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Designing Financial Support for SMEs during Crises: the Role of Bank Lending

Tianxi Wang* Xuan Wang †

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Abstract

When designing schemes to help SMEs survive crises, the government typically faces asymmetric information, so that it cannot target the SMEs most worth saving. We show that the government can exploit the information in the borrower loan demand to improve policy targets compared with existing programmes. If the aim is employment protection, optimal policy should fully subsidise the funding cost of *only* those SMEs whose loan size is below a threshold. If the aim is economic efficiency, the government should target SMEs whose loan size is above a threshold. In general, public policy should utilise private sectors' information and expertise.

Keywords: pandemic crisis, bank lending, unemployment, information asymmetry, small and medium-sized enterprises

JEL Codes: G21, G38, D82, H81

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1 Introduction

During crisis times such as COVID-19, how can the government best support small and medium-sized enterprises (SMEs) with limited resources?[1] Careful consideration of the government support policy needs to address a mix of adverse selection and moral hazard issues. First, the public support scheme should target entrepreneurs with positive net present value (NPV) projects, but the government is unlikely to observe the SME quality. Second, the public support scheme should target those entrepreneurs who lack their own financial resources, which, again, the government is unlikely to observe. Third, a typical moral hazard issue may arise: once the entrepreneurs obtain the funding, they may divert it for purposes other than the productive use of their enterprises. The public support scheme should prevent such malpractices. Our idea is that the government can utilise the information and expertise of private-sector banks to better handle these issues. More specifically, the government can use the information contained in the SME demand for loans to improve its target and rely on banks' expertise to mitigate SME borrowers' moral hazard. Our results suggest that there is scope to enhance the efficacy of various existing government support schemes for SMEs amid the COVID-19 crisis, such as the UK "Bounce-back Loans" scheme (BBL).

In our model, a continuum of entrepreneurs each owns and manages an SME. Each entrepreneur incurs a cost to survive the pandemic, and each has a certain quantity of their own funds. This quantity and the surviving cost together characterise the entrepreneur's type. The surviving cost represents the quality of the SME. If this cost is higher than the value conditional on survival, then this SME is not worth saving. If an entrepreneur cannot cover the surviving cost with their own funds, we call this entrepreneur *in deficit*. Those in deficit can plug the gap by borrowing from banks. In addition, the government has a limited budget allocated to help SMEs survive. To most effectively use the public funds, the government may wish to target some specific types of entrepreneurs. For example, the government should only help those in deficit. The difficulty is that the type is the entrepreneur's private information. *Ex post*, after receiving funding, entrepreneurs can misuse it for purposes other than the survival of their businesses. We model this moral hazard issue following the classic Holmstrom and Tirole (1997). We study how the government can utilise private sector banks to alleviate the adverse selection and moral hazard issues.

To begin with, the government can rely on banks to deal with the moral hazard issue and target entrepreneurs in deficit. For these two purposes, the government could simply pass funds to banks, which pool the public funds with their own funds to lend to the entrepreneurs. We call this the Simple Policy. Bank funding is typically more expensive than entrepreneurs' self-funding. Hence, only entrepreneurs in deficit will demand loans and receive, indirectly, help from the government. Moreover, banks have the expertise to tackle the moral hazard issue of entrepreneurs by enforcing the pledged income and leaving

[1]The SME sector is important to the broader economy. For example, over 99% of all businesses in the European Union are SMEs, which account for over 65% of the total workforce, and similarly in the UK and the US.

a wedge between the pledged income and the present value of the borrowers' enterprises, as Holmstrom and Tirole (1997) show. The Simple Policy can hence both deal with the moral hazard issue and help only the entrepreneurs in deficit . We shall shortly see that many existing Covid-19 support schemes in the developed economies bear the flavour of the Simple Policy. However, the problem is that it has no target among entrepreneurs in deficit. By enlarging the supply of funds, the policy reduces the cost of bank funding to SMEs, but this reduction benefits equally *all* of the types that have access to bank funding. Unless the public funding is large enough to fill the deficits of all the positive-NPV SMEs – which we assume it is not – the government can do better by improving its targeting based on the entrepreneur loan demand.

The target depends on the objective of the government. We consider two objectives: employment protection or efficiency maximisation. If the government's objective is to protect employment, it should save as many SMEs as possible. Therefore, the policy should help the SMEs with the smallest deficits. The government, however, does not observe this deficit. Nevertheless, since bank funding is more costly than self-funding (as well-known in the corporate finance literature), entrepreneurs will only borrow bank funding exactly to fill their shortfalls. Their demand for loans, therefore, mirrors their funding deficits. Hence, the optimal employment protection policy is to help *only* those entrepreneurs whose loan size is below a threshold. We show that the optimal policy should fully subsidise the funding cost of their loans. The intuition is as follows. Consider the operation of reducing the subsidy for the funding cost per entrepreneur. Given the government funding budget, the threshold of loan size therefore increases. This operation selects in entrepreneurs with large deficits. However, it crowds out entrepreneurs with small deficits whose NPVs are so low that they cannot afford the unsubsidised part of the funding cost. Because the funding deficits of those selected in are all larger than the deficits of those crowded out, the net effect of this operation is a reduction in the number of SMEs that are saved.

With a given public budget, by targeting entrepreneurs with small funding shortfalls, the government can save as many businesses as possible to protect employment. However, this policy does not consider the difference in the NPVs of the SMEs to be saved. If the government's objective is to maximise efficiency, it should target projects with a high NPV. Again, the government can base its targeting on the entrepreneur loan demand. Whereas in the previous case, the government uses the information of loan demand to infer the shortfall, in this case, it uses the information to infer the NPV. This is because the entrepreneur loan demand is positively correlated with their NPVs. Indeed, only those SMEs with sufficiently high NPVs can afford the high funding cost of a large loan. Therefore, if the government aims to improve economic efficiency, it should target those entrepreneurs whose loan size is above a threshold, which is opposite to the optimal policy when the government aims to protect employment.

Our research suggests there is scope for improvement in various existing financial support schemes to help SMEs survive the COVID-19 pandemic. Most existing schemes aim to protect employment[2] and take the form of interest subsidies for loans up to a limit. For example, in euro area countries, the maximum amount per borrower is typically 25% of the borrower's turnover in 2019 or twice the wage bill in 2019. In the UK, the Bounce-back Loans have an upper limit of £50,000 per claimant. The US Paycheck Protection Programme (PPP) has a large loan size cap of $10 million. Under these schemes, firms are free to borrow additional loans via the normal lending channel if their demand for bank funding exceeds the limit set by the government loan support programmes. Therefore, SMEs with all levels of funding demand will benefit from these schemes.

Based on our study, the government can save more SMEs and protect more jobs if it increases the cap but *only* allows SMEs whose *total* loan size is below the cap to use the scheme. Those SMEs whose total loan size is above the cap are disqualified. Among the disqualified borrowers, only part of them will fail due to lack of public support, and thus, these SMEs are selected *out*. On the other hand, the increase in the cap selects *in* SMEs that would otherwise not be saved. We prove that the number of SMEs selected in is larger than those selected out, and hence, in net, more SMEs and jobs are saved. The intuition is as follows: In equilibrium, the funding demand of the SMEs who would survive even without public support is equal to the aggregate private supply of funding. Thus, the aggregate demand for funding from those SMEs that would not survive without the public support scheme is equal to the funding supplied by this scheme. Moreover, the funding demanded by the selected-in SMEs is smaller than the increased cap, while that of the selected-out SMEs is larger than the cap (exactly why they are disqualified). Consequently, the funding demand of each selected-in SME is smaller than that of each selected-out SME. Given the quantity of public funding, the number of SMEs selected in is thus greater than that selected out.

2 Related Literature

A strand of literature on how the government can better support SMEs has quickly sprung up since the outbreak of COVID-19. Our core message is that the government can utilise private sector banks to tackle the entrepreneur's moral hazard issues and alleviate its information constraint. In particular, despite its information constraint, we show that the government can exploit the information contained in the SME loan demand to help target the SMEs most worth saving.

Like our paper, Goodhart et al. (2020), Kahn and Wagner (2020), and Vardoulakis (2020) also emphasise information friction. Goodhart et al. (2020) build a multi-sector equilibrium model with small businesses that have private information on their profitability. Goodhart et al. (2020) focus on the distorting effect of loan guarantees on lenders' moni-

[2] This can be evidenced by the US Department of the Treasury's introductory article on the US Paycheck Protection Programme (https://home.treasury.gov/policy-issues/coronavirus/assistance-for-small-businesses/paycheck-protection-program), as well as the UK HM Treasury's news article on government-backed financial support schemes (https://www.gov.uk/government/news/government-backed-loans-help-thousands-of-businesses-to-protect-jobs-during-pandemic). As the UK Chancellor of the Exchequer, Rishi Sunak, put it: 'I said we would do whatever it takes to protect jobs and livelihoods and that (government-backed loan support) is exactly what we have done.'

toring incentives, and the authors propose a screening device to deter certain unprofitable businesses from accessing the loan support, and the critical policy trade-off is between short-term employment stabilisation and long-run allocative efficiency. Since we do not consider banks' incentives, Goodhart et al. (2020)'s work complements our model. Vardoulakis (2020) considers a central bank that wants to reduce bank funding cost by purchasing their loans but banks have private information on their loan quality. Vardoulakis (2020) shows that the central bank can design a multi-tier loan pricing facility to reduce its adverse selection problem. Moreover, the welfare gain of the multi-tier scheme can be measured based on three sufficient statistics. Kahn and Wagner (2020) consider both direct liquidity provision to businesses by the central bank and traditional liquidity provision by banks. The central bank has better knowledge of the externalities during a pandemic, whereas the private banks have informational advantages over the businesses. Kahn and Wagner (2020) show which liquidity provision is preferred is determined by the variance of firm characteristics. Relating to Kahn and Wagner (2020), Li and Li (2020) investigate direct financing to non-financial firms by a central bank that faces information asymmetry. They show the distortionary effects of such direct financing can be mitigated by central bank lending through informed banks. Different from these papers, our model emphasises how the government can target SME borrowers based on the size of loan demand.

