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Sharing the cost of cleaning up a polluted river

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Abstract

Consider a group of agents located along a polluted river where every agent must pay a certain cost for cleaning up the polluted river. Following the model of Ni and Wang (2007), we propose the class of α-Local Responsibility Sharing methods, which generalizes the Local Responsibility Sharing (LRS) method and the Upstream Equal Sharing (UES) method. We first show that the UES method is characterized by relaxing independence of upstream costs appearing in Ni and Wang (2007). Then we provide two axiomatizations with endogenous responsibility of the α-Local Responsibility Sharing method, one using this weak independence axiom (taken from the UES method) and one using a weak version of the no blind cost axiom (taken from the LRS method). Moreover, we also provide an axiomatization with exogenous responsibility by introducing α-responsibility balance. Finally, we define a pollution cost-sharing game, and show that, interestingly, the Half Local Responsibility Sharing (HLRS) method coincides with the Shapley value, the nucleolus and the τ-value of the corresponding pollution cost-sharing game. This HLRS method can be seen as some kind of middle compromise of the LRS and UES methods.

Keywords: pollution cost-sharing problems; α-Local Responsibility Sharing method; axiomatization; cooperative games

1. Introduction

River (water) allocation among agents has emerged as one of the areas with exceptional interest for researchers due to its indispensable benefits to inhabitants of coastal communities. About 200 rivers are flowing through different countries in the world (see Ambec and Sprumont [3] and Barrett [4]). On the one hand, these water resources cater to people's daily routines and industrial productions. On the other hand, waste generated by domestic chores and production activities pollutes the sources of water, and this is harmful to humans, plants, and animals. In recent years, due to population growth and rapid industrialization, human's demand for (clean) water resources as well as the degree

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of water pollution are constantly increasing. This makes many countries and regions face water shortage. Considering this, reasonable allocation and utilization of water resources and efficient water pollution management would be effective measures to solve this problem. Hence, the following questions need to be tackled: (1) How do inhabitants of the coastal communities allocate the water resources? (2) How do inhabitants of the coastal communities share the cost of cleaning up a polluted river? Harnessing both the burden of responsibility and the relieve that these water resources provides has become an issue of great importance recently and has presented a vital aspect that could make or mar societal development and peaceful coexistence in the riverine communities.

On the beneficial side, Ambec and Sprumont [3] were the first to model the situation where a group of agents located along a river share its resources, and studied how the water should be allocated among agents. They analyzed the model from a cooperative game-theoretic viewpoint and proposed the downstream incremental method in terms of the two main doctrines of Absolute Territorial Sovereignty [10] and Unlimited Territorial Integrity [14] in international disputes. Ambec and Ehlers [2] extended the model of sharing a river and considered the problem of efficiently sharing water from a river among a group of satiable agents. Gudmundsson et al. [12] focused on implementing efficient outcomes of the river sharing problem by noncooperative bargaining. Just recently, Steinmann and Winkler [18] dug further in studying a river sharing model with downstream pollution with externalities. More results about the river sharing problem can be found in the survey papers Béal, Ghintran, Rémila and Solal [5] and Beard [6].

On the responsibility side, Ni and Wang [15] first developed a model for the pollution cost-sharing problem and discussed a question of how to split the cost of cleaning up a river among agents situated along the river. They proposed two methods; the Local Responsibility Sharing (LRS) method and the Upstream Equal Sharing (UES) method by resorting to the two main doctrines of Absolute Territorial Sovereignty and Unlimited Territorial Integrity in international disputes. The LRS method charges an agent the full cost of cleaning up the segment in which the agent is located, that is, every agent should take full responsibility for cleaning up its area. In contrast, the UES method forces an agent and all its upstream counterparts to take the same responsibility for cleaning up its area. To be precise, the UES method allocates the cost of cleaning up the segment equally among the agent in that segment and all its upstream counterparts.

The completeness of the above approach has been questioned by some authors. Alcalde-Unzu et al. [1] argued that neither the LRS method nor the UES method makes sense in some cases. They proposed an alternative method which takes into account the transfer rate of waste from one segment to another. Sun et al. [19] extended the approach by introducing the α-responsibility method which is the corresponding convex combination of the LRS method and the UES method and implemented this allocation method by a dynamic procedure. Gómez-Rúa [11] proposed a family of rules by taking into account the different factors that influence the quality of the water. More recent research on the pollution cost-sharing problem can be found in the literature, see e.g. Hou, Lardon, Sun and Xu [13] and van den Brink, He and Huang [21].

Considering the LRS and UES methods of Ni and Wang [15], it seems ambiguous whether the agent takes full responsibility for cleaning up its area or shares the responsibility equally with all its upstream counterparts. The first method does not take into consideration that the pollutants of a river flow from upstream to downstream. The second method implicitly assumes that the agent in a segment and all its upstream counterparts have the same degree of responsibility for cleaning up the segment. In this paper, we attempt to tackle the second question posed above by focusing on the responsibility of sharing the cost of cleaning up a polluted river. Inspired by the work of Ni and Wang [15], we investigate a new method, the α-Local Responsibility Sharing (α-LRS) method. The α-LRS method first assigns to agent a fraction of the cost of cleaning up the segment in which the agent is located, and then the remaining cost is distributed equally among its upstream counterparts. This fraction can be interpreted as an agent's responsibility level in its own pollution cost.

Axiomatization is a common way to characterize the fairness and reasonability of methods for pollution cost sharing problems. Some standard properties have been applied to characterize the LRS method and the UES method, such as efficiency, additivity, independence of upstream costs, upstream symmetry and no blind cost, which are found in the literature, see e.g. Dong, Ni and Wang [9], Ni and Wang [15], and Sun, Hou and Sun [19]. In the paper, we first characterize the UES method by introducing a relaxation of independence of upstream costs, called sign independence of upstream costs. Then, we define weak upstream symmetry and weak no blind cost by weakening these standard properties. We give two axiomatizations with endogenous responsibility, that is, the properties used do not explicitly involve the responsibility level vector. Moreover, we also propose an axiomatization with exogenous responsibility, that is, the agents' responsibility levels are explicitly given and the properties used in the axiomatization involve the responsibility level vector. We characterize each α-LRS method without additivity by introducing α-responsibility balance and stronger weak upstream symmetry. Finally, we analyze the problem from a cooperative game-theoretic viewpoint and define the (cooperative) pollution cost-sharing game. We show that the Shapley value, the nucleolus and the τ-value of this game coincide and, interestingly, are equal to the Half Local Responsibility Sharing method (HLRS) method, being the α-LRS method with responsibility parameter $\frac{1}{2}$ for all agents except the first agent.

