TI 2022-003/VIII
Tinbergen Institute Discussion Paper

Will all autonomous cars cooperate? Brands' strategic interactions under dynamic congestion

Revision: July 2022

Xiaojuan Yu[1,2]
Vincent A.C. van den Berg[2,3]
Erik T. Verhoef[2,3]
Zhi-Chun Li[4]

1 Zhongnan University of Economics and Law

2 Vrije Universiteit Amsterdam

3 Tinbergen Institute

4 Huazhong University of Science and Technology

Tinbergen Institute is the graduate school and research institute in economics of Erasmus University Rotterdam, the University of Amsterdam and Vrije Universiteit Amsterdam.

Contact: discussionpapers@tinbergen.nl

More TI discussion papers can be downloaded at https://www.tinbergen.nl

Tinbergen Institute has two locations:

Tinbergen Institute Amsterdam
Gustav Mahlerplein 117
1082 MS Amsterdam
The Netherlands
Tel.: +31(0)20 598 4580

Tinbergen Institute Rotterdam
Burg. Oudlaan 50
3062 PA Rotterdam
The Netherlands
Tel.: +31(0)10 408 8900

Will all autonomous cars cooperate?
Brands' strategic interactions under dynamic congestion

Xiaojuan Yu[a,b*], Vincent A. C. van den Berg[b,c], Erik T. Verhoef[b,c], Zhi-Chun Li[d]

[a] School of Business Administration, Zhongnan University of Economics and Law, Wuhan 430073, China

[b] Department of Spatial Economics, VU Amsterdam, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands

[c] Tinbergen Institute, Gustav Mahlerplein 117, 1082 MS Amsterdam, The Netherlands

[d] School of Management, Huazhong University of Science and Technology, Wuhan 430074, China

Pre-print version from May 2022 (original January 2022)

Abstract

Autonomous cars allow safe driving with a smaller headway than that required for normal human-driven cars, thereby potentially improving road capacity. To attain this capacity benefit, cooperation among autonomous cars is vital. However, the future market may have multiple car brands and the incentive for them to cooperate is unknown. This paper investigates the competition and cooperation between multiple car brands that, may provide both autonomous and normal vehicles. We develop a two-stage game-theoretic model to investigate brands' strategic interactions and evaluate, from both policy and organizational perspectives, the implications of their cooperation incentives and pricing competition. We compare four market structures: duopoly competition, perfect competition, a public welfare-maximizing monopoly, and a private profit-maximizing monopoly. Various parameters are evaluated, including factors such as the capacity benefits from cooperation, cooperation cost and price elasticity. This evaluation provides policy insights into actions that could be considered by regulators and organizations for the operation of autonomous cars.

Keywords: Autonomous cars; Cooperation strategy; Duopoly competition; Game theory; Regulatory policy

* Corresponding author.
Email addresses: x.yu@vu.nl (Xiaojuan Yu), v.a.c.vandenberg@vu.nl (Vincent van den Berg), e.t.verhoef@vu.nl (Erik Verhoef), smzcli@gmail.com (Z.-C. Li).

1. Introduction

With the development of vehicle automation, car manufacturers and software companies have presented prototypes of autonomous cars and announced that these vehicles, with their advanced self-driving capabilities, are anticipated to be available to the consumer mass market in the near future (e.g., Burns, 2013; Fagnant and Kockelman, 2015; Wadud et al., 2016; Nieuwenhuijsen et al., 2018; Krueger et al., 2019). This innovative transportation technology will undoubtedly alter the way that travelers travel and vehicle ownership, as well as when autonomous vehicles and normal cars coexist on the roads.

By their use of sensors and mutual information-exchange technology, autonomous cars can sense and may even be able to anticipate the braking and acceleration decisions of the cars on front, thereby reducing reaction time and vehicle spacing in comparison to manually driven vehicles. The smaller headway and intervehicle spacing will allow autonomous cars to increase the capacity of the existing roads, contributing to reducing travel time losses and potential schedule delays in peak periods. It is widely demonstrated in the literature that predictions of possible effects on road capacity vary, from almost no effect to a quintupling of capacity (e.g., Fernandes and Nunes, 2012; Shladover et al., 2012). To amplify this capacity-improvement benefit, employing cooperation through vehicle-to-vehicle cooperative technology is vital.

There may, however, be multiple autonomous car brands in future vehicle markets. They may include traditional automakers, such as Audi, Toyota, Ford, and Volvo, which are not interested in becoming hardware suppliers for the navigational intelligence incorporated in their vehicles, as well as information technology (IT) companies, such as Google and Baidu, for whom information technology, connectivity, and automotive engineering are an integrated development. There is a strong incentive for policy makers to encourage different car brands to cooperate due to the potential benefits of capacity improvement. Not all brands, however, have an incentive to cooperate: even though cooperation with another brand may raise the efficiency of road use and reduce travel time for their own car users, the same is true for competing brands, so competitors are strengthened. As cooperation with another brand would also raise competitors' attractiveness, it would be more profitable to prevent cooperation with another brand.

Research on aspects of the impact of autonomous driving is still in its infancy. Existing studies regarding the operation of autonomous cars have assumed a single system and implicitly focused on one single car brand (e.g., Van den Berg and Verhoef, 2016; Yu et al., 2022; Sun and Yin, 2021), thereby ignoring the competition and cooperation between different brands. We instead investigate the cooperation incentives and pricing competition between multiple brands and evaluate the associated welfare effects. These questions become particularly complex when travelers perceive different brands and car types as imperfect substitutes, for reasons of such unobservable characteristics as brand loyalty, post-sale services, and traveler preferences for particular aspects of vehicles. Our model accommodates this.

In addition, compared to normal human-driven cars, autonomous cars allow travelers to free up the time traditionally spent in driving-related tasks and enable them to perform leisure- or work-related activities in the vehicle (Haboucha et al., 2017; Pudāne et al., 2018). This can be expected to loosen scheduling constraints that travelers face at home and at work, allowing them to avoid the heaviest traffic more easily. Hence, departure time change appears to be one of the most important alterations in behavior once autonomous cars are introduced and it is of great importance to take travelers' departure time choices into account when studying the effects of these vehicles.

Several earlier studies have investigated travel equilibria involving autonomous cars. Regarding the dynamic congestion setting, previous studies mainly built on the bottleneck model to investigate morning commute dynamics in single- or multi-modal transportation systems, in which autonomous cars were supplied by a single brand. For instance, Van den Berg and Verhoef (2016) investigated travelers' departure time and travel mode choices when autonomous cars and normal cars both exist, taking the effects of autonomous cars on the capacity, value of time, and preference heterogeneity into consideration. As a result of the reduced value of time caused by autonomous cars, their users travel in the center of the peak period, and normal car users travel in the shoulders of the period. By differentiating travelers' home- and work-related activities in autonomous cars, Yu et al. (2022) and Pudāne (2020) developed models of dynamic bottleneck congestion to investigate the impacts of travelers' different activity choices in autonomous cars on aggregate travel patterns. They found that autonomous and normal cars always travel separately, and the specific travel orders depended heavily on utility functions. In their work, Lamotte et al. (2017) investigated how capacity should be allocated to autonomous and normal cars by assuming that autonomous car users are separated from conventional users in their use of road capacity and need to book their trip in advance. Liu (2018) and Zhang et al. (2019) studied the joint equilibrium of departure time and parking location choices when all travelers travel with autonomous cars. Considering the interaction between normal cars and shared autonomous cars, Tian et al. (2019) investigated dynamic departure patterns and endogenous penetration rates of shared autonomous cars under a parking space constraint. Tang et al. (2021) further examined how to regulate the market in the presence of parking space constraints and shared autonomous cars in a multi-modal transportation system, in which travelers could choose among shared autonomous cars, private regular cars, and public transit.

With respect to the network equilibrium problem, studies have proposed various network equilibrium models to take the effects of autonomous cars on capacity into account (e.g., Chen et al., 2016; Chen et al., 2017; Liu and Song, 2019; Zheng et al., 2020). Wu et al. (2020) discussed the traffic flow patterns under a linear traffic corridor with expressways for autonomous cars running alongside streets for manually driven cars, whereby a trip can consist of both self-driving on an express way and self-driving on a non-autonomous street. However, these studies only focused on a single brand of autonomous car, such that they acknowledge the impacts of autonomous cars on road capacity but do not explore the cooperation incentives among multiple brands.

In this paper, we use a bottleneck framework to investigate travelers' travel behavior, in which travelers choose departure times and travel modes to minimize their generalized price. To understand the incentives for different brands to cooperate, we then adopt a two-stage game-theoretic framework, in which brands choose cooperation strategies and pricing decisions to maximize their own profits. In the first stage, brands decide their cooperation strategies, taking the impact of the second, price-setting stage into account. Cooperation with another brand also makes competitors' autonomous cars more effective; a duopoly sees this as a downside, whereas a private or public monopolist owning both brands would regard this as an advantage. We therefore compare four types of market structure: duopoly competition, perfect competition, a public welfare-maximizing monopoly, and a private profit-maximizing monopoly. All have multiple product variants.

Our research is, from a methodological viewpoint, closely related to the growing body of literature on externality regulation in aviation and private roads when firms have market power (e.g., Daniel, 1995; de Palma and Lindsey, 2000; Brueckner, 2002; Pels and Verhoef, 2004; Van Dender, 2005; Zhang and Zhang, 2006; Silva and Verhoef, 2013; Van den Berg, 2013; Fu et al., 2018; Kuang et al., 2020). Although some of the results of social and monopolistic supply pricing that we draw upon reflect insights from earlier works, there are important differences. In particular, because in previous networks operators did not need to decide whether to employ within- or cross-brand cooperation to make full use of the capacity benefits, the nature of the interaction between congestion and market power in these earlier studies is different from that in our setting (where brands' cooperation strategies and the share of autonomous cars have an extensive impact on capacity). In addition, to the best of our knowledge, this paper is the first study to investigate the competition and cooperation between multiple car brands involving autonomous cars.

The main contribution of this study is an in-depth investigation of strategic interactions between different car brands and understanding of the incentives for different brands to cooperate, using a joint theoretical game model and dynamic congestion model. This study moves beyond prior studies that simply consider a single car brand in the market. It is found that a duopoly can have multiple equilibria in a cooperation strategy. Unless cooperation costs are too high, cross-brand cooperation is one of them, with the highest profits and highest welfare; but it is by no means certain that it is the Nash equilibrium that will prevail.

Second, by exploring the roles of different market structures, we compare the proposed duopoly model with a public welfare-maximizing monopoly model and a private profit-maximizing monopoly model. Duopolistic pricing only partly internalizes the externalities that each duopoly's car users impose upon one another, whereas a public monopolist and a private monopolist both fully internalize the congestion externalities on all users. Different from the multiple equilibria for duopoly markets, with a public monopolist or a private monopolist, the equilibrium is always unique: either cooperation across "brands" (both variants, then provided by one monopolist) or no cooperation at all (when the cooperation cost is high).

Third, in addition to presenting a model and solving for the equilibrium conditions, we provide exemplary and illustrative results of the effects of change on important organizational and travel parameters. These parameter scenario analyses provide additional insights for organizational and regulatory policy issues that can then be used for evaluation and justification of alternative decisions for regulators seeking to regulate car supply and traffic congestion to improve welfare.

The remainder of this paper is organized as follows. Section 2 presents the modelling framework, which includes travelers' travel decisions in dynamic congestion, and the brands' two-stage game model. Section 3 analyzes the strategic interactions between two brands, mainly discussing Bertrand pricing competition for imperfect substitutes and solving the equilibria for the brands' cooperation decisions. Section 4 compares other market structures, considering perfect competition, a public welfare-maximizing monopoly, and a private profit-maximizing monopoly. Section 5 presents numerical illustrations and sensitivity analysis. Section 6 concludes the paper.

2. Problem description and model formulation

We consider the situation in which, each morning, travelers travel from home to a workplace. They use the same, single road, which is subject to bottleneck congestion. Everybody travels by car, either in an autonomous or in a normal vehicle. This paper focuses on privately owned autonomous cars; shared autonomous cars are thus not considered. Despite shared autonomous cars being considered as more environmentally sustainable, privately owned autonomous cars may turn out to be preferred by consumers (Zhang et al., 2018) based on several recent autonomous vehicle preference survey results (Bansal et al., 2016; Krueger et al., 2016; Haboucha et al., 2017). We consider price-sensitive demand: if autonomous cars lower costs, demand will increase.

For the model development, we first consider a duopoly model with horizontally differentiated products. In this duopoly game, two competing brands in the market, denoted by 1 and 2, provide both autonomous and normal cars to potential travelers.[1] Travelers regard these cars as imperfect substitutes since there are other factors involved, such as loyalty, service levels, and consumer preferences.

Autonomous cars are expected to increase road capacity, especially when

[1] We use car types to distinguish autonomous cars and normal cars, and brands to denote cars from different brand firms.

vehicle-to-vehicle cooperative technology is employed. Nevertheless, as cooperation with another brand would also raise the competitor's attractiveness, not all brands have an incentive to cooperate. This paper focuses on brands' cooperation decisions for autonomous cars; cooperative communication between normal cars is thus ignored.

We consider that each brand has three cooperation options for its autonomous cars: not to cooperate at all, to cooperate only within own brand, and to cooperate across brands. That is, the set of the cooperation strategies for each brand is {not cooperate, cooperate within brand, cooperate across brands}. Within-brand cooperation means autonomous cars can only communicate with vehicles of their own brand. Cross-brand cooperation includes cooperation within own brand and cooperation between different brands, which means that autonomous cars can communicate with vehicles of their own brand and with vehicles of the competitor's brand. It is assumed that only when both brands want their vehicles to cooperate with the cars in the other brand do they cooperate successfully across brands. Brands' cooperation decisions have a significant impact on bottleneck capacity, and hence affect travelers' travel behavior.

Travelers' behavior is characterized by the dynamic congestion model: travelers are rational utility maximizers, who seek to minimize their generalized travel price by choosing their departure time and travel mode (autonomous versus normal manual driving). Brands' behaviors are characterized in a two-stage game-theoretic model. Both brands are profit-maximizing decision makers. In the first stage, they make their cooperation decisions separately and simultaneously. In the second stage, they compete for supply quantity and pricing to maximize their own profit, taking the decisions in the first stage into account. Section 4 will investigate other market structures.

For ease of reference, Table 1 below summarizes the notation used in this paper. The notation will also be introduced in the text.

2.1 Modelling travelers' travel decisions under imperfect substitutes

Travelers' departure time choices are characterized by Vickrey's (1969) bottleneck model, and travel mode choices by the Wardrop user equilibrium with imperfect substitutes.

The bottleneck model assumes that travelers dislike waiting in traffic congestion, and dislike arriving either earlier or later than the preferred arrival time, t^*. A traveler's travel cost consists of schedule delay cost and travel time cost.[2] Schedule delay cost is the cost due to arriving at a time different from the most preferred arrival time t^*. For travelers arriving early, the schedule delay cost is the product of how early they arrive, measured by $t^* - t$, and the schedule delay value of arriving early, denoted by β. For travelers arriving late, the schedule delay cost is the product of how late they arrive, measured by $t - t^*$, and the schedule delay value of arriving late, denoted by γ.

[2] For a complete review, see Arnott et al. (1993), Small (2015), and Li et al. (2020).

Table 1. Notation list.

Notations for brands' possible cooperation strategies	
NN	Neither brand 1 nor brand 2 employs cooperation between vehicles
YN	Brand 1 employs cooperation within own brand; brand 2 does not employ cooperation
NY	Brand 1 does not employ cooperation; brand 2 employs cooperation within own brand
YY	Brands 1 and 2 both employ cooperation, but only within own brand
$Y_X Y_X$	Brands 1 and 2 both employ cooperation across brands
Notations in the model	
t^*	Preferred arrival time
t	Arrival time
β	Value of scheduling delay early: the cost of arriving an hour earlier than t^*
γ	Value of scheduling delay late: the cost of arriving an hour later than t^*
α	Value of time (VOT) for normal car users: the cost of an hour of travel time
θ	VOT reduction parameter for autonomous car users
$TT[t]$	Travel time for travelers arriving at time t
$c_i[t]$	Travel cost for car type i users arriving at time t, $i \in \{a,n\}$
δ	Equilibrium travel cost for users with car type i
c_i	Equilibrium travel cost for users with car type i
N_{ij}	Number of users with car type i of brand j, $i \in \{a,n\}$, $j \in \{1,2\}$
s	Bottleneck capacity for normal cars
s_a	Bottleneck capacity for autonomous cars
$s_{a(m)}$	Averaged bottleneck capacity for autonomous cars under strategy profile m
T_f	Free-flow travel time
D_{ij}	Inverse demand function for car type i of brand j
A_{ij}	Maximum willingness to pay for car type i of brand j
b_{ij}	Parameters in the inverse demand functions
MU_{ij}	Per trip mark-up on car type i of brand j
MC_{ij}	Marginal automobile cost of car type i of brand j
MC_{cop}	Extra cost for within-brand or cross-brand cooperation
cop_j	Dummy variable: 0 denotes not cooperating, and 1 denotes cooperating
f_{ij}	Proportion of product i of brand j in all products i
$R_m[f_{a1}]$	Function determining capacity effects under strategy profile m
Π_j	Profit of brand j
λ_{ij}	Lagrangian multipliers referring to the travel equilibrium condition
MU_{ij}^r	Brand j 's best response function on the mark-up of car type i
MU_{ij}^{NE}	Bertrand-Nash equilibrium solutions for the mark up
$\Pi_{j,m}$	Profit of brand j under strategy profile m
W	Social welfare
B	Consumer benefit
ω	Relative efficiency

Travel time cost is the sum of the cost of queuing at the bottleneck and the free-flow travel cost. The cost of queuing equals the value of time (VOT) multiplied by the delay caused by queuing. As autonomous cars can drive themselves, the travel time can be used for other activities, which will lower the VOT for autonomous car users. Suppose the value of time for normal car users is α, and is $\theta \cdot \alpha$ for autonomous car users, where $0<\theta<1$ is the VOT reduction parameter. The queuing delay at the bottleneck equals the number of cars in the queue divided by the capacity of the bottleneck.

