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Support for Small Businesses amid COVID-19

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Abstract

A sizeable proportion of enterprises, especially SMEs, assisted by the government, will fail to repay. Should a screening mechanism then be applied to deter those most likely to default from seeking such financial assistance? The answer depends on the relative weights attached to the competitive objectives of stabilisation and allocative efficiency. For this purpose, we develop a two-sector equilibrium model featuring oligopolistic small businesses with asymmetric private information and a screening contract. The sector adversely affected in a pandemic can apply for government loans to reopen later. A pro-allocation government sets a harsh default sanction to deter entrepreneurs with bad projects thereby improving productivity in the long run, at the cost of persistent unemployment, whereas a pro-stabilisation government sets a lenient default sanction. The optimal default sanction balances the trade-off between allocation and stabilisation. Finally, we solve for the optimal default sanction numerically and conduct comparative statics.

Keywords: COVID-19, government guarantees, optimal default sanction, unemployment, productivity, adverse selection, private information, screening

JEL Codes: D82, E44, G38, H81

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1 Introduction

The combination of the Coronavirus pandemic and the policy measures of lockdown and quarantine introduced in response has had a drastic effect on the cash flows and solvency of businesses in the many countries affected, in particular on small, and medium-sized, enterprises (SMEs), who generally have had less own resources and no access to external finance, except through their banks. Fairlie (2020), for example, estimated that "the number of active business owners in the United States plummeted by 3.3 million or 22 percent over the crucial two-month window from February to April 2020. The drop in business owners was the largest on record, and losses were felt across nearly all industries and even for incorporated businesses."[1]

So, if the country was to avert an economic collapse, with a large proportion of its SME population being forced to shut up shop, the need was to get external financial assistance to them, and quickly. Given the massive numbers of SMEs in any country, approximately 5.9 million in the UK, over 99% of all businesses,[2] this could hardly be done, at least not quickly enough, directly from a government office, since they were not set up to do this; rather it had to be done via the existing relationship between SMEs and their main bank.

But a problem is that SMEs have a relatively high failure rate and are notoriously liable to fail to repay the due amount on their borrowing, principal and interest, becoming non-performing loans (NPLs) on bank books. The likelihood of credit extension to SMEs transforming into NPLs in the aftermath of the Coronavirus pandemic is, obviously, even greater, given the manifold uncertainties and changes to conditions and behaviour that the pandemic, and the policy response, have generated.

So if the banks themselves were to be left carrying the can for any significant share of the losses arising from the NPLs on such loans, they would have wished to be extremely careful in monitoring and checking which SMEs would be provided with such emergency credit, and which would be refused. Such monitoring, however, takes time and effort. Moreover, the banks might be more conservative in their own interests than would be socially, or politically, desirable.

For all these reasons, in the UK the government then decided that such emergency loans to SMEs, known as 'Bounce back Loans', (BBLs), would henceforth be 100% guaranteed by the government, i.e. that they would not count as NPLs or cause losses to the banks.[3] As a result there was no reason for the banks not to provide all-comers, who were demonstrably prior SMEs on their books, with loans immediately on the occasion of being asked. Nor was there any real likelihood that such credit expansion would be constrained by (regulatory) capital or liquidity requirements. Quantitative Easing (QE) had made, and would increasingly make, liquidity in fulsome supply. Since credit expansion in the face of the pandemic emergency was the government's intention, any shortage of bank capital that might occur would be offset, either perhaps by relaxing the rigour of the regulations and/or by restricting bank pay-outs in buybacks, dividends, and, even perhaps, in executive remuneration.

There is an upper limit of such BBLs, £50,000 per claimant. Such loans carry a fixed interest rate of 2.5 percent. Given these attractive terms there has, not surprisingly, been

[1] Also see Rana Foroohar (2020), 'Small business: a canary in the US economic coal mine', *Financial Times*, June 28.

[2] According to UK Parliament business statistics in 2019.

[3] Although the scheme requires lenders to pursue defaulters before calling in the government guarantee, it is difficult to believe that they will spend much time or effort in doing so.

a rapid and huge take-up, amounting to £28 bn as of end June, (*Sunday Times*, June 28, Business Section, p. 1, article by Peter Evans). Apart from the normal costs of bankruptcy, there are no special, or additional, sanctions on such borrowers who then default. In view of the massive uncertainties of the current situation, many might think it worth taking the gamble of borrowing the money, in order to see whether they might have a chance of being successful in the eventual recovery, even when they have private information that such chances are slim, and that they are more likely than not to fail.[4]

The government guarantees and favourable contractual terms have triggered a series of public comments on the likely massive defaults on the UK's emergency loans.[5] But why does it matter that many of these BBL loans will never get repaid, and that attempts to chase up defaulters will involve much cost and effort? One reason, obviously, is that it will add to future public sector debt and deficit, though one has to net off the increased revenue that such extra expenditure will provide in the short run. But there is another reason for concern, which is the likely misallocation of resources that will come from keeping alive enterprises that are sub-par, even zombie companies.[6]

Assuming that the government puts some weight on allocative efficiency, in order to raise productivity and output over future years, (as well as the desire to maintain current employment and output in the face of the pandemic and the lockdown), it will need to try to screen out unprofitable, and less profitable, potential borrowers. Recall that it has withdrawn this role from banks in pursuit of a swifter disbursement of funds, via 100% guarantees.

There are two ways of doing such screening, ex ante and ex post. Under ex ante screening, the government will only lend to SMEs with a proven record of past profitability. This works well if the future is going to be like the past; but one general assessment is that the pandemic has greatly changed the prospective conjuncture, so that future patterns of behaviour may differ a lot from those in the past. A second objection to ex ante screening is that it would generate many hard cases and, therefore, could evoke considerable political and social opposition initially. For example, the criterion that an SME has to be able to show profits in each of its last two accounting years would exclude an enterprise with huge profits in one of those years and a tiny loss in the other.

The second, ex post, form of screening involves the government imposing an additional (pecuniary) penalty on those failing to be sufficiently profitable to pay back the loan in full. This would work better if the potential borrowers had reasonably good (private) information on whether they were likely to succeed in the changed conjuncture of the recovery from the pandemic. This is the condition that we assume in our model in this paper. Even if this condition holds, it does, of course, have some further disadvantages.

[4]There may even be some who intend to divert such funds entirely to their own consumption, hoping to evade attention in the confusion and mass defaults that will ensue. But this is fraud, and should be pursued and constrained by the usual processes.

[5]See, for example, Andy Bounds and Daniel Thomas (2020), 'UK's emergency loans for small companies likely to bring rash of defaults, say bankers', *Financial Times*, May 10; Stephen Morris, George Parker, and Daniel Thomas (2020), 'UK banks warn 40%-50% of 'bounce back' borrowers will default', *Financial Times*, May 31; Jill Treanor (2020), 'RBS boss Sir Howard Davies calls for toxic coronavirus loans fund-Chairman warns of mass defaults by stricken small firms' *The Sunday Times*, May 3.

[6]See for example, Daniel Thomas and George Parker (2020), 'UK bailout schemes could create coronavirus debt trap, warn banks', *Financial Times*, May 25. Similarly, Douglas Elliott, of Oliver Wyman, wrote in April 2020 in a paper 'Top 5 Concerns about Policy in the New Era', "Right now, governments are making, guaranteeing, or encouraging very large sums of loans with weak or non-existent credit underwriting. That may be the only choice at the moment, but it is not a long-term solution. Propping up companies that cannot survive in the longer run without continuing support becomes a real drag on an economy."

Unsuccessful borrowers could try to avoid the extra penalty by moving abroad or hiding their income, as with student loan repayment, where the pay-back is only a fraction of the outlay. In a sense, penalizing the unsuccessful is akin to kicking a person when they are already down; so, while ex post screening/sanctions would generate less political opposition initially, it would, probably, have more so afterwards.

Both ex ante, and ex post, screening have disadvantages. Possibly partly for such reasons, the UK government decided not to do any such screening on its BBL scheme. It is now, almost certainly, too late to reverse that decision, since that would represent a retroactive adjustment to the scheme's conditions. But our concern in this paper is rather to assess the normative issue of whether, and what degree of, (ex post) screening would have been socially optimal, rather than to propose any positive change to current policies. Bygones are bygones, and policy had to be made under extreme pressure in the heat of the moment.

To provide a normative perspective on the social optimality of screening for the government loan schemes to support SMEs, we develop a two-sector equilibrium model of oligopolistic small businesses in the presence of a pandemic shock. The COVID-19 pandemic shock forces the adversely shocked sector to close while the other sector remains open. The government provides bounce back loans to the adversely shocked sector to reopen after the pandemic. Potential entrepreneurs that apply for government loans to reopen businesses have private information about their profitability. Those with lower profitability are likely to default on government loans. The government can choose to implement a default sanction to ameliorate such adverse selection.

Note that the only default penalty we consider is modelled as a monetary deduction from the defaulter's residual income, and the defaulter is still allowed to continue her business should she wish. And we interpret this default sanction as the government requiring the borrowers to provide a certain level of personal guarantee.[7] Thus, throughout the paper we do not consider formal bankruptcy procedures or liquidating the firms' businesses as the default punishment. The reason is that formal bankruptcy procedures are rarely pursued when the lender is the government whose intention is to provide credit for small businesses to retain workers during crises. Indeed, large-scale formal bankruptcy procedures or liquidating the small firms' businesses in the case of default amid a pandemic crisis would be considered too harsh and seen as a massive PR disaster for the government. The implication is that a lenient default sanction attracts unprofitable projects which may remain in existence for a long time.

If the government is pro-allocation, we show that the government can choose to implement a harsh default sanction to deter unprofitable potential entrepreneurs from applying for loans and reopening businesses, but this pro-allocation policy leads to unemployment in the short run, and demand shortage is persistent. If the government is pro-stabilisation, we show that the government can choose to implement a lenient default sanction or even no sanctions (i.e., 100% guarantees) to restore short-run full employment once the pandemic has passed. In this case, demand shortage is short-lived, but the economy is shifted to a lower long-run equilibrium due to misallocation. Moreover, we develop an analytic measure "Stabilisation Proclivity" to characterise the conditions under which the government is likely to be more pro-allocation or pro-stabilisation. We establish which are the key

[7] Note that we are not modelling publicly-listed big corporations, which are typically protected by limited liability in case of formal bankruptcy procedures, despite the social cost of bankruptcy. Instead, these are SMEs, many of which are sole traders not protected by limited liability. The government would spare them the formal bankruptcy procedures, so the government is unlikely to shut down the SMEs in case of default. However, due to the government's unique ability to tax, it can require the SME borrowers to provide a certain level of personal guarantee, so that in case of default on a particular loan programme, a fine is implemented while the business is still allowed to carry on.

parameters that will determine whether the government would want to be lenient, or tough, in trying to screen out the potentially less successful borrowers. These parameters include:-

a) The degree to which the government's monitoring/verification scheme works well;

b) The extent of market power of the entrepreneurs;

c) The project return of tail of really low profitability potential borrowers;

d) The discount rate to future outcomes being applied.

The higher is (a), (c) and (d) and the lower (b), the more the government will give a higher weight to current stabilisation, and the less, even none at all, to the efficient allocation objective.

We then provide numerical examples to assess the social optimum, and show how the optimal default sanction varies as these parameters change. We show that the optimal default sanction is intermediate. It is neither a harsh default sanction that only attracts the profitable businesses and rules out default completely, nor a lenient default sanction that attracts a large number of borrowers to restore full employment rapidly but leads to massive defaults.

We first vary the discount rate and show how the optimal default sanction changes accordingly. What we conclude is that society would have to be really rather extremely myopic, a very high rate of time preference, to eschew entirely the option of screening out borrowers with poor profitability prospects. In a crisis governments tend to be myopic. Act in haste, repent at leisure. But extreme pressure does cause extreme myopia.

Then, we conduct comparative statics on the optimal default sanction by changing the market power of the entrepreneurs, the quality of monitoring technology, and the overall quality of the inefficient entrepreneurial pool. Our findings are as follows. When the market power of the entrepreneurs decreases, it causes a noticeable shift of the optimal default sanction to a more lenient stance. An improvement in the overall quality of the inefficient entrepreneurial pool implies a more lenient stance. Particularly, when we improve the quality of the monitoring technology, it leads to a more lenient default sanction as the social optimum. What could justify the government's most generous credit guarantees for its loan support to SMEs is a near-perfect monitoring technology. Nevertheless, lenders are unlikely to conduct meticulous monitoring given the scale and the speed of this crisis.

As the pandemic crisis is likely to leave a long-run scar, we extend the model to consider a long-run scarring effect. To this end, we assume the short-run unemployment causes structural unemployment in the long run. Our numerical results show that, compared with the benchmark, the optimal default sanction becomes more lenient. The government in this case cares more about short-run stabilisation rather than allocative efficiency.

Finally, we investigate the implication of the interest rate Effective Lower Bound (ELB) for the optimal default sanction. This is relevant because currently, the interest rate is at an ultra-low level in major advanced economies. We show that when the monetary policy is constrained from decreasing the interest rate further, a harsh default sanction that causes persistent demand shortage even leads to involuntary unemployment in the sector that remained open during the pandemic. This is due to the aggregate demand externality of a multi-sector economy. Thus, at the interest rate ELB, our numerical results show that the optimal default sanction becomes more lenient than its counterpart when the interest rate is unconstrained. If the government fails to provide unemployment insurance, the aggregate demand externality is exacerbated, then the government is likely to take an even more lenient stance.

The remainder of the paper is structured as follows: In Section 2 we review the extremely rapidly growing associated literature on the effects of the pandemic on the economy, especially in relation to the objectives of stabilization and allocative efficiency. Then in Section 3 we set out a model, simplified as far as possible, in which we aim to explore the effects of introducing an (ex post) screening contract into a government scheme for financing companies adversely affected by enforced closure, to allow them to reopen, on the twin objectives of stabilization and allocative efficiency. Section 4 provides equilibrium characterisation and analysis. Section 5 solves for the optimal default sanction and conducts comparative statics. Section 6 concludes.

