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Going Through The Roof: On Prices for Drugs Sold Through Insurance

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Going Through The Roof: On Prices for Drugs Sold Through Insurance*

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Abstract. We offer a theory of how the combination of budget constraints and insurance drives up prices. A natural context for our theory is the health care market, where drug prices can be very high. Our model predicts that monopoly prices for orphan drugs are inversely related to the prevalence up until a maximum price. This is supported by empirical evidence in the literature. As a result, prices of drugs sold by a monopoly treating rare serious diseases are doomed to go sky high.

Key Words: Monopoly pricing, Insurance, Orphan Drugs

JEL Classification: D42, G22, I13, L12.

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1. Introduction

There is widespread concern about drug prices.[1] According to AHIP (2016)[2] almost half of the studied 150 specialty medications has a price of more than 100,000$ per patient per year. The yearly costs of some of those drugs exceed 500,000$. Clearly, this exceeds the budget of many households. This is also shown by Shrank *et al*. (2006), Gagne *et al*. (2014), and Kesselheim *et al*. (2016). They report that, at least in the US, a considerable group of patients cannot even afford to follow prescribed treatments because the drugs are too expensive. Many of these drugs treat rare diseases. Prices are especially high for medication treating rare diseases. For instance, Messori *et al.* (2010) and Medic *et al.* (2017) show that prices for orphan drugs[3] in European countries are inversely related to the prevalence of the disease.

We offer a simple theory which explains how budget constraints and optional insurance interact to drive up prices. In our model, this leads to a largely inverse relation between prices and prevalence. The market for health care is a perfect and relevant example of where this can occur. To simplify our exposition, we frame our model within this context.

Consider a monopoly selling an orphan drug and consider the effects of having that drug covered by insurance. First, it reduces financial risks. For simplicity, we ignore this by assuming risk neutral individuals.[4] Second, it allows individuals, who cannot afford the drug *ex post*, to buy insurance policy at a much lower price *ex ante*, see, *e.g.*, De Meza (1983) and Nyman (1999). In a perfectly competitive insurance market, if only 1 out of 10 individuals gets ill, the policy price is only 1/10th of the drug price. Those who can afford the insurance but not the drug, gain access to the drug through the insurance. They will choose to buy insurance provided that the price of the drug does not exceed its

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[1] See, *e.g.*, "Poll: Voters are angry about prescription drug costs", Opinion, The Hill, 05/24/2019, and "What to expect in EU health in the year ahead: drug pricing will dominate the debate", Politico Europe, 12/18/2019.

[2] AHIP (2016), *High-Priced Drugs: Estimates of Annual Per-Patient Expenditures for 150 Specialty Medications*. AHIP is the US trade association of the health insurance community.

[3] The definition of orphan drugs is country-specific, but always refers to cures for diseases which both very rare and are either life threatening or have other severe consequences for the quality of life. See, *e.g.*, McCabe *et al.* (2005), for orphan drugs' prevalence thresholds in the US, EU, UK, Japan, and Australia.

[4] Our main results continue to hold with risk averse individuals.

expected benefit to patients. In this way insurance expands the demand for the drug. Another consequence is that every time the prevalence rate of the disease is halved, the price of the drug can double without affecting the insurance price and, thus, the demand. We will see that the optimal price of the firm approximates this inverse relation to the prevalence of the disease.

Table 1 shows the lifetime prevalence (LTP) for some diseases which can be fatal or otherwise detrimental to a patient's well-being. *E.g.*, the odds that an individual will get multiple sclerosis (MS) is about 0.00189. Our model suggests that if the expected

Disease	**LTP**	**Source**
Breast Cancer	0.125	Feuer *et al*. (1993)
Congenital Heart Disease	0.01311 (children), 0.0612 (adults)	Marelli *et al*. (2014)
Active epilepsy	0.004	MacDonald *et al*. (2000)
Parkinson's Disease	0.002	MacDonald *et al*. (2000)
Amyotrophic Lateral Sclerosis	0.0008 – 0.0043	Johnston *et al*. (2006)
Multiple Sclerosis	0.00189	Eaton *et al*. (2010)
Crohn's Disease	0.00230	Eaton *et al*. (2010)
Pemphigus	0.00007	Eaton *et al*. (2010)
Primary Biliary Cirrhosis	0.00013	Eaton *et al*. (2010)

Table 1. Lifetime Prevalence (LTP) for selected diseases

disutility of MS is large compared to the average income in society, inclusion of MS medication in insurance could drive the price up by roughly a factor 500, with a maximum of the expected disutility of a MS patient.[5]

The intuition for our results is that health insurance expands what an individual can afford. This is relevant only if the benefit derived from the good exceeds the individual's ability to pay. With health, more than with common goods like apples, purchasing decisions are driven by what the patient can afford rather than by taste. Individuals only buy the policy or the drug if they have enough budget. This is illustrated by a considerable empirical literature which reports that individual income/budget is the most significant and powerful driver for different sorts of insurance (see, *e.g.*, Yuan and Jiang 2015 for China).

[5] Our model considers the case of a single effective drug, supplied by a monopolist, in the absence of countervailing buyer power.

This limits the scope of our analysis to diseases which are severe. Cures for throat aches or other minor complaints are unlikely to be affected by the mechanism described in this paper. Given that the disease is severe enough, the prevalence of the disease and its severity play different roles. The rarity of a disease affects the extent to which this mechanism increases the price. The expected benefit of the drug determines the maximum price the drug can obtain.

The literature offers other explanations for why drugs for rare diseases tend to be expensive. One explanation, and this is what pharmaceutical companies often claim, is that these drugs are expensive because they need to recover development costs in a smaller market (see, *e.g.*, Bosanquet *et al.*, 2003). This is a sunk cost fallacy. An alternative explanation is that high development costs lead to less entry, especially in smaller markets. As a result, firms in smaller markets are less likely to face competition, which results in higher prices. We are not satisfied with this explanation for two reasons. First, for the case of orphan drugs, most markets are a monopoly (see, *e.g.* Simoens, 2011), so there is little variation in the amount of competition. Second, Kesselheim (2016) does not find a positive correlation between development costs and prices. As development costs drive the entry decision, such a correlation should exist if the market entry decision causes the inverse relationship between prices and market size (disease prevalence).

There is a substantial literature on drug prices from other perspectives. A large part of this literature studies price controls and regulations; see, *e.g.*, Scott Morton (1997), Duggan and Scott Morton (2006 and 2010), Sood *et al.* (2009), Brekke *et al.* (2011). In our model we focus on a monopolist that is free of such controls. Another part analyses how drug manufacturers' market power is affected either by the size and bargaining power of private insurers (see, *e.g.*, Lakdawalla and Sood 2009, and Lakdawalla and Yin 2015) or through market competition (see, *e.g.*, Gaynor *et al.* 2000). In our model, we consider a simple case of a price-setting monopoly, without competition or countervailing market power. This makes our model especially relevant for orphan drugs. Lakdawalla (2018) provides an overview on these topics and many others, related to the economics of the pharmaceutical industry. Our model provides a theoretical explanation of why the drug prices and disease prevalence can be inversely related.

Finally, the access provided through insurance was discussed earlier by, among others, De Meza (1983) and Nyman (1999). However, unlike this earlier literature, we explore the consequences on the pricing of the products accessed via the insurance. Our findings clearly show the relevance of this approach.

The paper is organized as follows. In the following section, we provide an example which illustrates the main idea of the paper. The model is presented in Section 3 and is then analyzed in Section 4. Section 5 concludes and discusses some remaining issues. Appendix contains all proofs.

2. Example

Consider a unit mass of risk neutral individuals. Each individual has a budget w, and individual budgets are uniformly distributed on the $[0,1]$ interval. Each individual has a chance $q \in \left(0, \frac{1}{2}\right)$ of becoming ill, a *patient*. An individual who does not fall ill is *healthy* and receives utility $u^{\mathrm{H}}(w) = w$. The utility of an untreated patient is

$$u^{\mathrm{S}}(w) = u^{\mathrm{H}}(w) - s = w - s,$$

where s is the utility loss due to the disease. For this example, we assume that the consequences of getting this disease are severe, namely $s > 1$. Consequently, the individual's willingness to pay for preventing this loss is equal to his budget.

There is a cure for the disease, a drug, which is supplied by a monopolist. The monopolist has zero production cost and sets price p. Buying the drug cures the disease, preventing utility loss s. The utility of a patient who buys the drug is

$$u^{\mathrm{D}}(w, p) = u^{\mathrm{H}}(w - p) = w - p$$

Without insurance, an individual buys the drug if and only if he is a patient, if he can afford the drug, *i.e.* $p \leq w$, and if the price does not exceed the benefit, *i.e.* $p \leq s$. Thus, the demand for the drug in case of no insurance is

$$D^{\mathrm{NI}}(p) = q \, \mathbb{Pr}\{p \leq s, \, p < w\} = \begin{cases} q(1-p), & \text{if } p \leq 1 \\ 0, & \text{if } p > 1 \end{cases}$$

Maximizing monopoly profit $\pi^{\mathrm{NI}}(p) = pD^{\mathrm{NI}}(p)$ yields optimal monopoly price $p^{\mathrm{NI}} = \frac{1}{2}$ and a profit level of $\pi^{\mathrm{NI}}(p^{\mathrm{NI}}) = \frac{1}{4}q$.