Let $TT[t]$ denote the travel time of arriving at time *t*, which is the sum of the delay from queuing at the bottleneck and the free-flow travel time. The travel cost for arriving at time t with car type i, $c_i[t]$, can thus be expressed as:

$$
\begin{aligned}
&c_n[t]=\alpha \cdot TT[t]+\begin{cases}\beta(t^*-t), & \text{if } t\le t^* \\ \gamma(t-t^*), & \text{if } t>t^*\end{cases} \quad \text{with a normal car} \\
&c_a[t]=\theta\alpha \cdot TT[t]+\begin{cases}\beta(t^*-t), & \text{if } t\le t^* \\ \gamma(t-t^*), & \text{if } t>t^*\end{cases} \quad \text{with an autonomous car}
\end{aligned}, \tag{1}
$$

where subscript '*n*' denotes a normal car and '*a*' an autonomous car.

In equilibrium, the queueing time of normal car users grows at a rate of β/α for early arrivals and $-\gamma/\alpha$ for late arrivals. For autonomous car users, the queuing time grows at a rate of $\beta/(\theta\alpha)$ for early arrivals and $-\gamma/(\theta\alpha)$ for late arrivals. Following Arnott et al. (1989), Lindsey (2004), and Van den Berg and Verhoef (2011a, b, 2016), autonomous cars and normal cars will travel separately over time; that is, autonomous car users travel in the center of the peak period and normal car users travel in the shoulders of the peak period.

As autonomous cars and normal cars self-select to travel separately in time, the increase in the capacity of autonomous cars does not affect the capacity when normal cars pass the bottleneck. Note that the cooperation pattern of an autonomous car does not affect the users' VOT and schedule delays, so there is no temporal separation of sub-groups of autonomous car users. In line with convention, we define the following compound preference parameter $\delta=\beta\gamma/(\beta+\gamma)$. Following Van den Berg and Verhoef (2011a, b), the equilibrium travel cost, c_i, can be shown to be:

$$
\begin{aligned}
c_n&=\frac{\delta(N_{n1}+N_{n2})}{s}+\frac{\delta(N_{a1}+N_{a2})}{s_a}+\alpha T_f, \\
c_a&=\frac{\theta\delta(N_{n1}+N_{n2})}{s}+\frac{\delta(N_{a1}+N_{a2})}{s_a}+\theta\cdot\alpha T_f,
\end{aligned} \tag{2}
$$

where N_{ij} denotes the number of users with car type i of brand j $(i\in\{a,n\},\ j\in\{1,2\})$, s denotes the capacity of the bottleneck when normal cars pass, s_a is the capacity of the bottleneck when autonomous cars pass, and T_f is the free-flow travel time. The bottleneck capacity for autonomous cars depends on brands' cooperation strategies, which will be defined as $s_{a(m)}$ later (see Eq. (5)), with m representing a given strategy profile.

For the car brands' market, we consider the differentiated duopoly proposed by Dixit (1979), assuming that demands arise from the quadratic utility function. We consider all car types as imperfect substitutes to account for the fact that not all travelers choose the car with the most attractive generalized price, therefore allowing cars with different generalized prices to have travelers in equilibrium. The quadratic utility function gives rise to a linear demand structure, with equivalent inverse and direct demands, D_{ij}, which is a function of a brand's own choice of outputs, as well as the competitor's outputs:

$$\begin{pmatrix} D_{a1} \\ D_{n1} \\ D_{a2} \\ D_{n2} \end{pmatrix} = \begin{pmatrix} A_{a1} \\ A_{n1} \\ A_{a2} \\ A_{n2} \end{pmatrix} - \begin{pmatrix} b_{a_1a_1} & b_{a_1n_1} & b_{a_1a_2} & b_{a_1n_2} \\ b_{a_1n_1} & b_{a_2a_2} & b_{n_1a_2} & b_{n_1n_2} \\ b_{a_1a_2} & b_{n_1a_2} & b_{a_2a_2} & b_{a_2n_2} \\ b_{a_1n_2} & b_{n_1n_2} & b_{a_2n_2} & b_{n_2n_2} \end{pmatrix} \begin{pmatrix} N_{a1} \\ N_{n1} \\ N_{a2} \\ N_{n2} \end{pmatrix}. \tag{3}$$

Where the inverse demand D_{ij} measures the marginal willingness to pay for car type i of brand j in terms of the generalized price. The coefficients $A_{ij}\ (i \in \{a,n\},\ j \in \{1,2\})$ and $b_{kl}\ (k \in \{a_1,n_1,a_2,n_2\},\ l \in \{a_1,n_1,a_2,n_2\})$ are both positive parameters: A_{ij} is the intercept or the maximum marginal willingness to pay for car type i of brand j; and b_{kl} measures how much the inverse demand decreases when the number of corresponding car users increases, where $b_{kl}\ (l = k)$ measures the own effect (i.e., how much the marginal willingness to pay for product i of brand j decreases if there are more users of this product), and $b_{kl}\ (l \neq k)$ measures the cross effect (i.e., how much the marginal willingness to pay for product i of brand j decreases if there are more users of the substitutes).[3]

As travelers have idiosyncratic preferences for car brands and car types, they would have different preferences for competing cars when these have an equal generalized price. Travel equilibrium requires that the marginal willingness to pay for car type $i\ (i \in \{a,n\})$ of brand $j\ (j \in \{1,2\})$ equals the associated generalized price. The generalized price per trip consists of the travel cost, the mark-up on cars (over marginal cost), the automobile cost (i.e., the marginal resource cost of making a car trip), and the possible cost of employing cooperation between cars. The mark-up is determined by the brand firms and is expressed in a per-trip equivalent. It is assumed that within-brand cooperation and cross-brand cooperation have the

[3] For $\forall\, kl \neq ii\ (i = a_1, n_1, a_2, n_2)$, the corresponding cars are perfect substitutes if $b_{kl} = b_{ii}$, imperfect substitutes if $b_{kl} < b_{ii}$, and independent if $b_{kl} = 0$. If b_{kl} were negative, the products would be complements, a possibility that we shall ignore.

same cooperation cost, MC_{cop}, which is constant per trip.[4] The Wardrop user equilibrium conditions can thus be expressed as:

$$\begin{aligned} &D_{aj} = c_a + MU_{aj} + MC_{aj} + MC_{cop} \cdot cop_j, \quad j \in \{1,2\}. \\ &D_{nj} = c_n + MU_{nj} + MC_{nj}, \quad j \in \{1,2\}. \qquad , \\ &cop_j = \begin{cases} 1, \ cooperate\ within\ brand\ or\ across\ brands \\ 0, \ otherwise \end{cases} \end{aligned} \quad (4)$$

where MU_{aj} and MU_{nj} denote the mark-ups on autonomous cars and normal cars of brand j, respectively; MC_{aj} and MC_{nj} denote the marginal automobile cost of autonomous cars and normal cars produced by brand j, respectively; and cop_j is a dummy variable to characterize brand j's cooperation strategy, where 1 means cooperation between cars and the dummy is 0 otherwise.

2.2 Brands' possible cooperation strategies

Each brand decides its cooperation strategy separately and simultaneously. This decision carries a trade-off in the sense that cooperation not only reduces travel times for a brand's own autonomous car users, but also for the competing brand's autonomous car users, through increasing the road capacity and at the expense of an extra cooperation cost. Therefore, brands choose their cooperation strategies by carefully balancing these factors. As we assume that only when both brands want their vehicles to cooperate with the cars in the other brand they can cooperate successfully across brands, the theoretical possibility of cooperation with the other brand's vehicles, but not within own brand, is discarded. Consequently, five possible Nash equilibria of brands' cooperation strategies can be reached:

(not cooperate, not cooperate), in which neither brand 1 nor brand 2 employs cooperation between vehicles. For presentation purposes, we use *NN* to denote this case.

(cooperate within brand, not cooperate), in which brand 1 employs cooperation within its own brand and brand 2 does not employ cooperation. We use *YN* to denote this case.

(not cooperate, cooperate within brand), in which brand 1 does not employ cooperation, and brand 2 employs cooperation within its own brand. We use *NY* to denote this case.

(cooperate within brand, cooperate within brand), in which brands 1 and 2 both employ cooperation, but only within their own brand. We use *YY* to denote this case.

(cooperate across brands, cooperate across brands), in which brands 1 and 2 both employ cooperation within own brand and between the brands. We use $Y_X Y_X$ to denote this case.

More detailed analysis of these equilibria is carried out in section 3.2. As we discuss later, brands' cooperation strategies have a significant impact on travelers' travel cost and marginal willingness to pay, through changing the effective capacity of the road.

[4] Cooperation cost is similar to the economics term of "transaction cost", which can be understood as a transaction cost specific to employing cooperation between vehicles.

2.3 Impact of brands' cooperation strategies on capacity

We now introduce the impacts of autonomous cars on road capacity under different cooperation strategies. As all autonomous cars travel jointly in a mixed flow, and the order of autonomous car drivers is random and independent of the cooperation regimes between the cars, it is reasonable and intuitive to assume that there is a constant averaged capacity for autonomous cars that increases with the market share of cooperative autonomous cars. Incorporation of capacity interaction among driving orders of heterogeneous autonomous cars can be made in a further model extension.

Given a certain strategy profile m $(m \in \{NN, YN, NY, YY, Y_X Y_X\})$, let $s_{a(m)}$ denote the averaged capacity for autonomous cars under m. Compared to normal cars, autonomous cars can drive with shorter headways. Hence, for any m, $s_{a(m)} \geq s$ always holds.

Let f_{ij} denote the market share of i-type cars of brand j in all i-type cars, which satisfies $f_{ij} = N_{ij} \big/ (N_{i1} + N_{i2})$. Given the share of brand 1's autonomous cars, f_{a1}, and brands' strategy profile, m, we use $1/R_m[f_{a1}]$ to characterize the capacity effects of autonomous cars (so that $f_{a2} = 1 - f_{a1}$ need not be defined separately).[5] The averaged capacity for autonomous cars satisfies $s_{a(m)} = s / R_m[f_{a1}]$, which can be specified as:[6]

$$s_{a(m)} = \begin{cases} s / R_{NN}[f_{a1}], & \text{if } m = NN, \\ s / R_{YN}[f_{a1}], & \text{if } m = YN, \\ s / R_{NY}[f_{a1}], & \text{if } m = NY, \\ s / R_{YY}[f_{a1}], & \text{if } m = YY, \\ s / R_{Y_X Y_X}[f_{a1}], & \text{if } m = Y_X Y_X. \end{cases} \tag{5}$$

It should be noted that the expression of $R_m[f_{a1}]$ does not impact the formulation of the theoretical model. For ease of understanding, the theoretical properties of the averaged capacity proposed in this paper are summarized as follows.

Under no cooperation at all (*NN*) or cross-brand cooperation ($Y_X Y_X$), all autonomous cars are homogeneous in terms of facing the same capacity. Therefore, we assume a fixed increase in capacity compared to normal cars. Let $R_{Y_X Y_X}[f_{a1}] = k_{Y_X Y_X}$ and $R_{NN}[f_{a1}] = k_{NN}$, where $k_{Y_X Y_X}$ and k_{NN} are positive constants not exceeding 1. As a result, the road capacity for autonomous cars without cooperation, is $s_{a(NN)} = s / k_{NN}$, and for autonomous cars with cross-brand cooperation, is $s_{a(Y_X Y_X)} = s / k_{Y_X Y_X}$, with $s / k_{Y_X Y_X} \geq s / k_{NN}$.

Under *YN* or *NY*, the vehicles of one brand cooperate with those within its own brand

[5] $f_{a1} = N_{a1} / (N_{a1} + N_{a2})$. Since $f_{a1} + f_{a2} = 1$, the function of f_{a2} can always be transferred to the function of f_{a1}.

[6] We use $1/R$ to characterize the capacity effects because s is the denominator in the travel cost function (see Eq. (2)).

and the vehicles of the other brand do not. When there is mixing of autonomous cars with different cooperation regimes, the capacity effects may be less beneficial, as the strongest gains are likely to be realized when achieving a situation with exclusively fully cooperative autonomous cars. The literature demonstrates that autonomous cars increases the road capacity more when cooperative autonomous cars form the larger fraction of the fleet of vehicles (Tientrakool et al., 2011; Fernandes and Nunes, 2012; Shladover et al., 2012). This would make the averaged capacity for autonomous cars a convexly increasing function of the share of cooperative autonomous cars, i.e., $\partial s_{a(YN)} / \partial f_{a1}[f_{a1}] > 0$ and $\partial^2 s_{a(YN)}[f_{a1}] / \partial f_{a1}^2 \geq 0$. Therefore, following Van den Berg and Verhoef (2016), we assume that $R_{YN}[f_{a1}]$ is a concavely decreasing function of f_{a1}, which we ensure by using $\partial R_{YN}[f_{a1}] / \partial f_{a1} < 0$ and $\partial^2 R_{YN}[f_{a1}] / \partial f_{a1}^2 \leq 0$. Capacity under strategy profile NY, $s / R_{NY}[f_{a1}]$, follows directly from symmetry.

Under *YY*, the two brands both apply within-brand cooperation. From the symmetry between the brands, the averaged capacity depends on which brand of autonomous cars has a larger share. Note that $f_{a1} + f_{a2} = 1$, the increase in f_{a1} also means a reduction in f_{a2}. Accordingly, when $f_{a1} \leq 0.5$, the decrease in f_{a2} dominates, which leads to a decreasing averaged capacity. When $f_{a1} > 0.5$, the increase in f_{a1} dominates, which leads to an increasing averaged capacity. That is, as f_{a1} increases, the averaged capacity first decreases and then increases. Specifically, $s / R_{YY}[f_{a1}]$ is at its maximum when $f_{a1} = 0$ and $f_{a1} = 1$, and at its minimum when $f_{a1} = 0.5$. Therefore, we assume $\partial R_{YY}[f_{a1}] / \partial f_{a1} > 0$ for $f_{a1} \leq 0.5$, and $\partial R_{YY}[f_{a1}] / \partial f_{a1} < 0$ for $f_{a1} \geq 0.5$.

As a result of the limited information, more detailed expressions for $R_m[f_{a1}]$ are calibrated using simulation approach in the numerical model in Section 5.

2.4 Modelling brands' two-stage game

Each brand aims at maximizing its own profit by determining the mark-ups on its own autonomous and normal cars and deciding the cooperation strategy for its autonomous vehicles. The per-trip profit from a specific car type equals the number of cars multiplied by the mark-up. We consider that the two brands (1 and 2) compete for travelers in terms of pricing, keeping the other brands' prices fixed; i.e., there is Bertrand competition with imperfect substitutes. Brands' duopolistic behavior can then be characterized by the following two-stage game model.

Stage one: Each brand decides its cooperation strategy separately and simultaneously to maximize its own profit, while considering the effects on the second stage and the other brand's cooperation strategy as given.

Stage two: Each brand decides its mark-ups to maximize its profit, under the constraints of the Wardrop user equilibrium conditions in Eq. (4), and given the pre-committed cooperation strategy.

Given the pre-committed cooperation strategy, brand *j*'s profit, Π_j, is the sum of the profit from its autonomous cars and normal cars, which can be represented as:[7]

$$\Pi_j = MU_{aj} \cdot N_{aj} + MU_{nj} \cdot N_{nj} . \quad (6)$$

Using backward induction, we first find the second-stage Bertrand-Nash equilibrium for the static pricing (i.e., mark-up) competition game, taking the brands' cooperation strategies as given. By calculating the equilibrium mark-ups, we can obtain the brands' reduced-form profit functions, conditional on their cooperation strategies. The first stage then examines the brands' cooperation decisions, whereby they consider the effects on the outcome of the second, pricing stage, particularly in terms of profits.

3. Solving for equilibrium of the two-stage game

As we consider four alternative types and brands of car and the measurement of capacity effects is complicated, it is hard to obtain specific expressions for the travel equilibrium solutions analytically. In the following, we explore the analytical properties of the decisions in the two stages.

3.1 Stage two: pricing competition

In stage two, we take the brands' cooperation strategies as given, and as determined in stage one, and look for the Bertrand-Nash equilibrium in the mark-ups. This means that each brand takes the competitor's mark-ups as fixed, but recognizes that quantities will adjust to maintain equilibrium. Namely, given the pre-committed cooperation strategy, each brand, j, maximizes its own profit by setting the mark-ups (MU_{aj}, MU_{nj}). This profit maximization problem under constraint (4) is equivalent to solving the following Lagrangian:

$$\begin{aligned}\Pi_j &= MU_{aj} \cdot N_{aj} + MU_{nj} \cdot N_{nj} \\ &-\sum_{j=1}^{2} \lambda_{aj} \cdot \left(D_{aj}[N_{a1}, N_{n1}, N_{a2}, N_{n2}] - c_a[N_{a1}, N_{n1}, N_{a2}, N_{n2}] - MU_{aj} - MC_{aj} - MC_{cop} \cdot cop_j \right), \quad (7) \\ &-\sum_{j=1}^{2} \lambda_{nj} \cdot \left(D_{nj}[N_{a1}, N_{n1}, N_{a2}, N_{n2}] - c_n[N_{a1}, N_{n1}, N_{a2}, N_{n2}] - MU_{nj} - MC_{nj} \right)\end{aligned}$$

---

[7] For presentation purpose, here we omit the strategy indicator in the profit function in stage two, and will take it into account in stage one.

where λ_{aj} and λ_{nj} are Lagrangian multipliers referring to the Wardrop user equilibrium conditions in Eq. (4).

