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Equity Risk Factors for the Long and Short Run: Pricing and Performance at Different Frequencies

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ABSTRACT

We find that the outperformance for Fama-French factors compared to macroeconomic factors in terms of fitting the cross-section of expected returns disappears when accounting for horizon effects. In addition, we obtain novel empirical relations between macroeconomic factors and Fama-French factors at longer horizons. To obtain our results, we introduce a general linear multifactor asset pricing methodology that integrates systematic risk measured at different frequencies into a single pricing equation. Our setup allows for a setting where investors with different investment horizons may experience different levels of systematic risk, which could arise from delayed stock price reaction to systematic factor news.

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Factor investing is a highly relevant topic for investors as well as academics. Beardsley, Donnadieu, Sheridan, and Xu (2018) report that smart beta products[1] represent a global assets under management (AUM) of 430 billion USD. Smart beta AUM has been growing at a rate of 30% per year since 2012, while this was only 13% for standard passive strategies. Common factors, which typically form the basis of smart beta strategies, explain a large part of the time-series and cross-sectional variation in stock returns and offer sizable risk premia.[2] Institutional investors use these factors for both portfolio construction and performance evaluation (e.g. Dahlquist and Ødegaard, 2018).

There is extensive evidence suggesting a delay in reaction of certain stock prices to news about systematic factors. As a consequence of such delays, investors with different investment horizons may experience different levels of systematic risk (Kamara, Korajczyk, Lou, and Sadka, 2016). Factor models are often estimated using a monthly observation frequency. The investment horizon, however, varies across investors and does not need to be equal to the estimation horizon. For example, Lan, Moneta, and Wermers (2021) find a range of holding periods between 1 year and 4.5 years for US-domiciled actively managed equity mutual funds. To appropriately measure the relation between systematic factors and asset prices thus requires an integrated framework that allows for horizon effects.[3]

[1]Smart beta strategies can be characterized as simple, rules-based, and transparent strategies that have their conceptual origin in the Ross (1976) factor investing framework (Kahn and Lemmon, 2016).

[2]Fama and French (1993) and Asness, Moskowitz, and Pedersen (2013).

[3]Kamara, Korajczyk, Lou, and Sadka (2016) also note that horizon pricing effects could alternatively arise since investors may choose to trade infrequently when monitoring and trading costs are sufficiently large. See Bandi and Tamoni (2017), Dew-Becker and Giglio (2016) for additional reasons why horizon pricing effects may arise.

This paper contributes to the literature by introducing a general linear multifactor asset pricing methodology that integrates systematic risk measured at different frequencies into a single pricing equation. We achieve this by applying a frequency domain decomposition to the stochastic discount factor.[4] Our flexible setup applies to all linear multifactor asset pricing models, works with any frequency filter that produces orthogonal additive components, and allows us to investigate asset pricing relations across frequencies in a multifactor setting. We explore two applications by performing empirical tests within our setup on macroeconomic and Fama-French type factor models. These empirical tests leverage our frequency-specific asset pricing framework to obtain several novel insights into both factor pricing and model performance for these factor models.

The macroeconomic factors that we study are the term spread, expected and unexpected inflation, industrial production, and the default spread as in Chan, Chen, and Hsieh (1985) and Chen, Roll, and Ross (1986). The Fama-French type factors that we consider are the market, size, value, momentum, and liquidity factors as defined in Fama and French (1993), Carhart (1997), and Pastor and Stambaugh (2003).

We use the Christiano and Fitzgerald (2003) band-pass filter to isolate fluctuations corresponding to different frequency ranges. Band-pass filtering is the standard approach in the macroeconomics literature to study business cycle behavior. We consider a range of frequency bands to obtain an overview of the behavior of the different factors at different horizons. An advantage over compounding returns and aggregating factors over time,[5] is that filtering allows us to separate and

[4]Frequency domain techniques have been used previously in a similar setting by Dew-Becker and Giglio (2016) and Neuhierl and Varneskov (2021), for example.

[5]As in, for example, Cooper, Gulen, and Schill (2008) and Kamara, Korajczyk, Lou, and Sadka (2016).

describe the behavior for each horizon in isolation, as well as specifically identify factor horizon pricing consequences within the context of a single pricing equation.

Our empirical analysis uses a large cross-section of equity returns and contains monthly data for US equity portfolios. The test assets are 202 portfolios consisting of bivariate sorted portfolios and a set of industry portfolios which capture many cross-sectional anomalies (Giglio and Xiu, 2018; Feng, Giglio, and Xiu, 2020). Our sample period runs from January 1968 to December 2016. Beyond studying monthly data, we apply band-pass filtering to obtain fluctuations at wave lengths lower than 1 year (short-term), 1 to 3 years (shorter business cycles), 3 to 8 years (longer business cycles), and longer than 8 years (long-term trends). The cycle lengths for the business cycles are in line with Burns and Mitchell (1946) and Comin and Gertler (2006), and the long-term trends cycle length is based on Haug and Dewald (2004).

The frequency-specific asset pricing framework allows us to uncover a range of new findings regarding Fama-French type and macroeconomic factors.[6] Our asset pricing results show that the frequency-specific specification outperforms the monthly specification for both types of factor models. In addition, the superiority of the Fama-French factors compared to macroeconomic factors in terms of cross-sectional fit disappears when accounting for horizon effects.[7] Moreover,

[6]Surprisingly, although horizon effects have been investigated for Fama-French type factors (Adrian and Rosenberg (2008), Bandi, Garcia, Lioui, and Perron (2010), Bandi and Tamoni (2017), Boons and Tamoni (2015), Brennan and Zhang (2020), Cosemans (2011), Neuhierl and Varneskov (2021), and in particular Kamara, Korajczyk, Lou, and Sadka (2016).) little is known about how macroeconomic factors behave at different horizons. Moreover, as suggested by Andersen, Bollerslev, Diebold, and Wu (2005), interesting relations may exist between Fama-French type factors and macroeconomic factors depending on the frequency that we consider.

[7]The superior performance of the Fama-French model estimated on monthly data has been documented previously by Asness, Moskowitz, and Pedersen (2013) and Chan, Karceski, and Lakonishok (1998). Our finding is line with Fama (1981, 1990), who finds that the explanatory power of macroeconomic variables increases with the time horizon.

we find that different factors are priced at different frequencies and that risk premia differ across frequencies.

Our data analysis provides additional context to our findings. For the Fama-French factors, 75% of the variation falls within the short-term one-year pass band. In contrast, for some macroeconomic factors more than 30% of the variation falls within the long-term trend pass band of 8 years and longer. This stark difference provides a suggestion regarding the source of model performance difference at the monthly estimation frequency. As the horizon increases, the correlations between Fama-French type and macroeconomic factors increase from close to zero to absolute magnitudes of 0.4 to 0.6. This suggests that the factors contain more similar information at longer horizons, which resonates with the disappearance of the model performance differential when accounting for frequency pricing effects.

The result that the value factor is priced at the business cycle horizon is in line with Zhang (2005) and Petkova and Zhang (2005), who show that value stocks are riskier during downturns, and less risky during good times. In addition. Our findings shed additional light on Asness, Moskowitz, and Pedersen (2013), and show that when looking beyond the one-month horizon, macroeconomic factors could be a possible source driving common variation of value and momentum strategies.

Our results are robust to the filter version. We find comparable results when using the Christiano-Fitzgerald random walk filter, the Baxter-King filter, and a one-sided backward looking version of the Christiano-Fitzgerald random walk filter. The latter version shows that our results are robust to excluding forward-looking information from the filter.

The findings in this paper indicate that factor-based portfolio construction and performance evaluation should take into account the different behavior and pricing of factors at different horizons. As Cohen, Polk, and Vuolteenaho (2009) describe, exposure to factors on a long rather than a short horizon is relevant for managers making long-term investment decisions and for individuals following a buy-and-hold strategy, since they trade infrequently. Risk that appears in the short-term may not appear systematically in the long run for these investors. Long-term investors collect short-run risk premia while bearing less of the risk associated with these short-term factors in the long run, as illustrated in Kamara, Korajczyk, Lou, and Sadka (2016).

The remainder of the paper is organized as follows. Section I discusses research related to our approach to study factors at different frequencies. Section II presents the theoretical framework of our multifactor frequency-specific representation and Section III sets out the empirical asset pricing methodology. Section IV describes the data and Section V discusses return and factor behavior across frequencies. Section VI presents our asset pricing results and Section VII concludes.

I. Related Literature

Our paper contributes to existing literature that shows that the return measurement interval matters for the exposure, pricing, and explanatory power of different risk factors. Several papers investigate the properties and performance of equity factor models at different observation frequencies by compounding returns and aggregating factor data. Levhari and Levy (1977) and Handa, Kothari, and Wasley (1989) show that both factor exposures as well as price of risk estimates vary with the time horizon on which the CAPM is estimated.

A range of studies[8] provides further evidence of the sensitivity of beta estimates to the return intervals for different markets and different data frequencies. Handa, Kothari, and Wasley (1989), Kothari, Shanken, and Sloan (1995), and Campbell and Viceira (2002) advocate to consider longer horizon betas, as monthly and quarterly betas might either be affected by mispricing or not pick up delayed systematic shocks. Handa, Kothari, and Wasley (1993) show that the outcome of tests of asset pricing models can indeed be sensitive to the return measurement interval.

We use frequency filtering techniques that allow us to study asset return fluctuations at different horizons in isolation. One advantage of the frequency filtering approach is that it does not suffer from the disadvantage of using long-horizon compounded returns, since these still affected by large short-term shocks, such as the 2008 crisis. Another advantage is that it allows us to study horizon pricing effects within the context of a single asset pricing equation. The frequency filtering approach is standard in the macroeconomics literature and has received increasing attention in the finance literature.

Our paper contributes to an increasing body of papers using frequency filtering approaches. For example, Goldberg and Vora (1978) show that the CAPM market beta varies substantially across different cyclical components. Ortu, Tamoni, and Tebaldi (2013) propose a component-wise decomposition of time series based on the level of persistence to analyze long-run risk. Boons and Tamoni (2015) use the framework of Ortu, Tamoni, and Tebaldi (2013) and find that macroeconomic growth risk is a key determinant of variation in expected returns at the four-year horizon. Bandi and Tamoni (2017) represent the beta of an asset as a linear combination of frequency-specific

[8]Brailsford and Faff (1997), Brailsford and Josev (1997), Cohen, Maier, and Schwartz (1986), Frankfurter, Leung, and Brockman (1994), Gilbert, Hrdlicka, Kalodimos, and Siegel (2014), and Hawawini (1983).

betas and show that the business cycle component (2 to 4 years) can explain the cross-section of equity portfolios. Bandi, Chaudhuri, Lo, and Tamoni (2021) use a spectral factor model approach to show that traditional factor models restrict betas to be constant over frequencies, while this can hide horizon-specific pricing effects. In contrast to these papers, our approach works with any filter that produces orthogonal additive components, does not depend on assumptions regarding the state variable process, and applies to all linear multifactor asset pricing models.

The frequency filtering approach that we use is related to papers that study time variation in betas, which is empirically relevant for multifactor models (Ferson, 2013). For example, Andersen, Bollerslev, Diebold, and Wu (2006) and Lewellen and Nagel (2006) use a rolling window regression to incorporate time variation in beta estimates. These rolling window regressions may be seen as particular types of filters in light of Ang and Kristensen (2012), who develop a nonparametric kernel estimation approach to estimate time-varying parameters in conditional factor models. This method allows for various kernels, with rolling window estimation being the equal-weighted one-sided filter case of this estimator. They find that estimates obtained from a two-sided symmetric kernel fit result in a better fit than backward-looking filters. Our spectral approach is similar to the kernel approach of Ang and Kristensen (2012), while the kernel approach makes use of the time domain and our spectral approach makes use of the frequency domain.

Our paper is related to papers studying frequency-specific effects in the stochastic discount factor (SDF). Dew-Becker and Giglio (2016) use a frequency-specific SDF framework and derive frequency-specific prices of risk and show that low-frequency fluctuations in the economy are priced significantly. Neuhierl and Varneskov (2021) develop a model-free framework and show that frequency-specific effects are indeed present within the SDF.

II. Theoretical Framework

In this section, we introduce a general linear multifactor asset pricing methodology that integrates systematic risk measured at different frequencies into a single pricing equation. Our starting point is the standard linear multifactor asset pricing model[9]

$$\mathbb{E}[r_{i,t}] = \lambda' \beta_i, \tag{1}$$

where $r_{i,t}$ denotes the excess return on asset i at time t, β_i denotes the exposures of asset i to the set of factors, and λ denotes the prices of risk (see e.g. Cochrane, 2005). For this asset pricing model, we have the asset pricing equation

$$\mathbb{E}[m_{t+1} R_{i,t+1}] = 1, \tag{2}$$

where m_{t+1} denotes the SDF and $R_{i,t+1}$ the return on asset i at time t. Ross (1976) shows that there exists an SDF of the form

$$m_{t+1} = a + b' f_{t+1}, \tag{3}$$

that generates the pricing relation (1).[10]

To allow for frequency-specific pricing effects (as found by e.g. Neuhierl and Varneskov, 2021), we make use of a frequency filter that decomposes a covariance stationary time series y_t into a sum

[9] See, for example, Chen, Roll, and Ross (1986), Fama and French (1993, 2015), Baker and Wurgler (2006), and Ang, Hodrick, Xing, and Zhang (2006). Many potential factors have been identified to explain the cross-section of equity returns. Harvey, Liu, and Zhu (2016) report over 300 factors in various multifactor settings which attempt to explain the cross-section differences in returns, also referred to as "the factor zoo" (Cochrane, 2011).

[10] Specifically, $\beta_i = \text{var}(f_t)^{-1}\text{cov}(f_t, r_{i,t})$ and $\lambda = -\text{var}(m_t)/\mathbb{E}(m_t)$.

of orthogonal components capturing fluctuations from different frequencies. This provides the following decomposition.[11]

$$y_t = \sum_{\tau=1}^{\mathcal{T}} y_t^{(\tau)}, \tag{4}$$

where t denotes time, τ denotes frequency of component $y_t^{(\tau)}$, and $\mathcal{T}$ denotes the total number of non-overlapping frequency bands.[12] The higher the persistence of a component, the lower the frequency. Thus, the first component $y_t^{(1)}$ captures the high-frequency fluctuations whereas the last component $y_t^{(\mathcal{T})}$ captures the low-frequency fluctuations.

Using the orthogonal decomposition in (4), we decompose the excess returns $r_{i,t}$ and the factors f_t into orthogonal components. This results in the following orthogonal frequency decomposition,

$$r_{i,t} = \sum_{\tau=1}^{\mathcal{T}} r_{i,t}^{(\tau)} \qquad \text{and} \qquad f_t = \sum_{\tau=1}^{\mathcal{T}} f_t^{(\tau)}. \tag{5}$$

Since the frequency components are orthogonal, it holds that $\mathrm{cov}\big(r_{i,t}^{(\tau)}, f_t^{(\sigma)}\big) = 0$ for $\tau \neq \sigma$.

Making use of decompositions (5), we propose a frequency-specific representation of the SDF for linear multifactor asset pricing models.[13] This representation is linear in the horizon components and leads to a frequency-specific linear multifactor asset pricing equation. While our SDF

[11]The existence of the decomposition under the assumption of covariance stationarity can be shown using a spectral representation theorem or the Wold (1938) decomposition. See Internet Appendix I.A for further details.

[12]In a band-pass specification, the decomposition would be written as $y_t = \sum_{\tau=1}^{\mathcal{T}} y_t^{(p_{\tau-1}, p_\tau)}$, where $y_t^{(p_{\tau-1}, p_\tau)}$ denotes the component of y_t with period oscillations between $p_{\tau-1}$ and p_τ, with $p_0 = 0 \leq p_1 < \ldots < p_{\mathcal{T}} = \infty$. Here, $y_t^{(p_{\tau-1}, p_\tau)}$ corresponds to $y_t^{(\tau)}$ in the abbreviated notation of (4).

[13]This decomposition can be motivated by the frequency effects found in the SDF by Neuhierl and Varneskov (2021).

representation allows for horizon variation, it imposes no a priori structure. The proof of the following proposition is stated in Internet Appendix II.

PROPOSITION 1: *The frequency decomposition* (4) *allows the following linear, frequency-specific multifactor SDF,*

$$m_{t+1} = a + \sum_{\tau=1}^{\mathcal{T}} b^{(\tau)\prime} f_{t+1}^{(\tau)}. \tag{6}$$

This frequency-specific SDF corresponds to the pricing relation

$$\mathbb{E}[r_{i,t}] = \sum_{\tau=1}^{\mathcal{T}} \lambda^{(\tau)\prime} \beta_i^{(\tau)}, \tag{7}$$

where

$$\beta_i^{(\tau)} = \mathrm{Var}\left(f_{t+1}^{(\tau)}\right)^{-1} \mathrm{Cov}\left(f_{t+1}^{(\tau)},\ r_{i,t+1}^{(\tau)}\right) \tag{8}$$

denotes the frequency-specific regression betas of the frequency component of the excess return with respect to the frequency component of the factors and a constant, and $\lambda^{(\tau)}$ *denotes the frequency-specific prices of risk.*

III. Empirical Asset Pricing Applications

To investigate the type of insights that can be gained from our frequency-specific asset pricing method, we apply it to two well-known linear multifactor asset pricing models.