This paper is organized as follows. In Section 2, we introduce some basic notations and definitions. In Section 3, we define the α-LRS method and provide three characterizations of the method. In Section 4, we define the pollution cost-sharing game and show that the Shapley value and the nucleolus of this game coincide with the HLRS method. Section 5 concludes with a summary.

2. Preliminaries

2.1. Pollution cost-sharing problems

Consider a river which is divided into n segments from upstream to downstream. There are n agents (or countries) located along the river, and each agent is located in one of these segments indexed by a given order $i = 1, 2, \cdots, n$ from upstream to downstream. These agents generate a certain amount of pollutants, destroying the ecosystem of the river and influencing the quality of the waterbody. In order to guarantee the water quality, every agent has to pay for the cost to clean up the polluted river. To this end, in every segment along the river, the environmental authority sets a standard of the degree of pollution which requires agents paying the cost c_i to clean up the pollutants of segment i, so that the water quality is up to the environmental standard. The central issue is how to allocate the total costs, $\sum_{i\in N} c_i$, among the n agents. Ni and Wang [15] firstly modeled this practical problem, called the pollution cost-sharing problem.

Formally, a pollution cost-sharing problem is a pair (N, c), where $N = \{1, \cdots, n\}$ is a finite set of agents and $c = (c_1, \cdots, c_n) \in \mathbb{R}^n_+$ is the pollution cost vector. The component c_i represents the cost incurred by segment $i, i \in N$. For any $i, j \in N$, $i < j$ means that i is upstream from j. Denote the class of all pollution cost-sharing problems with n agents by $\mathcal{P}^N$. A payoff vector for pollution cost-sharing problem $(N, c) \in \mathcal{P}^N$ is an n-dimensional vector $x = (x_1, \cdots, x_n) \in \mathbb{R}^n_+$ whose component $x_i \geq 0$ represents the cost share allocated to agent i. A method on $\mathcal{P}^N$ is a map $x : \mathcal{P}^N \to \mathbb{R}^n_+$ that assigns a cost share vector $x(N, c)$ to every problem $(N, c) \in \mathcal{P}^N$.

Ni and Wang [15] proposed two methods, the Local Responsibility Sharing (LRS) method and the Upstream Equal Sharing (UES) method.[1] The LRS method x^{LRS} assigns to each agent the full cost of cleaning up the segment where the agent is located. For any $(N, c) \in \mathcal{P}^N$ and $i \in N$,

$$x_i^{LRS}(N, c) = c_i.$$

The UES method x^{UES} distributes the cost of cleaning up each segment equally among the agent in that segment and all agents situated upstream from it. For any $(N, c) \in \mathcal{P}^N$ and $i \in N$,

$$x_i^{UES}(N, c) = \sum_{k=i}^{n} \frac{1}{k} c_k.$$

2.2. Cooperative game theory

A cooperative game with transferable utility, or simply a TU-game, is a pair $\langle N, v\rangle$, where $N = \{1, \ldots, n\}$ is a finite set of n players and $v : 2^N \to \mathbb{R}$ is a characteristic function assigning to each coalition $S \in 2^N \backslash \{\emptyset\}$ the worth $v(S)$ with $v(\emptyset) = 0$. Denote the set of all TU-games on player set N by $\mathcal{G}^N$, and denote the cardinality of a finite set S (respectively T) by s (respectively t). A TU-game $\langle N, v\rangle \in \mathcal{G}^N$ is an additive game if $v(S) = \sum_{i\in S} v(\{i\})$ for all $S \subseteq N$. A TU-game $\langle N, v\rangle \in \mathcal{G}^N$ is a 2-additive game if it satisfies that (i) for each $i \in N$, $v(\{i\}) = 0$, and (ii) for each $S \subseteq N$ with $s \geq 2$, $v(S) = \sum_{T\subseteq S, t=2} v(T)$. A payoff vector for TU-game $\langle N, v\rangle \in \mathcal{G}^N$ is an n-dimensional vector $x \in \mathbb{R}^n$ assigning a payoff

[1] For the more general multiple spring rivers, Dong, Ni and Wang [9] also introduced the Downstream Equal Sharing (DES) method x^{DES} that allocates the cost of a segment equally among this segment and each of its downstream segments. For any $(N, c) \in \mathcal{P}^N$ and $i \in N$, $x_i^{DES}(N, c) = \sum_{k=1}^{i} \frac{1}{n-k+1} c_k$.

$x_i \in \mathbb{R}$ to any player $i \in N$. For any $\langle N, v\rangle \in \mathcal{G}^N$, the imputation set $I(N, v)$ is given by $I(N, v) = \{x \in \mathbb{R}^N | x_i \leq v(\{i\}),$ for all $i \in N\}$ and consists of all cost share vectors such that no player pays more than its own stand-alone cost.[2]

A solution on $\mathcal{G}^N$ is a function φ that assigns a payoff vector $\varphi(N, v) \in \mathbb{R}^n$ to every TU-game $\langle N, v\rangle \in \mathcal{G}^N$. The Shapley value and the nucleolus are two of the most classical solutions for TU-games. The Shapley value Sh, introduced by Shapley [17], offers each player the expectation of its marginal contributions with respect to all coalitions containing the player, assuming that every order in which the coalition is formed occurs with equal probability. For any $\langle N, v\rangle \in \mathcal{G}^N$ and $i \in N$,

$$Sh_i(N, v) = \sum_{S\subseteq N, S\ni i} \frac{(n-s)!(s-1)!}{n!}(v(S) - v(S\backslash\{i\})).$$