The other papers on government loan support are based on different types of frictions. Segura and Villacorta (2020) analyse different government interventions to support firms and develop a pecking order between direct transfers and indirect support through guarantees to new loans or reductions in the capital requirement. The critical friction in their paper is the moral hazard due to the borrower's unobserved effort cost. Philippon (2020) analyses how government interventions can improve efficiency when the decentralised economy amid COVID-19 is distorted by wage rigidity. Elenev et al. (2020) build a structural model calibrated with the US data to evaluate three government policies aimed at reducing corporate defaults and banking fragility. None of these papers is concerned with the government using loan demand to target certain types of firms despite information asymmetry.

More generally, our message is that public policy should utilise private sectors' information and expertise. Our paper therefore connects with the literature on public-private partnerships. Hart (2003) develops a model of public-private partnerships and provides the foundation for the determinants of the boundaries between public and private firms in an advanced capitalist economy. Akintoye et al. (2003) show how currency risk management methods can help the complex process of managing procurement via such partnerships. Satish and Shah (2009) outline the need for private sector participation in infrastructure and assess the various models of public-private participation. Cui et al. (2018) review the existing public-private partnership research to explore the status quo, trends, and gaps in infrastructure projects. While this literature emphasises the costs and risk sharing of public-private partnerships, we underscore the utilisation of private sectors' information and expertise.

3 Model

The economy is in a crisis, in which a continuum of entrepreneurs try to survive. Each entrepreneur owns and manages an SME that employs n workers. The type of an entrepreneur is characterised by two variables, (a, x), where a is the amount of the entrepreneur's own funds and x is the cost of surviving the crisis. We assume (a, x) distributes on $[0, \overline{a}] \times [\underline{x}, \overline{x}]$ with p.d.f. h. An entrepreneur's type is their private information. Funding the surviving cost is a necessary condition for survival. This condition satisfied, the chance of survival depends on the entrepreneur's effort choice. If they work hard, which incurs a private cost of c, their enterprise survives with probability p. If they shirk, the enterprise survives with probability $p - \Delta > 0$. The effort choice is private information.

All the surviving enterprises generate profits Π. Thus, the quality of the business projects is represented by the cost x. Making the profit variable will not qualitatively change the results. We assume that

$$p\Pi - c > (p - \Delta)\,\Pi. \tag{1}$$

That is, working hard is socially efficient. Let $V := p\Pi$ be the present value of a surviving SME conditional on the entrepreneur working hard. An SME is worth saving if and only if

$$V \geq x + c.$$

More specifically, if $V < x + c$, then $(p - \Delta)\,\Pi < x$ by Assumption (1). That is, if the SME is not worth saving conditional on the entrepreneur working hard, then it is not worth saving in the case of shirking.

If entrepreneurs' funding is not used for their businesses, it earns a gross interest rate normalised to 1. If $a < x$, the entrepreneur needs external funds to survive, and we call this entrepreneur *in deficit*. The only source of external funds, excluding the public budget, is from banks, which have a total of K units of funding for SMEs. We assume K is sufficiently scarce that in equilibrium bank funds are more expensive than entrepreneurs' own funds.

The government has a budget of G that can be used to help SMEs survive the crisis. At first glance, this might not so closely resemble many existing Covid-19 support schemes, which do not provide funds directly out of the public purse to SMEs but take the form of credit guarantees and interest subsidies. For example, the UK BBL scheme provides a 100% credit guarantee for a bank loan (i.e., "Bounce-bank Loans") up to the limit of £50,000 per SME. Moreover, the scheme pays the interest for the Bounce-back Loans for the first year, and afterwards it pays the interest in excess of 2.5%. However, our way of modelling the government scheme actually represents well this type of credit-guarantee and/or interest-subsidy schemes. The reason is as follows. For any government support scheme to help more SMEs survive, the scheme must enlarge the funding supply for them. If a scheme takes the form of credit guarantees and interest subsidies, it may appear that the enlargement of the supply of funding comes from the private sector. However, since the enlargement would not realise should the government scheme be absent, the extra funding must be induced by the return paid by the support scheme. Thus, to a large degree, the size of the enlargement is equal to the present value of the future payment from the scheme, which is what we capture by G in the model.

We start our analysis with the case in the absence of government funding and then investigate how government funding G can be most effectively deployed.

4 The market equilibrium - no government support

In the absence of government funding, entrepreneurs can only turn to their banks. In what follows, we determine banks' gross lending rate R and the types of entrepreneurs that can survive by borrowing from banks in equilibrium. Given we have normalised the gross return rate of entrepreneurs' self funding is 1, we interpret $R-1$ as the net funding cost of entrepreneurs. We shall focus on the case that bank funding is more expensive than self funding, i.e., $R > 1$. There are three conditions that characterise the types (a, x) of entrepreneurs who survive by borrowing from banks. First, because $R > 1$, only those entrepreneurs who are in deficit borrow from banks. That is, the types of borrowers satisfy:

$$x \leq a. \tag{2}$$

Second, as in Holmstrom and Tirole (1997), the moral hazard of the entrepreneurs' shirking generates a wedge between the present value of an enterprise and the value pledgeable to the bank. Let

$$c_\Delta \equiv \frac{c}{\Delta}.$$

An entrepreneur works hard only if their stake in the business is no less than c_Δ conditional on the project's survival; otherwise, they will shirk. Hence, the pledgeable income to the lender is no larger than $V - pc_\Delta$. Given the gross lending rate R, then the maximum amount that an entrepreneur could borrow from a bank is

$$\frac{V - pc_\Delta}{R} \equiv k_e.$$

A type (a, x) can find sufficient bank funding to make up the deficit if and only if the following incentive compatibility (IC) constraint is met:

$$k_e \geq x - a. \tag{3}$$

Third and lastly, not all the entrepreneurs eligible for bank funding find it worthwhile to weather through the crisis by borrowing bank funds. Given the cost of bank funding

$R > 1$, an entrepreneur of type (a, x) borrows a quantity of bank funding that just suffices to cover its shortfall $x - a > 0$. Then the entrepreneur must repay the bank $R(x - a)$, and the financial gains for running the enterprises only amount to $V - R(x - a) - c$. This value must be no smaller than the gain from giving up their enterprise and investing their own funds elsewhere; otherwise, the entrepreneur will not borrow from the bank. Thus, the individual rationality (IR) constraint for type (a, x) to borrow is $V - R(x-a) - c \geq a$, equivalent to

$$V - c \geq Rx - (R - 1)a. \tag{4}$$

A type (a, x) that survives the crisis by borrowing from banks must satisfy constraints (2), (3) and (4). Now we illustrate these types in a-x plane for the case of $R > 1$ in Figure 1 below.

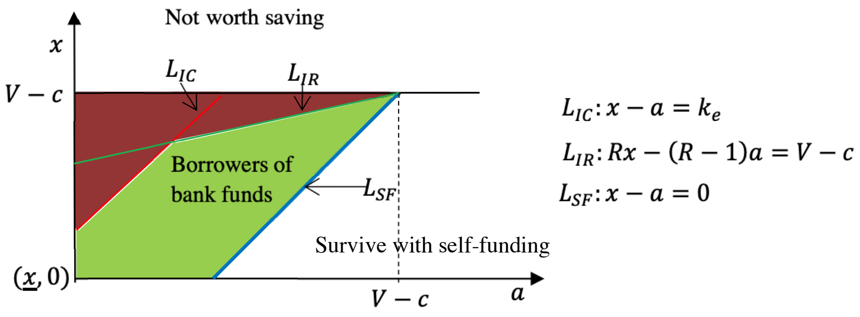


Figure 1: The types in the green area survive by borrowing bank funds if $R > 1$. The three non-axis boundary lines of the area, L_{SF}, L_{IC} and L_{IR} are defined by the constraints (2), (3) and (4) being binding.

The enterprises in the dark red area of Figure 1 have a positive NPV but fail to survive the pandemic. This is an efficiency loss, of which there are two sources, according to constraints (3) and (4). Intuitively, constraint (3) says that an entrepreneur must have an adequate stake to ensure the bank that they will no shirk. Constraint (4) says that the SME must have a large enough NPV to afford borrowing expensive bank funds. These two constraints point to two sources of efficiency loss: the former is that entrepreneurs cannot borrow sufficiently due to the IC constraint; the latter is that the bank funds are too expensive. Note that reducing the cost R of bank funding for the entrepreneurs only ameliorates the latter problem, but not so much the former.

The funding cost $R - 1$ is determined by market clearing. Any type (a, x) in the green area of Figure 1 demands $x - a$ units of bank funding. The total demand is thus

$$D(R) \equiv \iint_{\substack{k_e \geq x - a \geq 0 \\ V - c \geq Rx - (R-1)a}} (x - a) h(a, x) da dx.$$

We show in Lemma 1 below that this demand is a decreasing function of the price of bank funding R.

Lemma 1. *$D'(R) < 0$ for $R \geq 1$ and $\lim_{R\to\infty} D(R) = 0$.*

Proof: See Appendix A.

Typically, bank funding is more expensive than firms' self funding. Therefore, we assume bank funding is sufficiently scarce to command $R > 1$. That is, we assume:

$$K < D(1). \tag{5}$$

Immediately from Lemma 1 follows Proposition 1, which states that the price of bank funds R decreases with the quantity of their aggregate supply, K.