The models that we consider are:

- *Application 1*: the macroeconomic specification of Chen, Roll, and Ross (1986), with the monthly growth rate of industrial production (*IP*), the monthly change expected in expected inflation (*EI*), unexpected inflation (*UI*), the credit spread (*DEF*), and the term spread (*TS*) as factors.

- *Application 2*: a Fama-French type specification, which is an extension of the three-factor model of Fama and French (1993). The model has market (*MKT*), size (*SMB*), value (*HML*), momentum (*MOM*; Carhart, 1997), and a traded liquidity (*LIQ*; Pastor and Stambaugh, 2003) as factors.

Our choice for these models is motivated by the following aspects. By using these factor specifications, we include examples of macroeconomic factors (Application 1) as well as fundamental, technical, and market factors (Application 2). In this way, we cover all factor types, except statistical factors, as set out by Chan, Karceski, and Lakonishok (1998). In addition, by studying both Fama-French type factors and macroeconomic factors, we can shed additional light on the relation between both types of factors (as studied by Asness, Moskowitz, and Pedersen, 2013, and references therein).

We make use of Proposition 1 to obtain frequency-specific representations of these factor models. Since these representations still have a linear factor structure, we may apply the Fama and MacBeth (1973) procedure to test which of the factors in these models are priced. In the first step of this procedure, we use frequency-specific time-series regressions for each pass band τ and each portfolio i to obtain estimates of the frequency-specific exposures. It follows from Proposition 1 that the time-series step is the band spectrum regression of Engle (1974), where we use a different frequency filter. We see from (8) that we may estimate the frequency-specific betas[14] via

$$r_{i,t}^{(\tau)} = \beta_{0,i} + \beta_i^{(\tau)\prime} f_t^{(\tau)} + \varepsilon_{i,t}^{(\tau)}. \tag{9}$$

The second step of the procedure is to estimate the cross-sectional regression (7). Thus, for each time t we regress excess returns on all the frequency-specific estimated exposures of (9) for each horizon $\tau = 1, \ldots, \mathcal{T}$ and a constant. This leads to the following regression.

$$r_{i,t+1} = \lambda_{0,t} + \sum_{\tau=1}^{\mathcal{T}} \lambda_t^{(\tau)\prime} \widehat{\beta}_i^{(\tau)} + \eta_{i,t+1}, \tag{10}$$

where $\eta_{i,t+1}$ denotes the pricing error.

To compare the outcomes to the standard estimation setup, we also estimate (9) and (10) using unfiltered monthly data. We run these regressions using both static and rolling window estimation for the exposures. For the rolling window estimate, we estimate the conditional loadings $\widehat{\beta}_{i,t}$ for each month t based on the preceding W months and obtain $T - W$ time-varying exposure estimates. A common choice in the literature for monthly data is a window size of $W = 60$ months.[15]

When the estimated exposures are collinear, or when the exposure matrix is of reduced rank, the second stage may not be estimated correctly using the standard estimators (e.g., Kan and Zhang,

[14] That both the left-hand side and the right-hand side are taken within frequency band τ follows since by orthogonality of the frequency components it holds that $\operatorname{cov}\left(r_{i,t}^{(\tau)}, f_t^{(\sigma)}\right) = 0$ for $\tau \neq \sigma$. This is shown explicitly in the proof of Proposition 1.

[15] See, for example, Ang and Kristensen (2012), Fama and French (1993, 1997), and Kamara, Korajczyk, Lou, and Sadka (2016), who also employ a window size of 5 years.

1999; Kleibergen, 2009). We therefore make use of the rank test of Cragg and Donald (1997) to test the null hypothesis whether the matrix $[\mathbf{1}_N \boldsymbol{\beta}]$ has less than full rank. The matrix contains a constant and the exposure matrix. We perform this test for monthly static exposures, for each window of the monthly rolling window approach, and for the complete set of static frequency betas.

We test for the significance of the average prices of risk λ_t using bootstrapped standard errors. We use a circular block bootstrap (Politis and Romano, 1992) to estimate of the variance of the price of risk and use this to calculate the corresponding t-statistic.[16] The block size is determined based on the data-driven method of Politis and White (2004), which yields the optimal block size in a finite sample of given length. We set the block size at $\frac{1}{2}p_{\mathcal{T}-1}$, (i.e., the highest periodicity in the last pass band) such that for cycles longer than $p_{\mathcal{T}-1}$ years, the possible low correlation of half a cycle is preserved.[17] We construct $B = 1,000$ unfiltered samples and filter these to obtain statistics for filtered data. In Internet Appendix V.A we describe this bootstrap procedure in detail.

To assess the performance of the pricing models relative to each other, we use the test statistic derived by Rivers and Vuong (2002) and Hall and Pelletier (2011). Their statistic allows us to compare the goodness of fit of two non-nested models and is still valid under misspecification of the models. The test statistic is given by

$$J = \frac{Q^{(1)} - Q^{(2)}}{\hat{\sigma}_T/\sqrt{T}}, \tag{11}$$

[16]See Cameron and Trivedi (2005) section 11.2.5–11.2.6 for the details of constructing a bootstrap estimate of the standard error without asymptotic refinement.

[17]In our empirical setting, we set $p_{\mathcal{T}-1} = 96$ months. Therefore we set the block size of the bootstrap to 48 months.

where $Q^{(m)} = \frac{1}{T}\sum_{t=1}^{T} \eta_t^{(m)\prime}\eta_t^{(m)}$, where $\eta_t^{(m)}$ is the $N \times 1$ vector of pricing errors corresponding to model m for $m = 1,2$, and where $\hat{\sigma}_T$ is a consistent estimator of the asymptotic standard deviation of the difference between $Q^{(1)}$ and $Q^{(2)}$. The statistic is asymptotically normally distributed. We estimate the standard error $\hat{\sigma}_T/\sqrt{T}$ of the difference in pricing errors using a bootstrap procedure. Internet Appendix V.B describes this bootstrap procedure and provides further details regarding the test statistic (11).

IV. Data and Filter Choice

For our empirical analysis, we use monthly US data and our sample period runs from January 1968 until December 2016. A complete description of our data sources and availability can be found in Internet Appendix III.

A. Test asset return data

We use the 202 value-weighted equity portfolios of Giglio and Xiu (2018) and Feng, Giglio, and Xiu (2020), which consist of 25 size-value sorted portfolios, 17 industry sorted portfolios, 25 portfolios formed on operating profitability and investment activity, 25 size-variance sorted portfolios, 35 portfolios formed on size and net share issuance, 25 portfolios formed on size an accruals, 25 size-beta portfolios, and 25 size-momentum portfolios. The portfolios are obtained from the data library of Kenneth French. All sets of 25 portfolios are 5×5 bivariate sorted portfolios, the 35 portfolios are 5×7 bivariate sorted, and the industry portfolios are sorted based on their SIC code.[18] Our use of this set of 202 equity portfolio returns is motivated by the fact that it captures most well-known cross-sectional anomalies and exposures to various factors (Giglio and Xiu, 2018).

B. Factor data

The first multifactor model that we consider is the model of Chen, Roll, and Ross (1986) and Chan, Chen, and Hsieh (1985), which contains five macroeconomic factors. These factors are the monthly growth rate of industrial production, the monthly change expected in expected inflation, unexpected inflation, the credit spread, and the term spread. The data for these factors is obtained from the OECD and the St. Louis Fed. The definitions of these factors as well as details regarding their construction are given in Internet Appendix III.

The second multifactor model that we consider is a Fama-French type factor model. We consider the market factor (MKT), size factor (SMB), and value (HML) factor of Fama and French (1993). We further include the momentum factor (MOM) of Carhart (1997) and the traded liquidity factor (LIQ) of Pastor and Stambaugh (2003). The first four factors are obtained from the data library of Kenneth French. The traded liquidity factor is obtained from the website of Robert F. Stambaugh.[19]

[18]For a detailed description of the portfolio formation and factor construction, see http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[19]http://finance.wharton.upenn.edu/~stambaug/.

Table I
Summary statistics of the 202 equity excess portfolios returns

This table reports the descriptive statistics of excess returns on 202 equity portfolios from January 1968 to December 2016. For each portfolio set we report the mean, median, standard deviation (std.), minimum (min.) and maximum (max.) of the average returns within the set in percentage. The average standard deviation of each portfolio set is also reported. S/Val corresponds to size-value, ind. to industry, O/I to operating profitability-investment activity, S/Var to size-variance, S/N to size-net share issuance, S/A to size-accruals and S/M to size-momentum.

	25 S/Val	17 ind.	25 O/I	25 S/Var	35 S/N	25 S/A	25 S/B	25 S/M	All 202
Mean av. return	0.68	0.55	0.57	0.66	0.64	0.64	0.67	0.63	0.63
Median av. return	0.69	0.57	0.57	0.74	0.68	0.65	0.66	0.66	0.65
Std. av. return	0.21	0.10	0.20	0.32	0.23	0.15	0.15	0.33	0.23
Min. av. return	0.06	0.32	–0.03	–0.40	0.09	0.22	0.33	–0.06	–0.40
Max. av. return	1.01	0.70	0.96	1.09	0.96	0.85	0.90	1.24	1.24
Average std.	5.70	5.75	5.23	5.79	5.77	5.83	5.63	5.93	5.71

C. Descriptive statistics

Table I summarizes average excess returns of the 202 equity portfolios on a monthly frequency. Most of the variation in average returns can be found in the size-variance portfolios and the size-momentum portfolios. Most of the average excess returns are positive, although there are exceptions such as the small size-high variance, small size-low profitability, and low operating profitability-high investment portfolios, which have respective average excess returns of –0.40, –0.06 and –0.03. The highest average excess return corresponds to the small size-high momentum/low variance portfolios. In Internet Appendix III we present an overview of these statistics and the correlations between the excess returns of the portfolios. This correlation lies between 0.32 and 0.98.

The summary statistics of the macroeconomic variables and the Fama-French factors for the unfiltered monthly observations are given in panel A and B of Table II, respectively. The expected inflation is the least volatile macroeconomic factor, while the term spread is the most volatile macroeconomic factor. The macroeconomic factors are moderately correlated with correlations between –0.31 and 0.33, with the exception of the correlation between expected and unexpected inflation which is 0.58. We further find moderate correlation between the Fama-French factors with correlations between –0.28 and 0.29. The highest correlation is between the market and size factor. The correlation between the value and momentum factor is negative, in line with Asness, Moskowitz, and Pedersen (2013).

D. Filter choice

For our main analysis, we use the Christiano-Fitzgerald random walk (CF-RW) filter, which approximates the ideal band pass filter without losing observations at the begin and end of the sample. The CF-RW filter finds its origin in Baxter and King (1999), who develop a band-pass filter (BK-filter) that resembles the ideal band-pass filter closely at the cost of losing observations at the begin and end of the data set. Christiano and Fitzgerald (2003) handle the lost observations by expanding the time series, assuming that the underlying time series follows a random walk, such that the best pre-sample (post-sample) estimate is the first (last) data point.[20]

We choose to use the Christiano and Fitzgerald (2003) filter due to its flexible pass bands

[20]The performance of the CF-RW filter is very similar to the BK filter, see Figure IA.2b in the Internet Appendix for a comparison of the CF-RW filter and the BK-filter on the market factor. The correlation between these filtered series is 0.98.

Table II
Summary statistics of the macroeconomic variables and Fama-French factors

Panel A reports the summary statistics of the macroeconomic variables and Panel B of the Fama-French factors for the sample period January 1968 to December 2016. We report the monthly averages (mean) and corresponding standard deviation (std.) in percentages, and correlations.

Panel A: *Macroeconomic variables*								
				Correlation				
Variable		Mean	Std.	*IP*	*EI*	*UI*	*RP*	*TS*
Industrial Production	*IP*	0.18	0.73	1				
Expected Inflation	*EI*	0.00	1.24	0.03	1			
Unexpected Inflation	*UI*	0.01	1.08	-0.17	0.58	1		
Credit Spread	*DEF*	1.09	0.44	-0.31	-0.03	0.12	1	
Term Spread	*TS*	1.78	1.35	0.12	0.30	0.33	0.18	1
Panel B: *Fama-French factors*								
				Correlation				
Factor		Mean	Std.	*MKT*	*SMB*	*HML*	*MOM*	*LIQ*
Market	*MKT*	0.49	4.54	1				
Size	*SMB*	0.16	3.11	0.29	1			
Value	*HML*	0.38	2.90	-0.28	-0.21	1		
Momentum	*MOM*	0.63	4.34	-0.14	-0.02	-0.18	1	
Liquidity	*LIQ*	0.40	3.39	-0.01	0.00	0.04	-0.01	1

and clean separation.[21] It separates the time series $\{y_t : t = 1, \ldots, T\}$ into a cyclical component and a component capturing the remainder. Denoted in the time domain, the filtered time series is

[21] We do not consider a moving average (MA) filter, since it does not fully isolate the intended horizon as it does not produce orthogonal components. Figure IA.2a of the Internet Appendix shows a comparison between an MA filtered series and a CF-RW filtered series.

calculated as

$$\widetilde{y}_t = \widetilde{g}_{t-1} y_1 + g^*_{t-2} y_2 + \cdots + g^*_1 y_{t-1} + g^*_0 y_t + g^*_1 y_{t+1} + \ldots + g^*_{T-t-1} y_{T-1} + \widetilde{g}_{T-t} y_T, \tag{12}$$

for $t = 3, \ldots, T-2$. The band-pass filter weights g^*_ℓ correspond to an ideal pass band filter for the frequency range $[\omega_1, \omega_2]$, and are given by

$$g^*_\ell = \begin{cases} \frac{\sin(\omega_2 \ell) - \sin(\omega_1 \ell)}{\pi \ell} & \text{for } \ell \neq 0 \\ \frac{\omega_2 - \omega_1}{\pi} & \text{for } \ell = 0. \end{cases} \tag{13}$$

We describe the ideal band pass filter in more detail in Internet Appendix I.B. The frequency bounds $\omega_1 < \omega_2$ are defined as $\omega_j = 2\pi / p_j$, where p_j denotes the corresponding cycle length in months. For the beginning and the end of the sample, Christiano and Fitzgerald (2003) assume that y_t follows a random walk. They use end-point weights that are computed as linear functions of the other weights. The end-point weights of the CF-RW filter are given by

$$\widetilde{g}_{T-t} = -\left(\frac{1}{2} g^*_0 + \sum_{\ell=1}^{T-t-1} g^*_\ell \right) \tag{14}$$

and

$$\widetilde{g}_{t-1} = -\left(g^*_{t-2} + \ldots + g^*_1 + g^*_0 + g^*_1 + \ldots + g^*_{T-t-1} + \widetilde{g}_{T-t} \right). \tag{15}$$

The CF-RW filter is two-sided, and as such includes forward-looking information. While this should not be an issue when investigating cross-sectional relations (Ang and Kristensen, 2012), we assess the robustness by using a truncated one-sided version of the CF-RW filter, which only includes backward-looking data. Internet Appendix Section IV.C describes this one-sided filter.

V. Return and Factor Behavior at Different Frequencies

In this section, we analyze the properties of the filtered data. We first focus on the filtered data itself, and then continue with the relations between the different series. All filtering is done with the CF-RW filter, unless specified otherwise.

A. Filtered returns and factors

We apply the filter for different pass bands to obtain an orthogonal decomposition into short-, medium-, and long-term fluctuations. The periodicities that we use for the medium-term fluctuations, which we separate into shorter business cycles (1 to 3 years) and longer business cycles (3 to 8 years), are inspired by the business cycle definitions of Burns and Mitchell (1946) and Comin and Gertler (2006). Cycles longer than 8 years correspond to long-term trends, as in Haug and Dewald (2004). We consider short-term fluctuations as cycles shorter than 1 year. Table III reports an overview of the pass bands that we use in our empirical setting and the corresponding frequencies.

We filter the equity portfolio excess returns, the macroeconomic factors, and the Fama-French factors. Figure 1 shows an example of filtered output for the industrial production factor. Figure 1a shows the filtered industrial production monthly growth rate filtered between 3 to 8 years, as well as corresponding filter weights. The high frequency component is shown in Figure 1b. The density plots show that the filtered time series is a frequency-weighted series depending on past and future observations. The filter weights are asymmetric.

