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COMMIT TO A CREDIBLE PATH OF RISING CO_2 PRICES

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Abstract

CO_2 pricing is essential for an efficient transition to the green economy. Despite Daniel, Litterman and Wagner (2019)' claim that CO_2 prices should decline, CO_2 prices should rise over time. First, damages from global warming are proportional to economic activity and this makes CO_2 prices grow at the same rate as the economy. Second, even if uncertainty about the damage ratio is gradually resolved over time, this only slows down the price rise. Third, if CCS is allowed for, the optimal CO_2 price will rise before it declines but this decline does not occur until more than two centuries ahead. Fourth, damages are likely to be a very convex function of temperature which with rising temperature implies that CO_2 prices must grow faster than the economy. Fifth, internalizing the social benefits of learning by doing or a shift towards technical progress in renewable energy production requires a subsidy for renewable energy, not a temporary spike in CO_2 prices. Having high CO_2 prices upfront is an artefact of failing to separate out renewable energy subsidies from the carbon price. Finally, efficient intertemporal allocation of policy efforts implies that a temperature cap or cap on cumulative emissions requires that CO_2 prices must rise at a rate equal to the risk-adjusted interest rate, typically higher than the economic growth rate. Summing up, CO_2 prices must rise at a rate at least equal to the economic growth rate and at most to the risk-adjusted interest rate. They should not decline.

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Keywords: CO_2 prices, risk, damages

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1. Introduction

Global warming is perhaps the biggest market failure on the planet (1). People and corporations do not take account of their impact on global warming when they emit carbon. The solution of Pigou (2) is to charge emissions a price equal to the social cost of carbon (SCC). This SCC is the expected present discounted value of all present and future damages caused by emitting one additional ton of CO_2 today. Policy makers must evaluate the SCC under big uncertainties regarding the wealth of future generations and future global warming damages resulting from emissions today. This involves risky trade-offs between consumption today and damages from global warming to consumption in the distant future. Daniel et al. (3) analyze these trade-offs in the workhorse recursive dynamic asset pricing model consisting of a discrete-time decision tree with a finite horizon extended to allow for Epstein-Zin or EZ preferences to distinguish between aversion to risks and aversion to intertemporal fluctuations (4, 5) and climate change (the EZ-Climate model) and generate optimal carbon dioxide (CO_2) price paths based on probabilistic assumptions about climate damages. Their main insight is that it is optimal to have a high price today that is expected to decline over time as the "insurance" value of mitigation declines and technological change makes emission cuts cheaper.

We make our point by extending the continuous-time workhorse recursive dynamic asset pricing model to allow for emissions and global warming with uncertainty about the future size of the economy and skewed risks of global warming damages. Our alternative model uses Duffie-Epstein or DE preferences (6) to separate risk aversion from aversion to intertemporal fluctuations, which is the continuous-time equivalent of EZ preferences. This allows for a preference for early resolution of uncertainty and accords well with empirical evidence. Both our and the EZ-Climate model offer an alternative to the most often used dynamic integrated climate-economy DICE model (7). But in contrast to the EZ-model, our DE-Climate model does not have to split time up in large periods and does not implicitly enforce declining volatility over time, one of the factors driving their surprising conclusion of an eventual decline in the SCC. We allow for uncertainty about growth of the economy, uncertainty about the ratio of damages to GDP and the pricing of the associated risks. Yet we arrive at radically different conclusions about the time path of optimal CO_2 prices and reject the advice that CO_2 prices should decline.

We decompose the SCC in its driving elements and show first of all that declining volatility as in (3), even when there is a preference for early resolution of uncertainty, only slows down the increase in CO_2 prices temporarily and does not reverse it. And this effect is eventually swamped by the rise in prices associated with growth of the economy. Second, we show that only an unrealistically large-scale implementation of carbon capture and storage (CCS) can trigger a declining SCC over time, but that requires that almost the entire temperature anomaly is rolled back. Third, we offer robustness exercises which also point to a rising, not a declining CO_2 price.

There are other arguments why declining CO_2 prices are unlikely. It is not clear at all that learning will lead to declining uncertainty about damages over time. Also, learning by doing (8, 9) or directed technical change (10) does not require a spike of CO_2 price in the coming years but an upfront spike in renewable energy subsidies (11, 12). Moreover, damages are likely to rise much sharper with temperature than is commonly assumed (13) and damages may be catastrophic (14). These features lift the path of CO_2 prices up and make the path even steeper than the time path of GDP when temperature rises. Finally, temperature caps such as the 1.5 or 2 °C caps of the Paris Agreement require that CO_2 prices rise at a rate equal to the risk-adjusted interest rate, typically larger than the rate of the economic growth once risk is priced in (15, 16).

2. DE-Climate Model

The core of the DE-Climate model consists of the following elements. To make the trade-off between sacrifices in current consumption against less consumption due to global warming in the future, we use Duffie-Epstein preferences (6) which recursively defines a value function giving the expected welfare from time *t* onwards, i.e. V_t. This formulation distinguishes the coefficient of relative risk aversion *RA* from the inverse of the elasticity of intertemporal substitution *EIS*[1]. Policy makers prefer early (late) resolution of uncertainty if *RA* exceeds (is less than) 1/*EIS*. Econometric evidence on financial markets strongly suggests this separation and that *RA* exceeds 1/*EIS* (17, 18). As a consequence, the risk-adjusted interest rate incorporates a so-called "timing premium" (19). If *RA* = *1/EIS* as with the power utility function, policy makers are indifferent about the timing of the resolution of uncertainty and there is no timing premium in interest rates.

