TI 2020-079/VI
Tinbergen Institute Discussion Paper

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Debt sustainability when $r - g < 0$: no free lunch after all*

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December 7, 2020

Abstract

Interest rates on public debt have for several years now fallen short of GDP growth rates in much of the Western world. In his presidential address to the AEA Blanchard argued that this implies that there are no fiscal costs to high debt (Blanchard, 2019).[1] In this paper we argue that the safe rate is not the right interest rate to use for that comparison. We develop a General Equilibrium Asset Pricing model and econometrically estimate the relevant characteristics of the stochastic processes driving the primary surplus in relation to the growth rate of aggregate consumption and derive the proper risk premium. The resulting interest rate exceeds the growth rate. We then calculate the discounted value of future primary surpluses using the same stochastic process for the primary surplus and compare that to the market value of the (Dutch) public sector debt. We test various explanations for the gap between these two and derive the fiscal adjustment necessary to eliminate it (the "fiscal sustainability gap").

JEL codes: G12, H62, H63

Key Words: Debt valuation, sustainable deficits, fiscal adjustment gap

*Views expressed in this paper are our own and do not necessarily reflect those of De Nederlandsche Bank. We thank Karlijn Kersten of the Dutch ministry of Finance (AFEP) for help with data collection.

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[1] He did outline other reasons not to run large deficits in his address.

1 Introduction

So is there a free lunch after all? With interest rates on sovereign debt below growth rates in much of the Western world, traditional approaches to debt sustainability seem to suggest just that. If debt can be rolled over with interest just capitalized while public expenditure and revenues grow roughly in line with GDP, $r - g < 0$ seems to suggest the free lunch is there. Indeed Olivier Blanchard in his presidential address to the AEA suggested that debt has no fiscal cost anymore in these circumstances (Blanchard, 2019).[2] In this paper we argue that this is only true if public primary surpluses are either deterministic or, if stochastic, not correlated at all to consumption growth. However, we econometrically establish and quantify the pattern of correlation between public surpluses and consumption growth, calculate the appropriate risk premium and the associated risk adjusted interest rate using a general equilibrium asset pricing model. We find that there is in fact no free lunch, since the risk adjusted interest rate does exceed the growth rate. We use the same stochastic process to calculate the net present value of those surpluses and compare it with the market value of the debt. We test various explanations for the gap between these two quantities and derive the fiscal adjustment necessary to eliminate it (the "fiscal sustainability gap").

Traditionally, debt sustainability has been evaluated in a deterministic environment. The change in the debt level equals the interest payments minus the primary surplus, where the primary surplus equals government revenues minus non-interest expenditure. By iterating this relation forward and imposing a no-Ponzi condition (the so called terminal value condition, TVC), the current debt level then equals the sum of discounted primary surpluses. In a deterministic framework, future primary surpluses are discounted at the safe rate of interest. When debt to GDP ratios are assumed to be constant, this implies that primary surpluses are expected to grow at a rate equal to the growth rate of GDP. Currently, the interest rate in both the US and the EU is well below the growth rate of GDP. But we show this does not imply an infinite value of discounted future primary surpluses once one recognizes that these surpluses are stochastic and thus should not be discounted using the safe rate of interest. We calculate the appropriate risk premium for the Netherlands and apply our findings to the sustainability of the Dutch sovereign debt in both pre- and post corona times.

The Netherlands is an interesting case since the large public support programs have led to a sudden very substantial increase in both public deficits and public debt, like in most countries globally; but in the Netherlands the switch from a relatively low and declining debt ratio to a debt ratio well over the targets of the European Stability and Growth Pact is giving rise to sustainability questions. A predictable discussion is looming about whether sustainability at the new debt ratio requires further fiscal cutbacks and if so how fast deficits then should return to normal.

Like much of the current literature we view the value of government debt as a claim on future primary surpluses. When the future surpluses permanently grow faster than the in-

[2] He carefully adds the qualification "if the future is like the past".

terest rate, the sum of discounted primary surpluses, and therefore the value of government debt, would be infinite if discounted back using that particular interest rate. Blanchard (2019) and Bohn (2008) show that on average, growth rates have indeed exceeded interest rates in the United States. However, traditional analysis did (and often does) not take into account that even though government debt itself can be risk-free, in that it has at least very little default risk, the claim on primary surpluses is risky. Future primary surpluses are uncertain and are correlated with economic activity. The claim on primary surpluses can be compared with a stock that gives a claim on dividends. The dividend growth rate is in expectation higher than the interest rate, but the value of the stock itself is finite. The reason for this is that the dividend payouts are pro-cyclical and therefore should be discounted with a risk premium, i.e. the equity risk premium. How high the discount rate should be depends on the degree of procyclicality.

Our analysis is partially motivated by the debt sustainability analysis of Netherlands Bureau for Economic Policy Analysis (with the Dutch acronym CPB) (CPB, 2019) and subsequent discussions in Dutch policy circles about the implications of interest rates below growth rates. The CPB defines the sustainability gap of the Dutch government as the permanent change in primary surpluses that makes the value of the government debt equal to value of discounted stream of primary surpluses. To discount future primary surpluses, the CPB does argue that surpluses are risky and therefore a risk premium should be included in the discount rate. The discount rate is then set equal to the return on household wealth.

But asset pricing theory tells us that the value of the claim on primary surpluses (and therefore the appropriate discount rate) depends on the correlation with economic activity. The value of an uncertain cash-flow that pays out in crisis times, when marginal utility is low, is worth more than a cash-flow with the same expected value that pays out in booms. Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2020b) also take this view and apply arbitrage pricing theory (APT; see Ross (1976)) to the valuation of the stream of primary surpluses in the US. Moreover they find like many others have done that primary surpluses are procyclical and conclude that the appropriate discount rate contains a considerable risk premium.

We also start from the recognition that future primary surpluses are stochastic. However, we choose a different methodology than Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2020a) use to calculate the risk premium. Instead of using the very general utility-free APT asset pricing model, we choose to set up a consumption-based asset pricing model. In a consumption-based asset pricing model, the value of an asset is directly linked to the correlation of the return of that asset and the return on consumption. While such consumption-based asset pricing models require more stringent conditions than the very general APT approach, the explicit use of a preference structure has the advantage of providing more intuition in what drives risk premia. We extend the standard pure exchange economy (Lucas Jr, 1978) by introducing a government, similar to Bohn (1991), with a number of further modifications to address the Mehra-Prescott puzzle that plagues traditional applications of consumption-based asset pricing models.[3]

After developing the theoretical model, we calibrate and estimate the model to fit data of the Netherlands. We derive the proper (time varying) risk premium corresponding to the stochastic processes followed by primary surpluses and consumption growth and use the result in our valuation of the stream of future primary surpluses.[4] When we compare this NPV of the stream of primary surpluses to the current market value of the public debt, we find that the discounted value of the stream of primary surpluses is roughly half of the market value of the debt. Jiang et al. (2020b) find a similar (but much more extreme) result and label this "the debt valuation puzzle". We then use empirical evidence to investigate five alternative views on how such a gap should be interpreted. First we discuss (and dismiss) default risk, then a bubble in debt prices (cf Brunnermeier, Merkel, and Sannikov (2020)), the possibility of a convenience yield on public debt (i.e. investors are willing to accept very low returns because of other uses for debt than purely as an investment vehicle), the gap as a harbinger of future inflation (Cochrane (2018)), or a case of shifting risk to future tax payers (Bohn (1999)). We reject the default risk and convenience yield arguments; and we find no econometric evidence for the bubble hypothesis, find no evidence for the future inflation outcome and thus are left with the shifting of risk to future tax payers hypothesis.

On the presumption that that is considered undesirable, we calculate the permanent increase in the primary surplus that would make the discounted value of the surpluses equal to the market value of debt for our base year 2018, in which the primary surplus was slightly above average, this gap is around 0.28% of GDP. Taking into account projections of the deficits of the corona crisis, this number substantially increases. The market value of debt increased, government revenues decreased and the government introduced a large spending package. We estimate the sustainability gap to be between 0.39% and 0.52% in this case, depending on how fast the government manages to turn the corona crisis induced primary deficits back into primary surpluses. This will critically depend on how fast the economic recovery is and whether there will be a second wave of infections.

Finally our results indicate the importance of taking general equilibrium (GE) effects into account. We also present required fiscal adjustment calculations based on a partial equilibrium (PE) approach, using the risk premia calculated for the base run in all scenario's. This PE approach is closer to what is done in practice in the sort of Government agency that does comparable calculations as part of its policy advice. These PE estimates of the required adjustments are substantially larger than our general equilibrium calculations because the PE effects ignore the impact of the required fiscal adjustment itself on the risk premium. Since fiscal adjustment in our setup leads to lower risk premia, the GE

[3]In particular we use the Duffie-Epstein continuous-time variant of Epstein-Zin preferences and we introduce jump- or catastrophe risk, and finally, also leverage.

[4]To check our results for robustness, we also develop and estimate an APT based asset pricing model fitted to Dutch data, similar to the approach followed by Jiang et al. (2020b). We get similar risk premia for the discounting of future primary surpluses, and we again find that once the proper risk premia are used, interest rates do exceed growth rates and the TVC is not violated. We report the results of this robustness check in Appendix D.

estimate of the required fiscal adjustment is substantially lower than the PE calculations indicate.

2 Related literature

Our paper contributes to literature in several ways. Firstly, we connect to literature that test the inter-temporal budget constraint (ITBC) for the government (Blanchard, 2019; Bohn, 2007; Hamilton, 1986; Jiang et al., 2020b; Peter, Roberds, & Sargent, 1991). Blanchard (2019) has sparked the recent re-emergence of the debate on the government budget constraint, arguing that (safe) discount rates have been below economic growth rates at least half of the time for the US. He concludes that when debt can be rolled over permanently, this basically does away with the budget constraint for the government and that debt in that case does not have fiscal costs. Admittedly Blanchard (2019) also argues that, while interest rates below growth rates effectively imply the absence of an ITBC for the government, running high fiscal deficits is risky nevertheless because of the possibility of multiple equilibria. Depending on the level of public debt, investors might believe debt will become more risky and refuse to roll it over without getting a risk premium, driving up discount rates above the growth rate of the economy and thereby introducing the ITBC once again. This observation begs the question what low risk free interest rates really mean for public debt sustainability.

Commencing the debate on how to incorporate risk in a sustainability analysis, Abel, Mankiw, Summers, and Zeckhauser (1989) early on suggested that an economy's dynamic efficiency (for which interest rates should exceed growth rates) should be assessed adding a risk premium to the interest rate, since the debate is about the marginal productivity of capital being below the growth rate and shares are a claim on risky future output.[5] Bohn (1995) specifically addressed the sustainability of public deficits question when safe rates are below the growth rate of the economy. He argued that the discount rate that should be used in a sustainability analysis depends on the probability distribution of future spending and revenue flows, and consequently of future debt levels, and showed that the government has to satisfy the inter-temporal budget constraint regardless of the safe rate of interest. His basic point is that although current debt may be safe, future debt levels are stochastic if the primary surplus is a stochastic process and this requires a risk premium both when discounting future primary surpluses and when assessing the no-Ponzi condition requiring debt to grow at less than "the" rate of interest. More recently Jiang et al. (2020b) make the same point claiming that using the risk free rate as discount rate in assessing debt sustainability is fundamentally wrong because debt represents a claim on future primary surpluses and these are cointegrated with GDP. But rolling over such a claim at the risk free rate requires a zero risk premium, which is counterfactual. In fact they argue for an even higher discount rate than would follow from surpluses being a constant fraction of GDP because the primary surplus to GDP ratio is procyclical, and therefore a claim on

[5] See also Blanchard's well known textbook with Stanley Fischer for an extensive discussion of dynamic efficiency (Blanchard & Fischer, 1989).

that surplus has negative insurance value. Summing up, not including a risk premium in the debt sustainability analysis underestimates the riskiness of a claim on the primary surplus and basically does away with decades of asset pricing literature. We follow this path and use an asset pricing model that is calibrated on the primary surplus for the Netherlands, thereby incorporating the riskiness of a claim on the primary surplus into our sustainability calculations, although we use a different approach to the calculation of what that premium should be than the one followed in Jiang et al. (2020b).

We also contribute to the burgeoning literature that describes the link between economic growth and the fiscal deficit. Sargent and Wallace (1982) provide an early example on the interaction between private debt and fiat money. Anand and Van Wijnbergen (1989) use a similar deterministic framework to assess consistency of fiscal deficits incorporating the Central Bank into the fiscal analysis and linking it to inflation and seigniorage revenues building on Phelps (1973). A more flexible stochastic framework that incorporates uncertainty and risk is used by Budina and Van Wijnbergen (2008), but they also discount at the safe rate of interest.

Our work is not about whether fiscal theory of the price level holds, it solely assesses the riskiness of a claim on the primary surplus, and incorporates these risk and uncertainty characteristics into an asset pricing model of the public debt. As such, this paper is also related to the literature on consumption-based asset pricing. The pure exchange asset pricing model was developed by Lucas Jr (1978). In this equilibrium setting, the risk-free rate and the equity premium are endogenously determined within the model. Mehra and Prescott (1985) show that with reasonable assumptions on preferences and data on consumption volatility, the model is not able to generate a reasonable risk-free rate and equity premium. Rietz (1988) and Barro (2006) propose disaster risk as a potential solution of the equity premium puzzle. Barro (2009) extends the model with Epstein-Zin preferences. Epstein-Zin preferences separate the elasticity of intertemporal substitution from the risk-aversion coefficient and together with disaster risk this model generates a reasonable equity premium. We adopt the framework of Barro (2009) for two reasons. First, it is important to match the historical observed interest rate and equity premium, since these directly affect discount rates within the model. Second, using a disaster risk framework we can also capture the response of the primary surplus in crisis times. Wachter (2013) allows for time-varying disaster risk to explain excess stock market volatility, we do not consider this extension.

Finally, we link the fundamental value of government debt to the market value of government debt, and formally test for the presence of rational bubbles on financial markets using the test presented in Phillips, Shi, and Yu (2015) for the presence of (possibly multiple) bubbles. A rational bubble reflects a belief that an asset's price depends on a variable that is not a part of the relevant fundamental value of that asset. There is a large stream of literature on asset pricing bubbles starting with seminal work by Samuelson (1958); Tirole (1985), Blanchard and Watson (1982) and West (1987) develop theories of and tests for rational bubbles. In principle rational bubbles can persist when interest rates fall short of growth rates (Brunnermeier, 2008). And we do find that the market value of the Dutch public debt exceeds the properly discounted value of future primary surpluses, although we find a discrepancy that is less extreme than the one found by Jiang et al. (2020b) for the US. One explanation could be that the government debt is riding an bubble. However the outcome of our bubble tests do not support that interpretation.

3 Traditional debt Sustainability assessment

The analysis of debt sustainability traditionally starts out from the inter-temporal government budget constraint identity linking debt B_t, the interest rate r and the government's primary surplus PS_t, where the primary surplus equals government revenues T_t minus non-interest expenditure G_t.

$$\dot{B}_t = rB_t - PS_t \tag{1}$$

Here $\dot{B}_t$ denotes the derivative of B_t with respect to time. This simply says that the government follows a time path of expenditure G_t financed by (a time path of) taxes T_t and/or debt issue B_t where debt holders are paid out an interest rate r. By integrating the equation forward in time we obtain:

$$B_t = \int_t^u e^{-r(s-t)} PS_s ds + e^{-r(u-t)} B_u. \tag{2}$$

This is transformed into a real inter-temporal government budget constraint (ITBC) by imposing the no-Ponzi game terminal value condition (TVC):

$$\lim_{u \to \infty} e^{-r(u-t)} B_u = 0, \tag{3}$$

and then taking the limit of the RHS of (2) to get:

$$B_t = \int_t^\infty e^{-r(s-t)} PS_s ds. \tag{4}$$

We can easily transform this in a condition on debt-output ratio's $b_t = \frac{B_t}{Y_t}$ and primary surplus ratios $ps_t = \frac{PS_t}{Y_t}$ to get:

$$b_t = \int_t^\infty e^{-(r-g)(s-t)} ps_s ds. \tag{5}$$

With g the real growth rate of GDP in period t. Equation (5) is at the basis of all standard approaches to debt sustainability used widely, also outside academics at institutions like the World Bank, the IMF or government think tanks like the Dutch NBEA (Anand & Van Wijnbergen, 1989; Debrun, Ostry, Willems, & Wyplosz, 2019; Draper & Armstrong, 2007).[6] And there is nothing wrong with this analysis as long as we can assume a non-stochastic environment.

Unfortunately, the TVC condition (3) is not going to be satisfied when $r \leq g$, in that case the integrals are not finite. However, in a stochastic environment that does not

[6] Netherlands Bureau for Economic Analysis, called CPB for Centraal Plan Bureau in the Netherlands.

necessarily imply that there is no effective ITBC. Transforming (5) into an equation that is useful in a stochastic environment requires the use of the so called *Stochastic Discount Factor* (SDF) or *pricing kernel*, the standard approach to pricing any asset in the modern theory of asset pricing. In the following section we set up an asset pricing model to analyze the budget conditions in a stochastic setting.

Once one calculates the parameters of the proper SDF process and estimates the NPV of all future primary surpluses using that SDF process, a new puzzle may arise as pointed out by Jiang et al. (2020b). Even when using discount rates properly reflecting the stochastic nature of the primary surplus-to-GDP ratio and hence of future debt levels, one may have to confront the fact that in reality the equality does not hold. Jiang et al. (2020b) demonstrate that this is *in extremis* the case for the US: they find large *negative* values for the RHS of equation (5) while the market value of the US debt is obviously positive. We find a less extreme but still substantial inequality for the Netherlands. Different authors reach different conclusions on what such an inequality would imply. Brunnermeier et al. (2020) suggest that this inequality reflects the government riding an asset price bubble; i.e. investors in US treasuries fail to impose the TVC condition (3). Jiang et al. (2020b) postulate a variety of solutions to the puzzle they find, including violation of the TVC and large convenience yields, but in the end reject all of them. Cochrane (2020) notes the possibility of such an inequality emerging, stating that it reflects a discrepancy between the market value of the debt and its value to investors and consumers, and sees it as a harbinger of future inflation eroding the real value of the debt, in line with the Fiscal Theory of the Price Level (cf Woodford (1995)). Finally Bohn (1999) argues that governments issuing debt at low rates while the underlying primary surpluses are risky essentially means that governments by exploiting their ability to issue debt at high prices in effect shift risk to future tax payers. We return to this question when discussing our results.

