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Bankruptcy Codes and Risk Sharing of Currency Unions

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Abstract

Since the Eurozone Crisis of 2010-12, a critical debate on the viability of a currency union has focused on the role of a fiscal union in adjusting for country heterogeneity. However, a fully-fledged fiscal union may not be politically feasible. This paper develops a two-country general equilibrium model to examine the benefits of the bankruptcy code of a capital markets union - in the absence of a fiscal union - as an alternative mechanism to improve the financial stability and welfare of a currency union. When domestic credit risks are present, I show that a lenient bankruptcy code in the cross-border capital markets union removes the pecuniary externality of banking insolvency, so it leads to a Pareto improvement within the currency union. Moreover, the absence of floating nominal exchange rates removes a mechanism to neutralise domestic credit risks; I show that softening the bankruptcy code can recoup the lost benefits of floating nominal exchange rates. The model provides the financial stability and welfare implications of bankruptcy within a capital markets union in the Eurozone.

Keywords: Equilibrium default, bankruptcy code, fiscal union, capital markets union, financial stability, bank credit and inside money, price-level and exchange rate determinacy, liquidity-intermediary asset pricing

JEL Codes: E42, F33, G15, G21

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1 Introduction

The establishment of a currency union in Europe has long begged the question of what constitutes a fiscal union capable of making cross-country transfers within the currency union (see Friedman 1997; Goodhart 1997, 1998).[1] Without such fiscal integration in light of country heterogeneity, Friedman (1997) raised concerns that the adoption of the euro could amplify country heterogeneity and create divergence and, in turn, lead to political disunity. The Eurozone Crisis may appear to validate these concerns. Following the crisis, the core and the peripheral Eurozone have exhibited diverging financial stability risks and economic fundamentals (see Figure A1). Thus, over 20 years since the euro's creation, a critical debate following the Eurozone Crisis has centred on the ability of a fiscal union to improve the viability of sharing a single currency. Meaningful work on fiscal unions has been timely produced (see Farhi and Werning 2017; Kehoe and Pastorino 2017). However, pragmatically, a fully-fledged fiscal union might not be politically feasible (see a detailed discussion in the Nobel Lecture by Sargent 2012). The question then arises: when such a fiscal union is absent for the time being, what else can be done to improve the welfare and financial stability of currency unions?

The goal of this paper, therefore, is to design the bankruptcy code of a capital markets union as an alternative mechanism to improve the financial stability and social welfare of a currency union, in the absence of a fiscal union. This is particularly pertinent now. Owing to the COVID-19 pandemic crisis and its heterogeneous effects on member countries felt across the Eurozone, the exorbitant debt build-up calls for a formal analysis on the welfare effects and financial stability impact of debt restructuring and bankruptcy specific to currency unions. Furthermore, the focal point on the bankruptcy code also contributes to the ongoing debate on the capital markets union as a close substitute for a fiscal union to improve the viability of the Eurozone (see Martinez, Philippon, and Sihvonen 2019). In 2016, the European Commission proposed a legal directive of a lenient cross-border insolvency law or bankruptcy code in Europe as a vital foundation of the capital markets union (hereinafter CMU). However, there have been few economic studies that analyse the welfare implication of such bankruptcy code adjustment and its relevance to the financial stability of the Eurozone.[2]

For this purpose, I develop a three-period two-country general equilibrium model with money, uncertainty, and equilibrium default. The innovation is to relate the cross-border insolvency reform, or bankruptcy code adjustment, to the capital markets union's functioning in improving the financial stability of a currency union. This model has the unique features of a single currency, banking, and bankruptcy codes. In the presence of domestic credit risks, I show that when the CMU bankruptcy code is sufficiently lenient to allow for some degree of state-dependent default in the cross-border capital markets, it removes the pecuniary externality due to banking insolvency. Therefore, a Pareto improvement is obtained despite the social cost of default. As such, CMU bankruptcy code leniency can be a close substitute for a fiscal union. The reason is that the endogenous default on cross-border financial securities that ensues from softening the union-wide bankruptcy code generates a liquidity transfer from the country in the good state to the country in the bad state. This liquidity transfer via equilibrium default adjusts for country heterogeneity and, in turn, shields the domestic banking sector from insolvency.

Moreover, to understand why bankruptcy leniency is specifically vital to a currency union, I

[1] One argument is that a single union-wide monetary authority may prove inadequate because countries in the Eurozone exhibit economic and financial heterogeneity; thus, a fiscal union is needed to adjust for this heterogeneity and improve financial stability.

[2] Section 6.3.2 provides a brief history and the institutional details of recent cross-border insolvency reforms in Europe.

conduct the counterfactual of *credible*[3] national currencies and the role of nominal exchange rates. I show that under relatively general conditions, competitive floating exchange rates can indeed neutralise credit risks across states and alleviate domestic banking stress. In a currency union where such a mechanism is absent, softening the union-wide bankruptcy code may recoup the lost benefits of flexible nominal exchange rates.

To formalise this theory, I model the issuance of the single currency via banks in a two-country and two-good endowment economy. Within the currency union, in each country, there are a continuum of households and a domestic commercial banking sector. Each country's households are risk-averse and are endowed with one type of consumption good in the first two periods. The endowment in the second period is state-contingent. Households borrow commercial loans from their domestic commercial banking sector to get money for all transactions. Households trade goods as they consume both home goods and foreign goods. They also trade *nominal* financial securities via the capital markets union for risk sharing, akin to Geanakoplos and Tsomocos (2002) and Peiris and Tsomocos (2015). More importantly, in this model, a union-wide central bank exists to issue the single currency as the only stipulated means of exchange via interbank credit to the two national banking sectors.

The backbone of the model is the role of the bank's balance sheets in creating and circulating the single currency against credit. This feature builds on the theory of inside money and outside money à la Shapley and Shubik (1977) and Dubey and Geanakoplos (1992, 2003b, 2006). *Inside money* is defined as money endogenously issued against an offsetting bank credit, and *outside money* refers to the initial monetary endowment that is free and clear of any debt obligation.[4] In my model, inside money is issued in the common currency the moment households apply for loans from their respective domestic commercial banking sectors. The domestic commercial banking sectors ultimately obtain the common currency from the union-wide central bank via interbank credit. Such liquidity creation by banks helps to establish the price-level determinacy and inflation determinacy, and therefore, the nominal exchange rate determination in the counterfactual where I consider national currencies. In equilibrium, domestic commercial bank sectors end up splitting the seigniorage with the union-wide central bank. As argued by Reis (2013), such a seigniorage split is a distinct feature of the central bank balance sheet of a currency union. Consequently, this model can generate strong real effects from nominal and financial forces.

As credit risks were at the forefront of the Eurozone Crisis, the second key ingredient of the model is non-bank borrowers' credit risks that give rise to non-performing loans (hereinafter NPLs) in equilibrium. As in Dubey, Geanakoplos, and Shubik 2005, I model the default cost as a non-pecuniary penalty. The stance of bankruptcy code is modelled as the harshness of the penalty per unit of default, or the *price* of default. The essence of this model is to explore the efficacy of softening the CMU bankruptcy code to reduce the volatilities of credit risks across states.

To this end, I design three regimes in a currency union, which I call hereinafter Regimes ID (Internal Devaluation), FU (Fiscal Union), and BL (Bankruptcy Leniency). I also consider the counterfactual regime with national currencies, which I call Regime FX (Foreign Exchange). I design these regimes mainly by varying the relative stance between the domestic bankruptcy code for bank lending and the CMU bankruptcy code for the cross-border capital markets. For each regime, I show analytically and numerically the implication of regime characteristics for allocation efficiency within the state, risk sharing, inflation, and

[3] "Credible" in this context means the sovereign does not intervene in the foreign exchange markets via quantitative measures and that exchange rate targeting is not in the national central bank's mandate.

[4] These distinctions can be traced at least as far as back to Gurley and Shaw (1960).

asset prices.

Let us consider **Regime ID** as the baseline. It is a currency union that rules out a fiscal union and sets a punitive CMU bankruptcy code. It resembles the status quo of the Eurozone that it lacks a fully-fledged fiscal union and meanwhile, the stance for cross-border default is tough. I show that NPLs arise endogenously and exhibit high volatility across states, which leads to banking insolvency in the bad state, and hence, a pecuniary externality. Since banking insolvency calls for a national bailout tax that would be levied in the bad state when the NPL is high, such national fiscal action resembles the actual fiscal austerity measures adopted in the Eurozone after crises. Prices in this regime would turn out suppressed due to higher transaction costs. Therefore, Regime ID is referred to as the *internal devaluation* regime. Then I consider **Regime FU** as a fiscal union for comparative statics. It is modelled as a currency union featuring cross-country transfers via a fiscal union. It would resemble a hypothetical Eurozone with a fully-fledged fiscal union.[5] In Regime FU, cross-border transfers can ensure that the domestic banking sectors survive. Consequently, fiscal union removes the pecuniary externality of banking insolvency, and welfare improves.

When a fiscal union is absent, however, **Regime BL** is considered. In this regime, the CMU bankruptcy code is set more leniently than the domestic bankruptcy code.[6] As a result, domestic households may default on the cross-border financial securities in the bad state. The option to default in the cross-border capital market provides extra liquidity, acting like "cross-country transfers" from the rich to the poor to alleviate the stress of the otherwise failing banks. Consequently, there is no pecuniary externality owing to banking insolvency, and via the associated price effects, welfare improves in a Pareto sense. The internal devaluation effect due to the pecuniary externality dissipates entirely. Nevertheless, there exists a lower bound for the CMU bankruptcy code. Surpassing the lower bound is shown to impede, rather than enhance risk sharing, leading to a social welfare even inferior to Regime ID.

To corroborate the role of Regime BL (bankruptcy leniency) in improving the viability of currency unions, the model needs to explain why CMU bankruptcy code adjustment is *specifically* vital for currency unions. Thus, I consider the counterfactual of credible national currencies, a question mostly unaddressed in the existing literature. **Regime FX** is such an extension. I prove that under very general conditions, competitive floating exchanges indeed adjust for and neutralise domestic credit risks. Accordingly, banks survive, and welfare improves. However, if a currency union is the *a priori* arrangement of member countries, in a parameterisation of the model, I show that Regime BL (bankruptcy leniency) obviates such needs for floating exchange rates to neutralise domestic credit risks. Necessarily, removing nominal exchange rates implies rigidities in country-level inflation. Since countries cannot rely on inflation as a form of "soft" default, the capital markets union should allow for actual default and acknowledge the underlying credit risks. Therefore, encouraging some degree of cross-border equilibrium default by softening the CMU bankruptcy code provides compensation for the lost benefits of nominal exchange rates.

[5] Regime FU's closest existing real-world equivalences are the US and China. In the US, different states use the same US dollar as the only stipulated means of exchange, and in China, different provinces share the same Chinese RMB. Both countries, if seen as currency union blocks in their own right, have their federal government or central government as the "fiscal union" to make cross-state or cross-province fiscal transfers (see the discussion about the US case in Sargent 2012).

[6] In practice, the default cost for unsecured lending takes a myriad of forms such as market/credit exclusion, sanctions, immediate liquidation, or harshness of the terms of debt restructuring. Take market/credit exclusion as an example: an ultra-tough bankruptcy code could mean when borrowers default, they are excluded from credit markets forever. A somewhat lenient bankruptcy code could mean upon default, per unit of default, defaulters are excluded from credit markets only for a finite period.

The rest of the paper is structured as follows: Section 2 reviews the related literature. Section 3 presents the currency union model, various regime equilibrium analyses, and the analytical results. Section 4 provides welfare analysis and numerical examples and offers the impetus for policy considerations. Section 5 investigates the credible national currency case. Section 6 discusses the results, policy implications, and two layers of institutional details (one on the Eurozone TARGET2 system and the other on the cross-border insolvency reforms in practice). Section 7 is a conclusion. The online appendix contains model extensions and all derivations, as well as additional proofs.

2 Related Literature

A burgeoning emergence of academic endeavour has started tackling this issue of currency union viability (see Brunnermeier and Reis 2015 for an excellent summary of recent theories). Broadly, there are three types of proposals, albeit not orthogonal to one another and non-exhaustive: 1) fiscal unions (e.g., Farhi and Werning 2017; Kehoe and Pastorino 2017; Lane 2012), 2) banking unions (e.g.,Martinez, Philippon, and Sihvonen 2019), and 3) union-wide safe assets (e.g., Lane 2012; Brunnermeier et al. 2016). This paper contributes to the existing work by considering an alternative financial regime, i.e., capital markets union (see Martinez et al. 2019). The innovation is that I consider the economics of bankruptcy within the capital markets union in the presence of credit risks, which are omitted in Martinez et al. (2019). Such a financial regime is plausible because the forces work through the invisible hand of the markets, which might encounter less political resistance in terms of implementation.

This paper also relates to the rich body of literature on optimal currency areas that starts with Mundell (1961); McKinnon (1963); Kenen (1969). Subsequently, the new open economy macro literature (salient examples include Obstfeld and Rogoff 2000; Gali and Monacelli 2005) builds micro-foundations and is applied to provide concrete welfare analysis on the monetary and fiscal issues in currency unions (see Gali and Monacelli 2008; Ferrero 2009; Aguiar et al. 2015; Farhi and Werning 2017; Kehoe and Pastorino 2017; Adam and Grill 2017). My paper complements this body of literature. It models the endogenous determination of the value of currencies and price level and introduces equilibrium default. This is relevant because NPLs and banking insolvency were at the forefront of the Eurozone debt crisis.

In terms of the broader message, my paper shares a similar kindred spirit to Adam and Grill (2017) and Goodhart, Peiris, and Tsomocos (2018) that equilibrium default can conditionally benefit currency unions. However, these two papers do not explain why cross-border bankruptcy leniency is particularly vital to sustaining a currency union. For example, the friction in Adam and Grill (2017) is the non-state contingent bond, which is not necessarily a friction specific to currency unions. It is known in the general equilibrium theory with incomplete markets that equilibrium default can improve risk sharing by increasing the asset span (see Zame 1993; Dubey et al. 2005). Different from Goodhart et al. (2018) and Adam and Grill (2017), my model explicitly models the arrangement of sharing the common currency. Meanwhile, I make available state-contingent nominal financial securities. This has the advantage of highlighting that equilibrium default improves risk sharing due to being bound by the common currency, rather than owing to a highly incomplete financial market.

To formalise a currency union with banking fragility (e.g., the Eurozone Crisis), the model should explicitly include the currency, banks, liquidity, and credit. Therefore, I choose an international finance modelling framework based on the seminal papers by Geanakoplos and Tsomocos (2002); Tsomocos (2008); Peiris and Tsomocos (2015). Geanakoplos and Tsomocos (2002) model a general equilibrium to unify international trade and finance. The authors prove the existence of the equilibrium. Parallel to Geanakoplos and Tsomocos (2002), Tsomocos (2008) proves generic determinacy and money non-neutrality of international monetary equilibria. The author obtains price-level determinacy and the endogenous determination of nominal exchange rates in a rich general equilibrium. Further enriching Geanakoplos and Tsomocos (2002), Peiris and Tsomocos (2015) develop an international finance model with incomplete markets and relax the assumption of fully committed debt repayment. These frameworks incorporate money and financial frictions into international trade, sharing a similar spirit to Manova (2012).

In this paper, I simplify and modify Peiris and Tsomocos (2015) to consider the special case of a currency union. Rather than assuming each country has one independent central bank as in Peiris and Tsomocos (2015), I assume that countries share the same central bank, and I also consider the risk-shifting of domestic commercial banks, which are not present in Peiris and Tsomocos (2015). These modifications allow me to isolate the impact of sharing a common currency on the seigniorage split between domestic commercial banks and the union-wide central bank (see Reis 2013).

Money and liquidity creation via bank credit are key features of this paper. The early formalisation of this mechanism can be found in the general equilibrium theory of money.[7] In this literature, there is an assumed requirement that money must be used to carry out transactions formalised through liquidity-in-advance constraints similar to Grandmont and Younes (1972, 1973); Lucas Jr and Stokey (1987). Inside money enters the economy against an offsetting obligation that guarantees its departure, and it is issued when borrowing agents apply for loans from the banks. The banking sector therefore can be either an intermediary of existing money or a creator of new inside money, as in Dubey and Geanakoplos (1992, 2003b, 2006), Bloise et al. (2005), Bloise and Polemarchakis (2006), Tsomocos (2003), and Goodhart et al. (2006, 2013).

Following the 2007-2009 Global Financial Crisis, there has been a revival of inside money modelling due to the renewed interest in banks' balance sheet transformation for credit extension and liquidity creation and the associated macro-financial outcomes. Recent advances include and are not limited to Bigio and Weill (2016), Brunnermeier and Sannikov (2016) , Faure and Gersbach (2017), Donaldson et al. (2018), Bianchi and Bigio (2020), Piazzesi and Schneider (2018), McMahon et al. (2018), Kiyotaki and Moore (2018a), Kiyotaki and Moore (2018b), Wang (2019), and Kumhof and Wang (2020). Sharing a similar spirit, liquidity creation is also much emphasised in the literature on banking (see Gorton and Pennacchi 1990; Diamond and Rajan 2001; Stein 2012; Hart and Zingales 2014; DeAngelo and Stulz 2015) and safe assets (see J Caballero and Farhi 2017).

As the second key ingredient of the model is endogenous default, this paper also connects with a large body of literature on strategic sovereign default. Although I do not explicitly consider the default decision by a separate government, the default decision of the atomistic households in a given country is interpreted as the aggregate default at the country level. Typically there are two ways of thinking about default at the country level: 1) strategic default via explicit default costs (e.g. Eaton and Gersovitz 1981; Aguiar and Gopinath 2006; Arellano 2008; Arellano and Ramanarayanan 2012; Na et al. 2018) and 2) default without explicit costs but driven by political considerations (e.g.Guembel and Sussman 2009; D'Erasmo and Mendoza 2016). This paper belongs to the first group. As argued in Eaton and Gersovitz (1981), strategic default is suitable to analyse the trade-off of country-level default, because any negative net worth criterion for a country-level default is essentially

[7] Even earlier, this mechanism was also much emphasised by early economists when the banking sector was just booming. Classic works by Macleod (1866), Wicksell (1906), Hahn (1920), Hawtrey (1923), Schumpeter (1954), Keynes (1931), Tobin (1963) and Minsky (1977) have all provided insight into this monetary operation and its macro-financial implications.

irrelevant.