2 Related Literature

Amid the COVID-19 pandemic, a new literature has quickly sprung up. A large number of real-time papers on COVID Economics have been complied by the CEPR, and various policy proposals have been collected in Baldwin and Weder di Mauro (2020). A growing number of the recent papers integrate epidemiological SIR or SIER models of contagion in economic settings. A non-exhaustive list includes Atkeson (2020), Berger, Herkenhoff and Mongey (2020), Bethune and Korinek (2020), Eichenbaum, Rebelo, and Trabandt (2020), Kaplan, Moll, and Violante (2020), Krueger, Uhlig, and Xie (2020), and Rampini (2020). Another set of papers focus on the role of government policies, banks' liquidity provision, the economic nature of the pandemic shock, and their impact on aggregate demand and supply as well as welfare (see for example an incomplete list of Hagedorn and Mitman (2020), Elenev, Landvoigt, and Van Nieuwerburgh (2020), Faria-e Castro (2020), Fornaro and Wolf (2020), Gonzalez-Uribe and Wang (2020), Guerrieri, Lorenzoni, Straub, and Werning (2020)), Kahn and Wagner (2020), Philippon (2020), and Segura and Villacorta (2020). Gonzalez-Uribe and Wang (2020) provide empirical evidence on the UK government loans and guarantees after the Global Financial Crisis and draw lessons for the most recent government loans and guarantees for small businesses going through the COVID-19 crisis. Li, Strahan, and Zhang (2020) empirically show that US banks faced the largest increase in liquidity demands, and they suggest banks were able to meet the demand partly due to the Fed's liquidity provision. Guerrieri, Lorenzoni, Straub, and Werning (2020) build a multi-sector infinite horizon model and show how a negative supply shock due to shutdown can translate into a demand shortage. As we shall shortly discuss, we combine the model in Guerrieri et al. (2020) with a screening contract to study the adverse selection issue relating to government loans and guarantees to expound a normative theory on how the government should set the contractual terms for its loan support to small businesses.

Notably, our paper relates to the group of COVID-19 literature on public liquidity provision (see, for example, Kahn and Wagner (2020), Segura and Villacorta (2020), Philippon (2020), and Elenev, Landvoigt, and Van Nieuwerburgh (2020)). Kahn and Wagner (2020) develop a theory to underpin the conditions under which direct provision of liquidity is preferable to the traditional distribution of liquidity, and vice versa. The main trade-off there is between externalities and informational advantages. In a similar spirit but with a different friction, Segura and Villacorta (2020) analyse different types of government interventions to support firms and develop a pecking order between direct transfers and indirect support through guarantees to new loans or reductions in the capital requirement. The critical friction in their paper is the moral hazard due to the borrower's unobserved effort cost. Our paper differs from Kahn and Wagner (2020) and Segura and Villacorta

(2020) in that we focus on setting the contractual terms of government loans to ameliorate adverse selection while endogenising its impact on persistent unemployment, which is not present in Kahn and Wagner (2020) and Segura and Villacorta (2020). More specifically, in Segura and Villacorta (2020), the optimal policy is derived from the deployment of fundings through the workings of financial intermediaries to reduce moral hazard. However, our paper focuses on adverse selection and re-entry frictions, and the optimal policy stems from the trade-off between allocation (reducing agency cost) and stabilisation (reducing unemployment). Another distinction between our paper and these two papers is that we model multiple goods and sectors, offering an additional perspective on the aggregate demand externality and its interaction with optimal policy.

Philippon (2020) analyses how government interventions can improve efficiency when the decentralised economy amid COVID-19 is distorted by wage rigidity. Sharing a similar spirit, our paper complements Philippon (2020). While both papers focus on the information asymmetry between the borrowers and the government, our paper differs in the following distinct ways. First, the key trade-off in our paper is between short-run unemployment stabilisation and long-run allocative efficiency, whereas Philippon (2020) presents a one-period model and hence does not highlight the short-run and long-run angle. Second, the short-run unemployment in our paper is a direct result of the government's default sanction, but the unemployment in Philippon (2020) arises due to downward wage rigidity. Third, whereas Philippon (2020) focuses on competitive equilibrium, we choose to model oligopolistic businesses, because our focus is on SMEs, which possess a certain degree of market power. Finally, Philippon (2020) assumes private creditors can observe firms' project returns, so that the government can implement optimal policy based on the government's observation of private creditors' action. Our paper takes the view that private creditors, e.g., banks, would not be able to monitor borrowers or acquire information. This is not just due to the scale and speed of the crisis, but also because the government's credit guarantees would leave little incentive for banks to make efforts to monitor and acquire information.

Relatedly, Elenev, Landvoigt, and Van Nieuwerburgh (2020) build a structural model calibrated with the US data to evaluate three government policies aimed at short-circuiting the interplay between corporate defaults and banking fragility. They find that the government loan schemes in the US are preventing the bulk of firm bankruptcies. Our paper differs from Elenev et al. (2020) in that we explicitly consider the agency problem between the government and the borrowing entrepreneurs or firms who possess private information. Moreover, we consider the potential adverse effect of government loan schemes and guarantees that cause zombification and the drop in long-term productivity. In Elenev et al. (2020), the authors consider conditional loan bridge schemes which require the government to possess information on the default probability of the borrowers. The implicit assumption is that the government is able to acquire information about the firms' profitability quickly. Given the scale and the speed of the pandemic crisis, we think such information acquisition may be difficult to carry out in practice and hence consider a different approach. In our paper, the government simply designs the contractual terms of government loan schemes as a screening device that can deter unprofitable entrepreneurs from applying for loans in the first place, as a way of reducing subsequent default.

Furthermore, our paper emphasises and explicitly models the policy trade-off between stabilisation and allocation. On stabilisation, Oi (1962) shows that if labour has a high degree of fixity, firms would be better off to maintain their labour force rather than to risk high replacement demands in the future. In a similar spirit, Caggese (2007) finds that with negative productivity shocks, a financial constraint binds which reduces variable capital and leads to inefficient productions. This underscores the importance of favourable government loans to support firms in the downturn. Barrero, Bloom, and Davis (2020) estimate that that COVID-19 shock caused 3 new hires for every 10 layoffs, that 32-42% of COVID-induced layoffs will be permanent, suggesting a slow absorption of labour into new jobs. Fairlie (2020) also provides timely and early evidence on the impacts of social distancing restrictions and demand shifts from COVID-19 on small businesses. In contrast, on allocation, Blattner, Farinha, and Rebelo (2019) show that cheap credit and zombification caused a decline in productivity. Acharya, Crosignani, Eisert, and Eufinger (2020) document the effect of cheap credit and zombification on firm markups and inflation. Acharya, Eisert, Eufinger, and Hirsch (2019) show that the ECB's Outright Monetary Transactions programme induced zombie lending by banks, Schivardi, Sette, and Tabellini (2017) also provide evidence in Italy. Similarly, using French data Lelarge, Sraer, and Thesmar (2010) show that government loan guarantees significantly increase the firms' probability of default. D'Acunto, Tate, and Yang (2018) study the US case and detect underperformance of firms given that the government assumes most of the costs of default. Jordà, Kornejew, Schularick, and Taylor (2020) provide long-run evidence on the economic costs of corporate debt booms and inefficient debt restructuring. Our paper contributes to these two groups of literature by presenting a model on how to balance the policy trade-off between them.

In terms of modelling, we build our model along the lines of the multi-sector case of Guerrieri, Lorenzoni, Straub, and Werning (2020)). In both papers, the pandemic shock is modelled as a shutdown of the adversely affected sector while the other sector stays open. Borrowing the insight from Guerrieri et al. (2020), we also make use of the demand shortage and aggregate demand externality the shutdown of one sector can cause the other sector if the two sectors produce complements.

However, our paper departs from Guerrieri et al. (2020) in several important dimensions. First, unlike Guerrieri et al. (2020) who focus on competitive equilibrium, we model oligopolists in a Cournot equilibrium so that entrepreneurs make positive profits, which play a critical role in our design of the default sanction as a screening contract for entrepreneurs' profitability. Second, our main friction is the adverse selection stemming from the reentering entrepreneurs' private information, which is not present in Guerrieri et al. (2020). Third, the main focus and key research questions are both different. The central question in Guerrieri et al. (2020) is under what conditions a negative supply shock can cause aggregate demand shortage, while in our paper, we are specifically investigating government loans and guarantees that support small businesses during a pandemic. This is a policy issue in which the trade-off between reducing unemployment and reducing the agency cost of adverse selection naturally emerges. Moreover, the theoretical contribution of our paper is to combine insights from the industrial organisation (IO) literature (see Shapiro (1989)) with financial contracting, which we shall shortly discuss.

Finally, given the information friction in our model and the use of a screening contract, our model connects with the literature on private information and screening that follows a rich tradition (see Rothschild and Stiglitz 1978, Stiglitz and Weiss (1981), and Bester (1985) as classic examples, and more recently Dubey and Geanakoplos (2002) and Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019)). Particularly, Boot, Thakor, and Udell (1991) find that private information increases collateral usage. However, entrepreneurs in our model do not pledge collateral because their primary reason to borrow loans is to retain workers, and employees cannot be used as collaterals. Moreover, amid COVID-19, the government's loan schemes charge exceptionally low interest rates. This environment intensifies the adverse selection associated with the entrepreneurs' private information, and hence, we design the default sanction as a screening device.

Also related is the vast literature on financial contracting, notably with the agency costs between bondholders and shareholders in many classic papers (see for example Jensen and Meckling (1976), Fama (1978), and Smith Jr and Warner (1979)). The agency issue in our model exists between the lender, i.e., the government, and the borrowing entrepreneurs who have private information. We reduce the agency cost by having the government implement a default sanction, which can be interpreted as the personal guarantee the borrowing entrepreneurs need to provide. Thus, the default sanction plays a similar role as bond covenants that serve to mitigate the agency issues as widely analysed in the financial contracting literature.

Unlike the abovementioned literature on private information, screening, and financial contracting, our concern is not only on designing the financial contract to reduce the agency cost, but also about the unintended and undesirable consequences of reducing the agency cost in a macroeconomic setting. After all, the focus of our paper is on how to trade off the pandemic-induced stabilisation issues as a result of reducing the agency cost against long-run productivity. We endogenise the social cost of reducing the agency costs as the near-term surge in unemployment and reduction in production and bring to the forefront the policy trade-off between reducing near-term unemployment and increasing long-term productivity.

3 Model

3.1 Environment

The economy has three types of agents: entrepreneurs, workers, and the government. It lasts for three periods (t=0,1,2). Date $t = 0$ is when the pandemic occurs, date $t = 1$ refers to the short run, and date $t = 2$ is interpreted as the long run. There are two sectors, sector I and sector J, that produce different goods. Each sector has N entrepreneurs who hire workers to carry out a project to produce sector-specific goods. Entrepreneurs in each sector form an oligopoly and thus have market power in setting prices. There are Q workers in the economy, and $\frac{Q}{2}$ workers specialise in each sector. Each worker is endowed with 1 unit of labour supplied to an entrepreneur inelastically. Each entrepreneur's maximum production capacity is to employ $\frac{Q}{2N}$ workers. In the absence of the pandemic shock, full employment would be achieved in both sectors. Both entrepreneurs and workers consume goods from both sectors and exhibit the same constant elasticity of substitution preferences. Note that we abstract away physical capital investment for productions. The reason is that during crises, governments typically provide loans to small businesses to retain workers rather than fund investments. Market loans for capital investment are relatively accessible since capital, unlike workers, can be pledged as collateral, which may obviate the need for government loans (see Gonzalez-Uribe and Wang (2020)).

In times of pandemics, e.g., COVID-19, sector I is adversely shocked and forced to close. The government provides bounce back loans to the adversely shocked sector to reopen after the pandemic. If the borrowing entrepreneurs default, the government can choose to implement a default sanction. If the government chooses zero default sanctions, then the government assumes all the cost of default and is said to provide full guarantees for the bounce back loans. Figure 1 outlines the flow of funds of the economy.

Figure 1: Flow of funds

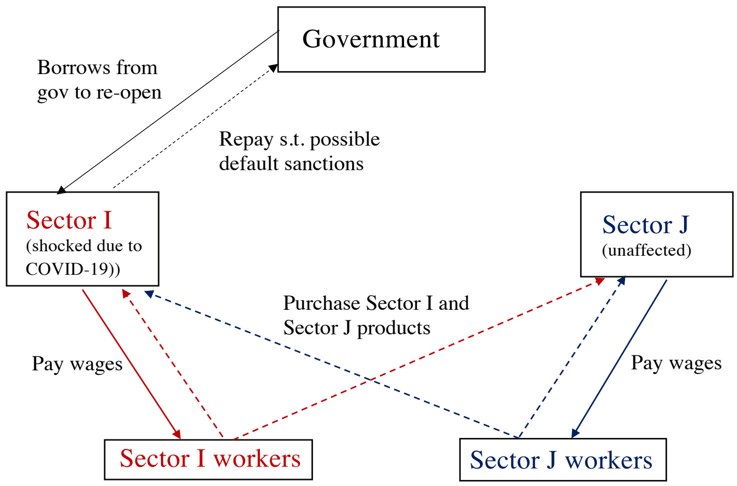
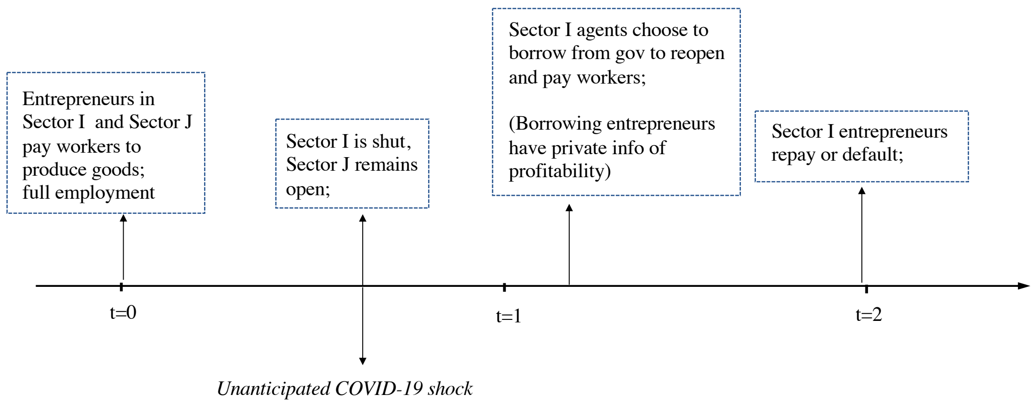


Figure 2 illustrates the timeline, and we divide each of the first two dates into two sub-periods. At the start of $t = 0$, entrepreneurs in sector I use their previous sales revenues to pay wages for the labour hired for production in period $t = 0$. Then, an unanticipated short-lived pandemic shock occurs at the end of $t = 0$ and hits sector I. Sector I is forced to close while sector J remains open. Thus, at the end of $t = 0$, the entrepreneurs in sector I have not produced anything and become workers, and the economy's spending falls onto sector J.