4 An Asset Pricing Model

We take as a starting point a Lucas-tree exchange economy (cf Lucas Jr (1978)) and extend it with a government like in Bohn (1991). In the Lucas exchange economy, the representative agent is endowed with an amount of perishable goods each period. The endowment Y_t at each period can be seen as the output of a fruit tree, where every period the amount of fruit is stochastic. Endowment in this economy can be allocated to private consumption C_t and government spending G_t: $Y_t = C_t + G_t$.

4.1 The representative agent

Since we are interested in valuing the stream of future primary surpluses in a stochastic setting, we must use an asset pricing model, but one that is in line with empirical evidence. Bohn (1991) basically extends the standard Lucas asset pricing model by adding a government and its ITBC. But we know already since Mehra and Prescott (1985) that this model, with time separable utility, cannot match observed asset prices. We also assume an infinitely lived representative agent, but different from Bohn (1991), one that has the continuous-time equivalent of Epstein-Zin (EZ) preferences, also called Stochastic Differential Utility (Duffie & Epstein, 1992b; Epstein & Zin, 1989). This type of utility specification is more in line with empirical evidence (Epstein & Zin, 1991) and allows us to separate the relative risk aversion coefficient γ from the elasticity of intertemporal substation ϵ. Utility V_t is then recursively defined:

$$V_t = E_t\Big[\int_t^\infty f(C_s, V_s)ds\Big]$$

where

$$f(C,V) = \frac{\beta}{1-1/\epsilon}\frac{C^{1-1/\epsilon} - \Big((1-\gamma)V\Big)^{\frac{1}{\zeta}}}{\Big((1-\gamma)V\Big)^{\frac{1}{\zeta}-1}} \quad \text{for } \epsilon \neq 1 \tag{6}$$

with $\zeta = \dfrac{1-\gamma}{1-1/\epsilon}$.

β is the pure rate of time preference.

In addition to introducing EZ-preferences we follow Barro (2009) and Wachter (2013) and allow for disasters in the consumption process to arrive at a reasonable equity premium but also to capture the proper consumption and government spending dynamics during crisis times. Consumption has the following form:

$$\frac{dC_t}{C_{t-}} = \mu dt + \sigma^C dW_t^C + (e^{Z_C} - 1)dN_t. \tag{7}$$

μ is the constant growth rate of consumption, σ^C is the constant volatility, W_t^C is a standard Brownian motion and N_t is a Poisson process with constant arrival rate λ. The random variable Z_C controls the jump size and $e^{Z_C} - 1$ can be interpreted as the percentage loss of consumption when a jump takes place. We use a distribution with $E[e^{Z_C} - 1] < 0$ such that jumps are on average negative. From this consumption equation, we can derive the stochastic discount factor π_t and the interest rate r for this economy.

Let r be the (endogenous) risk-free interest rate. We show in appendix A that the stochastic discount factor π_t has the following dynamics:

$$\frac{d\pi_t}{\pi_{t-}} = \Big(-r - \lambda E[e^{-\gamma Z_C} - 1]\Big)dt - \gamma\sigma^C dW_t^C + (e^{-\gamma Z_C} - 1)dN_t. \tag{8}$$

The stochastic discount factor pins down the price of assets in the economy. Consider a risk-free asset M_t with continuous return μ_M. The asset is not correlated with the stochastic discount factor and does not suffer from disaster risk. No arbitrage then implies that the return on this asset must be equal to the risk-free interest rate: $\mu_M = r$.

The safe interest rate in this general equilibrium model depends on several factors:

$$\begin{aligned} r = &\underbrace{\beta + \mu/\epsilon - \frac{1}{2}\gamma(1+1/\epsilon)(\sigma^C)^2}_{\text{Standard interest rate}} \\ &\underbrace{-(\gamma - 1/\epsilon)\lambda\frac{E[e^{(1-\gamma)Z_C} - 1]}{1-\gamma} - \lambda E[e^{-\gamma Z_C} - 1]}_{\text{Jump Risk}}. \end{aligned} \tag{9}$$

First, the interest rate is increasing in the pure rate of time preference β . A more impatient agent demands a higher interest rate in return for postponing consumption. Another way of saying the same thing is that if future goods are less attractive (higher rate of time preference), their price goes down (higher interest rate). In addition, if there is positive growth, marginal utility of the representative agent will be lower in the future than it is now. Therefore the interest rate is increasing in the growth rate μ. The elasticity of intertemporal substitution ϵ controls the magnitude of this effect: we have μ/ϵ in the expression for r. Lastly, risk depresses the safe rate of interest since the agent is risk-averse and future consumption is uncertain, so certainty equivalent consumption falls when σ^C goes up. The third term in equation (9) captures the effect of risk in normal times and depends on risk aversion and the volatility of consumption. The last two terms capture jump risk.

The representative agent has a claim on consumption and therefore the total wealth of the agent equals: $S_t = E_t\left[\int_t^\infty \frac{\pi_s}{\pi_t} C_s ds\right]$. The return on wealth S_t is risky and it therefore contains a risk premium. We can calculate the risk premium of the claim on consumption:

$$RP_C = \underbrace{\gamma(\sigma^C)^2}_{\text{Standard risk}} + \underbrace{\lambda E\left[\left(e^{-\gamma Z_C} - 1\right)\left(1 - e^{Z_C}\right)\right]}_{\text{Jump risk}}. \tag{10}$$

The derivations are given in appendix A.

The first term in the expression for the risk premium is the conventional term capturing consumption risk related to the diffusion term of the geometric Brownian motion component driving consumption; obviously RP_C increases in the volatility of that diffusion process, σ^C, and in the degree of risk aversion γ. The second term captures the impact of risk due to the arrival rate and randomness in the jump size.

4.2 Dividends and the equity premium

We model dividends as levered consumption: $D_t = C_t^\phi$, as in Abel (1999) and Wachter (2013). When $\phi = 1$, consumption and dividends are equally risky. Leverage occurs when $\phi > 1$. In the case of leverage, dividends fall more than consumption when a disaster strikes, which is consistent with the data (Longstaff & Piazzesi, 2004). The process of dividends then follows:

$$\frac{dD_t}{D_{t-}} = \left(\phi\mu + \frac{1}{2}\phi(\phi-1)(\sigma^C)^2\right)dt + \phi\sigma^C dW_t^C + (e^{\phi Z_C} - 1)dN_t. \tag{11}$$

In the same way as for the consumption risk premium, we can derive an expression for the endogenous equity premium, which is the excess return on the claim on dividends. The derivations are given in appendix A. Denote the value of the claim on dividends by F_t: $F_t = E_t\Big[\int_t^\infty \frac{\pi_s}{\pi_t} D_s ds\Big]$. The equity premium EP equals:

$$EP = \underbrace{\phi\gamma(\sigma^C)^2}_{\text{Standard risk}} + \underbrace{\lambda E\Big[\Big(e^{-\gamma Z_C} - 1\Big)\Big(1 - e^{\phi Z_C}\Big)\Big]\Big)}_{\text{Jump risk}}. \tag{12}$$

The difference with the risk premium of the consumption claim is the leverage coefficient ϕ. When $\phi > 1$, dividends are a levered claim on consumption and therefore the equity premium is higher than the risk premium on the consumption claim. By modeling dividends as levered consumption, we are able to capture that dividends are more volatile than consumption.

4.3 The government

To finance government spending G_t, the government levies lump-sum taxes T_t. Both G_t and T_t are stochastic processes. Additionally, we allow the government to issue debt. We assume the debt is risk-free, so the government pays the risk-free interest rate over existing debt. Note that we could allow for default risk, but when calculating the sustainability gap or more in general in a sustainability analysis one should start from the assumption that the government is able to repay its debt so as to avoid circular reasoning. This gives the following debt dynamics:

$$dB_t = rB_t dt + \Big(G_t - T_t\Big)dt. \tag{13}$$

So when the government decides to run a primary (i.e. non-interest) deficit, the debt will grow faster than the interest rate. We can write the budget constraint for the representative agent as follows:

$$C_t = Y_t - T_t - (G_t - T_t). \tag{14}$$

So when the primary surplus is equal to zero, consumption equals endowment minus taxes. However, when the government decides to spend more than it receives from taxes, the representative agent finances the deficit and reduces consumption commensurately.

It is straightforward to show that if the No-Ponzi condition $\lim_{u\to\infty} E_t\Big[\frac{\pi_u}{\pi_t} B_u\Big] = 0$ is satisfied, then government debt must be equal to the SDF-discounted value of the current and future government surpluses taking correlation with the SDF into account (cf Appendix B):

$$B_t = E_t\Big[\int_t^\infty \frac{\pi_s}{\pi_t}(T_s - G_s)ds\Big]. \tag{15}$$

We have not specified the processes for spending and tax income yet, but from this formula it is clear that the correlation between these processes and consumption is key for the value of government debt. If the surpluses are pro-cyclical, they will be discounted at a rate higher than the interest rate and vice versa. Equation (15) is the basis for our sustainability analysis. We will both calculate the market value of Dutch government debt and evaluate the expectation and integral to obtain the value of the discounted surpluses. The gap between the two can be interpreted as the sustainability gap.

4.4 The Primary Surplus

Define the primary surplus by $PS_t = T_t - G_t$. Just to illustrate our approach with a simplified example, consider first the case where ps_t, the primary surplus to consumption ratio, is constant and equal to $\overline{ps} : PS_t = \overline{ps}C_t$. Equation (15) shows that the value of government debt is the discounted value of government surpluses. To find the right discount rate for this formulation, as an intermediate step we can look at the total wealth of the agent (S_t). The representative agent has a claim on the consumption stream and therefore we must have that $S_t = E_t\left[\int_t^\infty \frac{\pi_s}{\pi_t} C_s ds\right]$. In appendix A we show that the consumption claim should be discounted at a rate equal to the risk-free rate from equation (9) plus the consumption risk premium from equation (10):

$$S_t = \int_t^\infty \exp\Big\{-(r + RP_C)(s-t)\Big\}E_t[C_s]ds. \tag{16}$$

In the simple case where the primary surplus is strictly proportional to the consumption claim, we get that the value of government debt equals:

$$B_t = \overline{ps}S_t = \int_t^\infty \exp\Big\{-(r + RP_C)(s-t)\Big\}E_t[PS_s]ds. \tag{17}$$

We emphasize that discounting future primary surpluses with the same rate as the rate appropriate for the consumption claims is only correct when consumption and the primary surplus are perfectly correlated.

But in reality the primary surplus is not a constant fraction of consumption. Define the primary surplus to consumption ratio as $ps_t = \frac{PS_t}{C_t}$. It is straightforward to show that when ps_t is pro-cyclical, the discount rate should be higher than the risk-free rate plus the consumption risk premium that was derived under the assumption that the primary surplus to consumption ratio is constant. Therefore, we are interested in estimating separate processes for consumption and for ps_t to empirically assess the relevant correlation parameter. That process should reflect that the two processes are not perfectly correlated at any given moment in time, but that the ratio between the primary surplus and GDP (and therefore in this simplified model to consumption) tends to a constant ratio in steady state, as we document in the next section. In section 5.1 we show in the data the ps process is both mean reverting and procyclical. We incorporate these empirical characteristics by representing ps_t as an Ornstein-Uhlenbeck mean reversion process:

$$dps_t = \theta(\overline{ps} - ps_t)dt + \rho\sigma^{PS}dW_t^C + \sqrt{1-\rho^2}\sigma^{PS}dW_t^{PS} + (e^{Z_{PS}} - 1)dN_t. \tag{18}$$

The first term captures the mean-reversion of ps_t. θ controls the speed of mean reversion to $\overline{ps}$. The second and third term are Gaussian shocks. We use two Brownian motions since ps_t is assumed to be correlated to C_t. σ_{PS} denotes the volatility of ps_t and ρ the correlation coefficient for diffusion risk between ps_t and C_t.

Finally we assume that the ps_t ratio also jumps when a consumption disaster strikes. In this specification $e^{Z_{PS}} - 1$ denotes the drop in percentage points of ps_t when a disaster strikes. Note that the Poisson process (the zero-one jump) in (18) and (7) is the same but the random variables representing the size of the jump are different in the two equations. Thus the two processes jump at the same time but possibly with different jump sizes, as happened during the 2009 financial crisis (cf figure 2). Furthermore we assume that the two jump size variables (Z_C, Z_{PS}) have a bivariate normal distribution with mean $[\mu_{Z_C} \quad \mu_{Z_{PS}}]'$ and variance-covariance matrix:

$$\begin{bmatrix} \sigma^2_{Z_C} & \rho_Z \sigma_{Z_C} \sigma_{Z_{PS}} \\ \rho_Z \sigma_{Z_C} \sigma_{Z_{PS}} & \sigma^2_{Z_{PS}} \end{bmatrix}. \tag{19}$$

5 Estimation and Calibration

5.1 Data

We base our analysis on annual data over the period 1969 until 2018. In order to estimate and calibrate the model for the Dutch economy, we need the Dutch primary surplus, aggregate private consumption and the total public debt outstanding. For both the primary surplus and consumption data for the Dutch economy we rely on yearly national accounts over the period 1969-2018, available at Statistics Netherlands (CBS). The primary surplus is constructed using the interest rate expenses and the EMU-saldo for the general government.[7] In addition we also obtain household consumption from the national accounts. The GDP deflator is used as a measure of economy wide inflation. Since our model is in real terms, we use the GDP deflator to obtain real consumption growth.

Finally we need a measure for the value of outstanding public debt. Public debt is traditionally measured by simply adding the principals of all instruments outstanding (i.e. the face value). This is a misleading measure however, in spite of its commonality, because it equally counts principals coming due in the near and the far future. By looking at the current day market value instead of the face value of government debt, we get a measure of the current value of all future cash-flow obligations of the government to properly account for interest cost as the rate of return on the government debt portfolio (Hall & Sargent, 2011). Statistics Netherlands (CBS) publishes the market value of government debt. The market value of total government debt outstanding at the end of 2018 equals €459.797 billion versus its €405.773 billion euro face value. At the end of 2018, the market value actually exceeds the face value as most Dutch debt traded against a negative interest rate at that date.

[7] We correct the EMU-saldo for a once-off in 1995 when the entire debt of social housing co-operations was scrapped, see https://zoek.officielebekendmakingen.nl/kst-24725-2.html.

Figure 1: Primary Surplus to Consumption Ratio

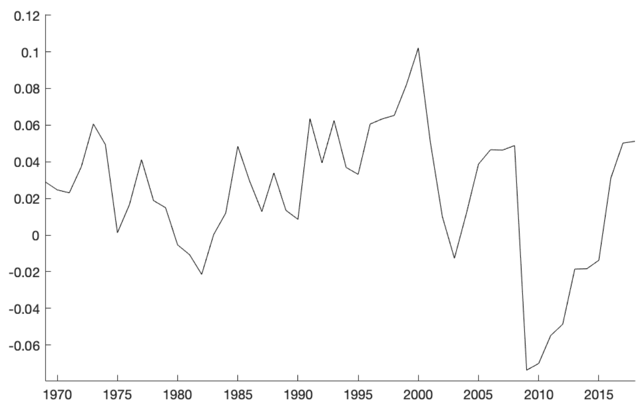


Figure 1 plots the primary surplus to consumption ratio. The sample average of ps_t equals 2.24% (1969-2018). One key feature is that the ratio seems to be mean reverting. Investors apparently do not allow the government to run a deficit for a too long time period.

Figure 2 below shows the correlation between the change in ps_t and consumption growth. It indeed seems that the primary surplus ratio is pro-cyclical so the discount rate for the primary surplus should be even higher than the discount rate for the consumption claims. Figure 2 also indicates that during the financial crisis (2009) consumption and the primary surplus ratio dropped simultaneously.

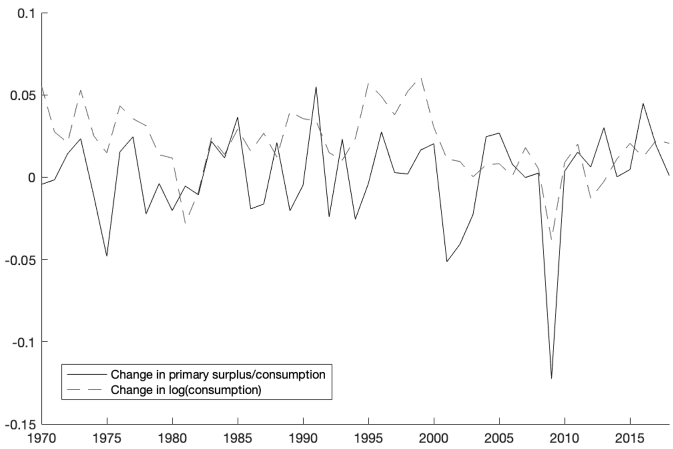
5.2 Non-crisis-time estimation

The next step is to estimate (7) and (18) for the Netherlands. Unfortunately it is not possible to estimate the complete equation on one sample because we did not experience enough jumps: there is only one crisis (the financial crisis of 2008/2009) in our sample. Therefore we follow a two-step procedure: we first estimate the process for the primary surplus to consumption ratio in normal times and then calibrate the jump part of the equation.

To estimate the primary surplus in non-crisis times we therefore shorten the sample to exclude the financial crisis and only estimate the process using data between 1969 and 2008. The sample average over this period without any disasters equals 3.22%. We therefore set $\overline{ps}$ equal to this sample average. The sample average over the non-crisis sample is almost 1% higher compared to the sample average including the financial crisis. However, when we later add jump risk the long-run expectation of ps_t will be lower again since jumps have a negative effect on the ratio. To estimate the process without jumps, we use the following econometric specification:

$$\begin{bmatrix} \Delta ps_t \\ \Delta \log(C_t) \end{bmatrix} = \begin{bmatrix} \beta_1(\overline{ps} - ps_{t-1}) \\ \beta_2 \end{bmatrix} + \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{bmatrix}. \tag{20}$$

Figure 2: Change in Primary Surplus to Consumption Ratio ($ps_t - ps_{t-1}$) and consumption growth ($\log(C_t) - log(C_{t-1})$).