Suppose now that a competitive insurance market is available, where individuals can buy full insurance against premium r, $r = qp$. In return, insured patients receive the drug for free. Insured individual receives utility

$$u^{\mathrm{I}}(w, r) = u^{\mathrm{H}}(w - r) = w - r$$

Individuals with budget $w \in [0, r)$ can afford neither drug nor insurance. They remain uninsured and untreated if ill. Individuals with budget $w \in [r, p)$ can only afford the insurance, and individuals with budget $w \in [p, 1]$ can afford both.

If $p < s$, the expected benefit of the insurance, qs, exceeds its cost, $r = qp$. That means that all individuals with budget $w \in [r, p)$ become insured and, therefore, acquire the drug from the monopolist. All individuals with budget $w \in [p, 1]$, if there are any, either buy the policy when healthy or buy the drug if they become ill. In both cases, they get the drug when ill.

If $p > s > 1$, individuals buy neither insurance (because its cost exceeds the benefit) nor drug (its price is above the budget). Thus, the total demand for the drug in case of insurance is

$$D^{\mathrm{I}}(p) = q\,\mathbb{Pr}\{p \leq s,\ qp \leq w\} = \begin{cases} q(1 - qp), & \text{if } qp \leq 1 \text{ and } p \leq s \\ 0, & \text{otherwise} \end{cases}$$

Maximizing monopoly profit $\pi^{\mathrm{I}}(p) = pD^{\mathrm{I}}(p)$ yields optimal monopoly price $p^{\mathrm{I}} = \min\left\{\frac{1}{2q}, s\right\}$ and a profit level of

$$\pi^{\mathrm{I}}(p^{\mathrm{I}}) = \begin{cases} \frac{1}{4}, & \text{if } 2qs > 1 \\ qs(1 - qs), & \text{otherwise} \end{cases}$$

This leads to two important observations.

First, if s is large enough ($s \geq \frac{1}{2q}$), the insurance premium $r = \frac{1}{2}$, is equal to the drug's price in the case of no insurance, $p^{\mathrm{NI}} = \frac{1}{2}$. Therefore, for large enough values of s, insurance does not help additional people to get access to the drug. Regardless of whether insurance is available, the only people who have access to the drug are those who have a budget of ½ or more. However, in the case without insurance they pay $p^{\mathrm{NI}} = \frac{1}{2}$ *conditional* upon being ill. With insurance, they pay $r = \frac{1}{2}$ *regardless* of their health. Consequently, insurance raises the revenue of the firm by factor $1/q$.

Second, this simple example shows that the optimal monopoly drug price p^{I} in case of insurance and the incidence[6] q are inversely related, provided $p^{\mathrm{I}} \leq s$. Every time the incidence is halved, the optimal price doubles, until p^{I} reaches its upper bound s. The reason for this result is straightforward. The optimal price is wholly determined by the budget constraints of the population, and the insurance relaxes it. If the probability of

[6] In the static environment of this paper, both the incidence and the prevalence equal q.

becoming ill is halved, the policy price is also halved. This creates additional demand, resulting in stronger incentives for the firm to raise its price. Effectively, the policy price r plays the same role as the drug price p does without insurance. Eventually, when the incidence is so low that $\frac{1}{2q} > s$, the optimal price is s.

3. Model

The model generalizes the example of the previous section. As before, there is a mass of individuals. Each individual faces an exogeneous probability $q \in (0,1)$ of becoming ill, a *patient*, as a result of a severe disease. Otherwise the individual is *healthy*. In contrast to the previous example, individuals do not only differ in income but also in the health damage caused by the disease if ill. There is a monopolist, a supplier of a drug that is the only cure for the disease. The drug cures the disease immediately and fully.[7] The monopolist sets price p to maximize its profits and has no costs of production. Finally, there may exist an insurance market. If it exists, individuals can also buy a policy that covers the treatment cost, *i.e.*, drug price p, in case of illness. Policy price r is competitive

$$r \stackrel{\text{def}}{=} qp$$

so that insurance companies get zero profit. If an insured becomes a patient, he acquires the drug and the insurance company pays the bill.

Ex ante, individuals are characterized by $(w, s) \in \Re_+^2$, where w is the budget of an individual and s is his utility loss in case of getting the disease and not being cured by the drug. Both w and s are independent random variables that follow distributions $F_w(x)$ and $F_s(x)$ on $[0, \infty)$. The corresponding distribution density functions and hazard rates exist and are denoted by

$$f_w(x) \stackrel{\text{def}}{=} \frac{d}{dx} F_w(x),\ f_s(x) \stackrel{\text{def}}{=} \frac{d}{dx} F_s(x), \text{ and}$$

$$\lambda_w(x) \stackrel{\text{def}}{=} \frac{f_w(x)}{1-F_w(x)},\ \lambda_s(x) \stackrel{\text{def}}{=} \frac{f_s(x)}{1-F_s(x)}$$

We assume both $f_w(x)$ and $f_s(x)$ are continuous, and we denote population means $\mathbb{E}[w]$ and $\mathbb{E}[s]$ by:

$$\bar{s} \stackrel{\text{def}}{=} \mathbb{E}[s] = \int_0^\infty s\, dF(s) \text{ and } \bar{w} \stackrel{\text{def}}{=} \mathbb{E}[w] = \int_0^\infty w\, dF(w)$$

[7] The assumption of full recovery is not essential for our results but simplifies the analysis.

We assume that, on average, s is much larger than w, *i.e.*, $\bar{s} \gg \bar{w}$. Under this assumption, individuals suffer a much greater loss of utility in case of illness than the loss of the average population budget (for some individuals, however, it can be true that $s < \bar{w}$ or $w > \bar{s}$). In other words, we consider a severe disease which is likely to be fatal or results in the loss of basic functionalities. Whenever necessary we make additional assumptions on the distributions.

Individuals receive the following payoffs. Healthy uninsured individuals receive u^{H}, where

$$u^{\mathrm{H}}(w) \stackrel{\text{def}}{=} w$$

Uninsured patients who do not buy the drug receive utility u^{S}:

$$u^{\mathrm{S}}(w,s) \stackrel{\text{def}}{=} u^{\mathrm{H}}(w) - s = w - s$$

Uninsured patients who buy the drug are cured, and receive utility u^{D}:

$$u^{\mathrm{D}}(w,p) \stackrel{\text{def}}{=} u^{\mathrm{H}}(w-p) = w - p$$

Finally, insured individuals are always cured if ill and receive expected utility u^{I}:

$$u^{\mathrm{I}}(w,r) \stackrel{\text{def}}{=} qu^{\mathrm{D}}(w,r) + (1-q)u^{\mathrm{H}}(w-r) = w - r$$

The timing of the model is as follows. First, individuals learn their budgets, w. Second, if an insurance market exists, individuals decide whether to buy the insurance. Third, individuals learn whether they are patients and, if so, they learn s. Fourth, insured patients get the drug. Uninsured patients decide whether to buy the drug. Finally, payoffs are realized.

In the next section, we compute and compare the monopoly prices in the absence of the insurance market, and when the insurance market is present.

4. Analysis

Consider the case without insurance. Let the monopoly charge price $p > 0$ for the drug. Only individuals with $w > p$ can afford the drug, and only individuals with $s > p$ are willing to buy the drug.[8] Thus, the demand for the drug is

$$D^{\mathrm{NI}}(p) = q\,\mathbb{Pr}\{s > p,\ w > p\} = q\big(1 - F_w(p)\big)\big(1 - F_s(p)\big)$$

and the monopoly profit in this case is

[8] In the demand expressions that follow, we use strict inequalities for notational simplicity.

$$\pi^{\mathrm{NI}}(p) \stackrel{\text{def}}{=} D^{\mathrm{NI}}(p)(p-c) = q\big(1-F_w(p)\big)\big(1-F_s(p)\big)(p-c)$$

Maximizing $\pi^{\mathrm{NI}}(p)$ yields the following result.

Proposition 1.

In the absence of the insurance market, the profit-maximizing monopoly price p^{NI} always exists, is generically unique, increases in c, and satisfies the following condition:

$$p^{\mathrm{NI}} = \frac{1}{\lambda_w(p^{\mathrm{NI}})+\lambda_s(p^{\mathrm{NI}})} + c \tag{1}$$

When the hazard rates $\lambda_w(p)$ and $\lambda_s(p)$ are non-decreasing, equation (1) uniquely determines p^{NI} (for exponential distributions, where $\lambda_w(p)$ and $\lambda_s(p)$ are constant, (1) defines p^{NI} explicitly). Optimal monopoly price p^{NI} is increasing in c and independent of q, which is both the incidence and prevalence of the disease in our model. Incidence q only determines the size of the drug market. In contrast, q plays a crucial role when an insurance market is present.

