Will all autonomous cars cooperate? Brands’ strategic interactions under dynamic congestion

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Abstract

Autonomous cars allow safe driving with a smaller headway than that required for normal human-driven cars, thereby potentially improving road capacity. To attain this capacity benefit, cooperation among autonomous cars is vital. However, the future market may have multiple car brands and the incentive for them to cooperate is unknown. This paper investigates competition and cooperation between multiple car brands, which may offer both autonomous and normal cars. In particular, we develop a two-stage game theoretic model to investigate brands’ strategic interactions and evaluate, from both policy and organizational perspectives, the implications of their cooperation incentives and pricing competition. We compare four market structures: duopoly competition, perfect competition, a public welfare-maximizing monopoly, and a private profit-maximizing monopoly. Various parameters are evaluated, including factors such as price elasticity, capacity effects, and cooperation cost. This evaluation provides policy insights into actions that could be considered by regulators and organizations for the operation of autonomous cars.

Keywords: Autonomous cars; Cooperation strategy; Duopoly competition; Game theory; Regulatory policy

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1. Introduction

With the development of vehicle automation, car manufacturers and software companies have presented prototypes of autonomous cars and announced that autonomous cars with advanced self-driving capabilities are anticipated to be available to the consumer mass market in the near future (e.g., Burns, 2013; Fagnant and Kockelman, 2015; Wadud et al., 2016; Nieuwenhuijsen et al., 2018; Krueger et al., 2019). This innovative transportation technology will undoubtedly alter travelers’ way of traveling and vehicle ownership, as well as when autonomous vehicles and normal cars coexist on the roads.

By their use of sensors and mutual information-exchange technology, autonomous cars can sense and possibly anticipate the braking and acceleration decisions of the leading cars, thereby reducing reaction time and vehicle spacing in comparison to manually driven vehicles. The smaller headway and intervehicle spacing will allow autonomous cars to increase the capacity of existing roads, contributing to reducing travel time losses and potential schedule delays in peak periods. It is widely demonstrated in the literature that predictions of possible effects on road capacity vary, from almost no effect to a quintupling of capacity (e.g., Fernandes and Nunes, 2012; Shladover et al., 2012). To amplify this capacity-improvement benefit, employing cooperation through vehicle-to-vehicle cooperative technology is vital.

There may, however, be multiple autonomous car brands in future vehicle markets. They may include traditional automakers, such as Audi, Toyota, Ford, and Volvo, which are not interested in becoming hardware suppliers for the navigational intelligence incorporated in their vehicles, but also IT companies, such as Google and Baidu, for whom information technology, connectivity, and automotive engineering are an integrated development. There is a strong incentive for policy makers to stimulate different car brands to cooperate due to the potential benefits of capacity improvement. Not all brands, however, have an incentive to cooperate: even though cooperation with another brand may raise the efficiency of road use and reduce travel time for their own car users, the same is true for competing brands, so competitors are strengthened. As cooperation with another brand would also raise competitors’ attractiveness, it would be more profitable to prevent cooperation with another brand.

Research on aspects of the impact of autonomous driving is still in its infancy. Existing studies regarding the operation of autonomous cars have assumed a single system and implicitly focused on one single car brand (e.g., Van den Berg and Verhoef, 2016; Yu et al., 2019; Sun and Yin, 2021), thereby ignoring the competition and cooperation between different brands. We instead investigate the cooperation incentives and pricing competition between multiple brands and evaluate the associated welfare effects. These questions become particularly complex when travelers perceive different brands and car types as imperfect substitutes, for reasons of such unobserved characteristics as brand loyalty, post-sale services, and traveler preferences for particular aspects of cars. Our model accommodates this.
In addition, compared to normal human-driven cars, autonomous cars allow travelers to free up the time traditionally spent in driving-related tasks and enable them to perform leisure- or work-related activities in the vehicle (Haboucha et al., 2017; Pudâne et al., 2018). This can be expected to loosen scheduling constraints that travelers face at home and at work, allowing them to avoid the heaviest traffic more easily. Hence, departure time change appears to be one of the most important alterations in behavior after introducing autonomous cars and it is of great importance to take travelers’ departure time choices into account when studying the effects of these vehicles.

Several earlier studies have investigated travel equilibria involving autonomous cars. Regarding the dynamic congestion setting, previous studies mainly built on the bottleneck model to investigate morning commute dynamics in single- or multi-modal transportation systems, in which autonomous cars were supplied by a single brand. For instance, Van den Berg and Verhoef (2016) investigated travelers’ departure time and travel mode choices when autonomous cars and normal cars both exist, taking the effects of autonomous cars on the capacity, value of time, and preference heterogeneity into consideration. As a result of the reduced value of time caused by autonomous cars, their users travel in the center of the peak period, and normal car users travel in the shoulders of the period. By differentiating travelers’ home- and work-related activities in autonomous cars, Yu et al. (2019) and Pudâne (2020) developed models of dynamic bottleneck congestion to investigate the impacts of travelers’ different activity choices in autonomous cars on aggregate travel patterns. They found that autonomous and normal cars always travel separately, and the specific travel orders depended heavily on utility functions. In their work, Lamotte et al. (2017) investigated how capacity should be allocated to autonomous and normal cars by assuming that autonomous car users are separated from conventional users in their use of road capacity and need to book their trip in advance. Liu (2018) and Zhang et al. (2019) studied the joint equilibrium of departure time and parking location choices when all travelers travel with autonomous cars. Considering the interaction between normal cars and shared autonomous cars, Tian et al. (2019) investigated dynamic departure patterns and endogenous penetration rates of shared autonomous cars under a parking space constraint. Tang et al. (2021) further examined how to regulate the market in the presence of parking space constraints and shared autonomous cars in a multi-modal transportation system, in which travelers could choose among shared autonomous cars, private autonomous cars, and public transit. With respect to the network equilibrium problem, studies have proposed various network equilibrium models to take the effects of autonomous cars on capacity into account (e.g., Chen et al., 2016; Chen et al., 2017; Liu and Song, 2019; Zheng et al., 2020). Wu et al. (2020) discussed the traffic flow patterns under a linear traffic corridor with both expressways for autonomous cars and manual-driving streets, where a trip can consist of both self-driving on expressways and manual-driving on non-autonomous streets. However, these studies only focused on a single brand of autonomous car, such that they acknowledge the impacts of autonomous cars on road capacity.
but do not explore the cooperation incentives between multiple brands.

In this paper, we first use a bottleneck framework to investigate travelers’ travel behavior, in which travelers choose departure times and travel modes to minimize their generalized price. To understand the incentives for different brands to cooperate, we then adopt a two-stage theoretical game framework, in which brands choose cooperation strategies and pricing decisions to maximize their own profits. In the first stage, brands decide their cooperation strategies, taking the impact of the second, price-setting stage into account. Cooperation with another brand also makes competitors’ autonomous cars more effective; a duopoly sees this as a downside, whereas a private or public monopolist owning both brands regards this as an advantage. We therefore compare four types of market structure: duopoly competition, perfect competition, a public welfare-maximizing monopoly, and a private profit-maximizing monopoly. All have multiple product variants.

Our research is, from a methodological viewpoint, closely related to the growing body of literature on externality regulation in aviation and private roads when firms have market power (e.g., Daniel, 1995; de Palma and Lindsey, 2000; Brueckner, 2002; Pels and Verhoef, 2004; Van Dender, 2005; Zhang and Zhang, 2006; Silva and Verhoef, 2013; Van den Berg, 2013; Fu et al., 2018; Kuang et al., 2020). Although some of the results of social and monopolistic supply pricing that we draw upon reflect insights from earlier works, there are important differences. In particular, because in previous networks operators did not need to decide whether to employ within- or cross-brand cooperation to make full use of the capacity benefits, the nature of the interaction between congestion and market power in these earlier studies is different from that in our setting (where brands’ cooperation strategies and the share of autonomous cars have an extensive impact on capacity). In addition, to the best of our knowledge, this paper is the first study to investigate the competition and cooperation between multiple car brands involving autonomous cars.

The main contribution of this study is an in-depth investigation of strategic interactions between different car brands and understanding of the incentives for different brands to cooperate, using a joint theoretical game model and dynamic congestion model. This study moves beyond prior studies that simply consider a single car brand in the market. It is found that a duopoly can have multiple equilibria in a cooperation strategy. Unless cooperation costs are too high, cross-brand cooperation is one of them, with the highest profits and highest welfare; but it is by no means certain that it is the Nash equilibrium that will prevail.

Second, by exploring the roles of different market structures, we compare the proposed duopoly model with a public welfare-maximizing monopoly model and a private profit-maximizing monopoly model. Duopolistic pricing only partly internalizes the externalities that each duopoly’s car users impose upon one another, whereas a public monopolist and a private monopolist both fully internalize the congestion externalities on all users. Different from the multiple equilibria for duopoly markets, with a public monopolist or a private monopolist, the equilibrium is always unique: either cooperation across “brands”
(both variants, then offered by one monopolist) or no cooperation at all (when the cooperation cost is high).

Third, in addition to presenting a model and solving for the equilibrium conditions, we provide exemplary and illustrative results of the effects of change on important organizational and travel parameters. These parameter scenario analyses provide additional insights for organizational and regulatory policy issues that can then be used for evaluation and justification of alternative decisions for regulators seeking to regulate car supply and traffic congestion to improve welfare.

The remainder of this paper is organized as follows. Section 2 presents the modelling framework, which includes travelers’ travel decisions in dynamic congestion, and the brands’ two-stage game model. Section 3 analyzes the strategic interactions between two brands, mainly discussing Bertrand pricing competition for imperfect substitutes and solving the equilibria for the brands’ cooperation decisions. Section 4 compares other market structures, considering perfect competition, a public welfare-maximizing monopoly, and a private profit-maximizing monopoly. Section 5 presents numerical illustrations and sensitivity analysis. Section 6 concludes the paper.

2. Problem description and model formulation

We consider the situation where each morning, travelers travel from home to a workplace. They use the same, single road, which is subject to bottleneck congestion. Everybody travels by car, either in an autonomous or in a normal vehicle. This paper focuses on privately owned autonomous cars; shared autonomous cars are thus not considered. Despite shared autonomous cars being considered as more environmentally sustainable, privately owned autonomous cars may turn out to be preferred by consumers (Zhang et al., 2018) based on several recent autonomous vehicle preference survey results (Bansal et al., 2016; Krueger et al., 2016; Haboucha et al., 2017). We consider price-sensitive demand: if autonomous cars lower costs, demand will increase.

For the model development, we first consider a duopoly model with horizontally differentiated products. In this duopoly game, two competing brands in the market, denoted by 1 and 2, offer both autonomous and normal cars to potential travelers. Travelers regard these cars as imperfect substitutes since there are other factors involved, such as loyalty, service levels, and consumer preferences for other particular aspects.

Autonomous cars are expected to increase road capacity, especially when vehicle-to-vehicle cooperative technology is employed. Nevertheless, as cooperation with another brand would also raise the competitor’s attractiveness, not all brands have an incentive to cooperate. This paper focuses on brands’ cooperation decisions for autonomous cars; cooperative communication between normal cars is thus ignored.

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1 We use car types to distinguish autonomous cars and normal cars, and brands to distinguish cars from different brand firms.
### Table 1. Notation list.