At the start of $t = 1$, the pandemic has passed. The prior entrepreneurs can choose to reopen businesses or choose to remain as workers earning wage income. If they choose to reenter, they need to borrow government loans to pay wages at the start of $t = 1$. The prior entrepreneurs have projects with different returns and they have private information on these projects. Those with lower profitability are more likely to default at the end of date $t = 1$.

Given the scale and the speed of the pandemic, we assume that the government is unable to acquire information on the entrepreneurs' profitability in time. However, the government may implement, should it wish, a sanction if the borrowing entrepreneurs default. As one might expect, if the government implements a lenient default sanction or no sanctions at all, inefficient entrepreneurs with unprofitable projects may try to borrow and reenter. If the government implements a harsh default sanction, it is possible to deter those with low profitability, but this policy may increase the unemployment rate. We will discuss the policy trade-off in more detail shortly after we set up the model formally.

Figure 2: Timeline



3.2 Entrepreneurs

We first characterise the entrepreneurs' optimisation in the absence of the pandemic shock. In the absence of the pandemic shock, the entrepreneurs simply solve for a static profit maximisation problem while taking into account their price impact.

Let,

$W_E(n) \equiv$ the nominal wage paid by entrepreneur n in sector E, and E $\in$ {I,J},

$P_E \equiv$ the price of goods in sector E,

$q_E(n) \equiv$ the quantity of goods produced by entrepreneur n in sector E,

$q_E \equiv$ the total quantity of goods in sector E,

$h_E(n) \equiv$ the labour demand by entrepreneur n in sector E,

and production technology is given as $q_E(n) = \sigma h_E(n)$. We call σ the project return, and $\sigma = 1$ in the absence of the pandemic shock.

Formally, entrepreneurs maximise the profits of $\Pi_E(n)$ subject to the production technology as below. Because entrepreneur n has market power, they take into account their impact on prices. Indeed, entrepreneurs in each sector act oligopolistically. Therefore, in their objective function, the price P_E, i.e., $P_E\big(q_E(n) + \sum_{m\neq n} q_E(m)\big)$, is a function of the quantity.

$$\underset{q_E(n),h_E(n)}{Max} \quad \Pi_E(n) = P_E\big(q_E(n) + \sum_{m\neq n} q_E(m)\big)\Big(q_E(n)\Big) - W_E(n)\Big(h_E(n)\Big),$$

subject to

$$q_E(n) = \sigma h_E(n).$$

We introduce ϵ_E as the price elasticity of demand, and by definition, it is expressed as follows,

$$\epsilon_E = -\frac{P_E}{q_E}\frac{\partial q_E}{\partial P_E}.$$

As we assume constant elasticity of substitution preferences, ϵ_E is the same across goods, and we shall drop the indexing E hereafter ($\epsilon_E = \epsilon$). We focus on symmetric equilibria so that each entrepreneur sets $q_{Et}(n) = \frac{q_{Et}}{N}$. Entrepreneur n's optimality condition leads to

$$W_E(n) = \sigma(1 - \frac{1}{N\epsilon})P_E, \tag{1}$$

where we assume the parameter space $\epsilon > 1$.

Note that wages are sector-specific, so we now drop the indexing n in the wage notation for normal times. Substitute in $\sigma = 1$, it follows that $W_E = (1 - \frac{1}{N\epsilon})P_E$. In the subsequent equilibrium characterisation we normalise goods I price to 1.

Due to her market power, entrepreneur n's profits are positive. Given the maximum hiring capacity is $\frac{Q}{2N}$, entrepreneur n employs the labour of $h_E(n) = \frac{Q}{2N}$, so we can now drop the indexing for labour and denote it as h instead. Entrepreneur n's profits $\Pi_E(n)$ can be reexpressed as

$$\Pi_E(n) = \frac{q_E(n)}{N\epsilon} = \frac{h}{N\epsilon}.$$

3.2.1 Pandemic shock

We now turn to an unanticipated short-lived pandemic shock that occurs at the end of $t = 0$. At the start of $t = 0$, entrepreneur n in sector I uses her previous sales revenues to pay wages for the labour hired for production in $t = 0$. Then, after the pandemic shock, sector I is forced to close while sector J remains open. Thus, at the end of $t = 0$, entrepreneur n has not produced anything and becomes a worker.

At the start of $t = 1$, the pandemic has passed. The prior entrepreneurs totalling N can choose to reopen the businesses and rehire labour for production. Moreover, we assume a new set of workers with new ideas totalling a fraction ϕ of industry cap can also enter and become entrepreneurs. Since sector I was completely shut at $t = 0$, the industry gap at $t = 1$ is simply N. The parameter ϕ reflects industry dynamism. To motivate the long-run scarring of the pandemic, we do not force the industry gap to close at $t = 2$. In the very long run ($t = 2$), if there is still industry gap, i.e., $N > M$, where M denotes the number of reentering entrepreneurs at $t = 1$, then $\frac{1}{\kappa}\phi(N - M)$ workers with new ideas could fill in the industry gap, where κ ($\geq \phi$) is assumed to be an increasing and concave function of short-run unemployment and it reflects long-run scarring of the pandemic: a large κ could result in insufficient new entries in the long run, and hence, long-run structural unemployment is a possibility.

In the short run, if this set of workers and the prior entrepreneurs choose to reenter, they need to borrow the government's bounce back loans of F_1 to pay for labour at the start of $t = 1$. Among the N prior entrepreneurs, a fraction of them totalling $(1 - \alpha)N$ have inefficient projects of different returns that are lower than the pre-pandemic level (i.e. $\sigma(n) < 1$). The rest of these prior entrepreneurs (αN) and the normal set of workers with new ideas (ϕN) are profitable; they have good projects of the same return as the pre-pandemic level. We assume $\phi + \alpha < 1$ to reflect that the pandemic worsens the overall profitability of the adversely shocked sector in the short run. When the fraction of prior entrepreneurs with inefficient projects manage to obtain government loans and reopen businesses, aggregate productivity will drop due to misallocation, i.e., inefficient production. The issue at hand is to design a mechanism that can deter borrowers with inefficient projects while taking into account the impact on unemployment.

Specifically, if the prior entrepreneurs decide not to reenter, they stay as workers and receive wages. If they reenter at the start of $t = 1$, they carry out a project that produces output as $\sigma(n)h_1$. And $\sigma(n)$ is the project return of entrepreneur n, which represents her profitability. For the profitable entrepreneurs, their project return is equal to 1, the same as the pre-pandemic level. For the unprofitable entrepreneurs, we assume their project return $\sigma(n)$ follows a uniform distribution $\sigma(n) \sim \mathcal{U}(\sigma_B, \bar{\sigma})$, where,

$$\begin{aligned} \bar{\sigma} &= 1 - \frac{1}{N\epsilon}, \\ \sigma_B &= \frac{2N - 2/\epsilon}{(1-\gamma)Q}. \end{aligned} \tag{2}$$

As we shall show in the equilibrium characterisation, the parameters σ_B and $\bar{\sigma}$ ensure the following. In the absence of sanctions, the entrepreneur with the lowest profitability among the inefficient entrepreneurs will try to reenter, borrow government loans, and pay wages for production, and after production, her low revenues are simply insufficient for repayment. The entrepreneur with the highest profitability among the inefficient entrepreneurs will try to reenter, borrow government loans, and pay workers for production, but after production, her revenues are just enough to repay the loan obligations.[8] However, as we shall explain shortly, because she can divert a fraction of her funds due to an imperfect monitoring and verification technology, she will default on the government loans nevertheless.

Importantly, $\sigma(n)$ is the private information of the reentering entrepreneurs. They set the nominal wage to be the same as in $t = 0$, i.e., $W_{I1} = W_{I0}$. Consequently, *ex ante* different types of reentering entrepreneurs are indistinguishable. We assume limited pledgeability of output, so when the project return is realised at the end of $t = 1$, if the entrepreneur defaults, the government only takes away a fraction γ of the entrepreneurs' remaining funds. We interpret γ as the quality of the government's verification or monitoring technology.[9]

First, we characterise entrepreneur n's default decision assuming zero sanctions from the government. When the project return is sufficiently high, so that if entrepreneur n were to declare default after production, the amount of funds the government garnishes would be higher than or equal to the loan obligations, i.e., $\gamma\sigma(n)h_1 \geq F_1$, then the entrepreneur would choose to repay fully. When the project return is sufficiently low that the revenues are insufficient to repay loans, i.e., $\sigma(n)h_1 < F_1$, she is simply unable to repay the loans and has to default after production. However, there is an intermediate region for the project return that the entrepreneur will default even when she has enough revenues. We call this type of default *strategic default*. With a relatively high project return, even if the revenues are large enough to repay loans in full, if the pledged amount is low that the government takes away a relatively small amount of funds in the case of default,

[8] In our numerical examples, we also vary $\bar{\sigma}$ and σ_B widely for robustness check.

[9] In reality, usually the banks perform the tasks of pursuing delinquent borrowers. An increase in γ can mean a better incentive for the banks to monitor borrowers' creditworthiness or stronger creditors' protection in the case of debt restructuring.

i.e.,$\gamma\sigma(n)h_1 < F_1 \leq \sigma(n)h_1$, then the entrepreneur will nevertheless default strategically. Let $\mathbb{I}$ be the indicator for default, i.e., $\mathbb{I} = 1$ means default and $\mathbb{I} = 0$ means repayment. Lemma 1 summarises the endogenous choice of default.

Lemma 1. (default decision): Assuming zero default sanctions and conditional on reentering, (3) summarises the decision to default.

$$\mathbb{I} = \left\{ \begin{array}{cc} 0, & \text{if } F_1 \leq \gamma\sigma(n)h_1 \\ 1 \ \ \text{(strategic)}\ , & \text{if } \gamma\sigma(n)h_1 < F_1 \leq \sigma(n)h_1 \\ 1, & \sigma(n)h_1 < F_1 \end{array} \right\}. \tag{3}$$

Lemma 1 indicates that the higher the project return $\sigma(n)$, the lower the likelihood of default. To deter the inefficient entrepreneurs from borrowing and, in turn, reduce default, the government may implement a default sanction as a screening contract such that when the entrepreneurs default at the end of $t = 1$, a monetary deduction is taken from her residual income. Because the re-entering entrepreneurs have the outside option of being a worker earning wage income, the residual income of any defaulting entrepreneur is positive (no less than the outside option), and a monetary deduction is possible.

Let λ_1 be the sanction. At the start of $t = 1$, entrepreneur n forms conditional expectation of her proceeds and evaluate them against her outside option. Suppose entrepreneur n has reentered and suppose that she defaults and becomes a worker at the end of $t = 1$, she can divert a fraction of her revenues totalling $(1-\gamma)\sigma(n)h_1$ as her residual income. And due to the sanction, the total amount of money she will receive at the end of $t = 1$ amounts to $(1-\gamma)\sigma(n)h_1 - \lambda_1$. However, had she chosen not to reenter and instead remained as a worker at the start of $t = 1$, she would have received wages of $\mathbb{E}_1(W_{I1}|\mathbb{I}_d = 1, \lambda_1)$, where $\mathbb{I}_d = 1$ indicates remaining as a worker. Therefore, if the following incentive constraint (4) holds, she will choose to apply for government loans and reopen businesses. In the case of the equality sign, we assume the entrepreneur chooses to reenter, and we call this entrepreneur the marginal entrepreneur,

$$\underbrace{(1-\gamma)\sigma(n)h_1 - \lambda_1}_{\text{default}} \geq \underbrace{\mathbb{E}_1(W_{I1}|\mathbb{I}_d = 1, \lambda_1)}_{\text{outside option}}. \tag{4}$$

The left-hand side of the incentive constraint (4) states the gains if the entrepreneur defaults, and the right-hand side is the value of her outside option. As (4) is conditional on her choosing to default, her benefits of repayment must be smaller or at most equal to her gains in the case default, so (5) must hold.

$$\underbrace{(1-\gamma)\sigma(n)h_1 - \lambda_1}_{\text{default}} \geq \underbrace{\sigma(n)h_1 - F_1}_{\text{repay}}. \tag{5}$$

Note that as the government increases the sanction λ_1, the inefficient entrepreneurs with low profitability may be deterred from reentering, reducing the aggregate default. This relationship is proved to be monotonic in Proposition 1 after we define the equilibrium.

3.3 Government

The government's choice variable is the default sanction λ_1, and the government commits to the sanction. At the start of $t = 1$, the government provides loans of F_1 to each borrowing entrepreneur to reopen businesses,[10] where,

$$F_1 = \frac{W_{I1}Q}{2N}.$$

At the end of $t = 1$, some borrowing entrepreneurs default. Let df_1 be the total amount of default and Λ_1 be the total money collected from sanctions. The government uses the money collected via sanctions plus any borrowing if needed to cover default as in (6).

$$df_1 \leq B_1 + \Lambda_1. \tag{6}$$

If the money collected via default sanctions is insufficient to cover default, i.e., $df_1 > \Lambda_1$, then the government borrows money by issuing a one-shot undated consol of B_1 to the workers and entrepreneurs, and the government only pays the interest in future periods by raising an equivalent amount of taxation. As we show shortly after defining the equilibrium, the agents in the economy have sufficient savings to lend to the government after the pandemic. Note that if the government chooses to implement zero default sanctions, the government essentially provides full guarantees for its loan scheme. As the government increases the default sanctions, the government guarantees decrease accordingly.