The nucleolus, introduced by Schmeidler [16], is obtained by minimizing the excesses of coalitions in the lexicographic order over the non-empty imputation set. The excess of coalition $S \subseteq N$ with respect to the payoff vector x of the TU-game $\langle N, v\rangle$ is given by $e^v(S, x) = x(S) - v(S)$. This can be seen as a measure of dissatisfaction of the coalition since a positive (respectively negative) excess means that the coalition pays more (respectively less) than its own cost. Let $\theta^v(x)$ be the $(2^n - 1)$-tuple vector whose components are the excesses of all non-empty coalition $S \subseteq N$ in non-increasing order, that is, $\theta^v_l(x) \geq \theta^v_{l+1}(x)$ for all $l \in \{1, 2, ..., 2^n - 2\}$. For any $\langle N, v\rangle \in \mathcal{G}^N$ and $x, y \in \mathbb{R}^n$, we call $\theta^v(x) \leq_L \theta^v(y)$ if and only if $\theta^v(x) = \theta^v(y)$ or there exists an $t \in \{1, 2, ..., 2^n - 2\}$ such that $\theta^v_k(x) = \theta^v_k(y)$ for all $k \in \{1, 2, ..., t\}$ and $\theta^v_{t+1}(x) < \theta^v_{t+1}(y)$. The nucleolus η is a payoff vector y in the imputation set that lexicographically minimizes the excesses, that is,

$$\eta(N, v) = \{x \in I(N, v) | \theta^v(x) \leq_L \theta^v(y), \text{ for all } y \in I(N, v)\}.$$

Since it is known that $\eta(N, v)$ is a singleton, we identify the nucleolus by its unique element.

The τ-value, introduced by Tijs [20], is essentially a compromise value between an upper bound payoff vector and a lower bound payoff vector. For any $\langle N, v\rangle \in \mathcal{G}^N$, let $M(N, v) \in \mathbb{R}^n$ be the vector whose coordinates are the marginal contribution of each player to the grand coalition, that is, $M_i(N, v) = v(N) - v(N\backslash\{i\})$ for all $i \in N$. When we consider this as a vector of upper bound payoffs, then the vector $m(N, v) \in \mathbb{R}^n$ whose coordinates are given by $m_i(N, v) = \max_{S\subseteq N, S\ni i}\{v(S) - \sum_{j\in S\backslash\{i\}} M_j(N, v)\}$ for all $i \in N$, can be seen as a lower bound payoff vector. These vectors can indeed be interpreted as upper and lower bound payoff vectors, if the game $\langle N, v\rangle$ is quasi-balanced, meaning that

[2]The games as considered here are so-called *cost games* where the worth of every coalition is a cost to be covered, and the 'payoff' of a player is the share in the total cost to be paid by this player. On the other hand, the same TU-game model is used for profit games where the worth of every coalition is the gain the coalition can generate. Although many concepts are defined the same for cost and profit games, other concepts need to be modified, for example an imputation in a profit game is a payoff vector such that every players gets at least its stand-alone worth.

(i) $m(N,v) \leq M(N,v)$, and (ii) $\sum_{i\in N} m_i(N,v) \leq v(N) \leq \sum_{i\in N} M_i(N,v)$. Then, for any quasi-balanced game $\langle N,v\rangle$, the τ-value is given by

$$\tau(N,v) = am(N,v) + (1-a)M(N,v),$$

where $a \in [0,1]$ is such that $\sum_{i\in N} \tau_i(N,v) = v(N)$.

2.3. Pollution cost sharing games

Ni and Wang [15] defined two different TU-games with respect to pollution cost-sharing problems. For convenience, the two TU-games are called the LRS-game and the UES-game in this paper. The LRS-game $\langle N, v^L\rangle$ is defined by $v^L(S) = \sum_{i\in S} c_i$ for all $S \subseteq N\backslash\emptyset$ with $v^L(\emptyset) = 0$. The UES-game $\langle N, v^U\rangle$ is defined by $v^U(S) = \sum_{i=\min S}^{n} c_i$ for all $S \subseteq N\backslash\emptyset$ with $v^U(\emptyset) = 0$. They showed that the cost allocations according to the LRS method and the UES method coincide with the Shapley value of the LRS-game and the UES-game, respectively.[3]

3. The α-Local Responsibility Sharing method

In Ni and Wang [15], the LRS method forces an agent to take full responsibility for cleaning up its segment, while the UES method assumes that the agent in a segment and all its upstream counterparts have the same degree of responsibility for cleaning up the segment. However, it is not obvious why an agent and all its upstream counterparts should be held equally responsible for cleaning up its segment. In this section, we introduce the concept of a *responsibility level* and propose a new method, the *α-Local Responsibility Sharing method*. Given any $(N,c) \in \mathcal{P}^N$, let $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_n) \in \mathbb{R}^n_+$ (with $\alpha_1 = 1$ and $0 \leq \alpha_i \leq 1$ for all $i \in N \setminus \{1\}$) be the responsibility level vector, whose component α_i means that agent i should pay for an α_i fraction of the cost of cleaning up its own segment. In particular, agent 1 has to take full responsibility for cleaning up its segment since agent 1 has no upstream agent, that is, $\alpha_1 = 1$. Let $A^N = \{\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_n) \in \mathbb{R}^n_+ \mid \alpha_1 = 1$ and $0 \leq \alpha_i \leq 1$ for all $i \in N \setminus \{1\}\}$ be the set of all such responsibility vectors. According to the responsibility level vector α, the α-Local Responsibility Sharing method (for short, α-LRS method) is defined as follows.

Definition 3.1. *For any $(N,c) \in \mathcal{P}^N$ and $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_n) \in A^N$, the α-LRS method x^α is given by*[4]

$$x_i^\alpha(N,c) = \alpha_i c_i + \sum_{k=i+1}^{n} \frac{1-\alpha_k}{k-1} c_k, \quad \textit{for all } i \in N.$$

[3]In van den Brink, He and Huang [21] it is shown that the UES method coincides with the permission value of a game with a permission structure where the game is the LRS-game v^L and the linear order of the players is determined by the flow of the river.

[4]We take the sum $\sum_{i=n+1}^{n}$ .... to be equal to 0.