<table>
<thead>
<tr>
<th>Notations for brands’ possible cooperation strategies</th>
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<tbody>
<tr>
<td>NN</td>
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<tr>
<td>YN</td>
</tr>
<tr>
<td>NY</td>
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<tr>
<td>YY</td>
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<td>Y1Y2</td>
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<table>
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<tr>
<th>Notations in the model</th>
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<tr>
<td>$t^*$</td>
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<td>$t$</td>
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<tr>
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<td>$\gamma$</td>
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<td>$\alpha$</td>
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<td>$\theta$</td>
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| $TT(t)$ | Travel time for travelers arriving at time $t$ |
| $c_i(t)$ | Travel cost for car type $i$ users arriving at time $t$, $i \in \{a, n\}$ |
| $c_i$    | Equilibrium travel cost for users with car type $i$ |
| $\delta$ | Compound preference parameter: $\delta = \beta \gamma / (\beta + \gamma)$ |
| $N_{ij}$ | Number of users with car type $i$ of brand $j$, $i \in \{a, n\}$, $j \in \{1, 2\}$ |
| $s$      | Bottleneck capacity for normal cars |
| $s_a$    | Bottleneck capacity for autonomous cars |
| $s_{a(w)}$ | Averaged bottleneck capacity for autonomous cars under strategy profile $m$ |
| $T_f$    | Free-flow travel time |
| $D_{ij}$ | Inverse demand function for car type $i$ of brand $j$ |
| $A_{ij}$ | Maximum willingness to pay for cars of type $i$ and brand $j$ |
| $b_{ij}$ | Parameters in the inverse demand functions |
| $MU_{ij}$ | Per trip mark-up on car type $i$ of brand $j$ |
| $MC_{ij}$ | Marginal automobile cost of car type $i$ of brand $j$ |
| $MC_{co}$ | Extra cost for within-brand or cross-brand cooperation |
| $\text{cop}_{ij}$ | 0-1 variable: 0 denotes not cooperating, and 1 denotes cooperating |
| $f_{ij}$ | Proportion of product $i$ of brand $j$ in all products $i$ |
| $R_m[f_{ij}]$ | Function determining capacity effects under strategy profile $m$ |
| $\Pi_j$ | Profit of brand $j$ |
| $\lambda_{ij}$ | Lagrangian multipliers referring to the travel equilibrium condition |
| $MU^R_{ij}$ | Brand $j$’s reaction function on the mark-up of car type $i$ |
| $MU^{NE}_{ij}$ | Bertrand-Nash equilibrium solutions for the mark up |
| $\Pi_{j,m}$ | Profit of brand $j$ under strategy profile $m$ |
| $W$ | Social welfare |
| $B$ | Consumer benefit |
| $\omega$ | Relative efficiency |
We consider that each brand has three cooperation options for its autonomous cars: not to cooperate at all, to cooperate only within own brand, and to cooperate across brands. Within-brand cooperation means autonomous cars can only communicate with vehicles of own brand. Cross-brand cooperation includes cooperation within own brand and cooperation between different brands, which means that autonomous cars can communicate with vehicles of own brand and vehicles of competitor’s brand. Brands’ cooperation decisions have significant impacts on road capacity, and hence affect travelers’ travel behavior.

Travelers’ behavior is characterized by a dynamic congestion model: travelers are rational utility maximizers, who seek to minimize their generalized travel price by choosing their departure time and travel mode (autonomous versus normal manual driving). Brands’ behaviors are characterized in a two-stage theoretic game model. Both brands are profit-maximizing decision makers. In the first stage, they make their cooperation decisions separately. In the second stage, they compete for supply quantity and pricing to maximize their own profit, taking the decisions in the first stage into account. Section 4 will investigate other market structures.

For ease of reference, Table 1 above summarizes the notation used in this paper. The notation will also be introduced in the text.

2.1 Modelling travelers’ travel decisions under imperfect substitutes

Travelers’ departure time choices are characterized by the bottleneck model, and travel mode choices are characterized by the Wardrop user equilibrium with imperfect substitutes.

The bottleneck model assumes that travelers dislike waiting in traffic congestion, and dislike arriving either earlier or later than the preferred arrival time, $t^*$. A traveler’s travel cost consists of schedule delay cost and travel time cost. Schedule delay cost is the cost due to arriving at a time different from the most preferred arrival time $t^*$. For people arriving early, the schedule delay cost is the product of how early they arrive, measured by $t-t^*$, and the schedule delay value of arriving early, denoted by $\beta$. For people arriving late, the schedule delay cost is defined similarly, and $\gamma$ denotes the schedule delay value of arriving late.

Travel time cost is the sum of the cost of queuing at the bottleneck and the free-flow travel cost. The cost of queuing equals the value of time (VOT) multiplied by the delay caused by queuing. As autonomous cars can drive themselves, the travel time can be used for other activities, which will lower the VOT for autonomous car drivers. Suppose the value of time for normal car drivers is $\alpha$, and is $\theta \cdot \alpha$ for autonomous car drivers, where $0 < \theta < 1$ is the VOT reduction parameter. The queuing delay at the bottleneck equals the number of cars in the queue divided by the capacity of the bottleneck.

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2 For a complete review, see Arnott et al. (1993), Small (2015), and Li et al. (2020).
Let $TT[t]$ denote the travel time of arriving at time $t$, which is the sum of the delay from queuing at the bottleneck and the free-flow travel time. The travel cost for arriving at time $t$ with car type $i$, $c_i[t]$, can thus be expressed as:

$$c_i[t] = \alpha \cdot TT[t] + \left\{ \begin{array}{ll} \beta(t^* - t), & \text{if } t \leq t^* \\
\gamma(t - t^*), & \text{if } t > t^* \end{array} \right. \begin{array}{l}
\text{with a normal car}
\end{array}$$

$$c_a[t] = \theta \alpha \cdot TT[t] + \left\{ \begin{array}{ll} \beta(t^* - t), & \text{if } t \leq t^* \\
\gamma(t - t^*), & \text{if } t > t^* \end{array} \right. \begin{array}{l}
\text{with an autonomous car}
\end{array},$$  \hfill (1)

where subscript ‘$n$’ denotes a normal car and ‘$a$’ denotes an autonomous car.

In equilibrium, the queueing time of normal car users grows at a rate of $\beta/\alpha$ for early arrivals and $-\gamma/\alpha$ for late arrivals. For autonomous car users, the queueing time grows at a rate of $\beta/(\theta \alpha)$ for early arrivals and $-\gamma/(\theta \alpha)$ for late arrivals. Following Arnott et al. (1989), Lindsey (2004), and Van den Berg and Verhoef (2011a, b, 2016), autonomous cars and normal cars will travel separately over time; that is, autonomous car users travel in the center of the peak period and normal car users travel in the shoulders of the peak period.

As autonomous cars and normal cars self-select to travel separately in time, the increase in the capacity of autonomous cars does not affect the capacity when normal cars pass the bottleneck. Note that the cooperation pattern of an autonomous car does not affect the users’ values of time and schedule delays, so that there is no temporal separation of sub-groups of autonomous car users. Following conventions, we define the following compound preference parameter $\delta = \beta \gamma / (\beta + \gamma)$. Following Van den Berg and Verhoef (2011a, b), the equilibrium travel cost, $c_e$, can be shown to be:

$$c_e = \frac{\delta(N_{a1} + N_{a2})}{s_a} + \frac{\delta(N_{n1} + N_{n2})}{s} + \alpha T_f, \begin{array}{l}
\text{with normal cars}
\end{array}$$

$$c_a = \frac{\delta(N_{a1} + N_{a2})}{s_a} + \frac{\delta(N_{n1} + N_{n2})}{s} + \theta \alpha T_f, \begin{array}{l}
\text{with autonomous cars}
\end{array},$$  \hfill (2)

where $N_{ij}$ denotes the number of users with car type $i$ of brand $j$ ($i \in \{a,n\}$, $j \in \{1,2\}$), $s$ denotes the capacity of the bottleneck when normal cars pass, $s_a$ denotes the capacity of the bottleneck when autonomous cars pass, and $T_f$ denotes the free-flow travel time. The bottleneck capacity for autonomous cars depends on brands’ cooperation strategies, which will be defined as $s_{a(m)}$ later (see Eq. (5)), with $m$ representing a given strategy profile.

For the car brands’ market, we consider the differentiated duopoly proposed by Dixit (1979), assuming that demands arise from the quadratic utility function. We consider all car types as imperfect substitutes to account for the fact that not all travelers choose the car with the most attractive generalized price, therefore allowing cars with different generalized prices to have travelers in equilibrium. The quadratic utility function gives rise to a linear demand
structure, with equivalent inverse and direct demands, $D_{ij}$, which is a function of a brand’s own choice of outputs, as well as the competitor’s outputs:

$$
\begin{pmatrix}
D_{a1} \\
D_{a1} \\
D_{a2} \\
D_{a2}
\end{pmatrix}
= 
\begin{pmatrix}
A_{a1} \\
A_{a1} \\
A_{a2} \\
A_{a2}
\end{pmatrix}
- 
\begin{pmatrix}
b_{a1n1} a_{n1} b_{a2n2} a_{n2} \\
b_{a1n1} a_{n1} b_{a2n2} a_{n2} \\
b_{a1n2} a_{n2} b_{a2n2} a_{n2} \\
b_{a1n2} a_{n2} b_{a2n2} a_{n2}
\end{pmatrix}
\begin{pmatrix}
N_{a1} \\
N_{a1} \\
N_{a2} \\
N_{a2}
\end{pmatrix}.
\tag{3}
$$

Where the inverse demand $D_{ij}$ measures the marginal willingness to pay for car type $i$ of brand $j$ in terms of the generalized price. The coefficients $A_{ij}$ ($i \in \{a,n\}, j \in \{1,2\}$) and $b_{ij}$ ($k \in \{a,n_1,a_2,n_2\}$, $l \in \{a,n_1,a_2,n_2\}$) are both positive parameters. $A_{ij}$ is the intercept or the maximum marginal willingness to pay for car type $i$ of brand $j$. Parameter $b_{ij}$ measures how much the inverse demand decreases when the number of corresponding car users increases, where $b_{ij} (l = k)$ measures the own effect (i.e., how much the marginal willingness to pay for product $i$ of brand $j$ decreases if there are more users of this product), and $b_{ij} (l \neq k)$ measures the cross effect (i.e., how much the marginal willingness to pay for product $i$ of brand $j$ decreases if there are more users of the substitutes).

As travelers have idiosyncratic preferences for car brands and car types, they would have different preferences for competing cars when these have an equal generalized price. Travel equilibrium requires that the marginal willingness to pay for car type $i$ ($i \in \{a,n\}$) of brand $j$ ($j \in \{1,2\}$) equals the associated generalized price. The generalized price per trip consists of the travel cost, the mark-up on cars (over marginal cost), the automobile cost (i.e., the marginal resource cost of making a car trip), and the possible cost to employ cooperation between cars. The mark-up is determined by brand firms and is expressed in a per-trip equivalent. It is assumed that within-brand cooperation and cross-brand cooperation have the same cooperation cost, $MC_{cop}$, which is constant per trip. The equilibrium can thus be expressed as:

$$
D_{aj} = c_a + MU_{aj} + MC_{aj} + MC_{cop} \cdot cop_j, \quad j \in \{1,2\},
$$

$$
D_{nj} = c_n + MU_{nj} + MC_{nj}, \quad j \in \{1,2\},
$$

$$
cop_j = \begin{cases} 
1, \text{ cooperate within brand or across brands} \\
0, \text{ otherwise}
\end{cases}
$$

where $MU_{aj}$ and $MU_{nj}$ denote the mark-ups on autonomous cars and normal cars of brand $j$, respectively; $MC_{aj}$ and $MC_{nj}$ denote the marginal automobile cost of autonomous cars

$^3$ For $\forall kl \neq ii$ ($i = a_1, n_1, a_2, n_2$), the corresponding cars are perfect substitutes if $b_{il} = b_{ii}$, imperfect substitutes if $b_{il} < b_{ii}$, and independent if $b_{il} = 0$. If $b_{il}$ were negative, the products would be complements, a possibility that we shall ignore.
and normal cars produced by brand $j$, respectively; $cop_j$ is a dummy variable to characterize brand $j$’s cooperation strategy, where 1 means cooperation between cars and the dummy is 0 otherwise.

