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Sectoral slowdowns in the UK: Evidence from transmission probabilities and economic linkages

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Abstract

This paper studies shock transmission across macroeconomic sectors in the UK, using data from the Bank of England's Flow of Funds statistics. We combine two different approaches to quantify the spread of shocks to assess whether sectors with large bilateral economic linkages as measured through network data have a greater statistical likelihood of shock transmission between them. The combination of both approaches reveals the Monetary Financial Institutions sector's role as shock absorber, and identifies the most important channels of shock transmission. The inferential discrepancies between network data and the actual spillovers highlight the contribution of the proposed methodology.

Keywords: Flow of Funds, contagion, epidemiology, intersectoral networks, Gibbs sampling, Bayesian priors

JEL classification codes: E37, E32, E01, G01

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I Introduction

This paper studies the transmission of sectoral shocks in the United Kingdom economy, using an epidemiological modeling framework that combines insights from a network-based approach and a statistical approach. We are interested in understanding whether macroeconomic sectors that are highly interconnected in a network sense through large flows of assets and liabilities are necessarily statistically more likely to spread shocks in an economic sense. By considering both approaches together, we are able to distinguish between different sources that contribute to the systemic risk in the economy.

Economists have historically held the view that when studying the effect of aggregate shocks on the macroeconomy, the effect of shocks at the micro level, i.e., to individuals, firms or sectors, could be ignored (e.g., Lucas, 1995). More recently, however, a literature has emerged that demonstrates a role for micro-level shocks as the origin of aggregate fluctuations. Very broadly, the literature studying the transmission of sectoral shocks within a country (or, equivalently, the transmission of country-level shocks to other countries, or the transmission of firm-level shocks to the rest of the economy) can be divided into two main approaches. The first approach focuses on the underlying network structure that describes the various interactions between sectors of the economy.[1] The second approach uses sector-specific (or firm or country-specific) variables and statistical/econometric methods such as factor models, vector autoregressions (VARs) or cluster analysis to analyze the transmission of shocks between sectors/firms/countries (e.g., Canova, 2005; Foerster et al., 2011; Billio et al., 2012; Diebold and Yılmaz, 2014; Mitchener and Richardson, 2019). Another approach that falls into this latter category is the epidemiological model, formalized by Kermack and McKendrick (1927) to study how a disease spreads within a population. In economics this model

[1]Some important contributions to this literature on macroeconomic and financial networks include Allen and Gale (2000), Gabaix (2011), Acemoglu et al. (2012), Carvalho and Gabaix (2013), Elliott et al. (2014) and more recently Baqaee and Farhi (2019), Liu (2019), Acemoglu and Azar (2020), and Dhyne et al. (2020).

has been used, for example, by Toivanen (2013) and Glasserman and Young (2015) to model how shocks spread across European banks, and Demiris et al. (2014) to study the spillover of currency crises across countries.

In an epidemiological model where all sectors (or, depending on the application, firms, countries, or individuals) are allowed to have heterogeneous contact rates, the number of parameters grows quadratically with the number of sectors in the economy. Depending on the number of events (labeled as contaminations and recoveries) that have occurred in the data, this dimensionality can mean that in practice some of the parameters cannot be identified from the data. This problem is similar to the issue that appears in large-dimensional VAR models, where the number of parameters grows rapidly with the number of lags and variables. Researchers have used Bayesian techniques (known as Bayesian VAR) to improve the inference and the forecasting performance of their VAR models, as well as to avoid overfitting or other ill-behaved estimation problems. This is often done using Bayesian priors that have an effect comparable to the frequentist notion of regularization (Bańbura et al., 2010; Koop, 2013; Ghosh et al., 2019).

In a spirit similar to Bayesian VAR models, this paper proposes to inform the parameters of an epidemiological model using Bayesian priors. However, instead of shrinking the contamination probabilities to zero, as would be the case with LASSO or ridge-type regularizers often used in Bayesian VAR models, in our Bayesian framework contamination probabilities are "steered" towards values determined from an external data source in an intuitive way.[2] For the construction of the priors it is assumed that contact rates between sectors, also known as contamination probabilities, are proportional to the strength of the actual

[2]The effect on the parameter estimates in our approach shares similarities with regularization approaches such as LASSO because the inclusion of priors helps to distinguish between more and less likely channels of shock transmission. It does not, however, necessarily lead to lower variances (although this can still occur), which is often one of the other objectives of regularization methods.

economic ties between pairs of sectors and the strength of the sector that is targeted.[3] To incorporate the fact that economic ties relate proportionally to the contamination probabilities, but are likely not one-to-one, we allow for and jointly estimate sector-specific scaling factors that measure the proportionality between the priors and the posterior contamination probabilities. The prior information encapsulates the idea that sectors that have only a weak economic connection are less likely to have a large contamination probability between them, and that an infected sector will have a harder time infecting a very large sector relative to a small sector. As such, these priors are consistent with the mechanism proposed in the network-based models of Allen and Gale (2000), Baqaee and Farhi (2019), Liu (2019), and Acemoglu and Azar (2020), where shocks spread through the economic system because (in the case of our paper) sectors are directly connected through, e.g., lending and borrowing.

By using Bayesian priors that are informative regarding the economic ties between sectors, and by updating the estimates of the contamination probabilities using observed changes in the economic conditions of the sector, we can explore whether a statistical model like the epidemiological model that quantifies (statistical) shock transmission and a network-based model that measures the direct economic linkages between sectors imply similar channels of shock transmission in the system of sectors studied. For example, if a sector appears likely to infect another sector in the epidemiological setting, is it then also the case that these two sectors have strong economic ties? The combined approach furthermore helps to distinguish whether economic shock transmission seems to be driven by actual economic linkages or if there is an alternative mechanism at play. If the estimates indicate that the priors are relatively unimportant, it suggests that other factors are more relevant to explain how shocks transmit between sectors, such as, for example, the so-called animal spirits.

[3] This is analogous to the assumption made by Garas et al. (2010) in their network-based epidemiological model.

Concretely this paper employs two related but different datasets to estimate both a traditional and a Bayesian epidemiological model for the seven major macroeconomic sectors in the United Kingdom, and a sector representing the rest of the world. The first data set describes the assets and liabilities between these sectors and is used to construct the prior distributions. The second data set describes the total assets and liabilities held by these sectors. The epidemiological model captures how negative net worth growth shocks spread through the system. By having the balance sheets and measuring the bilateral flows between these eight sectors, these two datasets are ideally suited for studying both economic and statistical linkages, and for then estimating a Bayesian model, for which it is essential that the priors incorporate other information than the information that is used for the updating of the posterior distributions.

Some of our main findings are the following. We initially find that based on the traditional epidemiological approach, the Insurance and Pension Funds sector in the UK is highly susceptible to shocks from all other sectors in the economy. Incorporating information on the actual economic linkages, however, we find that the main source of contagion to the Insurance and Pension Funds sector is the Household sector, in that weakness in the Household sector will likely lead to a deterioration of economic conditions in the Insurance and Pension Funds sector. The bilateral network structure thus reveals that while in principle multiple sectors could be the catalyst for slow growth in the Insurance and Pension Funds sector, it is most likely the Household sector via the close economic connection between these two sectors. Only through the combined approach can one obtain this insight. The combination of approaches also highlights an important role for the Monetary Financial Institutions sector as a shock absorber by revealing that this sector is susceptible to shocks from various sectors in the system, while it is unlikely to generate spillovers to any of the other sectors, except for the Rest of the World sector. Regarding the main question of interest, the conclusion is that often when there is a significant economic connection from one to another sector, the Bayesian epidemiological model confirms that economic shock transmission between these sectors is likely. However, there are some connections for which this is not the case. For example, the data on bilateral economic connections shows a strong connection from the Government sector to the Insurance and Pension Funds sector, while this is not reflected in the combined approach. Overall, the results show that economic connections go a long way in explaining shock transmission but are not the sole mechanism at play.

The paper proceeds as follows. The next section is the methodology section, describing the basic statistical epidemiological model, the network-based priors for the epidemiological model and the methodology this paper proposes that combines both approaches. The empirical application follows, with Section III describing the data and Section IV providing the estimation results. The paper then concludes.

II Estimation of the Bayesian epidemiological model

II.a A simple epidemiological model

One way of modelling the transmission of shocks across the sectors is via the two-level epidemiological model with immediate recovery, used, for example, in Janssens et al. (2017).[4] The main idea of this model is that a sector is assumed to be in one of three states: susceptible (S), initial slow growth (D) and protracted slow growth (C). In the slow growth states, a sector is 'infectious' and is possibly able to push other – currently susceptible – sectors into the initial slow growth state. A sector in the initial slow growth state either recovers and enters the susceptible state again, or, if it fails to recover, enters the protracted slow growth state. Similarly, once having entered the protracted slow growth state, a sector either recovers and becomes susceptible again, or fails to recover and stays in the protracted state. The states are assumed observable. In the application below, changes in each sector's net worth will be used to classify the state it is in.

[4]There is nothing special about our choice of this model as a baseline to represent the traditional epidemiological approach. Any Markov-based epidemiological model can be used, but this section follows the notation and assumptions of the specific model mentioned.

In this model, there are four main sets of parameters that are of interest and describe the transmission mechanisms of shocks between sectors. The first set contains the probabilities that the i sectors are able to recover from an initial slow growth period, denoted by p_i, for $i = 1, \ldots, N$ with N the total number of sectors in the economy. The second set of parameters contains the probabilities of each sector recovering from a protracted slow growth period, analogously denoted q_i. Third are the set of contamination probabilities p_{ij}, describing the probability that sector i infects sector j, i.e., the probability that sector i 'contaminates' sector j so that sector j enters the initial slow growth state in the next period. Finally, there is the parameter called 'nature', denoted n, that quantifies the probability of exogenous contamination. Combining these two contamination probabilities, the overall probability of sector i becoming contaminated at a given point in time is given by $\tilde{p}_{it} = 1 - (1 - n)\prod_{j \in \text{Infectious}_{t-1}}(1 - p_{ji})$, where the set Infectious_{t-1} contains all sectors that were infectious (i.e., in an initial or protracted slow growth period) at time $t-1$, and the product of an empty set is defined to be equal to 1.

From the estimated parameters, the expected number of sectors that a sector will contaminate in an otherwise susceptible system can be computed; this quantity is called the basic reproduction number in the epidemiological literature and is denoted by R_0. We can distinguish between the R_0 measured from the moment the sector enters an initial slow growth period, R_0^d, versus when the sector is in a protracted slow growth period, R_0^c:

$$R_0^c = \frac{1}{q_i} \sum_{j \neq i} p_{ij} \tag{1}$$

$$R_0^d = \sum_{j \neq i} p_{ij} + (1 - p_i) \frac{1}{q_i} \sum_{j \neq i} p_{ij} \tag{2}$$

The parameters in this model can be obtained via maximum likelihood estimation using data describing the state each sector i, for $i = 1, \ldots, N$, is in at time $t = 1, \ldots, T$. Maximum likelihood estimates are equivalent to the mode of the Bayesian posterior when using an uninformative prior; thus making it an intuitive starting point for the Bayesian approach we will use. Therefore, the maximum likelihood parameters and their standard errors are computed as the mode and standard errors of the posterior distribution, averting the need for asymptotic approximations such as the delta method. More details on the posterior distributions of the parameters will be given in the next subsection.

II.b Constructing the priors and updating the posteriors

Alternatively, when one also observes actual economic linkages between sectors (as opposed to only observing the states each individual sector is in), a sector's contagiousness can be characterized via the underlying network structure of the assets and liabilities connecting all sectors. Adopting the methodology of Garas et al. (2010), this network structure can be used to determine the network-based contamination probabilities between all sectors over time. These are computed as follows: consider sectors $i = 1, \ldots, N$. Let liab_{ij} denote the liabilities from sector i to sector j. Note that liabilities from sector i to sector j are equal to assets from sector j to sector i. Thus for each sector i over the whole network, we can compute aggregate liabilities and assets, respectively, as:

$$\text{liab}_i = \sum_{j \neq i} \text{liab}_{ij}, \text{ and } \text{asset}_i = \sum_{j \neq i} \text{liab}_{ji}. \tag{3}$$

Next, define $w^{ij}_{\text{tot}} = \text{liab}_{ij} + \text{liab}_{ji}$, with the interpretation of implied bilateral contact rates, and $\tilde{w}^{i}_{\text{tot}} = \text{liab}_i + \text{asset}_i$, representing the economic importance of sector i.

Following Garas et al. (2010), the contamination probabilities take the form:

$$p_{ij} \propto \frac{w^{ij}_{\text{tot}}}{\tilde{w}^{j}_{\text{tot}}} \tag{4}$$

which says that the probability of sector i contaminating sector j is proportional to the ratio of the bilateral contact rate to sector j's economic importance.

We now propose using a Bayesian approach to link the network-based contamination probabilities of Equation (4) with the simple epidemiological model described above. The motivation behind this Bayesian approach is as follows. For two sectors i and j that share large bilateral flows, the numerator of the ratio in Equation (4) will be higher and thus it will be more likely that a slow growth period will spread from sector i to sector j. However, if sector j is a large sector relative to the overall shared assets and liabilities between the two sectors, the denominator of the ratio will be large, thereby reducing the likelihood of spread. Thus the contamination probability of Garas et al. (2010) is a combination of these two effects – the magnitude of the bilateral flows versus the relative size of one of the sectors to the other. This information can be used to inform the estimation of the contamination probabilities in the epidemiological model, which are based on the statistical occurrence of contaminations, i.e., the sequence of slow growth periods across multiple sectors, irrespective of the relative size of the sectors to the overall economy and without regard to the bilateral flows. Specifically, this information will be incorporated by using the network-based contamination probabilities in Equation (4) as priors for the estimation of the contamination probabilities of the simple epidemiological model described above.

As can be seen from Equation (4), the ratio of the bilateral contact rates only pins down the contamination probabilities up to a scale factor. In the interpretation of Garas et al. (2010), this scale is time-varying and represents the intensity of the crisis that has hit the global economy. In the setting of the epidemiological model that we adopt, all initial slow growth periods are assumed to have the same intensity over time and their severity is determined by which and how many sectors are hit by the shock, how contagious these sectors are and how quickly these sectors are able to recover. However, even though the contamination probabilities are assumed to be time-invariant, the methodology still has to deal with the fact that we only have prior information on the contamination probabilities up to a scale. Therefore, we postulate that the network-based contamination probabilities are of the form $p_{ij} = z_i q_{ij}$ with

$$q_{ij} = \frac{w_{\text{tot}}^{ij}}{\tilde{w}_{\text{tot}}^{j}}, \tag{5}$$

so as to allow for some flexibility by letting the scale-factor be sector-specific, but not time-varying and specify z_i as a random parameter that is sampled alongside the other parameters in the model.[5] By allowing the scale factors to vary across sectors, the parameters z_i have an intuitive interpretation: if z_i is low for a sector, it means that despite sector i having a large economic connection to another sector j (i.e., a large q_{ij}), the linkage does not necessarily result in a large contamination probability (p_{ij}). Relating this to the discussion in the Introduction, if z_i is low, it indicates that for this sector, actual economic connections are less relevant for explaining shock transmission.

