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Tax Curvature

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Abstract

In a Mirrleesian environment, a monopsonist sets hourly wages and individuals choose how many hours to work. Labor market outcomes do not only depend on the level and slope of the income tax function, but also on its curvature. A more concave tax schedule raises the elasticity of labor supply, which boosts wages. Consequently, optimal marginal tax rates for low-skilled workers are declining in income. I derive an optimal tax formula in terms of sufficient statistics that accounts for the impact of tax curvature on labor market outcomes.

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1 Introduction

Firms exercise significant market power when setting wages. How much market power they can exert depends critically on the elasticity of labor supply (Robinson, 1933, Manning, 2003). This statistic is not policy-invariant. I show that this insight has an implication for tax policy. In particular, if wages are set by a monopsonist and individuals choose how many hours to work, the government can use the *curvature* of the tax function to boost wages of low-skilled workers by raising their elasticity of labor supply. This is achieved by setting marginal tax rates that are declining in income. Consequently, the optimal tax schedule at the bottom of the income distribution is concave.

To reach this conclusion, I study a Mirrleesian environment where individuals differ in their ability, which is not observed by the government. Individuals take their hourly wage and the tax schedule as given and optimally choose how many hours to work. Wages are not determined competitively as in Mirrlees (1971), but instead set by a monopsonist that observes ability and maximizes profits taking into account how wages affect individual labor supply. The government has a preference for redistribution and levies a nonlinear tax on labor earnings and a confiscatory tax on profits.

Labor market outcomes do not only depend on the level and slope of the tax function (the zeroth and first derivative), but also on its curvature (the second derivative). The latter determines how responsive individuals are to wage changes. To illustrate, suppose the tax function is convex and marginal tax rates increase steeply with income. Individuals then have weak incentives to work longer hours following an increase in the hourly wage. A low elasticity of labor supply, in turn, implies that a profit-maximizing monopsonist sets low wages. Consequently, a local increase in the *second* derivative of the tax function reduces the hourly wage, hours worked and labor earnings.

A government that is interested in redistribution can exploit this feature to boost wages of low-skilled workers. I characterize the second-best allocation and show that the optimal tax schedule is concave at the bottom of the income distribution. Declining marginal tax rates at the bottom raise the labor supply elasticity of low-skilled workers, which positively affects their wages. In fact, the government sets the curvature of the tax function in such a way that the monopsonist does not extract any rents from hiring the least-skilled workers. If the tax system is optimized, the monopsonist pays the least-skilled workers a wage equal to their productivity.

The finding that optimal marginal tax rates for low-skilled workers are declining in income is a local result. I also derive an optimal tax formula that holds at each point in the income distribution. To that end, I study the welfare effects of increasing the *second* derivative of the tax function just below a particular income level, and decreasing it right above. This reform induces a local increase in the marginal tax rate, cf. Saez (2001) and Golosov et al. (2014). The optimal tax formula is obtained by setting the sum of the welfare-relevant effects from this reform equal to zero. Compared to existing results from the literature, the additional 'sufficient statistics' that characterize optimal tax policy are the impact of tax curvature on labor earnings and hourly wages, and the impact of the level and slope of the tax function on hourly wages and profits.

A number of recent papers study optimal redistributive taxation in an environment where firms have market power, either in the market for goods (Kaplow, 2019, Boar and Midrigan, 2020, Kushnir and Zubrickas, 2020, Eeckhout et al., 2021, Gürer, 2021) or, as in the current paper, the market for labor (Hariton and Piaser, 2007, da Costa and Maestri, 2019, Hummel, 2021). Hariton and Piaser (2007) and da Costa and Maestri (2019) study a model where firms do not observe ability and screen workers through nonlinear compensation contracts. Hummel (2021) assumes firms do observe ability and offer workers a combination of earnings and labor effort. The main difference with these studies is that in the current paper, the monopsonist sets hourly wages and individuals choose how many hours to work. An important implication is that, unlike in the aforementioned studies, labor market outcomes depend on the curvature of the tax function, which is the main focus of this paper.

It is well known that measures of labor supply or earnings responses to wage or tax changes, such as the elasticity of taxable income (ETI), depend on the curvature of the tax function. See, among others, Saez (2001) and Jacquet and Lehmann (2021) for a discussion of this issue. A key difference is that in my model, the curvature of the tax function affects labor market outcomes directly and not only the behavioral responses to wage or tax changes. Slemrod and Kopczuk (2002) derive an optimal elasticity of taxable income in an environment where the government can affect this elasticity using administrative instruments. By contrast, in my model the government uses the curvature of the tax function to affect the elasticity of labor supply.

The remainder of this paper is organized as follows. Section 2 presents the model and analyzes the impact of tax curvature on labor market outcomes. Section 3 studies the implications for optimal marginal tax rates at the bottom. Section 4 derives an optimal tax formula that holds at each point in the income distribution. Finally, Section 5 concludes.

2 Model

There is a continuum of individuals who differ in their ability (or skill) $n \in [n_0, n_1]$, which measures how much output an individual produces per hour worked. The cumulative distribution of ability is denoted by $F(n)$ with density $f(n)$. Individuals supply labor on the intensive margin to a single monopsonist. The monopsonist observes ability and sets the hourly wage at each ability level in order to maximize profits. The government does not observe individual ability but only their labor earnings. It has a preference for redistribution and levies a nonlinear tax on labor earnings. To focus exclusively on labor income taxation, I assume profits are taxed at a confiscatory rate. The timing is as follows.

1. The government chooses the tax schedule $T(\cdot)$ on labor income $z(n) = w(n)l(n)$.
2. The monopsonist sets the hourly wage $w(n)$ at each ability level $n \in [n_0, n_1]$.
3. Individuals choose how many hours $l(n)$ to work.

Working backward, an individual with ability n takes the hourly wage $w(n)$ and the tax schedule $T(\cdot)$ as given. Preferences over consumption c and labor effort l are described by a separable utility function $U(c, l) = u(c) - \phi(l)$, with $u'(\cdot), \phi'(\cdot), \phi''(\cdot) > 0$ and $u''(\cdot) \leq 0$. The individual chooses how many hours to work in order to maximize utility:

$$v(n) = \max_{l} \Big\{ u(w(n)l - T(w(n)l)) - \phi(l) \Big\}. \tag{1}$$

The first-order necessary condition determines the optimal choice of labor effort $l(n)$:

$$u'(w(n)l(n) - T(w(n)l(n)))w(n)(1 - T'(w(n)l(n))) = \phi'(l(n)). \tag{2}$$

At the optimum, the marginal benefits of working an extra hour (on the left-hand side) are equal to the marginal costs (on the right-hand side). Equation (2) pins down the number of hours worked as a function of the hourly wage.