To find the best response of brand *j* to any action of the other brand (-*j*), we fix $MU_{i,(-j)}$ and take the derivatives of Π_j with respect to MU_{aj}, MU_{nj}, N_{a1}, N_{n1}, N_{a2}, and N_{n2}. Thus, brand *j*'s best response function for the mark-up on car type *i*, MU_{ij}^r, is given by (see Appendix A):[8]

$$MU_{ij}^r\left[MU_{a(-j)}, MU_{n(-j)}\right] = \frac{\partial c_i}{\partial N_{ij}} N_{ij} + \frac{\partial c_{(-i)}}{\partial N_{ij}} N_{(-i)j} - \frac{\partial D_{ij}}{\partial N_{ij}} N_{ij} - \frac{\partial D_{(-i)j}}{\partial N_{ij}} N_{(-i)j} - \lambda_{i(-j)} \cdot \left(\frac{\partial c_i}{\partial N_{ij}} - \frac{\partial D_{i(-j)}}{\partial N_{ij}} \right) - \lambda_{(-i)(-j)} \cdot \left(\frac{\partial c_{(-i)}}{\partial N_{ij}} - \frac{\partial D_{(-i)(-j)}}{\partial N_{ij}} \right) \quad (8)$$

$$= \frac{\partial c_i}{\partial N_{ij}} \cdot (N_{ij} - \lambda_{i(-j)}) + \frac{\partial c_{(-i)}}{\partial N_{ij}} \cdot (N_{(-i)j} - \lambda_{(-i)(-j)}) - \frac{\partial D_{ij}}{\partial N_{ij}} N_{ij} - \frac{\partial D_{(-i)j}}{\partial N_{ij}} N_{(-i)j} + \lambda_{i(-j)} \frac{\partial D_{i(-j)}}{\partial N_{ij}} + \lambda_{(-i)(-j)} \frac{\partial D_{(-i)(-j)}}{\partial N_{ij}}, \quad (9)$$

with

$$\lambda_{ij} = \frac{\begin{pmatrix} N_{i(-j)} \cdot \left((\frac{\partial D_{i(-j)}}{\partial N_{ij}} - \frac{\partial c_i}{\partial N_{ij}})(\frac{\partial D_{(-i)j}}{\partial N_{(-i)j}} - \frac{\partial c_{(-i)}}{\partial N_{(-i)j}}) - (\frac{\partial D_{i(-j)}}{\partial N_{(-i)j}} - \frac{\partial c_i}{\partial N_{(-i)j}})(\frac{\partial D_{(-i)j}}{\partial N_{ij}} - \frac{\partial c_{(-i)}}{\partial N_{ij}}) \right) \\ + N_{(-i)(-j)} \cdot \left((\frac{\partial D_{(-i)(-j)}}{\partial N_{ij}} - \frac{\partial c_{(-i)}}{\partial N_{ij}})(\frac{\partial D_{(-i)j}}{\partial N_{(-i)j}} - \frac{\partial c_{(-i)}}{\partial N_{(-i)j}}) - (\frac{\partial D_{(-i)(-j)}}{\partial N_{(-i)j}} - \frac{\partial c_{(-i)}}{\partial N_{(-i)j}})(\frac{\partial D_{(-i)j}}{\partial N_{ij}} - \frac{\partial c_{(-i)}}{\partial N_{ij}}) \right) \end{pmatrix}}{(\frac{\partial D_{ij}}{\partial N_{ij}} - \frac{\partial c_i}{\partial N_{ij}})(\frac{\partial D_{(-i)j}}{\partial N_{(-i)j}} - \frac{\partial c_{(-i)}}{\partial N_{(-i)j}}) - (\frac{\partial D_{ij}}{\partial N_{(-i)j}} - \frac{\partial c_i}{\partial N_{(-i)j}})(\frac{\partial D_{(-i)j}}{\partial N_{ij}} - \frac{\partial c_{(-i)}}{\partial N_{ij}})}, \quad (10)$$

where superscript 'r' represents the best response.

Here, $(-i)$ denotes a car type other than i, and $(-j)$ denotes a brand other than j. Eqs. (8) and (9) implicitly define the best-response function for the mark-ups. The first two terms in Eq. (8) are the marginal external costs on users of brand j (including autonomous and normal cars), imposed by brand j's car type i users.[9] The sum of the last four terms gives the duopolistic mark-up. The third and fourth terms are the mark-ups for a private monopolist to supply a single autonomous car ($i = a$) or a normal car ($i = n$), respectively. The last two terms capture a correction for the competition from the competitor's imperfect substitutes. These effects are more complicated. As in Van den Berg (2013), who consider two parallel facilities, the closer the substitutes, the larger this correction: stronger competition leads to lower prices. With independent demands, the price is at its highest; with perfect

[8] In game theory, the best response is the strategy (or strategies) which produces the most favorable outcome for a player, taking other players' strategies as given.

[9] Marginal external cost is the marginal social cost external to the user's choice, i.e., the derivative of total cost minus the user's own usage cost.

substitutes, the price is at its lowest. The numerical example in section 5 illustrates that these insights also apply in the current context.

Eq. (9) implies that duopolistic pricing only partly internalizes the congestion externalities on own brand car users. Cournot competition would result in full internalization of congestion externalities among a brand's own customers, a result that is well known from the aviation literature (Brueckner, 2002; Pels and Verhoef, 2004). With Bertrand competition, an operator takes into account that pricing for marginal external congestion costs within the group of its own customers becomes less effective, as some of the customers will switch to the competitor, which would again increase congestion (e.g., Silva and Verhoef, 2013). However, in contrast to the standard Bertrand duopoly, both brands can now ask positive mark-ups, for two reasons. The first is that they internalize the congestion externality on their own brand and collect the revenue from the associated mark-up. The second is that they provide imperfect substitutes, which softens competition. This result corresponds with the findings of Small and Verhoef (2007) for private toll roads with imperfect substitutes, and Silva and Verhoef (2013) for duopolistic airlines.

It can be seen that one brand's optimal mark-up decision depends on the mark-up decision of the other brand. Given the best-response functions, the Nash-Bertrand equilibrium for the mark-ups, $(MU_{aj}^{NE}, MU_{nj}^{NE})$, is at the intersection of these best response functions, which can be implicitly expressed as:

$$\begin{pmatrix} MU_{aj}^{NE} \\ MU_{nj}^{NE} \end{pmatrix} = \begin{pmatrix} MU_{aj}^{r}[(MU_{a(-j)}^{NE}, MU_{n(-j)}^{NE})] \\ MU_{nj}^{r}[(MU_{a(-j)}^{NE}, MU_{n(-j)}^{NE})] \end{pmatrix},\ j \in \{1,2\}, \tag{11}$$

where superscript '*NE*' denotes the Nash-Bertrand equilibrium solution.

Once the capacity function, $s_{a(m)}$, and inverse demand function, D_{aj}, are given, we can determine the solutions for MU_{ij}^{NE} through Eqs. (8)-(11). The corresponding profits can be directly solved through Eq. (7). Due to the complex capacity effects, closed-form solutions and economic interpretations are hard to obtain. We use the simulation method to solve them in the numerical examples in section 5, and provide intuitive interpretations there.

3.2 Stage one: Nash equilibrium for brands' cooperation strategies

In this stage, we turn to the brands' decisions on cooperation strategy. Recall that each brand's strategy set is {not cooperate, cooperate within brand, cooperate across brands}. In a Nash equilibrium, each brand is assumed to know the equilibrium strategies of the other brand, and no one has anything to gain by changing only one's own strategy.

Let $\Pi_{j,m}$ denote the profit of brand j under strategy profile *m*, which equals the sum of the profit of autonomous cars and the profit of normal cars (see Eq. (6)). Because it is only when two brands both choose to cooperate across brands that all autonomous cars can be cooperative successfully, the situations (not cooperate, cooperate across brands), (cooperate within brand, cooperate across brands), (cooperate across brands, not cooperate), and (cooperate across brands, cooperate within brand) will never be the Nash equilibrium in practice, but they are helpful in determining the equilibrium of the game. For presentation purposes, we use *NB* to denote brand 1 not cooperating and brand 2 wishing to cooperate across brands; *WB* for brand 1 cooperating within own brand and brand 2 wishing to cooperate across brands; and conversely for *BN* and *BW*. Consider that when only one brand wants vehicles to cooperate across brands, that brand will instead choose the remaining strategy that has the higher profit. The payoffs under these strategy profiles thus satisfy:

$$(\Pi_{1,NB},\Pi_{2,NB}) = \begin{cases} (\Pi_{1,NY},\Pi_{2,NY}) & \text{if } \Pi_{2,NY} \geq \Pi_{2,NN} \\ (\Pi_{1,NN},\Pi_{2,NN}) & \text{otherwise} \end{cases}, (\Pi_{1,WB},\Pi_{2,WB}) = \begin{cases} (\Pi_{1,YY},\Pi_{2,YY}) & \text{if } \Pi_{2,YY} \geq \Pi_{2,YN} \\ (\Pi_{1,YN},\Pi_{2,YN}) & \text{otherwise} \end{cases},$$
$$(\Pi_{1,BN},\Pi_{2,BN}) = \begin{cases} (\Pi_{1,YN},\Pi_{2,YN}) & \text{if } \Pi_{1,YN} \geq \Pi_{1,NN} \\ (\Pi_{1,NN},\Pi_{2,NN}) & \text{otherwise} \end{cases}, (\Pi_{1,BW},\Pi_{2,BW}) = \begin{cases} (\Pi_{1,YY},\Pi_{2,YY}) & \text{if } \Pi_{1,YY} \geq \Pi_{1,NY} \\ (\Pi_{1,NY},\Pi_{2,NY}) & \text{otherwise} \end{cases}. \quad (12)$$

The accompanying payoff matrix for the proposed game is given in Table 2, in which the appropriate cell of the matrix represents the profits to the brand firm when a particular pair of strategies is chosen. The pricing stage, stage two as described above, follows next.

Table 2. Payoff matrix for the first-stage of the game.

		Brand 2		
		Not cooperate	Within brand	Across brands
Brand 1	Not cooperate	$(\Pi_{1,NN},\Pi_{2,NN})$	$(\Pi_{1,NY},\Pi_{2,NY})$	$(\Pi_{1,NB},\Pi_{2,NB})$
	Within brand	$(\Pi_{1,YN},\Pi_{2,YN})$	$(\Pi_{1,YY},\Pi_{2,YY})$	$(\Pi_{1,WB},\Pi_{2,WB})$
	Across brands	$(\Pi_{1,BN},\Pi_{2,BN})$	$(\Pi_{1,BW},\Pi_{2,BW})$	$(\Pi_{1,Y_XY_X},\Pi_{2,Y_XY_X})$

To find the Nash equilibria, we substitute the profits obtained from stage two into Table 2, and check each strategy pair to see if it has the property that each brand's strategy maximizes its payoff given the other brand's strategies. If neither brand can increase its payoff by choosing a strategy different from its current one, the corresponding strategy profile is a Nash equilibrium. Through examining each strategy profile in turn, we can find the following possible equilibrium outcomes.

- If $\Pi_{1,Y_XY_X} \geq \max[\Pi_{1,NB},\Pi_{1,WB}]$ and $\Pi_{2,Y_XY_X} \geq \max[\Pi_{2,BN},\Pi_{2,BW}]$, (cooperate across brands, cooperate across brands) is a Nash equilibrium.
- If $\Pi_{1,NN} \geq \Pi_{1,YN}$ and $\Pi_{2,NN} \geq \Pi_{2,NY}$, (not cooperate, not cooperate) is a Nash equilibrium.
- If $\Pi_{1,NY} \geq \Pi_{1,YY}$ and $\Pi_{2,NY} \geq \Pi_{2,NN}$, (not cooperate, cooperate within brand) is a Nash equilibrium.

- If $\Pi_{1,YN} \geq \Pi_{1,NN}$ and $\Pi_{2,YN} \geq \Pi_{2,YY}$, (cooperate within brand, not cooperate) is a Nash equilibrium.
- If $\Pi_{1,YY} \geq \Pi_{1,NY}$ and $\Pi_{2,YY} \geq \Pi_{2,YN}$, (cooperate within brand, cooperate within brand) is a Nash equilibrium.

It should be noted that a duopoly may have multiple Nash equilibria for this two-stage game, which is illustrated in the numerical examples in section 5. For ease of understanding, the interrelationship between the two stages is summarized in Fig. 1.

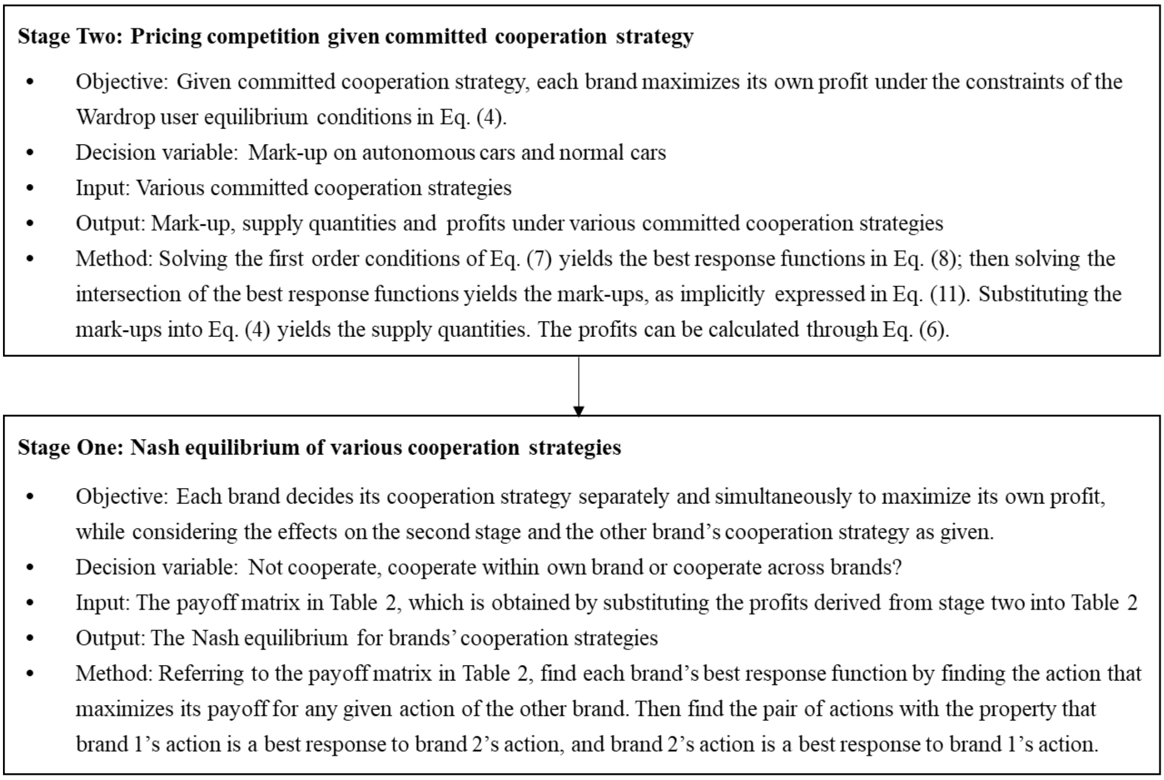


Fig. 1. A diagrammatical representation of the two-stage model.

4. Other market structures

To investigate further how competition and market power work for brand firms, travelers and society, this section turns to three other market structures: perfect competition, a public welfare-maximizing monopoly, and a private profit-maximizing monopoly. The public and private monopolists each own two brands but coordinate these brands' strategies to maximize a composite objective (aggregate welfare, or aggregate profits). The two "brands" are, therefore, still offered as imperfect substitute goods on the market.

4.1 Perfect competition

With two brands and two car types, one would not normally expect to see perfect competition in general, but we still consider this case as the absence of monopolistic demand-related mark-ups and congestion charges provides an important and natural benchmark. The best way to imagine the conditions under which perfect competition would prevail is when each of the four brand-type combinations could be supplied by large numbers of small firms.

Perfect competition leads to marginal cost pricing and thus zero mark-ups. The demands are determined by the following Wardrop user equilibrium conditions:

$$\begin{aligned} &D_{aj}[N_{a1},N_{n1},N_{a2},N_{n2}]=c_a[N_{a1},N_{n1},N_{a2},N_{n2}]+MC_{aj}+MC_{cop}\cdot cop_j, \quad j\in\{1,2\},\\ &D_{nj}[N_{a1},N_{n1},N_{a2},N_{n2}]=c_n[N_{a1},N_{n1},N_{a2},N_{n2}]+MC_{nj}, \quad j\in\{1,2\}. \end{aligned}, \qquad (13)$$

where D_{ij} is given by Eq. (3). As capacity effects do not affect the cost difference between driving autonomous cars and normal cars (see Eq. (2)), travelers will ignore these effects when choosing car types under perfect competition. Given the expression of the inverse demand function, one can obtain closed-form solutions for N_{a1}, N_{n1}, N_{a2}, and N_{n2}.