2.2 Brands’ possible cooperation strategies

Each brand decides its cooperation strategy separately. This decision carries a trade-off in the sense that cooperation increases the road capacity and performance of the traffic system, at the expense of extra cooperation cost. Therefore, brands choose their cooperation strategies by carefully balancing these factors. Assume that only when both brands want their vehicles to cooperate with the cars in the other brand can they cooperate successfully across brands. The theoretical possibility of cooperation with the other brand’s vehicles, but not within own brand, is discarded. Consequently, five possible strategies can be reached in the equilibrium:

- (not cooperate, not cooperate), in which neither brand 1 nor brand 2 employs cooperation between vehicles. For presentation purpose, we use $NN$ to denote this case.
- (cooperate within brand, not cooperate), in which brand 1 employs cooperation within own brand and brand 2 does not employ cooperation. We use $YN$ to denote this case.
- (not cooperate, cooperate within brand), in which brand 1 does not employ cooperation, and brand 2 employs cooperation within own brand. We use $NY$ to denote this case.
- (cooperate within brand, cooperate within brand), in which brands 1 and 2 both employ cooperation, but only within own brand. We use $YY$ to denote this case.
- (cooperate across brands, cooperate across brands), in which brands 1 and 2 both employ cooperation within own brand and between the brands. We use $YX$ to denote this case.

More detailed analysis on these strategies will be carried out in section 3.2. As we discuss later, brands’ cooperation strategies have a significant impact on travelers’ travel cost and willingness to pay, by changing the effective capacity of the road.

2.3 Impact of brands’ cooperation strategies on capacity

We now introduce the impacts of autonomous cars on road capacity under different cooperation strategies. As all autonomous cars travel jointly in a mixed flow, and the order of autonomous car drivers is random and independent of the cooperation regimes between the cars, it is reasonable and intuitive to assume that there is a constant averaged capacity for autonomous cars that increases with the market share of cooperative autonomous cars. Incorporation of capacity interaction among driving orders of heterogeneous autonomous cars can be made in a further model extension.

Given a certain strategy profile $m$ ($m \in \{NN, YN, NY, YY, YX, YX\}$), let $s_{a(m)}$ denote the averaged capacity for autonomous cars under $m$. Compared to normal cars, autonomous cars can drive with shorter headways. Hence, for any $m$, $s_{a(m)} \geq s$ always holds.
Let $f_{ij}$ denote the market share of $i$-type cars of brand $j$ in all $i$-type cars, which satisfies $f_{ij} = \frac{N_{ij}}{(N_{i1} + N_{i2})}$. Given the share of brand 1’s autonomous cars $f_{a1}$, and brands’ strategy profile $m$, we use $R_m[f_{a1}]$ to characterize the capacity effects caused by autonomous cars (so that $f_{a2} = 1 - f_{a1}$ need not be defined separately). The averaged capacity for autonomous cars thus satisfies $s_{a(m)} = s / R_m[f_{a1}]$.\(^5\)

The capacity effects under different strategy profiles can be formulated as follows.

Under no cooperation at all $(NN)$ or cross-brand cooperation $(Y_Y Y_Y)$, all autonomous cars are homogeneous in terms of facing the same capacity. Therefore, we assume a fixed increase in capacity compared to normal cars. Let $R_{NN}[f_{a1}] = k_{NN}$ and $R_{NN}[f_{a1}] = k_{NN}$, where $k_{NN}$ and $k_{NN}$ are positive constants not exceeding 1. For autonomous cars without cooperation, the road capacity is $s_{a(\text{NN})} = s / k_{NN}$, and for autonomous cars with cross-brand cooperation, the capacity is $s_{a(\text{YY})} = s / k_{YY}$, with $s / k_{YY} \geq s / k_{NN}$.

Under $YN$ or $NY$, the vehicles of one brand cooperate with those within its own brand and the vehicles of the other brand do not. When there is mixing of autonomous cars with different cooperation regimes, the capacity effects may be less beneficial, as the strongest gains are likely to be realized when achieving a situation with exclusively fully cooperative autonomous cars. This would make the capacity for autonomous cars a convex function of the share of cooperative autonomous cars. The averaged capacity for autonomous cars under $YN$ is thus $s / R_{YN}[f_{a1}]$, with $\partial R_{YN}/\partial f_{a1} < 0$ and $\partial^2 R_{YN}[f_{a1}]/\partial f_{a1}^2 \leq 0$. Capacity under strategy profile $NY$ follows directly from symmetry. We calibrate these functions in detail for the numerical simulations presented in section 5.

Under $YY$, the two brands both apply within-brand cooperation. From the symmetry between brands, it is intuitive that $R_{YY}[f_{a1}] = R_{YY}[1 - f_{a1}]$ holds. Hence, the averaged capacity for autonomous cars, $s / R_{YY}[f_{a1}]$, will be symmetric around $f_{a1} = 0.5$. Moreover, $R_{YY}[f_{a1}]$ first increases and then decreases, i.e., $\partial R_{YY}[f_{a1}]/\partial f_{a1} > 0$ for $f_{a1} \leq 0.5$, and $\partial R_{YY}[f_{a1}]/\partial f_{a1} < 0$ for $f_{a1} \geq 0.5$. It is clear that $s / R_{YY}[f_{a1}]$ is at its maximum when $f_{a1} = 0$ and $f_{a1} = 1$, and at its minimum when $f_{a1} = 0.5$.

Accordingly, for strategy profile $m$, the averaged capacity for autonomous cars can be specified as:

\(^4\) $f_{a1} = N_{a1} / (N_{a1} + N_{a2})$. Since $f_{a1} + f_{a2} = 1$, the function of $f_{a2}$ can always be transferred to the function of $f_{a1}$.

\(^5\) We use $1/R$ to characterize the capacity effects because $s$ is the denominator in the travel cost function (see Eq. (4)).
\[ s_{a(m)} = \begin{cases} 
\frac{s}{k_{NN}}, & \text{if } m = NN, \\
\frac{s}{R_{YN}[f_{a1}]}, & \text{if } m = YN, \\
\frac{s}{R_{NY}[f_{a1}]}, & \text{if } m = NY, \\
\frac{s}{R_{YY}[f_{a1}]}, & \text{if } m = YY, \\
\frac{s}{k_{YY}}, & \text{if } m = YY, 
\end{cases} \tag{5} \]

with

\[ 0 < k_{YY} \leq \max\{R_{YN}[f_{a1}], R_{NY}[f_{a1}], R_{YY}[f_{a1}]\} \leq k_{NN} \leq 1. \tag{6} \]

Eq. (6) means that cross-brand cooperation produces the highest capacity and no cooperation at all the lowest, although it still exceeds the capacity for non-autonomous vehicles. Due to the limited information about the averaged capacity for autonomous cars with multiple brands, more detailed expressions for \( R_{m}[f_{a1}] \) will be calibrated using simulation in section 5.

### 2.4 Modelling brands’ two-stage game

Each brand aims at maximizing its own profit by determining the mark-ups on its own autonomous and normal cars and deciding the cooperation strategy for its autonomous cars. The per-trip profit from a specific car type equals the number of cars times the mark-up. We consider that the two brands (1 and 2) compete for travelers in terms of pricing, keeping the other brands’ prices fixed; i.e., there is Bertrand competition with imperfect substitutes. The duopolistic brands’ behavior can then be characterized by the following two-stage game model.

**Stage one**: Each brand (1 and 2) decides its cooperation strategy to maximize its profit, under the constraint of travelers’ travel equilibrium.

**Stage two**: Each brand chooses its mark-up to maximize its profit, under the constraint of travelers’ travel equilibrium, and given the pre-committed cooperation strategy.

Given pre-committed cooperation strategy, brand \(j\)’s profit, \( \Pi_j \), is the sum of the profit from its autonomous cars and normal cars, which can be represented as:

\[ \Pi_j = MU_{aj} \cdot N_{aj} + MU_{nj} \cdot N_{nj}. \tag{7} \]

Using backward induction, we first find the second-stage Bertrand-Nash equilibrium for the static pricing (i.e., mark-up) competition game, taking the brands’ cooperation strategies as given. By calculating the equilibrium mark-ups, we can obtain the brands’ reduced-form profit functions, conditional on their cooperation strategies. The first stage then looks at brands’ cooperation decisions, where they consider the effects on the outcome of the second,

---

6 For presentation purpose, here we omit the strategy indicator in the profit function in stage two, and will take it into account in stage one.
pricing stage, particularly for profits. Fig. 1 summarizes the computational steps for the two-stage game.

Fig. 1. Computational steps for the two-stage game.

3. Solving for equilibrium in the two-stage game

As we consider four alternative types and brands of car and the measurement of the capacity effects is complicated, it is hard to obtain specific expressions for the travel equilibrium solutions analytically. In the following, we explore the analytical properties of the decisions in the various stages.

3.1 Stage two: pricing competition

In stage two, we take brands’ cooperation strategies as given, and as determined in stage one, and look for the Bertrand-Nash equilibrium in the mark-ups. This means that each brand takes the competitor’s mark-ups as fixed, but recognizes that quantities will adjust to maintain equilibrium. Namely, given the pre-committed cooperation strategy, each brand, \( j \), maximizes its own profit by setting the mark-ups \((MU_{aj}, MU_{aj})\). This profit maximization problem under constraint (4) is equivalent to solving the following Lagrangian:
\[ \Pi_j = MU_{aj} \cdot N_{aj} + MU_{nj} \cdot N_{nj} \]
\[ -\sum_{j=1}^{2} \lambda_{aj} \left( D_{aj}[N_{a1}, N_{a1}, N_{a2}, N_{a2}] - c_a[N_{a1}, N_{a1}, N_{a2}, N_{a2}] - MU_{aj} - MC_{aj} - MC_{cop} \cdot cop \right), \]  
\[ -\sum_{j=1}^{2} \lambda_{nj} \left( D_{nj}[N_{n1}, N_{n1}, N_{n2}, N_{n2}] - c_a[N_{n1}, N_{n1}, N_{n2}, N_{n2}] - MU_{nj} - MC_{nj} \right) \]

where \( \lambda_{aj} \) and \( \lambda_{nj} \) are Lagrangian multipliers referring to the travel equilibrium condition in Eq. (4).

Taking the derivatives of \( \Pi_j \) with respect to \( MU_{aj} \), \( MU_{nj} \), \( N_{a1} \), \( N_{a1} \), \( N_{a2} \), and \( N_{a2} \), we can obtain brand \( j \)'s reaction function for the mark-up on car type \( i \), \( MU_{ij}^R \) (see Appendix A):

\[ MU_{ij}^R = \frac{\partial c_i}{\partial N_{ij}} N_{ij} + \frac{\partial c_{(i)}(N_{(i)}j)}{\partial N_{ij}} N_{ij} - \frac{\partial D_{ij}}{\partial N_{ij}} N_{ij} - \frac{\partial D_{(i)j}}{\partial N_{ij}} N_{(i)j} \]
\[ - \lambda_{(i)j} \left( \frac{\partial c_{(i)}}{\partial N_{ij}} - \frac{\partial D_{(i)j}}{\partial N_{ij}} \right) - \lambda_{(i)(-i)} \left( \frac{\partial c_{(i)}}{\partial N_{ij}} - \frac{\partial D_{(i)(-i)}}{\partial N_{ij}} \right) \]
\[ = \frac{\partial c_i}{\partial N_{ij}} \cdot (N_{ij} - \lambda_{(i)j}) + \frac{\partial c_{(i)}}{\partial N_{ij}} \cdot (N_{(i)j} - \lambda_{(i)(-i)}) - \frac{\partial D_{ij}}{\partial N_{ij}} N_{ij} - \frac{\partial D_{(i)j}}{\partial N_{ij}} N_{(i)j} \]
\[ + \lambda_{(i)j} \frac{\partial D_{(i)j}}{\partial N_{ij}} + \lambda_{(i)(-i)} \frac{\partial D_{(i)(-i)}}{\partial N_{ij}}, \]  
\[ (9) \]

with

\[ \lambda_{ij} = \frac{\begin{bmatrix} N_{(i)j} & \left( \frac{\partial D_{(i)j}}{\partial N_{ij}} - \frac{\partial c_{(i)}}{\partial N_{ij}} \right) \left( \frac{\partial c_{(i)}}{\partial N_{ij}} - \frac{\partial D_{(i)(-i)}}{\partial N_{ij}} \right) \end{bmatrix}}{\left( \frac{\partial D_{ij}}{\partial N_{ij}} - \frac{\partial c_i}{\partial N_{ij}} \right) \left( \frac{\partial c_i}{\partial N_{ij}} - \frac{\partial D_{(i)j}}{\partial N_{ij}} \right) - \left( \frac{\partial D_{ij}}{\partial N_{ij}} - \frac{\partial c_{(i)}}{\partial N_{ij}} \right) \left( \frac{\partial c_{(i)}}{\partial N_{ij}} - \frac{\partial D_{(i)(-i)}}{\partial N_{ij}} \right)} \]  
\[ (11) \]

Here, \((-i)\) denotes a car type other than \(i\), and \((-j)\) denotes a brand other than \(j\).