Now consider the case where individuals can buy insurance. Individuals with budget $w \in (r, p)$ can either buy the policy, which yields utility $u^{\mathrm{I}}(w, r)$, or remain uninsured, which yields expected utility

$$(1-q)u^{\mathrm{H}}(w) + q\,\mathbb{E}[u^{\mathrm{S}}(w)] = w - q\bar{s}$$

Clearly, these individuals only buy insurance if $r \le q\bar{s}$, *i.e.*, if $p \le \bar{s}$. Expected utility of individuals with $w > p$ from remaining uninsured is

$$(1-q)u^{\mathrm{H}} + q\,\mathbb{E}[\max\{u^{\mathrm{S}}, u^{\mathrm{D}}\}] = w - q\,\mathbb{E}[\min\{s, p\}] > w - qp = u^{\mathrm{I}}$$

These individuals do not buy insurance. They buy the drug only if they become ill *and* the realized value of s is larger than p. In other words, not buying insurance at price $r = qp$ has an option value: a patient whose s turns out to be smaller than p prefers not to buy the drug.

Thus, the demand for drugs from the insured for $p \le \bar{s}$ is

$$D^{\mathrm{I}}(p) = q\,\mathbb{Pr}\{qp < w < p\} = q\big(F_w(p) - F_w(qp)\big) \tag{2}$$

and $D^{\mathrm{I}}(p) = 0$ for $p > \bar{s}$. The demand for drugs from the uninsured is

$$D^{\mathrm{U}}(p) = D^{\mathrm{NI}}(p) = q\big(1-F_w(p)\big)\big(1-F_s(p)\big) \tag{3}$$

for any p. Note that $D^{\mathrm{U}}(p) > 0$ even if $p > \bar{s}$. This demand comes from wealthy uninsured patients whose health damage s is high. The profits from selling to respectively insured and uninsured patients are:

$$\pi^{\mathrm{I}}(p) = \begin{cases} q\big(F_w(p) - F_w(qp)\big)(p-c), & \text{for } p \le \bar{s} \\ 0, & \text{for } p > \bar{s} \end{cases}, \text{ and}$$

$$\pi^{\mathrm{U}}(p) = \pi^{\mathrm{NI}}(p) = q\big(1 - F_w(p)\big)\big(1 - F_s(p)\big)(p-c)$$

The total monopoly profit is, therefore

$$\pi^{\mathrm{T}}(p) \stackrel{\text{def}}{=} \begin{cases} \pi^{\mathrm{I}}(p) + \pi^{\mathrm{NI}}(p), & \text{if } p \le \bar{s} \\ \pi^{\mathrm{NI}}(p), & \text{if } p > \bar{s} \end{cases}$$

Figure 1 illustrates which individual types buy the drug through insurance (contributing to $D^{\mathrm{I}}(p)$) and which types buy the drug on their own (contributing to $D^{\mathrm{U}}(p)$).

If insurance is available, the monopoly has two options. First, it can charge price $p > \bar{s}$ (so that no one buys insurance) and sell only to uninsured patients. Second, it can charge price $p \le \bar{s}$ and benefit from the insurance market too. Both options can be optimal. The latter case, $p \le \bar{s}$, is the most interesting situation. A sufficient condition for this case is that the monopoly price without insurance, p^{NI}, does not exceed the average benefit of the drug, $\bar{s}$. If $p^{\mathrm{NI}} > \bar{s}$, then it is possible that the monopolist prefers selling only to wealthy patients. The relationship between p^{NI} and $\bar{s}$ depends on the production cost of the drug, c.

Lemma 1.

For given distributions $F_s(x)$ and $F_w(x)$, there is a threshold cost level $\bar{c} \in (0, \bar{s})$ such that $p^{NI} > \bar{s}$ when $c > \bar{c}$, $p^{NI} < \bar{s}$ when $c < \bar{c}$, and $p^{NI} = \bar{s}$ when $c = \bar{c}$.

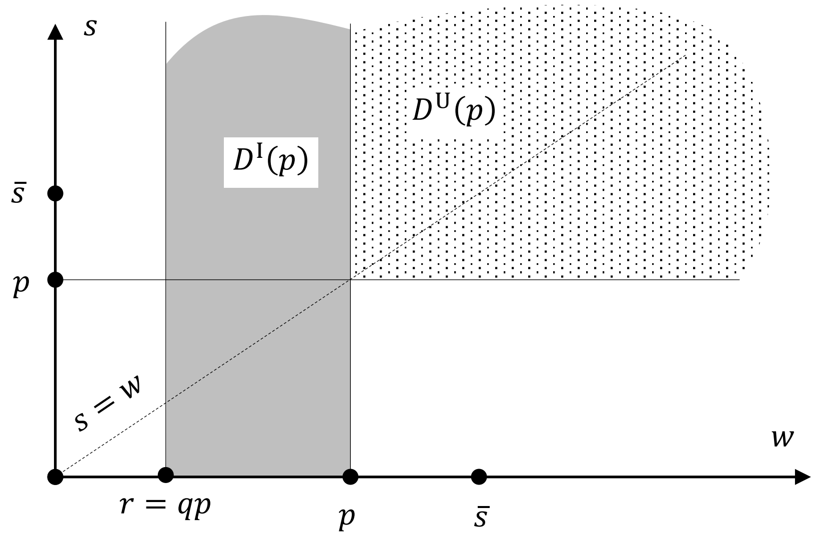

Figure 1. Composition of demands $D^{\mathrm{U}}(p)$ (dotted area) and $D^{\mathrm{I}}(p)$ (shaded area), for $p \leq \bar{s}$. When $p > \bar{s}$, area for $D^{\mathrm{I}}(p)$ disappears.

For illustration, we provide an explicit expression for $\bar{c}$ for exponential distributions, where $\lambda_w(p)$ and $\lambda_s(p)$ are constant. In this case, $\lambda_s = \frac{1}{\bar{s}}$, $\lambda_w = \frac{1}{\bar{w}}$, $p^{\mathrm{NI}} = \frac{1}{\lambda_w+\lambda_s} + c$, so that

$$\bar{c} = \frac{1}{\lambda_s} - \frac{1}{\lambda_w+\lambda_s} = \frac{\bar{s}^2}{\bar{s}+\bar{w}}$$

When $\bar{s}$ is large, $\bar{c}$ is also large, condition $c < \bar{c}$ is likely to hold, and monopoly price p^{NI} is below $\bar{s}$. This is our leading example that we consider below. For the completeness of the analysis, we also consider the case of high production cost, $c > \bar{c}$, afterwards.

4.1. Case $p^{\mathrm{NI}} \leq \bar{s}$

Unless production cost c is too high, the original monopoly price p^{NI} in case of no insurance is lower than the expected health damage $\bar{s}$ of a patient. In this case, even without changing its price, monopoly gets additional demand and, therefore, additional profit when insurance is available. It is easy to see that choosing any price $p > \bar{s}$ is strictly sub-optimal with insurance. Indeed, for any $p > \bar{s}$:

$$\pi^{\mathrm{T}}(p) = \pi^{\mathrm{NI}}(p) \leq \pi^{\mathrm{NI}}(p^{\mathrm{NI}}) < \pi^{\mathrm{T}}(p^{\mathrm{NI}})$$

due to $\pi^{\mathrm{I}}(p) > 0$ for $p \in (c, \bar{s}]$. Thus, an optimal monopoly price is bounded by $\bar{s}$. We denote it by p^{I}:

$$p^{\mathrm{I}} \stackrel{\mathrm{def}}{=} \arg\max_{p\in[c,\bar{s}]} \pi^{\mathrm{T}}(p) \tag{4}$$

Price p^{I} is the optimal monopoly price when $p^{\mathrm{NI}} < \bar{s}$.

In case of rare diseases, when q is small, opening the insurance market always leads to an increase of the monopoly price, as the following proposition states.

Proposition 2.

For all q small enough, if $p^{NI} < \bar{s}$ then $p^{I} > p^{NI}$, and if $p^{NI} = \bar{s}$ then $p^{I} = \bar{s}$, i.e., $\exists \bar{q} > 0$: $\forall q \in (0, \bar{q})$: if $p^{NI} < \bar{s}$ then $p^{I} \in (p^{NI}, \bar{s}]$, and if $p^{NI} = \bar{s}$ then $p^{I} = \bar{s}$.