We could estimate these two processes separate using OLS, but instead we jointly estimate them using a seemingly unrelated regressions (SUR) model. The SUR model has two advantages: it is more efficient if $\epsilon_{1,t}$ and $\epsilon_{2,t}$ are correlated and it directly gives an estimation of the variance-covariance matrix. Define the $1 \times n-1$ data vectors $\boldsymbol{\Delta ps} = [\Delta ps_2 \ \ldots \ \Delta ps_n]'$, $\boldsymbol{ps} = [ps_1 \ \ldots \ ps_{n-1}]'$, $\boldsymbol{\Delta \log(C_t)} = [\Delta \log(C_2) \ \ldots \ \Delta \log(C_n)]$. Let $\mathbf{0}$ and $\mathbf{1}$ denote a vector of respectively zeros and ones and denote by $\boldsymbol{\epsilon_1}$ and $\boldsymbol{\epsilon_2}$ the error vectors. We first estimate the following system using OLS:

$$\begin{bmatrix} \boldsymbol{\Delta ps} \\ \boldsymbol{\Delta \log(C_t)} \end{bmatrix} = \begin{bmatrix} \overline{ps} - \boldsymbol{ps} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} \boldsymbol{\epsilon_1} \\ \boldsymbol{\epsilon_2} \end{bmatrix}. \tag{21}$$

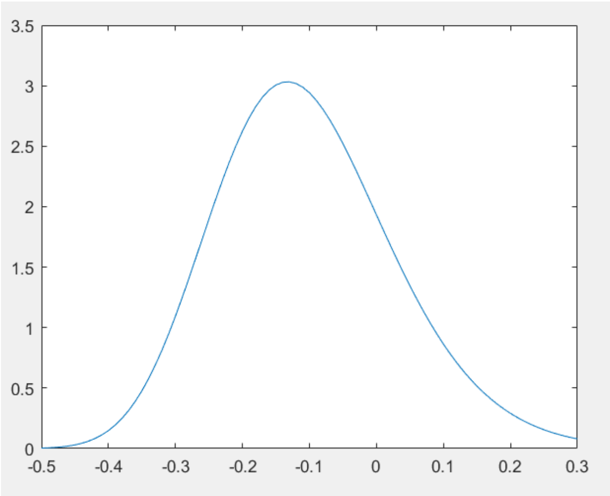
We then obtain an estimate for the contemporaneous variance-covariance matrix: $\hat{\Sigma} = \begin{bmatrix} \hat{\sigma_{11}} & \hat{\sigma_{12}} \\ \hat{\sigma_{12}} & \hat{\sigma_{22}} \end{bmatrix}$ where $\hat{\sigma_{ij}} = \frac{1}{n-2}\boldsymbol{\epsilon_i}'\boldsymbol{\epsilon_j}$. We use this estimate to perform a Feasible Generalized Least Squares regression and re-estimate β and Σ. We obtain the following estimates: $\hat{\beta}_1 = 0.455$, $\hat{\beta}_2 = 0.0235$ and $\hat{\Sigma} = \begin{bmatrix} 4.58 \times 10^{-4} & 1.80 \times 10^{-4} \\ 1.80 \times 10^{-4} & 3.74 \times 10^{-4} \end{bmatrix}$.

From these regression results, we get: $\theta = \hat{\beta}_1 = 0.455$, $\mu = \hat{\beta}_2 = 0.0235$, $\sigma_{PS} = \sqrt{4.58 \times 10^{-4}} = 0.021$, $\sigma_C = \sqrt{3.74 \times 10^{-4}} = 0.019$ and $\rho = \frac{1.80 \times 10^{-4}}{\sigma_C \sigma_{PS}} = 0.44$. Lastly, we use 2018 as our base year. Therefore we set the initial consumption level $C_0 = 341.458$ billion euros and $ps_0 = 0.0511$.

5.3 Jump calibration

Following Barro (2009) we allow for disasters in the consumption process as specified in equation (7). We therefore need to calibrate both the distribution of consumption losses Z_C and the arrival rate λ.

Figure 3: Pdf of $e^{Z_c} - 1$ with $\mu_{Z_C} = -12\%$ and $\sigma_{Z_C} = 15\%$.



We use the consumption data for the Netherlands as compiled by Barro and Ursúa (2008) to make an educated assumption on the arrival rate and economic impact of disasters. Barro and Ursúa (2008) identify four disasters for the Netherlands over the period 1870-2009. Each resulted in a steep consumption decline. The first in the list of Barro and Ursúa (2008) was in 1893 (9.8%), subsequent disasters happened in 1918 (44%), 1935 (4.5%) and 1944 (55.4%). These dates refer respectively to the panic of 1893, the first world war, the great depression and the second world war. For the period between 2009 and 2020 we can identify two further events, namely the financial crisis and the corona crisis. Based on our own dataset real consumption declined 3.8% during the financial crisis, and is estimated to decline around 7.6% in 2020 due to the coronavirus (DNB, 2020).

As a result, we count six disasters over a period of roughly 150 years. This equates to a 4% probability of a disaster occurring in any given year, therefore we set $\lambda = 0.04$. The average consumption loss of the six disasters for the Netherlands is approximately 20%. For the expected jump size we take a conservative calibration of 10% in the base-scenario, as wars are arguably extreme scenarios and are relatively over-represented in our sample. The jump sizes follow a log-normal distribution: $e^{Z_C} - 1$ where Z_C follows a normal distribution with parameters μ_{Z_C} and $\sigma^2_{Z_C}$. We choose the parameters $\mu_{Z_C} = -12\%$ and $\sigma_{Z_C} = 15\%$ which gives an expected disaster size of $e^{\mu_{Z_C} + \frac{1}{2}\sigma^2_{Z_C}} - 1 = -10.3\%$, with pdf given in figure 3.

This is in line with the calibration Barro and Ursúa (2008) arrive at: they use simulations based on 87 consumption disasters in several countries with a consumption contraction higher than 10% and find that the probability of moving from normalcy to a disaster (defined as a consumption decline of more than 10%) is approximately 2.9% for OECD countries. The estimated mean disaster size equates to a drop of 22% in consumption. For non-war disasters the decline in consumption is slightly lower (17%). These arrival rate estimates are likely to have a downward bias given both the financial crisis and the corona crisis are not included in the sample, therefore we use 4% for λ as discussed earlier. But our expected jump size is smaller than the one assumed in Barro and Ursúa (2008).

We set the leverage parameter ϕ equal to 2.5, similar to Wachter (2013). This implies that when consumption would drop with 10% after a disaster, dividends would drop with $1-0.9^{2.5}=23\%$. This is in line with the Great Depression and the 2008/2009 financial crisis, in which stock prices and dividends fell much more than consumption.

Finally, we need to calibrate the distribution of primary surplus disasters $e^{Z_{PS}}-1$ and the correlation between consumption and primary surplus disasters. The variables (Z_C, Z_{PS}) have a bivariate normal distribution with mean $[\mu_{Z_C} \quad \mu_{Z_{PS}}]'$ and variance-covariance matrix:

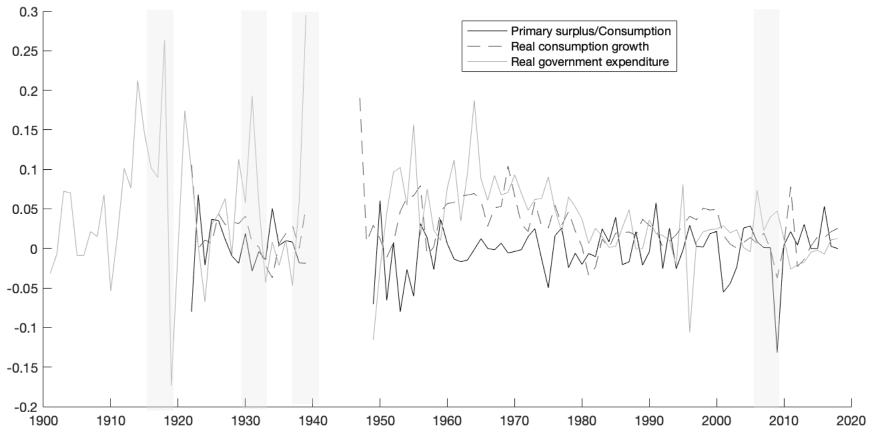
$$\begin{bmatrix} \sigma^2_{Z_C} & \rho_Z\sigma_{Z_C}\sigma_{Z_{PS}} \\ \rho_Z\sigma_{Z_C}\sigma_{Z_{PS}} & \sigma^2_{Z_{PS}} \end{bmatrix}. \tag{22}$$

The sign of $\mu_{Z_{PS}}$ should reflect that in case of a disaster when household consumption goes down, government spending (through social benefits) goes up and tax income goes down, resulting in a larger budget deficit i.e. we then get a decline in the primary surplus. Government spending data for the Netherlands support this claim: figure 4 clearly shows an increase in G_t in all disaster periods (shaded areas). In the financial crisis the primary surplus to consumption ratio dropped substantially more than consumption, but it is not entirely clear that this is in general the case. For the corona crisis, the projections of the Dutch Central Bank (DNB) imply that consumption drops with 7.6%. The primary surplus to GDP ratio is expected to go from a projected surplus of 2.5% to -5.7% (DNB, 2020), an 8.3 percent-point decline. Since consumption is about 45% of GDP in the Netherlands, this is approximately an 18.5 percent-point decline in the primary-surplus to consumption ratio. However, for larger consumption disasters it does not seem to be realistic to assume that the primary surplus to consumption ratio always drops much more than consumption, since simply there might not be enough fiscal space left. We therefore assume that $\mu_{Z_{PS}}=-15\%$. We set the volatility of the jump size distribution $\sigma_{Z_{PS}}=10\%$. This implies that when a jump occurs, the primary surplus to consumption ratio ps_t drops in expectation with $e^{\mu_{Z_{PS}}+\frac{1}{2}\sigma_{Z_{PS}}}-1=-13.5$ percentage points. We do not have a clear prior on the correlation ρ_Z between the two random variables and therefore assume in our base case that $\rho_Z=0$. Note that even when $\rho_Z=0$, there is still correlation between C_t and ps_t since the jumps occur at the same moment and the expected jump size is negative for both variables. Since these parameters are hard to calibrate, we will later vary $\mu_{Z_{PS}}$ and ρ_Z in robustness checks.

5.4 Utility calibration

The last step in the calibration procedure is to pin down the preference parameters: the rate of time preference β, the elasticity of intertemporal substitution ϵ and the relative risk aversion coefficient γ. These parameters are calibrated to match the risk-free rate and the equity premium. Dimson, Marsh, and Staunton (2011) calculate the long-term realized equity premium for several countries over the period 1990-2010 and find that the geometric average of the equity premium in the Netherlands was 4.2%. This is the excess return over

Figure 4: Primary Surplus to Consumption Ratio, consumption growth and government spending



Note: Figure 4 shows the change in the primary surplus to consumption ratio ($ps_t - ps_{t-1}$), the real consumption growth $\log(C_t)) - \log(C_{t-1})$ and the change in real government expenditure $\log(G_t)) - \log(G_{t-1})$ based on national accounts data from 1900-2018. Nominal series are corrected for inflation by dividing by the CPI as the GDP deflator is unavailable for the period 1900-1950. Shaded areas represent disasters (WWI, the great depression, WWII, and the financial crisis from left to right). Source: Authors calculations, Statistics Netherlands.

the return on Treasury bills: very short-term fixed income securities. The average return on Treasury bills has been 0.8% for the Netherlands. We use this number as target for our risk-free rate. This results in the following calibration: $\beta = 1.3\%$, $\epsilon = 1.5$ and $\gamma = 5.5$ which leads to $r = 0.78\%$ and $EP = 4.23\%$. The risk premium of the consumption claim with this calibration equals $RP_C = 2.05\%$. Finally, table 1 gives an overview of the calibration parameters used in our main specification.

6 Results: Valuing the Dutch public sector debt

In this section we present our key results on debt valuation based on the calibration/estimation procedures outlined in the previous section. We numerically evaluate the integral to calculate the value of the discounted future primary surpluses and compare it to the market value of the outstanding debt. The details of the integration procedure are given in appendix C. To illustrate the procedure, we first perform the calculations under the assumption of a constant ps-to-C ratio. Next we analyse the same question on the more realistic assumption of a stochastic ps-to-C ratio using the econometric results from section 5. The logical next step is then to calculate what we call the fiscal sustainability gap for a number of different scenarios, the gap closing of which would lead to equality between the market value of the debt and the properly discounted value of primary surpluses. We do that in

Table 1: Calibration and estimation parameters

Parameter		Value
Growth rate C	μ	2.35%
Volatility C	σ_c	1.9%
Mean reversion ps	θ	0.455
Volatility ps	σ_{ps}	2.1%
Correlation ps, C	ρ	0.44
Arrival rate	λ	0.04
Jump size parameter C	μ_{Z_C}	-12%
Jump size volatility C	σ_{Z_C}	15%
Jump size parameter ps	$\mu_{Z_{PS}}$	-15%
Jump size volatility ps	$\sigma_{Z_{PS}}$	10%
Jump size correlation ps, C	ρ_Z	0
Leverage parameter	ϕ	2.5
Rate of time preference	β	1.3%
Intertemporal substitution	ϵ	1.5
Risk aversion	γ	5.5
Real risk-free rate	r	0.78%
Risk premium C	RP_C	2.05%
Equity premium	EP	4.23%
Initial consumption	C_0	€341.458 (billion)
Initial level ps	ps_0	5.1%
Average ps (normal times)	$\overline{ps}$	3.2%

section 8 below.

6.1 Debt Valuation with a constant primary surplus to consumption ratio

We start with the simple case where we assume that the primary surplus to consumption ratio ps_t is constant. The sample average of the primary surplus to consumption ratio is 2.24%. This case is useful since the closed form solutions give a transparent insight in the factors influencing the value of the discounted primary surpluses. With a constant ps_t, the primary surplus PS_t is proportional to consumption C_t and the discount rate for the primary surpluses is equal to the *risk-free interest rate* plus the *consumption risk premium* minus the *expected growth rate of consumption*. Using our base case calibration, i.e. with the parameters specified in table 1, we obtain a risk-free rate of 0.78%, a risk premium of 2.05% and an expected growth rate (taking into account possible jumps) of 1.94% which leads to an effective discount rate of 0.89%. Note that this discount rate can be seen as a lower bound since, as we have seen in the previous section, in reality, the ps_t ratio is pro-cyclical and therefore the risk premium based on the assumption of a fixed ratio of the primary surplus to consumption is too low, as we will see in the next section.

The initial value of household consumption in 2018 equals €341.458 billion. The value of the discounted primary surplus is then equal to $\frac{2.24\%\times 341.458\times 10^{9}}{0.89\%} = 858$ billion euros. This is 187% of the market value of government debt in 2018 (€460 billion). A ratio of 100% would imply that the value of discounted primary surpluses is exactly equal to the market value. So when the primary surplus would indeed be a constant fraction of consumption, the value of the discounted surpluses would actually exceed the market value of government debt. However, this approach underestimates the risk within the primary surplus process and therefore we now move to the case in which the primary surplus to consumption ratio is a stochastic process.

6.2 Debt Valuation when the ratio of the primary surplus to consumption/GDP is stochastic

When the primary surplus to consumption ratio itself is stochastic, the correlation between this ratio and consumption matters for the valuation of the primary surpluses. As we saw, the primary surplus to consumption ratio is procyclical, i.e. it is low in crisis times. Therefore the consumption risk premium of 2.05% that we looked at in subsection 6.1 is too low for discounting future expected primary surpluses.

Using our econometric analysis of the stochastic processes driving primary surpluses and consumption growth we derive the proper risk premia that accurately reflect the correlation between the two processes. Since ps_t is a mean-reverting process (and therefore PS_t mean-reverts around C_t), the rate that is used to discount future primary surpluses depends on the maturity and thus varies over time: see figure 5.

Using our base calibration and the time varying risk premium we just presented, we find that in 2018 the value of the claim on primary surpluses equals 218 billion euros when primary surpluses are assumed to be stochastic. This is 47% of the market value of debt in 2018.

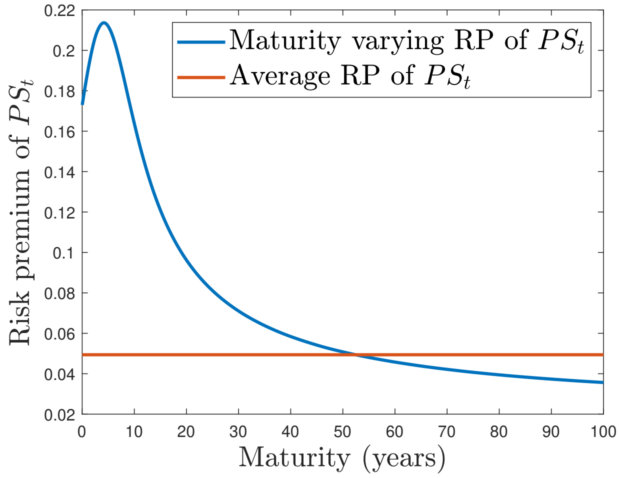
We consider it useful to also have an informative summary measure of the RP time series available. To that end, in order to say something about "the" risk premium associated with the PS_t process we can calculate an average risk premium RP_{PS} defined as as the solution to the following problem:

$$\int_t^{\infty} e^{-(r+RP_{PS})(s-t)} E_t[PS_s]ds = 218 \times 10^9. \tag{23}$$

We find that RP_{PS} is 4.94%, which is indeed substantially higher than the risk premium on the consumption claim. This is not very surprising, since future primary surpluses are subject to disaster risk and can actually become negative. RP_{PS} is represented by the red line in figure 5.

