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On existence of private unemployment insurance with advance information on future job losses

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Abstract

We study the existence of a profitable unemployment insurance market in a dynamic economy with adverse selection rooting in information on future job losses. The new feature of the model is that the insurer and workers interact repeatedly. Repeated interactions make it possible to threaten workers with exclusion from future insurance benefits after a default on insurance premia. With exclusion, not only the insurance against the fundamental risk, but also against future bad news about job losses matters. In contrast to conventional wisdom, we find that private unemployment insurance in the US can be profitable for a relatively short exclusion length of one year. To stimulate the emergence of a private unemployment insurance market, policy makers can facilitate the creation of a registry that archives past defaults on insurance premia.

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1 Introduction

Unemployment is one of the most severe risks that many people face. Even with unemployment benefits provided by the government, it is well documented that becoming unemployed forces people to substantial and undesirable cuts in their consumption expenditures (e.g., Gruber, 1997). Furthermore, Braxton, Herkenhoff, and Phillips (2020) provide evidence that the unemployed resort to costly self-insurance by defaulting on their debt with the consequence of long-lasting detrimental effects on their credit score and financial liberty. In the light of this, why isn't there a thriving private unemployment insurance market in the US?

The profitability of insurance in existing markets is plagued by two types of private information frictions, moral hazard and adverse selection. However, Shavell (1979) demonstrates that moral hazard does not impede the existence of insurance. The conventional explanation for the missing insurance market is therefore an adverse selection problem that is grounded in individual's private foreknowledge about a job loss. Employing the static binary loss model originally proposed by Rothschild and Stiglitz (1976), Hendren (2017) argues that unemployment insurance policies are too adversely selected to be profitable.

In the Rothschild and Stiglitz model, risk-averse agents have private advance information on their idiosyncratic endowment risk, which leads to an adverse selection problem. Conditional on this information, but before the endowment shock realizes, they decide whether to accept an insurance contract offered by a risk-neutral insurer or to merely consume their endowments in autarky. This decision occurs at a single instance, which is why we refer to it as a single-interaction model. This parsimonious framework abstracts, by construction, from long-run interactions between agents and the insurer. We show that allowing for long-run interactions matters for the existence of a profitable unemployment insurance market and offers new insights on how to overcome the negative effects of adverse selection for market existence.

To study the profitability of private unemployment insurance, we propose a dynamic version of the binary loss model. Private advance information arrives every period and is modeled as a private signal that informs agents on their subsequent endowment shock, with an adverse shock in case of unemployment. After the signal realizes, agents decide in each period whether to sustain the insurance contract on offer. With repeated interactions – and this is the crucial difference to the single-interaction case – the insurer has the possibility to threaten agents with exclusion from future insurance in case they decide to reject the contract and default to the no-trade allocation. Exclusion fundamentally alters agents' incentives to participate in the insurance market. Not only the current, but also the future benefits of insurance matter, when agents decide whether to accept an insurance contract offered to them, which increases their willingness to pay for insurance.

The benefits of insurance in the dynamic economy are larger than in a single-interaction economy because they include additional insurance possibilities. In a dynamic economy, agents can insure not only the fundamental endowment shocks, but also the future private signal realizations, that is, "bad news" received in the future. Insurance against bad news cannot be provided in the single-interaction environment because today's news have already been realized. As a consequence, the threat of exclusion from future insurance is powerful in the dynamic context. It can even prompt agents who don't face any unemployment risk in the current period to purchase insurance. Inclusion of these agents in insurance is important for profitability because they constitute good risks. In the single-interaction model, these safe agents are not willing to pay for insurance at all and drop out of the insurance scheme. Including these agents is also relevant because they constitute a non-negligible group in reality. Many individuals can be almost certain of not losing their jobs, at least in the short run, for example, because of their senior rank.

As our main theoretical result, we provide a no-trade condition for the dynamic economy with the length of exclusion as the key parameter, which generalizes the earlier findings. The possibility of exclusion is what distinguishes our dynamic framework from previous work; without exclusion, the no-trade condition reduces to the one provided in Hendren (2017). Intuitively, the longer the agents are excluded from future insurance, the higher their willingness to pay for insurance today is. This makes the provision of insurance more profitable to the insurer and is reflected in a more restrictive no-trade condition in the dynamic economy.

We employ our theoretical model to study the issue of the missing unemployment insurance (UI) market in the US. While the missing market is a universal phenomenon, our focus on this country is motivated by the fact that the importance of private advance information on future job losses is well documented for the US. Stephens (2004) and Hendren (2017) find that individual subjective job-loss expectations carry predictive power for subsequent job losses, even when public information available to an econometrician is taken into account. Through the lens of our model, we revisit the issue of the missing private unemployment insurance market in the US and ask: do repeated interactions and market exclusion matter for its existence?

To address this question, we inform our theoretical model with Hendren (2017)'s estimates of individuals' willingness to pay for unemployment insurance and the costs of adverse selection. Through the lens of his single-interaction model – or equivalently, of our dynamic model without exclusion from insurance in the future – his estimates yield costs of adverse selection that by far exceed agents' willingness to pay, hindering the existence of a profitable insurance market. This changes when we allow for exclusion. Already if agents face the threat to be excluded for one year after a default, unemployment insurance can be provided at a profit.

Bond and Krishnamurthy (2004) emphasize the role of exclusion for well-functioning unsecured credit markets. After filing for private bankruptcy, individuals face a loss in financial liberty of up to 10 years according to Chapter 7. Compared to this exclusion length, an exclusion length of one year to render the UI market profitable does not appear long. A relatively short exclusion length suffices to render the insurance market profitable because the benefits of future insurance are quantitatively important. Thereby, the future benefits of insurance stem predominantly from insurance against bad news that were not accounted for in the previous literature.

The quantitative importance of exclusion for the existence of unemployment insurance does not depend on whether we consider an economy with infinitely or finitely lived agents. Considering also a life-cycle version of the model is relevant because the threat of exclusion is of no concern to agents who are retiring, which impedes profitability. However, we find that a similar length of exclusion suffices here for profitable insurance provision. The insurer can exploit that agents' willingness to pay for insurance differs with respect to age. The insurer provides less attractive UI to the young cohorts – who value insurance the most – and extracts more resources from them to subsidize the better insurance offered to individuals nearing retirement, which prevents the contract from unraveling.

Our robust quantitative results imply that adverse selection rooting in individual foreknowledge of future job loss alone is unlikely to be the cause of the missing unemployment insurance market. The threat of a relatively short exclusion period is enough to make the provision of insurance profitable even when UI policies are adversely selected. Arguably, there isn't a thriving private UI market in the US but – unlike in case of adverse selection as the sole reason for the missing market – our quantitative and theoretical results have clear policy implications for how to kick-start the market. To make the exclusion threat credible, it is helpful to have a registry system that collects information on defaults of UI policies, similar to the successful registry with information on credit defaults. Creating and maintaining such a registry entails fixed costs, and economic policy can contribute to overcoming them. One difference to the credit market is that we show that exclusion is not only important for the well-functioning of the UI market but already indispensable for its existence. Thus, the launch of the registry system must be pre-announced and in place when the first UI policies are sold.

Related literature In a related paper, Braxton, Herkenhoff, and Phillips (2020) study optimal public UI in a dynamic economy with temporary exclusion from the credit market of defaulting agents due to search frictions. Our contribution is to highlight the importance of contractual exclusion for the existence of a profitable UI market hampered by adverse selection stemming from private information on future job losses. Our paper is closely related to Hendren (2017) who also studies the conditions for profitable unemployment insurance. We generalize his theoretical results in a dynamic economy to accentuate the role of market exclusion and insurance against bad news for the existence of a profitable UI market.

Our paper also contributes to the literature on the optimal design of unemployment insurance over the life cycle. While the previous literature mainly focusses on moral hazard as the private information friction, we offer new insights into the design of optimal insurance because we investigate the role of adverse selection. Michelacci and Ruffo (2015) find that younger and not older workers should receive more generous UI because the implicit costs of moral hazard are mitigated by long-term career concerns in their case. We find that unemployment insurance for older workers should be subsidized because exclusion from future insurance is less relevant for them.

Our theoretical approach shares similarities with the literature on the welfare effects of advance information in efficient risk sharing. Hirshleifer (1971) and Schlee (2001) show that advance information can make risk-averse agents ex-ante worse off if such information leads to an evaporation of risks that otherwise could have been shared in a competitive equilibrium with full insurance. Allowing also for the insurance of news as well as fundamental risk, Denderski and Stoltenberg (2020) analyze the social value of better public information when agents have also private advance information about future income shocks. None of these papers study the role of advance information and exclusion for the existence of profitable UI market.

In Section 2, we present the model. In Section 3, we provide and discuss our theoretical results on existence of insurance. Section 4 contains a quantitative application of theory to unemployment insurance and the last section concludes.

2 Environment

Time is infinite and there is a unit mass of agents who, when employed, earn income y.[1] Each period, agents receive a private signal n about their probability to become unemployed and suffer an income loss l, $0 < l < y$, with their income dropping to $y - l$ (throughout, we use *unemployment* and *income loss* interchangeably). The shock to their endowment realizes before agents consume. The signal is i.i.d. across agents and over time with two realizations, *good* or *bad news*, $n \in \{g, b\}$. When $n = b$, agents become unemployed with probability one. For $n = g$, the probability of an income loss is $0 < p < 1$. The probability to receive bad news is $0 < \mu < 1$.

To facilitate comparison with earlier work, we consider contracts that prescribe consumption that solely depends on the current realizations of the signal and the endowment shock. A consumption allocation prescribed by such a contract is denoted by $c = \{c_{gy}, c_{gl}, c_{bl}\}$, with c_{gy} as consumption in case of no loss, and c_{gl} (c_{bl}) as consumption in the event of unemployment after receiving a good (bad) private signal. Thus, the insurance contracts and the corresponding consumption allocations are memoryless.

