left(u^A - c^A - \left(u^B - c^B\right)\right)$ has endpoints $\left(u^B - c^B, t\right) = \left(0, \frac{1}{3}\left(u^A - c^A\right)\right)$ and $\left(u^B - c^B, t\right) = \left(u^A - c^A, 0\right)$ in Figure 4. Without going into details, we state the market configurations per assumption.

- Assumption 22 implies duopolies with positive market shares under UU and UP, while constrained monopoly for company A under PU and PP with marginal cost pricing for B independent of uniform or personalized pricing.
- Assumption 23 implies no duopolies with positive market shares and constrained monopolies for company A under UU, UP, PU and PP with marginal cost pricing for B independent of uniform or personalized pricing.

Results for Assumption 22

As mentioned, Assumption 22 implies duopolies with positive market shares under UU and UP, while constrained monopolies for company A under PU and PP. This implies that the expressions derived in Table 1 remain valid for UU and UP. Given marginal cost pricing for firm B, the other expressions for constrained monopoly can be easily derived for PU and PP. We summarize all equilibrium expressions in Table 2.

The closed-form solutions for profits in Table 2 provide payoffs of the bi-matrix game that represents endogenous adoption of pricing technologies: either uniform pricing (U) or personalized pricing (P). We obtain the following modified game.

	U	P
U	$\frac{[3t+u^A-c^A-\left(u^B-c^B\right)]^2}{18t}, \frac{[3t-(u^A-c^A)+u^B-c^B]^2}{18t}$	$\frac{\left[t+u^A-c^A-\left(u^B-c^B\right)\right]^2}{8t}, \frac{\left[3t-\left(u^A-c^A\right)+u^B-c^B\right]^2}{16t}$
P	$u^A - c^A - \left(u^B - c^B\right), 0$	$u^A - c^A - \left(u^B - c^B\right), 0$

Inspection of this game reveals that adopting the personalized pricing technology dominates adopting a uniform technology for company A,[36] while it weakly dominates for company B.

[36] Denote $C = u^A - c^A - \left(u^B - c^B\right) > 0$. By assumption, $t > \frac{1}{3}C$ implies

$$\pi_{PU} > \pi_{UU} \iff 18tC - (3t + C)^2 = -9t^2 + 12Ct - C^2 > 2C^2 > 0,$$
$$\pi_{PP} > \pi_{UP} \iff 8tC - (t + C)^2 = -t^2 + 6Ct - C^2 > \tfrac{8}{9}C^2 > 0.$$

Therefore, joint adoption of the personalized pricing technology is the equilibrium in weakly dominant strategies, but company A adopting while B does not emerges as a second Nash equilibrium. We state this result without further proof.

Proposition 24 *Let Assumption 22 hold. For company A, personalized pricing dominates uniform pricing, while it is a weak dominance for company B. Joint adoption of the personalized pricing technology and only company A adopting are both equilibrium. Moreover, less-efficient company B never benefits from competition in personalized pricing compared to competition in uniform pricing, whereas more-efficient company A always benefits.*

Results for Assumption 23

As mentioned, Assumption 23 implies constrained monopolies for company A under UU, UP, PU and PP with marginal cost pricing for B independent of uniform or personalized pricing. This implies that the expressions derived in modified Table Table 2 remain valid for PU and PP. Given marginal cost pricing for firm B, the other expressions for constrained monopoly can be easily derived for UU and UP. We summarize all equilibrium expressions in Table 3.

The closed-form solutions for profits in Table 2 provide payoffs of the bi-matrix game that represents endogenous adoption of pricing technologies: either uniform pricing (U) or personalized pricing (P). We obtain the following modified game.

	U	P
U	$u^A - c^A - \left(u^B - c^B\right) - t, 0$	$\frac{t+u^A-c^A-\left(u^B-c^B\right)}{2}, 0$
P	$u^A - c^A - \left(u^B - c^B\right), 0$	$u^A - c^A - \left(u^B - c^B\right), 0$

Inspection of this game reveals that adopting the personalized pricing technology dominates adopting a uniform technology for constrained monopolist A,[37] while adopting does not matter for company B because this company has no change to become active and make a positive profit. Therefore, joint adoption of the personalized pricing technology is once more an equilibrium in weakly dominant strategies, but company A adopting while B does not is a second Nash equilibrium. We state this result without further proof.

Proposition 25 *Let Assumption 23 hold. For company A, personalized pricing dominates uniform pricing, while it is a weak dominance for company B. Joint adoption of the personalized pricing technology and only company A adopting are both equilibrium. Moreover, less-efficient company B never benefits from competition in personalized pricing compared to competition in uniform pricing, whereas more-efficient company A always benefits.*

[37] Denote $C = u^A - c^A - \left(u^B - c^B\right) > 0$. By assumption, $t < \frac{1}{3}C$ implies $C > t$ and $\pi_{PP} = C = \frac{1}{2}(C + C) > \frac{1}{2}(t + C) = \pi_{UP}$.

	UU	PP	PU	UP
p^A	$c^A+t+\frac{u^A-c^A-\left(u^B-c^B\right)}{3}$	$c^A+t+u^A-c^A-\left(u^B-c^B\right)-2tx$	$c^A+t+u^A-c^A-\left(u^B-c^B\right)-2tx$	$c^A+\frac{t}{2}+\frac{u^A-c^A-\left(u^B-c^B\right)}{2}$
p^B	$c^B+t-\frac{u^A-c^A-\left(u^B-c^B\right)}{3}$	c^B	c^B	$c^B-\frac{t}{2}-\frac{u^A-c^A-\left(u^B-c^B\right)}{2}+2tx$
x^A	$\frac{1}{2}+\frac{u^A-c^A-\left(u^B-c^B\right)}{6t}$	1	1	$\frac{1}{4}+\frac{u^A-c^A-\left(u^B-c^B\right)}{4t}$
x^B	$\frac{1}{2}-\frac{u^A-c^A-\left(u^B-c^B\right)}{6t}$	0	0	$\frac{3}{4}-\frac{u^A-c^A-\left(u^B-c^B\right)}{4t}$
π^A	$\frac{\left[3t+u^A-c^A-\left(u^B-c^B\right)\right]^2}{18t}$	$u^A-c^A-\left(u^B-c^B\right)$	$u^A-c^A-\left(u^B-c^B\right)$	$\frac{\left[t+u^A-c^A-\left(u^B-c^B\right)\right]^2}{8t}$
π^B	$\frac{\left[3t-\left(u^A-c^A\right)+u^B-c^B\right]^2}{18t}$	0	0	$\frac{\left[3t-\left(u^A-c^A\right)+u^B-c^B\right]^2}{16t}$

Table 2: Assumption 22 implies the following prices (p), market shares (x) and profits (π) per company for each market configuration of AI pricing technologies, where U means uniform pricing and P personalized pricing.