Table III
Overview of the pass bands

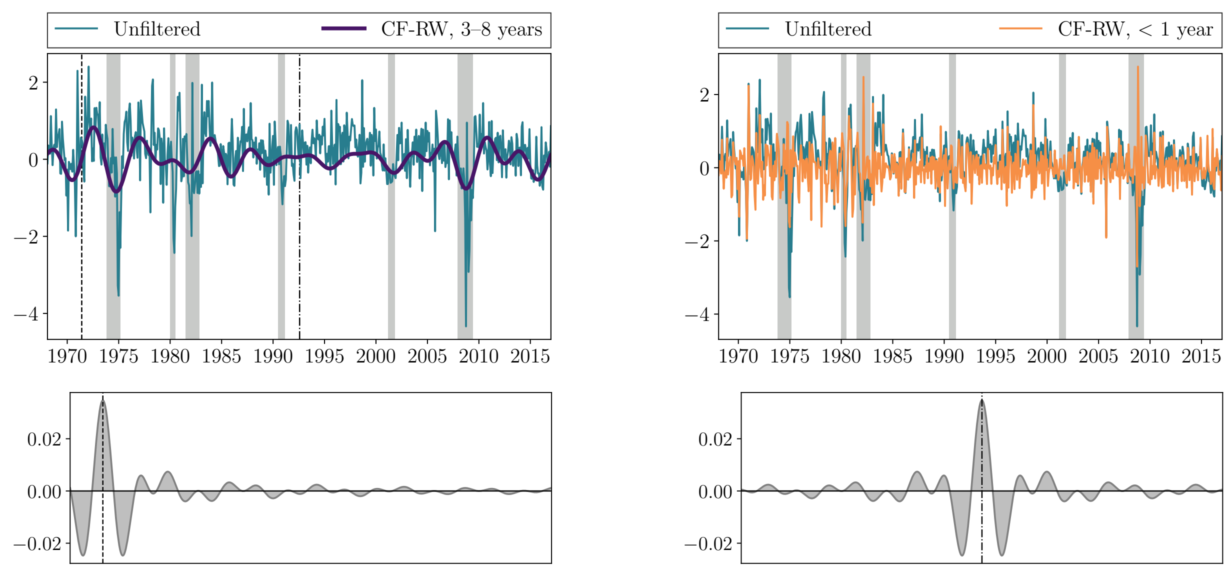
This table shows the labeling we use for each frequency pass band and periodicity in months (years).

Label	Periodicity		ω_1	ω_2
Short-term; high frequency	$<$ 12 months	($<$ 1 year)	$2\pi/12$	∞
Medium-term; short business cycle	12–36 months	(1–3 years)	$2\pi/36$	$2\pi/12$
Medium-term; long business cycle	36–96 months	(3–8 years)	$2\pi/96$	$2\pi/36$
Long-term; low frequency	$>$ 96 months	($>$ 8 years)	0	$2\pi/96$

Figure 1. Filtered industrial production. The industrial production factor (*IP*) is filtered using the Christiano-Fitzgerald random walk (CF-RW) filter. Figure 1a shows the filtered factor for the pass band 3–8 years and Figure 1b shows fluctuations $<$ 1 year. The grey areas represent NBER recessions. The plots below are density plots of the filter weights for a 3–8 year filter, corresponding to the two observations of the dashed and dash dotted line.

(A) Long business cycle component (3–8 years) and the original industrial production factor.

(B) High frequency component (<1 year) and the original industrial production factor.



In Table IV we report summary statistics for the filtered factors. Overall, we see a declining standard deviation as the frequency decreases. The percentage variance shows that the credit spread and term spread are relatively low frequency factors, since the lower frequency components (3 to 8 years and greater than 8 years) contain over 30% of the monthly variation. This is in line with Faria and Verona (2018), who find that the business cycle component of the term spread contains the majority of the variation. For Fama-French factors and both expected- and unexpected inflation, we see that over 75% of the monthly variation is captured in the less-than-one-year pass band.

To verify whether the cross-frequency components are orthogonal in-sample, we look at the correlation between the factor components. Table IA.2 in the Internet Appendix shows these correlations. The correlations between the cross-frequency components are generally zero, although for the lower frequencies (3 to 8 years and greater than 8 years) there is some correlation, but generally lower than 0.15 in absolute terms. This slight frequency leakage at lower frequencies can be attributed to the relatively limited sample length in terms of the periodicity for the lower frequencies.

B. Correlation structure

B.1. Correlations between factors and returns

The 202 portfolio excess returns and the macroeconomic factors show limited correlation on a monthly (unfiltered) basis, with correlations between –0.1 and 0.1, similar to the values found by Chen, Roll, and Ross (1986). For the Fama-French factors these correlations are higher, between –0.5 and 0.5, and for the market factor even higher at 0.8, similar to the values found by Fama and

Table IV

Descriptive statistics of the filtered factors

This table reports the descriptive statistics for the macroeconomic factors (panel A) and Fama-French factors (panel B) for the sample which runs from January 1968 to December 2016, for monthly unfiltered data and filtered data. The mean, standard deviation and percentage variance (in comparison to monthly data) are reported.

Panel A: *Macroeconomic factors*						Panel B: *Fama-French factors*					
	Unf.	<1y	1–3y	3–8y	>8y		Unf.	<1y	1–3y	3–8y	>8y
			Mean						*Mean*		
IP	0.18	0.00	0.00	-0.01	0.18	*MKT*	0.49	0.00	0.02	-0.00	0.48
EI	-0.00	0.00	0.00	-0.00	-0.00	*SMB*	0.16	0.00	0.00	-0.02	0.17
UI	0.01	0.00	0.00	0.00	0.01	*HML*	0.38	0.00	-0.01	-0.04	0.43
DEF	1.09	0.00	0.00	0.01	1.08	*MOM*	0.63	0.00	0.01	0.02	0.60
TS	1.78	0.00	0.00	-0.00	1.78	*LIQ*	0.40	0.00	0.01	0.04	0.36
			Standard deviation						*Standard deviation*		
IP	0.73	0.55	0.32	0.32	0.15	*MKT*	4.54	4.07	1.63	1.00	0.66
EI	1.24	1.23	0.15	0.04	0.02	*SMB*	3.11	2.85	0.97	0.50	0.65
UI	1.08	0.99	0.35	0.20	0.12	*HML*	2.90	2.51	1.08	0.79	0.60
DEF	0.44	0.10	0.17	0.25	0.31	*MOM*	4.34	3.93	1.54	0.86	0.65
TS	1.35	0.58	0.51	0.75	0.77	*LIQ*	3.39	3.11	1.07	0.58	0.62
			% Variance						*% Variance*		
IP		57.12	19.49	18.63	4.35	*MKT*		80.38	12.80	4.84	2.09
EI		98.37	1.43	0.08	0.04	*SMB*		83.81	9.67	2.59	4.35
UI		83.88	10.56	3.51	1.30	*HML*		75.33	13.96	7.50	4.24
DEF		4.94	14.71	31.42	48.67	*MOM*		82.08	12.54	3.92	2.26
TS		18.47	14.28	30.82	32.96	*LIQ*		84.33	9.91	2.97	3.31

French (1993) and Carhart (1997).

These correlations are highly similar for the short-term fluctuations. For the medium-term fluctuations, the correlation patterns are different. The correlations between the ten factors and the portfolio returns increase in absolute value for 1 to 3 years, but remains low for factors such as industrial production and the term spread. This changes for 3 to 8 years and fluctuations greater than 8 years, where the correlation is on average about 0.3 to 0.4. The correlation structure between the portfolio excess returns themselves is preserved across frequencies, but weakens slightly as the frequency decreases.

B.2. Correlations between factors of the same type

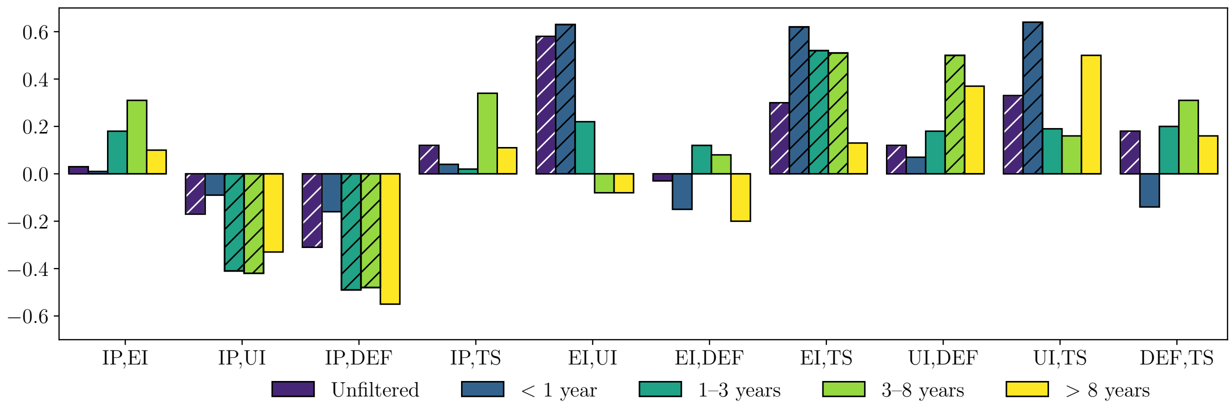
Figure 2a shows the correlations between the macroeconomic factors and Figure 2b shows the correlations between the Fama-French factors. The figures report the correlations for the unfiltered data as well as for each pass band. The correlations for the unfiltered data correspond to those reported in Table II and are similar to those reported by Chen, Roll, and Ross (1986), Fama and French (1993), and Carhart (1997). Overall, for each of the different pass bands, there is less correlation between the Fama-French factors than between the macroeconomic factors.

B.3. Correlations between factors of different types

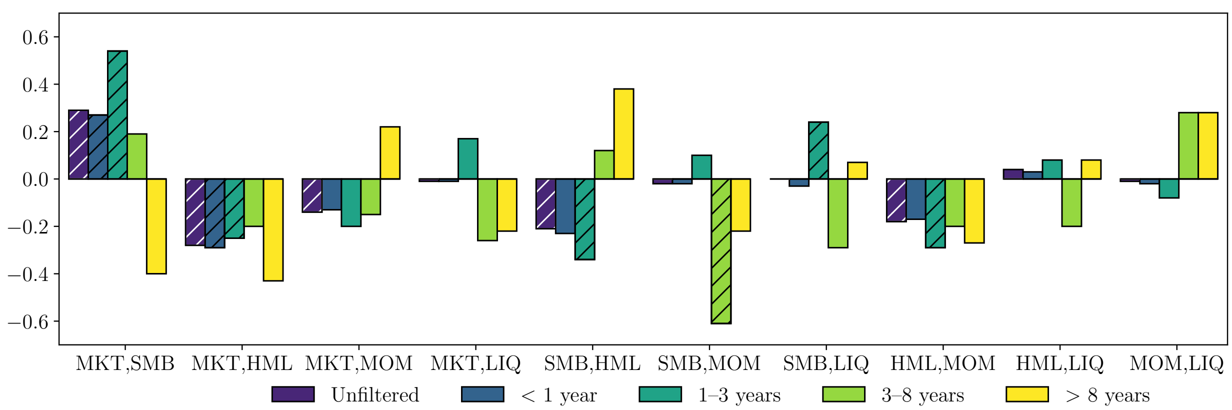
Next, we investigate the correlation between the individual Fama-french factors and the individual macroeconomic factors. This is motivated by Asness, Moskowitz, and Pedersen (2013), who refer to macroeconomic factors as a potential source of common variation that could be driving

Figure 2. Correlations across frequencies (within factor types). Figure 2a shows the correlation between the macroeconomic factors for each pass band and Figure 2b shows the correlation between the Fama-French factors for each pass band, for the sample which runs from January 1968 to December 2016. Shaded bars represent significance at the 5% significance level.

(A) Correlations macroeconomic factors.



(B) Correlations Fama-French factors.



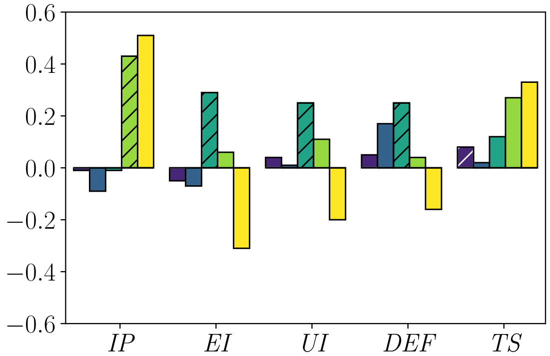
value and momentum returns. Each panel of Figure 3 shows the correlation for a Fama-French factor with each of the macroeconomic factors for different pass bands. Similar to Asness, Moskowitz, and Pedersen (2013) and Daniel and Titman (2012), we find that the macroeconomic and Fama-French factors are not highly correlated at the monthly frequency.

Figure 3a shows that the correlation between the market factor and the industrial production factor increases as the frequency decreases. This changes from zero correlation on a monthly basis to a correlation of 0.43 for cycles between 3 to 8 years and a correlation of 0.51 for fluctuations with a horizon greater than 8 years. For each Fama-French factor, we find substantial correlation between the specific Fama-French factor and the macroeconomic factors for shorter business cycles (cycles between 1 and 3 years). Figure 3d shows that the momentum factor and the term spread become negatively correlated for cycles greater than 3 years. Figure 3e shows that liquidity is negatively correlated to expected inflation for cycles between 3 to 8 years.

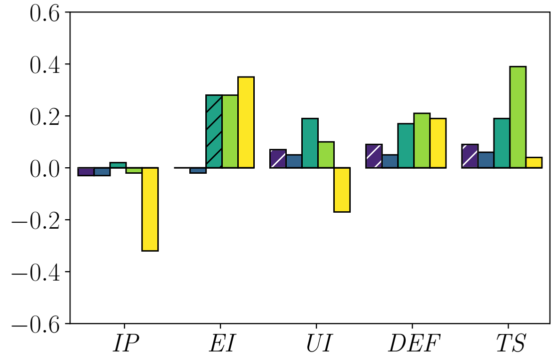
In summary, we find that the default spread is negatively correlated to the momentum factor for most frequencies. These findings show that, when taking into account the different frequencies, macroeconomic variables can indeed be related to value and momentum returns, but also to the other Fama-French factors. Our frequency-specific approach, has therefore allowed us to uncover additional relations relative to the monthly observation frequency results of Asness, Moskowitz, and Pedersen (2013). Our findings show that indeed, macroeconomic factors could be a possible source driving common variation of value and momentum strategies.

Figure 3. Correlations across frequencies (across factor types). The figures show the correlation between each Fama-French factor and the macroeconomic factors for the data ranging from January 1968 to December 2016. Shaded bars represent significance at the 5% significance level.

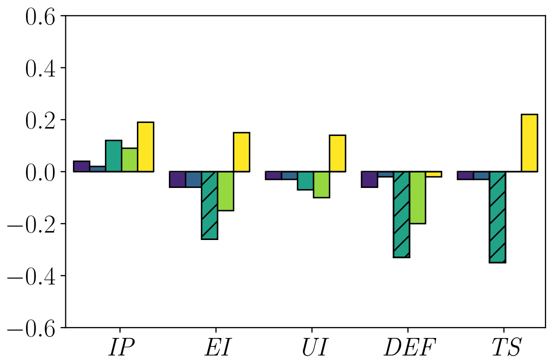
(A) Market factor.



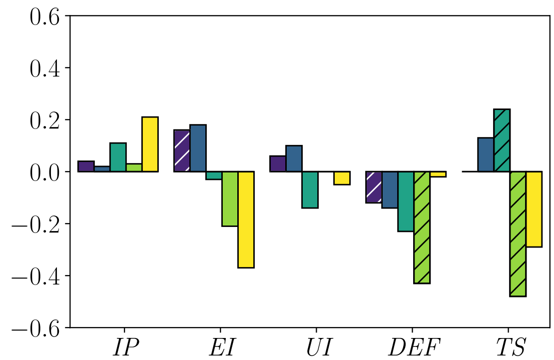
(B) Size factor.



(C) Value factor.



(D) Momentum factor.



(E) Liquidity factor.