Agents discount future utilities with $0 < \beta < 1$ and have identical expected-utility preferences over consumption streams. The instantaneous utility function $u : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is strictly increasing, strictly concave and satisfies the Inada conditions. In particular, we define $\tilde{w}(c)$ to be the lifetime expected utility implied by a particular allocation before any risk has been resolved:

$$\tilde{w}(c) \equiv (1-\beta)\mathbb{E}\sum_{t=0}^{\infty}\beta^t u(c_t) = \mu u(c_{bl}) + (1-\mu)\left[pu(c_{gl}) + (1-p)u(c_{gy})\right], \tag{1}$$

Each period after the realization of the private signal, but before the endowment shock occurs, agents have the option to default to autarky. In case of default, agents consume their endowments and – which is the new element here – are excluded from insurance for $N \geq 0$ periods when they default. The individual rationality constraint of good-signal agents can be compactly written as:

$$\begin{aligned}&(1-\beta)\left[pu(c_{gl}) + (1-p)u(c_{gy})\right] + \beta\tilde{w}(c) \geq \\ &(1-\beta)\left[pu(y-l) + (1-p)u(y)\right] + \beta(1+\beta+...\beta^{N-1})U^{Aut} + \beta^{N+1}(1+\beta+...)\tilde{w}(c),\end{aligned}$$

[1] Opting for an infinite horizon allows us to analytically characterize the necessary and sufficient condition for the absence of trade. In Section 4, we also study a life-cycle economy.

or:

$$(1-\beta)\left[pu(c_{gl})+(1-p)u(c_{gy})\right]+\beta(1-\beta^N)\tilde{w}(c)\geq \\ (1-\beta)\left[pu(y-l)+(1-p)u(y)\right]+\beta(1-\beta^N)U^{Aut},$$

with

$$U^{Aut}=\mu u(y-l)+(1-\mu)\left[pu(y-l)+(1-p)u(y)\right]. \tag{2}$$

With memoryless contracts and i.i.d. private signals, the continuation value $\tilde{w}(c)$ depends only on the future, but not on the current realization of signal and endowment.[2]

For the following, it is convenient to express allocations in terms of utility. Let $C: R \rightarrow R^+$ be the inverse of the utility function u. With u strictly increasing and strictly concave, C is strictly increasing and strictly convex. A memoryless utility allocation is then denoted by $h=\{u(c_{gy}),u(c_{gl}),u(c_{bl})\}=\{h_{gy},h_{gl},h_{bl}\}$ with the corresponding memoryless consumption allocation as $c=\{C(h_{gy}),C(h_{gl}),C(h_{bl})\}$. Equivalently, we will write $\tilde{w}(h)$ instead of $\tilde{w}(c)$. The set of implementable (or constrained feasible) allocations is defined as follows.

Definition 1 (Implementable allocations) *An allocation $h=\{h_{gy},h_{gl},h_{bl}\}$ is* implementable *if the following statements hold for all periods $t\geq 0$.*

1. *h is resource feasible*

$$\mu C(h_{bl})+(1-\mu)[pC(h_{gl})+(1-p)C(h_{gy})]\leq \mu(y-l)+(1-\mu)[p(y-l)+(1-p)y], \tag{3}$$

2. *h is incentive compatible*

$$(1-\beta)h_{bl}+\beta\tilde{w}(h)\geq(1-\beta)h_{gl}+\beta\tilde{w}(h), \tag{4}$$

$$(1-\beta)\left[ph_{gl}+(1-p)h_{gy}\right]+\beta\tilde{w}(h)\geq(1-\beta)\left[ph_b+(1-p)u(0)\right]+\beta\tilde{w}(h), \tag{5}$$

3. *h is individually rational*

$$(1-\beta)\left[ph_{gl}+(1-p)h_{gy}\right]+\beta(1-\beta^N)\tilde{w}(h)\geq(1-\beta)\left[pu(y-l)+(1-p)u(y)\right] \\ +\beta(1-\beta^N)U^{Aut}, \tag{6}$$

$$(1-\beta)h_{bl}+\beta(1-\beta^N)\tilde{w}(h)\geq(1-\beta)u(y-l)+\beta(1-\beta^N)U^{Aut}. \tag{7}$$

Note that the two incentive compatibility constraints simplify to:

$$h_{bl}\geq h_{gl},$$

$$ph_{gl}+(1-p)h_{gy},\geq ph_{bl}+(1-p)u\left(0\right),$$

which resemble the corresponding conditions in Hendren (2013). The two individual rationality constraints (6)-(7) are different than in his paper when there is exclusion, that is, $N>0$.

[2] With contracts that are contingent on the history of endowment and signal realizations, this is not the case. We discuss the relevance of history-dependent contracts for existence of profitable insurance in Section 3.2.

Per construction, the no-trade allocation, $\{u(y), u(y-l), u(y-l)\}$, is implementable. As Hendren (2013), we focus on implementable allocations to study the existence of a profitable insurance market. The question is whether there exists an alternative implementable allocation to autarky that is cost efficient. More formally, such an allocation is defined as follows.

Definition 2 (Cost-efficient allocation) *A cost-efficient allocation h^* is implementable and maximizes the slack on the resource constraint:*

$$\begin{aligned} h^* = \arg \max_{h_{bl}, h_{gl}, h_{gy}} \pi = [(1-\mu)p + \mu](y-l) + (1-\mu)(1-p)y \\ - \mu C(h_{bl}) - (1-\mu)[pC(h_{gl}) + (1-p)C(h_{gy})]. \end{aligned} \tag{8}$$

A cost-efficient allocation is the allocation that a profit-maximizing monopolist insurer chooses whose choices are constrained by individual rationality and incentive constraints.[3]

3 Analysis

In this section, we deliver our main theoretical result on the necessary and sufficient no-trade condition with repeated interactions and a fixed length of exclusion.

3.1 Existence of insurance

As an intermediate step before the main theoretical result, we characterize cost-efficient allocations with trade, for which any two elements differ from autarky. We show that if such allocations exist, they feature insurance, which creates slack on the resource constraint.

Lemma 1 (Cost-efficient allocation with trade) *Let $h^* = \left\{h^*_{gl}, h^*_{bl}, h^*_{gy}\right\}$ be a cost-efficient allocation with trade. Then, the following statements hold.*

(i) Incentive constraints of bad-signal agents hold with equality, so that the utility of agents who incur an income loss is equalized across signal realizations, $h^*_{bl} = h^*_{gl} = h^*_l$, and the incentive constraints of good-signal agents are slack.

(ii) Individual rationality constraints of good-signal agents hold with equality, the individual rationality constraints of bad-signal agents are slack.

(iii) h^* is characterized by $u(y-l) < h^*_l \leq h^*_{gy} < u(y)$.

The proof is provided in Appendix A.1. The logic of the proof can be summarized as follows. For part (i), agents with good signals have a higher outside option value than the bad-signal agents which is reflected in the cost-efficient allocation due to individual rationality constraints, and so their expected utility exceeds the one of the bad-signal agents. Thus, only agents with a bad signal have an incentive to report a good signal realization but not vice versa.

[3] The online appendix to Hendren (2013) offers a formal discussion of this. The question of insurance market existence boils down to whether a monopolist insurer can incur profits or not. This circumvents the issue of potentially non-existent competitive Nash equilibria. Other papers studying monopolistic provision of insurance include Stiglitz (1977) and Chade and Schlee (2012).

For part (ii), convexity of resource costs implies that it is optimal to equalize the consumption of all agents who suffer an income loss. Excluding the bad-signal agents from the contract completely and only transferring resources within the pool of good signal agents is prevented by private information, as captured in the incentive constraints (4)-(5).[4] Perfect insurance, $h_{gy} = h_l$, minimizes the resource costs to satisfy incentive constraints but might be prevented by the individual rationality constraint of good-signal agents as part of the transfers is diverted to bad signal agents. Agents with a bad signal benefit from the insurance contracts directly by receiving transfers from the more fortunate good-signal agents and indirectly because the continuation value with insurance is higher than without trade. These two benefits together render the individual rationality constraints of bad-signal agents slack. Lemma 1 therefore implies that the problem of finding a cost-efficient allocation simplifies to:

$$\max_{h_{gy},h_l} [(1-\mu)p + \mu](y-l) + (1-\mu)(1-p)y \\ - [\mu + (1-\mu)p]\, C(h_l) - (1-\mu)(1-p)C(h_{gy}), \tag{9}$$

subject to individual rationality constraints with that of the good-signal agents holding with equality. As autarky is implementable and yields zero profits, the insurer has no incentive to choose an allocation that requires more resources, implying that an optimal allocation satisfies resource feasibility, though not necessarily with strict equality. We proceed to our main theoretical result and show under which conditions autarky is the only implementable allocation and therefore also the cost-efficient allocation.

Theorem 1 (No Trade) *The autarky allocation $h = \{u(y), u(y-l), u(y-l)\}$ is the only implementable allocation if, and only if*

$$\frac{u'(y-l)}{u'(y)} \leq \underbrace{T_s(p,\mu)T_r(p,\mu,\beta,N)}_{T_d(p,\mu,\beta,N)}, \tag{10}$$

with $T_s(p,\mu)$ as the single-interaction pooled price ratio:

$$T_s(p,\mu) = \frac{\mu + (1-\mu)\,p}{(1-\mu)\,p} = \frac{\mathbb{E}\,[P|P \geq p]}{1-\mathbb{E}\,[P|P \geq p]}\frac{1-p}{p},$$

with $T_r(p,\mu,\beta,N)$ as diminishing factor resulting from repeated interactions:

$$T_r(p,\mu,\beta,N) = \left\{\frac{[1-\beta\mu-\beta^{N+1}(1-\mu)]\,p}{[1-\beta\mu-\beta^{N+1}(1-\mu)]\,p + \beta(1-\beta^N)\mu}\right\},\ 0 < T_r \leq 1,\ \frac{\partial T_r}{\partial N} < 0,$$

and the repeated-interactions pooled price ratio is weakly lower than its single interaction counterpart, $0 < T_d(p,\mu,\beta,N) \leq T_s(p,\mu)$ with equality if, and only if $N = 0$.

The proof is provided in Appendix A.2. The no-trade condition (10) generalizes earlier findings by allowing for repeated interactions in the insurance market. For $N = 0$, the condition can be

[4] In Appendix B.2, we show that when signals are publicly observable, insurance can always be provided because the insurer can exclude bad signal agents from the market completely.

rearranged, yielding:

$$\frac{p}{1-p} \times \frac{u'(y-l)}{u'(y)} \leq \frac{\mathbb{E}[P|P \geq p]}{1-\mathbb{E}[P|P \geq p]}, \tag{11}$$

which is the condition provided in Hendren (2013, 2017) for a single interaction. The ratio of marginal utilities $u'(y-l)/u'(y)$ is the marginal rate of substitution of consumption absent any transfers, which captures the good-signal agents' willingness to pay for insurance. The ratio $p/(1-p)$ is the insurer's actuarially fair cost of transferring resources between the good signal agents without and with a loss. Finally, on the right hand side of (11), we have the true cost of such a transfer, one that accounts for the additional resources going to bad signal agents. With adverse selection due to private information, the true costs exceed the actuarial fair costs such that

$$\frac{\mathbb{E}[P|P \geq p]}{1-\mathbb{E}[P|P \geq p]} \frac{1-p}{p} > 1.$$

The main message from Theorem 1 is that the no-trade condition becomes more restrictive for $N > 0$, that is, it is easier to sustain insurance. With $N > 0$, opting for autarky becomes costlier because agents also sacrifice a part of their future benefits of insurance. This can be seen by re-arranging the no-trade condition as follows:

$$\underbrace{\left[\frac{1}{T_r(p,\mu,\beta,N)}\right]}_{\geq 1} WTP \leq T_s(p,\mu).$$

On the left-hand side, the willingness to pay increases due to repeated interactions because $T_r(p,\mu,\beta,N) \leq 1$ is decreasing in N. The tightness of the no-trade condition depends on the appeal of future insurance benefits to good-signal agents. Thereby, the increased attractiveness of the insurance contract with repeated interactions does not simply stem from extending the benefits in case of a single interaction to multiple periods, but also from additional insurance possibilities.