	UU	PP	PU	UP
p^A	$u^A - \left(u^B - c^B\right) - t$	$c^A + t + u^A - c^A - \left(u^B - c^B\right) - 2tx$	$c^A + t + u^A - c^A - \left(u^B - c^B\right) - 2tx$	$c^A + \frac{t}{2} + \frac{u^A - c^A - \left(u^B - c^B\right)}{2}$
p^B	c^B	c^B	c^B	c^B
x^A	1	1	1	1
x^B	0	0	0	0
π^A	$u^A - c^A - \left(u^B - c^B\right) - t$	$u^A - c^A - \left(u^B - c^B\right)$	$u^A - c^A - \left(u^B - c^B\right)$	$\frac{t + u^A - c^A - \left(u^B - c^B\right)}{2}$
π^B	0	0	0	0

Table 3: Assumption 23 implies the following prices (p), market shares (x) and profits (π) per company for each market configuration of AI pricing technologies, where U means uniform pricing and P personalized pricing.

F Appendix: Extensions and Applications

In this appendix, we discuss the technical details of the extensions and applications of Section 5. Extensions of the function g, as defined in (2) of Appendix A.1, play a crucial role.

F.1 Discontinuity

Dropping continuity from Assumption 1 has no real consequences other than lacking existence of a root x^* such that $g(x^*) = 0$. This may or may not exclude the special case case of symmetric competition at x^* in Proposition 2. To see this, under the remainder of Assumption 1 and condition (D) for duopoly, either such discontinuities of g with downward drops (while never upward due to monotonicity) occur at locations other than at the unique x^* for which $g(x^*) = 0$ (still due to monotonicity), or a discontinuity occurs at some x^D where the function g drops from positive to negative and either $g\left(x^D\right) > 0$ or $g\left(x^D\right) < 0$ implying x^* does not exist. For the latter situation and e.g. in case of $g\left(x^D\right) > 0$, company A's market segment becomes the closed interval $\left[0, x^D\right]$, while B's market segment $\left(x^D, 1\right]$ remains half-open. In general, and in particular for the other relaxations of Assumption 1 that we discuss below, discontinuities in the reservation price functions imply either the same cases or less cases to consider than under continuity and we will not discuss discontinuity anymore.

F.2 A General Framework: multidimensional characteristics and oligopoly

One aim of this appendix is to seek generality and to bridge the gap between Hotelling's model under Assumption 1 and several aspects of reality, of which e.g. multidimensional characteristics of big data, oligopoly and incomplete market coverage are important ones.

F.2.1 The model

Multidimensional characteristics can be incorporated by replacing the interval $[0, 1]$ by a compact set $X \subset \mathbb{R}^m$, $m \geq 1$. Each dimension can be either a continuous or discrete variable. The distribution of characteristics is described by a cumulative multivariate distribution function $F : X \to [0, 1]$. For example, gender-specific income distributions $F(x_1, x_2)$, where $x_1 \in \mathbb{R}_+$ represents income and $x_2 \in \{0, 1\}$ gender defined as a binary variable.

The extension to $n \geq 1$ companies in oligopoly (and monopoly) starts with the index of companies, denoted as i, that runs from 0 to n with the convention that not buying is equivalent to company 0, which is needed to capture incomplete market coverage. The reservation price functions on the extended domain and the enlarged set of agents are defined as $r^i : X \longrightarrow \mathbb{R}$ with normalization $r^0(x) = 0$.[38] Company i produces against constant marginal costs $c^i \geq 0$

[38] This framework can accomodate for heterogeniety of outside options and corresponding consumer specific surplus of not buying. For example, first-time buyers of a car and second-time buyers who have the option to

with convention $c^0 = 0$, and offers maximal consumer surplus $r^i(x) - c^i$. A unique global maximum of r^i at $\hat{x}^i \in X$, $i = 1, \ldots, n$, can be interpreted as the location of company i as an e.g. Hotelling (1929). For example, under $X = [0, 1]$ and Assumption 1 in the main text the global maximum of the function r^A on $[0, 1]$ is is company A's location $\hat{x}^A = 0$. Our framework can also accommodate multiple global and local maxima associated with economic situations where the notion of location is ambiguous.

F.2.2 Characterization of equilibrium in Personalized Prices

Recall from the main text that under Assumption 1 in Hotelling's model, both firms offer positive consumer surplus to each consumer; a positive (negative) value of the function g at x indicates that the potential maximal consumer surplus for consumer x can be attained at company A (B); g expresses the gap in maximal consumer surpluses between the best and second-best options for consumer x; and g is convenient in indicating which company will attract consumer x under competition in personalized prices. In what follows, we will modify the function g by making it company specific, denoted g^i. The latter gap function will be defined from the perspective of buying at company i (or not buying when the superscript is 0) and it expresses the gap between company i's maximal consumer surplus and the second-highest maximal consumer surplus of all other companies (including 0). This can be accomplished by putting all candidate maximal consumer surpluses of all other companies under the maximum function. So, we define the function $g^i : X \to \mathbb{R}$, $i = 0, \ldots, n$, as

$$g^i(x) = r^i(x) - c^i - \max_{j=0,\ldots,n; j \neq i} \left\{ r^j(x) - c^j \right\}, \tag{9}$$

where $r^0(x) - c^0 = 0$ by convention. The following technical result holds.

Lemma 26 *Let $x \in X$. There exists at least one index i for which $g^i(x) \geq 0$. Moreover, index i is unique if and only if $g^i(x) > 0$ and for other $j \neq i$ it holds that $g^j(x) < 0$.*

Proof
The set of indices is finite. Therefore, there exists an $i \in \arg\max_{j=0,\ldots,n} \left\{ r^j(x) - c^j \right\}$. For any such i,

$$r^i(x) - c^i = \max_{j=0,\ldots,n; j \neq i} \left\{ r^j(x) - c^j \right\} \geq \max_{j=0,\ldots,n; j \neq i} \left\{ r^j(x) - c^j \right\}$$

implies $g^i(x) \geq 0$. In case this i is unique, the $\geq$ must be strict (otherwise a second index in arg max would exist contradicting uniqueness). Hence, $r^i(x) - c^i > r^j(x) - c^j$ for all $j \neq i$. This implies for any such $j \neq i$ that

$$g^j(x) = r^j(x) - c^j - \max_{k=0,\ldots,n; k \neq j} \left\{ r^k(x) - c^k \right\} = r^j(x) - c^j - \left(r^i(x) - c^i \right) < 0.$$

postpone replacement. All it takes is specifying function $r^0 : X \to \mathbb{R}$, which we do not pursue in order to stay closer to the literature.

If $g^i(x) > 0$ for some i, then $r^i(x) - c^i > \max_{j=0,\ldots,n; j \neq i} \{r^j(x) - c^j\}$ implies $\arg\max_{j=0,\ldots,n} \{r^j(x) - c^j\} = \{i\}$, the unique index i. QED

Corollary 27 *Under $X = [0,1]$ and Assumption 1 in Hotelling's model, $g^0(x) < 0$ for all $x \in [0,1]$, $g^A(x) > 0$ is equivalent to $g(x) > 0$ and $g^B(x) > 0$ is equivalent to $g(x) < 0$, where g is defined in* (2).

