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Quantifying time-varying forecast uncertainty and risk for the real price of oil

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Abstract

We propose a novel and numerically efficient quantification approach to forecast uncertainty of the real price of oil using a combination of probabilistic individual model forecasts. Our combination method extends earlier approaches that have been applied to oil price forecasting, by allowing for sequentially updating of time-varying combination weights, estimation of time-varying forecast biases and facets of miscalibration of individual forecast densities and time-varying inter-dependencies among models. To illustrate the usefulness of the method, we present an extensive set of empirical results about time-varying forecast uncertainty and risk for the real price of oil over the period 1974-2018. We show that the combination approach systematically outperforms commonly used benchmark models and combination approaches, both in terms of point and density forecasts. The dynamic patterns of the estimated individual model weights are highly time-varying, reflecting a large time variation in the relative performance of the various individual models. The combination approach has built-in diagnostic information measures about forecast inaccuracy and/or model set incompleteness, which provide clear signals of model incompleteness during three crisis periods. To highlight that our approach also can be useful for policy analysis, we present a basic analysis of profit-loss and hedging against price risk.

Keywords: Oil price, Forecast density combination, Bayesian forecasting, Instabilities, Model uncertainty

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1 Introduction

The adverse macroeconomic implications of oil price fluctuations have been known since the 1970's, making the real price of crude oil a key variable in both macroeconomic forecasting and structural analysis (e.g. Hamilton, 1983, 2009; Kilian, 2009; Ravazzolo and Rothman, 2013; Baumeister and Hamilton, 2019). For instance, central banks, private sector forecasters and international organizations view the price of oil as one of the key variables in generating macroeconomic projections and in assessing macroeconomic risks. Not only are oil price forecasts important for generating macroeconomic projection, but for some sectors of the economy, such as airlines, utilities and automobile manufacturers, they are also crucial for how they operate their business. Common to all, however, is the notion that the price of oil is not easy to forecast. Hamilton (2009) documents that the statistical regularities of changes in the real price of oil have historically tended to be (1) permanent, (2) difficult to predict, and (3) governed by very different regimes at different points in time. He further argues that the price of oil seems to follow a random walk without drift. It is therefore not very surprising that it is widely accepted to either use the current spot price or the price of oil futures contracts as the forecast of the price of oil.

More recently, academic and professional researchers have explored numerous alternative models and methods in order to forecast the most likely future realization of the oil price (e.g. Alquist and Kilian, 2010; Alquist et al., 2013; Baumeister and Kilian, 2012, 2015; Manescu and Robays, 2016; Bernard et al., 2018; Pak, 2018; Garratt et al., 2019; Baumeister et al., 2020). These papers have primarily been focusing on evaluating point forecasts and have concluded that although a random walk is hard to beat in out-of-sample oil price forecasting exercises, careful attention to the economic fundamentals that are driving energy markets can lead to practical improvements in forecasts.

In this paper, we provide a novel and numerically efficient quantification approach to forecast uncertainty of the real price of crude oil using a combination of probabilistic individual model forecasts. The proposed Forecast Density Combination (FDC) model is based on a probabilistic and econometric interpretation of the Bayesian Predictive Synthesis (BPS) model due to McAlinn and West (2019) and McAlinn et al. (2020). BPS is a coherent Bayesian framework for evaluation, calibration, and data-informed combination of multiple forecast densities.[1] The proposed combination method extends earlier approaches that have been applied to oil price forecasting models, by allowing for three key features. First, the method features time-varying combination weights, and explicitly factors into the model combination the inherent uncertainty surrounding the estimation of the combination weights. Second, it explicitly allows modelling and estimation of time-varying forecast biases and facets of miscalibration of individual forecast densities and time-varying interdependencies among models. Third, diagnostic analysis of model set incompleteness is provided and learning from previous forecast mistakes, which serves to improve model specification. Finally, it is worth emphasizing that the model is computationally efficient to handle using relatively standard Markov Chain Monte Carlo with Normal/Kalman Filters instead of more involved methods like Sequential Monte Carlo including Particle Filtering (Billio et al., 2013; Casarin et al., 2020).

We make use of the proposed combination model and provide an extensive set of empirical results about time-varying out-of sample forecast performance, forecast uncertainty and risk for the real price of oil. Following Garratt et al. (2019) recent replication of Baumeister and Kilian (2015), we use real-time monthly data, where the full sample covers the period 1973:01-2017:12.[2] Our starting point is to document substantial changes over time in both the mean and volatility pattern for the log-level of the real price of oil, a pattern that also transfers into considerable time-variation in the shape of the data densities. We then proceed by showing six main results, which extends the current literature on forecasting the price of oil.

First, our combination approach systematically outperforms all benchmarks we compare it to, both in terms of point and density forecasts. The competing benchmark models, range from the six state-of-the-art individual forecasting models used in Baumeister and Kilian

[1] As detailed in McAlinn and West (2019), BPS provides a foundational Bayesian perspective for forecast density combination, based on the earlier literature on Bayesian "agent/expert opinion analysis" (West, 1984; Genest and Schervish, 1985; West, 1988, 1992; West and Crosse, 1992). In a forecast density combination context, a decision maker regards multiple models or forecasters as providers of "forecast data" to be used in her priorposterior updating. Note that the approach applies whether the forecast densities arise from sets of models, forecasters, agencies, or institutions.

[2] Garratt et al. (2019) provide a database which includes updated monthly real-time data vintages for oil market variables similar to those used in Baumeister and Kilian (2012, 2015).

(2015), including the commonly used naive no-change model, to alternative combination approaches such as equal weights for the individual models and Bayesian Model Averaging (BMA). While the gains from the model combination relative to the alternative models are limited at the one month horizon, substantial gains in relative forecast accuracy are obtained at all other horizons. At the six month horizon, the magnitude of reduction in terms of root mean squared forecasting error (RMSFE) and log score relative to the no-change model exceeds 10% and are credibly different. For longer horizons, the gains are substantially larger, with reductions in the range of 30% and 40% for RMSFE and 50% and 110% for log score, at the 12 and 24 months horizon, respectively.

Second, we find that the favourable forecast performance from the proposed approach is not specific for certain time periods but it holds throughout the evaluation period. Large time variation is found in the relative performance of the various individual models and alternative model combinations. For instance, in line with Baumeister and Kilian (2012), it is seen that the VAR model performs well up until 2010, but then does relatively poorly over the subsequent period, thereby corroborating recent results in Baumeister et al. (2020).

Third, an important feature of the approach is that it allows for time-varying individual model weights that sequentially adapt according to the recent relative forecast performance of each model within the model combination set. At all forecast horizons, we document a considerable time variation in the weights attached to each model, also reflecting the large time-variation in the individual models forecasting performance. One key feature is that the weights are not restricted to be a convex combination in the unit interval - like most of the combination methods that are currently used within the econometrics literature - but are instead specified as a general linear combination and are thereby permitted to evolve along the real line. This has two advantages. First, allowing for both positive and negative weights means that it's possible to hedge against any potential forecast risk in recent periods. Second, a declining weight on one model does not necessarily imply an increasing weight on another model. Instead, model weights are permitted to change in accordance to the forecasts of the individual models. Such features are clearly desirable to a wide range of practitioners. For instance, just as a financial portfolio manager hedges against risk by assigning a negative weight to an asset, the proposed approach is able to automatically assign a negative weight to mitigate the impact of forecast risk, such as forecast bias. Such natural behaviour is not possible under combination models in which the model weights are restricted to be convex combinations, e.g. BMA or equal weighting methods.

Fourth, another important feature of the approach is that it has a built-in time-varying intercept that is absent from simpler combination methods such as BMA. It is well known within the econometrics literature on forecast combinations that BMA assumes that the true model is included in the model set. By allowing for an intercept component that can adapt during episodes of low frequency signals from a set of forecasting models, our combination approach is able to better mitigate effects of model set misspecifciaton, i.e., model set incompleteness. We show that this has been particularly important since 2010, as the intercept term for all forecast horizons gradually starts to increase before abruptly dropping during the oil price collapse of 2014.

Fifth, the combination approach has built-in diagnostic information measures about forecast inaccuracy and/or model set incompleteness, which is also absent from simpler combination methods such as BMA. This is measured by the estimated time-varying model combination residual, which provides clear signals of model incompleteness during three crisis periods. This type of diagnostic information gives important signals about specifying models and model set improvements.

Finally, we present a basic analysis of profit-loss and hedging against price risk to highlight the models potential for policy analysis. As a measure of optimal hedge ratio, the Minimum Variance Hedge (MVH) ratio is used which fluctuates between 0.1-0.4 at most horizons. However, notable spikes occur around the turn of the century as well as the two oil price collapses of 2009 and 2014.

Our paper is related to the recent resurgence in interest in forecast comparison, calibration, and combination of density forecasts in macroeconomics, econometrics, and statistics. Prominent new developments range from combining predictive densities using weighted linear combinations of prediction models, evaluated using various scoring rules (e.g. Hall and Mitchell, 2007; Amisano and Giacomini, 2007; Jore et al., 2010; Hoogerheide et al., 2010; Kascha and Ravazzolo, 2010; Geweke and Amisano, 2011, 2012; Gneiting and Ranjan, 2013; Aastveit et al., 2014), to more complex combination approaches that allows for time-varying weights with possibly both learning and model set incompleteness (e.g. Koop and Korobilis, 2012; Billio et al., 2013; Casarin et al., 2015; Pettenuzzo and Ravazzolo, 2016; Del Negro et al., 2016; Aastveit et al., 2018; McAlinn and West, 2019; McAlinn et al., 2020; Takanashi and McAlinn, 2020; Casarin et al., 2020). Despite these research activities on several macroeconomic and financial variables, there are currently, to the best of our knowledge, no studies on how to quantify forecast uncertainty associated with the dynamic behaviour of the real price of crude oil.

The contents of this paper is structured as follows. In Section 2 we present our choice of Basic Forecast Density Combination model using Bayesian inference. In Section 3 a summary of the models used is given. Empirical results are presented in Section 4. Section 5 contains conclusions and suggestions for more research.