The endowment of the economy Y_t follows a Geometric Brownian motion with drift μ and volatility σ and includes additional terms to allow for disaster shocks with constant mean arrival rate λ. The size of the shocks is a random variable with time-invariant distribution. Consumption thus equals $C_t = (1 - A_t) Y_t / (1 + D_t),$ where A_t denotes the fraction of output used for abatement and D_t is the damage ratio. The time path of business-as-usual emissions E_t is exogenous. Actual emissions are $(1 - u_t) E_t,$ where u_t denotes the abatement rate. Without carbon capture and sequestration (CCS), the upper bound of the abatement rate is 1 in which case all emissions are fully abated. With CCS, it is possible to abate more than 100% in which case we impose an upper bound on the abatement rate of $\bar{u} > 1.$ If $u > 1$, carbon is removed from the atmosphere). The cost function is $A_t = c_0 e^{-c_1 t} u_t^{c_2}$, where $c_0 > 0,$ c_1 is the exponential decline in costs over time due to technological progress and $c_2 > 1.$ Temperature is a linear function of cumulative carbon emissions (20). Finally, the damage

[1] 1/EIS can also be interpreted as a coefficient measuring aversion to intertemporal fluctuations.

ratio is a convex function of temperature and shocks that take some time to have their have their full impact and follow a skewed distribution to reflect "tail" risk (cf. Eq. [4] in Methods).

More details of the DE-Climate model and the way in which optimal climate policy is formulated are given in Appendix A.

3. Results

In our base calibration, we choose $RA = \gamma = 7$, $EIS = \eta = 1.5$ and the rate of impatience $\beta = 2\%$ per year. These are values that are typically used in the asset pricing literature with Epstein-Zin preferences (e.g. table 1 in (16)) and we will refer to these values as the market-based calibration. The full calibration of the model is given in Table 1.

Fig. 1 presents the three core variants using our base calibration. The first variant with the blue lines (diamonds) has constant volatility and a timing premium included in the discount rate. It also excludes CCS ($u < 1$). Without carbon removal from the atmosphere, abatement reduces emissions but abatement efforts cannot extract previously emitted CO_2 from the atmosphere. The second variant indicated by the orange lines (dots) shares in Fig. 1 all assumptions with the first variant, except that we allow volatility to decline gradually over time in line with the assumptions made by Daniel et. al. (3). And finally, the third variant indicated by the yellow lines (triangles) allows for carbon removal from the atmosphere, as in Daniel et. al. (3). Only then do we arrive at an eventually negative slope of the SCC and thus of the optimal CO_2 price: the yellow line most closely matches Daniel et. al. (3) (cf. panel (d)).

Panel (d) in Fig.1 shows the optimal SCC for all three variants. In the first, the SCC always rises over time because damages grow with GDP. If we introduce a steady decline in volatility to match the assumptions made implicitly by Daniel et. al. (3), the SCC still rises over the entire interval considered, albeit at a slower pace (orange dotted line). So, despite a preference for early resolution of uncertainty, carbon prices still rise with declining volatility. The declining SCC only emerges if CCS from the atmosphere is introduced, like it is in (3). Along the yellow line (triangles), the SCC first increases as economic activity grows over time, but once CCS gets underway and the temperature starts to fall, the SCC starts declining again. Marginal damages are increasing in temperature, which explains why when the temperature anomaly is almost fully reversed (cf. panel (c)), the SCC actually goes down. A strong conclusion emerges: even declining volatility and associated decline in risk is not enough to generate a declining SCC; CCS on a grand but unrealistic scale, almost completely rolling back the temperature anomaly, is needed to get SCC to decline.

Both the Kyoto and Paris agreements envisage stabilizing temperatures, not a reversal to pre-industrial times.

The other panels in Fig.1 show the mechanism. Without carbon capture on the stock of carbon, i.e. $u < 1$, net emissions $E_t(1-u_t)$ fall to zero and stay there (cf. panel (a)). The declining volatility variant (orange dots) is interestingly different from the baseline variant (blue diamonds): emissions stay higher initially. The SCC incorporates future damages and therefore the declining volatility in the future is already priced into the initial SCC. Since the externality is smaller in the declining volatility variant, the abatement efforts are also smaller, and emissions are somewhat larger. The temperature stabilizes at almost half a degree higher in the declining volatility variant (panel (c)).

The yellow line (triangles) shows the consequences of CCS. We implement CCS by allowing u to increase beyond its ceiling 1. When u exceeds 1, net emissions turn negative and carbon is removed from the atmosphere. That happens in the second half of our time interval (cf. panel (b)) and continues until all the temperature increases since 1900 have been reversed (cf. panel (c)). CCS drives the abatement rate up to over 150% and thus brings the temperature anomaly down to zero eventually. This is the only case where we see the SCC decline eventually, and it happens only from 2200 onwards. Moreover, this requires huge and unrealistic amounts of CCS, which are unlikely to be brought on due to untested technologies and the obstacle of NIMBY politics.

Eq. [7], Methods decomposes the optimal CO_2 price path into two components: one rising component of the price due to growing economic activity (a weighted geometric average of aggregate consumption and the endowment, panel (a)) and a component that is not related to economic growth (panel (b)). The SCC is then the product of the lines in panels (a) and (b). A flat line in panel (a) implies that the price grows at the rate of economic activity. For the first variant with constant volatility, the non-growth component rises until 2100 and then stays flat. Until 2100 temperature is still increasing and the SCC grows faster than economic activity. After 2100, the temperature curve flattens, and the growth rate of the SCC is equal to the growth rate of economic activity. Declining volatility affects the carbon price, but clearly the growth in economic activity dominates this effect. Only allowing for CCS pushes the non-growth-related component sufficiently fast towards zero to eventually lead to declining carbon prices, but only when the temperature anomaly is almost fully reversed (cf. the yellow line in Fig. 2 (a)).