The maximum of a finite number of maximal consumer surpluses for consumer x always exists. For the corresponding maximizing index, say i, the gap function is nonnegative. In case of a unique maximum for x, index i must be unique and this can only occur in case the maximal consumer surplus for consumer x at company i strictly exceeds that of all other companies. It then follows that i's gap function at consumer x is positive and those of all other companies are negative. So, if a gap function value is positive at x, its superscript i indicates the unique and socially most efficient company with respect to potential maximal consumer surplus for consumer x.

It may be that index i is equal to 0 in which case no company is able to offer positive maximal consumer surplus to consumer x who is then best off not to buy. We may forego analyzing all trivial cases $g^0(x) > 0$ and market coverage will not be reached in equilibrium. Formally, define $X^0 = \{x \in X | g^0(x) > 0\}$ and we concentrate our analysis on its complement $X \backslash X^0$. As a final remark, market coverage can be defined as the condition $g^0(x) < 0$ for all $x \in X$.

Proposition 2 and 4 and 6 were analyzed under market coverage in equilibrium, we extend the analysis to multidimensional characteristics, oligopoly and incomplete market coverage. In the following result, we consider company i and j as defined by

$$i \in \arg\max_{k=1,\ldots,n} \left\{r^k(x) - c^k\right\} \qquad \text{and} \qquad j \in \arg\max_{k=0,\ldots,n; k \neq i} \left\{r^k(x) - c^k\right\}.$$

Proposition 28 *Let $x \in X \backslash X^0$.*

1. *If $g^i(x) > 0$ and $i, j \neq 0$: $\quad p^i(x) = c^i + g^i(x) > c^i, \quad p^j(x) = c^j, \quad$ and x buys at i,*
2. *If $g^i(x) = g^j(x) = 0$: $\quad p^i(x) = c^i, \quad p^j(x) = c^j, \quad$ and x buys at i or j.*

All other equilibrium personalized prices $p^k(x)$ are indeterminate. Company i's profit from selling to consumer x is equal to $g^i(x)$ with percentage markup $g^i(x)/p^i(x)$. Moreover, social welfare at x is maximal.

Proof

The combined approaches of Simon and Zame (1990) and Armstrong and Vickers (2001) imply that competition in personalized prices is equivalent to a first-price auction in utilities with *i*) $u^h \in (-\infty, \infty)$, $h = 0, \ldots, n$, *ii*) utility functions $r^h(x) - c^h - u^h$ of winning the auction and 0 otherwise, *iii*) the highest utility wins and *iv*) the auctioneer is modeled as an additional player who chooses the winner in case of a draw. (The auctioneer replaces consumer x who also makes an endogenous choice.)

Case 1. Then, $r^i(x) - c^i > r^j(x) - c^j > 0$. Consider the strategy profile $u^i = u^j = r^j(x) - c^j$, $u^k \leq r^j(x) - c^j$ for $k \neq i, j$ and the auctioneer declares i as the winner in case of ties. By Simon and Zame (1990) this strategy profile is the unique Nash equilibrium of the auction. For consumer x utility u^i equals $r^i(x) - p^i(x)$ and u^j equals $r^j(x) - p^j(x)$, we obtain

$$p^i(x) = r^i(x) - \left(r^j(x) - c^j\right) = c^i + g^i(x) > c^i \qquad \text{and } p^j(x) = c^j.$$

Similarly, u^k equals $r^k(x) - p^k(x)$ and $u^k \leq r^j(x) - c^j$ imply

$$p^k(x) \geq r^k(x) - \left(r^j(x) - c^j\right) = c^k + \left[r^k(x) - c^k - \left(r^j(x) - c^j\right)\right],$$

where the last term is either 0 or negative. Hence, $p^k(x)$ is indeterminate and a range of equilibrium prices below marginal costs exists for $k \neq i, j$, which requires a setting of $n \geq 3$. In equilibrium, company k knows it will not produce and its loss-making price is void as long as it does not attract consumer x.

Case 2. Then, $r^i(x) - c^i = r^j(x) - c^j > 0$ and company i and j are symmetric. The same arguments apply as in Case 1. Because $g^i(x) = 0$, $p^i(x) = c^i + g^i(x) = c^i$. In equilibrium, the randomization by the auctioneer between i and j is indeterminate too.

Finally, company i's profit from selling to consumer x is $p^i(x) - c^i = g^i(x)$ with percentage markup $\left(p^i(x) - c^i\right)/p^i(x) = g^i(x)/p^i(x)$, provided $c^i > 0$. QED

The proof of this extended result is written to exploit the equivalence to bidding utility in an auction in which the auctioneer is also an active player and this combines the approaches as pioneered in Simon and Zame (1990) and Armstrong and Vickers (2001). It reiterates the economic logic that is extensively discussed in the main text and shows that it remains valid in the generalized model. It implies that the company who has the ability to offer the highest maximal consumer surplus also has the ability to outbid the others in terms of utility offered to consumer x in personalized prices. In equilibrium this company will do so and the consumer will buy from this company. The personalized price it sets is equal to consumer x's reservation price for its product minus the maximal consumer surplus of its closest competitor. Formally, we obtain that company i's equilibrium personalized price is given by

$$p^i(x) = c^i + g^i(x) = r^i(x) - \max_{k=0,\ldots,n;k\neq i}\left\{r^k(x) - c^k\right\}.$$

In the first case company i makes a positive profit from selling to consumer x, because $p^i(x) - c^i = g^i(x) > 0$, while in the second case profit will be 0. The percentage markup from selling to consumer x is $g^i(x)/p^i(x) \geq 0$ in both cases. The price of its closest competitor j is pushed down to marginal cost pricing, the lowest loss-avoiding price that j can offer to consumer x

and that achieves the second-highest maximal consumer surplus attainable for this consumer. In equilibrium the company with the highest maximal consumer surplus serves consumer x and therefore social welfare is maximized in equilibrium. The second case defines the boundaries between the market areas for company i and j. Such market areas can be defined as the $X^i = \{x \in X | g^i(x) > 0\}$ and $X^j = \{x \in X | g^j(x) > 0\}$. Then, for continuous reservation price functions, the equalities $g^i(x) = g^j(x) = 0$ define the boundary between these areas. As a final remark, note that j may be 0 in either case. Then, our conventions $r^0(x) = c^0 = 0$ imply $p^0(x) = 0$ and consumer surplus $r^0(x) - p^0(x) = 0$. This covers all boundaries separating companies' market areas and areas without market coverage, i.e., boundaries between X^i and X^0.

Nash equilibrium also produces indeterminacies when extending the analysis beyond duopoly to $n \geq 3$. These indeterminacies are similar to those reported in e.g. Funaki et al. (2020) and Rhodes and Zhou (2021). In equilibrium, only the personalized prices for company i and j are pinned down. All other companies know their personalized prices cannot attract the consumer unless they charge sufficiently low loss-making prices below marginal costs that would win over the consumer, i.e., $p^k(x) < r^k(x) - (r^j(x) - c^j) < c^k$. Of course, in equilibrium no company will set such prices. However, each $p^k(x) \geq r^k(x) - (r^j(x) - c^j)$ for company $k \neq i, j$ is an equilibrium price that reflects indeterminacy of the Nash equilibrium concept and that implies a range of prices below marginal costs. Note that this indeterminacy also occurs in case company k is another closest competitor next to j, because then equilibrium only imposes that either j or k has to offer the second-highest maximal consumer surplus to consumer x. The latter also occurs in duopoly in case $j = 0$ and $k \neq 0$. As we will discuss later, multinomial logit demand can be seen as an equilibrium refinement that excludes this indeterminacy and results in equilibrium personalized prices above marginal costs.