Using the network-based contamination probabilities of Garas et al. (2010) to construct priors for the epidemiological model ensures that a higher probability density is attributed to larger values of contamination probabilities if the respective sectors also have a meaningful economic connection and a lower probability density results if the receiving sector is very large relative to the magnitude of the bilateral relationship. In effect, what this prior information does is encapsulate the ideas that (a) sectors that have only a weak economic connection are less likely to have a large contamination probability between them, and (b) an infected sector is going to have a harder time infecting a very large sector relative to a small sector. The Bayesian approach also mitigates another drawback of the purely epidemiological approach: that a large number of parameters need to be estimated, increasing quadratically in the number of sectors. Depending on the time span for which the data employed is available, some of these parameters may not be identified, or are estimated on the basis of very few events.[6] By using a Bayesian approach to incorporate the network

[5]Results for the restricted case when $z_i = 1$ are provided in the Supplemental Appendix; this also allows us to infer from the results the necessity of estimating z_i along with the other parameters.

[6]For example, for the contamination probability from sector i to j to be identifiable, an event should have taken place at least once where sector i was in a slow growth period (and therefore infectious) and sector j was in a susceptible state at the same time, i.e., sector i must have had ample opportunity to infect sector j in the observable data.

structure as additional information when estimating the contamination probabilities, identification of these parameters is aided, similar to the role of regularization methods in a frequentist setting.

As a first step, priors need to be specified for all parameters that are going to be estimated, including both the recovery probabilities and the contamination probabilities. The time series of network-based contamination probabilities q_{ij} is constructed for all sectors in the network by computing the expression in Equation (4) at each point in time. Because the network-based contamination probabilities are time-varying, we denote them by $q_{ij,t}$, where the subscript t refers to the time dimension. Next, these network-based contamination probabilities $q_{ij,t}$, $t = 1, ..., T$ are used to construct a prior. To obtain the exact choice of the prior for every contamination probability $p_{ij} = z_i q_{ij}$, a Beta distribution is fitted to the constructed time series $q_{ij,t}$, $t = 1, \ldots, T$ for every $i = 1, \ldots, N$ and $j = 1, \ldots, N$, $i \neq j$, resulting in the implied $(\alpha_{ij},\ \beta_{ij})$ parameters.[7] For the parameters z_i, $i = 1, \ldots, N$, uninformative priors are used. The Bayesian analysis will attribute higher probability mass to large values of p_{ij} when (a) there are more successful contaminations from sector i to j in a statistical sense, and/or when (b) α_{ij} is larger, so when the $q_{ij,t}$-statistic of Garas et al. (2010) indicates that the connection between sector i and j is 'economically' relevant. Furthermore, the more stable the economic relationship between sectors i and j over time (a low variance of the time series $q_{ij,t}$), the larger the coefficients α_{ij} and β_{ij} will be and thus, the more influential the network-based information of Garas et al. (2010) will be to the shape of the posterior of the contamination probabilities $p_{ij} = z_i q_{ij}$'s.

[7] There are two reasons why the Beta$(\alpha,\ \beta)$-distribution is most suitable as prior distribution. First, the support of the Beta distribution is between zero and one, and can take many shapes, making this distribution suitable to model priors on parameters that represent probabilities. Second, in the special case that in the data sector i is always the only infectious sector when sector j becomes infected, $z_i = 1$, and exogenous contamination by nature is ruled out ($n = 0$), it can be shown that the Beta distribution would be the conjugate prior of p_{ij}, motivating this prior as a natural choice.

Priors are also needed for the recovery probabilities p_i and q_i for all sectors. However, a similarly intuitive way to define priors on the probability of recovering from slow growth episodes (either initial or protracted) is not available, because the network structure of the sectors does not provide any information about their intrinsic likelihood of recovering from these episodes without making additional assumptions. Turning to the epidemiological model, the assumed Markov structure implies that the recovery of a sector is not affected by what happens in the other sectors, so in that sense the probability of recovery is not affected by the underlying network structure. Note that this is different from the probability of becoming contaminated in a certain period; this probability depends on all other sectors that were contagious the previous sector. This independence also implies that the estimation of p_i and q_i is independent of the estimates of the p_{ij}'s. Therefore, a non-informative prior (Beta(1,1), which coincides with the uniform distribution) can be used for these recovery probabilities, such that the posterior distributions are given by

$$\pi(p_i|y) \propto p_i^{\sum_t I(X_{it}=S \text{ and } X_{it-1}=D)+1}(1-p_i)^{\sum_t I(X_{it}=C \text{ and } X_{it-1}=D)+1} \tag{6}$$

and

$$\pi(q_i|y) \propto q_i^{\sum_t I(X_{it}=S \text{ and } X_{it-1}=C)+1}(1-q_i)^{\sum_t I(X_{it}=C \text{ and } X_{it-1}=C)+1} \tag{7}$$

where $I(\cdot)$ is the indicator function, 'S' refers to the susceptible state, 'D' to the initial slow growth state, and 'C' to the protracted slow growth state. Since the Bayesian analysis also uses an uninformative prior for p_i and q_i, and these recovery probabilities are independent of the p_{ij} (contamination probability) estimates, maximum likelihood estimation and the Bayesian estimation procedure will give the same posterior distributions for the set of parameters $\{p_i, q_i\}_{i=1}^N$.

Parameter n, the exogenous contamination probability, will also be estimated using an uninformative prior (Beta(1,1)). However, given that n cannot be estimated independently from the sector-specific contamination probabilities, p_{ij}, the estimates of n obtained from the

Bayesian estimation procedure will differ from those obtained from the maximum likelihood estimation procedure.

Appendix A describes how one can sample from the posterior distributions and how to sample the scale-parameters z_i when they are not assumed to be equal to one. A combination of Gibbs sampling and the Metropolis-Hastings algorithm is used. It should be stressed that the priors are constructed from a different data set than the data used to update the posterior distribution, as should be the case in Bayesian inference. Both data sets will be introduced below in the next section.

III Data

We use two datasets from the Office for National Statistics (ONS) in the United Kingdom: the UK Economic Accounts: Flow of Funds data and the ONS Enhanced Financial Accounts (Flow of Funds) data, containing quarterly time series balance sheet data for the total financial accounts from 1993-2019.[8] Both datasets consist of the aggregate asset (total due from all the other sectors) and liability (total due to all the other sectors) positions of nine different sectors: (i) Public Corporations, (ii) Private Non-Financial Corporations, (iii) Monetary Financial Institutions (MFIs), (iv) Financial Institutions Other than MFIs and Insurance Corporations and Pension Funds (henceforth referred to as 'Other Financial Intermediaries'), (v) Insurance Corporations and Pension Funds, (vi) Central Government, (vii) Local Government, (viii) Households and Non-Profit Institutions Serving Households (henceforth referred to as 'Households') and (ix) Rest of the World.[9] The first data set is described by the ONS as the "official" dataset. The latter dataset is described by the ONS as "experimental" – using the first dataset as its starting point but supplemented with

[8]These data are available at `https://www.ons.gov.uk/economy/nationalaccounts/uksectoraccounts`. For an extensive description of these data, see Bowers and Foster (2015), Bowers et al. (2015) and Burrows et al. (2015).

[9]Due to the presence of the "Rest of the World" sector, the Flow of Funds data represent a closed macroeconomic system.

Solvency II, commercial, and survey data – and has the added advantage of containing the bilateral asset and liability amounts between every pair of sectors. Thus this latter dataset provides information on the economic linkages that may enhance or augment the inference gleaned from the aggregate official Flow of Funds dataset. The subset of assets and liabilities that cannot be attributed to sector-pairs by ONS, labeled as 'unknown', is not used in the analysis. Furthermore, because of its experimental nature, the ONS Enhanced Financial Accounts will be used only in the cases that the analysis requires inference on bilateral connections between sectors. In all other cases, we rely on the official Flow of Funds data.

We exploit the tight connection between Local and Central Government and aggregate these two sectors' assets and liabilities into one 'Government' sector. This aggregation reduces the number of sectors and as such the total number of parameters to be estimated in the epidemiological model discussed above. Using the asset/liability information between the eight sectors, it is possible to determine the net worth (assets minus liabilities) of each sector. The development of the net worth over time will be used to determine the states of the sectors for the epidemiological model.

Figure 1 displays the contributions of the different sectors to the total amount of assets (a) and liabilities (b) in the economy, based on the UK Economic Accounts: Flow of Funds data set. The different sectors in the model vary considerably in their contribution to the overall economy as well as in the role they play in lending to and borrowing from other sectors. In 2019Q2 for example, the Public Corporations sector held about 0.34% of total liabilities and only 0.05% of total assets. The largest sector, on the other hand, the Monetary Financial Institutions sector, had around 28.68% of total assets and 28.22% of total liabilities. The Households sector had a large share, 14.46%, of total assets, but only 4.17% of total liabilities. These results also indicate large differences in the leverage ratio between sectors. In this same period, the Public Corporations sector had a ratio of total liabilities to total assets of 7.12, while the ratio for the Household sector was 0.29.

[Insert Figure 1]

III.a Descriptive statistics of the network structure

The flows of assets and liabilities between the eight sectors can be characterized in a network sense as a weighted directed graph, where the sectors are nodes and the edges show these flows. As an example, Figure 2 shows the asset-liability network in the second quarter of 2019 with each sector represented by a node in the network and arrows corresponding to assets received by that sector from other sectors. Asset-liability connections that are smaller than 0.25% of the total assets in the economy are not visualized to improve readability. This threshold corresponds to approximately 100 billion pounds. The values inside the nodes describe the intra-sector transactions (assets and liabilities within the sector). An arrow that enters sector j from sector i shows the assets received from sector j, that is, the arrow represents a liability of sector i to sector j. Thus, assets are denoted in the same color as the node they belong to. Given the chosen (100bn pound) threshold, it is interesting to note that four sectors seem to have very few inward arrows, namely the Public Corporations, Private Non-Financial Corporations, Other Financial Intermediaries and Government sectors. In particular, the Public Corporations sector has no asset connections (inward pointing arrows) larger than 100 billion pounds, and only has a single liability connection (outward pointing arrow) above 100bn pounds, with the Government sector. Recall from Figure 1 that it is also the smallest sector in terms of both total assets and total liabilities. The Private Non-Financial Corporations sector has the lowest net worth (total assets minus total liabilities) in this period, while the Household sector has the highest net worth value.

[Insert Figure 2]

It is also apparent from Figure 2 that assets primarily flow to the Insurance Corporations and Pension Funds, Monetary Institutions, and Household sectors, as well as to the Rest of the World.

III.b Descriptive statistics of slow growth periods

Figure 3 visualizes the states of each sector over time, where white depicts the periods of positive net worth growth (that is, the susceptible state), grey indicates periods of initial slow growth and black indicates that a sector is in a protracted slow growth state. The figure already suggests relevant heterogeneity between sectors: some (e.g., the Public Corporations sector) have incurred long periods of protracted slow growth (recognizable as wide black bars), while others (e.g., the Households sector) experienced at most three consecutive periods of protracted slow growth.[10] The sectors in the latter category will likely have higher recovery probabilities from protracted slow growth periods than the former.

[Insert Figure 3]

Figure 4 depicts the coincidence of sectors experiencing protracted slow growth in each quarter over time. As can be seen from this figure, the largest number of sectors experiencing a prolonged period of slow growth at the same time is five. This happened in five quarters, during three distinct periods, namely 1994Q2-Q4, 2002Q3 (early 2000s recession), and 2008Q3 (Great Recession). Other notable periods are 2018Q3-Q4 (developments around Brexit such as failure to ratify the withdrawal agreement) and 1989Q2 (early 1990s recession) with four sectors in a protracted slow growth state at the same time.

[Insert Figure 4]

IV Results

In this section we present the empirical results, first by applying the epidemiological and Garas et al. (2010) methods separately to the different data sets. Next, the approaches are combined in the way described in Section II and results from the combined approach are presented.

[10]This heterogeneity also likely reflects differences in policy response to slowdowns in different sectors.

IV.a Epidemiological model

The parameter estimates for the epidemiological model are provided in Table 1. The top portion of the table contains the probabilities that slow growth in a row sector will transmit to the column sector in the subsequent period. The probability of exogenous contamination, n, is estimated to be 0.0003 (the posterior mode, although the distribution is left skewed – the posterior mean is 0.003 with a standard deviation of 0.003). This implies that at any point in time, entering a slow growth period due to an exogenous source is extremely unlikely. Based on the statistical model alone, this parameter thus seems unnecessary for modeling spillovers in the UK.[11] In Table 1, the estimates of p_{ij} for which the posterior attributes less than 10% probability mass to the region $0 < p_{ij} < 0.05$ are marked in bold, to provide an indication of which contamination probabilities deviate significantly from zero.[12]

[Insert Table 1]

The estimates for p_i and q_i, the probability that a sector recovers from an initial period of slow growth and protracted slow growth, respectively, sketch a worrisome picture. Once a sector experiences an initial period of slow growth (defined as one consecutive period of negative net worth growth), the Insurance and Pension Funds sector, the Other Financial Intermediaries sector and the Public Corporations sector have estimated probabilities of recovering below 0.5, meaning that for these sectors, the probability they will have another period of negative net worth growth is greater than the probability of recovery. For six out of eight sectors, the estimate of q_i is below p_i, implying that once these sectors enter a protracted

[11] Janssens et al. (2017) find a probability of exogenous contamination for the US of 0.07 (standard deviation of 0.013), which is considerably higher.

[12] The use of t-statistics and/or the assumption of asymptotic normality to evaluate significance of the p_{ij} parameters in the different models presented in Table 1, Table S1 and Table 3 is not appropriate due to the highly right-skewed posterior distributions and the support (between 0 and 1) of the distributions. Therefore, this alternative measure of significance is employed.

slow growth period, they are even less likely to recover than when they are in an initial slow growth period. The sectors that are most likely to recover from slow growth periods are the Households sector, the Private Depository Institutions sector and the Government sector. Only in around 30-34% of the cases will these sectors slide from an initial slow growth period into a protracted slow growth period. Furthermore, the first two of these sectors will spend on average only 1.67 (=1/0.567) periods in a protracted slow growth period. This perhaps reflects the historical tendency for policymakers to be quick to provide economic stimulus to the Household sector to assist with its recovery. Recall also from Section III.b that this sector never experienced more than three consecutive protracted slow growth periods. The Other Financial Intermediaries sector has the highest probability of recovering once it enters a protracted slow growth period. Recall that these recovery probability estimates p_i and q_i will remain the same throughout the different models estimated in this paper, as discussed in the Methodology section.

[Insert Figure 5]

Based on their basic reproduction numbers, R_0, the Public Corporations, Rest of the World, Government and Household sectors are the most infectious. Notice that there is quite a discrepancy between the mode and mean estimates of R_0 in Table 1, caused by the posterior distribution of the R_0 estimates being highly left-skewed. The overall infectiousness of all sectors is visualised in Figure 5, showing all contamination probabilities greater than 0.2.[13] Sectors with many arrows pointing into them are most likely to be affected by slow growth in other sectors while those with many arrows pointing away from them are most likely to transmit slow growth episodes to other sectors. The thickness of the lines represents the magnitude of the estimated mode of the contamination probabilities (a thicker arrow is a

[13] There is nothing special about the choice of 0.2 as a cutoff for visualization in the figure; it was chosen to balance the figure's (and later figures') legibility with the representation of the connections. A lower (higher) threshold would of course depict more (fewer) connections.

higher probability) and the size of the sector node reflects the estimated basic reproduction numbers when a sector initially enters a slow growth period.