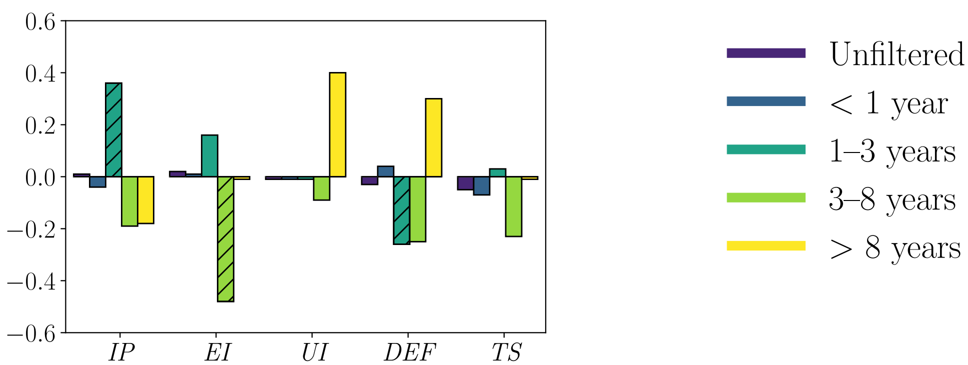
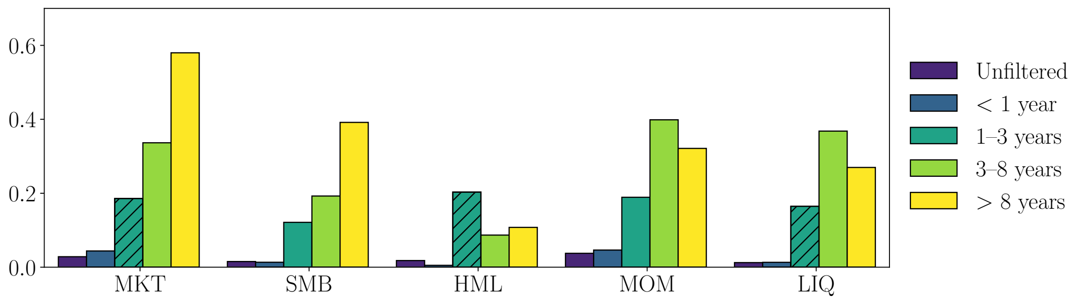


Figure 4. *R*-squared of the Fama-French factors regressed on the macroeconomic factors. This figure represents the regression between the Fama-French factors and the macroeconomic factors for the sample which runs from January 1968 to December 2016. Shaded bars represent significance at the 5% significance level based on 1,000 bootstrapped R^2.



B.4. Explaining Fama-French factors with macroeconomic factors

In light of Asness, Moskowitz, and Pedersen (2013), who suggest macroeconomic factors as a possible source driving common variation of value and momentum strategies, we further analyze the relation between the Fama-French factors and the macroeconomic factors. We do this by regressing each filtered Fama-French factor $f_t^{(\tau)}$ on the full set of filtered macroeconomic factors and a constant as follows.

$$f_t^{(\tau)} = \gamma_0 + \gamma_{IP}^{(\tau)} IP_t^{(\tau)} + \gamma_{EI}^{(\tau)} EI_t^{(\tau)} + \gamma_{UI}^{(\tau)} UI_t^{(\tau)} + \gamma_{DEF}^{(\tau)} DEF_t^{(\tau)} + \gamma_{TS}^{(\tau)} TS_t^{(\tau)} + \varepsilon_t^{(\tau)}. \tag{16}$$

We run this regression for unfiltered monthly data as well as for filtered data corresponding to each frequency pass band τ (i.e. as a band spectrum regression). For long-horizon regressions such as these, the asymptotic distribution of the t-statistics will be non-standard. We therefore use the rescaled t-statistic, computed as $t/\sqrt{T}$ (Valkanov, 2003). The distribution of the rescaled t-statistic is simulated using the bootstrap method described in Goetzmann and Jorion (1993). We use this method to obtain p-values and R^2 confidence intervals.[22]

Figure 4 shows the time series R^2 of (16) for each frequency pass band. We see that according to the model fit, the value factor seems to be the best explained by the macroeconomic factors compared to the other Fama-French factors. The R^2 values increase as the frequency decreases, showing that the Fama-French factors can be better explained by the macroeconomic factors at lower frequencies. We focus on the estimation results for short-term business cycle fluctuations (between 1 and 3 years) as reported in Table V, since these have a particularly interesting economic interpretation. The results show that the R^2 is significant for the market factor, value factor, and liquidity factor with values of 0.186, 0.203, and 0.165, respectively. At this frequency, the value factor is negatively related to the term spread.

The short-term business cycle frequency relation between the value factor and macroeconomic variables, as well as the negative relation with the term spread, is in line with the literature. As Fama and French (1989) point out, the term spread tracks short-term business cycle fluctuations. Specifically, the term spread is found to be low during good economic conditions, and high in times of bad economic conditions. Additionally, Zhang (2005) and Petkova and Zhang (2005) document that value firms are more burdened with unproductive capital, making them more prone to economic downturns. Value stocks are therefore riskier during bad economic times, and less risky during good economic times, suggesting business cycle dependency.[23] Finally, the results

[22]Engle (1974) points out that applying a filter to both the left- and right-hand side in a regression satisfying the assumptions of OLS, will lead to estimates that are still unbiased but inefficient due to the serial correlation in the disturbance terms.

[23]Empirical evidence has been documented in Petkova (2006) and Hahn and Lee (2006), who find a significant

Table V

Coefficient estimates of each Fama-French factor regressed on the macroeconomic factors for fluctuations between 1 and 3 years

This table reports the coefficient estimates of each Fama-French factor regressed on the macroeconomic factors for the sample which runs from January 1968 to December 2016, with the corresponding rescaled t-statistics, $t/\sqrt{T}$, reported underneath between parentheses. In the column $t/\sqrt{T}$, the empirical p-values are reported using 1,000 bootstrap replications. We provide the bootstrapped R^2 as well as the 95% bootstrap confidence interval. The coefficient estimates correspond to filtered data with fluctuations between 1 and 3 years.

	constant	γ_{IP}	γ_{EI}	γ_{UI}	γ_{DEF}	γ_{TS}	R^2
Market (MKT)	0.000	1.047	2.319	1.183*	2.858**	−0.333	0.186**
$t/\sqrt{T}$	0.012	0.171	0.186	0.242	0.275	−0.097	(0.021, 0.177)
Empirical p-value	0.240	0.179	0.243	0.064	0.024	0.510	
Size (SMB)	0.000	0.441	1.176	0.462	1.035	0.053	0.121
$t/\sqrt{T}$	0.004	0.117	0.153	0.153	0.161	0.025	(0.022, 0.178)
Empirical p-value	0.678	0.337	0.260	0.277	0.227	0.871	
Value (HML)	0.000	0.188	−0.885	0.225	−1.569*	−0.541**	0.203**
$t/\sqrt{T}$	−0.013	0.047	−0.108	0.070	−0.229	−0.238	(0.018, 0.156)
Empirical p-value	0.179	0.711	0.415	0.589	0.086	0.049	
Momentum (MOM)	0.000	−0.382	−1.722	−0.710	−2.701*	1.272***	0.189
$t/\sqrt{T}$	0.008	−0.066	−0.146	−0.154	−0.275	0.391	(0.03, 0.215)
Empirical p-value	0.379	0.598	0.482	0.229	0.084	0.002	
Liquidity (LIQ)	0.000	1.085**	0.769	0.397	−0.802	−0.068	0.165**
$t/\sqrt{T}$	0.010	0.267	0.093	0.122	−0.116	−0.030	(0.017, 0.159)
Empirical p-value	0.333	0.033	0.457	0.318	0.337	0.812	

***, **, * significant at 1, 5, 10% significance level, respectively.

also show a significant relation between the term spread and the momentum factor.

relation between the value factor and innovations/change in the term spread in monthly data, respectively. Taking into account different frequencies, Kang, In, and Kim (2017) find a significant relation between the short-business cycle component of the value factor and the change in term spread.

VI. Asset Pricing Results

A. Factor exposure estimation results

We start by analyzing the first stage horizon-specific exposure estimates. These results are reported in Table VI. We find that for monthly data, a large portion of the market, size, and value exposures is significant. The proportion of significant exposures increases as the frequency decreases. The results suggest that horizon effects are present in the exposures. The exposures vary across horizons and differ in standard deviation and significance levels across horizons, especially for the macroeconomic factors and for the size and value factor.

Panel B of Table VI reports the average time series R^2 over the 202 portfolios. These R^2 values show that the fit improves for the macroeconomic model as the horizon becomes longer, in line with Gençay, Selçuk, and Whitcher (2001). For the Fama-French model, we find that the average fit remains stable across horizons. The correlations between factor exposures across horizons are generally moderate to low, indicating that there are indeed horizon-specific effects present in the exposures. Internet Appendix Table IA.3 reports the correlations between the frequency-specific betas.

In order to test the validity of our second stage results, we check whether the total exposure matrix is full rank by the Cragg and Donald (1997) rank test. Panel C of Table VI shows these results, indicating that we can reject the null hypothesis of less than full rank for the 202 mixed portfolios for the static full sample (static) exposures on a monthly basis for the Fama-French model, but

Table VI

Descriptive statistics of the horizon-specific loadings

This table reports descriptive statistics of the 202 first stage regressions. Panel A reports the average loading and standard deviation over the portfolios for exposures at different horizons for the sample which runs from January 1968 to December 2016, for both the macroeconomic and Fama-French factors. We also report the proportion of significant exposures at the 5% level, based on Newey-West standard errors with 48 lags. Panel B reports the average time series R^2. Panel C reports the bootstrapped p-value of the rank test, with H_0: less than full rank. "Monthly" corresponds to monthly data, the pass bands correspond to filtered data and "all frequencies" corresponds to all pass bands combined. Factors and returns are filtered using the CF-RW filter.

Panel A: Loadings and significance

		Macroeconomic factors						Fama-French factors					
	Freq.	cons.	*IP*	*EI*	*UI*	*DEF*	*TS*	cons.	*MKT*	*SMB*	*HML*	*MOM*	*LIQ*
Average	Monthly	0.00	0.02	−0.53	0.44	0.45	0.35	0.00	1.02	0.42	0.17	−0.05	0.01
	$<$ 1y	0.00	−0.55	−0.48	−0.06	7.50	1.13	0.00	1.02	0.41	0.17	−0.05	0.01
	1–3y	0.00	1.40	3.12	1.53	3.68	−0.59	0.00	1.05	0.41	0.14	−0.06	0.00
	3–8y	0.00	2.06	−4.96	1.28	0.98	0.11	0.00	1.00	0.48	0.21	−0.01	0.08
	$>$ 8y	0.01	0.48	−6.84	−2.78	0.05	0.59	0.00	0.95	0.49	0.07	−0.00	0.07
Standard deviation	Monthly	0.01	0.15	0.16	0.15	0.30	0.11	0.00	0.13	0.47	0.25	0.16	0.04
	$<$ 1y	0.00	0.20	0.19	0.14	1.62	0.40	0.00	0.13	0.47	0.24	0.16	0.05
	1–3y	0.00	0.50	1.17	0.48	1.05	0.34	0.00	0.15	0.48	0.24	0.17	0.08
	3–8y	0.00	0.49	4.15	0.73	0.66	0.21	0.00	0.16	0.50	0.32	0.21	0.19
	$>$ 8y	0.01	1.18	8.42	1.62	0.36	0.20	0.00	0.19	0.50	0.40	0.23	0.17
Proportion significant	Monthly	0.05	0.00	0.85	0.42	0.02	0.70	0.39	1.00	0.95	0.89	0.66	0.29
	$<$ 1y	0.00	0.33	0.69	0.00	1.00	0.85	0.00	1.00	0.95	0.88	0.64	0.32
	1–3y	0.00	0.97	1.00	1.00	1.00	0.86	0.00	1.00	0.94	0.87	0.73	0.61
	3–8y	0.01	1.00	0.82	0.92	0.88	0.64	0.06	1.00	0.93	0.91	0.81	0.84
	$>$ 8y	0.83	0.80	0.84	0.97	0.68	0.99	0.91	1.00	0.97	0.96	0.90	0.87

Panel B: Average first stage time series R^2

	Monthly	$<$ 1y	1–3y	3–8y	$>$ 8y
Macroeconomic	0.020	0.041	0.192	0.287	0.493
Fama-French	0.864	0.862	0.900	0.882	0.862

Panel C: Rank test p-value

	Monthly	All frequencies
Macroeconomic	0.367	0.005
Fama-French	$<$ 0.001	0.003

we cannot reject the null hypothesis for the macroeconomic model.[24] Consequently, the prices of risk attached to the macroeconomic factor exposures estimated with the traditional approach on unfiltered monthly data will not be informative. For the frequency-specific specification, we can reject the null hypothesis of less than full rank for both models.

B. Cross-sectional asset pricing regression results

Before we proceed with the frequency-specific asset pricing results, we first run the standard asset pricing regression for the unfiltered monthly data, based on both full sample exposures and rolling window exposures. The window size for the rolling window exposures is 60 months. Standard errors are based on 1,000 bootstrapped samples. The standard setup provides a reference point against which to compare the frequency-specific results.

The monthly cross-sectional asset pricing results (Table VII) show that the Fama-French model performs better than the macroeconomic factor model in terms of cross-sectional fit,[25] and mean absolute pricing error. For the macroeconomic factors we see that expected inflation plays a significant role using static exposures, while for rolling window exposures none of the macroeconomic exposures are significant. For the Fama-French factors, both value and momentum are significantly priced for both static and rolling window exposures. These findings are in line with Asness,

[24] We cannot reject the null hypothesis of less than full rank for the 202 mixed portfolios for rolling window exposures with a window size of 5 years for both models.

[25] We check for factor structure in the residuals of the first stage by using the FACCHECK measure described in Kleibergen and Zhan (2015). We find that the factor structure in the residuals is less present for the Fama-French models compared to the macroeconomic model on all frequencies and becomes less severe on lower frequencies. Hence comparison of the second stage R^2 of the two models may not be appropriate. We therefore also compare the models based on the pricing error of the models.

Table VII

Cross-sectional regressions for 202 mixed portfolios, unfiltered monthly data

This table reports second-stage cross-sectional regressions of monthly excess US equity portfolio returns on the estimated loadings and a constant over the sample which runs from January 1968 to December 2016,

$$r_{i,t+1} = \lambda_{0,t} + \sum_j \lambda_{j,t} \widehat{\beta}_{j,i,t} + \eta_{i,t+1}$$

where the loadings are estimated over the full sample (static exposures) and with a rolling window of five years in the first stage. The table reports the annualized time series averages of the λ_j with t-statistics obtained by the bootstrap procedure between parentheses underneath, and the annualized average risk premium $\left(\lambda_{j,t} \times \widehat{\beta}_{j,i,t}\right)$ in percentage for each factor j. We further report the average cross-sectional (adjusted) R^2, the mean absolute pricing error (MAPE, between brackets underneath the R^2) and the sum of squared pricing errors ($Q^{(m)}$, between brackets underneath the adjusted R^2). ***, **, and * denote significance at the 1%, 5%, and 10% level, respectively, based on 1,000 bootstrapped samples.

Panel A: *Macroeconomic factor model*

	IP	*EI*	*UI*	*DEF*	*TS*	$\overline{\lambda}_0$	R^2 [MAPE]	Adj. R^2 $\left[Q^{(m)}\right]$
Static exposures								
$\overline{\lambda}$	−2.782	8.099**	9.243*	−0.274	0.971	7.693**	0.276	0.257
t-statistic	(−1.039)	(2.150)	(1.749)	(−0.176)	(0.187)	(2.374)	[2.136]	[9.211]
Risk premium	−0.062	−4.280	4.032	−0.123	0.339			
Rolling window exposures								
$\overline{\lambda}$	−0.785	−0.777	0.621	−0.392	−0.366	6.478***	0.292	0.274
t-statistic	(−0.743)	(−0.582)	(0.353)	(−1.041)	(−0.234)	(2.940)	[2.139]	[7.973]
Risk premium	1.257	0.290	0.312	−1.786	1.747			

Panel B: *Fama-French factor model*

	MKT	*SMB*	*HML*	*MOM*	*LIQ*	$\overline{\lambda}_0$	R^2 [MAPE]	Adj. R^2 $\left[Q^{(m)}\right]$
Static exposures								
$\overline{\lambda}$	−0.145	1.039	5.538***	9.016***	−1.770	6.798***	0.422	0.407
t-statistic	(−0.060)	(0.561)	(3.019)	(4.582)	(−0.362)	(2.929)	[1.831]	[4.599]
Risk premium	−0.148	0.438	0.957	−0.426	−0.021			
Rolling window exposures								
$\overline{\lambda}$	−3.502	2.107	3.385**	4.372***	2.929	10.510***	0.417	0.402
t-statistic	(−1.560)	(1.158)	(2.281)	(2.842)	(1.427)	(4.544)	[1.874]	[5.145]
Risk premium	−3.595	0.975	0.582	−0.183	0.008			

Moskowitz, and Pedersen (2013), Chan, Karceski, and Lakonishok (1998), and Connor (1995). The average market price of risk is negative for both specifications, which is consistent with the literature.[26] As Cochrane (2005) notes, the intercept should be zero when using excess portfolio returns. Internet Appendix Table IA.4 shows the cross-sectional results for the Fama-French model when imposing a zero-intercept restriction. The average price of market risk is positive in these specifications.

We now proceed with the frequency-specific specification and discuss the additional insights gained from our approach. Table VIII shows the results using frequency-specific exposures based on filtered data. We report t-statistics based on bootstrapped standard errors. The results show that the model performance differential between the macroeconomic and Fama-French models in terms of cross-sectional R^2 and average pricing error is smaller compared to the difference in cross-sectional fit on a monthly basis.[27]

The risk premium estimates in the macroeconomic model show that on the short horizon expected inflation, unexpected inflation, and the term spread are significantly priced. The term spread is significantly priced at the 1 to 3 year horizon, while credit spread is significant at the 3 to 8 years horizon. For horizons longer than 8 years, none of the factors are significant.