The monopsonist sets the wage $w(n)$ at each ability level in order to maximize profits. It takes the tax schedule $T(\cdot)$ as given, but has to take into account that wages affect hours worked, cf. equation (2). The monopsonist thus maximizes aggregate profits $\Pi = \int_{n_0}^{n_1} \pi(n) f(n) dn$, where the profits from employing a worker with ability n are

$$\pi(n) = \max_{w,l,\lambda} \Big\{ (n-w)l + \lambda \big[u'(wl - T(wl)) w (1 - T'(wl)) - \phi'(l) \big] \Big\}. \tag{3}$$

Here, λ denotes the multiplier on the constraint (2). Combining the first-order conditions with respect to the wage and hours worked gives, after rearranging,

$$\frac{w(n)}{n} = \frac{e_{lw}(n)}{1 + e_{lw}(n)}. \tag{4}$$

At the optimum, the markdown of wages relative to productivity depends on the (firm-level) elasticity $e_{lw}(n)$ of hours worked with respect to the hourly wage, which varies across the skill distribution. Ignoring function arguments to save on notation, the latter can be found by implicitly differentiating the first-order condition (2):

$$e_{lw} = \frac{dl}{dw}\frac{w}{l} = \frac{w}{l}\frac{u''wl(1-T')^2 + u'(1-T') - u'wlT''}{-u''w^2(1-T')^2 + \phi'' + u'w^2T''}. \tag{5}$$

A few remarks are in place. First, because there is a single monopsonist, the *firm-level* elasticity of labor supply is equal to the aggregate, or *market-level* elasticity of labor supply: both are given by e_{lw}.[1] The assumption of a single monopsonist if of coure extreme, but captures that the labor supply curve each firm faces is less than perfectly elastic (Manning, 2003). Second, the elasticity of labor supply depends on properties of the tax function, in particular the level T (which enters u' and u''), the slope T' and the *curvature* T''. This leads to the following result.

Proposition 1. *A local increase in the curvature of the tax function $T(\cdot)$ at earnings $z(n)$ leads to a lower equilibrium wage $w(n)$, hours worked $l(n)$ and labor earnings $z(n)$.*

Proof. An increase in T'' lowers the elasticity e_{lw} of labor supply, as the numerator (denominator) in equation (5) decreases (increases). Equation (4) then implies the equilibrium wage decreases. At an interior optimum, the labor supply curve (2) is upward sloping: $e_{lw} > 0$. Consequently, hours worked l and labor earnings $z = w \times l$ decline as well. □

According to Proposition 1, a local increase in the *second* derivative of the tax function $T(\cdot)$ at earnings $z(n)$ reduces the hourly wage, hours worked and labor earnings of individuals with ability n. The reason is that an increase in the curvature of the tax function makes individual labor supply less responsive to a change in the hourly wage. To illustrate, suppose the tax schedule is convex and marginal tax rates are steeply increasing in income. In that case, individuals have weak incentives to

[1]By contrast, if labor markets are perfectly competitive, the market-level elasticity of labor supply is given by equation (5), whereas the firm-level elasticity of labor supply is infinite. In that case, equation (4) implies there is no markdown: $w(n) = n$.

work longer hours following an increase in the hourly wage. Put differently, the elasticity of labor supply e_{lw} is low. A low elasticity of labor supply makes it attractive for the monopsonist to pay low wages as well, cf. equation (4). Consequently, a local increase in the curvature of the tax function reduces the hourly wage, hours worked and labor earnings. The following example illustrates this.

Example 1. *Suppose the individual utility function is $U(c,l) = c - \frac{l^{1+1/\varepsilon}}{1+1/\varepsilon}$ and the tax schedule has a constant rate of progressivity $p \in (0,1)$: $T(z) = z - \frac{1-\tau}{1-p} z^{1-p}$.*[2] *Equilibrium labor supply follows from equation (2): $l(n) = (1-\tau)^{\frac{\varepsilon}{1+p\varepsilon}} w(n)^{\frac{(1-p)\varepsilon}{1+p\varepsilon}}$. Hence, the elasticity of labor supply is $e_{lw} = \frac{(1-p)\varepsilon}{1+p\varepsilon}$ and the equilibrium wage is, cf. equation (4):*

$$w(n) = (1-p)\frac{\varepsilon}{1+\varepsilon} n. \tag{6}$$

A higher rate of tax progressivity $p \in (0,1)$ means that marginal tax rates are more quickly increasing in income. This makes it less attractive for individuals to work longer hours if their wage increases: the elasticity of labor supply e_{lw} is decreasing in p. An increase in the rate of tax progressivity thus amplifies the negative impact of market power on wages by making individuals less responsive to wage changes.

It is useful to point out that the effect described in Proposition 1 and illustrated with the example above differs from the wage-moderating effect of tax progressivity. The latter states that a local increase in the *marginal* tax rate lowers the equilibrium wage. This is a robust prediction in models where labor markets are imperfectly competitive: it holds in the context of union bargaining (Hersoug, 1984), search frictions (Pissarides, 1985) and efficiency wages (Pisauro, 1991). The main difference with Proposition 1 is that the latter concerns the impact of the second (as opposed to the first) derivative of the tax function. In the aforementioned studies, labor market outcomes only depend on the level and slope of the tax function. Consequently, a local change in the curvature of the tax function has no impact. By contrast, a change in the curvature does affect labor market outcomes if, as in my model, a monopsonist sets hourly wages and individuals choose how many hours to work.

Turning to the optimal tax problem, the government levies a confiscatory tax on profits and chooses the tax schedule $T(\cdot)$ on labor earnings to maximize the following welfare function:

$$\mathcal{W} = \int_{n_0}^{n_1} \Psi(v(n)) f(n) dn. \tag{7}$$

The function $\Psi(\cdot)$ is an increasing, weakly concave transformation of individual utilities. Together with the concavity in the individual utility function $u(\cdot)$, it determines the government's preferences for redistribution from high-skilled to low-skilled workers. The government chooses the tax schedule $T(\cdot)$ to maximize social welfare (7), taking into account how a change in the tax function affects labor market outcomes and subject to the budget constraint

$$\int_{n_0}^{n_1} T(z(n)) f(n) dn + \int_{n_0}^{n_1} \pi(n) f(n) dn = G. \tag{8}$$

[2] According to Heathcote et al. (2017), this specification provides a good approximation of the current US tax schedule. They estimate a value of $\hat{p} = 0.181$.