This visualization shows us that the Rest of the World and the Insurance and Pension Funds sector are relatively likely to be pushed into a slow growth state by almost all of the other sectors in the economy. This is depicted by seven inward arrows for both sectors, implying that all other sectors have a probability of contaminating these sectors that is above our cut-off value of 0.2. This means that in the observable data, whenever any sector experienced a period of negative net worth growth, more than 20% of the time the Rest of the World sector and/or the Insurance and Pension Funds sector experienced a decline in net worth growth in the following period. This high probability of entering a period of negative net worth growth seems particularly worrisome in combination with the low recovery probabilities of the Insurance and Pension Funds sector, and the large overall infectiousness of the Rest of the World sector as measured by its reproduction numbers (R_0^D and R_0^C). Also highly susceptible to contamination are the Other Financial Intermediaries sector and the Monetary Financial Institutions sector; both have five probabilities of being contaminated that are larger than 0.2. Least susceptible to contamination are the Households and the Public Corporations sectors. This is an interesting finding, corroborating the earlier observation in Section III.a that these sectors have relatively few liability and asset arcs towards the other sectors in the economy and are thus relatively more insulated.

Most notable from the individual contamination probabilities are the estimated 48.6% probability that the Rest of the World sector will contaminate the Insurance and Pension Funds sector (standard deviation of 0.102) and the 54.6% probability that slow growth in the Household sector will be followed by slow growth in the Private Non-Financial Corporations sector (standard deviation of 0.195). In general, however, many contamination probabilities are estimated to be low (i.e., 26 contamination probabilities are equal to or below 0.2) and often with large standard errors. This motivates why introducing additional information through the specification of priors may be a valuable resource for learning more about the transmission probabilities in this economy. For example, as one of the G7 countries, it is plausible that the UK economy leads many of the economies of the rest of the world. In the above Granger causal set-up, the results suggest that the Rest of the World sector is highly susceptible to infections from sectors in the UK, although the actual spillovers (measured by economic linkages) from most of the sectors in the UK to the rest of the world may be limited. The combined Bayesian approach will be able to identify to which sectors the Rest of the World sector is most susceptible, based on both statistical occurrences and economic linkages.

IV.b Priors: network-based contamination probabilities

Table 2 reports the implied mean and standard deviation of the contamination probabilities, estimated by fitting a beta distribution to the network-based contamination probabilities of Garas et al. (2010) as described in Section II.b; these time-varying probabilities are visualized in Figure 6. Note that these estimates serve as inputs (i.e., determine the prior distribution) for the Bayesian approach described in Section II.

[Insert Table 2]

As can be seen from Table 2, the mean (unscaled) contamination probabilities obtained from the methodology of Garas et al. (2010) are in the range of [0, 0.787]. As noted above, the Garas et al. (2010) contamination probabilities are only identified up to a scaling factor; however, the range [0,0.787] is comparable to the contamination probabilities obtained via the maximum likelihood method as presented in Table 1 when taking into account the uncertainty around the parameter estimates, which could motivate the choice of fixing the scaling factor to 1 for all sectors.[14]

[14]The Supplemental Appendix presents the estimation results obtained under the assumption that the scaling factor $z_i = 1$ for all sectors, but in the main paper we focus on the joint estimation of the scaling factor for each sector along with the other parameters.

Figure 6 shows that most network-based contamination probabilities have remained relatively stable over time. Some interesting exceptions include the build-up in exposure of the Other Financial Intermediaries sector towards the Monetary Financial Intermediaries sector during the period 2002-2009. The probability of the Other Financial Intermediaries sector infecting the Monetary Financial Institutions sector gradually increases from 0.4 to 0.7 during that period and a similar but reversed pattern is visible for the probability of the Other Financial Intermediaries sector infecting the Rest of the World sector. The interpretation of this finding is that during the build-up to the financial crisis, the Other Financial Intermediaries sector saw a shift from having a high exposure to the Rest of the World sector to having high exposure to the Monetary Financial Institutions sector, a pattern that reversed after its peak in 2009.

[Insert Figure 6]

The findings from the network-based approach of Garas et al. (2010) seem to share a number of similarities with the epidemiological model estimates for the UK Economic Accounts: Flow of Funds data set. For example, the sectors that are most susceptible to a change in net worth of the Private Non-Financial Corporations sector are the Monetary Financial Institutions, the Rest of the World and the Insurance and Pension Funds sectors, as measured by the contamination probabilities in Table 1. These three sectors are also most connected to the Private Non-Financial Corporations sector according to the network-based contamination probabilities in Table 2. Some other similarities are that the Government sector is second most likely to be infected by the Public Corporations sector in the statistical epidemiological model; it is the most likely in the analysis based on the network data. Similarly, the Insurance and Pension funds sector is highly susceptible to shocks from the Household sector in both methodologies.

However, not all levels of contagiousness and contamination probabilities from the statistical epidemiological approach coincide with levels of interconnectedness and connection 'strength' from the network approach. For example, according to the estimates in Table 1, the Rest of the World sector is highly contagious towards the Insurance and Pension Funds sector, with a contamination probability of 0.486. According to the network-based contamination measure, the Insurance and Pension Funds sector should be much less susceptible to the Rest of the World sector (contamination probability of 0.06); taken together these results imply that important spillovers may occur even in the absence of significant economic linkages. As a second example of differences between the two approaches, in the statistical epidemiological model of Table 1, both the Rest of the World sector and the Insurance and Pension Funds sector were very likely to be contaminated by all the other sectors in the economy. In the network-based approach of Table 2, the Insurance and Pension Funds sector only seems to be highly exposed to the Household sector (estimated probability of 0.486), and modestly to the Private Non-Financial Corporations and Government sectors (probabilities of 0.201 and 0.283, respectively). The Rest of the World does show high exposure towards multiple sectors, although fewer than seen from the maximum likelihood estimates, including the Private Non-Financial Corporations, Monetary Financial Institutions, and Other Financial Intermediaries sectors, with network-based contamination probabilities of 0.537, 0.460 and 0.422 respectively.

IV.c Results Bayesian procedure

This subsection presents results that are obtained by estimating sector-specific scaling factors, z_i, along with the contamination probabilities and recovery probabilities, i.e., $p_{ij} = z_i \frac{w_{\text{tot}}^{ij}}{\tilde{w}_{\text{tot}}^{j}} = z_i q_{ij}$, using Beta-distributed priors for the parameters q_{ij} where the mean and standard deviations of the Beta distribution are given in Table 2. The methodology underpinning these results is described in Appendix A and Section II.[15]

[15]In the Supplemental Appendix, detailed estimates and discussion are provided regarding the special case where the scaling factor is fixed and set to $z_i = 1$ for all sectors. This special case forces the scale of the contamination probabilities of Garas et al. (2010) to correspond one-to-one with the contamination probabilities of the epidemiological model.

[Insert Table 3]

The estimation results are summarized in Table 3. The exogenous contamination probability n has a posterior mode (standard deviation) of 0.11 (0.05), showing that the model attributes a considerably larger role to contamination by exogenous factors than the model that ignores network information. This large probability of exogenous contamination highlights the importance of this parameter in an economic epidemiological model. For all the parameters (except the recovery probabilities due to their independence), the variance of the posterior distribution is larger than when the scaling factor is fixed to be equal to one, as displayed in Table S1 in the Supplemental Appendix. This larger variance comes from the uncertainty that is introduced by estimating the N additional parameters (the scaling factors z_i, for $i = 1, ..., N$).

The estimates of the scaling factor show there is some heterogeneity across sectors; for seven sectors, the estimate of z_i is above one, while only for the Public Corporations sector, is it below one. The differences between the z_i's of the different sectors highlight the fact that the importance of the between-sector economic connection for the estimation of the overall contamination probabilities between sectors is sector-specific. Given that for most sectors the posterior mode of the scaling factor, z_i, is estimated to be different than one, it follows that the simplifying assumption of setting z_i equal to one would introduce a bias in all contamination probabilities, albeit for most sectors this bias is small, depending on the actual deviation of the z_i mode from one. For example, the Public Corporations sector has a posterior mode for its scaling factor equal to 0.622 (standard deviation 0.566), implying that the estimated non-scaled probabilities will overstate the overall infectiousness of the Public Corporations sector.

Visualizations comparing the prior and posterior distributions for all bilateral sector contamination probabilities and recovery probabilities are given in the Figures B1 and B2 respectively, in Appendix B. For a selection of cases (16 of the 56 contamination probabilities), the prior distribution indicates that there is little or no evidence of an economic connection, and this lack of connection typically results in the posterior distribution also attaching most posterior weight to values of contamination probabilities close to zero. Stated differently, the data on observed slowdowns does not contain enough evidence to update the network-based prior to a different direction and as such, these priors have a similar effect as a LASSO regularizer. For example, in the first column, containing the probabilities that the Public Corporations sector becomes contaminated, almost all contamination probabilities (except for contamination by the Government sector) have prior and posterior distributions with probability mass piled up at zero. Note that the maximum likelihood estimates of these contamination probabilities were also already relatively low, see Table 1.

To help better understand the role of the prior distributions on the epidemiological estimates, we highlight an example of two other cases that can occur, shown in Figure 7: (a) the posterior distribution suggests lower contamination probabilities to be more likely than the prior does, and (b) the posterior suggests higher contamination probabilities to be more likely than the prior does. In the first case, in Figure 7(a), depicting the probability of the Government sector contaminating the Monetary Financial Institutions sector, the posterior (black line) implies a lower contamination probability than the prior (red, dashed line). This occurs when the network-based contamination probabilities are higher than the ones implied by the epidemiological model. This means that the economic linkages that connect the two sectors (i.e., as measured by the assets and liabilities between them) provide more support for a high contamination probability than what actually seems to be statistically observed in the data on slow growth periods. The reverse is the case for the contamination probability from the Public Corporations sector to the Households sector in Figure 7(b). These two examples show that through application of our procedure, we identify both cases where the information on economic connections indicates less and more shock transmission, respectively, than supported by the data on net worth growth shock transmission alone.

[Insert Figure 7]

Comparing Table 3 with Table 1 and the network representation of these obtained estimates in Figure 8 with Figure 5, the most important difference is the change in the overall level of contagiousness of the sectors, summarized in the R_0^d statistics (the basic reproduction number following an initial slow growth period). The overall infectiousness of the sectors has largely decreased, as a result of also using information regarding the underlying network structure of the sectors and their actual economic connections. Note that due to the use of uninformative priors for the recovery probabilities and the independence between the recovery probabilities and the other parameters, the estimation of the recovery probabilities is not affected. This invariance of the recovery probabilities implies that the decrease in overall infectiousness is driven by the decrease in contamination probabilities; for example, many of the contamination probabilities are no longer above 0.20, the threshold chosen in both directed graphs. Furthermore, it is noticeable that the Public Corporations sector is no longer the single highly infectious sector; there are six sectors with comparably high basic reproduction numbers.

[Insert Figure 8]

While the differences in overall contagiousness between sectors are not very pronounced after introducing the scaling factor and the prior distributions, the methodology does highlight several strong connections between sectors. Consequently, the model estimates suggest that measures aimed at avoiding the spread of negative growth shocks between sectors in the UK can focus on the most important economic connections between sectors (the largest p_{ij}'s), rather than looking at the biggest spreaders as measured by the basic reproduction number R_0. Three connections will be discussed below. In the Supplemental Appendix we show that these connections also are present in a model without the estimated scaling factor, and are thus robust to introducing this model feature.

First of all, there is a high probability of the Public Corporations sector pushing the Government sector into a slow growth period, with a mode (standard deviation) posterior estimate equal to 0.323 (0.253). This sensitivity of the Government sector to changes in net worth growth of the Public Corporations sector was already visible in the (pure) epidemiological approach, but is more pronounced thanks to the inclusion of information about the economic connections between these two sectors. While these two sectors are intrinsically closely connected, being aware of this connection is still important for mitigating systemic shock propagation, since negative net worth growth in the Government sector has a high probability of spilling over to the Insurance and Pension Funds sector and the Rest of the World sector.

The second important connection is represented by the high contamination probability from the Households sector to the Insurance and Pension Funds sector (with mode (standard deviation) probability of 0.442 (0.253)) and from the Insurance and Pension Funds sector to the Households sector (with mode (standard deviation) probability of 0.532 (0.252)). The pure epidemiological approach only highlighted the strong connection from the Households to the Insurance and Pension Funds sectors, but introducing data on economic connections suggests a strong *bilateral* connection. Given the high overall contagiousness of the Insurance and Pension Funds sector, this seems to be a connection that if reduced in strength, would also reduce the overall connectedness of the system.

Third, we observe a strong bilateral connection between the Monetary Financial Institutions and Rest of the World sectors, as well as between the Other Financial Intermediaries and Rest of the World sectors. These bilateral connections only become evident through the introduction of the network data to the estimation procedure. The Monetary Financial Institutions sector is one of the less contagious sectors of the economy because of its high recovery probabilities. In fact, Figure 8 indicates a role for the Monetary Financial Institutions sector as shock absorber, by virtue of being highly susceptible to shocks of other sectors such as the Households and Other Financial Intermediaries sectors but having high recovery probabilities and generally low probabilities of spilling shocks to other sectors in the economy.

The posterior distributions of R_0^d and R_0^c are visualized in Figure 9. It is notable that the posterior distributions of the basic reproduction numbers R_0^d and R_0^c of the Public Corporations sector do attach considerable probability mass to values above 10, while for the other sectors, this is not the case. Thus, while the modes of the R_0 values of most sectors are comparable, very high values of R_0 are considerably more plausible for the Public Corporations sector than for the other sectors.

[Insert Figure 9]

V Conclusion

This paper has combined statistical and network-based epidemiological models through Bayesian estimation techniques. Using network information obtained from the financial accounts of the eight major macroeconomic sectors in the UK, priors have been formed for contamination probabilities between these sectors. These priors attribute more probability mass to large values of the contamination probability between two sectors if these sectors are economically connected through large assets/liability flows. Next, the probabilities are updated based on observed 'statistical' contaminations that have taken place, i.e., if sector i entered a slow growth period in the period after sector j was already in a slow grow period, we say that sector j contaminated sector i and hence increase the probability going into the next iteration.

This Bayesian epidemiological model bridges the gap between the approaches of Garas et al. (2010) and a standard epidemiological model and leads to important new insights. First of all, in general, the combination of both approaches leads to results that emphasize that sometimes important spillovers between sectors can occur, even in the absence of a significant economic connection and that sometimes important economic connections do not mean that spillovers of net worth growth slowdowns are inevitable. While the actual economic connection can be a good starting point in evaluating spillovers between sectors in an economy, it should not be the single tool used in analysis. For example, the pure network-based approach overstates the infectiousness of the Public Corporations sector, as does the pure statistical approach.

However, combining both sources of information highlights that most of the contagiousness of the Public Corporations sector runs through the Government sector and the overall contagiousness of the Public Corporations sector is limited. The combined approach also highlights that there is not a single highly contagious sector, but rather, there are several strong connections in the system that can lead to spread of negative shocks. This finding differs from that using the pure statistical approach, that finds many low, but non-zero contamination probabilities. As such, introducing network data into the pure statistical approach works as a regularizer. For example, while the epidemiological approach suggested high susceptibility of the Insurance and Pension Funds sector towards and from almost all other sectors in the economy, the combined approach of this paper highlights that in particular, the very strong dependence between the Households sector and the Insurance and Pension Funds Corporations sector is notable; slow growth in the Households sector will in many cases (44% of the time) push the Insurance and Pension Funds sector into a state of slow growth too. Given the low recovery probability of the Insurance and Pension Funds sector, and its infectiousness towards other sectors, the linkage from the Households sector to this sector is important for understanding the transmission of shocks within the complete system.

By closely comparing prior and posterior distributions of the parameters in the epidemiological model, one can assess whether the risk of contamination between sectors arises because of the underlying network structure connecting the respective sectors, or because of another factor that has caused many contaminations to occur in the past. This can be useful for policymakers both in seeking to pre-emptively stave off the development of a potential crisis and in determining to which sectors scarce resources should be directed in the event of an economic slowdown. Such models may also prove useful in the analysis of banking networks, where a comparison of the economic connections between banks (e.g., derived from their trading network structure) with the spillovers that have actually occurred over time might be used to understand drivers of systemic risk. We leave this for further research.