For the Fama-French model we find a significant price of risk for momentum for horizons up to 1 year and for size and value at the 3 to 8 years horizon. The value factor being priced at the

[26]Fama and French (1992, 1993), Jagannathan and Wang (1996), Petkova (2006), Liu and Zhang (2008), Da and Schaumburg (2011) and Eiling (2013) report negative market premia estimates on monthly and quarterly data, Boons and Tamoni (2015) reports a negative market price of risk on lower frequencies as well.

[27]Internet Appendix Table IA.4 panel B shows that, similar to the results for the unfiltered version, the frequency-specific results are again not strongly affected by restricting the intercept to zero.

business cycle horizon is in line with Zhang (2005) and Petkova and Zhang (2005). The market factor does not play a significant role at the 5% level on any of the horizons, but the average price of risk is negative at the 3 to 8 years horizon.[28] These results are in line with the findings of Kamara, Korajczyk, Lou, and Sadka (2016), who show that market, size, and value have long-run risk factor characteristics, while liquidity and momentum exhibit characteristics of short-term risk factors. Our results also align with Koijen, Lustig, and Van Nieuwerburgh (2017) and Bandi, Chaudhuri, Lo, and Tamoni (2021), who find that the temporal dimension of risk is relevant for the cross-section of expected returns.

To compare the cross-sectional fit of the frequency-specific specification to the monthly (unfiltered) specification, we compute the adjusted cross-sectional R^2. For the frequency-specific models, we find a 10% to 20% higher fit compared to their monthly counterpart. In addition, the mean squared pricing error and sum of squared pricing errors are lower for the frequency-specific models.

To test whether this improvement in goodness of fit is significant, we perform the test of Rivers and Vuong (2002) and Hall and Pelletier (2011) for non-nested models. The results are reported in Table IX. We find that the macroeconomic frequency-specific model outperforms its monthly counterparts, for both static and rolling window exposures. Comparing the macroeconomic and Fama-French factor models, we find that the Fama-French model significantly outperforms the macroeconomic factor model when using unfiltered monthly data. For the frequency-specific specification, this difference diminishes and is insignificant. These results indicate that the macroeco-

[28] A possible explanation for this negative market risk premium estimate is that the market portfolio acts as a hedge portfolio against not-included state variables (Fama, 1996).

Table VIII
Cross-sectional regressions for 202 mixed portfolios, filter-based exposures

This table reports second-stage cross-sectional regressions of monthly filtered excess US equity portfolio returns on the spectral loadings and a constant for the sample which runs from January 1968 to December 2016,

$$r_{i,t+1} = \lambda_{0,t} + \sum_{\tau=1}^{\mathcal{T}} \lambda_t^{(\tau)\prime} \widehat{\beta}_i^{(\tau)} + \eta_{i,t+1}$$

where the spectral loadings are estimated over the full sample for each pass band τ. The table reports the annualized time series averages of the frequency-specific $\lambda_j^{(\tau)}$ with t-statistics obtained by the bootstrap procedure between parentheses underneath, and the annualized average risk premium $\left(\lambda_{j,t}^{(\tau)} \times \widehat{\beta}_{i,j}^{(\tau)}\right)$ in percentage for each factor j. We further report the average cross-sectional (adjusted) R^2, the mean absolute pricing error (MAPE, between brackets underneath the R^2) and the sum of squared pricing errors ($Q^{(m)}$, between brackets underneath the adjusted R^2). ***, **, and * denote significance at the 1%, 5%, and 10% level, respectively, based on 1,000 bootstrapped samples.

Panel A: *Macroeconomic factor model*

	IP	*EI*	*UI*	*DEF*	*TS*	$\overline{\lambda}_0$	R^2 [MAPE]	Adj. R^2 $[Q^{(m)}]$
$\overline{\lambda}^{(<1y)}$	1.063	6.411**	6.759***	0.024	4.062***	7.265**	0.541	0.490
t-statistic	(0.950)	(2.580)	(2.650)	(0.087)	(3.095)	(2.267)	[1.554]	[3.951]
Risk premium	−0.583	−3.046	−0.437	0.182	4.592			
$\overline{\lambda}^{(1-3y)}$	0.355	−0.217	1.499	0.314	1.718**			
t-statistic	(0.390)	(−0.409)	(1.223)	(0.547)	(2.139)			
Risk premium	0.495	−0.677	2.294	1.156	−1.008			
$\overline{\lambda}^{(3-8y)}$	−0.508	−0.257	0.439	−1.921**	−0.525			
t-statistic	(−0.462)	(−0.403)	(0.489)	(−1.989)	(−0.161)			
Risk premium	−1.045	1.274	0.561	−1.879	−0.057			
$\overline{\lambda}^{(>8y)}$	0.282	0.115	0.216	0.231	−0.430			
t-statistic	(0.337)	(0.171)	(0.288)	(0.187)	(−0.136)			
Risk premium	0.135	−0.783	−0.599	0.012	−0.253			

Table VIII—*continued*

Panel B: *Fama-French factor model*								
	MKT	*SMB*	*HML*	*MOM*	*LIQ*	$\bar{\lambda}_0$	R^2 [MAPE]	Adj. R^2 $[Q^{(m)}]$
$\bar{\lambda}^{(<1y)}$	2.765	−0.770	−0.799	12.893***	1.484	8.031***	0.555	0.506
t-statistic	(0.834)	(−0.341)	(−0.325)	(3.215)	(0.292)	(3.511)	[1.598]	[2.914]
Risk premium	2.825	−0.318	−0.139	−0.606	0.013			
$\bar{\lambda}^{(1-3y)}$	0.051	0.422	1.516	−3.116	−1.873			
t-statistic	(0.016)	(0.217)	(0.732)	(−1.078)	(−0.953)			
Risk premium	0.053	0.173	0.209	0.176	−0.009			
$\bar{\lambda}^{(3-8y)}$	−5.312*	2.770**	2.538**	−3.238	2.467			
t-statistic	(−1.720)	(1.973)	(2.269)	(−1.104)	(1.186)			
Risk premium	−5.291	1.337	0.525	0.037	0.186			
$\bar{\lambda}^{(>8y)}$	0.463	−0.272	0.732	2.380	0.587			
t-statistic	(0.176)	(−0.135)	(0.375)	(1.046)	(0.255)			
Risk premium	0.440	−0.133	0.050	−0.006	0.044			

nomic factors contain pricing information at lower frequencies that is not picked up by the analysis on unfiltered monthly data.

C. Robustness checks

We test for robustness with respect to the additional assumptions made for the CF-RW filter by comparing the cycles extracted with the CF-RW filter with those extracted by the Baxter-King (BK) filter (Baxter and King, 1999). Internet Appendix Figure IA.2b shows the filtered market factor using the BK-filter and the CF-RW filter. We find that the cycles extracted by the BK filter are very similar to the CF-RW filter, with a correlation of 0.95.

Table IX
Test results non-nested models

This table reports the test statistic (Rivers and Vuong, 2002; Hall and Pelletier, 2011) J which tests two non-nested models (model 1 and model 2) based on the squared pricing error $Q^{(m)}$. The standard deviation is constructed with 1,000 bootstrap samples. The test statistic follows asymptotically a normal distribution. For each model, we report the factors and estimation method of the exposures, static or rolling window, where rolling window estimation is done with a window size of 60 months. ***, **, and * denote significance at the 1%, 5%, and 10% level, respectively. Bold font refers to model m with a significantly lower squared pricing error (at 5%).

Model 1	*Model 2*	$Q^{(1)} - Q^{(2)}$	J
Macroeconomic, frequency-specific	Macroeconomic, static unfiltered	−5.260**	−2.189
Macroeconomic, frequency-specific	Macroeconomic, rolling window unfiltered	−4.022**	−2.117
Fama-French, frequency-specific	Fama-French, static unfiltered	−1.685	−1.431
Fama-French, frequency-specific	Fama-French, rolling window unfiltered	−2.231*	−1.851
Macroeconomic, static unfiltered	**Fama-French, static unfiltered**	4.612**	2.230
Macroeconomic, rolling window unfiltered	**Fama-French, rolling window unfiltered**	2.828**	2.396
Macroeconomic, frequency-specific	Fama-French, frequency-specific	1.037	1.192

We also perform the pricing analysis using a one-sided backward-looking filter and find that our results are robust to the exclusion of forward-looking information. Internet Appendix Table IA.5 reports these results.

As an additional robustness check, we vary the block size of the bootstrap setting to construct standard errors. These results are reported in Internet Appendix Table IA.6. The results of Table VIII (t-statistics) are not affected by choosing a smaller block size of three months.

VII. Conclusions

This paper introduces a general linear multifactor asset pricing methodology that integrates systematic risk measured at different frequencies into a single pricing equation. Our flexible setup applies to all linear multifactor asset pricing models, works with any frequency filter that produces orthogonal additive components, and allows us to investigate asset pricing relations across frequencies in a multifactor setting.

We use our approach in two applications and perform empirical tests within our setup on macroeconomic and Fama-French type factor models. Our frequency-specific asset pricing framework allows us to obtain several new insights into both factor pricing and model performance for these factor models.

The macroeconomic factor model includes the monthly growth rate of industrial production, the monthly change expected in expected inflation, unexpected inflation, the credit spread, and the term spread. The Fama-French type model includes the market, size, value, momentum, and liquidity factors. We estimate monthly (unfiltered) and frequency-specific versions of these models on 202 US equity portfolios. We use data from January 1968 until December 2016.

We document the behavior of the factors across frequencies. We find that the correlation between macroeconomic factors and Fama-French factors increases with the horizon. The correlation between the market factor and the monthly growth rate of industrial production increases from almost zero correlation on a monthly basis to a correlation of 0.4 for horizons between 3 and 8 years, and to a correlation above 0.5 for fluctuations greater than 8 years. In addition, we find that for horizons between 1 and 3 years, about 20% of the variation in the value factor can be explained by macroeconomic factors. This sheds additional light on the analysis of Asness, Moskowitz, and Pedersen (2013), who found no relation when studying macroeconomic factors as a possible source of common variation driving value and momentum returns on monthly data.

The frequency-specific approach shows the existence of horizon effects in both the loading estimates and risk premia for macroeconomic and Fama-French factors. Our results indicate that at the short horizon, expected inflation, unexpected inflation, and the term spread play a significant role in explaining the cross-section of returns. At business cycle horizons, lower frequency factors such as the credit spread and term spread play significant roles. For the Fama-French model, we find that momentum is priced for horizons up to 1 year, and size and value are priced at the 3 to 8 years horizon. The value factor being priced at the business cycle horizon is in line with Zhang (2005) and Petkova and Zhang (2005).

We perform bootstrap versions of a non-nested model test to compare the goodness of fit for the different specifications. In line with the literature, we find that using monthly unfiltered data, the Fama-French model outperforms the macroeconomic model significantly. For the macroeconomic model, the frequency-specific specification significantly outperforms the monthly specification in terms of goodness of fit. Given the improved fit for the frequency-specific macroeconomic model, our test no longer shows any significant difference in goodness of fit for the frequency-specific versions of the macroeconomic and Fama-French models. These results indicate that the macroeconomic factors contain pricing information at lower frequencies that is not picked up by the analysis on unfiltered monthly data.

The results in this paper show empirically that there are frequency-specific effects in two well-known linear multifactor asset pricing models. Our findings are relevant for both portfolio construction and performance evaluation (e.g., Cohen, Polk, and Vuolteenaho, 2009; Dahlquist and Ødegaard, 2018), since long-term investors collect short-term risk premia while bearing less of the risk associated with these short-term factors in the long run (e.g., Kamara, Korajczyk, Lou, and

Sadka, 2016).

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Internet Appendix for "Equity Risk Factors for the Long and Short Run: Pricing and Performance at Different Frequencies"

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In this document we provide supplementary material to Van der Zwan, Hennink, and Tuijp (2021).

I. Time series decomposition and the ideal band pass filter

A. Decomposition of the time series

Under the assumption that a time series y_t is covariance stationary, the spectral representation theorem (see, e.g., Parzen (1961) and Priestley (1981) for more details) states that y_t can be written as

$$y_t = \sum_{\tau=1}^{\mathcal{T}} v_\tau \cos(2\pi\omega_\tau t) + u_\tau \sin(2\pi\omega_\tau t), \qquad v_\tau, u_\tau \sim N(0, \sigma_\tau^2) \text{ i.i.d.} \tag{IA.1}$$

where σ_τ^2 corresponds to the spectral density function at frequency ω_τ. In other words, any covariance stationary time series can be decomposed into multiple orthogonal components capturing fluctuations at different frequencies. That is,

$$y_t = \sum_{\tau=1}^{\mathcal{T}} y_t^{(p_{\tau-1}, p_\tau)} = \sum_{\tau=1}^{\mathcal{T}} y_t^{(\tau)}, \tag{IA.2}$$

where $y_t^{(p_{\tau-1},p_\tau)} \equiv y_t^{(\tau)}$. p_τ denotes the length of one cycle, e.g., the periodicity. Thus, $y_t^{(p_{\tau-1},p_\tau)}$ denotes the component of y_t with period oscillations between $p_{\tau-1}$ and p_τ, with $p_0 = 2 \leq p_1 < ... < p_{\mathcal{T}} = \infty$. The frequency and periodicity are related as $\omega_\tau = 2\pi/p_\tau$. It follows that the set $\{1,...,\mathcal{T}\}$ spans the whole frequency domain—that is, the components are non-overlapping and contain fluctuations from every persistence level. Since these pass bands are non-overlapping span and all pass bands together contain fluctuations from every level, it also holds that

$$\mathrm{var}(y_t) = \sum_{\tau=1}^{\mathcal{T}} \mathrm{var}(y_t^{(\tau)}). \tag{IA.3}$$

Further, from this theory it follows that the ideal band pass filter can be employed to extract those frequency components. We describe the ideal band pass filter in section IB.

The above mentioned spectral representation theorem can be related to the approach of Ortu, Severino, Tamoni, and Tebaldi (2020) in the following way. Under the assumption that a time series y_t is covariance stationary, the Wold decomposition theorem states that y_t can be written as

$$y_t = \sum_{\ell=0}^{\infty} \psi_\ell \varepsilon_{t-\ell}, \tag{IA.4}$$

where ε_t denotes a white noise process and ψ_ℓ denotes the absolute summable Wold coefficients, such that the spectral functions of y_t are well-defined. Then, Theorem 2 in Ortu, Severino, Tamoni, and Tebaldi (2020) shows that we can alternatively write the time series y_t as (IA.2).

Since the portfolio (excess) returns and factors meet the aforementioned requirement, e.g., they are covariance stationary, we can decompose both into a sum of horizon-specific components. Further, having this representation, it follows that the covariance between factors and returns can also be decomposed such that (see, e.g., Boons and Tamoni, Appendix B2 for a proof of this result in context of the Wold decomposition),

$$\operatorname{cov}(r_{i,t}, f_t) = \sum_{\tau=1}^{\mathcal{T}} \operatorname{cov}\left(r_{i,t}^{(\tau)}, f_t^{(\tau)}\right). \tag{IA.5}$$

In other words, there are no cross-component covariance terms across different horizons between the excess returns and the factors. This follows from the fact that the components $r_{i,t}^{(\tau)}$ and $f_t^{(\sigma)}$ capture information from different frequencies for $\tau \neq \sigma$.

B. The ideal band pass filter

We briefly describe the ideal band pass filter. A more detailed derivation of the ideal band pass filter can be found in for example, Sargent (1987), chapter 11.6. One can decompose a covariance stationary time series $\{y_t : t = 1, ..., T\}$ orthogonally such that both components lie in a different frequency range, that is,

$$y_t = \widetilde{y}_t + \eta_t, \tag{IA.6}$$

where $\widetilde{y}_t$ contains the desired frequency range $[a, b]$, where $a < b$. The frequency a (b) refers to fluctuations with a period larger than p_a (p_b). For example, in monthly data when $a = 2\pi/12$ and $b = 2\pi/36$, $p_a = 12$ and $p_b = 36$, i.e., the periodicity of the filtered component is between 1 and 3 years. The component η_t contains all the frequencies outside of $[a, b]$. It is a eminent result the filtered time series can be written as a moving average of the original time series (for example, Sargent (1987), p. 256–260),

$$\widetilde{y}_t = G(L)y_t, \tag{IA.7}$$

where $G(L)$ corresponds to the ideal pass band filter with the following form

$$G(L) = \sum_{\ell=-\infty}^{\infty} g_\ell^* L^\ell, \tag{IA.8}$$

with $L^\ell(y_t) \equiv y_{t-\ell}$. Here, the ideal band pass filter weights g_ℓ^* are given by

$$g_\ell^* = \begin{cases} \frac{\sin(\omega_2 \ell) - \sin(\omega_1 \ell)}{\pi \ell} & \text{for } \ell \neq 0 \\ \frac{\omega_2 - \omega_1}{\pi} & \text{for } \ell = 0. \end{cases} \tag{IA.9}$$

Using the fact that a time series can be represented by a combination of infinite amount of sine and cosines functions of various frequencies (Baron Fourier, 1878), the ideal band pass filter of (IA.8) corresponds to the following frequency response function

$$G\left(e^{-i\omega}\right) = \begin{cases} 1 & \text{for } \omega \in [a, b] \\ 0 & \text{otherwise.} \end{cases} \tag{IA.10}$$

The frequency response function is defined in the frequency domain. From this perspective, one can see that this filter can perfectly maintain the the fluctuations between the pass band $[a,b]$ of the original time series y_t without amplifying them, while suppressing the frequencies outside this pass band. However, employing this ideal band pass filter requires an infinite amount of data. We therefore use an approximation to this ideal band pass filter.