2 A basic and numerically efficient Forecast Density Combination approach

A basic probabilistic approach to combine forecast information from different sources proceeds as follows. Let y_t be the economic variable of interest; let $\tilde{\boldsymbol{y}}_{\boldsymbol{t}}' = (\tilde{y}_{1,t}, \ldots, \tilde{y}_{n,t})$ be forecasts for this variable from $i = 1, ..., n$ models. In a simulation context $\tilde{y}_{i,t}$ is a draw from the forecast distribution with conditional density $p(\tilde{y}_{i,t}|I_{i,t-1}, M_i)$ given information set $I_{i,t-1}$ and model M_i. Let $\boldsymbol{v}_{\boldsymbol{t}}' = (v_{0,t}, \ldots, v_{n,t})$ be latent continuous random variable parameters where $v_{1,t}, \ldots, v_{n,t}$ will be used to combine the model forecasts and the role of $v_{0,t}$ is discussed below. The decomposition of the joint density of $y_t, \boldsymbol{v}_t, \tilde{\boldsymbol{y}}_t$ for the case of continuous random variables is given as:

$$p(y_t|I_{t-1}, M) = \int\int p(y_t|\boldsymbol{v}_t, \tilde{\boldsymbol{y}}_{\boldsymbol{t}})p(\boldsymbol{v}_t|\tilde{\boldsymbol{y}}_{\boldsymbol{t}})p(\tilde{\boldsymbol{y}}_{\boldsymbol{t}}|I_{t-1}, M)d\boldsymbol{v}_t d\tilde{\boldsymbol{y}}_{\boldsymbol{t}}, \tag{1}$$

where I_{t-1} is the joint information set of all models and M the union of all models. We label $p(y_t|\boldsymbol{v}_t, \tilde{\boldsymbol{y}}_{\boldsymbol{t}})$ as the combination density; $p(\boldsymbol{v}_t|\tilde{\boldsymbol{y}}_{\boldsymbol{t}})$ as the variable parameter density and $p(\tilde{\boldsymbol{y}}_{\boldsymbol{t}}|I_{t-1}, M)$ as the joint forecast density of the different models. Note that the integrals are of dimension $n + 1$ and n.

A key step is to give specific content to the different densities. For the case of BPS it follows that:

$$
\begin{aligned}
p(y_t|\boldsymbol{v}_t, \tilde{\boldsymbol{y}}_t) &= n(y_t|v_{0,t} + \sum_{i=1}^{n} v_{i,t}\tilde{y}_{i,t}, \sigma_t^2), &(2)\\
p(\boldsymbol{v}_t|\boldsymbol{v}_{t-1}, \boldsymbol{\Sigma_t}) &= n(\boldsymbol{v}_t|\boldsymbol{v}_{t-1}, \boldsymbol{\Sigma_t}), &(3)\\
p(\tilde{\boldsymbol{y}}_t|I_{t-1}, M) &= \prod_{i=1}^{n} p(\tilde{y}_{i,t}|I_{i,t-1}, M_i). &(4)
\end{aligned}
$$

We emphasize that the combination density is a multivariate normal one with a time-varying constant $v_{0,t}$ in the conditional mean. This specification adds flexibility to the model combination and allows for forecast adjustments to shocks and regime changes in the data series while σ_t^2 allows for time-varying volatility. The parameter $\boldsymbol{\Sigma_t} = \sigma_t^2 \boldsymbol{W_t}$ and $\boldsymbol{W_t}$ is a diagonal matrix with elements w_{it} given below.

A first feature of this approach, compared to standard combination models like BMA, is an analysis of the dynamic behaviour of the error ε_t implied by the combination density. It is given as:

$$\varepsilon_t = y_t - (v_{0,t} + \sum_{i=1}^{n} v_{i,t}\tilde{y}_{i,t}). \tag{5}$$

The forecast error of the ith model is usually defined as $y_t - \tilde{y}_{i,t}$ due to, for instance, sudden shocks in the series and model misspecification. Given the conditional mean of the combination density in (2), it is seen that the BPS error in (5) can be viewed as weighted combination of model forecast errors from each of the individual models.

We also investigate the dynamic behaviour of the error $\varepsilon_{i,t}$ using only model M_i, which is given as:

$$\varepsilon_{i,t} = y_t - (v_{0,t} + v_{i,t}\tilde{y}_{i,t}), \tag{6}$$

A second feature of the approach is the possibility to learn about the contribution of the different individual model forecasts in the combination. Learning is specified as a random walk process of the continuous latent variable parameters $\boldsymbol{v}_t = (v_{0,t}, ...v_{n,t})'$, see equation (3). We note that the weights may become negative which in some cases helps in dynamic averaging. That is, the proposed approach is able to automatically assign a negative weight which may mitigate the impact of forecast risk, such as forecast bias. Such natural behaviour is not possible under combination models in which the model weights are restricted to be convex combinations, e.g. BMA or equal weighting methods.

The joint forecast density of the different models $p(\tilde{\boldsymbol{y}}_{\boldsymbol{t}}|I_{t-1}, M)$ is, due to the conditional independence assumption, the product of the individual forecast densities.

Given the specified probability model, there exists a system of equations that has been labeled a latent dynamic factor model by McAlinn and West (2019) and McAlinn et al. (2020). However, we interpret this system as a multivariate regression model with generated regressors $\tilde{y}_t$ and latent time-varying parameters $v_{i,t}$. By construction, this equation system can be represented in the form of a generalized linear State Space Model where the explanatory variables $\tilde{y}_{i,t}$ are not given data but generated draws from the forecast distributions of the n models:

$$y_t = v_{0,t} + \sum_{i=1}^{n} v_{i,t}\tilde{y}_{i,t} + \varepsilon_t, \varepsilon_t \sim NID(0, \sigma_t^2), \tag{7}$$

$$v_{i,t} = v_{i,t-1} + \varepsilon_{v,t}, \varepsilon_{v,t} \sim NID(0, \sigma_{v,t}^2 = \sigma_t^2 w_t), i = 0, \ldots, n. \tag{8}$$

The time-varying volatility parameters σ_t^2 and $\sigma_{v,t}^2$ play important roles in this model as smoothness parameters: $\sigma_t^2 = \frac{\delta\sigma_{t-1}^2}{\gamma_t}$ is a betagamma volatility model in which $\delta \in (0, 1]$ is a volatility discount factor and $\gamma_t \sim \text{Beta}(\frac{\delta h_{t-1}}{2}, \frac{(1-\delta)h_{t-1}}{2})$ is an independent Beta innovation such that $h_t = \delta h_{t-1} + 1$ and $\mathbb{E}[\gamma_t|h_{t-1}] = \delta$ at all dates $t = 1, \ldots, T$. The weight $w_t = \frac{1-\beta}{\beta} w_{t-1}$ is a component discount term in which $\beta \in (0, 1]$ is a state discount factor. For details, see West and Harrison (2006), Sect. 6.3.2 and Sect. 10.8.

In Figure 1 we show in a roadmap the connections between the components of the model. We distinguish between two figure shapes: rectangles which contain data and forecasts from different models and their combination; circles which contain latent time-varying regression parameters and the unobserved random parameters from the stochastic volatility process which have to be filtered/integrated out.

Bayesian estimation procedure using MCMC. The analytic solution of the integrals specified in the probability model (1)-(4) is often not known. We make use of simulation methods in order to deal with this problem. We also make use of Bayesian inference specifying prior information on the parameters of the stochastic volatility processes and the time-varying equation parameters which can be interpreted as unobserved states. Apart from the fundamental choice for Bayesian inference, there exists a practical reason in our case. Using simulation-based Bayesian inference the generated forecast draws from the

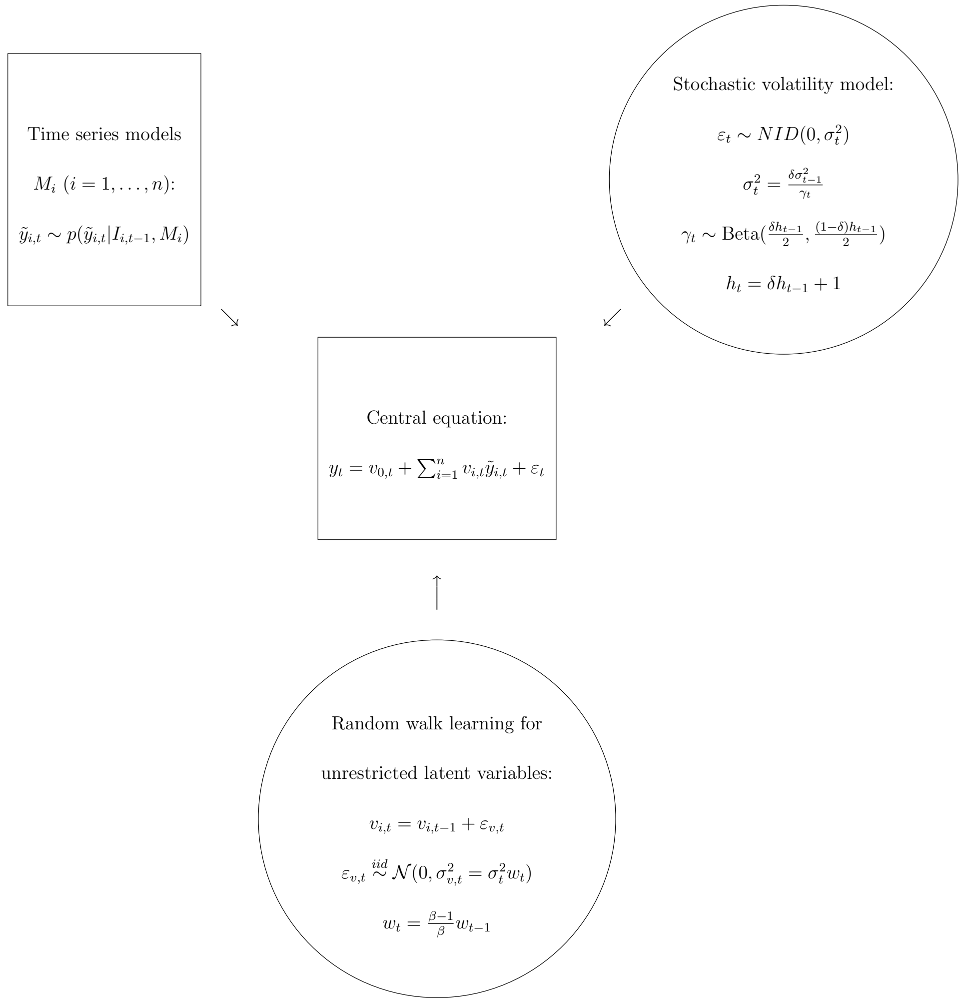

Figure 1: FDC model in generalized linear state space form. Given data, rectangles indicate model forecasts and combined forecasts. Circles refer to latent time-varying regression parameters and the random parameters from the stochastic volatility process where filtering/integration is used.

different models are computationally directly carried forward to the estimation of the combination density. Thus, the uncertainty in the forecasts of the different models carries directly into the uncertainty of the combination forecasts. In contrast, frequentist methods like method of moments or maximum likelihood proceed in a two-step fashion by substituting the point forecasts of the different models in the combination equation and as such the second stage results suffer from the generated regressor problem. That is, uncertainty measures of the final forecasts are sensitive to the estimation of the first stage, see Pagan

(1984) for background details.