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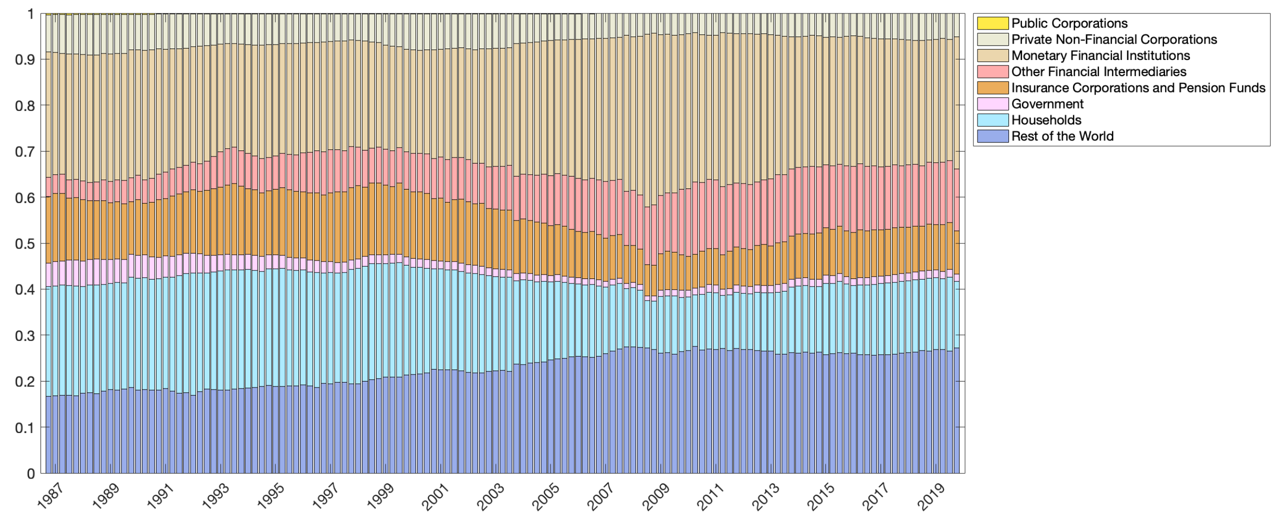
Figure 1: Shares of assets and liabilities of each sector over time

(a) Asset share of each sector over time (assets per sector divided by the total amount of assets in the economy)



(b) Liability share of each sector over time (liabilities per sector divided by the total amount of liabilities in the economy)

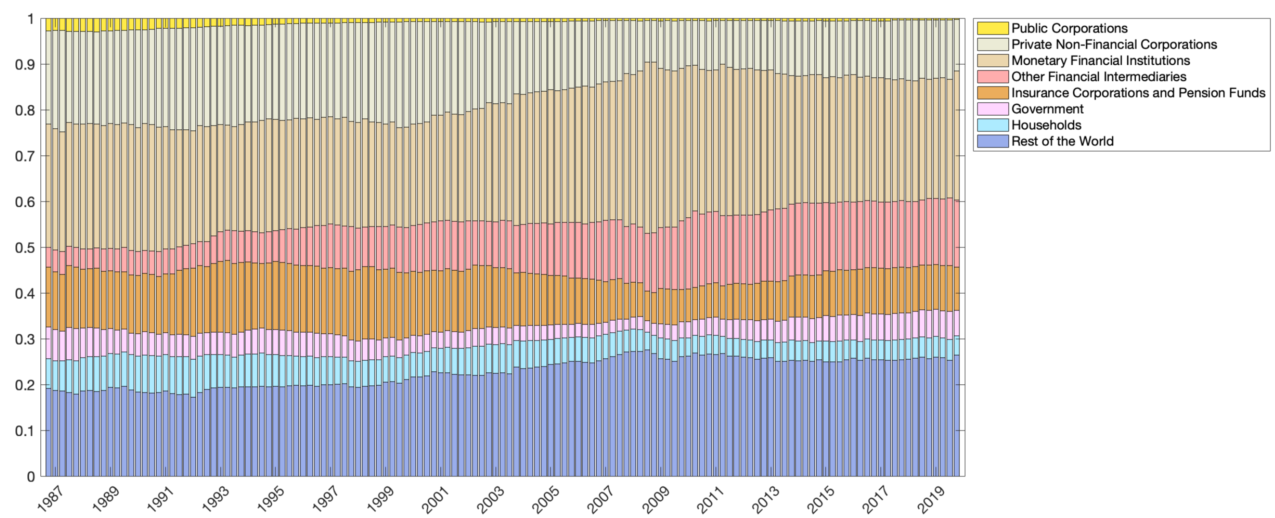


Figure 2: Visualization of the asset-liability network of the eight sectors of the United Kingdom in 2019Q2. Only connections larger than 100 billion pounds are shown. Values within the node describe the intra-sector transactions (asset and liability transfers within the sector). An arrow directed from sector i to sector j represents a liability of sector i to sector j, and thus an asset of sector j. Thus, liability connections are given the same color as the node these assets belong to.

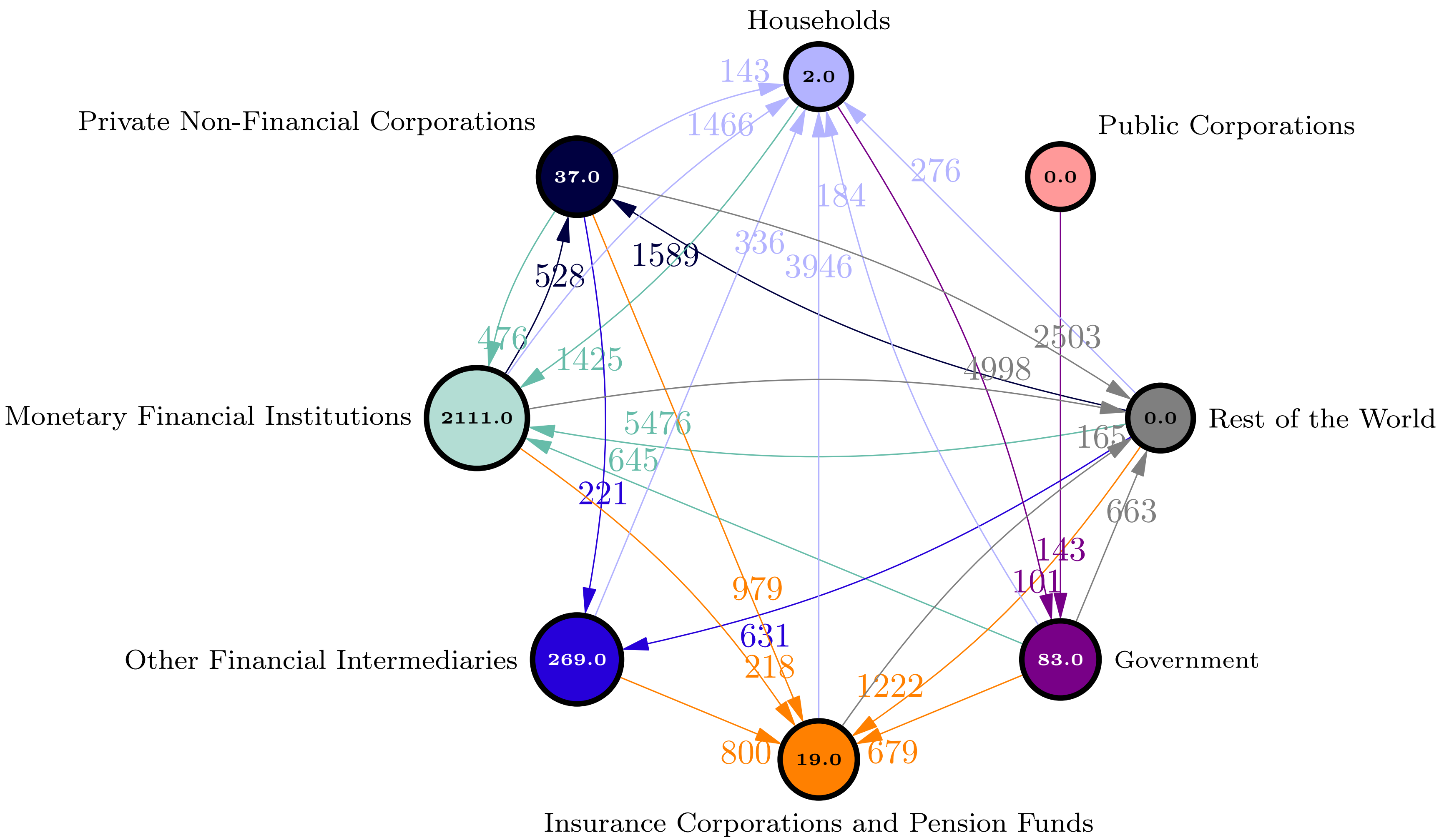


Figure 3: Development of (initial) slow growth episodes (grey) and protracted slow growth periods (black) over time for each sector.

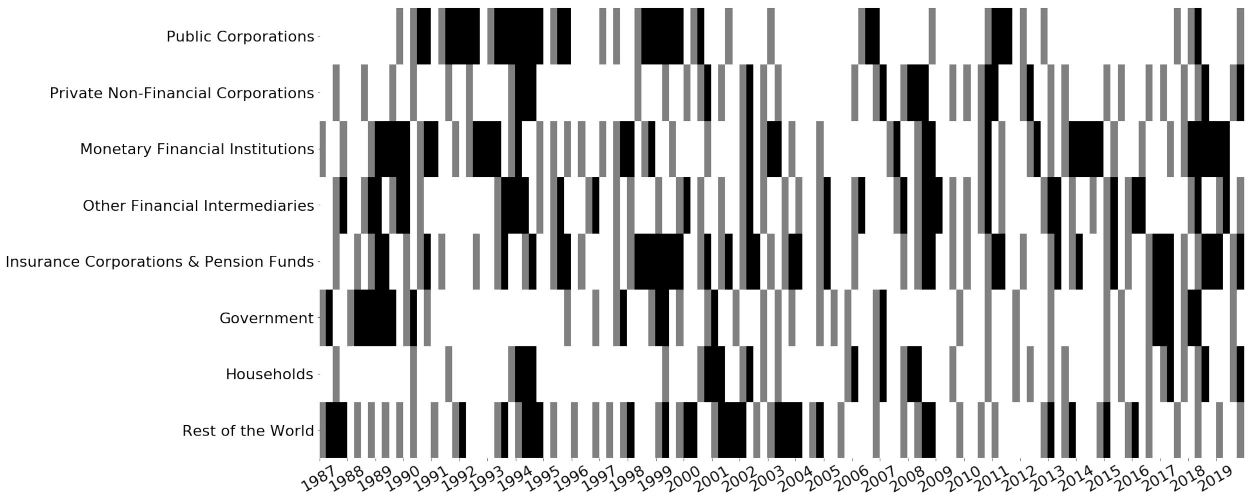


Figure 4: The number of protracted slow growth episodes over time, relative to a total of 8 sectors

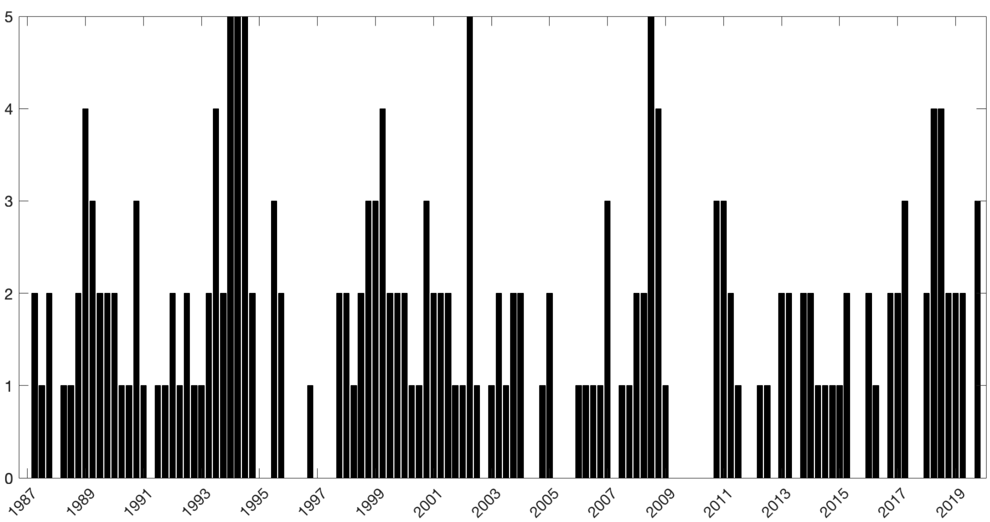


Figure 5: Representation of transmission probabilities between sectors (depicted by a connection between the sector nodes when the contamination probability p_{ij} in Table 1 is larger than 0.20), and basic reproduction numbers R_0^d upon initial entry into a slow growth period (depicted by the size of the sector node – a larger node has a higher reproduction number, with the value given in the node), based on the mode estimates of Table 1.

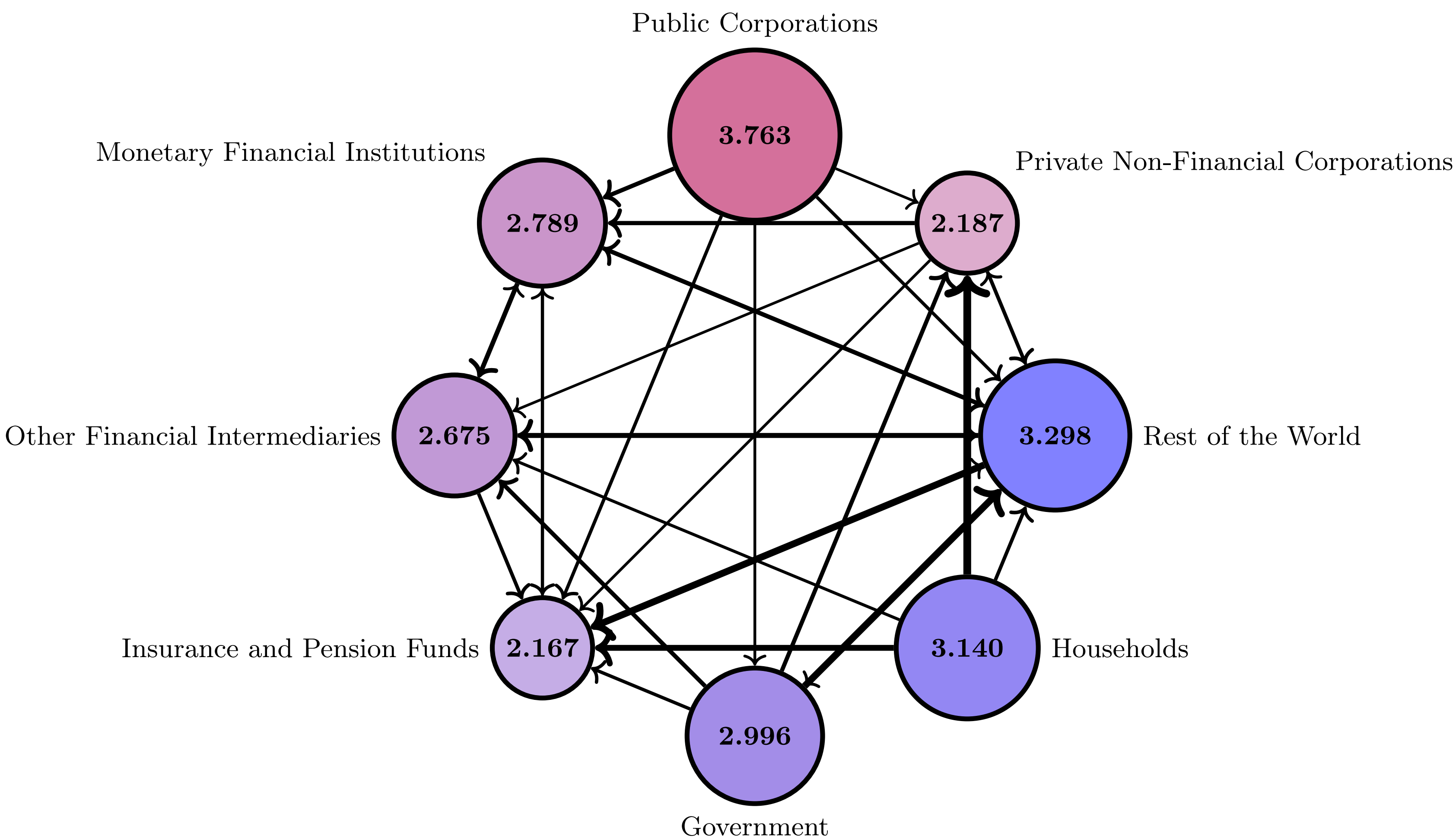


Figure 6: Proxies for the contamination probabilities, $p_{ij,t} \propto \frac{w^{ij}_{tot,t}}{\tilde{w}^{j}_{tot,t}}$ *over time. In each graph, the colored lines show the probabilities* $p_{ij,t}$ *that sector* i *(the sector name indicated by the graph title) will infect sector* j *(the sector name can be found by comparing the color in the legend to the color of the line in the graph) over time as determined by the network-based contamination probability measure of Garas et al. (2010).*

