TI 2022-004/VIII
Tinbergen Institute Discussion Paper

Autonomous cars and activity-based bottleneck model: How do in-vehicle activities determine aggregate travel patterns?

Xiaojuan Yu[1,2]
Vincent A.C. van den Berg[2,3]
Erik T. Verhoef[2,3]

1 Zhongnan University of Economics and Law

2 Vrije Universiteit Amsterdam

3 Tinbergen Institute

Autonomous cars and activity-based bottleneck model: How do in-vehicle activities determine aggregate travel patterns?

Xiaojuan Yu[a,b*], Vincent A. C. van den Berg[b,c], Erik T. Verhoef[b,c]

[a] School of Business Administration, Zhongnan University of Economics and Law, Wuhan 430073, China

[b] Department of Spatial Economics, VU Amsterdam, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands

[c] Tinbergen Institute, Gustav Mahlerplein 117, 1082 MS Amsterdam, The Netherlands

Abstract

When traveling in an autonomous car, the travel time can be used for performing activities other than driving. This paper distinguishes users' work-related and home-related activities in autonomous cars and proposes an activity-based bottleneck model to investigate travelers' behavior in the morning commute, shedding light on how the scope to undertake in-vehicle activities affects travelers' trip-timing preferences and decisions, and therewith social welfare. These welfare effects can be expected to depend on the optimality of both the market for trips, and the market for vehicles. We therefore consider different supply regimes for automobiles, and un-priced congestion versus queue-eliminating road pricing. We reveal analytically the relationship between users' various in-vehicle activities and trip timing choices by autonomous and normal car users. Three supply regimes for autonomous cars are investigated: welfare-maximizing public supply, competitive marginal cost supply, and profit-maximizing private supply. Pricing rules under different supply regimes are compared analytically, and the relative efficiencies in terms of the welfare gains are compared numerically. Results show that travelers' in-vehicle activity choices have significant impacts on the travel patterns, congestion externality, supply decisions and the associated welfare effects.

Keywords: Activity based modelling; Autonomous cars; Bottleneck model; Private vs public supply; Traffic congestion

---

* Corresponding author.
Email addresses: x.yu@vu.nl (Xiaojuan Yu), v.a.c.vandenberg@vu.nl (Vincent van den Berg), e.t.verhoef@vu.nl (Erik Verhoef).

1. Introduction

Autonomous cars are vehicles that can drive themselves without human operation.[1] The emergence of autonomous cars is expected to provide an alternative way to alleviate the losses caused by traffic congestion. In particular, autonomous cars allow users to use their travel time more productively. Autonomous car users can perform actions other than driving; for example, work activities, such as processing mail or writing documents, and behaviors associated with being at home, such as relaxing or leisure (Haboucha et al., 2017; Pudāne et al., 2018; Correia et al., 2019). Unlike the pure deadweight loss for normal car users, travel time in autonomous cars could become a closer substitute for time spent at home and/or at the workplace. This raises an interesting and important issue: how do commuters' in-vehicle activity choices (leisure or work) affect their trip-timing preferences and travel decisions? Research on such behavioral impacts of autonomous driving is, however, still in its infancy.

In the literature, studies investigating in-vehicle activities in autonomous cars can be classified into two categories: the static time allocation model (Pudāne et al., 2018; Fosgerau, 2019; Tscharaktschiew and Evangelinos, 2019) and dynamic scheduling model (Van den Berg and Verhoef, 2016; Fosgerau, 2019; Tian et al., 2019; Pudāne, 2020; Zhang et al., 2020). The static time allocation model regards time as a quantity, and thus ignores the timing of trips. The dynamic scheduling model, which was first introduced by Vickrey (1969), has been recognized as an important tool for analyzing the issues related to trip timing. Several studies have adopted the dynamic scheduling model to investigate the morning commute problem with autonomous cars (e.g., Van den Berg and Verhoef, 2016; Liu, 2018; Tian et al., 2019; Su and Wang, 2020). In this paper, we also employ a dynamic scheduling model, as we focus on the interrelated trip timing decisions for autonomous and normal car users in the morning peak period, and analyze the resulting congestion effects and welfare effects.

Existing dynamic congestion models investigating autonomous car users' in-vehicle activities mainly build on Vickrey's (1969) bottleneck model, in which the benefits of performing activities in the vehicle are modeled as a reduction in the value of time (VOT) (Van den Berg and Verhoef, 2016; Tian et al., 2019; Zhang et al., 2020), or a change in the value of scheduling delay early and late (Pudāne, 2020). The main assumption behind this approach is that users' value of travel time and the value of schedule delay are constants, i.e., the marginal cost of travel time and the marginal costs of arriving early and late at work are,

---

[1] The Society of Automotive Engineers (SAE) defines 6 levels of driving automation ranging from Level 0 (fully manual) to Level 5 (fully autonomous). https://www.synopsys.com/automotive/autonomous-driving-levels.html. This paper mainly focuses on autonomous cars with full automation.

respectively, assumed to be constants α, β, and γ. Such scheduling preferences are referred to as $\alpha-\beta-\gamma$ preferences (e.g., Knockaert et al., 2016).

However, some empirical studies have confirmed that individuals' scheduling preferences vary strongly over the morning peak. In particular, the values of schedule delay and values of travel time are strongly time-dependent (see, e.g., Ettema and Timmermans, 1997, 2003; Liu et al., 2007; Tseng and Verhoef, 2008). In this regard, bottleneck models with time-varying marginal utilities have been presented in the literature to investigate commuters' travel choices (Vickrey, 1973; Fosgerau and de Palma, 2012; Fosgerau and Lindsey, 2013; Li et al., 2014), the value of travel time variability (Tseng and Verhoef, 2008; Fosgerau and Engelson, 2011; Börjesson et al., 2012), and congestion charging issues (Li et al., 2017; Li and Zhang, 2020). Such time-varying scheduling models are referred to as activity-based bottleneck models in Li et al. (2014, 2017). Nevertheless, these studies mainly involve auto-only or auto-bus systems, and the marginal utility in the vehicle is normalized to zero since there is no distinction between different in-vehicle activities.

As travelers are allowed to perform various productive tasks when traveling in an autonomous car, these vehicles are expected to reduce the marginal disutility caused by travel time. Nevertheless, the extent to which this marginal disutility will change is currently far from clear. Several studies have used stated choice experiments to estimate the effects of travelers' in-vehicle activities on their preferences from the perspective of the value of travel time savings. According to Steck et al. (2018), the value of travel time savings of an autonomous car is 31% less than that of a normal car. Correia et al. (2019) observed that people working in autonomous vehicles have a lower value of travel time saving compared to those in normal cars; however, there is no difference in travel time spent by people on leisure between an autonomous car and a normal car. Correia et al. (2019) further used the static time allocation model to explain this phenomenon. There is still little if any experience from either field trials or laboratory experiments of how in-vehicle utility varies by activity and time of day. This study is the first quantitative approach taken to analyze the relationship between in-vehicle activities and travelers' trip-timing preferences and decisions under time-varying scheduling preferences and dynamic congestion.

Against this background, we propose an activity-based two-mode model to investigate the relationship between users' various in-vehicle activities and travel patterns under time-varying scheduling preferences and dynamic congestion. Before making their travel decision, all travelers are assumed to be ex-ante identical. The proposed model distinguishes home-related and work-related in-vehicle activities, and allows different in-vehicle activities to yield different marginal utilities. As a result of the variability of the marginal utility of activities over time, autonomous and normal car users not only show heterogeneity in scheduling utilities, but also in their preferred arrival times. This heterogeneity is endogenous in the model, due to travelers choosing endogenously between a normal and an autonomous car. To the best of our knowledge, endogenous heterogeneity in preferred arrival times between autonomous and normal car users has received very little attention so far. Studies accounting for time-varying marginal activity utilities usually assume that all travelers are homogeneous. Only Li et al. (2017) considered a linear time-varying marginal utility function to investigate the step-tolling problem with heterogeneous users. In their setting, however, the utility in the vehicle is normalized to zero and there is no behavior, such as the purchase of an autonomous vehicle, which changes a user's marginal utilities.

Our central focus is to establish how and to what extent the scope to undertake different types of in-vehicle activities affects trip-timing preferences and decisions. It is important also to understand whether, and to what extent, the answers will be context specific. Two important factors in that respect are whether or not there are congestion policies in place, and under which market conditions the supply of autonomous cars will occur. These two factors are crucial because they are both important determinants of the prevailing market prices in the two (inter-related) markets of interest: the market for vehicles, and the market for trips that can be made with those vehicles. We consider the potential importance of these two context variables by looking at conceptual endpoints on the imaginary scales that can be envisaged: for trips, the two benchmarks of unpriced congestion versus optimally priced congestion; and for vehicles, purely competitive supply versus monopolistic private, and public supply. Considering only these stylized extremes allows us to explore the potential relevance of these context variables, while maintaining focus on the main question that we seek to address by keeping the number of cases to consider limited.

This paper sheds light on the travel equilibrium in a bottleneck system with autonomous and normal cars and, considers various in-vehicle activity choices. Autonomous and normal car users present heterogeneity in their preferred arrival times, due to the productive use of travel time in autonomous cars. Different possible types of equilibrium patterns of travelers may arise; for example, in terms of the order of traveling. Although the clear distinction in various discretely different travel patterns is a natural consequence of a deterministic modeling approach, the patterns will be strongly indicative of the temporal clustering of traveler types that we may also expect to observe in reality. The same will be true for the efficiency and welfare measures that we can derive. It is found that in the absence of road pricing, autonomous and normal car users may travel simultaneously or sequentially. With respect to the congestion effects, the marginal external benefit of switching from a normal to an autonomous car may be positive or negative.

This paper also brings new theoretical and practical insights to road pricing. Charging drivers for the delay or congestion they cause is a well-known concept among economists and traffic engineers (Pigou, 1920; Walters, 1961; Vickrey, 1969). As is well established in the literature, queuing is a pure loss with bottleneck congestion. Time-varying tolling removes queuing, and users arrive in the order of the value of schedule delay (Arnott et al., 1987; Lindsey, 2004; Van den Berg and Verhoef, 2011a, b). It is widely recognized that heterogeneous travelers may exhibit large behavioral differences in departure time choices during peak hours and in response to congestion policies (Arnott et al., 1987, 1994; Lindsey, 2004; Van den Berg and Verhoef, 2011a, b; Li et al., 2017; Silva et al., 2017). Considering time-varying scheduling preferences and endogenous heterogeneity, we find that the adoption of autonomous cars and the time-varying marginal utilities obtained from performing various in-vehicle activities will induce autonomous and normal car users to arrive sequentially or alternatingly[2]. The associated welfare depends heavily on the equilibrium patterns and is also significantly impacted. These results differ from those of Van den Berg and Verhoef (2016) and Fosgerau (2019), in which road pricing does not affect travelers' traveling orders and all users arrive simultaneously.

This paper also aims to evaluate the welfare effects of different car supply regimes. We compare three supply markets of autonomous cars: perfect competition, private monopoly, and public supply. To this end, we build a zero-profit marginal cost supply model, a profit-maximizing model, and a welfare-maximizing model; with the quantities supplied and the prices charged as decision variables. The introduction of autonomous cars may attract individuals who would not travel with only normal cars, and this results in an increase in travel demand which in turn may worsen congestion. Therefore, it is necessary to consider price-sensitive travel demand. The interaction between road pricing and car supply regimes is explicitly investigated, and the pricing rule under different combinations of road pricing and car supply schemes are obtained and compared analytically.

This study is different from the existing studies in the following major aspects. First, we adopt a variant of the Vickrey (1973) bottleneck model of the morning commute, in which individuals derive time-of-day varying utilities from being at home and being at work. By distinguishing travelers' home-related and work-related activities in autonomous cars, we

[2] Here, "alternatingly" means that autonomous cars travel first, then normal cars, and finally autonomous cars again.

allow the various in-vehicle activities to produce different marginal utilities, which are both time-varying. Our model produces heterogeneity in general scheduling preferences and preferred arrival times between autonomous and normal car users. Compared to the pre-existing heterogeneity in previous studies that considered the morning commute problem with multi-class users (see e.g., Arnott et al., 1987, 1994; Lindsey, 2004; Van den Berg and Verhoef, 2011a, b;), heterogeneity in this paper is endogenous and follows from the choice between autonomous cars and normal cars. Second, travel equilibrium and the associated welfare effects under road pricing are also explored; and, as explained, we consider three different car supply regimes. These extensive differences allow us to gain new insights into travelers' trip-timing decisions and the resulting congestion effects and welfare effects, for a bi-modal bottleneck system with autonomous cars. Table 1 below summarizes the differences between earlier models and highlights our contribution to the literature on dynamic traffic models with autonomous cars.

Table 1. Contributions to the dynamic congestion models with autonomous cars.

Citations	Scheduling preference	Activity choices	Heterogeneity	Mode choice	Car supply	Road pricing	Elastic demand
Van den Berg and Verhoef (2016)	$\alpha-\beta-\gamma$	×	α	√	√	×	×
Fosgerau (2019)	Time-varying	√	α	×	×	×	×
Zhang et al. (2020)	$\alpha-\beta-\gamma$	×	α	√	×	√	×
Pudāne (2020)	$\alpha-\beta-\gamma$	√	α,β,γ	×	×	×	×
This paper	Time-varying	√	$h[t], v[t], w[t], t^*$	√	√	√	√

The remainder of the paper is organized as follows. Section 2 proposes an activity-based bottleneck model to investigate travelers' travel equilibrium in the absence of road pricing, considering users' various in-vehicle activity choices in autonomous cars. Section 3 compares three supply regimes of autonomous cars: welfare-maximizing public supply, competitive marginal cost supply, and profit-maximizing private monopolistic supply. Section 4 investigates the interaction between queue-eliminating road pricing and car supply regimes. Numerical illustrations are presented in section 5 and section 6 concludes the paper.

2. Activity-based bottleneck model with autonomous cars

2.1 Assumptions and notation

Table 2. Nomenclature.

$h[t]$	Marginal utility of being at home at time t
$w[t]$	Marginal utility of being at work at time t
$v_H[t]$	Marginal utility of performing home-related activities in autonomous cars at time t
$v_W[t]$	Marginal utility of performing work-related activities in autonomous cars at time t
θ_H	Parameter determining the marginal utility of in-vehicle home activities
θ_W	Parameter determining the marginal utility of in-vehicle work activities
$H^i[t]$ $(i=a,n)$	Willingness to pay to spend a unit of time at home rather than in the vehicle at time t
$W^i[t]$ $(i=a,n)$	Willingness to pay to spend a unit of time at work rather than in the vehicle at time t
t_D	Departure time of a traveler from home
t_A	Arrival time of a traveler at the workplace
T_{ff}	Free flow travel time
t^{i*}	Preferred arrival time for car type i users, $i=a,n$
$T[t]$	Travel delay at arrival time t
$c^i[t_D,t_A]$	Travel cost for someone who departs at t_D and arrives at t_A, $i=a,n$
$c^i[t]$	Travel cost for arriving at t with car type i, $i=a,n$
NT	In the absence of road pricing
$c^i_{NT}[N^a,N^n]$	Equilibrium travel cost for car type i users without road pricing, $i=a,n$
N^i	The number of users with car type i, $i=a,n$
MC^i	Marginal automobile cost of car type i, $i=a,n$
MU^a	Mark-up on autonomous cars
$D[N^a+N^n]$	Inverse demand function
MEC^i	Marginal external cost imposed by car type i users, $i=a,n$
MEB	Marginal external benefit incurred by switching to an autonomous car
t_s, t_e	Starting and ending time of the queue (toll)
t_{an}, t_{na}	Intersection of the travel delay (toll) curves for different car type users
SW	Social welfare
Π	Profit of the private monopolist
CP	With queue-eliminating congestion pricing
$p^i_{CP}[t]$	Travel price for car type i users at arrival time t with road pricing, $i=a,n$
$\tau[t]$	Toll at arrival time t
$p^i_{CP}[N^a,N^n]$	Equilibrium travel price for car type i users with road pricing, $i=a,n$
t^i_s, t^i_e	Starting and ending time for the traveling in the out-of-equilibrium continuation of the iso-price functions
$\overline{\tau^i}$	Average toll for car type i users, $i=a,n$
ω	Relative efficiency

We begin our exposition by listing our assumptions and introducing our notation. We consider a morning commuting schedule that consists of three activities (i.e., being at home, being in the vehicle and being at work) and one trip (i.e., the journey from home to work). Every morning, travelers travel from home to a workplace connected by a single road that is subject to bottleneck congestion. Everybody travels either by autonomous car or in a normal car. We ignore other transport modes, such as public transport and biking. Travelers can gain utility from being at home, being in the vehicle and being at the workplace. All travelers are ex-ante identical. They choose arrival times and select a travel mode (automated vs manual driving) to minimize the generalized price.

This paper focuses on the effects of travelers' various in-vehicle activities and does not consider the potential increase in road capacity when autonomous cars are introduced (see, e.g., Wadud et al., 2016; Van den Berg and Verhoef, 2016; Liu and Song, 2019; Wang et al., 2021). In our modeling approach, although the increase in road capacity will shorten the duration of the travel periods and thus bring an external benefit, it does not impact travelers' traveling orders. Omitting such capacity effects thus allows us to focus on travelers' trip-timing preferences and travel decisions. The incorporation of the capacity effects can be made in a future model extension.

This section builds an activity-based bottleneck model to investigate the relationship between the travel times of autonomous and normal car users, and the associated congestion effects, taking users' in-vehicle activity choices into account. Next section will investigate users' mode choices and car suppliers' supply decisions under different supply regimes of autonomous cars. For ease of reference, Table 2 above summarizes the notation. The notation will also be introduced in the text.

2.2 Model set-up

We build our model on Vickrey (1973) and Tseng and Verhoef (2008), extending their modelling to a case in which travelers' behavior in autonomous cars may affect the marginal utilities of spending time in the vehicle compared to being at home or at work at the same moment. We focus on the equilibrium travel patterns with mixed autonomous and normal cars.

