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Taking Time Seriously: Implications for Optimal Climate Policy

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Taking Time Seriously: Implications for Optimal Climate Policy

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Abstract

Induced innovation and associated issues of path dependence and inertia are of critical importance in the transition to a carbon free economy. We develop a model that, instead of modeling these processes themselves, models the implications of these characteristics and in the process allows us to shed a more nuanced light on this transition phase, an often neglected task. The resulting policy recommendations emphasize the advantages of immediate action and show under what conditions optimal policy might differ from one sector to another. The model thus generates important and policy-relevant insights while seriously considering transition dynamics.
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1 Introduction

Successfully tackling climate change requires, in some shape or form, a transformation of our economy. Our aim in this paper is to develop a stylized model attempting to

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seriously formalize the dynamics of this transformation. While virtually every serious model of the climate economy includes time, most do so with a focus on the long-run (some notable exceptions are discussed below). While useful in its own right, this focus on the long-run has led to a neglect studying the process of change itself (Stern, 2018). Yet, throughout this transition, induced innovation and associated issues of path dependence and inertia – features we term the dynamic characteristics of emitting systems – are likely to be of critical importance. We develop a simple model that, instead of modeling these dynamic processes themselves, provides a stylized way of capturing their underlying termporal characteristics. Our model can be solved analytically and generates insights that are complementary to recent advances in the economic literature.

Our main contribution is best understood as a new formulation of abatement costs. While traditional abatement cost representations such as the one in the DICE model (Nordhaus, 1993) consider emissions costs as a simple unique function of the degree of abatement below a given reference ("business-as-usual") trajectory, we, additionally, focus on the rate of abatement. This therefore suggests that mitigation costs depend on two components: one reflecting the costs associated with the degree of abatement relative to the projected reference (and assumed least-cost) trajectory ("rigid costs") and another associated with the rate at which we transform the economy ("transitional costs").

Rigid (marginal) costs reflect abatement with no enduring consequences beyond the in-period emissions savings. Such costs are associated with a contemporaneous emissions reduction, but not more. As such, abatement efforts within a period are reversible and have no consequences for subsequent abatement costs. Transitional costs, however, have enduring consequences. Mitigation options for which emissions reductions in one period extend to subsequent periods without additional abatement costs thus fall into the category of transitional costs. Examples include building insulations, or other investments into long-lived, emitting infrastructure or capital stock. Investments in 'enabling' capacity also fall into this latter category. For example, investments in electric charging points facilitate subsequent emissions reductions at least as large as achieved in the first period. The same could be said of factories producing emissions-reducing capital, like wind turbines.[1]

In practice, we argue, it is likely that many abatement options contain some combination of both of these costs structures. The build-up of new low-carbon industries with significant learning-by-doing and scale economies, as witnessed for renewable energy, clearly takes this form – the up-front costs associated with initial emissions reductions were large, but have yielded longer term cost reductions with an enduring stream of emission savings. Thus, these two costs are interdependent. This is because what appear to be high and rigid costs of switching to cleaner sources at an early stage of transition may in reality be attributable to transitional investment costs, as they may lower the cost of long-run abatement. In other words, many ex-ante cost assessments and policy prescriptions may have misallocated costs by assuming technology costs to

[1] A given wind turbine would normally represent the first type of investment, to the extent that its costs were amortized over the lifetime of the emissions reductions it delivers. However, the collective of wind turbine investments and the associated supply chains clearly have elements of both types of costs, not least because the factory can continue producing wind turbines (a continuing stream of enhanced abatement) and because of the learning and scale economies involved.

be fixed and rigid, whereas in reality they often prove to be transitional.

Our model thus formalizes this straightforward dynamic intuition. When thinking about abatement costs, it is crucial to differentiate between rigid and transitional costs, i.e. between whether the costs result in "only" contemporary emissions reductions or whether they change the course of the economy, thus affecting long-term emissions and costs.

The policy implications from our model follow directly from this line of thought. For a system with purely rigid costs, it makes sense to abate up to the cost value that is justified by climate damages, mediated by a carbon price. In other words, this is the standard analysis found in traditional integrated assessment models (IAMs) such as the DICE model. However, for systems exhibiting mainly transitional costs, it makes more sense to transform the economy at a steady rate. With a fixed abatement target, this implies starting as early as possible to minimize the rate of change. Thus, adding transitional costs to the analysis underlines the case for immediate action. To be clear, whether (strong) immediate action is recommended thus depends on the level of influence of transitional costs on total abatement costs. This influence is best understood as the economy's capacity to respond endogenously to reduce long-run costs of a transition that involves induced innovation, path dependency, and inertia.

Intuitively, the motivation for immediate action can be rationalized by thinking about the consequences of the path dependency of technological change as well as inertia. Path dependent technological innovation may imply that investments into "dirty" technologies will yield higher returns than investments into "green" technologies, as these may be less productive to start off with. Without immediate intervention, however, the costs faced by society will increase the longer the intervention is postponed as the "productivity-distance" between "green" and "dirty" technologies will be larger down the line than it is today. This is the type of argument found in Acemoglu et al. (2012) who argue that optimal policy consists of research subsidies to "green" technologies (and a carbon price), i.e. immediate action. While path dependency can arise from a variety of sources (e.g. learning-by-doing, building on the shoulders of giants), it will result in a well-defined cost that society can correct for with immediate action.

The motivation for immediate action stemming from considering inertia is slightly more complicated as inertia gives rise to a more complex intertemporal problem. Failing to act today not only leaves future generations with a "dirtier world," but also with an economic system where the costs of deep emission reductions then are likely larger than they would be if initiated now. Here, two often conflated issues arise. First, delaying immediate action increases the costs incurred from damages. These costs, however, are highly uncertain (Pindyck, 2020) and likely heavy-tailed, making it difficult to equate marginal costs and benefits to calculate an optimal path of policy effort. Second, abatement costs are also likely heavy-tailed. Consider the extreme example of postponing climate policy until 2049, thus leaving one year to decarbonize the whole economy until 2050. Needless to say, the associated abatement costs would be extraordinarily high. Indeed, the resulting transition costs will be hard or impossible to price as they involve risks of a more systemic nature, including risks for financial stability (Bank of England, 2015). This suggests that costs arising from damages and abatement efforts are highly uncertain and hard to estimate because they are heavy-tailed (in the sense of Weitzman (2009)'s *Dismal Theorem*). Since transition risks arise when the economy transitions too quickly, it is relatively intuitive that such risks will be mitigated when starting the transition earlier. It follows that given inertia, the value of early policy action is amplified because it minimizes these "transition risks" (Bank of England, 2015).

If the level of influence of transitional costs on total abatement costs is crucial for policy (and if this level of influence is endogenously determined by the dynamic characteristics), it is straightforward to see that policy recommendations might differ from sector to sector (or from country to country). Indeed, the sector-specific climate policy recommendations will depend on the dynamic characteristics of said sector. For a carbon tax, for example, this implies that pricing in a sector will depend on the degree of induced innovation, inertia, and path dependence within that sector. While the policy recommendations stemming from our high-level model stop there, varying policies for different sectors suggest a more nuanced mix of policy instruments. While carbon taxes will surely remain of key importance, other policy instruments may act as complements to carbon prices, depending on factors including the sources of induced innovation in the sector. This complements the findings in Acemoglu et al. (2012) that the first-best policy consists of a carbon tax and subsidies.

Our paper relates to a large literature, within and outside of economics, working to model the climate economy. The most prominent approach to tackling this issue is to set-up a numerical IAM, as pioneered by William Nordhaus and his DICE model (Nordhaus, 1993). While this model has been updated repeatedly over time (Nordhaus and Boyer, 2000; Nordhaus, 2014), it has, at its core, remained the same, namely a Ramsey growth model with an optimal steady state as well as an optimal transition path, extended to include a carbon cycle, a set of climate equations mapping atmospheric carbon into the atmosphere, and an energy sector. Importantly, technological change is assumed to be exogenous. The policy recommendation is a climate policy ramp, indicating that emissions controls should increase over time. The model has been extended at various points in time, for example, to allow for regional disaggregation (Nordhaus and Yang, 1996), to account for within region inequalities (Dennig et al., 2015), or to endogenize technological change (Popp, 2004; Dietz and Stern, 2015). More recently, numerical IAMs have become increasingly technical and complicated.

A more transparent approach to modeling the climate economy are analytical IAMs (e.g. Golosov et al., 2014).[2] An interesting and relevant approach for our purposes therefore comes from Hassler et al. (2020), who extend Golosov et al. (2014)'s set-up to a multi-region model with one oil-producing region and many oil-important regions. Their approach is appealing because, even though it is very rudimentary and simple, it delivers interesting and important policy insights. Specifically, their results suggest that the costs of implementing a policy that is too stringent are small compared to the costs of implementing a policy that is too lax. Additionally, they emphasize the importance of all regions implementing climate mitigation policies because the costs stemming from some regions free-riding on the efforts of others are are large.