At the country level, default punishment can take a myriad of forms that range from credit or market exclusion (e.g., Eaton and Gersovitz 1981; Aguiar and Gopinath 2006; Arellano and Ramanarayanan 2012; Na et al. 2018) to sanctions (see the discussion in Bulow and Rogoff 1989), and from the loss of insurance opportunities (e.g., Bloise et al. 2017) to internal devaluation (e.g., Regime ID of this paper). In light of this consideration, in this paper, I do not model the various specific forms of punishment but assume a non-pecuniary default penalty à la Shubik and Wilson (1977) and Dubey et al. (2005). The intensity parameter λ of the default penalty is interpreted as the bankruptcy code in my model. Unlike Eaton and Gersovitz (1981); Aguiar and Gopinath (2006); Arellano and Ramanarayanan (2012); Na et al. (2018) that model default as a binary decision, my paper emphasises that the social cost of default depends on the severity of default; hence, partial default is also considered. Modelling partial default is also found in Calvo (1988); Bolton and Jeanne (2007); Corsetti and Dedola (2013); Adam and Grill (2017) and is in line with empirical evidence (see Trebesch and Zabel 2017) and quantitative findings (see Gordon and Guerron-Quintana 2018). I acknowledge that an alternative way of modelling default punishment would be to collateralise lending. However, because I model aggregate debt positions at a country level, seizing "collaterals" at a country level would imply further political frictions outside the scope of this paper. In light of this issue, I have only considered uncollateralised lending.

Finally, the extension of this model connects with the body of literature on the cost and benefit of flexible exchange rates and the nexus between nominal exchange rates and default. For example, Neumeyer (1998) acknowledges that the general belief that "excessive" exchange rate variability harms the economy is difficult to prove in a formal setting. However, the author shows that when the excess exchange rate risk is driven by political factors that influence monetary affairs, flexible exchange rates cause inefficiency. Guembel and Sussman (2004) use a market microstructure approach to obtain optimal exchange rates. The authors assume markets are incomplete so that the cost of flexible exchange rates stems from its volatility that impedes risk sharing. In the extension of my model in which national currencies are considered, I choose not to model any cost of flexible exchange rates but only consider the potential benefits. The reason is that I want to pin down the upper bound of the lost benefits by removing nominal exchange rates and see to what extent the CMU bankruptcy leniency can recoup such lost benefits. Indeed, a key benefit of flexible exchange rate in my model extension is to neutralise domestic credit risks such that banks remain solvent. The role of nominal exchange rates, therefore, is to provide a buffer for country-level default, an insight reminiscent of a key point from Uribe (2006).

3 The Model - Currency Union

The model is a simple two-country endowment economy with uncertainty, and both aggregate endowment risks and idiosyncratic income risks are present. There are two types of consumption goods available for international trade, and each country has only one type of consumption good. In each country reside a domestic commercial banking sector and a continuum of households. A union-wide central bank acts as the lender of last resort of issuing the common currency to the two national commercial banking sectors. The common currency is fiat because it does not enter utility functions. Households borrow from commercial banking sectors to obtain the common currency for transactions.

3.1 Model Description

The economy has three periods, $t \in T = \{0, 1, 2\}$, with date $t = 1$ having S states of nature which I index with $\boldsymbol{s} \in S = \{1, .., S\}$. Including date $t = 0$, there are $S + 1$ date-events in the set $S^* = \{0, 1, .., S\}$. Consumption happens at $t = 0, 1$, and date $t = 2$ is for any outstanding loan settlement. For simplicity there is no discounting. The two countries are indexed by $H \in \{I, J\}$ where trade occurs at prices denominated in a common currency. Country I has a measure 1 of households i and the commercial banking sector i, and country J has a measure 1 of households j and the commercial banking sector j.

Households in both countries are risk-averse and consumption goods are all perishable. In country I, households i are endowed with outside money m^i in the common currency and domestic consumption good e^i_{I0} at $t = 0$. At $t = 1$, households i are endowed with state contingent domestic consumption goods $e^i_I = (e^i_{I1}, .., e^i_{I\boldsymbol{s}}, .., e^i_{IS}) \in \mathbb{R}^S_+$. Similarly, in country J, households are endowed with outside money m^j in the common currency and domestic consumption good c^j_{J0} at $t = 0$. Households all have the option to obtain additional common currency from the banking sector via credit (inside money). At $t = 1$, households j are endowed with state contingent domestic consumption goods $e^j_J = (e^j_{J1}, .., e^j_{J\boldsymbol{s}}, .., e^j_{JS}) \in \mathbb{R}^S_+$. In every state of nature, the two types of goods are traded at nominal spot prices $p_I = (p_{I0}, p_{I1}, ..., p_{I\boldsymbol{s}}, ...p_{IS}) \in \mathbb{R}^{S^*}_+$ and $p_J = (p_{J0}, p_{J1}, ..., p_{J\boldsymbol{s}}, ..., p_{JS}) \in \mathbb{R}^{S^*}_+$ in the common currency. Given two types of contingent endowments and households' preferences, both aggregate endowment risks and idiosyncratic income risks can be captured.

To link cross-country trade and capital flows, I make available state-contingent *nominal* financial securities. These financial securities are akin to Arrow securities, but the payoff of the financial security for state $\boldsymbol{s}$ is 1 unit of the common currency, rather than 1 unit of good I or J. I assume the number of these financial securities is the same as the number of states, and I call these securities as the nominal Arrow securities. The set of state prices is denoted as $\pi = (\pi_1, ..\pi_l, .., \pi_S) \in \mathbb{R}^S_+$. These financial securities are traded on exchanges, so I have in mind this huge anonymous cross-border capital market in a currency union.[8] Therefore, cross-country lenders and borrowers do not have one-on-one interactions.

In addition to the financial contracts, there are inter-period domestic loan contracts and interbank loan contracts that provide liquidity in the common currency as the only stipulated means of exchange. The union-wide central bank lends interbank loans (μ^i_{CB}, μ^j_{CB}) to provide the common currency to the commercial banking sectors in the two countries. In each country, there is a domestic commercial bank sector that extends loans (μ^i_I or μ^j_J) to provide liquidity in the common currency to its respective domestic households, as in Assumption 1.

Assumption 1 (bank lending). *In terms of loans to non-bank sectors, commercial banks only grant loans to domestic non-bank sectors, but not foreign non-bank sectors.*

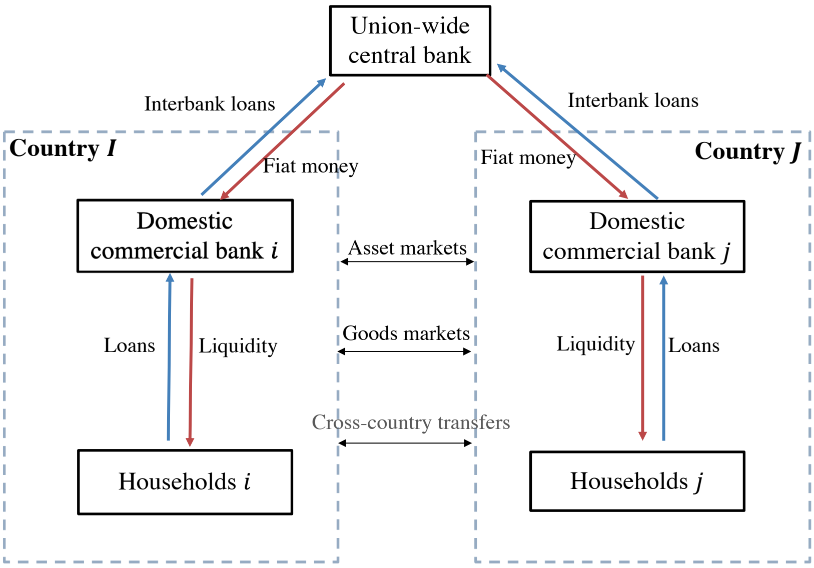
This assumption is based on the strong "home bias" of bank lending in the Eurozone widely documented in the empirical literature (see Acharya and Steffen 2015; Becker and Ivashina 2017; Gabrieli and Labonne 2018; Ongena et al. 2018). It also reflects the doom-loop in the Eurozone à la Brunnermeier et al. (2016) and Farhi and Tirole (2017) that banks in the Eurozone hold a disproportionately large amount of national debt.[9]

[8]The model baseline assumes state-contingent assets in order to highlight that even for a highly sophisticated capital market in a currency union, there is room for equilibrium default due to the nominal rigidity. I acknowledge that the capital markets union in the Eurozone is far from complete in reality, which, as existing theory suggests, gives a stronger scope for default to improve risk sharing. But assuming a highly incomplete market would weaken this paper's theoretical argument because such an assumption is not necessarily the friction specific to currency unions.

[9]As many existing works have endogenised this home bias or relationship lending either in the Eurozone context or in a broader context (see Acharya and Rajan 2013; Gennaioli et al. 2014; Uhlig 2014; Acharya et al. 2014; Farhi and Tirole 2017); therefore, I do not seek to provide further microfoundations for Assumption 1 in this paper.

The capital markets union takes the form of financial asset markets that facilitate cross-country capital flow. Following Shubik and Wilson (1977) and Dubey et al. (2005), market participants choose how much to deliver for asset payoffs, and the asset market is assumed an anonymous market with promises between different sellers not allowed to be distinguished even though they may deliver differently. This assumption implies that the expected delivery rates of the financial securities denoted as K are macro variables taken as given by the households, in the same tradition as the competitive market environment. All deliveries are pooled, and buyers of the pool for each financial security receive a pro-rata share of the net deliveries. Each ownership share of the pool of the financial security $\boldsymbol{s}$ receives a fraction $K_{\boldsymbol{s}} \in [0, 1]$ of the promised delivery in state $\boldsymbol{s}$.

Figure 1: Nominal flows of the economy



The model structure and agents' interactions are depicted in Fig (1). Linking the two member countries are the union-wide central bank and interbank markets, capital asset markets, goods markets, and possibly cross-country transfers via a fiscal/banking union.

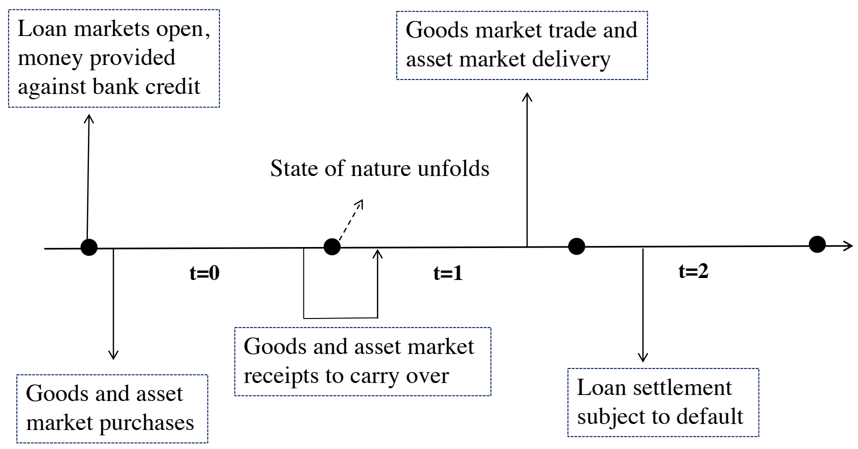
Assumption 2 (means of transaction). *Money is used to facilitate transactions due to a high searching cost and lack of double coincidence of wants.*

Assumption 2 together with the banking structure illustrated in Fig (1) implies that all transactions are carried out in the common currency and that households face liquidity-in-advance constraints akin to Lucas Jr and Stokey (1987). This assumption helps to make explicit the issuance of the single currency against credit.[10] Figure (2) shows the timeline. At $t = 0$, loan markets open so that fiat money in the common currency is issued against bank loans. Households use money to buy assets and imports, and they carry the monetary proceeds of selling assets and exports to $t = 1$. Uncertainty unfolds at $t = 1$, and assets deliver nominal payoffs and goods are traded. At $t = 2$, households use money at hand to

[10]For a detailed characterisation of money and credit in a more general setting, please see Gu et al. (2016) who incorporate frictions such as spatial or temporal separation, imperfect information and limited commitment.

settle outstanding loans. I make the sequence precise when I formally describe the budget constraints and the flow of funds.

Figure 2: Timeline



3.2 Country I

Country I's modelling is described in detail. Since the modelling of country J is symmetric to that of country I, I only formalise country I's problem for conciseness. For Country J's problem, please see the online appendix.

Households i

Households i consume at $t = 1, 2$ and derive utility from the two tradable goods, i.e. the domestic consumption good of $c^i_{Is^*}$ and the foreign consumption good of $c^i_{Js^*}$. Additionally, households i will suffer a non-pecuniary penalty if they default.

Let $(\forall \boldsymbol{s} \in S)$,

$v^i_{\boldsymbol{s}} \equiv$ the households' choice of repayment rate on domestic loans (i.e., the NPL rate is $1 - v^i_{\boldsymbol{s}}$),

$\theta^i_{\boldsymbol{s}} \equiv$ the long position in the asset markets,

$\phi^i_{\boldsymbol{s}} \equiv$ the short position in the asset markets,

$D^i_{\boldsymbol{s}} \equiv$ the choice of asset delivery,

$p_{\boldsymbol{s}} \equiv$ the union-wide price level index at $t = 1$,

$[d^i_{\boldsymbol{s}}]^+ \equiv$ the size of default on domestic loans in real terms, where

$$[d^i_{\boldsymbol{s}}]^+ = \frac{max[(1 - v^i_{\boldsymbol{s}})\mu^i_I, 0]}{p_{\boldsymbol{s}}},$$

$[f^i_{\boldsymbol{s}}]^+ \equiv$ the size of default in asset markets in real terms, where

$$[f^i_{\boldsymbol{s}}]^+ = \frac{max[\phi^i_{\boldsymbol{s}} - D^i_{\boldsymbol{s}}, 0]}{p_{\boldsymbol{s}}}.$$

Formally households' preference is given as follows:

$$\underbrace{Max}_{\mu_I^i,\theta^i,\phi^i,c_I^i,c_J^i,q_I^i,b_J^i,v^i,D^i} \quad E_0\Big\{U^i\big(c_{I0}^i, c_{J0}^i, c_{I\boldsymbol{s}}^i, c_{J\boldsymbol{s}}^i\big) - \lambda^i[d_{\boldsymbol{s}}^i]^+ - \lambda[f_{\boldsymbol{s}}^i]^+\Big\},$$

where the preference over consumption goods $U(\cdot)$ is assumed to be homothetic, strictly increasing, concave, and differentiable. The disutility from default is separable from consumption utility and is linear in the amount of default. The λ^i denotes the domestic default penalty harshness, modelled as a proxy for the domestic bankruptcy code throughout the paper. I denote the CMU bankruptcy code of the cross-border capital market as λ. Default can be either strategic or due to ill fortune, but creditors cannot observe why borrowers default. The households evaluate their marginal benefit from default and their marginal cost of default. If the former is larger than the latter, households default strategically even if there are resources at hand.

Households i choose the amount of domestic loans of μ_I^i to borrow, the long position of θ^i in the cross-border capital market, the short position of ϕ^i, the amount of domestic goods of c_I^i to consume, the amount of importing goods of c_J^i to consume, the amount of exporting goods of q_I^i, the amount of money of b_J^i to spend on imports, the loan repayment rate of v^i, and total asset delivery of D^i on their short positions.

Let Δ denote any unused money from the corresponding flow of funds constraint, let $\eta_0^i, \eta_{1\boldsymbol{s}}^i, \eta_{2\boldsymbol{s}}^i$ be the shadow price of the corresponding constraint, let r_I be the domestic loan rate and $\tau_{I\boldsymbol{s}}$ be the domestic tax rate, let $K_{\boldsymbol{s}}$ be the aggregate delivery rates of the nominal Arrow security $l = \boldsymbol{s}$, and let $\delta_{\boldsymbol{s}}^i$ be any *potential* cross-country transfer. When a fiscal union is absent, then $\delta_{\boldsymbol{s}}^i$ is simply set to 0 for all states. Households i are subject to the following budget sets and flow of funds constraints.

At $t = 0$:

$$b_{J0}^i + \sum_{l=1}^{S} \pi_l \theta_l^i \le \frac{\mu_I^i}{1 + r_I} + m^i. \qquad \eta_0^i \quad (1)$$

At $t = 1\ \forall \boldsymbol{s} \in S$:

$$b_{J\boldsymbol{s}}^i(1 + \tau_{I\boldsymbol{s}}) + (D_{\boldsymbol{s}}^i - K_{\boldsymbol{s}}\theta_{\boldsymbol{s}}^i) + \phi_{\boldsymbol{s}}^i \tau_{I\boldsymbol{s}} \le \Delta(1) + \sum_{l=1}^{S} \pi_l \phi_l^i + p_{I0} q_{I0}^i + p_{I\boldsymbol{s}} q_{I\boldsymbol{s}}^i + \delta_{\boldsymbol{s}}^i. \qquad \eta_{1\boldsymbol{s}}^i \quad (2)$$

At $t = 2$:

$$v_s^i \mu_I^i \le \Delta(2)_{\boldsymbol{s}}. \qquad \eta_{2\boldsymbol{s}}^i \quad (3)$$

And for $\boldsymbol{s}^* \in S^*$, the feasibility constraints are satisfied, i.e., $c_{I\boldsymbol{s}^*}^i \le e_{I\boldsymbol{s}^*}^i - q_{I\boldsymbol{s}^*}^i$ and $c_{J\boldsymbol{s}^*}^i \le \frac{b_{J\boldsymbol{s}^*}^i}{p_{J\boldsymbol{s}^*}}$.

Condition (1) states that at $t = 0$, households choose to apply for an inter-period loan[11] of

[11] The modelling of the inter-period loan reflects the reality that the bank's asset is typically less liquid

μ_I^i from the domestic commercial banking sector at the loan rate r_I to obtain inside money. Households i use the money inflow from the domestic commercial banking sector, plus any outside money of m^i to buy financial securities from the cross-border capital market and imports (*money as a means of transaction*). At the same time, households i receive monetary income from selling securities and exports for a total of $\sum_{l=1}^{S} \pi_l \phi_l^i + p_{I0} q_{I0}^i$ and carry it over into $t = 1$ (*money as a store of value*).

Condition (2) states that at $t = 1$, households i use the monetary income from $t = 0$ and export income of $t = 1$ plus any unused money Δ(1) and cross-country fiscal transfer of $\delta_{\boldsymbol{s}}^i$ (if any) to spend on the imports of $b_{J\boldsymbol{s}}^i$ and to deliver the net monetary payoff of $D_{\boldsymbol{s}}^i - K_{\boldsymbol{s}} \theta_{\boldsymbol{s}}^i$ for the security $l = \boldsymbol{s}$. Moreover, import expenditures and cross-country borrowing are subject to a state-contingent tax levied by the national government for a possible bank bailout. The underlying assumption is that bank bailout is costly. The real cost of bank bailout is reflected through the distortionary bailout tax.[12]

At $t = 2$, households use the residual money from $t = 1$ to settle the domestic loan and choose how much to repay or default (see Condition (3)). This loan settlement constraint is equivalent to the transversality condition in infinite horizon models.