The specification of the model discussed so far leads to the formulation of the likelihood of a generalized linear State Space model. Our Bayesian inferential procedure requires to choose prior values for the discount factors $\delta \in (0,1]$, $\beta \in (0,1]$ and priors for the initial values of the time-varying parameters $(v_{i,t}, \sigma_t^2)$ at $t = 0$. The role of discount factor $\beta \in (0,1]$ is to operate on the parameter evolution via $w_t = \frac{1-\beta}{\beta} w_{t-1}$. Setting $\beta = 1$ implies a constant coefficients model, i.e. $w_t = 0$, while $\beta \in (0,1]$ is consistent with time-varying coefficients. The parameter $\delta \in (0,1]$ operates on the volatility evolution via $\sigma_t^2 = \frac{\delta \sigma_{t-1}^2}{\gamma_t}$ in which γ_t are Beta distributed innovations. Relevant choices of the discount factors β and δ are, of course, always context dependent. In our application, we opted for the value 0.9. As a simple robustness check, we computed LPS and RMSFE values for the average of the triple (0.85, 0.9, 0.95). The results were consistent with the single value chosen.[3]

The prior on the latent time-varying parameters is conditional normal $v_{i,0}|\sigma_0^2 \sim N(m_0, C_0 \frac{\sigma_0^2}{s_0})$ where the hyperparameter: m_0 controls the mean, C_0 controls variance and h_0 and s_0 jointly control mean and variance of the measurement volatility, s_0 indirectly effects the variance of parameters. The initial prior on the measurement variance is marginal inverted gamma $\sigma_0^2 \sim IG(h_0/2, h_0 s_0/2)$, see also (McAlinn and West, 2019, Appendix A.1). Following McAlinn and West (2019), specific choices for hyper parameters are $m_0 = (0, 1/n, \ldots, 1/n)'$, $C_0 = 10^{-4}\mathbf{I}_p$, $s_0 = 0.002$ and $h_0 = 10$.

Algorithmic outline using Kalman Filter and MCMC We emphasize that the proposed method makes use of a three-step Monte Carlo procedure instead of the usual two-step method. The extra step is due to the generation of random draws from the forecast distributions of the different individual models.

- **Forecast from n models.** Generate draws from the forecast distributions from the n different models which gives $\tilde{y}_{i,t}, i = 1, \ldots, n$

- **Latent variable parameters**. Use the Kalman update with initial value $v_{i,0}, i = 1, \ldots, n$, and generate variable parameters $v_{i,t}, i = 1, \ldots, n$ from the random walk process.

[3] Alternatively, we could take a uniform prior on the bounded interval.

- **SV parameters.** Given draws $\tilde{y}_{i,t}, i = 1, \ldots, n$, $v_{i,t}, i = 0, \ldots, n$, generate a draw of the SV parameters from inverted Gamma distribution.

Forecasting proceeds as follows: Given generated $v_{i,t}, i = 0, \ldots, n$, generated SV values and generated $\tilde{y}_{i,t}, i = 1, \ldots, n$, use (7) to generate a one step forecast value y_{t+1}. Repeating this process gives a synthetic sample of future values and a forecast density at time $t + 1$.

3 Individual models and alternative combinations

Let S_t denote the spot price of crude oil at date t. Forecasts are obtained using a general stochastic volatility model with Student's t-distributed errors given as

$$S_{t+h|t} - \hat{S}_{t+h|t} = \epsilon_{t+h|t}, \quad \epsilon_{t+h|t} \sim T(\mu, e^{h_{t+h|t}}, \nu), \tag{9}$$

$$h_{t+h|t} = \mu + \phi(h_{t+h-1|t} - \mu) + \zeta_{t+h|t}, \quad \zeta_{t+h|t} \sim NID(0, \omega^2), \tag{10}$$

in which $|\phi| < 1$ and $\hat{S}_{t+h|t}$ denotes a point forecast of the real oil price, which is set equal to the conditional mean of the posterior predictive density.[4]

Individual models Following Baumeister and Kilian (2015), the point forecasts of the real price of oil, $\hat{S}_{t+h|t}$, are obtained using six state-of-the-art oil price forecasting models. The names and acronyms are listed in Table 1. Next, we summarize the specifications.

The set of models starts with a no-change forecast, with acronym NC,

$$\hat{S}_{t+h|t} = S_t. \tag{11}$$

The second model includes the changes in the price index of non-oil industrial raw materials and is denoted by CRB:

$$\hat{S}_{t+h|t} = S_{t|t}(1 + \pi_t^{h,rm} - \mathbb{E}_t[\pi_{t+h}^{(h)}]). \tag{12}$$

in which $\pi_t^{h,rm}$ denotes the percent change of an index of the spot price of industrial raw materials (other than oil) over the preceding h months and is obtained from the Commodity

[4]The model is estimated using the Metropolis-within-Gibbs sampling algorithm described in Chan et al. (2014). In this exercise we retain a sample of 10000 draws from the posterior distribution after discarding the first 5000 draws as a burn-in.

Table 1: List of individual forecasting models and various forecast density combination approaches and their acronyms.

Model	Description
NC	No-change model
CRB	Changes in the price index of non-oil industrial raw materials
Futures	West Texas Intermediate (WTI) oil futures prices
Spread	Spread between the spot prices of gasoline and crude oil
TVspread	Time-varying parameter model of the gasoline and heating oil spreads
VAR	Vector autoregression
Equal	Equal weighted linear combinations of forecast densities
BMA	Bayesian Model Averaging
BMA2	Bayesian Model Averaging with a two-year rolling window
BPS	Bayesian Predictive Synthesis

Research Bureau, and $\pi_{t+h}^{(h)}$ denotes the expected rate of inflation over the next h-periods which is proxied by recursively constructed averages of past U.S. consumer price inflation data. This model is based on the intuition that there are broad-based predictable shifts in the demand for globally traded commodities.

The third model includes West Texas Intermediate (WTI) oil futures prices and is denoted by Futures:

$$\hat{S}_{t+h|t} = S_{t|t}(1 + f_t^{WTI,h} - s_t^{\mathrm{WTI}} - \mathbb{E}_t[\pi_{t+h}^{(h)}]), \tag{13}$$

in which $f_t^{WTI,h}$ is the log of the current WTI oil futures price for maturity h and s_t^{WTI} is the log the WTI spot price. This model reflects idea that many practitioners and policy institutions rely on the price of oil future contracts in generating forecasts of the nominal price of oil.

The fourth model includes the spread between spot prices of gasoline and crude oil and is denoted by Gasoline:

$$\hat{S}_{t+h|t} = S_{t|t}\exp(\hat{\beta}[s_t^{\mathrm{gas}} - s_t^{\mathrm{WTI}}] - \mathbb{E}_t[\pi_{t+h}^{(h)}]), \tag{14}$$

in which s_t^{WTI} is the log of the nominal U.S. spot price of gasoline. This model reflects the idea of many market practitioners believing that a rising spread between the price of gasoline and the price of oil signals upward pressure on the price of oil.

The fifth model is a time-varying parameter model of the gasoline and heating oil spreads and is denoted by TVSpread:

$$\hat{S}_{t+h|t} = S_{t|t} \exp(\hat{\beta}_{1,t}[s_t^{\text{gas}} - s_t^{\text{WTI}}] + \hat{\beta}_{2,t}[s_t^{\text{heat}} - s_t^{\text{WTI}}] - \mathbb{E}_t[\pi_{t+h}^{(h)}]), \tag{15}$$

in which s_t^{heat} is the log of the nominal U.S. spot price of heating oil which is obtained from the EIA and the time-varying parameters evolve according to a random walk with independent Gaussian white noise errors. This model reflect the idea by Verleger (2011) that the price of oil is likely to be determined by the refined product in highest demand. He argues that in the US this product has been alternating between gasoline and heating oil.

The sixth model is an oil market Vector Autoregressive Model and is denoted by VAR:

$$y_t = b + \sum_{i=1}^{p} B_i y_{t-i} + e_t, \tag{16}$$

where y_t is a 4×1 vector of variables including: the percent change in global crude oil production; global real economic activity index of Kilian (2009)[5]; the log of the real price of oil - $\hat{S}_{t+h|t} = \exp(\hat{y}_{3,t+h|t})$ and global above-ground crude oil inventories. This model can be viewed as the reduced-form representation of the global oil market structural VAR model developed by Kilian and Murphy (2014).

Combination models We use five of the individual models listed in Table 1 to compose model combinations: CRB, Futures, Spread, TVspread and VAR. The NC model is then used as a benchmark model to compare relative forecast performance of both the individual models and the combination models.

[5] Hamilton (2021) argues that an alternative measure, derived from world industrial production, is a better indicator of global real economic activity. Baumeister et al. (2020) compare various measures of global real economic activity in terms of their out-of-sample forecasting performance for the real price of oil. They find that models based on alternative measures of global economic conditions such as world industrial production or a common factor extracted from a panel of real commodity prices lead to substantially better forecasts. However, due to the lack of available real-time data vintages, we refrain from using these alternative measures of global economic conditions.