[2] Golosov et al. (2014) develop an analytically tractable model that models the energy sector with a linear impulse response function of economic production to carbon emissions. Relying on only four assumptions, their model provides an analytical solution to the optimal carbon tax problem that shows that only the discounting, damages, and carbon depreciation parameters matter for the externality costs of carbon emissions. The optimal per-unit tax on emissions is therefore equal to the marginal externality cost of emissions. This outcome implies that there is no use for very high tax rates until we know that damages will be very high. This model has been extended various times (e.g. Hassler and Krusell, 2012; Traeger, 2015). See Hassler and Krusell (2018) for a more complete overview of "environmental macroeconomics."

Our paper provides an alternative simple model resulting in complementing policy recommendations. Indeed, the workings of our model are even simpler since it is not a general equilibrium model (which of course also comes with drawbacks). Importantly, however, Hassler et al. (2020) assume that technological change is exogenous as well as a relatively high elasticity of substitution between energy sources. In regard to the latter, the authors specifically state that such elasticities are realistic over the medium-run (or even long-run). While this is likely to be true over such time horizons, the model thus neglects the important transition phase to said medium-term. Throughout this transition, these elasticities of substitutions are likely to be nowhere near as high (as the authors themselves admit) due to issues of induced innovation, path dependence and inertia. Our model, by modeling technological change as endogenous as well as allowing for inertia and path dependence, thus provides a more realistic framework to model this transition, which is the most relevant for policy purposes at present time.

The most complete approaches to modeling the climate economy aim to correct for two market failures, environmental externalities and knowledge spillovers. Most prominently, Acemoglu et al. (2012) address this by showing that carbon taxes and research subsidies are optimal in the fight against increasing emissions.[3] Their two-sector model of directed technical change clearly endogenizes technical change and "allows for" path dependence. Yet, their main results (described above) also rely on an unusually large elasticity of substitution between "green" and "dirty" technologies. While this again may be a realistic assumption for the long-term, it is currently not very realistic due to issues such as, for example, inertia, which these models neglect.

Most relevant to our paper is the nascent but growing literature that has started to incorporate adjustment costs – inertia – into models of the climate economy by noting that an economy cannot adjust to the changes required for a carbon free economy over night (e.g. Vogt-Schilb et al., 2018). Related to this is a growing literature on the optimal timing and speed of policies as abrupt changes may lead to stranded assets (see van der Ploeg and Rezai, 2020, for a review). For example, Rozenberg et al. (2020) show that different types of abatement policies lead to different short-term paths of emissions. Specifically, since carbon prices disproportionally affect owners of "dirty" capital, they tend to create stranded assets. Other policies such as investment standards, do not. There is also a literature exploring the "irreversibility" of investments into capital. For example, Baldwin et al. (2020) show how the irreversibility of investments into dirty sectors implies an accelerated transition to investments into clean sectors in order to avoid future stranded assets. While this type of work clearly does shift the focus to the process of change, they often neglect issues of path dependence. Yet, path dependence is just as important as inertia in the transformation to a new economy.

Our model is simpler in principle than any of the above models. However, it is more comprehensive and flexible in the sense that by focusing on the overall balance between rigid and transitional costs, it can encompass the implications of induced innovation, inertia and path dependence together, in a stylized and transparent manner. Moreover, by coupling this to a representation of climate damages – again, deliberately stylized but reflecting the essential characteristics of how cumulative emissions drive temperature change – we develop a full cost-benefit model which is indeed amenable to analytic solutions to illuminate the key dependencies.

Thus, we think of our model as a useful complement to the models just mentioned.

[3]See also Acemoglu et al. (2016).

These models either focus on long-term dynamics and hence neglect the transition phase, and/or focus on the transition by thinking about one issue (e.g. inertia) but then neglecting others (e.g. induced innovation and path dependency). Our simple model is able to reconcile these omissions with a fuller aggregated representation of these important dynamic characteristics. By clarifying the distinction between rigid and transitional components, and formalizing a distribution of costs between them, it provides what it is to our knowledge a significant development in assessing the economic characteristics of the dynamic transition to a carbon free economy.

The rest of the paper is organized as follows. Section 2 briefly motivates the importance of induced innovation, path dependency, and inertia in convincingly modeling the climate economy. Section 3 motivates the main model, which is presented in full in Appendix B. Section 4 presents our main results. Section 5 sketches an extension of the model for a two-sector economy. Section 6 concludes.

2 Why Dynamic Characteristics Matter

This section (very) briefly motivates the importance of induced innovation, inertia, and path dependence when thinking about modeling the climate economy.

Induced Innovation. That technological change is not exogenous is widely accepted within economics. It is also widely accepted that innovation is induced as the direction of innovation responds to changes in relative prices, as first proposed by Hicks (1932). Indeed, there is much empirical research documenting that changes in relative prices of energy inputs, for example, influence what technologies are being developed and adopted (see Popp (2019) for a review of the evidence of induced technical change).

Inertia. Within models of the climate economy, inertia is usually incorporated as an adjustment cost, most notable in REMIND (Bauer et al., 2016).[4] More generally, inertia can be defined as the cost of switching pathways. In other words, inertia implies that the transition to a green economy cannot happen "overnight." The simplest form of inertia arises from the lifetime of the capital stock. As an example, note that power stations may last decades, and roads or buildings even longer.

It is, within the technology systems literature, well-established that new technologies typically penetrate markets with a logistic S-curve dynamic (e.g. Rogers, 2010; Grubler et al., 2016). In other words, technological transitions take decades. Bento and Wilson (2016) review such dynamics for energy technologies, finding transition times ranging from ten years to five decades for new technologies to go from a 10% to a 90% market share. A classic example is the substitution of cars for horses, which took 12 years to go from a 10% to a 90% market share. The resulting S-curve is drawn in Figure 5 of Grübler et al. (1999).[5]

[4]The importance of adjustment costs in economics has been noted for a long time (e.g. Lucas Jr, 1967; Gould, 1968). Note that there is a literature on inertia in the climate system, i.e. the potential for a delay in increase in temperatures in response to higher CO2 concentration (e.g. Lemoine and Rudik, 2017). See Mattauch et al. (2020) and Lemoine and Rudik (2020) for a comment and response.

[5]Within economics, the observation that technology diffuses slowly has been noted for a long time (e.g. Griliches, 1957; Mansfield, 1961; Rosenberg, 1976). See Hall and Khan (2003) and Hall (2004) for surveys on the diffusion of new technological innovations.

For our purposes, technological inertia means that although there could be demand for certain technologies in the economy, it may not necessarily be possible to scale them up at the required rate to achieve certain climate goals. It is therefore pertinent to take inertia into account when modeling the climate economy as technologies cannot be instantaneously scaled up following policy incentives such as a carbon price.

Path Dependence. Path dependence refers to the fact that initial conditions matter for future outcomes. Innovation is intrinsically path dependent (Arthur and Polak, 2006), since knowledge builds upon itself without end. Hence, at any given point in time, the state of the economy reflects its previous evolution. Within the economics of climate change, path dependency is being studied both theoretically (e.g. Acemoglu et al., 2012) and empirically (e.g. Aghion et al., 2016). In a recent overview, Aghion et al. (2019) identify at least five determinants of path dependence: knowledge spillovers, network effects, switching costs, positive feedbacks, and complementarities.

As mentioned in the introduction, the literature often discusses the issue of climate change in terms of two market failures, environmental externalities and knowledge spillovers. It is often argued that carbon taxes are optimal to fight current (or immediate) externalities while subsidies to "green" technologies are meant deal with future externalities (e.g. Acemoglu et al., 2012). As such, there is a direct relationship between path dependence and the associated market failure of knowledge spillovers. However, both path dependence and especially inertia give rise to an incredibly complicated intertemporal problem, which may be considered a third market failure. The issue arising from this time dimension is more complex than thinking about the trade-off between future and current generations or how advanced certain types of technologies will be in relation to others in absence of "green" subsidies. It involves thinking of (possibly infinitely) many costs (which, as discussed in the introduction, possibly cannot be priced adequately) associated with (possibly infinitely) many different paths. While our model does not further address issues of market failures, it does underline the importance of considering issues such as inertia and path dependence.

The above makes clear that induced innovation, inertia, and path dependence are critical elements in any model modeling the climate economy. Indeed, thinking about these dynamic characteristics implies that abatement actions in one period have first-order consequences for the capacity and costs of emission reductions in subsequent periods. However, the above also clarifies that these processes are immensely complex and difficult to model accurately. The model we motivate in the next section therefore focuses on the implications of these dynamic characteristics, thus circumventing the issue of precisely modeling each of these processes.

3 Motivating the Model

The aim of this section is to formalize the simple intuition behind our model and its solution.[6] The full model and its analytic solution can be found in Appendix B. We present a simple and stylized one-sector model that abstracts from issues of fossil fuel resource depletion. Instead of modeling the dynamic characteristics discussed above,

[6] This motivation relies heavily on Grubb et al. (1995).

our model focuses on the implications of these processes. As such, this model promises to be simpler to understand than traditional IAMs and thus more accessible to policy makers. We extend the set-up to a two-sector model in section 5.

3.1 Set-Up

Let $E_{ref}(t)$, measured in petajoule (PJ) per year, denote the reference ("business-as-usual") trajectory of total energy required at time t and suppose that, at any given time, energy generation is divided between fossil fuel energy, $E_F(t)$, and very low carbon energy, $E_L(t)$. Therefore

$$\begin{aligned} E_F(t) &= (1-\epsilon(t))E_{ref}(t) \\ E_L(t) &= \epsilon(t)E_{ref}(t), \end{aligned}$$

where $\epsilon(t)$ denotes the fraction of global energy supplied from low carbon sources at time t.