Domestic Commercial Banking Sector i

Bank i is the domestic commercial banking sector in country I. Bank i extends loans to domestic households and provides liquidity for the households to make purchases. To ensure the liquidity bank i provides would have one-to-one convertibility to the common currency the union-wide central bank issues, bank i needs to borrow interbank loans from the union-wide central bank. In this sense, commercial banks act as the "creators of money" à la Tobin (1963), with the central bank being the ultimate fiat money issuer.[13]

Bank i needs to make the following choices. It needs to choose the amount of domestic liquidity of $\mu_I^i/(1 + r_I)$ to supply to the household, the amount of interbank loans μ_{CB}^i to borrow from the union-wide central bank to obtain the common currency, and the amount of interbank liquidity L^i to make available to ensure that the liquidity supplied to domestic households has a one-to-one convertibility to the common currency obtained from the central bank. Bank i maximises its franchise value, defined as the average payoff across states weighted by the risk-neutral probabilities. Formally,

$$\underbrace{Max}_{\mu_{CB}^i, \mu_I^i, L^i, \omega^i} \sum_{\boldsymbol{s}=1}^{S} z_{\boldsymbol{s}} \omega_s^i,$$

where ω_s^i is bank i's nominal profits for state $\boldsymbol{s}$ and $z_{\boldsymbol{s}}$ is the risk-neutral probability for state $\boldsymbol{s}$. Let ρ be the interbank rate, and let $R_{\boldsymbol{s}}^i$ be the bank's expected repayment rate of the households. Bank i is subject to the following flow of funds constraints:

than its liability, i.e., money in this case.

[12] The results are also robust to taxing exports and cross-bordering lending instead. Throughout the paper, I use bailout tax and bailout cost interchangeably. In the equilibrium characterisation, households would choose to default in the bad state even if there were no tax levy.

[13] In practice, when individual commercial banks supply loans they immediately write deposits as IOU notes for the borrowers, but the deposits are convertible to central bank reserves with the central bank being the lender of the last resort. Therefore, ultimately fiat money is issued by the central bank, and the commercial banks are a risk-shifting "pass-through" of central bank fiat money.

$$L^i \leq \frac{\mu^i_{CB}}{1+\rho}, \tag{4}$$

$$\frac{\mu^i_I}{1+r_I} \leq L^i, \tag{5}$$

$$\omega^i_{\boldsymbol{s}} = \Delta(4) + \Delta(5) + R^i_{\boldsymbol{s}}\mu^i_I - \mu^i_{CB}. \tag{6}$$

At $t = 0$, bank i borrows interbank loans from the union-wide central bank and obtains fiat money in the common currency, ready to be extended as interbank liquidity of L^i as in Condition (4).

Meanwhile, when bank i extends commercial loans of μ^i_I to the households, it must ensure the liquidity bank i provides against the bank loans has one-to-one convertibility to the fiat money issued by the central bank. This is shown in Condition (5) and Lemma 2, which shall prove that Condition (5) is binding whenever $\rho > 0$. Eq (6) states at $t = 2$, $\forall \boldsymbol{s} \in S$, bank i uses the households' loan repayment to pay back the interbank loans[14] to the union-wide central bank, and the difference between these two repayments adds to bank i's net cash flow, i.e., profits.

Depending on the NPL rate of $1 - v^i_{\boldsymbol{s}}$ for state $\boldsymbol{s} \in S$, bank i's nominal profits $\omega^i_{\boldsymbol{s}}$ could be negative and that bank i becomes insolvent. Given the characteristics of different regimes to be specified in Proposition 2, there may be cross-country transfers of $\delta^{bI}_{\boldsymbol{s}}$ or domestic government bailout funds of $T_{I\boldsymbol{s}}$ injected to bank i. I define $\omega^{i'}_{\boldsymbol{s}}$ as the after-bailout net cash inflow to bank i, $\forall \boldsymbol{s} \in S$, i.e., $\omega^{i'}_{\boldsymbol{s}} = \omega^i_{\boldsymbol{s}} + \delta^{bI}_{\boldsymbol{s}} + T_{I\boldsymbol{s}}$.

National Government i

National government i collects taxes from domestic households to build a state-contingent bailout fund of $T_{I\boldsymbol{s}}$. Assumption 3 implies that the national government will levy a tax to bail out the domestic banking sector, should the government foresee domestic banking insolvency in a particular state. The households and the domestic commercial banking sector are assumed uninformed at $t = 0$ of the national government's contingent action at $t = 1$. At $t = 1$, they take the government's action as given.

Assumption 3 (bailout). *The insolvency of the domestic banking sector incurs a high social cost.*

The social cost in Assumption 3 can be interpreted in two dimensions. When the domestic commercial banking sector incurs negative profits and becomes insolvent, it is unable to pay back the interbank loans to the union-wide central bank. One dimension of the social cost is that domestic banking insolvency means defaulting on the union-wide central bank. The implication is that the country as a whole may lose the membership of being in the currency union. The other dimension is that domestic banking insolvency would require enormous resources for the national government to restore its domestic banking system. Given such considerations, the national government will bail out the domestic banking system should it foresee domestic banking insolvency in a particular state.

[14] Interbank loans are modelled as non-defaultable. When the Euro Crisis emerged, the authorities arranged the form of the rescue to make sure that there was no default on the interbank loans that French and German banks had provided to the Greeks.

To collect the bailout fund, the national government levies taxes based on import expenditures and cross-country borrowing as in Eq (7), reflecting the point that in a bad state, the government resorts to fiscal austerity to bail out the domestic banking system.

$$T_{I\boldsymbol{s}} = p_{J\boldsymbol{s}} c^{i}_{J\boldsymbol{s}} \tau_{I\boldsymbol{s}} + \phi^{i}_{\boldsymbol{s}} \tau_{I\boldsymbol{s}}. \tag{7}$$

National government i uses the bailout funds to rescue the domestic commercial banking sector whenever the banking sector's nominal profits (adjusting for possible cross-country transfers) would drop to negative, i.e., the bank fails. In short, in the bad state the national government makes a state-contingent transfer to ensure $\omega^{i'}_{\boldsymbol{s}} = \omega^{i}_{\boldsymbol{s}} + \delta^{bI}_{\boldsymbol{s}} + T_{I\boldsymbol{s}} = 0$.

3.3 Union-wide Central Bank

The union-wide central bank lends interbank loans of μ^{h}_{CB}, $\forall h \in \{i, j\}$, and provides fiat money in the common currency to the two national commercial banking sectors. The union-wide central bank sets the interbank target rate of ρ as the policy rate.

To guarantee the determinacy of the price level, the union-wide central bank, through the flow of funds of the banking system, collects households' outside money as the seigniorage, but it does not redistribute the seigniorage within the same period. In this sense, the treatment of seigniorage in this model is non-Ricardian (Sims 1994; Buiter 1999). This approach follows Dubey and Geanakoplos (1992, 2006) and Tsomocos (2003). It resonates with the institutional separation between a central bank and a government and takes the view that price-level determinacy in equilibrium reflects the central bank mandate on price stability.

3.4 Equilibrium

The currency union equilibrium is defined as an allocation $(c^{i}_{Is^*}, c^{i}_{Js^*}, c^{j}_{Is^*}, c^{j}_{Js^*}, b^{j}_{Is^*}, b^{i}_{Js^*}, q^{i}_{Is^*}, q^{j}_{Js^*}, \theta^{h}_{l}, \phi^{h}_{l}, D^{h}_{\boldsymbol{s}}, \mu_I, \mu_J, \mu_{CB})$ with prices $(p_{I\boldsymbol{s}^*}, p_{J\boldsymbol{s}^*}, \pi_l, r_I, r_J, v^{h}_{\boldsymbol{s}}, R^{h}_{\boldsymbol{s}})$, given bankru--ptcy codes (λ^h, λ) and policy rate and fiscal rules $(\rho, \tau_H, \delta^h, \delta^{bH})$, $\forall \boldsymbol{s}^* \in S^*$, $\forall \boldsymbol{s} \in S$, $\forall l \in S$, $h \in \{i, j\}$, $H \in \{I, J\}$ such that agents maximise subject to liquidity-in-advance constraints and budget constraints, markets clear, and expectations are rational.

- Goods markets:

$$p_{I\boldsymbol{s}^*} q^{i}_{I\boldsymbol{s}^*} = b^{j}_{I\boldsymbol{s}^*},$$

$$p_{J\boldsymbol{s}^*} q^{j}_{J\boldsymbol{s}^*} = b^{i}_{J\boldsymbol{s}^*},$$

- Asset markets for the nominal Arrow security $\boldsymbol{s} = l$:

$$\sum_{h \in \{i,j\}} \phi^{h}_{l} = \sum_{h \in \{i,j\}} \theta^{h}_{l},$$

- Domestic loan markets:

$$\frac{\mu^{h}_{H}}{1 + r_H} = L^h,$$

- Interbank loan and money market:

$$1+\rho = \frac{\mu^i_{CB}+\mu^j_{CB}}{M},$$

- Rational expectation:

$$K_{\boldsymbol{s}} = \left\{ \begin{array}{cc} \frac{\sum_{h\in\{i,j\}} D^h_{\boldsymbol{s}}}{\sum_{h\in\{i,j\}} \phi^h_{\boldsymbol{s}}} & \text{if } \sum_{h\in\{i,j\}} \phi^h_{\boldsymbol{s}} > 0 \\ \text{arbitrary} & \text{if } \sum_{h\in\{i,j\}} \phi^h_{\boldsymbol{s}} = 0 \end{array} \right\},$$

$$R^h_{\boldsymbol{s}} = \left\{ \begin{array}{cc} v^h_{\boldsymbol{s}} & \text{if } 1-v^h_{\boldsymbol{s}} > 0 \\ \text{arbitrary} & \text{if } 1-v^h_{\boldsymbol{s}} = 0 \end{array} \right\}.$$

3.5 Equilibrium and Regime Characterisation

This subsection characterises the equilibrium and regimes. Suppose state s is a good state for country I and a bad state for country J, and state s' is a bad state for country I and a good state for country J, i.e. $e^i_{Is} > e^i_{Is'}$, $e^j_{Js} < e^j_{Js'}$. The subsequent analysis focuses on such asymmetric endowment shocks. Let $\gamma_{\boldsymbol{s}}$ the probability state $\boldsymbol{s}$ occurs, $\forall \boldsymbol{s} \in S$.

Lemmas 1-3 prove the binding conditions of the flow of funds constraints to ensure both nominal and real determinacy. Lemma 4 states the shadow price of the flow of funds constraint at $t=1$. Lemma 5 and Proposition 1 characterise the equilibrium. Proposition 2 designs regimes of a currency union.

Lemma 1. Binding conditions of the Liquidity-in-advance constraints.

If $r_I > 0$, then $\Delta(1) = 0$.

If $\rho > 0$, then $\Delta(4) = 0$.

Proof. See Appendix A.1.

Lemma 2. Interbank liquidity and the single currency convertibility.

If $\rho > 0$, then $\Delta(5) = 0$.

Proof. See Appendix A.2.

Remark: That (5) binds means that the interbank liquidity the domestic banking sector i extends to the households is pegged one-to-one to the common currency issued by the union-wide central bank. The convertibility is not imposed *a priori* but rather a result of the non-arbitrage conditions from the interbank market.

Lemma 3. No worthless money at the end.

If $r_I > 0$, then $\Delta_{\boldsymbol{s}}(3) = 0$.

If $\rho > 0$, then $\Delta_{\boldsymbol{s}}(6) = 0$.

Proof. See Appendix A.3.

Lemma 4. Heterogeneous tightness of nominal constraints.

In a currency union with trades in goods market and asset market, if $r_I, r_J > 0$ and no full default on loans, then $\eta^i_{1s} \neq \eta^i_{1s'}$ and/or $\eta^j_{1s} \neq \eta^j_{1s'}$.

Proof. *See Appendix A.4.*

Lemma 5. (zero credit risks and the loss of exchange rates): If in the currency union $\forall \boldsymbol{s} \in S$, $h \in \{i, j\}$, $v_{\boldsymbol{s}}^h = 1$, given markets are complete, domestic banking sectors break even for all states, i.e. $\omega_{\boldsymbol{s}}^h = 0$.

Proof. *See Appendix A.5.*

Claim. *With domestic credit risks, the loss of floating nominal exchange rates (i.e., currency unions) may translate into a currency crisis, disguised as a banking debt crisis, i.e.,* $\omega_{\boldsymbol{s}}^h < 0, \exists \boldsymbol{s} \in S, h \in \{i, j\}$.

The above claim implies that the banking sectors in a currency union become more vulnerable due to losing the flexibility of exchange rates. Having a floating exchange rate might neutralise domestic credit risks and prevent such crises. Not to jump ahead of myself, I shall revisit this claim with a formal argument and proof in Proposition 4 of the equilibrium analysis of the currency union and Proposition 6 in Section 5 in which I consider credible national currencies and the role of nominal exchange rates.

Given that in a currency union, zero domestic credit risks in all states of nature as in Lemma 5 is unlikely to hold in reality, in the subsequent analysis, I only focus on the cases when domestic credit risks are present in a currency union. I also do not consider the case of 100 % NPLs where there exists a state in which the household defaults on domestic loans completely. Formally, let,

$$\overline{\Lambda}^h = \{\lambda^h : v_{\boldsymbol{s}}^h = 1, \forall \boldsymbol{s} \in S, h \in \{i, j\}\},$$
$$\underline{\Lambda}^h = \{\lambda^h : v_{\boldsymbol{s}}^h = 0, \exists \boldsymbol{s} \in S, h \in \{i, j\}\}.$$

Thus, $\overline{\Lambda}^h$ covers the cases of full delivery of domestic loans in all states, and $\underline{\Lambda}^h$ covers the cases in which there exists a state of full default on domestic loans. In *all* the subsequent analysis I restrict λ^h to be an intermediate default penalty for domestic loans, i.e. for $h \in \{i, j\}$, $\lambda^h \notin \overline{\Lambda}^h$ and $\lambda^h \notin \underline{\Lambda}^h$.

Proposition 1. (the Fisher effect and money non-neutrality):

- **The Fisher effect**: Suppose for households i, $b_{J\boldsymbol{s}^*}^i > 0$, $\forall \boldsymbol{s}^* \in S^*$. Suppose further that households i have some money left over the moment the domestic loan comes due at $\boldsymbol{s}$, then in equilibrium,

$$1 + r_I = \Big(E_0\big(\frac{U^i_{c^i_{J\boldsymbol{s}}}}{U^i_{c^i_{J0}}}\big)\big(\frac{p_{J0}}{p_{J\boldsymbol{s}}}\big)\frac{1}{(1+\tau_{I\boldsymbol{s}})}\Big)^{-1},$$

where $U^i_{c^i_{J0}}$ and $U^i_{c^i_{J\boldsymbol{s}}}$ are household i's marginal utilities of consuming imports at $t = 0$ and in state $\boldsymbol{s}$. A similar expression of the Fisher effect obtains for country J as well.

Taking the logarithm of the above Fisher equation and interpreting it loosely, the nominal interest rate equals the real interest rate plus the expected inflation adjusted by any bailout tax. Any tax needed for bank bailout in a currency union also distorts real allocation and inflation. As the bailout tax puts downward pressure on inflation and the real interest rate, and it resembles fiscal austerity, I call this distortionary effect the *internal devaluation* effect.

- **Money non-neutrality**: Suppose $\rho > 0$, any change in ρ results in a different equilibrium in which some households' consumption is different.

  Remark: Even with flexible prices, money and default render monetary policy non-neutral.

  Proof. See Appendix A.6.

Corollary 1.1. (credit risks and seigniorage split): Suppose $\rho > 0$, in a currency union with idiosyncratic credit risks, suppose $v_s^i > \sum_{\boldsymbol{s}=1}^{S} z_{\boldsymbol{s}} v_{\boldsymbol{s}}^i > v_{s'}^i$ and $v_s^j < \sum_{\boldsymbol{s}=1}^{S} z_{\boldsymbol{s}} v_{\boldsymbol{s}}^j < v_{s'}^j$, $\forall \boldsymbol{s} \in S$, the term structure of interest rates incorporates credit risks and shows the seigniorage split between the central bank and commercial banks, and $\omega_s^i, \omega_{s'}^j > 0$.

In state s:

$$\rho M + \left(\frac{v_s^j}{\sum_{s=1}^{S} z_{\boldsymbol{s}} v_{\boldsymbol{s}}^j} - 1\right)\mu_{CB}^j + T_s^j = \sum_{h \in \{i,j\}} m^h - \omega_s^i. \tag{8}$$

In state s':

$$\rho M + \left(\frac{v_{s'}^i}{\sum_{s=1}^{S} z_{\boldsymbol{s}} v_{\boldsymbol{s}}^i} - 1\right)\mu_{CB}^i + T_{s'}^i = \sum_{h \in \{i,j\}} m^h - \omega_{s'}^j. \tag{9}$$

Proof. See Appendix A.7.

Corollary 1.2. (Monetary policy rate pass-through): For $H \in \{I, J\}, h \in \{i, j\}, \forall \boldsymbol{s} \in S$,

$$1 + r_H = \frac{1 + \rho}{\sum_{\boldsymbol{s}=1}^{S} z_{\boldsymbol{s}} v_{\boldsymbol{s}}^h}. \tag{10}$$

Corollary 1.1 states that both the liquidity creation by banks and the credit risks of households affect the term structure of the interest rates. The left-hand side of the Eqs (8) (9) is the union-wide central bank's interest rate revenue for issuing the common currency. It equates the total outside money minus the rents extracted by commercial banks, to be collected by the union-wide central bank at $t = 2$. Note that the union-wide central bank does not collect all the outside money as profits. This amount of profits collected by the central bank is called seigniorage. No matter how small it is, it serves to obtain price-level determinacy.[15] The term structure of interest rate in relation to the seigniorage can be interpreted as the nexus between fiat money and the fiscal sovereign. Indeed, Goodhart (1998) argues that seigniorage is part of the government's taxation plan, and as Tsomocos (2003) puts it, by collecting the seigniorage, "the government compels the acceptance of fiat money as a final discharge of debt".

Corollary 1.2 or Eq (10) shows the imperfect pass-through of the union-wide monetary policy hampered by credit risks. It states the borrowing cost at the national level equates to the the union-wide monetary policy rate adjusted for the expected domestic NPL rates. The implication is that a fall in the union-wide monetary policy rate does not necessarily translate to a loosened monetary condition at the national level, because the monetary policy pass-through is augmented with terms of financial contracts at the national level.

The model now sets out the regime designs of a currency union by varying the domestic and

[15] For a general proof of determinacy, please see Dubey and Geanakoplos (2006) and Tsomocos (2008).

CMU bankruptcy codes ($\lambda^h, \lambda, h \in \{i, j\}$) and then their respective welfare properties are ranked. Which regime the currency union falls under is endogenous to the relative harshness of domestic and the CMU bankruptcy code. Proposition 2 formalises the regime design.