Proposition 1. *Assuming condition (5) holds, R is determined by K through the following market clearing condition:*

$$D(R) = K. \tag{6}$$

$R > 1$ and decreases with K.

Hereinafter we call *outsiders* those types (a, x) that are outside the loan market, i.e., the types in the dark red area in Figure 1. These types have a positive NPV, but need the support of public funding to survive the crisis. The government has G units of public funds to support SMEs. What is the best way to deploy them?

5 The public funding support scheme

Given that the government does not observe entrepreneurs' types, it would not be able to achieve much if it simply distributes the public funds on its own. The government cannot differentiate entrepreneurs whose enterprises have a positive NPV from those whose enterprises do not. Nor can the government distinguish entrepreneurs who are in deficit of funds from those whose self-funding suffices to cover the surviving costs. In a nutshell, unable to observe entrepreneurs' characteristics, the government cannot achieve much on its own. Its funding scheme needs to utilise the expertise of banks. The simplest way to do so is to pass its funds to banks and let banks distribute the funds, i.e. the following simple policy.

Simple Policy (SP): The government lends public funds at zero net rate to banks, which pool the public funds with their own funds K to lend to the entrepreneurs.

The effect of SP is to enlarge the supply of bank funds from K to $K + G$ and thereby reduce its price R. We assume G is sufficiently small so that the net funding cost $R - 1$ for entrepreneurs is still positive, captured by the following equation.

$$G < D(1) - K. \tag{7}$$

The gross lending rate R is thus determined by the following market-clearing condition:

$$D(R) = K + G. \tag{8}$$

A reduction in R (due to the enlargement of bank funding from K to $K + G$) affects both the IC and the IR constraint. First, as R falls, external finance becomes cheaper and enterprises with smaller NPV can afford it. Hence, the IR constraint (4) is relaxed. This is represented by line L_{IR} In Figure (1) rotating clockwise around point O (coordinate $(x = V - c, a = V - c)$). Second, a reduced R also increases the present value of the pledgeable income k_e to the bank[3] and thus relaxes the IC constraint (3). This is represented by line L_{IC} in Figure (1) parallelly moving northwestward. These two effects are illustrated in Figure 2 below.

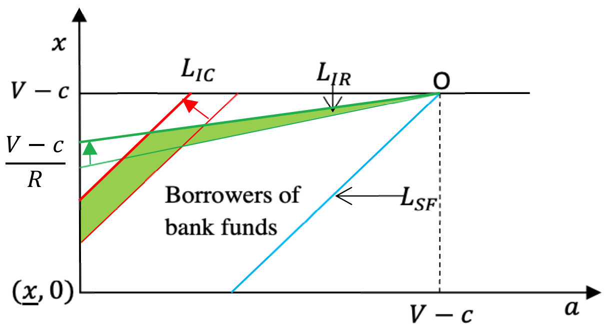


Figure 2: The effects of Simple Policy : compared to Figure 1, with R reduced due to bank funding supply enlarged, line L_{IR} rotates clockwise around point O and line L_{IC} moves northwestward. Hence, the green area marks the SMEs the policy saves.

With the Simple Policy, the entrepreneurs in the green area of Figure (2) are saved, in addition to those in the green area of Figure (1). By the market clearing condition (6), the aggregate demand of the latter group is K. Therefore, the aggregate demand of the former group is G, the public funding. That is,

$$\int_{\text{green area}} (x - a)h(a, x)dadx = G. \tag{9}$$

This equation determines the new value of R and hence the size of the green area.

While being simple, compared to the scenario of the government instinctively distributing funds on its own, the Simple Policy is an improvement. It exclude entrepreneurs whose enterprises have a negative NPV from using the public funds. It also exclude entrepreneurs who are not in deficit (as $R > 1$ still). Moreover, observe that if the public funding G is so abundant that condition (7) is violated, then the Simple Policy achieves what can be

[3]Since $k_e = (V - pc_\Delta)/R$ decreases with R.

done best. Suppose $G \geq D(1) - K$, then $R = 1$ in equilibrium. In this case, the IR line L_{IR} coincides with the positive NPV line, and the only source of inefficiency is the moral hazard issue of entrepreneurs, with which no financial support can help. Put differently, all the positive NPV enterprises that could be aided with financial support survive the crisis. Hence, if the government budget G is so abundant that $G \geq D(1) - K$, the Simple Policy is optimal.

Nevertheless, it is unlikely the government can provide such a large quantity of funds that the cost gap $R - 1$ between the internal funding and external funding disappears; in reality, condition (7) should hold. In this case, we show that there is scope of improvement over the Simple Policy. The issue is with the Simple Policy is that it subsidises entrepreneurs in proportion to the quantity of bank funds that they borrow. Thus, there is no refined target. The government can do better by improving targeting. The optimal policy depends on the government objective in designing the policy. We suppose that it can have two objectives. One is to minimise unemployment and thus to save as many enterprises as possible, and the other is to maximise economic efficiency.

5.1 The case of protecting employment

We have assumed that the government observes neither a nor x of any entrepreneur to highlight the information asymmetry in reality. However, in our benchmark case, we shall first characterise the optimal policy when the government observes entrepreneurs' types (a, x). Given that the government observes entrepreneurs' types, it seems that the government would require no help from the banks and should therefore operate the funding support policy on its own. Hence, in this hypothetical scenario, we assume the government implements its support policy disjointly from banks. It simply funds outsider entrepreneurs with positive NPV projects. It seems that in this hypothetical case, the government can achieve the maximum of job protection; after all, it observes entrepreneurs' types and can target exactly the types that it wants to help. However, we shall shortly show this is not necessarily true – the joint operation with banks under asymmetric information can do even better than the disjoint case with full information.

With full information, to save as many businesses as possible, the government should target those whose shortfall $x - a$ is small. Thus, there should be a cut-off s for the shortfall such that the government funds the shortfall of an outsider SME if and only if its shortfall $x - a$ is smaller than s. Of course, the government should only fund SMEs with positive NPVs. Therefore:

Optimal Full-Information Policy for Protecting Employment (OFI-E): The government finances the shortfall of those outsiders for whom the $x - a \leq s$ and $x \leq V - c$.

If $s > k_e$, then line $x - a = s$ is above line L_{IC} and this policy can be illustrated in Figure 3a as follows. The green area marks the businesses saved by the public funds.

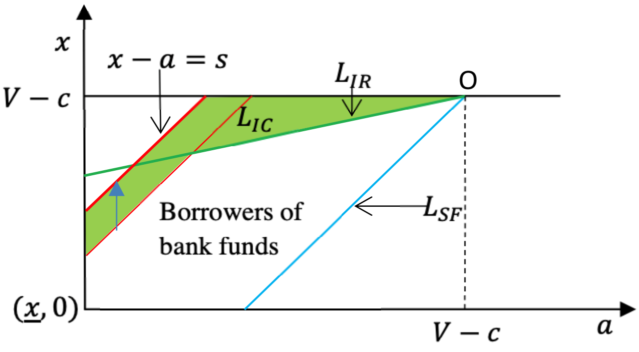


Figure 3a: The green area marks the SMEs saved by OFI-E if $s > k_e$.

If $s < k_e$, then line $x - a = s$ is below line L_{IC}. This policy can be illustrated as follows:

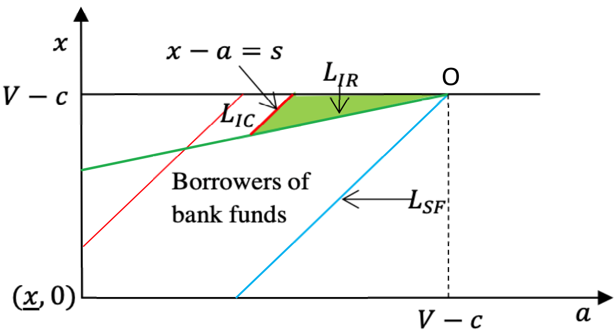


Figure 3b: The green area marks the SMEs saved by OFI-E if $s < k_e$.

The value of s is determined by the following budget condition:

$$\int_{\text{green area}} (x - a)h(a, x)dadx = G. \tag{10}$$

Obviously, threshold s increases with G. Hence, there exists a threshold G^* such that if the government budget $G \geq G^*$, then $s \geq k_e$ and Figure 3a is the case, whereas if $G < G^*$, then $s < k_e$ and Figure 3b is the case. We shall focus on the case where the government budget is limited, i.e., $G < G^*$.

The policy OFI-E certainly does better than the Simple Policy. Let us denote the green area in Figure 3b by Ω_3 and that in Figure 2 by Ω_2. Obviously, for any $(a, x) \in \Omega_3 - \Omega_2$,[4] the shortfall $x - a$ is smaller than that for any $(a, x) \in \Omega_2 - \Omega_3$. Given that $\int_{\Omega_2}(x - a)h(a, x)dadx = G = \int_{\Omega_3}(x - a)h(a, x)dadx$, it follows that $\int_{\Omega_2 - \Omega_3}(x - a)h(a, x)dadx = \int_{\Omega_3 - \Omega_2}(x - a)h(a, x)dadx$, which implies that $\int_{\Omega_2 - \Omega_3} h(a, x)dadx < \int_{\Omega_3 - \Omega_2} h(a, x)dadx$ and that $\int_{\Omega_2} h(a, x)dadx < \int_{\Omega_3} h(a, x)dadx$. That is, there are more entrepreneurs in Ω_3 than there are in Ω_2. Intuitively, because in general the entrepreneurs saved by Policy

[4] In general, for two sets A and B, $A - B = \{x | x \in A \text{ and } x \notin B\}$ and hence $A = (A - B) \cup (A \cap B)$.