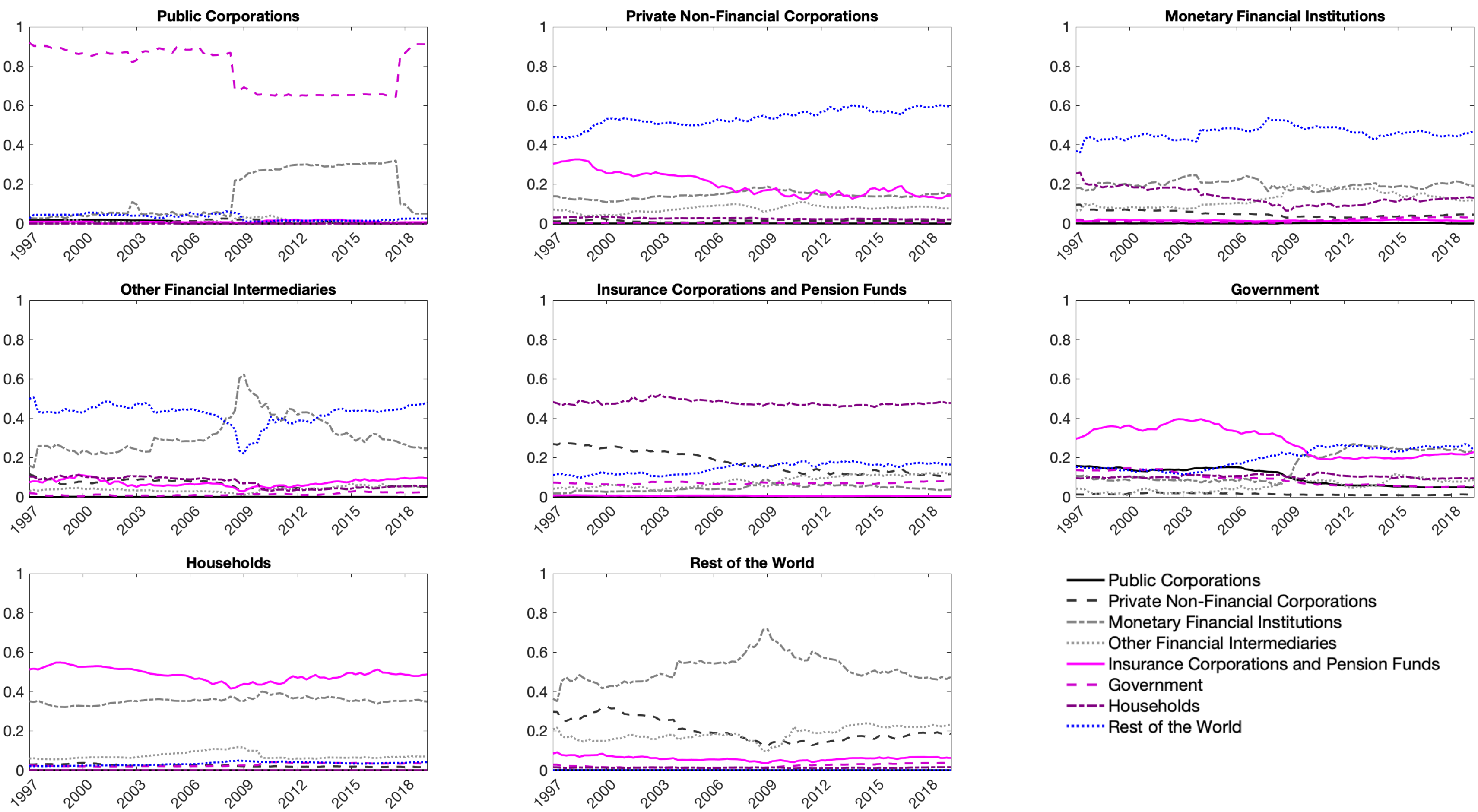
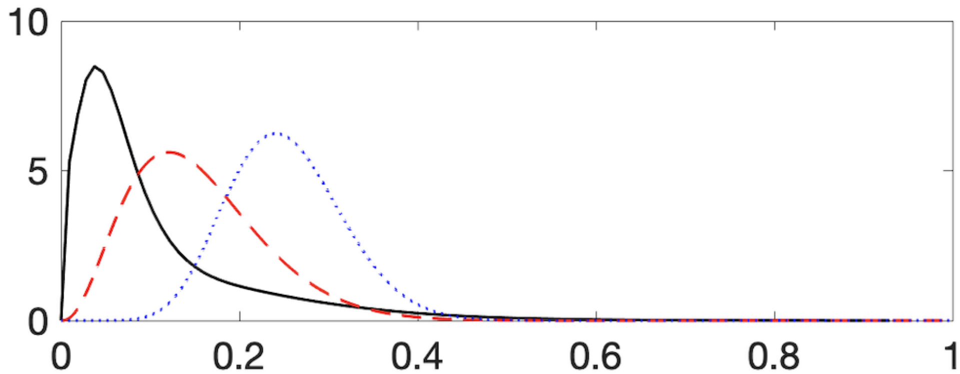


Figure 7: Prior (red, dashed) and posterior (black) distributions of some selected contamination probabilities based on the model with estimated sector-specific scaling factor. The blue dotted line represents the proposal distribution from which the posterior proposal draws are generated in the Metropolis-Hastings algorithm, showing good overlap with the posterior.

(a) Government to Monetary Financial Institutions



(b) Public Corporations to Households

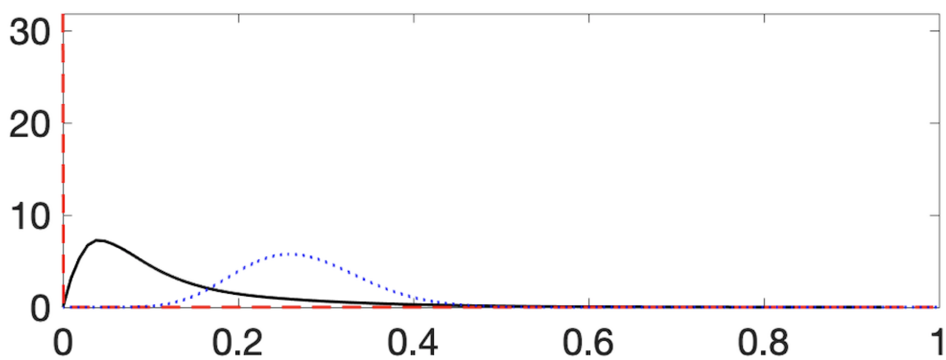


Figure 8: Representation of transmission probabilities p_{ij} between sectors (depicted by a connection between the sector nodes when the contamination probability p_{ij} in Table 3 is larger than 0.20) and basic reproduction numbers R_0^d upon initial entry into a slow growth period (depicted by the size of the sector node, with the value given in the node), based on the posterior mode estimates of Table 3.

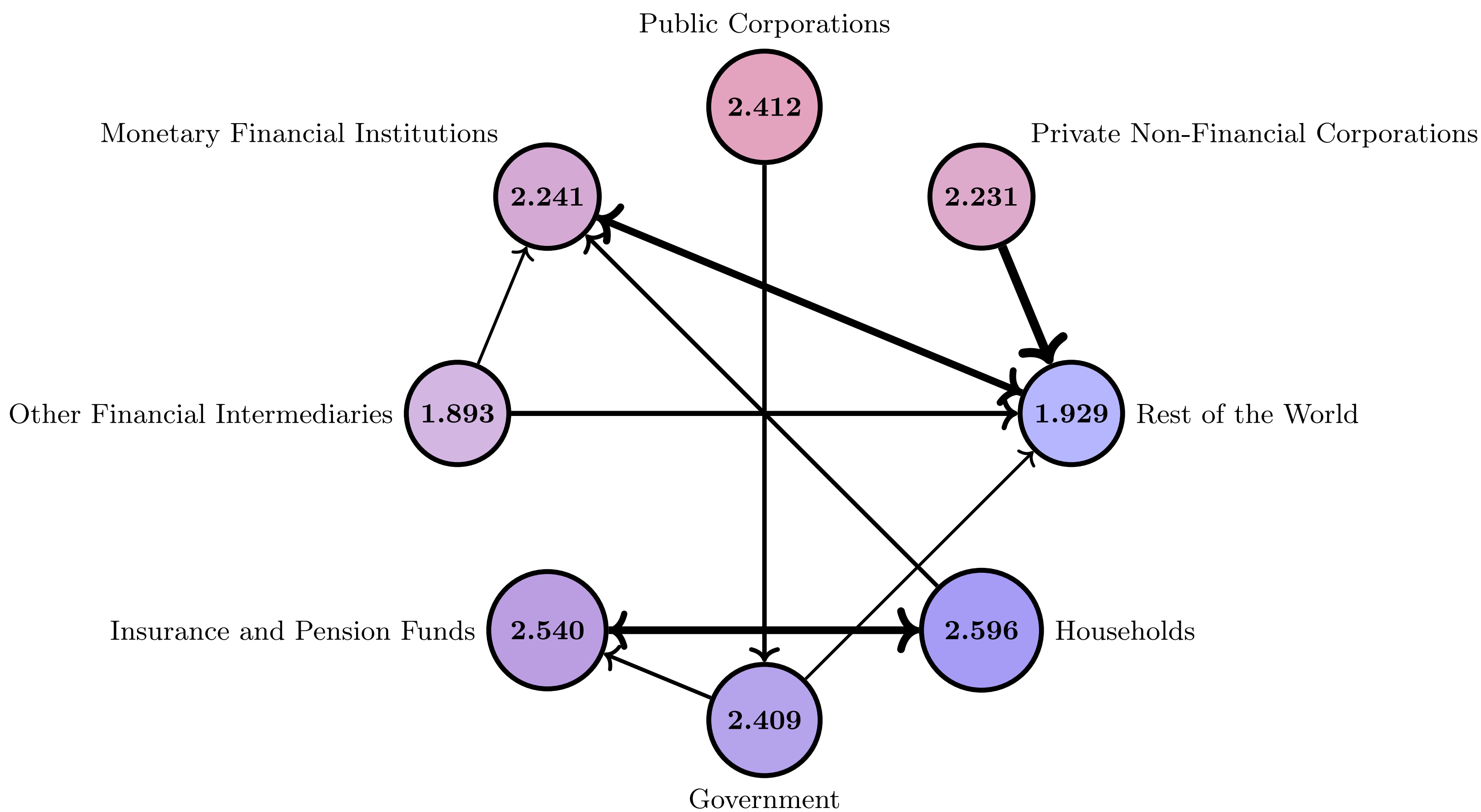
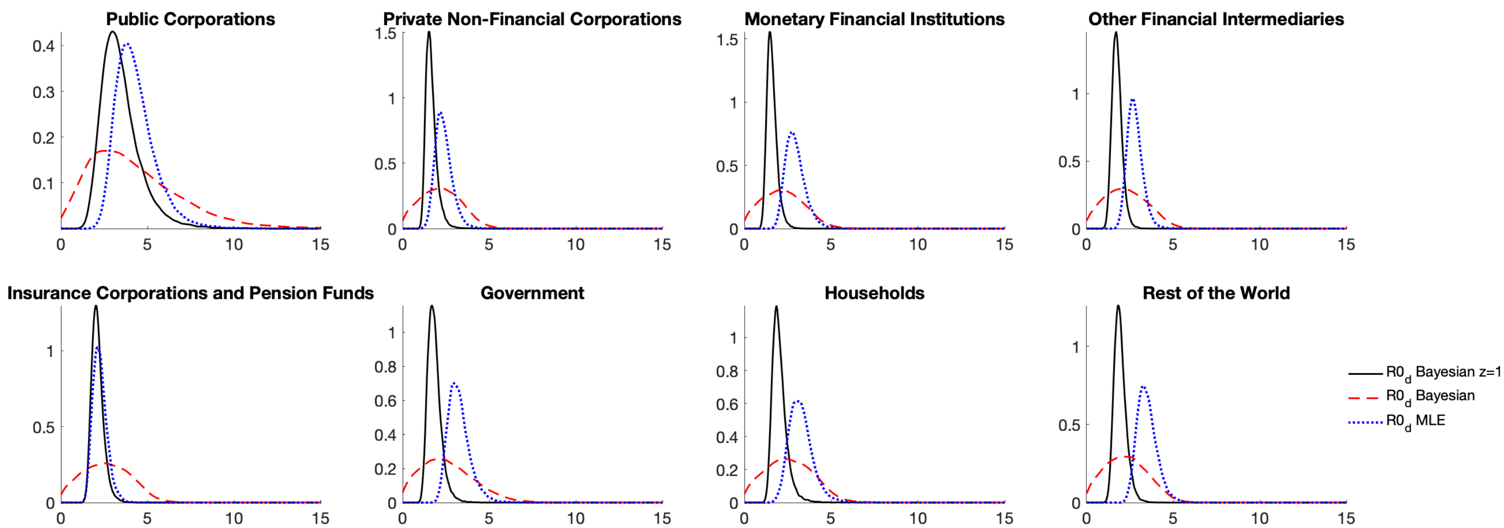


Figure 9: Posterior distributions of R_0^d (panel a) and R_0^c (panel b), the basic reproduction numbers, using an informative prior for the contamination probabilities and $z = 1$ for all sectors (black solid line), using an informative prior for the contamination probabilities and sector-specific scaling factors z_i (red dashed line) and an uninformative prior (the MLE density, blue dotted line). This basic reproduction number measures the expected number of contaminations from a sector from the moment it has entered an initial slow growth period (panel a), or from the moment it has entered a protracted slow growth period (panel b), in an otherwise uncontaminated system.