F.2.3 Market coverage and constrained monopoly

We continue with deriving two simple conditions, one for market coverage and one for constrained monopoly of company i. The market will be covered if every consumer buys at one of the companies in equilibrium. By Proposition 28, each consumer should have access to at least one company that is able to provide positive maximal consumer surplus to this consumer. Formally, $X^0 = \{x \in X | g^0(x) > 0\}$ is the set of consumers who have no access to a company who can provide positive maximal consumer surplus. Market coverage in equilibrium is achieved if this area is empty, i.e., $X^0 = \emptyset$. Rewriting $g^0(x) < 0$ for all $x \in X$ yields that the upper envelop of all maximal consumer surpluses has to be positive on X:

Corollary 29 *The market is covered in equilibrium if and only if* $\max_{k=1,\ldots,n} \left\{r^k(x) - c^k\right\} > 0$ *for all* $x \in X$.

In a constrained monopoly, the monopoly company can attract all consumers in $X\backslash X^0$ by its equilibrium personalized prices. Formally, company i is a constraint monopolist if its market share X^i in equilibrium equals $X\backslash X^0$. Rewriting $g^i(x) > 0$ for all $x \in X\backslash X^0$ yields the surface of company i's maximal consumer surplus is strictly above the surface of the upper envelop of maximal consumer surpluses of all its competitors on X/X^0.

Corollary 30 *Company i is constrained monopolist in equilibrium if and only if $r^i(x) - c^i > \max_{k=1,\ldots,n;k\neq i}\left\{r^k(x) - c^k\right\}$ for all $x \in X\backslash X^0$. Moreover, if also $r^i(x) - c^i > 0$ for all $x \in X$, then $X^i = X$.*

As a final remark, Proposition 28 does not require market coverage, monotonicity nor continuity as imposed by Assumption 1. It demonstrates that the qualitative insights of Proposition 2 and 4 and 6 extend beyond this assumption to incomplete market coverage, multidimensional consumer characteristics, oligopoly and monopoly, which is why we have included $n = 1$.

F.2.4 Tractable examples

In this subsection we provide two tractable examples. In the first example we illustrate how to characterize market segments in a three-dimensional space of consumer characteristics, while in the second one is based on an extension of the square city in Larralde et al. (2009).

Characterizing the boundaries of market areas boils down to analyzing $g^i(x) = 0$, $i = 0, \ldots, n$. For duopoly under market coverage, we have seen that the separation of market areas is identified at $r^A(x) - c^A = r^B(x) - c^B$. For $n \geq 3$ and market coverage, the boundaries of company j's market area require investigating $n - 1$ of such equalities, and to characterize all separators of all market areas requires repeating this for $n - 1$ companies. The task at hand is clearly defined, but without imposing additional structure on X and the reservation price functions, not much can be said. Therefore, we leave investigating oligopoly for future research and provide a trivial example of three symmetric companies on a unit simplex.

Example 31 *Consider an oligopoly of three symmetric companies, the three dimensional space $X = \left\{(x_1, x_2, x_3) \in \mathbb{R}^3_+ \middle| x_1 + x_2 + x_3 = 1\right\}$, locations $\hat{x}^1 = (1,0,0)$, $\hat{x}^2 = (0,1,0)$ and $\hat{x}^3 = (0,0,1)$, and linear reservation price functions $r^j(x) = 1 - t\left(\left|\hat{x}^j_1 - x_1\right| + \left|\hat{x}^j_2 - x_2\right| + \left|\hat{x}^j_3 - x_3\right|\right)$, where $t \in (0,1)$. We assume $t > 0$ to ensure market coverage. Then, the equilibrium personalized prices are given by*

$$p^1(x) = 1 - t(|1 - x_1| + x_2 + x_3) - \max\{1 - t(|1 - x_2| + x_1 + x_3), 1 - t(|1 - x_3| + x_1 + x_2)\}$$

and (the closure of) company 1's market area consists of the convex hull of location $\hat{x}^1$, $\left(\frac{1}{2}, \frac{1}{2}, 0\right)$, $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ and $\left(\frac{1}{2}, 0, \frac{1}{2}\right)$. The market areas for the other two companies are similar.

Square City

The following example imposes additional structure on X and the reservation price functions. It modifies Hotelling's model to a two-dimensional square city with heterogeneous adjustment costs that are quadratic. This model extends Larralde et al. (2009) and derives the boundary of market segments corresponding to the equilibrium in personalized prices.

Example 32 *Consider the modified Hotelling's model for a two-dimensional square city $X = [0,1]^2$ company A and B with locations $\hat{x}^A$ and $\hat{x}^B \neq \hat{x}^A$ in X and costless production $c^A = c^B = 0$. Adjustment costs are quadratic and integrated in the reservation price functions $r^i(x) = u^i - t^i\left((x_1 - \hat{x}_1^i)^2 + (x_2 - \hat{x}_2^i)^2\right)$, $i = A, B$. We assume $t^i < \frac{1}{2}u^i$ meaning both companies provide positive maximal consumer surplus to all consumers in X. Larralde et al. (2009) additionally assume $u^A = u^B = u$ and $t^A = t^B = t$. The boundary between A's and B's market segments is characterized by $r^A(x) - c^A = r^B(x) - c^B$. This equation specifies a central conic section that can be written in matrix form as*

$$\begin{bmatrix} x_1 & x_2 & 1 \end{bmatrix} \begin{bmatrix} t^B - t^A & 0 & t^A\hat{x}_1^A - t^B\hat{x}_1^B \\ 0 & t^B - t^A & t^A\hat{x}_2^A - t^B\hat{x}_2^B \\ t^A\hat{x}_1^A - t^B\hat{x}_1^B & t^A\hat{x}_2^A - t^B\hat{x}_2^B & u^A - u^B - t^A\left(\left(\hat{x}_1^A\right)^2 + \left(\hat{x}_2^A\right)^2\right) + t^B\left(\left(\hat{x}_1^B\right)^2 + \left(\hat{x}_2^B\right)^2\right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$$

see e.g. Ayoub (1993). There are two cases: homogeneous adjustment costs $t^A = t^B = t$ and heterogeneous adjustment costs $t^A \neq t^B$.