First, insurance is more relevant for good-signal agents because it becomes more likely they will need it. For one period, the probability of a good-signal agent suffering an income loss l is p, which constitutes the risk she likes to insure away. For N future periods, the probability for an agent who always receives a good signal to suffer an income loss at least once is $\text{Prob}(l) = 1-(1-p)^{N+1}$. Thus, in the infinite limit the probability to incur a loss at least once is one, which increases the attractiveness of insurance for good-signal agents.

Second, with repeated interactions, there can be insurance even in the case of $p = 0$, that is, when the good signal agents are certain of remaining employed in the current period. The case of $p = 0$ is a natural one to consider in the context of unemployment insurance because there are many formal and informal institutions (e.g., employment protection legislation, no-compete clauses and seniority of position) that temporarily eliminate the risk of unemployment for some agents. Such *safe* agents are not willing to pay for insurance in the static model because the signals have already been realized and can therefore no longer be insured which results in an infinitely large single-interaction pooled price ratio T_s:

$$\lim_{p \mapsto 0} \frac{\mathbb{E}\left[P|P \geq p\right]}{1 - \mathbb{E}\left[P|P \geq p\right]} \frac{1-p}{p} = \infty, \tag{12}$$

With repeated interactions and positive exclusion length, however, the value of repeated interactions pooled price ratio T_d for an arbitrarily small value of p is finite and given by:

$$\lim_{p \mapsto 0} T_d(p, \mu, \beta, N > 0) = \frac{\left[1 - \beta\mu - \beta^{N+1}(1-\mu)\right]}{(1-\mu)\beta(1-\beta^N)} > 1. \tag{13}$$

Hence, for some parameter values, safe agents might be willing to accept the insurance contract in the current period. The reason for them to do so is that rejecting the contract now foregoes the value of insurance against receiving *bad news* (receiving a bad signal) in the future.

3.2 No-trade theorem with repeated interactions: discussion

Theorem 1 considers private i.i.d. signals on future income losses for a given length of exclusion from the contract. In the following, we discuss qualitatively how the no-trade condition is affected by: persistent signals; random, instead of deterministic return from autarky (as in the literature on sovereign default (e.g. Arellano, 2008) or search for credit (e.g. Braxton et al., 2020); and contracts that depend not only on the current, but also on past endowment and signal realizations.[5]

Persistent signals In the model, the probability to receive a particular signal n' tomorrow does not depend on the signal realization n today, $\text{Prob}(n'|n) = \text{Prob}(n')$. Lemma 1 implies that the no-trade condition is primarily driven by the individual rationality constraints of good-signal agents. Therefore, depending on which signal becomes persistent, there are opposite effects on the no-trade condition. Consider first persistent good signals, $\text{Prob}(n' = g|n = g) > \text{Prob}(n' = g|n = b) = 1 - \mu$. Then, insuring against bad news becomes less important for good signal agents. Correspondingly, their willingness to pay for insurance decreases, rendering no-trade more likely. Assume alternatively that bad signals become persistent, $\text{Prob}(n' = b|n = b) > \text{Prob}(n' = b|n = g) = \mu$. Now the willingness of good-signal agents to pay for insurance increases because receiving a bad signal once has long-lasting negative consequences, making the insurance of bad news more relevant.

Random return after default After default, agents in our environment are allowed back into insurance for sure after being excluded for N periods. Assume alternatively that returning from autarky is possible every period but random with $0 \leq \theta \leq 1$ as the probability to return, implying that the expected time until re-entry (including the current period) is $1/\theta$. We elaborate on this case in Appendix A.3. We show that the no-trade condition has an equivalent

[5] In Section 4, we assess the quantitative importance of persistent signals, random return from autarky and history dependent contracts for the existence of unemployment insurance in the US.

structure to Theorem 1 and is given by:

$$\frac{u'(y-l)}{u'(y)} \leq T_s(p,\mu)T_r(p,\mu,\beta,\theta), \tag{14}$$

with the single-interaction pooled price ratio $T_s(p,\mu)$ as in Theorem 1 and:

$$T_r(p,\mu,\beta,\theta) = \frac{p\left[1-\beta(1-\theta)\mu\right]}{p\left[1-\beta(1-\theta)\mu\right]+\beta(1-\theta)\mu}.$$

Ceteris paribus, the higher the probability to return is, the more likely it is that autarky is the only implementable allocation. Observe that $T_r(p,\mu,\beta,N=0) = T_r(p,\mu,\beta,\theta=1) = 1$ and $\lim_{N\mapsto\infty} T_r(p,\mu,\beta,N) = T_r(p,\mu,\beta,\theta=0)$. Furthermore, there is an explicit relationship between N and θ that yields an identical no-trade condition:

$$\theta = \frac{\beta^N(1-\beta)}{1-\beta^{N+1}}.$$

History-dependent insurance contracts To ensure comparability with earlier work, we studied insurance contracts that prescribe consumption solely on the basis of the current signal and endowment realization. However, a risk neutral insurer can increase profits by offering insurance contracts that track the history of signals and endowments, for two reasons. First, these contracts allow to smooth consumption of agents not only across different states in the current period but also over time, which increases agents' willingness to pay for insurance and relaxes the individual rationality constraints. Second, the insurer can use both contemporaneous utilities h and continuation values $\tilde{w}$ to provide agents the incentives to truthfully report their private signal. Hence, agents can be screened to alleviate the costs of adverse selection. Taken together, history-dependent contracts further shrink the parameter region in which the no-trade allocation is the only implementable allocation. For this reason, the no-trade condition in equation (10) derived for memoryless contracts should be interpreted as the least restrictive condition for the absence of trade.

4 Quantitative results

In this section, we study the quantitative importance of repeated interactions and the length of exclusion for the existence of a private unemployment insurance market in the US. As our main result, we discover unemployment insurance to be profitable for a relatively short exclusion.

4.1 Estimates and calibrated parameters

We begin by reviewing estimates for the willingness to pay for unemployment insurance and the pooled-price ratio in the US. Afterwards, we describe how we calibrate the structural parameters of the model presented in the previous section to be consistent with these estimates.

We target Hendren (2017)'s annual estimates of the willingness-to pay (WTP) from the Panel Study of Income Dynamics (PSID) as well as the single interaction pooled price ratio

Table 1: Estimates and calibrated parameters: baseline

	Estimate/Parameter	Value
WTP	Willingness to pay	$\{1.29, 1.58, 1.87\}$
PPR	Pooled price ratio, $\inf[T_s]$	4.36
$\text{Prob}(U)$	Mean, subjective job loss distribution	0.0310
μ	Measure of bad-signal agents	0.0239
p	Job loss probability, good-signal	0.0073
β	Annual discount factor	0.9600

Notes: WTP for unemployment insurance with relative risk aversion $\sigma = \{1, 2, 3\}$. Pooled price ratio as point estimate for the minimum pooled price ratio (semi-parametric), $\inf[T_s]$, evaluated at the mass point $\text{Prob}(U) = 0.0310$.

(PPR) and the mean of the subjective job loss probability distribution based on the Health and Retirement Survey (HRS), $\text{Prob}(U)$, which are summarized in Table 1.

The willingness to pay estimates – for relative risk aversion $\sigma \in \{1, 2, 3\}$ – one-for-one correspond to the ratio of marginal utilities in the model. The single interaction pooled price ratio in the model is parameterized by p and μ. To identify these parameters, on top of the estimate of the pooled-price ratio, we use the mean of the subjective job loss probability distribution. In the model, there are two subjective unemployment probabilities, $\text{Prob}(U|n)$, conditional on the signal n. Thus, the mean of the job loss probability in the model is

$$\text{E}[\text{Prob}(U|n)] = \text{Prob}(U|n = g)\,\text{Prob}(n = g) \tag{15}$$

$$+ \text{Prob}(U|n = b)\,\text{Prob}(n = b) = p(1 - \mu) + \mu, \tag{16}$$

and we choose $\{\mu, p\}$ to solve

$$\text{E}[\text{Prob}(U|n)] \stackrel{!}{=} \text{Prob}(U) = 0.0310 \tag{17}$$

$$T_s(\mu, p) \stackrel{!}{=} \text{PPR} = 4.36, \tag{18}$$

which results in $p = 0.0073$ and $\mu = 0.0239$.[6] For the discount factor, we choose a standard annual value of $\beta = 0.96$, implying an annual real interest rate of four percent. The estimate of the pooled price ratio by far exceeds the willingness to pay in all three cases, clearly satisfying the no-trade condition for a single interaction (11). Correspondingly, Hendren (2017) concludes that unemployment insurance contracts in the US would be too adversely selected to be profitable.

4.2 Exclusion and the existence of profitable unemployment insurance

To assess the quantitative importance of repeated interactions for unemployment insurance, we compute the number of exclusion periods necessary for the existence of unemployment

[6] All the results in this section are robust to approximating the distribution of subjective beliefs with more than just two points. In Appendix B.3, we consider a version of the model with a distribution of beliefs that features safe, uninformed and bad signal agents.

insurance in the US. In the next step, we compare this number to alternative exclusion periods observed in reality.

Given p, μ, β, we compute the shortest exclusion needed for the existence of unemployment insurance, N_{min}, as follows:

$$N_{min} \equiv N : \frac{u'(y-l)}{u'(y)} - T_d(p, \mu, \beta, N) = 0.$$

The resulting values are reported in the first row of Table 2. Depending on risk aversion, the minimum number of exclusion periods for the existence of unemployment insurance varies between less than one year for $WTP = 1.87$ to approximately two and a half years for $WTP = 1.29$. In the second row of Table 2, we display the expected number of exclusion periods when return is random. Comparing the exclusion periods in the first and the second row, we find that the expected length of exclusion is slightly larger than in case of a fixed-length exclusion N_{min}. The reason for this is as follows. When return from autarky is random, there is a non-zero probability to return from autarky earlier than with a deterministic exclusion, making autarky more attractive to the agents. To compensate for this and to render insurance more attractive than autarky, the expected length of exclusion must be larger than in the deterministic case.[7] Why does a relatively short exclusion length suffice to overturn Hendren (2017)'s results? The probability to receive a good signal and become unemployed in the current period is relatively small with $(1-\mu)p = 0.0071$ but the probability to receive bad news and become unemployed in the future, $\mu \times 1 = 0.0239$, is over three times higher.

To see this even more clearly, consider alternatively the case of safe good signal agents, so that $p = 0$ instead. We then set $\mu = 0.0310$ to match the mean of the subjective unemployment beliefs distribution $\text{Prob}(U)$. With $p = 0$, the single-interaction pooled price ratio becomes arbitrarily large, in line with Equation (12), and profitable insurance cannot be provided for any finite willingness to pay estimates. In the dynamic model, however, there can be insurance, which stems – by construction — exclusively from insurance against bad news, as encapsulated by (13). To illustrate this, we compute the necessary exclusion length for $\sigma = 2$. Compared to the baseline, the exclusion length increases by 60% to $N_{min} = 1.89$.