In the morning, commuters can gain utility from being at home, being in the vehicle and being at work. Let $h[t]$ denote the marginal utility of being at home at time t, and $w[t]$ denote the marginal utility of being at work at time t. Each of the marginal utility functions are assumed to be continuous and smooth. The marginal utility of being in a normal car is normalized to zero. People adopting an autonomous car can spend time in the vehicle on activities other than driving, such as relaxing as they do at home or on professional activities such as those they perform at work. We call these in-vehicle home activities and in-vehicle work activities, respectively.

In order to reflect diverse substitution possibilities of time spent in an autonomous car, we allow for different in-vehicle activities to generate different marginal utilities for different activities. Although several studies have used experiments or survey data to estimate the value of travel time savings with an autonomous car (Steck et al., 2018; Correia et al., 2019), the evidence for how in-vehicle utility varies by activity and time of day is still limited. A natural assumption to make is that engaging in a home-related activity yields a fraction of the utility that would be gained from being at home at that moment. Similarly, engaging in a work-related activity yields another fraction (possibly different) of the utility that would be gained from being at work, again at the same moment. That is, the marginal utility of in-vehicle home activities, $v_H[t]$, is a proportion of $h[t]$, and the marginal utility of in-vehicle work activities, $v_W[t]$, is a proportion of $w[t]$:

$$v_H[t] = (1-\theta_H)\cdot h[t],\ v_W[t] = (1-\theta_W)\cdot w[t], \tag{1}$$

where θ_H and θ_W are assumed to be constants between 0 and 1. This assumption has also been adopted by Fosgerau (2019) and Pudāne (2020). Lower values of θ_H and θ_W mean that autonomous car users obtain higher marginal utilities from the associated in-vehicle activities.

Definition of in-vehicle activity choices. At any given time *t*, if performing home-related activities in autonomous cars benefits users more, i.e., $v_H[t] > v_W[t]$, they will choose home-related in-vehicle activity; otherwise, if $v_H[t] < v_W[t]$, they will choose work-related in-vehicle activity.

As we only consider three possible locations for the individual, travel behavior is determined entirely by differences in utility level. For users with car type *i* (*i* = *n, a*), let $H^i[t]$ denote the willingness to pay to spend a unit of time at home rather than in the vehicle at time *t*, and $W^i[t]$ denote the same for being at work versus in the vehicle. Hence, we obtain:

$$\begin{aligned} &H^n[t] = h[t],\ H^a[t] = h[t] - v_H[t] = \theta_H \cdot H^n[t], \\ &W^n[t] = w[t],\ W^a[t] = w[t] - v_w[t] = \theta_W \cdot W^n[t], \end{aligned} \tag{2}$$

where superscript '*n*' indicates a normal, manually driven car, and '*a*' indicates an autonomous car. Eq. (2) implies that a switch to an autonomous car shifts $H^n[t]$ downwards by a proportion of $(1-\theta_H)$, and $W^n[t]$ downwards by a proportion of $(1-\theta_W)$.

To motivate a trip from home to work, assume the marginal utility of spending time at home is non-increasing over time in the morning period, and the marginal utility of work is non-decreasing over time. This assumption has been justified by previous empirical studies (see, for example, Tseng and Verhoef, 2008; Börjesson et al., 2012), and was adopted in studies by Fosgerau and Engelson (2011), Fosgerau and Lindsey (2013) and Hjorth et al. (2015). Under this assumption, $H^i[t]$ is non-increasing and $W^i[t]$ is non-decreasing over the peak. At some moment in time, the value of $W^i[t]$ will equal the value of $H^i[t]$ to justify going to work at all. Let t_D denote the departure time of a traveler from home and t_A the arrival time of the traveler at the workplace. The free-flow travel time, which is the time needed from home to work without congestion, is denoted as T_{ff}. For presentation purpose, we introduce below the definition of preferred arrival time.

Definition of preferred arrival time. With a given free-flow travel time, T_{ff}, individuals with car type i would prefer to depart at moment t_D and arrive at moment t_A such that $H^i[t_D]=W^i[t_A]$ and $t_A=t_D+T_{ff}$. We use t^{i*} to denote the intersection of $H^i[t-T_{ff}]$ and $W^i[t]$, and call this the preferred arrival time.[3]

The relationship of the preferred arrival times between autonomous and normal car users is presented in Proposition 1.

Proposition 1. If $\theta_H<\theta_W$, $t^{a*}<t^{n*}$; if $\theta_W<\theta_H$, $t^{a*}>t^{n*}$; if $\theta_H=\theta_W$, $t^{a*}=t^{n*}$.

Proof. According to the above definition, the preferred arrival time is determined by $H^i[t^{i*}-T_{ff}]=W^i[t^{i*}]$. For normal car users, $h[t^{n*}-T_{ff}]=w[t^{n*}]$ is satisfied, and for autonomous car users, $\theta_H\cdot h[t^{a*}-T_{ff}]=\theta_W\cdot w[t^{a*}]$ is satisfied. Note that $h[t]$ is non-increasing over time and $w[t]$ is non-decreasing. At t^{n*}, when $\theta_H<\theta_W$, we have $\theta_H\cdot h[t^{n*}-T_{ff}]=\theta_H\cdot w[t^{n*}]<\theta_w\cdot w[t^{n*}]$. This means that at t^{n*}, autonomous car users will stay at the workplace, since the marginal willingness to pay for being at workplace exceeds

[3] Even though in reality there are official work start times, people routinely deviate from that. The essence of this issue is the trade-off between the willingness to pay for being at home and the willing to pay for being at the workplace (or the trade-off between the queueing time cost and scheduling delay cost). When the work time is subject to certain restrictions, the value of schedule delay become higher, but the essence of the modelling work remains the same.

their willingness to pay for being at home. Hence, $t^{a*} < t^{n*}$ holds. Similarly, when $\theta_H > \theta_W$, $\theta_H \cdot h[t^{n*} - T_{ff}] = \theta_H \cdot w[t^{n*} - T_{ff}] > \theta_W \cdot w[t^{n*}]$ holds. This indicates that autonomous car users will choose to stay at home at t^{n*}. Hence, $t^{a*} > t^{n*}$ holds. When $\theta_H = \theta_W$, all travelers have the same preferred arrival time, i.e., $t^{a*} = t^{n*}$.

Remark 1. Proposition 1 reveals that when the substitution possibilities of performing home- versus work- based activities in an autonomous car are perceived to be different, its adoption will lead to a shift in the preferred arrival time. Pudāne (2020, Chapter 3) derived, simultaneously and independently of us, a similar result for a model in the case of no congestion.[4] This property is important, since the heterogeneity in preferred arrival time has significant impacts on travel decisions, as we will see in Section 2.4. Fig. 1 illustrates the case under $\theta_W < \theta_H$ with $t^{a*} > t^{n*}$.

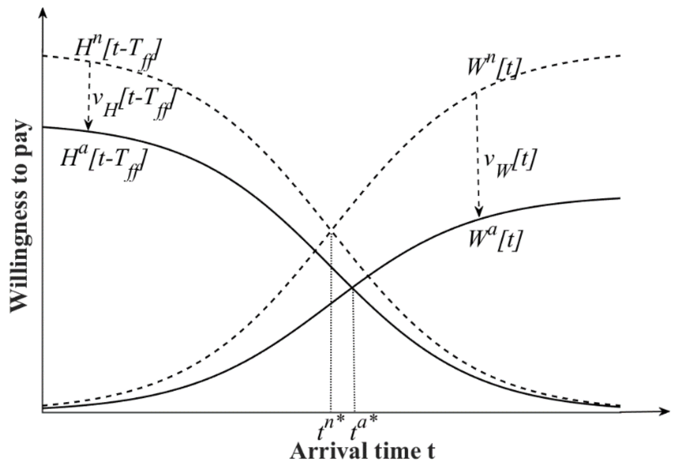


Fig. 1. Preferred arrival times under $\theta_W < \theta_H$.

The travel cost for travelers who depart at t_D and arrive at t_A involves the travel time cost and the schedule delay cost. According to Tseng and Verhoef (2008), the travel time cost involves travel delay cost and free-flow travel cost, which are jointly the integral of $H^i[t]$ between t_D and t_A. The schedule delay cost is the integral of $H^i[t] - W^i[t]$ between t_A and t^{i*} for arriving before t^{i*}, and the integral of $W^i[t] - H^i[t]$ between t^{i*} and t_A for arriving after t^{i*}. Let $T[t]$ denote the travel delay at the bottleneck for someone who arrives at t, which equals the queue length when joining the queue divided by the bottleneck capacity s. The travel cost for someone who departs at t_D and arrives at t_A, with $t_A = t_D + T[t_A] + T_{ff}$,

[4] But for dynamic congestion, Pudāne (2020) only considered $\alpha - \beta - \gamma$ preferences, in which autonomous car users' in-vehicle activities do not affect their preferred arrival time.

is (in line with Tseng and Verhoef, 2008):

$$c^i[t_D, t_A] = \int_{t_D}^{t_A} H^i[t]dt + \begin{cases} \int_{t_A}^{t^{i*}} (H^i[t] - W^i[t])dt, & if\ t_A < t^{i*} \\ \int_{t^{i*}}^{t_A} (W^i[t] - H^i[t])dt, & if\ t^{i*} < t_A \end{cases} \quad i = a, n. \tag{3}$$

Consequently, for any arrival time *t*, the travel cost with car type *i*, $c^i[t]$, can be further expressed as:

$$c^i[t] = \int_{t-T[t]-T_{ff}}^{t} H^i[t]dt + \begin{cases} \int_{t}^{t^{i*}} (H^i[t] - W^i[t])dt, & if\ t < t^{i*} \\ \int_{t^{i*}}^{t} (W^i[t] - H^i[t])dt, & if\ t^{i*} < t \end{cases} \quad i = a, n. \tag{4}$$

2.3 Joint travel time and travel mode choice equilibrium

We first investigate the no-toll case in which the road is not priced. We use 'NT' to denote this case. Section 4 will consider queue-eliminating road pricing.

In the dynamic equilibrium, travel costs for travelers with the same car type need to be constant over the arrival times used by these users, and can be no lower at other times. In equilibrium, we can, therefore, omit the arrival time indicator and write marginal costs and benefits as functions of the numbers of autonomous and normal cars. It should be noted that the specific expressions of the equilibrium travel cost differ in different cases. We use an implicit form, $c^i_{NT}[N^a, N^n]$, to denote collectively the equilibrium travel cost of users with car type i, where subscript '*NT*' represents no road pricing, N^a is the number of autonomous cars, and N^n is the number of normal cars.

The generalized price per trip is the sum of the travel cost, automobile cost and mark-up on the car (over the marginal cost). The automobile cost captures the marginal production cost of cars, which is assumed to be constant. The mark-up refers to the value that a supplier adds to the marginal production cost, so that the sum of mark-up and marginal cost equals the supply price. In this paper, the mark-up is determined by the car suppliers and is expressed as a per-trip equivalent. Normal cars are assumed to be provided under perfectly competitive conditions, and we normalize the automobile cost and mark-up for normal cars to zero. For autonomous cars, we use MC^a to denote the automobile cost and MU^a the mark-up per trip. MC^a and MU^a, therefore, reflect by how much the marginal production cost and mark-up of autonomous cars exceed those for normal cars; this difference may be negative, zero or positive.

We assume that users regard autonomous cars and normal cars as perfect substitutes, given that differences in the valuation of time in vehicles are already explicitly valued in our model. Hence, there is one aggregate inverse demand function, $D[N^a + N^n]$. In the mode choice equilibrium, travelers have no incentive to change their mode choice. That is, the generalized price of the chosen mode is at the minimum of the generalized prices available, and the generalized price of an unused mode is not below that minimum. As we consider price-sensitive demand, the inverse demand must be no higher than the generalized price for both vehicle types. The equilibrium can thus be mathematically expressed as:

$$\begin{cases} N^a \cdot \left(D[N^a + N^n] - (c_{NT}^a[N^a, N^n] + MC^a + MU^a) \right) = 0 \\ N^n \cdot \left(D[N^a + N^n] - c_{NT}^n[N^a, N^n] \right) = 0 \\ D[N^a + N^n] - (c_{NT}^a[N^a, N^n] + MC^a + MU^a) \le 0 \\ D[N^a + N^n] - c_{NT}^n[N^a, N^n] \le 0 \\ N^a \ge 0,\ N^n \ge 0 \end{cases} \quad . \tag{5}$$

Normal cars and autonomous cars are both used only if the equation system in (5) has an interior solution for N^n and N^a. Since the two vehicle types are assumed to be perfect substitutes, given MC^a and MU^a, functions $c_{NT}^n[N^a, N^n]$ and $c_{NT}^a[N^a, N^n]$ must vary with N^n and N^a in a suitable way to get $N^n > 0$ and $N^a > 0$. In particular, c_{NT}^a and c_{NT}^n have different responses to marginal changes of N^n and N^a. Overall, if the benefits from in-vehicle activities exceed the additional purchase cost, $MC^a + MU^a$, all users will choose autonomous cars. If these benefits fall short of $MC^a + MU^a$, all users will choose normal cars. For these corner solutions, the travel patterns reduce to those found in the regular bottleneck model with homogeneous users, and are therefore not interesting for further exploration in the current paper. We will therefore focus on interior solutions in the following analytical exposition, while the numerical analysis that follows will take the possibility of corner solutions into account.

2.4 Properties of the equilibrium travel patterns

In the following, we analyze the properties of the equilibrium; for example, in terms of the arrival intervals of autonomous and normal car users.

In equilibrium, the travel cost for a type should be constant over time as long as arrivals for that type occur. By taking the derivative of (4) to t, and solving $dc^i[t]/dt = 0$, we find:

$$\frac{dT[t]}{dt} = \begin{cases} 1 - \dfrac{W^n[t]}{H^n[t - T[t] - T_{ff}]}, & \textit{with a normal car} \\ 1 - \dfrac{\theta_W}{\theta_H} \cdot \dfrac{W^n[t]}{H^n[t - T[t] - T_{ff}]}, & \textit{with an autonomous car} \end{cases} . \tag{6}$$

Eq. (6) shows that when normal car users pass the bottleneck, the travel delay changes over the arrival time at a rate of $1 - W^n[t] / H^n[t - T[t] - T_{ff}]$, and when autonomous car users pass the bottleneck, the travel delay changes over the arrival time at a rate of $1 - \theta_W W^n[t] / \left(\theta_H \cdot H^n[t - T[t] - T_{ff}]\right)$.

According to Eq. (6), interior solutions for the mode choice equilibrium are more likely to happen when θ_H and θ_W differ more strongly. Indeed, when θ_H and θ_W are relatively close, the preferred arrival times, t^{a*} and t^{n*}, also become closer. Therefore, $c_{NT}^n[N^a, N^n]$ and $c_{NT}^a[N^a, N^n]$ have more similar responses to changes in N^a and N^n, which weakens the differences in impacts of N^a and N^n upon $c_{NT}^n[N^a, N^n]$ and $c_{NT}^a[N^a, N^n]$. Corner solutions are thus more likely to happen. For instance, when $\theta_H = \theta_W < 1$, due to the same preferred arrival times between autonomous and normal car users and the coinciding iso-cost curves as per Eq. (6), $c_{NT}^a[N^a, N^n] < c_{NT}^n[N^a, N^n]$ holds for any positive N^a and N^n. If then $c_{NT}^a[N^a, N^n] + MC^a + MU^a < c_{NT}^n[N^a, N^n]$, all users will choose autonomous cars. As stated, we will not consider these corner solutions in the analytical part of the paper as the analysis matches that of the standard bottleneck model. In the following, we focus on the travel patterns when autonomous and normal car users are both present.

Eq. (6) indicates that when $\theta_H \neq \theta_W$, $dT[t]/dt$ differs over car types. To keep the no-toll equilibrium, autonomous and normal car users are separated temporally. More specifically, when $\theta_H < \theta_W$, $1 - \dfrac{W^n[t]}{H^n[t - T[t] - T_{ff}]} \geq 1 - \dfrac{\theta_W}{\theta_H} \cdot \dfrac{W^n[t]}{H^n[t - T[t] - T_{ff}]}$ always holds. This means that the travel delay grows faster or shrinks more slowly for normal cars than for autonomous cars, because normal car users need a steeper increase (or flatter decrease after t^{a*}) to maintain the equilibrium. Specifically, if θ_H and θ_W are very different, $dT^n[t]/dt > 0$ and $dT^a[t]/dt < 0$ may happen. This means that the queue of autonomous cars has begun to dissipate, whereas normal cars are still building up the queue. Combining $t^{a*} < t^{n*}$ (see

Proposition 1), we can find that autonomous car users will travel first. By contrast, when $\theta_H > \theta_W$, $1-\frac{W^n[t]}{H^n[t-T[t]-T_{ff}]} \le 1-\frac{\theta_W}{\theta_H}\cdot\frac{W^n[t]}{H^n[t-T[t]-T_{ff}]}$ always holds, meaning that the travel delay grows more slowly or shrinks more quickly for normal car users. Combining $t^{a*} > t^{n*}$, we can find that normal car users will travel first. We summarize this property in Proposition 2 and Proposition 3.[5]

The proof of Proposition 2 is given in Appendix A. The derivations of the equilibrium arrival intervals and the associated travel costs involve a rather tedious and cumbersome process, and are thus provided in Appendix B.

Proposition 2. In the absence of road pricing, autonomous car and normal car users will never travel alternatingly[6]; that is, both groups have only a single arrival time window so that the group that travels first will not also travel last.

Proposition 3. In the absence of road pricing, autonomous car and normal car users may travel simultaneously or sequentially, depending on the relative values of θ_H and θ_W.

(i) Case 0: If $\theta_H = \theta_W$, all users will travel simultaneously.

(ii) Case 1: If $\theta_H < \theta_W$, autonomous car users will travel first.

(iii) Case 2: If $\theta_H > \theta_W$, normal car users will travel first.