4.2 Public welfare-maximizing monopoly

Under public welfare-maximizing monopoly, the public monopolist maximizes aggregate welfare by setting the mark-ups and quantities for the four types of car. Social welfare, *W*, is operationalized as a social surplus and is defined as the consumer benefit, $B[N_{a1},N_{n1},N_{a2},N_{n2}]$, minus total usage cost (or, equivalently, consumer surplus plus profit). Assuming that there are no income effects, consumer benefit is the line integral of the four inverse demand functions and is independent of the path used for the integration.

The associated social welfare maximization problem is:

$$\begin{aligned} \max_{\substack{MU_{ij},\,N_{ij}\\ i\in\{a,n\};\ j\in\{1,2\}.}} \quad & W=B[N_{a1},N_{n1},N_{a2},N_{n2}] \\ & -\sum_{i\in\{a,n\}}\sum_{j=1}^{2}(c_{ij}[N_{a1},N_{n1},N_{a2},N_{n2}])\cdot N_{ij}-\sum_{j=1}^{2}(MC_{aj}+MC_{cop}\cdot cop_j)\cdot N_{aj} \end{aligned}. \qquad (14)$$

Combining the user equilibrium conditions and solving the above welfare maximization problem yields the following mark-ups (see Appendix B):

$$MU_{ij}^{pub}=\frac{\partial c_i}{\partial N_{ij}}\cdot\sum_{j=1}^{2}N_{ij}+\frac{\partial c_{-i}}{\partial N_{ij}}\cdot\sum_{j=1}^{2}N_{(-i)j}=MEC_{ij}. \qquad (15)$$

The superscript '*pub*' denotes public welfare-maximizing pricing. Eq. (15) implies that the welfare-maximizing public mark-up on cars of type i and brand j equals the marginal external costs imposed by these car users, MEC_{ij}. It is isomorphic to the conventional Pigouvian congestion toll (see Pigou, 1920; Small and Verhoef, 2007). The public monopolist thus fully internalizes the externalities caused by all car users, as one might expect.

In terms of the cooperation strategy, as public welfare-maximizing pricing fully internalizes the congestion externality, the public monopolist is more likely to have an incentive to apply cross-brand cooperation. This is because the increase in the congestion costs for users with substitutes depresses their willingness to pay, and hence the welfare that the monopolist can extract from them for a given level of demand, on a dollar-by-dollar basis. The social planner would find it beneficial to apply cross-brand cooperation to reduce congestion more fully.

4.3 Private profit-maximizing monopoly

Private monopoly is at the other extreme of private supply. Under a private profit-maximizing monopoly, a private monopolist would maximize the aggregate profit in the market by setting all mark-ups and quantities for all cars:

$$\max_{\substack{MU_{ij}, N_{ij}, \\ i\in\{a,n\}, j\in\{1,2\}}} \quad \Pi = \sum_{i\in\{a,n\}} \sum_{j=1}^{2} MU_{ij} \cdot N_{ij} \,. \tag{16}$$

Solving the above profit maximization problem yields the following monopolistic mark-up:

$$MU_{ij}^{mon} = \frac{\partial c_i}{\partial N_{ij}} \cdot \sum_{j=1}^{2} N_{ij} + \frac{\partial c_{-i}}{\partial N_{ij}} \cdot \sum_{j=1}^{2} N_{(-i)j} - \frac{\partial D_{ij}}{\partial N_{ij}} N_{ij} + (A_{ij} - D_{ij} + \frac{\partial D_{ij}}{\partial N_{ij}} N_{ij}) \,, \tag{17}$$

where the superscript '*mon*' denotes the private monopoly. Here, the first two terms are the marginal external costs. The third term is the monopolistic mark-up from users with car type i of brand j. The fourth term is the mark-up due to the other three substitutes: it measures the effect that a higher price on ij has on increasing the demand for $-(ij)$, which raises the profit in these sub-markets. The closer the car type and brand substitutes (i.e., the closer $\left|\partial D_{ij} / \partial N_{-(ij)}\right|$ gets to $\left|\partial D_{ij} / \partial N_{ij}\right|$), the higher the mark-ups, since this increases the strength of the third effect.

Private monopolistic mark-up is generally higher than the duopolistic mark-up in Eq. (8). The monopolist not only charges a higher demand-related mark-up, as consumers switching to other product variants does not mean losing them in terms of revenue, but also fully internalizes congestion externalities. This is, in the first place, due to congestion externalities imposed on all travelers now being considered, not just on the subset served by the duopolies. In the second place, duopolies do not internalize the full externalities that customers impose upon one another, since customers switching to the other brand would still create congestion for the brand's remaining customers. A private monopolist may, furthermore, lead to a suboptimal cooperation strategy due to the distortion in mark-ups, especially when the cooperation cost is high.

5. Numerical simulations

To understand the mechanism of cooperation and competition between brands and the equilibrium results from the proposed game, we execute some numerical analyses using a simulation method. In the following, we first calibrate the parameter values for the base case and complete our base case numerical model, then carry out extensive sensitivity analyses. The model outcomes are sensitive to parameterization, underlining the importance of presenting these results.

5.1 Parameter specifications

In the base calibration, we consider two ex-ante symmetric brands.[10] Following Van den Berg and Verhoef (2016), we focus on petrol autonomous and normal cars. The schedule delay parameters are based on the ratios β/α=39/64 and γ/α=1521/640 established by Small (1982), as these are common in the literature. We consider a trip length of 20 km, with a free-flow travel time of 20 minutes. We use a VOT (α) of €10/h (van den Berg and Verhoef, 2011b) and assume a base value of θ of 0.8. The automobile cost of normal cars is normalized to zero and we use an an MC_a at €1.51 (van den Berg and Verhoef, 2016). It is assumed that cooperation will increase the automobile cost by 10%, which means that the cooperation cost per trip, MC_{cop}, is €0.151.

5.1.1 Calibration of the capacity effect functions

Although several studies have used experiments or simulation to estimate the effective capacity when autonomous cars and normal cars both exist, the evidence for how the effective capacity varies by cooperation regimes is still limited. This poses challenges for the calibration of the capacity effect functions. We approach this task by approximating the expression of the capacity functions applying some of the values predicted in the literature.[11]

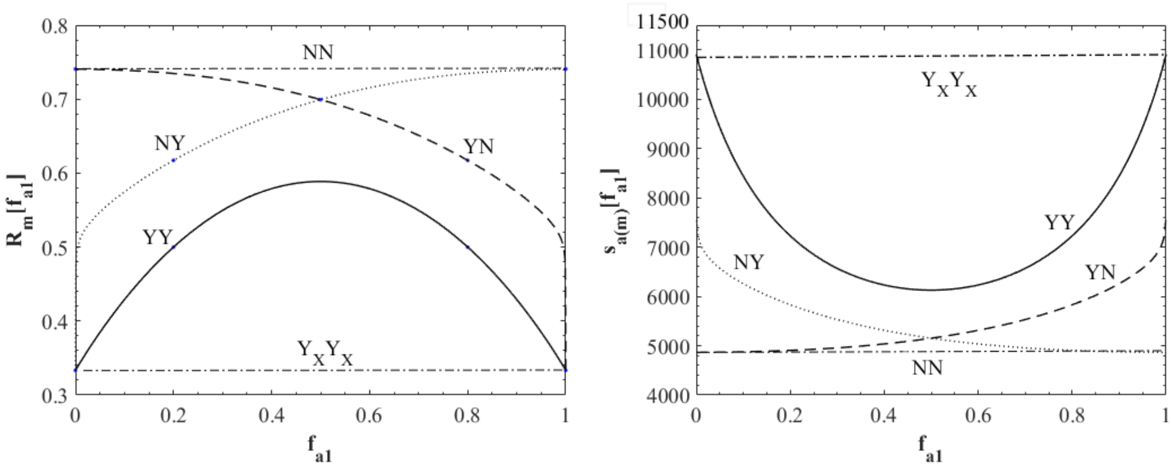
As is well established in the literature, the increase in road capacity is a non-linear function of the share of the autonomous cars that cooperate (Tientrakool et al., 2011; Fernandes and Nunes, 2012; Shladover et al., 2012). For example, it can be as low as 1% for autonomous cars that do not cooperate (Shladover et al., 2012) and as high as 414% with very efficient cooperation (Fernandes and Nunes, 2012). To distinguish the effects of different cooperation regimes, we assume that if all autonomous cars are non-cooperative, the averaged capacity is $s_{a(NN)}$=1.35s; if all autonomous cars are cooperative across brands, the averaged capacity is $s_{a(Y_X Y_X)} = 3s$; if brand 1 applies cooperation within its own brand and brand 2 does

[10] This assumption is not vital to the results; it simply helps the interpretation.
[11] In this paper, we ignore the patterns of the vehicle following and only consider the averaged capacity for all autonomous cars.

not, the averaged capacity for autonomous cars satisfies $s_{a(YN)}[0.8] = 1.62s$ and $s_{a(YN)}[0.5] = 1.43s$; and if both brands cooperate within their own brand, $s_{a(YY)}[0.8] = s_{a(YY)}[0.2] = 2s$ is satisfied. Fitting these values with polynomial functions leads to the following relationship:

$$R_m[f_{a1}] = \begin{cases} 1/1.35, & m = NN, \\ 1-0.18 f_{a1}^{2.41} - 0.26(1-f_{a1})^{-0.04}, & m = YN, \\ 1-0.18(1-f_{a1})^{2.41} - 0.26 f_{a1}^{-0.04}, & m = NY, \\ 1-0.67(f_{a1}^{1.70} + (1-f_{a1})^{1.70}), & m = YY, \\ 1/3, & m = Y_X Y_X. \end{cases} \quad (18)$$

This relationship is illustrated in Fig. 2. It can be seen that as f_{a1} increases, $s_{a(YN)}[f_{a1}]$ increases convexly, $s_{a(NY)}[f_{a1}]$ decreases convexly, and $s_{a(YY)}[f_{a1}]$ decreases first and then increases, which is at its minimum when $f_{a1} = 0.5$. This implies that the choice of $R_m[f_{a1}]$ appears to be reasonably in line with the theoretical properties in section 2.3. Because there is limited information about the expressions of $R_m[f_{a1}]$, extensive sensitivity analysis is carried out in section 5.3.1.



(a) The $R_m[f_{a1}]$ function (b) The $s_{a(m)}[f_{a1}]$ function

Fig. 2. Capacity effects of autonomous cars for the numerical model.

5.1.2 Calibration of travel cost and inverse demand functions

To calibrate the inverse demand functions, we consider perfect competition with normal cars and non-cooperative autonomous cars as the base case. For the base calibration, there are 9,000 users. The elasticity with respect to own generalized price is -0.35; with substitutes from the same brand or the same type of car, the cross-price elasticity is 0.2. For substitutes with different car types and brands, the cross-price elasticity is 0.1.[12] Under this calibration, the travel cost in the equilibrium with autonomous cars is €12.00, and with normal cars is

[12] The inverse demand functions are:
$D_{a1} = 103.6 - 0.0154N_{a1} - 0.0085N_{n1} - 0.0083N_{a2} - 0.0077N_{n2}, D_{a2} = 103.6 - 0.0083N_{a1} - 0.0077N_{n1} - 0.0154N_{a2} - 0.0085N_{n2},$
$D_{n1} = 106.31 - 0.0085N_{a1} - 0.0162N_{n1} - 0.0077N_{a2} - 0.0088N_{n2}, D_{n2} = 106.31 - 0.0077N_{a1} - 0.0088N_{n1} - 0.0085N_{a2} - 0.0162N_{n2}.$

€13.87. In the following analysis, the currency unit used is also €.

5.2 Base case analysis

Table 3 present the outcomes[13] of the base calibrations. We compare the outcomes under four market structures: perfect competition (MC), duopoly competition (Duopoly), public monopolist (Public), and private monopolist (Private). Under the base calibration, a public welfare-maximizing monopolist and a private profit-maximizing monopolist will choose cross-brand cooperation ($Y_X Y_X$).[14] We therefore only present the outcomes with $Y_X Y_X$ for these two regimes. Relative efficiency, ω, is defined as the welfare gain of a policy from the case without cooperation under perfect competition, divided by the gain from welfare-maximizing public pricing with cross-brand cooperation. A negative value thus reflects that welfare is below that under perfect competition without cooperation. As the two brands are ex-ante symmetric, outcomes under *YN* and *NY* are also symmetric, as presented in the fourth and fifth columns of Table 3. The payoff matrix for this two-stage game is shown in Table 4. Some main insights are summarized as follows.

Compared to non-cooperation, within-brand cooperation (*YY*) and cross-brand cooperation ($Y_X Y_X$) both raise the mark-ups on all cars, and more so for cross-brand cooperation. The reason is that cooperation lowers the travel cost for all travelers. As we consider price-sensitive demand, the decrease in the travel cost will, in turn, attract more car users and soften the competition. In contrast, if only one brand cooperates and the other does not (*YN* and *NY*), the cooperating brand tends to set a lower mark-up to compensate for the possible loss of customers to alternatives when the cooperation cost is added to the price and thus to attract more autonomous car users. Meanwhile, the non-cooperating brand benefits from the cooperating brand's cooperation without paying any cooperation cost and sets a higher mark-up on its autonomous cars. As a result, the mark-up on cooperative autonomous cars decreases, whereas that on non-cooperative autonomous cars rises.

According to the payoff matrix of this game in Table 4, there exist two pure Nash equilibria: (within brand, within brand) and (across brands, across brands). Cross-brand cooperation may seem the most likely to occur, as it has the highest profits and offers the greatest welfare, but it is by no means certain that it is the Nash equilibrium that will prevail, as it requires a simultaneous move. Indeed, if brand 1 does not cooperate at all, brand 2's best response is to cooperate within its own brand; if brand 1 cooperates within its own brand, brand 2's best response is to cooperate within its own brand; if brand 1 cooperates across brands, brand 2's best response is to cooperate across brands. By symmetry, the same best responses hold for brand 2. Consequently, within-brand cooperation and cross-brand cooperation are the two Nash equilibria. Cross-brand cooperation is more attractive and hence

---

[13] The results of interest are: the mark-ups (MU_{ij}), number of users (N_{ij}), travel cost (c_i), profit of each brand (Π_j), consumer benefit (B), welfare (W), and relative efficiency (ω).

[14] This is because cross-brand cooperation can reduce the congestion the most. Congestion is internal to welfare-maximizing and profit-maximizing monopolists, thereby improving the welfare or industry profit, respectively.

the dominating equilibrium.

In terms of relative efficiency, despite the high congestion levels, perfect competition performs best (apart from the welfare-maximizing public monopoly with $Y_X Y_X$), followed by duopoly competition, and worst performing is the private monopoly. This is because duopoly competition and private monopoly lead to marker power distortion, especially for private monopoly power. Under duopoly competition, cooperation across brands performs best, as it reduces congestion the most. With respect to the profit, Table 3 shows that duopolistic brands cannot capture as much surplus generated by high quality as a private monopolist, since the benefit partly goes to the competitor.

Table 3. Outcomes of the pricing competition stage

	NN		*YN*	*NY*	*YY*	$Y_X Y_X$			
	MC	Duopoly	Duopoly			MC	Duopoly	Public	Private
MU_{a1}	0	23.29	23.17	23.41	23.33	0	23.49	3.88	48.97
MU_{a2}	0	23.29	23.41	23.17	23.33	0	23.49	3.88	48.97
MU_{n1}	0	24.09	24.14	24.16	24.30	0	24.63	10.27	52.28
MU_{n2}	0	24.09	24.16	24.14	24.30	0	24.63	10.27	52.28
N_{a1}	2,279	1,744	1,753	1,740	1,758	2,331	1,793	2,553	1,255
N_{a2}	2,279	1,744	1,740	1,753	1,758	2,331	1,793	2,553	1,255
N_{n1}	2,221	1,703	1,707	1,705	1,712	2,279	1,722	1,770	1,056
N_{n2}	2,221	1,703	1,705	1,707	1,712	2,279	1,722	1,770	1,056
c_a	12.00	9.82	9.63	9.63	9.14	9.67	7.99	8.78	6.07
c_n	13.87	11.40	11.22	11.22	10.13	11.57	9.58	10.40	7.30
Π_1	0	81,628	81,804	81,936	82,616	0	84,533	28,104	116,654
Π_2	0	81,628	81,936	81,804	82,616	0	84,533	28,104	116,654
B	533,812	482,475	482,879	482,879	484,099	536,586	487,136	525,882	376,328
W	410,625	404,139	405,430	405,430	409,385	431,011	419,530	435,773	341,499
ω	0.00	-0.26	-0.21	-0.21	-0.05	0.81	0.35	1.00	-2.75

Table 4. Payoff matrix of the first stage of the game.