Suppose alternatively that p and μ as in the baseline model, but $N = 0$, such that insurance against bad news is irrelevant. Consider the increase in p necessary to render insurance profitable, again adjusting μ to match $\text{Prob}(U) = 0.031$. We find that p must be almost three times larger than in the baseline. Therefore, the profitability of insurance provision in the dynamic model stems indeed mainly from the possibility to provide insurance against bad news.

Are the minimum exclusion lengths long or short? Arguably, there is no thriving private unemployment insurance market in the US, which makes it difficult to say whether the number of exclusion periods in Table 2 are realistic. To put the numbers into perspective, we compare them to two real-world analogues: the loss in financial liberty resulting from *private bankruptcy* and the exclusion time after a *sovereign default*.

[7] By construction, *history-dependent insurance contracts* cannot yield lower profits than the memoryless contracts considered in the baseline. Quantitatively, we find that with such contracts an exclusion of a single year also suffices for existence not only in case of $\sigma = 3$ but also when $\sigma = 2$, for deterministic as well as for random exclusion (see Table 5 in Appendix B.4 for the exact numbers).

Table 2: Existence of insurance: minimum number of exclusion periods

	WTP, $u'(c_u)/u'(c_e)$, for various σ		
	1.29 ($\sigma = 1$)	1.58 ($\sigma = 2$)	1.87 ($\sigma = 3$)
Fixed exclusion, N_{min}	2.6803	1.1774	0.6962
Random exclusion, $\mathbb{E}[N_{min}]$	2.8906	1.2309	0.7207

Notes: Minimum number of exclusion periods for the existence of unemployment insurance, N_{min}, and expected exclusion length $\mathbb{E}[N_{min}] = 1/\theta - 1$ as functions of the willingness to pay, WTP.

In the US, individuals can file for *private bankruptcy* according to Chapter 7 (about 71% of filings) or Chapter 13 (29% of filings). In both cases, the bankruptcy appears on the individual's credit history for a given time period with the consequence that it is either impossible to receive unsecured credit or only possible at a high interest rate premium. The loss of access to financial markets closely resembles the idea of exclusion from insurance. The private bankruptcy entry appears in the individual credit history for ten (Chapter 7) or seven years (Chapter 13). In the light of theses numbers, a necessary exclusion length of up to three years does not appear to be unrealistically large.

Another possibility is to compare the exclusion numbers to the time until countries gain re-access to international financial markets after a *sovereign default*. For example, Schmitt-Grohé and Uribe (2017) in Chapter 13 provide estimates how long a sovereign default lasted on average for a sample of countries in the years 1974–2014. Excluding the period of default, it takes on average about 9 years until countries can borrow again some amount (partial re-access) and about 15 years until they can borrow an amount exceeding 1% of their GDP (full re-access). Both estimates are well above the length of the exclusion period necessary for the existence of profitable unemployment insurance that we compute.

4.3 Robustness exercises

In this section, we first study exclusion with persistent instead of i.i.d. signals. Then, we consider an economy with a finite instead of an infinite life span of agents.

4.3.1 Persistent signals

In the baseline specification, we assumed signals to be i.i.d. If signals are persistent, the existence of profitable insurance might require longer exclusion periods, in particular when good signals are persistent. Hendren (2017) does not study how subjective job-loss expectations change over time, and therefore does not provide estimation results on the persistency of the subjective expectations. In our model, the persistency of the subjective expectations is captured by the signal transition probabilities $\text{Prob}(n' = g|n = g)$ and $\text{Prob}(n' = b|n = b)$. One possibility to indirectly infer the signal transition probabilities is to use estimates of the transition in and out of employment and unemployment. For the US, Hobijn and Şahin (2009) estimate annual probabilities to remain in employment and unemployment, $\text{Prob}(E'|E)$ and $\text{Prob}(U'|U)$, of 0.90 and 0.07, respectively. Using the additional two moments, we identify

Table 3: Existence of insurance: minimum number of exclusion periods, persistent signals

	WTP, $u'(c_u)/u'(c_e)$, for various σ		
	1.29 ($\sigma = 1$)	1.58 ($\sigma = 2$)	1.87 ($\sigma = 3$)
Fixed exclusion, N_{min}	2.7022	1.1853	0.7004
Random exclusion, $\mathbb{E}[N_{min}]$	3.0331	1.2857	0.7529

Notes: Persistent signals.Minimum number of exclusion periods for the existence of unemployment insurance, N_{min}, and expected exclusion length $\mathbb{E}[N_{min}] = 1/\theta - 1$ as functions of the willingness to pay, WTP.

p, p_b and $\text{Prob}(n' = g|n = g), \text{Prob}(n' = b|n = b)$ via the following restrictions:

$$\text{Prob}(U) \stackrel{!}{=} p(1-\mu) + \mu p_b, \tag{19}$$

$$\text{PPR} \stackrel{!}{=} \frac{p(1-\mu) + \mu p_b}{(1-\mu)p}, \tag{20}$$

$$\text{Prob}(E'|E) \stackrel{!}{=} \text{Prob}(n' = g|n = g)(1-p), \tag{21}$$

$$\text{Prob}(U'|U) \stackrel{!}{=} \text{Prob}(n' = b|n = b)p_b + \text{Prob}(n' = g|n = g)p, \tag{22}$$

with p_b as the probability to become unemployed after receiving a bad signal (previously assumed to be one), and:

$$\mu = \frac{1 - \text{Prob}(n' = g|n = g)}{2 - \text{Prob}(n' = g|n = g) - \text{Prob}(n' = b|n = b)},$$

as the measure of bad-signal agents in the unique invariant distribution induced by the signal-transition probabilities. The last two restrictions link the perceived transitions of the individuals to the actual transitions in and out of employment and unemployment in US data.

Equally weighting the percentage deviations of model and data moments, we estimate $p = 0.0073$, $p_b = 1.0000$, $\text{Prob}(n' = g|n = g) = 0.9770$, and $\text{Prob}(n' = b|n = b) = 0.0629$. The overall fit is good with and an average percentage deviation of model and data moments of 3.88%.Thereby, three moments ($\text{Prob}(U)$, PPR, $\text{Prob}(U'|U)$) are matched (almost) exactly while $\text{Prob}(E'|E)$ is slightly larger than in the data.[8]

In Table 3, we display the minimum number of exclusion periods for the existence of profitable insurance with persistent signals.[9] Comparing these numbers to the exclusion periods for i.i.d. signals in Table 2, we find that the minimum number of exclusion periods increases but the increase is quantitatively small (in particular in case of deterministic exclusion). Thus, the conclusion that a relatively short exclusion length is enough to render unemployment insurance profitable is not driven by the assumption of i.i.d. signals in the baseline.[10]

The difference in exclusion length can be attributed to good-signal agents' lower willing-

[8] Further details on the estimation results can be found in Table B.4 in Appendix B.4.

[9] Here and in the finite horizon case below, the minimum number of exclusion is found numerically as the smallest number N that yields strictly positive profits for the insurer.

[10] The exclusion lengths we compute here are conservative estimates because the model overestimates the probability to stay employed. This weakens the insurance incentives of good-signal agents and therefore results in necessary exclusion lengths that are slightly upward biased.

ness to participate in insurance when signals are persistent. While the probability to become unemployed in the current period is identical with persistent and i.i.d. signals, the probability to become unemployed in the next period is slightly lower with persistent signals. With i.i.d. signals, the probability is $(1-\mu)p+\mu = 0.0310$, while for persistent signals it amounts to $\text{Prob}(n' = g|n = g) \times p + \text{Prob}(n' = b|n = g) \times 1 = 0.0301$.

4.3.2 Finite planning horizon

To gain further confidence in the computed exclusion lengths, we also discuss the relevance of a finite instead of an infinite planning horizon of the agents for the exclusion length necessary to provide insurance at a profit.

With an infinite horizon, an exclusion length of up to three years is found be enough to generate slack on the resource constraint. A natural question is whether this conclusion changes when agents' planning horizon is shorter than three years, for example, because of retirement. This is a relevant question because the employed individuals in Hendren (2017)'s HRS sample are up to 64 year old. The (full) retirement age in the US is 67, which makes considering a finite planning horizon relevant, at least for some of the individuals in the HRS sample. For this reason, we study a life-cycle economy.[11]

Agents work for K periods and then they retire. Each period a new generation is born.[12] All generations are of equal measure set to $\frac{1}{K}$ so that the total measure of all agents in the economy is equal to 1. In period t, the working age of a generation who entered the labour market in $t-k$ is k, and we will use k as generation index. An important feature of the life-cycle economy, absent in the infinite horizon baseline, is that the insurer can condition insurance premia and benefits on publicly observable age (but not on the history of signals and endowments), which allows for inter-generational transfers. As in the infinite horizon economy, the incentive compatibility constraints of the bad signal agents are binding, and for each k, the utility allocation comprises two elements, $h^k = \left\{h_l^k, h_{gy}^k\right\}, k \in \{1, ..., K\}$. Taking this into account, we define the working age k expected utility as:

$$\tilde{w}^k = (1-\beta)\left[(\mu + (1-\mu)p)\, h_l^k + (1-\mu)(1-p)h_{gy}^k\right],$$

while the expected utility in autarky in every period is independent of k and given by:

$$\tilde{U}^{Aut} = (1-\beta)\left[(\mu + (1-\mu)p)\, u(y-l) + (1-\mu)(1-p)u(y)\right].$$

With a finite working life, a cohort can not be excluded for longer than the remaining time until retirement. For example, a total exclusion length of two periods is only relevant for all agents that have at least two years until they retire. With exclusion of $N \geq 1$ future periods

[11] In the main text, we focus on the main features of this economy. In Appendix B.1, we provide further details, including the insurer's profit maximization problem that defines the cost-efficient allocation.

[12] An alternative possibility is to assume the insurer can transfer resources between periods for a single cohort of agents.