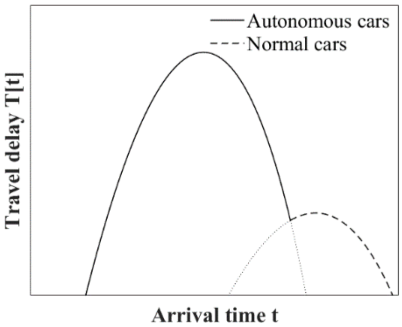
Remark 2. The equilibrium travel orders in Proposition 3 are different from the existing studies, such as Van den Berg and Verhoef (2016), Fosgerau (2019), Tian et al. (2019), and Pudāne (2020), in which autonomous cars travel in the center of the peak period and normal cars travel in the shoulders. The reason is that our model considers the variability of travelers' travel preferences over time, and thus in-vehicle activities will lead to heterogeneity in the preferred arrival time (see Proposition 1), which was ignored in the existing studies. Hence, it is of great significance to consider time-varying marginal activity utilities when investigating travelers' various activities in autonomous cars.

---

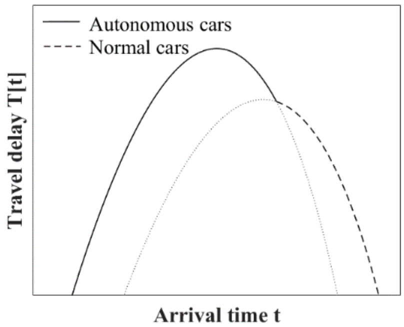
[5] Note that in this paper θ_H and θ_W are assumed to be constants. If we relax this assumption and consider time-varying θ_H and θ_W, autonomous car users may travel in the center and normal car users travel in the shoulders of the peak period. This travel pattern has been studied in Van den Berg and Verhoef (2016).

[6] We use the word "alternatingly" to mean that one type travels first, then the second, and finally the first type again.

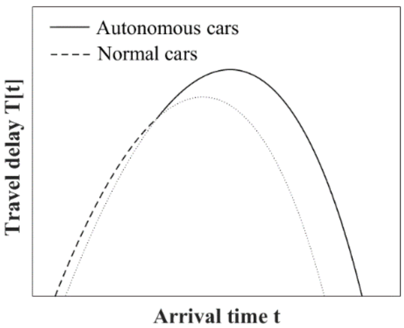
In what follows, we maintain the definition of cases given in Proposition 3. Fig. 2 illustrates these different types of equilibrium.[7] As stated, the travel delay in Fig. 2 can be interpreted as iso-cost curves in the absence of road pricing, with travel delay plotted along the vertical axis and arrival time along the horizontal axis. The upper envelope gives the equilibrium cost line. The dotted curves thus denote an out-of-equilibrium continuation of the iso-cost functions for the corresponding type. These indicate what the travel times would have to be, but are not, for users to be willing to travel at those moments outside their own equilibrium time window.



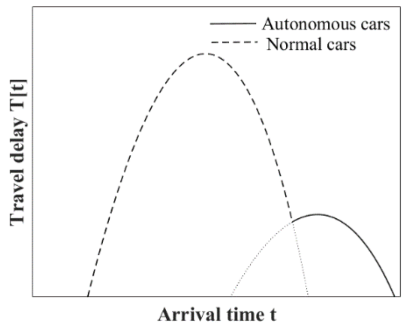
(a) Case 1 (double peaks)



(b) Case 1 (one peak)



(c) Case 2 (one peak)



(d) Case 2 (double peaks)

Fig. 2. Possible equilibrium travel patterns without road pricing.

Note: Case 1 happens when $\theta_H < \theta_W$, and case 2 happens when $\theta_H > \theta_W$. The number of peaks is determined by the share of autonomous cars and the difference between t^{a*} and t^{n*}.

The heterogeneity in preferred arrival times may lead to double peaks in the travel delay curves. More specifically, when $\theta_W < \theta_H$, normal car users travel first, due to $t^{a*} > t^{n*}$. As

[7] Taking the second derivative of Eq. (6) with respect to t yields $d^2T/dt^2 < 0$, implying that the travel delay function is concave. We ignore the less interesting case in which preferred arrival times for autonomous and normal car users are so markedly different that the two travel delay curves do not intersect.

θ_W increases, the two travel delay curves go from one intersection with double peaks, to one intersection with one peak. When θ_W exceeds θ_H, autonomous car users travel first. The two curves now go from one intersection with one peak, to one intersection with double peaks. Specially, if $\theta_W = \theta_H$, all uses will travel simultaneously, which can be easily obtained from Eq.(6). The number of peaks (i.e. one or two) is determined by the share of autonomous cars and by the relative size of θ_H and θ_W. If performing in-vehicle home and in-vehicle work activities yields markedly different marginal utilities, the preferred arrival times for autonomous and normal car users are also markedly different and the travel delay curves will have double peaks, as illustrated in Fig. 2a and Fig. 2d.

2.5 Properties of the congestion effects

In this bi-modal bottleneck system, with autonomous cars and normal cars interacting, all users will impose congestion externalities on the others. In line with Lindsey (2004), the congestion effects are defined as the derivative of type *i*'s generalized cost to the number of type *j* users. By taking the derivative of $c_{NT}^i[N^a, N^n]$ $(i = a, n)$ with respect to N^a and N^n, we find the own-congestion effects exceed the cross-congestion effects: travelers tend to impose higher external costs on travelers of their own type than on travelers with different preferences. We summarize this important property in Proposition 4. The proof of Proposition 4 is given in Appendix C.

Proposition 4. When $\theta_H \neq \theta_W$, $\frac{\partial c_{NT}^a}{\partial N^a} > \frac{\partial c_{NT}^a}{\partial N^n}$ and $\frac{\partial c_{NT}^n}{\partial N^n} > \frac{\partial c_{NT}^n}{\partial N^a}$.

Proposition 4 indicates that the travel cost is more sensitive to the number of own-type users. Indeed, when $\theta_H < \theta_W$, autonomous cars travel first. Increasing the number of cars will advance the starting time of the queue, and more so for increasing the number of autonomous cars, due to the earlier preferred arrival time. Hence, c_{NT}^a is more sensitive to N^a. Similarly, increasing the number of cars will also postpone the ending time of the queue, and more so for increasing the number of normal cars. Hence, c_{NT}^n is more sensitive to N^n. The same logic applies when $\theta_H > \theta_W$.

The marginal external cost imposed by users with car type *i*, MEC^i, is the sum of the congestion effects imposed on all other travelers, which can be expressed as:

$$MEC^i = N^a \cdot \frac{\partial c_{NT}^a[N^a, N^n]}{\partial N^i} + N^n \cdot \frac{\partial c_{NT}^n[N^a, N^n]}{\partial N^i}, \tag{7}$$

where the first term is the marginal external cost on autonomous car users, and the second term is the marginal external cost on normal car users. Taking the difference between the marginal external costs imposed by normal and autonomous cars, we obtain what we will refer to as the marginal external benefit of a normal car user switching to autonomous driving, *MEB*:

$$MEB = MEC^n - MEC^a. \tag{8}$$

Proposition 5 reveals the properties of the marginal external benefit. The proof is shown in Appendix D.

Proposition 5. For the bi-modal bottleneck system with time-varying marginal activity utilities, the marginal external benefit of switching from a normal to an autonomous car can be positive or negative, depending on the marginal utilities for different activities and the number of autonomous cars.

(i) When $\theta_H < \theta_W$,

$$MEB = \frac{(\theta_W - \theta_H) W^n[t_{an}] \cdot \left(N^a \cdot (\theta_W \cdot W^n[t_s] - \theta_H \cdot H^n[t_s - T_{ff}]) + N^n \cdot (W^n[t_e] - H^n[t_e - T_{ff}])\right)}{\theta_H \cdot \left(H^n[t_s - T_{ff}] - H^n[t_e - T_{ff}]\right) + \theta_W \cdot \left(W^n[t_{an}] - W^n[t_s]\right) + \theta_H \cdot \left(W^n[t_e] - W^n[t_{an}]\right)} \frac{1}{s},$$

$$\frac{\partial MEB}{\partial N^a} < 0, \quad \frac{\partial MEB}{\partial N^n} > 0.$$

(ii) When $\theta_H > \theta_W$,

$$MEB = \frac{(\theta_W - \theta_H) \cdot W^n[t_{na}] \cdot \left(N^a \cdot (\theta_W \cdot W^n[t_e] - \theta_H \cdot H^n[t_e - T_{ff}]) + N^n \cdot (W^n[t_s] - H^n[t_s - T_{ff}])\right)}{\theta_H \cdot \left(H^n[t_s - T_{ff}] - H^n[t_e - T_{ff}]\right) + \theta_H \cdot \left(W^n[t_{na}] - W^n[t_s]\right) + \theta_W \cdot \left(W^n[t_e] - W^n[t_{t_{na}}]\right)} \cdot \frac{1}{s},$$

$$\frac{\partial MEB}{\partial N^a} < 0, \quad \frac{\partial MEB}{\partial N^n} > 0.$$

(iii)When $\theta_H = \theta_W$, $MEB = 0$.

Proposition 5 reveals that increasing the number of autonomous cars will lower the marginal external benefit of switching from a normal to an autonomous car. This is intuitively because travelers tend to impose higher external costs on travelers of their own type than travelers with different preferences. Two limiting cases can be considered. First, when $N^n << N^a$, the overall marginal external cost caused by autonomous cars exceeds the marginal external cost caused by normal cars, and the marginal external benefit of switching from a normal to an autonomous car is negative. Second, when $N^n >> N^a$, the marginal external benefit is positive, since the overall marginal external cost caused by normal cars is higher than the overall marginal external cost caused by autonomous cars.

Remark 3. The observation in Proposition 5 is different from the 'negative MEB' found by Van den Berg and Verhoef (2016), which states that the marginal external benefit of switching from a normal to an autonomous car is always negative, regardless of the share of autonomous cars. The reason is that in Van den Berg and Verhoef (2016), the choice for an autonomous car lowers that traveler's VOT, and does not have an impact on the value of schedule delay. To maintain equilibrium, autonomous car users travel in the center of the peak period and normal car users travel in the shoulders. For the remaining normal car users, another driver's switch from a normal car to an autonomous vehicle leaves their travel cost unchanged. For autonomous car drivers, the own-congestion effect exceeds the cross-congestion effect, so that the switch will lead to an increase in user cost. Hence, the marginal external benefit of switching from a normal to an autonomous car is, in the absence of a capacity effect, always negative in that model. However, in our setting, in-vehicle activities not only affect the marginal utilities of time losses, but also the marginal utilities of schedule delay. As a result of heterogeneous scheduling preferences, own-congestion effects exceed the cross-congestion effects for both autonomous and normal car users. Hence, the number of autonomous cars and normal cars also plays an important role in deciding the marginal external benefit, and it may become positive even in the absence of a favorable capacity effect of autonomous driving. This implies that ignoring the various in-vehicle activities and time-varying scheduling preferences may overestimate the congestion effects of autonomous cars. Thus, the regulation of autonomous cars is needed in order to achieve optimally control of traffic congestion and improve welfare. For instance, the regulator could subsidize autonomous car users for the positive externality, and tax them for the negative externality.

3. Supply regimes of autonomous cars

Switching from a normal to an autonomous car may bring a positive or negative externality, but travelers are typically assumed to ignore the externalities they impose on others. This section investigates car suppliers' pricing decisions under three supply regimes of autonomous cars: welfare-maximizing public supply, competitive marginal cost supply, and private monopolistic supply. Normal cars are supplied under perfect competition and the mark-up on them is thus zero. We consider price-sensitive demand, such that if autonomous cars lower costs, demand and congestion will increase.

The decision variables of car suppliers include the number of autonomous cars N^a, the number of normal cars N^n, and the mark-up on autonomous cars MU^a. We are interested in suppliers' pricing decisions on autonomous cars, and the welfare effects under different supply regimes, compared to the social optimum. As road congestion is un-priced, the welfare-maximizing public pricing is second-best. In the next section, we will consider queue-eliminating road pricing and the associated car supply decisions.

3.1 Welfare-maximizing public supply of autonomous cars

A social planner aims to maximize social welfare by setting the mark-up on autonomous cars, hence affecting users' mode choices. Social welfare, SW, is defined as the total consumer benefit minus the travel costs and minus the automobile cost. When both vehicle types are used, i.e. $N^i > 0$, the welfare-maximizing problem can be mathematically expressed as:

$$\begin{aligned} &\max_{N^a,N^n,MU^a} \quad SW = \int_0^{N^a+N^n} D[n]dn - c_{NT}^a[N^a,N^n]\cdot N^a - c_{NT}^n[N^a,N^n]\cdot N^n - MC^a \cdot N^a \\ &s.t. \quad \begin{cases} D[N^a+N^n] = c_{NT}^a[N^a,N^n] + MC^a + MU^a \\ D[N^a+N^n] = c_{NT}^n[N^a,N^n] \end{cases} \end{aligned} \quad . \tag{9}$$

Solving the Lagrangian associated with Eq. (9) yields the following interior solutions of the mark-up:

$$MU^a = \frac{\partial c_{NT}^a}{\partial N^a}N^a + \frac{\partial c_{NT}^n}{\partial N^a}N^n - \left(\frac{-D' + \frac{\partial c_{NT}^n}{\partial N^a}}{-D' + \frac{\partial c_{NT}^n}{\partial N^n}} \right) \cdot \left(\frac{\partial c_{NT}^a}{\partial N^n}N^a + \frac{\partial c_{NT}^n}{\partial N^n}N^n \right). \tag{10}$$

The derivation of the pricing rule in Eq. (10) is provided in Appendix E. The first two terms in Eq. (10) are the marginal external costs imposed by autonomous car users. The last term corrects for the marginal cost of congestion caused via the substitution between autonomous and normal cars. This correction equals the marginal external costs imposed by normal cars, multiplied by a fraction $(-D' + \frac{\partial c_{NT}^n}{\partial N^a}) \Big/ (-D' + \frac{\partial c_{NT}^n}{\partial N^n})$.

Proposition 6. Under all travel pattern cases, the second-best mark-up on autonomous cars can be collectively expressed as

$$MU^a = MEC^a - \Delta N^n \big/ \Delta N^a \cdot MEC^n , \qquad (11)$$

with the last term measuring the substitution effects between autonomous and normal cars, and $\Delta N^n \big/ \Delta N^a \leq 1$.

Proof. The numerator term of the fraction $(-D' + \partial c_{NT}^n \big/ \partial N^a)$ is indeed the change in the constraint for normal cars due to a marginal change in N^a, whereas $(-D' + \partial c_{NT}^n \big/ \partial N^n)$ is the change in the constraint for normal cars due to a marginal change in N^n. The ratio $(-D' + \frac{\partial c_{NT}^n}{\partial N^a}) \Big/ (-D' + \frac{\partial c_{NT}^n}{\partial N^n})$ therefore gives $\Delta N^n \big/ \Delta N^a$, which can be interpreted as the change in the number of normal cars induced by a change in the number of autonomous cars. According to Proposition 4, $\frac{\partial c_{NT}^n}{\partial N^n} > \frac{\partial c_{NT}^n}{\partial N^a}$ holds. Hence, $(-D' + \frac{\partial c_{NT}^n}{\partial N^a}) \Big/ (-D' + \frac{\partial c_{NT}^n}{\partial N^n}) < 1$. The entire mark-up can thus be rewritten as Eq. (11), with the last term measuring the substitution effects, which corrects for the marginal external costs of congestion caused by substitution to normal cars when a mark-up is levied on autonomous cars.

Remark 4. Combining the properties of the congestion effects and the marginal external benefit in Proposition 4 and Proposition 5, we find that the second-best mark-up on autonomous cars may be positive or negative. Specifically, if the overall demand is perfectly inelastic, i.e., $\Delta N^n \big/ \Delta N^a = 1$, the mark-up on autonomous cars becomes $MU^a = MEC^a - MEC^n = -MEB$. The negative value here means the regulator needs to subsidize autonomous car users, so a negative tax, when they cause a positive externality.

3.2 Zero-profit marginal cost supply of autonomous cars

Perfect competition leads to supply against marginal and average cost, and thus a zero mark-up. Given the inverse demand function and marginal utility functions, solving Eq. (5) yields the solutions of the equilibrium numbers of autonomous and normal cars. Comparing the zero mark-up with the second-best mark-up in Eq. (10), the relationship between marginal cost supply and second-best public supply can be obtained, as presented in the following proposition.

Proposition 7. Compared to second-best public supply, marginal cost supply may lead to the under-supply or over-consumption of autonomous cars.

Remark 5. The above result differs from Van den Berg and Verhoef (2016), in which marginal cost supply always leads to over-consumption of autonomous cars when only considering the reduction in the VOT and ignoring the capacity effects. The reason is that in their setting, autonomous cars travel in the center of the peak period; switching from a normal car to an autonomous car thus raises the externality on autonomous car users themselves and does not affect normal car users. Therefore, the marginal external benefit is always negative. In our setting, the mark-up under a second-best public supply may be positive or negative.