Fig 3 shows the overriding impact of the growth rate on the SCC. Panel (a) illustrates that the growth of economic activity is the most important driver of the rising carbon prices. If μ is equal to zero, economic activity is on average shrinking due to the impact of the economic disasters, which leads to a declining path of the SCC. A 1% increase in the growth rate μ can have huge effects in the far future. The growth rate also affects the discount rate indirectly. Since the EIS in our calibration is larger than 1, an increase in the growth rate leads to a lower discount rate. Panel

(b) filters the growth of the economy out of the SCC, but due to a lower discount rate the purple line (squares) with the highest growth rate is still above the others.

When we have zero growth uncertainty, a faster resolution of uncertainty, more convex relationships between the damage ratio and temperature, lower risk aversion, a lower EIS, more ethical preferences or business-as-usual emissions rising in proportion to the endowment, the main policy message remains: the SCC must rise over time (see Appendix B).

4. Discussion

The policy recommendation from our results and robustness runs indicate that it is a best to have a steadily rising path of CO_2 prices, not declining prices. There are a few other concerns with the analysis in (3). They extend their model to allow for learning by doing in renewable energy production and suggest that contributes to a temporary spike and subsequent decline in the carbon price. However, careful economic analysis shows that this does not affect the optimal carbon tax. To get the right economic incentives, policy makers should implement a temporary renewable energy subsidy in which case this temporary spike in the carbon price is not there (e.g. Appendix C, (11) or (12)). Others have argued that in models with a fixed reserves of exhaustible fossil fuel intertemporal arbitrage implies that a constant tax on CO_2 emissions simply squeezes rents of the fossil fuel barons and has no effects on the time profile of emissions whatsoever; expectations of falling CO_2 taxes do postpone emissions and limit damages from global warming (22) as has also been suggested in (3). However, this result depends on some implausible features and the optimal CO_2 prices typically either rise or rise before they fall (23).

The pattern of a rising optimal carbon price is manifest in almost every integrated assessment of the economy and global warming. If on top of the normal growth uncertainty, risk of macroeconomic disasters and uncertainty about the damage ratio highlighted here and in (3), account is taken of climatic forms of uncertainty such as in the carbon stock and temperature dynamics or the uncertainty (24) or about tipping of the Greenland or Antarctic Ice Sheet or reversal of the Gulf Stream, the optimal response is in all cases a rising carbon price. If integrated assessment models are extended to allow for long-run risk in economic growth with temperature-induced tail risks, the temperature risk premium increases with temperature (25, 26) and it is even more difficult to get a declining carbon price. Although ambiguity aversion has surprisingly little impact on the optimal carbon price (27, 28), other than one might have been expected given the worst-case assumption that optimal requires one to taken when faced with the multiple priors framework (29), the optimal carbon price under ambiguity aversion still rises. Finally, if one allows for learning after a tipping point when it becomes known that the climate sensitivity has increased or carbon sinks have been weakened, the optimal response is to have a rising path of CO_2 prices before and a rising but higher path after the tipping point (30, Figure 4, Panel D).

The International Governmental Panel does not look for the optimal carbon price but seeks for the most efficient carbon pricing response that ensures that a temperature cap of 2 °C is achieved. Intertemporal efficiency then demands that the carbon price must grow at a rate equal to the risk-adjusted interest rate (e.g. Appendix D, (15) or (16)). This suggests that the carbon price grows even faster than the economic growth rate and thus definitely does not decline.

5. Conclusion

We have shown that plausible assumptions about volatility and CCS result in an inexorably rising path for the SCC. The policy recommendation that follow from that finding is that it is a best to start with a significant CO_2 price and at the same time commit to a steadily rising path of CO_2 prices, not to declining prices, contrary to what is argued by Daniel et al. (3). The EZ-Climate outcomes of declining prices are strongly suggestive of an inefficient intertemporal policy pattern, and require assumptions/policies that are almost certainly counterfactual (declining damage rate uncertainty) or equally almost certainly technologically out of reach (complete reversal of all increases in CO2 concentration back to pre-industrial levels through CCS). And anyhow, in (3) the decline set in long after the energy transition has taken place and the economy is longer using fossil fuel. This reduces the policy relevance of their results even further, even accepting all their assumptions, the resulting decline in the CO_2 prices only kicks in the very distant future.

There are good economic and political reasons to commit to a steadily rising path of CO_2 prices. Going beyond our paper, this recommendation gives further benefits: only by credibly committing to such a path are corporations going to make the long run and irreversible investments that are needed to transition to the carbon-free economy. Uncertainty about future prices and about the timing of a transition will cause corporations to hold back investments as carbon-intensive capital stock then acquires an option value (31). A practical problem is that politicians tend to procrastinate and postpone carbon pricing and prefer subsidies to higher prices in fear of losing office. This leads to adverse Green Paradox effects: such second-best policies induce owners of fossil fuel reserves to extract too quickly and accelerate emissions and global warming rather than slowing it down (32, 33). Such political distortions might prevent the path of CO_2 prices be not high enough upfront. Credible commitment to a steadily rising path of prices is thus of paramount importance.