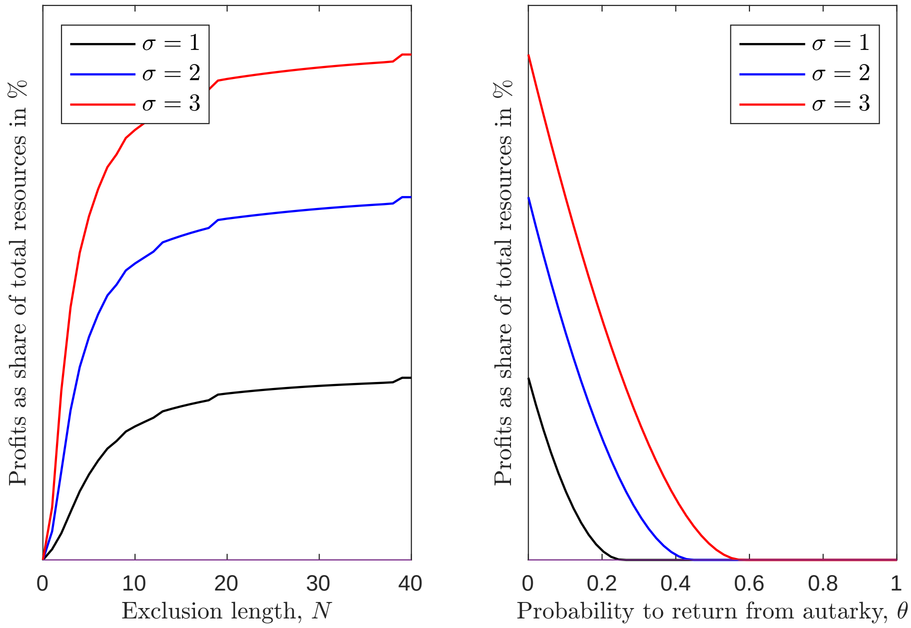

Figure 1: Insurer profits as share of total resources in % for different exclusion length (deterministic - left panel, random - right panel) and risk aversion, life-cycle economy.

and $k < K$, the individual rationality constraints read:[13]

$$(1-\beta)h_l^k + \sum_{t=k+1}^{\min\{k+N,K\}} \beta^{t-k}\tilde{w}^t \geq (1-\beta)u(y-l) + \sum_{t=k+1}^{\min\{k+N,K\}} \beta^{t-k}\tilde{U}^{Aut},$$

and:

$$(1-\beta)\left[ph_l^k + (1-p)h_{gy}^k\right] + \sum_{t=k+1}^{\min\{k+N,K\}} \beta^{t-k}\tilde{w}^t \geq (1-\beta)\left[pu(y-l) + (1-p)u(y)\right] + \sum_{t=k+1}^{\min\{k+N,K\}} \beta^{t-k}\tilde{U}^{Aut}.$$

When either $N = 0$ or $k = K$, these constraints involve instantaneous utilities only:

$$(1-\beta)h_l^k \geq (1-\beta)u(y-l), \tag{23}$$

$$(1-\beta)\left[ph_l^k + (1-p)h_{gy}^k\right] \geq (1-\beta)\left[pu(y-l) + (1-p)u(y)\right], \tag{24}$$

and the individuality rationality constraints are like the ones in the single-interaction economy. As in the baseline, the individual rationality constraints of the single-interaction model emerge for $N = 0$, but not only then. They also apply to the generation that will retire in the next period.

To numerically compute cost-efficient allocations, we set $y - l = 1$ and then compute l to

[13] For deterministic exclusion, we assume here N to be an integer. The formulae can be adapted to have $N \in \mathbb{R}_{\geq 0}$, for example, by taking an appropriately weighted average of the expected utility of the contract and the outside option in the last non-integer part of the exclusion period.

yield the ratio of marginal utilities reported as the willingness to pay in Table 1. The values of β, p and μ are taken from Table 1, the total working life is set to $K = 40$.

Cost-efficient allocations in the life-cycle economy are characterized by two key properties. As illustrated in Figure 2, the insurer extracts relatively more profits from the younger generations. The reason is that the younger generations have a higher willingness to pay for insurance because they can become unemployed at multiple times in the future. Correspondingly, agents that are close to retirement value unemployment insurance less, and it becomes relatively easier to extract resources from the younger generations. Second, as illustrated in Figure 3, the insurer provides better unemployment insurance to the older generations as indicated by a higher replacement rate or smaller relative difference between consumption of employed and unemployed agents. Agents that are close to retirement have a lower future value of insurance. To keep these agents in the insurance scheme, the insurer has to offer them relatively better unemployment insurance than the younger generations.

As illustrated in Figure 1, we find very similar total exclusion lengths for the finite horizon economy as the ones computed in Table 2. If exclusion is not possible, that is, when either $N = 0$ or $\theta = 1$, the insurer can not improve upon the no-trade allocation, and makes zero profits. However, we find that for $N \geq 1$ the insurer already incurs positive profits in case of deterministic exclusion for all three risk-aversion values (left panel). For the random return specification, the threshold values for θ are 0.24, 0.43 and 0.57, for $\sigma = \{1, 2, 3\}$ (right panel). These values imply expected necessary exclusion lengths of 3.08, 1.33 and 0.75 years, respectively.

5 Conclusions

The main takeaway from our analysis is that the future benefits of insurance matter for the existence of a profitable UI market. These benefits naturally emerge when insurer and workers meet repeatedly. We find these benefits to be sizeable, such that a relatively short exclusion suffices to render the UI market profitable in the US. Considering these future benefits, adverse selection alone is unlikely to explain the non-existing unemployment insurance market.

Unlike in case of adverse selection as the only reason for the missing market, our theoretical and quantitative results have clear policy implications for how to stimulate the existence of the private UI market. The threat of exclusion is pivotal and must be credible. A useful mechanism to ensure credibility is a registry system that collects information on UI premia defaults, similar to the successful system collecting information on credit defaults. However, creating such a registry system entails fixed costs, and economic policy can facilitate creating and maintaining the registry. The main difference to the credit market is that we show that exclusion is not only important for the well-functioning of the private UI market but already for its existence. Thus, the registry system must be pre-announced and up and running when the first UI policies are sold.

The main premise of our analysis is not limited to unemployment insurance. Indeed, we expect the ability of the insurer to exclude agents from insurance following their default, to improve the profitability of insurance in existing markets with adverse selection due to advance information. We leave the exploration of the effects of inclusion in those other markets for future research.

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A Proofs and Derivations

A.1 Proof of Lemma 1

We prove this lemma in several steps. To begin with, we show all implementable allocations must at least deliver the expected lifetime utility of autarky.

Step 1 *Let h be an implementable allocation. The following statements hold*

(i) Allocation h delivers a lifetime expected utility not worse than that of autarky, $\tilde{w}(h) \geq U^{Aut}$.

(ii) It is $\tilde{w}(h) = U^{Aut}$ if, and only if, h is the no-trade (autarky) allocation.

(iii) Let h be an implementable allocation with trade. Then, $h_{bl} \geq u(y-l)$ and the individual rationality constraint of the bad-signal agents is slack.

Proof. These statements are standard results. For this reason, we just sketch the proofs and omit the details. If (i) was not true, adding the two individual rationality constraints would result in h being not implementable. The if part of statement (ii) is by definition. For the only-if part, setting $\tilde{w}(h) = U^{Aut}$ collapses the individual rationality constraints to current period utilities which $\tilde{w}(h)$ and U^{Aut} are averages of (with identical weights). Thus, both individual rationality constraints can only be satisfied if $h_{gl} = h_{bl} = u(y-l)$ and $h_{gy} = u(y)$ to also meet $\tilde{w}(h) = U^{Aut}$. Hence, with trade it must be that $\tilde{w}(h) > U^{Aut}$. Statement (iii) is proven by contradiction, assuming $h_{bl} < u(y-l)$ together with (4) and (6) implies $h_{gl} < u(y-l)$ and $h_{gy} > u(y)$ which either violates resource feasibility or yields $\tilde{w}(h) < U^{Aut}$. ■

In the following step, we prove Part (i) of Lemma 1.

Step 2 *Let h be a cost-efficient allocation with trade. Incentive constraints of bad-signal agents hold with equality, so that utility for agents who incur an income loss is equalized across signal realizations, $h_{bl} = h_{gl} = h_l$, and the incentive constraints of good-signal agents are slack.*

Proof. Given the incentive compatibility constraint of bad-signal agents, we only need to consider implementable allocations with trade which have either $h_{bl} > h_{gl}$ or $h_{bl} = h_{gl} = h_l$.

For the first case, consider a cost-efficient allocation with trade, $h_0 = \{h_{gy}, h_{gl}, h_{bl}\}$, $h_{bl} > h_{gl}$. Thus, by assumption, h_0 is also implementable (satisfies all constraints). The resulting expected utility is:

$$\tilde{w}(h_0) = \underbrace{(1-\mu)(1-p)h_{gy}}_{\tilde{w}_y(h_0)} + \underbrace{(1-\mu)ph_{gl} + \mu h_{bl}}_{\tilde{w}_l(h_0)}$$

with $\tilde{w}_y(h_0)$ as the utility of agents without and $\tilde{w}_l(h_0)$ as the expected utility of agents with income loss. The resource costs $\tilde{C}(h_0)$ of this allocation are:

$$\tilde{C}(h_0) = \underbrace{(1-\mu)(1-p)C\left(h_{gy}\right)}_{\tilde{C}_y(h_0)} + \underbrace{(1-\mu)pC\left(h_{gl}\right) + \mu C\left(h_{bl}\right)}_{\tilde{C}_l(h_0)}$$

Next, let ε_b^{ir} be the utility slack in the bad-signal agents individual rationality constraint (7) and $\varepsilon_b^{ic} = h_{bl} - h_{gl}$ the utility slack in the incentive constraint of bad-signal agents (4). By assumption, $\varepsilon_b^{ir} > 0$ and $\varepsilon_b^{ic} > 0$. Consider a perturbation $h_1 = \{h_{gy}, h_{gl} + \delta_g, h_{bl} - \delta_b\}$ such that $\delta_g = \frac{\mu\delta_b}{(1-\mu)p}$ and the expected utility of the alternative allocation is given by

$$\tilde{w}(h_1) = \underbrace{(1-\mu)(1-p)h_{gy}}_{\tilde{w}_y(h_1)} + \underbrace{(1-\mu)p\left(h_{gl}+\delta_g\right) + \mu\left(h_{bl}-\delta_b\right)}_{\tilde{w}_l(h_1)} = \tilde{w}(h_0).$$

This perturbation keeps the incentive compatibility of good-signal agents satisfied (as it was already met by h_0, implementable by assumption, and now we are decreasing h_{bl} and increasing h_{gl}). It also trivially satisfies the good-signal agents individual rationality constraint (the continuation value $\tilde{w}(h_1) = \tilde{w}(h_0)$ is unchanged and we have increased h_{gl}). We need to ensure that the remaining two constraints are also satisfied. The individual rationality constraint of bad-signal agents is satisfied for $\delta_b \leq \varepsilon_b^{ir}/(1-\beta)$. The incentive compatibility constraint of bad-signal agents requires:

$$h_{bl} - \delta_b \geq h_{gl} + \delta_g \iff \delta_b \leq \varepsilon_l^{ic}\frac{(1-\mu)p}{(1-\mu)p+\mu}$$

Thus, $\delta_b \leq \min\left\{\frac{\varepsilon_l^{ir}}{1-\beta}, \varepsilon_l^{ic}\left[\frac{(1-\mu)p}{(1-\mu)p+\mu}\right]\right\}$ ensures that the perturbed allocation is implementable. The perturbed allocation requires resource costs $\tilde{C}(h_1)$:

$$\tilde{C}(h_1) = \underbrace{(1-\mu)(1-p)C\left(h_{gy}\right)}_{\tilde{C}_y(h_1)} + \underbrace{(1-\mu)pC\left(h_{gl}+\delta_g\right) + \mu C\left(h_{bl}-\delta_b\right)}_{\tilde{C}_l(h_1)}$$

The difference in required resources, for an arbitrarily small δ_b is

$$\begin{aligned}\tilde{C}(h_1) - \tilde{C}(h_0) &= (1-\mu)pC'(h_{gl})\delta_g - \mu C'(h_{bl})\delta_b \\ &= \mu\delta_b\left[C'(h_{gl}) - C'(h_{bl})\right] < 0,\end{aligned}$$

where the last inequality follows from strict convexity of resource costs with $h_{gl} < h_{bl}$, implying that h_0 cannot be a cost-efficient allocation. Thus, a cost-efficient allocation is characterized by binding incentive constraints of bad-signal signals and $h_{bl} = h_{gl} = h_l$. Using this, Step 1 and resource feasibility imply $h_l > u(y-l)$ and $h_{gy} < u(y)$. Finally, the incentive constraint of the good-signal agents can only be met with equality if $c_{gy} = 0$ now that we have established $h_{gl} = h_{bl}$. However, this can not be optimal because $u(c)$ satisfies Inada conditions, $\lim_{c\mapsto 0} u'(c) = \infty$ and hence an infinitely small redistribution back from c_l to c_{gy} yields infinite improvements in lifetime utility, increasing profits. Thus, setting $c_{gy} = 0$ can not be optimal and the good-signal agent incentive compatibility constraint must be slack. ■

So far we established that only incentive constraints of good-signal agents are slack as well as individual rationality constraints of bad-signal agents. Further, we have shown that $h_l > u(y-l)$ and $h_{gy} < u(y)$. In the remaining two steps, we demonstrate first that individual rationality constraints of good-signal agents are binding, completing the proof of Lemma 1, Part (ii), and afterwards that $h_{gy} \geq h_l$ at the cost-efficient allocation, completing the proof of Lemma 1, Part (iii).