The model implies that the cost of energy provision is (implicitly) the sum of low carbon energy, E_L, and the cost of fossil fuel energy, E_F. Each of these costs depend on the path of abatement, ϵ. It follows that the optimal control problem is to minimize the (discounted) total cost of energy provision plus the damages caused by climate change (which we model as a function of cumulative emissions at time t)

$$\min_{\{\epsilon\}} \int_0^T \left(C_L[E_L(t)] + C_F[E_F(t)] + D\left[E_F(0) + \int_0^t E_F(\tau)d\tau\right]\right) e^{-rt}dt, \tag{1}$$

where $C_L(\cdot)$ and $C_F(\cdot)$ denote the (\$/year) costs of low carbon and fossil fuel energy, respectively, $D(\cdot)$ denotes the (\$/year) damages caused by climate change, $T \geq 0$ is the time horizon, and $r > 0$ is the discount rate. Note that the dependence on time of each of the terms above implies that costs as well as damages may be functions of the *trajectory* of emissions abatement and not just the instantaneous level. For notational simplicity we suppress the time argument from now on.[7]

3.2 Defining C_L and C_F: Towards Transitional Costs

We express this dependence of costs on the instantaneous level of as well as the trajectory of emissions abatement as follows

$$C_L = c_{A,L}g_A(\epsilon) + c_{B,L}g_B(\dot{\epsilon}) \tag{2}$$
$$C_F = c_{A,F}g_A(1-\epsilon) + c_{B,F}g_B(\dot{\epsilon}), \tag{3}$$

where $\dot{\epsilon} = \frac{d\epsilon}{dt}$, c_A and c_B are scaling parameters, and $g_A(\cdot)$ and $g_B(\cdot)$ are functions depending on ϵ and $\dot{\epsilon}$, respectively.

Equation (2) tells us that the cost of low carbon energy, C_L, depends on two parts. The first ("the A-part") represents the idea that some investments in low carbon energies directly mitigate emissions. For example, the purchase of wind turbines directly generates a stream of emissions reductions. The second ("the B-part") suggests that

[7] We define $t = 0$ to be "today" in (1).

C_L also depends on the rate of change of switching to cleaner energy sources. The intuition behind this argument is that a new technology cannot suddenly erupt and take over a whole sector. It needs supply chains, factories, markets, etc. Consider the example of building a factory producing wind turbines. The factory itself does not abate emissions. However, a bigger factory will be associated with an increase in the rate at which wind turbines can be rolled out. Thus, as shown formally in Grubb et al. (2018), investments in factories, supply chains, infrastructures, and accelerated deployment carry costs that relate to the rate of growth of low carbon technologies (or, conversely, to the rate of emissions reduction).

Equation (3) depends on two similar terms. If we assume a fixed unit cost of fossil fuel production, the overall costs will scale with the size of the fossil fuel economy ("the A-part"). However, more ambitious scenarios – anything approaching 2C or lower – imply a more rapid reduction. This would create dislocations and stranded assets as well as varied social costs. Consequently, there is a cost related to the pace of decline of fossil fuels ("the B-part").

So far we have specified the model in terms of abatement ϵ, which could represent specific technologies and sectors. We return to this in section 5, where we explore the implications of our model in a two-sector set-up. For now, in order to align our notation with that commonly used in DICE and other IAMs, we note that the total energy from new low carbon sources multiplied by the average emissions factor of the rest of the energy system is equal to global emissions savings. Hence, it equates to the global abatement fraction $\mu(t)$ found in most IAMs.[8] Specifically, global emissions at time t, $m(t)$, are reduced below a reference trajectory as $m(t) = m_{ref}(t)(1 - \mu(t))$, where global abatement $\mu(t)$ is simply the global proportion of new low carbon energy.

Given that total energy costs are the sum of fossil fuel and low carbon energy costs, we can express them as follows

$$\begin{aligned} \text{Costs} \equiv C(t) &= c_{A,L} f_{A,L}(\mu) + c_{A,F} f_{A,F}(1-\mu) + c_{B,L} f_{B,L}(\dot{\mu}) + c_{B,F} f_{B,L}(\dot{\mu}) \quad (4) \\ &= c_A f_A(\mu) + c_B f_B(\dot{\mu}), \quad (5) \end{aligned}$$

where the second step follows by grouping the terms depending on μ and $\dot{\mu}$, respectively, $\dot{\mu} = \frac{d\mu}{dt}$, and $f_A(\cdot)$ and $f_B(\cdot)$ are functions depending on μ and $\dot{\mu}$, respectively. As in equations (2) and (3), total energy costs also depend on an "A-part" and a "B-part," the interpretation of which is similar. The "A-part" displays the dependence of total costs on the degree of abatement relative to a reference trajectory. We term these rigid costs because they reflect assumed fixed cost curves (for fossil fuel and low carbon sources) at any given point in time. The "B-part" shows that total costs also depend on the rate of abatement. We term these transitional costs. One way of interpreting transitional costs is that they facilitate future emission reductions – and hence 'multiply' overall emission savings arising from an investment in one period.

Note that the "A-part" in (5) derives from components that depend on μ and $(1 - \mu)$ (in (4)) whereas the "B-part" comes from components that are additive with

[8] In most IAMs the emission reduction is defined by a simple equilibrium within each period, which in models like DICE is determined in relation to an abatement cost curve of the form

$$\text{Abatement cost as a fraction of global GDP} = \Lambda(t) = c_A \mu(t)^\theta,$$

where $\mu(t)$ is the fractional abatement raised to some power $\theta > 0$ and scaled by $c_A \geq 0$. In Nordhaus (1993)'s DICE model, $\theta = 2.887$ and $c_A = 0.0686$.

respect to $\dot{\mu}$. Specifically, low carbon energy generation itself displaces the cost of fossil fuel generation: the closer that low carbon energy costs converge to the cost of fossil fuel – for example, by industrial learning and scale effects – the smaller the resulting c_A. Whereas an accelerated transition will not only cost more to ramp up new industries rapidly, it will also increase the likelihood and costs of stranded assets and other transitional costs arising from the disruption of incumbent and redundant infrastructures. It follows logically that the deeper and faster the transition, the more important the transitional costs become relative to the rigid costs.

The rigid cost component of total energy costs is quite traditional. Various versions of DICE express $f_A(\mu)$ as being quadratic or set the coefficient somewhat higher (see footnote 8). Since Grubb et al. (2018) show that learning-by-doing tends to reduce not only the scale but also the convexity of the marginal cost curve, we here define $f_A(\mu) = \mu^2$. The transitional cost component of total energy costs takes a similar form since transitional costs are convex and quadratic (see Grubb et al. (2018) for a detailed explanation of this). Thus, we set $f_B(\dot{\mu}) = \dot{\mu}^2$. We can therefore rewrite the above costs as follows

$$C(t) = c_A\mu^2 + c_B\dot{\mu}^2. \tag{6}$$

Again, c_A and c_B are parameters scaling the fractional abatement to the scale of the energy system. More broadly, c_A and c_B can be seen as representing the magnitudes of rigid and transitional abatement costs, respectively. The first term represents the ongoing cost element associated with a given degree of abatement relative to the high carbon baseline while the second term reflects transitional efforts associated with changing the level of abatement.

Recall that (1) consisted of three parts: the costs incurred from low carbon and fossil fuel energy usage as well as the damages from cumulative emissions. We have so far shown that the two costs can be re-written as two other terms, one depending on the level of abatement and the other on the rate of abatement. As can be seen in the minimization problem stated in Appendix B (equations (18) and (19)), two (quadratic) terms we are minimizing represent exactly the intuition found in (6). The third term, the damages, we motivate in section 3.4.

In section 2 we defined inertia as the costs of changing pathways and path dependence as the dependence of future outcomes on initial conditions. Given the model so far, we can now look at how inertia and path dependence arise in our model.

Inertia, if defined as the costs of changing pathways, is represented in our model as the dependence of costs on the rate of abatement.

Defining path dependency in our model is a bit more complex since in our stylized model, past histories (or initial conditions) don't matter. As such, path dependency in our model refers to the fact that abatement costs depend not only on the level, but also on the rate of abatement. Thus, different pathways from one abatement level to another one have different costs associated with them. Hence, we are able to model the manifestation of path dependency but not the actual process. Therefore, our model provides a stylized way of capturing underlying mechanisms, such as, for example, learning by doing, R&D, network effects, and economies of scale.

3.3 The Interdependence of Rigid and Transitional Costs

We model rigid and transitional costs as being interdependent. Hence, for example, a lower rigid component c_A and a higher transitional component c_B imply that abatement costs are increasingly dominated by the transitional cost of moving from one state to another, relative to the enduring costs of staying at any given distance from the business-as-usual reference path. Introducing this interdependence between rigid and transitional costs therefore implies expanding the understanding of the enduring effect of transitional costs to include the future stream of emission reductions arising from investment which induces innovation in low carbon technologies.