3.4 Workers

Workers consume goods in both sectors. Let us label workers by $i \in Q$. Let $c_{It}(i)$ be the consumption of sector I goods, and $c_{Jt}(i)$ be the consumption of sector J goods. Their preferences are represented by the utility function

$$\mathbb{E}_t \sum_{t=0}^{2} \beta^t U(c_{It}(i), c_{Jt}(i)), \tag{7}$$

where

$$U(c_{It}(i), c_{Jt}(i)) = \frac{1}{1-\delta}\Big((\frac{1}{2})^{\nu} c_{It}(i)^{1-\nu} + (\frac{1}{2})^{\nu} c_{Jt}(i)^{1-\nu}\Big)^{\frac{1-\delta}{1-\nu}}. \tag{8}$$

The utility function satisfies $U' > 0, U'' < 0$, and it features constant elasticity of substitution (CES) $1/\nu$ ($\nu < 1$) between the two sectors' goods bundles and constant intertemporal elasticity of substitution $1/\delta$.

In our benchmark case, we assume workers have access to real, zero net supply, one-period bonds, paying interest rate r_t to share labour income risks. The approximate real-world mapping of this assumption often takes the form of the government's unemployment benefits and various types of tax transfers. Risk sharing provides analytical convenience without the loss of generality; moreover, it allows us to exclusively focus on the short-run and long-run trade-off of the government's support loan scheme due to asymmetric information without introducing additional frictions.[11] The policy question, therefore, is

[10]In practice, such timely and almost real-time government loan support involves the banking sector issuing inside money against an offsetting credit, with the government providing guarantees. Indeed, in response to the COVID-19 crisis, many governments worldwide have unveiled large-scale loan stimulus programmes through the banking system.

[11]Later on, when we model interest rate ELB, we discuss the policy implication of relaxing the full insurance assumption.

how much more the government should care about short-run stabilisation in addition to providing unemployment benefits.

First, we characterise the workers' maximisation. Due to the equal weighting of goods in CES preferences, exploiting symmetry, the relative goods price is 1 and wages are the same across sectors. Let $a_t(i)$ be the one-period bonds each worker holds, p_{It} be the relative price of goods I, and r_t be the interest rate.[12] Each worker i maximises (7) subject to the budget constraint

$$p_{It}c_{It}(i) + c_{Jt}(i) + a_t(i) \leq w_t h_t(i) + (1 + r_{t-1})a_{t-1}(i). \tag{9}$$

The optimality condition gives the Euler equation for consumption goods J. Let U_{c_J} be the partial derivative of U with respect to c_{Jt}. Given homothetic preferences we have Gorman aggregation, so the individual's marginal rate of substitution between goods equals the relative price, which is a macro variable. Hence, if the Euler equation holds individually, and it also holds for the group as in (10).

$$U_{\boldsymbol{c}_J}(\boldsymbol{c}_{It}, \boldsymbol{c}_{Jt}) = \beta(1 + r_t)U_{\boldsymbol{c}_J}(\boldsymbol{c}_{It+1}, \boldsymbol{c}_{Jt+1}). \tag{10}$$

In the absence of the pandemic, $\boldsymbol{c}_{It} = \boldsymbol{c}_{It+1}, \boldsymbol{c}_{Jt} = \boldsymbol{c}_{Jt+1}$, so the interest rate $r = 1/\beta - 1$.

When the pandemic shock hits at date $t = 0$, sector I shuts down, so $\boldsymbol{c}_{I0} = 0$. The economywide consumption falls onto sector J, and $\boldsymbol{c}_{J0} = \frac{Q}{2}$.

We define the natural interest rate in this context as the interest rate in the Euler equation in the hypothetical case that enough profitable entrepreneurs reopen at $t = 1$ with the pre-pandemic level of project returns and no private information. Thus, the natural interest rate is the interest rate when the economy operates as if in full potential from date $t = 1$ onwards.

Let r_0^* be the natural interest rate for date $t = 0$,

$$\begin{aligned} 1 + r_0^* &= \frac{1}{\beta}\frac{U_{c_J}(0, c_{J0})}{U^*_{c_J}(c_{I1}, c_{J1})} \\ &= \frac{1}{\beta}\frac{U_{c_J}(0, Q/2)}{U_{c_J}(Q/2, Q/2)} \\ &= \frac{1}{\beta}(\frac{1}{2})^{\frac{\nu-\delta}{1-\nu}}. \end{aligned} \tag{11}$$

Lemma 2. With complete markets, and given the pandemic shock, $r_0^* < 1/\beta - 1$, and the supply shock causes a demand shortage at date $t = 0$ *iff*

$$\nu > \delta. \tag{12}$$

[12] The interest rate is in terms of goods J, rather than the real interest rate obtained by deflating the nominal interest rate by expected inflation rate from the price index of the two types of goods. Since during the pandemic sector I goods are not traded, we cannot observe its price, nor can we measure such price index. Indeed, as estimated in Cavallo (2020), the official CPI does not reflect the rapid changes in prices in various sectors due to COVID-19. So, both for simplicity, and in line with the most current developments, the interest rate in our context refers to the interest payment in terms of goods J.

Lemma 2 directly follows from (11). It is an insight from Guerrieri et al. (2020). It states that when the two sectors' goods are complements, the supply shock due to the pandemic causes a demand shortage. In a single-sector economy, a negative supply shock typically increases the current-period marginal utility of consumption, which increases the natural interest rate. Because a negative supply shock causes a demand boom in a single-sector at $t = 0$, the interest rate increases to equilibrate the economy. However, in our environment and also in the multi-sector case of Guerrieri et al. (2020), (12) implies the two goods are complements, and interestingly, the natural rate decreases. The reason is that a negative supply shock decreases the current-period marginal utility of consumption due to goods being complements. So, the negative supply shock causes a demand shortage for goods in the unshocked sector, and the interest rate needs to decrease to equilibrate the economy.

If the interest rate is downward rigid, for example, at the Effective Lower Bound (ELB) where the interest rate cannot adjust downward sufficiently, it leads to a decline in the demand for goods J, and hence, involuntary unemployment in sector J. Indeed, akin to *Drèze equilibrium*, when prices are downward rigid, the supply is rationed (Drèze (1975)). This result, specific to a multi-sector economy, will have nuanced policy implications for designing the screening contract and setting the optimal default sanction.

3.5 Equilibrium

The two-sector equilibrium with imperfect competition is defined as an allocation with prices, given the screening contract λ_1 such that

(i) the non-price-taking entrepreneurs engage in Cournot competition and choose their actions simultaneously taking into account their price impact,

(ii) agents maximise subject to re-entry frictions, the incentive constraint, and budget constraints, and

(iii) goods markets, labour markets, and loan markets clear, and expectations are rational.

4 Equilibrium Characterisation

With our equilibrium definition, first of all, Lemma 3 shows that the agents in the economy have sufficient savings to lend to the government after the pandemic. Due to the pandemic shock, there is zero production in sector I and the economy is unable to spend all of its nominal income on goods J alone at the end of $t = 0$. Consequently, agents in the economy end up having extra money as savings at the end of $t = 0$ and carry it forward. As proved in Appendix (A), this amount of savings is more than enough to invest in the government's one-shot issuance of the undated consol.

Lemma 3. At the end of $t = 1$, agents in the economy have sufficient savings to finance the government's borrowing of B_1.

Proof. Appendix (A)

Now let us suppose the government only cares about resource allocation and sets the default sanction λ_1 sufficiently harshly to deter unprofitable borrowers. We derive the harsh default sanction λ_{A1} so that it is a Nash equilibrium for the entrepreneurs with profitable projects to reenter and apply for loans, while those with inefficient projects stay as workers. Then, we suppose the government wants to ensure full employment by setting a lenient default sanction. We solve for such lenient sanction λ_{B1} so that it is a Nash equilibrium for the profitable entrepreneurs along with a fraction of inefficient entrepreneurs to reenter and the rest stay as workers, while ensuring full employment is achieved from $t = 1$ onwards. To derive the expressions for the harsh and lenient default sanctions, we first need to check monotonicity holds, put differently, that the harsher the sanction, the higher the project return of the marginal entrepreneur and the lower number of defaults.

Let M $(\leq N)$ be the number of reentering entrepreneurs. We define $u_{I1}(M)$ as the unemployment rate in sector I at date $t = 1$ as follows, and Proposition 1 proves monotonicity.

$$u_{I1}(M) = 1 - \frac{\frac{QM}{2N}}{\frac{Q}{2} + N - M}.$$

Proposition 1. (monotonicity): Whenever $df_1 > 0$ and $M \leq N$, an increase in the default sanction λ_1 leads to higher profitability of the marginal entrepreneur and fewer defaults.

Proof. Appendix (B).

Given monotonicity, Theorem 1 that shortly follows derives the harsh default sanction λ_{A1} and the lenient default sanction λ_{B1}. A harsh sanction indicates a pro-allocation government, and a lenient sanction indicates a pro-stabilisation government.

Theorem 1. (screening, default, and unemployment):

A. Suppose the government sets the sanction $\lambda_1 > \lambda_{A1}$, where

$$\lambda_{A1} = (1 - \frac{1}{N\epsilon})\Big((1-\gamma)\frac{Q}{2N} - (1 - u_I(\phi N + \alpha N))\Big),$$

the $(\phi + \alpha)N$ good entrepreneurs will reenter and all the inefficient entrepreneurs will stay out, and no one defaults.

Sector I's short-run unemployment rate is $u_{I1}(\phi N + \alpha N)$. If scarring $\kappa \in (\phi, +\infty)$, long-run unemployment rate is positive; if $\kappa = \phi$, full employment obtains in the long run.

B. Suppose the government sets the sanction as λ_{B1}, where

$$\lambda_{B1} = (1-\gamma)\big(\frac{\phi}{1-\alpha}\bar{\sigma} + \frac{1-\phi-\alpha}{1-\alpha}\sigma_B\big)\frac{Q}{2N} - (1 - \frac{1}{N\epsilon})\big(1 - u_I(N-1)\big),$$

then $(1 - \phi - \alpha)N$ inefficient projects will go ahead and these entrepreneurs will default, but sector I achieves full employment at $t = 1$ and beyond.

C. With no sanctions, all the prior entrepreneurs will apply for loans, and a fraction of profitable applicants are crowded out.

Proof. Appendix C.

Theorem 1 states that when the default sanction is sufficiently harsh, the government can keep all the $(1 - \alpha)N$ inefficient projects out. In this scenario, the economy suffers persistent unemployment, and if scarring is sufficiently large, positive long-run unemployment also prevails. However, if the government chooses a sufficiently lenient default sanction, it can restore full employment immediately at date $t = 1$ and beyond. However, in this case, the entrepreneurs with low profitability remain in existence, harming aggregate productivity in sector I in the long run. Furthermore, if the government does not impose default sanctions at all, all the $(1 - \alpha)N$ inefficient entrepreneurs will try to re-enter. This scenario will cause some of the $(\phi + \alpha)N$ profitable entrepreneurs to be excluded and further dampen aggregate productivity. In Theorem 1, the harsh default sanction $\lambda_1 > \lambda_{A1}$ and the lenient default sanction $\lambda_1 = \lambda_{B1}$ outline two extreme cases. It is possible that the optimal default sanction could be an intermediate case.

Before moving on to solve for the optimal default sanction, Proposition 2 characterises the interest rate dynamics via the aggregate Euler equations of these two polar cases.

Proposition 2. (sanctions, interest rates, & demand shortage): Suppose Inequality (12) holds,

A. Suppose $\lambda_1 > \lambda_{A1}$, $r_0 < 1/\beta - 1$, and

$$1 + r_0 = \frac{1}{\beta}\Big(\frac{1}{(\phi + \alpha)^{1-\nu} + 1}\Big)^{\frac{\nu-\delta}{1-\nu}}.$$

Interest rate remains below $1/\beta - 1$ for $t = 1$, and demand shortage is persistent.

B. Suppose $\lambda_1 = \lambda_{B1}$, $r_0 < 1/\beta - 1$ also holds, but from $t = 1$ onwards, the interest rate immediately returns to $1/\beta - 1$. Demand shortage is short-lived.

Proof. Appendix (D).

Proposition 2 states that when the government is pro-allocation and sets a harsh default sanction, the interest rate goes down and remains low persistently. This is a harsh sanction stifles production in the short run, and the supply shortage leads to demand shortage due to the complementarity of the two sectors. In equilibrium, the consumption for goods I gradually increases while the consumption for goods J remains the same. Given $\nu > \delta$, goods I and goods J are complements, so the marginal utility of consumption for goods J immediately drops before it gradually increases. Demand shortage is persistent. However, if the government is pro-stabilisation and sets a lenient sanction, the interest rate only goes down for date $t = 0$ due to the pandemic shock, and then it immediately returns to $1/\beta - 1$. Demand shortage is short-lived. The reason is that the government attracts enough entrepreneurs to re-enter at $t = 1$, and full employment is reached immediately, at the possible cost of lowering future productivity.

5 Optimal Default Sanctions

To solve for the optimal sanction, we first define the social welfare function and then allow the government to choose the number of reentering entrepreneurs M, where $M \in [(\phi + \alpha)N, N]$, subject to the decentralised optimality conditions of the entrepreneurs and workers. We shall shortly show M has a one-to-one mapping with the default sanction λ_1. The M^* that maximises the social welfare corresponds to the optimal default sanction λ_1^*.