		Brand 2		
		Not cooperate	Within brand	Across brands
Brand 1	Not cooperate	(81628,81628)	(81936,81804)	(81936,81804)
	Within brand	(81804,81936)	(82616,82616)	(82616,82616)
	Across brands	(81804,81936)	(82616,82616)	(84533,84533)

5.3 Sensitivity analysis

There is a very wide range of predictions for the capacity effects of autonomous cars in the literature. There is also little guidance in respect of the values of cooperation cost, MC_{cop}, the VOT reduction parameter, θ, and users' price elasticity. Therefore, it is vital to perform extensive sensitivity analyses with respect to these values.[15]

5.3.1 Varying the capacity effects of autonomous cars

There are conceptually two different channels via which the cooperation regimes affect the eventual impact of autonomous driving on bottleneck capacity: the benefits from cooperating across brands and the benefits from cooperating within brand. It is instructive to set up the sensitivity analysis such that these two channels can be clearly distinguished. To that end, we investigate through a sensitivity analysis of $s_{a(Y_X Y_X)}$ and via a sensitivity analysis of $s_{a(YY)}$, respectively.[16]

First, we look at the impact of the maximum increase in capacity due to cooperation by varying $s_{a(Y_X Y_X)}$ from $2.5s$ to $4.5s$. In the base case, the maximum capacity was $3s$.[17] Fig. 3a depicts the profit of each brand under different cooperation strategies under duopoly competition. It can be seen that the profit is highest under cross-brand cooperation, followed by within-brand cooperation. According to the resulting payoff matrixes, there always exist two Nash equilibria: (cooperate within brand, cooperate within brand) and (cooperate across brands, cooperate across brands). This means that the variation of the maximum capacity does not affect the Nash equilibria of brands' cooperation decisions. In Fig. 3a, the cooperation strategies under Nash equilibrium are depicted with solid curves, and the dotted curves represent the non-equilibrium strategies. Due to the highest profit, cross-brand cooperation is the dominating equilibrium and achieves social optimality; but again, it is by no means certain that it will prevail.

Fig. 3b compares the performances of different market structures. Relative efficiencies under duopoly competition with cross-brand cooperation, private monopoly, and perfect competition all increase with the maximum capacity.[18] Indeed, a higher maximum capacity improves the welfare gains for all regimes by reducing congestion, which lowers the welfare gain from optimal pricing in the denominator of the relative efficiencies. Conversely, when the two duopolies cooperate only within own brand, the relative efficiency declines, due to the decreasing capacity gains from within-brand cooperation.

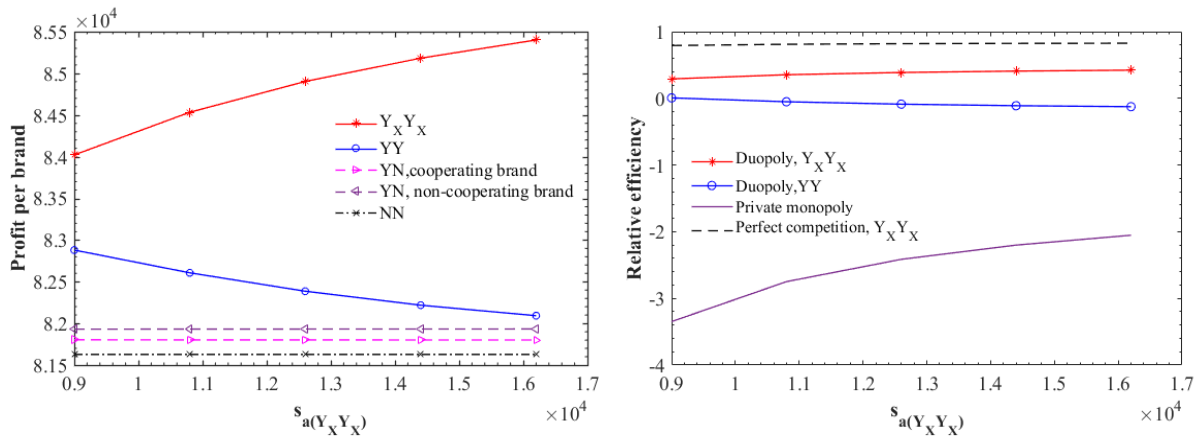
The effects of $s_{a(Y_X Y_X)}$ on the pricing competition are given in Appendix C.

[15] The effects of other parameters are in line with the theoretical discussion. Moreover, for these parameters, we have much more guidance from the literature. Hence, these parameters will not be discussed further here.

[16] The capacity effects under *NN* and *YN* (or *NY*) are less beneficial and less interesting.

[17] Maximum capacity refers to the capacity for autonomous cars when both brands choose to cooperate across brands, i.e., $s_{a(Y_X Y_X)}$.

[18] Public and private monopolists both choose cross-brand cooperation.



(a) Profit of each brand under duopoly (b) Relative efficiency

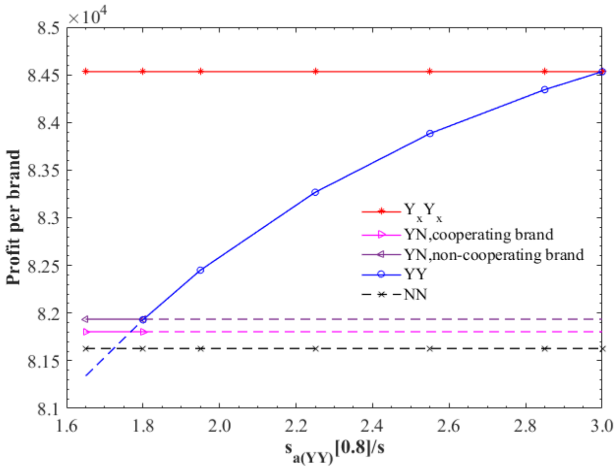
Fig. 3. Effects of $s_{a(Y_X Y_X)}$ on the profit and relative efficiency.

Note: In Fig. 3a, solid curves represent the Nash equilibrium for the cooperation strategies, and dotted curves represent the non-equilibrium strategies.

Next, we investigate the impact of the relative capacity benefit of within-brand cooperation under duopoly competition, by varying $s_{a(YY)}[0.8]$ from $1.65s$ to $3s$.[19] The capacity functions under other cooperation strategies remain the same as the base case. Fig. 4a shows that $s_{a(YY)}[0.8]/s$ below 1.8 leads to three Nash equilibria: (cooperate within brand, not cooperate), (not cooperate, cooperate within brand), and (cooperate across brands, cooperate across brands). When $s_{a(YY)}[0.8]/s$ exceeds 1.8, (cooperate within brand, cooperate within brand) and (cooperate across brands, cooperate across brands) become the two equilibria. This implies that the capacity benefit of within-brand cooperation has a significant impact on the Nash equilibrium of the cooperation decisions. Specifically, when $s_{a(YY)}[0.8]/s$ is low, brands have no incentive to apply within-brand cooperation. As $s_{a(YY)}[0.8]/s$ increases, the benefits of within-brand cooperation are strengthened.

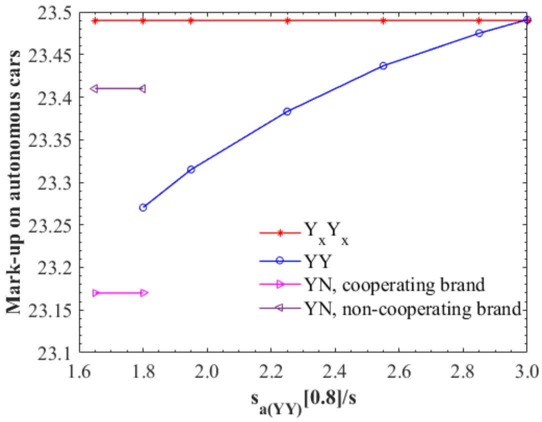
The effects on equilibrium mark-ups are shown in Fig. 4b and 4c. As $s_{a(YY)}[0.8]$ increases, the mark-up on autonomous cars and normal cars under within-brand cooperation both increase and approach those under cross-brand cooperation. The reason is that a higher $s_{a(YY)}[0.8]$ reduces competition through lowering the travel cost and attracting more car users. Specifically, when $s_{a(YY)}[0.8]$ approaches $3s$, within-brand cooperation becomes similar to cross-brand cooperation.

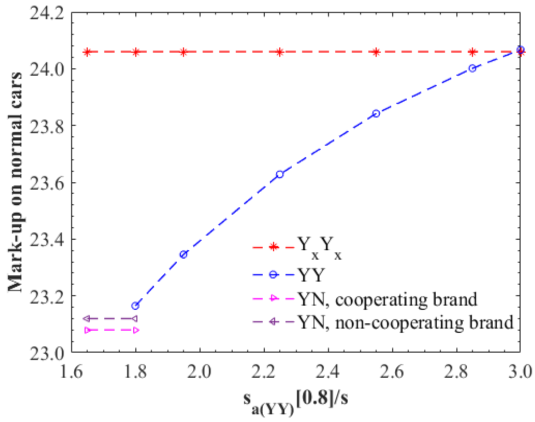
[19]Note that $s_{a(YY)}[0.8]$ is between $1.65s$ and $3s$. Specifically, $s_{a(YY)}[0.8]=1.65s$ means the effect of within-brand cooperation is more similar to non-cooperation, whereas $s_{a(YY)}[0.8]=3s$ means it is more similar to cross-brand cooperation. The effects on the relative efficiency is not investigated, since the results under other strategies are the same as the base case.



(a) Profit of each brand

Note: Solid curves represent Nash equilibrium strategies and dashed curves represent non-equilibrium strategies.





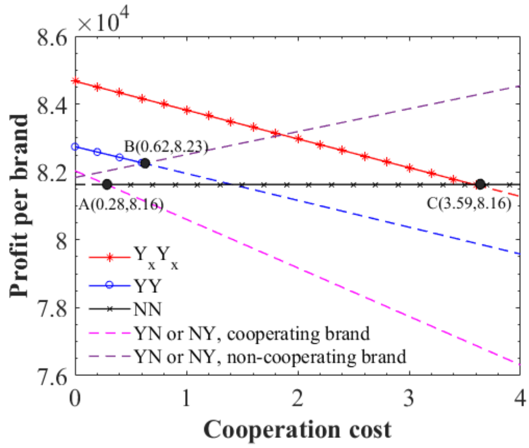
(b) Mark-up on autonomous cars (c) Mark-up on normal cars

Fig. 4. Effects of $s_{a(YY)}[0.8]$ on the profit and mark-ups under duopoly competition.

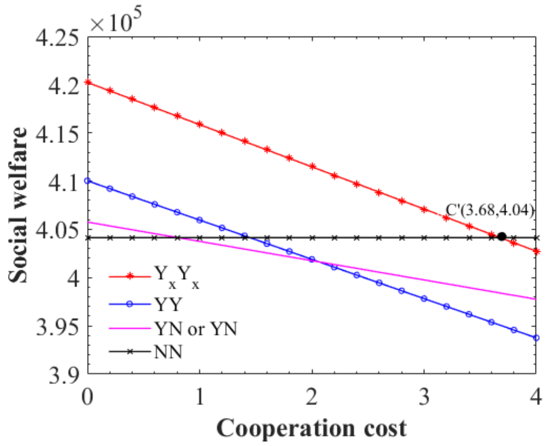
5.3.2 Varying the cooperation cost

We vary the cooperation cost from 0 to 4. Fig. 5a compares brands' profits with alternative cooperation strategies under duopoly competition. It can be seen that when only one brand applies cooperation and the other does not, a larger cooperation cost lowers the profit of the cooperating brand and raises that of the non-cooperating brand. Typically, this duopoly game has multiple Nash equilibria. Unless the cooperation cost is very high, cross-brand cooperation is one of these equilibria; but again, it is by no means certain that it will prevail. Specifically, when the cooperation cost exceeds 3.59, cross-brand cooperation stops being the Nash equilibrium. Nevertheless, the social welfare depicted in Fig. 5b shows that cross-brand cooperation will still be socially optimal until the cooperation cost exceeds 3.68. This suggests that profit-maximizing duopolistic firms may lead to below-optimal cooperation.

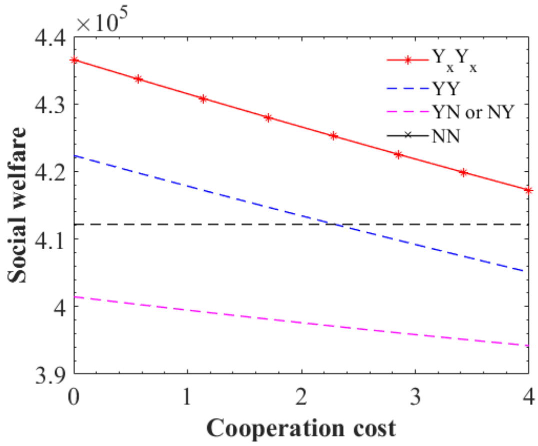
Fig. 5c and 5d depict the social welfare and industry profit under public monopoly and private monopoly, respectively. As cooperation cost increases, social welfare and industry profit both decrease. The public monopolist always chooses cross-brand cooperation, whereas the private monopolist stops cooperation when the cooperation cost exceeds 2.48. This indicates that compared to a social optimum cooperation strategy, duopoly competition and private monopoly both lead to too little cooperation, especially for the private monopolist.



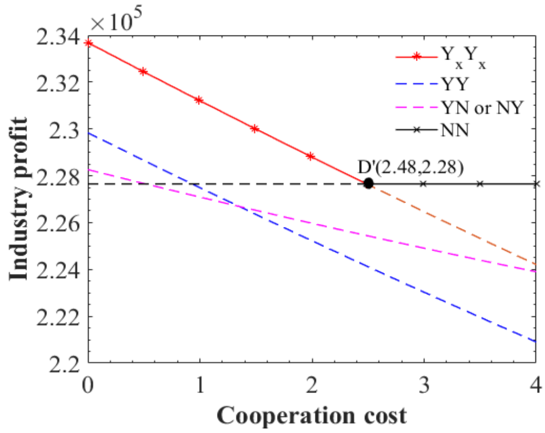
(a) Profit per brand under duopoly



(b) Social welfare under duopoly



(c) Social welfare under public monopoly



(d) Industry profit under private monopoly

Fig. 5. Effects of cooperation cost on the profit and social welfare.

Note: In Fig. 5a, 5b and 5d, solid curves represent the Nash equilibrium for the cooperation strategies, and dotted curves represent the non-equilibrium strategies.

Fig. 6 shows the interactions of the pricing competition between different car types and brands under the resultant equilibrium strategies. Fig. 6a shows that the public monopolist slightly reduces the mark-up on all cars, and overall congestion becomes lower as the cooperation cost increases. In contrast, with cross-brand cooperation, the private monopolist lowers the mark-up on autonomous cars as a result of the reduction in private monopolistic power, and slightly decreases that on normal cars because of the reduction in congestion. A jump occurs when the private monopolist changes from cross-brand cooperation to non-cooperation, as shown in Fig. 6b. Fig. 6c shows that for duopoly competition, a higher cooperation cost lowers the mark-up on autonomous cars and raises that on normal cars. This is because a higher cooperation cost makes autonomous cars less attractive and normal cars more appealing. Meanwhile, the reduction in congestion tends to lower the mark-up on all cars. The first effect remains dominant.

Fig. 7 depicts the relative efficiencies of equilibrium strategies under different market structures. It can be seen that the relative efficiency decreases with the cooperation cost for all market structures. This is because a higher cooperation cost leads to less cooperation between cars, a possible societal benefit of a private supply, making demand-related mark-ups, a societal disadvantage, relatively more important. The relative efficiency under private monopoly decreases most, due to private monopolistic power. For the different equilibrium strategies under duopoly competition, the welfare loss is the smallest when the two brands both choose cross-brand cooperation, followed by both choosing within-brand cooperation, and the largest when neither of them cooperates.

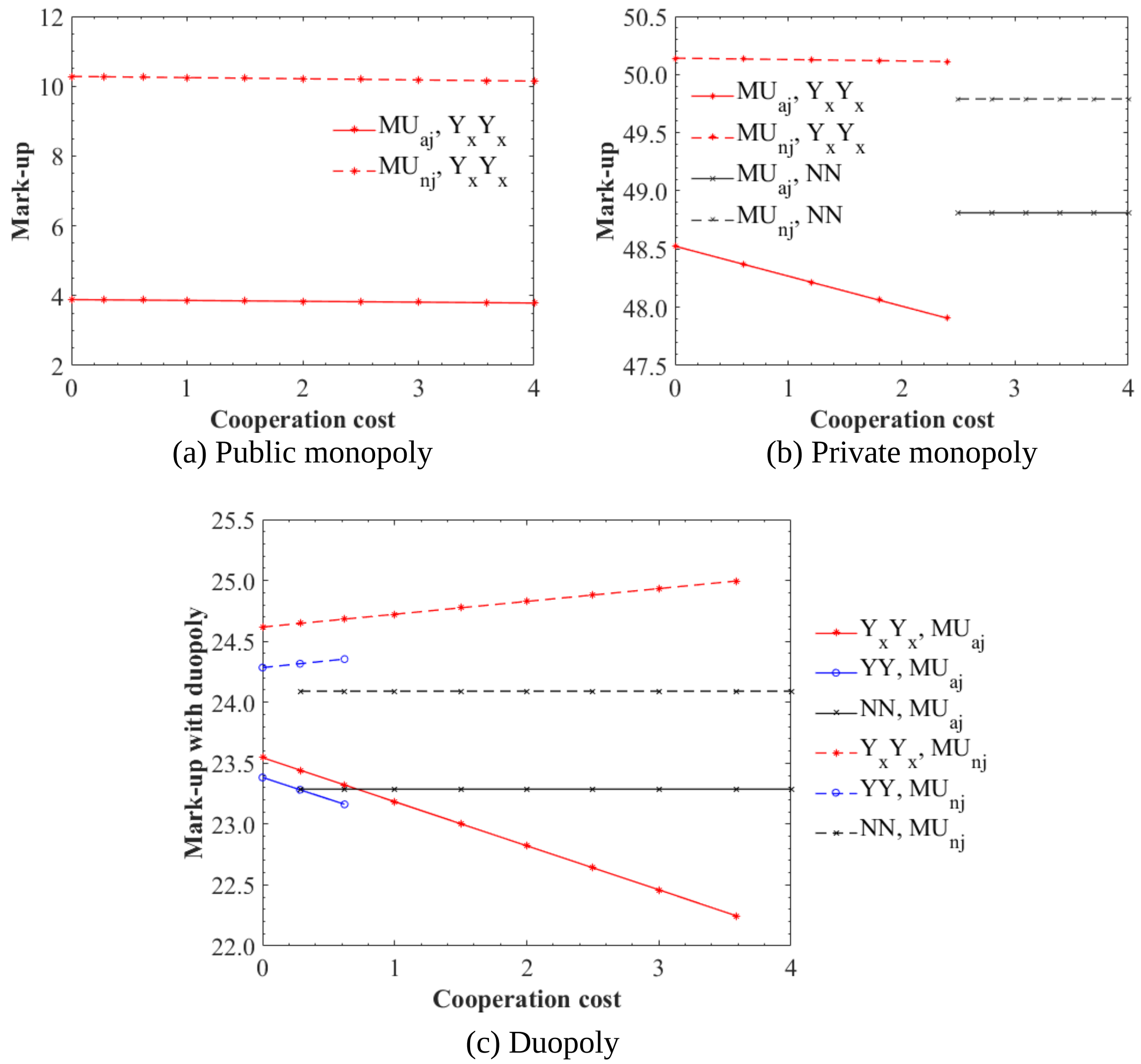


Fig. 6. Effects of cooperation cost on mark-ups under different market structures.