The assumption that rigid and transitional costs are interdependent is, we posit, entirely realistic. Consider, for example, power generation. The cost of renewables now revealed suggests that the enduring cost difference between low carbon and fossil fuel may be negligible. Transitional costs, on the contrary, are additive for low carbon and fossil fuel technologies, as shown in the model above. Thus, more generally, what appear to be high (and rigid) costs of switching to cleaner sources at an early stage of transition may in reality be attributable to transitional investment costs. Therefore, treating such costs as rigid in economic models of climate abatement may be a mistake because they also contribute to a lower future cost stream of emissions abatement. In other words, such costs (can) shift the path of abatement.

Theoretically, we model this interdependence of the two costs by transforming the cost function defined above and considering a weighting factor that measures the influence of the transitional cost to the total abatement cost. We term this the pliability of the system. Lower c_A and higher c_B hence increases the relative influence of transitional costs, i.e. investments which – now that the two terms are dependent – have an enduring impact in lowering the cost of subsequent emission reductions, relative to the ex-ante assumptions about costs and reference emissions.

We trade off the two cost components with reference to a total abatement cost over a fixed characteristic transition time $\hat{t}$. For $\hat{t}$ years ahead, we define the relationship between rigid and transitional costs by assuming that the cost of linear emissions reductions, at a constant rate, over $\hat{t}$ is invariant with respect to the split of costs between rigid and transitional costs. The abatement cost at time t is therefore

$$C(t) = \hat{c}_A \left((1-p)\mu^2 + p\frac{\hat{t}^2}{3}\dot{\mu}^2 \right), \tag{7}$$

where $\hat{c}_A$ is the value of c_A when the emitting system exhibits purely rigid costs, $\hat{t}$ is the characteristic transition time, and $0 \leq p \leq 1$ is the pliability of the system, reflecting the weight of the transitional cost on total abatement cost. Appendix A gives the derivation in full. Our model thus posits that total abatement costs, as displayed in (7), depend on two parts: the pliability of the system and the time scale which it takes the system to adjust accordingly.

The pliability of the system captures the influence of the transitional costs on total abatement costs and is best understood as the economy's capacity to respond endogenously to reduce long-run costs of a transition that involves induced innovation. The degree of pliability defines the extent to which the system can adjust to different outcomes in a path dependent way.

3.4 Climate Damages

To keep things simple, our model, similar to most economic models, links climate impacts to changes in temperature. In line with much of the literature, we assume that global damages increase in proportion to the square of temperature change. A central estimate is that 500 Gigatons of carbon (GtC) in cumulative emissions increase global temperatures by about 1 degree Celsius (Stocker et al., 2013, Figure SPM.10) so that

Annual damages from climate change at time t:

$$\text{Damages} \propto (\text{temperature above pre-industrial})^2 \approx \left(\frac{M(t)}{500\text{GtC}}\right)^2, \quad (8)$$

where $M(t)$ are the cumulative CO2 emissions in GtC at time t.[9] Contrary to common assumption, the time lag between emissions and resulting temperature changes are small. Indeed, Ricke and Caldeira (2014) estimate the median time lag (until the maximum warming occurs) to be just above ten years (see also Mattauch et al. (2020)).

4 Results

The problem we just motivated is stated formally and solved analytically in Appendix B. We now describe the calibration and main results stemming from said model.

4.1 Calibration

Costs of Climate Damages. We assume that damages from climate change are \$3 trillion per year after an additional one degree temperature increase (i.e. an additional 500 GtC in emissions). This estimate is towards the high end of estimates usually used in the economics literature, but is modest compared to estimates from outside of climate change economics which place more weight on risks, non-linear responses, and the welfare of future generations, for example.

Discount Rate. We assume the real discount rate to be 2.5% per year. This is a compromise between the 'prescriptive' and 'descriptive' rates, though leaning more towards the latter in that it leads to significant discounting of costs after a few decades. Note that when the emitting system is pliable ($p = 1$), the derivation of our model in Appendix B shows that the case for immediate action does not depend much on the choice of the discount rate.

Reference Emissions Growth. Over extended periods, global emissions growth exhibits a linear trend (though with significant fluctuations). Given that growth rates have declined sharply in the past few years (partly because of climate policy), we do not assume exponential growth of baseline emissions. Specifically, as shown in more detail in Appendix B, we model reference emissions as follows

$$m_{ref}(t) = m_0 + m_1 t, \quad (9)$$

[9] Note that (8) completes the formulation of our minimization problem in equations (18) and (19) in Appendix B (together with the intuition provided in (6)).

where $m_0, m_1 \geq 0$ are parameters. Specifically, m_1 is the linear growth rate of yearly emissions. Over the past few decades, the average increase in fossil fuels CO2 has been about 1.5% of 2010 emissions. We take as our reference ("business-as-usual") scenario a view in which global emissions rise at approximately 120 MtC per year. This, while lower than the historical trend, is well within the range of projections by the International Energy Agency.

Abatement Cost Parameters. There is a vast literature on abatement costs, but hardly any of it examines transitional costs. This is suboptimal since our main argument is that abatement costs consist of two interdependent costs: rigid and transitional ones. To address this issue, we estimate abatement cost parameters with reference to a 50% cut in global emissions from recent levels (2018). The procedure, described in detail in Appendix A, references abatement costs to the existing economics literature, integrated over a specific time period, but estimates the coefficients from two possible extremes of assumptions about the actual source of these costs: purely rigid abatement costs (i.e. $p = 0$) and purely transitional costs (i.e. $p = 1$).

In the first extreme (purely rigid exogenous abatement costs), we assume a 50% cut in global CO2 emissions from 2018 levels by 2050. The projected cost of this is 1.5% of projected GDP. This corresponds to central estimates from energy system models in recent comparative studies (e.g. Kriegler et al., 2015).

For the second extreme (purely transitional costs), we assume the same cutback of 50% in CO2 emissions by 2050. Assuming a linear trajectory of abatement, the resulting cost is the same over the fixed time period from 2018 to 2050. However, these costs are attributed to transitional costs as they are reorienting the energy system over these years.

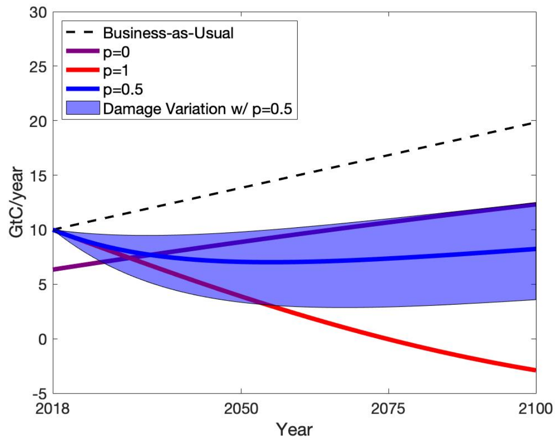
Note that the above implies that we fix a characteristic transition time ($\hat{t}$) to be 32 years, i.e. from 2018 to 2050.

4.2 Main Results

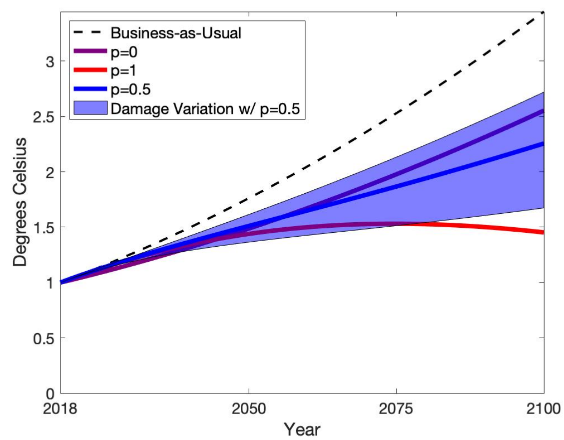
To simplify the interpretation of our results, we present our main results for three different scenarios. Specifically, Figure 1 displays annual emissions (in GtC per year), global mean temperature increases with respect to pre-industrial times (in degrees Celsius), annual damages from climate change (in trillion USD per year), and annual abatement investment (in trillion USD per year) for (i) a system with no pliability (i.e. $p = 0$), (ii) a fully pliable system (i.e. $p = 1$), and (iii) a semi-pliable system (i.e. a midway between the first two cases with $p = 0.5$).

Purely Rigid Costs, $p = 0$. The scenario with a fully non-pliable system is a system with purely rigid costs with no place for transitional costs. Indeed, from (7), we see that if $p = 0$, total abatement costs are $\hat{c}_A \mu^2$, which is the formulation the DICE model uses. And indeed, the policy recommendation implied by our model is similar to the climate-policy ramp observed in DICE. Annual abatement investment increases from below 500 billion USD per year to above 1.5 trillion USD per year in 2100.