5.1 Analytics

We assume the government assigns equal weights to everyone in the economy, so the social welfare function takes the form as the sum of the consumption utilities of all agents in the economy. Let $\boldsymbol{c}_{It} \equiv$ the total consumption of goods I, and let $\boldsymbol{c}_{Jt} \equiv$ the total consumption of goods J. Since all agents exhibit the same CES preferences and their utility function is homogeneous of degree 1, the social welfare function at date $t = 1$ takes the following form,

$$V_1 = \mathbb{E}_1 \sum_{t=1}^{2} \beta^{t-1} \frac{1}{1-\delta} \Big((\frac{1}{2})^{\nu} \boldsymbol{c}_{It}^{1-\nu} + (\frac{1}{2})^{\nu} \boldsymbol{c}_{Jt}^{1-\nu} \Big)^{\frac{1-\delta}{1-\nu}}. \tag{13}$$

Let $\sigma(n')$ denote the marginal entrepreneur's project return when the number of re-entries is M. Conditional on M, there is no uncertainty from date $t = 1$ onwards, so we drop the expectation sign hereafter. Given $\sigma(n)$ follows a uniform distribution $\sigma(n) \sim \mathcal{U}(\sigma_B, \bar{\sigma})$, it follows that

$$\sigma(n') = \big(1 - \frac{M - (\alpha+\phi)N}{(1-\alpha)N}\big)\bar{\sigma} + \frac{M - (\alpha+\phi)N}{(1-\alpha)N}\sigma_B, \tag{14}$$

so the default sanction that corresponds to M is (15); thus, M and λ_{M1} have a one-to-one mapping.

$$\lambda_{M1} = (1-\gamma)\Big(\big(1 - \frac{M-(\alpha+\phi)N}{(1-\alpha)N}\big)\bar{\sigma} + \frac{M-(\alpha+\phi)N}{(1-\alpha)N}\sigma_B\Big)\frac{Q}{2N} - (1-\frac{1}{N\epsilon})\big(1 - u_{I1}(M-1)\big). \tag{15}$$

First, given M, we solve the aggregate consumption of the two goods by substituting in the decentralised optimality conditions of the entrepreneurs and the workers. Thus, we can express the aggregate consumption $\boldsymbol{c}_{It}$ and $\boldsymbol{c}_{Jt}$ as functions of exogenous parameters, which include M, and hence, the social welfare function can be reexpressed as functions of M with other exogenous parameters. What then remains to be done is searching for the M that maximises the social welfare function. Given (15) that expresses the default sanction λ_M as a function of exogenous parameters that include the number of reentering entrepreneurs M, as long as the optimal M is obtained, we can derive the optimal sanction.

At $t = 1$, there are $(\alpha+\phi)N$ number of profitable entrepreneurs entering, and the number of inefficient entries amounts to $M - (\alpha+\phi)N$. Thus, the aggregate consumption of sector I goods in equilibrium is given in (16).

$$\boldsymbol{c}_{I1} = \frac{Q}{2N}\Big(\frac{(\sigma(n') + \bar{\sigma})}{2}(M - \phi N - \alpha N) + (\alpha+\phi)N\Big), \tag{16}$$

where $\sigma(n')$ is given in (14).

At $t = 2$, whether sector I reaches full employment partly depends on the scarring κ. If scarring κ is sufficiently small, the number of entries at $t = 2$ amounts to $N - M$, and there is no long-run unemployment. The aggregate consumption of goods I at $t = 2$ is thus expressed in (17). The aggregate consumption of goods J remains $\frac{Q}{2}$ throughout when the interest rate is unconstrained from the ELB.

$$\boldsymbol{c}_{I2} = \frac{Q}{2N}\Big(\frac{(\sigma(n') + \bar{\sigma})}{2}(M - \phi N - \alpha N) + (\alpha+\phi)N + \frac{\phi}{\kappa(u_{I1})}(N - M)\Big), \tag{17}$$

Therefore, substituting (16), (17), and $\boldsymbol{c}_{Jt} = \frac{Q}{2}$ in the social welfare function (13), the social welfare function can be expressed as a function of exogenous parameters that include M. Before we solve for the optimal M^*, we first develop an analytic measure conditional on M, which we call "Stabilisation Proclivity" (SP) to characterise the government's policy stance: the higher the Stabilisation Proclivity, the more likely government will set a lenient sanction, i.e., being closer to λ_{B1}; the lower the Stabilisation Proclivity, the more likely the government will set a harsh sanction, i.e., being closer to λ_{A1}. Formally, the Stabilisation Proclivity SP is expressed in (18).

$$SP = \frac{1}{\beta}\Big(\frac{2\phi}{\kappa(u_{I1})\big(\sigma(n') + \bar{\sigma} + (\sigma_B - \bar{\sigma})\frac{M-\phi N-\alpha N}{(1-\alpha)N}\big)} - 1\Big)^{-1}. \tag{18}$$

To understand the economic intuition of our measure, let us use $U(\boldsymbol{c}_{It}, \boldsymbol{c}_{Jt})$ denote the single-period utility, i.e., $U(\boldsymbol{c}_{It}, \boldsymbol{c}_{Jt}) = \frac{1}{1-\delta}\Big((\frac{1}{2})^\nu \boldsymbol{c}_{It}^{1-\nu} + (\frac{1}{2})^\nu \boldsymbol{c}_{Jt}^{1-\nu}\Big)^{\frac{1-\delta}{1-\nu}}$, then the first-order derivative of $V_1(M)$ for M is expressed follows:

$$\frac{\partial V_1(M)}{\partial M} = \frac{\partial U(\boldsymbol{c}_{I1}, \boldsymbol{c}_{J1})}{\partial M} + \beta\frac{\partial U(\boldsymbol{c}_{I2}, \boldsymbol{c}_{J2})}{\partial M}. \tag{19}$$

As we show in Proposition 3, the first term on the right-hand side of (19), the short-run social utility, increases with M i.e., $\frac{\partial U(\boldsymbol{c}_{I1},\boldsymbol{c}_{J1})}{\partial M} > 0$, and the second term on the right-hand side of (19), the long-run social utility, decreases with M if scarring is sufficiently small, i.e., $\frac{\partial \tilde{U}(\boldsymbol{c}_{I2},\boldsymbol{c}_{J2})}{\partial \tilde{M}} < 0$. When the first term dominates the second term, the government may be more pro-stabilisation, as it would prefer a larger M; when the second term dominates the first term, the government may be more pro-allocation. We develop the measure SP borrowing the concept of marginal rate of substitution. The marginal rate of substitution between short run and long run social utility is $-\Big(\frac{\partial \tilde{U}(\boldsymbol{c}_{I1},\boldsymbol{c}_{J1})}{\partial \tilde{M}}\Big)/\Big(\beta\frac{\partial \tilde{U}(\boldsymbol{c}_{I2},\boldsymbol{c}_{J2})}{\partial \tilde{M}}\Big)$. Let q_{It} be the sector I production at date t, and as we shall show in Proposition 3 shortly, the measure SP is derived from $-\Big(\frac{\partial q_{I1}}{\partial \tilde{M}}\Big)/\Big(\beta\frac{\partial q_{I2}}{\partial \tilde{M}}\Big)$, and it offers an indication whether the government prefers to be more pro-allocation or more pro-stabilisation.

Proposition 3. (Stabilisation Proclivity): Given (16), (17), and $M \in [(\phi+\alpha)N, N]$, and suppose the long-run scarring is bound from above that $\kappa(u_{I1}) < \frac{\phi}{\bar{\sigma}}$, then the short-run social utility increases with M, and the long-run social utility decreases with M, i.e.,

$$\frac{\partial U(\boldsymbol{c}_{It}, \boldsymbol{c}_{Jt})}{\partial M} > 0,$$

$$\frac{\partial U(\boldsymbol{c}_{I2}, \boldsymbol{c}_{J2})}{\partial M} < 0,$$

and the Stabilisation Proclivity measure (SP) is derived from the following:

$$-\frac{\partial q_{I1}/\partial M}{\beta \partial q_{I2}/\partial M} = \underbrace{\frac{1}{\beta}\Big(\frac{2\phi}{\kappa(u_{I1})\big(\sigma(n') + \bar{\sigma} + (\sigma_B - \bar{\sigma})\frac{M-\phi N-\alpha N}{(1-\alpha)N}\big)} - 1\Big)^{-1}}_{SP}. \tag{20}$$

Proof. Appendix (E).

Naturally, if SP is large, then it does not sacrifice much long-run output that the government takes a more pro-stabilisation stand; if SP is small, then the trade-off between stabilisation and allocation is weighing more on allocation, so the government may take a more pro-allocation approach. Note that SP depends on exogenous parameters such as $\beta, \bar{\sigma}, \sigma_B, N$, and ϕ, and some of these parameters involve deep parameters such as monitoring technology γ and price elasticity of demand ϵ. By investigating how SP responds to changes in these parameters, Proposition 4 characterises the government's policy stance on the trade-off between allocation and stabilisation.

Proposition 4. (policy stance): Keeping all other parameters unchanged, and suppose $\kappa(u_{I1}) < \frac{\phi}{\bar{\sigma}}$ continues to hold,

A. An improvement in the monitoring technology γ increases SP.

B. An increase in ϵ or N decreases the oligopoly rents and increases SP.

C. An increase in σ_B increases SP.

D. An increase in β decreases SP.

Proof. Appendix (F).

Proposition 4 characterises the government's policy stance on the trade-off between allocation and stabilisation. When the monitoring/verification technology improves, the government tends to be pro-stabilisation and set a more lenient default sanction. If the monitoring technology is poor, the government tends to set a harsher sanction to deter inefficient entrepreneurs. Indeed, an increase in γ in reality could involve banks providing guarantees to the government or the entrepreneurs putting up collateral. In these cases, quite intuitively, the government can safely set a lenient default sanction and take a more pro-stabilisation stance.

When the price elasticity of demand is high or the size of the entrepreneurial pool is large, the entrepreneurs' market power is low, their oligopolistic rents decrease and workers' wages increase. Consequently, the government tends to be pro-stabilisation. Moreover, the size of the industry increases scarring, which further pushes the government to be pro-stabilisation. Furthermore, an increase in σ_B increases SP, and a decrease in σ_B decreases SP. The reason is that as the lowest profitability σ_B decreases, the overall quality of the prior entrepreneurs worsens. Consequently, the government tends to be less pro-stabilisation and more pro-allocation.

Finally, perhaps quite intuitively, when the future utilities are discounted less, the government tends to be pro-allocation, and when the future utilities are discounted more, the government tends to be pro-stabilisation. Indeed, if a government is myopic, e.g., the government is more impatient than the public, then the government discounts the future utilities more; consequently, the government will take a very pro-stabilisation stance. For example, if the government cares about immediate re-elections, it is likely to set a lenient default sanction or no sanctions at all to promote short-term employment and sacrifice long-term productivity. The overall toll on social welfare could be substantial.

5.2 Numerical Examples

In this subsection, we assign numerical values to deep parameters in Table 1 as the benchmark case, and we provide numerical examples. As the model is stylised, conducting a large-scale calibration exercise is outside the scope of this paper; nevertheless, the numerical values are chosen to be as close to reality as possible.

The number of entrepreneurs in each sector is set to be 10 in normal times, and the total number of workers is 800. Thus, each small business firm employs 40 workers in normal times, and we believe this is a reasonable size for small businesses. The price elasticity of demand is chosen to be 1.2, so that in normal times the entrepreneur's profits is around 3.6 times the worker's wages. The constant elasticity of substitution ν is therefore around 0.83, and the intertemporal elasticity of substitution is set to be 1.43, so that $\nu > \delta$ holds, i.e., goods in sector I and sector J are complements. The fraction of new workers entering in each period is set to be 0.1 and the fraction of profitable prior entrepreneurs is set to be 0.5, so that if we were to incorporate the model to an infinite setting where each period would correspond to one year, it would take 5 years for sector I to reach full employment and fully restore production if the government sets a harsh default sanction to keep all unprofitable projects out. Given the severity of the COVID-19 pandemic, we think 5 years would be a plausible figure. The discount factor β is set to 0.97. And finally, we set the monitoring technology γ to be 0.7, so that when borrowing agents default on government loans, the government could only garnish 70% of the borrower's funds and the borrower diverts 30% of her funds. Moreover, we assume the scarring function takes the form $\kappa = ln(\iota U_I(M_1) + e)^\theta$, where $U_I(M_1)$ is the short-run unemployment rate, so that short-run unemployment could lead to insufficient entries in the long run and structural unemployment. Without the loss of generality, we set θ equal to ϕ such that when the short run obtains full employment, the long-run scarring disappears and date $t = 2$ sees no structural unemployment. The parameter ι is free, and in our robustness check, the results remain robust by considering a wide range of ι.

Table 1: Parameterisation

Discount factor	$\beta = 0.97$
Monitoring technology	$\gamma = 0.7$
Price elasticity of demand	$\epsilon = 1.2$
Intertemporal elasticity of subsitution	$\frac{1}{\delta} = 1.43$
The fraction of new workers entering	$\phi = 0.1$
The fraction of profitable prior entrepreneurs	$\alpha = 0.5$
Number of entrepreneurs in each sector	$N = 10$
Total number of workers	$Q = 800$

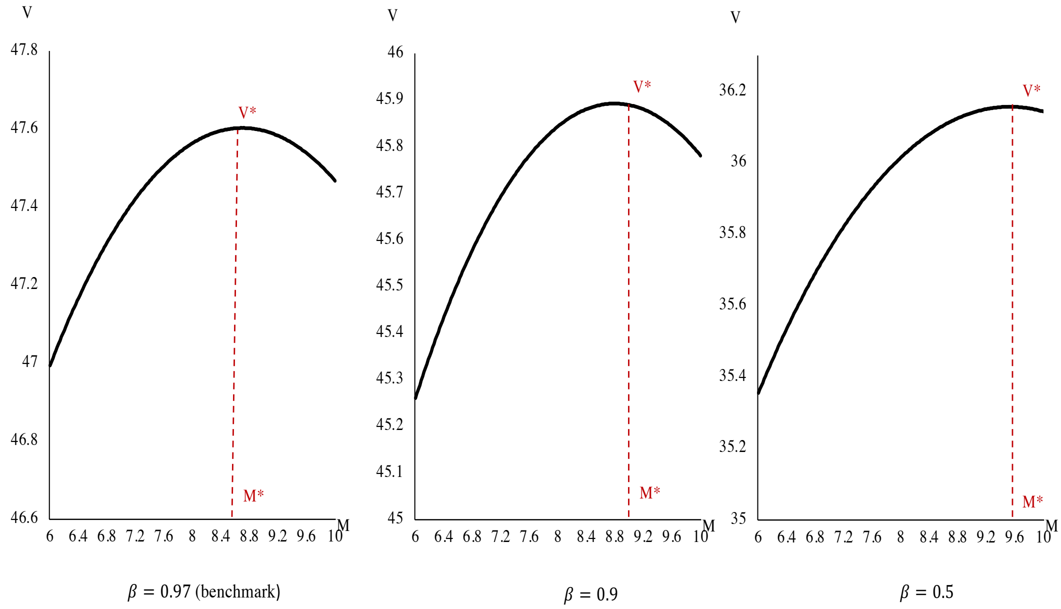
The parameterisation in Table 1 and our assumption of the distribution of the inefficient entrepreneurs' project return as in (2) imply that the highest project return among the inefficient entrepreneurs $\bar{\sigma}$ is around 0.92 and that the lowest project return among them σ_B is around 0.076. First, in our benchmark cases we assume away the long-run scarring by replacing $U_I(M_1)$ with 0 in the scarring function, and we search for the optimal default sanction numerically. Then, we add back the long-run scarring and compare the new optimal default sanction with our benchmark case.