By setting price $p \leq \bar{s}$, monopoly gets profit $\pi^{\mathrm{T}}(p) = \pi^{\mathrm{I}}(p) + \pi^{\mathrm{NI}}(p)$. Profit $\pi^{\mathrm{NI}}(p)$ attains its maximum at $p = p^{\mathrm{NI}}$. Profit $\pi^{\mathrm{I}}(p)$ from sales through insurance turns out to be increasing on $p \in [c, \bar{s}]$ when q is small. The reason is surprising: demand for insurance increases in the drug price p, until $p = \bar{s}$. Indeed, the derivative of (2) is positive for small q. To see why, consider Figure 2 and let the price increase by dp.

Due to the price increase, the firm loses the demand $D^{\mathrm{I}}(p)$ from insured in Area A, because these people cannot afford the insurance anymore. At the same time, the firm gains the demand $D^{\mathrm{I}}(p)$ in Areas B and C. Without the price increase, these individuals prefer to wait and see whether they become ill and, if so, how badly. With the price increase, they cannot afford the drug on their own and switch to insurance.

Now, the sizes of Areas B and C do not depend on the likelihood of getting ill, q. In contrast, Area A does depend on q: it shifts to the left and shrinks. Area A becomes negligible if q is small enough. Because $f_w(\cdot)$ is bounded, also the demand lost on Area A becomes negligible when q is small enough. Therefore, for q small enough, the price effect of areas B and C dominates that of area A and demand $D^{\mathrm{I}}(p)$ is upward sloping. As a result, profit $\pi^{\mathrm{I}}(p)$ increases in p for any $p \leq \bar{s}$. This gives the monopoly an additional incentive to raise its price if it is still below $\bar{s}$. Therefore, $p^{\mathrm{I}} > p^{\mathrm{NI}}$ if $p^{NI} < \bar{s}$ and $p^{\mathrm{I}} = \bar{\mathrm{s}}$ if $p^{NI} = \bar{s}$.

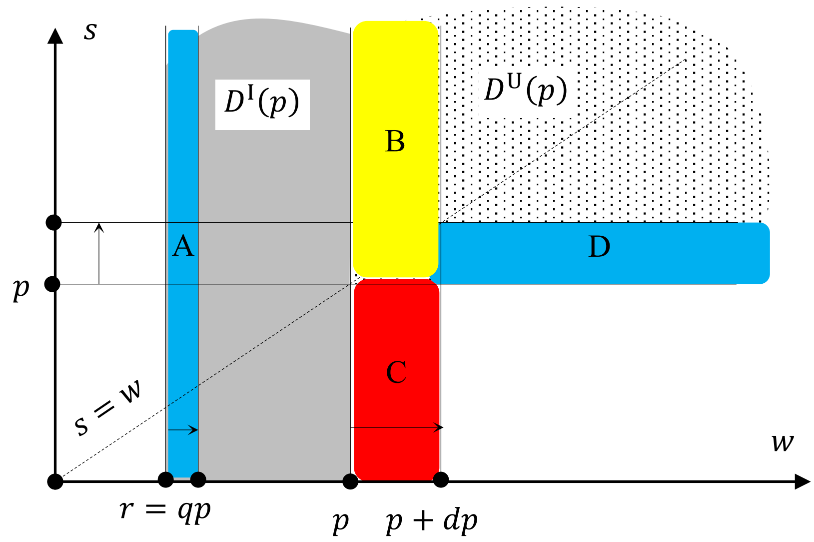

Figure 2. Effects of a marginal price change from p to $p + dp$ on demands $D^{\mathrm{I}}(p)$ and $D^{\mathrm{U}}(p)$.

Proposition 2 is an asymptotic result. However, $p^{\mathrm{I}} > p^{\mathrm{NI}}$ is a pretty common phenomenon, as the following proposition demonstrates.

Proposition 3.

Let $F_w(x)$ and $F_s(x)$ be twice differentiable and let hazard rates $\lambda_w(x)$ and $\lambda_s(x)$ be non-decreasing. If $p^{NI} < \bar{s}$ then $p^I \in (p^{NI}, \bar{s}]$, and if $p^{NI} = \bar{s}$ then $p^I = \bar{s}$.

The measure of marginal individuals in area A is proportional to $\lambda_w(qp)$ whereas in areas B and C it is proportional to $\lambda_w(p)$. For non-decreasing hazard rate (NDHR) distributions, $\lambda_w(qp) \leq \lambda_w(p)$. As a result, the total effect of a price change on $\pi^{\mathrm{I}}(p)$ is non-negative for any q.

Many frequently used continuous distributions are NDHR distributions, including the uniform, exponential, normal, and logistic distribution, and (for some parameter values) the gamma distribution, the Weibull distribution, power function distributions, the Pareto distribution (more on NDHR distributions can be found in Barlow *et al.*, 1963). Thus, irrespective of q, the availability of insurance increases the monopoly price for a large variety of distributions. But how large is the increase? The following proposition partly answers this question.

Proposition 4.

Let $F_s(x)$ *first-order stochastically dominate* $F_w(x)$*, i.e.,* $F_w(x) \geq F_s(x)$*, let* $f_s(x) \leq \frac{1}{x}$*, and let* $p^{NI} \leq \bar{s}$*. Then,* $p^I = \bar{s}$ *for all q small enough, i.e.,* $\exists \bar{q} > 0$*:* $\forall q \in (0, \bar{q})$*:* $p^I = \bar{s}$*.*

In case of very rare diseases for which q is small, the monopoly raises its price p^{I} all the way up to the expected benefit of getting rid of the decease, $\bar{s}$. In case of severe diseases, this amount $\bar{s}$ can be very large, well above the average population budget $\bar{w}$. Then the monopoly price goes through the roof.

The reason for this striking result is as follows. Consider any price $p < \bar{s}$, so that all individuals want to have access to the drug *ex ante*. If q is small, then almost all individuals will be able to afford insurance. Then, a marginal increase in p has two negative effects on monopoly profit: (i) some individuals cannot afford the insurance anymore (area A), and (ii) fewer uninsured are ill enough to buy it (area D). The first effect is negligible if q is small, and the second effect is proportional to $f_s(p)$. Therefore, if and q and f_s are small enough, as Proposition 4 specifies, the negative effects are dominated by the positive effects. The two positive effects are: (i) all uninsured with budget $w = p$ become insured, and those with small health damage, $s < p$, start acquiring the drug (area C); and (ii) every sale has a higher profit margin. A marginal increase in price is therefore profitable for any $p < \bar{s}$, resulting in optimal price $p^I = \bar{s}$.

The results of Proposition 2 and Proposition 4 apply to sufficiently rare diseases. We now study how prices and profits depend on q if the disease is more common, but detrimental to a person's well-being. To this end, we take an arbitrary CDF $F(x)$ with support $[0, \infty)$, continuous density $f(x)$ and the hazard rate $\lambda(x)$, and consider a family of utility loss distributions parametrized by $\alpha \in (0,1]$:

$$F_s(x|\alpha) \stackrel{\text{def}}{=} F(\alpha x) \tag{5}$$

The corresponding distribution density is $f_s(x|\alpha) = \alpha f(\alpha x)$ and the hazard rate is $\lambda_s(x|\alpha) = \alpha\lambda(\alpha x)$. It can be seen that in the limit when $\alpha \to 0$, the CDF $F_s(x|\alpha)$ of the utility loss s, its distribution density $f_s(x|\alpha)$ and the hazard rate $\lambda_s(x|\alpha)$ all converge to zero (pointwise for any x and uniformly for any $x \leq X < \infty$). The expected utility loss $\bar{s}$ in this limit goes to infinity. The following proposition states the result.

Proposition 5.

Let $F_s(x)$ be defined as $F_s(x|\alpha)$ in (5). In the limit when $\alpha \to 0$, insurance policy price $r^* = qp^I$ satisfies

$$r^* = \frac{1}{\lambda_w(r^*)} + qc \tag{6}$$

and the total monopoly profit $\pi^T(p^I)$ is

$$\pi^{\mathrm{T}}(p^I) = \frac{1-F_w(r^*)}{\lambda_w(r^*)} \tag{7}$$

The policy price r^* is increasing in q. When $F_w(x)$ is a NDHR distribution, then the optimal drug price p^I and the profit $\pi^T(p^I)$ are decreasing in q.

When the hazard rate $\lambda_s(x)$ is small, the utility loss s from the disease for almost all patients becomes very large and plays (almost) no role in determining optimal monopoly price p^{I}: all uninsured who can afford it buy the drug after becoming ill with probability almost one. The measure of individuals in area C in Figure 2 is small because $f_s(p)$ is small, and that of area D is small because $F_s(p)$ is small. This limit case effectively coincides with the example presented in Section 2, where $s > w$ for all individuals. Because all patients in area B have access to the medication for both prices, the optimal policy price r^* in (6) is determined by Area A alone.