Step 3 *Let h be a cost-efficient allocation with trade. Then the individual rationality constraint of the* *good-signal agents are binding.*

Proof. Suppose not, such that there is not only slack $\varepsilon_b^{ir} > 0$ but also a slack $\varepsilon_g^{ir} > 0$ in the good-signal agents individual rationality constraint (6). Let $h_0 = \{h_l, h_{gy}\}$ be the cost-efficient allocation with trade in that case. This cannot be the cost-efficient allocation because further resources can be saved by choosing an allocation $h_1 = \{h_l, h_{gy} - \delta\}$ as long as

$$\delta \leq \min\left\{\frac{\varepsilon_b}{\beta(1-\beta^N)(1-\mu)(1-p)}, \frac{\varepsilon_g}{1-\beta+\beta(1-\beta^N)(1-\mu)(1-p)}\right\},$$

which ensures that individual rationality is satisfied for good and bad-signal agents. Choosing such allocation decreases the resource costs by $(1-\mu)(1-p)\left[C(h_{gy}) - C(h_{gy}-\delta)\right]$, contradicting that the good-signal individual rationality constraints are not binding at the cost-efficient allocation. ■

Step 4 *Let h be a cost-efficient allocation with trade. Then, $h_{gy} \geq h_l$.*

Proof. Suppose not, and without loss of generality, let $h_l = h_{gy} + \varepsilon$ for $h_0 = \{h_l, h_{gy}\}$ with ε arbitrarily small. Then, an insurer can improve by deviating to $h_1 = \{h_{bl} = h_l, h_{gl} = h_l - \delta_{gl}, h_{gy} + \delta_{gy}\}$ with $\delta_{gl} = \frac{1-p}{p}\delta_{gy}$, and $\delta_{gy} \leq \frac{\varepsilon}{2}$. This perturbation doesn't affect the good-signal individual rationality constraint, but by convexity of resource costs it marginally increases the insurer's profits by $(1-\mu)(1-p)\left[C'(h_{gy}+\varepsilon) - C'(h_{gy})\right]\delta_{gy}$. ■

A.2 Proof of Theorem 1

The proof proceeds in two parts. First, we show that the no-trade condition is sufficient. In the second part, we show the no-trade condition is also necessary.

Part I: No-trade condition is sufficient We show that if the no-trade condition holds, autarky is the cost-efficient allocation. From Lemma 1, we consider the problem of choosing $\{h_{gy}, h_l\}$ to maximize (9) subject to the individual rationality constraints of good-signal agents (6):

$$\begin{aligned}\Big[(1-\beta)p + \beta(1-\beta^N)\left[\mu + (1-\mu)p\right]\Big]\, h_l \\ + \Big[(1-\beta)(1-p) + \beta(1-\beta^N)(1-\mu)(1-p)\Big]\, h_{gy} \geq U^{Aut}(g)\end{aligned}$$

The first-order conditions for h_{gy}, h_l are

$$\begin{aligned}(1-\mu)(1-p)C'(h_{gy}) &= \lambda_g^{IR}\Big[(1-\beta)(1-p) + \beta(1-\beta^N)(1-\mu)(1-p)\Big] \\ \left[\mu + (1-\mu)p\right]C'(h_l) &= \lambda_g^{IR}\Big[(1-\beta)p + \beta(1-\beta^N)\left[\mu + (1-\mu)p\right]\Big],\end{aligned}$$

with $\lambda_g^{IR} \geq 0$ as the multiplier on the constraint (6). The two first order conditions can be combined into a single first order condition

$$\frac{\mu + (1-\mu)\,p}{(1-\mu)\,p}\left\{\frac{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p}{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p + \beta(1-\beta^N)\mu}\right\} = \frac{C'(h_{gy})}{C'(h_l)} = \frac{u'(c_l)}{u'(c_{gy})}.$$

Next, if the no-trade condition holds, and using (10), it follows

$$\frac{u'(y-l)}{u'(y)} \leq \frac{\mu+(1-\mu)\,p}{(1-\mu)\,p}\left\{\frac{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p}{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p+\beta(1-\beta^{N})\mu}\right\}=\frac{u'(c_l)}{u'\left(c_{gy}\right)}.$$

From Lemma 1, we have $c_l \geq y-l$ and $c_{gy} \leq y$. With decreasing marginal utility, this is only possible if $c_l = y-l$ and $c_{gy} = y$. Hence, when the no-trade condition holds, the cost-efficient allocation is the no-trade allocation.

Part II: No-trade condition is necessary We show that if the no-trade condition is violated, there is an implementable allocation with trade which generates a slack in the resource constraint. Without loss of generality, consider a perturbed allocation $h = \left\{h_{gy} = u(y-\delta), h_l = u(y-l+\gamma)\right\}$ with $\delta > 0, \gamma > 0$ with δ, γ arbitrarily small. By definition, this is an allocation with trade which also satisfies all other results in Lemma 1, but we don't know if this perturbed allocation is resource feasible. Net resources saved by this perturbation are $(1-\mu)(1-p)\,\delta - [\mu+(1-\mu)p]\,\gamma$. Observe that when this is non-negative, this perturbed allocation is implementable. By Lemma 1, the allocation implies the following pattern of binding individual rationality constraints

$$\tilde{w}(g)^N = U^{Aut}(g)^N, \tag{25}$$

$$\tilde{w}(b)^N > U^{Aut}(b)^N. \tag{26}$$

Before the signal realizes, the relevant expected utilities for N future periods are

$$\tilde{w}^N = (1-\mu)\tilde{w}(g)^N + \mu\tilde{w}(b)^N$$

$$U^{Aut,N} = (1-\mu)U^{Aut}(g)^N + \mu U^{Aut}(b)^N$$

Let $\tilde{w}(g), \tilde{w}(b)$ and $U^{Aut}(g), U^{Aut}(b)$ be the lifetime utilities of good-and bad-signal agents as the corresponding limiting expressions $N \to \infty$ of both sides of the individual rationality constraints (25) and (26). The following relationships between lifetime utilities and the period-N utilities apply

$$\tilde{w}(g)^N = \tilde{w}(g) - \beta^{N+1}\tilde{w} \tag{27}$$

$$\tilde{w}(b)^N = \tilde{w}(b) - \beta^{N+1}\tilde{w} \tag{28}$$

$$U^{Aut}(g)^N = U^{Aut}(g) - \beta^{N+1}U^{Aut} \tag{29}$$

$$U^{Aut}(b)^N = U^{Aut}(b) - \beta^{N+1}U^{Aut}. \tag{30}$$

Let ϵ be the slack on the individual rationality constraint of bad-signal agents:

$$\tilde{w}(b)^N - U^{Aut}(b)^N = \epsilon > 0.$$

Individual rationality constraints of good-signal agents are binding, $\tilde{w}(g)^N - U^{Aut}(g)^N =$

0. Using (28), (30) as well as (27), (29), gives the following restrictions

$$\tilde{w}(b)^N - U^{Aut}(b)^N = \epsilon = \tilde{w}(b) - U^{Aut}(b) + \beta^{N+1}(U^{Aut} - \tilde{w})$$
$$\tilde{w}(g)^N - U^{Aut}(g)^N = 0 = \tilde{w}(g) - U^{Aut}(g) + \beta^{N+1}(U^{Aut} - \tilde{w}),$$

Using the definition of lifetime utilities of good and bad-signal agents in the insurance contract and in autarky, one eventually gets

$$\tilde{w} - U^{Aut} = \frac{\mu\epsilon}{1-\beta^{N+1}}.$$

For the individual rationality constraint of the bad-signal agents (26) combined with (28) and (30), it follows that:

$$\tilde{w}(b)^N - U^{Aut}(b)^N = \epsilon = (1-\beta)u'(y-l)\gamma + \beta(1-\beta^N)\frac{\mu\epsilon}{1-\beta^{N+1}}$$

such that ϵ can be written as

$$\epsilon = \frac{(1-\beta)\,(1-\beta^{N+1})\gamma}{1-\beta^{N+1}-\beta(1-\beta^N)\mu}u'\left(y-l\right). \tag{31}$$

For the binding individual rationality constraint of good-signal agents (25) we have:

$$0 = \tilde{w}\left(g\right)^N - U^{Aut}\left(g\right)^N = (1-\beta)\left[-\left(1-p\right)u'\left(y\right)\delta + pu'\left(y-l\right)\gamma\right] + \beta(1-\beta^N)\frac{\mu\epsilon}{1-\beta^{N+1}}$$

This implies:

$$\frac{\left(1-\mu\right)\left(1-p\right)\delta}{\left[\mu+\left(1-\mu\right)p\right]\gamma} = \frac{u'\left(y-l\right)}{u'\left(y\right)}\frac{\left(1-\mu\right)p}{\mu+\left(1-\mu\right)p}\frac{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p+\beta(1-\beta^N)\mu}{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p}. \tag{32}$$

Suppose that the no-trade condition is violated:

$$\frac{u'\left(y-l\right)}{u'\left(y\right)} > \frac{\mu+\left(1-\mu\right)p}{\left(1-\mu\right)p}\left\{\frac{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p}{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p+\beta(1-\beta^N)\mu}\right\}.$$

Using this, (32) boils down to:

$$\frac{\left(1-\mu\right)\left(1-p\right)\delta}{\left[\mu+\left(1-\mu\right)p\right]\gamma} > 1 \implies \left(1-\mu\right)\left(1-p\right)\delta - \left[\mu+\left(1-\mu\right)p\right]\gamma > 0,$$

thus, net resources are positive, and the no-trade condition is not only sufficient but also necessary.