II. Proof of Proposition 1

Proof of Proposition 1: Similar to the linear multifactor case of Cochrane (2005) chapter 6, we assume that the stochastic discount factor has a linear form in the frequency components, e.g.,

$$m_{t+1} = a + \sum_{\tau=1}^{\mathcal{T}} b^{(\tau)\prime} f_{t+1}^{(\tau)}. \tag{IA.11}$$

We further have the traditional asset pricing equation $\mathbb{E}[m_{t+1}R_{i,t+1}] = 1$. We now work out this pricing equation

$$\begin{aligned}
1 = \mathbb{E}[m_{t+1}R_{i,t+1}] &= \mathbb{E}[m_{t+1}]\mathbb{E}[R_{i,t+1}] - \mathrm{cov}(m_{t+1}, R_{i,t+1}) \\
\iff \mathbb{E}[R_{i,t+1}] &= \frac{1}{\mathbb{E}[m_{t+1}]} - \frac{\mathrm{cov}(m_{t+1}, R_{i,t+1})}{\mathbb{E}[m_{t+1}]} = \frac{1}{a} - \frac{\mathrm{cov}(m_{t+1}, R_{i,t+1})}{a} \\
\iff \mathbb{E}[r_{i,t+1}] \equiv \mathbb{E}[R_{i,t+1}] - R_{t+1}^{f} &= -\frac{\mathrm{cov}(m_{t+1}, R_{i,t+1})}{\mathbb{E}[m_{t+1}]} = -\frac{\mathrm{cov}(m_{t+1}, R_{i,t+1})}{a}
\end{aligned} \tag{IA.12}$$

where we use that $\mathbb{E}[m_{t+1}] = a$ and assuming that the mean of factors is captured in a, e.g., $\mathbb{E}[f_{t+1}] = 0$. In the last line, we use the pricing equation for the risk-free rate R_{t+1}^{f} that $1 = \mathbb{E}[m_{t+1}R_{t+1}^{f}] = \mathbb{E}[m_{t+1}]R_{t+1}^{f}$, since the risk-free rate is considered to be known. Thus, $R_{t+1}^{f} =$

$\frac{1}{\mathbb{E}[m_{t+1}]}$. We work out the covariance term in (IA.2),

$$
\begin{aligned}
\operatorname{cov}(m_{t+1}, R_{i,t}) &= \operatorname{cov}\left(a + \sum_{\tau=1}^{\mathcal{T}} b^{(\tau)\prime} f_{t+1}^{(\tau)}, R_{i,t+1}\right) \\
&= \operatorname{cov}\left(\sum_{\tau=1}^{\mathcal{T}} b^{(\tau)\prime} f_{t+1}^{(\tau)}, \sum_{\tau=1}^{\mathcal{T}} R_{i,t+1}^{(\tau)}\right) \\
&= \sum_{\tau=1}^{\mathcal{T}} b^{(\tau)\prime} \mathbb{E}\left[f_{t+1}^{(\tau)} R_{i,t+1}^{(\tau)}\right] + \sum_{\substack{s,\tau=1 \\ s\neq\tau}}^{\mathcal{T}} b^{(\tau)\prime} \mathbb{E}\left[f_{t+1}^{(\tau)} R_{i,t}^{(s)}\right] + \sum_{\substack{s,\tau=1 \\ s\neq\tau}}^{\mathcal{T}} b^{(s)\prime} \mathbb{E}\left[f_{t+1}^{(s)} R_{i,t+1}^{(\tau)}\right] \\
&= \sum_{\tau=1}^{\mathcal{T}} b^{(\tau)\prime} \mathbb{E}\left[f_{t+1}^{(\tau)} R_{i,t+1}^{(\tau)}\right],
\end{aligned}
$$

where the last line follows from the fact that we decompose the factors and returns as in (IA.2), that is, orthogonal across horizon. Therefore, theoretically, cross-component covariance terms are zero. We plug this result into (IA.12)

$$
\mathbb{E}[r_{i,t+1}] = -\frac{1}{a}\left(\sum_{\tau=1}^{\mathcal{T}} b^{(\tau)\prime} \mathbb{E}\left[f_{t+1}^{(\tau)} R_{i,t}^{(\tau)}\right]\right). \tag{IA.13}
$$

We now define the frequency-specific beta as

$$
\beta_i^{(\tau)} = \operatorname{Var}\left(f_{t+1}^{(\tau)}\right)^{-1} \operatorname{Cov}\left(f_{t+1}^{(\tau)},\ R_{i,t+1}^{(\tau)}\right) \tag{IA.14}
$$

for $\tau = 1, ..., \mathcal{T}$. We recover these frequency-specific betas in the equation above by multiplying the term in the summation by $\mathbb{E}\left[f_{t+1}^{(\tau)} f_{t+1}^{(\tau)\prime}\right] \mathbb{E}\left[f_{t+1}^{(\tau)} f_{t+1}^{(\tau)\prime}\right]^{-1}$, thus,

$$
\begin{aligned}
\mathbb{E}[r_{i,t+1}] &= -\frac{1}{a}\left(\sum_{\tau=1}^{\mathcal{T}} b^{(\tau)\prime}\, \mathbb{E}\left[f_{t+1}^{(\tau)} f_{t+1}^{(\tau)\prime}\right] \mathbb{E}\left[f_{t+1}^{(\tau)} f_{t+1}^{(\tau)\prime}\right]^{-1} \mathbb{E}\left[f_{t+1}^{(\tau)} R_{i,t}^{(\tau)}\right]\right) \\
&= -\frac{1}{a}\left(\sum_{\tau=1}^{\mathcal{T}} b^{(\tau)\prime}\, \mathbb{E}\left[f_{t+1}^{(\tau)} f_{t+1}^{(\tau)\prime}\right] \beta_i^{(\tau)}\right).
\end{aligned}
$$

We now define λ_0 and $\lambda^{(\tau)}$ as

$$
\begin{aligned}
\lambda_0 &\equiv \frac{1}{\mathbb{E}[m_{t+1}]} = \frac{1}{a} \\
\lambda^{(\tau)} &\equiv -\frac{1}{a}\mathbb{E}\left[f_{t+1}^{(\tau)} f_{t+1}^{(\tau)\prime}\right] b^{(\tau)} = -\lambda_0\, \mathbb{E}\left[f_{t+1}^{(\tau)} f_{t+1}^{(\tau)\prime}\right] b^{(\tau)},
\end{aligned}
$$

where $\lambda^{(\tau)}$ denotes the prices of risk at frequency τ. Hence, we have that

$$
\mathbb{E}[r_{i,t+1}] = \sum_{\tau=1}^{\mathcal{T}} \lambda^{(\tau)\prime} \beta_i^{(\tau)}. \tag{IA.15}
$$

Q.E.D.

III. Data

This section provides a description of the data and its sources, details on the construction of expected inflation used to construct two macroeconomic factors, and additional statistics on the test portfolios.

TABLE IA.1: Data description

Variable names, ranges and source of the data are provided in this table.

Variable	Sample range	Source
Consumer Price Index (CPI)	Jan 1955–Dec 2016	OECD
Corporate Bond yield, Moody's Aaa	Jan 1968–Dec 2016	St. Louis Fed
Corporate Bond yield, Moody's Baa	Jan 1968–Dec 2016	St. Louis Fed
Fama-French three factors (market, size, value)	Jan 1968–Dec 2016	Kenneth R. French
Industrial Production, seasonally adjusted	Jan 1968–Dec 2016	St. Louis Fed
Momentum factor	Jan 1968–Dec 2016	Kenneth R. French
One Month Treasury Bill	Jan 1955–Dec 2016	Kenneth R. French
Portfolio returns	Jan 1968–Dec 2016	Kenneth R. French
Liquidity factor (traded)	Jan 1968–Dec 2016	Lubos Pastor
Long-term Government Bond over 10 year (LGB)	Jan 1968–Dec 2016	St. Louis Fed
Three Month Treasury Bill yield (TB)	Jan 1968–Dec 2016	OECD

A. *Definitions of macroeconomic factors*

- Industrial production (IP): the factor is defined as the monthly growth rate[1] of industrial production.

- Expected inflation (EI): we define EI as the monthly change in the inflation expectation. Thus,

$$EI_t = \mathbb{E}_t[I_{t+1}] - \mathbb{E}_{t-1}[I_t], \tag{IA.16}$$

where I_t is the monthly inflation rate, measured as the natural log difference of the seasonally adjusted consumer price index (CPI) of period t. $\mathbb{E}_{t-1}[\cdot]$ denotes the expectation operator conditional on the information available at the end of month $t-1$. We follow the method of Fama and Gibbons (1984) as in Chen, Roll, and Ross (1986), Liu and Zhang (2008), and

[1] Chen, Roll, and Ross (1986) also consider the yearly growth rate to account for growth in the long run and to overcome the noise in monthly growth rates, however, we account for this by filtering.

Shen, Yu, and Zhao (2017) to construct expected inflation. The precise construction is given in section III.B of this Appendix.

- Unexpected inflation (UI): UI captures inflation information not captured by EI, so We define unexpected inflation (UI) as

$$UI_t = I_t - \mathbb{E}_{t-1}[I_t], \tag{IA.17}$$

where $I_t = \log \text{CPI}_t - \log \text{CPI}_{t-1}$ and $\mathbb{E}_{t-1}[I_t]$ (Fama and Gibbons, 1984).

- Credit spread (DEF): a yield spread capturing the default premium. We construct DEF following Liu and Zhang (2008) by taking the spread between the Baa corporate bond yield and the Aaa corporate bond yield. This differs in two ways from Chen, Roll, and Ross (1986). First, they consider returns and we consider the yield. Second, they choose a long-term government bond instead of Aaa corporate bond. Instead, we consider a Aaa corporate bond rather than a government bond, since the use of government bond data may unintendedly introduce a liquidity effect in the variable (e.g., Elton, Gruber, Agrawal, and Mann, 2001).

- Term spread (TS): we define the term spread as the spread between Treasury bonds with a maturity over ten years and the three-month Treasury bill.

B. Construction of expected inflation

We follow Fama and Gibbons (1984) to model the monthly expected inflation as in Chen, Roll, and Ross (1986), Liu and Zhang (2008) and Shen, Yu, and Zhao (2017). Following Fisher (1930), the expected inflation can be written as

$$\mathbb{E}_{t-1}[I_t] = R_t^f - \mathbb{E}_{t-1}[\mathrm{RHO}_t], \tag{IA.18}$$

where $I_t = \log \mathrm{CPI}_t - \log \mathrm{CPI}_{t-1}$, the risk-free rate is denoted as R_t^f (one-month treasury bill), and RHO_t is defined as the ex-post real return on treasury bills in period t, that is, $\mathrm{RHO}_t = R_t^f - I_t$. Fama and Gibbons (1984) employ a first order moving average MA(1) process to model the change in RHO_t, so that

$$\Delta \mathrm{RHO}_t = \varepsilon_t + \theta \varepsilon_{t-1}. \tag{IA.19}$$

Note that we need CPI and treasury bill data prior to the sample beginning, January 1968, to estimate θ at the beginning of the sample. We model the monthly difference in ex-post real return on treasury bills using an updating MA(1) model, e.g., expanding window estimation starting from January 1955 on wards. Using the fitted residual values of (IA.19), it follows that

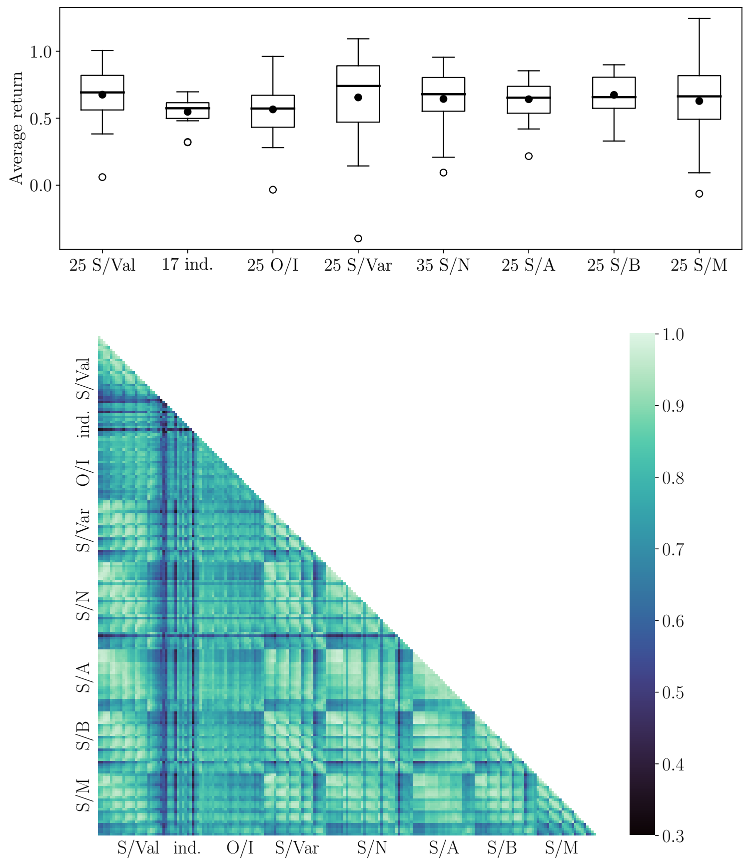
$$\mathbb{E}_{t-1}[\mathrm{RHO}_t] = \mathrm{RHO}_{t-1} - \widehat{\varepsilon}_t - \widehat{\theta}\widehat{\varepsilon}_{t-1}. \tag{IA.20}$$

We plug this into (IA.18) to obtain expected inflation, which we then use to create the variables expected inflation (yielding the first difference) and unexpected inflation.

C. Additional descriptive statistics equity returns

In this subsection we show some additional statistics on the 202 equity portfolios in excess of the risk-free rate on monthly frequency. These equity portfolios consist of seven sets of bivariate sorted portfolios and a set of industry portfolios. Figure IA.1 shows a visualization of Table 1 and

FIGURE IA.1: Boxplots of the average excess returns and correlation matrix of the excess returns of the 202 equity portfolios. This figure summarizes the average excess returns of the 202 portfolios for the sample which runs from January 1968 to December 2016. The solid thick line corresponds to the median, the black dot corresponds to the mean. The circle are "outliers". S/Val corresponds to size-value, ind. to industry, O/I to operating profitability-investment activity, S/Var to size-variance, S/N to size-net share issuance, S/A to size-accruals and S/M to size-momentum.



the correlation between the portfolios. The correlation of the portfolios lies between 0.3 and 0.98 for monthly data.

IV. Filter analysis

A. Filter choice

We present a comparison between the Christiano-Fitzgerald random walk (CF-RW) filter, a moving average (MA) filter and the Baxter-King (BK) filter. Figure IA.2 shows the market factor, filtered with three types of filters. To illustrate the difference between the CF-RW filter and a moving average filter, we zoom in on the period January 2005 to December 2010, which contains a crisis. Figure IA.2a zooms in on this period and shows the MA filter and CF-RW filter compared to the monthly factor. The moving average filter has 36 lags, e.g., a three-year moving average, and the CF-RW filter filtering out fluctuations between 1 to 3 years. The CF-RW filter is able to capture the sudden drop, whereas the moving average filter is not—it rather smooths out the loss.

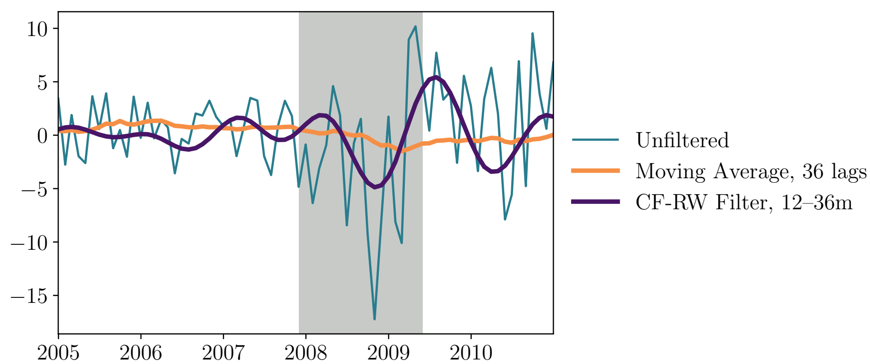
Figure IA.2b shows the performance of the Baxter-King filter (dashed) and the CF-RW filter, both filtering out fluctuations between 3 to 8 years over the whole sample. The filtered series follow a similar pattern, although at some points the BK filter seems to be less smooth at certain points (between 1990 and 2000). The correlation between the two series is high, 0.95, which is also the case for other factors and equity returns. Also note the required observation loss of 36 observations on both sides when using the BK filter. This indicates that both filters are able to pick up relevant and similar low-frequency information.