When $c = 0$, expression (6) which determines the policy price r^* (with insurance) is identical to expression (1) that determines the drug price p^{NI} (without insurance) at $\lambda_s = 0$, *i.e.*, $r^* = p^{\mathrm{NI}}$. Without insurance, all individuals with $w > p^{\mathrm{NI}}$ pay price p^{NI} *ex-post* and *only* when they become ill, *i.e.*, with probability q (they all have $s > p^{\mathrm{NI}}$ with probability almost one in the limit when $\alpha \to 0$). With insurance, the same individuals pay the same price $r^* = p^{\mathrm{NI}}$ but now *ex-ante* and for the policy, *i.e.*, *with probability one*. As a result, due to insurance, monopoly profit increases by the factor of $\frac{1}{q}$, the same result as in the example of Section 2. This explains the monotonicity of $\pi^{\mathrm{T}}(p^{\mathrm{I}})$ in q.

Considering the case where marginal costs are not negligible, equation (6) shows that the inverse relation between drug price and disease prevalence is a slight simplification. Rather, the profit margin on the drug is inversely related to the incidence, while the term qc shows that the drug's marginal costs are just shared by the insured. The upshot is that if marginal costs are relatively high, insurance creates less upward pressure on the price, and access to the drug increases: $r^* < p^{\mathrm{NI}}$.

Expressions (6) and (7) in Proposition 5 are only approximations for r^* and π^{I}. Moreover, for positive (and small) hazard rates $\lambda_s(x)$ they are only valid when $p^{\mathrm{I}} < \bar{s}$. Inevitably, for small incidental probabilities q, the drug price $\frac{r^*}{q}$ becomes larger than $\bar{s}$. Then, results of Proposition 4 hold (the assumptions of Proposition 4 always hold in the limit when $\alpha \to 0$). In this case, $p^{\mathrm{I}} = \bar{s}$. Combining Proposition 4 and Proposition 5 we conclude that for small hazard rates $\lambda_s(x)$, p^{I} can be approximated by p_A^{I}, where

$$p_A^{\mathrm{I}}(q) = \min\left\{\bar{s}, \frac{1}{q} r_A^*\right\}, \text{ and } r_A^* \text{ solves } r_A^* = \frac{1}{\lambda_w(r_A^*)} + qc$$

Figure 3 presents (numerically computed) functions $p^{\mathrm{I}}(q)$ and $p_A^{\mathrm{I}}(q)$ for $c = 0.1$ and with exponential distributions with $\lambda_w = 1$, which is a normalization, and $\lambda_s = 0.2$, (for these parameter values, $\bar{q} \approx 0.212$). Overall, the approximation is rather good; the relative difference between p^{I} and p_A^{I}, *i.e.*, the relative approximation error, is less than 18%. This result is in line with empirical findings that prices for orphan drugs are inversely related to the prevalence of the disease, see, *e.g.*, Messori et al. (2010) and Medic *et al.* (2017). Figure 3 also shows that for $\lambda_s > 0$, the policy price r^* is decreasing in q for its large values (in the example, for $q > 0.375$). In the limit when $\lambda_s \to 0$, the policy price r^* become strictly increasing for all q, as Proposition 5 claims.

Figure 3 also presents total monopoly profit $\pi^{\mathrm{T}}\left(p^{\mathrm{I}}(q)\right)$ as a function of q. For relatively large incidental probabilities when $p^{\mathrm{I}}(q) < \bar{s}$, it is decreasing in q, as Proposition 5 states. For small values of q, when $p^{\mathrm{I}}(q) = \bar{s}$, monopoly profit is almost linear in q: a higher incidence increases the number of the ill without any strong effects on r^*, which is already almost zero.

In the limit when $\alpha \to 0$ and $c = 0$, monopoly profit is independent of q: r^* is independent of q and satisfies (6): $r^*\lambda_w(r^*) = 1$. By (7), π^{T} is independent of q too. That is why both r^* and π^{T} are so flat in Figure 3 for large values of $q > \bar{q}$, when $c = 0.1$. For small incidental probabilities, when $p^{\mathrm{I}}(q) = \bar{s}$, both r^* and π^{T} are increasing in q. Indeed, $r^*(q) = q\bar{s}$ in this case, and at $q = 0$:

$$\frac{d}{dq}\pi^{\mathrm{T}}\left(p^{\mathrm{I}}(q)\right) = \left(1 - \left(1 - F_w(\bar{s})\right)F_s(\bar{s})\right)(\bar{s} - c) > 0$$

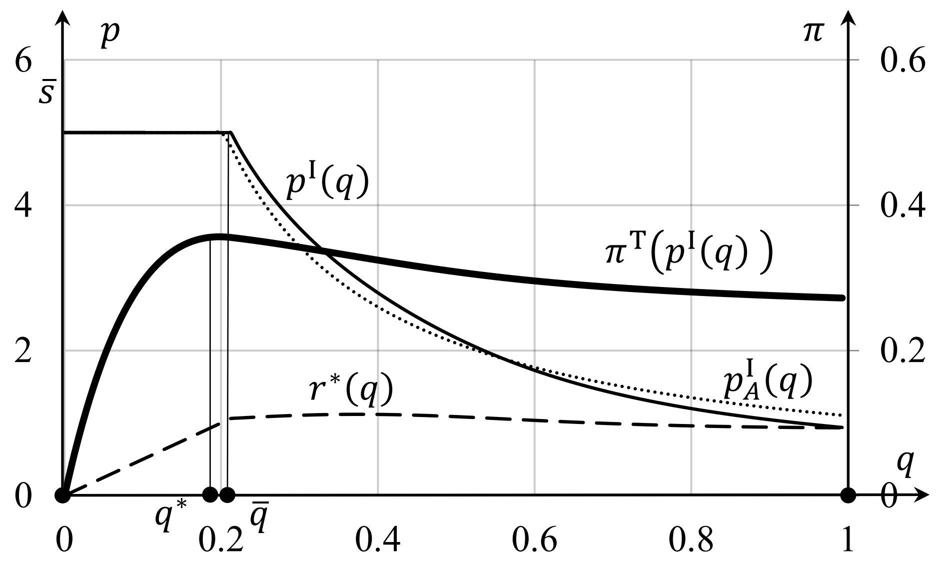

Figure 3. Optimal monopoly price $p^{\mathrm{I}}(q)$ (thin solid curve) and its approximation $p_A^{\mathrm{I}}(q)$ (dot-curve), policy price $r^*(q)$ (dash-curve), left axes, and total monopoly profit $\pi^{\mathrm{T}}\left(p^{\mathrm{I}}(q)\right)$ (thick curve), right axes, as functions of q for $\lambda_w = 1$, $\lambda_s = 0.2$, and $c = 0.1$.

Thus, the total monopoly profit $\pi^{\mathrm{T}}\left(p^{\mathrm{I}}(q)\right)$ attains its maximum at a certain incidental probability q^*, its value $q^* \approx 0.198$ is also shown in Figure 3.[9]

In the limit, when diseases are rare and severe, the optimal price is capped by $\bar{s}$. Suppose indeed that $p^{\mathrm{I}}(q) = \bar{s}$. Then, a marginal increase in q has two effects on demand. First, more people who have access to the drugs, possibly through insurance, become actual patients and will use the drug. This positive effect on demand is large, because almost the whole population has access to the drug. Second, because the price is already capped, it does not respond to the increase in q. Consequently, the insurance premium, $r = qp$ increases in q, reducing the number of people who can afford the insurance. This effect decreases demand. When q is very small, this negative effect on demand is of the second order, because it only affect the q share of individuals, namely those who fall ill. Therefore, demand and profits increase in q, when q is very small. When q is larger, the negative effect becomes stronger so that the demand for drugs, and even the monopoly profit may decline in q. In fact, according to Proposition 5, the monopoly profit

[9] According to our numerical simulations, q^* is of the order of λ_s/λ_w for small values of the latter ratio.

$\pi^{\mathrm{T}}\left(p^{\mathrm{I}}(q)\right)$ decreases in q when $p^{\mathrm{I}}(q) < \bar{s}$, *i.e.* when $q > \bar{q}$. Moreover, due to the envelope theorem, $\pi^{\mathrm{T}}\left(p^{\mathrm{I}}(q)\right)$ is continuously differentiable in q at $q = \bar{q}$. Hence, $\pi^{\mathrm{T}}\left(p^{\mathrm{I}}(q)\right)$ decreases in q at $q = \bar{q}$. This implies that monopoly profit attains its maximum in q when the resulting drug price is already capped by $p^{\mathrm{I}} = \bar{s}$. Figure 3 also illustrates that for $q \geq \bar{q}$, the negative effect of a larger incidence (a lower optimal price) dominates the positive effect (more drug sales), resulting in lower profits.

4.2. Case $p^{\mathrm{NI}} > \bar{s}$

Let us now consider the case $p^{\mathrm{NI}} > \bar{s}$ or, equivalently, $c > \bar{c}$. For $p^{\mathrm{NI}} \leq \bar{s}$, Proposition 3 shows that under quite general assumptions (NDHR distributions), the optimal drug price in case of insurance is p^{I} defined in (4), and it is higher than p^{NI}. Now we extend Proposition 3 to the case $p^{\mathrm{NI}} > \bar{s}$. We denote the optimal drug price by p^*. We show that when $p^{\mathrm{NI}} > \bar{s}$, the availability of insurance does not *increase* the optimal price above p^{NI}, but that it may *decrease* the optimal price back to $\bar{s}$.