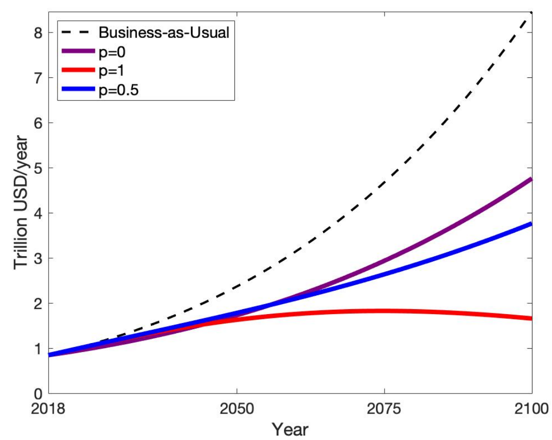
This initial effort manifests itself in a downward jump of annual emissions (with respect to the business-as-usual scenario). Yet, after this initial cutback in emissions,



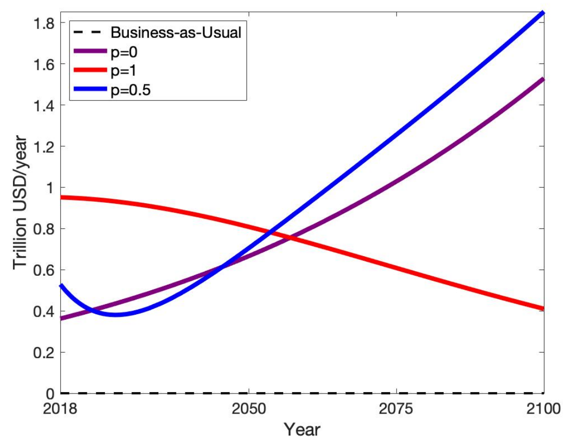
(a) Annual Emissions



(b) Global Mean Temperature Increase



(c) Annual Damages from Climate Change



(d) Annual Abatement Investment

Figure 1: Optimal Policy and Implications for $p = 1$, $p = 0.5$, and $p = 0$.

emissions continue to rise steadily throughout the century. This is because abatement in this scenario cannot keep up with the rising emissions from the business-as-usual/reference scenario. As cumulative emissions continue to rise, global mean temperatures are expected to rise above 2.5 degrees Celsius by 2100 (with no end in sight). Damages, as expected, also skyrocket over time, reaching more than 4 trillion USD per year by the end of the century.

Purely Transitional Costs, $p = 1$. The scenario with a fully pliable system is one with solely transitional costs and no place for rigid costs. The policy recommendation in a fully pliable system halves global annual emissions before 2050 and reaches no new emissions in around 2075. Afterwards, annual emissions become negative, implying that cumulative emissions will decrease. Since temperature increases are proportional to cumulative emissions, global mean temperatures will increase to just below 1.5 degrees Celsius until around 2075, and will decrease afterwards.

The optimal policy associated with this requires immediate action at a much higher level than seen in the other scenarios. Initial annual abatement investment in the fully pliable system are almost twice as high as the ones in the fully non-pliable system at almost 1 trillion USD per year. Contrary to the case above, optimal effort decreases over time, reaching less than 500 billion USD per year by 2100. Interestingly, a figure slightly below 500 billion USD per year is typically indicated by the International Energy Agency as the investment requirements to deliver the goals set out in the Paris Agreement. Damages, in this scenario, remain much lower than in the other two cases between one and two trillion USD per year between 2018 and 2100.

A Semi-Pliable System, $p = 0.5$. It is clear that neither of the two extreme cases presented above represents reality. Indeed, most systems will differ in their degrees of pliability. The third scenario thus presents a midway between the cases above. As can be seen from the results, this "midway scenario" leads to emissions, temperature increases, damages, and investments that lie in between the estimates from the two cases above.

The essential insight from our model is that pliability can greatly lower the overall costs associated with climate change, but only if the required effort is invested up-front so as to "change course." A pliable energy system implies that the costs of avoiding (possibly infinitely) high climate damages will end up being much lower than what standard economic approaches suggests.[10]

5 Two-Sector Extension

For a non-pliable system, where costs are mostly a function of abatement at a given point in time, it makes sense to abate up to the cost value that is justified by climate damages, mediated by a carbon price. However, with a pliable system in which costs depend on the rate of change of abatement, it makes sense to transform the economy at a steady rate instead. With a fixed abatement target, this implies starting as early as possible to minimize the rate of change. Indeed, it may even imply acting in multiple sectors simultaneously. The aim of this section is thus to think about our simple framework in a two-sector setting and explore further policy implications.

Two-Sector Extension. Consider an economy in with two sectors. One of these sectors has high transitional costs while the other has mostly rigid (exogenously) defined abatement costs. Including damages, we can write the total cost by extension of model (7) as follows:

$$\begin{aligned}\tilde{C}(t) \approx\ & \hat{c}_{A_1}\left((1-p_1)\mu_1^{\alpha_1} + p_1\frac{\hat{t}^2}{3}\dot{\mu}_1^2\right) + \hat{c}_{A_2}\left((1-p_2)\mu_2^{\alpha_2} + p_2\frac{\hat{t}^2}{3}\dot{\mu}_2^2\right) \\ & + d\left(\int_0^{\hat{t}} (1-\mu_1-\mu_2)\, m_{ref}\, d\tau\right)^2, \end{aligned} \tag{10}$$

[10]To ensure that the results just presented do not merely arise because of our model assumptions, Grubb and Wieners (2020) insert the cost function derived in this paper into the DICE model itself and observe similar impacts of pliability on optimal policy.

where $d \geq 0$ is the damage parameter. We assume that the first and second sector face costs that increase with their own levels of abatement, following power laws with exponents α_1 and α_2, respectively.[11] This is equivalent to assuming that each sector has its own marginal abatement cost curve.[12] There are now two control variables, the paths of both $\mu_1(\tau)$ and $\mu_2(\tau)$ over time. The optimal levels of abatement $\mu_1(\tau)$ and $\mu_2(\tau)$, at any time $\tau \geq 0$, are now interdependent because, even as abatement costs are unique to each sector, damages are determined by the cumulative level of emissions across both sectors, such that both problems cannot be solved independently.

While the N-sector problem with $N > 1$ cannot be solved analytically, and perhaps not even numerically, we demonstrate here how our analytic solution for the one-sector problem yields an approximate analytic solution to the N-sector problem. Suppose for simplicity that the economy consists of two sectors that contribute equally to cumulative emissions, such that each share is $1/2$. By considering both sectors individually and ignoring their interaction, we would compute damages inflicted by each sector by looking at the square of that sector's cumulative emissions. Total damages would be obtained by adding up sectoral damages. This 'sum of squares' approach would lead to $(1/2)^2 + (1/2)^2 = 1/2$, which deviates by a factor 2 from the square of the sum, which equals $1 = (1/2 + 1/2)^2$. Hence, in treating sectors individually we lose roughly a factor 2 in damages. For N sectors, similarly, we lose a factor N. A practical approach to solving the N-sector problem, therefore, would be to consider each sector in isolation, such that our analytic solution depending on that specific sector's pliability is applicable, while artificially 'scaling up' each sector's damages in order to account for the ignored interaction effects.

If the sectors vary in size, the scaling factors would depend on the size of the sector (smaller sectors should receive larger multiples). High pliability sectors (e.g. power) would take up all abatement not carried out by the other sector (e.g. steel), while low pliability sectors (e.g. steel) would carry out the difference between the optimal abatement in a totally non-pliable economy, and what the other sector (e.g. power) is contributing.

Implications for Optimal Policy. Jointly with the results presented in section 4.2, the results in this section suggest an additional implication for policy.

The optimal degree of effort – reflected for example in a carbon price – depends on pliability, which is our way of saying that it depends upon the degree of induced innovation, inertia, and path dependence. When these dynamics vary by sector, the carbon price per sector will also vary, depending on what we characterize as each sectors' own pliability and characteristic transition times. This finding is reminiscent of Vogt-Schilb et al. (2018) who find that optimal abatement investments differ across sectors in the economy.

These effects, of course, may be exacerbated, by learning-by-doing or any other factors that contribute to pliability. Indeed, it is possible that marginal abatement

[11] There are no simple analytical forms we can use for these curves that sum to a quadratic form for the total abatement, but one can find two power laws the sum of which is approximately the quadratic form over the region of interest.

[12] This is not the same as assuming that the quadratic increase in costs at the aggregate level stems from cost increases as one moves from one sector to another, following a marginal abatement cost curve. However, each sector may have its own sectoral marginal abatement cost curve, which aggregates to the whole economy curve. This aggregation is outside the scope of the present work.

costs are, *ex post*, zero or even negative for systems with high innovation which results in technologies cheaper than incumbent fossil fuels. Furthermore, if different regions face different sectoral challenges to reducing emissions, their carbon prices will also be different. This complements the policy recommendations in Hassler et al. (2020) who argue that having one region not participate in emissions abatement is too costly for the world. We thus extend this by showing that there might be a need for different carbon prices for different sectors and/or different regions in the world.

Given that optimal effort may differ by sector, it is likely, we posit, that the optimal policy instrument – or more likely, the optimal mix of instruments – may also vary. A carbon price, as is well known, can both accelerate diffusion of a low carbon technology and enhance its profitability, but only if it makes it plausibly profitable to private actors in the first place. However, many dimensions of the innovation process, particularly its earlier stages, involve public benefits that cannot reasonably accrue to private actors. This thus underlines the case by Acemoglu et al. (2012) for subsidies, in addition to a carbon price, for "green" technologies. As such, effective subsidies may establish a "positive" path dependency and undermine the incumbent path dependency.

6 Conclusion

We have developed and applied a stylized approach to exploring the implications of dynamic factors in global climate change cost-benefit assessment. We have explained dynamic factors as the combination of (a) induced innovation, whereby action in one period may reduce future mitigation costs, (b) inertia, which reflects costs of adjustment and infrastructure investments, and (c) their combination into wider phenomena of path dependency in emitting systems. The faster the transition being contemplated, the more important these dynamics elements may become.