In addition to the BPS model discussed in Section 2, we also consider Bayesian model averaging (BMA). BMA is a popular ensemble learning method that has been widely used within the econometrics literature (see, e.g. Aastveit et al. (2019) and references therein). When using BMA the individual forecast densities, $p(\tilde{y}_{t,h}|M_i, I_t)$, from model M_i are pooled into a combined posterior/forecast density, $p(\tilde{y}_{t,h}|I_t)$, given as

$$p(\tilde{y}_{t,h}|I_t) = \sum_{i=1}^{N} w_{i,t,h} p(\tilde{y}_{t,h}|M_i, I_t), \tag{17}$$

where the weights, $w_{i,t,h}$, are specified in one of two ways. In the first instance, following, among others, Amisano and Giacomini (2007); Hall and Mitchell (2007) and Jore et al. (2010), we use recursive weights based on the logarithmic score which take the form

$$w_{i,t,h} = \frac{\exp(\sum_{t=T_0}^{T-1-h} \ln p(\tilde{y}_{t,h}|M_i, I_t))}{\sum_{i=1}^{N} \exp(\sum_{t=T_0}^{T-1-h} \ln p(\tilde{y}_{t,h}|M_i, I_t))}, \tag{18}$$

in which T_0 denotes the start date of the forecast evaluation period and T denotes the end date of the period. In addition to this, we consider a version of BMA which uses a two-year rolling window when updating the weights.

Finally, we also consider equally weighted forecasts (equal) where we set the weight attached to each model to $w_{i,t,h} = 1/N$ in equation (17). In fact, such a simple combination of forecasts is commonly used, see e.g. Timmermann (2006), Stock and Watson (2006), Clark and McCracken (2010) and Baumeister and Kilian (2015), and is often found to outperform more sophisticated adaptive forecast combination methods.

4 Empirical results

In this section we present results from a real-time, out-of-sample forecast study, in which we generate both point and density forecasts of the real price of oil in the global market for crude oil. Following Garratt et al. (2019) recent replication of Baumeister and Kilian (2015), we use real-time monthly data, where the real price of oil in the global market is approximated by deflating the U.S. refiners acquisition cost for crude oil imports (IRAC) by the seasonally adjusted U.S. consumer price index for all urban consumers (CPI).[6] The

[6] Garratt et al. (2019) provide an updated version of the real-time data set used in Baumeister and Kilian (2012, 2015). The data set includes monthly real-time vintages of IRAC and CPI, as well as

full data cover the period 1973:01-2017:12. The initial forecasts discussed in Section 3 are estimated on data from 1973:01-1991:12 and forecasts are then made over the remaining data for the period 1992:012017:12 using real-time data vintages. When constructing the combinations, BPS requires an initial training data period which we set to 50 months. Since the first 24 months worth of forecasts account for differences in the forecast horizons, all forecasting models are evaluated on the same period of 1998:03-2017:12. Our objective is to forecast the final release of the real oil price data.

Our starting point is to document substantial changes over time in both the mean and volatility pattern for the log-level of the real price of oil, a pattern that also transfers into considerable time-variation in the shape of the data densities. We then proceed by showing the main results, which extend the current literature on forecasting the price of oil. This includes new results about the posterior-predictive density of the combined density forecast model which extend beyond the predictive mean to include uncertainty, along with such features as log score, weight behaviour over time, diagnostic measures of model and model set incompleteness and implied results about risk features. To facilitate the discussion of the results, we restrict the use of the word posterior-predictive. For instance, instead of referring each time to posterior-predictive means of the log scores we just refer to log scores when there is no doubt about the interpretation.

4.1 Typical data patterns of the real price of oil

We begin the analysis by examining the real price of oil in various transformations as shown in Figure 2. The shaded regions highlight various episodes of historical significance for the global market for crude oil: The 1979 oil crisis, the commencement of the IranIraq War in 1980, the disbandment of OPEC in 1985, the 1990/91 Persian Gulf War, the Asian Financial Crisis 1997/98, the oil price surge of mid 2003-08, the collapse of the oil price during the Great Recession and the oil price decline of mid-2014 to early 2015. The various transformations collectively highlight three typical data features in the period 1973-2018. First, the log-level series show substantial changes in the mean of the series which suggests

real-time vintages of variables used for estimating the various individual models, such as, e.g., world oil production, oil inventories and the global real economic activity index.

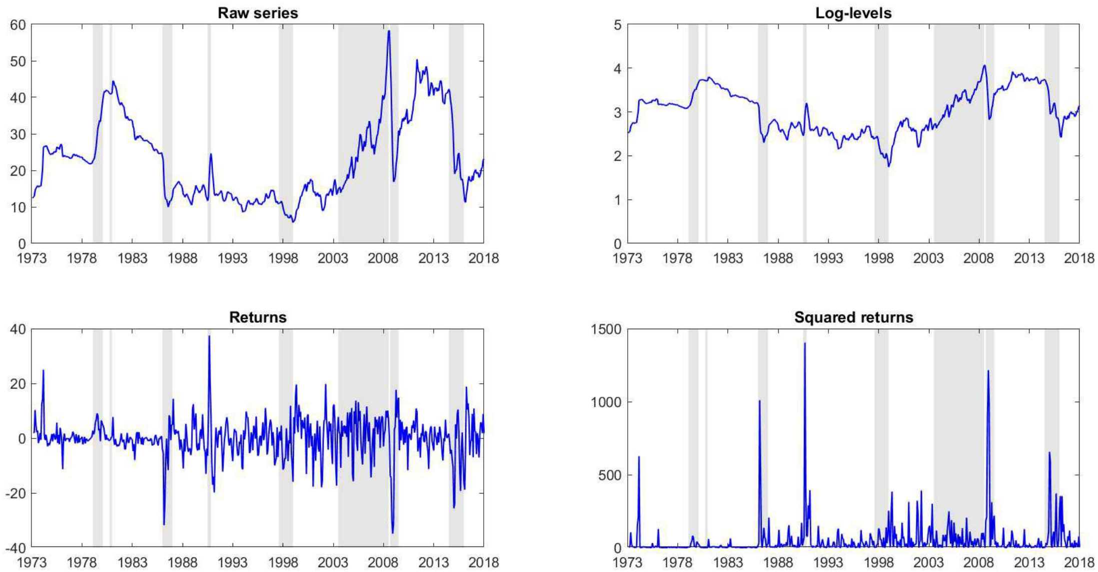

Figure 2: Real IRAC price of oil at monthly frequency over the period: 1973:01-2017:12.

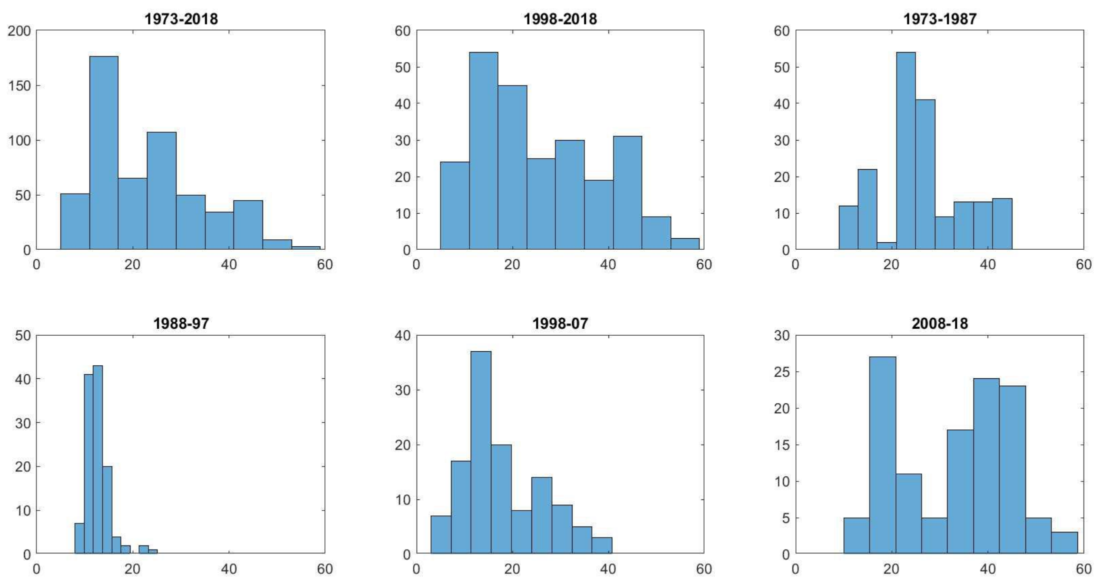

Figure 3: Distributions of the real IRAC price of oil in levels at monthly frequency over the period: 1973:01-2017:12, the forecast evaluation period: 1998:03-2017:12, and various sub-sample periods. The horizontal axis represents the real price of oil in USD and the vertical axis represents the pooled counts of observations in the respective bins.

that a time-varying autoregressive mean process may be beneficial. Second, the returns and squared returns series show volatility clustering suggesting that stochastic volatility is an important data feature to model. Third, using sub-period analysis, a changing mean and volatility pattern in the log-level series indicate that a time-invariant autoregressive mean model with SV may provide reasonably accurate forecasts over the initial data period, 1974-2002, as well as the periods 2010-2014 and 2015-2018. This confirms that in sub-periods stable patterns are present but periodic shocks at the mean level have occurred as indicated above. Further evidence of time-varying elements of the oil price distribution can be seen in Figure 3, which shows data distributions over the full data period and notable sub-periods: The forecast evaluation period (1998-2018), a period of turmoil (1973-1987), a period of tranquility (1988-97), the oil price surge (1998-07) and the most recent decade (2008-18). In each case, the horizontal axis represents the level of the real price of oil in USD, as also shown in the top left panel in Figure 2, pooled into nine bins. The vertical axis represents the pooled count of observations over the respective (sub-)periods. Substantial time-variation in the shape of the data distributions is seen. It is noteworthy that asymmetry, fat tails and bi-modality are important features of the data. This suggests that models which allow for non-linearities in both mean and variance, as well as fat-tails, such as in our proposed BPS framework, may provide forecast improvements over simpler linear models, such as the suite of individual forecasting models, and the commonly used equal weight and BMA combination schemes discussed in Section 3. In the next section we discuss the accuracy of the estimated densities compared to the different data distributions.

4.2 Forecast accuracy of individual models and combinations

Forecast results across the evaluation period are provided in Table 2. The upper panel provides density forecast results evaluated by the log score relative to a no-change model benchmark.[7] The lower panel provides point forecast results evaluated by the RMSFE relative to a no-change model benchmark.[8] For interpretation purposes, log score values

[7]See Gneiting and Raftery (2007) for background underlying the log score in assessing the accuracy of probabilistic forecasts.