3.3 Private monopolistic supply of autonomous cars

The private monopolist maximizes its profit, Π, which equals the mark-up per trip multiplied by the number of autonomous cars. The profit maximization problem is:

$$\max_{N^a, N^n, MU^a} \quad \Pi = MU^a \cdot N^a$$
$$s.t. \quad \begin{cases} D[N^a + N^n] = c_{NT}^a[N^a, N^n] + MC^a + MU^a \cdot \\ D[N^a + N^n] = c_{NT}^n[N^a, N^n] \end{cases} \tag{12}$$

Solving the Lagrangian associated with Eq. (12) yields:

$$MU^a = -N^a \cdot \left(D' - \partial c_{NT}^a / \partial N^a\right) + \frac{N^a \cdot \left(D' - \partial c_{NT}^a / \partial N^n\right)}{\left(D' - \partial c_{NT}^n / \partial N^n\right)} \cdot \left(D' - \partial c_{NT}^n / \partial N^a\right), \tag{13}$$

which can be further rewritten as:

$$MU^a = \left(\frac{\partial c_{NT}^a}{\partial N^a} - \frac{\partial c_{NT}^n}{\partial N^a}\right) N^a - \left(\frac{-D' + \frac{\partial c_{NT}^n}{\partial N^a}}{-D' + \frac{\partial c_{NT}^n}{\partial N^n}}\right)\left(\frac{\partial c_{NT}^a}{\partial N^n} - \frac{\partial c_{NT}^n}{\partial N^n}\right) N^a, \tag{14}$$

$$= \left(\frac{\partial c_{NT}^a}{\partial N^a} - \frac{\partial c_{NT}^n}{\partial N^a}\right) N^a - \frac{\Delta N^n}{\Delta N^a}\left(\frac{\partial c_{NT}^a}{\partial N^n} - \frac{\partial c_{NT}^n}{\partial N^n}\right) N^a. \tag{15}$$

The first term in Eq. (14) represents the change in the marginal willingness to pay for autonomous cars, caused by a change in the number of autonomous car users themselves. The last term corrects in order to include the change in marginal willingness to pay caused via the substitution between autonomous and normal cars, when an MU^a is levied on autonomous cars. This correction equals the marginal willingness to pay caused by a change in the number of normal cars, multiplied by a fraction that represents the change in the number of normal cars following a change in the number of autonomous cars. As normal cars are provided under perfect competition and the mark-up on them is zero, this fraction has the same expression as that for welfare-maximizing public supply and again gives $\Delta N^n / \Delta N^a$, as expressed in Eq. (15).

Proposition 8. In the absence of road pricing, under all equilibrium travel pattern cases, the private monopolistic mark-up on autonomous cars can be collectively expressed as:

$$MU^a = MEC^a - \frac{\Delta N^n}{\Delta N^a} MEC^n - \left(\frac{\partial c_{NT}^n}{\partial N^a} - \frac{\Delta N^n}{\Delta N^a}\frac{\partial c_{NT}^n}{\partial N^n}\right)(N^a + N^n), \tag{16}$$

with the second term measuring the substitution effects between autonomous cars and normal cars, and the last term measuring the private monopoly power.

4. Queue-eliminating road pricing

As is well established in the literature, queuing is a pure loss with bottleneck congestion, and can be removed with time-varying tolling (Arnott et al., 1987). This section will investigate the interaction between queue-eliminating road pricing and car supply regimes.

We first look at users' travel behavior under queue-eliminating road pricing. Since all queuing is eliminated, the travel delay $T[t]$ is zero. The travel price at arrival time t is the sum of the free-flow travel cost, the schedule delay cost, and the time-varying toll $\tau[t]$. The schedule delay cost is, as before, the integral of $H^i[t]-W^i[t]$ between t and t^{i*} for arrival before t^{i*}, and the integral of $W^i[t]-H^i[t]$ between t^{i*} and t when arriving after t^{i*}.

Let subscript '*CP*' denote the time-varying road pricing regime. At any arrival time, t, the travel prices, $p_{CP}^i[t]$, are:

$$\begin{aligned} p_{CP}^n[t] &= \int_{t-T_{ff}}^{t^{n*}} H^n[t]dt + \int_{t^{n*}}^{t} W^n[t]dt + \tau[t], \quad \textit{with a normal car}, \\ p_{CP}^a[t] &= \theta_H \cdot \int_{t-T_{ff}}^{t^{a*}} H^n[t]dt + \theta_W \cdot \int_{t^{a*}}^{t} W^a[t]dt + \tau[t], \quad \textit{with an autonomous car} \end{aligned} \quad . \tag{17}$$

4.1 Properties of the equilibrium travel patterns

In equilibrium, travel prices for all users are constant over time as long as arrivals occur. Taking the derivative of Eq. (17) with respect to t yields:

$$\frac{d\tau[t]}{dt}=\begin{cases}H^n[t-T_{ff}]-W^n[t], & \textit{with a normal car},\\ \theta_H H^n[t-T_{ff}]-\theta_W W^n[t], & \textit{with an autonomous car}.\end{cases} \tag{18}$$

Eq. (18) implies that when normal car users pass the bottleneck, the toll must change over the arrival time at a rate of $H^n[t-T_{ff}]-W^n[t]$, and when autonomous car users pass the bottleneck, the toll must change over the arrival time at a rate of $\theta_H H^n[t-T_{ff}]-\theta_W W^n[t]$ to decentralize the optimum.

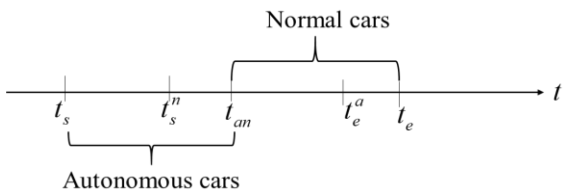
The equilibrium travel orders are similar to those in the absence of road pricing. However, it is now noteworthy that when θ_W is close to θ_H, autonomous car users travel in the shoulders of the peak period and normal car users travel in the center. Indeed, as θ_W approaches θ_H, the preferred arrival times for autonomous and normal car users becomes closer, but autonomous car users prefer a lower toll since they can reduce the cost of scheduling delay by performing tasks in the vehicle. As a consequence, there exist three possible relationships between the arrival intervals of autonomous and normal car users. We present this important finding in Proposition 9 and the arrival intervals and time points are defined in Fig. 3. Where t_s and t_e denote the starting and ending time of tolling, t_s^i and t_e^i denote the starting and ending time for the traveling in the out-of-equilibrium continuation of the iso-price functions, and t_{an} and t_{na} denote the transition times when the first type stops traveling and the second type starts traveling. The proof of Proposition 9 is given in Appendix F.

Proposition 9. With queue-eliminating road pricing, autonomous and normal car users may travel sequentially or alternatingly. Three possible equilibrium travel patterns may arise.

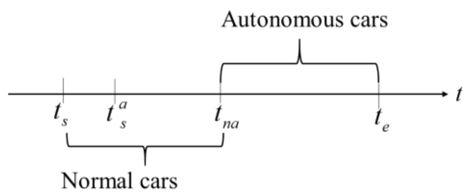
(i) Case 1: If $\dfrac{H^n[t_{an}-T_{ff}]}{W^n[t_{an}]}>\dfrac{(1-\theta_W)}{(1-\theta_H)}$ and $t_e>t_e^a$, autonomous car users will travel first.

(ii) Case 2: If $\dfrac{H^n[t_{na}-T_{ff}]}{W^n[t_{na}]}<\dfrac{(1-\theta_W)}{(1-\theta_H)}$ and $t_s<t_s^a$, normal car users will travel first.

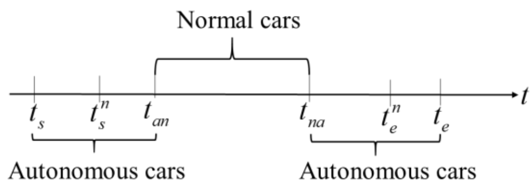
(iii) Case 3: If $\frac{H^n[t_{an}-T_{ff}]}{W^n[t_{an}]}>\frac{(1-\theta_W)}{(1-\theta_H)}$, $\frac{H^n[t_{na}-T_{ff}]}{W^n[t_{na}]}<\frac{(1-\theta_W)}{(1-\theta_H)}$, $t_s<t_s^n$ and $t_e>t_e^n$, normal car users will travel in the center and autonomous car users will travel in the shoulders of the peak period.



(a) Case 1



(b) Case 2



(c) Case 3

Fig. 3. Relationship between arrival intervals of autonomous and normal cars.

Note: The conditions for each case to happen are summarized in Proposition 9.

Remark 6. Proposition 9 suggests that under queue-eliminating road pricing, in-vehicle activities still play important roles in determining the aggregate travel patterns. More specifically, when θ_W is markedly lower than θ_H, normal car users travel first and there are double toll peaks. As θ_W increases, the two toll curves go from one intersection with double peaks, to one intersection with one peak. When θ_W approaches θ_H, the two toll curves have two intersections, in which case 3 happens. As θ_W continues to increase, normal cars travel first, and the two toll curves go from one intersection with one peak to one intersection with double peaks.

Using the same logic as in the absence of road pricing, we can obtain the equilibrium arrival intervals and the associated travel price. We again omit the arrival time indicator and use $p_{CP}^i[N^a,N^n]$ to denote the equilibrium travel price for users with car type *i* under queue-eliminating road pricing.

In the following, we investigate the pricing rule of autonomous cars under different supply regimes with queue-eliminating road pricing. We again compare three supply regimes:

welfare-maximizing public supply, marginal cost supply, and profit-maximizing private monopolistic supply.

4.2 Different supply regimes of autonomous cars

The congestion toll is the transfer of money from road users to the authority and is not included in welfare. Solving the associated welfare-maximizing problem, we can obtain the relationship between welfare-maximizing public supply and marginal cost supply, which is presented in Proposition 10. The proof is given in Appendix G.

Proposition 10. When the queuing delay at the bottleneck is replaced by a time-varying toll, marginal cost supply of autonomous cars achieves the social optimum.

Remark 7. As all queuing is eliminated, in equilibrium, the marginal social cost for certain car type users is constant and equals their travel price. The welfare-maximizing mark-up thus becomes zero: the optimal congestion price gives the optimal incentive to switch to an autonomous car if both types of car are priced at a marginal automobile cost. Accordingly, the marginal cost supply of autonomous cars produces a social optimum.

Solving the profit-maximization problem yields the optimal mark-up under private monopolistic supply. It is found that with a profit-maximizing private supply, the pricing rule of autonomous cars under queue-eliminating road pricing is similar to that in the absence of road pricing. The only difference is that now we use the travel price, p_{CP}^{i} $(i = a, n)$, to replace the travel cost c_{NT}^{i} $(i = a, n)$ in Eq. (13).

5. Numerical examples

This section presents two numerical examples to illustrate the proposed model and analysis: one for $\theta_H = \theta_W$ and one for $\theta_H > \theta_W$. We present the numerical outcomes under the competitive marginal cost supply, welfare-maximizing public supply, and private monopolistic supply of autonomous cars, without and with road pricing. After discussing the base cases, we turn to the sensitivity analyses. The model outcomes are rather sensitive to the parameters, underlining the importance of presenting this wide spectrum of results.

The numerical setting stays close to that in Van den Berg and Verhoef (2016) and focuses on petrol passenger cars. We consider a trip of 20 km with a free-flow travel time of 20 minutes, implying a free-flow speed of 60 km/h. The capacity of the bottleneck is 3,600 vehicles per hour. We use an MC_a of €1.51 (Van den Berg and Verhoef, 2016). We compare outcomes under two cases: $\theta_H = \theta_W = 0.8$ and $\theta_H = 0.8,\ \theta_W = 0.5$. The sensitivity analyses do indeed present a wide spectrum of outcomes as θ_H and θ_W vary over the entire range.

The marginal utility functions are constructed to ensure that the average travel delay cost per hour and the average travel time cost per hour approximates the values under $\alpha - \beta - \gamma$ preferences (see, e.g., Small, 1982; Kouwenhoven et al., 2014). For users with normal cars, we use the marginal utility functions as follows:

$$H^n[t] = 12 - 5t, W^n[t] = 8 + 10t. \tag{19}$$

Since such linear marginal utilities can be regarded as an approximation of non-linear marginal utility functions, this illustration can be considered as a first linear approximation of the outcomes with general scheduling preferences.

The inverse demand function is linear, and is calibrated to ensure that the price elasticity of demand is -0.35 in the base equilibrium, and when there are no autonomous cars, there are 9,000 users. This results in $D[N^a + N^n] = 59.27 - 0.0049 \cdot (N^a + N^n)$.

Under this base calibration, in the equilibrium without road pricing, the travel cost under zero free flow travel time is €11.72, and the generalized travel cost is €15.37. An equilibrium travel cost of €11.72 and a peak duration of 2.5 hour would for the Vickrey (1969) model mean that $\delta = 4.69$, which corresponds with $\gamma = 22.74$ and $\beta = 5.91$. The free-flow travel cost of €3.65 for 20 minutes produces an α of 10.95. This implies that the choice of the marginal utility functions appears to be reasonably in line with earlier studies.

For presentation purpose, we use *AV* to denote autonomous cars, *MC* to denote marginal cost supply, *Pub* to denote welfare-maximizing public supply, and *PM* to denote private monopolistic supply. Welfare-maximizing public supply of autonomous cars with road pricing naturally produces the social optimum. We are interested in the total number of users *N*, the number of autonomous car users N^a, the number of normal car users N^n, the equilibrium travel cost without road pricing c^i_{NT}, the equilibrium travel price with road pricing p^i_{CP}, the average toll for each autonomous car user $\overline{\tau^a}$, the average toll for each normal car user $\overline{\tau^n}$, social welfare *SW*, and relative efficiency ω. Relative efficiency for a certain regime is defined as the welfare improvement compared to the case without autonomous cars in the absence of road pricing, divided by the welfare improvement of public supply with queue-eliminating road pricing.[8] Relative efficiency cannot, therefore, exceed 1.

5.1 Base case with $\theta_H = \theta_W = 0.8$, implying equal preferred arrival time

The setting of $\theta_H = \theta_W = 0.8$ means autonomous and normal car users have the same preferred arrival time, i.e. $t^{a*} = t^{n*} = 0.38$. Travelers tend to perform home-related activity when the arrival time is earlier than 0.27, and tend to perform work-related activity when they arrive after 0.27. We are interested in the equilibrium outcomes under different supply regimes and the associated equilibrium travel patterns.

5.1.1 Equilibrium with $\theta_H = \theta_W = 0.8$

The outcomes under different supply regimes with $\theta_H = \theta_W = 0.8$ are given in Table 3. The introduction of autonomous cars lowers travel costs (prices). Even with a private monopolist, users are better off than they are without autonomous cars. Of course, users can individually never be worse off than before *given* the choices of others, as they can always all choose to continue using normal cars; in this case they are better off also after considering others' behavioral changes. Some interesting observations are summarized as follows.

Table 3. Outcomes with $\theta_H = 0.8, \theta_W = 0.8$, $t_a^* = t_n^* = 0.38$.

	Without road pricing			With road pricing	
	No *AV*	*MC*	*Pub & PM*	*Pub & MC*	*PM*
N	9,000	9,195	8,974	9,195	9,091
N^a	0	9,195	8,974	4,002	1,857
N^n	9,000	0	0	5,193	7,233
MU^a	0	0	1.55	0	0.73
c_{NT}^a (p_{CP}^a)	-	12.70	12.24	12.70	12.48
c_{NT}^n (p_{CP}^n)	15.37	-	-	14.21	14.72
$\overline{\tau^a}$	0	0	0	3.64	1.82
$\overline{\tau^n}$	0	0	0	9.27	8.56
SW	197,574	207,140	211,200	269,830	269,090
ω	0	0.13	0.19	1.00	0.99

[8]The relative efficiency measure is $\omega = (SW - SW^{No\ AV,NT})/(SW^{Pub,CP} - SW^{No\ AV,NT})$.

In the absence of road pricing, all supply regimes lead to the corner solution of 100% autonomous cars and cause the over-consumption of autonomous cars. For marginal cost supply, this is due to the zero mark-up and the lower generalized price of an autonomous car. For welfare-maximizing public supply and private monopolistic supply, the corner solution is due to the user equilibrium conditions for car ownership and for road use. In this situation, it is unprofitable for the private monopolist to apply a price premium, because the higher profit margin would be outweighed by a loss of autonomous car sales as individuals would switch to normal cars. As a result, the welfare-maximizing and profit-maximizing regimes have the same equilibrium, and thus result in the same relative efficiency.

With queue-eliminating road pricing, welfare-maximizing and private monopolistic supply both lead to interior solutions. Private monopolistic supply results in the under-supply of autonomous cars (1,857 vs 4,002), due to private monopolistic power. Since road pricing eliminates all congestion externality, the mark-up is still lower than that without road pricing (0.73 vs 1.55). In addition, the average toll for autonomous car users is less than that for normal car users. The reason is that normal car users pass the bottleneck in the central peak period, and autonomous car users pass the bottleneck in the shoulders of the peak period, as illustrated in Fig. 4b.

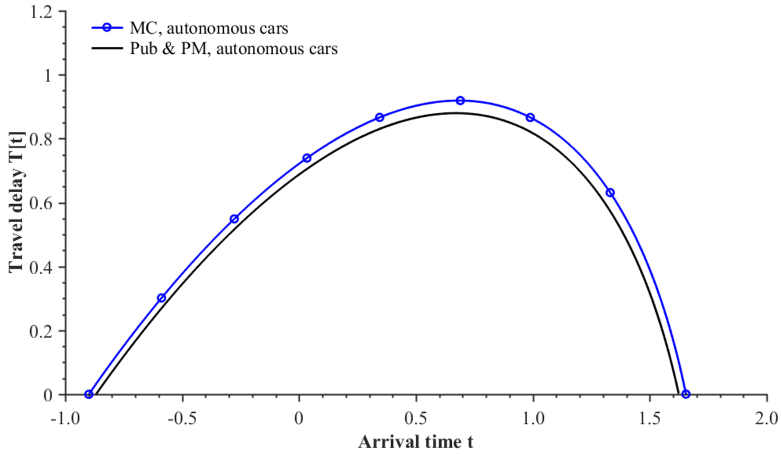
Road pricing is effective in improving welfare, and even with a private monopolist the relative efficiency is 0.99. This is intuitive, as queuing is eliminated. Without road pricing, perfect competition may be more harmful to users than private monopolistic supply. Indeed, although consumer surplus falls due to the private monopolist's pricing behavior, the welfare with a private monopolist still exceeds that under marginal cost supply, as congestion is reduced to a greater extent by having fewer travelers.