Total revenues from taxing labor income and profits must be sufficient to finance some exogenous spending G. The literature offers two methods to solve an optimal tax problem of this kind: the mechanism design approach (see, e.g., Mirrlees, 1971) and the tax perturbation approach (see, e.g., Saez, 2001 and Golosov et al., 2014). See Jacquet and Lehmann (2021) for a formal discussion and comparison of both methods. In what follows, I use the mechanism design approach to derive a property of marginal tax rates at the bottom of the income distribution (Section 3) and the tax perturbation approach to derive an optimal tax formula in terms of sufficient statistics that holds at each point in the income distribution (Section 4).

3 Declining marginal tax rates

Solving the optimal tax problem using the mechanism design approach requires finding the allocation $(v(n), \pi(n), l(n))$ for each $n \in [n_0, n_1]$ that maximizes welfare (7) subject to resource and incentive constraints. The resource constraint is obtained by inverting the relationship $v(n) = u(z(n) - T(z(n))) - \phi(l(n))$ with respect to $T(z(n))$ and using the property $z(n) = nl(n) - \pi(n)$. Substituting these in the government budget constraint (8) gives

$$\int_{n_0}^{n_1} \left[nl(n) - u^{-1}(v(n) + \phi(l(n))) \right] f(n)dn = G. \tag{9}$$

The incentive constraints, in turn, describe the optimizing behavior of the monopsonist and individuals. To derive the first of these, differentiate the expression for profits (3) with respect to ability n and apply the envelope theorem:

$$\pi'(n) = l(n). \tag{10}$$

To derive the second incentive constraint, differentiate the expression for individual utility (1) and again apply the envelope theorem:[3]

$$\begin{aligned} v'(n) &= u'(w(n)l(n) - T(w(n)l(n)))l(n)(1 - T'(w(n)l(n)))w'(n) \\ &= \phi'(l(n))l(n)\frac{w'(n)}{w(n)} = \phi'(l(n)) \left(\frac{\pi(n)}{nl(n) - \pi(n)} \right) b(n), \quad \text{where} \quad b(n) = l'(n). \end{aligned} \tag{11}$$

The first step uses equation (2) and the second step uses the relationship $\pi(n) = (n - w(n))l(n)$.[4]

As will be made clear below, in order to find the allocation that maximizes social welfare subject

[3] An alternative way to derive this incentive constraint is the following. Suppose an individual with hourly wage $w(n)$ chooses labor earnings (recall: the government only observes earnings). Incentive compatibility then requires

$$n = \arg\max_m \; u(z(m) - T(z(m))) - \phi\left(\frac{z(m)}{w(n)}\right).$$

Differentiating the objective with respect to ability n gives, at the truth-telling solution,

$$v'(n) = \phi'\left(\frac{z(n)}{w(n)}\right)\frac{z(n)}{w(n)^2}w'(n) = \phi'(l(n))l(n)\frac{w'(n)}{w(n)},$$

which coincides with the second line of equation (11).

[4] To see this, differentiate both sides with respect to ability to find $\pi'(n) = l(n) + (n - w(n))l'(n) - w'(n)l(n)$. Equation (10) then implies $w'(n)l(n) = (n - w(n))l'(n)$.

to resource and incentive constraints, it is important to take the constraint $\pi(n_0) \geq 0$ explicitly into account. Equation (10) then implies firms make non-negative profits from hiring each worker. Lastly, to make sure that the allocation can be implemented using a non-linear tax $T(z(n))$, labor earnings must satisfy the monotonicity constraint $z'(n) \geq 0$.[5] Differentiating $z(n) = nl(n) - \pi(n)$ and imposing equation (10), the latter requires $b(n) = l'(n) \geq 0$.

The problem of finding the optimal nonlinear tax schedule $T(\cdot)$ can now be written as a standard optimal control problem, with state variables $(v(n), \pi(n), l(n))$ and control variable $b(n)$. Solving this problem leads to the following result.

Proposition 2. *Optimal marginal tax rates for the least-skilled workers are declining in income. Put differently, the optimal tax schedule is concave at the bottom of the income distribution:* $T''(z(n_0)) < 0$.

Proof. The key behind this result lies in demonstrating that the constraint $\pi(n_0)$ is binding. To do so, write the optimal tax problem in terms of allocation variables:

$$\begin{aligned}
&\max_{[v(n),\pi(n),l(n),b(n)]_{n_0}^{n_1}} \quad \mathcal{W} = \int_{n_0}^{n_1} \Psi(v(n)) f(n) dn, \qquad (12)\\
\text{s.t.} \quad & \int_{n_0}^{n_1} \Big[nl(n) - u^{-1}(v(n) + \phi(l(n)))\Big] f(n) dn = G,\\
\forall n: \quad & v'(n) = \phi'(l(n)) \left(\frac{\pi(n)}{nl(n) - \pi(n)}\right) b(n),\\
\forall n: \quad & \pi'(n) = l(n),\\
\forall n: \quad & l'(n) = b(n),\\
\forall n: \quad & b(n) \geq 0,\\
& \pi(n_0) \geq 0.
\end{aligned}$$

Using integration by parts on the incentive constraints, the Lagrangian is given by

$$\begin{aligned}
\mathcal{L} = \int_{n_0}^{n_1} \Bigg[& \bigg(\Psi(v(n)) + \eta\Big(nl(n) - u^{-1}(v(n) + \phi(l(n))) - G\Big)\bigg) f(n)\\
& + \lambda(n)\phi'(l(n)) \left(\frac{\pi(n)}{nl(n) - \pi(n)}\right) b(n) + \lambda'(n)v(n) + \mu(n)l(n) + \mu'(n)\pi(n) + \chi(n)b(n)\\
& + \chi'(n)l(n) + \psi(n)b(n)\Bigg] dn + \lambda(n_0)v(n_0) - \lambda(n_1)v(n_1) + \mu(n_0)\pi(n_0) - \mu(n_1)\pi(n_1)\\
& + \chi(n_0)l(n_0) - \chi(n_1)l(n_1) + \xi\pi(n_0). \qquad (13)
\end{aligned}$$

[5] As demonstrated in Appendix A, monotonicity of labor earnings implies the first-order conditions of the profit maximization problem are both necessary and sufficient. If that is the case, the first-order condition for the utility maximization problem is necessary and sufficient as well.