Our model, we posit, can be seen as a complement to existing models focusing on the long-term balance of costs and benefits. In doing so, however, many of these models neglect transitional costs and have to make prior assumptions about the long-term costs of abatement (as expressed through rigid marginal abatement cost curves for future periods). Through its conceptual simplicity, our model also complements an emerging body of complex IAMs, which represent some dynamic processes with great technological detail, but which have, as a result, become very complex to understand and parametrize, with a risk of obscuring more general insights.

The policy insights from our simple model are complementary to existing models. They underline the importance of immediate action, but expand the scope of the benefits arising to the extent that early emission reductions will not only reduce climate impacts sooner, but may also facilitate a future stream of lower emissions – a climate-benefits-multiplier effect arising from these dynamic consequences. Also, because these dynamic multiplier effects may be expected to differ between different sectors, it follows that the optimal carbon price may vary between different sectors, because the degree of induced innovation, inertia and path dependence may be expected to vary between sectors. The same logic may extend to different forms of interventions which could, to different degrees, facilitate path-dependent adjustments, implying the potential relevance of using a mix of policy instruments.

The general and crucial insight from our model is that dynamic characteristics can have a radical impact on the optimal approach to tackling climate change. Thus, it is crucial for future work to find ways to realistically (and ideally simplistically) model dynamic characteristics in possibly more complete economic models than ours. Furthermore, we hope that this paper will incentivize other researchers to extend our ideas about the level and the rate of abatement into fuller economic models. While we applaud the recent push in economics to analyze the effects of second-best policies (e.g. Rezai and Van Der Ploeg, 2017; Hassler et al., 2020), this should be done with an explicit focus on the transition phase to the long-run where issues of induced innovation, path dependency, and inertia are of immense importance.

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A Transformation of Rigid and Transitional Costs to Pliability and Transition Time

The dynamic cost function expressed in terms of rigid and transitional components is hard to interpret intuitively, and in the paper we summarize its transformation in terms of pliability of the system – its ultimate capacity to reduce abatement costs – and the characteristic transition time of the system.

We define the latter with reference to a linear transition over a characteristic transition time from 2018 (the base year of our analysis) to 2050. The corresponding $\hat{t} = 32$ years after 2018 are well within the range of estimates of large-scale transitions in the global energy system. We define the relationship between rigid and transitional cost components by assuming that the total undiscounted cost of linear emission reductions, at rate $\bar{m}\%/year$, over time horizon $\hat{t}$ is invariant to the split of costs between rigid and transitional costs, i.e. we take

$$\begin{aligned} \mu &= (m_1 - \bar{m})t \\ \dot{\mu} &= (m_1 - \bar{m}). \end{aligned} \tag{11}$$

Integrating (undiscounted) abatement cost to time $\hat{t}$, we thus define (using equation (6))

$$\overline{C} = \int_0^{\hat{t}} c_A \mu^2 dt + \int_0^{\hat{t}} c_B \dot{\mu}^2 dt = (m_1 - \bar{m})^2 \left(c_A \frac{\hat{t}^3}{3} + c_B \hat{t} \right). \tag{12}$$

Define $c_A \equiv \hat{c}_A(1-p)$ and $c_B \equiv \hat{c}_B p$ for some $0 \leq p \leq 1$, which we denote the pliability of the system. Clearly, in a fully non-pliable system ($p = 0$), i.e. one with only rigid costs, we have

$$\overline{C}_{\text{rigid}} = (m_1 - \bar{m})^2 \hat{c}_A \frac{\hat{t}^3}{3}, \tag{13}$$

whereas in a fully pliable system ($p = 1$), i.e. one with only transitional costs, we have

$$\overline{C}_{\text{transitional}} = (m_1 - \bar{m})^2 \hat{c}_B \hat{t}. \tag{14}$$

These two equations define the extreme cases of entirely rigid or entirely transitional costs for a characteristic adjustment period $\hat{t}$. As the total abatement costs are fixed over $\hat{t}$, equations (13) and (14) help us to pin down the value of $\hat{c}_B$ in terms of $\hat{c}_A$ and $\hat{t}$ as follows

$$\hat{c}_B = \hat{c}_A \frac{\hat{t}^2}{3}. \tag{15}$$

Note that the trade-off between the two costs in equations (13) and (14) does not depend on the rate (since the slope ($\bar{m}$) drops out when we are compare rigid and transitional costs). This form is intuitively reasonable since the adjustment costs rise as the square of the rate, which is inversely proportional to the timescales over which adjustment costs are defined.

Given $c_A \equiv \hat{c}_A(1-p)$ and $c_B \equiv \hat{c}_B p$, we can now write total costs (see equation (6)) more generally, using the pliability of the system, i.e.

$$C = \hat{c}_A(1-p)\mu^2 + \hat{c}_B p \dot{\mu}^2. \tag{16}$$

Substituting for $\hat{c}_B$ (see (15)), we can write the overall cost for a system of pliability p as follows

$$C = \hat{c}_A \left((1-p)\mu^2 + p\frac{\hat{t}^2}{3}\dot{\mu}^2 \right). \tag{17}$$

This is the expression presented in (7). $\hat{c}_A$ now serves as the overall cost scaling factor, and $\hat{t}$ is the characteristic timescale of major system adjustments.

B The Model and its Analytic Solution

Optimization Problem. We here identify the optimal emissions path that minimizes the sum of abatement and climate damage costs. Cumulative emissions relative to pre-industrial times drive economic damages. Hence, we define cumulative emissions at time t, relative to pre-industrial times, as $M(t)$. We take $t = 0$ to mean today. As such, $M(0) = M_0$ represents current cumulative emissions (again, relative to pre-industrial times). Annual emissions are defined as $m(t) \equiv M'(t)$, where the apostrophe denotes a time derivative. In the same vein, $M''(t) = m'(t)$ represents the rate of change of annual emissions. As the path of $M(t)$ is sufficient to determine both $m(t)$ and $m'(t)$, we take $M(t)$ as our control variable.

The level of abatement $\mu(t)$ referred to in the main test is implied by the cumulative emissions path via $M'(t) = m(t) = gE_{ref}(t)(1-\mu(t))$, where $E_{ref}(t)$ denotes energy demand in the reference scenario, g is a conversion factor from energy to emissions, and $\mu(t)$ denotes abatement.[13]

Hence, we aim to solve an optimization problem involving $M(t)$ and its two derivatives, i.e.

$$\min_{\{M(t)\}_0^T} \int_0^T F(M(t), M'(t), M''(t), t)dt \qquad \text{such that } M(0) = M_0. \tag{18}$$

We are looking for an optimal path for all $0 \le t \le T$, denoted by $\{M(t)\}_0^T$. The lower limit of the integral is zero, representing today, while the upper limit $T \ge 0$ represents the time horizon of the optimization. We are particularly interested in the limit $T \to \infty$. The starting point of the path $\{M(t)\}_0^T$ is fixed at $M(0) = M_0$, representing cumulative emissions today. If we also consider today's marginal emissions as unchangeable, then we may similarly restrict $M'(0)$ to some fixed value. As we will show below, in regimes b and c (defined below), $M'(0) = m(0)$ is forced to match current annual emissions.

[13]The reason we work with $M(t)$ rather than $m(t)$ or $\mu(t)$ is because it is more convenient to work with derivatives (rather than integrals) of the control variable $M(t)$.

The integrand F depends on the control variable $M(t)$ and its first two derivatives $M'(t) = m(t)$ and $M''(t) = m'(t)$ as follows

$$F = e^{-rt}\left(\frac{d}{8}M(t)^2 + c_A\left(m_{ref}(t) - m(t)\right)^2 + 2c_B^\star\left(m'_{ref}(t) - m'(t)\right)^2\right), \tag{19}$$

where $d, c_A, c_B^\star \geq 0$ are parameters and $r \geq 0$ is the discount rate. The numerical constants 8 and 2 are chosen for convenience in the form of the final result, with the parameter c_B in the main text replaced by $c_B = 2c_B^\star$. Here, damages at time t are given by $D(t) = \frac{dM(t)^2}{8}$, as motivated in section 3.4.

The reference path of emissions per unit of time is denoted by $m_{ref}(t)$ and its rate of change is denoted by $m'_{ref}(t)$. As indicated in the main text, we consider the reference scenario of linearly increasing annual emissions, such that $m_{ref}(t) = m_0 + m_1 t$ and $m'_{ref}(t) = m_1$, where $m_0, m_1 \geq 0$ are parameters. In the reference scenario, annual emissions at time zero are m_0. Cumulative emissions in the reference scenario for $t \geq 0$ are

$$M_{ref}(t) \equiv M_0 + \int_0^t m_{ref}(s)ds. \tag{20}$$

Abatement $\mu(t)$ reduces emissions relative to the reference path as $m(t) = (1 - \mu(t))m_{ref}(t)$, but we do not use $\mu(t)$ in our formulation below as we, as discussed above, express everything in terms of cumulative emissions $M(t)$.