Eqs. (9) and (10) implicitly define the best-response function for the mark-ups. The first two terms in Eq. (9) are the marginal external costs on users of brand \(j\) (including autonomous and normal cars), imposed by brand \(j\)'s car type \(i\) users.\(^7\) The sum of the last four terms gives the duopolistic mark-up. The third and fourth terms are the mark-ups for a private monopolist to supply a single autonomous car \((i = a)\) or normal car \((i = n)\), respectively. The

\(^7\) MEC is the marginal social cost external to the user’s choice, i.e., the derivative of total cost minus the user’s own usage cost.
last two terms capture a correction for the competition from the competitor’s imperfect substitutes. These effects are more complicated. As in Van den Berg (2013), who considered two parallel facilities, the closer the substitutes, the larger this correction: stronger competition leads to lower prices. With independent demands, the price is highest; with perfect substitutes, the price is lowest. The numerical example in section 5 illustrates that these insights also apply in the current context.

Eq. (10) implies that duopolistic pricing only partly internalizes the congestion externalities on own brand car users. Cournot competition would result in full internalization of congestion externalities among a brand’s own customers, a result that is well known from the aviation literature (Brueckner, 2002; Pels and Verhoef, 2004). With Bertrand competition, an operator takes into account that pricing for marginal external congestion costs within the group of its own customers becomes less effective, as some of the customers will switch to the competitor, which would again increase congestion (e.g., Silva and Verhoef, 2013). However, in contrast to the standard Bertrand duopoly, both brands can now ask positive mark-ups, for two reasons. The first is that they internalize the congestion externality on their own brand and collect the revenue from the associated mark-up. The second is that they provide imperfect substitutes, which softens competition. This result corresponds with the findings of Small and Verhoef (2007) for private toll roads with imperfect substitutes, and Silva and Verhoef (2013) for duopolistic airlines.

It can be seen that one brand’s optimal output decision depends on the outputs of the other brand. Given the best-response functions, the Nash-Bertrand equilibrium for the mark-ups, \( (MU_{aj}^{NE}, MU_{nj}^{NE}) \), is at the intersection of these reaction functions, which can be implicitly expressed as:

\[
\begin{align*}
    MU_{aj}^{NE} &= (MU_{aj}^{R}(MU_{a(j)}^{NE}, MU_{m(-j)}^{NE})) , \\
    MU_{nj}^{NE} &= (MU_{nj}^{R}(MU_{a(-j)}^{NE}, MU_{m(-j)}^{NE})) , \\
\end{align*}
\]

(12)

Once the capacity effects function and inverse demand function are given, we can determine the solutions for \( MU_{ij}^{NE} \) through Eqs. (9)-(12). The corresponding profits can be directly solved. Closed-form solutions and economic interpretations are hard to obtain, due to the complex capacity effects. We use the simulation method to solve them in the numerical examples in section 5, and provide intuitive interpretations there.

3.2 Stage one: Nash equilibrium for brands’ cooperation strategies

In this stage, we turn to brands’ decisions on cooperation strategy. Each brand’s strategy set is \{not cooperate, cooperate within brand, cooperate across brands\}.

Let \( \Pi_{j,m} \) denote the profit of brand \( j \) under strategy profile \( m \), which equals the sum of the profit of autonomous cars and the profit of normal cars (see Eq. (7)). When only one
brand wants vehicles to cooperate across brands, cross-brand cooperation will not arise and that brand will instead choose the remaining strategy that has the higher profit. For presentation purposes, we use \( NB \) to denote brand 1 not cooperating and brand 2 wishing to cooperate across brands; \( WB \) for brand 1 cooperating within own brand and brand 2 wishing to cooperate across brands; and conversely for \( BN \) and \( BW \). The payoffs under these strategy profiles thus satisfy:

\[
\begin{align*}
(\Pi_{1, NB}, \Pi_{2, NB}) &= \begin{cases}
(\Pi_{1, NN}, \Pi_{2, NN}) & \text{if } \Pi_{1, NN} \geq \Pi_{2, NN} \\
(\Pi_{1, NY}, \Pi_{2, NY}) & \text{otherwise}
\end{cases}, \\
(\Pi_{1, BN}, \Pi_{2, BN}) &= \begin{cases}
(\Pi_{1, BN}, \Pi_{2, BN}) & \text{if } \Pi_{1, BN} \geq \Pi_{2, BN} \\
(\Pi_{1, BY}, \Pi_{2, BY}) & \text{otherwise}
\end{cases}.
\end{align*}
\]

As it takes both brands to reach cross-brand cooperation, profits under situations \( NB, WB, BW, \) and \( BN \) will never be realized in practice, but they are helpful in determining the equilibrium in the game. The accompanying payoff matrix for this cooperation game is given in Table 2, in which the appropriate cell of the matrix represents the profits to the brand firm when a particular pair of strategies is chosen. The pricing stage, stage two as described above, follows next.

**Table 2. Payoff matrix for the two-stage game.**

<table>
<thead>
<tr>
<th>Brand 1</th>
<th>Brand 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not cooperate</td>
<td>Within brand</td>
</tr>
<tr>
<td>Not cooperate</td>
<td>( (\Pi_{1, NN}, \Pi_{2, NN}) )</td>
</tr>
<tr>
<td>Within brand</td>
<td>( (\Pi_{1, NY}, \Pi_{2, NY}) )</td>
</tr>
<tr>
<td>Across brands</td>
<td>( (\Pi_{1, BN}, \Pi_{2, BN}) )</td>
</tr>
</tbody>
</table>

In a Nash equilibrium, no players can gain by unilaterally changing their own strategy. Substituting Eq. (13) into Table 2 and analyzing the payoff matrix in Table 2, we find:

- If \( \Pi_{1, NY} \geq \max[\Pi_{1, NB}, \Pi_{1, WB}] \) and \( \Pi_{2, BY} \geq \max[\Pi_{2, BN}, \Pi_{2, BB}] \), (cooperate across brands, cooperate across brands) is a Nash equilibrium.
- If \( \Pi_{1, NN} \geq \Pi_{1, NY} \) and \( \Pi_{2, NN} \geq \Pi_{2, NY} \), (not cooperate, not cooperate) is a Nash equilibrium.
- If \( \Pi_{1, NY} \geq \Pi_{1, NY} \) and \( \Pi_{2, NY} \geq \Pi_{2, NY} \), (not cooperate, cooperate within brand) is a Nash equilibrium.
- If \( \Pi_{1, NY} \geq \Pi_{1, NY} \) and \( \Pi_{2, NY} \geq \Pi_{2, NY} \), (cooperate within brand, not cooperate) is a Nash equilibrium.
- \( \Pi_{1, NY} \geq \Pi_{1, NN} \) and \( \Pi_{2, NY} \geq \Pi_{2, NN} \), (cooperate within brand, not cooperate) is a Nash equilibrium.
- If \( \Pi_{1, YY} \geq \Pi_{1, NY} \) and \( \Pi_{2, YY} \geq \Pi_{2, NY} \), (cooperate within brand, cooperate within brand) is a Nash equilibrium.
A duopoly may have multiple Nash equilibria for this two-stage game, which is illustrated in the numerical examples in section 5.

4. Other market structures

For comparison purposes, this section turns to three other market structures: perfect competition, a public welfare-maximizing monopoly, and a private profit-maximizing monopoly. The public and private monopolists each own two brands but coordinate these brands’ strategies to maximize a composite objective (aggregate welfare, or aggregate profits). The two “brands” are, therefore, still offered as imperfect substitute goods on the market.

4.1 Perfect competition

Perfect competition leads to marginal cost pricing and thus zero mark-ups. With two brands and two car types, one would not normally expect to see perfect competition in general, but we still consider this case as the absence of monopolistic demand-related mark-ups and congestion charges provides an important and natural benchmark. The best way to imagine the conditions under which perfect competition would prevail is when each of the four brand-type combinations could be supplied by large numbers of small firms. The demands are determined by the following Wardrop user equilibrium conditions:

\[
D_{aj}\left[N_{a1}, N_{a2}, N_{n1}, N_{n2}\right] = c_a[N_{a1}, N_{a2}, N_{n1}, N_{n2}] + MC_{aj} + MC_{cop} \cdot cop_j, \quad j \in \{1, 2\},
\]

\[
D_{nj}\left[N_{a1}, N_{a2}, N_{n1}, N_{n2}\right] = c_n[N_{a1}, N_{a2}, N_{n1}, N_{n2}] + MC_{nj}, \quad j \in \{1, 2\},
\]

where \( D_{aj} \) is given by Eq. (3). As capacity effects do not affect the cost difference between driving autonomous cars and normal cars (see Eq. (2)), travelers will ignore these effects when choosing car types under perfect competition. Given the expression of the inverse demand function, one can obtain closed-form solutions for \( N_{a1}, N_{a2}, N_{n1}, \) and \( N_{n2} \).

4.2 Public welfare-maximizing monopoly

We now turn to a public welfare-maximizing monopoly, in which a public monopolist maximizes aggregate welfare by setting the mark-ups and quantities for the four types of car. Social welfare, \( W \), is operationalized as a social surplus and is defined as the consumer benefit, \( B[N_{a1}, N_{a2}, N_{n1}, N_{n2}] \), minus total usage cost (or, equivalently, consumer surplus plus profit). Assuming that there are no income effects, consumer benefit is the line integral of the four inverse demand functions and is independent of the path used for the integration.

The associated social welfare maximization problem is:
Combining the user equilibrium conditions and solving the above welfare maximization problem yields the following mark-ups (see Appendix B):

\[
MU_{ij}^{\text{pub}} = \frac{\partial c_i}{\partial N_{ij}} \sum_{j=1}^{2} N_{ij} + \frac{\partial c_{-i}}{\partial N_{ij}} \sum_{j=1}^{2} N_{(-i)j} - MEC_{ij}.
\]  

The superscript ‘pub’ denotes public welfare-maximizing pricing. Eq. (16) implies that the welfare-maximizing public mark-up on cars of type \( i \) and brand \( j \) equals the marginal external costs imposed by these car users, \( MEC_{ij} \). It is isomorphic to the conventional Pigouvian congestion toll (see Pigou, 1920; Small and Verhoef, 2007). The public monopolist thus fully internalizes the externalities caused by all car users, as one might have expected.

In terms of the cooperation strategy, as public welfare-maximizing pricing fully internalizes the congestion externality, the public monopolist is more likely to have an incentive to apply cross-brand cooperation. This is because the increase in the congestion costs for users with substitutes depresses their willingness to pay, and hence the welfare that the monopolist can extract from them for a given level of demand, on a dollar-by-dollar basis. The social planner would find it beneficial to apply cross-brand cooperation to reduce congestion more fully.