Figure 3 displays the responses of the social utility to the number of entries, or equivalently, the default sanction, while assuming away scarring. The vertical axis is the value of total social utility denoted as V, which is defined in (13). The horizontal axis is the number of entries, each of which corresponds to a default sanction. In particular, the number of entries $M = 6$ corresponds to the harsh default sanction λ_{A1}, and the number of entries $M = 10$ corresponds to the lenient default sanction λ_{B1}. The number of entries monotonically decreases with the harshness of the default sanction. The first subplot is our benchmark case where the discount factor β is set to 0.97, the second subplot sets

$\beta = 0.9$, and the third subplot sets $\beta = 0.5$. The decrease in β can be interpreted as a more impatient or myopic government.

The first subplot of Figure 3 shows that in our benchmark case, the optimal default sanction is intermediate; it is more lenient than the harsh default sanction λ_{A1} but harsher than the lenient default sanction λ_{B1}.[13] The second subplot of Figure 3 assumes a lower discount factor $\beta = 0.9$. The optimal default sanction is also intermediate, but compared with the case of $\beta = 0.97$, it moves further away from the harsh default sanction λ_{A1}. This result suggests if the government is myopic, that it cares more about the short-term gains than the long-run productivity, the government will choose a more lenient default sanction than our benchmark case and attract more entries at $t = 1$ to reduce unemployment.

Figure 3: Social utility, optimal sanctions, and β



The third subplot of Figure 3 assumes an extremely low discount factor $\beta = 0.5$. The government's choice of default sanction further moves away from the harsh default sanction λ_{A1}. In contrast to the first two subplots, the optimal default sanction in this case is much closer to the lenient default sanction λ_{B1}. This result suggests that as the government becomes more impatient, it prefers to set more lenient contractual terms for its loans, or attach no conditionality at all, to support the short-term employment rebound.

Figure 4 illustrates the responses of sector I output to the default sanction in the benchmark case. The first subplot shows the responses of sector I output. Its horizon axis indicates the time that starts at $t = -1$ which corresponds to the pre-pandemic equilibrium. The dashed line corresponds to a pro-allocation government that sets the harsh default sanction λ_{A1}, the dotted line corresponds to a pro-stabilisation government that sets the lenient default sanction λ_{B1}, and the solid line corresponds to the optimal default sanction that maximises the social welfare. Let us observe the case of the pro-stabilisation government. As the pandemic occurs at date $t = 0$, illustrated by the dotted line, sector

[13] In our robustness check where we consider infinite horizons, the optimal default sanction also turns out intermediate, but it is much closer to the harsh case λ_{A1} than the three-period case. The reason is that in the three-period model, the long run is a single period, whereas in infinite horizons the long run sums up all the future periods after the short run, so the weights in the social welfare function for future allocations are larger, and the optimal default sanction is more sensitive to the discount rate.

I's output drops to zero before it goes back up at $t = 1$ at a lower level than the pre-pandemic equilibrium and remains there ever since. This is because the pro-stabilisation government uses a lenient default sanction to mop up the unemployment immediately after the pandemic passes.

Let us now turn to the dashed line of the first subplot of Figure 4, which indicates a pro-allocation government. Compared with the pro-stabilisation government, the output does not rebound as much at $t = 1$, because the short run suffers unemployment. The output gradually increases and overtakes the pro-stabilisation case only in the long run. At $t = 2$, the economy bounces back to a long-run equilibrium at exactly the same pre-pandemic level. This reason is simple. The pro-allocation government uses a harsh default sanction to deter all the inefficient entrepreneurs so that the long-run productivity can be restored fully.

Figure 4: Output and default sanctions

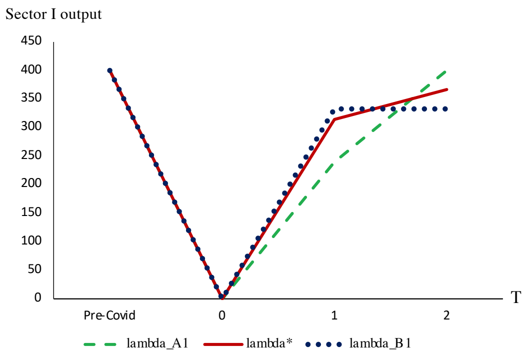
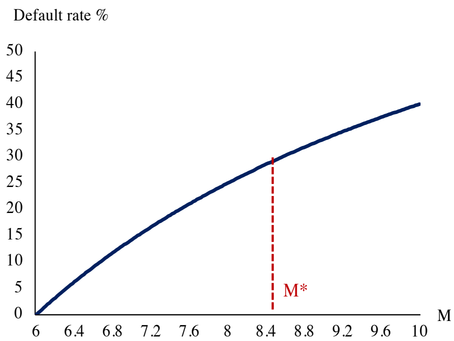


Figure 5: Optimal default

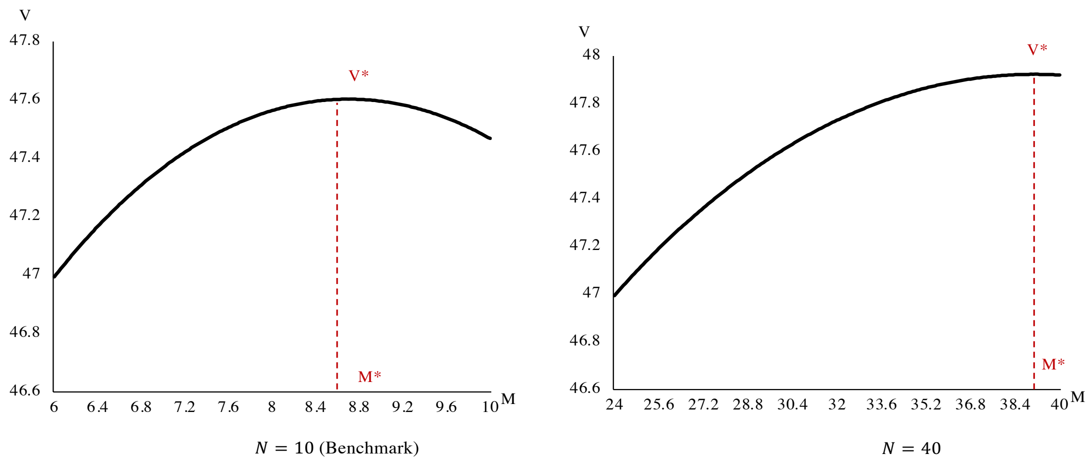


The case of the optimal default sanction is illustrated by the solid line. It is intermediate and is a result of balancing the trade-off between short-term employment and long-run productivity. The optimal case does not suggest a lenient default sanction to mop up unemployment completely, and it also tolerates some productivity loss in the long run so that the short-run unemployment rate is below the case of a harsh default sanction. The implication for the defaults, almost by definition, varies with the harshness of the sanction. As can be seen in Figure 5, the harsh default sanction which corresponds to $M = 6$ rules out default completely, and the lenient default sanction that corresponds to $M = 10$ leads to 40% of borrowers defaulting. The optimal default corresponds to M^* is again intermediate.

Next, we vary parameters to see how they alter the optimal default sanction. First, we increase γ from 0.7 to 0.95, which means an improvement in the lender's monitoring technology. As the second subplot of Figure A1 in Appendix (G)) illustrates, the optimal default sanction turns out the most lenient stance of λ_{B1}. Indeed, a near-perfect monitoring technology could justify the government's most generous credit guarantees for its bounce-back loans to SMEs. This is not surprising. If the government has a superior monitoring technology, it would not worry about the long-run misallocation of resources thanks to adverse selection but only focus on restoring short-run employment. However, this scenario is unlikely to hold during the pandemic. Given the scale and the speed of the crisis, lenders are unlikely to carry out meticulous monitoring in haste.

Then we increase the number of entrepreneurs in each sector from the benchmark case of 10 to 40. This parameter change lowers the market power of the entrepreneurs and reduces their oligopolistic rents. In turn, labour income share accounts for a larger fraction of total output. Therefore, as we can see in the second subplot of Figure 6, the optimal default sanction moves to a more lenient stance as stabilisation has become more pertinent.

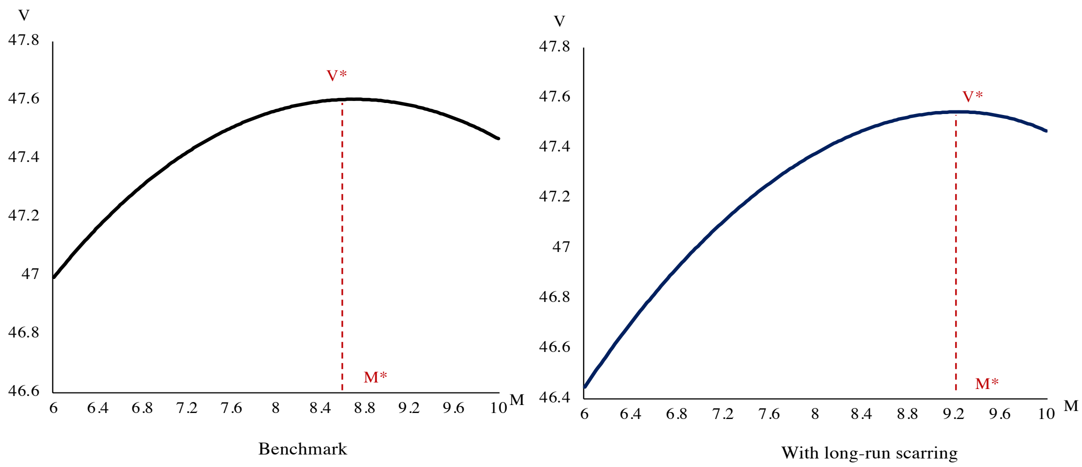
Figure 6: Social welfare, optimal sanctions, and N



We also increase the price elasticity of demand ϵ to 4 from the benchmark case of 1.2, which decreases the entrepreneurs' oligopolistic rents. As we show in Figure A2 in Appendix (G), it causes a shift to a more lenient stance, since lower rents imply a higher wage, which adds to the utility loss associated with unemployment. We then increase σ_B and ϕ to improve the overall quality of the inefficient entrepreneurial pool. This also results in a shift to a more lenient optimal default sanction (see Figure A3 and Figure A4 in Appendix (G)), as the utility loss associated with future productivity slow down becomes less severe.

Finally, we study the effects of long-run scarring by considering the scarring function. Figure 7 illustrates the comparative statics between our benchmark and the long-run scarring case. The overall welfare with long-run scarring is lower than the benchmark case; moreover, the optimal default sanction becomes more lenient. Naturally, if short-run unemployment due to COVID-19 leaves a long-run scar such that in the long-run structural unemployment occurs, the government will care about more short-run stabilisation rather than allocative efficiency.

Figure 7: Social welfare, optimal sanctions, and long-run scarring



5.3 Interest Rate Effective Lower Bound

In this subsection, we discuss the implication of the interest rate ELB for the optimal default sanction. The interest rate ELB limits monetary policy and constrains the interest from falling to prop up demand in the unshocked sector. Goods J produced by the unshocked sector are complements to goods I which are produced by the adversely shocked sector, so the marginal utility of consumption for goods J decreases, leading to a demand shortage. The interest rate should decrease to equilibrate the economy, leaving the supply of goods J unconstrained. When the interest rate cannot adjust downward, however, it causes an inward shift of the supply curve, and in turn, involuntary unemployment in sector J. This is a natural result in line with the *Drèze equilibrium* (Drèze (1975)) in that when the price, the interest rate in our case, is downwardly rigid, supply is constrained. Proposition 5 formalises our argument.

Proposition 5. (ELB, default sanctions, and involuntary unemployment): Suppose the economy is at the ELB where the interest rate cannot go below $1/\beta - 1$, demand shortage leads to *involuntary* unemployment at sector J.

Proof. Appendix (H).

The interest rate ELB causes involuntary unemployment in sector J. If the government sets a harsh default sanction, not only does sector I suffer persistent unemployment as in our benchmark case, but sector J hires fewer workers as well. This is an aggregate demand externality the government would want to avoid. Therefore, the social cost of a harsh default sanction may outweigh that of our benchmark.

Remark: At the interest rate ELB, the optimal default sanction may be more lenient than its counterpart when the interest rate is unconstrained.

To see why, let us observe in (21) how the single-period utility of consumption $U(\boldsymbol{c}_{It}, \boldsymbol{c}_{Jt})$ changes with respect to the number of entries M before $\bar{t}$ when full employment is reached in sector I, excluding the accidents when M falls on the "corner" of the $\bar{t}$ step-function.