We now discuss five parameters that we will vary to investigate the robustness of the results with respect to these parameters. The results are given in figure 6. First, the initial level of ps_t was above average in 2018. Panel (a) therefore investigates the effect of the initial value of ps_t on the value of the claim on primary surpluses. It is clear that the

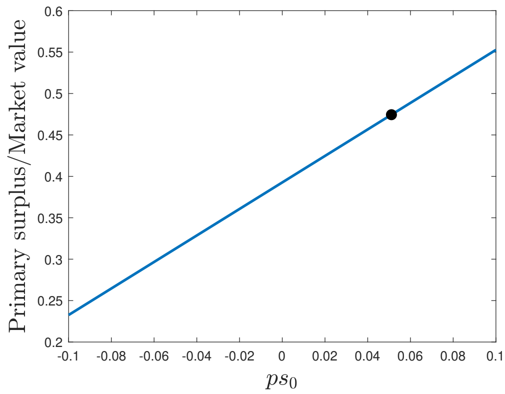
Figure 5: The risk premium of PS_t in 2018



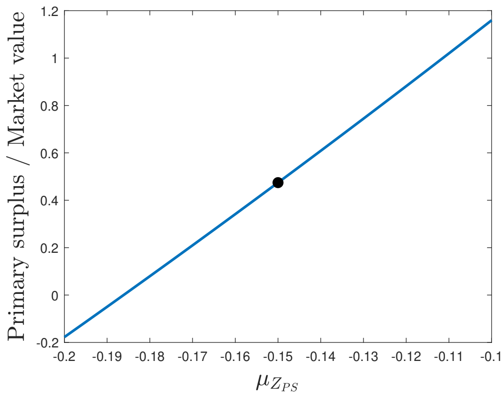
Note: Let the maturity be equal to u. First, $E_t[PS_{t+u}]$ and $E_t[\pi_{t+u}PS_{t+u}]$ are calculated. The maturity-varying risk premium is then calculated as: $-\log\Big(E_t[\pi_{t+u}PS_{t+u}]/E_t[PS_{t+u}]\Big)/u - r$. The average risk premium is calculated as the solution to equation (23).

initial value matters, in particular because its impact on the time path of ps_t comes early. Second, we vary the jump size parameter $\mu_{Z_{PS}}$ in panel (b). A value of $\mu_{Z_{PS}}$ below -19% could even lead to a negative value of the surpluses and therefore the results are quite sensitive to this parameter. Third, we look at the effect of the jump correlation parameter ρ_Z. Our base calibration assume $\rho_Z = 0$, but there is still correlation since jumps take place at the same time and are on average negative for both ps_t and C_t. Panel (c) shows that correlation of the jump sizes has a substantial effect on the value of the surpluses and again the value can become negative. Fourth, we consider the effect of the risk-free rate on the valuation of surpluses. We calibrate the real risk-free rate at 0.76%. In our model, the interest rate is constant but currently interest rates are at an all time low and real rates are currently even negative in Europe. Panel (d) shows that lower real rates have a large effect on the valuation of future surpluses. A real risk-free rate of approximately 0.25% would make the value of primary surpluses equal to the market value of government debt. Lastly, we have estimated the mean reversion parameter using data in non-crisis periods. It is however likely that recovery of the primary surpluses is faster after a crisis than in normal times. Especially for the corona crisis this is likely to be the case since several government measures are temporary and a (partial) rebound of the economy is expected when a vaccine is available. We therefore vary the mean-reversion parameter in panel (e). The figure shows that increasing θ from the base calibration leads to a higher value of the discounted primary surpluses. Summarizing, panels (a) and (c) indicate that our base estimate that the value of surpluses is 47% of the market value might be too optimistic since the initial value of ps_t is high in 2018 and because our assumption of the correlation between the jump size of the

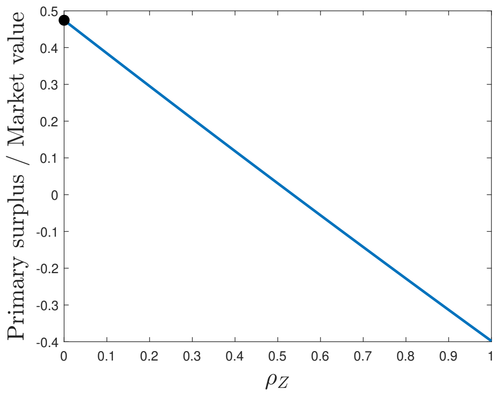
Figure 6: The value of the claim on primary surpluses as a percentage of the value of government debt



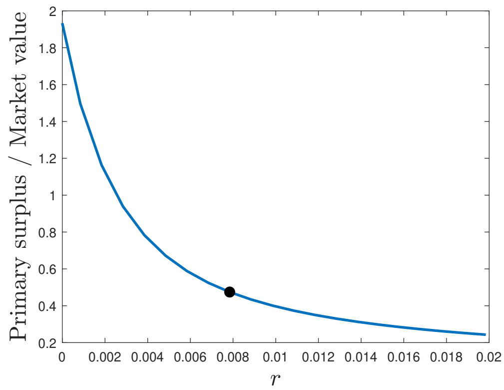
(a) Initial value of ps_t



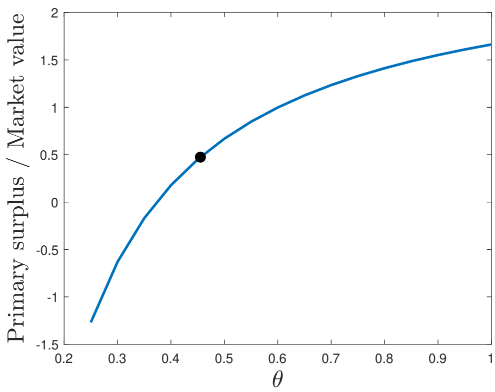
(b) Jump size parameter of ps_t



(c) Jump correlation parameter



(d) Risk-free interest rate



(e) Mean reversion parameter of ps_t

Note: We calculate the value of the discounted primary surpluses $E_t\left[\int_t^{\infty} \frac{\pi_s}{\pi_t}(PS_s)ds\right]$ for different parameter values. The value of the discounted primary surpluses is divided by the market value of government debt. The black dots indicate the outcome for the base calibration. The interest rate is an endogenous variable, but we vary the interest rate by changing the pure rate of time preference β.

primary surpluses and consumption is conservative. However, panels (d) and (e) show that the low interest rate environment and potential faster adjustment of the primary surpluses after a crisis work in the opposite direction. We therefore argue that our base results are reasonable with the following main conclusion: the value of the claim on surpluses is positive but substantially smaller than the market value of government debt.

6.3 Robustness check

As a robustness check, we replicate the APT based approach of Jiang et al. (2020b) in appendix D. This approach is not utility-based, but based on arbitrage pricing theory, as the name indicates. The stochastic discount factor is assumed to be a direct function of macro variables. The distribution of the macro variables is estimated using a VAR and the coefficients that relate the stochastic discount factor to the macro variables are estimated using observed asset prices. This approach is more flexible than our utility based approach, but is also less transparent. We find that the drawback of this approach is that the results are very dependent on technical restrictions that are imposed on the estimation of the stochastic discount factor. We find that depending on the restrictions, the value of a claim on primary surpluses at the end of 2018 equals either €121 or €286 billion. This is quite a difference between the two restrictions, but at least these results are in the same range as our estimate of €218 billion. All estimates lead to the same conclusion: the value of the claim on primary surpluses is below the market value of debt. That leads to the obvious question: what explains this difference, what is the solution to what Jiang et al. (2020b) call "the Debt Valuation Puzzle"?

7 Explaining the Debt Valuation Puzzle

Although the results from our analysis are nowhere near as puzzling as the sustainability gap in the US found by Jiang et al. (2020b),[8] the fundamental value of Dutch treasury paper does not align either with the market value of debt. We find that the market value of debt is substantially higher than its fundamental value based on the Dutch primary surplus process and using properly risk adjusted discount rates. There are a couple of factors that could explain at least a part of the gap between the value of the debt at market prices and the properly discounted present value of future primary surpluses.

However, one factor that cannot explain the discrepancy is default risk. Default risk because of this sustainability gap should be reflected in the market prices of the debt and can thus not explain any difference between that market value of the debt and the value of what effectively is its collateral (future primary surpluses). We next investigate four alternative explanations.

First, the bubble explanation comes from Brunnermeier et al. (2020), who incorporate a bubble in the FTPL equation to explain why countries with persistently negative pri-

[8]They find a discrepancy between the market value of the public debt and the discounted value of the primary surplus of more than 4 times GDP.

mary surpluses can pay an interest rate on government debt below the economy's growth rate. They show a bubble component can explain why countries with persistently negative primary surpluses have low inflation and can finance government expenditures without ever raising taxes to cover. Although the Dutch data suggest a gap that is an order of magnitude smaller than the gap found in the US, a similar mechanism could in principle explain the difference between the market value of the debt and our best estimate of the properly discounted future primary surpluses. Though formally testing for a bubble in asset prices is hard and full of pitfalls (see for instance Evans (1991)) we can use recent work by Phillips et al. (2015) to identify periods of explosive behaviour in Dutch Treasuries if any. Similar to Lamoen, Mattheussens, and Dröes (2017) we use the GSADF-test on Dutch Treasury paper and identify significant explosive behaviour. The GSADF-test procedure distinguishes a Brownian motion driving return behaviour from a explosive process. The main rationale of the GSADF-test can be explained as follows. The price of a financial asset can be seen as:

$$P_t = \sum_{i=0}^{\infty} \frac{1}{1+r_f}^{i} E_t(D_{t+i} + U_{t+i}) + B_t, \tag{24}$$

where the price reflects the expected value for future dividends (in the case of a bond the coupon payments) plus a set of unobserved fundamentals that could drive the return on the underlying asset. Rational investors would price the asset based on their expectations for the fundamental value of the bond. In case the GSADF test detects explosive behaviour this then has to follow from the bubble component (B_t). Moreover, explosive behaviour in one of the fundamentals would translate in a infinite price as the expected value of an explosive process is infinite.

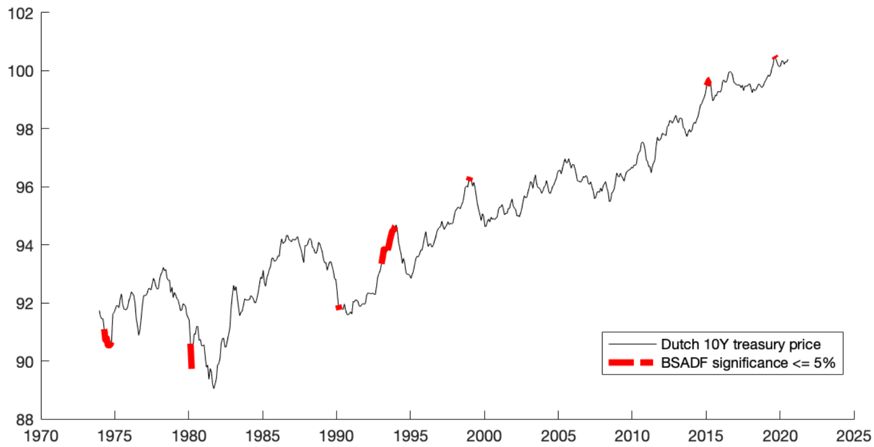
Our test results do indicate explosive behaviour, but only for short periods, indicating irrational investors in some periods (figure 7).[9] Moreover, in line with Lamoen et al. (2017) the GSADF test shows there are multiple periods of significant exuberant behaviour in the Dutch Treasury markets resulting in a t-value for the GSADF over the whole sample of 1.972 and a corresponding p-value of 0.001 indicating the presence of at least one (periodically collapsing) bubble.

The GSADF test, however, does not show clear evidence of a bubble building in the current period, like it does find for the early '90 (see figure 7). Moreover, we show the transversality condition is not violated when we appropriately adjust for the risk characteristics of a claim on the primary surplus (i.e we find $r > g$ once the right risk premium is included), hence a bubble is not a plausible explanation of the difference between the market and fundamental value of government debt.

Second, investors could pay a significant convenience yield for holding Dutch Treasury paper. The convenience yield is the government's expected return investors are willing to forgo in exchange of holding safe government debt, for example because of its usefulness

[9] We follow Lamoen et al. (2017) and run the GSADF test with a minimum window length of 36 observations and use 2000 Monte Carlo simulations to calculate significance. We include a constant but not a trend in the ADF tests and include one lag.

Figure 7: GSADF test results for Dutch 10Y Treasury price

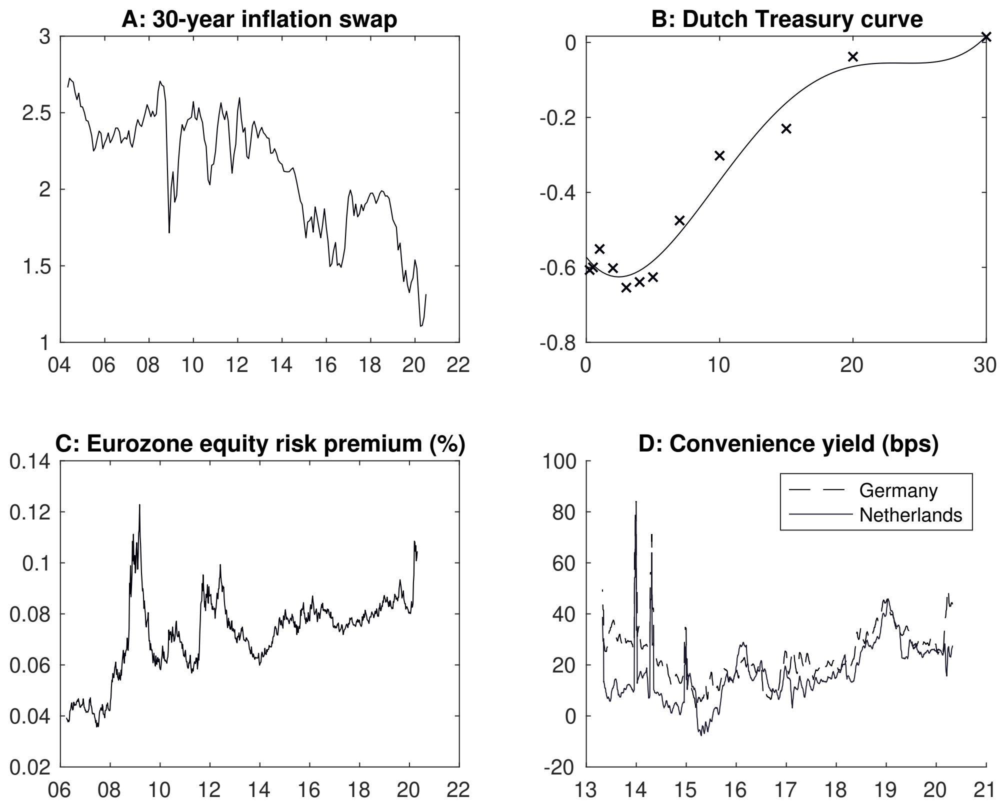


Note: Figure 7 shows the result from running the GSADF procedure as described in Phillips et al. (2015). The red lines show significant explosive behaviour at the 5% level in the Dutch 10Y treasury bond price between 1970 and 2020 on a monthly basis. The price is derived from the yield on the *on-the-run* bond by treating it as a zero-coupon bond. For the overall sample we obtain a t-value for the GSADF of 1.972 and a corresponding p-value of 0.001. *Source*: Thomson Reuters, Authors calculations

in repo transactions. Following the procedure used in Jiang et al. (2020b) we use the spread between AAA-rated Dutch treasury paper and AAA rated commercial paper with an (implicit) state guarantee as an estimate of the convenience yield on Dutch Treasury paper. Moreover, we check our indicator with the kfw-bund spread for robustness (De Santis & Stein, 2015) (see figure 8 panel D). It turns out the convenience yield currently is at most between 20-30bps across different maturities, and is a relatively constant factor. Although significant, this 20-30bps difference is not enough to bridge the gap between the market valuation and the fundamental value. When we increase the yield by 20-30bps across all the individual Dutch Treasury bonds outstanding, taking into account the bond's (modified) duration, that leads to a market value adjustment of at most €6 billion to €9 billion for the 20 and 30bps respectively, which translates to at most 2% of the total market value, a much smaller number than we find for the sustainability gap. Thus although we find evidence of a convenience yields, the numbers we find are way too small to explain the gap we find between market and fundamental values of the Dutch sovereign debt.

Third, following Cochrane (2018)'s interpretation based on the fiscal theory of the price level (FTPL), a sustainability gap based on market pricing could be a harbinger of expected future long-run inflation. According to Cochrane (2018) and Sims (2011) when the nominal market value is below the real value to consumers and investors, those economic agents substitute goods and services for government debt, i.e. aggregate demand declines. According to the FTPL, we should expect the price level in the economy to decline over the long run until the real value of nominal debt matches the real present value of a claim on the primary surplus. Similarly, a market value of long-term debt above the real value of future primary surpluses, as we find for the Netherlands, is interpreted as a harbinger of higher inflation in the future. We should expect that to coincide with higher long-run inflation expectations compared to the current period. A high value of debt at market prices than its fundamental value in this view presages its erosion by future inflation.

Figure 8: Inflation, interest rates, risk premia and convenience yield



Note: Figure 8 shows some suggestive evidence for different theories that could bridge the gap between market prices and the fundamental value of a claim on the primary surplus. Panel A shows a zero coupon 30-year inflation swap on Euro HICP. Panel B is the Dutch Treasury curve at the end of july 2020 with maturity on the x-axis ranging from 1 month to 30-years. Panel C shows the equity risk premium based on a Gordon Growth Model for the euro stoxx 50 Index in line with Geis, Kapp, and Kristiansen (2018). Finally, Panel D shows the convenience yield measured as the spread between AAA-rated Dutch Treasury paper and AAA-rated commercial paper with an implicit state guarantee in basis points (bps). As a robustness check we also add the Kfw-Bund spread in line with De Santis and Stein (2015). *Source*: Bloomberg, Authors calculations

Of course long-term inflation expectations might indeed fill in the gap between the market and fundamental value of government debt by signaling future erosion of the market value. However, we find no evidence this could actually be the case for the Netherlands.

Although there are no Dutch inflation-linked bonds, and as a consequence no inflation swaps on Dutch inflation, Eurozone market-based inflation expectations have followed a secular trend downward (see figure 8 panel A). Moreover, the nominal Dutch yield-curve does not signal particular high inflation scenarios as even the yield on a 30-year nominal bond does not exceed the short maturity T-bill yield by more than 0.6 percent. A high future inflation scenario would also contradict recent literature that finds a low Eurozone inflation risk premium as investors are seemingly pricing out upward shocks to inflation (Camba-Mendez & Werner, 2017; Campbell, Sunderam, & Viceira, 2009).

Finally fourth, Bohn (1999) offers another possible explanation. If the low interest rates (i.e. the high market value of debt) are a reflection of high risk aversion, running a high deficit will impose risks to future taxpayers; he shows that government debt may be safe for the debt holder, but that that implies it is risky for future taxpayers who are implicitly taking a short position in the safe government bond.[10] Indeed estimates for an equity risk premium signal high risk aversion under Eurozone investors (see panel C). Though the exact level of the equity risk premium is hard to pin down, there is some evidence the equity risk premium trended upwards over the last decade (Duarte & Rosa, 2015; Graham & Harvey, 2018). Moreover, Bekaert, Hoerova, and Duca (2013) and Bekaert and Hoerova (2014) show risk aversion can explain returns, although they also show lax monetary policy can reduce risk aversion significantly, somewhat reversing the prior argument of increasing risk aversion.

8 Fiscal adjustment and debt sustainability

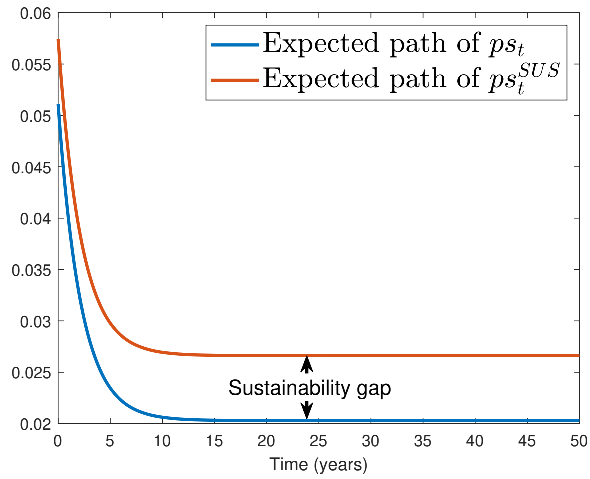
In the previous section we concluded that shifting risk to future tax payers is the most logical explanation for the debt valuation puzzle. It is then natural to ask what sort of fiscal adjustment would be necessary to avoid doing that. To that end we define the sustainability gap as the permanent increase in primary surpluses that makes the market value of debt equal to the value of the claim on primary surpluses (to be precise, the sustainability gap equals the increase in both ps_0 and $\overline{ps}$). In subsection 8.1 below we derive this measure for the pre-Corona end-of-our-sample period, December 2018. In the second subsection we analyse the post-Corona situation in our base case stochastic scenario but deriving an upper and a lower bound for the impact of the corona crisis depending on how fast the deficits go back to their pre-crisis time path (i.e. a fast (slow) return corresponding to a swift (sluggish) economic recovery).