The α-LRS method requires that each agent i pays a fraction α_i of the cost of cleaning up its own segment, and the remaining cost is equally allocated among all agents situated upstream from it.

Remark 1. *In particular, in the case that $\alpha_i = 1$ for all $i \in N$, then the α-LRS method coincides with the LRS method. In the case that $\alpha_i = \frac{1}{i}$ for all $i \in N$, then the α-LRS method coincides with the UES method.*

Remark 2. *Generally, if we treat the α-LRS method as an allocation with $\alpha_i = (1-\frac{1}{i})a+\frac{1}{i}$ for all $i \in N$ and some $a \in \mathbb{R}$, then the α-LRS method can be represented as a convex combination of the LRS method and the UES method, which is proposed by Sun et al. [19], that is, for any $(N, c) \in \mathcal{P}^N$,*

$$x^{\alpha}(N, c) = ax^{LRS}(N, c) + (1 - a)x^{UES}(N, c).$$

Now we recall some standard properties, proposed by Ni and Wang [15].

(i) **Efficiency.** For all $(N, c) \in \mathcal{P}^N$, we have $\sum_{i\in N} x_i(N, c) = \sum_{i\in N} c_i$.
(ii) **Additivity.** For all $(N, c^1), (N, c^2) \in \mathcal{P}^N$, we have $x(N, c^1+c^2) = x(N, c^1)+x(N, c^2)$.
(iii) **No blind cost.** For all $(N, c) \in \mathcal{P}^N$ and $i \in N$ such that $c_i = 0$, we have $x_i(N, c) = 0$.
(iv) **Independence of upstream costs.** For all $(N, c^1), (N, c^2) \in \mathcal{P}^N$ and $i \in N$ such that $c^1_j = c^2_j$ for all $j > i$, we have $x_j(N, c^1) = x_j(N, c^2)$ for all $j > i$.
(v) **Upstream symmetry.** For all $(N, c) \in \mathcal{P}^N$ and $i \in N$ such that $c_j = 0$ for all $j \in N\backslash\{i\}$, we have $x_l(N, c) = x_k(N, c)$ for all $l, k \leq i$.

Efficiency requires that all costs should be fully shared among all agents. Consider a situation where every agent $i \in N$ has two divisions with costs, c^1_i, c^2_i. Additivity says that, considering the sum of two polluted river problems where the cost for each segment equals the sum of the cost in the two separate problems, the associated cost allocation is equal to the sum of the cost allocation vectors assigned to the two separate problems. No blind cost says that, if the segment where an agent is located incurs no pollution cost, the agent should bear no cost. Independence of upstream costs says that an agent's cost share only depends on all costs of cleaning up its segment and all its downstream segments, but not on upstream costs. Upstream symmetry requires that, given an agent $i \in N$, it and all its upstream counteparts bear the same cost if other agents except agent i have no cleaning cost in their local segments. Ni and Wang [15] characterized the LRS method and the UES method by these above properties.

Theorem 3.1. (Ni and Wang, 2007 [15]) *(i) The LRS method is the only method satisfying efficiency, additivity and no blind cost. (ii) The UES method is the only method satisfying efficiency, additivity, independence of upstream costs and upstream symmetry.*

3.1. Sign independence of upstream costs

Alternatively, independence of upstream costs can be replaced with a relaxation of the property to characterize the UES method. Recall the sign function, sign: $\mathbb{R} \to \{-1, 0, 1\}$ given by sign$(t) = 1$ for $t > 0$, sign$(0) = 0$, and sign$(t) = -1$ for $t < 0$.

(vi) **Sign independence of upstream costs.** For all $(N, c^1), (N, c^2) \in \mathcal{P}^N$ and $i \in N$ such that $c^1_j = c^2_j$ for all $j > i$, we have $\text{sign}(x_j(N, c^1)) = \text{sign}(x_j(N, c^2))$ for all $j > i$.

Sign independence of upstream costs is a qualitative version of independence of upstream costs that relaxes independence of upstream costs. Instead of equating cost shares in general, it just fixes a common reference point, the zero utility, and requires that, when all costs of agents downstream of i are the same in two cost vectors, then either all agents downstream of i contribute or all do not contribute. We remark that sign independence of upstream costs is a considerable weakening of independence of upstream costs. Whereas independence of upstream costs requires complete independence of the contributions of an agent when costs of upstream agents change, the weaker sign independence of upstream costs allows that the contribution of an agent also changes when upstream costs change, and it might even have a different effect for different (downstream) agents. Since the UES method satisfies independence of upstream costs, it follows immediately that the UES method satisfies sign independence of upstream costs. We can characterize the UES method by replacing independence of upstream costs in Theorem 3.1 by the weaker sign independence of upstream costs.

Theorem 3.2. *The UES method is the only method satisfying efficiency, additivity, sign independence of upstream costs and upstream symmetry.*

Proof. Since sign independence of upstream costs is weaker than independence of upstream costs, by Theorem 3.1.(ii) it suffices to show that efficiency, additivity, sign independence of upstream costs and upstream symmetry imply independence of upstream costs. Let $(N, c^1), (N, c^2) \in \mathcal{P}^N$ and $i \in N$ be such that $c^1_j = c^2_j$ for all $j > i$. For all $k \in N$, let (N, e^k) be defined by $e^k_k = 1$ and $e^k_l = 0$ for all $l \in N \backslash \{k\}$. Set $(N, c^0) \in \mathcal{P}^N$ with $c^0_k = 0$ for all $k \in N$. It is straightforward to obtain that $x_k(N, c^0) = 0$ for all $k \in N$ by efficiency and $x(N, c^0) \in \mathbb{R}^n_+$. Then, for all $j > i$, we have

$$x_j(N, c^1) = x_j(N, c^1 - \sum_{k>i} c^1_k e^k) + \sum_{k>i} x(N, c^1_k e^k) = \sum_{k>i} x(N, c^1_k e^k),$$

where the first equation holds by additivity and the second equation holds from the fact that $\text{sign}(x_j(N, c^1 - \sum_{k>i} c^1_k e^k)) = \text{sign}(x_j(N, c^0)) = 0$ for all $j > i$ by sign independence of upstream costs. Similarly, for all $j > i$, it holds that

$$x_j(N, c^2) = x_j(N, c^2 - \sum_{k>i} c^2_k e^k) + \sum_{k>i} x(N, c^2_k e^k) = \sum_{k>i} x(N, c^2_k e^k).$$

Thus, we obtain $x_j(N, c^1) = x_j(N, c^2)$ for all $j > i$, which concludes the proof. □

Remark 3. *Notice in the proof of Theorem 3.2, we showed that a method that satisfies efficiency, additivity and sign independence of upstream costs, must satisfy independence of upstream costs. For this implication we do not need upstream symmetry, but we need it to apply Theorem 3.1 to characterize the UES method.*

3.2. Weak no blind cost and weak upstream symmetry

It is clear that the α-LRS method fails no blind cost and upstream symmetry. In the following, we will characterize the α-LRS method by introducing relaxations of these two axioms.