Note: Solid curves represent autonomous cars and dotted curves represent normal cars.

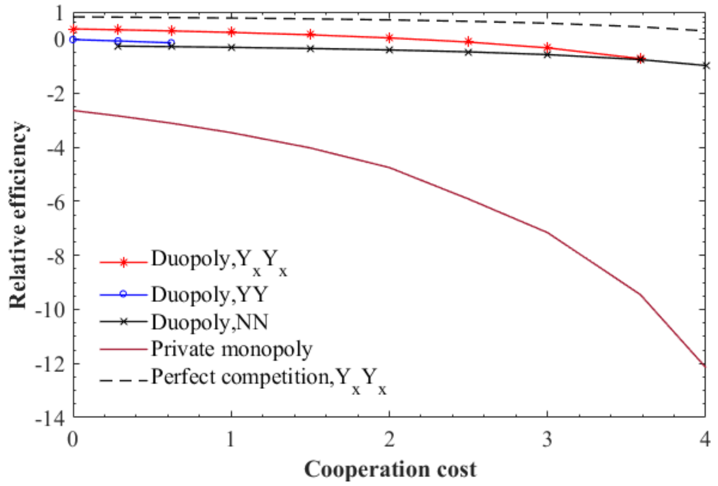


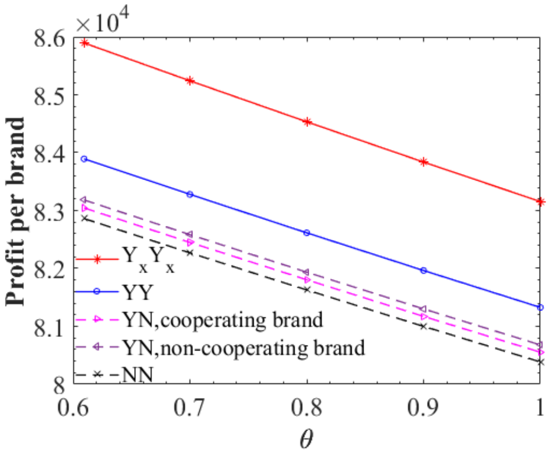
Fig. 7. Effects of cooperation cost on relative efficiencies.

5.3.3 Varying the VOT reduction parameter

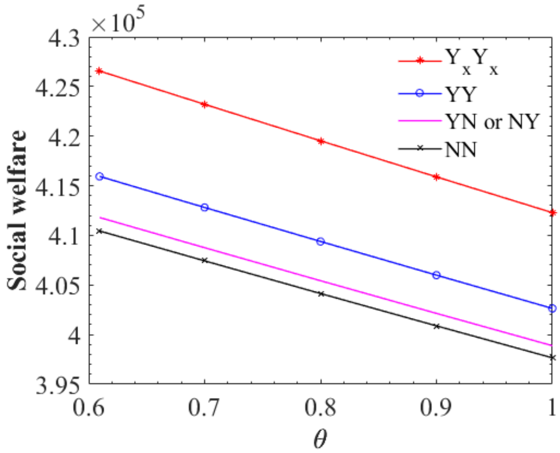
We vary the VOT reduction parameter, θ, from its theoretical minimum of β / α to 1. Fig. 8 shows that as θ increases, the profit per brand and the social welfare both decrease, due to the increasing cost of travelling. Brands benefit most from cross-brand cooperation, followed by solely within-brand cooperation, only one-brand cooperation, and lastly by no cooperation at all, as shown in Fig. 8a. This implies that θ does not affect the Nash equilibria for the cooperation strategies, (cooperate within brand, cooperate within brand) and (cooperate across brands, cooperate across brands), where cross-brand cooperation is more attractive and is the dominating equilibrium. Fig. 8b shows that cross-brand cooperation is also the social optimum given a duopoly.

For the pricing decisions under different market structures, Fig. 9 shows that, as θ increases, all suppliers tend to reduce the mark-up on autonomous cars and raise that on normal cars, because a larger θ means a lower willingness to pay for autonomous cars and a higher willingness to pay for normal cars. With a public monopolist, the mark-up on normal cars changes more than that on autonomous cars, implying that the increase in congestion imposed by normal cars is stronger than its reduction by the use of autonomous cars. In contrast, for private monopolistic pricing and duopolistic pricing, the mark-up on autonomous cars changes more, owing to competition and substitutional effects between different cars, which affects autonomous cars more.

Fig. 10 compares the changes in relative efficiencies. In this sensitivity, as θ increases, the performance of duopoly competition and private monopoly both become modestly better, owing to the increasing competition and reducing market power. Cross-brand cooperation also naturally performs better than within-brand cooperation. Under perfect competition, a higher θ leads more drivers to switch to normal cars and raises congestion. As a result, the relative efficiency decreases. It is clear from Fig. 10 that the impact of varying θ on relative efficiency is modest: the curves are all fairly flat.

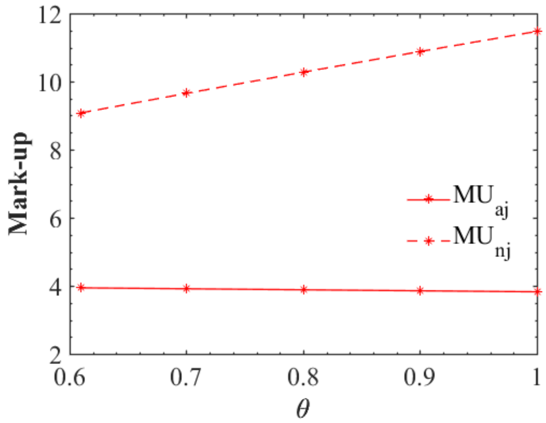


(a) Profit per brand

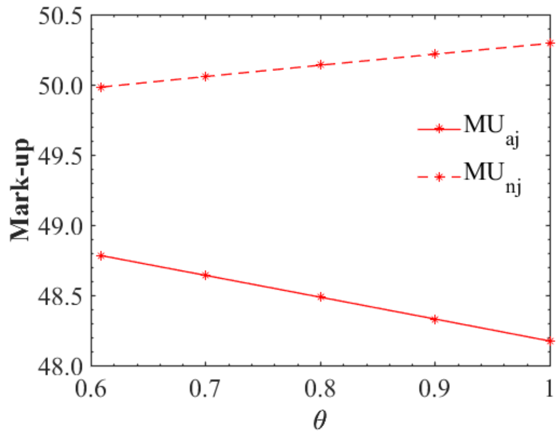


(b) Social welfare

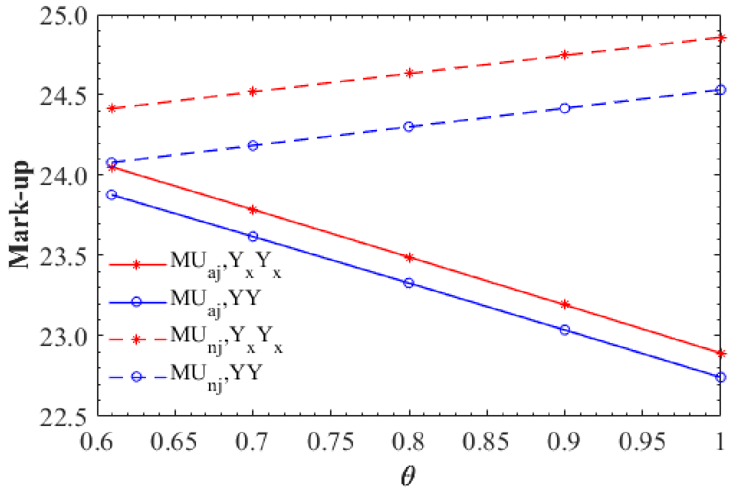
Fig. 8. Effects of θ on profit and social welfare under duopoly competition.



(a) Public monopoly



(b) Private monopoly



(c) Duopoly

Fig. 9. Effects of θ on mark-ups under different market structures.

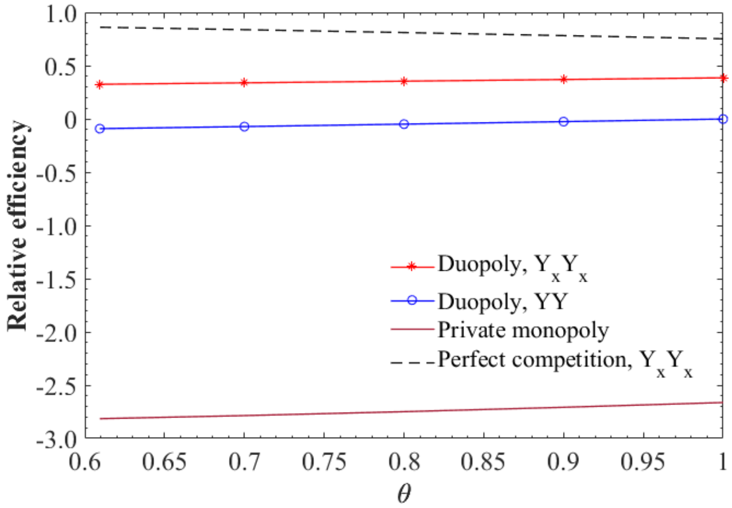


Fig. 10. Effects of θ on relative efficiencies.

5.3.4 Varying the price elasticity of demand

To investigate the effects of price elasticity on duopoly decisions, we increase and decrease the own-price elasticity and cross-price elasticities by the same percentage at the same time. In the base case, own-price elasticity was -0.35, and cross-price elasticities were 0.2 and 0.1. We now increase these elasticities to twice and three times the size, compared to the base case, as well as decreasing them to 1/3 and 1/2. To calibrate the inverse demand functions for each price elasticity, we make the demand curve tilt around the equilibrium in the base equilibrium without autonomous cars. Higher price elasticities thus lead to larger coefficients in the inverse demand function, which also means lower consumer benefit and social welfare.

The profit under cross-brand cooperation is again the highest, followed by within-brand cooperation. Consequently, elasticities do not change brands' equilibria for their cooperation strategies: (cooperate within brand, cooperate within brand) and (cooperate across brands, cooperate across brands). The equilibrium outcomes are summarized in Table 5.

Table 5 shows that greater price elasticity tends to lower the mark-up on all cars. Indeed, as travelers become more price-sensitive, a slight drop in the mark-up will lead to an increase in the demand for corresponding cars. Brands hence find it more beneficial to charge lower mark-ups to attract more car users and this softens the price and mark-ups.

In terms of relative efficiency, Table 5 shows that as demand becomes more sensitive, duopoly competition becomes more efficient, despite the declining welfare. This is because an increase in price elasticity raises the number of autonomous cars and normal cars, which, on the one hand, increases congestion and, on the other, lowers the marginal willingness to pay. The former effect implies that the societal benefit from private pricing increases and the latter that its downsides decrease. Relative efficiency thus increases with the price elasticity of demand.

Table 5. Outcomes with different price elasticities.

	1/3*base case		1/2*base case		2*base case		3*base case	
	YY	$Y_X Y_X$	YY	$Y_X Y_X$	YY	$Y_X Y_X$	YY	$Y_X Y_X$
MU_{aj}	68.40	68.56	45.87	46.03	12.02	12.19	8.23	8.40
MU_{nj}	70.57	70.90	47.45	47.78	12.68	13.01	8.77	9.09
N_{aj}	1,726	1,738	1,734	1,752	1,800	1,869	1,838	1,939
N_{nj}	1,682	1,685	1,690	1,695	1,753	1,772	1,790	1,818
Π_j	236,770	238,654	159,734	161,628	43,881	45,836	30,827	32,810
SW	1,180,089	1,190,138	794,928	805,003	215,811	226,057	150,645	160,949
ω	-2.25	-1.81	-1.11	-0.69	0.38	0.75	0.46	0.81

5.3.5 Asymmetric cooperation cost

The final sensitivity analysis considers asymmetric cooperation cost. We normalize the cooperation cost of brand 2 to zero, and vary the cooperation cost for brand 1, $MC_{cop,1}$, from 0 to 4.

The Nash equilibria for the two-stage game is summarized in Table 6. It can be seen that when $MC_{cop,1}$ is between 0.57 and 1.75, in addition to cross-brand cooperation, (not cooperate, cooperate within brand) is also a Nash equilibrium. This indicates that under asymmetric cooperation cost, a higher cooperation cost tends to reduce brands' incentive to cooperate. Specifically, when $MC_{cop,1}$ exceeds 1.75, (not cooperate, cooperate within brand) becomes the unique Nash equilibrium, and cross-brand cooperation stops being the Nash equilibrium.

Fig. 11 shows the effects of cooperation cost on relative efficiency. Under duopoly competition, relative efficiencies under $Y_X Y_X$ and YY both decrease with brand 1's cooperation cost, and the relative efficiency under NY is constant at -0.19. As a result, when brand 1's cooperation cost is low, cross-brand cooperation performs best; when brand 1's cooperation cost is high, NY performs best. Therefore, asymmetry of cooperation cost tends to reduce the efficiency of cooperation.

Table 6. Nash equilibria under asymmetric cooperation cost.

NE	$MC_{cop,1} < 0.57$	$0.57 \le MC_{cop,1} \le 1.75$	$MC_{cop,1} > 1.75$
(Not, not)	×	×	×
(Within, not)	×	×	×
(Not, within)	×	√	√
(Within, within)	√	×	×
(Across, across)	√	√	×
Number of NE	2	2	1

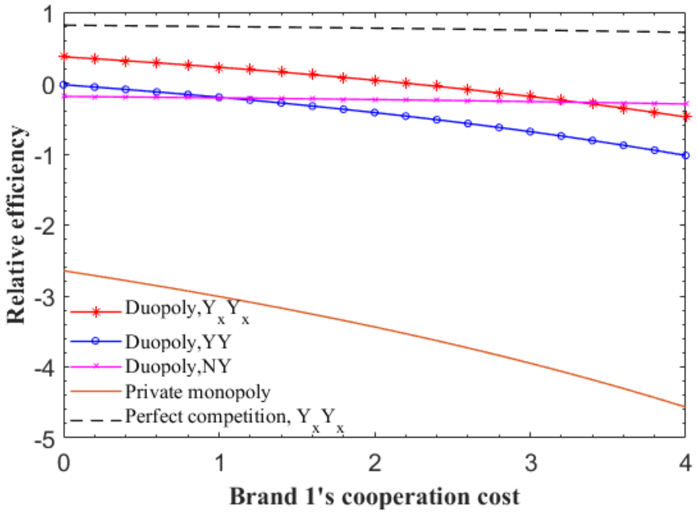


Fig. 11. Effects of asymmetric cooperation cost on relative efficiencies.

6. Conclusion

This paper investigated the strategic interactions of multiple car brands—which may provide both autonomous cars and normal cars—focusing on the question of whether brands will want their autonomous cars to cooperate within and/or across brands. This is an important question, as cooperation is what ensures that road capacity increases with the use of autonomous cars. In our paper, each brand has three options for its autonomous cars: cooperation within own brand only, cooperation across brands, or no cooperation between vehicles. We considered four market structures: duopoly competition, perfect competition, a public welfare-maximizing monopoly, and a private profit-maximizing monopoly. For duopoly competition, we built a two-stage game model, first with a choice of cooperation strategy and then Bertrand competition with imperfect substitutes.