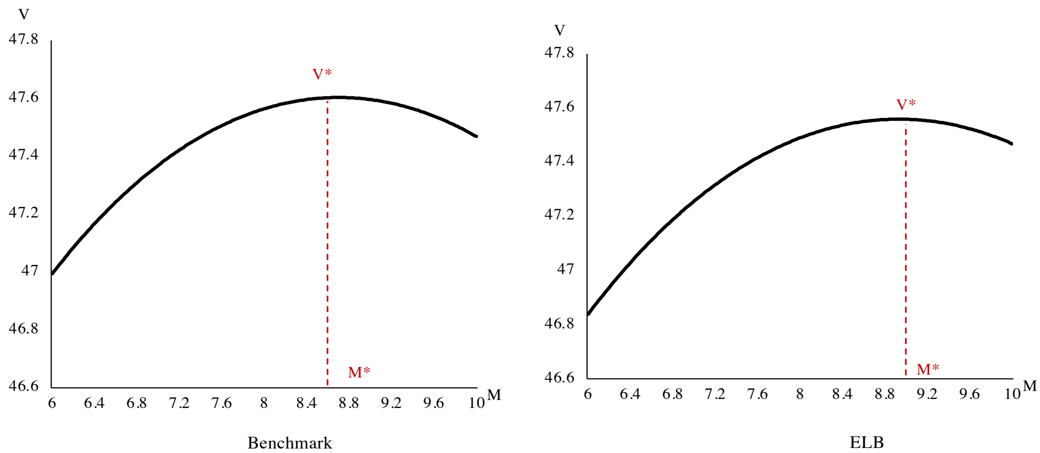
$$\frac{\partial U(\boldsymbol{c}_{It}, \boldsymbol{c}_{Jt})}{\partial M} = \underbrace{\frac{\partial U(\boldsymbol{c}_{It}, \boldsymbol{c}_{Jt})}{\partial \boldsymbol{c}_{It}} \frac{\partial \boldsymbol{c}_{It}}{\partial M}}_{X} + \underbrace{\frac{\partial U(\boldsymbol{c}_{It}, \boldsymbol{c}_{Jt})}{\partial \boldsymbol{c}_{Jt}} \frac{\partial \boldsymbol{c}_{Jt}}{\partial M}}_{Y}. \tag{21}$$

As shown in Proposition 3, the increase of M, or equivalently the leniency of the default sanction, leads to an increase in the single-period utility via increasing the marginal utility of consuming goods I, i.e., $X > 0$. At the ELB, the demand shortage causes the consumption of goods J to decrease, so the marginal utility of consuming goods I, $\frac{\partial U(\boldsymbol{c}_{It},\boldsymbol{c}_{Jt})}{\partial \boldsymbol{c}_{It}}$, is lower than its counterpart when the interest rate is unconstrained, so X is lower than the unconstrained case.

However, in our benchmark case, the increase of M causes no externality to sector J, so it does not change the single-period utility via the marginal utility of consuming goods J, i.e., $Y = 0$. In contrast, at the ELB, as we have shown in Proposition 5, for period $t < \bar{t}$, an increase of consumption of goods I leads to an increase of consumption of goods J, and it follows that the leniency of the default sanction increases the consumption of goods J, via the aggregate demand externality channel, i.e., $\partial \boldsymbol{c}_{Jt}/\partial M > 0$. Therefore, an increase of M, or equivalently the leniency of the default sanction, leads to an increase in the single-period utility via increasing the marginal utility of consuming goods J. So, at the ZLB, it follows that $Y > 0$, countervailing the reduction in X. Moreover, by increasing M, it takes fewer periods for sector I to reach full employment, shortening the time horizon that sector J suffers involuntary unemployment. Overall, the increase of M, or equivalently the leniency of the default sanction, may contribute to a higher utility gain at the ELB than the economy away from the ELB, suggesting that the government may take a more lenient stance in the ultra-low interest rate environment.

Indeed, as our numerical example illustrates in Figure 8, at the ELB, the overall welfare is lower than the benchmark case due to the involuntary unemployment in sector J, and the optimal default sanction turns out more lenient than the benchmark case. As the discount factor further decreases, this effect is expected to be more pronounced. Furthermore, as shown in Guerrieri et al. (2020), market incompleteness relaxes the conditions for a negative supply shock to cause the demand shortage; therefore, when the government fails to provide unemployment benefits to insure workers' against labour income risks, the ELB is likely to exacerbate the aggregate demand externality, and the optimal defaullt sanction would turn out even more lenient.

Figure 8: Social welfare, optimal sanctions, and ELB



6 Conclusion

This paper has assessed the government loan support for small businesses during a pandemic from a normative angle. We combine the insight of the IO literature and adverse selection in financial contracting in a macroeconomic framework. A two-sector equilibrium model has been developed featuring oligopolistic small businesses and a screening contract in the presence of a pandemic shock. The adversely affected sector with private information can apply for government loans to reopen businesses once the pandemic has passed. The government can implement a screening contract to ameliorate the adverse selection.

We have shown that if the government provides full guarantees for its loan support or implements a lenient default sanction, it can ensure a quick rebound to full employment once the pandemic has passed. However, the economy suffers a hit in long-run productivity. In contrast, if the government sets a harsh default sanction as a screening device, we have shown that it can successfully deter unprofitable businesses from borrowing, and long-run productivity can be restored to the pre-pandemic level. However, the economy suffers persistent unemployment in the near term. The optimal default sanction balances the trade-off between stabilisation and allocation.

To characterise the government's policy stance, we have developed an analytic measure "Stabilisation Proclivity". Numerically, we have demonstrated that the optimal default sanction is intermediate, and we have established which are the key parameters that will determine whether the government would want to be lenient, or tough, in trying to screen out the potentially less successful borrowers. Particularly on the discount rate, what we conclude is that society would have to be really rather extremely myopic, a very high rate of time preference, to eschew entirely the option of screening out borrowers with poor profitability prospects. We have also illustrated that a long-run scarring effect pushes the optimal default sanction to a more lenient stance. Furthermore, we show the implication of the interest rate Effective Lower Bound for the optimal default sanction. At the ELB, a harsh default sanction causes an aggregate demand externality, so the optimal default sanction becomes more lenient, and the government is likely to offer more generous credit guarantees.

We have kept the model deliberately simple to illustrate the various channels clearly. As the model remains stylised, we do not attempt to conduct a large-scale calibration exercise in this paper. However, we believe the model offers testable implications and lays the foundation for future quantitative structural modelling when more data series relating to COVID-19 and the associated government loan support become available.

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Appendices

A Proof of Lemma 3

Since $P_{I(t=-1)} = P_{J(t=-1)} = 1$, at the start of $t = 0$, workers in each sector get wages totalling $(1-\frac{1}{N\epsilon})\frac{Q}{2}$, and entrepreneurs in each sector have profits from last period totalling $\frac{Q}{2N\epsilon}$. They spend half of the sum on goods J at the end of $t = 1$. Consequently, at the start of $t = 1$, workers in each sector have extra money totalling $(1-\frac{1}{N\epsilon})\frac{Q}{4}$, sector I entrepreneurs have extra money totalling $\frac{Q}{4N\epsilon}$, and sector J entrepreneurs have extra money totalling $\frac{Q}{4N\epsilon}$. Therefore, the extra sum of money due to no spending on goods I amounts to $\frac{Q}{2}$.

Since $max(df_1) \leq F_1 N$, i.e., $max(df_1) \leq (1-\frac{1}{N\epsilon})\frac{Q}{2}$, and $\Lambda_1 > 0$, given that $df_1 = B_1 + \Lambda_1$, it follows that $B_1 < \frac{Q}{2} - \frac{Q}{2N\epsilon}$, and so, the extra money agents carry over from $t = 0$ is more than enough to invest in B_1.

□

B Proof of Proposition 1

Conditional on $df_1 > 0$, for the marginal entrepreneur n, the incentive constraint takes equality sign and can be re-expressed as

$$\lambda_1 = (1-\gamma)\sigma(n)h_1 - \mathbb{E}_1(W_{I1}|\mathbb{I}_d = 1, \lambda_1). \tag{22}$$

Suppose $M(\leq N)$ entrepreneurs-to-be apply for government loans. $(\phi + \alpha)N$ are good entrepreneurs, and $M - \phi N - \alpha N$ are inefficient ones. Given the uniform distribution assumption, it follows that

$$\sigma(n) = \big(1 - \frac{M-(\alpha+\phi)N}{(1-\alpha)N}\big)\bar{\sigma} + \frac{M-(\alpha+\phi)N}{(1-\alpha)N}\sigma_B. \tag{23}$$

The expected wage conditional on deviation at $t = 1$ needs to be adjusted by the out-of-the-equilibrium unemployment rate, i.e.,

$$\begin{aligned} E_1(W_1|\mathbb{I}_d = 1, \lambda_1) &= (1 - u_1(M-1))(1 - \frac{1}{N\epsilon}) \\ &= (\frac{\frac{Q(M-1)}{2N}}{\frac{Q}{2} + N - M + 1})(1 - \frac{1}{N\epsilon}), \end{aligned}$$

we can see that $\partial E_1(W_1|\mathbb{I}_d = 1, \lambda_1)/\partial M > 0$, and since $\partial M/\partial\sigma(n) < 0$ and given (23), it follows that

$$\partial E_1(W_1|\mathbb{I}_d = 1, \lambda_1)/\partial\sigma(n) < 0. \tag{24}$$

Given (22), and (24), we can see that $\partial\sigma(n)/\partial\lambda_1 > 0$.

Thus, when λ_1 increases, the number of re-entries M decreases, and fewer entrepreneurs with bad projects enter, so the number of defaulters decrease.

□

C Proof of Theorem 1

Proof of Theorem 1A.

Given $(1 - \frac{1}{N\epsilon}) = \bar{\sigma}$, $F_1 = \bar{\sigma} h_1$. The inefficient entrepreneur with the highest profitability is on the verge on strategically default in the absence of sanctions. Among the prior entrepreneurs, $(1-\alpha)N$ will default conditional on re-opening.

Now let us set $\lambda_1 > \lambda_{A1}$, where

$$\lambda_{A1} = (1 - \frac{1}{N\epsilon})((1-\gamma)\frac{Q}{2N} - (1 - u_I(\phi N + \alpha N))), \tag{25}$$

then for the entrepreneur with $\bar{\sigma}$, her benefits of default is smaller than her outside option. She is deterred from re-entering. By monotonicity, all the other inefficient entrepreneurs are deterred from re-entering as well. At date $t = 1$, the unemployment rate is $u_{I1}(\phi N + \alpha N)$. If scarring $\kappa > \phi$, long-run unemployment rate at $t = 2$ is positive; if scarring $\kappa = \phi$, full employment obtains in the long run.

□

Proof of Theorem 1B.

Since the government wants to obtain full employment, it needs to attract $(1 - \phi - \alpha)N$ prior entrepreneurs to apply for the loans. By our uniform distribution specification, the marginal entrepreneur's $\sigma(n)$ must satisfy the following:

$$\frac{\bar{\sigma} - \sigma(n)}{(1-\phi-\alpha)N} = \frac{\bar{\sigma} - \sigma_B}{(1-\alpha)N},$$

which is equivalent to

$$\sigma(n) = \frac{\phi}{1-\alpha}\bar{\sigma} + \frac{1-\phi-\alpha}{1-\alpha}\sigma_B.$$

Suppose λ_{B1} just satisfy the marginal entrepreneur's incentive constraint, i.e.,

$$(1-\gamma)\sigma(n)h_1 - \lambda_{B1} = (1 - \frac{1}{N\epsilon})(1 - u_I(N-1)),$$

thus,

$$\lambda_{B1} = (1-\gamma)\big(\frac{\phi}{1-\alpha}\bar{\sigma} + \frac{1-\phi-\alpha}{1-\alpha}\sigma_B\big)\frac{Q}{2N} - (1 - \frac{1}{N\epsilon})\big(1 - u_I(N-1)\big).$$

By monotonicity, the inefficient entrepreneurs with profitability higher than $\frac{\phi}{1-\alpha}\bar{\sigma} + \frac{1-\phi-\alpha}{1-\alpha}\sigma_B$ re-enter and default, and those with lower profitability stay out.

□

Proof of Theorem 1C.

Given the parameter $\sigma_B = \frac{2N-2/\epsilon}{(1-\gamma)Q}$, it follows that $(1-\gamma)\sigma_B h_1 = W_{I1}$ and $\sigma_B h_1 < F_1$. Thus, the inefficient entrepreneur with the lowest profitability will try to re-enter at the start of $t = 1$ in the absence of sanctions and then default at the end of $t = 1$.

Given that $\bar{\sigma} = 1 - \frac{1}{N\epsilon}$, it follows that $\gamma\bar{\sigma}h_1 < F_1 = \bar{\sigma}h_1$. Thus, the inefficient entrepreneur with the highest probability will try to re-enter at the start of $t = 1$ in the absence of sanctions and then strategically default at the end of $t = 1$. Among the prior entrepreneurs, conditional on getting the loans, a fraction $1-\alpha$ of them will default. As the total number of applicants amounts to $(\phi + 1)N$ and the industry size is N, some profitable applicants are crowded out.

□

D Proof of Proposition 2.

Proof of Proposition 2.A.

The following Euler equations hold for date t.

$$U_{\boldsymbol{c}_J}(\boldsymbol{c}_{It}, \boldsymbol{c}_{Jt}) = \beta(1 + r_t)U_{\boldsymbol{c}_J}(\boldsymbol{c}_{It+1}, \boldsymbol{c}_{Jt+1}),$$

equivalent to

$$1 + r_t = \frac{1}{\beta}\frac{U_{\boldsymbol{c}_J}(\boldsymbol{c}_{It}, \boldsymbol{c}_{Jt})}{U_{\boldsymbol{c}_J}(\boldsymbol{c}_{It+1}, \boldsymbol{c}_{Jt+1})},$$

Given the CES preference (7),

$$1 + r_t = \frac{1}{\beta}\frac{(\boldsymbol{c}_{It}^{1-\nu} + \boldsymbol{c}_{Jt}^{1-\nu})^{\frac{\nu-\delta}{1-\nu}}\boldsymbol{c}_{Jt}^{-\nu}}{(\boldsymbol{c}_{It+1}^{1-\nu} + \boldsymbol{c}_{Jt+1}^{1-\nu})^{\frac{\nu-\delta}{1-\nu}}\boldsymbol{c}_{Jt+1}^{-\nu}}. \tag{26}$$

We know $\boldsymbol{c}_{I0} = 0$ and $\boldsymbol{c}_{J0} = \frac{Q}{2}$. Given $\lambda_1 > \lambda_{A1}$, at date $t = 1$, only $(\alpha + \phi)N$ good entrepreneurs produce in sector I. It follows that $\boldsymbol{c}_{I1} = \frac{(\phi+\alpha)Q}{2}$ and $\boldsymbol{c}_{J1} = \frac{Q}{2}$. Substitute these values into (26), we obtain

$$1 + r_0 = \frac{1}{\beta}\Big(\frac{1}{(\phi + \alpha)^{1-\nu} + 1}\Big)^{\frac{\nu-\delta}{1-\nu}}. \tag{27}$$

Therefore, given $\nu > \delta$, $1 + r_0 < \frac{1}{\beta}$.