B. Orthogonality of the filter

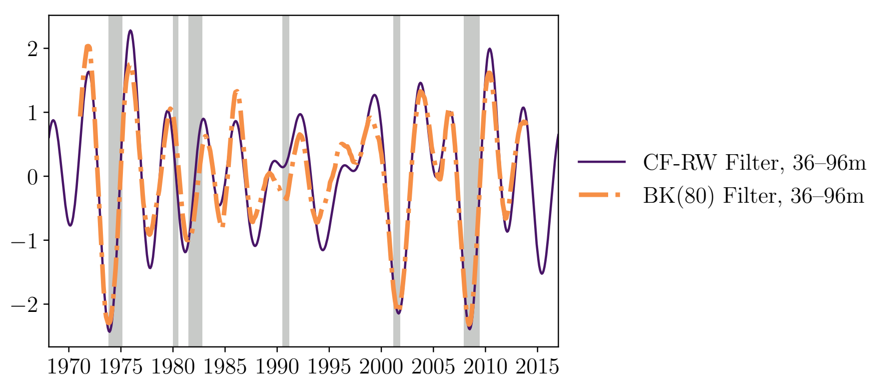
The orthogonal decomposition of a time series forms the basis of our theoretical framework. That is, the frequency components are orthogonal across horizons. Theoretically, the ideal band pass filter produces orthogonal components. However, we have a limited sample size. We therefore

FIGURE IA.2: Filtered market factor using different filters. The market factor is filtered using multiple filters and multiple pass bands. Figure IA.2a shows the monthly market factor, the Christiano-Fitzgerald random walk (CF-RW) filter with pass band 1 to 3 years and a moving average with 36 lags, between January 2005 and December 2010. Figure IA.2b shows the filtered market factor, using the Baxter-King (BK) filter of order 36 and the CF-RW filter, both between 3 to 8 years, over the whole sample. The grey areas represent NBER recessions.

(A) Market factor filtered with a moving average filter and the CF-RW filter.



(B) Market factor filtered with the BK- and CF-RW filter.



must assess the in-sample correlation between the components. Table IA.2 shows the in-sample correlations between the filtered factors. The correlation between the pass bands are generally zero or almost zero (< 0.05). For the lower frequencies the filter has more difficulty to isolate frequency specific information—the correlation between the components capturing 36–96 months and greater than 96 months are correlated, but generally lower than 0.15 in absolute terms.

C. One-sided Christiano-Fitzgerald filter

The CF-RW filter described in Section IV.E is a two-sided filter, which means it incorporates forward-looking information to filter the time series. Recall that this filter is defined as

$$\widetilde{y}_t = \widetilde{g}_{t-1}y_1 + g^*_{t-2}y_2 + \cdots + g^*_1 y_{t-1} + g^*_0 y_t + g^*_1 y_{t+1} + \ldots + g^*_{T-t-1}y_{T-1} + \widetilde{g}_{T-t}y_T. \qquad \text{(IA.21)}$$

Specifically, the filtered time series of (IA.21) at time t depends on both past $(y_1, \ldots, y_t)$ and future observations $(y_{t+1}, \ldots, y_T)$. To analyze a conditional model, we need a backward looking filter. Recall that (IA.21) is an approximation of the ideal band pass filter, which we can adjust such that it only uses past and contemporaneous information. The filtered time series is then calculated as

$$\widetilde{y}_t = \widehat{g}_{t-1}y_1 + g^*_{t-2}y_2 + \cdots + g^*_1 y_{t-1} + g^*_0 y_t, \qquad \text{(IA.22)}$$

where $\widehat{g}_{t-1} = -\left(g^*_0 + g^*_1 + \ldots + g^*_{t-2}\right)$. We refer to this filter as the CF-RW one-sided filter. This adjustment leads to a less ideal approximation of the optimal filter, which induces a “leakage effect”, which means that in some cases the neighboring frequencies of $[\omega_1, \omega_2]$ are not entirely filtered out. This is indeed the case, as the cross-component correlations are generally non-zero, but moderate. However, for neighboring pass bands, especially lower frequencies, the absolute correlation can increase to 0.5. The cross-sectional results using this filter are reported in Table IA.5.

TABLE IA.2: Correlation between the CF-RW filtered series

This table reports the correlations between the filtered macroeconomic and Fama-French factors across pass bands. The factors are filtered using the CF-RW filter.

		<12 months					12–36 months					36–96 months					>96 months				
		IP	*EI*	*UI*	*RP*	*TS*	*IP*	*EI*	*UI*	*RP*	*TS*	*IP*	*EI*	*UI*	*RP*	*TS*	*IP*	*EI*	*UI*	*RP*	*TS*
<12 months	*IP*	1.00																			
	EI	0.01	1.00																		
	UI	-0.09	0.63	1.00																	
	DEF	-0.16	-0.15	0.07	1.00																
	TS	0.04	0.62	0.64	-0.14	1.00															
12–36 months	*IP*	0.00	0.00	0.00	0.00	0.00	1.00														
	EI	0.00	0.01	0.01	0.01	0.01	0.18	1.00													
	UI	0.00	0.00	0.01	0.02	0.00	-0.41	0.22	1.00												
	DEF	0.00	0.00	0.00	0.01	0.00	-0.49	0.12	0.18	1.00											
	TS	0.00	0.00	0.00	0.00	0.00	0.02	0.52	0.19	0.20	1.00										
36–96 months	*IP*	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	-0.02	0.00	1.00									
	EI	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.01	-0.01	0.31	1.00								
	UI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	-0.42	-0.08	1.00							
	DEF	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.02	0.01	-0.48	0.08	0.50	1.00						
	TS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.34	0.51	0.16	0.31	1.00					
>96 months	*IP*	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.01	0.06	-0.02	0.01	0.06	1.00				
	EI	0.01	0.00	0.00	-0.01	0.00	0.02	-0.03	0.00	0.03	0.00	0.15	-0.41	-0.07	-0.12	-0.11	0.10	1.00			
	UI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.02	0.03	0.00	0.01	-0.33	-0.08	1.00		
	DEF	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	-0.01	0.00	-0.55	-0.20	0.37	1.00	
	TS	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.02	0.01	0.00	0.05	0.11	0.13	0.50	0.16	1.00

		<12 months					12–36 months					36–96 months					>96 months				
		MKT	*SMB*	*HML*	*MOM*	*LIQ*	*MKT*	*SMB*	*HML*	*MOM*	*LIQ*	*MKT*	*SMB*	*HML*	*MOM*	*LIQ*	*MKT*	*SMB*	*HML*	*MOM*	*LIQ*
<12 months	*MKT*	1.00																			
	SMB	0.27	1.00																		
	HML	-0.29	-0.23	1.00																	
	MOM	-0.13	-0.02	-0.17	1.00																
	LIQ	-0.01	-0.03	0.03	-0.02	1.00															
12–36 months	*MKT*	0.00	0.00	0.00	0.00	0.00	1.00														
	SMB	0.00	0.01	0.00	0.00	0.00	0.54	1.00													
	HML	0.00	0.00	0.00	0.00	0.00	-0.25	-0.34	1.00												
	MOM	0.00	0.00	0.00	0.00	0.00	-0.20	0.10	-0.29	1.00											
	LIQ	0.00	0.00	0.00	0.00	0.00	0.17	0.24	0.08	-0.08	1.00										
36–96 months	*MKT*	0.00	0.00	0.00	0.00	0.00	0.02	0.01	-0.02	0.01	0.03	1.00									
	SMB	0.00	0.00	0.00	0.00	0.00	0.04	0.01	-0.01	0.02	0.01	0.19	1.00								
	HML	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	-0.01	-0.01	-0.20	0.12	1.00							
	MOM	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.01	0.02	-0.15	-0.61	-0.20	1.00						
	LIQ	0.00	0.00	0.00	0.00	0.0	0.01	0.00	-0.03	0.02	0.05	-0.26	-0.29	-0.20	0.28	1.00					
>96 months	*MKT*	0.00	0.00	0.00	0.00	0.00	-0.04	0.00	0.01	-0.03	0.01	-0.05	0.13	0.07	-0.14	-0.18	1.00				
	SMB	0.00	0.00	0.00	0.00	0.00	0.03	0.00	-0.01	0.02	-0.01	0.05	-0.11	-0.07	0.12	0.17	-0.40	1.00			
	HML	0.00	0.00	0.00	0.00	0.00	0.03	0.01	-0.02	0.02	0.02	0.00	-0.08	-0.13	0.10	0.27	-0.43	0.38	1.00		
	MOM	0.00	0.00	0.00	0.00	0.00	-0.04	0.00	0.02	-0.03	-0.01	-0.04	0.14	0.12	-0.14	-0.24	0.22	-0.22	-0.27	1.00	
	LIQ	0.00	0.00	0.00	0.00	0.00	-0.02	-0.01	0.02	-0.01	-0.02	0.01	0.04	0.09	-0.04	-0.14	-0.22	0.07	0.08	0.28	1.00

V. Bootstrap procedures

A. *Bootstrapped standard errors*

We calculate the standard errors numerically by using bootstrap. Specifically, we use the circular block bootstrap approach of Politis and Romano (1992) to account for possible underlying dependence structure in the data. Our bootstrap approach works as follows. In each bootstrap, we draw blocks of 48 observations with replacement from our original monthly sample consisting of $T = 588$ observations from January 1968 to December 2016. Using these blocks, we construct new samples of length T for unfiltered excess returns and unfiltered factors. For each bootstrap sample b, we filter both the newly constructed data set, e.g., the excess returns and factors. We create in total $B = 1,000$ bootstrap samples. We then obtain standard errors for the estimates for the average effects/price of risk by calculating the standard deviation of this parameter estimation across B bootstrap samples.

Note that in the first stage of our analysis, e.g., exposure estimation, we work with filtered data. The reason for not bootstrapping filtered data is the assumptions made on the data in the bootstrap method. Circular block bootstrap and other bootstrap methods such as stationary block bootstrap and moving block bootstrap requires the data to be stationarity and weakly dependent. For low frequency filtered data, this can be a problem. For example, the case where we consider a block length of 30 observations and sample from filtered data containing fluctuations longer then 96 months. Since the block length is shorter than half a completion of a cycle, this block could capture a part of the longer-term trend, and is hence not stationary. This induces problems in the newly constructed bootstrapped sample b if for example the last observation of a block c and the first observation of the following block $c+1$ do not naturally follow each other. If c captures a long-term downward trend and $c+1$ captures a fluctuation capturing the top, the new sample contains a break since these blocks are disjoint. We therefore sample from the unfiltered data using blocks, which still preserve possible correlation structures, and then filter these new samples.

The bootstrapped standard errors are robust to alternative bootstrap methods such as moving block. Using conventional bootstrap methods, that is, drawing random individual observations, we observe that for lower frequencies the standard error estimates become less conservative. Our choice for circular block bootstrap rather than moving block bootstrap to avoid undersampling of the sample endpoints. Further, the bootstrapped standard errors are also robust to alternative block lengths. We used the data-driven block length selection method of Politis and White (2004) to select the block length of 48 months. They also recommend varying the block length to check its robustness, as this method also relies on a selection cut-off of five lags in the autocovariance.

B. Non-nested model testing

To assess the performance of the models, we use a test statistic derived by Rivers and Vuong (2002)) and Hall and Pelletier (2011) to compare two non-nested models estimated by using moment conditions. The test statistic is asymptotically normal distributed. This approach is similar to the method of Hansen and Hansen and Jagannathan (1997). The test essentially compares the Generalized Method of Moments (GMM) objective values of model $m = 1, 2$. The GMM objective denotes

$$Q^{(m)} = \min_{\theta} g(\theta)' W g(\theta), \tag{IA.23}$$

where $g(\theta)$ denotes the moment conditions and W the weighting matrix. GMM aims to minimize the pricing errors. Our setting can be seen as a special case with weighting matrix W being the identity matrix and $g(\theta)$ denoting the pricing errors of our model. $Q^{(m)}$ reduces then to the sum of squared pricing errors, e.g., $Q^{(m)} = \frac{1}{T}\sum_{t=1}^{T} \alpha_t^{(m)\prime} \alpha_t^{(m)}$, where $\alpha_t^{(m)}$ is the $N \times 1$ vector of pricing errors corresponding to model m. The test statistic then denotes

$$J = \frac{Q^{(1)} - Q^{(2)}}{\hat{\sigma}_T / \sqrt{T}}, \tag{IA.24}$$

where $\hat{\sigma}_T$ is a consistent estimator of the asymptotic standard deviation of the difference between $Q^{(1)}$ and $Q^{(2)}$. We obtain the standard error $\hat{\sigma}_T/\sqrt{T}$ by bootstrapping.

We use the bootstrap approach described above in Section V.A to obtain an estimation of asymptotic variance $\hat{\sigma}_T^2$ in the test statistic of Rivers and Vuong (2002) and Hall and Pelletier (2011). Specifically, in each bootstrap sample b we estimate the pricing errors of both models $m = 1,2$, calculate the GMM objective $Q^{(m)}$ for $m = 1,2$ and the corresponding GMM objective difference, $Q^{(1)} - Q^{(2)}$. We do this $B = 1{,}000$ times and calculate the standard error $\hat{\sigma}_T/\sqrt{T}$ of $Q^{(1)} - Q^{(2)}$ across B simulations.

VI. Additional Asset Pricing Results

Table IA.3 reports the cross-sectional correlation between the frequency-specific exposures. The correlation for both the macroeconomic and Fama-French model between the frequency-specific exposures is moderate, generally ranging from 0.05–0.50. Only for the market factor this correlation is high for the neighboring frequencies lower than 12 months and 12–36 months.

To assess the influence of including a constant in the second stage on the negative price of risk found for the market factor in our empirical setting, we consider the specification where we restrict the constant to be set at zero, e.g., $\lambda_0 = 0$. The results are shown for the Fama-French model in Table IA.4. Similar to Eiling (2013), we find that without an intercept, the market price of risk is positive in the monthly settings, indicating little variation in the market betas across the portfolios.

In the frequency-specific specification we not not find this difference—we find similar results for including a constant λ_0 and for restricting this constant to be zero.

The frequency domain filter that we use includes forward-looking information. We also test the results for this filter when truncating the future weights, i.e., not taking forward-looking information into account. See Section IV.C for a detailed description of this backward-looking filter. The resulting filtered time series are less smooth compared to conventional frequency domain filtering methods. The pricing results using the one-sided CF-RW filter and the conventional filter are reported in Table IA.5. We find similar results for both methods: similar point estimates and significance, goodness of fit and MAPE. Minor differences can be found for the macroeconomic data at lower frequencies, since the adjusted filter is not as smooth as the conventional filter. Thus, the cross-sectional results are not affected by incorporating forward-looking information.

Table IA.6 report the cross-sectional results of the 202 equity portfolios with standard errors obtained from the bootstrap procedure, with block size 3. The t-statistics are generally higher than our basis case of 48 months. Based on the 5% significance level, we find the same factors to be significant. Additionally, the market factor is significant at a 5% level. In short, our case of block size 48 months produce more conservative t-statistics, especially on lower frequencies, since it allows to pick up the possible low frequency correlation up to 48 months.

TABLE IA.3: Correlation between the frequency-specific exposures

This table reports the correlations between the frequency-specific exposure estimates over the four pass bands. The frequency-specific exposures are obtained by regressing the filtered portfolio returns on the filtered set of factors and a constant. Factors and returns are filtered using the CF-RW filter.