[8]The use of RMSFE relative to a no-change model benchmark is consistent with earlier studies in the oil price forecasting literature by Baumeister and Kilian (2012, 2015).

that are greater than zero indicate that the model outperforms the benchmark and vice versa. In contrast, RMSFE values that are less than one indicate that the model outperforms the benchmark and vice versa. To determine whether the forecast improvements or deteriorations are credibly different from zero, we report results from the Diebold-Mariano test with both 95% and 99% credible intervals.

Table 2: Density (Log Score) and point (RMFSE) forecast results relative to a no-change benchmark. Bold numbers indicate the best forecast performance at each horizon. One or two asterisks indicate that differences are, respectively, credibly different from zero according to the Diebold-Mariano test using 95% and 99% credible intervals.

Log Score									
Horizon	CRB	Futures	Spread	TVspread	VAR	Equal	BMA	BMA2	BPS
1	0,46	**1,69***	-0,55	-3,62	-14,26	-298,95**	-297,04**	-91,44**	-13,23
6	-0,21	3,11	-5,10*	-8,14	-29,57**	-161,18**	-158,22**	-2,55*	**12,59****
12	-12,50	10,02*	-16,56**	-19,14**	-28,74**	-141,46**	-139,63**	25,85*	**47,73****
24	-32,48**	26,10**	-16,91**	-35,25**	2,34	-152,15**	-142,21**	59,14**	**110,96****
RMSFE									
Horizon	CRB	Futures	Spread	TVspread	VAR	Equal	BMA	BMA2	BPS
1	0,95	0,99*	1,00	1,01	0,99	0,96*	0,96*	**0,90***	0,97
6	1,06*	0,97*	1,01	1,04	1,05*	0,99	0,99	0,96**	**0,89****
12	1,05	0,91**	1,01	1,02	1,04	0,96**	0,96**	0,88**	**0,71****
24	1,13**	0,89**	1,07	1,21**	1,01**	0,98	0,97**	0,78**	**0,57****

First focusing on density forecasts, we observe that the proposed BPS approach provides the best forecasts when forecasting the real price of oil beyond the immediate one-month-ahead forecast horizon, and that these improvements are credibly different from zero. The no-change model is difficult to beat when producing one month ahead density forecasts, however the performance difference between BPS and the no-change model is not credibly different from zero at this horizon. Interestingly, we find that the Futures model improves upon the no-change model at each forecast horizon, while the other individual models generally fail to outperform the benchmark; exceptions include the CRB model at the one-step-ahead horizon and the VAR at horizon 24. Due to aggregated forecast uncertainty, the equal weight and BMA combination methods do substantially worse than the no-change model at all horizons. By allowing for a more flexible weighting procedure we find that the two-year rolling window BMA2 not only outperforms both equal weighted and expanding window BMA approaches, but also improves upon the no-change benchmark at the longer forecast horizons of 12 and 24 months respectively.

Shifting focus to point forecasts, we observe that the proposed BPS model provides substantial improvements over the no-change benchmark at all forecast horizons, and that these improvements are credibly different from zero beyond the one month horizon. Consistent with Baumeister and Kilian (2012), we observe that the commodity price-based model improves upon the no-change model at the one-month horizon but does worse at longer horizons. Also in line with Baumeister and Kilian (2015), we find that the equal weights combination model outperforms the no-change benchmark at all forecast horizons, and that the importance of futures-based information improves with the forecast horizon. In line with the density forecast results, the point forecast accuracy of BMA is found to be similar to the equal weights model—a result that is commonly referred to as the "forecast combination puzzle" within the broader literature on model combinations of competing point forecasts. A simple theoretical explanation for this puzzle proposed in Claeskens et al. (2016) is that computed weights may result in biased forecasts (even when the original forecasts are unbiased), and a larger variance than in the deterministic case of equal weights. From a theoretical perspective, there is consequently no guarantee that a more sophisticated forecast combination methods will be better than the equal-weight case, or even improve on the original forecasts. For instance, Baumeister and Kilian (2015) report that point forecasts based on an equal weight combination model tend to be more accurate than those from a combination model in which weights are computed using the inverse root mean squared forecast error of the individual models - a finding that is corroborated in Garratt et al. (2019). That being said, we find that by specifying more dynamic weighting schemes in BMA2 and learning weights in BPS, we are able to generate greater forecast accuracy at all horizons.[9] Moreover, the proposed BPS approach provides substantial im-

[9]While not reported here, we have also studied the performance of standard TVP-SV AR and TVP-SV VAR models (Primiceri, 2005). In general the forecast accuracy of these models are inline with those of

provements beyond all individual and combination models. The size of these improvements is also increasing with the forecast horizon, in which oil prices are generally assumed to exhibit near random walk behavior. This suggests that the proposed BPS model may prove to be particularly useful for practitioners who are looking to hedge against oil price risk. We further explore this later in Section 4.4.

As a next step, we determine whether differences in forecast accuracy between models and model combinations hold throughout the time series periods for the different forecast horizons. To this end, we show the time patterns of cumulative log scores and RMSFEs relative to a no-change model benchmark in Figures 4 and 5 respectively. For interpretation purposes, values in Figure 4 that are greater than zero indicate that the model outperforms the benchmark and vice versa, while values in Figure 5 that are less than one indicate that the model outperforms the benchmark and vice versa.

The results in Figure 4 reveal considerable time variation in the relative performance of both individual and combination model specifications. In line with the full data period results, the proposed BPS model is competitive at the one-step horizon and outperforms the no-change benchmark, all individual models, and all combination models, at longer forecast horizons. In contrast, both equal weight and BMA models become progressively worse relative to the no-change benchmark, while the individual model specifications tend to cluster around similar values. Finally, while the recursive window BMA model (BMA2) does quite poorly when forecasting one-month-ahead, the forecast accuracy is competitive with the best individual model forecasts when forecasting six-months-ahead, and is second only to the BPS model at the longer 12 and 24 month horizons.

Turning our attention to the point forecast results, reported in Figure 5, we again find considerable time variation in the relative performance of each model specification. In line with the density forecast results, we find that the BPS model is competitive at the one-month horizon and provides substantial improvements at longer horizons. The fact that most of the models RMSFEs cluster around one at the one-month horizon suggests that the oil price exhibits near random walk behavior at this horizon. A notable exception occurs around the oil price collapse in 2009, during which the combination models provide notable

the constant coefficient VAR.

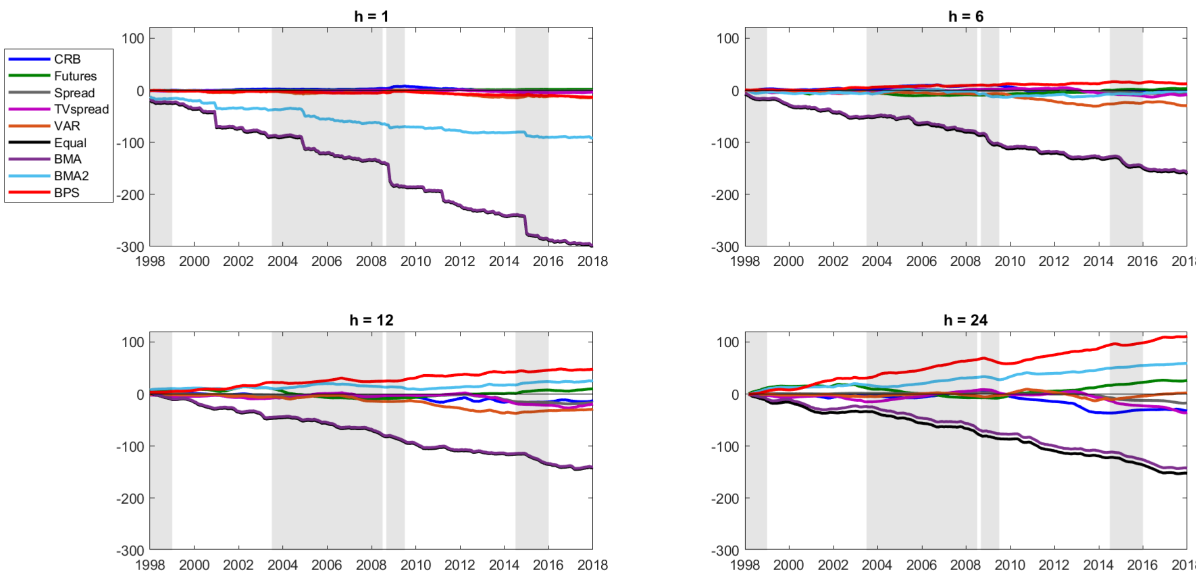

Figure 4: Time patterns of cumulative log scores relative to a no-change model benchmark over the forecast evaluation period: 1998:03-2017:12. Forecast horizons: 1, 6, 12 and 24 months ahead.