(a) R_0^d



(b) R_0^c

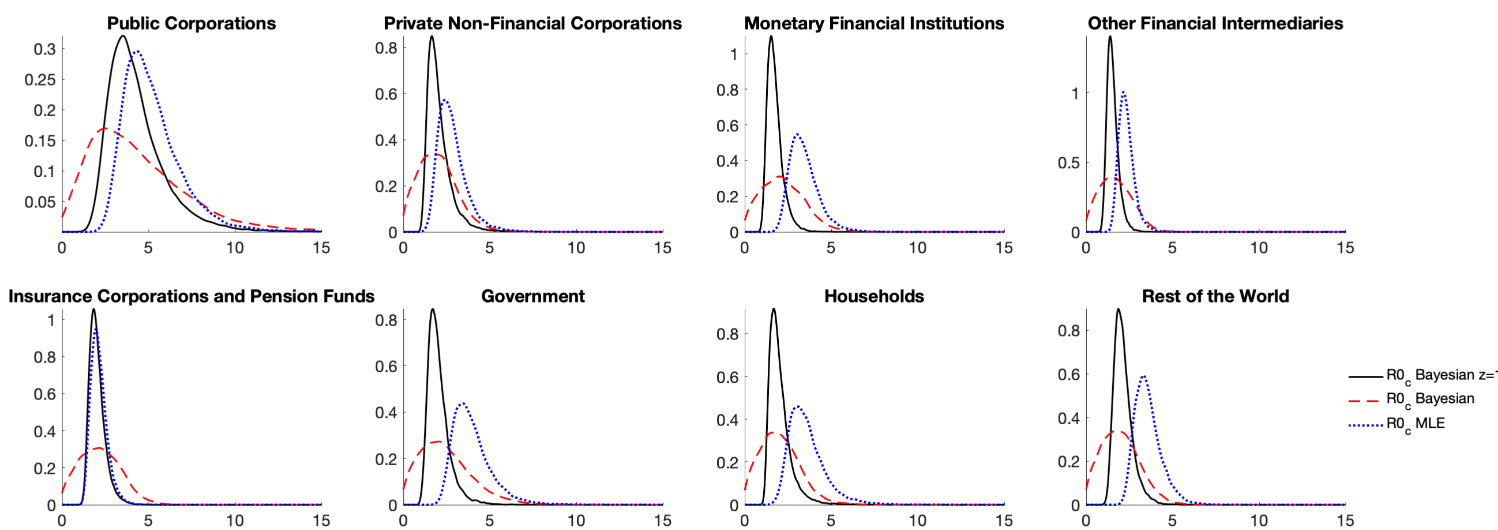


Table 1: Mode and standard error of posterior distributions based on uninformative priors (i.e., in this case, the mode coincides with the MLE estimate) of the contamination probabilities p_{ij}, recovery probabilities p_i and q_i and the basic reproduction numbers R_0 as in Equations (1)-(2). p_{ij} is the probability that slow growth in a row sector will transmit to the column sector in the subsequent period. R_0 estimates are based on simulations. Standard errors are in parentheses, based on the standard deviation of the posterior distribution. Estimates of p_{ij} for which the probability that $p_{ij} < 0.05$ is smaller than 0.1 are marked in bold.

p_{ij}	Public Corp.	Private Non-Financial Corp.	Monetary Financial Inst.	Other Financial Institutions	Insurance & Pension Funds	Government	Households	Rest of the World
Public Corp.		**0.205**	**0.318**	**0.125**	**0.255**	**0.216**	**0.161**	**0.251**
		(0.075)	(0.095)	(0.061)	(0.094)	(0.077)	(0.091)	(0.088)
Private Non-Financial Corp.	**0.179**		**0.312**	**0.206**	**0.214**	0.077	**0.187**	**0.274**
	(0.067)		(0.093)	(0.086)	(0.129)	(0.056)	(0.116)	(0.076)
Monetary Financial Inst.	**0.171**	0.090		**0.343**	**0.248**	**0.136**	0.073	**0.323**
	(0.059)	(0.042)		(0.085)	(0.085)	(0.052)	(0.047)	(0.065)
Other Financial Intermediaries	**0.134**	**0.192**	**0.220**		**0.270**	**0.196**	**0.152**	**0.320**
	(0.050)	(0.069)	(0.083)		(0.093)	(0.062)	(0.061)	(0.093)
Insurance & Pension Funds	0.077	**0.150**	**0.219**	**0.178**		**0.115**	**0.119**	**0.214**
	(0.040)	(0.050)	(0.065)	(0.071)		(0.045)	(0.050)	(0.056)
Government	**0.135**	**0.311**	**0.169**	**0.315**	**0.254**		**0.162**	**0.488**
	(0.060)	(0.092)	(0.076)	(0.075)	(0.114)		(0.078)	(0.109)
Households	**0.142**	**0.546**	**0.185**	**0.230**	**0.424**	0.110		**0.268**
	(0.072)	(0.195)	(0.092)	(0.098)	(0.128)	(0.071)		(0.095)
Rest of the World	0.114	**0.230**	**0.300**	**0.365**	**0.486**	**0.228**	**0.182**	
	(0.056)	(0.069)	(0.102)	(0.114)	(0.102)	(0.069)	(0.060)	
p_i	0.474	0.699	0.546	0.444	0.444	0.657	0.631	0.563
	(0.107)	(0.079)	(0.085)	(0.082)	(0.086)	(0.091)	(0.091)	(0.078)
q_i	0.322	0.567	0.417	0.656	0.513	0.500	0.567	0.553
	(0.083)	(0.121)	(0.081)	(0.087)	(0.081)	(0.110)	(0.120)	(0.091)
mean	4.295	2.368	2.981	2.801	2.277	3.259	3.267	3.501
R_0^d mode	3.763	2.187	2.789	2.675	2.167	2.996	3.140	3.298
st.dev.	(1.153)	(0.493)	(0.592)	(0.445)	(0.416)	(0.637)	(0.691)	(0.570)
mean	5.198	2.830	3.486	2.338	2.157	3.984	3.613	3.600
R_0^c mode	4.325	2.374	3.046	2.199	1.977	3.461	3.114	3.295
st.dev.	(1.682)	(0.861)	(0.868)	(0.449)	(0.478)	(1.143)	(1.091)	(0.774)

Table 2: Mean and standard deviation (between parentheses) of the prior distributions that are based on the network-based contamination probabilities of Garas et al. (2010). p_{ij} represents the probability that slow growth in a row sector will transmit to the column sector in the subsequent period.

p_{ij}	Public Corp.	Private Non-Financial Corp.	Monetary Financial Inst.	Other Financial Institutions	Insurance & Pension Funds	Government	Households	Rest of the World
Public Corp.		0.061	0.147	0.010	0.007	0.787	0.001	0.031
		(0.046)	(0.116)	(0.051)	(0.045)	(0.106)	(0.001)	(0.016)
Private Non-Financial Corp.	0.000		0.144	0.075	0.201	0.004	0.026	0.537
	(0.003)		(0.017)	(0.018)	(0.058)	(0.001)	(0.004)	(0.043)
Monetary Financial Inst.	0.000	0.049		0.124	0.016	0.016	0.136	0.460
	(0.001)	(0.014)		(0.039)	(0.002)	(0.010)	(0.040)	(0.031)
Other Financial Intermediaries	0.000	0.065	0.316		0.072	0.013	0.073	0.422
	(0.002)	(0.020)	(0.090)		(0.019)	(0.007)	(0.028)	(0.055)
Insurance & Pension Funds	0.000	0.177	0.044	0.079		0.071	0.478	0.145
	(0.002)	(0.055)	(0.017)	(0.030)		(0.005)	(0.013)	(0.027)
Government	0.103	0.014	0.155	0.055	0.283		0.103	0.197
	(0.044)	(0.004)	(0.075)	(0.031)	(0.073)		(0.008)	(0.054)
Households	0.000	0.024	0.355	0.073	0.486	0.028		0.034
	(0.000)	(0.005)	(0.017)	(0.016)	(0.032)	(0.008)		(0.008)
Rest of the World	0.000	0.203	0.516	0.185	0.060	0.022	0.014	
	(0.000)	(0.055)	(0.070)	(0.034)	(0.011)	(0.009)	(0.001)	

Table 3: Posterior estimates (mode and standard deviation) derived from the procedure as described in Appendix A of the contamination probabilities p_{ij}, recovery probabilities p_i and q_i and the basic reproduction numbers R_0 as in Equations (1)-(2). p_{ij} is the probability that slow growth in a row sector will transmit to the column sector in the subsequent period. The R_0 estimates are based on the Metropolis-Hastings draws from the posterior distributions of the contamination probabilities p_{ij}, scaling factors z_i and the recovery probabilities p_i and q_i and Equations 1 and 2. Estimates of p_{ij} for which the probability that $p_{ij} < 0.05$ is smaller than 0.1 are marked in bold.

p_{ij}	Public Corp.	Private Non-Financial Corp.	Monetary Financial Inst.	Other Financial Institutions	Insurance & Pension Funds	Government	Households	Rest of the World
Public Corp.		**0.087**	**0.090**	0.015	**0.104**	**0.323**	0.047	0.027
		(0.147)	(0.125)	(0.093)	(0.188)	(0.253)	(0.101)	(0.035)
Private Non-Financial Corp.	0.032		0.142	0.062	**0.163**	0.004	0.027	**0.621**
	(0.044)		(0.077)	(0.045)	(0.124)	(0.002)	(0.014)	(0.257)
Monetary Financial Inst.	0.003	0.038		0.165	0.016	0.017	0.117	**0.471**
	(0.002)	(0.032)		(0.094)	(0.010)	(0.016)	(0.075)	(0.250)
Other Financial Intermediaries	0.027	0.065	**0.229**		0.054	0.020	0.064	**0.364**
	(0.021)	(0.049)	(0.185)		(0.052)	(0.018)	(0.054)	(0.237)
Insurance & Pension Funds	0.010	**0.153**	0.052	0.081		0.086	**0.532**	0.131
	(0.005)	(0.105)	(0.033)	(0.061)		(0.040)	(0.252)	(0.080)
Government	0.083	0.014	0.054	0.096	**0.287**		0.086	**0.229**
	(0.092)	(0.012)	(0.098)	(0.082)	(0.220)		(0.071)	(0.156)
Households	0.006	0.024	**0.309**	0.065	**0.442**	0.021		0.033
	(0.011)	(0.015)	(0.188)	(0.045)	(0.253)	(0.018)		(0.022)
Rest of the World	0.001	**0.149**	**0.506**	**0.165**	0.065	0.027	0.017	
	(0.000)	(0.129)	(0.246)	(0.128)	(0.042)	(0.019)	(0.009)	
z_i	0.622	1.066	1.102	1.062	1.078	1.145	1.167	1.110
	(0.566)	(0.515)	(0.566)	(0.609)	(0.535)	(0.685)	(0.537)	(0.590)
p_i	0.474	0.699	0.546	0.444	0.444	0.657	0.631	0.563
	(0.107)	(0.079)	(0.085)	(0.082)	(0.086)	(0.091)	(0.091)	(0.078)
q_i	0.322	0.567	0.417	0.656	0.513	0.500	0.567	0.553
	(0.083)	(0.121)	(0.081)	(0.087)	(0.081)	(0.110)	(0.120)	(0.091)
mean	4.403	2.212	2.228	2.288	2.636	2.618	2.599	2.296
R_0^d mode	2.412	2.231	2.241	1.893	2.540	2.409	2.596	1.929
st.dev	(2.780)	(1.168)	(1.195)	(1.215)	(1.362)	(1.541)	(1.367)	(1.210)
mean	4.543	1.994	2.215	1.712	2.210	2.470	2.039	1.992
R_0^c mode	2.470	1.708	1.981	1.672	2.266	2.070	1.920	1.600
st.dev.	(3.019)	(1.150)	(1.244)	(0.933)	(1.190)	(1.556)	(1.162)	(1.092)

Appendix A. Metropolis-Hastings Sampling Algorithm

This section describes how posterior distributions of the parameters in the epidemiological model can be obtained when postulating that the network-based probabilities are of the form $p_{ij} = z_i q_{ij}$ with

$$q_{ij} = \frac{w_{tot}^{ij}}{\tilde{w}_{tot}^{j}}.$$

As in the main text of Section II, we impose prior distributions based on the observed network by first computing the ratio of bilateral contact rates for all combinations of sectors for each observed time period. By fitting a Beta-distribution on these series, for each q_{ij}, we can postulate a prior of the form Beta$(\alpha_{ij},\ \beta_{ij})$. We do not have any prior information on z_i, so we use an uninformative prior for these parameters. When sampling from the posterior distribution, however, we have to reject all draws that result in a p_{ij} larger than one or below zero, so as to obtain a posterior distribution truncated to only attribute positive probability mass to values between [0, 1].

First of all, we will make use of the principle of Gibbs sampling, which implies that we can sample a draw from the posterior of q_{ij} conditional on the previous draw of $\{z_i\}_{\forall i \neq j}$, n and all other contamination probabilities q_{kj}, $k \neq i, j$, and vice versa. Note here that conditional on the other parameters $\{z_i\}_{\forall i \neq j}$ and n, the kernel of q_{ij} only depends on the other contamination probabilities relating to sector j, that is, $q_{kj}, k \neq i, j$, but not all other contamination probabilities. Second, because the kernel of the posterior distributions of z_i, n and q_{ij} is not of known form, a Metropolis-Hastings algorithm with candidate-generating density functions is used to sample from the conditional distributions.

For z_i and n, a truncated normal distribution centered around the previous draw is used as the candidate-generating density. The variance of this normal distribution is a tuning parameter that can be chosen so as to obtain good acceptance ratios. For q_{ij}, the candidate-generating density is the Beta$(\alpha_{ij}+I_{ij},\ \beta_{ij}+N_{ij})$ density. This is the posterior distribution when z_i is set equal to one, exogenous contamination n is ruled out, and sector i is always the only sector that can contaminate sector j, in other words, this candidate-generating density ignores that there are other sectors (and exogenous factors) that could have played a role in infecting sector j. Here I_{ij} is the number of successful infections from sector i to sector j and N_{ij} the number of failed infections in the observed data. We do have to truncate this distribution so as to reject all draws for which $p_{ij} = z_i q_{ij} > 1$. All parameters have an individual acceptance probability, denoted by $\gamma_{(\cdot)}$, where the subscript indicates the respective parameter. To decide whether to accept a proposal draw from the candidate-generating function, a draw is generated from the uniform distribution: $u \sim U(0,1)$. If $u < \gamma_{(\cdot)}$, the proposed draw is used. Otherwise, the previous draw will be used. All probabilities $\gamma_{(\cdot)}$ will be defined below. This results in the following algorithm for obtaining draws from the posterior of n, z_i and q_{ij}, where the convention is adopted that the product over an empty set is equal to one:

1. Start with initial parameter draws for n, $\{z_i\}_{i=1}^N$ and $\{q_{ij}\}_{\forall i,j,i \neq j}$: $n^{(m)}$, $z_i^{(m)}$ and $q_{ij}^{(m)}$ for $i, j = 1, ..., N$, $j \neq i$. Set $m = 0$. N is the total number of sectors and m indexes the current draw.

2. Generate a proposal draw q_{ij}^* from the candidate-generating density $g(q_{ij})$ =Beta$(\alpha_{ij} + I_{ij},\ \beta_{ij} + N_{ij})$. If $p_{ij}^* = z_i^{(m)} q_{ij}^* > 1$, generate a new proposal draw. Accept this proposal draw with probability $\gamma_{q_{ij}}$ (defined below in Equation (A.1)). If the draw is accepted, $q_{ij}^{(m+1)} = q_{ij}^*$, else, $q_{ij}^{(m+1)} = q_{ij}^{(m)}$. Do this for all $i,\ j = 1, \ldots, N,\ i \neq j$.