The first-order condition with respect to the states $v(n)$, $\pi(n)$, $l(n)$ and the control $b(n)$ are:

$$v(n): \quad \left(\Psi'(v(n)) - \frac{\eta}{u'(c(n))}\right) f(n) + \lambda'(n) = 0, \tag{14}$$

$$\pi(n): \quad \lambda(n)\phi'(l(n))\frac{nl(n)}{(nl(n)-\pi(n))^2} b(n) + \mu'(n) = 0, \tag{15}$$

$$\begin{aligned} l(n): \quad & \eta\left(n - \frac{\phi'(l(n))}{u'(c(n))}\right) f(n) + \mu(n) + \chi'(n) \\ & + \lambda(n)\left[\phi''(l(n))\frac{\pi(n)}{nl(n)-\pi(n)} - \phi'(l(n))n\frac{\pi(n)}{(nl(n)-\pi(n))^2}\right] b(n) = 0, \end{aligned} \tag{16}$$

$$b(n): \quad \lambda(n)\phi'(l(n))\frac{\pi(n)}{nl(n)-\pi(n)} + \chi(n) + \psi(n) = 0. \tag{17}$$

Here, $c(n) = u^{-1}(v(n) + \phi(l(n)))$ denotes the consumption of an individual with ability n. The latter is increasing in ability, because monotonicity of labor earnings, i.e., $b(n) \geq 0$, implies labor effort $l(n)$ and utility $v(n)$ are increasing in ability as well.

The transversality conditions imply $\lambda(n_0) = \lambda(n_1) = \mu(n_1) = 0$ and $\mu(n_0) + \xi = 0$. Because $\Psi(\cdot)$ and $u(\cdot)$ are concave and because $v(n)$ and $c(n)$ are increasing in ability, equation (14) implies that $\lambda(n) \leq 0$ for all n with a strict equality only at the end-points.[6] Equation (15) then implies $\mu(n)$ is increasing in ability. From $\mu(n_1) = 0$, it follows that $\mu(n_0) < 0$ and hence, $\xi > 0$. The constraint $\pi(n_0) \geq 0$ thus holds with equality.

If $\pi(n_0) = 0$, it follows that $w(n_0) = n_0$ at the optimal tax system. Equation (4) then implies $e_{lw}(n_0) \to \infty$. This requires that the denominator in equation (5) approaches zero. Therefore, at the lowest skill level $-u''w^2(1-T')^2 + \phi'' + u'w^2T'' \to 0$. Because $u'' \leq 0$ and $\phi'' > 0$, it follows that the tax function is concave at the bottom of the income distribution: $T'' < 0$. □

The main insight from Proposition 2 is that the government can use the curvature of the tax function to boost wages of low-skilled workers. This is achieved by setting a tax schedule that is concave at the bottom of the income distribution. A more concave tax schedule, i.e., a decrease in the second derivative of the tax function, positively affects wages: see Proposition 1. As explained before, declining marginal tax rates make it attractive for individuals to work longer hours following an increase in the hourly wage. A high elasticity of labor supply, in turn, induces the monopsonist to pay high wages as well, cf. equation (4). A government that is interested in redistribution can exploit this feature to raise the wages of low-skilled workers, and finds it optimal to do so.

At the optimal tax system, the monopsonist does not extract any rents from hiring the least productive workers. Hence, despite that there is a single monopsonist, these workers get paid a wage equal to their productivity. The government can guarantee this is the case by setting the curvature of the tax function at the bottom of the income distribution in such a way that labor supply of the least-skilled workers becomes infinitely elastic: $e_{lw}(n_0) \to \infty$. Naturally, doing so requires that marginal tax rates at the bottom are declining in income: $T''(z(n_0)) < 0$.

[6] It can be verified that no solution exists where $v(n)$ and $c(n)$ do not vary with ability and hence, $\lambda(n) = 0$ for all n. This would require $b(n) = 0$ for all n, in which case $l(n)$ is constant as well, cf. equation (11). Equation (15) and the condition $\mu(n_1) = 0$ then imply $\mu(n) = 0$ for all n. From equation (16) and the transversality conditions $\chi(n_0) = \chi(n_1) = 0$ it follows that $\chi(n) \geq 0$ with a strict equality only at the end-points. Equation (17) then contradicts the requirement that the multiplier $\psi(n) \geq 0$.

The finding that optimal marginal tax rates are declining in income is a local result: it holds at the bottom of the income distribution. Unfortunately, using the mechanism design approach to derive optimal tax rules that that hold at each point in the income distribution and that can be meaningfully interpreted turns out to be particularly challenging. The next section attempts to derive such a result using an alternative approach to solve the optimal tax problem.

4 Optimal tax formula

The optimal tax problem can also be solved using the tax perturbation approach. See Saez (2001) and Golosov et al. (2014), among many others. The idea behind this approach is to study a perturbation, or reform of the nonlinear tax schedule $T(\cdot)$. Such a reform induces welfare-relevant effects. Optimal tax formulas can then be derived from the requirement that if the tax schedule is optimized, the welfare-relevant effects of the reform sum to zero.

Before studying a particular tax reform, it is useful to restate the labor market equilibrium conditions. In modified form, equations (2) and (4) can be written as

$$u'(z - T(z) - \alpha)z(1 - T'(z) - \beta) = \phi'(l)l, \tag{18}$$

$$n\left[u''(z - T(z) - \alpha)z(1 - T'(z) - \beta)^2 + \frac{\phi'(l)l}{z} - u'(z - T(z) - \alpha)z(T''(z) + \gamma)\right] = \phi'(l) + \phi''(l)l. \tag{19}$$

The first of these is obtained from multiplying equation (2) by l and the second from combining equations (4)–(5), using the property $z = w \times l$. These equations pin down equilibrium labor effort l and earnings z as a function of ability n and the reform parameters α, β and γ. These reform parameters can be used to study the impact of a local increase in the level, slope and curvature of the tax function, respectively. To illustrate, the effect of the tax curvature on earnings and labor effort is given by $dz/d\gamma$ and $dl/d\gamma$. The behavioral responses on equilibrium earnings and labor effort can be obtained by implicitly differentiating equations (18)–(19), evaluated at $\alpha = \beta = \gamma = 0$.[7] The effects on profits and wages, in turn, follow from the relationships $\pi = nl - z$ and $w = z/l$.

An important advantage of the tax perturbation approach is that it allows for a derivation of optimal tax formulas in terms of sufficient statistics (Chetty, 2009). In the current setting, these are: i) the income distribution, ii) behavioral responses and iii) welfare weights. Starting with the first, let $H(z)$ denote the cumulative distribution of earnings, with corresponding density $h(z)$. Monotonicity of labor earnings $z'(n) \geq 0$ implies that the distributions of earnings and ability are related through $H(z(n)) = F(n)$ and hence, $h(z(n))z'(n) = f(n)$. The behavioral responses, in turn, capture how an increase in the level, slope or curvature of the tax function affect labor market outcomes. With a slight abuse of notation, I denote by y_x the impact of a local increase in $x \in \{T, T', T''\}$ on outcome $y \in \{z, l, w, \pi\}$. As explained before, these behavioral responses can be obtained by implicitly differentiating equations (18)–(19) with respect to α, β and γ respectively, and using the relationships $\pi = nl - z$ and $w = z/l$.