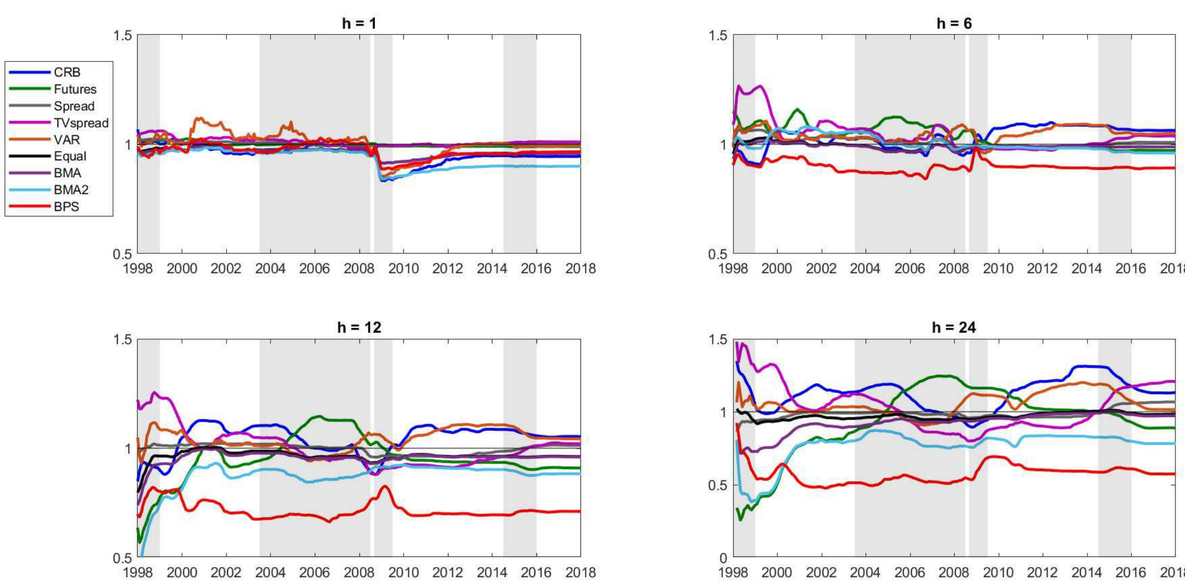

Figure 5: Time patterns of RMSFEs relative to a no-change model benchmark over the forecast evaluation period 1998:03-2017:12. Forecast horizons: 1, 6, 12 and 24 months ahead.

improvements over the no-change benchmark, however these gains gradually dissipate over the next few years. In contrast to the relatively similar forecast performance at the one-month horizon, the longer horizons exhibit much more dispersion. For instance, at each horizon, the futures price model produces generally superior forecasts relative to the no-change benchmark with the exception of the early to mid 2000's. This is in line with existing results that the real price of oil between mid 2003-08 was driven by unexpectedly high growth mainly in emerging Asia (Aastveit et al., 2015). We also find that the VAR model performs well up until 2010, but then does relatively poorly over the subsequent period, thereby corroborating recent results in Baumeister et al. (2020).

As a final step, we determine the usefulness of the proposed BPS combination method by providing an accurate approximation to the observed data distribution by plotting a histogram of the data over the forecast evaluation period together with the combined forecast densities from the BPS model in Figure 6. This is useful because economic decision makers, such as central bankers or portfolio managers, take forecasts of the price of oil into account when making their policy or investment decisions. As discussed in the previous section, we see substantial time-variation in the shape of the data distributions, with asymmetry, fat tails and bi-modality all being important data features. The additional insight from Figure 6 is that the proposed BPS combination method is able to provide a good approximation of the data distribution at each of the forecast horizons. To further investigate how good this approximation is from a statistical perspective, we present in Table 3 results from a two-sample Kolmogorov-Smirnov test between the empirical cumulative distribution function of the data and the cumulative distribution function of the BPS forecasts at each forecast horizon. The results show that we are unable to reject the null hypothesis that the data and forecast distributions differ at the 5% credible level.[10] This suggest that the BPS forecast density provides an accurate proxy for the empirical data distribution, and may consequently be useful to economic decision makers in practice.

[10] We have also studied absolute accuracy by testing if the density forecasts are correctly calibrated, using the test in Knüppel (2015). Forecasts from our BPS approach seem to be well-calibrated at all horizons (with the exception of $h = 6$), as the null hypothesis of correctly calibrated densities, cannot be rejected at the 10% credible level. For the other models and combination approaches, the null hypothesis are rejected at the 10% credible level at all horizons with the exception of h=1.

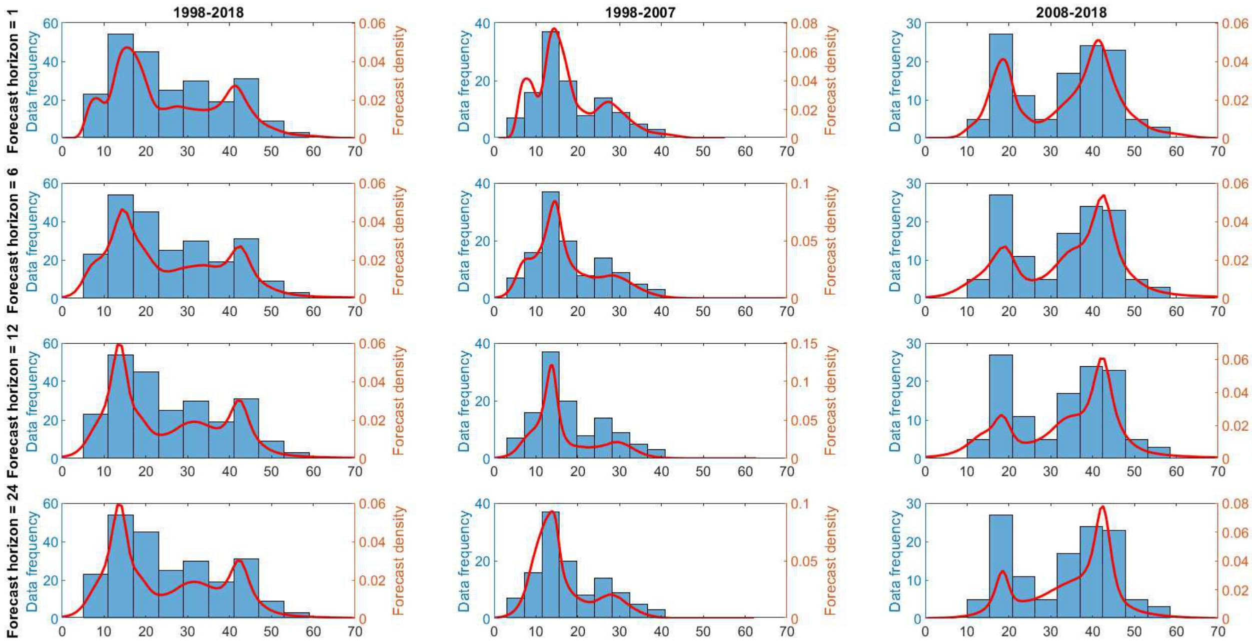

Figure 6: Forecast density combinations and data distributions of the real IRAC price of oil at monthly frequency over the forecast evaluation period: 1998:03-2017:12, and notable sub-periods. The blue histograms depict the data distributions of the real price of oil in levels over specified periods. The horizontal axis represents the level of the real price of oil in USD pooled into nine bins. The left vertical axis represents the pooled count of observations over the respective periods. The red curves depict a kernel estimate of the FDC using pooled draws from the FDC distribution across each of the periods. The right vertical axis shows the associated density values.

Table 3: P-values from a two-sample Kolmogorov-Smirnov test between the empirical cumulative distribution function of the data and cumulative distribution function of the BPS forecasts.

Forecast horizon	1	6	12	24
1998-2018	0.98	0.57	0.17	0.43
1998-2007	0.95	0.67	0.22	0.09
2008-2018	0.95	0.88	0.68	0.13

4.3 Learning about time-varying combination weights and model diagnostics

An important feature of BPS is that it allows for time-varying individual model weights that adapt according to the recent relative forecast performance of each model within the model combination set. The means of the densities of the individual model weights are shown in Figure 7. The general observation across all forecast horizons is that while considerable time variation exists, there are similarities among models. First, focusing on the one-step-ahead results we observe that the mean weights for both the CRB and futures price models tend to follow a similar trajectory, while the Futures, Spread and TV spread models respectively follow a similar path that is distinct from the former trajectory. It is particularly notable that the mean weights for the two series in the former group abruptly increase during the two oil price collapses of 2009 and 2014 respectively, but gradually decline in the subsequent years surrounding these events. In contrast, the mean weights for the three series in the latter group sharply declined during the oil price collapse of 2009, but then gradually increase, with all mean weights sharing roughly similar time patterns by the end of the period. Interestingly, the same weighting clusters are not observed at longer horizons. At the 12-month-ahead horizon, we find that each of the mean weights tends to follow a similar trajectory. One notable exception is the abrupt increase in the mean weight of the Futures model following the 2014 oil price collapse.

It is also worth noting that the BPS mean weights are not restricted to be a convex combination in the unit interval—like most of the combination methods that are currently used within the econometrics literature—but are instead specified as a general linear combination and are thereby permitted to evolve along the real line. While this may have a disadvantage relative to the natural interpretation of mean weights within convex combinations as representing a probability distribution over different possible models, specifying a linear combination offers two practical advantages. In the first instance, allowing for both positive and negative mean weights means that it's possible to hedge against any potential forecast risk in recent periods. Second, a declining mean weight on one model does not necessarily imply an increasing mean weight on another model. Instead, mean weights of models are permitted to change in accordance to the forecasts of the individual models.

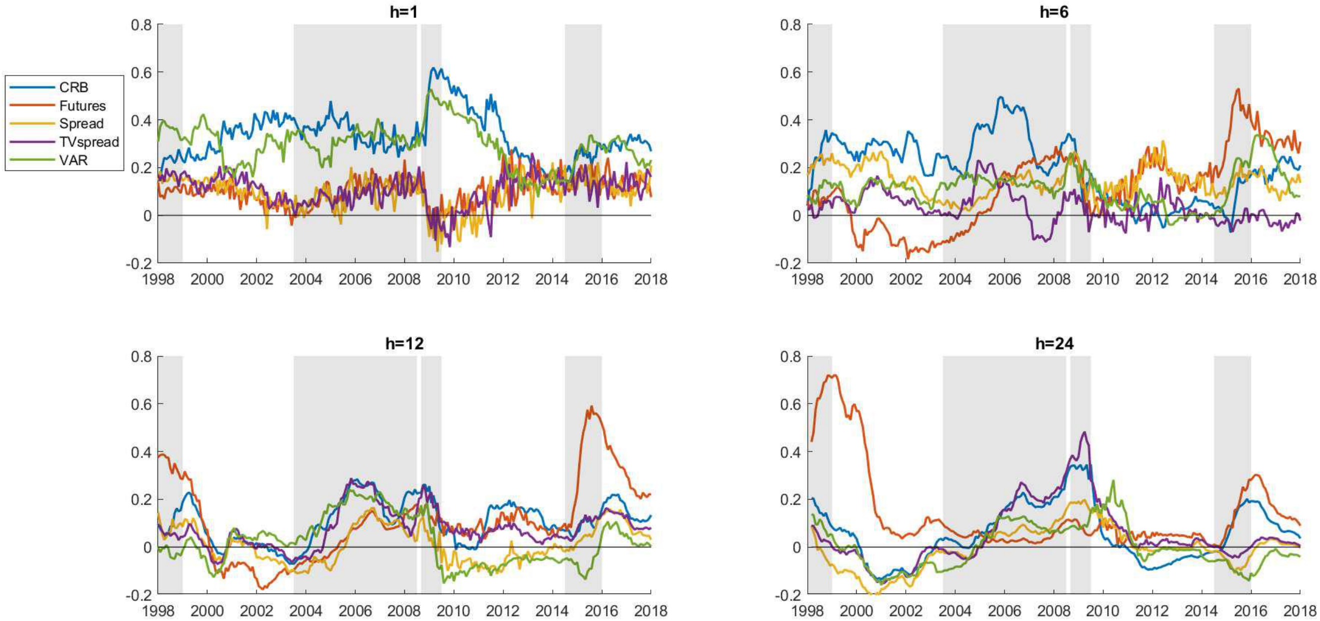

Figure 7: Time-varying posterior predictive mean of the individual model weights ($v_{i,t}$) in the BPS model, sequentially computed at each point in time over the forecast evaluation period 1998:03-2017:12.