8.1 How much should deficits be adjusted for debt sustainability: the Netherlands pre-Corona

For our base case of a stochastic primary-deficit-to-GDP ratio we find a valuation gap of 0.63% of consumption. Household consumption in the Netherlands is approximately 45%

[10] Jiang et al. (2020a) elaborate extensively on this theme.

Figure 9: The expected path of ps_t and the sustainability gap



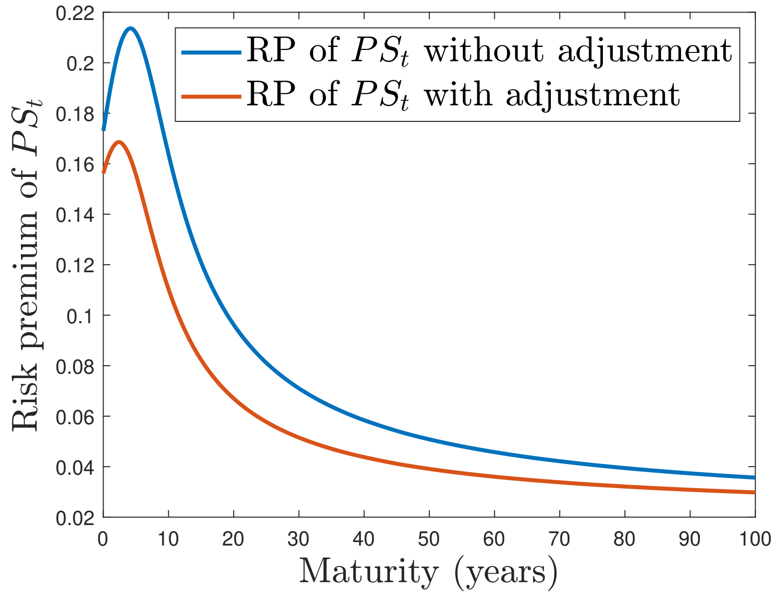
Note: The blue line indicates the expected path of ps_t over time. The orange line shows the expected path of the primary surplus ratio that makes the value of the claim on surpluses equal to the market value of government debt. The difference between the two lines equals the sustainability gap.

of GDP in 2018. If we express the sustainability gap as percentage of GDP instead of consumption (as the Dutch government thinktank CPB does), the gap equals 0.28%. It is useful to delve into that number a bit more by elaborating on its constituents, an analysis to which we turn now.

In the sustainable situation we have $ps_0^{SUS} = ps_0 + 0.63\%$ and $\overline{ps}^{SUS} = \overline{ps} + 0.63\%$. The expected paths of the primary surplus to consumption ratio in the initial case (ps_t) and in the sustainable case (ps_t^{SUS}) are illustrated in figure 9. In 2018, $ps_0 = 5.1\%$ is above the unconditional (or long-run) expectation: $E[ps_t] = 2.0\%$. Therefore ps_t is expected to decline over time. In the sustainable case, two things change. Figure 9 shows that the expected path of ps_t^{SUS} lies above the expected path of ps_t. This of course increases the value of the claim on primary surpluses. However, since $\overline{ps}^{SUS} > \overline{ps}$, the entire distribution of ps_t^{SUS} has shifted to the right compared to the distribution of ps_t. The probability of ps_t^{SUS} to get negative is smaller than the probability of ps_t to get negative. This does not only affect the expected value of the surpluses, but also the risk premium. The maturity-varying risk premium of ps_t^{SUS} is substantially lower compared to the initial case, as shown in figure 10. The average risk premium of the claim on ps_t^{SUS} equals 3.36%, which is also much smaller than the 4.94% risk premium of the claim on ps_t. So the fiscal adjustment necessary to eliminate the sustainability gap has two effects: it increases the expected value of future surpluses but due to the general equilibrium effect it also leads to a lower risk premium that is used to discount future surpluses.

Since the fiscal adjustment affects both risk premia and expected values, it is useful to take one step back and consider the partial equilibrium effect first: the sustainability gap under the assumption that the fiscal adjustment only affects the expected value for given

Figure 10: The risk premium of PS_t with and without fiscal adjustment.



Note: Let the maturity be equal to u. First, $E_t[PS_{t+u}]$ and $E_t[\pi_{t+u}PS_{t+u}]$ are calculated. The maturity-varying risk premium is then calculated as: $-\log\Big(E_t[\pi_{t+u}PS_{t+u}]/E_t[PS_{t+u}]\Big)/u - r$. The case without adjustment (blue line) corresponds to the base case. The case with adjustment (orange line) has initial primary surplus $ps_0^{SUS} = ps_0 + 0.63\%$ and $\overline{ps}^{SUS} = \overline{ps} + 0.63\%$.

time path of the risk-free real rate and risk premium. In step two we switch on the general equilibrium setting, endogenizing the time path of the risk premium. We implement this by solving for the sustainability gap that yields:

$$\int_t^{\infty} e^{-(r+4.94\%)(s-t)} E_t[PS_s]ds = 460 \times 10^9. \tag{25}$$

In other words, in this calculation we assume that the fiscal adjustment only has an effect on $E_t[PS_s]$ but does not change the risk premium of the primary surplus process. The sustainability gap is much larger in this partial equilibrium approach: 2.68% of consumption or 1.20% of GDP. The necessary fiscal adjustment is much larger since the general equilibrium impact on the risk premium is not yet taken into account. The lower time path for the risk premium in the fiscal adjustment scenario leads to a lower debt valuation gap compared to the partial equilibrium analysis and consequently a lower required fiscal adjustment. As we already saw earlier, in the full general equilibrium analysis the fiscal adjustment gap falls to 0.63% of consumption. Household consumption in the Netherlands is approximately 45% of GDP in 2018. If we express the sustainability gap as percentage of GDP instead of consumption (as the Dutch government thinktank CPB does), the gap equals 0.28%. Positive, but small.

Table 2: Sustainability gap under different scenario's

Scenario	ps_0	MV (b €)	Gap (% of GDP)	Gap (% of C)
Constant ps/c	2.24%	460	-	-
Stochastic ps/c , 2018	5.1%	460	0.28% (1.20%)	0.63% (2.68%)
Corona	-12.5%	507	0.52% (2.28%)	1.15% (5.07%)
Corona, direct adjustment	$E[ps_t]$, 2.03%	507	0.39% (1.67%)	0.87% (3.70%)

Note: Table 2 provides an overview of the sustainability gap under four different scenarios. The sustainability gap is defined as the increase in the primary surplus that needs to be implemented and sustained to satisfy equation (15) as a percentage of GDP or C respectively. The numbers between parentheses indicate the partial equilibrium estimates for given risk premia, using the approach in equation (25). The unbracketed numbers also take the GE impact on the risk premium into account. The first scenario shows the sustainability gap as % of GDP and of C when we assume a constant primary surplus to consumption ratio, and we set the starting value of the primary surplus ps_0 to its sample average (the gap then obviously equals 0). Second, we assume a stochastic primary surplus and set the initial value of the primary surplus to the latest observation in our sample (2018). Finally we obtain two corona scenarios in which we use the estimated impact of the corona crisis on the Dutch economy, but vary the speed of the subsequent economic recovery in the coming years (i.e. via a direct adjustment). ps_0 is the initial primary surplus used in our calculations, MV is the market value of the total debt outstanding as defined in Hall and Sargent (2011) in euro billion.

8.2 Application to the corona crisis deficits

When the corona crisis fully exploded in the Netherlands, the government responded with historically unheard of fiscal measures. The level of government debt jumped up. The market value of government debt by the end of Q2-2020 totalled €506.779 billion versus its €441.535 face value up from €457.270 billion market value (€394.670 billion face value) at the end of 2019. The primary surplus is projected to be -5.7% of GDP over 2020 (DNB, 2020), which is approximately -12.5% of household consumption. In our base year 2018 the primary surplus was 5.1% of consumption. We also use the projections for consumption from DNB (2020) to obtain the level of household consumption in 2020. Consumption was €354.862 billion in 2019 and is expected to drop with 7.6% which gives €327.9 billion as initial consumption level. Using €506.779 billion as market value of government debt, -12.5% as ps_0 and €327.9 billion as C_0 we obtain the following numbers.

The value of the discounted primary surpluses as percentage of the market value of the public debt (2020-Q2) now equals 17%. The sustainability gap (the permanent change in ps_t that makes market value and the discounted value of primary surpluses equal to each other) is also larger now, at 0.52% of GDP, and 1.15% of consumption, possibly a more relevant measure. The gap is larger since the market value of debt has increased, the consumption level has dropped and the initial primary surplus is very negative.

However this may be a high end estimate because using the stochastic process estimated for the primary surplus, with its relatively slow mean reversion, presumes that the Corona related deficit shock will take an expected 4.4 years to be turned into a surplus again and it will take some more years for "normal" deficits to re-occur. This is unlikely since the vast majority of explicit measures are once-off and therefore the associated deficit is likely to disappear faster. We therefore present another estimate where we do take into account the additional debt created during the corona crisis and the lower consumption level, but assume that the deficit process has immediately returned to its "normal times" range immediately after the initial debt shock: we set $ps_0 = E[ps_t]$, its long-run value in this scenario, resulting in a low end estimate. Since we then do not have many years of excessive deficits following the initial shock, the overall gap becomes much lower: in this rapid adjustment case the discounted future surpluses are worth €188 billion, which amounts to 37% of the market value of the debt and a resulting sustainability gap of 0.39% of GDP, or 0.87% of aggregate consumption. Higher than the no-corona gap of 0.28% of GDP, (0.63% of C) but much lower than the gap that results assuming the corona deficits will disappear at the regular pace embedded in the ps process instead of being triggered by once-off measures. All scenarios are summarized in table 2. In reality we will end up somewhere between scenario 3 and 4, at the time of writing it is yet too early to tell exactly where. Either way, as of September 2020, the required adjustments do seem manageable.

It is worth noting that the resulting relatively small required adjustments rely to a substantial extent on the General Equilibrium impact of the fiscal adjustment on the proper risk premium time pattern. We list between brackets the partial equilibrium effects that freeze the base run risk premium time pattern and use that for all the scenario's listed in the table. That procedure results in much higher required fiscal adjustments, as can be seen in the Table 2. This PE approach reflects common practice, as for example applied by the Dutch institution CPB, but is misleading because it ignores the endogeneity of the risk premium.

9 Conclusions

In this paper we addressed the question of whether safe rates of interest below growth rates imply an era of zero fiscal costs attached to deficits. Some have hinted that $r - g < 0$ means governments essentially face no inter-temporal budget constraints since they can grow out of any additional debt without having to raise primary surpluses (Blanchard (2019) is a recent exposition of this view).[11] Against this view we argue that to be sustainable the public debt should not exceed the discounted value of projected future surpluses, but the discount rate used to calculate the latter should reflect the stochastic characteristics of the primary surplus process.

Theory has the answer to the question of how to estimate that risk premium: it depends on the correlation between the stochastic process driving the primary surplus and consumption growth. Following the literature, we value the debt as a claim on future surpluses; if the ratio of these surpluses to consumption (and GDP) are procyclical, as one would expect and as we do find econometrically for the Netherlands, the claim pays out

[11]To be fair to Blanchard (2019), one should note that he gives other reasons than unsustainability against running large deficits even when $r < g$.

even less in bad times (more in good times) than a pure claim linked to consumption would need to pay out and should carry a correspondingly higher risk premium: public debt in the Netherlands viewed as a claim on future primary surpluses apparently has a negative insurance value.

We estimate the joint distribution of consumption growth and changes in the primary surplus and show that the two processes are indeed positively correlated. Not enough data is available to estimate the disaster risk parameters of the jump processes we need to incorporate in these stochastic processes to resolve the Mehra-Prescott puzzle; therefore we calibrate these parameters using historical data based partially on Barro (2009) and updated with recent Dutch data on the GDP consequences of the financial crisis and the current corona crisis. We test sensitivity to the parameter that captures correlation between the primary surplus and consumption in crisis times: the results are robust for different values.

To go from the econometrics to the risk premium we embed the processes in a simple general equilibrium asset pricing model calibrated to fit basic asset market pricing characteristics. In particular to resolve the Mehra-Prescott puzzle we incorporate catastrophe risk like in Barro and Ursúa (2008) and break the link between risk aversion and EIS by assuming the continuous-time variant of Epstein-Zin preferences (like in Duffie and Epstein (1992b)). This allows us to endogenously derive real rates and risk premia from the stochastic processes matching the data on consumption, primary surpluses and asset prices in the Dutch economy.

By way of example, we first calculate the ratio of the fundamental over market value of the debt under the assumption that the primary surplus is a constant fraction of consumption. This assumption leads to a risk premium of 2.05% and and a fundamental debt value that is higher the market value of the debt.

However once we take into account the procyclical nature of the primary surplus (and its implied negative insurance value), we get our best estimate of the risk premium: 4.94%, substantially higher than in the *constant PS/C ratio* case. This leads to a correspondingly higher growth-adjusted discount rate when calculating the fundamental value of the debt and a correspondingly lower ratio of the discounted surpluses (the fundamental value) over market value of 47%.

Taking the deficit impact of what we now know[12] about the fiscal impact of the corona crisis into account leads to a further decline of the fundamental debt value, to a shockingly low 17% of the market value if the higher deficits take as long to fade away as our econometric analysis suggests is the case for regular ps_t shocks. If we assume the deterioration consists exclusively of once-off measures and will therefore only last a single period, the debt value falls to a more palatable 37%.

We assess five potential explanations/implications of such a debt-sustainability gap: (A) government default risk; (B) the Government is riding a bubble; (C) there is a high convenience yield to Government debt leading to additional seigniorage-like revenues not captured in regular calculations; (D) the Cochrane/FTPL implication: a debt-sustainability

[12] Per end September 2020.

indicator below 1 is a harbinger of future inflation; or (E) A debt-sustainability gap and a government issuing debt at below the rate properly reflecting the risk characteristics of the primary surplus imply that the Government is shifting risk towards future tax payers. We reject (A) out of hand because default risk is reflected in market prices and can thus not explain any difference between that market value and its collateral; we then provided evidence that the data seem to rule out the the bubble and convenience yield hypotheses. We do not find evidence in favor of higher expected inflation in response to the recent increase in the sustainability gap so we are left with the shifting risk towards future taxpayers view.

And since both higher inflation and undue risk shifting towards the future are likely considered undesirable we calculate a fiscal sustainability gap corresponding to the debt sustainability gap: the permanent increase in the primary surplus needed to bring market and fundamental values of the debt equal to each other, which presumably forestalls both these possible two outcomes. Our base case with stochastic ps-to-GDP ratio would require a 0.28% permanent fiscal adjustment to restore debt-sustainability, while the post-corona deficits require a larger adjustment, in the range of 0.39% - 0.52% depending on how fast normality returns to the ps_t process.

A final remark concerns the importance of endogenizing the risk premium. We also present partial equilibrium effects where we ignore the impact of the fiscal adjustment on the risk premium, an approach that is more in line with the approach followed by the typical thinktanks doing this sort of calculations, like the Dutch NBEA (or CPB as its Dutch acronym goes). The required fiscal adjustments based on the PE approach are substantially larger than our GE calculations indicate because the PE approach ignores the offsetting effects of a lower risk premium once the fiscal measures are in fact taken.

There are several directions in which this paper can usefully be extended. First of all we intend to incorporate the Central Bank's balance sheet into the analysis of debt sustainability (for an early example of doing that cf Anand and Van Wijnbergen (1989)). One way of looking at the outburst of Large Scale Asset purchases by several CBs recently is that it amounts not to a change in the level of debt but to a maturity shortening; In a LSAP program, CBs buy long term debt and finance that by issuing short term commercial bank reserves held at the CB. In that sense LSAP programs come down to a maturity shortening of public debt, which should have an impact on the difference between the par value to the market value of the debt. A next step could be to introduce interest rate risk into the analysis. But none of those extensions will change the basic claim of this paper: safe rates below the growth rate do not imply a free lunch for the fiscal authorities. Debt sustainability analysis should be based on interest rates that properly reflect the risk characteristics of the future primary surpluses the outstanding debt is a claim on.

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A Asset prices

A.1 Hamilton-Jacobi-Bellman equation

The Hamilton-Jacobi-Bellman (HJB) equation for this problem equals:

$$0 = f(C_t, V_t) + V_C \mu C_t + \frac{1}{2} V_{CC} (\sigma^C)^2 + \lambda E\Big[V(e^{Z_C} C_{t-}) - V(C_{t-})\Big]. \tag{26}$$

Conjecture that $V(C_t) = \frac{g C_t^{1-\gamma}}{1-\gamma}$, substitute this form with its derivatives into the HJB equation and dividing by $g C_t^{1-\gamma}$ gives:

$$0 = \frac{\beta}{1 - 1/\epsilon}(g^{-1/\zeta} - 1) + \mu - \frac{1}{2}\gamma(\sigma^C)^2 + \lambda \frac{E[e^{(1-\gamma)Z_C} - 1]}{1-\gamma}, \tag{27}$$

with $\zeta = \frac{1-\gamma}{1-\frac{1}{\epsilon}}$. Solving for g yields:

$$g = \left(1 - \frac{1 - 1/\epsilon}{\beta}\left(\mu - \frac{1}{2}\gamma(\sigma^C)^2 + \lambda \frac{E[e^{(1-\gamma)Z_C} - 1]}{1-\gamma}\right)\right)^{-\zeta} \tag{28}$$

A.2 Stochastic Discount Factor

Duffie and Epstein (1992a) show that the stochastic discount factor for this problem equals: $\pi_t = \exp\{\int_0^t f_V(C_s, V_s) ds\} f_c(C_t, V_t)$.