Intuition. The optimization problem above indicates that we are interested in minimizing the net present value of the sum of damages and abatement costs, discounted at rate r. This net present value comprises three terms

1. The first term, which scales with d, represents damages to the economy at time t measured in units of currency, and is assumed to be proportional to the square of cumulative carbon emissions.
2. The second term, which scales with c_A, is sensitive to the amount of abatement at time t, which is proportional to the square of the deviation of the chosen path from the reference path $m_{ref}(t)$ of emissions per unit time, i.e. the square of $m_{ref}(t) - m(t)$, where the square ensures that we incur a non-negative cost for any deviation from the reference path.
3. The third term, which scales with $c_B^\star$, penalizes the curvature of the chosen emissions path away from the reference path, where the square of $m'_{ref}(t) - m'(t)$ again ensures that we incur a non-negative cost for any deviation from $m'_{ref}(t)$. For a pliable system, emissions reductions may not be expensive per se. Rather, it is the transformation of the system that is expensive. As such, "veering away" from the reference trajectory is penalized, but not explicitly the resulting distance.

Euler-Lagrange Equations. The optimization problem described above can be solved using standard Euler-Lagrange (EL) methods. The only non-standard feature is that the control variable $M(t)$ appears alongside both of its first and second derivative in the integrand F. For this reason, the standard EL equation is adjusted to include a third term as follows

$$0 = \frac{dF}{dM} - \frac{d}{dt}\frac{dF}{dM'} + \frac{d^2}{dt^2}\frac{dF}{dM''}. \tag{21}$$

Using the definition of F in (19) and explicitly writing out these terms we obtain

$$\begin{bmatrix} -4c_B^\star m_1 r^2 - 2c_A(m_0 r + m_1(rt-1)) & \frac{d}{4} & 2c_a r & 4c_B^\star r^2 - 2c_A & -8rc_B^\star & 4c_B^\star \end{bmatrix} \begin{bmatrix} 1 \\ M(t) \\ M'(t) \\ M''(t) \\ M'''(t) \\ M''''(t) \end{bmatrix} = 0,$$

which we here express as an inner product. This expression makes clear that in general we are faced with an inhomogenous linear ordinary differential equation (ODE) of fourth order. As is standard, the solution can be written as the sum of two solutions: one that solves the homogenous ODE and one that solves the inhomogenous ODE.

Solution to Inhomogenous ODE. The inhomogenous ODE can be solved by a linear function of time, which we write as

$$M(t) = B + bt, \tag{22}$$

where B and b are constants to be found. For this candidate solution $M(t)$, the second, third, and fourth derivatives are zero. Solving the resulting simplified ODE for B and b, we obtain

$$b = 8\frac{c_A m_1 r}{d} \qquad\qquad B = \frac{c_A(m_0 r - m_1)}{d/8} + 64 m_1 r^2 \left(\frac{c_B^\star}{4d} - \left(\frac{c_A}{d}\right)^2\right). \tag{23}$$

This simple solution already yields one important insight into the long-term behavior of our solution: in the long-run, cumulative emissions should be linear in time, such that annual emissions are optimally constant. Specifically, the optimal long-run constant level of emissions is given by the parameter b above. As can be seen, it decreases with the damage parameter d, but increases with the cost c_A, the discount rate r, and the increase of marginal emissions in the reference scenario, given by m_1. Depending on whether c_A is strictly positive or zero, costs incurred in the long-run are either unbounded or converge to a constant:

1. When c_A is strictly positive, optimal long-run emissions are strictly positive, as given by b above. Annual emissions being constant implies an ever-increasing cost per unit of time, as the distance between the level b and the reference path $m_{ref}(t)$ grows without bound. Hence, costs grow without bound. However, this ever-increasing cost is offset by the damages avoided. Further, discounting at rate r ensures that the net present value of both costs and damages remains finite.
2. For fully pliable systems, in which case $c_A = 0$, the optimal long-run level of emissions is zero, as can be seen simply by substituting $c_A = 0$ into the equation for b above. In this case, costs are driven entirely by the $c_B^\star$ term in F (see (19)). As $t \to \infty$, we obtain $(m'_{ref}(t) - m'(t))^2 = m_1^2$ because $m(t)$ is constant in this limit. Hence, for a fully pliable system, rather than growing without bound, optimal costs per unit of time converge to a constant value.

While the inhomogenous ODE determines the optimal long-term emissions path, the particular solutions to the homogeneous ODE determine the optimal course of action in the short-term. We discuss these next.

Solution to Homogenous ODE. To solve the homogenous ODE, note that each derivative in the ODE is simply multiplied by a constant. This suggests to look for solutions that are exponential in time. Indeed, the homogenous ODE of fourth order allows for four independent solutions taking the form

$$M(t) = \sum_{j=1}^{4} A_j \exp\left(\frac{a_j t}{2}\right), \tag{24}$$

where the parameters A_j and a_j remain to be determined for $j = 1, 2, 3, 4$. Substituting this candidate solution into the ODE and simplifying, we find that the constants a_j for each $j = 1, 2, 3, 4$ must solve the following fourth-order polynomial equation

$$\begin{bmatrix} d & 4c_A r & 4r^2 c_B^\star - 2c_A & -4rc_B^\star & c_B^\star \end{bmatrix} \begin{bmatrix} 1 \\ a_j \\ a_j^2 \\ a_j^3 \\ a_j^4 \end{bmatrix} = 0, \qquad j = 1, 2, 3, 4. \tag{25}$$

Generally, this equation is of fourth order, unless $c_B^\star = 0$, in which case it is only of second order (note the last two entries of the row vector).

Full Solution. The full solution is obtained by summing the solutions to the homogeneous and inhomogeneous ODEs, i.e.

$$M(t) = B + bt + \sum_{j=1}^{4} A_j \exp\left(\frac{a_j t}{2}\right), \tag{26}$$

where the parameters B and b are given by (23), the constants a_j for $j = 1, 2, 3, 4$ are the roots of the fourth-order polynomial equation given in (25), and the four constants A_j for $j = 1, 2, 3, 4$ remain to be determined by four boundary conditions, as discussed below. These boundary conditions will need to ensure that $M(0) = B + \sum_{j=1}^{4} A_j = M_0$, thereby putting a constraint on the A_j's.

Boundary Conditions. In general, the four constants A_j are determined by a total of four boundary conditions to be specified at either $t = 0$ or $t = T$. At $t = 0$, we impose $M(0) = M_0$, reflecting the fact that cumulative emissions (relative to pre-industrial times) at time zero are fixed. For systems with any transitional cost ($c_B > 0$), we also impose $M'(0) = M'_{ref}(0) = m_{ref}(0) = m_0$, because sudden jumps in emissions per unit of time would incur infinite costs. By imposing both boundary conditions, we ensure that the path of cumulative emissions $M(t)$ smoothly matches that of the reference trajectory of cumulative emissions $M_{ref}(t)$.

At $t = T$, we are faced with two free boundary conditions, as endpoint $M(T)$ and its derivative $M'(T)$ are left to be determined by the optimizer. However, in the limit as $T \to \infty$, which we consider below, two of the four homogenous solutions can be discarded (set to zero), as they blow up exponentially, thereby causing infinite damages. As such, only two constants A_j, for $j = 1, 2$ remain, which can be determined by the two boundary conditions at $t = 0$.

If $c_B = 0$, the ODE and polynomial equation are of second order. In this case, only a single boundary condition at $t = 0$ is required, which we take to be $M(0) = M_0$. In this case, a jump in marginal (but not cumulative) emissions at time zero is permitted. Whenever $c_B > 0$, we set $M(0) = M_0$ and $M'(0) = m_0$, such that both cumulative and marginal emissions at time zero match those of the reference trajectories $M_{ref}(t)$ and $m_{ref}(t)$.

Solutions Under Three Regimes. The optimal solution behaves quite differently, even qualitatively, depending on the numerical values of the parameters. Specifically, three regimes can be identified. We present the solution in each of three mutually exclusive and collectively exhaustive regimes: (a) $c_B = 0$, (b) $c_B \neq 0$, $c_A^2 \geq c_B d$, and (c) $c_B \neq 0$, $c_A^2 < c_B d$. These three regimes can be summarized as (a) no transitional costs, (b) non-zero but small transitional costs, and (c) large transitional costs. In each case, an analytic solution is possible, which can be found by (i) solving the (in general) fourth-order polynomial equation, (ii) discarding two of the four solutions to the homogenous ODE that correspond to the explosive solutions, and (iii) imposing the relevant boundary condition(s) at $t = 0$. We here only report the analytic solution in the case where $T = \infty$, which is economically the most relevant, and for which the solution takes the simplest possible form.

Regime a If $c_B^\star = 0$, such that the system contains no pliability whatsoever, the fourth-order ODE simplifies to a second-order ODE. The corresponding second-order polynomial equation allows for two unique roots, one positive and one negative. The positive root can be discarded as it corresponds to an explosive solution, such that we can set $A_2 = A_3 = A_4 = 0$, leaving only A_1 to be determined. The negative root is given by

$$a_1 = r - \sqrt{r^2 + \frac{d}{2c_A}}. \tag{27}$$

Note that $a_1 < 0$. (The other root contains a plus instead of a minus in front of the square root and is economically irrelevant.) Imposing the boundary conditions $M(0) = M_0$, the constant A_1 can be determined as

$$A_1 = M_0 - B, \tag{28}$$

where the value of B is given by (23) when $c_B^\star$ is set to zero (as $c_B = 0 \implies c_B^\star = 0$). In *Regime a*, we do not impose $M'(0) = m_0$ such that the optimal level of today's emissions, $M'(0)$, will generally differ from the reference level, m_0. This is unique to *Regime a*. For pliable systems in the two regimes below, a jump in marginal emissions is impossible.