5.1.2 Travel patterns with $\theta_H = \theta_W = 0.8$

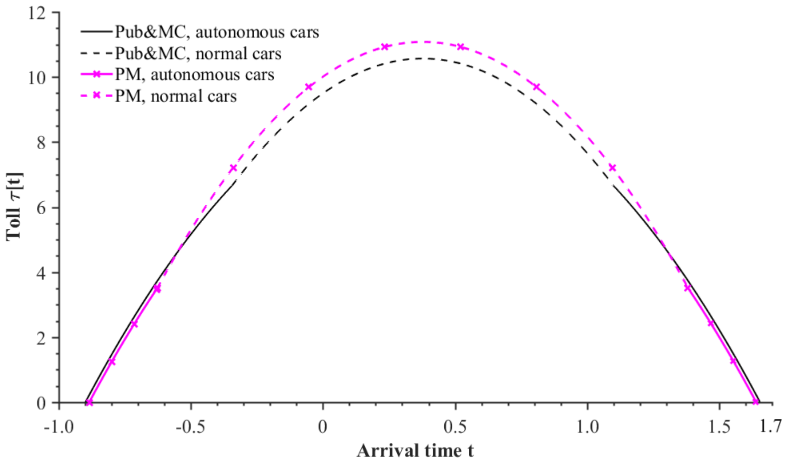
Fig. 4 shows the equilibrium travel patterns under different supply regimes: (a) without road pricing and (b) with queue-eliminating road pricing. For $\theta_H = \theta_W = 0.8$, the relative impact of autonomous driving on the ability to perform home and work activities in the car, as opposed to at the relevant location, are the same. Table 4 further presents the specific intervals of traveling and in-vehicle activity choices under different regimes.

As discussed, without road pricing, autonomous and normal car users travel sequentially. With road pricing, autonomous car users travel in the shoulders of the peak period to avoid a higher toll, and normal car users travel in the center to save the schedule delay cost. Fig. 4a shows that without road pricing, all users choose autonomous cars. The peak lasts longer for marginal cost supply than the peak for public and private monopolistic supply, owing to a larger number of vehicles. More specifically, under marginal cost supply, autonomous car users arrive in the interval [-0.90, 1.65], as presented in Table 4. Under welfare-maximizing public supply and private monopolistic supply, they arrive in the interval [-0.87, 1.62].

Fig. 4b shows that with road pricing, normal car users arrive in the center and autonomous car users in the shoulders of the peak period. The toll starts earlier and ends later under welfare-maximizing public supply than under a private monopoly, owing to the larger number of vehicles. As a result of the larger number of normal cars, the maximum toll with a private monopolist exceeds that with a welfare-maximizing public supplier.



(a) Without road pricing (All users choose autonomous cars)



(b) With road pricing (Autonomous cars travel in the shoulders of the peak period)

Fig. 4. Equilibrium travel patterns with $\theta_H = \theta_W = 0.8$.

For autonomous car users' in-vehicle activity choices, travelers tend to perform the home-related activity when the arrival time is earlier than 0.27, and the work-related activity if they arrive after 0.27. It can be observed from Table 4 that in the absence of road pricing, around 45% of the travel time is spent on the home-related activity to reduce the cost of schedule delay early, and 55% on the work-related activity to reduce the cost of schedule delay late. With queue-eliminating congestion pricing, half of the travel time is spent on the home-related activity.

Table 4. Arrival and in-vehicle activity intervals with $\theta_H = 0.8, \theta_W = 0.8$.

		Normal cars	Autonomous cars		
		Arrival interval	Arrival interval	In-vehicle home activity	In-vehicle work activity
NT	MC	-	[-0.90, 1.65]	[-0.90,0.27]	[0.27, 1.65]
	Pub & PM	-	[-0.87, 1.62]	[-0.87,0.27]	[0.27,1.62]
CP	MC & Pub	[-0.34, 1.10]	[-0.90, -0.34] [1.10, 1.65]	[-0.90, -0.34]	[1.10, 1.65]
	PM	[-0.63,1.38]	[-0.88, -0.63] [1.38,1.64]	[-0.88, -0.63]	[1.38,1.64]

5.2 Base case with $\theta_H = 0.8$, $\theta_W = 0.5$, implying that autonomous car users having a later preferred arrival time

5.2.1 Equilibrium with $\theta_H = 0.8$, $\theta_W = 0.5$

In this subsection, we consider $\theta_H = 0.8$, $\theta_W = 0.5$, with $t^{n*} = 0.38$ and $t^{a*} = 0.77$. Table 5 shows the equilibrium outcomes. All supply regimes lead to interior solutions. Marginal cost supply and welfare-maximizing public supply without road pricing lead to over-consumption of autonomous cars, whereas private monopolistic supply leads to under-supply, more so when combined with road pricing. Some main insights are summarized as follows.

(i) Although road pricing eliminates all congestion externality, private monopolistic mark-up is slightly higher than that without road pricing (3.88 vs 3.83). Owing to the higher travel cost difference between driving autonomous cars and normal cars, and the greater willingness to pay for autonomous cars, queue-eliminating road pricing now raises the private monopoly power.

(ii) Welfare-maximizing supply with congestion tolling and marginal cost supply without congestion tolling lead to the same equilibrium travel costs (prices) for users with the same type of car, although these two regimes have different supply decisions, even though social welfare is different because queuing occurs in only one of the two situations. This reflects that although road pricing affects the optimal split of different car types in addition to the impacts on travel patterns, it does not affect the first and last travelers' travel behavior. The travel intervals in Table 6 in subsection 5.2.2 further confirm this point.

(iii) For the congestion toll, owing to the larger congestion externality caused by normal cars, the average toll for normal cars again exceeds that for autonomous cars, and especially so for private monopolistic pricing. This is because the private monopoly power leads more users to choose normal cars, and the higher number of normal cars causes normal car users to pass the bottleneck in the central peak period. The travel patterns in Fig. 5b in subsection 5.2.2 further confirm this point.

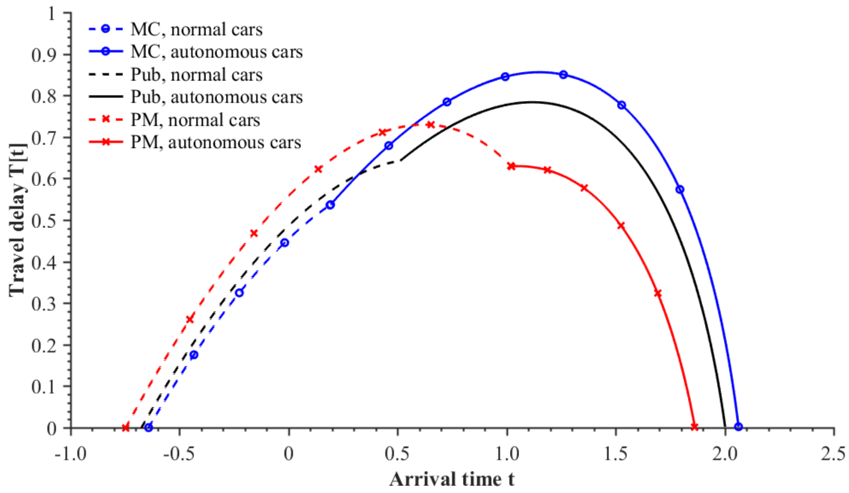
(iv) In terms of the relative efficiency, road pricing again performs better than not having road pricing. Without road pricing, marginal cost supply now performs better than private monopolistic supply (relative efficiency of 0.40 vs 0.35), and slightly worse than welfare-maximizing public supply (0.40 vs 0.41).

Table 5. Outcomes with $\theta_H = 0.8$, $\theta_W = 0.5$, $t^{n*}=0.38$, $t^{a*}= 0.77$.

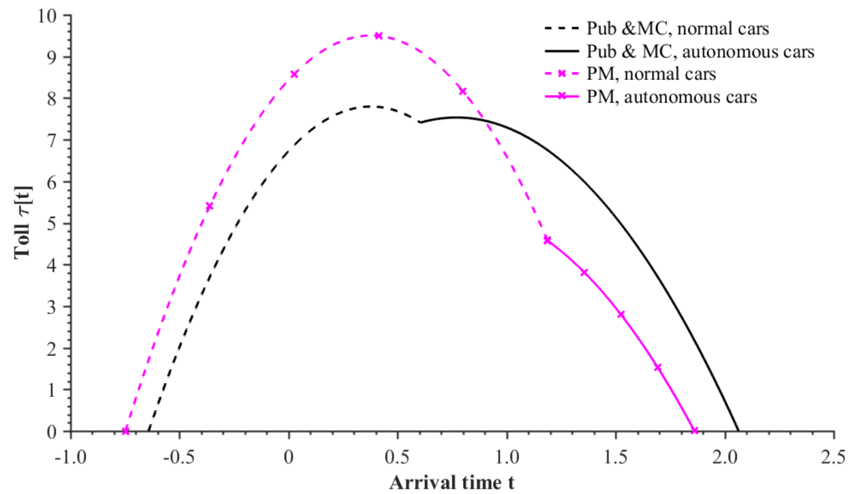
	Without road pricing				With road pricing	
	No AV	MC	Pub	PM	Pub & MC	PM
N	9,000	9,742	9,638	9,398	9,742	9,393
N^a	0	6,737	5,366	3,036	5,254	2,422
N^n	9,000	3,004	4,272	6,362	4,488	6,972
MU^a	-	0	1.21	3.83	0	3.88
c_{NT}^a (p_{CP}^a)	-	10.03	9.33	7.88	10.03	7.85
c_{NT}^n (p_{CP}^n)	15.37	11.54	12.04	13.22	11.54	13.24
$\overline{\tau^a}$	-	-	-	-	5.30	2.62
$\overline{\tau^n}$	-	-	-	-	5.65	6.97
SW	197,574	232,500	234,050	228,030	285,700	280,540
ω	0	0.40	0.41	0.35	1.00	0.94

5.2.2 Travel patterns with $\theta_H = 0.8$, $\theta_W = 0.5$

Fig. 5 depicts equilibrium travel patterns. Work-related activities in the vehicle reduces the cost of schedule delay late for autonomous car users. Thus, the preferred arrival time for normal car users is earlier than that for autonomous car users. In equilibrium, normal car users travel first.



(a) Without road pricing (Normal cars travel first)



(b) With road pricing (Normal cars travel first)

Fig. 5. Equilibrium travel patterns with $\theta_H = 0.8$, $\theta_W = 0.5$.

Although the starting and ending arrival times are the same for marginal cost supply without road pricing and welfare-maximizing public supply with road pricing, the equilibrium travel patterns are still different because car ownership numbers are different. More specifically, for the former regime, normal car users arrive in the interval [-0.64, 0.19] and autonomous car users arrive in the interval [0.19, 2.06], as presented in Table 6. In contrast, for the latter regime, normal car users arrive in the period [-0.64, 0.60] and autonomous car users arrive in the interval [0.60, 2.06].

Unlike the single peak in other regimes, travel delay or toll functions have double peaks under private monopolistic supply without congestion tolling, and under welfare-maximizing public supply with road pricing. The double peaks are induced by relatively similar numbers of autonomous and normal cars. This illustrates that the share of autonomous cars also affects the shape of the travel delay (or toll) functions.

For autonomous car users' in-vehicle activity choices, Table 6 shows that all travelers tend to perform work-related activity in the vehicle, to reduce the cost of schedule delay late.

Table 6. Arrival and in-vehicle activity intervals with $\theta_H = 0.8, \theta_W = 0.5$.

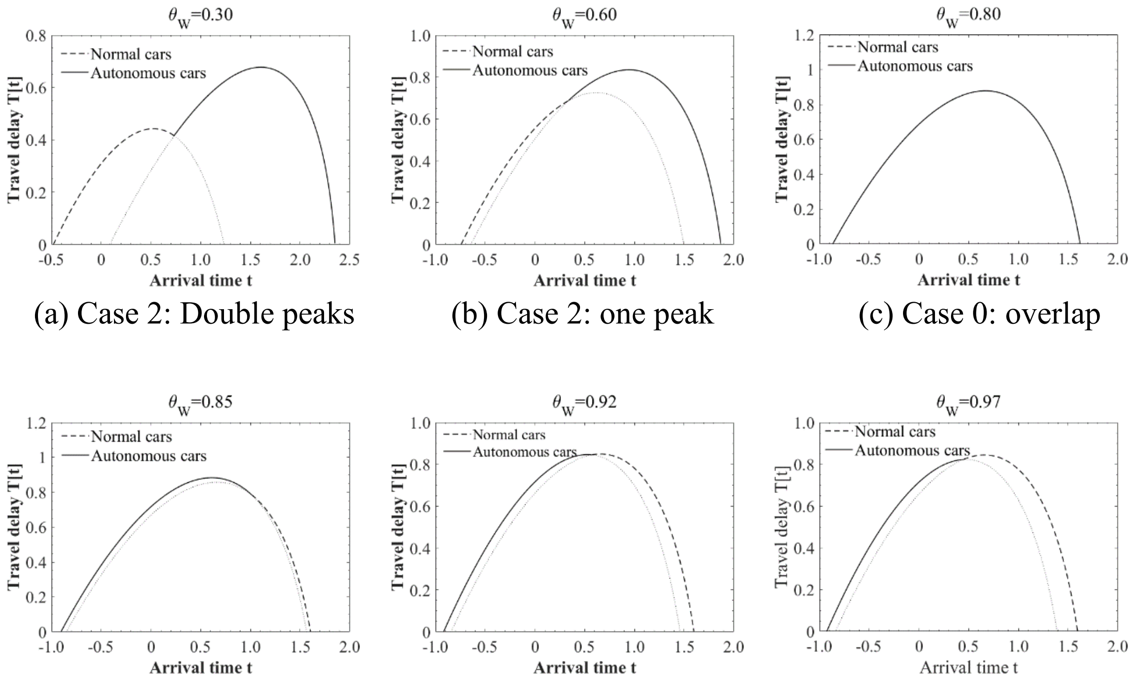
		Normal cars	Autonomous cars		
		Arrive interval	Arrive interval	In-vehicle home activity	In-vehicle work activity
NT	MC	[-0.64, 0.19]	[0.19, 2.06]	×	[0.19, 2.06]
	Pub	[-0.67, 0.51]	[0.51, 2.00]	×	[0.51, 2.00]
	PM	[-0.75, 1.02]	[1.02, 1.86]	×	[1.02, 1.86]
CP	MC & Pub	[-0.64, 0.60]	[0.60, 2.06]	×	[0.60, 2.06]
	PM	[-0.75, 1.19]	[1.19, 1.86]	×	[1.19, 1.86]

5.3 Varying θ_W

As there is little to no guidance from the literature on the values of θ_H and θ_W, it is vital to carry out extensive sensitivity analysis. To that end, we keep the value of θ_H at 0.8 and vary θ_W from 0.3 to 1.0. Subsections 5.3.1 and 5.3.2 present the changes in equilibrium travel patterns and relative efficiency, respectively.

5.3.1 Changes in travel patterns

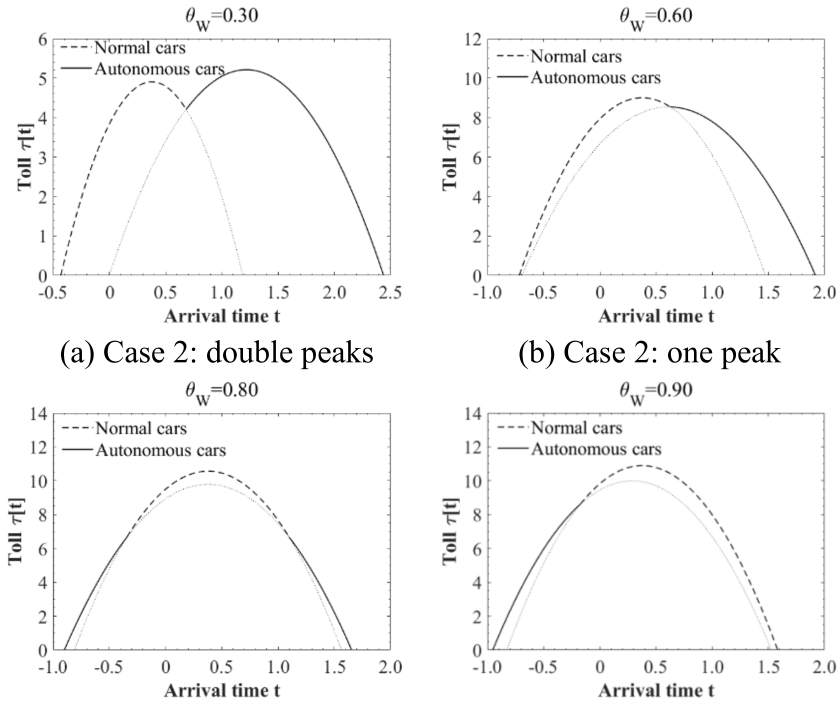
A smaller θ_W means a higher marginal utility of performing the work-related activity in the vehicle. Hence, as θ_W decreases, the percentage of travel time spent in the work-related activity increases, and reversely for the home-related activity. The travel delay or congestion toll curves for autonomous car users now move to the right. The travel patterns naturally confirm the insights from the theoretical model. Fig. 6 and Fig. 7 use welfare-maximizing public supply to illustrate the specific changes in the equilibrium travel patterns, in the absence of road pricing and under queue-eliminating road pricing, respectively.



(a) Case 2: Double peaks (b) Case 2: one peak (c) Case 0: overlap

(d) Case 1: one peak (e) Case 1: double peaks (f) Case 1: one peak

Fig. 6. Changes in travel patterns without road pricing.



(a) Case 2: double peaks (b) Case 2: one peak

(c) Case 3 (d) Case 1: one peak

Fig. 7. Changes in travel patterns with road pricing.

Fig. 6 confirms that in the absence of road pricing, when $\theta_W < \theta_H$, normal car users travel first. As θ_W increases, the two travel delay curves go from one intersection with double peaks, to one intersection with one peak. When $\theta_W > \theta_H$, autonomous car users travel first. The two curves now go from one intersection with one peak, to one intersection with double peaks, and then to one intersection with one peak.

Fig. 7 shows that under queue-eliminating road pricing, changes in equilibrium patterns are similar to those without tolling: with a smaller θ_W, normal car users travel first, and the reverse with a larger θ_W. Specially, when θ_W is close to θ_H, normal car users travel in the central peak and autonomous car users in the shoulders of the peak period.