Proposition 2. (domestic and CMU bankruptcy codes):

- If the CMU bankruptcy code is harsher than the domestic bankruptcy code, households fully deliver on financial assets traded in the cross-border capital market. i.e., for $h \in \{i, j\}$,
  - if $\lambda > \lambda^h$, $D^h_{\boldsymbol{s}} = \phi^h_{\boldsymbol{s}}$ at state s.
- If the CMU bankruptcy code is more lenient, households may default on financial assets in the cross-border capital market. i.e., for $h \in \{i, j\}$,
  - if $\lambda < \lambda^h$, $0 \le D^h_{\boldsymbol{s}} \le \phi^h_{\boldsymbol{s}}$ at state s.

Proof. See Appendix A.8.

Proposition 2 establishes the foundation for the design of the following three regimes. Formally, define $CA^H_{\boldsymbol{s}}$ as country H's current account net flow and $FA^H_{\boldsymbol{s}}$ as country H's capital account net flow at $t = 1$, i.e.,

$$
\begin{aligned}
CA^I_{\boldsymbol{s}} &= p_{I\boldsymbol{s}} q^i_{I\boldsymbol{s}} - p_{J\boldsymbol{s}} c^i_{J\boldsymbol{s}}, \\
FA^I_{\boldsymbol{s}} &= K_{\boldsymbol{s}} \theta^i_{\boldsymbol{s}} - D^i_{\boldsymbol{s}}, \\
CA^J_{\boldsymbol{s}} &= p_{J\boldsymbol{s}} q^j_{J\boldsymbol{s}} - p_{I\boldsymbol{s}} c^j_{I\boldsymbol{s}}, \\
FA^J_{\boldsymbol{s}} &= K_{\boldsymbol{s}} \theta^j_{\boldsymbol{s}} - D^j_{\boldsymbol{s}}.
\end{aligned}
$$

A positive CA means the current account is a running surplus, and a positive FA means international capital inflow, and vice versa. With these definitions, I state the following regime designs.

- **Regime ID (internal devaluation, baseline)**: $\lambda > \lambda^h$, $\delta^h_{\boldsymbol{s}} = \delta^{bH}_{\boldsymbol{s}} = 0$, $T_{H\boldsymbol{s}} = -\omega^h_{\boldsymbol{s}}$ whenever $\omega^h_{\boldsymbol{s}} < 0$, and $T_{H\boldsymbol{s}} = 0$ whenever $\omega^h_{\boldsymbol{s}} \ge 0$, where $h \in \{i, j\}$, $H \in \{I, J\}$, $\forall \boldsymbol{s} \in S$.

  Regime ID is the baseline currency union in which a punitive CMU bankruptcy code prevents default in the cross-border capital markets, and a fiscal union is also ruled out. The domestic bailout tax is levied in the respective bad state to bail out the domestic banking system.

- **Regime FU (fiscal union)**: A currency union supported by a fiscal union, and a punitive CMU bankruptcy code, i.e., $\lambda > \lambda^h, T_{H\boldsymbol{s}} = 0$, for $h \in \{i, j\}$, $H \in \{I, J\}$, $\forall \boldsymbol{s} \in S$. I consider two cases for Regime FU as follows.
  - **Regime FU.a**: A fiscal union that makes cross-country fiscal transfers of $\delta^h_{\boldsymbol{s}}$ directly between households, and $\delta^h_{\boldsymbol{s}} = -FA^H_{\boldsymbol{s}} - CA^H_{\boldsymbol{s}}$ and $\delta^{bH}_{\boldsymbol{s}} = 0$. It follows that $\sum_{h \in \{i,j\}} \delta^h_{\boldsymbol{s}} = 0$.[16]
  - **Regime FU.b**: A fiscal union that makes cross-country fiscal transfer of $\delta^{bH}_{\boldsymbol{s}}$ directly between domestic commercial banks, such that $\omega^h_{\boldsymbol{s}} + \delta^{bH}_{\boldsymbol{s}} \ge 0$ and $\sum_{H \in \{I,J\}} \delta^{bH}_{\boldsymbol{s}} = 0$, $\delta^h_{\boldsymbol{s}} = 0$. Regime FU.b can be interpreted as a banking union supported by a common fiscal entity.

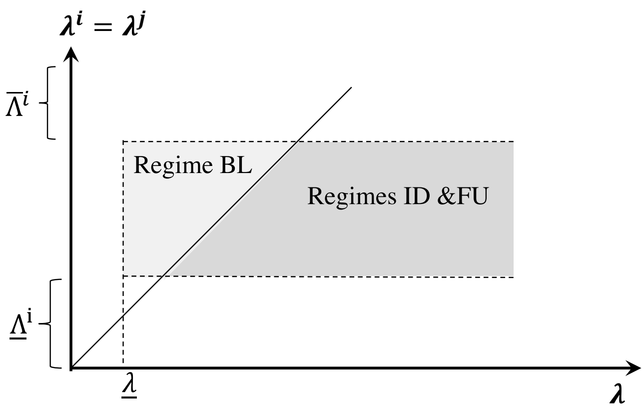
[16]Note that country H's Balance of Payment ($BoP^H_{\boldsymbol{s}}$) in state $\boldsymbol{s}$ is $BoP^H_{\boldsymbol{s}} = CA^H_{\boldsymbol{s}} + FA^H_{\boldsymbol{s}} + \delta^h_{\boldsymbol{s}}$.

- **Regime BL (bankruptcy leniency)**: $\underline{\lambda} < \lambda < \lambda^h$, $\delta_{\boldsymbol{s}}^h = \delta_{\boldsymbol{s}}^{bH} = 0$, and $T_{H\boldsymbol{s}} = 0$.

  Regime BL is a currency union with a more lenient CMU bankruptcy code, but a fiscal union is ruled out and no domestic bailout tax is levied. In this regime, the CMU bankruptcy code can induce endogenous default in the cross-border capital markets to emerge in equilibrium.

Note that the lower bound $\underline{\lambda}$ of the CMU bankruptcy code in Regime BL ensures the financial markets do not collapse. The reason is that if the CMU bankruptcy code is too lenient, households in both countries would fully default on the financial assets; hence, assets would not be traded at $t = 0$. To sum up, Fig (3) illustrates the regions of default penalty harshness and the corresponding regimes of the currency union. The horizontal axis denotes the CMU bankruptcy code λ, and the north-pointing vertical axis denotes domestic bankruptcy codes λ^i and λ^j. For the ease of illustration, $\lambda^i = \lambda^j$, but equality does not need to hold in general. Focusing on the intermediate domestic default penalty harshness, Regime ID belongs to the region where domestic bankruptcy codes are harsher than the CMU bankruptcy code, and Regimes ID and FU belong to the region where the CMU bankruptcy code is harsher than domestic bankruptcy codes.

Figure 3: Regimes and bankruptcy codes



3.6 Equilibrium Analysis

In this subsection, I show the welfare properties of each regime for allocations, risk sharing, and asset prices. In particular, propositions are given to demonstrate the mechanism in which a lenient bankruptcy code for the capital markets union could improve welfare. A caveat also highlights the possibility of cross-border default could impede international risk sharing if certain conditions are not met.

The intuition of the potential benefit of default in the cross-border capital market is that it provides extra liquidity for the borrower in the bad state such that risk sharing improves. Before formalising welfare improvement, we need to understand the mechanism of how a lenient CMU bankruptcy code can incentivise the borrower to grab the option to default in the bad state strategically. Suppose we are in Regime ID with a relatively lenient CMU bankruptcy code. The households in the bad state may fully default, whereas the other households in the other country may fully repay if they both have short positions on the Arrow security of this state. The reason being the poor households have a high marginal utility of consumption, which would outweigh the marginal cost of default. However, the other households are rich in this state, and the marginal utility of consumption is low, which would push down the marginal benefit of default. When the marginal benefit of default is less than the marginal cost of default, these rich households would fully deliver despite the poor households' full default. Therefore, although the poor households would fully default, the aggregate default rate on the nominal Arrow security of that state actually would fall between 0 and 1.

Moreover, the poor households may enter both the short and long positions of the nominal Arrow security of the bad state. The poor households would buy this Arrow security to insure against the bad shock, but they may also sell this Arrow security at the same time because selling gives the option to default fully. The option to default on financial securities provides extra liquidity leading to a possible increase in consumption or a higher domestic loan repayment rate, which implies an increase in the households' utility. An interior solution can be obtained because on the one hand selling more of the Arrow security leads to extra liquidity due to equilibrium default, and on the other hand, the market-clearing condition necessitates the poor households also buy more of this Arrow security, and buying incurs more cost of liquidity. Proposition 3 formalises the mechanism of default in the cross-border capital market. Later on, Proposition 5 builds on Proposition 3 and proves Pareto improvement.

Proposition 3. (strategic default on financial securities):

- When the CMU bankruptcy code is sufficiently lenient, households in the bad state may long and short the nominal Arrow security of that state at the same time, and fully default on this security.
- Consider the case where $S = \{1, 2\}$, let $\gamma_1 = \gamma_2$, $e_1^i > e_2^i$, and $e_1^j < e_2^j$. Suppose that in equilibrium $\lambda < p_2\eta_{12}^i < \lambda^i$, $\eta_{12}^j < \frac{\lambda}{p_2}$ and $\eta_{11}^i < \eta_{12}^i$ holds. Then, $\phi_2^i, \phi_2^j, \theta_2^i > 0$, $\theta_2^j = 0$, $D_2^i = 0, D_2^j = \phi_2^j$, and $0 < K_2 < 1$ whenever $\big(K_2 - \pi_2\big(v_2^i(1+r_I) - 1\big)\big)/p_{I2} > \pi_2 r_I/p_{I1}$. Similar logic follows for the other state.[17]

Proof. See Appendix B.1.

Corollary 3.1. When the CMU bankruptcy code is too lenient, it impedes international risk sharing in the currency union.

Corollary 3.2. As domestic bankruptcy codes become more lenient, the room to adjust the CMU bankruptcy code for the cross-border capital market decreases.

The insight of Proposition 3 is reminiscent of Example 2 in Dubey et al. (2005). A lenient CMU bankruptcy code encourages the households in the bad state to default fully. Even though the poor households have nominal inflows on hand for delivery, they do not deliver anything while the rich households deliver fully! Default in this case is strategic and makes endogenous the asset payoffs of the cross-border capital market. Assets are still traded despite strategic default: the households in the poor country enter both long and short positions of Arrow securities, and the households in the rich country only short Arrow securities of that state.

Note that financial securities are voluntarily traded despite the possibility of default, and no market participants are forced to buy or sell the financial securities in the cross-border capital market. In this sense, the market's invisible hand of provides "voluntary liquidity transfers" via endogenous default. This mechanism is in principle different from Regime FU where a fiscal union employs a visible hand to move nominal resources directly. However, a caveat remains for Regime BL (Corollary 3.1). Suppose now the CMU bankruptcy code

[17] In Section 4, an equilibrium with these characteristics is obtained.

λ is set ultra-low, i.e. $\lambda < p_{s'}\eta^j_{1s'}$ or $\lambda < p_s\eta^i_{1s}$, then Arrow securities are not traded. The currency union loses risk sharing altogether. Therefore, there exists a lower bound and an upper bound for the bankruptcy code of the capital markets union.

Moreover, for the currency union to retain risk sharing and for the aforementioned default to occur in the cross-border capital markets in equilibrium, the CMU bankruptcy code λ must fall into the interval $(p_{s'}\eta^j_{1s'}, \lambda^i) \cap (p_s\eta^i_{1s}, \frac{\lambda^j}{a_s})$ in equilibrium. As the domestic bankruptcy code λ^i or λ^j decrease, $||(p_{s'}\eta^j_{1s'}, \lambda^i) \cap (p_s\eta^i_{1s}, \lambda^j)||$ decreases. Thus, as domestic bankruptcy codes become more lenient, the range to set the CMU bankruptcy code shrinks (Corollary 3.2).

A key condition for Proposition 3 to go through is $\eta^j_{12} < \frac{\lambda}{p_2}$ in equilibrium (and its equivalent for state 1), which says the CMU bankruptcy code is strict enough to prevent default of the rich households. This condition ensures that defaultable Arrow securities are still traded in equilibrium even when the households of the poor country in the bad state fully default. I call this condition *within-union standard*. When the two countries' fundamentals differ exceptionally or when domestic bankruptcy code(s) are too lenient or discretionary, the "within-union standard" may fail to satisfy. In this case Regime BL causes asset trades to collapse.

In contrast to Regime BL, Regimes FU.a and FU.b advocate using the visible hand of a common fiscal entity to make cross-country transfers. Whereas a fully-fledged fiscal union in a currency union may be controversial and politically infeasible, in practice, there have been small steps towards building union-wide transfer funds, for example, the concept of a banking union in the Eurozone. Therefore, it is of interest to investigate the properties of Regimes FU.a and FU.b.

Let $var(1 - v^{h,FU.a}_{\boldsymbol{s}})$ be the variance of the NPL rate of households h in Regime FU.a, let $var(1 - v^{h,FU.b}_{\boldsymbol{s}})$ be that of households h in Regime FU.b, and $var(1 - v^{h,ID}_{\boldsymbol{s}})$ that of Regime ID, where $h \in \{i, j\}$. Lemma 6 says the domestic credit risk volatility across states is smaller in Regime FU.a than in Regime FU.b and Regime ID.

Lemma 6 (credit risk volatility):

- A fiscal union that mediates transfers between households can reduce domestic credit risk volatility across states.
- In Regime FU.a, for $h \in \{i, j\}, H \in \{I, J\}$, suppose $\delta^h_{\boldsymbol{s}} = -FA^H_{\boldsymbol{s}} - CA^H_{\boldsymbol{s}}$, it follows that $\delta^i_{\boldsymbol{s}} + \delta^j_{\boldsymbol{s}} = 0$, and moreover,

$$var(1 - v^{h,FU.a}_{\boldsymbol{s}}) \leq var(1 - v^{h,FU.b}_{\boldsymbol{s}}),$$
$$var(1 - v^{h,FU.a}_{\boldsymbol{s}}) \leq var(1 - v^{h,ID}_{\boldsymbol{s}}).$$

Proof. See Appendix B.2.

Proposition 3 and Lemma 6 equip the currency union with distinct institutional features for the common objective to reduce domestic banking stress. Proposition 4 formalises the mechanisms whereby this objective is achieved.

Proposition 4. (capital flow and banking crisis): Suppose $\lambda^h \notin \overline{\Lambda}^h$ and $\lambda^h \notin \underline{\Lambda}^h$, for $h \in \{i, j\}, H \in \{I, J\}, \forall \boldsymbol{s} \in S$.

- In Regime ID, the volatility of domestic credit risks and international capital flow can lead to domestic banking insolvency.
  - If $\lambda > \lambda^h$ and $\delta_{\boldsymbol{s}}^h = \delta_{\boldsymbol{s}}^{bH} = 0$, whenever $v_{\boldsymbol{s}}^h < \sum_{\boldsymbol{s}=1}^{S} z_{\boldsymbol{s}} v_{\boldsymbol{s}}^h$, then $\omega_{\boldsymbol{s}}^h < 0$ and $T_{H\boldsymbol{s}} = -\omega_{\boldsymbol{s}}^h$.

- In Regime FU.a, international capital flows do not drive domestic banking insolvency.
  - If $\lambda > \lambda^h$, $\delta_{\boldsymbol{s}}^{bH} = 0$, and $T_{H\boldsymbol{s}} = 0$, setting $\delta_{\boldsymbol{s}}^h = -FA_{\boldsymbol{s}}^H - CA_{\boldsymbol{s}}^H$, then $\omega_{\boldsymbol{s}}^h = 0$.

- In Regime FU.b, the banking union funds alleviate domestic banking stress.
  - If $\lambda > \lambda^h$, $\delta_{\boldsymbol{s}}^h = 0$, and $T_{H\boldsymbol{s}} = 0$, as long as $\omega_{\boldsymbol{s}}^j + \omega_{\boldsymbol{s}}^i \geq 0$, a banking union fund of $\delta_{\boldsymbol{s}}^{bH}$ can be set to transfer between bank i and bank j such that $\omega_{\boldsymbol{s}}^h + \delta_{\boldsymbol{s}}^{bH} \geq 0$ and $\sum_{H\in\{I,J\}} \delta_{\boldsymbol{s}}^{bH} = 0$.

- In Regime BL, equilibrium default in the cross-border capital markets may prevent domestic banking insolvency.
  - If $\underline{\lambda} < \lambda < \lambda^h$, $\delta_{\boldsymbol{s}}^h = \delta_{\boldsymbol{s}}^{bH} = 0$, and $T_{H\boldsymbol{s}} = 0$, under the conditions in Proposition 3 on strategic default, $\omega_{\boldsymbol{s}}^h = 0$.

Proof. See Appendix B.3.

Corollary 4.1. Under Proposition 4, Regimes FU and BL obviate the need for national bailout taxes whereas Regime ID needs it.

Proposition 4 shows whether cross-border capital flow may lead to domestic banking stress under various regimes in a currency union. In Regime ID, if domestic bailout funds are unavailable, the domestic commercial banking sector fails in the bad state.

In Regime FU.a, the visible hand of a fiscal union sets the amount of cross-country fiscal transfers mediated directly between households in different countries. Such transfers remove the stress from capital flows on member countries' domestic banking system. Domestic commercial banks survive even in the bad state. In Regime FU.b, the fiscal union makes transfers directly between commercial banks of different countries, subject to the total banking union fund constraint $\omega_{\boldsymbol{s}}^i + \omega_{\boldsymbol{s}}^j \geq 0$. However, Regime FU.a faces no such constraint. This constraint is in line with Bolton and Oehmke (2018) that study bank resolution of global banks and show that the loss-absorbing capital is shared across jurisdictions but faces implementation constraints.

In Regime BL, despite cross-country fiscal transfers being unavailable, a softened CMU bankruptcy code can help domestic banks bypass the pressure from international capital flow such that banks survive. The intuition is that a softened CMU bankruptcy code gives the domestic households the choice to default on their cross-country borrowing in the bad state. Because the domestic bankruptcy code is tougher than the CMU bankruptcy code in this regime, the marginal cost of default on domestic bank loans is higher than the household's marginal benefit of default. Rationally, the households choose not to default on domestic loans even in the bad state. The takeaway is the relative stance of domestic and CMU bankruptcy codes can change incentives on the margin and shift domestic credit risks to cross-border capital markets, relieving domestic banks from distress. By adjusting the CMU bankruptcy code, cross-border default provides a "voluntary" liquidity transfer via the capital markets, in the absence of a fiscal union.

With different extents of domestic banking stress and accordingly distinct needs for national level bailout tax, the regimes designed above bear different implications for allocation efficiency, risk sharing, and asset prices. Under the conditions of Propositions 3 and 4, the corollaries below formalise these implications for $h \in \{i, j\}$, and the online appendix contains their formal proofs.

Corollary 4.2 (allocation efficiency within state):

- In Regime ID, optimal allocation does not obtain in $t = 0$ due to domestic credit risks and the cost of liquidity; optimal allocation within state does not obtain in $t = 1$ due to the bailout cost causing the internal devaluation effect.