The derivatives of f are:

$$\begin{aligned} f_V(C, V) &= \beta\zeta\Big\{(1 - 1/\zeta)\Big((1-\gamma)V\Big)^{-1/\zeta} C^{1-1/\epsilon} - 1\Big\}, \\ f_C(C, V) &= \frac{\beta C^{-1/\epsilon}}{\Big((1-\gamma)V\Big)^{1/\zeta - 1}}. \end{aligned} \tag{29}$$

Substitute once more $V(C_t) = \frac{g C_t^{1-\gamma}}{1-\gamma}$ to obtain:

$$\begin{aligned} f_V(C_t, V_t) &= \beta\zeta\Big\{(1 - 1/\zeta) g^{-1/\zeta} - 1\Big\} \\ f_C(C_t, V_t) &= \beta g^{1-1/\zeta} C_t^{-\gamma} \end{aligned} \tag{30}$$

Therefore $\pi_t = \exp\left(\beta\zeta\Big\{g^{-\frac{1}{\zeta}}\Big(1 - \frac{1}{\zeta}\Big) - 1\Big\} t\right) \beta g^{1-\frac{1}{\zeta}} C_t^{-\gamma}$ and

$$\begin{aligned} \frac{d\pi_t}{\pi_{t-}} &= \beta\zeta\Big(g^{-1/\zeta}(1 - 1/\zeta) - 1\Big)dt + \frac{dC_t^{-\gamma}}{C_{t-}^{-\gamma}} \\ &= \beta\zeta\Big(g^{-1/\zeta}(1 - 1/\zeta) - 1\Big)dt - \gamma\Big(\mu - \frac{1}{2}(1+\gamma)(\sigma^C)^2\Big)dt - \gamma\sigma^C dW_t^C \\ &\quad + (e^{-\gamma Z_C} - 1)dN_t. \end{aligned} \tag{31}$$

Substituting g yields:

$$\frac{d\pi_t}{\pi_{t-}} = \Big(-\beta - \mu/\epsilon + \frac{1}{2}\gamma(1+1/\epsilon)(\sigma^C)^2 + (\gamma - 1/\epsilon)\lambda\frac{E[e^{(1-\gamma)Z_C} - 1]}{1-\gamma}\Big)dt \\ - \gamma\sigma^C dW_t^C + (e^{-\gamma Z_C} - 1)dN_t \tag{32}$$

A.3 Interest rate

Now consider a risk-free asset M_t with interest rate r. By no arbitrage, $\pi_t M_t$ should be a martingale.

$$\frac{d\pi_t M_t}{\pi_{t-} M_t} = \Big(r - \beta - \mu/\epsilon + \frac{1}{2}\gamma(1+1/\epsilon)(\sigma^C)^2 + (\gamma - 1/\epsilon)\lambda\frac{E[e^{(1-\gamma)Z_C} - 1]}{1-\gamma}\Big)dt \\ - \gamma\sigma^C dW_t^C + \Big(e^{-\gamma Z_C} - 1\Big)dN_t \tag{33}$$

This implies that the risk-free interest rate equals:

$$r = \beta + \mu/\epsilon - \frac{1}{2}\gamma(1+1/\epsilon)(\sigma^C)^2 - (\gamma - 1/\epsilon)\lambda\frac{E[e^{(1-\gamma)Z_C} - 1]}{1-\gamma} - \lambda E[e^{-\gamma Z_C} - 1]. \tag{34}$$

We can rewrite the stochastic discount factor as:

$$\frac{d\pi_t}{\pi_{t-}} = \Big(-r - \lambda E[e^{-\gamma Z_C} - 1]\Big)dt - \gamma\sigma^C dW_t^C + (e^{-\gamma Z_C} - 1)dN_t. \tag{35}$$

A.4 Consumption Risk Premium

Consider total wealth of the agent S_t. The representative agent owns the consumption stream C_t and therefore we must have that $S_t = \int_t^\infty E_t\Big[\frac{\pi_s}{\pi_t}C_s\Big]ds$. The wealth consumption ratio equals $\frac{S_t}{C_t}$. Now consider the dynamics of wealth S_t including consumption payments:

$$dS_t + C_t dt = \Big(\mu + \frac{C_t}{S_t}\Big)S_t dt + \sigma^C S_t dW_t^C + (e^{Z_C} - 1)S_{t-}dN_t. \tag{36}$$

The return on wealth including consumption payments multiplied with the pricing kernel must be a martingale:

$$d\pi_t S_t + \pi_t C_t dt = \Big\{\mu + \frac{C_t}{S_t} - r - \lambda E[e^{-\gamma Z_C} - 1] - \gamma(\sigma^C)^2\Big\}\pi_t S_t dt \\ + (1-\gamma)\sigma^C \pi_t S_t dW_t^C + \Big(e^{(1-\gamma)Z_C} - 1\Big)\pi_{t-}S_{t-}dN_t. \tag{37}$$

The no arbitrage condition then implies that:

$$\frac{C_t}{S_t} = r + \lambda E[e^{-\gamma Z_C} - 1] + \gamma(\sigma^C)^2 - \mu - \lambda E[e^{(1-\gamma)Z_C} - 1]. \tag{38}$$

The risk premium is equal to the expected return on the claim of consumption in excess of the interest rate:

$$RP_C = \mu + \frac{C_t}{S_t} + \lambda E[e^{Z_C} - 1] - r = \gamma(\sigma^C)^2 + \lambda E\Big[\Big(e^{-\gamma Z_C} - 1\Big)\Big(1 - e^{Z_C}\Big)\Big]. \tag{39}$$

A.5 Total Wealth

Therefore $\frac{C_t}{S_t} = r + rp - (\mu + \lambda E[e^{Z_C} - 1])$. We can now rewrite S_t as:

$$\begin{aligned} S_t &= \frac{S_t}{C_t} C_t = \frac{1}{r + rp - (\mu + \lambda E[e^{Z_C} - 1])} C_t \\ &= \int_t^\infty \exp\Big\{ - (r + rp)(s - t) \Big\} \exp\Big\{ (\mu + \lambda E[e^{Z_C} - 1])(s - t) \Big\} C_t ds \\ &= \int_t^\infty \exp\Big\{ - (r + rp)(s - t) \Big\} E_t[C_s] ds. \end{aligned} \tag{40}$$

A.6 Equity Premium

The value of the claim on dividends equals $F_t = \int_t^\infty E_t\Big[\frac{\pi_s}{\pi_t} D_s\Big] ds$. The price-dividend ratio equals $\frac{F_t}{D_t}$. We now use a similar derivation as for the consumption risk premium. First, the dynamics of the cum-dividend paying claim on dividends are:

$$dF_t + D_t dt = \Big(\phi\mu + \frac{1}{2}\phi(\phi - 1)(\sigma^C)^2 + \frac{D_t}{F_t}\Big) F_t dt + \phi\sigma^C F_t dW_t^C + (e^{\phi Z_C} - 1) F_{t-} dN_t. \tag{41}$$

The dynamics of the product with the pricing kernel are:

$$\begin{aligned} d\pi_t F_t + \pi_t D_t dt = &\Big\{ \phi\mu + \frac{1}{2}\phi(\phi - 1)(\sigma^C)^2 + \frac{D_t}{F_t} - r - \lambda E[e^{-\gamma Z_C} - 1] - \phi\gamma(\sigma^C)^2 \Big\} \pi_t F_t dt \\ &+ (\phi - \gamma)\sigma^C \pi_t F_t dW_t^C + \Big(e^{(\phi-\gamma)Z_C} - 1 \Big) \pi_{t-} F_{t-} dN_t. \end{aligned} \tag{42}$$

The no arbitrage martingale condition then yields:

$$\frac{D_t}{F_t} = r + \lambda E[e^{-\gamma Z_C} - 1] + \phi\gamma(\sigma^C)^2 - \phi\mu - \frac{1}{2}\phi(\phi - 1)(\sigma^C)^2 - \lambda E[e^{(\phi-\gamma)Z_C} - 1]. \tag{43}$$

The equity premium therefore equals:

$$EP = \phi\mu + \frac{1}{2}\phi(\phi-1)(\sigma^C)^2 + \frac{D_t}{F_t} + \lambda E[e^{\phi Z_C} - 1] - r = \phi\gamma(\sigma^C)^2 + \lambda E\Big[\Big(e^{-\gamma Z_C} - 1\Big)\Big(1 - e^{\phi Z_C}\Big)\Big]. \tag{44}$$

B Government debt

From the point of view of the representative agent, the debt gives a claim on the government surpluses. Debt is risk-free, so it must have the following dynamics: $dB_t = r_B B_t dt$ and it pays a dividend $T_t - G_t$. Again using the no-arbitrage assumption, we must have that

$\pi_t B_t^d$ is a martingale, where $dB_t^d = dB_t + (T_t - G_t)dt$, i.e. B_t^d is the cum-dividend debt price. We can write out the dynamics of $\pi_t B_t^d$:

$$\begin{aligned}
&d(\pi_t B_t^d) = d(\pi_t B_t) + \pi_t(T_t - G_t)dt = \Big(r_B - r - \lambda E[e^{-\gamma Z_C} - 1]\Big)\pi_t B_t dt \\
&+ (T_t - G_t)\pi_t dt - \gamma\sigma^C \pi_t B_t dW_t^C + (e^{-\gamma Z_C} - 1)\pi_{t-} B_t dN_t \\
&= \Big(r_B - r - \lambda E[e^{-\gamma Z_C} - 1] + \frac{T_t - G_t}{B_t}\Big)\pi_t B_t dt \\
&- \gamma\sigma^C \pi_t B_t dW_t^C + (e^{-\gamma Z_C} - 1)\pi_{t-} B_t dN_t.
\end{aligned} \tag{45}$$

This is a martingale if $r_B = r + \frac{G_t - T_t}{B_t}$. Therefore the dynamics of government debt are:

$$dB_t = rB_t dt + \Big(G_t - T_t\Big)dt. \tag{46}$$

By integrating $d\pi_t B_t^d = d\pi_t B_t + \pi_t(T_t - G_t)dt$ we obtain $\pi_t B_t^d = \pi_t B_t + \int_0^t \pi_s(T_s - G_s)ds$. Using the martingale property of $\pi_t B_t^d$ we get for $u > t$:

$$\pi_t B_t + \int_0^t \pi_s(T_s - G_s)ds = E_t\Big[\pi_u B_u + \int_0^u \pi_s(T_s - G_s)ds\Big] \tag{47}$$

Some rearranging yields:

$$B_t = E_t\Big[\frac{\pi_u}{\pi_t}B_u + \int_t^u \frac{\pi_s}{\pi_t}(T_s - G_s)ds\Big]. \tag{48}$$

We now impose the No-Ponzi condition:

$$\lim_{u\to\infty} E_t\Big[\frac{\pi_u}{\pi_t}B_u\Big] = 0. \tag{49}$$

By taking the limit it then follows that government debt should be equal to the discounted value of dividends:

$$B_t = E_t\Big[\int_t^\infty \frac{\pi_s}{\pi_t}(T_s - G_s)ds\Big]. \tag{50}$$

C Numerical implementation

To calculate the value of government debt, we use the following formula: $B_t = E_t\Big[\int_t^\infty \frac{\pi_s}{\pi_t} PS_s ds\Big]$. By Fubini's theorem, we can also use: $B_t = \int_t^\infty E_t\Big[\frac{\pi_s}{\pi_t} PS_s\Big]ds$. Furthermore, we can normalize π_t equal to 1, which gives: $B_t = \int_t^\infty E_t\Big[\pi_s PS_s\Big]ds$. Using the Ito product rule, we derive the following process for PS_t:

$$\begin{aligned}
dPS_t = d(ps_t C_t) =& \Big(\mu PS_t + \theta(\overline{ps}C_t - PS_t) + \rho\sigma^{PS}\sigma^C C_t\Big)dt + \Big(\sigma^C PS_t + \rho\sigma^{PS} C_t\Big)dW_t^C \\
&+ \sqrt{1-\rho^2}\sigma^{PS} C_t dW_t^{PS} + \Big((e^{Z_C} - 1)PS_{t-} + (e^{Z_{PS}} - 1)e^{Z_C} C_{t-}\Big)dN_t.
\end{aligned} \tag{51}$$

Next, consider the product of PS_t with the pricing kernel:

$$\begin{aligned} d\pi_t PS_t = &\Big\{\Big(\mu - \gamma(\sigma^C)^2 - r - \lambda E[e^{-\gamma Z_C} - 1]\Big)\pi_t PS_t + \theta(\overline{ps}\pi_t C_t - \pi_t PS_t) \\ &+ (1-\gamma)\rho\sigma^{PS}\sigma^C\pi_t C_t\Big\}dt + \Big((1-\gamma)\sigma^C\pi_t PS_t + \rho\sigma^{PS}\pi_t C_t\Big)dW_t^C \\ &+ \sqrt{1-\rho^2}\sigma^{PS}\pi_t C_t dW_t^{PS} + \Big\{\Big(e^{(1-\gamma)Z_C} - 1\Big)\pi_{t-}PS_{t-} + (e^{Z_{PS}} - 1)e^{(1-\gamma)Z_C}\pi_{t-}C_{t-}\Big\}dN_t. \end{aligned} \tag{52}$$

The drift of this product equals:

$$\begin{aligned} \mathcal{D}_{\pi PS} = &\Big(\mu - \gamma(\sigma^C)^2 - r - \lambda E[e^{-\gamma Z_C} - 1]\Big)\pi_t PS_t + \theta(\overline{ps}\pi_t C_t - \pi_t PS_t) \\ &+ (1-\gamma)\rho\sigma^{PS}\sigma^C\pi_t C_t + \lambda E[e^{(1-\gamma)Z_C} - 1]\pi_t PS_t + \lambda E\Big[(e^{Z_{PS}} - 1)e^{(1-\gamma)Z_C}\Big]\pi_t C_t. \end{aligned} \tag{53}$$

Note that $\mathcal{D}_{\pi PS}$ depends on $\pi_t C_t$. Define by $\mathcal{D}_{\pi C}$ the drift of the product $\pi_t C_t$:

$$\mathcal{D}_{\pi C} = \Big(\mu - \gamma(\sigma^C)^2 - r - \lambda E[e^{-\gamma Z_C} - 1] + \lambda E[e^{(1-\gamma)Z_C} - 1]\Big)\pi_t C_t. \tag{54}$$

We can then find $E_t[\pi_s PS_s]$ by solving the following system of differential equations forward:

$$\begin{aligned} d\pi_t C_t &= \mathcal{D}_{\pi C}dt \\ d\pi_t PS_t &= \mathcal{D}_{\pi PS}dt. \end{aligned} \tag{55}$$

This differential equation can easily be solved numerically. To calculate the integral, we then use a trapezoid rule with a time step of 0.01 years. We cut off the integral at $t_{max} = 750$ years.

D Alternative approach: utility-free asset pricing

As a robustness check, we apply the approach of Jiang et al. (2020b) to data of the Netherlands. This approach does not base the stochastic discount on a utility function. Instead, the stochastic discount factor is assumed to be a function of macro variables and is estimated to match observed asset prices. We first postulate the vector state variables that is included in the stochastic discount factor. The distribution of this vector is estimated using a VAR. After that, a theoretical asset pricing model is derived. With the asset pricing model in hand, we then discuss the estimation of the stochastic discount factor and the valuation of the claim on primary surpluses.

D.1 Estimating the VAR

The state variables that we include in the stochastic discount factor are: inflation (π_t), real GDP growth (x_t), the nominal yield on government bonds with a maturity of 1 year ($y_t(1)$), the spread between the nominal 5 year yield and the nominal 1 year yield on government bonds (spr_t), the log price-dividend ratio (pd_t), nominal dividend growth (d_t), growth of the tax to GDP ratio $\left(\Delta\log(\tau_t) \text{ where } \tau_t = \frac{T_t}{GDP_t}\right)$ and growth of spending to GDP ratio $\left(\Delta\log(g_t) \text{ where } g_t = \frac{G_t}{GDP_t}\right)$. The reason that we split the primary surplus into tax income and spending is that this allows us to separately price the tax and spending claim. Furthermore, both now cannot get negative which implies that we can take logarithms, which is not possible with the primary surplus. We then define the demeaned vector of state variables:

$$\tilde{z}_t = \begin{bmatrix} \pi_t - \overline{\pi} \\ x_t - \overline{x} \\ y_t(1) - \overline{y(1)} \\ spr_t - \overline{spr} \\ pd_t - \overline{pd} \\ d_t - \overline{d} \\ \Delta\log(\tau_t) \\ \Delta\log(g_t) \end{bmatrix}. \tag{56}$$

It would be natural to estimate the VAR $\tilde{z}_t = \tilde{\Psi}\tilde{z}_{t-1} + u_t$. However, the main variables of interest, τ_t and g_t are mean-reverting. This mean reversion also implies that the long run growth rate of $\log(\tau_t)$ and $\log(g_t)$ are zero. This is also true in our data-set and therefore the last two elements are not demeaned. A government can only sustain to increase the spending to GDP level temporarily. After a few years spending must decrease again to balance the budget. To capture this mean-reversion, we also include the level of both log tax to GDP and log spending to GDP. In our main method mean reversion was also captured within the primary surplus process. We for example allowed the change in primary surplus to consumption to depend on the level to capture mean reversion. We cannot just add $\log(\tau_t)$ and $\log(g_t)$ to the VAR, since this will cause a multicollinearity problem. We therefore first estimate

$$\tilde{z}_t = \tilde{\Psi}\tilde{z}_{t-1} + \beta_1\Big(\log(\tau_{t-1}) - \overline{\log(\tau)}\Big) + \beta_2\Big(\log(g_{t-1}) - \overline{\log(g)}\Big) + \tilde{u}_t, \tag{57}$$

where $\tilde{u}_t \sim N(0,\tilde{\Sigma})$ and $\tilde{u}_t = \tilde{\Sigma}^{\frac{1}{2}}\tilde{\epsilon}_t$ where $\tilde{\epsilon}_t \sim N(0,I)$, $\tilde{\Sigma} = \tilde{\Sigma}^{\frac{1}{2}}(\tilde{\Sigma}^{\frac{1}{2}})'$.

Note that $\log(\tau_t) = \Delta\log(\tau_t) + \log(\tau_{t-1})$. Therefore, we can define:

$$z_t = \begin{bmatrix} \tilde{z}_t \\ \log(\tau_t) - \overline{\log(\tau)} \\ \log(g_t) - \overline{\log(g)} \end{bmatrix}, \quad \Psi = \begin{bmatrix} \tilde{\Psi} & \beta_1 & \beta_2 \\ \tilde{\Psi}_7 & \beta_1(7)+1 & \beta_2(7) \\ \tilde{\Psi}_8 & \beta_1(8) & \beta_2(8)+1 \end{bmatrix},$$
$$\text{and } \Sigma^{\frac{1}{2}} = \begin{bmatrix} \tilde{\Sigma}^{\frac{1}{2}} & \mathbf{0}_{2\times 8} & \\ \tilde{\Sigma}^{\frac{1}{2}}_7 & 0 & 0 \\ \tilde{\Sigma}^{\frac{1}{2}}_8 & 0 & 0 \end{bmatrix}, \tag{58}$$

where $\tilde{\Psi}_x$ contains the x^{th} row of $\tilde{\Psi}$, similar notation is used for $\tilde{\Sigma}^{\frac{1}{2}}$ and $\beta_1(x)$ is the x^{th} element of the vector β_1. Then we can write the evolution of our system of state variables as:

$$z_t = \Psi z_{t-1} + \Sigma^{\frac{1}{2}} \epsilon_t. \tag{59}$$

Lastly, define $\Sigma = \Sigma^{\frac{1}{2}}(\Sigma^{\frac{1}{2}})'$.