		<1 year					1–3 years					3–8 years					>8 years				
		IP	*EI*	*UI*	*RP*	*TS*	*IP*	*EI*	*UI*	*RP*	*TS*	*IP*	*EI*	*UI*	*RP*	*TS*	*IP*	*EI*	*UI*	*RP*	*TS*
>1 year	IP	1.00																			
	EI	0.19	1.00																		
	UI	0.17	-0.45	1.00																	
	DEF	-0.08	-0.21	-0.24	1.00																
	TS	-0.39	-0.71	0.02	0.23	1.00															
1–3 years	IP	-0.21	-0.41	0.00	0.47	0.54	1.00														
	EI	-0.27	-0.61	0.07	0.43	0.65	0.52	1.00													
	UI	-0.23	-0.53	0.04	0.60	0.53	0.77	0.68	1.00												
	DEF	-0.28	-0.47	-0.12	0.61	0.55	0.75	0.65	0.71	1.00											
	TS	-0.00	0.39	0.00	-0.59	-0.16	-0.36	-0.48	-0.55	-0.51	1.00										
3–8 years	IP	-0.10	-0.05	-0.11	-0.03	0.19	0.19	0.16	0.10	0.10	0.13	1.00									
	EI	-0.01	-0.26	0.27	-0.02	0.32	0.12	0.40	0.24	0.12	-0.03	-0.16	1.00								
	UI	0.04	0.36	-0.05	-0.35	-0.28	-0.32	-0.52	-0.54	-0.27	0.26	-0.20	-0.28	1.00							
	DEF	-0.28	-0.37	-0.14	0.26	0.41	0.42	0.54	0.51	0.48	-0.17	0.55	0.18	-0.64	1.00						
	TS	-0.28	-0.37	-0.09	0.41	0.19	0.13	0.16	0.24	0.33	-0.40	-0.39	-0.34	0.05	-0.10	1.00					
>8 years	IP	0.21	0.59	-0.23	-0.39	-0.54	-0.40	-0.61	-0.55	-0.59	0.28	0.16	-0.33	0.23	-0.24	-0.34	1.00				
	EI	-0.15	-0.44	0.22	0.16	0.35	0.13	0.32	0.28	0.21	-0.32	-0.46	0.50	-0.04	-0.08	0.30	-0.52	1.00			
	UI	0.32	0.53	-0.15	-0.28	-0.53	-0.34	-0.58	-0.50	-0.50	0.23	-0.07	-0.16	0.17	-0.24	-0.35	0.72	-0.33	1.00		
	DEF	-0.17	0.15	0.02	-0.07	-0.10	-0.16	-0.17	-0.12	-0.17	0.00	-0.35	-0.01	0.33	-0.30	0.17	0.14	0.26	-0.15	1.00	
	TS	-0.11	-0.34	0.05	0.35	0.22	0.28	0.25	0.33	0.42	-0.31	-0.02	-0.19	0.02	0.11	0.50	-0.57	0.16	-0.74	0.08	1.0

		<1 year					1–3 years					3–8 years					>8 years				
		MKT	*SMB*	*HML*	*MOM*	*LIQ*	*MKT*	*SMB*	*HML*	*MOM*	*LIQ*	*MKT*	*SMB*	*HML*	*MOM*	*LIQ*	*MKT*	*SMB*	*HML*	*MOM*	*LIQ*
>1 year	MKT	1.00																			
	SMB	0.20	1.00																		
	HML	-0.38	0.12	1.00																	
	MOM	-0.31	-0.15	0.07	1.00																
	LIQ	0.16	-0.08	-0.15	0.01	1.00															
1–3 years	MKT	0.86	0.25	-0.34	-0.32	0.10	1.00														
	SMB	0.14	0.97	0.18	-0.14	-0.10	0.14	1.00													
	HML	-0.39	0.06	0.93	0.11	-0.18	-0.35	0.12	1.00												
	MOM	-0.23	-0.04	-0.02	0.90	-0.09	-0.20	-0.05	0.06	1.00											
	LIQ	0.22	0.14	-0.21	-0.06	0.34	0.28	0.08	-0.27	-0.01	1.00										
3–8 years	MKT	0.66	0.24	-0.37	-0.28	0.16	0.69	0.16	-0.39	-0.16	0.26	1.00									
	SMB	0.03	0.88	0.25	-0.14	-0.16	0.10	0.88	0.21	-0.06	0.00	0.05	1.00								
	HML	-0.49	0.03	0.90	0.11	-0.25	-0.46	0.11	0.86	0.01	-0.29	-0.40	0.19	1.00							
	MOM	-0.48	-0.07	0.19	0.83	-0.09	-0.48	-0.03	0.22	0.73	-0.21	-0.36	0.08	0.29	1.00						
	LIQ	0.10	-0.36	-0.00	0.02	0.11	-0.01	-0.34	0.06	-0.01	-0.02	0.06	-0.31	0.10	-0.11	1.00					
>8 years	MKT	0.20	-0.08	-0.16	-0.14	-0.18	0.20	-0.10	-0.16	-0.03	-0.06	0.41	-0.19	-0.07	-0.14	0.25	1.00				
	SMB	0.12	0.92	0.26	-0.19	-0.19	0.21	0.92	0.18	-0.10	0.02	0.11	0.90	0.16	-0.06	-0.35	-0.07	1.00			
	HML	-0.58	0.13	0.70	0.13	-0.30	-0.48	0.19	0.66	0.10	-0.16	-0.25	0.27	0.78	0.34	-0.20	0.03	0.21	1.00		
	MOM	0.21	0.11	-0.15	0.56	-0.09	0.26	0.06	-0.13	0.59	0.11	0.31	0.04	-0.12	0.44	0.03	0.14	0.04	0.03	1.00	
	LIQ	0.04	-0.24	-0.04	0.06	0.45	0.02	-0.26	-0.02	-0.03	0.11	-0.11	-0.30	-0.18	-0.07	0.26	-0.06	-0.28	-0.42	-0.09	1.0

TABLE IA.4: Cross-sectional regressions for 202 mixed portfolios without intercept

This table reports second-stage cross-sectional regressions of monthly filtered excess US equity portfolio returns on the spectral loadings for the sample which runs from January 1968 to December 2016,

$$r_{i,t+1} = \lambda_{0,t} + \sum_{\tau=1}^{\mathcal{T}} \lambda_t^{(\tau)\prime} \widehat{\beta}_{i,t}^{(\tau)} + \varepsilon_{i,t}^{\tau}.$$

where the loadings are estimated over the full sample and a rolling window of five years. In the frequency-specific case, the factors and portfolios are filtered using the one-sided backward-looking CF-RW filter. The table reports the annualized time series averages of the λ_j with t-statistics obtained by the bootstrap procedure between parentheses underneath, and the annualized risk premium in percentage. We further report the average cross-sectional (adjusted) R^2, the mean absolute pricing error $\left(\text{MAPE, between brackets underneath the } R^2\right)$ and the sum of squared pricing errors $\left(Q^{(m)}\text{, between brackets underneath the adjusted } R^2\right)$. ***, **, and * denote significance at the 1%, 5%, and 10% level, respectively, based on 500 bootstrapped samples.

Panel A: *Fama-French factor model, unfiltered monthly exposures*								
	MKT	*SMB*	*HML*	*MOM*	*LIQ*	$\overline{\lambda}_0$	R^2 [MAPE]	Adj. R^2 $\left[Q^{(m)}\right]$
Static exposures								
$\overline{\lambda}$	0.744	7.115***	10.312***	−2.903	6.370***	0.000	0.747	0.740
t-statistic	(0.430)	(4.237)	(6.504)	(−0.513)	(3.187)	(−)	[1.847]	[5.325]
Risk premium (%)	0.313	1.229	−0.487	−0.035	6.501			
Rolling window exposures								
$\overline{\lambda}$	2.080	4.750***	5.260***	3.387	6.614***	0.000	0.741	0.734
t-statistic	(1.110)	(3.168)	(3.506)	(1.343)	(3.233)	(−)	[1.876]	[6.169]
Risk premium (%)	0.958	0.794	−0.232	−0.045	6.638			
Panel B: *Fama-French factor model, filter-based exposures*								
	MKT	*SMB*	*HML*	*MOM*	*LIQ*	$\overline{\lambda}_0$	R^2 [MAPE]	Adj. R^2 $\left[Q^{(m)}\right]$
$\overline{\lambda}^{(<1y)}$	7.104*	−1.347	−1.573	15.765***	−0.988	0.000	0.809	0.788
t-statistic	(1.897)	(−0.548)	(−0.557)	(4.100)	(−0.160)	(−)	[1.590]	[3.351]
Risk premium (%)	7.258	−0.556	−0.274	−0.741	−0.009			
$\overline{\lambda}^{(1-3y)}$	2.017	0.600	1.251	−3.752	−1.706			
t-statistic	(0.509)	(0.257)	(0.546)	(−1.288)	(−0.752)			
Risk premium (%)	2.109	0.246	0.172	0.212	−0.008			
$\overline{\lambda}^{(3-8y)}$	−4.401	3.236**	3.038**	−2.093	2.275			
t-statistic	(−1.334)	(2.093)	(1.972)	(−0.655)	(0.957)			
Risk premium (%)	−4.384	1.561	0.629	0.024	0.171			
$\overline{\lambda}^{(>8y)}$	1.118	−0.400	1.943	−0.066	1.977			
t-statistic	(0.391)	(−0.184)	(0.920)	(−0.027)	(0.850)			
Risk premium (%)	1.063	−0.196	0.133	0.000	0.147			

TABLE IA.5: Cross-sectional regressions for 202 mixed portfolios, filter-based exposures, adjusted backward-looking CF-RW filter

This table reports second-stage cross-sectional regressions of monthly filtered excess US equity portfolio returns on the spectral loadings for the sample which runs from January 1968 to December 2016,

$$r_{i,t} = \lambda_{0,t} + \sum_{\tau=1}^{\mathcal{T}} \lambda_t^{(\tau)\prime} \widehat{\beta}_i^{(\tau)} + \varepsilon_{i,t}^{\tau}.$$

where the spectral loadings are estimated over the full sample for each pass band τ. The factors and portfolios are filtered using the one-sided backward-looking CF-RW filter described in Section IV.C. The table reports the annualized time series averages of the frequency-specific $\lambda_j^{(\tau)}$ with t-statistics obtained by the bootstrap procedure between parentheses underneath, and the annualized average risk premium $\left(\lambda_{j,t}^{(\tau)} \times \widehat{\beta}_{i,j}^{(\tau)}\right)$ in percentage for each factor j. We further report the average cross-sectional (adjusted) R^2, the mean absolute pricing error (MAPE, between brackets underneath the R^2) and the sum of squared pricing errors $\left(Q^{(m)}\right.$, between brackets underneath the adjusted $\left.R^2\right)$. ***, **, and * denote significance at the 1%, 5%, and 10% level, respectively, based on 500 bootstrapped samples.

Panel A: *Macroeconomic factor model*

	IP	*EI*	*UI*	*DEF*	*TS*	$\overline{\lambda}_0$	R^2 [MAPE]	Adj. R^2 $\left[Q^{(m)}\right]$
$\overline{\lambda}^{(<1y)}$	0.782	8.301***	4.802	1.005	5.686*	7.402**	0.544	0.494
t-statistic	(0.583)	(3.215)	(1.618)	(0.938)	(1.934)	(2.382)	[1.566]	[3.468]
Risk premium (%)	−0.295	−4.285	1.740	1.587	2.732			
$\overline{\lambda}^{(1-3y)}$	0.017	0.236	2.765	0.077	3.284*			
t-statistic	(0.012)	(0.215)	(1.557)	(0.071)	(1.906)			
Risk premium (%)	0.021	0.334	4.171	0.210	−1.333			
$\overline{\lambda}^{(3-8y)}$	−2.836	−1.035	−0.342	−1.434*	−6.134			
t-statistic	(−1.542)	(−0.800)	(−0.197)	(−1.892)	(−1.117)			
Risk premium (%)	−5.514	0.359	−0.505	−0.931	0.114			
$\overline{\lambda}^{(>8y)}$	1.795	0.575	1.195	0.812	4.034			
t-statistic	(1.323)	(0.602)	(0.863)	(0.421)	(0.850)			
Risk premium (%)	2.496	−1.353	−0.931	0.155	1.426			

Panel B: *Fama-French factor model*

	MKT	*SMB*	*HML*	*MOM*	*LIQ*	$\overline{\lambda}_0$	R^2 [MAPE]	Adj. R^2 $\left[Q^{(m)}\right]$
$\overline{\lambda}^{(<1y)}$	5.742	−2.122	−1.406	16.173***	−1.353	8.632***	0.562	0.513
t-statistic	(1.483)	(−0.788)	(−0.505)	(3.340)	(−0.248)	(3.768)	[1.535]	[2.718]
Risk premium (%)	5.867	−0.883	−0.243	−0.783	−0.013			
$\overline{\lambda}^{(1-3y)}$	−3.013	−0.181	4.425	−6.431	−3.408			
t-statistic	(−0.665)	(−0.063)	(1.572)	(−1.510)	(−1.029)			
Risk premium (%)	−3.146	−0.076	0.662	0.369	−0.031			
$\overline{\lambda}^{(3-8y)}$	−6.643	4.307**	2.003*	−3.623	5.101			
t-statistic	(−1.190)	(2.059)	(1.964)	(−0.562)	(1.407)			
Risk premium (%)	−6.790	1.977	0.409	0.047	0.350			
$\overline{\lambda}^{(>8y)}$	1.154	0.196	−0.947	2.931	1.402			
t-statistic	(0.243)	(0.058)	(−0.284)	(0.566)	(0.403)			
Risk premium (%)	1.126	0.096	−0.083	−0.001	0.110			

TABLE IA.6: Cross-sectional regressions for 202 mixed portfolios, filter-based exposures, alternative bootstrap (three month block size)

This table reports second-stage cross-sectional regressions of monthly filtered excess US equity portfolio returns on the spectral loadings for the sample which runs from January 1968 to December 2016,

$$r_{i,t} = \lambda_{0,t} + \sum_{\tau=1}^{\mathcal{T}} \lambda_t^{(\tau)\prime} \widehat{\beta}_i^{(\tau)} + \varepsilon_{i,t}^{\tau}.$$

where the spectral loadings are estimated over the full sample for each pass band τ. The table reports the annualized time series averages of the frequency-specific $\lambda_j^{(\tau)}$ with t-statistics obtained by the bootstrap procedure between parentheses underneath, and the annualized average risk premium $\left(\lambda_{j,t}^{(\tau)} \times \widehat{\beta}_{i,j}^{(\tau)}\right)$ in percentage for each factor j. We further report the average cross-sectional (adjusted) R^2, the mean absolute pricing error (MAPE, between brackets underneath the R^2) and the sum of squared pricing errors $\left(Q^{(m)}\right.$, between brackets underneath the adjusted $\left.R^2\right)$. ***, **, and * denote significance at the 1%, 5%, and 10% level, respectively, based on 1,000 bootstrapped samples. The bootstrap block size is set at 3 months.

Panel A: *Macroeconomic factor model*

	IP	*EI*	*UI*	*DEF*	*TS*	$\overline{\lambda}_0$	R^2 [MAPE]	Adj. R^2 $[Q^{(m)}]$
$\overline{\lambda}^{(<1y)}$	1.063	6.411**	6.759**	0.024	4.062**	7.265**	0.541	0.490
t-statistic	(1.015)	(2.288)	(2.411)	(0.039)	(2.13)	(2.303)	[1.554]	[3.951]
Risk premium (%)	−0.583	−3.046	−0.437	0.182	4.592			
$\overline{\lambda}^{(1-3y)}$	0.355	−0.217	1.499	0.314	1.718**			
t-statistic	(0.511)	(−0.278)	(1.433)	(0.540)	(2.027)			
Risk premium (%)	0.495	−0.677	2.294	1.156	−1.008			
$\overline{\lambda}^{(3-8y)}$	−0.508	−0.257	0.439	−1.921***	−0.525			
t-statistic	(−0.619)	(−0.279)	(0.395)	(−2.849)	(−0.287)			
Risk premium (%)	−1.045	1.274	0.561	−1.879	−0.057			
$\overline{\lambda}^{(>8y)}$	0.282	0.115	0.216	0.231	−0.430			
t-statistic	(0.472)	(0.162)	(0.254)	(0.527)	(−0.310)			
Risk premium (%)	0.135	−0.783	−0.599	0.012	−0.253			

Panel B: *Fama-French factor model*

	MKT	*SMB*	*HML*	*MOM*	*LIQ*	$\overline{\lambda}_0$	R^2 [MAPE]	Adj. R^2 $[Q^{(m)}]$
$\overline{\lambda}^{(<1y)}$	2.765	−0.770	−0.799	12.893***	1.484	8.031***	0.555	0.506
t-statistic	(0.635)	(−0.244)	(−0.232)	(2.706)	(0.281)	(3.540)	[1.598]	[2.914]
Risk premium (%)	2.825	−0.318	−0.139	−0.606	0.013			
$\overline{\lambda}^{(1-3y)}$	0.051	0.422	1.516	−3.116	−1.873			
t-statistic	(0.010)	(0.117)	(0.839)	(−1.322)	(−0.996)			
Risk premium (%)	0.053	0.173	0.209	0.176	−0.009			
$\overline{\lambda}^{(3-8y)}$	−5.312**	2.77**	2.538**	−3.238	2.467			
t-statistic	(−2.088)	(2.014)	(2.369)	(−1.035)	(0.967)			
Risk premium (%)	−5.291	1.337	0.525	0.037	0.186			
$\overline{\lambda}^{(>8y)}$	0.463	−0.272	0.732	2.38	0.587			
t-statistic	(0.159)	(−0.157)	(0.441)	(1.239)	(0.276)			
Risk premium (%)	0.440	−0.133	0.050	−0.006	0.044			

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