  At $t = 0$,

$$\frac{U^i_{c^i_{I0}}}{U^i_{c^i_{J0}}} = \frac{U^j_{c^j_{I0}}}{U^j_{c^j_{J0}}} \frac{1}{(1 + r_I)(1 + r_J)},$$

  and at $t = 1$, $\boldsymbol{s} \in S$

$$\frac{U^i_{c^i_{I\boldsymbol{s}}}}{U^i_{c^i_{J\boldsymbol{s}}}} = \frac{U^j_{c^j_{I\boldsymbol{s}}}}{U^j_{c^j_{J\boldsymbol{s}}}} \frac{1}{(1 + \tau_{I\boldsymbol{s}})(1 + \tau_{J\boldsymbol{s}})}.$$

- In Regime FU, optimal allocation efficiency does not obtain at $t = 0$ due to domestic credit risks and the cost of liquidity; optimal allocation within state obtains at $t = 1$.

  At $t = 0$,

$$\frac{U^i_{c^i_{I0}}}{U^i_{c^i_{J0}}} = \frac{U^j_{c^j_{I0}}}{U^j_{c^j_{J0}}} \frac{1}{(1 + r_I)(1 + r_J)},$$

  and at $t = 1$, $\boldsymbol{s} \in S$

$$\frac{U^i_{c^i_{I\boldsymbol{s}}}}{U^i_{c^i_{J\boldsymbol{s}}}} = \frac{U^j_{c^j_{I\boldsymbol{s}}}}{U^j_{c^j_{J\boldsymbol{s}}}}.$$

- In Regime BL, default in the cross-border capital markets obviates the need for national bailout tax, and optimal allocation does not obtain at $t = 0$ due to the cost of interbank liquidity; optimal allocation within state obtains at $t = 1$.

  At $t = 0$,

$$\frac{U^i_{c^i_{I0}}}{U^i_{c^i_{J0}}} = \frac{U^j_{c^j_{I0}}}{U^j_{c^j_{J0}}} \frac{1}{(1 + \rho)^2},$$

  and at $t = 1$, $\boldsymbol{s} \in S$

$$\frac{U^i_{c^i_{Is}}}{U^i_{c^i_{Js}}} = \frac{U^j_{c^j_{Is}}}{U^j_{c^j_{Js}}}.$$

We can observe that the wedge between the households' marginal rate of substitution across goods distorts allocation efficiency within state. Regime ID has the highest wedge due to domestic borrowing costs and the bailout cost, and the domestic borrowing cost incorporates the cost of liquidity and commands domestic credit risk premia. Regime BL has the lowest wedge which only results from the interbank transaction cost. Note that the transaction cost of money is Regime BL is just the interbank transaction cost, and it is lower than the borrowing cost in Regime FU. This is because the borrowing cost in Regime FU commands the credit risk premium of domestic loans, but that of Regime BL precludes it owing to the shield of default in the cross-border capital markets.

Corollary 4.3. (risk sharing):

- In Regime ID, optimal risk sharing does not obtain due to domestic credit risks, the cost of liquidity, and the bailout tax.

$$\frac{U^i_{c^i_{Is}}}{U^i_{c^i_{Is'}}} = \frac{U^j_{c^j_{Is}}}{U^j_{c^j_{Is'}}} \frac{1}{(1+\tau_{Js})(1+\tau_{Is})(1+r_I)(1+r_J)},$$ [18]

- In Regime FU, optimal risk sharing does not obtain due to borrowing costs which command the domestic credit risk premium and incorporate the cost of liquidity.

$$\frac{U^i_{c^i_{Is}}}{U^i_{c^i_{Is'}}} = \frac{U^j_{c^j_{Is}}}{U^j_{c^j_{Is'}}} \frac{1}{(1+r_I)(1+r_J)},$$

- In Regime BL when default in the cross-border capital markets obviates the need for national bailout tax, optimal risk sharing does not obtain due to two sources of inefficiency: the cost of interbank liquidity and the default premium in the capital markets union.

$$\frac{U^i_{c^i_{Is}}}{U^i_{c^i_{Is'}}} = \frac{U^j_{c^j_{Is}}}{U^j_{c^j_{Is'}}} \frac{K_s K_{s'}}{(1+\rho)^2},$$

The above expressions tell us that the wedge between the households' marginal rate of substitution across states distorts risk sharing, and that the wedge in Regime ID is the highest because of the extra distortion stemming from the bailout tax. Between Regime FU and Regime ID, however, it is not obvious whose wedge is higher. Both wedges in these two regimes incorporate the interbank transaction cost and credit/default risk premia.

Corollary 4.4. (asset pricing): Financial, monetary, and fiscal factors all affect the stochastic discount factor.

[18] The MRS across states for goods J is similar.

- In Regime ID, state prices are affected by the domestic credit risks, the cost of liquidity, and the bailout tax directly.

$$\pi_s = \gamma_s \frac{U^i_{c^i_{Js}}/p_{Js}}{U^i_{c^i_{I0}}/p_{I0}} = \gamma_s \frac{U^j_{c^j_{Is}}/p_{Is}}{U^j_{c^j_{J0}}/p_{J0}(1+\tau_{Js})(1+r_J)},$$

- In Regime FU, state prices are only directly affected by the domestic credit risks and the cost of liquidity.

$$\pi_s = \gamma_s \frac{U^i_{c^i_{Js}}/p_{Js}}{U^i_{c^i_{I0}}/p_{I0}} = \gamma_s \frac{U^j_{c^j_{Is}}/p_{Is}}{U^j_{c^j_{J0}}/p_{J0}(1+r_J)},$$

- In Regime BL, state prices are affected by the interbank transaction cost and the cross-border default premium directly.

$$\pi_s = \gamma_s \frac{U^i_{c^i_{Js}}/p_{Js}}{U^i_{c^i_{I0}}/p_{I0}} = \gamma_s \frac{U^j_{c^j_{Is}}/p_{Is}K_s}{U^j_{c^j_{J0}}/p_{J0}(1+\rho)},$$

Asset prices are typically suppressed by the transaction wedge. The interbank transaction cost, domestic loan credit risk premium, cross-border default premium, and the bailout cost all constitute the transaction wedge. We can observe from Corollary 4.4 that the state prices in Regime ID are typically lower than those in Regime FU and Regime BL, because Regime ID is distorted by the bailout tax as the extra transaction wedge. Since the total transaction cost in Regime ID is the highest, in this regime one unit of currency tomorrow is worth the lowest level of consumption goods today. Thus, using internal devaluation to sustain a currency union would also put downward pressure on asset prices. This mechanism is confirmed by the numerical examples in Section 4.2.

4 Welfare and Numerical Examples

4.1 Equilibrium Default and Welfare

The analysis so far suggests that both cross-country transfers and default in the capital markets union can reduce the transaction wedge distorting allocations and prices. Table 1 provides a summary of the transaction wedge in Corollaries 4.1-4.4. The dash "-" indicates the relative degree of inefficiency. The more dashes assigned, there are more sources of distortions to allocations, risk sharing, and asset prices.

Let us observe that compared with Regime ID, Regime BL's allocations, risk sharing, and asset prices are all much less distorted. The reason being bankruptcy leniency in the capital markets union can reduce the transaction cost for both countries in the currency union; thus, naturally, one expects Pareto improvement. Proposition 5 formalises this claim.

Table 1: Regime comparison

	Regime ID benchmark	**Regime FU** fiscal union	**Regime BL** bankruptcy leniency
Allocation within state	- - -	- -	-
Risk sharing	- - -	-	-
Asset prices	- - -	-	-
Austerity tax	yes	no	no
Banking crises	yes	no	no

[a] "-" indicates the relative degree of inefficiency.

Proposition 5. (Endogenous default and Pareto improvement):

- In the absence of a fiscal union, CMU bankruptcy leniency can Pareto improve a currency union.
- Consider the case where $S = \{1, 2\}$, let $\gamma_1 = \gamma_2$, $e^i_1 > e^i_2$, and $e^j_1 < e^j_2$. Consider Regime ID equilibrium, i.e. $\lambda^i, \lambda^j < \lambda$ and $v^i_2, v^j_1 < 1$. Now reduce λ to λ' such that $\lambda^i, \lambda^j > \lambda'$, whenever $\big(1 - \pi_2\big(v^i_2(1 + r_I) - 1\big)\big)/p_{I2} > \pi_2 r_I/p_{I1}$ and $\big(1 - \pi_1\big(v^j_1(1 + r_J) - 1\big)\big)/p_{J1} > \pi_1 r_J/p_{J2}$, it leads to a utility increase for both households i and j on the margin, while the expected utilities of domestic banking sectors remain unchanged.

Proof. See Appendix B.4.

Interestingly, Regime BL achieves Pareto improvement by introducing an additional friction. Typically when a friction is added, there is one more imperfection, and one might expect a worse outcome. Here, it is exactly the opposite. The currency union modelled so far bears two imperfections: the interbank transaction cost of money and the possibility to default on domestic loans, i.e., domestic credit risks. Suppose directly removing these two imperfections is not possible; Regime BL simply introduces the possibility to default on financial assets as a third imperfection. The third imperfection is introduced in such a way that the most "detrimental" imperfection, i.e., state-varying domestic credit risks, is not manifested in equilibrium, leading to a superior outcome. Indeed, the possibility to default on financial assets as in Regime BL can exactly shift borrowers' incentive on the margin to bypass the state-varying domestic credit risks, such that banks survive, obviating the need for the national bailout tax.

I hasten to add that the conditions identified in Proposition 5 are only sufficient but not necessary. The proof of Proposition 5 does not require setting the bailout tax to zero, which would provide an additional liquidity boost at $t = 1$ and increase consumption. However, as the softened bankruptcy code shifts the default incentive on the margin, the economy eventually shifts to the Regime BL equilibrium where the bailout tax is not even needed. Therefore, one can expect to further relax the conditions in Proposition 5 for Pareto improvement to occur. As analytic solutions are not available, the exact sufficient and necessary conditions are difficult to pin down, so I move onto numerical examples next.

4.2 Numerical Examples

As the model is stylised, I do not attempt to carry out a fully-fledged calibration exercise in this paper. The following numerical examples are provided to better illustrate the model's property. This subsection assumes two states (state s and state s') in $t = 1$ and solves the model numerically. Table 2 exhibits the exogenous parameters and functional forms.

4.2.1 CMU bankruptcy leniency and Pareto Improvement

Table 3 shows the numerical solutions of the endogenous variables of Regime ID, Regimes FU.a and FU.b, and Regime BL.[19] Let W^I denote country I's utility of consuming goods in both periods, SU^I denote country I's social utility, i.e., the consumption utility minus the social cost of default, and df denote the default rate. Regime ID leads to the least desirable equilibrium among the four regimes considered: the default rate on domestic loans in the bad state is high, asset prices are suppressed, domestic commercial bank loan rates are much higher than the policy rate due to a high credit risk premium, and bailout tax turns out positive as the domestic bank would become insolvent in the bad state. Hence, both the allocation welfare and social utility of Regime ID are quite low.

Table 2: Exogenous variables

Preference	$U^i = \alpha_0^i ln(c_{I0}^i) + (1-a_0^i) ln(c_{J0}^i) + \beta^i E_0[\alpha_{\boldsymbol{s}}^i ln(c_{I\boldsymbol{s}}^i) + (1-a_{\boldsymbol{s}}^i) ln(c_{J\boldsymbol{s}}^i)]$
	$U^j = \alpha_0^j ln(c_{J0}^j) + (1-a_0^j) ln(c_{I0}^j) + \beta^j E_0[\alpha_{\boldsymbol{s}}^j ln(c_{J\boldsymbol{s}}^j) + (1-a_{\boldsymbol{s}}^j) ln(c_{I\boldsymbol{s}}^j)]$
	$\alpha_0^i = \alpha_0^j = 0.5 \quad \alpha_s^i = \alpha_{s'}^j = 0.7 \quad \alpha_{s'}^i = \alpha_s^j = 0.3$
Prob. of states	$\gamma_s = \gamma_{s'} = 0.5$
Discount factor	$\beta^i = \beta^j = 1$
Endowment	$e_{I0}^i = e_{J0}^j = 10 \quad e_{Is}^i = e_{Js'}^j = 15 \quad e_{Is'}^i = e_{Js}^j = 5$
Bankruptcy code	$\lambda^i = \lambda^j = 0.1036 \quad \lambda^C = 0.098 \quad \lambda^A = \lambda^B = 0.12$
Outside money	$m^i = m^j = 0.1$
Policy rate	$\rho = 0.01$

Table 3: Endogenous variables

Regime ID Equilibrium (Baseline)					
$W^I = 3.2921$	$SU^I = 3.2496$	$df_{s'}^i = 13.02\%$	$\pi_{\boldsymbol{s}} = 0.4807$	$r^I = 8.03\%$	$\tau_{Is'} = 6.45\%$
Regime FU.a Equilibrium					
$W^I = 3.2943 \uparrow$	$SU^I = 3.2879 \uparrow$	$df_{\boldsymbol{s}}^i = 3.8\% \downarrow$	$\pi_{\boldsymbol{s}} = 0.4875 \uparrow$	$r^I = 4.49\% \downarrow$	$\tau_{Is'} = 0\% \downarrow$
Regime FU.b Equilibrium					
$W^I = 3.2943 \uparrow$	$SU^I = 3.2711 \uparrow$	$df_{s'}^i = 6.75\% \downarrow$	$\pi_{\boldsymbol{s}} = 0.4889 \uparrow$	$r^I = 4.53\% \downarrow$	$\tau_{Is'} = 0\% \downarrow$
Regime BL Equilibrium					
$W^I = 3.2947 \uparrow$	$SU^I = 3.2916 \uparrow$	$df_{s'} = 1.63\% \downarrow$	$\pi_{\boldsymbol{s}} = 0.4934 \uparrow$	$r^I = 1\% \downarrow$	$\tau_{Is'} = 0\% \downarrow$

[a] Arrows indicate the direction of travel compared with Regime ID.

Regime FU.a is a fiscal union with no banking union. Default rates on domestic loans are the same across states, and are lower than Regime ID, the internal devaluation regime; hence, the domestic commercial bank loan rates are lower. Moreover, asset prices, social utility, and allocation welfare all improve upon Regime ID, and the bailout tax is not needed. Regime FU.b, a fiscal union with a banking union, exhibits a similar improvement as Regime FU.a.

Regime BL assumes away cross-country fiscal transfers, but a lenient CMU bankruptcy code allows for default in the cross-border capital markets. Regime BL's default rate is of asset

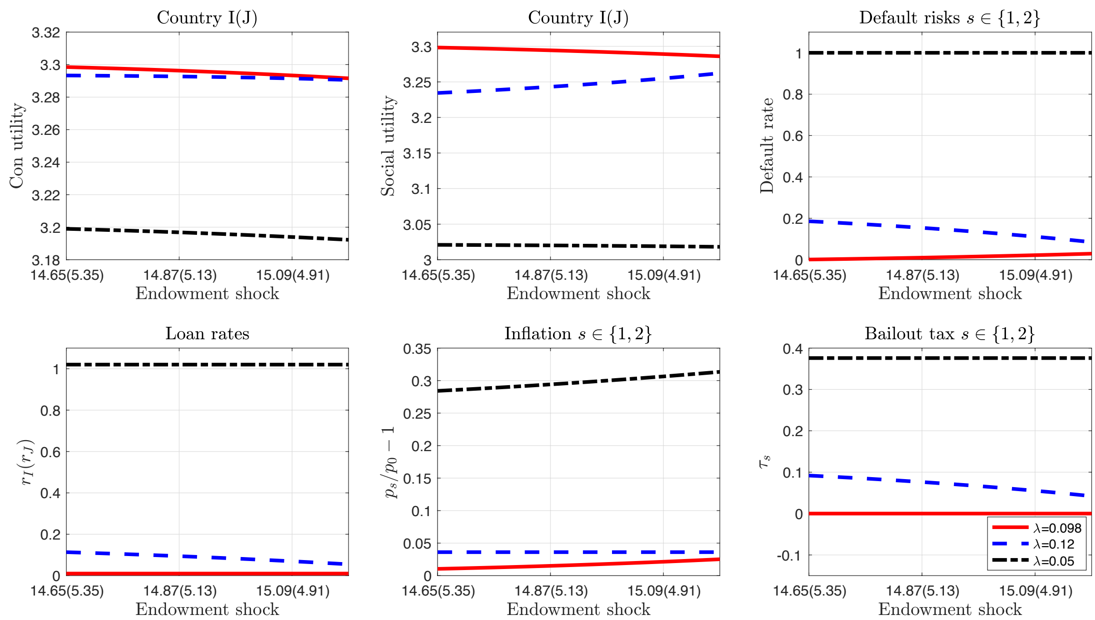
[19] As country J is symmetric to country I, here I only display the prices and allocations of country I for conciseness.

payoffs. Since CMU bankruptc leniency induces the default risks to move away from domestic loans to alleviate domestic banking stress, the domestic loan rate is equal to the monetary policy rate. Similar to Regime FU.a and Regime FU.b, allocation welfare, social utility, and asset prices are all higher than in Regime ID, and the bailout tax is also not needed in Regime BL.[20]

4.2.2 When Does Bankruptcy Leniency Not Work?

Nevertheless, a caveat remains on the welfare-improving role of CMU bankruptcy leniency. As Corollary 3.1 states, when the CMU bankruptcy code λ is set too low, no households in either country would ever repay cross-country borrowing, leading to a collapse of the cross-border capital markets. Figure 4 provides numerical support to this claim. The horizontal axis represents the state-contingent endowments of the two states in each country. The solid line represents the economy when the CMU bankruptcy code is set to 0.098, only slightly more lenient than the domestic bankruptcy codes ($\lambda^h = 0.1036, h \in \{i, j\}$). The dashed line shows the economic allocation and prices when the CMU bankruptcy code is set to 0.12, much harsher than the domestic bankruptcy codes. The dashed line essentially represents Regime ID. The dash-dotted line represents financial autarky, where an ultra-low CMU bankruptcy code ($\lambda = 0.05$) causes the collapse of financial markets and impedes risk sharing.

Figure 4: CMU VS domestic bankruptcy codes



As illustrated in Figure 4, the case with a slightly softened CMU bankruptcy code obtains the best allocation and social utility. In this case, assets are defaultable but are still traded, domestic borrowing costs in both countries are low, and neither country needs to levy bailout taxes since banks do not experience insolvency even in the bad state. A harsher CMU bankruptcy code, represented by the dashed line, shifts the equilibrium to Regime ID. Domestic borrowing costs are higher and bailout taxes are levied. The worst case among those considered here is financial autarky. It is when cross-border default backfires due to an ultra-lenient CMU bankruptcy code. In this case, no financial assets are traded for risk sharing, and the limited risk sharing is provided by money and state-dependent inflation, which

[20] I also check by varying income risks around the initial parameterisation of $e^h_{Hs} (h \in \{i, j\}, H \in \{I, J\})$ set out in Table 2, the results remain robust.

acts as a shock “absorber”. Because of the impediment to risk sharing, countries in the bad state are poorer than the case with risk sharing, hence, their marginal utility of consumption increases, implying a high marginal benefit of defaulting on domestic loans. It turns out quite dramatically that, given the domestic bankruptcy codes as $\lambda^h = 0.1036, h \in \{i, j\}$, both countries default 100 % on their domestic loans in their respective bad states. Banks would become insolvent if not for the national governments' bailout. As a consequence, borrowing costs and default risks are exorbitantly high, and bailout tax rates increase to around 37.6 %. Because limited risk sharing implies a collapse of goods trade at $t = 1$, inflation in both states soars up.