For homogeneous adjustment costs the quadratic terms cancel and we obtain

$$\left(\hat{x}_1^B - \hat{x}_1^A\right)x_1 + \left(\hat{x}_2^B - \hat{x}_2^A\right)x_2 = \tfrac{1}{2t}\left(u^A - u^B\right) + \tfrac{1}{2}\left[\left(\hat{x}_1^A\right)^2 + \left(\hat{x}_2^A\right)^2\right] - \tfrac{1}{2}\left[\left(\hat{x}_1^B\right)^2 + \left(\hat{x}_2^B\right)^2\right].$$

So, the boundary is a line segment in X. The slope $-\left(\hat{x}_1^B - \hat{x}_1^A\right)/\left(\hat{x}_2^B - \hat{x}_2^A\right)$ of this line segment depends upon the locations $\hat{x}^A$ and $\hat{x}^B$. How far the line segment is away from the origin $(0,0)$ depends upon the right-hand side and is determined by the difference in utilities at each locations and the difference in distances of locations from the origin. If we consider $\hat{x}^B$ as an origin, there are three different subcases corresponding to one of three quadrants defined by origin $\hat{x}^B$ where $\hat{x}^A$ can be located, namely the 2^{nd}, 3^{rd} $\left(\hat{x}^A < \hat{x}^B\right)$ and 4^{th} quadrant (the 1^{st} quadrant is a relabeling of the 3^{rd}). In the 2^{nd} and 4^{th} quadrant, the slope is negative and the line segment is upward sloping, while the slope is negative and the line segment downward sloping in the 3^{rd} quadrant. We forego investigating the intersection with X.

For heterogeneous adjustment costs the quadratic expression is a central conic section, see e.g. Ayoub (1993). It specifies an elipse in $\mathbb{R}^2$ with center

$$(c_1, c_2) = \left(\frac{t^A - t^B}{t^A\hat{x}_1^A - t^B\hat{x}_1^B}, \frac{t^A - t^B}{t^A\hat{x}_2^A - t^B\hat{x}_2^B}\right).$$

It can be rewritten as

$$\left(t^A - t^B\right)(x_1 - c_1)^2 + \left(t^A - t^B\right)(x_2 - c_2)^2 + \frac{\Delta}{\left(t^A - t^B\right)^2} = 0, \text{ or } (x_1 - c_1)^2 + (x_2 - c_2)^2 = \frac{-\Delta}{\left(t^A - t^B\right)^3},$$

where

$$\Delta = \left(t^B - t^A\right)\left[\left(t^B - t^A\right)\left(u^A - u^B - t^A\left(\left(\hat{x}_1^A\right)^2 + \left(\hat{x}_2^A\right)^2\right) + t^B\left(\left(\hat{x}_1^B\right)^2 + \left(\hat{x}_2^B\right)^2\right)\right) - \left(t^A\hat{x}_1^A - t^B\hat{x}_1^B\right)^2 - \left(t^A\hat{x}\right.\right.$$

is the determinant of the 3×3 *matrix. The simplified quadratic expression describes a circle with center* (c_1, c_2) *and the squared radius equal to the right-hand side. Of course, only the intersection of this circle with* X *matters.*

F.3 Nonmonotonicity

As a first extension to nonmonotonicty and to demonstrate the merits of our approach, we consider Hotelling's model with arbitrary locations, denoted as $\hat{x}^A$ and $\hat{x}^B$, and linear adjustment costs $t\,|\hat{x}^i - x|$ for $i = A, B$, see e.g. Hotelling (1929). The corresponding reservation price functions are piecewise-linear and single-peaked at the companies' locations. Hotelling's model is build upon two implicit assumptions, namely that the company-specific adjustment costs to buy at company i are *symmetric* around company i's most-valued consumer $\hat{x}^i$ and that these adjustment costs are also *homogeneous* across companies. We extend Hotelling's model to asymmetric company-specific linear adjustment costs to the left and right of each peak, and heterogeneity between companies with respect to these asymmetric linear adjustment costs. The special case of symmetry and homogeneity is reported as a corrollary.

Formally, consider functions r^A and r^B that are single-peaked with associated peaks at $\hat{x}^A \in [0,1]$ respectively $\hat{x}^B \in [0,1]$, where peaks represent locations and $\hat{x}^A \leq \hat{x}^B$.[39] The necessary and sufficient conditions for constrained monopoly with full market coverage become more demanding, namely the necessary and sufficient condition for e.g. such a monopoly of company A requires $g^A(x) > 0$ for all $x \in [0,1]$, i.e., the entire curve of the function $r^A(x) - c^A$ lies above the curve of $r^B(x) - c^B$ and the horizontal axis.[40] Translating this key insight in simple conditions, such as condition (A) under Assumption 1, is already daunting and requires additional structure. To see this, assume linearity of adjustment costs too, which implies the single-peaked functions are *piecewise linear*. Then, the necessary and sufficient conditions on $[0,1]$ require boundary condition (A) and (full coverage) $r^A(x) - c^A > 0$ for all $x \in [0,1]$ as in Proposition 4, and additionally $r^A\left(\hat{x}^B\right) - c^A > r^B\left(\hat{x}^B\right) - c^B$ at company B's location and a second boundary condition $r^A(1) - c^A > r^B(1) - c^B$, i.e., company A is able to offer more maximal consumer surplus than company B at B's most-valued consumer $\hat{x}^B$ and the consumer at $x = 1$. For single-peaked functions that lack piecewise linearity examples can be constructed to show that these conditions are insufficient and it is outside the scope of this study to explore this further. To summarize, the conditions for constrained monopoly become more demanding under single-peaked reservation prices than under Assumption 1.

[39] A single-peaked function is increasing for x to the left of the peak and it is decreasing for x to the right.
[40] Note that under monotonicty of Assumption 1 this reduces to the boundary condition (A).

The necessary and sufficient conditions for duopoly are the complement of the necessary and sufficient conditions for constrained monopoly. If the latter constraints are more demanding if monotonicity is dropped, then the necessary and sufficient conditions for duopoly are less restrictive than under Assumption 1. Therefore, duopoly with piecewise-linear reservation price functions is more common. Interestingly, a novel duopoly case emerges. Consider the above three conditions for a monopoly of company A but with $r^A\left(\hat{x}^B\right) - c^A < r^B\left(\hat{x}^B\right) - c^B$ instead. Then company A is able to offer higher maximal consumer surplus to consumers near both endpoints, while company B is able to offer higher maximal consumer surplus to consumers in the neighborhood of its most-valued consumer $\hat{x}^B$, i.e., $g^A(0), g^B\left(\hat{x}^B\right), g^A(1) > 0$ and under continuity these properties also hold in a nonempty neighborhood. Then Proposition 28 ensures a duopoly in which company A has disconnected market segments near the endpoints of the market and company B a market segment around its targeted consumer $\hat{x}^B$.

F.3.1 Piecewise-Linearity and Single-Peakedness: Constrained Monopoly

In this subsection we characterize the relative easier case of constrained monopoly in case of adjustment costs are linear, asymmetric around company i's most-valued consumer $\hat{x}^i$ and heterogeneous across companies. In the next subsection, we will use the necessary and sufficient conditions under which constrained monopoly emerges to provide guidance in the duopoly case.

Our main result characterizes constrained monopoly.