OFI-E have a smaller shortfall than those saved by the Simple Policy, more entrepreneurs are saved by the former than by the latter. That is precisely the point of OFI-E: To use the limited budget to help the most entrepreneurs, the government should prioritise those with small shortfalls.

Now let us turn to the model economy where the government observes neither a nor x of any entrepreneur. The government cannot directly target entrepreneurs with small shortfalls. For that, a partnership with banks can help, which we refer to as the public-private partnership. Because bank funding is expensive (i.e., $R > 1$), entrepreneurs will only borrow the quantity of bank funding that just suffices to cover their shortfalls of $x - a$. Put differently, entrepreneurs' demand for loans perfectly reflects their shortfalls. Therefore, to target entrepreneurs with smallest shortfalls, the government should target those with the smallest loan demand. The optimal policy with the public-private partnership (PPP) thus takes the following form:

Optimal Policy with the PPP for Protecting Employment (PPP-E(y)): If the total borrowing b of an entrepreneur is no bigger than a threshold b^*, the government subsidises them by yb.

While this form of policy has two parameters b^* and y, they are connected by the budget constraint similar to (9), namely, the aggregate shortfall of entrepreneurs saved by the policy is equal to G, as detailed below. Hence, there is only one degree of freedom and we can index the policy with y. We are going to find the optimal value of y and thereby pin down the optimal policy. For the time being, note that by the IC condition (3), the borrowing scale $b \leq k_e$. Therefore, for the policy PPP-E to impose a meaningful qualification criterion, $b^* < k_e$.

For this purpose, let us examine which entrepreneurs are added to the survival group by the policy. First, with this policy, an entrepreneur who borrows b from a bank will have $(1 + y)\, b$ units of funds in hand. To qualify for the benefit of the policy, the borrowing scale $b \leq b^*$. Therefore, a type (a, x) entrepreneur can survive because of the policy if and only if

$$(1 + y)\, b^* \geq x - a. \tag{11}$$

Given the borrowing scale $b = (x - a) \,/\, (1 + y) \leq b^*$, we have $b < k_e$ and the IC constraint is satisfied.

Second, the policy affects the IR constraint. A type (a, x) entrepreneur needs to borrow $(x - a) \,/\, (1 + y)$. Following the argument that leads to (4), they are willing to borrow if and only if $V - R\,(x - a) \,/\, (1 + y) \geq a + c$, or equivalently,

$$V - c \geq \frac{R}{1 + y} x - \frac{R - (1 + y)}{1 + y} a. \tag{12}$$

These two conditions characterise types (a, x) of the outsiders that the policy saves, which are illustrated in the green area of Figure (4) below. For the time being, we ignore the effect of government policy on banks' lending rate R.

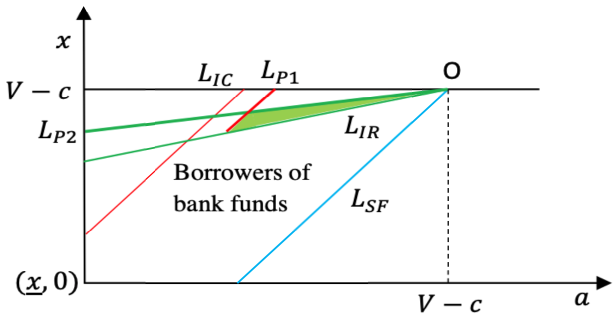


Figure 4: If $y < R - 1$, the types that are saved by PPP-E(y) compose the green area, where Line L_{P1} is defined by the equality form of (11), and Line L_{P2} by the equality form of (12).

The aggregate demand for surviving funds, of these added types and those who survive in the absence of public funding (in the green area of Figure 1) is $K + G$. As we have argued for (9) in connection with the Simple Policy, the market clearing condition implies the following:

$$\int_{\text{green area}} (x - a)h(a, x)dadx = G. \tag{13}$$

Hence, as was noted, the two policy parameters y and b^* are not independent. An increase in the scale of subsidy for each entrepreneur y must be accompanied by the decrease in the threshold b^*.

Now we consider the optimal scale of the subsidy y. First, we need $y \leq R-1$ to ensure non-arbitrage; otherwise, the policy causes borrowing to deliver a net gain to the borrower and all entrepreneurs will use the policy, which then defeats its purpose of targeting. Second, for a given y strictly less than $R - 1$, we show the net effect of a marginal increase in y is an increase in the number of SMEs saved. To prove this claim, we use Figure 4 to see how a marginal increase in y affects the positions lines L_{P1} and L_{P2}. The marginal increase in y induces Line L_{P2} in Figure 4 to rotate clockwise around point O $(x = V - c, a = V - c)$, according to (12). This movement alone would increase the green area in Figure 4 and select in more types to be saved. As said above, the increase in y is companied by a decrease in b^*. Therefore, Line L_{P1} in Figure 4 must move to the right, which squeezes out some types from being saved. These effects can be illustrated in Figure 5. The selected-in types are in the blue area, while the types in the red area are squeezed out.

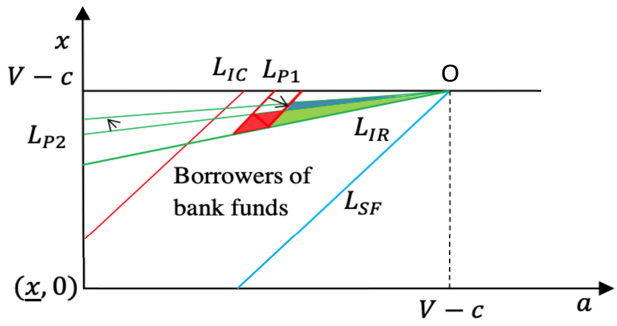


Figure 5: The marginal effect of increasing government subsidy y

From Figure 5, it is obvious that the shortfalls $(x - a)$ of the entrepreneurs squeezed out are all larger than the shortfalls of those selected in. Given that the aggregate shortfall is fixed equal to G, more entrepreneurs are saved due to the marginal increase in y. Hence,

The optimal value of y in the PPP-E(y) is $R - 1$. That is, the government should fully subsidize the funding cost of bank loans whose size is smaller than b^*.

Given $y = R - 1$, conditions (11) and (12) become:

$$Rb^* \geq x - a, \tag{14}$$

$$V - c \geq x. \tag{15}$$

Lemma 2. *$Rb^* < k_e$, that is, not all entrepreneur borrowers benefit from the policy.*

Proof: See Appendix B.

The intuition of Lemma 2 is that if all entrepreneurs benefit from the policy, it becomes the simple policy and the lending rate becomes $R = 1$. This is a contradiction to (7).

Thus far, we have ignored the effect of the policy PPP-E on the bank lending rate R. In practice, it may well be that the public funding G is so small relative to the aggregate private funding K that the policy barely moves R.[5] In this case, the effect of the optimal PPP-E(y^*), that is, PPP-E$(R - 1)$, is illustrated in Figure 6 as follows.

[5] Indeed, in reality, the government support policy encourages banks to extend loans, and the government budget is in most cases used to subsidise bank lending. To move the bank lending rate R, the regulator can relax bank capital requirement, as was the case in the Eurozone during the pandemic crisis. This point is outside the scope of this paper, but we acknowledge that adjusting macroprudential policy is another potential avenue to support SMEs.

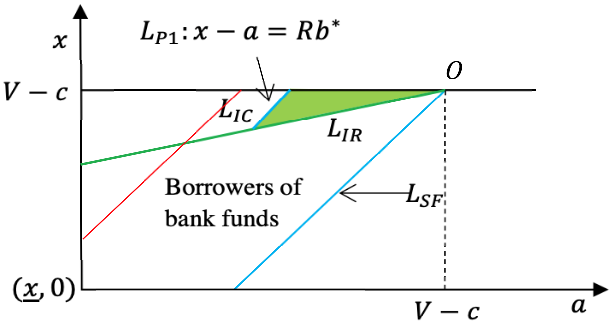


Figure 6: The effect of PPP-E$(R-1)$ if the effect of G on R is negligible.

Figure 6 is exactly the same as Figure 3b. It follows that the effect of the PPP-E$(R-1)$ is the same as that of OFI-E, which assumes the government has the full information regarding entrepreneurs' types. Hence,

Proposition 2. *If the public funding G has a negligible effect on the bank lending rate R, then optimal policy with the PPP under information asymmetry achieves the same effect as the full information case when the government operates on its own.*

Proposition 2 demonstrates the power of the PPP in overcoming information asymmetry: With the help of private banks, the government can achieve as much as it could in the absence of information asymmetry, if its policy has only a negligible impact on the lending rate of bank loans.

In the case where this impact is non-negligible, there is a possibility that the PPP can do even better under information asymmetry than the full information case when the government operates on its own. This happens when the gross lending rate R *increases* with the policy PPP-E. This possibility might sound counterintuitive, but let us establish its existence. By Lemma 2, in the presence of PPP-E, the surviving entrepreneurs can be classified into three groups, as illustrated in Figure (7), where line XBC is the line L_{P1}: $x - a = Rb^*$, line $X'B'C'$ is the IC line L_{IC} and line OBB' is the IR line L_{IR}. Area A_1 – i.e. triangle OBC – consists of entrepreneurs whose IR constraint is met only because of the policy. Area A_2 consists of entrepreneurs whose IR constraint is met in the absence of the policy and who are benefiting from the policy; and Area A_3 – i.e. quadrilateral $BXX'B'$ – consists of entrepreneurs whose IR is met in the absence of the policy, but are not benefiting from the policy.