Monotonicity of $T_r(p,\mu,\beta,N)$ in N. Let's rewrite the definition of $T_r(p,\mu,\beta,N)$ as:

$$T_r(p,\mu,\beta,N) = \frac{f(p,\mu,\beta,N)}{f(p,\mu,\beta,N)+g(\mu,\beta,N)}$$

with: $f(p,\mu,\beta,N) = \left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p$ and $g(\mu,\beta,N) = \beta(1-\beta^N)\mu$. Then, $\frac{\partial T_r}{\partial N} = \frac{\frac{\partial f}{\partial N}(f+g)-\left(\frac{\partial f}{\partial N}+\frac{\partial g}{\partial N}\right)f}{[f+g]^2} = \frac{\frac{\partial f}{\partial N}g-\frac{\partial g}{\partial N}f}{[f+g]^2}$. We have:

$$\begin{aligned}\frac{\partial f(p,\mu,\beta,N)}{\partial N} &= -\beta^{N+1}(1-\mu)p\ln(\beta),\\ \frac{\partial g(\mu,\beta,N)}{\partial N} &= -\beta^{N+1}\mu\ln(\beta).\end{aligned}$$

Thus:

$$\frac{\partial f}{\partial N}g - \frac{\partial g}{\partial N}f = -\beta^{N+1}\ln(\beta)\mu p(1-\beta) < 0,$$

as we have assumed $0<\beta<1$, $\mu>0$ and $p>0$.

A.3 Random return from autarky

An alternative possibility to model exclusion from insurance comes from the literature of sovereign default. With probability $0 \leq \theta \leq 1$, agents are offered an insurance contract again after defaulting as in Eaton and Gersovitz (1981) and Arellano (2008). With a constant probability to return each period, the average number of total exclusion periods (including teh current period) is $\mathrm{E}(N) = 1/\theta$. Incentive constraints and resource feasibility are unaffected by this change, and so is the definitions of lifetime utility $\tilde{w}(h)$. The value of autarky U^{Aut} now satisfies the following recursive equation:

$$U^{Aut} = (1-\beta)\left\{\mu u(y-l) + (1-\mu)\left[pu(y-l)+(1-p)u(y)\right]\right\} + \beta\theta\tilde{w} + \beta(1-\theta)U^{Aut} \quad (33)$$

Individual rationality constraints of good and bad-signal agents now read

$$\underbrace{(1-\beta)\left[pu\left(y-l+\gamma_g\right)+(1-p)u\left(y-\delta\right)\right]+\beta(1-\theta)\tilde{w}}_{\tilde{w}(g)} \geq \underbrace{(1-\beta)\left[pu(y-l)+(1-p)u(y)\right]+\beta(1-\theta)U^{Aut}}_{U^{Aut}(g)}, \quad (34)$$

and

$$\underbrace{(1-\beta)u(y-l+\gamma)+\beta(1-\theta)\tilde{w}}_{\tilde{w}(b)} \geq \underbrace{(1-\beta)u(y-l)+\beta(1-\theta)U^{Aut}}_{U^{Aut}(b)}. \quad (35)$$

In this economy, the no-trade condition is as follows.

Theorem 2 (No Trade) *The autarky allocation $\{y, y-l, y-l\}$ is the only implementable allocation if and only if*

$$\frac{u'(y-l)}{u'(y)} \leq \underbrace{T_s(p,\mu)T_r(p,\mu,\beta,\theta)}_{T_d(p,\mu,\beta,\theta)}$$

with $T_s(p,\mu)$ as the single-interaction pooled price ratio

$$T_s(p,\mu) = \frac{\mu + (1-\mu)\, p}{(1-\mu)\, p} = \frac{\mathbb{E}\,[P|P \geq p]}{1 - \mathbb{E}\,[P|P \geq p]} \frac{1-p}{p}$$

and with $T_r(p,\mu,\beta,\theta)$ as a diminishing factor resulting from repeated interactions

$$T_r(p,\mu,\beta,\theta) = \frac{p\,[1-(1-\theta)\beta\mu]}{p+(1-p)\beta(1-\theta)\mu}, \quad 0 < T_r \leq 1$$

and $0 < T_d(p,\mu,\beta,\theta) \leq T_s(p,\mu)$ as the repeated-interactions pooled price ratio for $0 \leq \theta \leq 1$.

Proof. The proof follows analogous steps as the one of Theorem 1.

Sufficiency We arrive at the following constraint on the FOC of the profit maximisation problem of an insurer (the right hand side equality):

$$\frac{u'\,(y-l)}{u'\,(y)} \leq \frac{\mu + (1-\mu)\, p}{(1-\mu)\, p} \frac{p\,[1-\beta(1-\theta)\mu]}{p\,[1-\beta(1-\theta)\mu] + \beta(1-\theta)\mu} = \frac{u'\,(c_l)}{u'\left(c_{gy}\right)},$$

Imposing the no-trade condition (the left hand side inequality) implies this can only be satisfied by $c_l = y - l$ and $c_{gy} = y$.

Necessity As in the proof of Theorem 1, we consider an allocation with trade, $h = \left\{h_{gy} = u(y-\delta), h_l = u(y-l+\gamma)\right\}$, which, if implementable, yields the following for the individual rationality constraints:

$$\tilde{w}(g) = U^{Aut}(g) \tag{36}$$

$$\tilde{w}(b) - U^{Aut}(b) = \epsilon > 0. \tag{37}$$

Following similar steps as in the proof of Theorem 1, the slack ϵ in the bad-signal agents individual rationality constraint of bad-signal agents is

$$\epsilon = \frac{(1-\beta)}{1-\beta(1-\theta)\mu} u'\,(y-l)\,\gamma. \tag{38}$$

We get the counterpart to (32) from the binding individual rationality constraints of the good-signal agents:

$$\begin{aligned} 0 &= \tilde{w}\,(g) - U^{Aut}\,(g) \\ &= (1-\beta)\left[-(1-p)\,u'\,(y)\,\delta + pu'\,(y-l)\,\gamma\right] + \beta(1-\theta)\mu\epsilon. \end{aligned}$$

Using (38), we get:

$$\frac{(1-\mu)\,(1-p)\,\delta}{[\mu+(1-\mu)p]\,\gamma} = \frac{u'\,(y-l)}{u'\,(y)} \frac{(1-\mu)\,p}{\mu+(1-\mu)\,p} \frac{p\,[1-\beta(1-\theta)\mu]}{p\,[1-\beta(1-\theta)\mu]+\beta(1-\theta)\mu}. \tag{39}$$

Assuming that the no-trade condition is violated yields

$$\frac{u'(y-l)}{u'(y)} > \frac{\mu+(1-\mu)\,p}{(1-\mu)\,p}\frac{p\left[1-\beta(1-\theta)\mu\right]}{p\left[1-\beta(1-\theta)\mu\right]+\beta(1-\theta)\mu},$$

combining with (39) implies that the allocation h generates positive resource savings. Hence, autarky cannot be the cost-efficient allocation, concluding the proof. ■

Equivalence between deterministic and random exclusion Comparing Theorem 1 and Theorem 2, we can establish equivalence between the two formulations if the diminishing factors $T_r(p,\mu,\beta,\theta)$ and $T_r(p,\mu,\beta,N)$ are equal. The two factors for fixed-length and random-length exclusion are

$$T_r(p,\mu,\beta,N) = \left\{\frac{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p}{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p+\beta(1-\beta^N)\mu}\right\}$$

and

$$T_r(p,\mu,\beta,\theta) = \frac{p\left[1-\beta(1-\theta)\mu\right]}{p\left[1-\beta(1-\theta)\mu\right]+\beta(1-\theta)\mu}.$$

The equivalence between the two is that $T_r(p,\mu,\beta,N=0) = T_r(p,\mu,\beta,\theta=1) = 1$ and $\lim_{N\mapsto\infty} T_r(p,\mu,\beta,N) = T_r(p,\mu,\beta,\theta=0) = \frac{p(1-\beta\mu)}{p(1-\beta\mu)+\beta\mu}$. The explicit relationship between N and θ is:

$$\theta = \frac{\beta^N(1-\beta)}{1-\beta^{N+1}},$$

which is confirmed by direct evaluation of the two no-trade conditions.

B Additional results

B.1 A life-cycle economy

We fill in the details on the incentive compatibility constraints and the cost minimisation problem of the insurer in the deterministic economy and then we discuss how to adjust the individual rationality constraints to arrive at the random exclusion formulation.

Deterministic exclusion The future utilities don't depend on the current private signal hence the incentive compatibility constraints are similar to those in the baseline version of the model, for each tenure k:

$$h^k_{bl} \geq h^k_{gl},$$
$$ph^k_{gl} + (1-p)h^k_{gy} \geq ph^k_{bl} + (1-p)u(0).$$

The insurer solves:

$$\max_{\left\{h_l^k, h_{gy}^k\right\}_{k=1}^K} \quad [\mu + (1-\mu)p]\, C\,[u(y-l)] + (1-\mu)(1-p)C\,[u(y)]$$
$$-\frac{1}{K}\sum_{k=1}^{K}\left[[\mu + (1-\mu)p]\, C(h_l^k) + (1-\mu)(1-p)C(h_{gy}^k)\right],$$

subject to incentive compatibility and individual rationality constraints.

Random exclusion The incentive compatibility and the resource constraint slack functions are unchanged when agents randomly return from autarky. For the individual rationality constraints, it's convenient to define the expected remaining lifetime utilities of the contract and autarky, as follows. When $k = 1$,

$$\hat{w}^k = (1-\beta)\left[((1-\mu)\,p + \mu)\,h_l^k + (1-\mu)\,(1-p)\,h_{gy}^k\right],$$

and for $1 \le k < K$

$$\hat{w}^k(1-\beta)\left[((1-\mu)\,p + \mu)\,h_l^k + (1-\mu)\,(1-p)\,h_{gy}^k\right] + \beta\hat{w}^{k+1}.$$

These can be used to define corresponding objects for autarky, when $k = 1$,

$$\hat{U}^{Aut,k} = (1-\beta)\left[((1-\mu)\,p + \mu)\,u(y-l) + (1-\mu)\,(1-p)\,u(y)\right]$$

and for $1 \le k < K$

$$\hat{U}^{Aut,k} = (1-\beta)\left[((1-\mu)\,p + \mu)\,u(y-l) + (1-\mu)\,(1-p)\,u(y)\right]$$
$$+\beta\left(\theta\hat{w}^{k+1} + (1-\theta)\hat{U}^{Aut,k+1}\right)$$

The individual rationality constraints for $k < K$ are:

$$(1-\beta)h_l^k + \beta(1-\theta)\hat{w}^{k+1} \ge (1-\beta)u(y-l) + \beta(1-\theta)\hat{U}^{Aut,k+1},$$
$$(1-\beta)\left(ph_l^k + (1-p)h_{gy}^k\right) + \beta(1-\theta)\hat{w}^{k+1} \ge$$
$$(1-\beta)\left(pu(y-l) + (1-p)u(y)\right) + \beta(1-\theta)\hat{U}^{Aut,k+1}.$$

B.2 Can individual rationality alone limit insurance possibilities?