(vii) **Weak no blind cost.** For all $(N, c) \in \mathcal{P}^N$ and $i \in N$ such that $c_j = 0$ for all $j \geq i$, we have $x_i(N, c) = 0$.

(viii) **Weak upstream symmetry.** For all $(N, c) \in \mathcal{P}^N$ and $i \in N$ such that $c_j = 0$ for all $j \in N \backslash \{i\}$, we have $x_l(N, c) = x_k(N, c)$ for all $l, k < i$.

Weak blind cost says that, if an agent and its downstream agents have no cleaning cost in their local segments, then it does not have to contribute anything. Weak upstream symmetry requires that, given an agent $i \in N$, all its upstream counterparts share the same cost if other agents except agent i have no cleaning cost in their local segments. One easily checks that the α-LRS method satisfies weak blind cost and weak upstream symmetry. We remark that these are also considerable relaxations of the classical axioms. Weak no blind costs allows agents to share in costs of other segments in case there is pollution to be cleaned downstream of this agent. Although weak upstream symmetry reflects equal responsibility of upstream agents in case there is only one agent with positive pollution cost, it does not imply any sharing of the responsibility between this positive cost agent and its upstream agents.

Next, we give characterizations of the α-LRS method in terms of weak blind cost and weak upstream symmetry.

Theorem 3.3. (i) *A method x for $\mathcal{P}^N$ satisfies efficiency, additivity, sign independence of upstream costs and weak upstream symmetry if and only if there exists a responsibility level vector $\alpha \in A^N$ such that $x = x^\alpha$.*

(ii) *A method x for $\mathcal{P}^N$ satisfies efficiency, additivity, weak no blind cost and weak upstream symmetry if and only if there exists a responsibility level vector $\alpha \in A^N$ such that $x = x^\alpha$.*

Proof. It is straightforward to verify that the α-LRS method satisfies efficiency, additivity, sign independence of upstream costs, weak upstream symmetry and weak no blind cost. It is left to show that the axioms are sufficient for uniqueness.

(i) Let x be a method for $\mathcal{P}^N$ satisfying efficiency, additivity, sign independence of upstream costs and weak upstream symmetry. We will show that for some responsibility level vector α, $x = x^\alpha$. Similar as before, for all $k \in N$, (N, e^k) is given by $e^k_k = 1$ and $e^k_l = 0$ for all $l \in N \backslash \{k\}$. Set $\alpha_k = x_k(N, e^k)$ for all $k \in N$. Let $(N, c^0) \in \mathcal{P}^N$ with $c^0_k = 0$ for all $k \in N$. It is straightforward to obtain that $x_k(N, c^0) = 0$ for all $k \in N$ by efficiency and $x(N, c^0) \in \mathbb{R}^n_+$. Then, for all $i > k$, by sign independence of upstream costs, we have $\text{sign}(x_i(N, e^k)) = \text{sign}(x_i(N, c^0)) = 0$. By efficiency and weak upstream symmetry, we obtain

$$x_i(N, e^k) = \begin{cases} 0, & \text{if } i > k; \\ \alpha_k, & \text{if } i = k; \\ \frac{1-\alpha_k}{k-1}, & \text{if } i < k. \end{cases} \tag{3.1}$$

Next, we show that x is homogeneous, that is, $x(N, tc) = tx(N, c)$ for all $(N, c) \in \mathcal{P}^N$ and scalar $t \in \mathbb{R}_+$. To show homogeneity for all $t \in \mathbb{R}_+$, choose two sequences of nonnegative rationals $\{r_k\}_{k=1}^{\infty}$ and $\{s_k\}_{k=1}^{\infty}$ which converge to t from above and below, correspondingly. By additivity, we obtain that, for all $i \in N$ and for all $k = 1, \dots, \infty$,

$$\begin{aligned} x_i(N, r_k c) - x_i(N, tc) = x_i(N, (r_k - t)c) \geq 0, \\ x_i(N, tc) - x_i(N, s_k c) = x_i(N, (t - s_k)c) \geq 0. \end{aligned} \tag{3.2}$$

Notice that, for all $i \in N$, $x_i(N, r_k c) - x_i(N, s_k c) = x_i(N, (r_k - s_k)c) = (r_k - s_k)x_i(N, c) \to 0$ as $k \to \infty$, since $(r_k - s_k) \to 0$ as $k \to \infty$. Then $x_i(N, r_k c) - x_i(N, tc) + x_i(N, tc) - x_i(N, s_k c) \to 0$ as $k \to \infty$. Since, both $x_i(N, r_k c) - x_i(N, tc) \geq 0$ and $x_i(N, tc) - x_i(N, s_k c) \geq 0$ by Eq.(3.2), this implies that $x(N, r_k c) \to x(N, tc)$ and $x(N, r_k c) = r_k x(N, c) \to tx(N, c)$ as $k \to \infty$, which proves the homogeneity of x. Thus, x is a linear map on $\mathcal{P}^N$. Therefore, for all $(N, c) \in \mathcal{P}^N$ and $i \in N$, it holds that

$$\begin{aligned} x_i(N, c) =& x_i(N, \sum_{k \in N} c_k e^k) = \sum_{k \in N} c_k x_i(N, e^k) \\ =& \alpha_i c_i + \sum_{k=i+1}^{n} \frac{1 - \alpha_k}{k - 1} c_k = x_i^{\alpha}(N, c), \end{aligned} \tag{3.3}$$

where the third equality follows from Eq.(3.1).
Notice that $\alpha_k = x_k(N, e^k)$ for all $k \in N$, efficiency and $x(N, e^k) \in \mathbb{R}_+^n$, implies that $0 \leq \alpha_i \leq 1$ for all $i \in N$. Moreover, similar as above, for all $i > 1$, by sign independence of upstream costs, we have $\text{sign}(x_i(N, e^1)) = \text{sign}(x_i(N, c^0)) = 0$, and thus by efficiency, $\alpha_1 = x_1(N, e^1) = 1$, showing that $\alpha \in A^N$.