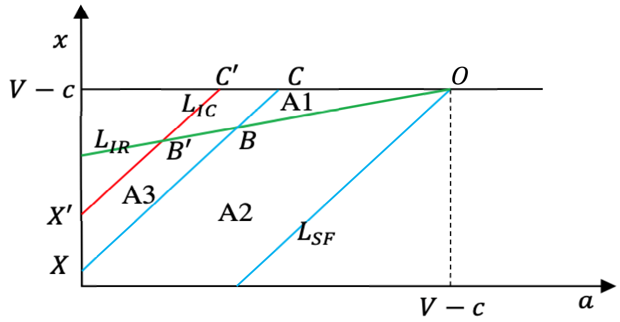


Figure 7: The three groups of entrepreneurs with policy PPP-E($R-1$)

Entrepreneurs of type (a, x) in Areas A_1 and A_2, as receiving the subsidy from the the policy, each demand a loan of $(x-a)/R$. Entrepreneurs in Area A_3 each demand a loan of $x-a$. Therefore, the aggregate demand for bank loans is $\int_{A_1 \cup A_2}(x-a)/R \cdot h(a,x)dadx + \int_{A_3}(x-a)h(a,x)dadx$. The new equilibrium loan price R is thus determined by

$$\int_{A_1 \cup A_2} \frac{x-a}{R} h(a,x)dadx + \int_{A_3}(x-a)h(a,x)dadx = K. \tag{16}$$

In the absence of the policy, the aggregate demand iss $\int_{A_2}(x-a)hdadx + \int_{A_3}(x-a)hdadx$. Compared to this formula, the policy produces two effects on the aggregate demand. One, entrepreneurs in Area A_1 are added, which alone would increase the aggregate demand. The other, entrepreneurs in Area A_2 borrow less from the banks, which alone would decrease the aggregate demand. If the p.d.f. $h(a,x)$ is sufficiently small in area A_2 and is sufficiently large in Area A_1, then former effect will dominate the latter and the policy will increase the aggregate demand for loans. As a result, it raises the funding cost R.[6]

By raising R relative to the case without any government intervention, policy PPP-E($R-1$) saves more SMEs than OFI. Let us start with Figure (3b), which illustrates the types that OFI saves while the bank lending rate R is unchanged (as it is implemented disjointly from the loan market). Now consider the effect of a rise in R. As R increases, Line L_{IC} shifts southeastward and Line L_{IR} rotates anti-clockwise around point O, as illustrated in Figure (8). The shifts of these two lines squeeze out types in the red area of Figure (8). Given that the aggregate shortfall is equal to $K+G$, squeezing out these types generates space for more types to be saved. That is, L_{P1} shifts leftward, selecting in the types in the blue area of Figure (8). Observe that the types squeezed out (i.e. those in the red area of Figure (8)) all have a greater shortfall $x-a$ than those selected in (i.e. those in the blue area). Therefore, the net effect is that the optimal policy with the PPP under asymmetric information saves more SMEs than the government works alone with the full information.

[6]This might look counterintuitive because the government policy increases the funding supply, which should in theory decrease the funding cost $R-1$. However, note that only the entrepreneurs in A_3 pay this funding cost, whereas the entrepreneurs in Areas A_1 and A_2 have this cost fully subsidized by the policy.

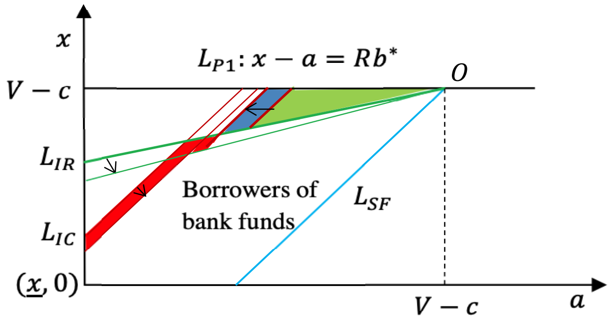


Figure 8: The effect of PPP-E($R - 1$) if it increases R, whereby the IR line L_{IR} rotates anti-clockwise, the IC line L_{IC} moves southeastwards and the qualification line L_{P1} moves to the left, relative to Figure 3b. Hence, compared to OFI-E, types in the red area are squeezed out, those in the blue selected in.

Certainly, policy PPP-E($R - 1$) does not always raise the funding cost R. Indeed, if the public budget G is sufficiently small, that cannot happen. To see this point, let us go back to Figure (7). Suppose $G \approx 0$, then, given $y = R - 1$, we must have $b^* \approx 0$; that is, the policy helps very few entrepreneurs. Hence, the size of A_1 is close to zero, and so is the positive effect of the types in this area on the aggregate demand for bank loans. We also find another condition under which policy PPP-E($R - 1$) reduces R, as shown in the following proposition.

Proposition 3. *Compared to the case without any government intervention, policy PPP-E($R - 1$) decreases R if the budget G is smaller than a threshold, or if H is the uniform distribution and*

$$2\,(V - pc_{\Delta}) < V - c. \tag{17}$$

Proof: See Appendix C.

Of course, if G is sufficiently small, then its impact on R is negligible, which brings us back to the case described by Proposition 2. If R is substantially decreased, then what happens is the opposite of the case illustrated by Figure (8), as is illustrated in Figure 9. More specifically, compared to OFI-E, the types that are squeezed out by PPP-E($R - 1$) all have a shortfall smaller than those selected. Hence, the PPP under the information asymmetry performs worse than the government with full information but with no partnerships with banks.

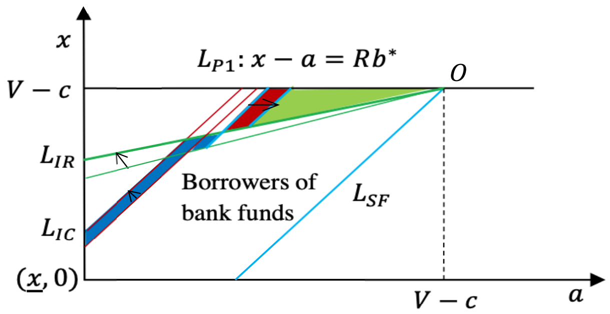


Figure 9: The effect of policy PPP-E($R-1$) if it substantially reduces R, whereby the IR line L_{IR} rotates clockwise, the IC line L_{IC} moves northwestwards, and the qualification line L_{P1} moves to the right, relative to Figure 3b. Hence, compared to OFI-E, types in the red area are crowded out, those in blue selected in.

Observe, however, that while in this case of decreasing R, PPP-E($R-1$) performs worse than OFI-E, it is still doing better than the Simple Policy. To see this point, compare Figure 9, which illustrates the effect of PPP-E($R-1$) , with Figure 2, which illustrates the effect of the Simple Policy. The latter saves the outsider entrepreneurs only by decreasing R, which is the only reason the lines L_{IR} and L_{IC} moves. In addition to that, PPP-E($R-1$) also saves outsiders by fully subsidising their funding cost, i.e. those in the triangle area demarcated by lines L_{IR}, $x = V - c$ and L_{P1}. Given that in both cases, the aggregate shortfall of the policy-save types is G, it must be true that the PPP-E induces a smaller decrease in R than the Simple Policy. Then, compared to the Simple Policy, the types that PPP-E($R-1$) squeezes out are in an area similar to the red area of Figure (8), while those that it selects in are the the triangle area demarcated by lines L_{IR}, $x = V - c$ and L_{P1}. Observe that the squeezed-out types all have a greater shortfall than the selected-in ones. Hence, PPP-E($R-1$) saves more entrepreneurs than the Simple Policy. Intuitively, while the types in the triangle area have a small shortfall, their low NPVs stop them from borrowing bank funds – this is why they are above the IR line. The Simple Policy, by reducing the funding cost R by a little bit (given G is small), does not help much. The substantial help comes from PPP-E($R-1$) , which fully subsidises their funding costs.

Let us finish the analysis of this section by considering two issues in implementing policy PPP-E($R-1$). First, note that $R-1$ is the funding cost for the entrepreneurs, not the interest expense. Given an entrepreneur survives with probability p, the gross loan rate is R/p, and the net loan rate that is observed in the loan contract is $r_o = R/p - 1$. That subsidy for the funding cost $y = R - 1$ means the government pays an interest expense of $(R-1)/p$, which is smaller than the whole interest expense r_o. To implement PPP-E($R-1$) precisely, the government needs to observe either the default risk $1-p$ or the funding cost $R-1$. In practice, the government will not be able to observe p. If it asks banks to report p, then banks may misreport its value and induce the government to shoulder more interest expense. However, there are reliable ways of measuring the funding cost $R-1$. In equilibrium, banks should obtain the same profit margin $R-1$

from extending loans of different risk profiles. We can assume the default risk of the safest loans is 0 (i.e., $p = 1$), such as the mortgage lending to high-quality borrowers (e.g., those with stable incomes, low loan-to-value ratios). Then, the net loan rate r_{safe} of this type of safe loans is equal to the funding cost $R - 1$. From $r_{safe} = R - 1$ and $r_o = R/p - 1$, it follows that $p = (r_{safe} + 1) / (r_0 + 1)$. The subsidised interest rate $(R - 1)/p$ should hence be $r_{safe}(r_0 + 1) / (r_{safe} + 1)$.

Second, the subsidy offered by PPP-E($R - 1$) is a concave function of loan size: It plummets to zero if $b > b^*$. Therefore, the implementation of the policy requires that either entrepreneurs can be prevented from borrowing multiple loans, or their overcall borrowing scale can be observed (so the policy can be based on it). If neither requirement is met, any attempt to make the subsidy a concave function of the loan size could be exploited by entrepreneurs borrowing many small loans. To avoid this exploitation, the subsidy should be either a linear or convex function of the loan size. In the case where the government's objective is to save as many SMEs as possible, there is no point in making the subsidy a convex function of the loan size. As a result, the only way is to distribute the subsidy in proportion to the loan size. That leads us back to the Simple Policy. Hence, the Simple Policy is optimal if entrepreneurs can borrow multiple loans and their overall borrowing is unobserved.