Regime b If $c_B^\star \neq 0$, $c_A^2 \geq c_B^\star d$, such that transitional costs are non-zero but small in relative terms, the fourth-order polynomial allows for four distinct roots. Two roots are positive and can be discarded from economic arguments, i.e. we set $A_3 = A_4 = 0$. The two remaining (negative) roots are given by

$$a_1 = r - \sqrt{r^2 + \frac{c_A}{c_B^\star} + \frac{\sqrt{c_A^2 - c_B^\star d}}{c_B^\star}} \qquad a_2 = r - \sqrt{r^2 + \frac{c_A}{c_B^\star} - \frac{\sqrt{c_A^2 - c_B^\star d}}{c_B^\star}}, \tag{29}$$

where each displayed square root is a real number because $c_A^2 \geq c_B^\star d$ by assumption in the current regime. Imposing the boundary conditions $M(0) = M_0$ and $M'(0) = m_0$, meaning both cumulative and marginal emissions at time zero must match those of the reference trajectory, we find the two constants A_1 and A_2 as follows

$$A_1 = \frac{2(m_0 - b) + a_2(B - M_0)}{a_1 - a_2} \qquad A_2 = \frac{2(m_0 - b) + a_1(B - M_0)}{a_2 - a_1}. \tag{30}$$

Regime c If $c_B^\star \neq 0$, $c_A^2 < c_B^\star d$, such that transitional costs are large in relative terms, i.e. the system is dominated by pliability, the fourth-order polynomial equation allows for four distinct, complex-valued, roots. To avoid the emissions path exploding as $t \to \infty$, we pick the two roots with negative real parts. Hence, we may set $A_3 = A_4 = 0$. The two negative roots a_1 and a_2 differ by only a single sign, such that we can denote them by $a_1 = a_+$ and $a_2 = a_-$, where $a_\pm$ is defined as

$$\begin{aligned} a_\pm \equiv r &- \frac{1}{\sqrt{2}}\sqrt{r^2 + \frac{c_A}{c_B^\star} + \sqrt{\frac{d}{c_B^\star} - \left(\frac{c_A}{c_B^\star}\right)^2 + \left(r^2 + \frac{c_A}{c_B^\star}\right)^2}} \\ &\pm \frac{i}{\sqrt{2}} \frac{\sqrt{\frac{d}{c_B^\star} - \left(\frac{c_A}{c_B^\star}\right)^2}}{\sqrt{r^2 + \frac{c_A}{c_B^\star} + \sqrt{\frac{d}{c_B^\star} - \left(\frac{c_A}{c_B^\star}\right)^2 + \left(r^2 + \frac{c_A}{c_B^\star}\right)^2}}}, \end{aligned} \tag{31}$$

where $i = \sqrt{-1}$ is the imaginary unit, and every displayed square root is a real (positive) number, because $c_A^2 < c_B^\star d$ in the current regime. A quick inspection reveals that both $a_\pm$ have negative real parts as desired. Imposing the boundary conditions $M(0) = M_0$ and $M'(0) = m_0$, we find that the constants A_1 and A_2 are identical in form to those in *Regime b*, namely

$$A_1 = \frac{2(m_0 - b) + a_2(B - M_0)}{a_1 - a_2} \qquad A_2 = \frac{2(m_0 - b) + a_1(B - M_0)}{a_2 - a_1}. \tag{32}$$

However, the numerical values of these constants differ from *Regime b*, because the two roots a_1 and a_2, which appear in the numerator and denominator, are now complex values. Hence, A_1 and A_2 are also complex valued. Naturally, the cumulative emissions path $M(t)$ for all time t remains real valued. After some tedious but straightforward trigonometric algebra, the optimal cumulative emissions trajectory $M(t)$ can be rewritten in trigonometric terms as

$$M(t) = B + bt + \exp\left(\frac{\hat{a}t}{2}\right)\left[\frac{2(m_0 - b) + \hat{a}(B - M_0)}{\tilde{a}} \sin\left(\frac{\tilde{a}t}{2}\right) + (M_0 - B)\cos\left(\frac{\tilde{a}t}{2}\right)\right],$$

where b and B are as in (23), while $\hat{a}$ and $\tilde{a}$ are real numbers coming from the real and imaginary parts of a_1 above. Explicitly, we have

$$\hat{a} = r - \frac{1}{\sqrt{2}}\sqrt{r^2 + \frac{c_A}{c_B^\star} + \sqrt{\frac{d}{c_B^\star} - \left(\frac{c_A}{c_B^\star}\right)^2 + \left(r^2 + \frac{c_A}{c_B^\star}\right)^2}} \tag{33}$$

and

$$\tilde{a} = \frac{1}{\sqrt{2}} \frac{\sqrt{\frac{d}{c_B^\star} - \left(\frac{c_A}{c_B^\star}\right)^2}}{\sqrt{r^2 + \frac{c_A}{c_B^\star} + \sqrt{\frac{d}{c_B^\star} - \left(\frac{c_A}{c_B^\star}\right)^2 + \left(r^2 + \frac{c_A}{c_B^\star}\right)^2}}}. \tag{34}$$

The intuition for *Regime c* is that, when "steering" is expensive, it might be beneficial to "oversteer" before correcting (steering back) later, which explains the appearance of trigonometric functions in the solution: emissions oscillate towards the long-term optimum. For a fully pliable system in which case $c_A = 0$, it is optimal to decarbonize the economy completely at some finite time, and even go into negative marginal emissions (capturing carbon dioxide from the atmosphere), also at some finite time, while oscillating (with exponentially decreasing amplitudes) towards a fully decarbonized limit.

In all three regimes, the optimal marginal emissions path $M'(t)$ is implied by the optimal cumulative emissions path $M(t)$ via a straightforward differentiation with respect to time. Further, in all cases an analytic solution remains possible even for a finite optimization horizon T, but the resulting expressions are more involved, because it no longer holds that two out of four roots from the fourth-order polynomial can be discarded (all four roots are relevant in this case). The resulting expressions are available from the authors upon request.

Required Effort at Time Zero. Having obtained the optimal path of cumulative emissions $M(t)$ in three regimes, we can also compute the optimal level of effort, measured in units of currency, at time zero. We compute this quantity by summing the cost terms in F (see (19)) evaluated at time zero, i.e. we compute

$$c_A \left(m_{ref}(0) - m(0)\right)^2 + 2c_B^\star \left(m'_{ref}(0) - m'(0)\right)^2, \tag{35}$$

which we compute for each regime. In *Regime a*, only the first term is relevant as $c_B^\star = 0$. In *Regime b* and *Regime c*, only the second term is relevant, because the boundary conditions enforce $m_{ref}(0) = m(0)$. For simplicity, we present results only to leading order d, c_A, and $c_B^\star$. (Otherwise, we would have to examine page-long expressions.) For *Regime a*, we find that the optimal level of effort at time zero, to leading order d, equals

$$\frac{(m_1 + r(m_0 + rM_0))^2}{c_A r^6} \left(\frac{d}{8}\right)^2 + O\left(d^3\right). \tag{36}$$

That is, the optimal level of effort is of second order in d. The optimal level of effort today is extremely sensitive to the discount rate r, which appears to the power of six in the denominator. Higher d, m_0, m_1, and M_0 also induce more effort. Higher abatement cost, c_A, actually suppresses effort at time zero, presumably because the decision-maker optimally "smears-out" the total effort over a longer period of time.

In *Regime b*, which takes $c_B^\star > 0$ while $c_A > 0$ is still dominant, the result is strikingly different. We consider the limit where d and $c_B^\star$ are small, so that inequality $c_A^2 > c_B^\star d$ of *Regime b* remains satisfied. Hence, the system is dominated by c_A cost, but there is nonetheless a small pliable component. In this case, we find that the optimal level of effort at time zero, to leading order in d and $c_B^\star$, is

$$\frac{m_1(m_1 + r(m_0 + rM_0))^2}{2r^2} \frac{c_B^\star}{c_A} d + O\left(d^2\right) + O\left(c_B^{\star 3/2}\right). \tag{37}$$

While the numerator remains largely similar, apart from an additional multiplicative factor m_1, there are three noticeable differences compared to the optimal effort under *Regime a*. First, the effect of the damage parameter d is now of first rather than second order. Hence, even for small damages, early action is optimal even for a system with only a small pliable component. Indeed, this may change the optimal level of effort by an order of magnitude. Second, the optimal level of effort increases with the cost parameter $c_B^\star$. The more expensive it is to change course, the better it is to do it early. Third, sensitivity to discount rates is greatly reduced, appearing in the denominator with a square rather than a sixth power. Since early action is required anyways, the effect of the discount rate may be less pronounced.

In *Regime c*, which assumed $c_B^\star > 0$ and $c_A^2 < c_B^\star d$, we must be careful regarding the series expansion to make sure the inequality is not violated. The resulting expression is not very insightful, and hence is not presented here.