[7] When implicitly differentiating equations (18)–(19), it is important to take the dependency of $T(z)$, $T'(z)$ and $T''(z)$ on earnings z into account. The behavioral responses then capture the changes along the actual tax schedule, taking into account its higher-order derivatives. See Jacquet et al. (2013) for a discussion of this issue.

Lastly, the welfare weight of an individual with earnings z is

$$g(z) = \frac{1}{\eta}\Psi'(v(\hat{n}(z)))u'(z - T(z)), \tag{20}$$

where η is the multiplier on the government budget constraint and $\hat{n}(z)$ denotes the ability level that corresponds to earnings z. In words, the welfare weight $g(z)$ measures by how much social welfare increases if an individual with earnings z receives an additional unit of after-tax income. These weights summarize in a reduced-form way the government's preferences for redistribution. Because both $\Psi(\cdot)$ and $u(\cdot)$ are concave, the welfare weights are declining in income z.

Figure 1 graphically illustrates the reform that is used to derive an optimal tax formula. The black, dotted line shows the original tax schedule. For simplicity, it is drawn as a straight line. The red, solid line shows the perturbed tax schedule after the reform is implemented. Below earnings z', the two tax functions are the same. In the small interval $[z', z' + \zeta]$, the government increases the curvature (i.e., the second derivative) of the tax function by dT''. This is shown by the convex part of the solid line. Following the increase in the tax curvature, the marginal tax rates above earnings $z' + \zeta$ increase by an amount equal to $dT' = dT''\zeta$. As a result, the perturbed tax schedule is steeper than the original tax schedule: see Figure 1. In the interval $[z' + \delta, z' + \delta + \zeta]$ with $\delta \gg \zeta$, the government reverses the increase in the curvature by lowering the second derivative of the tax function by dT''. This is shown by the concave part of the solid line. Following this reversal, the marginal tax rates of the perturbed and original tax schedule are the same at earnings above $z' + \delta + \zeta$: the dotted and solid line are parallel. However, the reform does increase the tax burden for individuals with earnings above this level by an amount equal to $dT = dT'\delta = dT''\zeta\delta$.[8]

To keep track of the welfare-relevant effects associated with the tax reform, it is useful to write the Lagrangian of the government's optimization problem

$$\mathscr{L} = \int_{z_0}^{z_1} \Big[\Psi(u(z - T(z)) - \phi(l)) + \eta\big(T(z) + \pi - G\big)\Big] h(z)dz, \tag{21}$$

where $z_0 = z(n_0)$ and $z_1 = z(n_1)$. Note that labor effort l and profits π vary along the earnings distribution as well and that earnings, labor effort and profits all depend on the level, slope and curvature of the tax function. The first part of the Lagrangian (21) states the objective, integrated over the income (as opposed to the ability) distribution. The second part captures the government budget constraint, again integrated over the income distribution, with associated multiplier η.

The reform graphically illustrated in Figure 1 generates three types of welfare-relevant effects. In what follows, I discuss each of these in turn and then turn to derive and explain the optimal tax formula. First, there are behavioral responses due to a change in the curvature of the tax function in the small

[8]While Figure 1 graphically illustrates the reform, it is also possible to give a formal definition. Let $R(z)$ denote the difference between the perturbed and the original tax schedule. This reform function is given by

$$R(z) = \begin{cases} 0 & \text{if } z \le z', \\ \frac{1}{2}dT''(z - z')^2 & \text{if } z \in (z', z' + \zeta], \\ -\frac{1}{2}dT''\zeta^2 + dT''\zeta(z - z') & \text{if } z \in (z' + \zeta, z' + \delta], \\ -\frac{1}{2}dT''\zeta^2 + dT''\zeta(z - z') - \frac{1}{2}dT''(z - (z' + \zeta))^2 & \text{if } z \in (z' + \delta, z' + \delta + \zeta], \\ dT''\zeta\delta & \text{if } z > z' + \delta + \zeta. \end{cases}$$

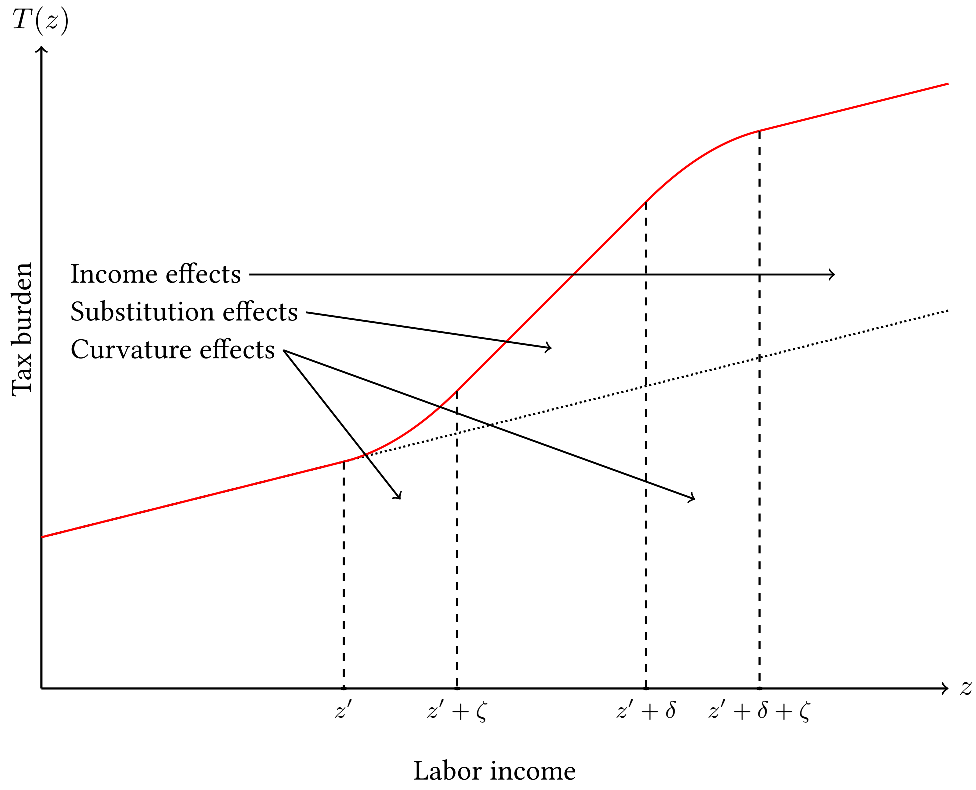

Figure 1: Tax reform and behavioral responses

intervals $[z', z' + \zeta]$ and $[z' + \delta, z' + \delta + \zeta]$. In Figure 1, these are labeled 'curvature effects'. Ignoring function arguments to save on notation, the welfare impact of raising the curvature in the interval $[z', z' + \zeta]$ by an amount equal to dT'' is

$$\int_{z'}^{z'+\zeta} \left[\Big(\Psi' \big(u'(1 - T') z_{T''} - \phi' l_{T''} \big) + \eta \big(T' z_{T''} + \pi_{T''} \big) \Big) h \right] (z) dz \times dT''. \tag{22}$$