4.3 Private profit-maximizing monopoly

A private monopolist would maximize the aggregate profit in the market by setting all mark-ups and quantities for all cars:

\[
\max_{MU_{ij}, N_{ij}} \Pi = \sum_{i \in \{a,a\}, j \in \{1,2\}} \sum_{j=1}^{2} MU_{ij} \cdot N_{ij}.
\]  

Solving the above profit maximization problem yields the following monopolistic mark-up:

\[
MU_{ij}^{\text{mon}} = \frac{\partial c_i}{\partial N_{ij}} \sum_{j=1}^{2} N_{ij} + \frac{\partial c_{-i}}{\partial N_{ij}} \sum_{j=1}^{2} N_{(-i)j} - \frac{\partial D_{ij}}{\partial N_{ij}} \cdot N_{ij} + (A_{ij} - D_{ij} + \frac{\partial D_{ij}}{\partial N_{ij}} \cdot N_{ij}),
\]  

where the superscript ‘mon’ denotes the private monopoly. Here, the first two terms are the marginal external costs. The third term is the monopolistic mark-up from users with car type
of brand \( j \). The fourth term is the mark-up due to the other three substitutes: it measures the effect that a higher price on \( ij \) has on increasing the demand for \(-ij\), which raises the profit on these sub-markets. The closer the car type and brand substitutes (i.e., \( \left| \frac{\partial D_y}{\partial N_{-y}} \right| \)) gets closer to \( \left| \frac{\partial D_y}{\partial N_{y}} \right| \), the higher the mark-ups, since this increases the strength of the third effect.

Comparing Eq. (9) and Eq. (18), we can find the difference in the pricing rule between duopoly competition and a private monopoly:

\[
MU_{yij}^{mon} - MU_{yij}^{R} = (\frac{\partial c_i}{\partial N_y} - \frac{\partial D_y}{\partial N_{(-i,-j)}})(N_y + \lambda_{y(-j)}) + (\frac{\partial c_i}{\partial N_{(-i)}} - \frac{\partial D_y}{\partial N_{(-i,-j)}})(N_{(-i),y} + \lambda_{(-i)yx}(y-x)). \tag{19}
\]

The private monopolistic mark-up is intuitively higher than the duopolistic mark-up. The monopolist not only charges a higher demand-related mark-up, as consumers switching to other product variants does not mean losing them in terms of revenue, but also fully internalizes congestion externalities. This is, in the first place, due to congestion externalities imposed on all travelers now being considered, not just on the subset served by the duopolies. In the second place, duopolies do not internalize the full externalities that customers impose upon one another, since customers switching to the other brand would still create congestion for the brand’s remaining customers. Eq. (19) also implies that the closer the substitutes, the smaller the difference in the pricing of the two regimes.

A private monopolist may lead to a suboptimal cooperation strategy due to the distortion in mark-ups, especially when the cooperation cost is high.

### 5. Numerical simulations

The previous sections presented analysis of the decision mechanisms and other properties of the proposed model. To further understand the comparative static properties of the proposed model, we carry out some numerical analyses using a simulation method. In the following, we calibrate the parameter values for the base case and complete our base case numerical model. The calibration assumes that the two brands are ex-ante symmetric. This assumption is not vital to the results; it simply helps the interpretation. After discussing the base case, we turn to sensitivity analyses, where we change some of the more important parameters. In addition to providing valuable insight into the equilibrium and welfare impacts of these factors, these exercises are also important because there is little to no guidance from the literature on the values of these parameters. By this method, we can derive some insightful observations concerning key policy and organizational implications.

#### 5.1 Calibration of capacity parameters

As discussed, when autonomous cars travel, the averaged road capacity depends heavily
on the share of autonomous cars and their cooperation regimes. This poses challenges for calibration. Because there is limited information about the specific effects, it is important for a meaningful simulation to calibrate the capacity effects carefully. We approach this task by approximating the shape of the capacity function applying some of the values predicted in the literature.

As is well established in the literature, the average increase in the capacity is a non-linear function of the share of the autonomous cars that cooperate (Tientrakool et al., 2011; Fernandes and Nunes, 2012; Shladover et al., 2012). We should distinguish the effects of different cooperation regimes. To that end, we first assume that if all autonomous cars are non-cooperative, the averaged capacity is \( s_{a(\text{NN})} = 1.35s \); if all autonomous cars are cooperative across brands, the averaged capacity is \( s_{a(\text{YY})} = 3s \).\(^8\) We further assume that if brand 1 applies cooperation within own brand and brand 2 does not, the averaged capacity for autonomous cars satisfies \( s_{a(\text{YN})}[0.8] = 1.62s \) and \( s_{a(\text{YN})}[0.5] = 1.43s \). Meanwhile, if both brands cooperate within own brand, we assume \( s_{a(\text{YY})}[0.8] = s_{a(\text{YY})}[0.2] = 2s \). By fitting these values with polynomial functions, we obtain the following capacity effects:

\[
R_m[f_{a_1}] = \begin{cases} 
1/1.35, & m = \text{NN}, \\
1-0.18 f_{a_1}^{2.41} - 0.26(1-f_{a_1})^{-0.04}, & m = \text{YN}, \\
1-0.18(1-f_{a_1})^{2.41} - 0.26 f_{a_1}^{-0.04}, & m = \text{NY}, \\
1-0.67(f_{a_1}^{1.70} + (1-f_{a_1})^{1.70}), & m = \text{YY}, \\
1/3, & m = Y_X Y_X.
\end{cases}
\] (20)

Consistent with the theoretical model, the capacity effect function, \( R[f_{a_1}] \), is concave, as depicted in Fig. 2a. Consider that the capacity for normal cars is 3,600. The averaged capacity function for autonomous cars, \( s_{a(\text{m})}[f_{a_1}] \), is convex, as depicted in Fig. 2b.

\(^8\) Here we ignore the patterns of the vehicle following and only consider the averaged capacity for all autonomous cars.

![Fig. 2. Capacity effects of autonomous cars for the numerical model.](image)
5.2 Calibration of user cost and inverse demand functions

Following Van den Berg and Verhoef (2016), we focus on petrol autonomous and normal cars. The schedule delay parameters are based on the ratios $\beta/\alpha=39/64$ and $\gamma/\alpha=1521/640$ established by Small (1982), as these are common in the literature. We consider a trip length of 20 km, with a free-flow travel time of 20 minutes. We use a VOT ($\alpha$) of 10 and assume a base value of $\theta$ of 0.8. We normalize the automobile cost of normal cars to zero and use an $MC_a$ at €1.51 (Van den Berg and Verhoef, 2016). We also assume that cooperation will increase the automobile cost by 10%, which means that the cooperation cost per trip, $MC_{cop}$, is €0.151.

To calibrate the inverse demand functions, we use marginal cost (MC) pricing with normal cars and non-cooperative autonomous cars as the base case. For the base calibration, there are 9,000 users. The capacity is 3,600 for normal cars and 4,860 (1.35s) for autonomous cars. The elasticity with respect to own generalized price is -0.35; with substitutes from the same brand or the same type of car, the cross-price elasticity is 0.2. For substitutes with different car types and brands, the cross-price elasticity is 0.1.9 Under this calibration, the generalized price in the equilibrium with autonomous cars is €13.51, and with normal cars is €13.87.

5.3 Base case

Table 3 gives the outcomes10 for the base calibrations. We compare the outcomes under four market structures: perfect competition (MC), duopoly competition (Duopoly), public monopolist (Public), and private monopolist (Private). Under the base calibration, a public welfare-maximizing monopolist and a private profit-maximizing monopolist will choose full cooperation ($Y_XX$).11 We therefore only present the outcomes with $Y_XX$ for these two regimes. Relative efficiency, $\omega$, is defined as the welfare gain of a policy from the case without cooperation under marginal cost pricing, divided by the gain from welfare-maximizing public pricing with cross-brand cooperation. A negative value thus reflects that welfare is below that under perfect competition without cooperation. As the two brands are ex-ante symmetric, outcomes under $YN$ and $NY$ are also symmetric, as presented in the fourth and fifth columns in Table 3.

We first look at the mark-ups under duopoly competition. The results show that, compared to non-cooperation, within-brand cooperation ($YY$) and full cooperation ($Y_XX$) both

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9 The inverse demand functions are:

\[
D_{nx} = 103.6 - 0.0154N_{nx} - 0.0083N_{nx} - 0.0083N_{nx} - 0.0077N_{nx} - 0.0154N_{nx} - 0.0085N_{nx}, \\
D_{nx} = 106.31 - 0.0085N_{nx} - 0.0162N_{nx} - 0.0077N_{nx} - 0.0088N_{nx} - 0.0085N_{nx} - 0.0162N_{nx}.
\]

10 The results of interest are: the mark-ups ($MU_j$), number of users ($N_j$), travel cost ($c_j$), profit of each brand ($\Pi_j$), consumer benefit ($B$), welfare ($W$), and relative efficiency ($\omega$).

11 This is because cross-brand cooperation brands can reduce the congestion the most. Congestion is internal to welfare-maximizing and profit-maximizing monopolists, thereby improving the welfare or industry profit.
raise the mark-ups on all cars, and more so for the latter. In contrast, if only one brand cooperates and the other does not (YN and NY), the mark-up on cooperative autonomous cars declines, whereas that on non-cooperative autonomous cars rises. The non-cooperating brand thus benefits from the implementation of cooperative cars by the other brand. The mark-ups on normal cars still increase, but less so than those under YY and YxYx. Indeed, cooperation lowers the travel cost for all users by increasing the capacity of autonomous cars. As we consider price-sensitive demand, the decrease in the travel cost will, in turn, attract more car users, which will increase congestion and decrease willingness to pay. These two effects jointly determine the mark-ups, in which the former tends to raise them and the latter to reduce them. For YY and YxYx, the congestion effects induced by the increasing demand dominate, which leads to increased mark-up on both autonomous cars and normal cars. The same logic applies to the decrease in the mark-ups on normal cars with YN and NY. However, for the mark-ups on autonomous cars in YN and NY, the cooperating brand tends to set a lower mark-up to compensate for the possible loss of customers when the full cooperation cost is added to the price and thus to attract more autonomous car users. The non-cooperating brand benefits from the cooperating brand’s cooperation without paying any cooperation cost and sets a higher mark-up on autonomous cars.

Table 3. Outcomes under the base calibration.

<table>
<thead>
<tr>
<th></th>
<th>NN</th>
<th>YN</th>
<th>NY</th>
<th>YY</th>
<th>YxYx</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MC Duopoly</td>
<td>Duopoly</td>
<td>Duopoly</td>
<td>MC Duopoly</td>
<td>Public</td>
</tr>
<tr>
<td>MUa1</td>
<td>0</td>
<td>23.29</td>
<td>23.17</td>
<td>23.41</td>
<td>23.33</td>
</tr>
<tr>
<td>MUa2</td>
<td>0</td>
<td>23.29</td>
<td>23.41</td>
<td>23.17</td>
<td>23.33</td>
</tr>
<tr>
<td>MUb1</td>
<td>0</td>
<td>24.09</td>
<td>24.14</td>
<td>24.16</td>
<td>24.30</td>
</tr>
<tr>
<td>MUb2</td>
<td>0</td>
<td>24.09</td>
<td>24.14</td>
<td>24.16</td>
<td>24.30</td>
</tr>
<tr>
<td>Na1</td>
<td>2,279</td>
<td>1,744</td>
<td>1,753</td>
<td>1,740</td>
<td>1,758</td>
</tr>
<tr>
<td>Na2</td>
<td>2,279</td>
<td>1,744</td>
<td>1,740</td>
<td>1,753</td>
<td>1,758</td>
</tr>
<tr>
<td>Nb1</td>
<td>2,221</td>
<td>1,703</td>
<td>1,707</td>
<td>1,705</td>
<td>1,712</td>
</tr>
<tr>
<td>Nb2</td>
<td>2,221</td>
<td>1,703</td>
<td>1,705</td>
<td>1,707</td>
<td>1,712</td>
</tr>
<tr>
<td>\Pi_1</td>
<td>0</td>
<td>81,628</td>
<td>81,804</td>
<td>81,936</td>
<td>82,616</td>
</tr>
<tr>
<td>\Pi_2</td>
<td>0</td>
<td>81,628</td>
<td>81,936</td>
<td>81,804</td>
<td>82,616</td>
</tr>
<tr>
<td>B</td>
<td>533,812</td>
<td>482,475</td>
<td>482,879</td>
<td>482,879</td>
<td>484,099</td>
</tr>
<tr>
<td>W</td>
<td>410,625</td>
<td>404,139</td>
<td>405,430</td>
<td>405,430</td>
<td>409,385</td>
</tr>
<tr>
<td>\omega</td>
<td>0.00</td>
<td>-0.26</td>
<td>-0.21</td>
<td>-0.21</td>
<td>-0.05</td>
</tr>
</tbody>
</table>
As for the cooperation strategy in duopolies, there exist two pure Nash equilibria: (within brand, within brand) and (across brands, across brands). Cross-brand cooperation is more attractive and hence the dominating equilibrium. As illustrated in the payoff matrix of this game in Table 4, if brand 1 does not cooperate at all, brand 2’s best response is to cooperate within own brand; if brand 1 cooperates within brand, brand 2’s best response is to cooperate within brand; if brand 1 cooperates across brands, brand 2’s best response is cross-brand cooperation. By symmetry, the same best responses hold for brand 2. Consequently, full cooperation and within-brand cooperation are the two Nash equilibria. Cross-brand cooperation may seem the most likely to occur, as it has the highest profits and gives the greatest welfare, but it is by no means certain that it is the Nash equilibrium that will prevail, as it requires a simultaneous move.