D.2 Asset pricing model

Instead of deriving the stochastic discount factor from a utility function, we assume that the stochastic discount factor is a direct function of our state variables. Define the market price of risk vector $\Lambda_t = \Lambda_0 + \Lambda_1 z_t$. Λ_0 is a 1×10 vector which contains the constant market prices of risk, while Λ_1 is a 10×10 matrix with coefficients that determine the time-varying market prices of risk. Each element of the vector Λ_t corresponds to the same element of the risk vector ϵ_t. For now, we assume that Λ_0 and Λ_1 are known and we derive the implied asset prices. Later, Λ_0 and Λ_1 are estimated in order to match observed asset prices. The nominal stochastic discount factor is assumed to be exponentially affine: $M_{t+1} = \exp\{m_{t+1}\} = \exp\Big\{ - y_t(1) - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\epsilon_{t+1}\Big\}$.

D.2.1 Nominal Yields

Denote by $P_t^B(h)$ the price of bond with maturity h at time t. We conjecture and later verify that $\log\Big(P_t^B(h)\Big) = A^B(h) + B^B(h)'z_t$. The price must satisfy the Euler equation:

$$\begin{aligned} P_t^B(h+1) &= E_t\Big[M_{t+1}P_{t+1}^B(h)\Big] = E_t\Big[\exp\Big\{m_{t+1} + \log\Big(P_{t+1}^B(h)\Big)\Big\}\Big] \\ &= E_t\Big[\exp\Big\{ - y_t(1) - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\epsilon_{t+1} + A^B(h) + B^B(h)'z_{t+1}\Big\}\Big] \\ &= E_t\Big[\exp\Big\{ - y_t(1) - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\epsilon_{t+1} + A^B(h) + B^B(h)'\Big(\Psi z_t + \Sigma^{\frac{1}{2}}\epsilon_{t+1}\Big)\Big\}\Big]. \end{aligned} \tag{60}$$

The only randomness comes from ϵ_{t+1}. Calculating the expectations gives:

$$\begin{aligned} P_t^B(h+1) = \exp\Big\{ &- y_t(1) + A^B(h) + B^B(h)'\Psi z_t + \frac{1}{2}B^B(h)'\Sigma B^B(h) \\ &- B^B(h)'\Sigma^{\frac{1}{2}}(\Lambda_0 + \Lambda_1 z_t)\Big\}. \end{aligned} \tag{61}$$

Define $A^B(h+1) = -y_t(1) + A^B(h) + \frac{1}{2}B^B(h)'\Sigma B^B(h) - B^B(h)'\Sigma^{\frac{1}{2}}\Lambda_0$ and $B^B(h+1) = B^B(h)'\Psi - B^B(h)'\Sigma^{\frac{1}{2}}\Lambda_1$. Then we have:

$$P_t^B(h+1) = \exp\{A^B(h+1) + B^B(h+1)z_t\}, \tag{62}$$

which verifies the conjecture. We normalize $P_t^B(0) = 1$, and therefore initialize $A^B(0) = 0$ and $B^B(0) = 0$. Nominal bond yields are given by $y_t(h) = -\log\Big(P_t^B(h)\Big)/h = -\frac{A^B(h)}{h} - \frac{B^B(h)}{h} z_t$.

D.2.2 Equity premium

The gross return on the stock market equals $R_{t+1} = \frac{P_{t+1}^S + D_{t+1}}{P_t}$ where P_t^S is the stock price and D_t is the nominal dividend payout. Denote by $r_t = \log(R_t)$ the log return:

$$\begin{aligned} r_{t+1} &= \log\Big(\frac{P_{t+1}^S + D_{t+1}}{P_t^S}\Big) = \log\Big(D_{t+1}\Big(1 + \frac{P_{t+1}^S}{D_{t+1}}\Big)\Big) - \log(P_t^S) \\ &= \log(D_{t+1}) + \log\Big(1 + \exp\{pd_{t+1}\}\Big) - \log(P_t^S) \\ &= \log(D_{t+1}) - \log(D_t) + \log\Big(1 + \exp\{pd_{t+1}\}\Big) - \log(P_t^S) + \log(D_t) \\ &= d_{t+1} + \log\Big(1 + \exp\{pd_{t+1}\}\Big) - pd_t. \end{aligned} \tag{63}$$

Using a first order Taylor approximation around $\overline{pd}$, we obtain $\log\Big(1 + \exp\{pd_{t+1}\}\Big) \approx \kappa_0 + \kappa_1 pd_{t+1}$ where $\kappa_1 = \frac{e^{\overline{pd}}}{1+e^{\overline{pd}}}$ and $\kappa_0 = \log(1 + \exp\{\overline{pd}\}) - \kappa_1 \overline{pd}$. This yields:

$$r_{t+1} = \kappa_0 + \kappa_1 pd_{t+1} + d_{t+1} - pd_t. \tag{64}$$

Returns must satisfy the Euler equation:

$$\begin{aligned} 1 = E_t\Big[M_{t+1}R_{t+1}\Big] &= E_t\Big[\exp\{m_{t+1} + r_{t+1}\}\Big] = E_t\Big[\exp\Big\{-y_t(1) - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\epsilon_{t+1} + r_{t+1}\Big\}\Big] \\ &= \exp\Big\{-y_t(1) - \frac{1}{2}\Lambda_t'\Lambda_t + \kappa_0 - pd_t\Big\} \times E_t\Big[\exp\Big\{-\Lambda_t'\epsilon_{t+1} + \kappa_1 pd_{t+1} + d_{t+1}\Big\}\Big]. \end{aligned} \tag{65}$$

Note that the distributions of d_{t+1} and pd_{t+1} are implied by the VAR. We can write $d_{t+1} = \overline{d} + e_6'\Psi z_t + e_6'\Sigma^{\frac{1}{2}}\epsilon_{t+1}$ and $pd_{t+1} = \overline{pd} + e_5'\Psi z_t + e_5'\Sigma^{\frac{1}{2}}\epsilon_{t+1}$ where e_i is a 10×1 column vector of zeros except for the $i-th$ place, which is a 1. Therefore we obtain:

$$\begin{aligned} &E_t\Big[\exp\Big\{-\Lambda_t'\epsilon_{t+1} + \kappa_1 pd_{t+1} + d_{t+1}\Big\}\Big] \\ &= E_t\Big[\exp\Big\{-\Lambda_t'\epsilon_{t+1} + \kappa_1\overline{pd} + \overline{d} + (\kappa_1 e_5 + e_6)'\Psi z_t + (\kappa_1 e_5 + e_6)'\Sigma^{\frac{1}{2}}\epsilon_{t+1}\Big\}\Big] \\ &= \exp\Big\{\kappa_1\overline{pd} + \overline{d} + (\kappa_1 e_5 + e_6)'\Psi z_t + \frac{1}{2}\Lambda_t'\Lambda_t \\ &+ \frac{1}{2}(\kappa_1 e_5 + e_6)'\Sigma(\kappa_1 e_5 + e_6) - (\kappa_1 e_5 + e_6)'\Sigma^{\frac{1}{2}}\Lambda_t\Big\}. \end{aligned} \tag{66}$$

Taking the logarithm on both sides and rearranging gives:

$$\kappa_0 + \kappa_1\overline{pd} + \overline{d} + (\kappa_1 e_5 + e_6)'\Psi z_t - pd_t - y_t(1) + \frac{1}{2}(\kappa_1 e_5 + e_6)'\Sigma(\kappa_1 e_5 + e_6) = (\kappa_1 e_5 + e_6)'\Sigma^{\frac{1}{2}}\Lambda_t. \tag{67}$$

This is equivalent to:

$$E_t[r_{t+1}] - y_t(1) + \frac{1}{2}V_t[r_{t+1}] = -Cov_t[m_{t+1}, r_{t+1}]. \tag{68}$$

The left hand side equals the excess return on equity over the risk-free rate corrected for Jensen's inequality. We refer to the left hand side as the equity premium in the data since it does not depend on the stochastic discount factor. The right hand side equals minus the covariance between the stochastic discount factor and the return on equity and can be interpreted as the equity premium given the asset pricing model.

D.2.3 Price-Dividend Ratio

We can write the $P_t^S = \sum_{h=0}^{\infty} P_t^S(h)$ where $P_t^S(h)$ is the price of a strip that pays out a unit of dividend D_t in h periods. Similar to the bond prices, we conjecture and later verify that $\log\Big(P_t^S(h)/D_t\Big) = A^S(h) + B^S(h)'z_t$. Using the Euler equation and dividing by D_t, we get:

$$\begin{aligned}
\frac{P_t^S(h+1)}{D_t} &= E_t\Big[M_{t+1}\frac{P_{t+1}^S(h)}{D_t}\Big] = E_t\Big[M_{t+1}\frac{P_{t+1}^S(h)}{D_{t+1}}\frac{D_{t+1}}{D_t}\Big] \\
&= E_t\Big[\exp\Big\{m_{t+1} + \log\Big(P_{t+1}^S(h)/D_{t+1}\Big) + d_{t+1}\Big\}\Big] \\
&= E_t\Big[\exp\Big\{-y_t(1) - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\epsilon_{t+1} + A^S(h) + B^S(h)'z_{t+1} + d_{t+1}\Big\}\Big] \\
&= E_t\Big[\exp\Big\{-y_t(1) - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\epsilon_{t+1} + A^S(h) + B^S(h)'\Big(\Psi z_t + \Sigma^{\frac{1}{2}}\epsilon_{t+1}\Big) \\
&\quad + \overline{d} + e_6'\Psi z_t + e_6'\Sigma^{\frac{1}{2}}\epsilon_{t+1}\Big\}\Big].
\end{aligned} \tag{69}$$

Calculating the expectations gives:

$$\begin{aligned}
\frac{P_t^S(h+1)}{D_t} &= \exp\Big\{-y_t(1) + \overline{d} + A^S(h) + \Big(e_6 + B^S(h)\Big)'\Psi z_t \\
&\quad + \frac{1}{2}\Big(e_6 + B^S(h)\Big)'\Sigma\Big(e_6 + B^S(h)\Big) - \Big(e_6 + B^S(h)\Big)'\Sigma^{\frac{1}{2}}(\Lambda_0 + \Lambda_1 z_t)\Big\}.
\end{aligned} \tag{70}$$

Define $A^S(h+1) = -y_t(1) + \overline{d} + A^S(h) + \frac{1}{2}\Big(e_6 + B^S(h)\Big)'\Sigma\Big(e_6 + B^S(h)\Big) - \Big(e_6 + B^S(h)\Big)'\Sigma^{\frac{1}{2}}\Lambda_0$ and $B^S(h+1) = \Big(e_6 + B^S(h)\Big)'\Psi - \Big(e_6 + B^S(h)\Big)'\Sigma^{\frac{1}{2}}\Lambda_1$. This verifies the conjecture. Since $P_t^S(0) = D_t$, we set $A^S(0)$ and $B^S(0)$ equal to 0.

D.2.4 Government Taxation Claim

Define by P_t^T the price of a claim that pays dividends T_t every time period where T_t is the nominal level of government spending. Again, we can write $P_t^T = \sum_{h=0}^{\infty} P_t^T(h)$ where

$P_t^T(h)$ is the price of a strip that pays out a unit of dividend T_t in h periods. Government taxation growth equals:

$$\Delta \log(T_{t+1}) = \Delta \log(\tau_{t+1}) + \pi_{t+1} + x_{t+1}. \tag{71}$$

Conjecture $\log\Big(P_t^T(h)/T_t\Big) = A^T(h) + B^T(h)'z_t$. Dividing the Euler equation by T_t yields:

$$\begin{aligned}\frac{P_t^T(h+1)}{T_t} &= E_t\Big[M_{t+1}\frac{P_{t+1}^T(h)}{T_t}\Big] = E_t\Big[M_{t+1}\frac{P_{t+1}^T(h)}{T_{t+1}}\frac{T_{t+1}}{T_t}\Big] \\ &= E_t\Big[\exp\Big\{m_{t+1} + \log\Big(P_{t+1}^T(h)/T_{t+1}\Big) + \Delta\log(\tau_{t+1}) + \pi_{t+1} + x_{t+1}\Big\}\Big] \\ &= E_t\Big[\exp\Big\{-y_t(1) - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\epsilon_{t+1} + A^T(h) + B^T(h)'z_{t+1} \\ &+ \Delta\log(\tau_{t+1}) + \pi_{t+1} + x_{t+1}\Big\}\Big] \\ &= E_t\Big[\exp\Big\{-y_t(1) - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\epsilon_{t+1} + A^T(h) + B^T(h)'\Big(\Psi z_t + \Sigma^{\frac{1}{2}}\epsilon_{t+1}\Big) \\ &+ \overline{\pi} + \overline{x} + (e_1+e_2+e_7)'\Psi z_t + (e_1+e_2+e_7)'\Sigma^{\frac{1}{2}}\epsilon_{t+1}\Big\}\Big].\end{aligned} \tag{72}$$

Now we calculate the expectations:

$$\begin{aligned}\frac{P_t^T(h+1)}{T_t} &= \exp\Big\{-y_t(1) + \overline{\pi} + \overline{x} + A^T(h) + \Big(e_1+e_2+e_7+B^T(h)\Big)'\Psi z_t \\ &+ \frac{1}{2}\Big(e_1+e_2+e_7+B^T(h)\Big)'\Sigma\Big(e_1+e_2+e_7+B^T(h)\Big) \\ &- \Big(e_1+e_2+e_7+B^T(h)\Big)'\Sigma^{\frac{1}{2}}(\Lambda_0 + \Lambda_1 z_t)\Big\}.\end{aligned} \tag{73}$$

This yields

$$\begin{aligned}&A^T(h+1) = -y_t(1) + \overline{\pi} + \overline{x} + A^T(h) + \frac{1}{2}\Big(e_1+e_2+e_7+B^T(h)\Big)'\Sigma\Big(e_1+e_2+e_7+B^T(h)\Big) \\ &- \Big(e_1+e_2+e_7+B^T(h)\Big)'\Sigma^{\frac{1}{2}}\Lambda_0 \\ &B^S(h+1) = \Big(e_1+e_2+e_7+B^T(h)\Big)'\Psi - \Big(e_1+e_2+e_7+B^T(h)\Big)'\Sigma^{\frac{1}{2}}\Lambda_1.\end{aligned} \tag{74}$$

Using the initial conditions $P_t^T(0)/T_t = 1$ we set $A^T(0) = B^T(0) = 0$. This enables us to calculate the value of the claim om future tax revenues. The derivation for the claim on future government spending is identical and therefore omitted.

D.3 Data

Our sample period is from 1974 to 2018. To measure inflation in the Netherlands we use the GDP deflator. Real GDP growth is then calculated using nominal GDP and the GDP

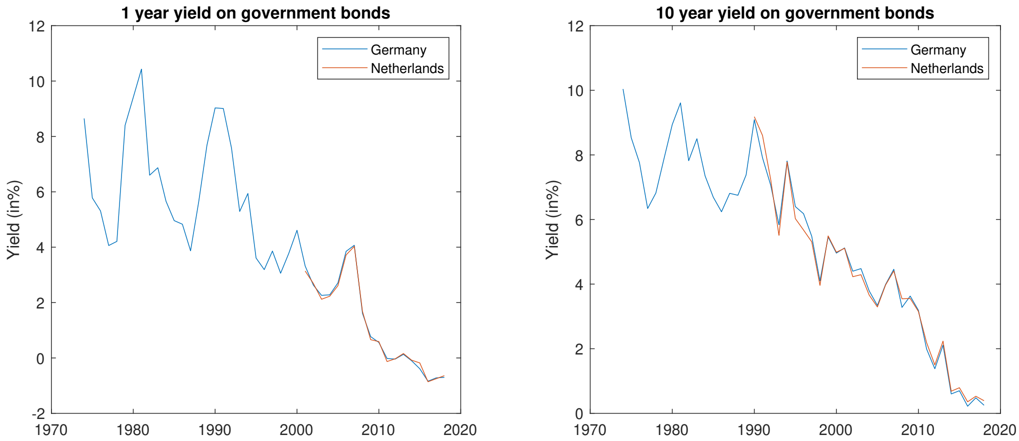

Figure 11: Comparison between Dutch and German interest rates.

deflator. All growth rates are calculated as the first difference of the logarithm. Data for the GDP deflator, nominal GDP, government revenues and government spending is obtained from Statistics Netherlands (Centraal Bureau voor de Statistiek).

Data for Dutch bond yields is not available for the entire period. Data availability for German bond yields is better which is the reason that we use German yields as a proxy for Dutch yields. We obtain German bond yields with maturities from 1 up to 30 years from the Bundesbank. Yields are end of year (31-th of December). 1 to 10 year yields are available for the entire sample period. 11 to 20 year yields are available since 1986 and 21 to 30 year yields since 2000. Figure 11 shows that during the period that data is available for both countries, Dutch and German yields are very close to each other. Therefore German yields are a good proxy for Dutch yields.