5.3.2 Relative efficiency

In addition to travel patterns, the associated social welfare impacts under different supply regimes are also strongly affected by travelers' in-vehicle activities. Fig. 8 depicts the relative efficiencies under all supply regimes. As there can be changes in travel patterns (i.e., different cases), relative efficiencies do not change monotonically, or smoothly, with θ_W. The kinks and jumps are either caused by a change in travel patterns, or a change from interior to corner solutions.

In the absence of road pricing, relative efficiencies under all supply regimes generally decrease with θ_W, especially when θ_W is below 0.8. Indeed, the increase in θ_W means a smaller benefit from in-vehicle activities, and thus reduces the welfare under all regimes. Compared to the no-toll cases, welfare is impacted less when queuing is eliminated by time-varying tolling. This means that relative efficiency is determined more by the change in the welfare gains of the no-toll regimes (numerator of relative efficiency). As a result, relative efficiencies without road pricing decrease with θ_W.

However, we also find the relative efficiency increases over a certain range with θ_W. Indeed, when θ_W exceeds 0.8, the increase in θ_W lowers both the number of autonomous cars and the congestion externality that autonomous car users impose, which tends to raise efficiency. It also raises the number of normal cars and the congestion externality normal car users impose, which tends to lower efficiency. Under public and private monopolistic supply, the relative efficiency starts to increase when θ_W exceeds 0.91. The reason is that when θ_W is lower than 0.91, the latter effect dominates and as θ_W exceeds 0.91, the former effect dominates. Due to the private monopoly power, the relative efficiency under private supply is generally lower than that under public supply. Nevertheless, it can be observed that when θ_W is between 0.72 and 0.82, the second-best public supply and private supply have the same relative efficiency. This is because all users choose autonomous cars, and the private monopolist finds it unprofitable to apply a private monopolist premium.

With road pricing, as the congestion externality is eliminated, the performance of private supply depends on the degree of monopoly power. Fig. 8 shows that the relative efficiency increases with θ_W only when θ_W is below 0.87. This is because a higher θ_W lowers the number of autonomous cars and reduces private monopoly power. Nevertheless, when θ_W exceeds 0.87, autonomous car users travel first. The increase in θ_W lowers the toll for them and thus attracts more users to choose autonomous cars. This makes more autonomous car users subject to the monopoly power of the private monopolist, which leads to slightly decreased relative efficiency.

As queuing is a pure loss, road pricing performs considerably better than no road pricing throughout Fig. 8. In the absence of road pricing, when θ_W exceeds 0.8, the over-supply caused by the zero mark-up under perfect competition is more harmful than the under-supply caused by private monopoly power. This means that private monopolistic pricing may perform better than perfect competition.

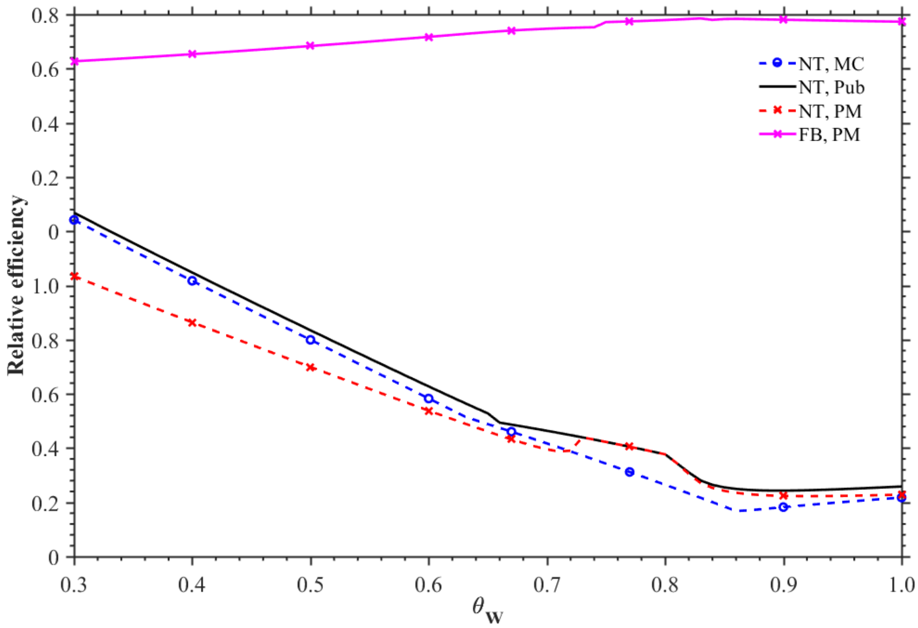
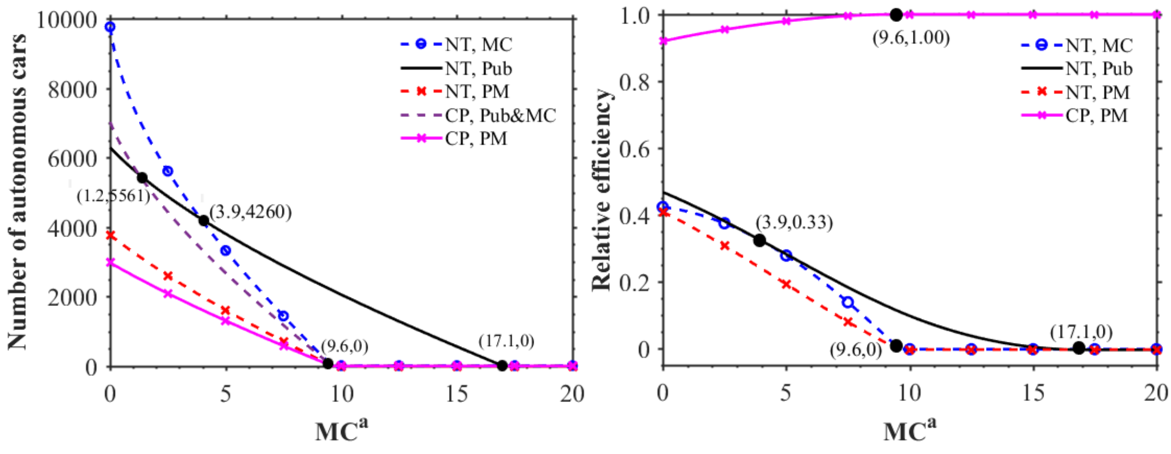


Fig. 8. Effects of θ_W on relative efficiency.

5.4 Varying MC^a

We now consider the effects of the marginal automobile cost of autonomous cars, MC^a, with $\theta_H = 0.8$, $\theta_W = 0.5$. Fig. 9a shows that, as one might expect, the number of autonomous cars decreases with MC^a under all supply regimes. The curve for "CP, Pub" gives the social optimal outcome. Since road pricing eliminates queuing, suppliers stop providing autonomous cars earlier with congestion tolling than without tolling (for the marginal cost of 9.6 vs 17.1). The zero mark-up under marginal cost supply without road pricing leads to over-consumption of autonomous cars, whereas private monopolistic supply leads to under-supply, especially when combined with road pricing. For welfare-maximizing public supply without tolling, an MC^a below 1.2 leads to under-supply, and over-consumption when MC^a exceeds 1.2. When MC^a exceeds 3.9, the mark-ups become negative. This reflects the need for a subsidy to attract more users to switch to autonomous cars.

A larger marginal cost of automobiles naturally reduces social welfare, also under optimum pricing and tolling. The relative efficiency depends on the relative strength of the welfare loss from the evaluated scheme and that under the welfare-maximizing supply with road pricing. Fig. 9b shows that without road pricing, relative efficiency decreases with MC^a. In particular, as MC^a increases, the relative efficiency of marginal cost supply approaches welfare-maximizing public supply when MC^a is lower than 3.9, and approaches private monopolistic supply when MC^a exceeds 3.9. In contrast, with road pricing, the relative efficiency of private monopolistic supply increases with MC^a, as a higher MC^a lowers the number of autonomous cars, and the mark-up is less distortive.



(a) Number of autonomous cars (b) Relative efficiency

Fig. 9. Effects of MC^a.

5.5 Varying the price elasticity of demand

Finally, we consider the effects of users' price elasticity of demand on the relative efficiency of different congestion and supply regimes. We keep $\theta_H = 0.8$, $\theta_W = 0.5$ and vary the absolute value of the demand elasticity from 0 to 20, by making the demand curve tilt around the equilibrium in the base equilibrium without autonomous cars.

Fig. 10 shows that, without road pricing, all relative efficiencies decrease with the elasticity. As demand becomes more price-sensitive, the numbers of autonomous and normal cars both increase, even though the sensitivity analyses are constructed such that the equilibrium prices and quantities are the same in the base equilibrium for each demand elasticity. This raises the occurrence of congestion and thus lowers the relative efficiency of welfare-maximizing public supply and marginal cost supply, and more so for marginal cost supply due to the zero mark-up. For private monopolistic supply, although the increase in the number of autonomous car users tends to raise the private monopoly power, the increase in the total number of car users lowers users' willingness to pay. As the latter effect dominates, the private monopoly power decreases. The increase in congestion continues to dominate over the reduction in private monopoly power. Consequently, the relative efficiency of private monopolistic supply decreases with price elasticity, and is slightly lower than with welfare-maximizing public supply.

With road pricing, the relative efficiency under private monopolistic supply decreases slightly with price elasticity, because the first-best welfare gain (denominator of relative efficiency) increases more than the welfare gain under private monopolistic supply.

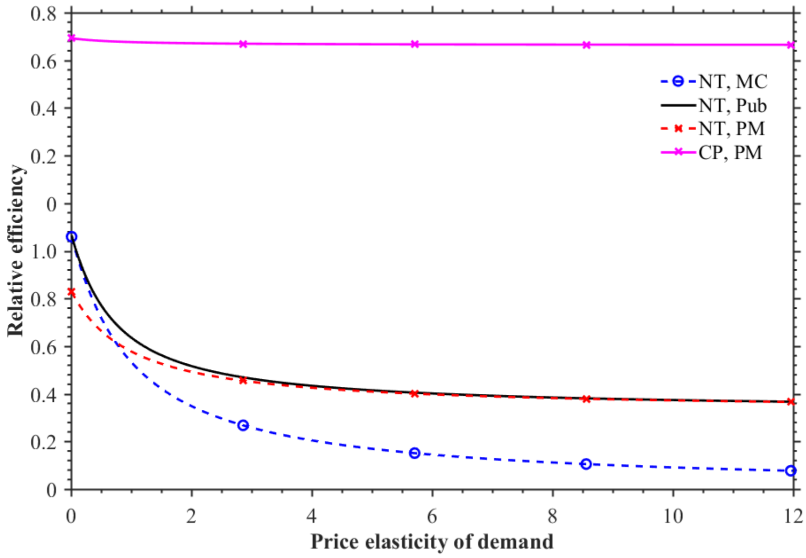


Fig. 10. Effects of demand elasticity on relative efficiency.

6. Conclusion

This paper investigated the effects of travelers' in-vehicle activities in autonomous cars using an activity-based bottleneck modelling framework. The marginal utilities of being at home, being in the vehicle and being at work were assumed to be time-dependent during the day. We distinguished in-vehicle home and in-vehicle work activities via allowing different in-vehicle activities to produce different marginal utilities. Travelers' travel equilibrium was explored analytically in the absence of road pricing and under queue-eliminating road pricing. We further considered three supply regimes of autonomous cars: welfare-maximizing public supply, competitive marginal cost supply, and private monopolistic supply. The proposed model could serve as a useful tool for evaluating various supply regimes and policies at the strategic level for a bi-modal bottlenck system with autonomous cars.

Some important findings and new insights were obtained. First, performing home-related (work-related) activities in the vehicle will advance (postpone) autonomous car users' preferred arrival times (Proposition 1). This result is different from existing studies using the traditional bottleneck model with piecewise constant marginal activity utilities, in which travelers' in-vehicle activities do not affect their preferred arrival times. Second, in the absence of road pricing, autonomous and normal car users may arrive sequentially or simultaneously; under queue-eliminating road pricing, autonomous and normal car users may travel sequentially or alternatingly (Proposition 2, Proposition 3 and Proposition 9). The resulting travel patterns are different from those found in current studies (see, for example, Van den Berg and Verhoef, 2016; Tian et al., 2019; Pudāne, 2020), as our model allows for heterogeneity in both scheduling preferences and preferred arrival times. Ignoring the distinction between in-vehicle home and in-vehicle work activities may cause bias in evaluating travelers' travel behavior. Third, travelers' own-congestion effects exceed the cross-congestion effects: travelers tend to impose higher external costs on travelers of their own type than travelers with different preferences (Propostion 4). The marginal external benefit of switching from a normal to an autonomous car may be positive or negative, depending on the marginal utilities for different activities and the number of autonomous cars (Proposition 5). Compared to the 'negative heterogeneity effects' in Van den Berg and Verhoef (2016), in which in-vehicle activities are modeled as a reduction in the value of time losses, we find that ignoring time-varying scheduling preference may over-estimate the congestion effects caused by autonomous cars. Fourth, the supply decisions on autonomous cars under different supply and road pricing regimes were obtained analytically and compared (Propositions 6-8, Proposition 10).

By evaluating the relative efficiency of alternative suppliers' provision decisions, we find that welfare-maximizing public supply with road pricing shows the best performance, which is the social optimum and leads to zero mark-up on autonomous cars, followed by private monopolistic supply with road pricing. This implies that competive marginal cost supply with queue-eliminating road pricing achieves the social optimum, and the over-supply of cars under welfare-maximizing mark-up in the absence of road pricing is more harmful than the distortion of private monopoly power with road pricing. In the absence of road pricing, marginal cost supply may also be more harmful than the private monopolistic supply of autonomous cars.

This research studied travelers' trip-timing preferences and decisions in the morning commute when both autonomous cars and normal cars are present. Our findings indicate several important managerial implications. Some of these concern the suppliers of autonomous cars. First, ignoring travelers' time-varying scheduling preferences and the various in-vehicle activities may lead to biases in estimating the benefits of introducing autonomous cars. Since the benefits from in-vehicle activities have a significant impact on travelers' willingness to pay for autonomous cars, we would suggest car manufactures and car suppliers collect more information about travelers' in-vehicle activity preferences and the factors that may improve travelers' marginal utilities in the vehicle. Second, since increasing the number of autonomous cars tends to lower the marginal external benefit of switching from a normal to an autonomous car, large car suppliers may find it advantageous to take the congestion effects caused by autonomous cars into account when deciding their supply decisions.

In terms of policy making, this study provides guidelines for regulators seeking to regulate car provision and traffic congestion to improve welfare. First, the marginal external benefit of switching from a normal to an autonomous car will become negative if the number of autonomous cars exceeds a certain level. This means that regulators should not blindly encourage autonomous driving but instead make informed trade-offs that take such effects into consideration. Second, on the supply side, as prefect competition may lead to over-consumption or under-supply of autonomous cars, we advise governments to base their policy with regard to the adoption of autonomous cars on solid modeling of the respective market, so that counter-productive interventions – e.g., providing subsidies when marginal external benefits are already negative – can be prevented. In the absence of road pricing, perfect competition may be more harmful than the private monopolistic supply of autonomous cars, which further underlines the importance of conducting careful market analysis before implementing industrial policies: even the desired direction of an intervention is not necessarily clear beforehand. Third, to eliminate the congestion externality further, we would suggest that governments implement time-varying congestion pricing. By doing so, perfect competition achieves the social optimum.

This study is the first quantitative approach to analyzing the relationship between in-vehicle activities and travel patterns under time-varying scheduling preferences and dynamic congestion. It provides new insights into the issues of autonomous cars and dynamic congestion. However, some important questions deserve attention in future research efforts. First, empirical evidence on how in-vehicle utility varies by activity and time of day is still limited. In this paper, we have assumed that travelers derive a constant fraction of the utility from either home-related or work-related activities. If other forms of marginal utility functions are adopted, the framework of the modeling approach and analysis remains but the travel patterns may be different. For instance, if we relax the assumption that θ_H and θ_W are constants and consider time-varying $\theta_H(t)$ and $\theta_W(t)$, the peak of the travel delay function with autonomous cars is likely to exceed the travel delay peak with normal cars. Hence, it is possible that autonomous cars travel in the center of the bottleneck, and normal car users travel in the shoulders, as discussed in Van den Berg and Verhoef (2016). In order to make use of the proposed model for practical applications in reality, there is a need to estimate and measure the marginal utilities for different activities, using stated or revealed preference data (see, e.g., Tseng and Verhoef, 2008; Hjorth et al., 2015; Correia et al., 2019). Second, it seems plausible that the marginal utility obtained from the same in-vehicle activity may differ from person to person. In any case, the parameters θ_H and θ_W are likely to vary over individuals. Then, travelers self-select arrival times according to their endogenous heterogeneity in preferred arrival times and scheduling preferences. Overall, travelers will be divided into more separate groups. Third, there may also be pre-existing heterogeneity between travelers, which may lessen the endogenous heterogeneity effect caused by autonomous cars. In this situation, some normal and autonomous car users may have the same marginal utility functions and preferred arrival times, implying that they travel mixed and reach the bottleneck at the same time. Fourth, when autonomous and normal cars are mixed, the proportion of autonomous cars will have a significant impact on traffic velocity and road capacity. The proposed model can be extended to incorporate the potential change in capacity. In particular, in future autonomous car markets, there may exist a range of car suppliers and individuals may regard different cars as imperfect rather than perfect substitutes. The competition and cooperation between the suppliers will be an interesting topic for future research.

References

Arnott, R., de Palma, A., Lindsey, R. (1987). *Schedule Delay and Departure Time Decisions with Heterogeneous Commuters*. University of Alberta, Department of Economics.

Arnott, R., De Palma, A., Lindsey, R. (1994). The welfare effects of congestion tolls with heterogeneous commuters. *Journal of Transport Economics and Policy*, 28(2), 139-161.

Börjesson, M., Eliasson, J., Franklin, J. P. (2012). Valuations of travel time variability in scheduling versus mean–variance models. *Transportation Research Part B: Methodological*, *46*(7), 855–873.