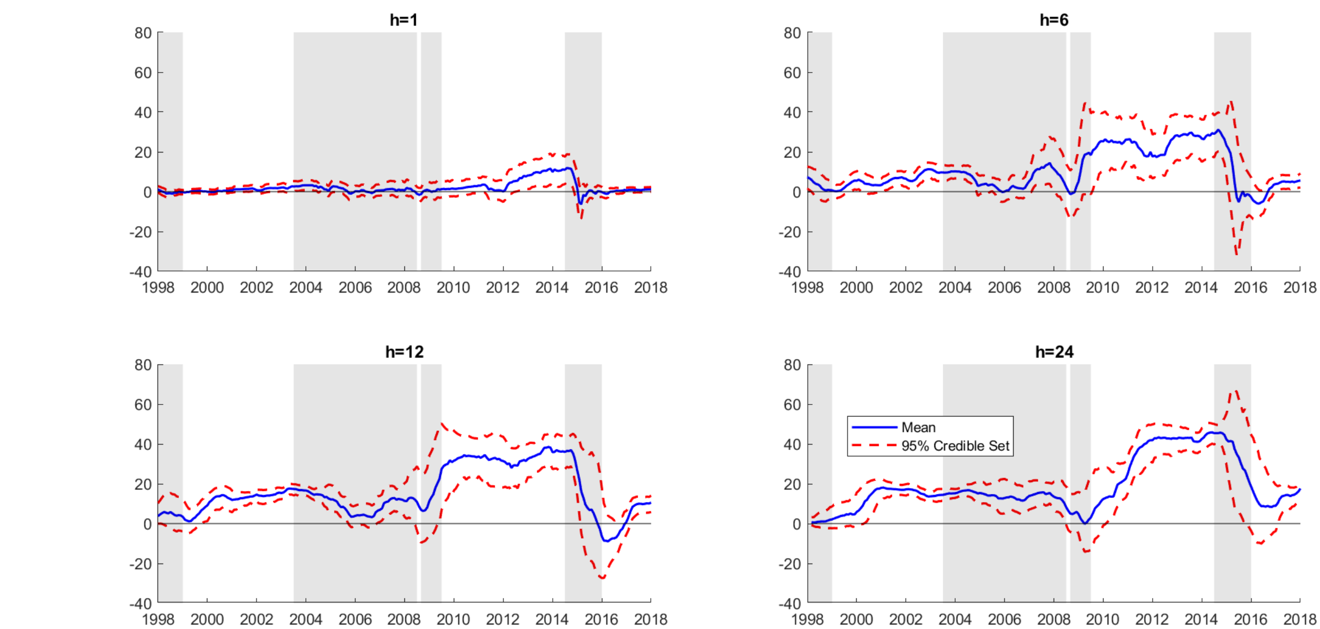

Figure 8: Time-varying posterior predictive mean of the intercept coefficient weight ($v_{0,t}$) in the BPS model, sequentially computed at each point in time over the forecast evaluation period 1998:03-2017:12.

Such features are clearly desirable to a wide range of practitioners. For instance, just as a financial portfolio manager hedges against risk by assigning a negative mean weight to an asset, the BPS approach is able to automatically assign a negative mean weight to mitigate the impact of forecast risk, such as forecast bias. Such natural behaviour is not possible under combination models in which the mean weights are restricted to be convex combinations, e.g. BMA or equal weighting methods.

Another important feature of BPS is that it has a built-in time-varying intercept that is absent from simpler combination methods such as BMA. It is well known within the econometrics literature on forecast combinations that BMA assumes that the true model is included in the model set. However, the model set could be misspecified due to incompleteness. By allowing for an intercept component that can adapt during episodes of low frequency signals from a set of forecasting models, BPS is able to better mitigate the effects of this problem. For instance, from the previous section, we know that there exists a model within the combination set–e.g. the VAR model–that provided superior one-step-ahead forecasts of the real price of oil up to and during the oil price drop of Great Recession relative to the no-change benchmark. After 2010, however, none of the models forecasted the oil price collapse of 2014. This suggests that there exists a degree of model set incompleteness since 2010, and we expect that this is reflected in the estimated BPS intercept for the one-step-ahead forecast. This is exactly what we observe in Figure 8 which shows the mean intercept terms at each forecast horizon. The one-step-ahead mean intercept is around zero up until 2010, when it starts to gradually increase before abruptly dropping and becoming negative during oil price collapse of 2014. Moreover, as shown previously in Figure 5, this feature allows BPS to improve upon the no-change benchmark despite the relatively weak signals stemming from the models during this period. Similar scenarios can be observed in the remaining estimates, which each exhibit a hump shaped response for the respective estimated mean intercepts over the period 2008 to 2014.

The final important feature of BPS is that it has built-in diagnostic information measures about forecast inaccuracy and/or model set incompleteness which is also absent from simpler combination methods such as BMA. We first present this diagnostic measure for σ_t^2 for the model set in Figure 9. It shows clearly that during three crisis periods this measure increases. In Figure A1 in the supplementary material we also provide estimates of this diagnostic measure for each individual model, $\sigma^2_{i,t}$, within the BPS framework. It is seen from the longer term forecasts that none of the individual models is capable to accurately forecast crises, in particular the VAR does poorly in crisis periods estimated over short as well as long horizons. This type of diagnostic information gives important signals about specifying model and model set improvements. We leave this as a topic for further research.

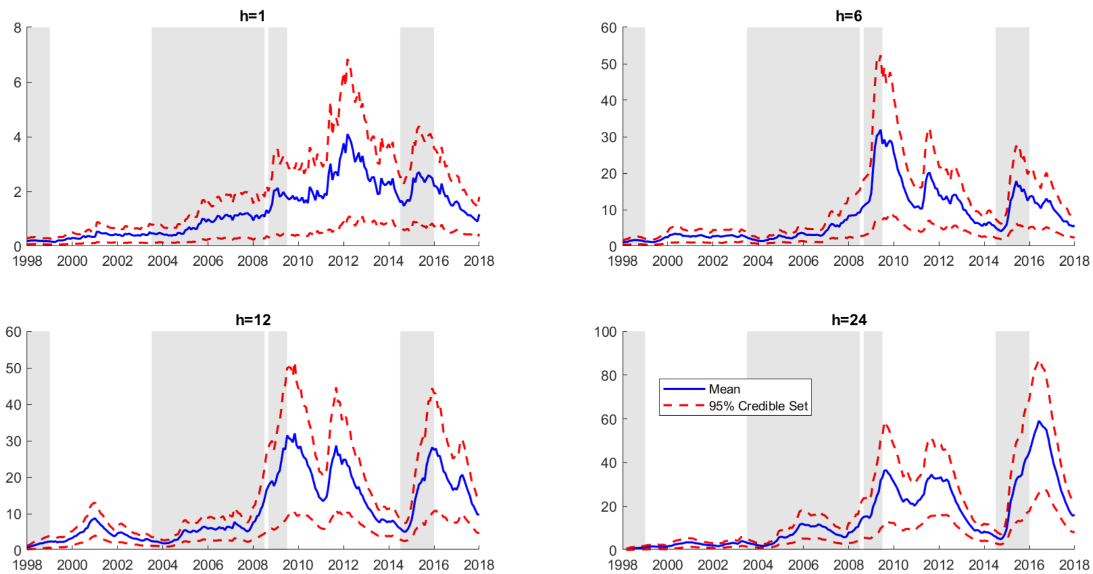

Figure 9: Time-varying variance, measured as the posterior predictive mean of the measurement variance, in the BPS model (σ^2_t), sequentially computed at each point in time over the forecast evaluation period 1998:03-2017:12. The red dotted line show the 95% credible bands.

4.4 Risk analysis

In this Subsection we analyse the risk and return properties of investing in the global market for crude oil using the BPS modelling approach as an investment tool. The means of the profit and loss distribution including 95% credibility regions associated with the forecasted spot prices from BPS are shown in Figure 10. For interpretation purposes, we highlight that positive mean values indicate a profit and negative mean values indicate a loss. There are no periods of profits or losses that are credibly different from zero, nor is there any observable serial correlation pattern across the entire data period, thus corroborating the widely held view that, in the short run, oil prices exhibit random walk behavior. That being said, there are periods in which BPS does well, and not so well. For instance, the means signify notable profits could have been made during the two oil price collapses of 2008-09 and 2014-15, however they then tend to gradually revert to zero and the credible set contains negative values.

Figure 10: Means of the profit and loss distribution (profit positive and loss negative) associated with the forecasts from the BPS model, sequentially computed at each point in time over the forecast evaluation period 1998:03-2017:12. The red dotted line show the 95% credible bands.

To quantify the measure of risk of loss associated with the profit and loss distributions, we next compute the value-at-risk (VaR). The VaR is widely used by regulators and practitioners in the financial industry to measure the quantity of assets needed to cover possible losses. The implied 1% and 5% VaR for the BPS model profit and loss distribution over the forecast evaluation period are shown in Figure A2 in the supplementary material. The vertical axis are in percent. For interpretation purposes, this means that, for instance, a one-month 1% VaR of -2 means that there is a 1% chance of a 2% loss during the one-month time frame.