Suppose that $c = \bar{c}$ so that $p^{\mathrm{NI}} = \bar{s}$. Under conditions of Proposition 3 or Proposition 4, $p^{\mathrm{I}} = \bar{s}$. In this case, the optimal drug price is $p^* = p^{\mathrm{I}} = \bar{s}$. Suppose now that c increases so that $p^{\mathrm{NI}} > \bar{s}$. With insurance, the monopoly has two options. First, it can set $p > \bar{s}$. Because insurance is too expensive, as $r = qp > q\bar{s}$, no one is willing to buy insurance. Consequently, demand is unaffected by the availability of insurance, and the best price above $\bar{s}$ is p^{NI}. Hence, if $p^* > \bar{s}$, then $p^* = p^{\mathrm{NI}}$. Second, it can lower its price to $p \in [c, \bar{s}]$. For these prices, price $p = p^{\mathrm{I}}$ is optimal, and the latter is $p^{\mathrm{I}} = \bar{s}$, so that $p^* = \bar{s}$.

Thus, when $p^{\mathrm{NI}} > \bar{s}$, the monopoly only needs to compare two candidate prices: $\bar{s}$ and p^{NI}. Whether the higher profit margin at $p^* = p^{\mathrm{NI}} > \bar{s}$ or additional sales (via the insurance) at $p^* = p^{\mathrm{I}} = \bar{s}$ is the best option, depends on c. When c is just above $\bar{c}$, p^{NI} is just above $p^{\mathrm{I}} = \bar{s}$, by continuity. Then, selling through the insurance market creates a jump in drug sales whereas the effect on the profit margin is negligible. In this case, $p^* = \bar{s}$ is the best option. However, $p^* = p^{\mathrm{NI}}$ is clearly the best option when $c \geq \bar{s}$. Thus, there exists some marginal cost threshold $c^* \in (\bar{c}, \bar{s})$, such that $p^* = \bar{s}$ is optimal if $c < c^*$, and $p^* = p^{\mathrm{NI}}$ is optimal if $c > c^*$. The following proposition summarizes these results.

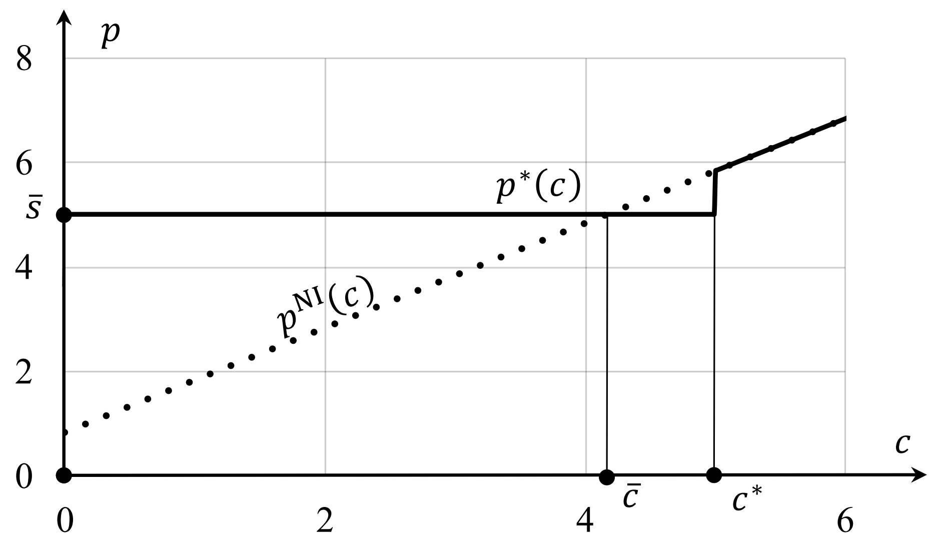

Figure 4. Optimal monopoly prices $p^{\mathrm{NI}}(c)$ (dotted line) and $p^*(c)$ (solid curve) as functions of unit production cost c, for $\lambda_w = 1$, $\lambda_s = 0.2$, and $q \leq 0.2$.

Proposition 6.

Let hazard rates $\lambda_w(x)$ and $\lambda_s(x)$ be non-decreasing. Then, there exists a threshold $c^* \in (\bar{c}, \bar{s})$ such that if $c > c^*$ then $p^* = p^{NI} > \bar{s}$, and if $c \in (\bar{c}, c^*)$ then $p^* = p^I = \bar{s}$.

Figure 4 illustrates Proposition 6 for exponential distributions with $\lambda_w = 1$, $\lambda_s = 0.2$, and $q \leq 0.2$. Monopoly price p^{NI} increases linearly in c (due to (1)) and, according to Lemma 1, becomes $p^{\mathrm{NI}} > \bar{s}$ when $c > \bar{c} \approx 4.17$. When cost c is high, namely when $c > c^* \approx 4.998$, optimal monopoly price $p^* = p^{\mathrm{NI}} > \bar{s} = 5$. Monopoly sells only to the wealthy patients. Opening the insurance market does not make sense in this setting. The monopolist does not change its price, and no individual buys insurance because its price $r^* = qp^*$ is higher than the expected utility loss $q\bar{s}$.

For $c \in (\bar{c}, c^*)$, opening the insurance market induces the monopoly to lower its price from $p^{\mathrm{NI}} > \bar{s}$ to $p^* = p^{\mathrm{I}} = \bar{s}$. By doing so, monopoly gets less revenue from wealthy uninsured patients, but makes additional sales through insurance to the less-wealthy insured patients. For $c < \bar{c}$, according to Proposition 4, opening the insurance market induces the monopoly to raise its price from $p^{\mathrm{NI}} < \bar{s}$ to $p^* = p^{\mathrm{I}} = \bar{s}$. In both cases, the drug price is $\bar{s}$ and is independent of c. Only when $c > c^*$, the drug price is $p^{\mathrm{NI}} > \bar{s}$ and is increasing in c. In case of extremely severe diseases, when consumer willingness to pay is very large, $c < \bar{c} < c^*$ is likely to occur, and $p^{\mathrm{I}} = \bar{s}$ is the outcome.

5. Discussion and Conclusion

This paper shows how the combination of budget constraints and optional insurance drives up prices, and why prices relate inversely to the incidence of the insured event. A natural fit is the health market. When drugs are covered by insurance, demand for the drugs is largely dependent on the insurance premium rather than the drug price. This gives the producing firm incentives to increase its price. As a result, when the health damage of the disease is large and its incidence is low, the policy price becomes about as large as the drug price before insurance is available. The drug price is then inversely proportional to the incidental probability until it hits its ceiling, the expected health damage.

These results are obtained by assuming that an individual's budget is the determinant of his purchasing decisions. This keeps the model simple. Nonetheless, our asymptotic results are quite robust and hold in a large variety of settings.

For instance, health insurance policies may also include a deductible (a fixed amount per year that is not insured), a co-payment (a fixed amount per treatment that is not insured), or a coinsurance (a percentage of the expenses that is not insured). Our model can easily accommodate these changes. In all these cases, the drug price hits the ceiling, the expected benefit of the drug, when the incidence q and the hazard rate λ_s vanish. With coinsurance β, for example, when the drug price is p, the competitive policy price is $r = qp(1-\beta)$. In addition, insured patients need to pay βp. Thus, only individuals with budget

$$w > qp(1-\beta) + \beta p = (q(1-\beta) + \beta)p$$

can afford insurance. Therefore, our asymptotic results (when $q, \alpha \to 0$) for optimal monopoly price p^{I} continue to hold if we replace incidence q by q^{CI}, where

$$q^{\mathrm{CI}} \stackrel{\mathrm{def}}{=} q(1-\beta) + \beta = q + (1-q)\beta$$

In the example of Figure 3, optimal monopoly price is $p^{\mathrm{I}} = \bar{s}$ as long as $q^{\mathrm{CI}} < \bar{q} \approx 0.212$. Coinsurance reduces the drug price when $q^{\mathrm{CI}} > \bar{q}$.

With deductible d, when the drug price is p, the competitive policy price is $r = q(p-d)$. In addition, insured patients need to pay d. Thus, only individuals with budget

$$w > q(p - d) + d = qp + (1 - q)d$$

can afford insurance. Also in this case, monopoly profit monotonically increases in price until the latter reaches the boundary $\bar{s}$, and all asymptotic results of our model continue to hold.

The model can be extended by introducing a probability t that the drug successfully treats the disease. In this case, the price ceiling is determined by the expected benefit of the drug, $t\bar{s}$. Correlation between budget w and health damage s can also be introduced into the model. Then, the *ex-ante* expected health damage for an individual may depend on his budget, and the monopoly needs to account for this. Nevertheless, if the expected damage is large for all budgets, the optimal price will be large as well. Alternatively, health care providers (drug gatekeepers) can be introduced into the model, who only prescribe the drug if the realized health damage exceed a certain threshold (*e.g.*, the price of the drug). In this case, all individuals are facing the same unavoidable background risk (of becoming sufficiently ill), and all our results continue to hold.