Proposition 33 *If the reservation price functions r^A and r^B are piecewise-linear and single-peaked with peaks $0 \leq \hat{x}^A \leq \hat{x}^B \leq 1$ and $\max\left\{r^A(x) - c^A, r^B(x) - c^B\right\} > 0$ for all $x \in [0,1]$ (market coverage), then under*

1. *(A), $r^A(0) - c^A > r^B(0) - c^B$ and $r^A\left(\hat{x}^B\right) - c^A > r^B\left(\hat{x}^B\right) - c^B$: A is constrained monopolist and personalized prices are given in Proposition 4.*
2. *(B), $r^A(1) - c^A > r^B(1) - c^B$ and $r^B\left(\hat{x}^A\right) - c^B > r^A\left(\hat{x}^A\right) - c^A$: B is constrained monopolist and personalized prices are given in Proposition 4.*

Moreover, social welfare at x is maximal.

Proof of Proposition 33

Suppose $\hat{x}^A \leq \hat{x}^B$ throughout this proof. By assumption, $g^0(x) < 0$ for all $x \in [0,1]$ and we may proceed with $g^A(x) = g(x)$. Note that A's graph of $r^A(x) - c^A$ consists of the increasing line piece with endpoints $\left(0, r^A(0) - c^A\right)$ and $\left(\hat{x}^A, r^A\left(\hat{x}^A\right) - c^A\right)$ on $\left[0, \hat{x}^A\right]$ and the decreasing line piece with endpoints $\left(\hat{x}^A, r^A\left(\hat{x}^A\right) - c^A\right)$ and $\left(1, r^A(1) - c^A\right)$ on $\left[\hat{x}^A, 1\right]$. Similarly, B's graph of $r^B(x) - c^B$ consists of the increasing line piece with endpoints $\left(0, r^B(0) - c^B\right)$ and $\left(\hat{x}^B, r^B\left(\hat{x}^B\right) - c^B\right)$ on $\left[0, \hat{x}^B\right]$ and the decreasing line piece with endpoints $\left(\hat{x}^B, r^B\left(\hat{x}^B\right) - c^B\right)$

and $\left(1, r^B(1) - c^B\right)$ on $\left[\hat{x}^B, 1\right]$. Therefore we distinguish the subintervals $\left[0, \hat{x}^A\right]$, $\left[\hat{x}^B, 1\right]$ and $\left[\hat{x}^A, \hat{x}^B\right]$, which is degenerated if $\hat{x}^A = \hat{x}^B$.
Recall that a constrained monopoly of A mplies $g(x) > 0$ for all $x \in [0, 1]$. Hence, a necessary condition for this to hold is that this inequality holds on any (finite) subset of $[0, 1]$ and we take $\left\{0, \hat{x}^B, 1\right\}$ and arrive at the inequalities as stated in the proposition. In order to show that these conditions are also sufficient, we first show the following result.

Claim 34 *If* $g(0), g\left(\hat{x}^B\right), g(1) > 0$*, then* $g(x) > 0$ *for* $x \in \left[\hat{x}^A, \hat{x}^B\right]$*, in particular* $g\left(\hat{x}^A\right) > 0$.

Proof of Claim
For $x \in \left[\hat{x}^A, \hat{x}^B\right]$, A's line piece with endpoints $\left(\hat{x}^B, r^A\left(\hat{x}^B\right) - c^A\right)$ and $\left(1, r^A(1) - c^A\right)$ is decreasing and we have $r^A(x) - c^A > r^A\left(\hat{x}^B\right) - c^B$ for $x \in \left(\hat{x}^A, \hat{x}^B\right)$. Similarly, B's line piece with endpoints $\left(0, r^B(0) - c^B\right)$ and $\left(\hat{x}^B, r^B\left(\hat{x}^B\right) - c^B\right)$ is increasing and we have $r^B\left(\hat{x}^B\right) - c^B > r^B(x) - c^B$ for $x \in \left(\hat{x}^A, \hat{x}^B\right)$. Combining these two inequalities with the necessary condition at $\hat{x}^B$ yields $r^A(x) - c^A > r^A\left(\hat{x}^B\right) - c^B > r^B(x) - c^B$ for $x \in \left[\hat{x}^A, \hat{x}^B\right]$, hence, $g(x) > 0$ for $x \in \left[\hat{x}^A, \hat{x}^B\right]$. This proves the claim.

Next, we consider the remaining intervals $\left[\hat{x}^B, 1\right]$ and $\left[0, \hat{x}^A\right]$ and show $g(x) > 0$ on these intervals.
Case $x \in \left[\hat{x}^B, 1\right]$: Because $g\left(\hat{x}^B\right), g(1) > 0$, it is straightforward to verify that A's decreasing line piece with endpoints $\left(\hat{x}^B, r^A\left(\hat{x}^B\right) - c^A\right)$ and $\left(1, r^A(1) - c^A\right)$ lies above B's decreasing line piece with endpoints $\left(\hat{x}^B, r^B\left(\hat{x}^B\right) - c^B\right)$ and $\left(1, r^B(1) - c^B\right)$, i.e., $g(x) > 0$ for all $x \in \left[\hat{x}^B, 1\right]$.
Case $x \in \left[0, \hat{x}^A\right]$: Because $g(0), g\left(\hat{x}^A\right) > 0$ (the claim), it is straightforward to verify that A's increasing line piece with endpoints $\left(0, r^A(0) - c^A\right)$ and $\left(\hat{x}^A, r^A\left(\hat{x}^A\right) - c^A\right)$ lies above B's increasing line piece with endpoints $\left(0, r^B(0) - c^B\right)$ and $\left(\hat{x}^A, r^B\left(\hat{x}^A\right) - c^B\right)$, i.e., $g(x) > 0$ for all $x \in \left[0, \hat{x}^A\right]$. Combining the claim and these two cases implies $g(x) > 0$ for all $x \in [0, 1]$. The equilibrium is characterized in Proposition 4. QED

Piecewise linearity and single-peakedness provide enough additional structure to ensure that checking e.g. $g(x) > 0$ at three particular consumers, namely B's most-valued consumer and the consumers located at the endpoints of the interval, suffices to ensure $g(x) > 0$ for all consumers in the market. The proof distinguishes the intervals $\left[0, \hat{x}^A\right]$, $\left[\hat{x}^A, \hat{x}^B\right]$ and $\left[\hat{x}^B, 1\right]$. On the outer intervals the line pieces describing reservation price functions are parallel. On the middle interval the line piece of company A is decreasing and the one for company B is increasing. Therefore, $g\left(\hat{x}^B\right) > 0$ at company B's most-valued consumer automatically implies $g\left(\hat{x}^A\right) > 0$ at company A's most-valued consumer. Because of this latter implication, the three conditions reduce to Condition (A) for the boundary case of most-valued consumers that are located at the endpoints, which we state as the following result.

Corollary 35 *If additionally company-specific adjustment costs are symmetric and homogeneous across companies, i.e.,* $d^i(x) = t\,|\hat{x}^i - x|$ *for* $i = A, B$*, then the three conditions for a constrained monopoly of* A *reduce to* (A)*.*

The three conditions for a constrained monopoly of company A include Condition (A) and all three conditions are necessary for such a monopoly. These conditions are therefore more demanding than Condition (A) for most-valued consumers located at endpoints and this latter condition is no longer the complement of the condition for duopoly, i.e. Condition (A). It also means that if Condition (A) holds while at least one of the other two conditions fails that we must end up in a duopoly with positive market segments for both comapanies, which is the topic of the next subsection.