Some important findings and new insights are obtained. First, duopoly competition or a private monopoly may lead to too little cooperation. Indeed, cooperation with another brand may lower the travel times of its own autonomous car users; but it also does the same for the competing brand, thereby strengthening the element of competition. A duopoly sees this as a downside, whereas a public monopolist owning all brands regards this as an advantage. A private monopoly may also lead to below-optimal cooperation, due to monopolistic market power. Second, a duopoly may, furthermore, have multiple equilibria in the cooperation strategy. Unless cooperation costs are very high, cross-brand cooperation is one of these equilibria; but it is by no means certain that it will prevail. No cooperation, and solely within-brand cooperation, are also often Nash equilibria. It is only when cooperation costs are too high that a unique Nash equilibrium obtains for (1) no cooperation at all (for symmetric brands) or (2) for one brand cooperating within its own brand and the other brand not cooperating (for asymmetric brands). In contrast, for the public welfare-maximizing monopolist and the private profit-maximizing monopolist, the equilibrium is always unique:

either cooperation across brands or no cooperation at all (when the cooperation cost is high). Third, the relative benefits of within-brand cooperation also have significant impact on the Nash-equilibrium of brands' cooperation strategies. Specifically, when within-brand cooperation increases capacity slightly, in addition to cross-brand cooperation, one firm cooperating within its own brand and the other not cooperating is also a Nash equilibrium. As road capacity is improved more effectively, within-brand cooperation and cross-brand cooperation become the Nash equilibria. Finally, in contrast to the standard Bertrand duopoly, in this paper both brands can ask positive mark-ups, since they internalize the congestion externality on their own users and provide imperfect substitutes to soften the competition.

The proposed model can serve as a useful tool for analyzing the impacts of competition between multiple car brands, and technical cooperation while driving between the autonomous vehicles they supply. Based on our findings, product differentiation and market characteristics play an important role in evaluating the welfare impacts of suppliers' cooperation and pricing strategies. Despite the advantages that vehicle-to-vehicle cooperative technology may bring for the product quality of a firm's own clientele, a potential rise in the competitors' attractiveness may lower the benefits from cooperation as perceived by firms. Our analysis has shown that the collective result of firms' trade-offs in deciding about cooperation between autonomous vehicles within and between brands will not perfectly overlap with the benefits and costs that should be traded off from a societal perspective. Required market corrections depend on aspects such as the level of cooperation cost, a-symmetries in cooperation costs, and the relative benefit of within-brand cooperation. The regulator should thus not blindly encourage cross-brand cooperation, but instead make informed trade-offs that take such effects into consideration. As far as pricing is concerned, we found that large car suppliers may find it advantageous to take at least part of the congestion externality and the substitution effects between different car types into account when making their supply decisions.

Although our study sheds light on the joint application of game-theoretic modelling and dynamic congestion modelling in analyzing a transport system with a mix of autonomous and normal human-driven cars, it can be further extended in several directions. First, this paper considered capacity effects in a general way by ignoring the different headways in mixed traffic and only investigated the averaged capacity of autonomous cars with different brands. In order to make use of the proposed model for practical applications in reality, there is a need to use traffic flow theory or stochastic process theory to take the capacity interaction between different headways into account (e.g., Van Wee et al., 2013; Zhou et al., 2020). Second, the proposed model only considered private autonomous cars. However, with the development of vehicle automation and the sharing economy, a mix of normal human-driven cars, autonomous cars, and shared autonomous cars can be expected to coexist in the next few decades, affecting the way of travel (e.g., Haboucha et al., 2017; Tian et al., 2021). Autonomous cars, especially when combined with shared use, may contribute to reducing or solving some of the most intractable urban problems, such as traffic congestion, road traffic accidents, and inefficient use of urban spaces. It would be interesting to investigate the cooperation and competition among private autonomous cars, shared autonomous cars, and normal human-driven cars. Third, in this paper, all users are assumed to be ex-ante identical, and users with an autonomous car have the same scheduling preference. Hence, all autonomous car users travel jointly in a mixed flow and have the same departure rate and equilibrium travel cost. It seems plausible that the extent to which using an autonomous car would reduce the VOT will vary by brands, such as a cheaper brand vehicle would reduce the VOT less than a more costly brand vehicle. This would mean that brands will have a significant impact on travelers' departure time choices and travel costs. Therefore, it would be interesting and necessary to consider the heterogeneity caused by brand differences into account in a future study. Finally, the choice of where to park an autonomous car and the option of renting it out when not using it (for instance, via Uber or Lyft) are interesting topics.

References

Arnott, R., de Palma, A., & Lindsey, R. (1989). Schedule delay and departure time decisions with heterogeneous commuters. *Transportation Research Record*, 1197, 57-67.

Arnott, R., de Palma, A., & Lindsey, R. (1993). A structural model of peak-period congestion: A traffic bottleneck with elastic demand. *The American Economic Review*, 83(1), 161-179.

Bansal, P., Kockelman, K. M., & Singh, A. (2016). Assessing public opinions of and interest in new vehicle technologies: An Austin perspective. *Transportation Research Part C: Emerging Technologies*, 67, 1-14.

Brueckner, J. K. (2002). Airport congestion when carriers have market power. *American Economic Review*, 92(5), 1357-1375.

Burns, L. D. (2013). A vision of our transport future. *Nature*, 497(7448), 181-182.

Chen, Z., He, F., Yin, Y., & Du, Y. (2017). Optimal design of autonomous vehicle zones in transportation networks. *Transportation Research Part B: Methodological*, 99, 44-61.

Chen, Z., He, F., Zhang, L., & Yin, Y. (2016). Optimal deployment of autonomous vehicle lanes with endogenous market penetration. *Transportation Research Part C: Emerging Technologies*, 72, 143-156.

Daniel, J. I. (1995). Congestion pricing and capacity of large hub airports: A bottleneck model with stochastic queues. *Econometrica: Journal of the Econometric Society*, 63(2), 327-370.

de Palma, A., & Lindsey, R. (2000). Private toll roads: Competition under various ownership regimes. *The Annals of Regional Science*, 34(1), 13-35.

Dixit, A. (1979). A model of duopoly suggesting a theory of entry barriers. *The Bell Journal of Economics*, 10(1), 20-32.

Fagnant, D. J., & Kockelman, K. (2015). Preparing a nation for autonomous vehicles: opportunities, barriers and policy recommendations. *Transportation Research Part A: Policy and Practice*, 77, 167-181.

Fernandes, P., & Nunes, U. (2012). Platooning with IVC-enabled autonomous vehicles: Strategies to mitigate communication delays, improve safety and traffic flow. *IEEE Transactions on Intelligent Transportation Systems*, 13(1), 91-106.

Fu, X., van den Berg, V. A.C., & Verhoef, E. T. (2018). Private road networks with uncertain demand. *Research in Transportation Economics*, 70, 57-68.

Haboucha, C. J., Ishaq, R., & Shiftan, Y. (2017). User preferences regarding autonomous vehicles. *Transportation Research Part C: Emerging Technologies*, 78, 37-49.

Krueger, R., Rashidi, T. H., & Dixit, V. V. (2019). Autonomous driving and residential location preferences: Evidence from a stated choice survey. *Transportation Research Part C: Emerging Technologies*, 108, 255-268.

Krueger, R., Rashidi, T. H., & Rose, J. M. (2016). Preferences for shared autonomous vehicles. *Transportation Research Part C: Emerging Technologies*, 69, 343-355.

Kuang, Z., Lian, Z., Lien, J. W., & Zheng, J. (2020). Serial and parallel duopoly competition in multi-segment transportation routes. *Transportation Research Part E: Logistics and Transportation Review*, 133, 101821.

Lamotte, R., de Palma, A., & Geroliminis, N. (2017). On the use of reservation-based autonomous vehicles for demand management. *Transportation Research Part B: Methodological*, 99, 205-227.

Li, Z. C., Huang, H. J., & Yang, H. (2020). Fifty years of the bottleneck model: A bibliometric review and future research directions. *Transportation Research Part B: Methodological*, 139, 311-342.

Lindsey, R. (2004). Existence, uniqueness, and trip cost function properties of user equilibrium in the bottleneck model with multiple user classes. *Transportation Science*, 38(3), 293-314.

Liu, W. (2018). An equilibrium analysis of commuter parking in the era of autonomous vehicles. *Transportation Research Part C: Emerging Technologies*, 92, 191-207.

Liu, Z., & Song, Z. (2019). Strategic planning of dedicated autonomous vehicle lanes and autonomous vehicle/toll lanes in transportation networks. *Transportation Research Part C: Emerging Technologies*, 106, 381-403.

Nieuwenhuijsen, J., de Almeida Correia, G. H., Milakis, D., van Arem, B., & van Daalen, E. (2018). Towards a quantitative method to analyze the long-term innovation diffusion of automated vehicles technology using system dynamics. *Transportation Research Part C: Emerging Technologies*, 86, 300-327.

Pels, E., & Verhoef, E. T. (2004). The economics of airport congestion pricing. *Journal of Urban Economics*, 55(2), 257-277.

Pigou, A. C. (1920). *The Economics of Welfare*. Macmillan, London.

Pudāne, B. (2020). Departure time choice and bottleneck congestion with automated vehicles: Role of on-board activities. *European Journal of Transport and Infrastructure Research*, 20(4), 306-334.

Pudāne, B., Molin, E. J. E., Arentze, T. A., Maknoon, Y., & Chorus, C. G. (2018). A time-use model for the automated vehicle-era. *Transportation Research Part C: Emerging Technologies*, 93, 102-114.

Shladover, S. E., Su, D., & Lu, X.-Y. (2012). Impacts of cooperative adaptive cruise control on freeway traffic flow. *Transportation Research Record*, 2324(1), 63-70.

Silva, H. E., & Verhoef, E. T. (2013). Optimal pricing of flights and passengers at congested airports and the efficiency of atomistic charges. *Journal of Public Economics*, 106, 1-13.

Small, K. A. (1982). The scheduling of consumer activities: Work trips. *The American Economic Review*, 72(3), 467-479.

Small, K. A. (2015). The bottleneck model: An assessment and interpretation. *Economics of Transportation*, 4(1-2), 110-117.

Small, K. A., & Verhoef, E. T. (2007). *The Economics of Urban Transportation*. Routledge.

Sun, X., & Yin, Y. (2021). An auction mechanism for platoon leader determination in single-brand

cooperative vehicle platooning. *Economics of Transportation*, 28, 100233.
Tang, Z. Y., Tian, L. J., & Wang, D. Z. W. (2021). Multi-modal morning commute with endogenous shared autonomous vehicle penetration considering parking space constraint. *Transportation Research Part E: Logistics and Transportation Review*, 151, 102354.
Tian, L. J., Sheu, J. B., & Huang, H. J. (2019). The morning commute problem with endogenous shared autonomous vehicle penetration and parking space constraint. *Transportation Research Part B: Methodological*, 123, 258-278.
Tian, Z., Feng, T., Timmermans, H. J. P., & Yao, B. (2021). Using autonomous vehicles or shared cars? Results of a stated choice experiment. *Transportation Research Part C: Emerging Technologies*, 128, 103117.
Tientrakool, P., Ho, Y. C., & Maxemchuk, N. F. (2011). Highway capacity benefits from using vehicle-to-vehicle communication and sensors for collision avoidance. *2011 IEEE Vehicular Technology Conference* (VTC Fall).
van den Berg, V. A. C. (2013). Serial private infrastructures. *Transportation Research Part B: Methodological*, 56, 186-202.
van den Berg, V. A. C., & Verhoef, E. T. (2011a). Congestion tolling in the bottleneck model with heterogeneous values of time. *Transportation Research Part B: Methodological*, 45(1), 60-78.
van den Berg, V. A. C., & Verhoef, E. T. (2011b). Winning or losing from dynamic bottleneck congestion pricing? The distributional effects of road pricing with heterogeneity in values of time and schedule delay. *Journal of Public Economics*, 95(7-8), 983-992.
van den Berg, V. A. C., & Verhoef, E. T. (2016). Autonomous cars and dynamic bottleneck congestion: The effects on capacity, value of time and preference heterogeneity. *Transportation Research Part B: Methodological*, 94, 43-60.
Van Dender, K. (2005). Duopoly prices under congested access. *Journal of Regional Science*, 45(2), 343-362.
van Wee, B., Annema, J. A., & Banister, D. (Eds.). (2013). *The Transport System and Transport Policy: An Introduction*. Edward Elgar Publishing, Cheltenham, UK, pp. 125-159.
Vickrey, W. (1969). Congestion theory and transport investment. *The American Economic Review*, 59(2), 251–260.
Wadud, Z., MacKenzie, D., & Leiby, P. (2016). Help or hindrance? The travel, energy and carbon impacts of highly automated vehicles. *Transportation Research Part A: Policy and Practice*, 86, 1-18.
Wu, W., Zhang, F., Liu, W., & Lodewijks, G. (2020). Modelling the traffic in a mixed network with autonomous-driving expressways and non-autonomous local streets. *Transportation Research Part E: Logistics and Transportation Review*, 134, 101855.
Yu, X., van den Berg, V. A. C., & Verhoef, E. T. (2022). Autonomous cars and activity-based bottleneck model: How do in-vehicle activities determine aggregate travel patterns?. *Transportation research part C: Emerging Technologies*, 139, 103641.
Zhang, A., & Zhang, Y. (2006). Airport capacity and congestion when carriers have market power. *Journal of Urban Economics*, 60(2), 229-247.
Zhang, W., Zhao, H., & Jiang, R. (2018). Impact of capacity drop on commuting systems under uncertainty. *Journal of Advanced Transportation*, 2018, 6809304.
Zhang, X., Liu, W., Waller, S. T., & Yin, Y. (2019). Modelling and managing the integrated morning-evening commuting and parking patterns under the fully autonomous vehicle environment. *Transportation Research Part B: Methodological*, 128, 380-407.

Zheng, F., Liu, C., Liu, X., Jabari, S. E., & Lu, L. (2020). Analyzing the impact of automated vehicles on uncertainty and stability of the mixed traffic flow. *Transportation Research Part C: Emerging Technologies*, 112, 203-219.

Zhou, Y., Ahn, S., Wang, M., & Hoogendoorn, S. (2020). Stabilizing mixed vehicular platoons with connected automated vehicles: An H-infinity approach. *Transportation Research Part B: Methodological*, 132, 152-170.

Acknowledgments

The work described in this paper was jointly supported by grants from the National Key Research and Development Program of China (2018YFB1600900), the National Natural Science Foundation of China (71525003, 71890970/71890974), and the NSFC-EU joint research project (71961137001). Any remaining errors are ours.

Appendix A. Pricing rule under duopoly competition.

We illustrate with brand 1. According to Eq. (7), the associated Lagrangian is:

$$\begin{aligned}\Pi_1 = {} & MU_{a1}\cdot N_{a1} + MU_{n1}\cdot N_{n1} \\ & -\lambda_{a1}\cdot\left(D_{a1}[N_{a1},N_{n1},N_{a2},N_{n2}] - c_a[N_{a1},N_{n1},N_{a2},N_{n2}] - MU_{a1} - MC_{a1} - MC_{cop}\cdot cop_1\right) \\ & -\lambda_{a2}\cdot\left(D_{a2}[N_{a1},N_{n1},N_{a2},N_{n2}] - c_a[N_{a1},N_{n1},N_{a2},N_{n2}] - MU_{a2} - MC_{a2} - MC_{cop}\cdot cop_2\right) \\ & -\lambda_{n1}\cdot\left(D_{n1}[N_{a1},N_{n1},N_{a2},N_{n2}] - c_n[N_{a1},N_{n1},N_{a2},N_{n2}] - MU_{n1} - MC_{n1}\right) \\ & -\lambda_{n2}\cdot\left(D_{n2}[N_{a1},N_{n1},N_{a2},N_{n2}] - c_n[N_{a1},N_{n1},N_{a2},N_{n2}] - MU_{n2} - MC_{n2}\right)\end{aligned} \quad \text{(A1)}$$

Taking the derivatives of Eq. (A1) with respect to MU_{a1}, MU_{n1}, N_{a1}, N_{n1}, N_{a2} and N_{n2} yields:

$$\frac{\partial \Pi_1}{\partial MU_{a1}} = N_{a1} + \lambda_{a1} = 0 \quad \Rightarrow \lambda_{a1} = -N_{a1}, \quad \text{(A2)}$$

$$\frac{\partial \Pi_1}{\partial MU_{n1}} = N_{n1} + \lambda_{n1} = 0 \quad \Rightarrow \lambda_{n1} = -N_{n1}, \quad \text{(A3)}$$

$$\frac{\partial \Pi_1}{\partial N_{a1}} = MU_{a1} - \lambda_{a1}\cdot\left(\frac{\partial D_{a1}}{\partial N_{a1}} - \frac{\partial c_a}{\partial N_{a1}}\right) - \lambda_{a2}\cdot\left(\frac{\partial D_{a2}}{\partial N_{a1}} - \frac{\partial c_a}{\partial N_{a1}}\right) - \lambda_{n1}\cdot\left(\frac{\partial D_{n1}}{\partial N_{a1}} - \frac{\partial c_n}{\partial N_{a1}}\right) - \lambda_{n2}\cdot\left(\frac{\partial D_{n2}}{\partial N_{a1}} - \frac{\partial c_n}{\partial N_{a1}}\right) = 0, \quad \text{(A4)}$$

$$\frac{\partial \Pi_1}{\partial N_{a2}} = -\lambda_{a1}\cdot\left(\frac{\partial D_{a1}}{\partial N_{a2}} - \frac{\partial c_a}{\partial N_{a2}}\right) - \lambda_{a2}\cdot\left(\frac{\partial D_{a2}}{\partial N_{a2}} - \frac{\partial c_a}{\partial N_{a2}}\right) - \lambda_{n1}\cdot\left(\frac{\partial D_{n1}}{\partial N_{a2}} - \frac{\partial c_n}{\partial N_{a2}}\right) - \lambda_{n2}\cdot\left(\frac{\partial D_{n2}}{\partial N_{a2}} - \frac{\partial c_n}{\partial N_{a2}}\right) = 0, \quad \text{(A5)}$$

$$\frac{\partial \Pi_1}{\partial N_{n1}} = MU_{n1} - \lambda_{a1}\cdot\left(\frac{\partial D_{a1}}{\partial N_{n1}} - \frac{\partial c_a}{\partial N_{n1}}\right) - \lambda_{a2}\cdot\left(\frac{\partial D_{a2}}{\partial N_{n1}} - \frac{\partial c_a}{\partial N_{n1}}\right) - \lambda_{n1}\cdot\left(\frac{\partial D_{n1}}{\partial N_{n1}} - \frac{\partial c_n}{\partial N_{n1}}\right) - \lambda_{n2}\cdot\left(\frac{\partial D_{n2}}{\partial N_{n1}} - \frac{\partial c_n}{\partial N_{n1}}\right) = 0, \quad \text{(A6)}$$

$$\frac{\partial \Pi_1}{\partial N_{n2}} = -\lambda_{a1}\cdot\left(\frac{\partial D_{a1}}{\partial N_{n2}} - \frac{\partial c_a}{\partial N_{n2}}\right) - \lambda_{a2}\cdot\left(\frac{\partial D_{a2}}{\partial N_{n2}} - \frac{\partial c_a}{\partial N_{n2}}\right) - \lambda_{n1}\cdot\left(\frac{\partial D_{n1}}{\partial N_{n2}} - \frac{\partial c_n}{\partial N_{n2}}\right) - \lambda_{n2}\cdot\left(\frac{\partial D_{n2}}{\partial N_{n2}} - \frac{\partial c_n}{\partial N_{n2}}\right) = 0. \quad \text{(A7)}$$