Table 4. Payoff matrix for the two-stage game.

<table>
<thead>
<tr>
<th>Brand 2</th>
<th>Not cooperate</th>
<th>Within brand</th>
<th>Across brands</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brand 1 Not cooperate</td>
<td>(81628,81628)</td>
<td>(81936,81804)</td>
<td>(81936,81804)</td>
</tr>
<tr>
<td>Within brand</td>
<td>(81804,81936)</td>
<td>(82616,82616)</td>
<td>(82616,82616)</td>
</tr>
<tr>
<td>Across brands</td>
<td>(81804,81936)</td>
<td>(82616,82616)</td>
<td>(84533,84533)</td>
</tr>
</tbody>
</table>

In terms of relative efficiency, despite the high congestion levels, perfect competition performs best (apart from the welfare-maximizing public monopoly with $Y_xY_x$), followed by duopoly competition, and worst performing is private monopoly. This is due to the market power distortion of duopolistic and private monopolistic pricing. Under duopoly competition, cooperation across brands performs best, as it reduces congestion the most. Table 4 shows that as the benefit partly goes to the competitor, duopolistic brands cannot capture as much surplus generated by high quality as a private monopolist.

5.4 Sensitivity analysis

The numerical simulation for the base calibration has (cooperate within brand, cooperate within brand) and (cooperate across brands, cooperate across brands) as the two Nash equilibria for the duopoly cooperation strategies, the latter being the dominating one. In this section, we test the sensitivity of the results and the accompanying mark-ups with respect to the cooperation cost, $MC_{coop}$, the VOT reduction parameter, $\theta$, the size and shape of the capacity effect function, and users’ price elasticity.\[\text{\footnote{The effects of other parameters are in line with the theoretical discussion. Moreover, for these parameters, we have much more guidance from the literature. Hence, these parameters will not be discussed further here.}}\]

We first investigate two symmetric brands. Subsection 5.4.1 will vary the cooperation
cost, $MC_{cop}$, and subsection 5.4.2 the VOT reduction parameter, $\theta$. Subsection 5.4.3 considers the impacts of the maximum increase in capacity caused by autonomous cars. Subsection 5.4.4 analyzes the effects of the capacity effect function. Subsection 5.4.5 varies price elasticity of demand. In subsection 5.4.6, we turn to considering asymmetric cooperation cost.

5.4.1 Varying the cooperation cost

It is plausible that there are costs of cooperation, but there is great uncertainty about this parameter. In this subsection, we vary the cooperation cost, $MC_{cop}$, from 0 to 4. As we shall see, the cooperation cost substantially affects suppliers’ equilibrium cooperation strategies.

Fig. 3. Effects of cooperation cost on equilibria under different markets.

Fig. 3a compares brands’ profits with alternative cooperation strategies under duopoly competition. For both within-brand cooperation and full cooperation, the two brands have the same supply outcome due to symmetry. When both brands apply cooperation, the profit per brand decreases with the cooperation cost. In contrast, when only one brand applies cooperation and the other does not, a larger cooperation cost lowers the profit of the cooperating brand and raises that of the non-cooperating brand.
Rewriting the profits in Fig. 3a into their payoff matrices, we can obtain the Nash equilibria for the cooperation strategies, which are shown with solid curves in Fig. 3a and further summarized in Table 5. The dotted curves in Fig. 3a represent the non-equilibrium strategies. Table 5 shows which outcome is a Nash equilibrium over which range of the cooperation cost. We see that typically there are multiple Nash equilibria of the duopoly game. Unless cooperation costs are very high, full cooperation is one of these equilibria; but again, it is by no means certain that it will prevail. No cooperation and within-brand cooperation are also often Nash equilibria.

Fig. 3 also reveals that duopoly competition and private monopoly may lead to below-optimal cooperation. Given that we have a duopoly, Fig. 3a shows that when the cooperation cost exceeds 3.59, full cooperation stops being the Nash equilibrium. Fig. 3b shows that full cooperation will still be socially optimal under duopoly competition until the cooperation cost exceeds 3.68. In contrast, the public monopolist stops full cooperation only when the cooperation cost exceeds 5.14, whereas the private monopolist has already stopped when it exceeded 2.48. Consequently, compared to a social optimum cooperation strategy, duopoly competition and private monopoly may lead to too little cooperation, especially for the private monopolist.

Table 5. Nash equilibria under a duopoly.

<table>
<thead>
<tr>
<th>NE</th>
<th>$MC_{cop} &lt; 0.28$</th>
<th>$0.28 \leq MC_{cop} &lt; 0.62$</th>
<th>$0.62 &lt; MC_{cop} \leq 3.59$</th>
<th>$MC_{cop} &gt; 3.59$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Not, not)</td>
<td>×</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>(Within, not)</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>(Not, within)</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>(Within, within)</td>
<td>√</td>
<td>√</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>(Across, across)</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>×</td>
</tr>
<tr>
<td>Number of NE</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

In addition to the equilibrium for the cooperation strategies, we are also interested in the interactions of the pricing competition between different car types and brands. A higher cooperation cost will naturally make autonomous cars less attractive and normal cars more appealing. Induced switching from autonomous cars to normal cars then means: (1) a lower equilibrium price for autonomous cars and a higher equilibrium price for normal cars; and (2) undetermined congestion effects, which are jointly determined by heterogeneity effects, capacity effects, and price-sensitive demand. The first effect tends to lead brands to reduce the mark-up on autonomous cars in order to attract more users and to raise the mark-up on normal cars. For the second effect, capacity effects tend to reduce the mark-up on autonomous cars (due to the reduction in congestion) and raise that on normal cars, and conversely for the heterogeneity effects; price-sensitive demand tends to reduce all mark-ups to attract more car
users. As discussed in the theoretical section, duopolistic pricing is jointly determined by the above two effects: welfare-maximizing public pricing internalizes the congestion externality for all users, and private monopolistic pricing internalizes both competition and congestion.

Fig. 4 depicts the specific pricing decisions under each equilibrium. Fig. 4a shows that the public monopolist slightly lowers the mark-up on all cars, and overall congestion becomes lower as the cooperation cost increases. In contrast, with full cooperation, the private monopolist lowers the mark-up on autonomous cars as a result of the reduction in private monopolistic power, and slightly lowers that on normal cars because of the reduction in congestion. A jump occurs when the private monopolist changes from full cooperation to non-cooperation, as shown in Fig. 4b. Fig. 4c shows that for duopoly competition, a higher cooperation cost lowers the mark-up on autonomous cars and raises that on normal cars. This is because a higher cooperation cost makes autonomous cars less attractive and normal cars more appealing. Meanwhile, the reduction in congestion tends to lower the mark-up on all cars. The first effect remains dominant.

Fig. 4. Effects of cooperation cost on equilibrium mark-ups.

Fig. 5 compares the relative efficiencies of equilibria strategies under different market structures. Relative efficiency decreases with the cooperation cost for all market structures
and does so most for private monopoly. This is intuitive because a higher cooperation cost leads to less cooperation between cars, a possible societal benefit of a private supply, making demand-related mark-ups, a societal disadvantage, relatively more important. Welfare-maximizing public pricing with cross-brand cooperation performs best, followed by marginal cost pricing, duopolistic pricing, with private monopolistic pricing showing the worst performance. For the different equilibrium strategies under duopoly competition, the welfare loss is the smallest when the two brands both choose cross-brand cooperation, followed by both choosing within-brand cooperation, and the largest when neither of them cooperates.

Fig. 5. Effects of cooperation cost on relative efficiencies.

5.4.2 Varying the VOT reduction parameter

We next vary the VOT reduction parameter, \( \theta \), from its theoretical minimum of \( \beta / \alpha \) to 1. The base value was 0.8. We find that \( \theta \) does not affect the Nash equilibria for the cooperation strategies, (cooperate within brand, cooperate within brand) and (cooperate across brands, cooperate across brands), where full cooperation is more attractive and is the dominating equilibrium. This is because for all \( \theta \) between \( \beta / \alpha \) and 1, brands benefit most from cross-brand cooperation, followed by solely within-brand cooperation, only one-brand cooperation, and lastly by no cooperation at all, as shown in Fig. 6a. Fig. 6b shows that full cooperation is also the social optimum given a duopoly. Nonetheless, as explained, this does not mean that cross-brand cooperation will prevail.
We now turn again to equilibrium mark-ups under different market structures. Similar to what we found for cooperation cost, a larger $\theta$ means a lower willingness to pay for autonomous cars and a higher willingness to pay for normal cars. The specific congestion effects are undetermined. The difference is that a larger $\theta$ means a higher value of time and hence also reduces the negative externality imposed by autonomous car drivers themselves, which tends to lower the mark-up on autonomous cars.
Fig. 7 shows that, as $\theta$ increases, all suppliers tend to reduce the mark-up on autonomous cars and raise that on normal cars. With a public monopolist, the mark-up on normal cars changes more than that on autonomous cars, implying that the increase in congestion imposed by normal cars is stronger than the reduction by the use of autonomous cars. In contrast, for private monopolistic pricing and duopolistic pricing, the mark-up on autonomous cars changes more, owing to competition and substitutional effects between different cars, which affects autonomous cars more.

Fig. 8 compares the changes in relative efficiencies. In this sensitivity, as $\theta$ increases, the performance of duopoly competition and private monopoly both become modestly better, owing to the increasing competition and reducing market power. Full cooperation also naturally performs better than within-brand cooperation. Under perfect competition, a higher $\theta$ leads more drivers to switch to normal cars and raises congestion. As a result, the relative efficiency decreases. It is clear from Fig. 8 that the impact of varying $\theta$ on relative efficiency is modest: the curves are all relatively flat.

![Fig. 8. Effects of $\theta$ on relative efficiencies.](image)

5.4.3 Varying the maximum capacity

Next, we look at the impacts of the maximum increase in capacity due to cooperation, by ranging the maximum capacity from 2.5s to 4.5s.\(^\text{13}\) Since profit is highest with full cooperation, followed by within-brand cooperation, as shown in Fig. 9, there always exist two Nash equilibria: (cooperate within brand, cooperate within brand) and (cooperate across brands, cooperate across brands). These findings are not affected by the variation of maximum capacity. Cross-brand cooperation is again the dominating equilibrium and achieves social optimality.

\(^{13}\) The maximum capacity will change the shape of $R(f_{s1})$. The higher the maximum capacity, the more convex the $R(f_{s1})$.
Fig. 9. Effects of maximum capacity on profit (per brand).

Fig. 10. Effects of maximum capacity on equilibrium mark-ups.

(a) Duopoly

(b) Public monopoly

(c) Private monopoly
Fig. 10 depicts the effects on mark-ups. Fig. 10a shows that with cross-brand cooperation, the mark-ups on autonomous cars and normal cars both increase with the maximum capacity, whereas when two brands cooperate solely within own brand, the mark-ups decrease. For cross-brand cooperation, this is because a larger maximum capacity reduces the travel costs more and attracts a greater number of car users, which softens the competition between brands. For cooperation within own brand, due to symmetry, the demand for autonomous cars of different brands is the same, which causes the eventual capacity of autonomous cars to achieve the lowest value of the parabolas (see Fig. 2b). A higher maximum capacity thus means a lower equilibrium capacity for autonomous cars, which leads to opposite outcomes compared to the case of cross-brand cooperation. Fig. 10 (b-c) suggests that public monopolist and private monopolist structures also tend to lower the mark-up on autonomous cars, and slightly raise that on normal cars, due to the resulting congestion effects.