4.2.3 CMU Bankruptcy Stance

It is fair to say the case of financial autarky is engineered by an unrealistically ultra-lenient CMU bankruptcy code because it would imply that a currency union has an extremely weak institution to enforce repayment and implement default punishment. But what if the CMU bankruptcy code is just slightly softened ($\lambda = 0.098$), as the solid line represents in Figure 4, and then we additionally soften the union-wide bankruptcy by a small increment, how would the economy behave? Figure 5 conducts such an experiment.

Figure 5: CMU bankruptcy code and equilibrium default

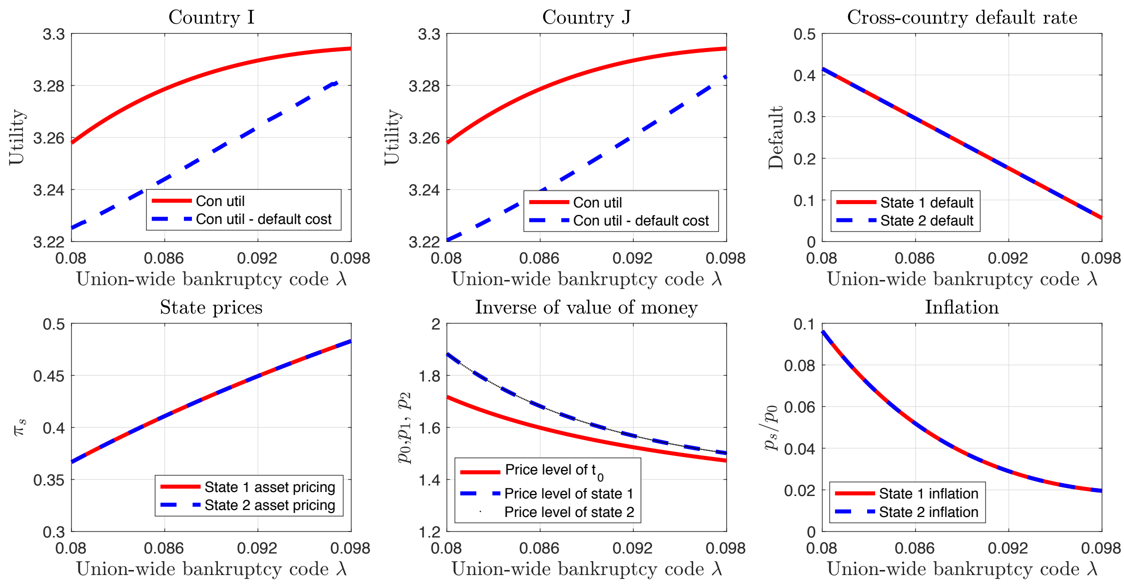
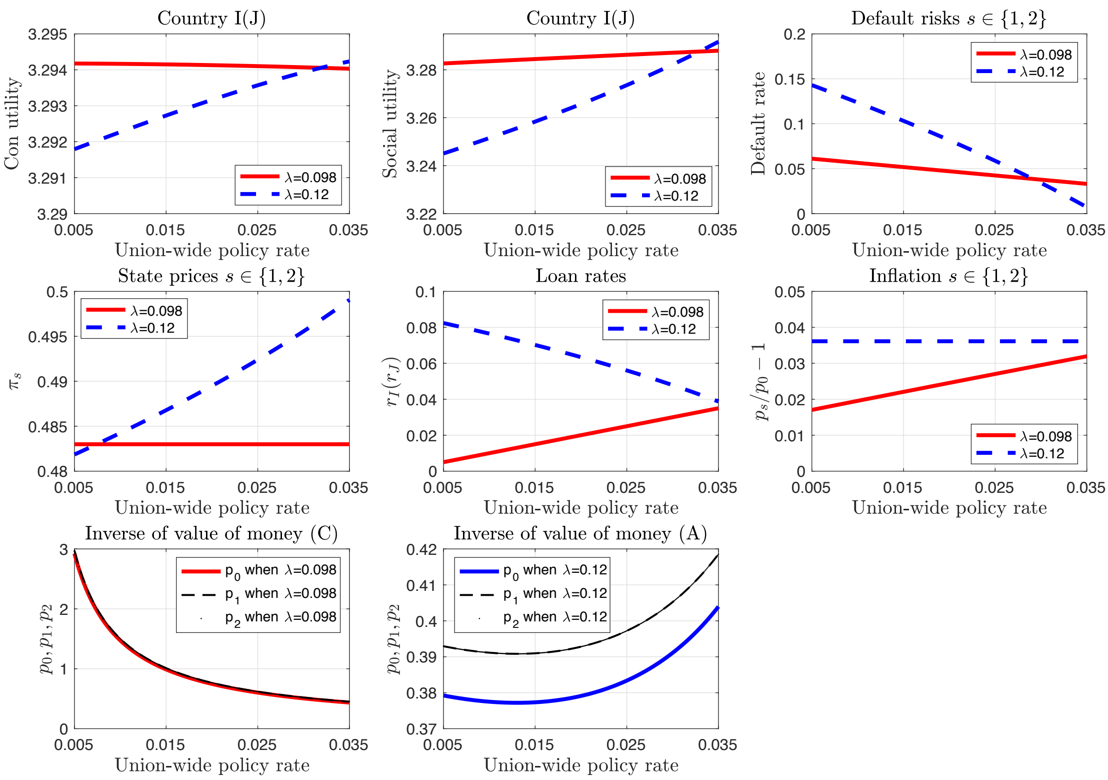


Figure 5 varies the CMU bankruptcy code (i.e., the horizontal axis λ) by small amounts but ensures it is slightly more lenient than domestic bankruptcy codes. Therefore, what we observe is the comparative statics within Regime ID with default in the cross-border capital markets. As λ decreases, the further left horizontally, the marginal cost of default decreases and the substitution effect dominates in equilibrium. Further to the left, therefore, default risks in the capital markets union increase, pushing down asset prices and generating inflationary pressure. As default risks in the cross-border capital markets increase, the value of the common currency decreases. The modelling of inside money and endogenous default naturally enables the model to produce the endogenous relationship between default and the value of money.

4.2.4 Union-wide Monetary Policy

Albeit a currency union has a single monetary authority and the “one size” of its policy tool does not fit all, it remains to be of interest to study the properties of such monetary regime, particularly the interplay of credit risks and value of the currency. This is a relevant exercise because after the Eurozone crisis, the monetary authority has kept rates low based on the classic argument about stimulating aggregate demand; however, the union-wide output recovery has been sluggish and goods prices remain subdued despite large quantities of reserve injection via asset purchase programmes. This phenomenon has sparked off a heated debate on "monetary hysteresis". Figure 6 conducts such policy experiment. It suggests that when the Eurozone has limited cross-country transfers and a tough stance on cross-border default, the classic demand effect of low policy rates might not prevail, because we also need to consider the related impact on the demand and supply of credit and liquidity.

Figure 6: Union-wide monetary policy



The first two rows in Figure 6 display the equilibrium of Regime BL (the solid line) and that of Regime ID (the dashed line), while varying the union-wide monetary policy rate (the horizontal axis). Counterintuitively, when the currency union is in the internal devaluation regime (Regime ID), a lower policy rate actually worsens allocation and tends to push down the price level of goods (see the last subplot of Figure 6), which could shed light on the slow growth and subdued prices in the Eurozone after the crisis. In Regime ID, a lower policy rate, i.e., further to the left on the horizontal axis, leads to an ample amount of interbank liquidity. On the margin, the influx of interbank liquidity (reserves) *ex ante* transmits to a lower repayment pressure on domestic borrowers *ex post*. Since a finite-horizon setup prevents commercial banks from "evergreening" their loans, in the bad state, this propagation causes a rise in NPLs. Therefore, domestic borrowing costs rise and stifle goods trade. Moreover, the rise in domestic borrowing costs hampers domestic credit extension, leading to a contraction in the price levels of goods. This result connects with a growing empirical literature on banks' risk-taking when the monetary policy rate remains low.[21]

We can also observe, however, when the currency union is in Regime BL (the solid line),

[21]See for example, Heider et al. (2019), Dell'Ariccia et al. (2017), Jiménez et al. (2014), and Maddaloni and Peydró (2011).

the effect of monetary policy is more conventional. A lower policy rate decreases borrowing costs and improves allocation welfare, although the social cost of default decreases.

5 A Counterfactual: Regime FX

So far, the model has shown the possibility of welfare improvement due to CMU bankruptcy leniency in the cross-border capital markets. However, the model has not explicitly explained why CMU bankruptcy leniency is *particularly* vital in a currency union. To understand this question, one needs to know what benefits a currency union has given up and whether cross-border bankruptcy leniency can recoup these benefits.

Therefore, in this section I consider the counterfactual of credible national currencies priced by competitive floating exchange rates. I call this scenario Regime FX. Not to complicate the model exceedingly, I do not consider the transition from a currency union to national currencies but only conduct comparative statics of equilibria of a currency union and of national currencies. I also do not consider the scenario in which national governments intervene in the foreign exchange market, nor do I assume heterogeneous bankruptcy codes between countries. Consequently, the floating exchange rate movements in this section are purely competitive due to income shocks, rather than due to different policy or institutional stances.

Regime FX considers a frictionless foreign exchange market that opens at $t = 0$ and $t = 1$. As in Regime ID, Regime FX assumes a harsh CMU bankruptcy code and no cross-country fiscal transfers. It only considers the friction of possible domestic credit risks stemming from idiosyncratic income shocks. Rather than having a union-wide central bank, each country now has its own national central bank that issues its own national currency. Moreover, each country issues its own *currency-specific Arrow securities* whose payoffs are in the respective national currencies. Outside money and accordingly seigniorage in each country are also denominated in the respective national currencies. Linking the countries are asset markets, goods markets, and particularly frictionless foreign exchange markets.

Therefore, compared with a currency union, this regime has an additional market, i.e., the foreign exchange market, and for each state, there are two types of nominal Arrow securities issued by country I and country J respectively. Rather than having a common interbank loan and money market, this regime has two interbank loans and money markets, one in each country. The foreign exchange market is assumed to meet twice. At $t = 0$, it opens immediately after the loan markets, and at $t = 1$. it opens before import purchases. Since all the other features remain the same as in the benchmark regime, I do not describe Regime FX's model setup in detail for conciseness. The online appendix provides the formalisation of Regime FX.[22]

Proposition 6 (credit risk neutralisation): With competitive floating exchange $\chi_{\boldsymbol{s}}$, set the bailout tax to zero, the domestic credit risk turns out state invariant, i.e., $v^h_{\boldsymbol{s}} = v^h, \forall \boldsymbol{s} \in S, h \in \{i, j, \}, H \in \{I, J\}$.

Proof. See Appendix B.5.

The proof of Proposition 6 B.5 shows that at $t = 2$, household i's loan settlement equation can be rearranged as $v^i_{\boldsymbol{s}}\mu^i_I = \frac{\mu^i_I}{1+r_I} + m^i$, and household j's loan settlement equation can be rearranged as $v^j_{\boldsymbol{s}}\mu^j_J = \frac{\mu^j_J}{1+r_J} + m^j$. We can observe that currency I, which is issued against bank loan μ^i_I in country I at $t = 0$, ends up repaying the very same loan against which

[22]For completeness, the online appendix also contains classic results such as PPP and UIP, as well as proofs.

the currency itself is issued. Similarly, currency J ends up repaying the very same loan against which currency J is issued. Because the face value of the loans is state invariant, the loan repayments must also be state invariant. Therefore, currency exits the system exactly from its own originating country. This is made possible by the FX markets and the floating exchange rates.

To see the intuition of the proof, let us observe the following equation, which is the loan repayment in country I.

$$v_s^i \mu_I^i = \sum_{l=1}^{S} \pi_{Il} \phi_{Il}^i + p_{I0} q_{I0}^i + p_{Is} q_{Is}^i - \phi_{Is}^i - f_{IJs}^i. \tag{11}$$

Household i can increase the loan repayment by ϵ through either decreasing f_{IJs}^i or increasing $p_{Is}q_{Is}^i$ at $t = 1$. Since ϕ_{Is}^i is the position of asset sales of the Arrow security $l = s$ at $t = 0$, it cannot be adjusted at $t = 1$. Suppose household i sells ϵ less f_{IJs}^i, this leads currency I to appreciate, χ_s decreases and $\chi_s f_{JIs}^j$ decreases by ϵ. Thus, country J ends up having ϵ less currency I to buy imports from country I, and $p_{Is}q_{Is}^i$ therefore decreases by ϵ. Overall, the right-hand side of (11) remains unchanged. This is a contradiction. If household i increases $p_{Is}q_{Is}^i$ by ϵ instead, that means the amount of currency I used to purchase household i's exports has to increase by ϵ and χ_s increases, leading to a depreciation. The same contradiction arises. Similar logic obtains if household i wants to achieve $v_s^i < v_{s'}^i$, which also causes contradiction.

Corollary 6.1. (capital flow and banks' survival): Competitive floating exchange rates χ and $= \chi_{\boldsymbol{s}}$ prevents domestic banking insolvency, $\omega_{\boldsymbol{s}}^h = 0$, $\forall \boldsymbol{s} \in S$, $h \in \{i, j\}$, and there is no need for national bailout tax.

Corollary 6.2. Under the conditions of Proposition 5, if currency union is the regime *a priori*, equilibrium default due to CMU bankruptcy leniency obviates the need for floating exchange rates to alleviate domestic banking stress.

Corollary 6.3. (Balance of Payments): Under competitive floating exchange rates and when FX-in-advance binds, current account and capital account exactly balance, $CA_{\boldsymbol{s}}^H + FA_{\boldsymbol{s}}^H = 0, H \in \{I, J\}, \boldsymbol{s} \in S$.

The online appendix provides formal proofs for the above corollaries. Corollaries 6.1 and 6.3 hold under very general conditions. However, in an ideal scenario with zero domestic credit risks, banks get loans repaid in full for all states of nature, and given markets are complete, the role of floating exchange rates is trivialised (Lemma 5). In other words, if countries do not have domestic credit risks in all states of nature, even without a fiscal union or CMU bankruptcy leniency, sharing a common currency brings no particular harm to the health of the banking sector.

As the floating exchange rate neutralises domestic credit risks, one would expect a lower transaction wedge in Regime FX than in Regime ID, because there is no need to level national bailout tax. The following numerical examples confirm the welfare improvement of Regime FX, and the online appendix contains the formalisation of its effect on allocation efficiency within state, risk sharing, and asset pricing.

5.1 Numerical Examples

Table (4) displays the numerical result of credible national currencies and compares it with those of currency unions. Note that the functional forms and exogenous variables are the same as in the baseline regime in a currency union. The unique endogenous variables in the national currency case are the exchange rates at $t = 0$ and $t = 1$.

The national currency case obtains a superior equilibrium to the internal devaluation regime of a currency union. This is because competitive floating exchange rates neutralise domestic credit risks and banks survive for all states, such that bailout taxes are not required. Exchange rate determination obtains via the FX markets. At country I's good state $s = 1$, currency I depreciates and at country I's bad state $s = 2$, currency I appreciates. Therefore, the domestic liquidities flowing back to country I for domestic loan repayment turn out the same for both states, domestic credit risk is state-invariant, and domestic banks do not encounter insolvency due to the shortage of liquidity causing credit risk volatilities. Nevertheless, the equilibrium of the national currency case is slightly inferior to the cross-border default regime in a currency union, suggesting default in the cross-border capital markets in a currency union can obviate the need for floating exchange rates.

Table 4: National currencies and currency unions

Currency union with internal devaluation (Regime ID)					
$W^I = 3.2921$	$SU^I = 3.2496$	$df^i_{s'} = 13.02\%$	$r^I = 8.03\%$	$\tau_{Is'} = 6.45\%$	$\chi = NA$
Currency union with cross-border default (Regime BL)					
$W^I = 3.2947 \uparrow$	$SU^I = 3.2916 \uparrow$	$df_{s'} = 1.63\% \downarrow$	$r^I = 1\% \downarrow$	$\tau_{Is'} = 0\% \downarrow$	$\chi = NA$
National currencies and floating exchange rates (Regime FX)					
$W^I = 3.2943 \uparrow$	$SU^I = 3.2713 \uparrow$	$df_{s'} = 3.35\% \downarrow$	$r^I = 1\% \downarrow$	$\tau_{Is'} = 0\% \downarrow$	$\chi_0 = 1$
					$\chi_1 = 1.07$
					$\chi_2 = 0.94$

[a] Arrows indicate the direction of travel compared with Regime ID.

6 Discussions and policy implications

6.1 Discussion of results

The first key result of the model is that softening the bankruptcy code to allow for default in the cross-border capital markets can improve a currency union in the absence of a fiscal union. Because cross-border equilibrium default provides an extra boost of liquidity for the borrowing country in the bad state. This extra liquidity acts like "cross-country liquidity transfers" to adjust for the country heterogeneity in a currency union, and consequently it proves to be a close substitute for a fiscal union. The key difference is that a fiscal union resembles a supranational entity that uses a visible hand to move nominal resources between countries. In contrast, cross-border default works through the invisible hand of markets: default is a choice and assets are conditionally traded voluntarily.

This key difference can be regarded as the strength of CMU bankruptcy leniency compared to a fiscal union. Indeed, the fiscal union modelled in Regime FU is assumed to be benevolent. Once the model relaxes the benevolence assumption, it would add further support to Regime BL (bankruptcy leniency) as a feasible approach to sustain currency unions.

However, the limitation of Regime BL (bankruptcy leniency) is that it relies on the assumption that the institutional qualities or the economic fundamentals of member countries in the currency union do not differ significantly. Suppose the two countries have divergent economic fundamentals to start with, for example, country J in its good state is much poorer than country I in its respective good state. Then in state s', i.e., country I's bad state and country J's good state, such a lenient CMU bankruptcy code may encourage country J to even fully default in its good state. In this scenario, capital markets union collapses, impeding international risk sharing and leading to a welfare loss. In this case, the within-union standard fails, and there is simply no room to adjust for CMU bankruptcy code. If the model is extended to incorporate production and capital accumulation for multi-periods, the issue becomes more acute. Adjusting the bankruptcy code would need to be timely because a delay would lead countries to fall out of the within-union standard endogenously due to the persistent internal devaluation effect. Moreover, even if their fundamentals do not differ too much, if country I has a lower institution quality, i.e., its domestic bankruptcy code λ^i is much weaker than that of country J, Corollary 3.2 suggests that there simply may not be any space left to adjust the union-wide bankruptcy code λ.