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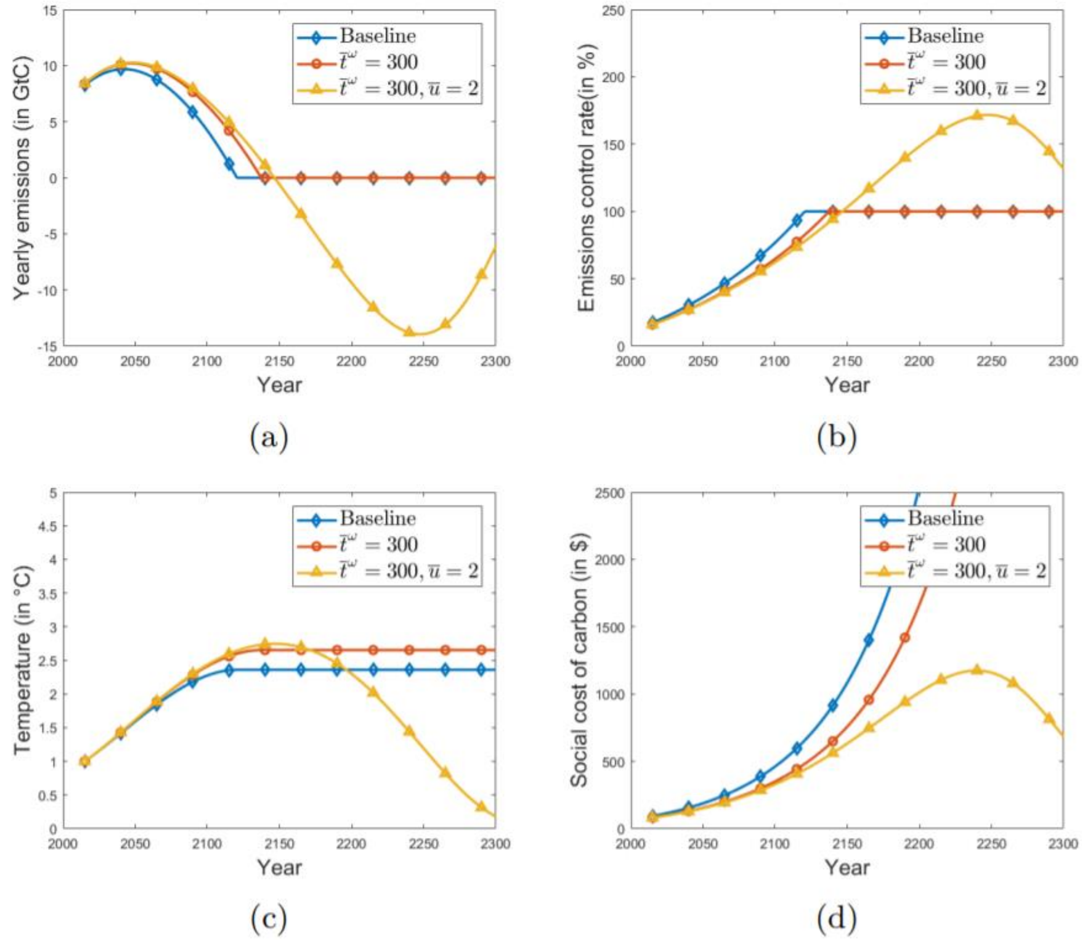

Fig. 1. Resolution of uncertainty and carbon removal under market-based calibration: expected time paths for emissions, the emissions control rate or abatement rate, temperature, and the optimal CO_2 price. The blue diamonds indicate the market-based variant with a constant volatility of the damage ratio. The orange dots indicate the variant with a volatility of the damage ratio declining linearly to zero in 300 years. Despite a preference for early resolution of uncertainty, carbon prices still rise with declining volatility albeit that carbon prices grow less rapidly (cf. orange dots with blue diamonds). With declining volatility of the damage ratio and additionally allowing the abatement rate to be larger than 1 (i.e. carbon removal), the carbon price rises and declines, but the decline does not occur until after 2200 (yellow triangles).

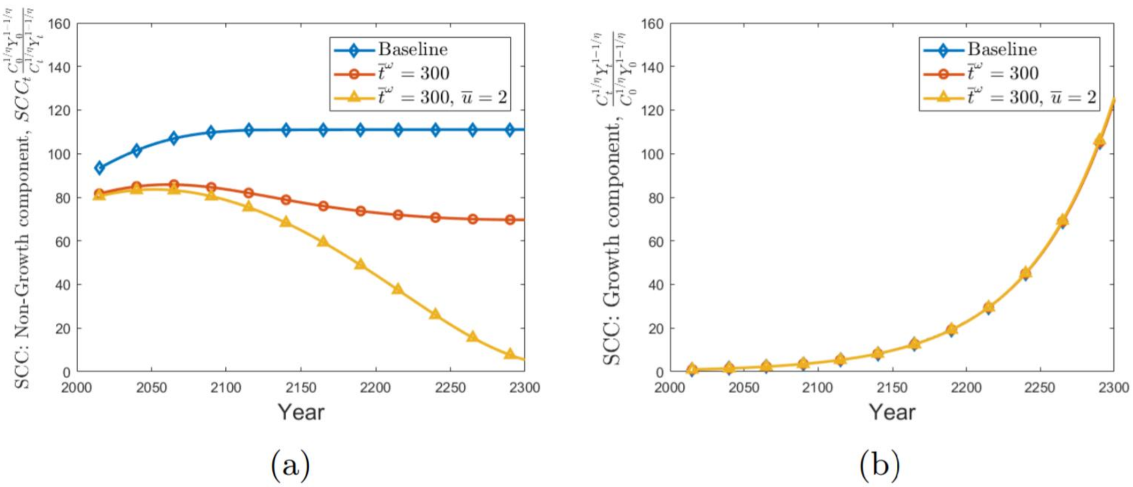

Fig. 2. Decompositions of the optimal CO_2 price over time under market-based calibration. This figure decomposes the CO_2 price into a growth related and non-growth-related term using Eq. [7]. The SCC or CO_2 price is equal to the product of panels (a) and (b) The temperature-related component of the price in Eq. [7] (panel (a)) rises initially a little over time if the volatility of the damage ratio is constant due to an increase in temperature (see blue diamonds) but declines if volatility declines over time and uncertainty is resolved (see orange dots) and continues to decline even further if CCS is permitted too (see yellow triangles). The growth-related component rises steadily over time. The main message is that the growth effect (panel (b)) dominates the temperature-related effect (panel (a)), so that the optimal CO_2 price must rise over time.