3. Generate a proposal draw z_i^*. The candidate-generating density from which to generate this draw has to be truncated at zero from below and at $1/\max_{j=1,\ldots,N \neq i}\{q_{ij}^{(m+1)}\}$ from above. We use a truncated normal distribution centered around the previous draw, $z_i^{(m)}$, $g(z_i^* | z_i^{(m)}, \{q_{ij}^{(m+1)}\}_{j=1,\ldots,N \neq i})$, defined below in Equation (A.2). Accept this proposal draw with probability γ_{z_i}, defined below in Equation (A.3). If the draw is accepted, $z_i^{(m+1)} = z_i^*$, else, $z_i^{(m+1)} = z_i^{(m)}$. Do this for all $i = 1, \ldots, N$.

4. Generate a proposal draw n^*. The candidate-generating density from which to generate this draw has to be truncated at zero from below and at one from above. We use a truncated normal distribution centered around the previous draw, $n^{(m)}$, $g(n^*|n^{(m)})$, defined in Equation (A.4). Accept this draw with probability γ_n, defined below in Equation (A.5), If the draw is accepted, $n^{(m+1)} = n*$, else, $n^{(m+1)} = n^{(m)}$.

5. Set $m = m + 1$ and return to step 2.

In the algorithm above, the following functions are used:

$$\gamma_{q_{ij}} = \min\left(\frac{f(q_{ij}^*|\{z_i^{(m)}\}_{\forall k,k\neq i}, n^{(m)}, \{q_{kj}\}_{\forall k,k\neq i})g(q_{ij}^{(m)})}{f(q_{ij}^{(m)}|\{z_i^{(m)}\}_{\forall k,k\neq i}, n^{(m)}, \{q_{kj}\}_{\forall k,k\neq i})g(q_{ij}^*)}, 1\right), \tag{A.1}$$

where $f(q_{ij}|\{z_i^{(m)}\}_{\forall k,k\neq i}, n^{(m)}, \{q_{kj}\}_{\forall k,k\neq i})$ is the kernel of the posterior distribution of q conditional on n, given by

$$f(q_{ij}|\{z_i^{(m)}\}_{\forall k,k\neq i}, n^{(m)}, \{q_{kj}\}_{\forall k,k\neq i}) \propto \prod_{(t):\ i\in\text{Infectious}_{t-1},\ j\in\text{Susceptible}_{t-1}} \tilde{p}_{jt}^{I(j:\ S\to D)}(1-\tilde{p}_{jt})^{I(j:\ S\to S)} q_{ij}^{\alpha_{ij}}(1-q_{ij})^{\beta_{ij}},$$

where $\tilde{p}_{jt} = 1-(1-n)\prod_{k\in\text{Infectious}_{t-1}}(1-z_i q_{kj})$ and the notation $(t):\ i \in \text{Infectious}_{t-1},\ j \in \text{Susceptible}_{t-1}$ contains the set of all time instances where sector j was susceptible to contamination and sector i was contagious. $I(j:\ S \to D)$ and $I(j:\ S \to S)$ are indicator functions indicating whether the sector j was contaminated (switched to state D) or remained susceptible.

$$g(z_i^*|z_i^{(m)}, \{q_{ij}^{(m+1)}\}_{j=1,\ldots,N\neq i}) = \frac{\phi\left(\frac{z_i^*-z_i^{(m)}}{\sigma_z}\right)}{\Phi\left(\frac{1/\max\{q_{ij}^{(m+1)}\}-z_i^{(m)}}{\sigma_z}\right)-\Phi\left(\frac{-z_i^{(m)}}{\sigma_z}\right)}, \tag{A.2}$$

where σ_z is a tuning parameter that will be adjusted to obtain a good acceptance ratio of the Metropolis-Hastings sampler.

$$\gamma_{z_i} = \min\left(\frac{f(z_i^*|\{q_{ij}^{(m+1)}\}_{i,j=1,...,N,i\neq j}, n^{(m)}, \{z_k^{(m)}\}_{\forall k,k\neq i})g(z_i^{(m)}|z_i^*, \{q_{ij}^{(m+1)}\}_{j=1,...,N,i\neq j})}{f(z_i^{(m)}|\{q_{ij}^{(m+1)}\}_{i,j=1,...,N,i\neq j}, n^{(m)}, \{z_k^{(m)}\}_{\forall k,k\neq i})g(z_i^*|z_i^{(m)}, \{q_{ij}^{(m+1)}\}_{j=1,...,N,i\neq j})}, 1\right) \tag{A.3}$$

and

$$f(z_i|\{q_{ij}\}_{i,j=1,...,N,i\neq j}, n, z_{k\forall k,k\neq i}) \propto \prod_{(t,j),\ j\neq i:\ i\in \text{Infectious}_{t-1},\ j\in \text{Susceptible}_{t-1}} \tilde{p}_{jt}^{I(j:\ S\to D)}(1-\tilde{p}_{jt})^{I(j:\ S\to S)}$$

is the kernel of the posterior distribution of z_i conditional on $\{q_{ij}\}_{i,j=1,...,N,i\neq j}$ and n. Here $\tilde{p}_{jt} = 1-(1-n)\prod_{i\in\text{Infectious}_{t-1}}(1-z_iq_{ij})$ and the notation $(t,j),\ j\neq i:\ i\in\text{Infectious}_{t-1},\ j\in\text{Susceptible}_{t-1}$ contains the set of all time instances and sectors $j\neq i$ where sector i had the opportunity to infect a sector.

$$g(n^*|n^{(m)}) = \frac{\phi\left(\frac{n^*-n^{(m)}}{\sigma_n}\right)}{\Phi\left(\frac{1-n^{(m)}}{\sigma_n}\right) - \Phi\left(\frac{-n^{(m)}}{\sigma_n}\right)} \tag{A.4}$$

σ_n is a tuning parameter that will be adjusted to obtain a good acceptance ratio of the Metropolis-Hastings sampler.

$$\gamma_n = \min\left(\frac{f(n^*|\{q_{ij}^{(m+1)}\}_{i,j=1,...,N,i\neq j}, \{z_i^{(m+1)}\}_{i=1,...,N})g(n^{(m)}|n_i^*)}{f(n^{(m)}|\{q_{ij}^{(m+1)}\}_{i,j=1,...,N,i\neq j}, \{z_i^{(m+1)}\}_{i=1,...,N})g(n^*|n^{(m)})}, 1\right). \tag{A.5}$$

$f(n^*|\{q_{ij}^{(m+1)}\}_{i,j=1,...,N,i\neq j}, \{z_i^{(m+1)}\}_{i=1,...,N})$ denotes the kernel of the posterior distribution of n conditional on the other parameter estimates, given by:

$$f(n|\{q_{ij}\}_{i,j=1,...,N,i\neq j}, \{z_i\}_{i=1,...,N}) \propto \prod_{(t,j)} \tilde{p}_{jt}^{I(j:\ S\to D)}(1-\tilde{p}_{jt})^{I(j:\ S\to S)},$$

with all notation defined above.

The resulting acceptance ratios of the employed Metropolis-Hastings sampler vary from 15% to 80% for the contamination probabilities before scaling q_{ij}.[16]

Because the contamination probabilities and recovery probabilities enter the likelihood independently, the posterior distribution of the recovery probabilities is not affected by the change in the definition of the contamination probabilities. Sampling of the recovery probabilities therefore can be done independently from the contamination probabilities. Again, we postulate a non-informative prior (Beta(1,1), which coincides with the uniform distribution), such that the posterior distributions are given by

$$\pi(p_i|y) \propto p_i^{\sum_t I(X_{it}=S \text{ and } X_{it-1}=D)+1}(1-p_i)^{\sum_t I(X_{it}=C \text{ and } X_{it-1}=D)+1} \tag{A.6}$$

and

$$\pi(q_i|y) \propto q_i^{\sum_t I(X_{it}=S \text{ and } X_{it-1}=C)+1}(1-q_i)^{\sum_t I(X_{it}=C \text{ and } X_{it-1}=C)+1}. \tag{A.7}$$

When adjusting this sampling algorithm for the maximum likelihood case (that is, when using uninformative priors), one simply replaces the sector-specific prior parameters α_{ij} and β_{ij} by 1 for all $i, j = 1, \ldots, N$. Furthermore, both in the case where there is no scale factor z_i and in the maximum likelihood case, z_i is just fixed to 1 for all sectors i and the step where z_i is sampled is omitted.

[16]Roberts et al. (1997) have shown that for a Gaussian target distribution, optimal acceptance ratios are between 23-50%. Although our target distribution is not Gaussian, the range of 23-50% is an often-used rule of thumb for all types of Metropolis-Hastings samplers.

Appendix B. Prior-posterior plots of Bayesian procedure with scaling factor

Figure B1: Prior (red dashed) and posterior (black) distribution functions for the contamination probabilities for the Bayesian epidemiological model with scaling factor this paper estimates. The prior distributions are based on the network-based contamination probabilities of Garas et al. (2010). The proposal distribution (blue dotted) represents the distribution used in the Metropolis-Hastings algorithm to sample proposal draws from. p_{ij} *represents the probability that slow growth in a row sector will transmit to the column sector in the subsequent period. x-axis runs from 0 to 1 in all figures.*

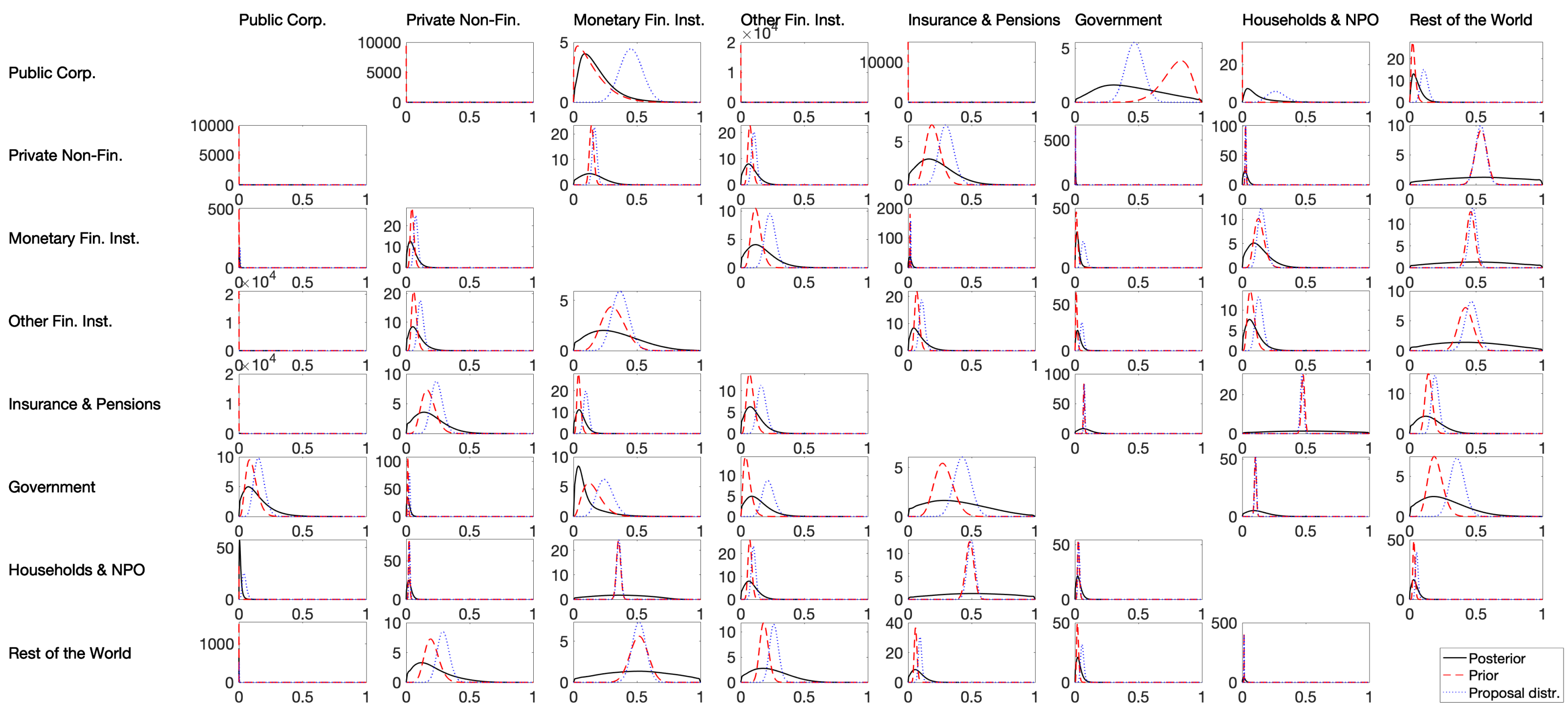
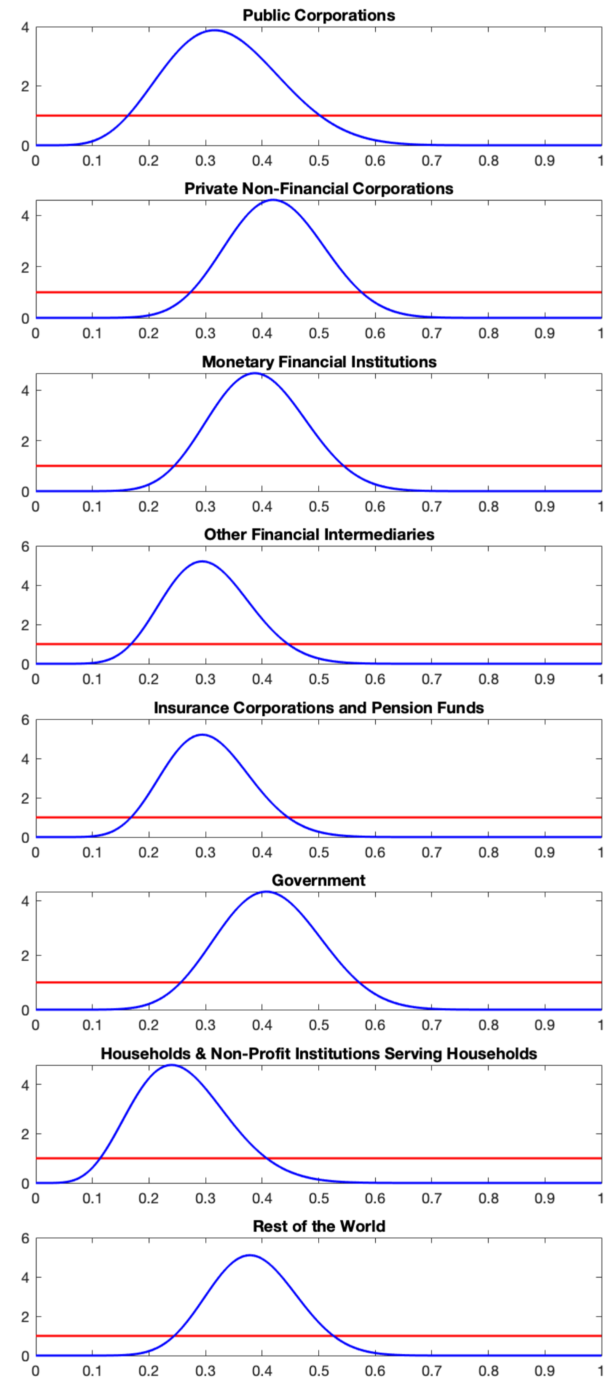
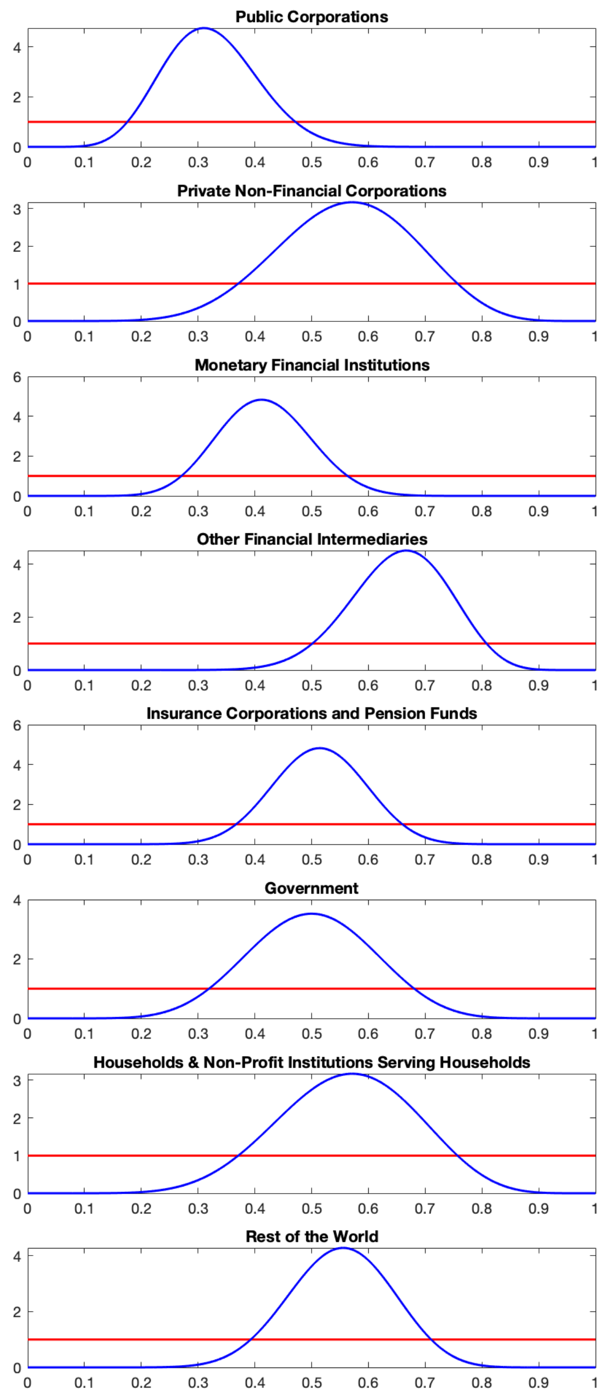