F.3.2 Piecewise-Linearity and Single-Peakedness: Duopoly

The analysis of theprevious subsection implies that at least three additional subcases for duopoly arise, of which we only analyse one subcase in this subsection for reasons of brievety.[41] Our main result for duopoly is as follows.

Proposition 36 *If the reservation price functions* r^A *and* r^B *are piecewise-linear and single-peaked with peaks* $0 \le \hat{x}^A \le \hat{x}^B \le 1$ *and* $\max\left\{r^A(x) - c^A, r^B(x) - c^B\right\} > 0$ *for all* $x \in [0,1]$ *(market coverage), then under*

1. (D) : *there exists a unique* $x^* \in (0,1)$*, A's market segment is* $[0, x^*)$*, B's market segment is* $(x^*, 1]$ *and personalized prices are given in Proposition 2. Moreover,*

   $$\begin{array}{llll} i) & x^* \in \left(0, \hat{x}^A\right], & \text{if} & r^A\left(\hat{x}^A\right) - c^A \le r^B\left(\hat{x}^A\right) - c^B, \\ ii) & x^* \in \left(\hat{x}^A, \hat{x}^B\right), & \text{if} & r^A\left(\hat{x}^A\right) - c^A > r^B\left(\hat{x}^A\right) - c^B \text{ and } r^B\left(\hat{x}^B\right) - c^B > r^A\left(\hat{x}^B\right) - c^A, \\ iii) & x^* \in \left[\hat{x}^B, 1\right), & \text{if} & r^B\left(\hat{x}^B\right) - c^B \le r^A\left(\hat{x}^B\right) - c^A; \end{array}$$

2. $(A), r^A(0) - c^A > r^B(0) - c^B$ *and* $r^A\left(\hat{x}^B\right) - c^A < r^B\left(\hat{x}^B\right) - c^B$: *there exist* $x^L, x^U \in (0,1)$*, A's disconnected market segments are* $\left[0, x^L\right)$ *and* $\left(x^U, 1\right]$*, B's market segment is* $\left(x^L, x^U\right)$ *and personalized prices are given in Proposition 2. Moreover,*

   $$\begin{array}{lll} i) & \hat{x}^A < x^L < \hat{x}^B < x^U, & \text{if} \quad r^A\left(\hat{x}^A\right) - c^A > r^B\left(\hat{x}^A\right) - c^B, \\ ii) & x^L \le \hat{x}^A < \hat{x}^B < x^U, & \text{if} \quad r^A\left(\hat{x}^A\right) - c^A \le r^B\left(\hat{x}^A\right) - c^B. \end{array}$$

Moreover, social welfare at x *is maximal.*

Proof of Proposition 36

The first paragraph of the proof of Proposition 33 also applies here. Furthermore, the function g is continuous, because the piecewise linear functions r^A and r^B are continuous. This is important

[41] There are three subcases for (A) and we only analyze the subcase $g(0) > 0$ and $g\left(\hat{x}^B\right) < 0$ and forego reporting results for the subcase $g(0) < 0$ and $g\left(\hat{x}^B\right) > 0$ and the subcase $g(0) < 0$ and $g\left(\hat{x}^B\right) < 0$, which seem the least plausible.

for applying the Intermediate Value Theorem whenever appropriate. The proof consists of two main cases that are related to (A) and (D) that have several subcases.

Case (A), $r^A(0) - c^A > r^B(0) - c^B$ and $r^A(\hat{x}^B) - c^A < r^B(\hat{x}^B) - c^B$: These conditions translate into $g(0), g(1) > 0$ and $g(\hat{x}^B) < 0$. For $x \in [\hat{x}^B, 1]$, $g(\hat{x}^B) < 0$, $g(1) > 0$, continuity of g and the Intermediate Value Theorem imply the existence of an $x^U \in (\hat{x}^B, 1)$ such that $g(x^U) = 0$. Moreover, x^U is unique because the intersection of two non-parallel lines is unique. We cannot derive properties with respect to the sign of $g(\hat{x}^A)$ and we must distinguish the subcases $g(\hat{x}^A) \geq 0$ and $g(\hat{x}^A) \leq 0$.

Subcase $g(\hat{x}^A) \geq 0$, or $r^A(\hat{x}^A) - c^A \geq r^B(\hat{x}^A) - c^B$: For $x \in [\hat{x}^A, \hat{x}^B]$, $g(\hat{x}^A) \geq 0$ and $g(\hat{x}^B) < 0$, continuity of g and the Intermediate Value Theorem imply the existence of an $x^L \in [\hat{x}^A, \hat{x}^B)$ such that $g(x^L) = 0$. Moreover, x^L is unique because the intersection of two non-parallel lines is unique. For $x \in [0, \hat{x}^A]$, it is without loss of generality to continue with $g(\hat{x}^A) > 0$. Then, $g(0) > 0$ and $g(\hat{x}^A) > 0$ imply that A's increasing line piece with endpoints $(0, r^A(0) - c^A)$ and $(\hat{x}^A, r^A(\hat{x}^A) - c^A)$ lies above B's increasing line piece with endpoints $(0, r^B(0) - c^B)$ and $(\hat{x}^A, r^B(\hat{x}^A) - c^B)$, i.e., $g(x) > 0$ for all $x \in [0, \hat{x}^A]$. Combining all three subintervals implies the existence of an $x^L \in [\hat{x}^A, \hat{x}^B)$ and an $x^U \in (\hat{x}^B, 1)$ such that $g(x) > 0$ for all $x \in [0, x^L) \cup (0, x^U]$, which form A's disjoint market segments, and $g(x) < 0$ for all $x \in (x^L, x^U)$, which is B's connected market segment.

Subcase $g(\hat{x}^A) \leq 0$, or $r^A(\hat{x}^A) - c^A \leq r^B(\hat{x}^A) - c^B$: For $x \in [0, \hat{x}^A]$, similar arguments imply the existence of a unique $x^L \in (0, \hat{x}^A]$ such that $g(x^L) = 0$ and $g(x) < 0$ for all $x \in [\hat{x}^A, \hat{x}^B]$. Combining all three subintervals implies the existence of an $x^L \in (0, \hat{x}^A]$ and an $x^U \in (\hat{x}^B, 1)$ such that $g(x) > 0$ for all $x \in [0, x^L) \cup (0, x^U]$ and $g(x) < 0$ for all $x \in (x^L, x^U)$.

Next, we analyze condition (D). Then, $g(0) > 0$, $g(1) < 0$ and the Intermediate Value Theorem implies the existence of an $x^* \in (0, 1)$ such that $g(x^*) = 0$. We cannot derive properties with respect to the signs of $g(\hat{x}^A)$ and $g(\hat{x}^B)$. Therefore, we must distinguish four subcases.