The second key result is that adjusting the bankruptcy code in a currency union can obviate the need for nominal exchange rates to neutralise domestic credit risks. However, nominal exchange rates could potentially incur (unmodelled) costs, such as competitive devaluation because of discretionary intervention by national governments. Such considerations would render CMU bankruptcy leniency in a currency union a more attractive regime than national currencies.

Furthermore, this model sets the CMU bankruptcy code of the capital markets union conditional on domestic bankruptcy codes, which are taken as structural parameters. In contrast, Franks and Sussman (2005) constructed an endogenous evolutionary theory of bankruptcy codes, in which bankruptcy codes can emerge either due to freedom of contracting of market participants or because of state activism in law-making. In this paper, I do not model the endogenous emergence of the bankruptcy codes, but I acknowledge that in practice the softening of the bankruptcy code in the capital markets union could emerge via either market forces or law-making, or a combination of both.

6.2 Policy implications and implementation

The immediate policy recommendation is that a currency union needs a capital markets union that features a slightly softened bankruptcy environment. Softening the cross-border bankruptcy environment can take measures to encourage more reorganisation rather than immediate liquidation of assets when companies become insolvent, to shorten the period of credit market exclusion for the defaulters, or to allow for debt discharges and adjustments. To increase the space to design the CMU bankruptcy code to induce benign default in the capital markets union, the domestic bankruptcy code, however, should not be too lenient. Thus, when a fiscal union is absent in a currency union, member countries should strengthen their domestic institutions by toughening the domestic bankruptcy code for domestic borrowing, and at the same time mutually agree on a softened CMU bankruptcy code for the cross-border capital market.

As for the implementation of the CMU bankruptcy code, it should be common knowledge to all market participants and should be *strictly* enforced. In this environment, market participants *ex ante* should know precisely the severity of the default punishment conditional on the size of default. Accordingly, default risks are priced in when assets are traded prior to the realisation of uncertainty.[23] This kind of implementation, therefore, ensures orderly

[23] For example, suppose the CMU bankruptcy code takes the form of credit exclusion as the default punishment. Market participants should know how many periods the defaulter is to be excluded given the size of default. Once a borrower announces default, the defaulter should be excluded from credit to the extent that market participants have come to expect.

default. It is essential to note the distinction between this type of orderly default and the unanticipated default whose risk is not correctly priced in.

The model is kept stylised such that the results and transmission mechanisms are clear; therefore, the model does not specify the exact form of default punishment. A fully-fledged and more realistic quantitative model could certainly provide further micro-foundations for default punishment, such as credit exclusion, debt restructurings, or sanctions. Nevertheless, no matter what the exact form of default punishment is, as long as it feeds into the marginal rate of substitution across consumptions, the essence of the current stylised model remains.

6.3 Institutional Details

6.3.1 The TARGET2 System

The model abstracted away from the national central banks; thus, it is difficult to find an exact mapping between the model and the TARGET2 system, which is the Eurozone payment system connecting national central banks (NCB)) and the European Central Bank (ECB). However, the two key results of this model can still shed light on the ongoing debate on the imbalance of the TARGET2 system (see Sinn 2011; Sinn and Wollmershäuser 2012; Whelan 2014).

TARGET is the acronym for *Trans-European Automated Real-time Gross Settlement Express Transfer System*. In the TARGET2 system, all Eurosystem banks maintain reserve accounts with their national central banks, which are ultimately connected by ECB as the lender of last resort. The TARGET2 system equips the national central banks with some degree of money creation power such that the creation of the euro via ECB becomes less rigid. Therefore, the TARGET2 system allows some national central banks to hold TARGET2 claims and some other national central banks to hold TARGET2 liabilities.

During the Eurozone Crisis, capital flight from the weak periphery mainly took the form of depositors moving deposits from weak banks in the periphery into the stronger banks in the core. This transfer of funds would have bankrupted many banks in the periphery, but it was recycled back through the TARGET2 system. In this sense, the TARGET2 system provides some cross-country transfers already, partly resembling the fiscal transfers in Regime FU (fiscal union) of my model. Therefore, in reality, the TARGET2 system has likely reduced the bailout costs of the Eurozone Crisis and lightened the internal devaluation effect in Regime ID of this model in the short run.

Nevertheless, the imbalance of the TARGET2 system is not completely the same as the cross-country fiscal transfers in Regime FU. This is because the imbalance of TARGET2 system takes the form of claims and liabilities. These positions are debt positions that eventually need to be settled, whereas Regime FU makes fiscal transfers that are free and clear of any debt obligation. If the debt positions on the TARGET2 system need to be repaid in full eventually, it would merely imply the delay of national bailout costs and internal devaluation. If the debt positions on the TARGET2 system were to carry priced-in credit risks and allow for default, a similar welfare-improving benefit could arise as that of Regime BL (bankruptcy leniency) in this paper.

6.3.2 Cross-border Insolvency Reforms in Practice

Now let us turn to the practical interpretations of bankruptcy leniency in conjunction with the institutional details of cross-border insolvency in the Eurozone. Private contracting tends to take a punitive stance towards cross-border bankruptcy in Europe. This is partly owing to legal uncertainty. Thus, practical steps have been taken in the European Union for the cross-border insolvency reform. In 2012, the European Commission proposed to recast the 2000 Insolvency Regulation with the main purpose to help identify the competent jurisdiction and applicable law insolvency proceedings. As the 2000 Insolvency Regulation carries legal uncertainty with some of its key concepts defined in general terms (see Sussman 2006). Then the 2015 recast regulation provided further clarification on the "centre of main interest" (COMI). More importantly, in 2016, the European Commission proposed to adopt a directive on business restructuring and giving the troubled businesses a second chance, softening the traditionally punitive stance on default.

In spirit, the 2016 directive is similar to Chapter 11 of the US Bankruptcy Code, and it seems broadly consistent with the policy implication of my model. Nevertheless, in practice, the cross-border insolvency reforms in EU or the Eurozone are extremely complicated because the national-level bankruptcy codes are very diverse (see Davydenko and Franks 2008). My model speaks to this issue because my result on softening the CMU bankruptcy code is exactly conditional on national-level bankruptcy codes and takes into consideration of their heterogeneity. Thus, my result does not lead to the conclusion of an unconditional top-down harmonisation.

7 Conclusion

This paper has proposed an international finance model specifically for currency unions to address the following question: what alternative arrangements can improve the financial stability and viability of currency unions when a fiscal union is absent? The model is able to show that one alternative arrangement is a financial regime that features a softened bankruptcy environment in the cross-border capital markets within a currency union.

There are two contributions of this proposed theory. First, the model shows that when domestic credit risks are present and when there is no fiscal union, endogenous default in the capital markets union within a currency union can lead to a Pareto improvement and enhance the viability of a currency union. Second, the model answers the question of why bankruptcy leniency for the cross-border capital markets is particularly vital for currency unions. That is, under very general conditions competitive floating exchange rates are shown to neutralise domestic credit risks and improve welfare, and sharing a common currency loses such benefits of exchange rates. However, equilibrium default in the cross-border capital markets can recoup the lost benefits of exchange rates. Therefore, when countries join a currency union with an incomplete fiscal union or banking union, the bankruptcy code needs to adjust.

I acknowledge that the endogenous determination of default punishment and the endogenous adoption of currency unions should be the subject of further research. The bankruptcy code λ is the key parameter of default punishment, and it is essentially the price of default. Suppose countries take the price of default as given and "demand" default, then the system would need the "supply" of default to endogenise the price. This issue sits right at the intersection between law, finance, and political economy. The underlying political-economic consideration is likely to stimulate further exciting research (see Foarta 2018 and Perotti and Soons 2020).

In conclusion, a broader role of this paper is an initial attempt to bridge the gap between the value of money, exchange rate determination and bankruptcy codes in international finance. This opens new avenues for research on the endogenous emergence of dominant international currencies and global financial cycles.

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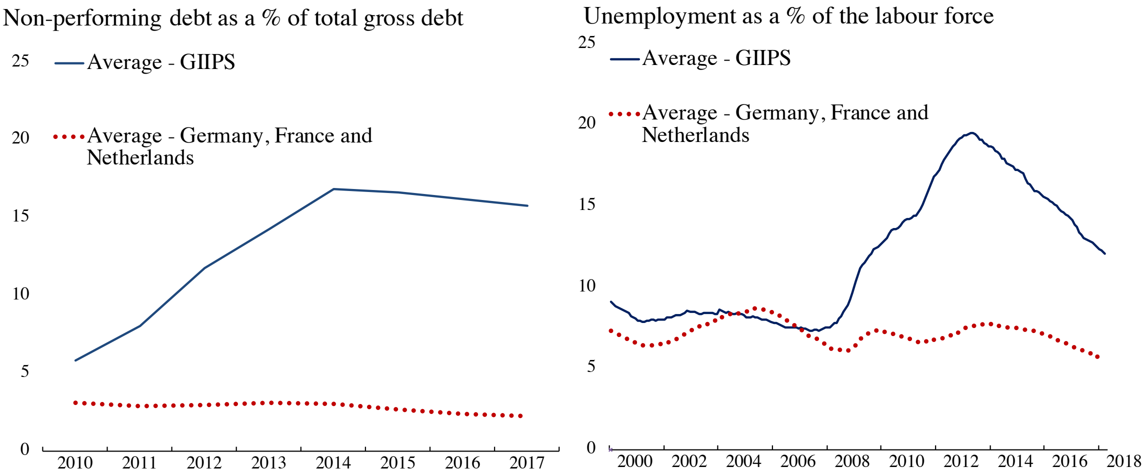
Appendices

Figure A1: Country heterogeneity in the Eurozone

(a) Non-performing loans

(b) Unemployment rate



Source: Statistical Data Warehouse, ECB and OECD Economic Surveys: Euro Area 2018. Average of first three quarters. GIIPS: Greece, Ireland, Italy, Portugal and Spain.

Source: Eurostat database and OECD Economic Surveys: Euro Area 2018, calculated as unweighted average. GIIPS: Greece, Ireland, Italy, Portugal and Spain.

A Equilibrium and Regime Characterisation

A.1 Proof of Lemma 1.

Proof. First, I identify the condition for household i's liquidity-in-advance constraint to bind. Suppose $r_I > 0$ and suppose further household i does not spend on all the money at hand, i.e. $\Delta(1) > 0$. The household can borrow ϵ less loan, but it only leads to $\epsilon/(1+r_I)$ reduction on the money at hand from (1).

Moreover, from (3), the reduction of loan needing to repay is $v_{\boldsymbol{s}}^i\epsilon$, and the pecuniary benefit due to the reduction of default cost amounts to $(1-v_{\boldsymbol{s}}^i)\epsilon$. Thus, there is extra money $(v_{\boldsymbol{s}}^i - 1/(1+r_I)+1-v_{\boldsymbol{s}}^i)/\epsilon = r_I\epsilon/(1+r_I)$ at $t=2$. Household i can either use this extra money to consume more or increase loan repayment, and both lead to welfare improvement. This is a contradiction; hence, $\Delta(1) = 0$.

Second, I prove the binding condition for bank i's liquidity-in-advance constraint. Suppose bank i does not extend all the money at hand as interbank liquidity to household i, i.e. $\Delta(4) > 0$. Bank i can borrow ϵ less interbank loan, but it only leads to $\epsilon/(1+\rho)$ reduction on the money at hand in (6). The reduction of loan needing to repay is ϵ, which means there is an extra money of $\rho\epsilon/(1+\rho)$ in (6) that adds to bank i's cash flow, increasing its utility. This is a contradiction. Hence, $\Delta(4) = 0$. □

A.2 Proof of Lemma 2

Proof. Suppose $\Delta(5) > 0$, bank i can reduce the interbank liquidity L^i by ϵ, given (4) binds, it means bank i borrows $(1+\rho)\epsilon$ less interbank loans. From (6), the money at hand is reduced by ϵ, but the interbank loan needing to repay is reduced by $(1+\rho)\epsilon$. This means there is an extra money of $\rho\epsilon$ that adds to bank i's cash flow at $t=2$, increasing its utility. This is a contradiction. Hence, $\Delta(5) = 0$. □

A.3 Proof of Lemma 3.

Proof. Suppose $\Delta_{\boldsymbol{s}}(3) > 0$, then household i can either increase repayment rate $v_{\boldsymbol{s}}^i$ or borrow ϵ more at the interest rate of r_I. If household i increases $v_{\boldsymbol{s}}^i$, it leads to an immediate increase in utility because the cost of default decreases This improvement of welfare means a contradiction. If household i borrows ϵ more, then household i has an extra money of $\epsilon/(1+r_I)$ and uses it to buy more imports or sell less exports, leaving all other actions unchanged, without violating the inequality (3), because the household has enough money at hand to repay the extra loan. The improvement of welfare means a contradiction.

Similarly contradiction arises if $\Delta_{\boldsymbol{s}}(6) > 0$. Therefore, $\Delta_{\boldsymbol{s}}(3) = 0$ and $\Delta_{\boldsymbol{s}}(6) = 0$. □

A.4 Proof of Lemma 4.

Proof. Suppose $\phi^i_s > 0, \theta^i_{s'} > 0$. From household i's perspective, the FOC for ϕ^i_s leads to $\pi_s \frac{U^i_{c^i_{I0}}}{p_{I0}} = (1+\tau_{Is})\gamma_s \eta^I_{1s}$. FOC for $\theta^i_{s'}$ leads to $\pi_{s'} \frac{U^i_{c^i_{J0}}}{p_{J0}} = \gamma_{s'}\eta^i_{1s'}$. Suppose $\eta^i_{1s} = \eta^i_{1s'}$, and also the FOCs for c^i_{I0}, c^i_{J0}, and μ^i_I lead to $\frac{U^i_{c^i_{J0}}}{p_{J0}} = (1+r_I)\frac{U^i_{c^i_{I0}}}{p_{I0}}$, it follows $\frac{\pi_s}{(1+\tau_{Is})\gamma_s} = \frac{\pi_{s'}(1+r_I)}{\gamma_{s'}}$. Hence,

$$\pi_s \gamma_{s'} > \pi_{s'} \gamma_s. \tag{12}$$

On the other hand, from household j's perspective, household j's FOC for θ^j_s leads to $\pi_s \frac{U^j_{c^j_{I0}}}{p_{I0}} = \gamma_s \eta^j_{1s}$, FOC for $\phi^j_{s'}$ gives $\pi_{s'} \frac{U^j_{c^j_{J0}}}{p_{J0}} = \gamma_{s'}(1+\tau_{Js'})\eta^j_{1s'}$. Given $\eta^j_{1s} = \eta^j_{1s'}$, and also the FOCs for $c^j_{J0}, c^j_{I0}, \mu^j_J$ lead to $\frac{U^j_{c^j_{I0}}}{p_{I0}} = (1+r_J)\frac{U^j_{c^j_{J0}}}{p_{J0}}$, it follows that $\frac{\pi_s(1+r_J)}{\gamma_s} = \frac{\pi_{s'}}{(1+\tau_{Js'})\gamma_{s'}}$. Hence,

$$\pi_1 \gamma_2 < \pi_2 \gamma_1. \tag{13}$$

We can see that (13) contradicts (12).

Suppose $\theta^i_s > 0, \phi^i_{s'} > 0$, by the same logic, a contradiction also arises. Therefore, it is not possible that $\eta^i_{1s} = \eta^i_{1s'}$ and $\eta^j_{1s} = \eta^j_{1s'}$. □

A.5 Proof of Lemma 5

Proof. It follows from the banks' FOC that $\forall \boldsymbol{s} \in S$, $h \in \{i,j\}$, $\omega^h_{\boldsymbol{s}} = (\frac{v^h_{\boldsymbol{s}}}{\sum_{\boldsymbol{s}=1}^S \eta_{\boldsymbol{s}} v^h_{\boldsymbol{s}}} - 1)\mu^h_{CB}$. Given the assumption that $v^h_{\boldsymbol{s}} = 1$, $\forall \boldsymbol{s} \in S$, $\omega^h_{\boldsymbol{s}} = 0$. □

A.6 Proof of Proposition 1

- The Fisher effect

  Denote η^i_0, $\eta^i_{\boldsymbol{s}1}$, and $\eta^i_{\boldsymbol{s}2}$ as the shadow prices of the three flow of funds constraints of household i. Suppose that household i has some money left over the moment the inter-period domestic loan comes due. From household i's FOCs for c^i_{J0} and μ^i_I, it follows that $\eta^i_0 = (1+r_I)E_0\eta^i_{\boldsymbol{s}2}$, namely, $\frac{U^i_{c^i_{J0}}}{p_{J0}} = (1+r_I)E_0\frac{U^i_{c^i_{I\boldsymbol{s}}}}{p_{I\boldsymbol{s}}}$. And also household i's FOCs for $c^i_{I\boldsymbol{s}}$ and $c^i_{J\boldsymbol{s}}$ give $\frac{U^i_{c^i_{I\boldsymbol{s}}}}{p_{I\boldsymbol{s}}} = \frac{U^i_{c^i_{J\boldsymbol{s}}}}{p_{J\boldsymbol{s}}(1+\tau_{I\boldsymbol{s}})}$. It follows that

$$1 + r_I = \Big(E_0(\frac{U^i_{c^i_{J\boldsymbol{s}}}}{U^i_{c^i_{J0}}})(\frac{p_{J0}}{p_{J\boldsymbol{s}}})\frac{1}{(1+\tau_{I\boldsymbol{s}})(1+\iota_{I\boldsymbol{s}})}\Big)^{-1}.$$

- Money non-neutrality

  For the general method of proof on money non-neutrality, please see Tsomocos (2001, 2003). Here a specific proof by contradiction is provided. Combine household i's FOCs for c^i_{I0}, c^i_{J0}, and μ^i_I, we obtain

$$\frac{U^i_{c^i_{J0}}}{p_{J0}} = (1+r_I)\frac{U^i_{c^i_{I0}}}{p_{I0}}, \tag{14}$$

  combine household j's FOCs for c^j_{J0}, c^j_{I0}, and μ^j_J, we obtain

$$\frac{U^j_{c^j_{I0}}}{p_{I0}} = (1+r_J)\frac{U^j_{c^j_{J0}}}{p_{J0}}. \tag{15}$$

Suppose the union-wide central bank increases the interbank rate ρ. Suppose consumption and default remain unchanged. Based on banks' FOCs that $1+r_I = \frac{1+\rho}{\sum_{s=1}^S z_s v^i_s}$ and $1+r_J = \frac{1+\rho}{\sum_{s=1}^S z_s v^j_s}$, we can see that r_I and r_J both increase. Eq(14) implies $\frac{p_{I0}}{p_{J0}}$ has to increase, whereas Eq(15) implies that $\frac{p_{J0}}{p_{I0}}$ has to increase. This is a contradiction. □

A.7 Proof of Corollary 1.1

Proof. Let z_s be the risk-neutral probabilities $\forall s \in S$. Bank i's FOC has $1+r_I = \frac{1+\rho}{\sum_{s=1}^S z_s v^i_s}$, $\omega^i_s = v^i_s \mu^i_I - \mu^i_{CB} = (\frac{v^i_s}{\sum_{s=1}^S z_s v^i_s} - 1)\mu^i_{CB}$. Given the assumption $v^i_s > \sum_{s=1}^S z_s v^i_s$, and $v^i_{s'} < \sum_{s=1}^S z_s v^i_s$, then $\omega^i_s > 0$, and $\omega^i_{s'} < 0$. Similarly, for bank j, $\omega^j_{s'} > 0$ and $\omega^j_s < 0$.