So far, we have analysed the government's optimal policy if the objective is to protect employment. In the following subsection, we consider the case where the government objective is to maximise economic efficiency.

5.2 The case of maximising efficiency

In this case, given the same shortfall, $x - a$, higher NPV projects should take priority. Again, we first consider the full-information scenario in which the government observes the type (a, x) of entrepreneurs. In this scenario, as in the preceding subsection, the government funds should be used to finance the shortfall of the outsiders. The first-best policy is to find an area Ω in the dark red region of Figure 1 to solve the following problem:

$$\begin{aligned} &\max_{\Omega} \int\int_{\Omega} (V - c - x) h(a, x) \, da dx, \\ &\text{s.t.} \int\int_{\Omega} (x - a) h(a, x) \, da dx = G. \end{aligned}$$

This problem may be complex mathematically, but economic intuition can greatly simplify it. Helping an enterprise survive the crisis is analogous to making an investment. If the government invests funds of $x - a$ to help an enterprise survive, the social value of this investment is $V - c - x$. The return rate of this investment is thus $(V - c - x)/(x - a)$. The entrepreneurs with higher return rates should therefore be prioritised. The first-best value-maximising policy is thus given as follows:

Optimal Full-information Policy for Value Maximisation (OFI-V): The government finances the shortfall of those outsiders whose types (a, x) satisfy:

$$\frac{V - c - x}{x - a} \geq \lambda^*.$$

Because $(V - c - x)/(x - a) = \lambda$ is equivalent to

$$x - \frac{\lambda}{1 + \lambda} a = \frac{V - c}{1 + \lambda},$$

all the ISO-return curves are straight lines that pass point O $(V - c, V - c)$; in particular, L_{IR} is an ISO-return line. Moreover, the greater the return rate, the steeper the slope of the ISO-return line. The threshold λ^* determines the number of entrepreneurs that the policy saves, whose aggregate shortfall is equal to G. Therefore, if G is larger than a certain threshold, then λ^* is small enough that the slope $\lambda^* / (1 + \lambda^*)$ of the boundary ISO-return line is smaller than the slope of the IR line L_{IR}. And the effect of OFI-V is illustrated as follows.

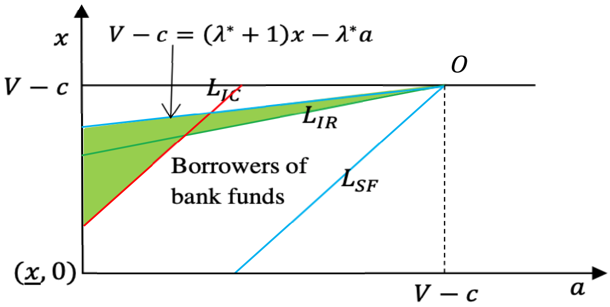


Figure 10a: The types that OFI-V saves are in the green area if G is larger than a threshold.

If G is smaller than the threshold, the slope $\lambda^* / (1 + \lambda^*)$ of the boundary ISO-return line is larger than the slope of the IR line L_{IR}. The effect of OFI-V is illustrated as follows.

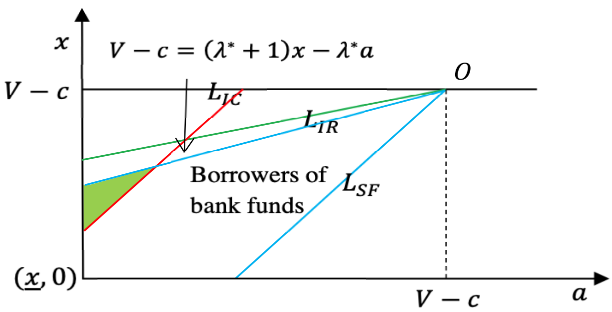


Figure 10b: The types that OFI-V saves are in the green area if G is smaller than a threshold.

The threshold, λ^*, is determined, again, by the following budget constraint.

$$\int_{\text{green area}} (x-a)h(a,x)dadx = G. \tag{18}$$

We now consider the possibility whether in partnership with banks, the government can design a better policy than the Simple Policy, based on, again, entrepreneurs' loan demand. The position of the green area in both Figures (10a) and (10b) suggests that the government should priotise entrepreneurs with a large enough shortfall $x-a$, and therefore, should target entrepreneurs with a large enough loan demand. Intuitively, this is because the size of an SME's loan demand is positively correlated with its NPV. Only when the SME has a sufficiently high NPV can it afford the funding cost of a large loan. Therefore, the PPP policy for value maximisation should hence subsidise the borrowers of large loans. This is the opposite of the optimal policy when the government objective is to protect employment. If we restrict to the case that the subsidy rate should be a constant for the qualified loans, then the policy should take the following form:

PPP Policy for Value Maximisation (PPP-V(b^*)**)**: The government subsidy to loans of size b is as follows:

$$s(b) = \max\{y(b-b^*), 0\}.$$

Namely, if the bank loan size b is no larger than the threshold b^*, entrepreneurs receive no benefits, but if it is above the threshold, the funding cost of the overrun is partly paid by the government.

Observe that if $b^* = 0$, the subsidy is in proportion to the bank funding, and hence, the policy is equivalent to the Simple Policy. On the other hand, it must hold that $b^* < k_e$, since k_e is the most entrepreneurs can borrow. Lastly, as with PPP-E, the two parameters of the policy, b^* and y, are connected by a clearing condition similar to (18) and not independent. Also, the condition $y \leq R-1$ holds in order not to give entrepreneurs incentives to overborrow. For the same purpose, unlike PPP-E, here PPP-V does not subsidise the entire loan if the loan's size is greater than a threshold, but subsidise only the part of the loan that is beyond the threshold. More specifically, if an entrepreneur's shortfall $x-a < b^*$, they have no incentives to borrow more than the threshold for the sake of benefiting from the policy: Borrowing more than the threshold incurs a positive cost of $(R-1-y)(b-b^*) + (R-1)(b^* - (x-a))$.

Only entrepreneurs with a shortfall $x-a \geq b^*$ are affected by the policy. Consider such an entrepreneur. If they borrow b, the government will offer them $y(b-b^*)$. Hence, the shortfall is filled if $b + y(b-b^*) = x-a$, or

$$(1+y)\,b - yb^* = x-a. \tag{19}$$

The maximum the entrepreneur can borrow b is k_e. The incentive compatibility constraint, for this group of entrepreneurs, is thus given as follows:

$$\widetilde{k} \geq x - a, \tag{20}$$

where $\widetilde{k} := (1+y)\,k_e - yb^*$. The equality case of (20) gives one boundary line of types (a, x) that can benefit from the policy, that is,

$$L_{P1} := \left\{(a, x)\,|\widetilde{k} = x - a\right\}. \tag{21}$$

As $b^* < k_e$, we have $\widetilde{k} > k_e$. That is, the original IC line $(x - a = k_e)$ moves left to obtain the new IC line L_{P1}.

As for the IR constraint, from (19), a type (a, x) entrepreneur needs to borrow $b = (x - a + yb^*)\,/\,(1+y)$ to fill the shortfall. Following the argument that leads to (4), the entrepreneur willing to borrow if and only if $V - R\,(x - a + yb^*)\,/\,(1+y) \geq a + c$, or equivalently,

$$V - c - \frac{Ry}{1+y} b^* \geq \frac{R}{1+y} x - \frac{R - (1+y)}{1+y} a. \tag{22}$$

The equality case of (22) gives the other boundary line L_{P2} on the (a, x) phase plane, that is,

$$L_{P2} := \left\{(a, x)\,|\frac{(1+y)\,(V-c)}{R} - yb^* = x - \frac{R-(1+y)}{R} a\right\}. \tag{23}$$

If we assume that the effect of the policy on the funding cost R is negligible, then its effect is illustrated as follows.

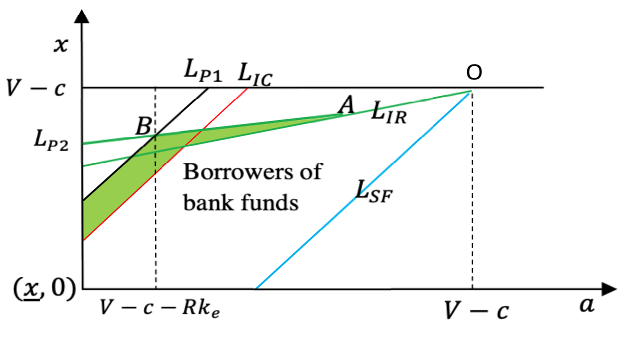


Figure 11: The effect of PPP-V(y) if its effect on R is negligible: The types in the green area are saved. Lines L_{IR} and L_{P1} cross at point A, lines L_{P1} and L_{P2} at point B.

Similar to the budget constraint (18) of the OFI-V, the parameters of the PPP-V (y, b^*) satisfy the usual market-clearing condition:

$$\int_{\text{green area}} (x-a)h(a,x)dadx = G. \tag{24}$$

An implication of this equation is $\partial y/\partial b^* > 0$: If the threshold b^* of the subsidy increases, there will be fewer entrepreneurs who receive the subsidy, and given the budget, each will receive more, so y increases.