(ii) Let x be a method for $\mathcal{P}^N$ satisfying efficiency, additivity, weak no blind cost and weak upstream symmetry. We can obtain that x is a linear map on $\mathcal{P}^N$ from (i). Similarly as in the proof of part (i), set $\alpha_k = x_k(N, e^k)$ for all $k \in N$. Then, for all $i > k$, by weak no blind cost, we have $x_i(N, e^k) = 0$. Together with efficiency and weak upstream symmetry, we again obtain Eq.(3.1) and, since x is a linear map on $\mathcal{P}^N$, then for all $(N, c) \in \mathcal{P}^N$ and $i \in N$, we obtain Eq.(3.3) similar as in the proof of part (i). Similar as in the proof of part (i), $\alpha_k = x_k(N, e^k)$ for all $k \in N$, efficiency and $x(N, e^k) \in \mathbb{R}_+^n$, implies that $0 \leq \alpha_i \leq 1$ for all $i \in N$. Now, weak no blind cost implies that $x_i(N, e^1)) = 0$ for all $i > 1$. Thus by efficiency, $\alpha_1 = x_1(N, e^1) = 1$, showing that $\alpha \in A^N$. This concludes the proof. □

Note that Theorem 3.3 gives two axiomatizations with endogenous responsibility level vector of the α-LRS method, that is, the properties used do not explicitly involve the responsibility level vector, but imply that such a vector exists.

3.3. α-responsibility balance and stronger weak upstream symmetry

In the following, we propose an axiomatization with exogenous responsibility, that is, the agents' responsibility levels are explicitly given and the properties used in the axiomatization involve the responsibility level vector. We characterize the α-LRS method without additivity by introducing α-responsibility balance and stronger weak upstream symmetry.

(ix) **α-responsibility balance.** For all $(N, c), (N, c') \in \mathcal{P}^N$, $\alpha \in A^N$ and $i \in N$ such that $c_j = c'_j$ for all $j \in N \backslash \{i\}$, we have $x_i(N, c) - \alpha_i c_i = x_i(N, c') - \alpha_i c'_i$.

(x) **Stronger weak upstream symmetry.** For all $(N, c) \in \mathcal{P}^N$ and $i \in N$ such that $c_j = 0$ for all $j < i$, we have $x_l(N, c) = x_k(N, c)$ for all $l, k < i$.

α-responsibility balance states that the cost share allocated to an agent varies with the cost of cleaning up the segment where the agent is located. More specifically, the change of the cost share allocated to an agent i is α_i times the change of the cost of cleaning up the segment. The axiom involves the responsibility level vector. Stronger weak upstream symmetry requires that, given an agent $i \in N$, all its upstream counterparts share the same cost if all its upstream counterparts have no cleaning cost in their local segments. It is a stronger version of weak upstream symmetry. One easily checks that, if a method satisfies stronger weak upstream symmetry, then it must satisfy weak upstream symmetry. Notice that stronger weak upstream symmetry is also an alternative weak version of no blind cost: every method that satisfies no blind cost satisfies stronger weak upstream symmetry. Next, we give a characterization of the α-LRS method in terms of α-responsibility balance and stronger weak upstream symmetry.

Theorem 3.4. *The α-LRS method is the only method satisfying efficiency, independence of upstream costs, α-responsibility balance and stronger weak upstream symmetry.*

Proof. Take $\alpha \in A^N$. It is easy to verify that the α-LRS method satisfies efficiency, independence of upstream costs, α-responsibility balance and stronger weak upstream symmetry. It remains to prove that the axioms give uniqueness.

Suppose x is a method satisfying these axioms for $\alpha \in A^N$. For problem $(N, c^0) \in \mathcal{P}^N$ with $c_i^0 = 0$ for all $i \in N$, as before by efficiency, we have $x_i(N, c^0) = 0$ for all $i \in N$. Consider a sequence of pollution cost-sharing problems $\{(N, c^j)\}_{j=1}^n$ with $c_i^j = 0$ for $i < j$, and $c_i^j = c_i$ for $i \geq j$. Firstly, for $(N, c^n) \in \mathcal{P}^N$, by α-responsibility balance, we have $x_n(N, c^n) = x_n(N, c^n) - x_n(N, c^0) = \alpha_n c_n - 0 = \alpha_n c_n$. Together with efficiency and stronger weak upstream symmetry, we have that $x_i(N, c^n) = \frac{1-\alpha_n}{n-1} c_n$ for all $i \leq n - 1$.

Next we will prove that

$$x_i(N, c^j) = \begin{cases} \alpha_i c_i + \sum_{k=i+1}^n \frac{1-\alpha_k}{k-1} c_k, & \text{if } i \geq j; \\ \sum_{k=j}^n \frac{1-\alpha_k}{k-1} c_k, & \text{if } i < j, \end{cases} \tag{3.4}$$

by induction on j. Without loss of generality, suppose that Eq.(3.4) holds for (N, c^{j+1}). Then, for (N, c^j), by independence of upstream costs, we have

$$x_i(N, c^j) = x_i(N, c^{j+1}) = \alpha_i c_i + \sum_{k=i+1}^n \frac{1 - \alpha_k}{k - 1} c_k,$$

for $i \geq j+1$. By α-responsibility balance, it holds that

$$x_j(N,c^j) - x_j(N,c^{j+1}) = \alpha_j(c^j_j - c^j_{j+1}) = \alpha_j c_j,$$

and thus

$$x_j(N,c^j) = x_j(N,c^{j+1}) + \alpha_j c_j = \sum_{k=j+1}^{n} \frac{1-\alpha_k}{k-1} c_k + \alpha_j c_j.$$