Our model focuses on the case where individuals have a choice whether they are insured or not. When health insurance is in part mandatory or state-provided, as it is in many European countries, insurance in our model should be seen as a supplementary insurance rather than the mandatory package. Whether prevalence affects the price in the mandatory insurance depends on the bargaining power of the authority deciding upon inclusion of the drugs and that of the firm. If the authority works with a maximum budget for a disease, a similar effect may well occur.

We do not provide a social welfare analysis of our results; it does not make much sense without accounting for individual risk attitudes. Yet, our results speak loudly to the R&D investment decision of the pharmaceutical firms, as the expected profits of a new drug depend significantly on whether it is likely to be included into (national) health insurance plans. Moreover, they also speak to (*i*) the incentives of pharmaceutical companies to invest into lobbing activity for including their drugs into those plans, and to (ii) the need for countervailing buyer power by the insurance companies or the government if the potential drug price increase (due the inclusion in health insurance packages) exceeds what a social planner would consider optimal.

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Appendix

Proof of Proposition 1.

Function $\pi^{\mathrm{NI}}(p)$ is continuously differentiable on $p \in [c, \infty)$, positive on $p \in (c, \infty)$, satisfies $\pi^{\mathrm{NI}}(c) = 0$ and $\pi^{\mathrm{NI}}(p) = 0$ when $p \to \infty$ (this is so because the assumed existence of the means $\overline{w}$ and $\bar{s}$ implies that distributions $F_w(x)$ and $F_s(x)$ approach one sufficiently fast when $x \to \infty$). Therefore, there is a maximizer p^{NI} of $\pi^{\mathrm{NI}}(p)$, $p^{\mathrm{NI}} \in (c, \infty)$, which satisfies the F.O.C.:

$$0 = \frac{d\pi^{\mathrm{NI}}}{dp} = q\big(1 - F_w(p)\big)\big(1 - F_s(p)\big)\big(\lambda_w(p) + \lambda_s(p)\big)\left(\frac{1}{\lambda_w(p)+\lambda_s(p)} + c - p\right)$$

from which (1) follows.

When (1) has multiple solutions (*i.e.*, multiple extreme points of π^{NI}), we denote them by p^*, the associated profit levels $\pi^{\mathrm{NI}}(p^*)$ are generically (w.r.t. cost parameter c) distinct so that p^{NI} is generically unique. The monotonicity of $p^{\mathrm{NI}}(c)$ is rather standard and can be shown as follows. Let $c_0 \geq 0$ and define function

$$G(p, c) \stackrel{\text{def}}{=} \pi^{\mathrm{NI}}\left(p^{\mathrm{NI}}(c_0)\right) - \pi^{\mathrm{NI}}(p)$$

on $p \in [c_0, p^{\mathrm{NI}}(c_0)]$. Since demand $D^{\mathrm{NI}}(p)$ strictly decreases in p, function $G(p, c)$ strictly increases in c:

$$\frac{\partial G}{\partial c}(p, c) = D^{\mathrm{NI}}(p) - D^{\mathrm{NI}}\left(p^{\mathrm{NI}}(c_0)\right) > 0 \text{ for } p < p^{\mathrm{NI}}(c_0)$$

Because $G(p, c_0) \geq 0$ by construction, $\frac{\partial G}{\partial c} > 0$ implies that $G(p, c_1) > 0$ for any $c_1 > c_0$ and any $p \in \left[c_0, p^{\mathrm{NI}}(c_0)\right)$. Hence, any price $p \in \left[c_0, p^{\mathrm{NI}}(c_0)\right)$ is suboptimal. It follows that $p^{\mathrm{NI}}(c_1) \geq p^{\mathrm{NI}}(c_0)$.

By the definition $p^{\mathrm{NI}}(c)$, the F.O.C. above holds for price $p = p^{\mathrm{NI}}(c_0)$ at $c = c_0$. Since its RHS is increasing in c, the F.O.C. necessarily fails for price $p = p^{\mathrm{NI}}(c_0)$ at $c = c_1 > c_0$, so that price $p = p^{\mathrm{NI}}(c_0)$ does not satisfy the F.O.C. and cannot be optimal. Therefore, $p^{\mathrm{NI}}(c_1) > p^{\mathrm{NI}}(c_0)$. This ends the proof. ■

Proof of Lemma 1.

Optimal price $p^{\mathrm{NI}}(c)$ is strictly increasing, satisfies (1) and $p^{\mathrm{NI}}(c) > c$. Hence, there is a number $\bar{c}$, $\bar{c} < \bar{s}$, such that $p^{\mathrm{NI}}(\bar{c}) = \bar{s}$. Then, $p^{\mathrm{NI}}(c) > \bar{s}$ for $c > \bar{c}$ and $p^{\mathrm{NI}}(c) < \bar{s}$ for $c \in (0, \bar{c})$. If $\bar{c} = 0$, then $p^{\mathrm{NI}}(c) > \bar{s}$ for any $c > 0$. ■

Proof of Proposition 2.

We define per-accident monopoly profit functions as follows:

$$\tilde{\pi}^{\mathrm{NI}}(p,c) \overset{\text{def}}{=} \frac{1}{q}\pi^{\mathrm{NI}}(p) = \big(1-F_w(p)\big)\big(1-F_s(p)\big)(p-c)$$

$$\tilde{\pi}^{\mathrm{I}}(p,c,q) \overset{\text{def}}{=} \frac{1}{q}\pi^{\mathrm{I}}(p) = \big(F_w(p)-F_w(qp)\big)(p-c)$$

$$\tilde{\pi}^{\mathrm{U}}(p,c) \overset{\text{def}}{=} \frac{1}{q}\pi^{\mathrm{U}}(p) = \tilde{\pi}^{\mathrm{NI}}(p,c)$$

$$\tilde{\pi}^{\mathrm{T}}(p,c,q) \overset{\text{def}}{=} \frac{1}{q}\pi^{\mathrm{T}}(p) = \tilde{\pi}^{\mathrm{I}}(p,c,q) + \tilde{\pi}^{\mathrm{NI}}(p,c)$$

Optimal monopoly prices p^{NI} and p^{I}, which maximize $\pi^{\mathrm{NI}}(p)$ and $\pi^{\mathrm{T}}(p)$ correspondingly, also maximize $\tilde{\pi}^{\mathrm{NI}}$ and $\tilde{\pi}^{\mathrm{T}}$. We show below that for small values of q, $\tilde{\pi}^{\mathrm{I}}(p,c,q)$ increases on $p \in [c,\bar{s}]$, *i.e.*, $\frac{\partial \tilde{\pi}^{\mathrm{I}}}{\partial p} > 0$. Combined with the F.O.C. $\frac{\partial \tilde{\pi}^{\mathrm{NI}}}{\partial p}(p^{\mathrm{NI}}) = 0$ we obtain $\frac{\partial \tilde{\pi}^{\mathrm{T}}}{\partial p}(p^{\mathrm{NI}}) > 0$. Therefore, $p^{\mathrm{I}} \in (p^{\mathrm{NI}}, \bar{s}]$ for $p^{\mathrm{NI}} < \bar{s}$ and $p^{\mathrm{I}} = \bar{s}$ for $p^{\mathrm{NI}} = \bar{s}$, and the result follows.

Let us consider the marginal profit function $\frac{\partial \tilde{\pi}^{\mathrm{I}}}{\partial p}(p,c,q)$ on $p \in [c,\bar{s}]$:

$$\frac{\partial \tilde{\pi}^{\mathrm{I}}}{\partial p}(p,c,q) = \big(F_w(p)-F_w(qp)\big) + \big(f_w(p)-qf_w(qp)\big)(p-c) \tag{8}$$

Because $f_w(x)$ is bounded, $\frac{\partial \tilde{\pi}^{\mathrm{I}}}{\partial p}(p,c,q)$ uniformly converges to $\frac{\partial \tilde{\pi}^{\mathrm{I}}}{\partial p}(p,c,0) > 0$ in the limit when $q \to 0$. Hence, there exists a $\bar{q} > 0$ such that $\frac{\partial \tilde{\pi}^{\mathrm{I}}}{\partial p}(p,c,q) > 0$ for all $q \in (0,\bar{q})$ and all $p \in [c,\bar{s}]$. This ends the proof. ■

Proof of Proposition 3.

The proof follows a similar idea as the proof of Proposition 2 does: we show that for all $q \in (0,1)$, $\frac{\partial \tilde{\pi}^{\mathrm{I}}}{\partial p} > 0$ so that $\tilde{\pi}^{\mathrm{I}}(p,c,q)$ increases on $p \in [c, p^{\mathrm{NI}}]$. Consequently, $p^{\mathrm{I}} \in (p^{\mathrm{NI}}, \bar{s}]$ for $p^{\mathrm{NI}} < \bar{s}$ and $p^{\mathrm{I}} = \bar{s}$ for $p^{\mathrm{NI}} = \bar{s}$, and the result follows.