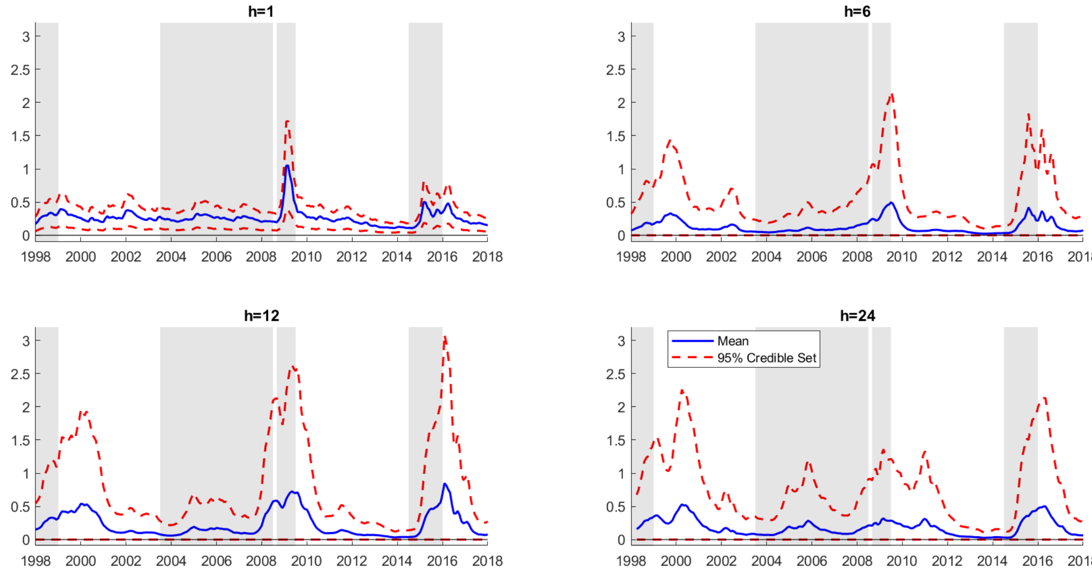

Figure 11: The minimum variance hedge ratio (MVH) from the BPS model, sequentially computed at each point in time over the forecast evaluation period 1998:03-2017:12. The red dotted line show the 95% credible bands.

Faced with such risks when operating in global oil markets, firms and portfolio managers naturally face the decision whether or not to hedge against unanticipated fluctuations in the price of oil. For instance, a petroleum company may wish to hedge its purchase price of crude oil by purchasing a futures contract. A widely used strategy for computing the optimal number of contracts needed to hedge a position is the ratio of the product of the optimal hedge ratio and the units of the position being hedged, to the size of a futures contract. The most common optimal hedge ratio is the Minimum Variance Hedge (MVH) ratio which aims to minimize the variance of the position's value. It is calculated as the product of (1) the correlation coefficient between the changes in the spot and futures prices, $\rho_{S,F}$, and (2) the ratio of the standard deviation of the changes in the spot price, σ_S, to the standard deviation of the futures price, σ_F.

Means of the MVH ratios, including 95% credibility regions, are shown for the forecasted spot price from the BPS model in Figure 11. Since the Brent price of crude oil is often used as a global price benchmark, we have used Brent futures data from 1992:1-2017:12

as provided by Garratt et al. (2019). The results show that the means of the optimal hedge ratio differ based on the forecast horizon. For instance, the mean of the MVH ratio tends to fluctuate between 0.2-0.4 at the one-step-ahead horizon, compared to 0-0.1 at the six-step-ahead horizon. That being said, at each horizon we observe notable spikes occur around the turn of the century as well as the two oil price collapses of 2009 and 2014.

5 Conclusion

Given the typical data pattern of the real price of oil over a substantial time period and some well-known models that describe these data, discussed in Section 3 and Section 4, we have successfully specified a basic probabilistic model structure and corresponding state space equation system based on the Bayesian Predictive Synthesis approach. Compared to more standard approaches like BMA, our BPS approach contains important extensions about diagnostic analysis of model set incompleteness and time-varying learning weights in the combination. This approach also leads to the use of numerically efficient Markov Chain methods in order to evaluate the Forecast Density Combination.

Applying a Bayesian procedure to estimate this model, we have obtained an extensive set of empirical results about time-varying forecast uncertainty and risk for the real price of oil over the period 1974-2018. This yielded substantial gains in forecast accuracy from point and, in particular, density forecasts using model combinations compared to individual models. These forecast gains are confirmed by exploring the estimated forecast time patterns, especially in the long term like 12-24 months, which is relevant information when forecasts are used for policy decisions. Dynamic patterns of the estimated individual model weights showed the relative contribution of individual models in the forecast combination. In addition, time patterns of diagnostic information about model incompleteness were obtained which give information on possible improvements about model specifications. We ended our analysis by showing results of time-varying risk in the oil price forecasts and presented a basic analysis of profit-loss and hedging against price risk.

The research presented can be extended in several directions and for classes of many economic data sets that are of interest for forecasters and policy makers. Exchange rate forecasting and risk analysis using sets of countries is an obvious example. Using microdata to strengthen the information contained in macroeconomic forecasts and using large sets of finance data for dynamic portfolio analysis are further research topics with potential interesting policy implications.

We end with a remark on the possible connections between typical data patterns of economic variables of interest, the complexity of an FDC model and the class of Monte Carlo simulation algorithms which has to be used for the numerical evaluation of the densities involved. The literature of this field is extensive and still expanding; for a recent survey we refer to Aastveit et al. (2019). Which FDC approach is the most useful to apply depends on data patterns and model specification. This is an interesting topic of future research but beyond the scope of the present paper.

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SUPPLEMENTARY MATERIAL

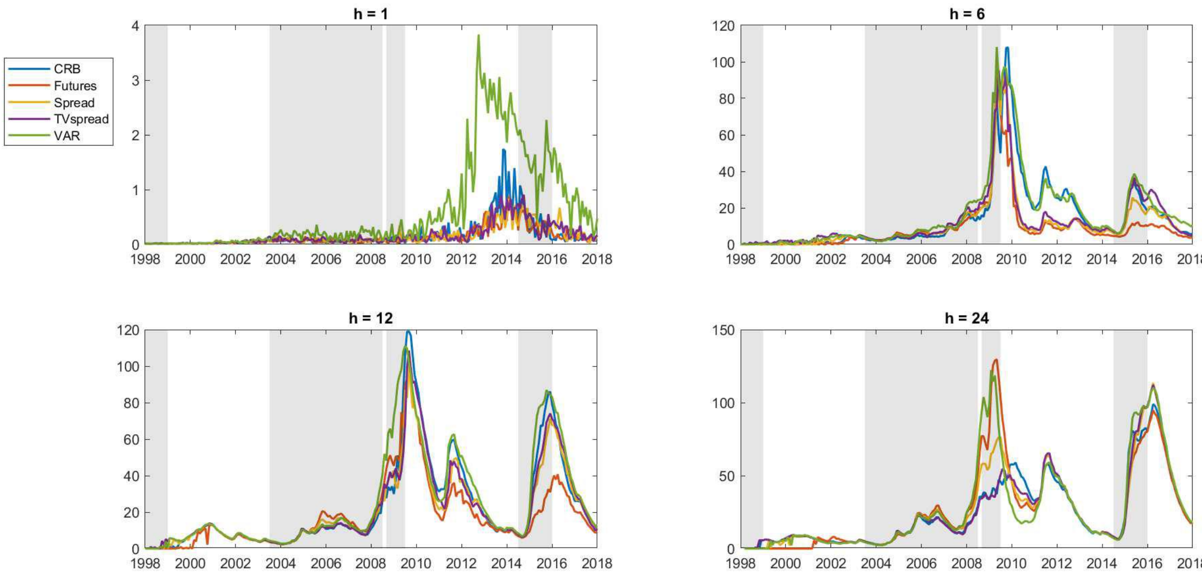

Figure A1: Time-varying variance, measured as the posterior predictive mean of the measurement variance, for individual models in the BPS model ($\sigma^2_{i,t}$), sequentially computed at each point in time over the forecast evaluation period 1998:03-2017:12.

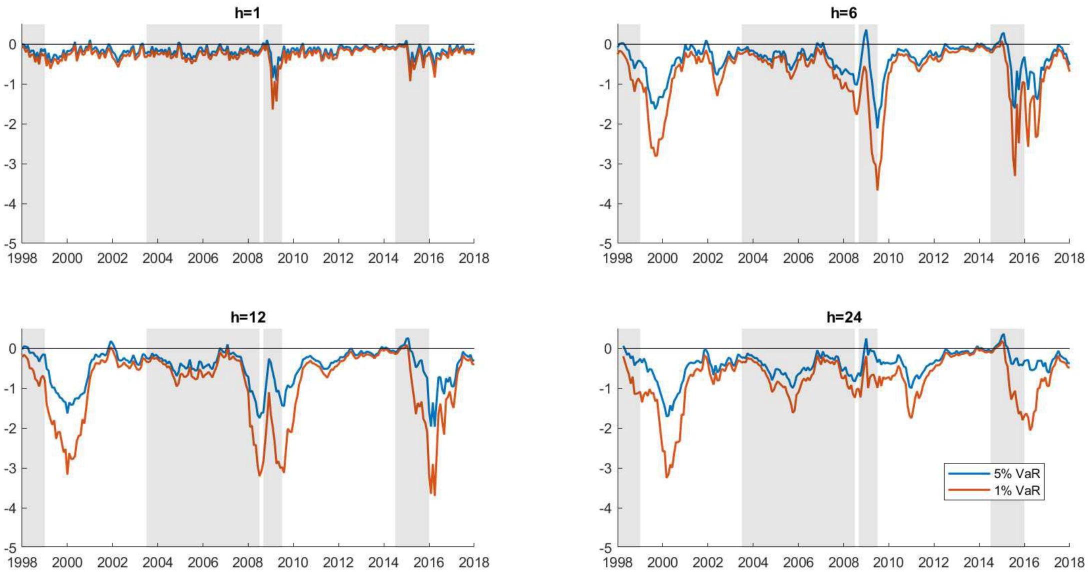

Figure A2: Value at Risk (VaR) of the profit-and-loss distributions from the BPS model, sequentially computed at each point in time over the forecast evaluation period 1998:03-2017:12.