Lastly, data for the stock market is obtained from Datastream. To capture the entire Dutch stock market we use the TOTMKNL index from the Datastream Global Equity Indices database. This index is a reprentative index for the entire Dutch stock market. Weightings are based on market capitalisation. We obtain the price (P_t) and the dividend yield (DY_t) from the index. The Price-Dividend ratio (PD_t) is simply the inverse of the dividend yield. To calculate dividend growth, we use the following formula:

$$\begin{aligned}\log(D_{t+1}) - \log(D_t) &= \log(\frac{D_{t+1}}{P_{t+1}}P_{t+1}) - \log(\frac{D_t}{P_t}P_t) \\ &= \log(DY_{t+1}) - \log(DY_t) + \log(P_{t+1}) - \log(P_t).\end{aligned} \tag{75}$$

D.4 VAR Estimation Results

Estimating the VAR gives the following coefficient matrix:

$$\Psi = \begin{bmatrix} 0.64 & 0.23 & 0.14 & 0.00 & -0.01 & -0.01 & 0.10 & 0.01 & -0.04 & -0.03 \\ 0.03 & 0.30 & 0.14 & 0.82 & 0.03 & -0.02 & 0.03 & -0.02 & 0.08 & 0.01 \\ -0.25 & 0.29 & 0.76 & 0.11 & -0.02 & -0.01 & 0.04 & 0.04 & 0.05 & -0.03 \\ 0.12 & -0.18 & -0.05 & 0.44 & 0.00 & 0.01 & -0.08 & 0.00 & 0.06 & -0.05 \\ -3.92 & -5.64 & 0.63 & 7.57 & 0.62 & 0.29 & 0.37 & -0.90 & -0.16 & -0.66 \\ 3.25 & 3.36 & 0.44 & -2.78 & 0.28 & -0.11 & 0.06 & -1.18 & -0.66 & 1.45 \\ 0.11 & 0.41 & 0.10 & -0.44 & -0.03 & 0.02 & 0.09 & 0.03 & -0.42 & 0.25 \\ 0.09 & -0.54 & -0.37 & -1.13 & -0.08 & -0.03 & -0.26 & -0.04 & 0.47 & -0.65 \\ 0.11 & 0.41 & 0.10 & -0.44 & -0.03 & 0.02 & 0.09 & 0.03 & 0.58 & 0.25 \\ 0.09 & -0.54 & -0.37 & -1.13 & -0.08 & -0.03 & -0.26 & -0.04 & 0.47 & 0.35 \end{bmatrix}. \tag{76}$$

Note that there is indeed mean-reversion in the tax and spending to GDP ratio: $\Psi_{7,9} = -0.42$ and $\Psi_{8,10} = -0.65$. The correlation matrix is given by:

$$100 \times \Sigma^{\frac{1}{2}} = \begin{bmatrix} 0.95 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.17 & 1.28 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.38 & 0.41 & 1.06 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.12 & -0.05 & -0.37 & 0.46 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2.90 & 6.60 & 0.24 & 0.67 & 13.35 & 0 & 0 & 0 & 0 & 0 \\ 2.91 & 2.44 & -1.87 & -1.63 & -3.45 & 6.45 & 0 & 0 & 0 & 0 \\ 0.01 & 0.19 & -0.20 & -0.37 & 3.00 & 0.21 & 1.83 & 0 & 0 & 0 \\ -0.14 & -1.66 & -0.31 & 0.01 & -0.46 & -0.55 & 0.46 & 1.94 & 0 & 0 \\ 0.01 & 0.19 & -0.20 & -0.37 & 3.00 & 0.21 & 1.83 & 0.00 & 0 & 0 \\ -0.14 & -1.66 & -0.31 & 0.01 & -0.46 & -0.55 & 0.46 & 1.94 & 0 & 0 \end{bmatrix}. \tag{77}$$

D.5 Stochastic Discount Factor Estimation

In the previous section we assumed that the stochastic discount factor was known and derived the asset prices implied by the stochastic discount factor. In this section we fit the stochastic discount factor to the data. Λ_0 and Λ_1 have too many elements to properly estimate; moreover several types of risk are not priced in the market and are therefore hard to estimate. Therefore we restrict the market price of risk of inflation, GDP growth, the price-dividend ratio and the change in the tax to GDP and spending to GDP ratio to be equal to zero.

We do allow for a non-zero market price of risk of the short rate, the interest rate spread and dividend growth. The price of risk of the short rate and the spread can be estimated using bond yield data, the price of risk of dividend growth can be estimated using stock market data. Therefore the third, fourth and sixth element of Λ_0 are allowed to be non-zero. We try two different restrictions on Λ_1.

Restriction 1: We allow the entire third, fourth and sixth row of Λ_1 to be non-zero. This implies that the time-varying market price of risk for the short rate, spread and dividend growth can depend on any of the state variables.

Restriction 2: We allow the fourth and sixth row of Λ_1 to be non-zero. Additionally, we allow the third and fourth element of third row to be non-zero. This estimation is a bit more restrictive and is the default restriction of Jiang et al. (2020b).

Both options seem plausible and we will analyze the results of both options. To estimate the unknown coefficients, we minimize the following function:

$$\begin{aligned}\min_{\Lambda_0,\Lambda_1} \Bigg\{ &\sum_{t=1}^{T}\sum_{i=1}^{30}\left(y_t(h) + \frac{A^B(h)}{h} + \frac{B^B(h)}{h}z_t\right)^2 \\ &+ \sum_{t=1}^{T}\left(E_t[r_{t+1}] - y_t(1) + \frac{1}{2}V_t[r_{t+1}] + Cov_t[m_{t+1}, r_{t+1}]\right)^2 \\ &+ \sum_{t=1}^{T}\left(PD_t - \sum_{h=0}^{\infty} e^{A^S(h)+B^S(h)'z_t}\right)^2\Bigg\}\end{aligned} \tag{78}$$

where $y_t(h)$ are the actual German bond yields and PD_t are actual observed price-dividend ratios. Several elements of Λ_0 and Λ_1 are restricted to 0, as discussed before. This minimization ensures that the bond yields, equity premium and price-dividend ratio in the model are as close as possible to the data. We additionally include the restriction that the expected yield curve (the yield curve with $z_t = \mathbf{0}$) is weakly increasing. This restriction is necessary to make sure that the yield curve is not decreasing after 30 years. This is not implied by the minimization problem since yields for bonds with maturities longer than 30 years are not available.

D.6 Stochastic Discount Factor Estimation Results

The estimation results of the stochastic discount factor are the following.
Restriction 1: $\Lambda_0 = [0\ \ 0\ \ -0.47\ \ -0.004\ \ 0\ \ 0.77\ \ 0\ \ 0\ \ 0\ \ 0]'$ and

$$\Lambda_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -26.17 & 29.53 & -25.769 & -54.52 & -2.30 & -1.52 & -0.72 & 2.36 & 5.65 & -3.22 \\ 10.06 & -5.35 & -24.28 & -93.60 & -1.29 & 1.27 & -7.25 & 7.90 & 14.70 & -11.52 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -12.83 & -25.29 & -9.53 & 39.89 & -2.54 & 2.36 & 5.28 & -30.18 & -9.02 & 10.33 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{79}$$

Restriction 2: $\Lambda_0 = [0 \quad 0 \quad -0.91 \quad 0.55 \quad 0 \quad 0.74 \quad 0 \quad 0 \quad 0 \quad 0]'$ and

$$\Lambda_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -20.02 & -188.37 & 0 & 0 & 0 & 0 & 0 & 0 \\ -14.40 & 12.53 & -28.51 & 68.21 & -3.34 & 1.04 & -1.94 & 9.22 & 21.61 & -15.26 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -9.53 & -30.42 & -8.70 & 30.25 & -2.24 & 2.73 & 6.29 & -30.55 & -9.38 & 10.53 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{80}$$

We now look at how well the model is able to fit asset prices. Figures 12 and 14 show that the model is able to replicate the equity premium and the price dividend ratio in the data very well for both restriction assumptions. For the yield curves, restriction 1 (figure 13) performs somewhat better than restriction 2 (figure 15) since restriction 1 has more free parameters. Overall, the expected yield curve using both restrictions seems reasonable. The expected yield curve using restriction 1 flattens out a bit faster for very long maturities, which is more realistic. The model seems to overestimate long yields in the more recent years (for example 2005 and 2015). Our main takeaway from these results is that the model is not perfect but generates plausible yield curves and that the differences between restriction 1 and 2 are small.

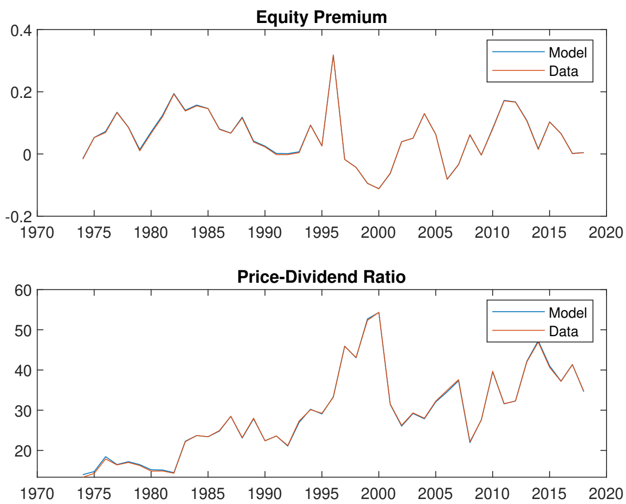

Figure 12: The observed and model-implied equity premium and price dividend ratio over time. These results are based on the estimated stochastic discount factor with restriction 1.

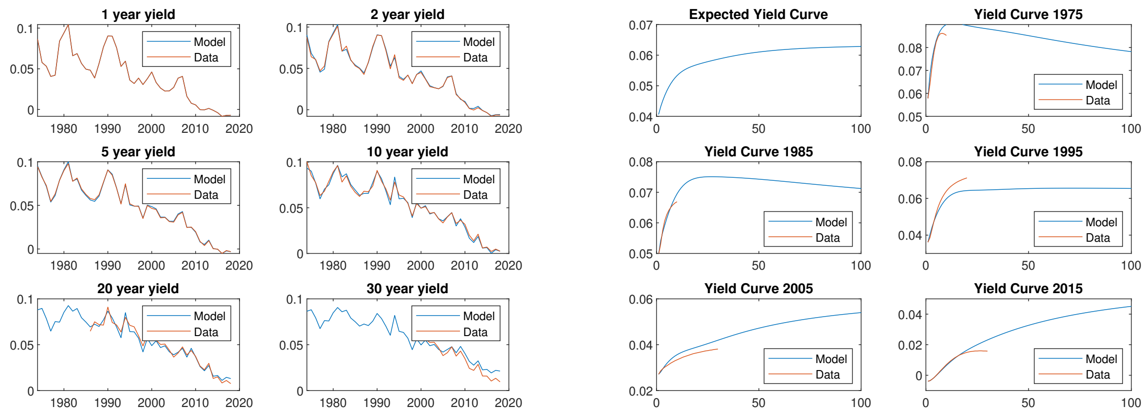

Figure 13: The observed and model-implied yields for different maturities and at different time periods. These results are based on the estimated stochastic discount factor with restriction 1.

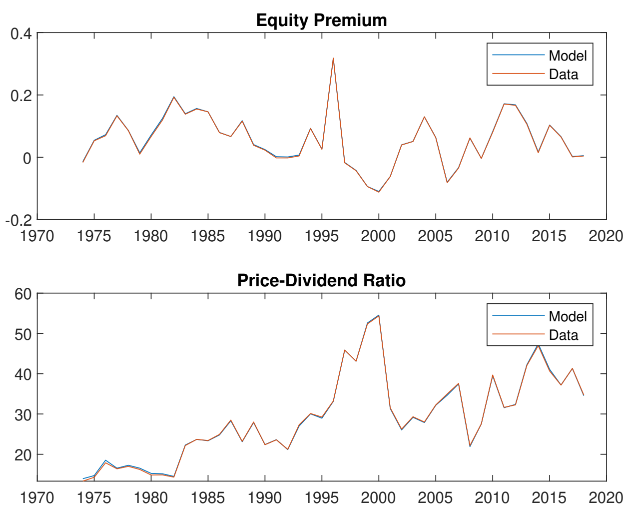

Figure 14: The observed and model-implied equity premium and price dividend ratio over time. These results are based on the estimated stochastic discount factor with restriction 2.

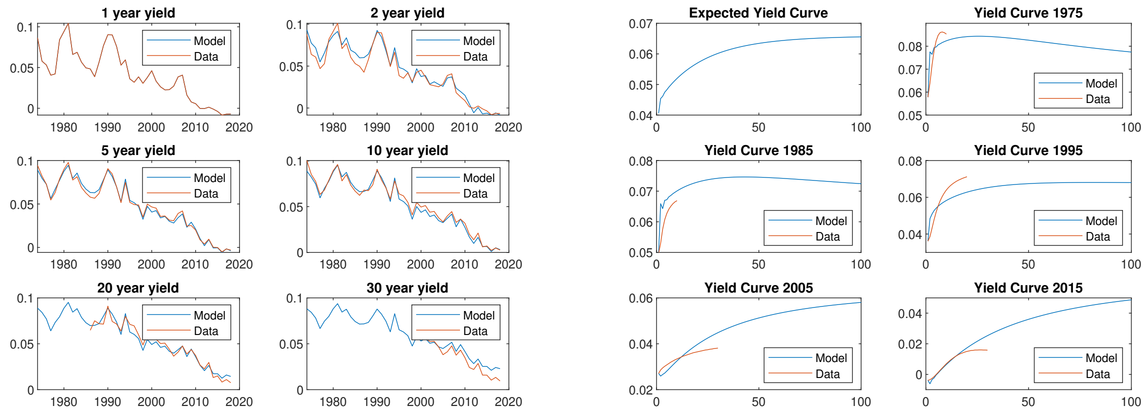

Figure 15: The observed and model-implied yields for different maturities and at different time periods. These results are based on the estimated stochastic discount factor with restriction 2.

D.7 Valuation of claim on primary surpluses

In this section we use the estimated stochastic discount factor and our asset pricing model to value the claim on primary surpluses. Figures 16 and 17 show the price dividend ratios for the tax claim, spending claim, GDP claim and dividend claim (stock) and additionally the value of the claim on primary surpluses scaled with GDP.

A couple of things stand out when looking at the price dividend ratios. First, the PD ratio of the T and G claim are very close to the PD ratio of GDP. This implies that the two processes are subject to similar risk as GDP. The PD ratio of GDP lies above the PD ratio of stocks, so a claim on dividends is riskier than a claim on GDP. For the first half of the sample, the PD ratio of GDP (and thus of T and G) follows the PD ratio of stocks, but after that the PD ratio of GDP starts increasing (because of low interest rates) while the PD ratio of stocks remains quite flat.

If we compare the PD ratios between restriction 1 and restriction 2, the results are quite similar. The PD ratio of both the tax claim and the spending claim are around 120. However, this is not the case if we look at the value of the claim on primary surpluses scaled with GDP. This value equals 0.16 in 2018 using restriction 1 against 0.38 using restriction 2. So a relatively small change in assumptions with minor effects on the yield curves and price dividend ratios does have quantitatively a large effect on the valuation of the primary surpluses. Slight changes in the value of the T claim and the G claim can have large effects on the value of the difference.

Using restriction 1, the value of the T claim in 2018 equals 40417 vs 38497 billion euros using restriction 2 which is a 4.75% decrease. For the G claim, the value with restriction 1 is 40296 vs 38211 billion euros using restriction 2 which is a 5.17% decrease. The value of the primary surplus claim using restriction 1 equals 120.97 vs. 285.80 billion euros with restriction 2 (136% increase).

Since the value of the spending claim (G claim) is decreasing more than the value of the tax claim (T claim), the value of the primary surplus claim (the difference between the T claim and G claim) is higher with restriction 2 compared to restriction 1. The main point is that relatively small changes in the valuation of the T and G claim (4.75% decrease in T claim vs. 5.17% decrease in G claim) can lead to large changes in the value of the primary surplus claim (136% increase).

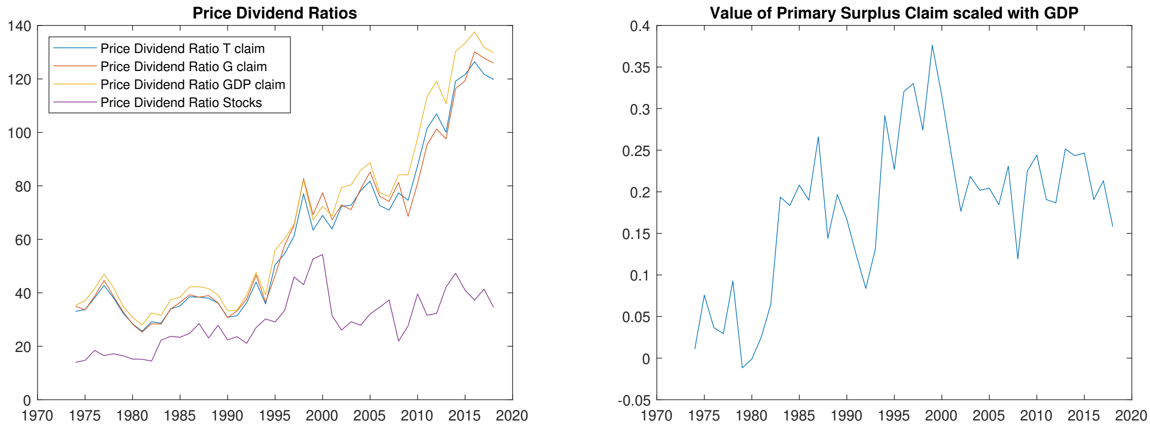

Figure 16: The left figure shows the price dividend ratios for the tax claim, government spending claim, GDP claim and dividend claim (stock) over time. The right figure shows the value of the primary surplus claim scaled with GDP. These results are based on the estimated stochastic discount factor with restriction 1.

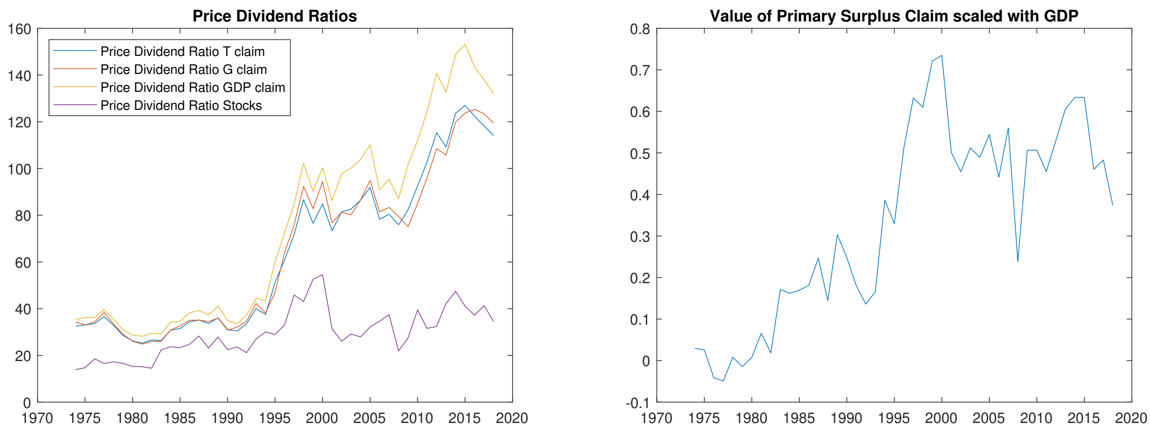

Figure 17: The left figure shows the price dividend ratios for the tax claim, government spending claim, GDP claim and dividend claim (stock) over time. The right figure shows the value of the primary surplus claim scaled with GDP. These results are based on the estimated stochastic discount factor with restriction 2.