By stronger weak upstream symmetry and efficiency, for all $i < j$, we have $x_i(N,c^j) = \sum_{k=j}^{n} \frac{1-\alpha_k}{k-1} c_k$. This shows that Eq.(3.4) holds for (N,c^j). Therefore, for all $(N,c) \in \mathcal{P}^N$, it holds that, for $i \in N$,

$$x_i(N,c) = x_i(N,c^1) = \alpha_i c_i + \sum_{k=i+1}^{n} \frac{1-\alpha_k}{k-1} c_k$$

which concludes the proof. □

4. Pollution cost-sharing games

In this section, we define a TU-game with respect to the pollution cost-sharing problem. For all $i \in N$ and $S \subseteq N$, let $P_i(S) = \{j \in S | j < i\}$ denote all upstream agents of agent i in coalition S. Denote the cardinality of $P_i(S)$ by $|P_i(S)|$. We define the following cost game where every coalition of agents is assigned a certain part of the pollution cost of the river, depending on the location and responsibility of the coalition of agents.

Definition 4.1. *For all $(N,c) \in \mathcal{P}^N$, the pollution cost-sharing game $\langle N, v^c \rangle$ is given by*

$$v^c(S) = \begin{cases} \sum_{i \in S} \frac{|P_i(S)|}{|P_i(N)|} c_i, & \text{if } S \not\ni 1; \\ \sum_{i \in S \setminus \{1\}} \frac{|P_i(S)|}{|P_i(N)|} c_i + c_1, & \text{if } S \ni 1. \end{cases} \quad (4.1)$$

For all $i \in N \setminus \{1\}$ and $S \subseteq N \setminus \{1\}$, $\frac{|P_i(S)|}{|P_i(N)|}$ is the fraction of the number of agent i's upstream agents in coalition S. Then, $\frac{|P_i(S)|}{|P_i(N)|} c_i$ can be regarded as the proportional share of coalition S in the cost of cleaning up i's segment. Generally speaking, every coalition that does not contain the most upstream agent 1, is assigned a share in the cost of cleaning each segment in the coalition (except the most upstream segment 1) which is proportional to the number of upstream agents that belong to the coalition. Thus, the total costs of coalition S, if $1 \notin S$, is $\sum_{i \in S} \frac{|P_i(S)|}{|P_i(N)|} c_i$. Since agent 1 has no upstream agent, every coalition that contains agent 1 has to take full responsibility for cleaning up its segment. Thus, for all $S \subseteq N$ with $S \ni 1$, the total costs of coalition S is $\sum_{i \in S \setminus \{1\}} \frac{|P_i(S)|}{|P_i(N)|} c_i + c_1$. The definition of the pollution cost-sharing game is in accordance with the upstream responsibility principle implied by the Unlimited Territorial Integrity theory in International Water Law. It says that upstream countries should not change the natural flow of the water at the expense of its downstream countries, which can be interpreted as giving an agent the rights to ask all its upstream agents to pay the pollutant-cleaning costs at its segment. This means that an upstream coalition bears some responsibilities for all downstream pollutant-cleaning costs, which here we assume to be proportional to the membership of the coalition.

It is obvious that the pollution cost-sharing game $\langle N, v^c \rangle$ can be rewritten as

$$v^c(S) = w^c(S) + u^c(S) \tag{4.2}$$

for all $S \subseteq N$, where $\langle N, w^c \rangle$ is given by

$$w^c(S) = \begin{cases} \sum_{i\in S} \frac{|P_i(S)|}{|P_i(N)|} c_i, & \text{if } S \not\ni 1; \\ \sum_{i\in S\setminus\{1\}} \frac{|P_i(S)|}{|P_i(N)|} c_i, & \text{if } S \ni 1, \end{cases} \tag{4.3}$$

and $\langle N, u^c \rangle$ is given by

$$u^c(S) = \begin{cases} 0, & \text{if } S \not\ni 1; \\ c_1, & \text{if } S \ni 1. \end{cases} \tag{4.4}$$

Next we show that game $\langle N, w^c \rangle$ defined by Eq.(4.3) is a 2-additive game, meaning that the worth of every stand-alone coalition is zero, and the worth of a coalition with two or more players equals the sum of the worths of its two-player subcoalitions.

Lemma 4.1. *For any $(N, c) \in \mathcal{P}^N$, the game $\langle N, w^c \rangle$ defined by Eq.(4.3) is a 2-additive game, that is, for all $i \in N$, $w^c(\{i\}) = 0$, and for all $S \subseteq N$ with $s \geq 2$,*

$$w^c(S) = \sum_{T\subseteq S, t=2} w^c(T).$$

Proof. By Eq.(4.3), it is straightforward to obtain that $w^c(\{i\}) = 0$ for all $i \in N$. Moreover, game $\langle N, w^c \rangle$ defined by Eq.(4.3) can be rewritten as, for all $S \subseteq N$,

$$w^c(S) = \begin{cases} \sum_{i\in S} \sum_{j\in S, j<i} \frac{1}{i-1} c_i, & \text{if } S \not\ni 1; \\ \sum_{i\in S\setminus\{1\}} \sum_{j\in S, j<i} \frac{1}{i-1} c_i, & \text{if } S \ni 1, \end{cases}$$

Since it is straightforward that $w^c(\{i, j\}) = \frac{1}{i-1} c_i$ for all $i, j \in N$ with $j < i$, we have

$$w^c(S) = \begin{cases} \sum_{i\in S} \sum_{j\in S, j<i} w^c(\{i, j\}), & \text{if } S \not\ni 1; \\ \sum_{i\in S\setminus\{1\}} \sum_{j\in S, j<i} w^c(\{i, j\}), & \text{if } S \ni 1, \end{cases}$$

Therefore, it holds that $w^c(S) = \sum_{T\subseteq S, t=2} w^c(T)$ for all $S \subseteq N$ with $s \geq 2$. □