The optimal value of (y, b^*) of the PPP-V solves the following problem

$$\max_{y,b^*} \int_{\text{Green area}} (V-c-x)h(a,x)dadx, \text{ s.t. } (24).$$

To begin with, we establish that the PPP-V can never do better than the OFI-V. Observe Figure 11. The triangle-area demarcated by L_{IR}, L_{P1} and $a=0$ is beneath the ISO-return line L_{IR}, while the triangle demarcated by L_{IR}, L_{P1} and L_{P2} is above it. Therefore, the types in the first triangle-area all have a higher return rate $(V-c-x)/(x-a)$ than those in the second one. However, the former are not saved by the PPP-V, whereas the latter are. In contrast, the OFI-V saves a higher-returned type before it saves a lower-returned one. Hence, the PPP under the asymmetric information can never do better than the government on its own with the full information. That is different to the preceding case. The reason for this difference is that as a signal, the loan demand is much more informative of the entrepreneur's shortfall than it is of their NPV. As a result, it offers not as much help in overcoming the asymmetric information in the present case as it does in the preceding case.

While the optimal PPP-V is dominated by the OFI-V, it still dominates the Simple Policy. We prove this claim by showing that the optimal $b^* > 0$ because the Simple Policy is a special case of the PPP-V at $b^* = 0$. Let us observe that if $b^* = 0$, then point A in Figure 11 coincides with point O $(V-c, V-c)$, and hence, line L_{P2} is an ISO-value line of the subsidy return function $(V-c-x)/(x-a)$. Now we consider a marginal rise in b^* from $b^* = 0$. To see how the rise in b^* moves lines L_{P1} and L_{P2}, we examine how it moves points A and B of Figure 11. Point A is the intersection of line L_{P2} and line L_{IR} and has coordinates $a = V-c-Rb^*$ and $x = V-c-(R-1)b^*$. The rise in b^* therefore moves A down along line L_{IR}. Point B is the intersection of lines L_{P1} and L_{P2} and has a coordinate $a = V-c-Rk_e$, which independent of (y, b^*), and hence it slides along the vertical line $a = V-c-Rk_e$ up or down. As A moves down along line L_{IR}, if point B is unmoved, the green area in Figure 11 will diminish, which will violate condition (24). To keep the condition holding, therefore, point B should slide straight up along line $a = V-c-Rk_e$. Altogether, the effect of the increase in b^* from $b^* = 0$ is illustrated as follows:

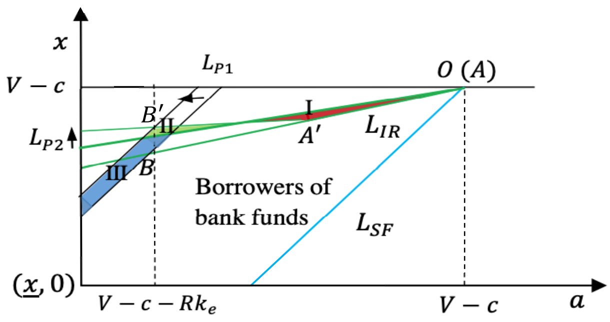


Figure 12: With a marginal increase in b^* from $b^* = 0$, point A moves from the position of point O to A', point B to B'.

We can see types in Area I are squeezed out, types in Areas II and III selected in. Given that at $b^* = 0$, line L_{P2} is an ISO-return line, the types in Area II have the same subsidy return rate $(V - c - x) / (x - a)$ as those in Area I to the first order effect. Given Area III is beneath the ISO-return line (i.e. line L_{P2} with $b^* = 0$), types in this area all have a higher subsidy return rate than those in Area I. Therefore, the types selected in (i.e. those in Areas II and III) have a subsidty return rate no smaller than those squeezed out (i.e. types in Area I) do. Given condition (24), the total subsidy to the former group is equal to that to the latter. Therefore, the net effect of squeezing out types in Area I and selecting in types in Areas II and III is an increase in value. That is, the marginal rise in b^* from $b^* = 0$ increases efficiency. Hence,

Proposition 4. *The optimal* $b^* > 0$, *thus, the optimal PPP-V does better than the Simple Policy.*

From the argument above, starting from any feasible value of b^*, a rise in b^* moves point A downwards along line L_{IR} and therefore, to keep condition (24) holding, point B must move up. The vertical coordinate of point B is $x = V - c - Rk_e + \widetilde{k}$. Therefore, we have $\partial \widetilde{k} / \partial b^* > 0$. Moreover, recall that $\partial y / \partial b^* > 0$ due to condition (24). It follows that $\partial \widetilde{k} / \partial y > 0$.

This result is important for our next result. When the budget G is extremely small, it should target those entrepreneurs that have the highest returns, i.e., entrepreneurs in the stripe between lines $x - a = \widetilde{k}$ and $x - a = k_e$. Put differently, we want that stripe as wide as possible, which means we want $\widetilde{k}$ as big as possible. Given $\partial \widetilde{k} / \partial y > 0$ and $y \leq R - 1$, this suggests the optimal $y = R - 1$. This intuition is confirmed by the following proposition.

Proposition 5. *If* $G \approx 0$, *then the optimal* $y = R - 1$. *The government finances all the funding cost of the part of bank loans that is above the threshold.*

Proof: See Appendix D.

6 Implications for Existing Covid-19 Policies

Presently, many government programmes aim to help SMEs survive the pandemic in order to protect employment. In this section, we use the results that we have obtained above as a benchmark to consider whether the government can do better than their current schemes. Typically, these programmes place a cap on the size of loans that it subsidises. However, for any additional amount that surpasses the cap, SMEs are allowed to borrow via the normal lending channel from the banks. For example, according to British Business Bank, a state-owned economic development bank established by the UK government with the aim to increase the supply of credit to SMEs and provide business advice, the UK BBL scheme has an upper limit of £50,000 per claimant, providing their turnover is £200,000 or over and have been trading before 1st March 2020; however, if the borrower wants to take out a further loan via the banks' own loan scheme, it is fine to do so.[7] Similarly, the US Paycheck Protection Programme does not prevent the firms from borrowing additional loans via their banks' normal lending channels. Thus, the total borrowing of an entrepreneur is not constrained by the size cap under the government schemes. In particular, businesses with an overall large loan demand can benefit from the programmes. This is against the optimal employment-protecting policy that we study in Subsection 5.1. The optimal policy, i.e. PPP-E$(R-1)$, should *fully* subsidise the funding costs of a SME if and only if its *overall* loan demand is below a threshold (which depends on the budget allocated for the programme).

To clearly see what difference our optimal policy can make relative to the existing schemes, we pick the U.K. BBL scheme as an example and formalise it within our framework. The policy can be reasonably represented by the government providing a costless loan to SMEs up to a cap δ. The required repayment depends on the assumption of whether the government takes into account the risk of the project. If the government takes it into account and shields itself from taking on the project risk, the entrepreneurs are required to pay back δ/p, and there is no net value transferred. If the government takes on the risk, the required payment is δ and the government transfers value $(1-p)\,\delta$ to each of the entrepreneurs. This value transfer gives all entrepreneurs incentives to use the scheme, including those who have a negative NPV, which is a waste of public funding. Therefore, the second case is doing worse than the first one. Due to this reason, we pick the first case in the comparison to the PPP-E.

To find the effect of the BBL, we examine how it affects the three constraints (2), (3) and (4) that characterise the types that survive by borrowing bank funds. First, the BBL gives (at most) a quantity δ of funds to an entrepreneur. Hence, if $x \le a+\delta$, the entrepreneur can self-fund the survival cost. The self-funding line L_{SF} is thus defined by equation $x = a+\delta$ now. Second, the net value of the repayment to the government loan is δ. To induce an entrepreneur to work hard, their stake in the enterprise can be no smaller than pc_Δ. Hence, what is left for the bank is no larger than $V-\delta-pc_\Delta$. The largest bank fund that an entrepreneur can borrow now changes to

[7]This has been confirmed via the authors' freedom of information request from the British Business Bank. Also see British Business Bank https://www.british-business-bank.co.uk/.

$$
\begin{aligned}
k_e' &= \frac{V - \delta - pc_\Delta}{R} \\
&= k_e - \delta / R.
\end{aligned}
$$

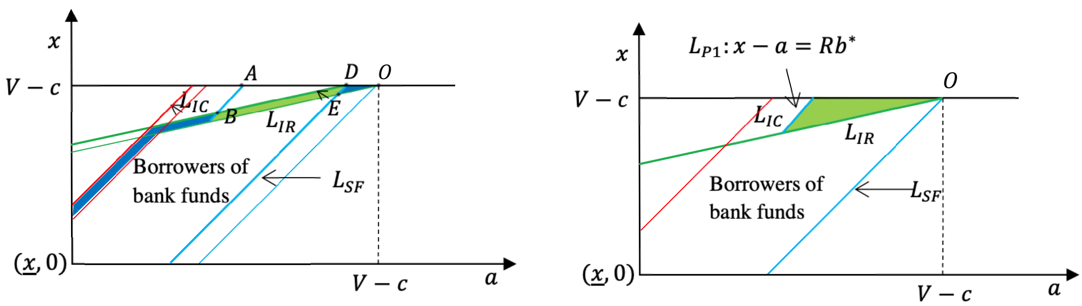
An entrepreneur in deficit can survive the crisis if and only if the shortfall $x - (a + \delta)$ is no greater than this threshold k_e', or equivalently

$$x - a \leq k_e + \delta \left(1 - 1/R\right).$$

The binding case of this constraint defines the new IC line L_{IC}. Lastly, if an entrepreneur uses the BBL and borrows bank funding to cover the shortfall $x - (a + \delta)$, their payoff is $V - \delta - R\left(x - (a + \delta)\right) - c$. This payoff should not be smaller than a, the payoff the entrepreneur obtains by giving up the enterprise and investing their wealth elsewhere. This IR constraint is equivalent to

$$Rx - (R - 1)\, a \leq V - c + (R - 1)\, \delta.$$

The binding case of this constraint defines the new IR line L_{IR}. Altogether, these effects are illustrated in Figure 13a.