Subcase $g(\hat{x}^A) > 0$ and $g(\hat{x}^B) < 0$: For $[\hat{x}^A, \hat{x}^B]$, repeating the previous argument yields existence of $x^* \in (\hat{x}^A, \hat{x}^B)$ such that $g(x^*) = 0$. Because it is an intersection of line pieces x^* is unique relative to $(\hat{x}^A, \hat{x}^B)$. For $[0, \hat{x}^A]$, Then, $g(0) > 0$ and $g(\hat{x}^A) > 0$ imply that A's increasing line piece lies above B's increasing line piece, i.e., $g(x) > 0$ for all $x \in [0, \hat{x}^A]$. For $[\hat{x}^B, 1]$, $g(\hat{x}^B) < 0$ and $g(1) < 0$ imply that A's decreasing line piece lies below B's decreasing line piece, i.e., $g(x) < 0$ for all $x \in [\hat{x}^B, 1]$. Combining these results implies there exists a unique $x^* \in (\hat{x}^A, \hat{x}^B)$ such that $g(x) > 0$ for all $x \in [0, x^*)$ and $g(x) < 0$ for all $x \in (x^*, 1]$.

Subcase $g(\hat{x}^A) \leq 0$ and $g(\hat{x}^B) < 0$: For $[0, \hat{x}^A]$, $g(0) > 0$, $g(\hat{x}^A) \leq 0$ and the Intermediate Value Theorem imply the existence of an $x^* \in (0, \hat{x}^A]$ such that $g(x^*) = 0$, which is unique relative to $[0, \hat{x}^A]$. For $[\hat{x}^A, \hat{x}^B]$, similar arguments as in the proof of Proposition 33 applied to $g(\hat{x}^A) \leq 0$, r^A decreasing and r^B increasing imply that $g(x) < 0$ for all $[\hat{x}^A, \hat{x}^B]$. For $[\hat{x}^B, 1]$, similar arguments as before imply $g(x) < 0$ for all $x \in [\hat{x}^B, 1]$. Combining these results implies there exists a unique $x^* \in [0, \hat{x}^A]$ such that $g(x) > 0$ for all $x \in [0, x^*)$ and $g(x) < 0$ for all $x \in (x^*, 1]$.

Subcase $g(\hat{x}^A) > 0$ and $g(\hat{x}^B) \geq 0$: Is the mirror image of the previous case.

Subcase $g(\hat{x}^A) \leq 0$ and $g(\hat{x}^B) \geq 0$: Impossible, because for $x \in [\hat{x}^A, \hat{x}^B]$, similar arguments as before applied to $g(\hat{x}^A) \leq 0$, r^A decreasing and r^B increasing imply that $g(x) < 0$ for all $(\hat{x}^A, \hat{x}^B]$. QED

Piecewise linearity and single-peakedness provide additional structure but still allow a plurality of subcases. Condition (D) ensures the existence of a unique boundary that separates two market shares, but this boundary can belong to either one of the intervals $[0, \hat{x}^A]$, $[\hat{x}^A, \hat{x}^B]$ and $[\hat{x}^B, 1]$. Only if we additionally assume that each company is able to attract its most-valued consumer we obtain that the boundary of market shares lies between these two consumers, which is case $ii)$ of condition (D). Next, condition (A) and two other conditions allow for two boundaries on company B's connected market segment, called x^L and x^U. Company B's market segment separates two market segments of company A. One of the conditions states that company B is able to attract its most-valued consumer $\hat{x}^B$ and B's market segement surrounds this consumer, i.e. $x^L < \hat{x}^B < x^U$. The lower bound can belong to either $[0, \hat{x}^A]$, or $[\hat{x}^A, \hat{x}^B]$. Only if we additionally assume that also company A is able to attract its most-valued consumer we obtain that the lower boundary of B's market share lies between these two consumers, which is case $i)$ of the conditions containing (A). The assumption that each company is able to attract its most-valued consumer seems plausible and realistic. However, our analysis hints at a possible explosion of cases when the Hotelling's model is extended to asymmetric company-specific adjustment costs and heterogeneity between companies beyond linear adjustment costs.

Next, we consider the special case of most-valued consumers that are located at the endpoints. From the constrained monopoly case we know that in that case condition (A) is the necessary and sufficient condition for a constrained monopoly of company A. Hence, for duopoly it rules out the case that has condition (A) as one of its three conditions and only the duopoly case corresponding to condition (D) is valid for this special case, which we state as the following result.

Corollary 37 *If additionally company-specific adjustment costs are symmetric and homogeneous across companies, i.e., $d^i(x) = t|\hat{x}^i - x|$ for $i = A, B$, then the conditions for a duopoly reduce to Condition (D) with a unique $x^* \in (\hat{x}^A, \hat{x}^B)$.*

We conclude this subsection with an example that illustrates a situation with asymmetric company-specific adjustment costs that are heterogeneous across companies in which the order of market segments is A, B, A.

Example 38 *Consider the situation in which company A sells a product or service that all consumers more or less value the same, i.e., negligible adjustment costs, say taking public transport* *to the airport and company B sells a luxury product, say a shuttle service to the airport, that targets a certain group of consumers who value this service higher but also have substantial symmetric linear adjustment costs. Suppose* $\hat{x}^A \in \left(0, \frac{1}{2}\right)$, $\hat{x}^B = \frac{1}{2}$, *negligible asymmetric adjustment costs can be modeled as* $r^A(0) - c^A = r^A(1) - c^A = 1$ *and* $r^A\left(\hat{x}^A\right) - c^A = 1 + \varepsilon$, *where* $\varepsilon > 0$ *is sufficiently small, and* $r^B(x) - c^B = 2 - 4\left|\frac{1}{2} - x\right|$. *For explanatory reasons, we only characterize the border case* $\varepsilon = 0$. *Application of Proposition 28 implies*

$$g^A(x) = -1 + 4\left|\tfrac{1}{2} - x\right|, \qquad g^B(x) = 1 - 4\left|\tfrac{1}{2} - x\right| \qquad \textit{and } g^0(x) < 0.$$

Then, $g^B(x) > 0$ *for* $x \in \left(\frac{1}{4}, \frac{3}{4}\right)$ *and* $g^A(x) > 0$ *whenever either* $x \in \left[0, \frac{1}{4}\right)$ *or* $x \in \left(\frac{3}{4}, 1\right]$. *The equilibrium personalized prices are given by*

$$p^A(x) = \begin{cases} c^A + 4\left|\frac{1}{2} - x\right| - 1, & \textit{if } x \in \left[0, \frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right], \\ c^A, & \textit{if } x \in \left(\frac{1}{4}, \frac{3}{4}\right), \end{cases}$$

$$p^B(x) = \begin{cases} c^B + 1 - 4\left|\frac{1}{2} - x\right|, & \textit{if } x \in \left[0, \frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right], \\ c^B, & \textit{if } x \in \left(\frac{1}{4}, \frac{3}{4}\right). \end{cases}$$

Obviously, the order of market segments is A, B, A *with company A selling to two disconnected market segments. For* $\varepsilon > 0$, *the qualitative insights remain valid and only the boundaries and price functions require quantitative adjustments.*

Note that this example is robust in the sense that minor perturbations of the function values for A at $x = 0$ *and* $x = 1$, *B's location* $\hat{x}^B$, *the utility* u^B *offered at the peak and not-too-asymmetric adjustment costs adjustments also preserve the qualitative insights.*