Here, the entire function between square brackets is evaluated at earnings z. To understand this expression, note that a change in the tax curvature affects earnings z, labor effort l and profits π for individuals with earnings in the interval $[z', z' + \zeta]$. The impact on the Lagrangian is then obtained by integrating $\frac{dL}{dT''} \times dT'' \times h$ from z' until $z' + \zeta$, where L is the term in square brackets below the integral sign of equation (21). Working out $\frac{dL}{dT''}$ and accounting for the impact of tax curvature on earnings z, labor effort l and profits π gives the result from equation (22).

Equation (22) can be simplified in a number of steps. First, use equation (2) to substitute out for $\phi' = u' w (1 - T')$. Second, the relationship $z = w \times l$ implies $z_{T''} = w_{T''} l + w l_{T''}$ and hence, $w_{T''} l = z_{T''} - w l_{T''}$. Third, a local increase in the curvature of the tax function has no impact on profits: $\pi_{T''} = 0$.[9] Fourth, use the definition of the welfare weight (20). Rearranging gives

$$\eta \times \int_{z'}^{z'+\zeta} \left[\Big(T' z_{T''} + g(1-T') l w_{T''} \Big) h \right](z) dz \times dT''. \tag{23}$$

By analogous reasoning, the welfare impact of a reduction in the curvature of the tax function in the small interval $[z'+\delta, z'+\delta+\zeta]$ by an amount equal to dT'' is

$$-\eta \times \int_{z'+\delta}^{z'+\delta+\zeta} \left[\Big(T' z_{T''} + g(1-T') l w_{T''} \Big) h \right](z) dz \times dT''. \tag{24}$$

Second, the tax reform raises the marginal tax rate in the interval $[z'+\zeta, z'+\delta]$: see Figure 1 and recall that $\delta \gg \zeta$. A higher marginal tax rate generates behavioral responses on earnings z, labor effort l and profits π. The impact on welfare associated with these 'substitution effects' is

$$\int_{z'+\zeta}^{z'+\delta} \left[\Big(\Psi' \big(u'(1-T') z_{T'} - \phi' l_{T'} \big) + \eta \big(T' z_{T'} + \pi_{T'} \big) \Big) h \right](z) dz \times \zeta dT''. \tag{25}$$

This expression is obtained by integrating $\frac{dL}{dT'} \times dT' \times h$ from $z'+\zeta$ until $z'+\delta$, where L is the term in square brackets below the integral sign of equation (21), as before. The term $\frac{dL}{dT'}$ accounts for the equilibrium responses of earnings z, labor effort l and profits π to a change in the marginal tax rate. To arrive at equation (25), note that the reform increases the marginal tax rate for individuals with earnings in the interval $[z'+\zeta, z'+\delta]$ by an amount equal to $dT' = \zeta dT''$.

The above expression can be simplified in a similar way as before. In particular, use the property $\phi' = u' w (1-T')$ and the definition of the welfare weight (20). Moreover, the relationship $z = w \times l$ implies that $z_{T'} = w_{T'} l + w l_{T'}$. Substituting this in equation (25) gives

$$\eta \times \int_{z'+\zeta}^{z'+\delta} \left[\Big(T' z_{T'} + \pi_{T'} + g(1-T') l w_{T'} \Big) h \right](z) dz \times \zeta dT''. \tag{26}$$

Third, the reform increases the tax burden for individuals with earnings above $z'+\delta+\zeta$ by an amount $dT = dT'\delta = dT''\zeta\delta$. This generates two types of welfare-relevant effects. First, the reform mechanically transfers income from individuals with earnings above this level to the government budget. Second, a change in the tax burden generates behavioral responses on earnings z, labor effort l and profits π. Figure 1 labels these 'income effects'. The total impact on welfare can be found by integrating $\frac{dL}{dT} \times dT \times h$ for earnings above $z'+\delta+\zeta$:

$$\int_{z'+\delta+\zeta}^{z_1} \left[\Big(\big(-\Psi' u' + \eta \big) + \Psi' \big(u'(1-T') z_T - \phi' l_T \big) + \eta \big(T' z_T + \pi_T \big) \Big) h \right](z) dz \times \zeta\delta dT'', \tag{27}$$

where the increase in the tax burden is $dT = \zeta\delta dT''$. Note that, when computing $\frac{dL}{dT}$ from equation (21), there is both a mechanical effect (first term) as well as behavioral responses (other terms) to an

[9] To see this, note that profits can be written as

$$\pi = \max_{l,z} \Big\{ nl - z \quad \text{s.t.} \quad u'(z - T(z) - \alpha) z (1 - T'(z) - \beta) = \phi'(l) l \Big\},$$

where I introduced the reform parameters α and β in the constraint (2). A change in any of these generally has an impact on profits. However, because γ does not show up, a local increase in the curvature does not affect profits.

increase in the tax burden. To simplify this expression, use the relationship $\phi' = u'w(1-T')$, the expression for the welfare weight (20) and the property $z = w \times l$. Differentiating the latter with respect to T gives $z_T = w_T l + w l_T$. Equation (27) can then be written as

$$\eta \times \int_{z'+\delta+\zeta}^{z_1} \left[\Big(1 - g + T' z_T + \pi_T + g(1-T') l w_T \Big) h \right](z) dz \times \zeta \delta dT''. \tag{28}$$

If the tax system is optimized, any reform should have no impact on social welfare. Put differently, the sum of the three welfare-relevant effects associated with the reform graphically illustrated in Figure 1 is equal to zero. This leads to the following result.