Our main theorem reveals that individual rationality and private information together restrict insurance possibilities. To clarify the importance of both frictions, we show that individual rationality alone does not prevent the existence of insurance in our environment. Suppose alternatively that agents receive *public* signals on their income-loss probability that have the same properties as the private ones. The cost-efficient allocation maximizes the slack on resource constraints subject to implementability given now only by resource feasibility and individual rationality. In the following proposition, we show that there always exists insurance.

Proposition 1 (Public information) *Autarky is not the cost-efficient allocation with public signals.*

Proof. To see why autarky cannot be a cost-efficient allocation, consider an insurance scheme that involves only transfers between good-signal agents, so that $h_{gy} = u(y-\delta)$ and $h_{gl} = u(y-l+\gamma_g)$, $\delta, \gamma_g > 0$, in line with resource feasibility:

$$(1-p)\delta \geq p\gamma_g.$$

The scheme provides insurance, leads to higher utility by concavity, to lower resource costs by convexity, and therefore at least weakly dominates autarky in terms of resources saved. What remains to be shown is that the scheme is also implementable. First, such a scheme affects the individual rationality constraints of good-signal agents as follows

$$\begin{aligned}
&(1-\beta)\left[pu'(y-l)\gamma_g - (1-p)u'(y)\delta\right] + \beta(1-\beta^N)(1-\mu)\left[pu'(y-l)\gamma_g - (1-p)u'(y)\delta\right] \\
=&\left[u'(y-l) - u'(y)\right]\left[(1-\beta) + \beta(1-\beta^N)(1-\mu)\right]p\gamma_g \\
&\geq 0.
\end{aligned}$$

The second line uses resource feasibility, the third one that $\left[(1-\beta)+\beta(1-\beta^N)(1-\mu)\right]$, p, γ_g are strictly positive; The positive sign then follows for $l > 0$, and strict concavity of utility. Per construction, such a scheme is consistent with resource feasibility and also satisfies individual rationality constraints of bad-signal agents who remain at autarky. Thus, autarky cannot be a cost-efficient allocation. ■

Any insurance transfer between good-signal agents increases their lifetime utility. The increase in lifetime utility also increases the continuation value of bad-signal agents and thus indirectly their lifetime utility. Thus, individual rationality constraints of bad-signal agents are satisfied even when they receive no transfers from good-signal agents. The key difference is that with public signals, incentive constraints are absent and such a transfer scheme is implementable. In other words, the true costs of transfers between good-signal agents equal the actuarial fair costs. With private information, this transfer scheme would violate the incentive constraints of bad-signal agents, increasing the costs of transfers beyond their actuarial fair costs. As a consequence, individual rationality alone cannot explain the absence of a profitable private insurance market.

B.3 Finer approximation of the distribution of p with safe and uninformed agents

Hendren (2017) uses a three-point approximation to the underlying distribution of subjective beliefs by introducing *uninformed* agents whose belief on their job loss is equal to the unconditional probability of unemployment, $\text{Prob}(U)$. We follow his example and enrich the type space in our model accordingly. Furthermore, we consider *safe* agents who are certain of remaining employed in the current period. As in the baseline, there are also bad-signal agents that become unemployed for sure. More precisely, we now consider $p \in \{0, 0.031, 1\}$; μ_s and μ_b are the shares of safe and bad-signal agents in the population, respectively, and $1-\mu_s-\mu_b$ is the share of the uninformed agents. To render the distribution of subjective beliefs similar to

Table 4: Existence of insurance: minimum exclusion length, 3-point distribution of p

	WTP, $u'(c_u)/u'(c_e)$, for various σ		
	1.29 ($\sigma = 1$)	1.58 ($\sigma = 2$)	1.87 ($\sigma = 3$)
Fixed exclusion, N_{min}	2.6782	1.1724	0.6915
Random exclusion, $\mathbb{E}[N_{min}]$	2.8998	1.2286	0.7273

Notes: 3-point distribution of p. Minimum number of exclusion periods for the existence of unemployment insurance, N_{min}, and expected exclusion length $\mathbb{E}[N_{min}] = 1/\theta - 1$ as functions of the willingness to pay, WTP.

the baseline, the shares of safe and bad-signal agents are chosen to satisfy:

$$\text{Prob}(U) \stackrel{!}{=} (1 - \mu_s - \mu_b)\,\text{Prob}(U) + \mu_b \tag{40}$$

$$\begin{aligned} \text{Var}(p) \stackrel{!}{=} {} & \mu_s(0 - \text{E}[\text{Prob}(U|n)])^2 \\ & + (1 - \mu_s - \mu_b)(\text{Prob}(U) - \text{E}[\text{Prob}(U|n)])^2 \\ & + \mu_b(1 - \text{E}[\text{Prob}(U|n)])^2, \end{aligned} \tag{41}$$

where $\text{Var}(p) = 0.0230$ is the variance of subjective beliefs in the baseline approximation.This procedure yields $\mu_s = 0.7419$ and $\mu_b = 0.0238$. We report the necessary exclusion lengths for existence of insurance for this richer specification of subjective beliefs in Table 4. Comparing the necessary exclusion length to the ones is Table 2, there is hardly any difference between the richer and the baseline specification. Remarkably, the transfers to the unemployed agents are not stemming only from the uninformed agents. Instead, also the safe agents transfer resources, which confirms that the insurance of future news is key for the higher profitability of insurance in the dynamic model.

B.4 Additional tables & figures

Table 5: Existence of insurance: minimum exclusion length, history-dependent contracts

	WTP, $u'(c_u)/u'(c_e)$, for various σ		
	1.29 ($\sigma = 1$)	1.58 ($\sigma = 2$)	1.87 ($\sigma = 3$)
Fixed exclusion, N_{min}	2.1868	0.7737	0.3616
Random exclusion, $\mathbb{E}[N_{min}]$	2.3344	0.8349	0.4085

Notes: History-dependent insurance contracts. Minimum number of exclusion periods for the existence of unemployment insurance, N_{min}, and expected exclusion length $\mathbb{E}[N_{min}] = 1/\theta - 1$ as functions of the willingness to pay, WTP.

Table 6: Parameter estimates and moments: persistent signals

	I. Parameter estimates			
	p_b	p	$\text{Prob}(n' = g \mid n = g)$	$\text{Prob}(n' = b \mid n = b)$
	1.0000	0.073	0.9770	0.0629
	II. Moments: data versus model			
	PPR	$\text{Prob}(U)$	$\text{Prob}(E' \mid E)$	$\text{Prob}(U' \mid U)$
Data	4.3600	0.0310	0.9000	0.0700
Model	4.3599	0.0311	0.9699	0.0700

Notes: Persistent signals. Parameter estimates, moments in the data and in the model. Parameter vector minimizes the equally weighted percentage deviations of data and model moments.

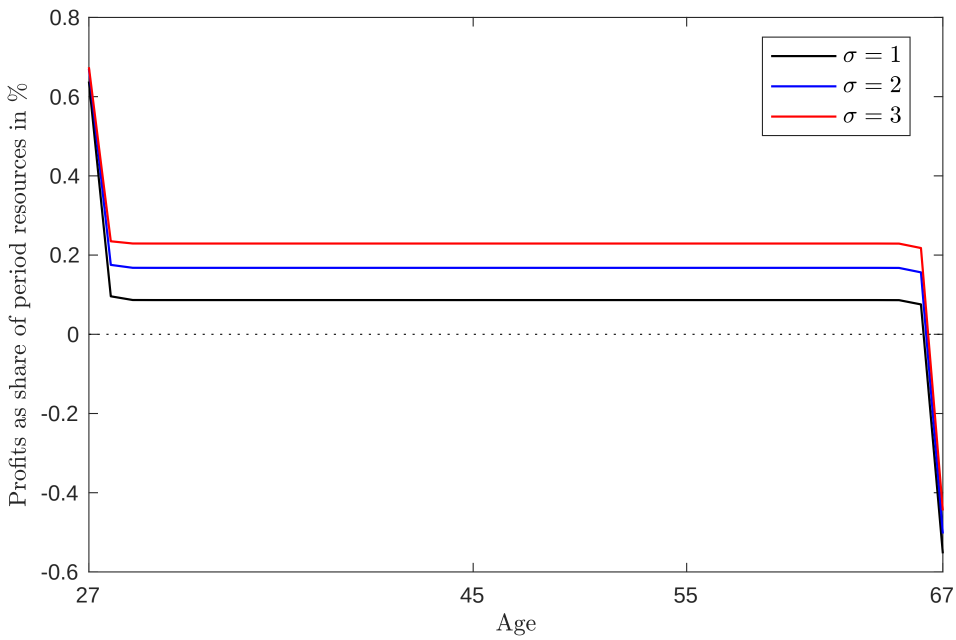

Figure 2: Insurer profits per cohort (as percentage point share in cohort's total resources) for different risk aversion, life-cycle economy, lifetime exclusion, $N_k = K - k$.

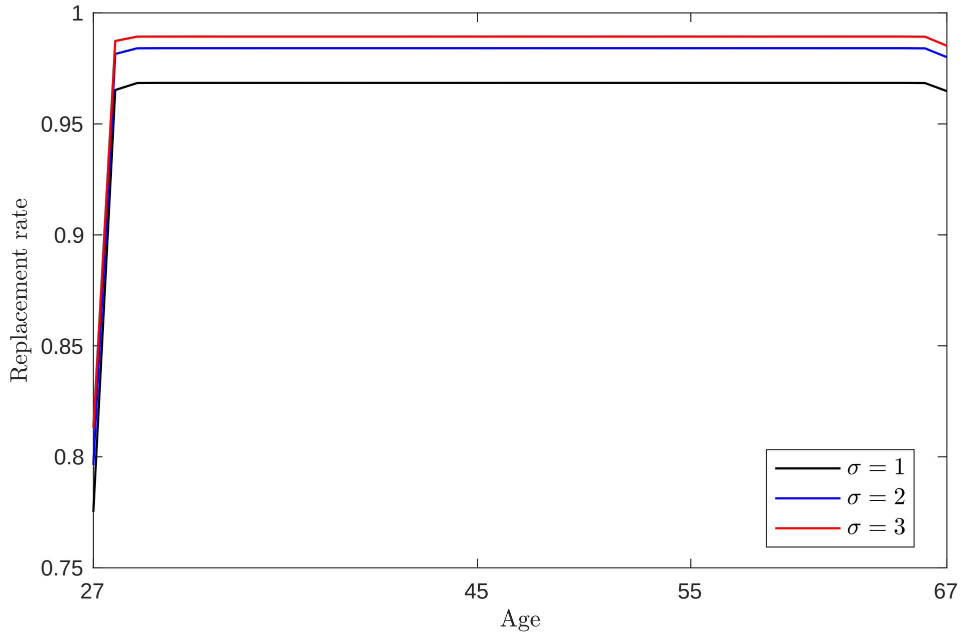

Figure 3: Replacement rate, defined as the ratio of consumption of unemployed agents over that of employed agents at each tenure in the labour market, c_l^k / c_{gy}^k, for different risk aversion, life-cycle economy, lifetime exclusion, $N_k = K - k$.