Figure B2: Prior (red) and posterior (blue) distributions for recovery probabilities in the epidemiological model.

(a) Probability of recovering from an initial slow growth period p_i



(b) Probability of recovering from a protracted slow growth period q_i



Sectoral slowdowns in the UK: Evidence from transmission probabilities and economic linkages

SUPPLEMENTAL APPENDIX (FOR ONLINE PUBLICATION ONLY)

Eva F. Janssens and Robin L. Lumsdaine

Supplemental Appendix

Estimations with priors, without scaling factor

This section discusses the results obtained from the Bayesian procedure without an estimated scaling factor, that is, assuming that $z_i = 1$ for all sectors, as is consistent with the procedure described in Section II. This assumption forces the scale of the contamination probabilities of Garas et al. (2010) to correspond one-to-one with the contamination probabilities of the epidemiological model. Table S1 reports the mode and standard deviation of the posterior parameter distributions that result from this procedure when using Beta-distributed priors for the parameters p_{ij}, with the mean and standard deviations of the Beta distribution given in Table 2 of the main paper. The probability of exogenous contamination, n, has a posterior mode of 0.081, and a posterior standard deviation of 0.017. Note that the posterior distribution attributes substantially more probability mass to higher values of n than was estimated in the maximum likelihood setting, but is quite comparable to the estimate of 0.11 (0.05) obtained when estimating the scale factors z_i instead of fixing them to equal one.

Visualizations comparing the prior and posterior distributions for all bilateral sector contamination probabilities and recovery probabilities are given in Figure S1 below and Figure B2 of the Appendix to the main paper. Note that the posterior distributions of the recovery probabilities are unchanged, as they are estimated independently from the other parameters and are not affected by scaling factor z_i. Comparing the prior and posterior distributions of the contamination probabilities in Figure S1, we see that typically, the posterior is more dispersed than the prior distribution. This is similar to the results in the main paper where the scaling factors were estimated along with the other parameters.

Next, analogous to the representation for the maximum likelihood estimates (Figure 5), the Bayesian estimates are visualized in a directed graph, see Figure S2. Comparing this to Figure 5, there are several differences. The overall infectiousness of the sectors largely has decreased, as a result of also using information regarding the underlying network structure of the sectors and their actual economic connections. Note that the use of uninformative priors for the recovery probabilities and the independence between the recovery probabilities and the other parameters means that the estimation of the recovery probabilities is not affected. Given that the recovery probabilities thus remain unaffected, the decrease in overall infectiousness must be driven by the decrease in contamination probabilities; for example, many of the contamination probabilities in Figure S2 are no longer above 0.20, the threshold chosen in both directed graphs.

The most contagious sector based on the combined approach is the Public Corporations sector, with large contagiousness towards the Government sector, and low recovery probabilities ($p_i = 0.474,\ q_i = 0.322$, see the first column of Table S1), similar to the results in the maximum likelihood setting. This is notable considering that it is the smallest sector in the economy and that its size is directly used in the computation of the prior distributions. The second-most contagious sector is the Insurance and Pension Funds sector, driven by its large contamination probabilities towards the Households sector and the Private Non-Financial Corporations sector, also in combination with low recovery probabilities ($p_i = 0.444,\ q_i = 0.513$). The high infectiousness of the Public Corporations sector in this model is clearly driven by fixing the scaling factor z_i to one, because, as seen in the main paper, when the scaling factor of the Public Corporations was estimated it turned out to be below one, leading to a lower overall infectiousness of this sector. Furthermore, for the other sectors, the scaling parameter was estimated to be above one. Consequently, the results in the main paper demonstrate that when the scaling factor is included as an additional parameter and allowed to be estimated, the Public Corporations sector is not particularly more infectious than several other sectors in the economy.

Because the network representation figures (Figures 5 and S2) have focused only on the mode of the posterior distributions, Figure 9 in the main paper shows posterior densities of the basic reproduction numbers R_0^d and R_0^c, for all sectors, simulated from the estimated transmission probabilities p_{ij}. The densities are simulated first from the p_{ij} estimates of the epidemiological model (akin to using an uninformed prior) and second using an informed prior that encodes the network information. As expected, with the informed prior there is less parameter uncertainty, resulting in a tighter posterior distribution. Furthermore, because the network information suggests fewer economically relevant connections than the statistical approach does, in general the estimated posterior distributions in Figure 9 have a smaller mode and mean when using the network-informed priors versus the maximum likelihood estimates. Interestingly, for the Public Corporations sector the posterior distributions in both cases (MLE and Bayes) are very similar, despite the fact that the underlying p_{ij}'s, that is, the probabilities corresponding to the sectors towards which the Public Corporations sector is contagious, have changed substantially. In Figure 5, the Public Corporations sector had many (relatively thin) outward arrows; in the informed-prior setting of Figure S2, it is only contagious towards the Government sector, albeit with a large contamination probability of 0.414.

Table S1: Posterior estimates (mode and standard deviations) of the contamination probabilities p_{ij}, recovery probabilities p_i and q_i and the basic reproduction numbers R_0, derived from the procedure as described in Section III.d. p_{ij} is the probability that slow growth in a row sector will transmit to the column sector in the subsequent period. The R_0 estimates are based on simulations from the posterior distributions of the contamination probabilities p_{ij} and the recovery probabilities p_i and q_i and Equations 1 and 2. The prior distribution of the p_{ij}-parameters is a Beta distribution with means and standard errors of the distribution given in Table 2. Estimates of p_{ij} for which the probability that $p_{ij} < 0.05$ is smaller than 0.1 are marked in bold.

p_{ij}	Public Corp.	Private Non-Financial Corp.	Monetary Financial Inst.	Other Financial Institutions	Insurance & Pension Funds	Government	Households	Rest of the World
Public Corp.		**0.048**	**0.077**	0.021	**0.037**	**0.414**	0.019	0.030
		(0.066)	(0.076)	(0.071)	(0.108)	(0.078)	(0.046)	(0.014)
Private Non-Financial Corp.	0.013		**0.148**	**0.074**	**0.134**	0.004	0.026	**0.518**
	(0.032)		(0.017)	(0.018)	(0.054)	(0.001)	(0.004)	(0.041)
Monetary Financial Inst.	0.001	0.038		**0.061**	0.016	0.008	**0.098**	**0.453**
	(0.001)	(0.014		(0.043)	(0.002)	(0.011)	(0.030)	(0.030)
Other Financial Intermediaries	0.019	0.028	**0.114**		**0.064**	0.009	0.052	**0.275**
	(0.015)	(0.020)	(0.070)		(0.019)	(0.008)	(0.024)	(0.053)
Insurance & Pension Funds	0.007	**0.111**	0.030	**0.043**		**0.073**	**0.470**	**0.081**
	(0.015)	(0.045)	(0.014)	(0.031)		(0.005)	(0.013)	(0.027)
Government	**0.098**	0.011	**0.051**	**0.051**	**0.180**		**0.103**	**0.160**
	(0.040)	(0.004)	(0.061)	(0.036)	(0.066)		(0.008)	(0.051)
Households	0.001	0.023	**0.353**	**0.076**	**0.493**	0.027		0.035
	(0.008)	(0.005)	(0.017)	(0.016)	(0.032)	(0.008)		(0.009)
Rest of the World	0.000	**0.146**	**0.444**	**0.116**	**0.060**	0.013	0.014	
	(0.000)	(0.047)	(0.060)	(0.038)	(0.011)	(0.011)	(0.001)	
p_i	0.474	0.699	0.546	0.444	0.444	0.657	0.631	0.563
	(0.107)	(0.079)	(0.085)	(0.082)	(0.086)	(0.091)	(0.091)	(0.078)
q_i	0.322	0.567	0.417	0.656	0.513	0.500	0.567	0.553
	(0.083)	(0.121)	(0.081)	(0.087)	(0.081)	(0.110)	(0.120)	(0.091)
mean	3.449	1.657	1.589	1.758	2.117	1.838	2.026	1.935
R_0^d mode	2.951	1.524	1.489	1.695	2.016	1.716	1.860	1.834
st.dev	(1.146)	(0.334)	(0.293)	(0.290)	(0.344)	(0.390)	(0.432)	(0.350)
mean	4.287	2.068	1.740	1.540	2.034	2.043	2.024	2.139
R_0^c mode	3.593	1.634	1.532	1.411	1.841	1.713	1.665	1.889
st.dev.	(1.666)	(0.742)	(0.453)	(0.336)	(0.465)	(0.639)	(0.648)	(0.533)

Figure S1: Prior (red dashed) and posterior (black) distribution functions for the contamination probabilities for the Bayesian epidemiological model without scaling this paper estimates. The prior distributions are based on the network-based contamination probabilities of Garas et al. (2010) and posteriors are updated as described in Appendix A. The proposal distribution (blue dotted) represents the distribution used in the Metropolis-Hastings algorithm to sample proposal draws from. p_{ij} represents the probability that slow growth in a row sector will transmit to the column sector in the subsequent period. x-axis runs from 0 to 1 in all figures.

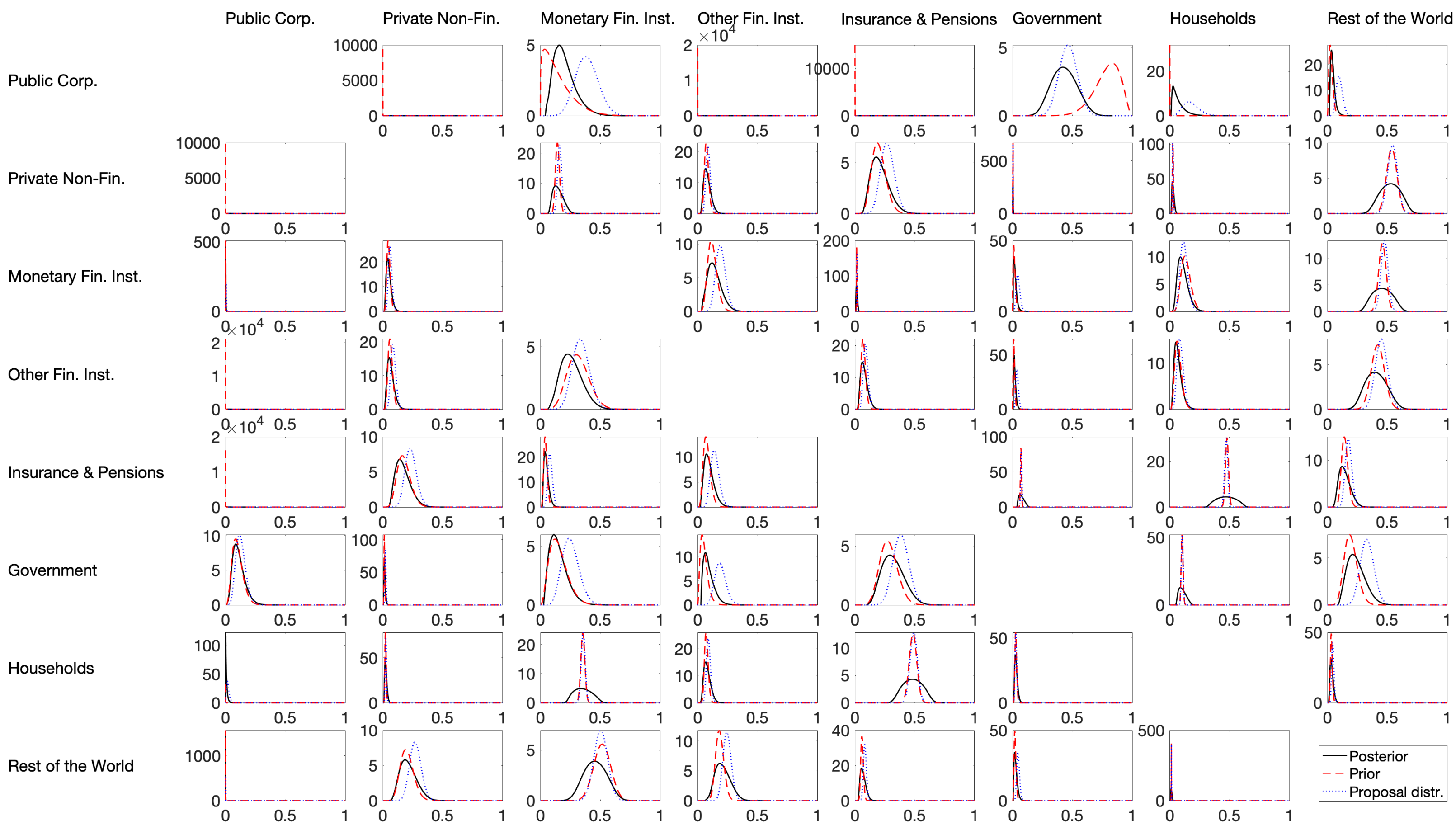


Figure S2: Representation of transmission probabilities p_{ij} between sectors (depicted by a connection between the sector nodes when the contamination probability p_{ij} in Table S1 is larger than 0.20) and basic reproduction numbers R_0^d upon initial entry into a slow growth period (depicted by the size of the sector node, with the value given in the node), based on the posterior mode estimates of Table S1.