Proposition 3. *At the optimal tax system, the following condition must hold at each point $z' \in [z_0, z_1]$ in the income distribution:*

$$\begin{aligned}
0 = &- \frac{d}{dz} \left[\Big(T'(z') z_{T''}(z') + g(z')(1 - T'(z')) \hat{l}(z') w_{T''}(z') \Big) h(z') \right] \\
&+ \Big(T'(z') z_{T'}(z') + \pi_{T'}(z') + g(z')(1 - T'(z')) \hat{l}(z') w_{T'}(z') \Big) h(z') \\
&+ \int_{z'}^{z_1} \Big(1 - g(z) + T'(z) z_T(z) + \pi_T(z) + g(z)(1 - T'(z)) \hat{l}(z) w_T(z) \Big) h(z) dz.
\end{aligned} \tag{29}$$

Here, it is explicitly taken into account which terms vary along the income distribution and $\hat{l}(z)$ denotes the labor effort associated with earnings z.

Proof. Add equations (23), (24), (26) and (28) and set the resulting expression equal to zero. Dividing by $\eta \times \zeta \times dT''$ gives

$$\begin{aligned}
&\frac{1}{\zeta} \int_{z'}^{z'+\zeta} \left[\Big(T' z_{T''} + g(1-T') l w_{T''} \Big) h \right](z) dz - \frac{1}{\zeta} \int_{z'+\delta}^{z'+\delta+\zeta} \left[\Big(T' z_{T''} + g(1-T') l w_{T''} \Big) h \right](z) dz \\
&+ \int_{z'+\zeta}^{z'+\delta} \left[\Big(T' z_{T'} + \pi_{T'} + g(1-T') l w_{T'} \Big) h \right](z) dz \\
&+ \int_{z'+\delta+\zeta}^{z_1} \left[\Big(1 - g + T' z_T + \pi_T + g(1-T') l w_T \Big) h \right](z) dz \times \delta = 0.
\end{aligned} \tag{30}$$

Next, take the limit as $\zeta \to 0$:

$$\begin{aligned}
&\left[\Big(T' z_{T''} + g(1-T') l w_{T''} \Big) h \right](z') - \left[\Big(T' z_{T''} + g(1-T') l w_{T''} \Big) h \right](z'+\delta) \\
&+ \int_{z'}^{z'+\delta} \left[\Big(T' z_{T'} + \pi_{T'} + g(1-T') l w_{T'} \Big) h \right](z) dz \\
&+ \int_{z'+\delta}^{z_1} \left[\Big(1 - g + T' z_T + \pi_T + g(1-T') l w_T \Big) h \right](z) dz \times \delta = 0.
\end{aligned} \tag{31}$$

The first term in square brackets is evaluated at earnings z' and the second term is evaluated at earnings

$z' + \delta$. To proceed, divide equation (31) by δ and rearrange

$$
\begin{aligned}
&-\frac{\big[(T' z_{T''} + g(1-T') l w_{T''})h\big](z'+\delta) - \big[(T' z_{T''} + g(1-T') l w_{T''})h\big](z')}{\delta} \\
&+ \frac{1}{\delta}\int_{z'}^{z'+\delta} \left[\Big(T' z_{T'} + \pi_{T'} + g(1-T') l w_{T'}\Big)h\right](z)dz \\
&+ \int_{z'+\delta}^{z_1} \left[\Big(1 - g + T' z_T + \pi_T + g(1-T') l w_T\Big)h\right](z)dz = 0.
\end{aligned} \tag{32}
$$

Taking the limit as $\delta \to 0$:

$$
\begin{aligned}
&-\frac{d}{dz}\left[\Big(T' z_{T''} + g(1-T') l w_{T''}\Big)h\right](z') + \left[\Big(T' z_{T'} + \pi_{T'} + g(1-T') l w_{T'}\Big)h\right](z') \\
&+ \int_{z'}^{z_1} \left[\Big(1 - g + T' z_T + \pi_T + g(1-T') l w_T\Big)h\right](z)dz = 0.
\end{aligned} \tag{33}
$$

The first term is the derivative of the expression in square brackets with respect to earnings, evaluated at z'. To arrive at the result from Proposition 3, note that equation (33) must hold for a reform considered at each income level $z' \in [z_0, z_1]$. As a final step, equation (29) makes explicit which terms vary across the earnings distribution. □

Equation (29) gives an optimal tax formula in terms of sufficient statistics that holds at each point in the income distribution. To understand the link with Figure 1, recall that the reform increases the tax curvature below a particular income level and decreases it right above. This reform leads to a local increase in the marginal tax rate and an increase in the tax burden for all individuals with earnings above this level. The first term of equation (29) captures the difference between the welfare effects of increasing the tax curvature and subsequently decreasing it. Upon making the intervals arbitrarily small, this essentially boils down to taking a derivative. The second term captures the welfare-relevant effects associated with a local increase in the marginal tax rate. Naturally, these effects are proportional to the density of the income distribution at the point where the marginal tax rate is increased. Lastly, the third term integrates the welfare impact of a higher tax burden over all individuals who see their tax burden increase as a result of the reform.

It is useful to contrast the result from Proposition 3 with the optimal tax formula that holds if labor markets are competitive. See, for instance, Saez (2001) and Golosov et al. (2014). Under perfect competition, labor market outcomes are not affected by a local increase in the curvature of the tax function: $z_{T''} = w_{T''} = 0$. Moreover, the wage of an individual with ability n is $w(n) = n$ and firms make zero profits: $\pi(n) = 0$. Consequently, wages and profits do not respond to a change in the level or slope of the tax function: $w_T = w_{T'} = \pi_T = \pi_{T'} = 0$. Equation (29) then becomes:

$$
0 = T'(z') z_{T'}(z') h(z') + \int_{z'}^{z_1} \Big(1 - g(z) + T'(z) z_T(z)\Big) h(z) dz. \tag{34}
$$

Apart from differences in presentation, this optimal tax formula coincides with the one derived in Saez (2001) and Golosov et al. (2014), among others.

Compared to the competitive benchmark, the additional 'sufficient statistics' that show up in the optimal tax formula (29) are the effects of tax curvature on earnings and hourly wages and the effects of the tax burden and the marginal tax rate on hourly wages and profits. Intuitively, the additional responses of earnings and profits to tax changes have budgetary effects, as the government taxes both labor income and profits. In equation (29), the budgetary effects are captured by $T' z_{T''}$, $\pi_{T'}$ and π_T. By contrast, the wage responses $w_{T''}$, $w_{T'}$ and w_T have an effect on individuals' disposable incomes. This effect is proportional to l, the number of hours worked, and the after-tax rate $1 - T'$. Because changes in disposable income affect individual utilities, the welfare effects associated with wage changes in equation (29) are weighted by the individual welfare weights g.