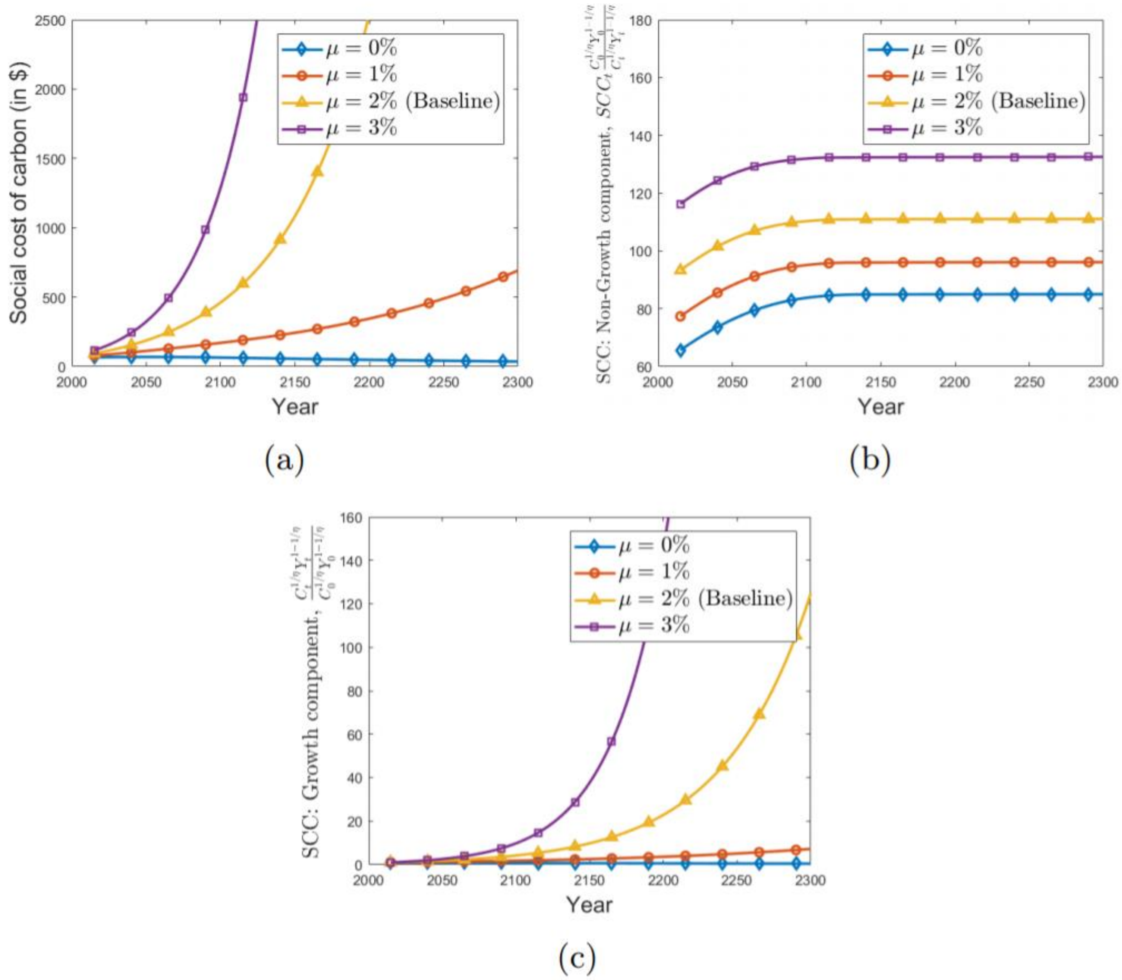

Fig. 3. Effect of the growth rate on the optimal CO_2 price or SCC. Panel (a) illustrates that the growth of economic activity is the most important driver of rising carbon prices. If the drift $\mu = 0$, economic activity shrinks on average due to the economic disasters, which leads to a declining path of the carbon price. A positive drift μ leads to rising carbon prices. The growth rate also curbs the discount rate, as the *EIS* in our calibration is larger than 1, and thus pushes up the carbon price. Panel (b) filters the growth of the economy out of the SCC, but due to a lower discount rate and *EIS* > 1 the purple line (squares) with highest drift is still above the others. Panel (c) gives the growth-related component of the SCC according to Eq. [7].

Preferences	Market based: $RA = \gamma = 7$, $EIS = \eta = 1.5$, impatience = β = 2%/year Ethical: $RA = \gamma = 2$, $EIS = \eta = 0.5$, impatience = β = 0.1%/year
Economy	Initial endowment: Y_0 = 80 trillion US dollars GBM: drift = μ = 2%/year, volatility = σ^Y = 3%/√year Disaster shocks: shape parameter of gamma distribution $\alpha = 10.5$ Mean size of disaster shocks: E[J] = 8.7%, Mean arrival rate of disaster shocks = λ = 0.035/year
BAU emissions	Initial flow of global emissions in BAU scenario: E_0 = 10 GtC/year Initial growth of BAU emissions: g_0^E = 1.8%/year Decline of the growth rate of BAU emissions: δ_E = 2.7%/year
Abatement cost	Current cost of full decarbonization: c_0 = 7.41% of initial GDP Rate of technological progress: c_1 = 1.9%/year Convexity parameter of the cost function: $c_2 = 2.6$ Maximum abatement: baseline $\bar{u} = 1$, CCS variant $\bar{u} = 2$
Temperature	Initial temperature: T_0 = 1 °C Transient climate response to cum. emissions: $TCRCE = \chi$ = 1.8 °C/TtC
Damage ratio	Convexity coefficient with respect to temperature: $\theta_T = 0.56$ Skew parameter for shocks: $\theta_\omega = 2.7$ Mean reversion of shocks: υ = 0.2/year Initial and mean steady-state value of shocks: $\omega_0 = \bar{\omega} = 0.21$ Variant with constant volatility: $\sigma_0^\omega = 0.05$, $\bar{t} \to \infty$ Variant gradual resolution of uncertainty: $\sigma_0^\omega = 0.05$, $\bar{t}$ = 300 years

Table 1. Calibration (with initial year 2015). Since damages and abatement expenditures are initially a small fraction of endowment, consumption is approximately 99.5% of endowment in 2015. Our calibration of Y_0 = 80 trillion US dollars follows (7) by calculating world consumption in US dollars using purchasing power parity instead of using exchange rates. Business as usual emissions are calibrated exogenously to match the DICE calibration (7) in the first century and then stabilize. The abatement cost function is also set up to match the DICE calibration (7).