Moving onto $t = 2$, as another ϕN good entrepreneurs enter sector I at $t = 2$. It follows that

$$1 + r_1 = \frac{1}{\beta}\Big(\frac{(\alpha + \phi)^{1-\nu} + 1}{min(1, (\alpha + \phi + \frac{1}{\kappa}\phi))^{1-\nu} + 1}\Big)^{\frac{\nu-\delta}{1-\nu}}, \tag{28}$$

so we can see that $r_1 < \frac{1}{\beta} - 1$. Low interest rate is persistent.

□

Proof of Proposition 2.B.

Given $\lambda_1 = \lambda_{B1}$, full employment obtains from $t = 1$ onwards. By a similar logic in the proof of Proposition 2A, we can show $r_0 < \frac{1}{\beta} - 1$. As production remains constant from $t = 1$ onwards, it follows that $r_1 = \frac{1}{\beta} - 1$.

□

E Proof of Proposition 3

First we work out the partial derivative of the single-period utility as follows:

for $t = 1$

$$\begin{aligned}\frac{\partial U(\boldsymbol{c}_{I1}, \boldsymbol{c}_{J1})}{\partial M} =& \frac{1}{1-\nu}\Big((\frac{1}{2})^{\nu}\boldsymbol{c}_{I1}^{1-\nu} + (\frac{1}{2})^{\nu}\boldsymbol{c}_{J1}^{1-\nu}\Big)^{\frac{\nu-\delta}{1-\nu}}(\frac{1}{2})^{\nu}(1-\nu)\boldsymbol{c}_{I1}^{-\nu} \\ & \Big(\frac{Q}{4N}\big(\sigma(n') + \bar{\sigma} + (\sigma_B - \bar{\sigma})\frac{M - \phi N - \alpha N}{(1-\alpha)N}\big)\Big) > 0.\end{aligned}$$

At date $t = 2$, given $\kappa(u_{I1}) < \frac{\phi}{\bar{\sigma}}$ and (14),

$$\begin{aligned}\frac{\partial U(\boldsymbol{c}_{I2}, \boldsymbol{c}_{J2})}{\partial M} =& \frac{1}{1-\nu}\Big((\frac{1}{2})^{\nu}\boldsymbol{c}_{I2}^{1-\nu} + (\frac{1}{2})^{\nu}\boldsymbol{c}_{J2}^{1-\nu}\Big)^{\frac{\nu-\delta}{1-\nu}}(\frac{1}{2})^{\nu}(1-\nu)\boldsymbol{c}_{I2}^{-\nu} \\ & \Big(\frac{Q}{4N}\big(\sigma(n') + \bar{\sigma} + (\sigma_B - \bar{\sigma})\frac{\tilde{M} - \phi N - \alpha N}{(1-\alpha)N}\big) - \frac{Q\phi}{2N\kappa(u_{I1})}\Big) < 0.\end{aligned}$$

It is straightforward to derive that

$$-\frac{\partial q_{I1}/\partial M}{\beta \partial q_{I2}/\partial M} = \underbrace{\frac{1}{\beta}\Big(\frac{2\phi}{\kappa(u_{I1})\big(\sigma(n') + \bar{\sigma} + (\sigma_B - \bar{\sigma})\frac{M-\phi N-\alpha N}{(1-\alpha)N}\big)} - 1\Big)^{-1}}_{SP}.$$

□

F Proof of Proposition 4

Let $X = \frac{\kappa(u_{I1})}{2\phi}(\sigma(n') + \bar{\sigma} + (\sigma_B - \bar{\sigma})\frac{M-\phi N-\alpha N}{(1-\alpha)N})$, and given (14) we can further simplify X as

$$X = \frac{\kappa(u_{I1})}{\phi}\big(\bar{\sigma} + (\frac{M}{(1-\alpha)N} - \frac{\alpha+\phi}{1-\alpha})(\sigma_B - \bar{\sigma})\big),$$

where $\kappa(u_{I1}) = \kappa\left(1 - \frac{\frac{Q}{2N}M}{\frac{Q}{2}+N-M}\right)$, and $\partial\kappa/\partial N > 0$.

For $M \in ((\phi+\alpha)N, N)$, $\frac{M}{(1-\alpha)N} - \frac{\alpha+\phi}{1-\alpha} > 0$. Given $\bar{\sigma}$, the lower σ_B is, the lower X is, and the smaller SP is.

Note that,

$$\bar{\sigma} = 1 - \frac{1}{N\epsilon},$$

$$\sigma_B = \frac{2N - 2/\epsilon}{(1-\gamma)Q}.$$

X can be further simplified as

$$X = \frac{\kappa(u_{I1})}{\phi}(1 - \frac{1}{N\epsilon} - (\frac{M}{(1-\alpha)N} - \frac{\alpha+\phi}{1-\alpha})(1 - \frac{1}{N\epsilon})(1 - \frac{2N}{(1-\gamma)Q})).$$

Thus, $\partial X/\partial \epsilon > 0$, $\partial X/\partial \gamma > 0$, and $\partial X/\partial N > 0$,

Since $SP = \frac{1}{\beta(\frac{1}{X}-1)}$, SP increases with X and decreases with β, i.e., an increase in ϵ, γ, σ_B, or N leads to an increase with SP, whereas an increase in β decreases SP.

□

G Other Comparative Statics

Figure A1: Social welfare, optimal sanctions, and γ

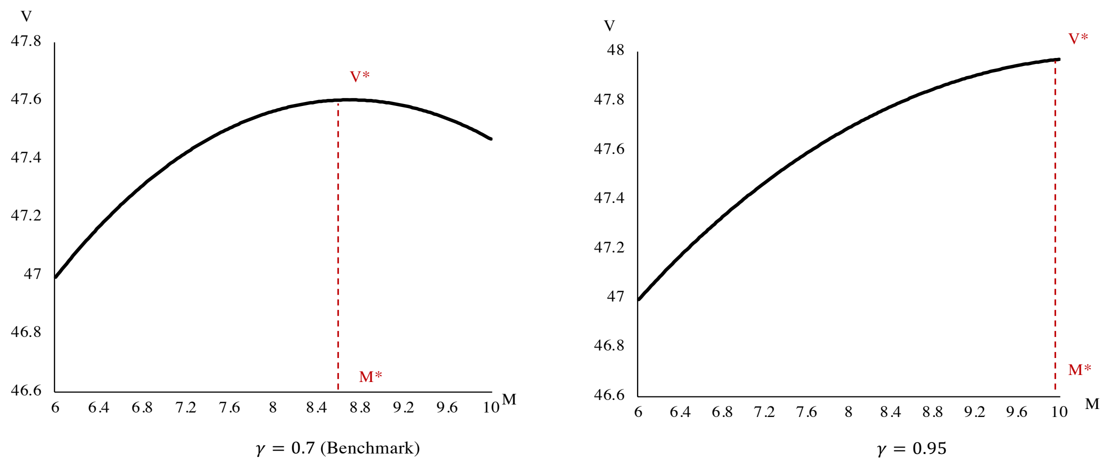


Figure A2: Social welfare, optimal sanctions, and ϵ

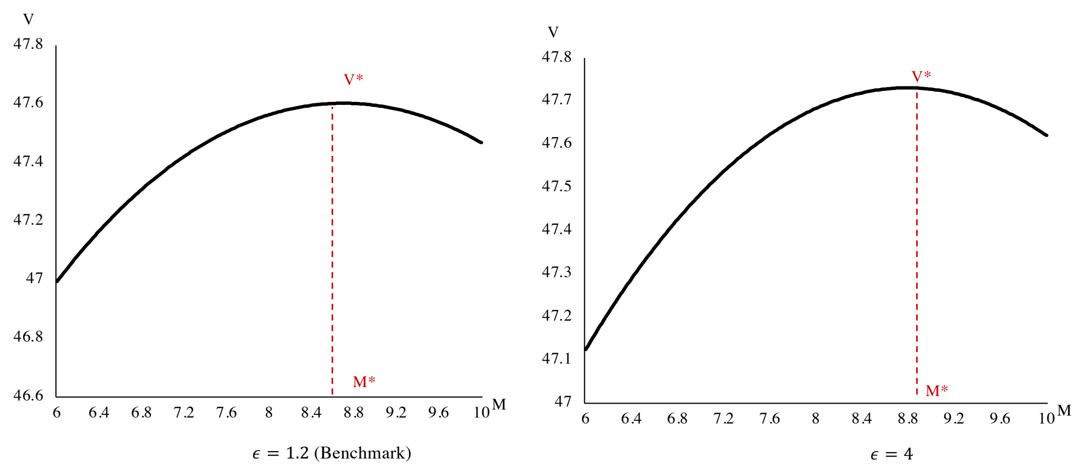


Figure A3: Social welfare, optimal sanctions, and σ_B

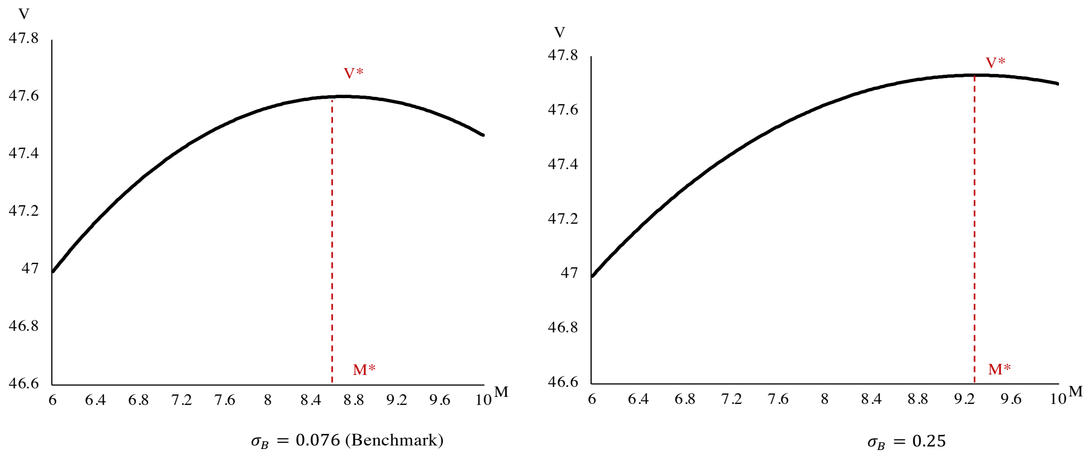
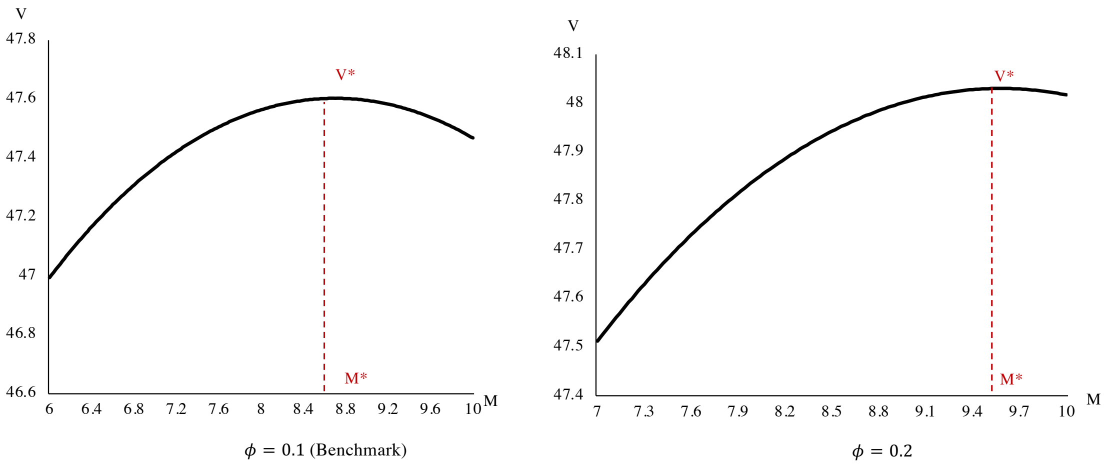


Figure A4: Social welfare, optimal sanctions, and ϕ



H Proof of Proposition 5

As shown in the Proof of Proposition 2, the Euler equation gives

$$1 + r_t = \frac{1}{\beta} \frac{U_{\boldsymbol{c}_J}(\boldsymbol{c}_{It}, \boldsymbol{c}_{Jt})}{U_{\boldsymbol{c}_J}(\boldsymbol{c}_{It+1}, \boldsymbol{c}_{Jt+1})}.$$

Suppose $\bar{t}$ is when sector J reaches full employment,

$$1 + r_{\bar{t}-1} = \frac{1}{\beta} \frac{U_{\boldsymbol{c}_J}(\boldsymbol{c}_{I\bar{t}-1}, \boldsymbol{c}_{J\bar{t}-1})}{U_{\boldsymbol{c}_J}(\boldsymbol{c}_{I\bar{t}}, \boldsymbol{c}_{J\bar{t}})}.$$

Since at the ELB, the interest rate is bound by $1/\beta - 1$, it follows that

$$1 = \frac{U_{\boldsymbol{c}_J}(\boldsymbol{c}_{I\bar{t}-1}, \boldsymbol{c}_{J\bar{t}-1})}{U_{\boldsymbol{c}_J}(\boldsymbol{c}_{I\bar{t}}, \boldsymbol{c}_{J\bar{t}})}. \tag{29}$$

And because $\boldsymbol{c}_{I\bar{t}-1} < \boldsymbol{c}_{I\bar{t}}$, $\partial U_{\boldsymbol{c}_J}(\boldsymbol{c}_{It}, \boldsymbol{c}_{Jt})/\partial c_{It} > 0$, and $\partial U_{\boldsymbol{c}_J}(\boldsymbol{c}_{Jt}, \boldsymbol{c}_{Jt})/\partial c_{Jt} < 0$, for (29) to hold, $\boldsymbol{c}_{J\bar{t}-1} < \boldsymbol{c}_{J\bar{t}}$ has to hold. Before $\bar{t}$, a decrease of consumption of goods I corresponds to a decrease of consumption of goods J. Accordingly, solving backward, it follows $\boldsymbol{c}_{J0} < \boldsymbol{c}_{J1} ... < \boldsymbol{c}_{J\bar{t}-1}$. Moreover, $\bar{c}_{J\bar{t}} = Q/2$ which corresponds to full employment in sector J, so for periods before $\bar{t}$ sector J suffers involuntary unemployment.

□