From Eq. (A4) and Eq. (A6), we can obtain:

$$MU_{a1} = \frac{\partial c_a}{\partial N_{a1}} N_{a1} + \frac{\partial c_n}{\partial N_{a1}} N_{n1} - \frac{\partial D_{a1}}{\partial N_{a1}} N_{a1} - \frac{\partial D_{n1}}{\partial N_{a1}} N_{n1} + \lambda_{a2} \cdot \left(\frac{\partial D_{a2}}{\partial N_{a1}} - \frac{\partial c_a}{\partial N_{a1}} \right) + \lambda_{n2} \cdot \left(\frac{\partial D_{n2}}{\partial N_{a1}} - \frac{\partial c_n}{\partial N_{a1}} \right),$$
$$MU_{n1} = \frac{\partial c_a}{\partial N_{n1}} N_{a1} + \frac{\partial c_n}{\partial N_{n1}} N_{n1} - \frac{\partial D_{a1}}{\partial N_{n1}} N_{a1} - \frac{\partial D_{n1}}{\partial N_{n1}} N_{n1} + \lambda_{a2} \cdot \left(\frac{\partial D_{a2}}{\partial N_{n1}} - \frac{\partial c_a}{\partial N_{n1}} \right) + \lambda_{n2} \cdot \left(\frac{\partial D_{n2}}{\partial N_{n1}} - \frac{\partial c_n}{\partial N_{n1}} \right). \quad \text{(A8)}$$

Substituting Eq. (A2) and Eq. (A3) into Eq. (A5) and Eq. (A7) yields:

$$N_{a1} \cdot \left(\frac{\partial D_{a1}}{\partial N_{a2}} - \frac{\partial c_a}{\partial N_{a2}} \right) + N_{n1} \cdot \left(\frac{\partial D_{n1}}{\partial N_{a2}} - \frac{\partial c_n}{\partial N_{a2}} \right) = \lambda_{a2} \cdot \left(\frac{\partial D_{a2}}{\partial N_{a2}} - \frac{\partial c_a}{\partial N_{a2}} \right) + \lambda_{n2} \cdot \left(\frac{\partial D_{n2}}{\partial N_{a2}} - \frac{\partial c_n}{\partial N_{a2}} \right),$$
$$N_{a1} \cdot \left(\frac{\partial D_{a1}}{\partial N_{n2}} - \frac{\partial c_a}{\partial N_{n2}} \right) + N_{n1} \cdot \left(\frac{\partial D_{n1}}{\partial N_{n2}} - \frac{\partial c_n}{\partial N_{n2}} \right) = \lambda_{a2} \cdot \left(\frac{\partial D_{a2}}{\partial N_{n2}} - \frac{\partial c_a}{\partial N_{n2}} \right) + \lambda_{n2} \cdot \left(\frac{\partial D_{n2}}{\partial N_{n2}} - \frac{\partial c_n}{\partial N_{n2}} \right). \quad \text{(A9)}$$

Solving Eq. (A9) yields:

$$\lambda_{a2} = \frac{N_{a1} \cdot \left((\frac{\partial D_{a1}}{\partial N_{a2}} - \frac{\partial c_a}{\partial N_{a2}})(\frac{\partial D_{n2}}{\partial N_{n2}} - \frac{\partial c_n}{\partial N_{n2}}) - (\frac{\partial D_{a1}}{\partial N_{n2}} - \frac{\partial c_a}{\partial N_{n2}})(\frac{\partial D_{n2}}{\partial N_{a2}} - \frac{\partial c_n}{\partial N_{a2}}) \right) + N_{n1} \cdot \left((\frac{\partial D_{n1}}{\partial N_{a2}} - \frac{\partial c_n}{\partial N_{a2}})(\frac{\partial D_{n2}}{\partial N_{n2}} - \frac{\partial c_n}{\partial N_{n2}}) - (\frac{\partial D_{n1}}{\partial N_{n2}} - \frac{\partial c_n}{\partial N_{n2}})(\frac{\partial D_{n2}}{\partial N_{a2}} - \frac{\partial c_n}{\partial N_{a2}}) \right)}{(\frac{\partial D_{a2}}{\partial N_{a2}} - \frac{\partial c_a}{\partial N_{a2}})(\frac{\partial D_{n2}}{\partial N_{n2}} - \frac{\partial c_n}{\partial N_{n2}}) - (\frac{\partial D_{a2}}{\partial N_{n2}} - \frac{\partial c_a}{\partial N_{n2}})(\frac{\partial D_{n2}}{\partial N_{a2}} - \frac{\partial c_n}{\partial N_{a2}})}. \quad \text{(A10)}$$

$$\lambda_{n2} = \frac{N_{a1} \cdot \left((\frac{\partial D_{a1}}{\partial N_{a2}} - \frac{\partial c_a}{\partial N_{a2}})(\frac{\partial D_{a2}}{\partial N_{n2}} - \frac{\partial c_a}{\partial N_{n2}}) - (\frac{\partial D_{a1}}{\partial N_{n2}} - \frac{\partial c_a}{\partial N_{n2}})(\frac{\partial D_{a2}}{\partial N_{a2}} - \frac{\partial c_a}{\partial N_{a2}}) \right) + N_{n1} \cdot \left((\frac{\partial D_{n1}}{\partial N_{a2}} - \frac{\partial c_n}{\partial N_{a2}})(\frac{\partial D_{a2}}{\partial N_{n2}} - \frac{\partial c_a}{\partial N_{n2}}) - (\frac{\partial D_{n1}}{\partial N_{n2}} - \frac{\partial c_n}{\partial N_{n2}})(\frac{\partial D_{a2}}{\partial N_{a2}} - \frac{\partial c_a}{\partial N_{a2}}) \right)}{\left((\frac{\partial D_{n2}}{\partial N_{a2}} - \frac{\partial c_n}{\partial N_{a2}})(\frac{\partial D_{a2}}{\partial N_{n2}} - \frac{\partial c_a}{\partial N_{n2}}) - (\frac{\partial D_{n2}}{\partial N_{n2}} - \frac{\partial c_n}{\partial N_{n2}})(\frac{\partial D_{a2}}{\partial N_{a2}} - \frac{\partial c_a}{\partial N_{a2}}) \right)}. \quad \text{(A11)}$$

Substituting Eqs. (A10) and (A11) into Eq. (A8), we can obtain the mark-up of brand 1 on autonomous cars and normal cars, as expressed in Eqs. (8)-(10). Similarly, we can derive λ_{a1}, λ_{n1}, MU_{a2}^r and MU_{n2}^r.

Appendix B. Pricing rule under a public welfare-maximizing monopoly

The consumer benefit can be expressed as:

$$B[N_{a1}, N_{n1}, N_{a2}, N_{n2}] = \int_{(0,0,0,0)}^{(N_{a1}, N_{n1}, N_{a2}, N_{n2})} \left(D_{a1}[\cdot]dx_{a1} + D_{n1}[\cdot]dx_{n1} + D_{a2}[\cdot]dx_{a2} + D_{n2}[\cdot]dx_{n2} \right)$$
$$= N_{a1} \cdot \left(A_{a1} - \frac{b_{a_1a_1} N_{a1} - b_{a_1n_1} N_{n1} + b_{a_1a_2} N_{a2} + b_{a_1n_2} N_{n2}}{2} \right) + N_{n1} \cdot \left(A_{n1} - \frac{b_{a_1n_1} N_{a1} + b_{n_1n_1} N_{n1} + b_{n_1a_2} N_{a2} + b_{n_1n_2} N_{n2}}{2} \right)$$
$$+ N_{a2} \cdot \left(A_{a2} - \frac{b_{a_1a_2} N_{a1} + b_{n_1a_2} N_{n1} + b_{a_2a_2} N_{a2} + b_{a_2n_2} N_{n2}}{2} \right) + N_{n2} \cdot \left(A_{n1} - \frac{b_{a_1n_2} N_{a1} + b_{n_1n_2} N_{n1} + b_{a_2n_2} N_{a2} + b_{n_2n_2} N_{n2}}{2} \right) \quad \text{, (B1)}$$

implying that $\partial B / \partial N_{ij} = D_{ij}$.

The associated Lagrangian is:

$$
\begin{aligned}
W = B[N_{a1}, N_{n1}, N_{a2}, N_{n2}] - \sum_{i\in\{a,n\}} \sum_{j=1}^{2} (c_{ij}[N_{a1}, N_{n1}, N_{a2}, N_{n2}] + MC_{ij}) \cdot N_{ij} - \sum_{j=1}^{2} MC_{cop} \cdot cop_j \cdot N_{aj} \\
- \lambda_{a1} \cdot \left(D_{a1}[N_{a1}, N_{n1}, N_{a2}, N_{n2}] - c_a[N_{a1}, N_{n1}, N_{a2}, N_{n2}] - MU_{a1} - MC_{a1} - MC_{cop} \cdot cop_1 \right) \\
- \lambda_{a2} \cdot \left(D_{a2}[N_{a1}, N_{n1}, N_{a2}, N_{n2}] - c_a[N_{a1}, N_{n1}, N_{a2}, N_{n2}] - MU_{a2} - MC_{a2} - MC_{cop} \cdot cop_2 \right) \cdot \\
- \lambda_{n1} \cdot \left(D_{n1}[N_{a1}, N_{n1}, N_{a2}, N_{n2}] - c_n[N_{a1}, N_{n1}, N_{a2}, N_{n2}] - MU_{n1} - MC_{n1} \right) \\
- \lambda_{n2} \cdot \left(D_{n2}[N_{a1}, N_{n1}, N_{a2}, N_{n2}] - c_n[N_{a1}, N_{n1}, N_{a2}, N_{n2}] - MU_{n2} - MC_{n2} \right)
\end{aligned}
\tag{B2}
$$

Substituting Eq. (B1) into Eq. (B2) and taking the derivatives of Eq. (B2) with respect to MU_{ij}, N_{ij}, and λ_{ij} ($i \in \{a,n\}$, $j \in \{1,2\}$) yields:

$$\frac{\partial W}{\partial MU_{a1}} = \lambda_{a1} = 0; \frac{\partial W}{\partial MU_{a2}} = \lambda_{a2} = 0; \frac{\partial W}{\partial MU_{n1}} = \lambda_{n1} = 0; \frac{\partial W}{\partial MU_{n2}} = \lambda_{n2} = 0; \tag{B3}$$

$$\frac{\partial W}{\partial N_{ij}} = D_{ij} - \frac{\partial c_a}{\partial N_{ij}} N_{aj} - \frac{\partial c_a}{\partial N_{ij}} N_{a(-j)} - \frac{\partial c_n}{\partial N_{ij}} N_{n(-j)} - \frac{\partial c_n}{\partial N_{ij}} N_{nj} - MC_{a1} - MC_{cop} \cdot cop_1 . \tag{B4}$$

Substituting the user equilibrium conditions in Eq. (4) into Eq. (B4) yields:

$$MU_{ij}^{pub} = \frac{\partial c_a}{\partial N_{ij}} N_{aj} + \frac{\partial c_a}{\partial N_{ij}} N_{a(-j)} + \frac{\partial c_n}{\partial N_{ij}} N_{n(-j)} + \frac{\partial c_n}{\partial N_{ij}} N_{nj} , \tag{B5}$$

as shown in Eq. (15).

Appendix C. Effects of the maximum capacity on the pricing competition

Fig. 12 depicts the effects of $s_{a(Y_X Y_X)}$ on equilibrium mark-ups. Fig.12a shows that with cross brand cooperation, the mark-ups on autonomous cars and normal cars both increase with the maximum capacity, whereas when two brands cooperate solely within own brand, the mark-ups decrease. For cross-brand cooperation this is because a larger maximum capacity reduces the travel costs more and attracts a greater number of car users, which softens the competition between brands. For cooperation within own brand, due to symmetry, the demand for autonomous cars of different brands is the same, which causes the eventual capacity of autonomous cars to achieve the lowest value of the parabolas (see Fig. 2b). A higher maximum capacity thus means a lower equilibrium capacity for autonomous cars, which leads to opposite outcomes compared to the case of cross brand cooperation. Fig 12 (b-c) suggests that public monopolist and private monopolist structures also tend to lower the mark up on autonomous cars, and slightly raise that on normal cars, due to the resulting congestion effects.

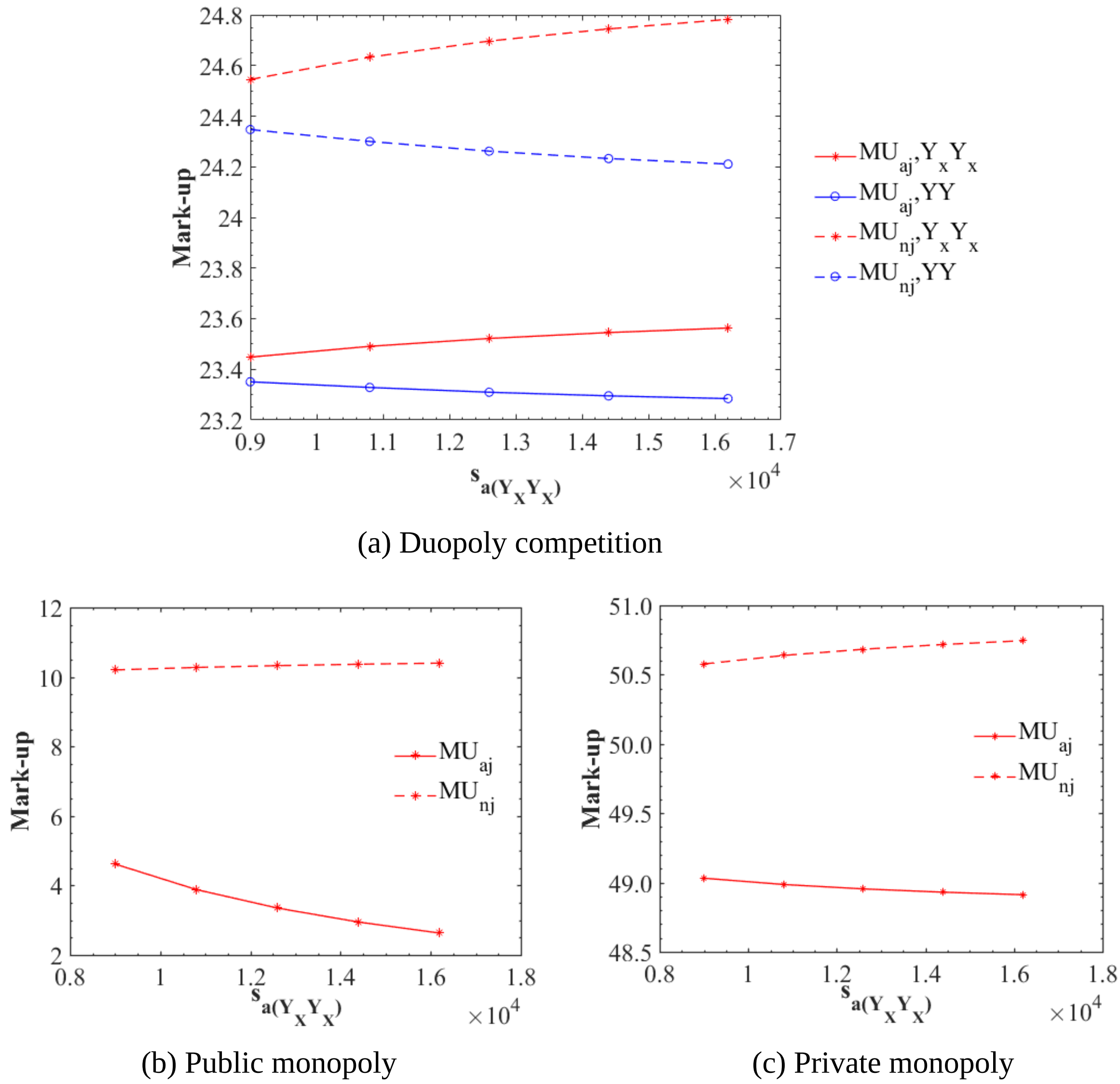


Fig. 12. Effects of $s_{a(Y_X Y_X)}$ on mark-ups under different market structures.