Appendix A: More Details on the DE-Climate Model and Derivation of the Optimal SCC

All agents have identical preferences and endowments, so all the agents can be replaced by one representative agent. If $RA = \gamma$ and $EIS = \eta$, DE preferences of this agent follow recursively from

$$V_t = \max_{u_t} E_t\left[\int_t^\infty f(C_s, V_s)ds\right] \quad \text{with} \quad f(C,V) = \frac{\beta}{1-1/\eta}\frac{C^{1-1/\eta} - \left((1-\gamma)V\right)^{\frac{1}{\zeta}}}{\left((1-\gamma)V\right)^{\frac{1}{\zeta}-1}} \quad \text{for} \quad \zeta \neq 1, \tag{1}$$

where $\zeta = (1-\gamma)/(1-1/\eta)$ and $\beta > 0$ denotes the utility discount rate or rate of time impatience. The endowment follows the stochastic process

$$dY_t = \mu Y_t dt + \sigma^Y Y_t dW_t^Y - JY_t dN_t, \tag{2}$$

where W_t^Y is a standard Wiener process, μ and σ^Y are the drift and volatility of the Geometric Brownian motion component, N_t is a Poisson process with mean arrival rate λ, and J is a random variable and is the share of output destroyed if a disaster hits the economy. We assume that $x = 1 - J$ has a power distribution with probability density $f(x) = \alpha x^{\alpha-1}$, so $E\left[x^n\right] = \alpha/(n+\alpha)$ and $0 \leq E\left[J\right] = 1/(1+\alpha) \leq 1$ (cf. (34)). For all moments to exist, we assume that $\gamma < \alpha$.

Since temperature depends linearly on cumulative emissions, the dynamics of temperature are

$$dT_t = \chi(1-u_t)E_t dt, \tag{3}$$

where χ denotes the transient climate response to cumulative emissions or *TCRCE*. Business-as-usual emissions E_t grow at the decreasing rate $g_t^E = g_0^E e^{-\delta_E t}$, where $g_0^E > 0$ and $\delta_E > 0$ are constants. The damage ratio is given by

$$D_t = T_t^{1+\theta_T}\omega_t^{1+\theta_\omega} \quad \text{with} \quad d\omega_t = \upsilon(\bar{\omega} - \omega_t)dt + \sigma_t^\omega dW_t^\omega, \tag{4}$$

where ω_t follows a Vasicek (or Ornstein-Uhlenbeck) process with short-run volatility σ_t^ω, mean reversion υ and long-run mean $\bar{\omega}$, and W_t^ω is a standard Wiener process. Here θ_T controls the convexity with respect to temperature and θ_ω controls the skew of the shocks hitting the damage ratio. We have $\sigma_t^\omega = \max\left[(1-t/\bar{t}^\omega)\sigma_0^\omega, 0\right]$, so that volatility starts with σ_0^ω and falls to zero after $\bar{t}^\omega$ years to capture gradual resolution of damage uncertainty. Volatility is constant if $\bar{t}^\omega \to \infty$.

The Hamilton-Jacobi-Bellman equation for this stochastic optimal control problem is

$$0=\max_{u_t}\left\{f(C_t,V_t)+W_Y\mu Y_t+\frac{1}{2}W_{YY}(\sigma^Y Y_t)^2+W_t+W_T\chi(1-u_t)E_t+W_\omega\upsilon(\bar{\omega}-\omega_t)\right.$$
$$\left.+\frac{1}{2}W_{\omega\omega}\max\left[\left((1-t/\bar{t}^{\,\omega})\sigma_0^\omega\right)^2,0\right]+\lambda E\left[W\left((1-J)Y_{t-},T_t,\omega_t,t\right)-W(Y_{t-},T_t,\omega_t,t)\right]\right\},$$

[6]

where the value function $V_t=W(Y_{t-},T_t,\omega_t,t)$ depends on the three state variables and time and its partial derivatives are denoted by subscripts. We conjecture and have verified that the value function is of the form $V_t=g_tY_t^{1-\gamma}/(1-\gamma)$ with $g_t=h(T_t,\omega_t,t)$ and rewrite [6] accordingly as

$$0=\min_{u_t}\left\{\beta\zeta\left(g_t^{-1/\zeta}\left(\frac{C_t}{Y_t}\right)^{1-1/\eta}-1\right)g_t+(1-\gamma)\left(\mu-\frac{1}{2}\gamma(\sigma^Y)^2+\lambda\frac{E\left[(1-J)^{1-\gamma}\right]-1}{1-\gamma}\right)g_t+h_t\right.$$
$$\left.+h_T\chi(1-u_t)E_t+h_\omega\upsilon(\bar{\omega}-\omega_t)+\frac{1}{2}h_{\omega\omega}\max\left[\left((1-t/\bar{t}^{\,\omega})\sigma_0^\omega\right)^2,0\right]\right\}.$$

[6′]

The first-order optimality condition gives the optimal risk-adjusted CO_2 price, i.e.

$$SCC_t=-\chi\frac{\partial W_t/\partial T_t}{f_C(C_t,V_t)}=\left(\frac{\chi}{(1-\gamma)\beta}\frac{-h_T}{g_t^{1-1/\zeta}}\right)\left(C_t^{1/\eta}Y_t^{1-1/\eta}\right).$$

[7]

If *EIS* = 1, the optimal CO_2 price is proportional to aggregate consumption *C*. In general, the second part of Eq. [7] indicates that this price is proportional to a weighted geometric average of aggregate consumption and the endowment with the weight to consumption equal to 1/*EIS*. The first part of Eq. [7] indicates that this price depends on the shape of the reduced-form value function. With convex enough damages, this term increases in temperature. We solve the stochastic dynamic programming problem with finite differences and as check also by least squares and Monte Carlo simulation, see (35). The calibration of our model is in Table 1.