Fig. 11 compares the performances of different market structures in the equilibrium. It can be observed that relative efficiencies under duopoly competition with full cooperation ($Y_X, Y_X$), private monopoly, and perfect competition all increase with the maximum capacity.\textsuperscript{14} Intuitively, a higher maximum capacity improves the welfare gains for all regimes by reducing congestion, which lowers the welfare gain from optimal pricing in the denominator of the relative efficiencies. Although the increase in capacity lowers competition and raises market power, the reduction in congestion still dominates, especially so for a private monopoly. Conversely, when the two duopolies cooperate only within own brand ($Y_X, Y_X$), the relative efficiency declines, due to the decreasing equilibrium capacity for autonomous cars passing the bottleneck.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig11.png}
\caption{Effects of maximum capacity on relative efficiencies.}
\end{figure}

\textsuperscript{14} Public and private monopolists both choose cross-brand cooperation.
5.4.4 Varying the capacity effects under cooperation solely within brand

In this sensitivity analysis, we vary the value of \( s_{u[YY]}[0.8] \) from 1.65s to 3s, and keep the capacity function under other cooperation strategies the same as the base case.\(^{15}\) The lower value of \( s_{u[YY]}[0.8] = 1.65s \) means that only within-brand cooperation is more similar to non-cooperation, whereas \( s_{u[YY]}[0.8] = 3s \) means it is more similar to cross-brand cooperation.

Fig. 12 depicts the effects of \( s_{u[YY]}[0.8] \) on brands’ profit, where the solid curves represent Nash equilibrium strategies and dotted curves represent non-equilibrium strategies. By analyzing the profit matrices under various strategies in Fig. 12, we find that when \( s_{u[YY]}[0.8]/s \) is lower than 1.8, there are three Nash equilibria: (cooperate within brand, not cooperate), (not cooperate, cooperate within brand), and (cooperate across brands, cooperate across brands). As \( s_{u[YY]}[0.8]/s \) increases, the advantage of cooperation is strengthened. Accordingly, (cooperate within brand, cooperate within brand) and (cooperate across brands, cooperate across brands) become the two equilibria, with cross-brand cooperation the dominating one.

![Fig. 12. Effects of \( s_{u[YY]}[0.8] \) on the profit of each brand.](image)

The effects on equilibrium mark-ups are shown in Fig. 13. As \( s_{u[YY]}[0.8] \) increases, the effects of only within-brand cooperation become closer to those of cross-brand cooperation. Mark-ups on autonomous and normal cars under within-brand cooperation both increase and approach those under full cooperation. This is because a higher \( 1/R_{YY}[0.8] \) reduces competition by lowering the travel cost and attracting more car users.

\(^{15}\) In this way, we vary the capacity effect of autonomous vehicles without cooperation relative to cross-brand cooperation.
5.4.5 Varying the price elasticity of demand

To investigate the effects of price elasticities on duopoly decisions, we increase and decrease the own-price elasticity and cross-price elasticities by the same percentage at the same time. In the base case, own-price elasticity was -0.35, and cross-price elasticities were 0.2 and 0.1. We now increase these elasticities to twice and three times the size, compared to the base case, as well as decreasing them to 1/3 and 1/2. To calibrate the inverse demand functions for each price elasticity, we make the demand curve tilt around the equilibrium in the base equilibrium without autonomous cars. Higher price elasticities thus lead to larger coefficients in the inverse demand function, which also means lower consumer benefit and social welfare.

The profit under cross-brand cooperation is again the highest, followed by within-brand cooperation. Consequently, elasticities do not change brands’ equilibria for their cooperation strategies: (cooperate within brand, cooperate within brand) and (cooperate across brands, cooperate across brands). The equilibrium outcomes are shown in Table 6.

Table 6. Outcomes with different price elasticities.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1/3*base case</th>
<th>1/2*base case</th>
<th>2*base case</th>
<th>3*base case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>YY</td>
<td>Y_x Y_x</td>
<td>YY</td>
<td>Y_x Y_x</td>
</tr>
<tr>
<td>MU_{aj}</td>
<td>68.40</td>
<td>68.56</td>
<td>45.87</td>
<td>46.03</td>
</tr>
<tr>
<td>MU_{aj}</td>
<td>70.57</td>
<td>70.90</td>
<td>47.45</td>
<td>47.78</td>
</tr>
<tr>
<td>N_{aj}</td>
<td>1,726</td>
<td>1,738</td>
<td>1,734</td>
<td>1,752</td>
</tr>
<tr>
<td>N_{aj}</td>
<td>1,682</td>
<td>1,685</td>
<td>1,690</td>
<td>1,695</td>
</tr>
<tr>
<td>Π_{ij}</td>
<td>236,770</td>
<td>238,654</td>
<td>159,734</td>
<td>161,628</td>
</tr>
<tr>
<td>SW</td>
<td>1,180,089</td>
<td>1,190,138</td>
<td>794,928</td>
<td>805,003</td>
</tr>
<tr>
<td>φ</td>
<td>-2.25</td>
<td>-1.81</td>
<td>-1.11</td>
<td>-0.69</td>
</tr>
</tbody>
</table>
As expected, the larger the absolute value of the price elasticity, the closer the substitutes and the lower the mark-ups. Intuitively, as travelers become more price-sensitive, a slight drop in the mark-up will lead to an increase in the demand for corresponding cars. Brands hence find it more beneficial to charge lower mark-ups to attract more car users and this softens the price and mark-ups.

Duopoly competition becomes more efficient as demand becomes more sensitive, despite the declining welfare. This is because an increase in price elasticity raises the number of autonomous cars and normal cars, which, on the one hand, raises congestion and, on the other, lowers the willingness to pay. The former effect implies that the societal benefit from private pricing increases and the latter that its downsides decrease. Relative efficiency thus increases with the demand elasticity for solely within-brand cooperation and full cooperation.

5.4.6 Asymmetric cooperation cost

The final sensitivity analysis considers asymmetric cooperation costs. We normalize the cooperation cost of brand 2 to zero, and vary the cooperation cost for brand 1, $MC_{cop,1}$, from 0 to 4.

Using the same logic as in section 5.4.1, we summarize the Nash equilibria for the duopoly competition in Table 7. When the cooperation cost for brand 1 exceeds 1.75, cross-brand cooperation stops being the Nash equilibrium. This will again lead to below-optimal cooperation, since the social welfare under full cooperation is still the highest.

A private monopoly may also lead to below-optimal cooperation. Indeed, the public monopolist will choose full cooperation until the cooperation cost reaches 13.27; the private monopolist stops full cooperation when the cooperation cost exceeds 4.75.

<table>
<thead>
<tr>
<th>Table 7. Nash equilibria under asymmetric cooperation cost.</th>
</tr>
</thead>
<tbody>
<tr>
<td>NE</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>(Not, not)</td>
</tr>
<tr>
<td>(Within, not)</td>
</tr>
<tr>
<td>(Not, within)</td>
</tr>
<tr>
<td>(Within, within)</td>
</tr>
<tr>
<td>(Across, across)</td>
</tr>
<tr>
<td>Number of NE</td>
</tr>
</tbody>
</table>

Fig. 14 shows the effects of cooperation cost on relative efficiency. Under duopoly competition, relative efficiencies under $XYX$ and $YY$ both decrease with brand 1’s cooperation cost, and the relative efficiency under $NY$ is constant at -0.19. As a result, when brand 1’s cooperation cost is low, cross-brand cooperation performs best; when brand 1’s cooperation...
cost is high, NY performs best.

![Graph](image)

**Fig. 14.** Effects of cooperation cost on relative efficiency.

### 6. Conclusion

This paper investigated the strategic interactions of multiple car brands—which may provide both autonomous cars and normal cars—focusing on the question of whether brands will want their autonomous cars to cooperate within and/or across brands. This is an important question, as cooperation is what ensures that road capacity increases with the use of autonomous cars. In our paper, each brand has three options for its autonomous cars: cooperation within own brand only, cooperation across brands, or no cooperation between vehicles. We considered four market structures: duopoly competition, perfect competition, a public welfare-maximizing monopoly, and a private profit-maximizing monopoly. For duopoly competition, we built a two-stage game model, with first a choice of cooperation strategy and then Bertrand competition with imperfect substitutes.

Duopoly competition or a private monopoly may lead to too little cooperation. For a duopoly, cooperation with another brand may raise the efficiency of road use by a brand’s own cars and thus lower the travel times of its own autonomous car users; but it does the same for the competing brand, thereby strengthening the element of competition. A duopoly sees this as a downside, whereas a private or public monopolist owning all brands regards this as an advantage. A private monopoly may also still lead to below-optimal cooperation, due to monopolistic market power.

A duopoly may, furthermore, have multiple equilibria in the cooperation strategy. Unless cooperation costs are very high, cross-brand cooperation is one of these equilibria; but it is by no means certain that it will prevail. No cooperation, and solely within-brand cooperation, are also often Nash equilibria. It is only when cooperation costs are too high that a unique Nash equilibrium obtains for (1) no cooperation at all (for symmetric brands) or (2) for one brand
cooperating within own brand and the other brand not cooperating (for asymmetric brands). In contrast, for the public welfare-maximizing monopolist and the private profit-maximizing monopolist, the equilibrium is always unique: either cooperation across brands or no cooperation at all (when the cooperation cost is high).

Apart from the cooperation cost, the shapes of the capacity function will also change the equilibria for brands’ cooperation strategies. Our numerical simulations suggest that when within-brand cooperation increases capacity slightly, in addition to cross-brand cooperation, one firm cooperating within own brand and the other not cooperating is also a Nash equilibrium. As cooperation within own brand increases capacity more effectively, cross-brand cooperation and within-brand cooperation become the Nash equilibria.

Although our study sheds light on the joint application of theoretical game modelling and dynamic congestion modelling in analyzing a transport system with a mix of autonomous and normal human-driven cars, it can be further extended in several directions. First, this paper considered capacity effects in a general way by ignoring the different headways in mixed traffic and only investigated the averaged capacity of autonomous cars with different brands. In order to make use of the proposed model for practical applications in reality, there is a need to use traffic flow theory or stochastic process theory to take the capacity interaction between different headways into account (e.g., Van Wee et al., 2013; Zhou et al., 2020). Second, the proposed model only considered private autonomous cars. However, with the development of vehicle automation and the sharing economy, a mix of normal human-driven cars, autonomous cars, and shared autonomous cars can be expected to coexist in the next few decades, affecting the way of travel (e.g., Haboucha et al., 2017; Tian et al., 2021). Autonomous cars, especially when combined with shared use, may contribute to reducing or solving some of the present most intractable urban problems, such as traffic congestion, road traffic accidents, and inefficient use of urban spaces. It would be interesting to investigate the cooperation and competition among private autonomous cars, shared autonomous cars, and normal human-driven cars. Third, the choice of where to park an autonomous car and the option of renting it out when not using it (for instance, via Uber or Lyft) are interesting topics. Finally, a public transit mode, such as a metro line, could be added to examine a multi-mode transportation system in which autonomous cars could act as competitors or complements to mass transit.

References


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Appendix A. Pricing rule under duopoly competition.