Correia, G. H. de A., Looff, E., Van Cranenburgh, S., Snelder, M., Van Arem, B. (2019). On the impact of vehicle automation on the value of travel time while performing work and leisure activities in a car: Theoretical insights and results from a stated preference survey. *Transportation Research Part A: Policy and Practice*, *119*, 359–382.

Ettema, D. F., Timmermans, H. J. (1997). *Activity-based Approaches to Travel Analysis*. New York: Pergamon.

Ettema, D., Timmermans, H. (2003). Modeling departure time choice in the context of activity scheduling behavior. *Transportation Research Record*, *1831*(1), 39–46.

Fosgerau, M. (2019). *Automation and the Value of Time in Passenger Transport*. In International Transport Forum Discussion Papers (No. 2019/10). OECD Publishing.

Fosgerau, M., de Palma, A. (2012). Congestion in a city with a central bottleneck. *Journal of Urban Economics*, *71*(3), 269–277.

Fosgerau, M., Engelson, L. (2011). The value of travel time variance. *Transportation Research Part B: Methodological*, *45*(1), 1–8.

Fosgerau, M., Lindsey, R. (2013). Trip-timing decisions with traffic incidents. *Regional Science and Urban Economics*, *43*(5), 764–782.

Haboucha, C. J., Ishaq, R., Shiftan, Y. (2017). User preferences regarding autonomous vehicles. *Transportation Research Part C: Emerging Technologies*, *78*, 37–49.

Hjorth, K., Börjesson, M., Engelson, L., Fosgerau, M. (2015). Estimating exponential scheduling preferences. *Transportation Research Part B: Methodological*, *81*, 230–251.

Knockaert, J., Verhoef, E. T., Rouwendal, J. (2016). Bottleneck congestion: Differentiating the coarse charge. *Transportation Research Part B: Methodological*, *83*, 59–73.

Kouwenhoven, M., de Jong, G. C., Koster, P., Van den Berg, V. A. C., Verhoef, E. T., Bates, J., Warffemius, P. M. (2014). New values of time and reliability in passenger transport in the Netherlands. *Research in Transportation Economics*, *47*, 37–49.

Li, Z. C., Lam, W. H. K., Wong, S. C. (2014). Bottleneck model revisited: An activity-based perspective. *Transportation Research Part B: Methodological*, *68*, 262–287.

Li, Z. C., Lam, W. H. K., Wong, S. C. (2017). Step tolling in an activity-based bottleneck model. *Transportation Research Part B: Methodological*, *101*, 306–334.

Li, Z. C., Zhang, L. (2020). The two-mode problem with bottleneck queuing and transit crowding: How should congestion be priced using tolls and fares? *Transportation Research Part B: Methodological*, *138*, 46–76.

Lindsey, R. (2004). Existence, uniqueness, and trip cost function properties of user equilibrium in the bottleneck model with multiple user classes. *Transportation Science*, *38*(3), 293–314.

Liu, H. X., He, X., Recker, W. (2007). Estimation of the time-dependency of values of travel time and its reliability from loop detector data. *Transportation Research Part B: Methodological*, *41*(4), 448–461.

Liu, W. (2018). An equilibrium analysis of commuter parking in the era of autonomous vehicles. *Transportation Research Part C: Emerging Technologies*, *92*, 191–207.

Liu, Z., Song, Z. (2019). Strategic planning of dedicated autonomous vehicle lanes and autonomous vehicle/toll lanes in transportation networks. *Transportation Research Part C: Emerging Technologies*, *106*, 381–403.

Pigou, A. C. (1920). *The Economics of Welfare.* London: Palgrave Macmillan.

Pudāne, B. (2020). Departure time choice and bottleneck congestion with automated vehicles: Role of on-board activities. *European Journal of Transport and Infrastructure Research*, *20*(4), 306–334.

Pudāne, B., Molin, E. J. E., Arentze, T. A., Maknoon, Y., Chorus, C. G. (2018). A time-use model for the automated vehicle-era. *Transportation Research Part C: Emerging Technologies*, *93*, 102–114.

Silva, H. E., Lindsey, R., de Palma, A., Van den Berg, V. A. C. (2017). On the existence and uniqueness of equilibrium in the bottleneck model with atomic users. *Transportation Science*, *51*(3), 863–881.

Small, K. A. (1982). The scheduling of consumer activities: Work trips. *The American Economic Review*, *72*(3), 467 – 479.

Steck, F., Kolarova, V., Bahamonde-Birke, F., Trommer, S., Lenz, B. (2018). How autonomous driving may affect the value of travel time savings for commuting. *Transportation Research Record*, *2672*(46), 11–20.

Su, Q., Wang, D. Z. W. (2020). On the morning commute problem with distant parking options in the era of autonomous vehicles. *Transportation Research Part C: Emerging Technologies*, *120*, 102799.

Tian, L. J., Sheu, J. B., Huang, H. J. (2019). The morning commute problem with endogenous shared autonomous vehicle penetration and parking space constraint. *Transportation Research Part B: Methodological*, *123*, 258–278.

Tscharaktschiew, S., Evangelinos, C. (2019). Pigouvian road congestion pricing under autonomous driving mode choice. *Transportation Research Part C: Emerging Technologies*, *101*, 79–95.

Tseng, Y. Y., Verhoef, E. T. (2008). Value of time by time of day: A stated-preference study. *Transportation Research Part B: Methodological*, *42*(7–8), 607–618.

Van den Berg, V. A. C., Verhoef, E. T. (2011a). Winning or losing from dynamic bottleneck congestion pricing? The distributional effects of road pricing with heterogeneity in values of time and schedule delay. *Journal of Public Economics*, *95*(7–8), 983–992.

Van den Berg, V. A. C., Verhoef, E. T. (2011b). Congestion tolling in the bottleneck model with heterogeneous values of time. *Transportation Research Part B: Methodological*, *45*(1), 60–78.

Van den Berg, V. A. C., Verhoef, E. T. (2016). Autonomous cars and dynamic bottleneck congestion: The effects on capacity, value of time and preference heterogeneity. *Transportation Research*

Part B: Methodological, *94*, 43–60.
Vickrey, W. (1969). Congestion theory and transport investment. *The American Economic Review*, *59*(2), 251–260.
Vickrey, W. (1973). Pricing, metering, and efficiently using urban transportation facilities. *Highway Research Record*, *476*, 36–48.
Wadud, Z., MacKenzie, D., Leiby, P. (2016). Help or hindrance? The travel, energy and carbon impacts of highly automated vehicles. *Transportation Research Part A: Policy and Practice*, *86*, 1–18.
Walters, A. A. (1961). The theory and measurement of private and social cost of highway congestion. *Econometrica: Journal of the Econometric Society*, *29*(4), 676–699.
Wang, J., Lu, L., Peeta, S., He, Z. (2021). Optimal toll design problems under mixed traffic flow of human-driven vehicles and connected and autonomous vehicles. *Transportation Research Part C: Emerging Technologies*, *125*, 102952.
Zhang, F., Liu, W., Lodewijks, G., Waller, S. T. (2020). The short-run and long-run equilibria for commuting with autonomous vehicles. *Transportmetrica B: Transport Dynamics*. 1-28.

Acknowledgments

We thank three anonymous referees for their helpful comments and constructive suggestions on an earlier draft of the paper. We also thank Professor Zhi-Chun Li of Huazhong University of Science and Technology for the helpful comments. The work described in this paper was jointly supported by grants from the National Key Research and Development Program of China (2018YFB1600900), the National Natural Science Foundation of China (71525003, 71890970/71890974), and the NSFC-EU joint research project (71961137001). Any remaining errors are ours.

Appendix A. Proof of Proposition 2

We use the contradiction method. We only illustrate for the case in which autonomous car users travel in the center and normal car users travel in the shoulders of the peak period, as shown in Fig. 11. Suppose the travel pattern in Fig. 11 happens. Assume autonomous car users travel in the interval $[t_{na}, t_{an}]$, and normal car users travel in the intervals $[t_s, t_{na}]$ and $[t_{an}, t_e]$.

At arrival time t_{na}, the slope of the travel delay function for autonomous car users is larger than that for normal car users. From Eq. (6), we obtain:

$$1-\frac{\theta_W}{\theta_H}\cdot\frac{W^n[t_{na}]}{H^n[t_{na}-T[t_{na}]-T_{ff}]}>1-\frac{W^n[t_{na}]}{H^n[t_{na}-T[t_{na}]-T_{ff}]}, \tag{A1}$$

which can be further simplified as:

$$\theta_W<\theta_H . \tag{A2}$$

Meanwhile, at arrival time t_{an}, the slope of the travel delay function for autonomous car users is smaller than that for normal car users. We obtain:

$$1-\frac{\theta_W}{\theta_H}\cdot\frac{W^n[t_{an}]}{H^n[t-T[t_{an}]-T_{ff}]}<1-\frac{W^n[t_{an}]}{H^n[t-T[t_{an}]-T_{ff}]}, \tag{A3}$$

which implies:

$$\theta_W>\theta_H . \tag{A4}$$

It is clear that Eq. (A2) contradicts Eq. (A4). Consequently, the above case will not happen. Using the same logic, we can prove it is also impossible for normal car users to travel in the center and autonomous car users to travel in the shoulders of the bottleneck.

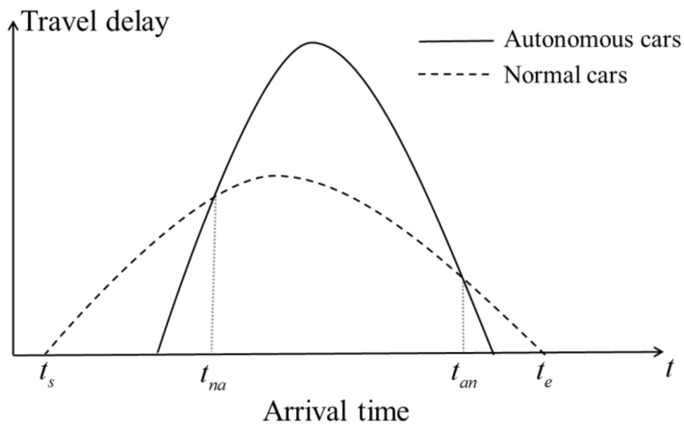


Fig. 11. Autonomous car users travel in the center of the bottleneck.

Appendix B. Solving the travel time equilibrium without road pricing

We use case 1 as an example, and the derivation of other cases can be solved by the same logic.

In case 1, autonomous car users arrive in the interval $[t_s,t_{an}]$, and normal car users arrive in the interval $[t_{an},t_e]$. In equilibrium, travelers with the same car type should have the same travel cost regardless of their arrival times; we thus have $c^a[t_s]=c^a[t_{an}]$ and $c^n[t_{an}]=c^n[t_e]$.

Consequently, the equilibrium solutions of t_s, t_e, t_{an}, $T[t_{an}]$, c_{NT}^a and c_{NT}^n can be solved from the following system:

$$\begin{cases} \theta_H \cdot \int_{t_s-T_{ff}}^{t_{an}-T[t_{an}]-T_{ff}} H^n[t]dt = \theta_W \cdot \int_{t_s}^{t_{an}} W^n[t]dt, \\ \int_{t_{an}-T[t_{an}]-T_{ff}}^{t_e-T_{ff}} H^n[t]dt = \int_{t_{an}}^{t_e} W^n[t]dt, \\ t_e - t_{an} = N^n / s,\ t_{an} - t_s = N^a / s, \\ c_{NT}^n = \int_{t_e-T_{ff}}^{t^{n*}} H^n[t]dt - \int_{t_e}^{t^{n*}} W^n[t]dt, \\ c_{NT}^a = \theta_H \cdot \int_{t_s-T_{ff}}^{t^{a*}} H^n[t]dt - \theta_W \cdot \int_{t_s}^{t^{a*}} W^n[t]dt. \end{cases} \tag{B1}$$

The first two equations are derived by substituting the expressions of $c^a[t_s]$, $c^a[t_{an}]$, $c^n[t_{an}]$ and $c^n[t_e]$ in Eq. (4) into $c^a[t_s]=c^a[t_{an}]$ and $c^n[t_{an}]=c^n[t_e]$. Therefore, once the marginal utility functions are given, we can determine the arrival time choices through solving Eq. (B1) directly.

Appendix C. Proof of Proposition 4

We illustrate with $\theta_H < \theta_W$. According to the first two equations in the equation system (B1), we have:

$$c_{NT}^n = \int_{t_e-T_{ff}}^{t^{n*}} H^n[t]dt - \int_{t_e}^{t^{n*}} W^n[t]dt,\ c_{NT}^a = \theta_H \cdot \int_{t_s-T_{ff}}^{t^{a*}} H^n[t]dt - \theta_W \cdot \int_{t_s}^{t^{a*}} W^n[t]dt\,. \tag{C1}$$

Taking the derivate of c_{NT}^n and c_{NT}^a in Eq. (C1) with respect to N^a and N^n yields:

$$\begin{aligned} &\frac{\partial c_{NT}^n}{\partial N^n} = \left(W^n[t_e] - H^n[t_e - T_{ff}]\right) \cdot \frac{\partial t_e}{\partial N^n},\ \frac{\partial c_{NT}^a}{\partial N^n} = \left(\theta_W \cdot W^n[t_s] - \theta_H \cdot H^n[t_s - T_{ff}]\right) \cdot \frac{\partial t_s}{\partial N^n}, \\ &\frac{\partial c_{NT}^a}{\partial N^a} = \left(\theta_W \cdot W^n[t_s] - \theta_H \cdot H^n[t_s - T_{ff}]\right) \cdot \frac{\partial t_s}{\partial N^a}, \frac{\partial c_{NT}^n}{\partial N^a} = \left(W^n[t_e] - H^n[t_e - T_{ff}]\right) \cdot \frac{\partial t_e}{\partial N^a}. \end{aligned} \tag{C2}$$

Because all users travel in the period $[t_s, t_e]$, autonomous car users travel in the period $[t_s, t_{an}]$ and normal car users travel in the period $[t_{an}, t_e]$, we have:

$$\begin{aligned} &t_e - t_s = (N^a + N^n)/s,\ t_e - t_{an} = N^n / s,\ t_{an} - t_s = N^a / s \text{ and} \\ &\frac{\partial t_e}{\partial N^n} = \frac{\partial t_s}{\partial N^n} + \frac{1}{s}; \frac{\partial t_e}{\partial N^a} = \frac{\partial t_s}{\partial N^a} + \frac{1}{s} \end{aligned} \tag{C3}$$

Combining Eq. (C3) and the first two equations of Eq. (B1), we obtain:

$$\frac{\partial t_s}{\partial N^n}=\frac{\theta_H\cdot\left(H^n[t_e-T_{ff}]-W^n[t_e]\right)\cdot\dfrac{1}{s}}{\theta_H\cdot\left(H^n[t_s-T_{ff}]-H^n[t_e-T_{ff}]\right)+\theta_H\left(W^n[t_e]-W^n[t_{an}]\right)+\theta_W\cdot\left(W^n[t_{an}]-W^n[t_s]\right)}<0; \tag{C4}$$

$$\frac{\partial t_e}{\partial N^a}=\frac{\left(\theta_H H^n[t_s-T_{ff}]-\theta_W\cdot W^n[t_s]\right)\cdot\dfrac{1}{s}}{\theta_H\cdot\left(H^n[t_s-T_{ff}]-H^n[t_e-T_{ff}]\right)+\theta_H\left(W^n[t_e]-W^n[t_{an}]\right)+\theta_W\cdot\left(W^n[t_{an}]-W^n[t_s]\right)}>0; \tag{C5}$$

$$\frac{\partial t_s}{\partial N^a}=\frac{\partial t_e}{\partial N^a}-\frac{1}{s}=\frac{\theta_H\cdot\left(H^n[t_e-T_{ff}]-W^n[t_e]\right)+(\theta_H-\theta_W)\cdot W^n[t_{an}]}{\theta_H\cdot\left(H^n[t_s-T_{ff}]-H^n[t_e-T_{ff}]\right)+\theta_H\left(W^n[t_e]-W^n[t_{an}]\right)+\theta_W\cdot\left(W^n[t_{an}]-W^n[t_s]\right)}\frac{1}{s}<0; \tag{C6}$$

$$\frac{\partial t_e}{\partial N^n}=\frac{\partial t_s}{\partial N^n}+\frac{1}{s}=\frac{\left(\theta_H\cdot H^n[t_s-T_{ff}]-\theta_W\cdot W^n[t_s]\right)+(\theta_W-\theta_H)\cdot W^n[t_{an}]}{\theta_H\cdot\left(H^n[t_s-T_{ff}]-H^n[t_e-T_{ff}]\right)+\theta_H\left(W^n[t_e]-W^n[t_{an}]\right)+\theta_W\cdot\left(W^n[t_{an}]-W^n[t_s]\right)}\cdot\frac{1}{s}>0. \tag{C7}$$

Taking the difference between Eq. (C6) and Eq. (C4) yields:

$$\frac{\partial t_s}{\partial N^a}-\frac{\partial t_s}{\partial N^n}=\frac{(\theta_H-\theta_W)\cdot W^n[t_{an}]\cdot\dfrac{1}{s}}{\theta_H\cdot\left(H^n[t_s-T_{ff}]-H^n[t_e-T_{ff}]\right)+\theta_H\left(W^n[t_e]-W^n[t_{an}]\right)+\theta_W\cdot\left(W^n[t_{an}]-W^n[t_s]\right)}<0. \tag{C8}$$

Taking the difference between Eq. (C7) and Eq. (C5) yields:

$$\frac{\partial t_e}{\partial N^n}-\frac{\partial t_e}{\partial N^a}=\frac{(\theta_W-\theta_H)\cdot W^n[t_{an}]/s}{\theta_H\cdot H^n[t_s-T_{ff}]-\theta_W\cdot W^n[t_s]+(\theta_W-\theta_H)\cdot W^n[t_{an}]-\theta_H\cdot\left(H^n[t_e-T_{ff}]-W^n[t_e]\right)}>0. \tag{C9}$$

Substituting Eqs (C4)-(C7) into Eq. (C2) and taking the different between $\partial c_{NT}^n/\partial N^n$ and $\partial c_{NT}^a/\partial N^a$, and the difference between $\partial c_{NT}^a/\partial N^a$ and $\partial c_{NT}^a/\partial N^n$ yield:

$$\frac{\partial c_{NT}^n}{\partial N^n}-\frac{\partial c_{NT}^n}{\partial N^a}=(W^n[t_e]-H^n[t_e-T_{ff}])\cdot(\frac{\partial t_e}{\partial N^n}-\frac{\partial t_e}{\partial N^a})>0. \tag{C10}$$

$$\frac{\partial c_{NT}^a}{\partial N^a}-\frac{\partial c_{NT}^a}{\partial N^n}=(\theta_W\cdot W^n[t_s]-\theta_H\cdot H^n[t_s])\cdot(\frac{\partial t_s}{\partial N^a}-\frac{\partial t_s}{\partial N^n})>0. \tag{C11}$$

Hence, $\partial c_{NT}^n/\partial N^n>\partial c_{NT}^n/\partial N^a$ and $\partial c_{NT}^a/\partial N^a>\partial c_{NT}^a/\partial N^n$ are satisfied for $\theta_H<\theta_W$. Using the same logic, we can prove that when $\theta_H>\theta_W$, $\partial c_{NT}^a/\partial N^a>\partial c_{NT}^a/\partial N^n$ and $\partial c_{NT}^n/\partial N^n>\partial c_{NT}^n/\partial N^a$ are still satisfied.