Appendix B: Sensitivity of Policy Simulations

In Fig. B1 we show that if we eliminate all uncertainty from the growth in the endowment and only have damage ratio uncertainty as in (3), the price still grows with time. This merely decreases the growth adjusted discount rate (since EIS>1) and increases in the expected growth of the economy, which leads to higher carbon prices. In Fig. B2 we take an intermediate case: volatility declines in one case at the base case speed (in 300 years to zero) and in the second case at a faster clip (in 100 years to zero). Again, the SCC rises over time in both cases, the more rapid decline in volatility does not reverse our basic tenet, the SCC needs to go up over time. However, the temperature-related component of the expression for the optimal SCC given in [7] now falls less markedly due to the offsetting effects of declining volatility and increasing temperature. Fig. B3 indicates that more convex relationships between the damage ratio and temperature lead to higher paths of the CO_2 price. The time pattern of the SCC does not really change (panel (b)).

Neither does changing the preference parameters individually: our basic conclusions remain valid. In Fig. B4 we show the impact of lowering the degree of risk aversion. This has almost no impact on the SCC since the impact on the growth-adjusted discount rate (which goes down with a lower RA and lower risk premia) is largely offset by an increase in the certainty-equivalent time path of future output given its expected value. Note that this result depends on the assumption that EIS>1. If we would have assumed that EIS<1, then decreasing RA would lead to a higher growth adjusted discount rate and a lower SCC. Changing the EIS while keeping RA constant does have a significant impact, as Fig. B5 shows: a lower EIS has no impact on the conversion from expected value to its certainty-equivalent values, but it does lead to higher growth adjusted discount rate and thus a lower SCC, but again without changing the time pattern: the SCC is still going up over time.

Figs. B6 and B7 present results for an *ethics-based* calibration of preferences. We follow (1) in taking an ethical stance against discounting the welfare of future generations and follow (20) in setting $RA = 2$ and $EIS = 0.5$, which corresponds to a coefficient of intergenerational inequality aversion of 2 (i.e. $1/EIS$). The main message is unaffected in spite of the switch to ethical preferences: declining volatility of the damage ratio does not lead to a declining path of carbon price unless it accompanied by an unrealistically large-scale adaptation of carbon removal that eliminates the whole temperature anomaly.

For clarity and simplification, we have assumed that business-as-usual emissions are exogenous. In Fig. B8 we show as a robustness check that our results regarding the rising path of carbon prices are unaffected if business-as-usual emissions are the product of the endowment and a declining carbon intensity and therefore are also influenced by endowment shocks.