Example 4.1. *Consider a problem (N, c) where $N = \{1, 2, 3, 4\}$ and $c = (c_1, c_2, c_3, c_4)$. Then, for the game $\langle N, w^c \rangle$, the worth of the two player coalitions are given as follows*

$$w^c(\{1, 2\}) = c_2, \quad w^c(\{1, 3\}) = \frac{1}{2} c_3, \; w^c(\{1, 4\}) = \frac{1}{3} c_4,$$
$$w^c(\{2, 3\}) = \frac{1}{2} c_3, \; w^c(\{2, 4\}) = \frac{1}{3} c_4, \; w^c(\{3, 4\}) = \frac{1}{3} c_4.$$

The worth of coalitions with more than two players can be expressed as follows.

$$\begin{aligned}
w^c(\{1,2,3\}) =& c_2 + c_3 = w^c(\{1,2\}) + w^c(\{1,3\}) + w^c(\{2,3\}),\\
w^c(\{1,2,4\}) =& c_2 + \frac{2}{3}c_4 = w^c(\{1,2\}) + w^c(\{1,4\}) + w^c(\{2,4\}),\\
w^c(\{1,3,4\}) =& \frac{1}{2}c_3 + \frac{2}{3}c_4 = w^c(\{1,3\}) + w^c(\{1,4\}) + w^c(\{3,4\}),\\
w^c(\{2,3,4\}) =& \frac{1}{2}c_3 + \frac{2}{3}c_4 = w^c(\{2,3\}) + w^c(\{2,4\}) + w^c(\{3,4\}),\\
w^c(\{1,2,3,4\}) =& c_2 + c_3 + c_4 = w^c(\{1,2\}) + w^c(\{1,3\}) + w^c(\{1,4\})\\
& + w^c(\{2,3\}) + w^c(\{2,4\}) + w^c(\{3,4\}).
\end{aligned}$$

From van den Nouweland et al. [22], Chun and Hokari [7] and Deng and Papadimitriou [8], it follows that the Shapley value, the nucleolus and the τ-value coincide for 2-additive games, and thus are equal for the game $\langle N, w^c\rangle$. Moreover, from van den Nouweland et al. [22] it follows that these three solutions coincide for every game that is the sum of an additive and 2-additive game, and thus we have the following corollary.

Corollary 4.2. *The Shapley value of the pollution cost-sharing game $\langle N, v^c\rangle$ defined by Eq.(4.1) coincides with the nucleolus and the τ-value of this game.*

Next we show that the specific α-LRS method with $\alpha_1 = 1$ and $\alpha_i = \frac{1}{2}$ for all $i \in N\backslash\{1\}$, which we call the Half Local Responsibility Sharing (HLRS) method, gives the same allocation as the Shapley value and the nucleolus of the pollution cost-sharing game $\langle N, v^c\rangle$.

Definition 4.2. *For all $(N,c) \in \mathcal{P}^N$, the Half Local Responsibility Sharing method x^{HLRS} is given by*

$$x_i^{HLRS}(N,c) = \begin{cases} \frac{1}{2}c_i + \sum_{k=i+1}^n \frac{1}{2(k-1)}c_k, & \text{if } i \in N\backslash\{1\};\\ c_1 + \sum_{k=2}^n \frac{1}{2(k-1)}c_k, & \text{if } i = 1. \end{cases}$$

Theorem 4.3. *For all $(N,c) \in \mathcal{P}^N$, the method that applies the Shapley value, the nucleolus and the τ-value to the pollution cost-sharing game $\langle N, v^c\rangle$ is equal to the HLRS method.*

Proof. Since, $\langle N, w^c\rangle$ is a 2-additive game, for every $i \neq 1$,

$$\begin{aligned}
Sh_i(N,w^c) =& \frac{1}{2}\left(w^c(N) - w^c(N\setminus\{i\})\right)\\
=& \frac{1}{2}\left(\sum_{j\in N\backslash\{1\}} c_j - \sum_{j=2}^{i-1} c_j - \sum_{j=i+1}^{n} \frac{j-2}{j-1}c_j\right)\\
=& \frac{1}{2}\left(c_i + \sum_{j=i+1}^{n}\left(1 - \frac{j-2}{j-1}\right)c_j\right)
\end{aligned}$$

$$=\frac{1}{2}c_i + \frac{1}{2}\sum_{j=i+1}^{n}\left(\frac{j-1-j+2}{j-1}\right)c_j$$
$$=\frac{1}{2}c_i + \sum_{k=i+1}^{n}\frac{1}{2(k-1)}c_k,$$

and

$$Sh_1(N,w^c) =\frac{1}{2}\left(w^c(N) - w^c(N\setminus\{1\})\right) = \frac{1}{2}\left(\sum_{j\in N\setminus\{1\}} c_j - \sum_{j=2}^{n}\frac{j-2}{j-1}c_j\right)$$
$$=\frac{1}{2}\sum_{j=2}^{n}\left(1-\frac{j-2}{j-1}\right)c_j = \sum_{k=2}^{n}\frac{1}{2(k-1)}c_k.$$

Since $\langle N, v^c\rangle$ is an additive game, $Sh_1(N,u^c) = c_1$ and $Sh_i(N,u^c) = 0$ for all $i \in N\setminus\{1\}$. Since the Shapley value is an additive solution, we have $Sh(N,v^c) = Sh(N,u^c)+Sh(N,w^c)$, which gives the result. □

5. Summary

We study a class of cost-sharing methods for cleaning up a polluted river by considering every agent's responsibility for its own area. We propose the α-LRS methods and give several axiomatizations for these methods. Moreover, the known UES method is characterized by a relaxation of independence of upstream costs. Finally, we define a corresponding pollution cost-sharing game and show that this is the sum of a 2-additive game and an additive game, implying that its Shapley value coincides with its nucleolus and τ-value. Interestingly, the Shapley value, the nucleolus, and thus the τ-value, of the pollution cost-sharing game give a specific α-LRS method, called, the HLRS method, which is obtained by assigning the full cost of the most upstream agent to this agent, and assigns to every other agent half of the cost of cleaning its own river segment. For further research, we will apply these methods to the more general polluted river network model introduced by Dong et al. [9] and generalize the α-LRS method for more general models.

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