Appendix D. Proof of proposition 5

Substituting Eq. (C2) into Eq. (7), we can find the marginal external cost imposed by users with car type i, MEC^i:

$$MEC^a = N^a \cdot (\theta_W \cdot W^n[t_s] - \theta_H \cdot H^n[t_s]) \cdot \frac{\partial t_s}{\partial N^a} + N^n (W^n[t_e] - H^n[t_e - T_{ff}]) \cdot (\frac{\partial t_s}{\partial N^a} + \frac{1}{s}). \quad \text{(D1)}$$

$$MEC^n = N^a \cdot (\theta_W \cdot W^n[t_s] - \theta_H \cdot H^n[t_s]) \cdot \frac{\partial t_s}{\partial N^n} + N^n \cdot (W^n[t_e] - H^n[t_e - T_{ff}]) \cdot (\frac{\partial t_s}{\partial N^n} + \frac{1}{s}). \quad \text{(D2)}$$

Taking the difference between Eq. (D1) and Eq. (D2) yields:

$$\begin{aligned} MEB &= MEC^n - MEC^a \\ &= \left(N^a \cdot (\theta_W \cdot W^n[t_s] - \theta_H \cdot H^n[t_s]) + N^n \cdot (W^n[t_e] - H^n[t_e - T_{ff}])\right) \cdot (\frac{\partial t_s}{\partial N^n} - \frac{\partial t_s}{\partial N^a}) \end{aligned} \quad \text{(D3)}$$

Substituting Eq. (C8) into Eq. (D3) yields:

$$\begin{aligned} MEB &= MEC^n - MEC^a \\ &= \frac{(\theta_W - \theta_H) \cdot W^n[t_{an}] \cdot \left(N^a \cdot (\theta_W \cdot W^n[t_s] - \theta_H \cdot H^n[t_s]) + N^n \cdot (W^n[t_e] - H^n[t_e - T_{ff}])\right)}{\theta_H \cdot \left(W^n[t_e] - H^n[t_e - T_{ff}]\right) + \left(\theta_H \cdot H^n[t_s - T_{ff}] - \theta_W \cdot W^n[t_s]\right) + (\theta_W - \theta_H) \cdot W^n[t_{an}]} \cdot \frac{1}{s}, \end{aligned} \quad \text{(D4)}$$

as expressed in Proposition 5. As $H^i[t]$ is non-increasing and $W^i[t]$ is non-decreasing over t, the denominator of Eq. (D4) is positive. Hence,

$$\begin{aligned} \frac{\partial MEB}{\partial N^a} &= \frac{(\theta_W - \theta_H) \cdot W^n[t_{an}] \cdot (\theta_W \cdot W^n[t_s] - \theta_H \cdot H^n[t_s]) / s}{\theta_H \cdot \left(W^n[t_e] - H^n[t_e - T_{ff}]\right) + \left(\theta_H \cdot H^n[t_s - T_{ff}] - \theta_W \cdot W^n[t_s]\right) + (\theta_W - \theta_H) \cdot W^n[t_{an}]} < 0 \\ \frac{\partial MEB}{\partial N^n} &= \frac{(\theta_W - \theta_H) \cdot W^n[t_{an}] \cdot (W^n[t_e] - H^n[t_e - T_{ff}]) / s}{\theta_H \cdot \left(W^n[t_e] - H^n[t_e - T_{ff}]\right) + \left(\theta_H \cdot H^n[t_s - T_{ff}] - \theta_W \cdot W^n[t_s]\right) + (\theta_W - \theta_H) \cdot W^n[t_{an}]} > 0 \end{aligned} . \quad \text{(D5)}$$

This completes the proof of Proposition 5 (i). Using the same logic, we can find the marginal external benefit under $\theta_H > \theta_W$.

Appendix E. Derivation of second-best public pricing of autonomous cars

The Lagrangian associated with the welfare-maximizing problem in Eq. (9) is:

$$\begin{aligned} \max_{N^a, N^n, MU^a} \quad SW = & \int_0^{N^a + N^n} D[n] dn - c_{NT}^a[N^a, N^n] \cdot N^a - c_{NT}^n[N^a, N^n] \cdot N^n - MC^a \cdot N^a \\ & - \lambda^a \cdot \left(D[N^a + N^n] - c_{NT}^a[N^a, N^n] - MC^a - MU^a\right) \\ & - \lambda^n \cdot \left(D[N^a + N^n] - c_{NT}^n[N^a, N^n]\right) \end{aligned} . \quad \text{(E1)}$$

Taking the derivatives of Eq. (E1) to N^a, N^n, MU^a, λ^a and λ^n yields:

$$\frac{\partial SW}{\partial N^a} = D - \frac{\partial c_{NT}^a}{\partial N^a} \cdot N^a - c_{NT}^a - \frac{\partial c_{NT}^n}{\partial N^a} \cdot N^n - MC^a - \lambda^a \cdot \left(D' - \frac{\partial c_{NT}^a}{\partial N^a}\right) - \lambda^n \cdot \left(D' - \frac{\partial c_{NT}^n}{\partial N^a}\right) = 0, \quad \text{(E2)}$$

$$\frac{\partial SW}{\partial N^n} = D - \frac{\partial c_{NT}^a}{\partial N^n} \cdot N^a - c_{NT}^n - \frac{\partial c_{NT}^n}{\partial N^n} \cdot N^n - \lambda^a \cdot \left(D' - \frac{\partial c_{NT}^a}{\partial N^n}\right) - \lambda^n \cdot \left(D' - \frac{\partial c_{NT}^n}{\partial N^n}\right) = 0, \quad \text{(E3)}$$

$$\frac{\partial SW}{\partial MU^a} = \lambda^a = 0, \tag{E4}$$

$$\frac{\partial SW}{\partial \lambda^a} = -\left(D[N^a + N^n] - c_{NT}^a[N^a, N^n] - MC^a - MU^a\right) = 0, \tag{E5}$$

$$\frac{\partial SW}{\partial \lambda^n} = -\lambda^n \cdot \left(D[N^a + N^n] - c_{NT}^n[N^a, N^n]\right) = 0. \tag{E6}$$

Substituting Eq. (E4) and Eq. (E6) into Eq. (E3) yields:

$$\lambda^n = \frac{-\frac{\partial c_{NT}^a}{\partial N^n} \cdot N^a - \frac{\partial c_{NT}^n}{\partial N^n} \cdot N^n}{\left(D' - \frac{\partial c_{NT}^n}{\partial N^n}\right)}. \tag{E7}$$

By substituting Eq. (E4), Eq. (E5) and Eq. (E7) into Eq. (E2), we can obtain the MU^a under second-best public supply, as expressed in Eq. (10). The same logic applies to profit-maximizing private monopolistic supply.

Appendix F. Proof of Proposition 9

This proof consists of two parts: (i) we first prove that under queue-eliminating road pricing, the travel pattern that autonomous car users travel in the center and normal car users travel in the shoulders of the peak period will never happen. We again use the contradiction method. Assume autonomous car users travel in the interval $[t_{na}, t_{an}]$, and normal car users travel in the intervals $[t_s, t_{na}]$ and $[t_{an}, t_e]$, as shown in Fig. 12.

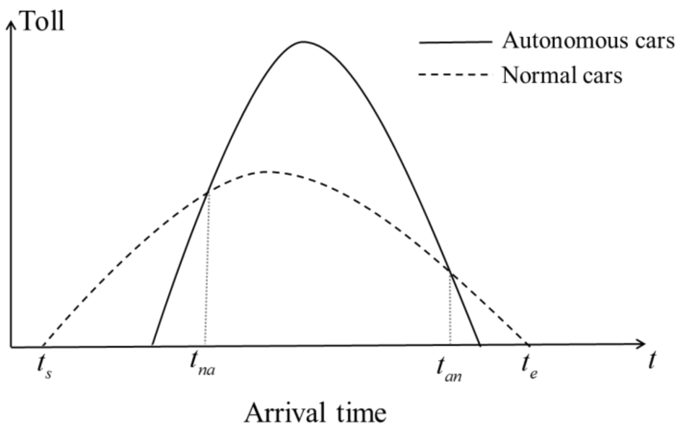


Fig. 12. Autonomous car users travel in the center of the bottleneck.

At arrival time t_{na}, the slope of the toll function for autonomous car users is larger than that for normal car users. From Eq.(18), we obtain:

$$\theta_H H^n[t_{na}-T_{ff}]-\theta_W W^n[t_{na}]>H^n[t_{na}-T_{ff}]-W^n[t_{na}]>0\,. \tag{F1}$$

Eq. (F1) can be further written as:

$$1<\frac{H^n[t_{na}-T_{ff}]}{W^n[t_{na}]}<\frac{1-\theta_W}{1-\theta_H}\text{, i.e., }\ \theta_W<\theta_H\,. \tag{F2}$$

Meanwhile, at arrival time t_{an}, the slope of the toll function for autonomous car users is smaller than that for normal car users. We obtain:

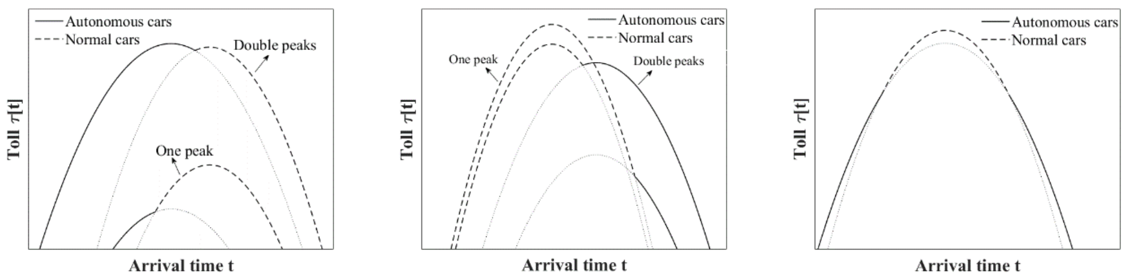
$$\theta_H H^n[t_{an}-T_{ff}]-\theta_W W^n[t_{an}]<H^n[t_{an}-T_{ff}]-W^n[t_{an}]<0\,. \tag{F3}$$

This implies:

$$1<\frac{W^n[t_{an}]}{H^n[t_{an}-T_{ff}]}<\frac{(1-\theta_H)}{(1-\theta_W)}\text{, i.e., }\ \theta_H<\theta_W\,. \tag{F4}$$

As Eq. (F2) contradicts Eq. (F4), the travel pattern in Fig. 12 will not happen.

(ii) We summarize the possible equilibrium travel patterns and the associated conditions for each case to happen in Fig. 13 and Table 7. The toll in Fig. 13 can be interpreted as iso-price curves. The specific travel patterns are jointly determined by the preference conditions and the time conditions, as presented in Table 7.



(a)Case 1 (Double/single peak) (b)Case 2 (Double/single peak) (c)Case 3 (two intersections)

Fig. 13. Possible equilibrium travel patterns under queue-eliminating road pricing.

Table 7. Conditions for each equilibrium travel pattern to happen.

Equilibrium	Case 1	Case 2	Case 3
Figures	Fig. 13a	Fig. 13b	Fig. 13c
Travel period (a)#	$[t_s, t_{an}]$	$[t_{na}, t_e]$	$[t_s, t_{an}]$ & $[t_{na}, t_e]$
Travel period (n)#	$[t_{an}, t_e]$	$[t_s, t_{na}]$	$[t_{an}, t_{na}]$
Preference conditions	$\frac{H^n[t_{an}-T_{ff}]}{W^n[t_{an}]} > \frac{(1-\theta_W)}{(1-\theta_H)}$	$\frac{H^n[t_{na}-T_{ff}]}{W^n[t_{na}]} < \frac{(1-\theta_W)}{(1-\theta_H)}$	$\frac{H^n[t_{an}-T_{ff}]}{W^n[t_{an}]} > \frac{(1-\theta_W)}{(1-\theta_H)}$ & $\frac{H^n[t_{na}-T_{ff}]}{W^n[t_{na}]} < \frac{(1-\theta_W)}{(1-\theta_H)}$
Time conditions*	$t_e > t_e^a$	$t_s < t_s^a$	$t_s < t_s^n, t_e > t_e^n$

(a) represents autonomous cars and (n) represents normal cars.

* t_s^i and t_e^i denote the starting and ending time for the traveling in the out-of-equilibrium continuation of the iso-price functions.

Appendix G. Proof of Proposition 10

The proof of Proposition 10 consists of two parts: (i) In equilibrium, the marginal social cost for certain car type users equals the generalized travel price. We again take case 1 as an example. In case 1, autonomous car users arrive in the interval $[t_s, t_{an}]$, and normal car users arrive in the interval $[t_{an}, t_e]$. In equilibrium, travelers with the same car type should have the same travel price regardless of their arrival times. The toll at arrival time *t* is the difference between the equilibrium travel price and the sum of the schedule delay cost and free-flow travel cost, which can be expressed as:

$$\tau[t] = \begin{cases} \tau^a[t] = \theta_H \cdot \int_{t_s - T_{ff}}^{t-T_{ff}} H^n[t]dt - \theta_W \cdot \int_{t_s}^{t} W^n[t]dt, \quad \textit{with an autonomous car}, \\ \tau^n[t] = \int_{t}^{t_e} W^n[t]dt - \int_{t^{n*}}^{t_e - T_{ff}} H^n[t]dt, \quad \textit{with a normal car}. \end{cases} \tag{G1}$$

The total social cost for all users, *TSC*, is:

$$TSC = \int_{t_s}^{t_{an}} (p_{CP}^a[t] - \tau^a[t]) \cdot s dt + \int_{t_{an}}^{t_e} (p_{CP}^n[t] - \tau^n[t]) \cdot s dt\,. \tag{G2}$$

By taking the derivative of the total social cost in Eq. (G2) to the number of autonomous car users, N^a, we find the marginal social cost of autonomous car users, MSC^a:

$$
\begin{aligned}
MSC^a = \frac{\partial TSC}{\partial N^a} &= (p_{CP}^a[t_{an}] - \tau^a[t_{an}]) \cdot s \cdot \frac{\partial t_{an}}{\partial N^a} - (p_{CP}^a[t_s] - \tau^a[t_s]) \cdot s \cdot \frac{\partial t_s}{\partial N^a} \\
&+ (p_{CP}^n[t_e] - \tau^n[t_e]) \cdot s \cdot \frac{\partial t_e}{\partial N^a} - (p_{CP}^n[t_{an}] - \tau^n[t_{an}]) \cdot s \cdot \frac{\partial t_{an}}{\partial N^a} \\
&= p_{CP}^a - \tau^a[t_{an}] \cdot s \cdot \frac{\partial t_{an}}{\partial N^a} + \tau^n[t_{an}] \cdot s \cdot \frac{\partial t_{an}}{\partial N^a} = p_{CP}^a
\end{aligned} \quad . \qquad \text{(G3)}
$$

Eq. (G3) implies that the marginal social cost for autonomous car users equals their equilibrium travel price.

Similarly, the marginal social cost for normal car users, MSC^n, satisfies:

$$
MSC^n = \frac{\partial TSC}{\partial N^n} = p_{CP}^n \, . \qquad \text{(G4)}
$$

Eq. (G3) and (G4) indicate that even with heterogeneous autonomous and normal car users, queue-eliminating tolling still achieves a social optimum when the tolls for the very first and the very last driver are zero. Using the same logic, we can find for the equilibria in case 2 and case 3, the marginal social costs again equal the corresponding travel prices.

(ii) The mark-up under welfare-maximizing public supply is zero.

The Lagrangian associated with the welfare-maximizing public supply problem is:

$$
\begin{aligned}
&\max_{N^a, N^n, MU^a, \lambda^a, \lambda^n} \quad SW = \int_0^{N^a+N^n} D[n]dn - TSC[N^a, N^n] - MC^a \cdot N^a \\
&-\lambda^a \cdot \left(D[N^a + N^n] - p_{CP}^a[N^a, N^n] - MU^a - MC^a \right) - \lambda^n \cdot \left(D[N^a + N^n] - p_{CP}^n[N^a, N^n] \right)
\end{aligned} \; .
$$

(G5)

Solving Eq. (G5) yields:

$$
MU^a = MSC^a - p_{CP}^a - \frac{\left(D' - \frac{\partial p_{CP}^n}{\partial N^a} \right)}{\left(D' - \frac{\partial p_{CP}^n}{\partial N^n} \right)} \cdot \left(MSC^n - p_{CP}^n \right) = 0 \, . \qquad \text{(G6)}
$$

Eq. (G6) implies that a marginal cost supply of autonomous cars achieves the social optimum.