At state s, the union-wide central bank's profits $\omega^{CB}_s = \mu^i_{CB} - \frac{\mu^i_{CB}}{1+\rho} + v^j_s \mu^j_s - \frac{\mu^j_{CB}}{1+\rho} + T^j_s$.

Rearranging the algebra and plugging in the market clearing condition for interbank loans and money, as well as bank j's FOCs,

$$\begin{aligned}
\omega^{CB}_s =& \mu^i_{CB} - \frac{\mu^i_{CB}}{1+\rho} + v^j_s \mu^j_J - \frac{\mu^j_{CB}}{1+\rho} + T^j_s \\
=& \mu^i_{CB} - \frac{\mu^i_{CB}}{1+\rho} + v^j_s \mu^j_J - \frac{1+\rho}{1+r_J}\mu^j_J + \frac{1+\rho}{1+r_J}\mu^j_J - \frac{\mu^j_{CB}}{1+\rho} + T^j_s \\
=& \frac{\rho}{1+\rho}(\mu^i_{CB} + \mu^j_{CB}) + (v^j_s - \frac{(1+\rho)\sum_{s=1}^S z_s v^j_s}{1+\rho})\mu^j_J + T^j_s \\
=& \frac{\rho}{1+\rho}(\mu^i_{CB} + \mu^j_{CB}) + (v^j_s - \frac{(1+\rho)\sum_{s=1}^S z_s v^j_s}{1+\rho})\frac{(1+\rho)}{\sum_{s=1}^S z_s v^j_s}\frac{\mu^j_{CB}}{1+\rho} + T^j_s \\
=& \rho M + (\frac{v^j_s}{\sum_{s=1}^S z_s v^j_s} - 1)\mu^j_{CB} + T^j_s,
\end{aligned}$$

Moreover, via the flow of funds and budget constraints of the households and commercial banks, total outside money that flows into the union-wide central bank equals $\sum_{h\in\{i,j\}} m^h - \omega^i_s$. Thus,

$$\rho M + (\frac{v^j_s}{\sum_{s=1}^S z_s v^j_s} - 1)\mu^j_{CB} + T^j_s = \sum_{h\in\{i,j\}} m^h - \omega^i_s.$$

Similar proof follows for state s',

$$\rho M + (\frac{v^i_{s'}}{\sum_{s=1}^S z_s v^i_s} - 1)\mu^i_{CB} + T^i_{s'} = \sum_{h\in\{i,j\}} m^h - \omega^j_{s'}.$$

□

A.8 Proof of Proposition 2

Proof. At country I's bad state s' when credit risks are present, suppose $\lambda > \lambda^i$, it means the marginal cost of default on financial securities is larger than the marginal cost of default on domestic loans. At the bad state s', household i's FOC for $v^i_{s'}$ gives $\frac{\lambda^i}{p_{s'}} = \frac{U^i_{c^i_{Is'}}}{p_{Is'}}$, it follows that

$$\frac{\lambda}{p_{s'}} > \frac{U^i_{c^i_{Is'}}}{p_{Is'}}. \tag{16}$$

The left-hand side of (16) is the marginal cost of default on financial securities and the right-hand side of (16) is the marginal benefit. Since the marginal cost of default on financial securities is larger than the marginal benefit, household i fully delivers on cross-country borrowing, i.e. $D^i_{s'} = \phi^i_{s'}$.

At the good state household i's FOC for v^i_s gives $\frac{\lambda^i}{p_s} \geq \frac{U^i_{c^i_{Is}}}{p_{Is}}$. Since $\lambda > \lambda^i$, it follows that $\frac{\lambda}{p_s} > \frac{U^i_{c^i_{Is}}}{p_{Is}}$. Hence, $D^i_s = \phi^i_s$.

Overall, $\forall \boldsymbol{s} \in S$, $D^i_{\boldsymbol{s}} = \phi^i_{\boldsymbol{s}}$.

Now suppose the cross-country bankruptcy code is more lenient, i.e. $\lambda < \lambda^i$. It means the marginal cost of default on financial securities is smaller than the marginal cost of default on domestic loans. At the bad state, $\lambda \leq \frac{U^i_{c^i_{Is'}}}{p_{Is'}}$, and $\frac{\lambda^i}{p_{s'}} \geq \frac{U^i_{c^i_{Is'}}}{p_{Is'}}$. It is less costly to default on financial securities, so household would default on financial securities first instead of domestic loans, i.e. $D^i_{s'} < \phi^i_{s'}$.

At the good state household i's FOC for v^i_s gives $\frac{\lambda^i}{p_{s'}} \geq \frac{U^i_{c^i_{Is'}}}{p_{Is'}}$, since $\lambda < \lambda^i$, the relationship between $\frac{\lambda}{p_{s'}}$ and $\frac{U^i_{c^i_{Is'}}}{p_{Is'}}$ is ambiguous. The marginal cost of default on financial securities can be larger than, or equal to, or smaller than the marginal benefit. Therefore, $D^i_s \leq \phi^i_s$.

Overall, $\forall \boldsymbol{s} \in S$, $D^i_{\boldsymbol{s}} \leq \phi^i_{\boldsymbol{s}}$. The same logic follows for the case of country J. □

B Equilibrium Analysis

B.1 Proof of Proposition 3

Proof. Since $\frac{\bar{\lambda}}{p_2} = \eta^i_{11}$, $\frac{a_2\bar{\lambda}}{p_2} = \eta^i_{22}$ holds. Given $\eta^j_{12} < \frac{\lambda}{p_2}$ and $a_2\lambda < a_2\bar{\lambda} < \lambda^i$, then $\eta^j_{12} < \frac{\lambda}{p_2} < \eta^i_{12}$. These conditions are the on-the-verge conditions between the households' marginal benefit of default and their marginal cost of default on Arrow securities. Thus, if holding short positions, household i defaults on Arrow security $l = 2$ fully while household j fully delivers the payoff of Arrow security $l = 2$.

Due to risk-aversion, it is straightforward to see that household i buys Arrow security $l = 2$ because $e^i_1 > e^i_2$, $\eta^i_{11} < \eta^i_{12}$, and household j sells Arrow security $l = 2$ because $e^j_1 < e^j_2$, $\eta^j_{11} > \eta^j_{12}$. However, to see why household i also sells Arrow security $l = s'$ at the same time whenever $\big(K_2 - \pi_2\big(v^i_2(1 + r_I) - 1\big)\big)/p_{I2} > \pi_2 r^l/p_{I1}$ holds, I show via proof by contradiction.

Suppose $\phi^i_2 = 0$ is the equilibrium outcome. Suppose now household i sells ϵ amount of Arrow security $l = 2$. According to the market clearing condition of Asset markets, household i would need to buy ϵ amount of Arrow security $l = 2$. This means household i needs to borrow $\pi_2\epsilon$ more money at $t = 0$ while needing to pay back $\pi_2\epsilon v^i_2(1 + r_I)$ at state 2, so the extra monetary cost is $\pi_2\epsilon\big(v^i_2(1+r_I)-1\big)$. However, because household i can default fully on the ϵ amount of Arrow security sold, the extra money inflow due to default amounts to $K_2\epsilon$, where $K_2 = 1 - \frac{\epsilon}{\sum_{h\in i,j}\phi^h_2}$. Thus, the total money inflow $K_2\epsilon - \pi_2\epsilon\big(v^i_2(1 + r_I) - 1\big)$ is positive. For the good state 1, $\eta^i_{21} < \frac{\lambda^i}{p_1}$ holds, $v^i_1 = 1$, so the extra monetary cost is $\pi_2\epsilon r_l$. If $\Big(K_2 - \pi_2\big(v^i_2(1 + r_I) - 1\big)\Big)/p_{I2} > \pi_2 r^l/p_{I1}$ holds, then $\nabla U^i(\cdot)(K_2\epsilon - \pi_2\epsilon\big(v^i_2(1+r_I)-1\big)) > \nabla U^i(\cdot)(\pi_2\epsilon r^l)$ holds, leading to an overall increase in household i's expected utility. This is inconsistent with $\phi^i_2 = 0$ as an equilibrium outcome. □

B.2 Proof of Lemma 6

Proof: Given $\lambda > \lambda^h$, $h \in \{i, j\}$. at state s, $FA^I_s = -\phi^i_s$, then $\delta^i_s = \phi^i_s - p_{Is}q^i_{Is} + p_{Js}c^i_{Js}$, and $\delta^j_s = -\theta^j_s - p_{Js}q^j_{Js} + p_{Is}c^j_{Is}$. Because $\phi^i_s = \theta^j_s$, $p_{Is}q^i_{Is} = p_{Is}c^j_{Is}$, and $p_{Js}c^i_{Js} = p_{Js}q^j_{Js}$, it follows that

$\delta_s^i + \delta_s^j = 0$. Moreover, invoking Lemma 1, $v_s^i \mu_I^i = \frac{\mu_J^j}{1+r_J} + m^j + p_{Is} q_{Is}^i - p_{Js} c_{Js}^i - \phi_s^i + \delta_s^i = \frac{\mu_J^j}{1+r_J} + m^j$, and $v_s^j \mu_J^j = \frac{\mu_I^i}{1+r_I} + m^i$.

Similarly, for state s', $\delta_{s'}^i + \delta_{s'}^j = 0$, $v_{s'}^i \mu_I^i = \frac{\mu_J^j}{1+r_J} + m^j$, $v_{s'}^j \mu_J^j = \frac{\mu_I^i}{1+r_I} + m^i$.

Therefore, $v_s^i = v_{s'}^i = v^i$, and $v_s^j = v_{s'}^j = v^j$, i.e. $var(1 - v_{\boldsymbol{s}}^{h,B.a}) = 0$.

□

B.3 Proof of Proposition 4

Proof: Commercial bank h's FOC gives $\omega_{\boldsymbol{s}}^h = (v_{\boldsymbol{s}}^h - \sum_{\boldsymbol{s}=1}^S z_{\boldsymbol{s}} v_{\boldsymbol{s}}^h)\mu_H^h$. In Regime ID, $\lambda > \lambda^h$ and $\delta_{\boldsymbol{s}}^h = 0$ for $h \in \{i,j\}$, $\forall \boldsymbol{s} \in S$. Given $v_{\boldsymbol{s}}^h < \sum_{\boldsymbol{s}=1}^S z_{\boldsymbol{s}} v_{\boldsymbol{s}}^h$, $\omega_{\boldsymbol{s}}^h < 0$. In Regime FU.a, $\lambda > \lambda^h$, and $\delta_{\boldsymbol{s}}^h = -FA_{\boldsymbol{s}}^H - CA_{\boldsymbol{s}}^h$, as shown in Proof of Lemma 6 $v_{\boldsymbol{s}}^h = v^h$. It follows $v_{\boldsymbol{s}}^h - \sum_{\boldsymbol{s}=1}^S z_{\boldsymbol{s}} v_{\boldsymbol{s}}^h = 0$ and $\omega_{\boldsymbol{s}}^h = 0$.

For the case of Regime BL, the conditions are more exacting. Given the conditions in Proposition 3, i.e. consider the case where $S = \{1,2\}$, let $\gamma_1 = \gamma_2$, $e_1^i > e_2^i$, and $e_1^j < e_2^j$. Suppose that in equilibrium $\lambda < p_2\eta_{12}^i < \lambda^i$, $\eta_{12}^j < \frac{\lambda}{p_2}$ and $\eta_{11}^i < \eta_{12}^i$ holds. Suppose $\big(K_2 - \pi_2\big(v_2^i(1+r_I) - 1\big)\big)/p_{I2} > \pi_2 r_I/p_{I1}$ holds, then, $\phi_2^i, \phi_2^j, \theta_2^i > 0$, $\theta_2^j = 0$, $D_2^i = 0$, $D_2^j = \phi_2^j$, and $0 < K_2 < 1$. Assume similar conditions for the other state. Thus, households purchase and sell the Arrow security of their respective bad state in order to default fully on the Arrow security, without defaulting on domestic loans. It follows that $v_{\boldsymbol{s}}^h = 1$ and $\omega_{\boldsymbol{s}}^h = 0$, where $\boldsymbol{s} \in \{1,2\}$.

□

B.4 Proof of Proposition 5

Proof. First I show decreasing λ to λ' leads to utility increase for both household i and household j on the margin.

In the Regime ID equilibrium, $p_2\eta_{12}^i = \lambda^i < \lambda$, it follows that

$$\frac{\lambda}{p_2} > \eta_{12}^i. \tag{17}$$

The marginal cost of default on financial securities is larger than the marginal benefit, so household i fully delivers on assets. Given $\eta_{11}^i < \eta_{12}^i$, $\phi_2^i = 0$ and $\theta_2^i > 0$.

Now let us decrease λ to λ', and $\lambda' < p_2\eta_{12}^i = \lambda^i$. It follows that

$$\frac{\lambda'}{p_2} < \eta_{12}^i. \tag{18}$$

Equation (18) states the marginal cost of default on financial securities is larger than the marginal benefit, so household i would fully default if holding short positions.

Suppose now household i sells an infinitesimal ϵ of Arrow security $l = 2$, i.e. $\phi_2^i = \epsilon$. According to the market clearing condition of asset markets, household i would need to buy ϵ amount of Arrow security $l = 2$. This means household i needs to borrow $\pi_2\epsilon$ more money at $t = 0$ while needing to pay back $\pi_s \epsilon v_2^i(1+r_I)$ at $t = 2$, so the extra monetary cost is $\pi_2\epsilon(v_2^i(1+r_I)-1)$. However, because household i fully defaults on the ϵ amount of Arrow security sold, so the extra money inflow due to default amounts to $K_2\epsilon$, where $K_2 = 1 - \frac{\epsilon}{\sum_{h\in\{i,j\}} \phi_s^h}$. Thus the total money inflow at state 2 is $K_s\epsilon - \pi_2\epsilon(v_2^i(1+r_I)-1)$. For the good state 1, the monetary cost is $\pi_s \epsilon r_I$. Since $\Big(K_2 - \pi_2\big(v_2^i(1+r_I) - 1\big)\Big)/p_{I2} > \pi_2 r_I/p_{I1}$ holds, then on the margin $\nabla U^i(\cdot)(K_2\epsilon - \pi_2\epsilon\big(v_2^i(1+r_I) - 1\big)) > \nabla U^i(\cdot)(\pi_2\epsilon r_I)$ holds, leading to an overall increase in household i's expected utility.

Similarly, setting $\lambda = \lambda^{ID}$ leads to an overall increase in household j's expected utility on the margin.

Second I show that the expected utilities of commercial banks are zero and remain unchanged. From Proposition 4, $\omega_{\boldsymbol{s}}^h = (v_{\boldsymbol{s}}^h - \sum_{\boldsymbol{s}=1}^S z_{\boldsymbol{s}} v_{\boldsymbol{s}}^h)\mu_H^h$, $h \in \{i, j\}$, $H \in \{I, J\}$. It follows that $U^h(\cdot) = \sum_{\boldsymbol{s}=1}^S z_{\boldsymbol{s}} \omega_{\boldsymbol{s}}^h = \sum_{\boldsymbol{s}=1}^S (z_{\boldsymbol{s}}(v_{\boldsymbol{s}}^h - \sum_{\boldsymbol{s}=1}^S z_{\boldsymbol{s}} v_{\boldsymbol{s}}^h)\mu_H^h) = (\sum_{\boldsymbol{s}=1}^S z_{\boldsymbol{s}} v_{\boldsymbol{s}}^h - \sum_{\boldsymbol{s}=1}^S z_{\boldsymbol{s}} \sum_{\boldsymbol{s}=1}^S z_{\boldsymbol{s}} v_{\boldsymbol{s}}^h)\mu_H^h = 0$. □

B.5 Proof of Proposition 6

Proof. Simplify the households' flow of funds constraints, we have the loan repayment equation below:

$$v_{\boldsymbol{s}}^i \mu_I^i = \sum_{l=1}^S \pi_{Il} \phi_{Il}^i + p_{I0} q_{I0}^i + p_{I\boldsymbol{s}} q_{I\boldsymbol{s}}^i - f_{IJ\boldsymbol{s}}^i - \phi_{I\boldsymbol{s}}^i. \tag{19}$$

Since $\sum_{l=1}^S \pi_{Il} \phi_{Il}^i + p_{I0} q_{I0}^i = \sum_{l=1}^S \pi_{Il} \theta_{Il}^j + b_{I0}^j = \chi f_{JI}^j - \Delta_2^j = f_{IJ}^i - \Delta_2^j$. Substitute it in (19), we have

$$v_{\boldsymbol{s}}^i \mu_I^i = f_{IJ}^i - \Delta_2^j + p_{I\boldsymbol{s}} q_{I\boldsymbol{s}}^i - f_{IJ\boldsymbol{s}}^i - \phi_{I\boldsymbol{s}}^i. \tag{20}$$

With market clearing conditions $\theta_{I\boldsymbol{s}}^j = \phi_{I\boldsymbol{s}}^i$, $f_{IJ\boldsymbol{s}}^i = f_{JI\boldsymbol{s}}^j \chi_{\boldsymbol{s}}$ and $p_{I\boldsymbol{s}} q_{I\boldsymbol{s}}^i = b_{I\boldsymbol{s}}^j$, and also $\theta_{I\boldsymbol{s}}^j + f_{JI\boldsymbol{s}}^j \chi_{\boldsymbol{s}} + \Delta_2^j = b_{I\boldsymbol{s}}^j$, (20) becomes

$$v_{\boldsymbol{s}}^i \mu_I^i = f_{IJ}^i. \tag{21}$$

equivalent to

$$v_{\boldsymbol{s}}^i \mu_I^i = \frac{\mu_I^i}{1 + r_I} + m^i. \tag{22}$$

Similarly for country J

$$v_{\boldsymbol{s}}^j \mu_J^j = \frac{\mu_J^j}{1 + r_J} + m^j. \tag{23}$$

Thus, $v_{\boldsymbol{s}}^i = v^i$ and $v_{\boldsymbol{s}}^j = v^j$. □