We illustrate with brand 1. Following Eq. (8), the associated Lagrangian is:
\[ \Pi_i = MU_{a1} \cdot N_{a1} + MU_{a2} \cdot N_{a2} 
\]
\[ - \lambda_{a1} \left( D_{a1} [N_{a1}, N_{a2}, N_{a3}, N_{a4}, N_{a5}, N_{a6}] - c_a [N_{a1}, N_{a2}, N_{a3}, N_{a4}, N_{a5}, N_{a6}] - MU_{a1} - MC_{a1} - MC_{a2} \cdot cop_{1} \right) \]
\[ - \lambda_{a2} \left( D_{a2} [N_{a1}, N_{a2}, N_{a3}, N_{a4}, N_{a5}, N_{a6}] - c_a [N_{a1}, N_{a2}, N_{a3}, N_{a4}, N_{a5}, N_{a6}] - MU_{a2} - MC_{a2} - MC_{a2} \cdot cop_{2} \right) \]
\[ - \lambda_{a1} \left( D_{a1} [N_{a1}, N_{a2}, N_{a3}, N_{a4}, N_{a5}, N_{a6}] - c_a [N_{a1}, N_{a2}, N_{a3}, N_{a4}, N_{a5}, N_{a6}] - MU_{a1} - MC_{a1} \right) \]
\[ - \lambda_{a2} \left( D_{a2} [N_{a1}, N_{a2}, N_{a3}, N_{a4}, N_{a5}, N_{a6}] - c_a [N_{a1}, N_{a2}, N_{a3}, N_{a4}, N_{a5}, N_{a6}] - MU_{a2} - MC_{a2} \right) \]

Taking the derivatives of Eq. (A1) with respect to \( MU_{a1}, MU_{a1}, N_{a1}, N_{a2} \) and \( N_{a2} \) yields:

\[ \frac{\partial \Pi_i}{\partial N_{a1}} = N_{a1} + \lambda_{a1} = 0 \quad \Rightarrow \quad \lambda_{a1} = -N_{a1}, \quad (A2) \]

\[ \frac{\partial \Pi_i}{\partial N_{a2}} = N_{a2} + \lambda_{a2} = 0 \quad \Rightarrow \quad \lambda_{a2} = -N_{a2}, \quad (A3) \]

\[ \frac{\partial \Pi_i}{\partial N_{a1}} = MU_{a1} - \lambda_{a1} \left( \frac{D_{a1}}{\partial N_{a1}} - \frac{D_{a1}}{\partial N_{a1}} \right) - \lambda_{a2} \left( \frac{D_{a2}}{\partial N_{a2}} - \frac{D_{a2}}{\partial N_{a2}} \right) - \lambda_{a1} \left( \frac{D_{a1}}{\partial N_{a1}} - \frac{D_{a1}}{\partial N_{a1}} \right) - \lambda_{a2} \left( \frac{D_{a2}}{\partial N_{a2}} - \frac{D_{a2}}{\partial N_{a2}} \right) = 0, \quad (A4) \]

\[ \frac{\partial \Pi_i}{\partial N_{a2}} = -\lambda_{a2} \left( \frac{D_{a2}}{\partial N_{a2}} - \frac{D_{a2}}{\partial N_{a2}} \right) - \lambda_{a1} \left( \frac{D_{a1}}{\partial N_{a1}} - \frac{D_{a1}}{\partial N_{a1}} \right) - \lambda_{a2} \left( \frac{D_{a2}}{\partial N_{a2}} - \frac{D_{a2}}{\partial N_{a2}} \right) - \lambda_{a1} \left( \frac{D_{a1}}{\partial N_{a1}} - \frac{D_{a1}}{\partial N_{a1}} \right) = 0, \quad (A5) \]

\[ \frac{\partial \Pi_i}{\partial N_{a1}} = MU_{a1} - \lambda_{a1} \left( \frac{D_{a1}}{\partial N_{a1}} - \frac{D_{a1}}{\partial N_{a1}} \right) - \lambda_{a2} \left( \frac{D_{a2}}{\partial N_{a2}} - \frac{D_{a2}}{\partial N_{a2}} \right) - \lambda_{a1} \left( \frac{D_{a1}}{\partial N_{a1}} - \frac{D_{a1}}{\partial N_{a1}} \right) - \lambda_{a2} \left( \frac{D_{a2}}{\partial N_{a2}} - \frac{D_{a2}}{\partial N_{a2}} \right) = 0, \quad (A6) \]

From Eq. (A4) and Eq. (A6), we can obtain:

\[ MU_{a1} = \frac{\partial c_{a1}}{\partial N_{a1}} N_{a1} + \frac{\partial c_{a1}}{\partial N_{a1}} N_{a1} - \frac{D_{a1}}{\partial N_{a1}} + \lambda_{a2} \left( \frac{D_{a1}}{\partial N_{a1}} - \frac{D_{a1}}{\partial N_{a1}} \right) + \lambda_{a2} \left( \frac{D_{a2}}{\partial N_{a2}} - \frac{D_{a2}}{\partial N_{a2}} \right), \quad (A8) \]

\[ MU_{a1} = \frac{\partial c_{a1}}{\partial N_{a1}} N_{a1} + \frac{\partial c_{a1}}{\partial N_{a1}} N_{a1} - \frac{D_{a1}}{\partial N_{a1}} + \lambda_{a2} \left( \frac{D_{a1}}{\partial N_{a1}} - \frac{D_{a1}}{\partial N_{a1}} \right) + \lambda_{a2} \left( \frac{D_{a2}}{\partial N_{a2}} - \frac{D_{a2}}{\partial N_{a2}} \right) \]

Substituting Eq. (A2) and Eq. (A3) into Eq. (A5) and Eq. (A7) yields:

\[ N_{a1} \left( \frac{D_{a1}}{\partial N_{a1}} - \frac{D_{a1}}{\partial N_{a2}} \right) + N_{a1} \left( \frac{D_{a1}}{\partial N_{a2}} - \frac{D_{a1}}{\partial N_{a2}} \right) = \lambda_{a2} \left( \frac{D_{a2}}{\partial N_{a2}} - \frac{D_{a2}}{\partial N_{a2}} \right) + \lambda_{a2} \left( \frac{D_{a2}}{\partial N_{a2}} - \frac{D_{a2}}{\partial N_{a2}} \right), \quad (A9) \]

Solving Eq. (A9) yields:

\[ \lambda_{a1} = \frac{N_{a1} \left( \frac{D_{a1}}{\partial N_{a1}} - \frac{D_{a1}}{\partial N_{a2}} \right) + \left( \frac{D_{a1}}{\partial N_{a1}} - \frac{D_{a1}}{\partial N_{a2}} \right) - \lambda_{a2} \left( \frac{D_{a2}}{\partial N_{a2}} - \frac{D_{a2}}{\partial N_{a2}} \right) - \lambda_{a2} \left( \frac{D_{a2}}{\partial N_{a2}} - \frac{D_{a2}}{\partial N_{a2}} \right)}{\left( \frac{D_{a1}}{\partial N_{a1}} - \frac{D_{a1}}{\partial N_{a2}} \right) + \lambda_{a2} \left( \frac{D_{a2}}{\partial N_{a2}} - \frac{D_{a2}}{\partial N_{a2}} \right) - \lambda_{a2} \left( \frac{D_{a2}}{\partial N_{a2}} - \frac{D_{a2}}{\partial N_{a2}} \right)} \]

\[ \lambda_{a1} = \frac{N_{a1} \left( \frac{D_{a1}}{\partial N_{a1}} - \frac{D_{a1}}{\partial N_{a2}} \right) + \left( \frac{D_{a1}}{\partial N_{a1}} - \frac{D_{a1}}{\partial N_{a2}} \right) - \lambda_{a2} \left( \frac{D_{a2}}{\partial N_{a2}} - \frac{D_{a2}}{\partial N_{a2}} \right) - \lambda_{a2} \left( \frac{D_{a2}}{\partial N_{a2}} - \frac{D_{a2}}{\partial N_{a2}} \right)}{\left( \frac{D_{a1}}{\partial N_{a1}} - \frac{D_{a1}}{\partial N_{a2}} \right) + \lambda_{a2} \left( \frac{D_{a2}}{\partial N_{a2}} - \frac{D_{a2}}{\partial N_{a2}} \right) - \lambda_{a2} \left( \frac{D_{a2}}{\partial N_{a2}} - \frac{D_{a2}}{\partial N_{a2}} \right)} \]
Substituting Eqs. (A10) and (A11) into Eq. (A8), we can obtain the mark-up of brand 1 on autonomous cars and normal cars, as expressed in Eqs. (10)-(11). Similarly, we can derive $\lambda_{11}$, $\lambda_{n1}$, $MU^R_{a2}$ and $MU^R_{n2}$.

**Appendix B. Pricing rule under a welfare-maximizing monopoly**

The consumer benefit can be expressed as:

$$\mathcal{B}[N_{a1}, N_{a2}, N_{n2}, N_{a1}] = \int_{(0,0,0)}^{(N_{a1}, N_{a2}, N_{n2}, N_{a1})} \left( D_{a1}[\cdot]dx_{a1} + D_{a2}[\cdot]dx_{a2} + D_{n1}[\cdot]dx_{n1} + D_{n2}[\cdot]dx_{n2} \right) ,$$

(\text{B1})

implying that $\partial \mathcal{B}/\partial N_{ij} = D_{ij}$.

The associated Lagrangian is:

$$W = \mathcal{B}[N_{a1}, N_{a2}, N_{n2}, N_{a1}] - \sum_{i \in \{a,n\}} \sum_{j=1}^{2} (c_{ij}[N_{a1}, N_{a2}, N_{n2}, N_{ij}] + MC_{ij} \cdot N_{ij}) - \sum_{j=1}^{2} MC_{cop} \cdot \text{cop}_j \cdot N_{ij}$$

- $\lambda_{a1} \cdot (D_{a1}[N_{a1}, N_{a2}, N_{n2}, N_{a1}] - c_{a1}[N_{a1}, N_{a2}, N_{n2}, N_{a1}] - MU_{a1} - MC_{a1} - MC_{cop} \cdot \text{cop}_1)$
- $\lambda_{a2} \cdot (D_{a2}[N_{a1}, N_{a2}, N_{n2}, N_{a2}] - c_{a2}[N_{a1}, N_{a2}, N_{n2}, N_{a2}] - MU_{a2} - MC_{a2} - MC_{cop} \cdot \text{cop}_2)$
- $\lambda_{n1} \cdot (D_{n1}[N_{a1}, N_{a2}, N_{n2}, N_{n1}] - c_{n1}[N_{a1}, N_{a2}, N_{n2}, N_{n1}] - MU_{n1} - MC_{n1})$
- $\lambda_{n2} \cdot (D_{n2}[N_{a1}, N_{a2}, N_{n2}, N_{n2}] - c_{n2}[N_{a1}, N_{a2}, N_{n2}, N_{n2}] - MU_{n2} - MC_{n2})$

(B2)

Substituting Eq. (B1) into Eq. (B2) and taking the derivatives of Eq. (B2) with respect to $MU_{ij}$, $N_{ij}$, and $\lambda_{ij}$ ($i \in \{a,n\}, j \in \{1,2\}$) yields:

$$\frac{\partial W}{\partial MU_{a1}} = \lambda_{a1} = 0; \quad \frac{\partial W}{\partial MU_{a2}} = \lambda_{a2} = 0; \quad \frac{\partial W}{\partial MU_{n1}} = \lambda_{n1} = 0; \quad \frac{\partial W}{\partial MU_{n2}} = \lambda_{n2} = 0;$$

(B3)

$$\frac{\partial W}{\partial N_{ij}} = D_{ij} - \frac{\partial c_{ij}}{\partial N_{ij}} N_{ij} - \frac{\partial c_{ia}}{\partial N_{ij}} N_{i(-j)} - \frac{\partial c_{ia}}{\partial N_{ij}} N_{i(-j)} - \frac{\partial c_{ia}}{\partial N_{ij}} N_{i(-j)} - MC_{a1} - MC_{cop} \cdot \text{cop}_1 .$$

(B4)

Substituting the user equilibrium condition into Eq. (B4) yields:

$$MU_{ij} = \frac{\partial c_{ij}}{\partial N_{ij}} N_{ij} + \frac{\partial c_{ja}}{\partial N_{ij}} N_{j(-i)} + \frac{\partial c_{ja}}{\partial N_{ij}} N_{j(-i)} + \frac{\partial c_{ja}}{\partial N_{ij}} N_{j(-i)} ,$$

(B5)

as shown in Eq. (16).