(a) BBL effect when government takes on no risks

(b) Effect of the PPP-E given small G

Figure 13: Comparing the effect of BBL with that of the PPP-E.

Compared to the optimal policy PPP-E$(R - 1)$, the BBL selected in entrepreneurs in the blue stripes and the blue triangle ODE in Figure 13a, while crowding out the entrepreneurs in the white triangle ABD. Observe that the former ones have a shortfall all greater than the latter. Therefore, given the budget G, the BBL saves fewer entrepreneurs than the PPP-E.

In practice, to implement PPP-E$(R - 1)$, the government can set a more generous cap than the BBL does, but only allow entrepreneurs with the overall loan demand no bigger than the gap to access the scheme. This policy, compared to the BBL, squeezes out the entrepreneurs in the blue stripes in Figure 13a. The funding saved will be employed to rescue those in the white triangle area ABD. In terms of the net effect, a larger number of SMEs will be rescued, and more jobs will be protected, compared with the BBL.

7 Conclusion

This paper develops a model to assess the government loan support schemes to the small and medium-sized businesses during crisis times. Two layers of asymmetric information and a typical moral hazard issue arise. The government is unlikely to observe SMEs' quality or the actual amount of financial resources SME entrepreneurs have, and once the entrepreneurs obtain the funding, a typical moral hazard issue may arise of funding diversion for other purposes. The model shows that the government can rely on the expertise of private-sector banks and simultaneously exploit the information contained in the SMEs' demand for loans to improve the target of its loan support programmes.

The model finds that if the government targets to save the most jobs, the optimal employment protection policy is to help *only* those entrepreneurs whose loan size is below a threshold, and that the government pay all the funding costs of these small-sized loans. On the other hand, if the government targets maximising economic efficiency, its target is the opposite of employment protection. To maximise economic efficiency, the government should only extend loan support to those whose loan size is above a threshold. To tackle information asymmetry, the government essentially bases its targeting on borrowers' loan demand to infer the borrowing entrepreneurs' funding shortfall or the NPVs of the underlying businesses.

The design of financial support for SMEs during crises is particularly pertinent now since governments worldwide have implemented and are implementing various loan support programmes to SMEs to save jobs. As we have shown, such schemes do not effectively exploit the information contained in the borrowers' demand for loans, and hence, they lack targeting and do not maximise employment protection. We have shown how to enhance the efficacy of the existing government loan support schemes.

Although the context of the model is around loan support to SMEs during a crisis, the model conveys a much more general message: for sophisticated policy design, the government should utilise the information and expertise of private agents to improve its policy target.

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Appendices

A Proof of Lemma 1

An increase in R has the following effects, referring back to Figure 1. Line L_{SF} stays unchanged. Line L_{IC} moves southeastward closer to line L_{SF}, since $k_e = (V - pc_{\Delta})/R$ decreases with R. Finally, line L_{IR} rotates anti-clockwise around point O ($x = V - c, a = V - c$), closer to line L_{SF}, since the intersect of the line with $x-$axis, $\frac{V-c}{R}$ decreases with R. Therefore, the area of borrowing entrepreneurs is diminished and fewer entrepreneurs borrow from banks, and thus, the total demand for loans, D, decreases.

When $R \to \infty$, both lines L_{IC} and L_{IR} converge to L_{SF} and the area of the borrowing entrepreneurs goes to 0, i.e., $D \to 0$. $\square$

B Proof of Lemma 2

Suppose on the contrary, all borrowers benefit from the policy. Then, the policy OPE is equivalent to the case with the simple policy, because each borrower gets a government fund in proportion to the bank funding that they demand. As a result, the market clearing condition (8) holds. Moreover, as all the entrepreneurs benefit from the policy, the funding cost to them is zero. That is, (8) holds at $R = 1$, which, however, contradicts with assumption (7). $\square$

C Proof of Proposition 2

By Lemma 2, in the presence of OPE, surviving entrepreneurs can be classified into three groups, as illustrated in the following diagram, where line OBC is the policy line $x - a = Rb^*$; Area A_1 is demarcated by line OBC, L_{IR} and line $x = V - c$, consisting of entrepreneurs whose IR constraint is met only because of the policy; Area A_2 is demarcated by line OBC, L_{IR}, L_{SF} and $a = 0$, consisting of entrepreneurs who would borrow even in the absence of the policy and are benefiting from the policy; and Area A_3 is demarcated by line OBC, L_{IR}, the two axes, and L_{IC}, consisting of entrepreneurs who would borrow in the absence of the policy and are not benefiting from the policy.

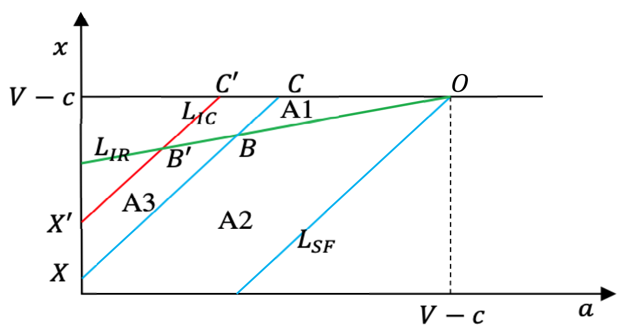

Figure C: The effect of OPE

Entrepreneurs of areas A_1 and A_2 benefit from the policy and hence borrow only $(x-a)/R$ from banks, while those of area A_3 do not, and they use bank funding to cover the whole shortfall $x-a$. Let $t := Rb^*$. Then, the aggregate demand for bank funding is

$$\widetilde{D}(R,t) := \int_{A_1 \cup A_2} \frac{x-a}{R} + \int_{A_3} x-a,$$

and the new lending rate R is determined by $\widetilde{D}(R,t) = K$. Certainly $\widetilde{D}(R,0)$ is equal to $D(R)$ given in (8) and $\widetilde{D}'_R < 0$. As $\partial R/\partial t = -\widetilde{D}'_t/\widetilde{D}'_R$, to prove that $\partial R/\partial t < 0$, that is, the policy decreases R, it suffices to show that $\widetilde{D}'_t < 0$. We find that

$$\widetilde{D}'_t = t\left(\frac{1}{R}\int_{\text{line OC}} hdadx - \int_{\text{line OB}} hdadx\right). \tag{25}$$

Obviously, at $G = 0$, points B and C coincide and $\widetilde{D}'_t = t\left(\frac{1}{R}-1\right)\int_{\text{line OC}} hdadx < 0$. As a result, $\widetilde{D}'_t < 0$ for G is small enough and the first part of the proposition is proved. To prove the second part, note that if H is the uniform distribution, the term in the bracket of (25) has the same sign as $\frac{1}{R}l_{OC} - l_{OB}$, where l denote the length. Certainly $l_{BC}/l_{OB} < l_{B'C'}/l_{O'B'}$ and hence $l_{OC}/l_{OB} < l_{O'C'}/l_{O'B'}$. Therefore, $\frac{1}{R}l_{OC} - l_{OB} < 0$ if $\frac{1}{R}l_{O'C'} - l_{O'B'} < 0$, which, given $l_{O'C'} = \sqrt{2}\left(V-c-\frac{1}{R}\left((V-pc_\Delta)\right)\right)$ and $l_{O'B'} = \sqrt{2}\left(V-c-(V-pc_\Delta)\right)$, is equivalent to

$$\frac{R+1}{R}\left(V-pc_\Delta\right) < V-c. \tag{26}$$

Because $R > 1$ and thus $(R+1)/R < 2$, inequality (26) holds if condition (17) holds. Therefore, if this condition holds and H is the uniform distribution, then always $\widetilde{D}'_t < 0$ and hence the policy decreases the lending rate R.

Q.E.D

D Proof of Proposition 5

Start with $y = R-1$. Given this is the highest possible value of y, we just need to prove the decrease from y causes a loss of efficiency.

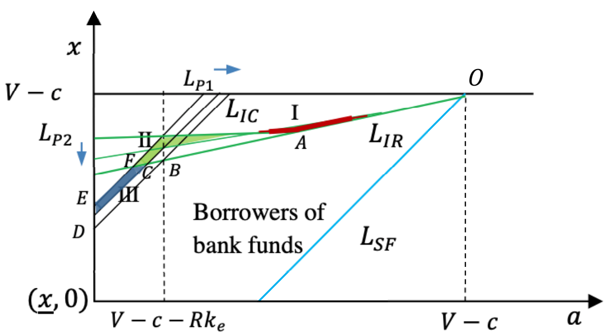

Figure D: A marginal decrease in y at $y = R-1$.

The effect of a marginal decrease in y can be illustrated by Figure D, with the movement of lines L_{P1} and L_{P2}. That is, the entrepreneurs in the blue area (III) and green area (II) are squeezed out, those in the red triangle area (I) added in. If $G \approx 0$, point A and point C approach point B indefinitely. As a result, the green area is infinitely small relative to the blue area and its contribution to the effect of the decrease in y is negligible. Observe that L_{IR} passes point $(V - c, V - c)$. Hence it is a ISO-value line of the return function of the subsidy $(V - c - x) / (x - a)$. Therefore, the return rates of entrepreneurs in the red triangle, which sits above line L_{IR}, are all lower than those of entrepreneurs in the blue stripe, which lies below line L_{IR}. Therefore, the efficiency gains by squeezing in the former group of entrepreneurs (area I) is smaller than the efficiency loss by squeezing out the latter (area III). As a result, a marginal decrease in y from $y = R - 1$ causes an efficiency loss.

Q.E.D.