Unfortunately, implementing the optimal tax formula (29) using estimates of sufficient statistics is a very challenging task. To the best of my knowledge, there are no estimates available of the impact of tax curvature on labor market outcomes. Moreover, to implement the optimal tax formula (29), one also requires knowledge of how these statistics vary across the earnings distribution. The result does, however, make clear what forces shape optimal policy and which statistics determine the welfare impact of tax reforms if labor market outcomes depend on the curvature of the tax function.

5 Conclusion

If a monopsonist sets hourly wages and individuals choose how many hours to work, labor market outcomes do not only depend on the level and the slope of the tax function, but also on its curvature. I use that insight to obtain the following three results. First, a local increase in the curvature (i.e., the second derivative) of the tax function reduces the hourly wage, hours worked and labor earnings. Intuitively, a more convex or less concave tax schedule lowers the elasticity of labor supply, which induces the monopsonist to pay lower wages. Second, the optimal tax schedule is concave at the bottom of the income distribution. Declining marginal tax rates at the bottom make low-skilled workers more responsive to wage changes, which leads the monopsonist to pay higher wages. Third, I derive an optimal tax formula that accounts for the impact of tax curvature on labor market outcomes. Compared to existing results in the literature, the additional 'sufficient statistics' that characterize the optimal tax system are the impact of tax curvature on earnings and hourly wages, and the impact of the tax burden and the marginal tax rate on hourly wages and profits.

How important the effects of tax curvature are on labor market outcomes is an open question, but one that can be investigated empirically. One approach would be to construct a measure of tax curvature $T''(z)$ throughout the income distribution, which can then be used as an explanatory variable in a regression framework with wages, hours worked or labor earnings as the dependent variable.[10] Another approach would be to regress any of these outcomes on (instrumented) measures of an individual's own tax burden, marginal tax rate *and* the marginal tax rate an individual would face if the individual's earnings increase or decrease. The model from this paper predicts that a higher (lower) marginal tax rate of one's 'neighbor' in the income distribution with slightly higher (lower) earnings negatively affects wages, hours worked and labor earnings. I leave an empirical investigation of this issue as a topic for future research.

[10] If the tax system is piecewise linear, $T''(z)$ is not defined everywhere. One can, however, calculate $\frac{T'(z_{n+1})-T'(z_n)}{z_{n+1}-z_n}$ for two earnings levels z_{n+1} and z_n close to each other, without taking the limit as $z_{n+1} - z_n \to 0$.

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A Monotonicity of labor earnings

This appendix first demonstrates that monotonicity of labor earnings, i.e., $z'(n) \geq 0$, implies the second-order conditions in both the profit and utility maximization problem are satisfied. To do so, note that the profit maximization problem can be written as

$$\pi(n) = \max_{l,z} \Big\{ nl - z \quad \text{s.t.} \quad u'(z - T(z))z(1 - T'(z)) = \phi'(l)l \Big\}. \tag{35}$$

Here, the constraint is obtained by multiplying both sides of equation (2) by l and using the relationship $z = w \times l$. Inverting this constraint gives $l = \tilde{l}(z)$. Substituting this in the objective results in an unconstrained maximization problem

$$\pi(n) = \max_{z} \Big\{ n\tilde{l}(z) - z \Big\}. \tag{36}$$

The first-order condition is

$$n\tilde{l}'(z) - 1 = 0. \tag{37}$$

The second-order condition is $n\tilde{l}''(z) \leq 0$. Upon working out $\tilde{l}'(z)$ using the implicit function theorem on the constraint in equation (35), equations (37) and (4) coincide.

Equation (37) determines equilibrium labor earnings $z(n)$ as a function of ability. By the implicit function theorem,

$$z'(n) = -\frac{\tilde{l}'(z)}{n\tilde{l}''(z)} = -\frac{1}{n^2\tilde{l}''(z)}. \tag{38}$$

Consequently, $z'(n) \geq 0$ if and only if the second-order condition $n\tilde{l}''(z) \leq 0$ is satisfied. Monotonicity of labor earnings $z(n) = nl(n) - \pi(n)$, in turn, implies that $z'(n) = nl'(n) + l(n) - \pi'(n) = nl'(n) = nb(n) \geq 0$, where I use the incentive constraint (10). Therefore, if $b(n) \geq 0$, it follows that labor earnings are monotone in ability and the first-order conditions in the profit maximization problem are both necessary and sufficient.

Next, consider the utility maximization problem. The individual's first-order condition (2) is sufficient if the objective (1) is concave in hours worked. This is the case if

$$u''w^2(1 - T')^2 - u'w^2T'' - \phi'' \leq 0, \tag{39}$$

where I ignore function arguments to save on notation. To demonstrate that the first-order conditions of the profit maximization problem imply this condition holds, write the first-order condition of the utility maximization problem as

$$\Gamma(w,l) = u'(wl - T(wl))w(1 - T'(wl)) - \phi'(l) = 0. \tag{40}$$

Next, combine the first-order conditions of the profit maximization problem (3) with respect to the wage w and hours worked l to find

$$(n - w)\Gamma_w(w,l) + l\Gamma_l(w,l) = 0. \tag{41}$$

Working out $\Gamma_w(w,l)$ and $\Gamma_l(w,l)$ gives

$$(n - w)(u''wl(1 - T')^2 + u'(1 - T') - u'wlT'') + l(u''w^2(1 - T')^2 - \phi'' - u'w^2T'') = 0. \tag{42}$$

Using the relationship $u'w(1 - T') = \phi'$ and collecting terms,

$$nu'wlT'' = n(u'(1 - T') + u''wl(1 - T')^2) - \phi' - \phi''l. \tag{43}$$

To proceed, multiply the final expression by $w/(nl)$ and again use $u'w(1 - T') = \phi'$,

$$u'w^2T'' = \frac{\phi'}{l} + u''w^2(1 - T')^2 - \frac{\phi'w}{nl} - \frac{\phi''w}{n}. \tag{44}$$

Substitute this equation in the second-order condition for the utility maximization problem (39). Simplifying gives

$$-\phi'' - \frac{\phi'}{l} + \frac{\phi'w}{nl} + \frac{\phi''w}{n} = \left(\frac{w}{n} - 1\right)\left(\phi'' + \frac{\phi'}{l}\right) \leq 0. \tag{45}$$

Because the wage is weakly below productivity and $\phi', \phi'' > 0$, it follows that the second-order condition for utility maximization is satisfied. To sum up, adding the requirement $b(n) \geq 0$ in the government's optimization problem guarantees that labor earnings are monotone in ability: $z'(n) \geq 0$. The latter, in turn, implies that the first-order conditions of the profit maximization problem are both necessary and sufficient. If that is the case, the second-order condition for the utility maximization problem is satisfied as well.