Evaluating (8) at $q = 1$ yields $\frac{\partial \tilde{\pi}^{\mathrm{I}}}{\partial p}(p,c,1) = 0$. In the rest of the proof, we show that $\frac{\partial^2 \tilde{\pi}^{\mathrm{I}}}{\partial p \partial q} < 0$ so that $\frac{\partial \tilde{\pi}^{\mathrm{I}}}{\partial p}(p,c,q) > \frac{\partial \tilde{\pi}^{\mathrm{I}}}{\partial p}(p,c,1) = 0$ for $q < 1$. Differentiating (8) yields:

$$\frac{\partial^2 \tilde{\pi}^{\mathrm{I}}}{\partial p \partial q} = -\Big(pf_w(qp) + \big(f_w(qp) + qpf_w'(qp)\big)(p-c)\Big)$$

Since $\lambda_w(x)$ is non-decreasing, we obtain

$$\lambda_w'(x) = \frac{f_w'(x)}{(1-F_w(x))} + \frac{f_w^2(x)}{\big(1-F_w(x)\big)^2} \geq 0 \to f_w'(x) \geq -\lambda_w(x) f_w(x)$$

so that

$$\frac{\partial^2 \tilde{\pi}^{\mathrm{I}}}{\partial p \partial q} \leq -f_w(qp)\left(p + \left(1 - qp\lambda_w(qp)\right)(p-c)\right)$$

Next, as $p\lambda_w(qp) < p\lambda_w(p)$ due to $q < 1$, we obtain

$$\frac{\partial^2 \tilde{\pi}^{\mathrm{I}}}{\partial p \partial q} < -f_w(qp)\left(p + \left(1 - p\lambda_w(p)\right)(p-c)\right)$$

Finally, for NDHR distributions, (1) uniquely determines p^{NI}. Since the RHS of (1) is not increasing in p^{NI}, it follows that for all $p \in (c, p^{\mathrm{NI}}]$:

$$p \leq \frac{1}{\lambda_w(p)+\lambda_s(p)} + c \rightarrow \lambda_w(p) < \frac{1}{p-c}$$

Hence,

$$\frac{\partial^2 \tilde{\pi}^{\mathrm{I}}}{\partial p \partial q} < -f_w(qp)\left(p + \left(1 - p\frac{1}{p-c}\right)(p-c)\right) = -f_w(qp)(p-c) < 0$$

This ends the proof. ■

Proof of Proposition 4.

We use the notation developed in the proof of Proposition 2. Let us consider the marginal profit function $\frac{\partial \tilde{\pi}^{\mathrm{T}}}{\partial p}$. Using $F_s(p) \leq F_w(p) < 1$ and $f_s(p) \leq \frac{1}{p}$, we write $\frac{\partial \tilde{\pi}^{\mathrm{T}}}{\partial p}$ at $q = 0$ as follows:

$$\begin{aligned}\frac{\partial \tilde{\pi}^{\mathrm{T}}}{\partial p} &= \left(1 - \left(1 - F_w(p)\right)F_s(p)\right) + \left(1 - F_w(p)\right)\left(\lambda_w(p)F_s(p) - f_s(p)\right)(p-c) \\ &> \left(1 - F_s(p)\right) - \left(1 - F_w(p)\right)f_s(p)(p-c) \\ &> \left(1 - F_w(p)\right)\left(1 - f_s(p)(p-c)\right) > \left(1 - F_w(p)\right)\left(1 - \frac{1}{p}(p-c)\right) \\ &= \left(1 - F_w(p)\right)\frac{c}{p} > 0\end{aligned}$$

Hence, $\frac{\partial \tilde{\pi}^{\mathrm{T}}}{\partial p}(p, c, 0) > 0$. Since $\frac{\partial \tilde{\pi}^{\mathrm{T}}}{\partial p}$ is uniformly continuous in q on $p \in [c, \bar{s}]$, there exists $\bar{q} > 0$ such that $\frac{\partial \tilde{\pi}^{\mathrm{T}}}{\partial p} > 0$ for any $q \in (0, \bar{q})$. This, in turn, implies that optimal monopoly price is $p^{\mathrm{T}} = \bar{s}$ for $q \in (0, \bar{q})$. This ends the proof. ■

Proof of Proposition 5.

We explicitly add parameter α to the argument lists of the monopoly profit function π^{T}:

$$\pi^{\mathrm{T}}(p, \alpha) = q\left(F_w(p) - F_w(qp)\right)(p-c) + q\left(1 - F_w(p)\right)\left(1 - F_s(p|\alpha)\right)(p-c)$$

In the limit when $\alpha \rightarrow 0$, for any $X > 0$ and any $p \leq x$:

$$\pi^{\mathrm{T}}(p, 0) = q\left(1 - F_w(qp)\right)(p-c) \text{ and} \tag{9}$$

$$\frac{\partial \pi^{\mathrm{T}}}{\partial p}(p, 0) = qf_w(qp)\left(\frac{1}{\lambda_w(qp)} + qc - qp\right) = qf_w(qp)\left(\frac{1}{\lambda_w(r)} + qc - r\right)$$

from which (6) follows. Plugging (6) into (9) yields (7).

Monotonicity of r can be shown in the same way as the monotonicity of p^{NI} is shown in the proof of Proposition 1. When $F_w(x)$ is a NDHR distribution, the monotonicity of π^{T} follows immediately from (7). To show the monotonicity of p^{I}, we set $r^* = qp^{\mathrm{I}}(q)$ in (6) and differentiate it w.r.t. q. This results in:

$$\frac{dp^{\mathrm{I}}}{dq} = -\frac{\lambda_w(r^*)+r^*\lambda_w'(r^*)}{q^2\left(\lambda_w^2(r^*)+\lambda_w'(r^*)\right)} < 0 \tag{10}$$

This ends the proof. ■

Remark. Optimal price $p^{\mathrm{I}}(q)$ is decreasing in q under a weaker condition than the NDHR of $F_w(x)$. Analyzing (10) results in the following sufficient condition:

$$\lambda_w'(x) > -\min\left\{\lambda_w^2(x), \frac{1}{x}\lambda_w(x)\right\}$$

Similarly, profit π^{T} is decreasing in q when $\lambda_w'(x) > -\lambda_w^2(x)$.

Proof of Proposition 6.

Let us define function $H(c)$ as follows:

$$H(c) \stackrel{\text{def}}{=} \pi^{\mathrm{T}}(p^{\mathrm{I}}) - \pi^{\mathrm{NI}}(p^{\mathrm{NI}})$$

It has the following properties. First, $H(\bar{c}) > 0$, because $\pi^{\mathrm{T}}(p^{\mathrm{I}}) > \pi^{\mathrm{NI}}(p^{\mathrm{NI}})$ at $c = \bar{c}$. Second, $H(\bar{s}) < 0$, because $\pi^{\mathrm{NI}}(p^{\mathrm{NI}}) > \pi^{\mathrm{T}}(p^{\mathrm{I}}) = 0$ at $c = \bar{s}$. Third, $H'(c) < 0$ on $[\bar{c}, \bar{s}]$, because, due to the envelope theorem:

$$\frac{d}{dc}\pi^{\mathrm{T}}(p^{\mathrm{I}}) = \frac{\partial}{\partial c}\left(\left(D^{\mathrm{NI}}(p^{\mathrm{I}}) + D^{\mathrm{I}}(p^{\mathrm{I}})\right)(p^{\mathrm{I}} - c)\right) = -D^{\mathrm{NI}}(p^{\mathrm{I}}) - D^{\mathrm{I}}(p^{\mathrm{I}})$$

$$\frac{d}{dc}\pi^{\mathrm{NI}}\left(p^{\mathrm{NI}}(c)\right) = -\frac{\partial}{\partial c}\left(D^{\mathrm{NI}}\left(p^{\mathrm{NI}}(c)\right) - c\right) = -D^{\mathrm{NI}}\left(p^{\mathrm{NI}}(c)\right)$$

so that

$$H'(c) = D^{\mathrm{NI}}\left(p^{\mathrm{NI}}(c)\right) - D^{\mathrm{I}}(p^{\mathrm{I}}) - D^{\mathrm{NI}}(p^{\mathrm{I}}) \le -D^{\mathrm{I}}(p^{\mathrm{I}}) < 0$$

for $c \in [\bar{c}, \bar{s}]$. Hence, there exists a unique number $c^* \in (\bar{c}, \bar{s})$ defined by $H(c^*) = 0$ such that $H(c) > 0$ on $(\bar{c}, c^*)$ and $H(c) < 0$ on $(c^*, \bar{s})$. This ends the proof. ■