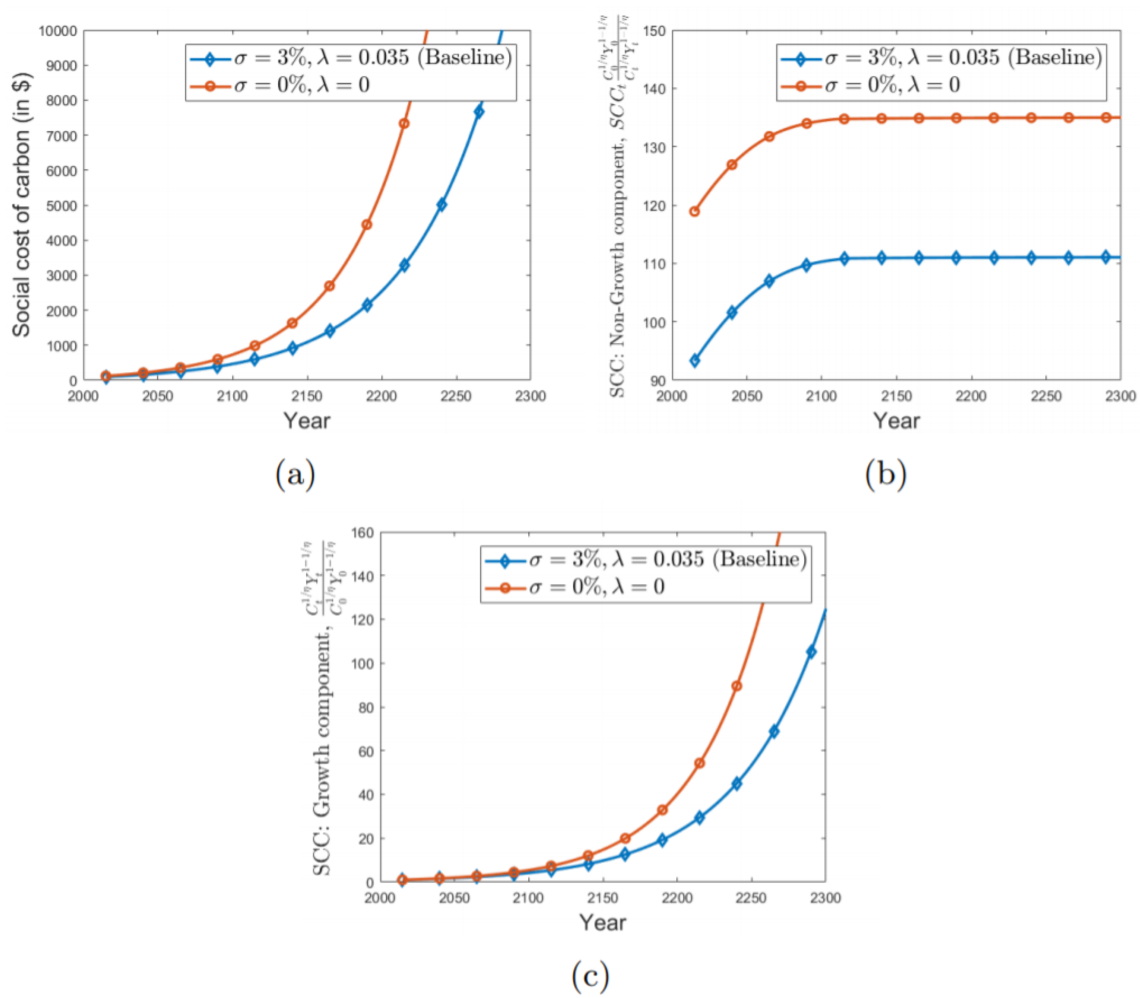

Fig. B1. Effects Without Macroeconomic Volatility or Macroeconomic Disaster Risk on Carbon Pricing. If all macroeconomic volatility and disaster risks are removed for our base run with constant volatility of the damage ratio, the optimal CO_2 prices increase. The reason for that is that in our calibration *EIS* > 1. According to Eq. [7], panel (b) gives the non-growth component and panel (c) gives the growth-related component of the CO_2 price.

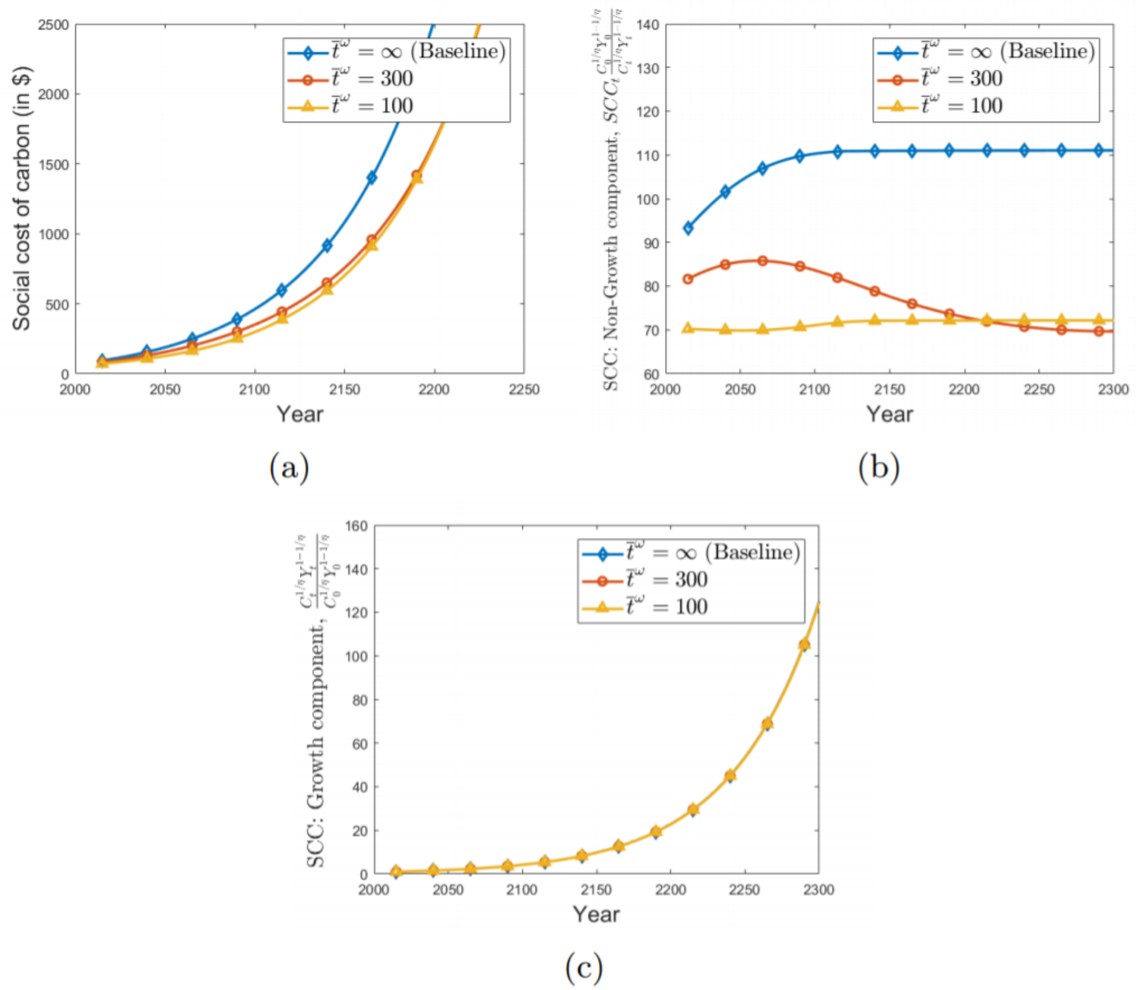

Fig. B2. More rapid decline of the volatility of the damage ratio. The effects of letting volatility of the damage ratio decline in one (yellow diamonds) rather than three centuries (orange dots) shows that the temperature-related component of the expression for the optimal SCC given in Eq. [7] now falls less markedly. In fact, panel (b) indicates that the optimal carbon price corrected for growth in economic activity now rises slightly, rather than declining eventually. This is the case since the effects of rising temperature and declining volatility in the first century approximately cancel out. Panel (c) givens the growth-related component of the SCC.

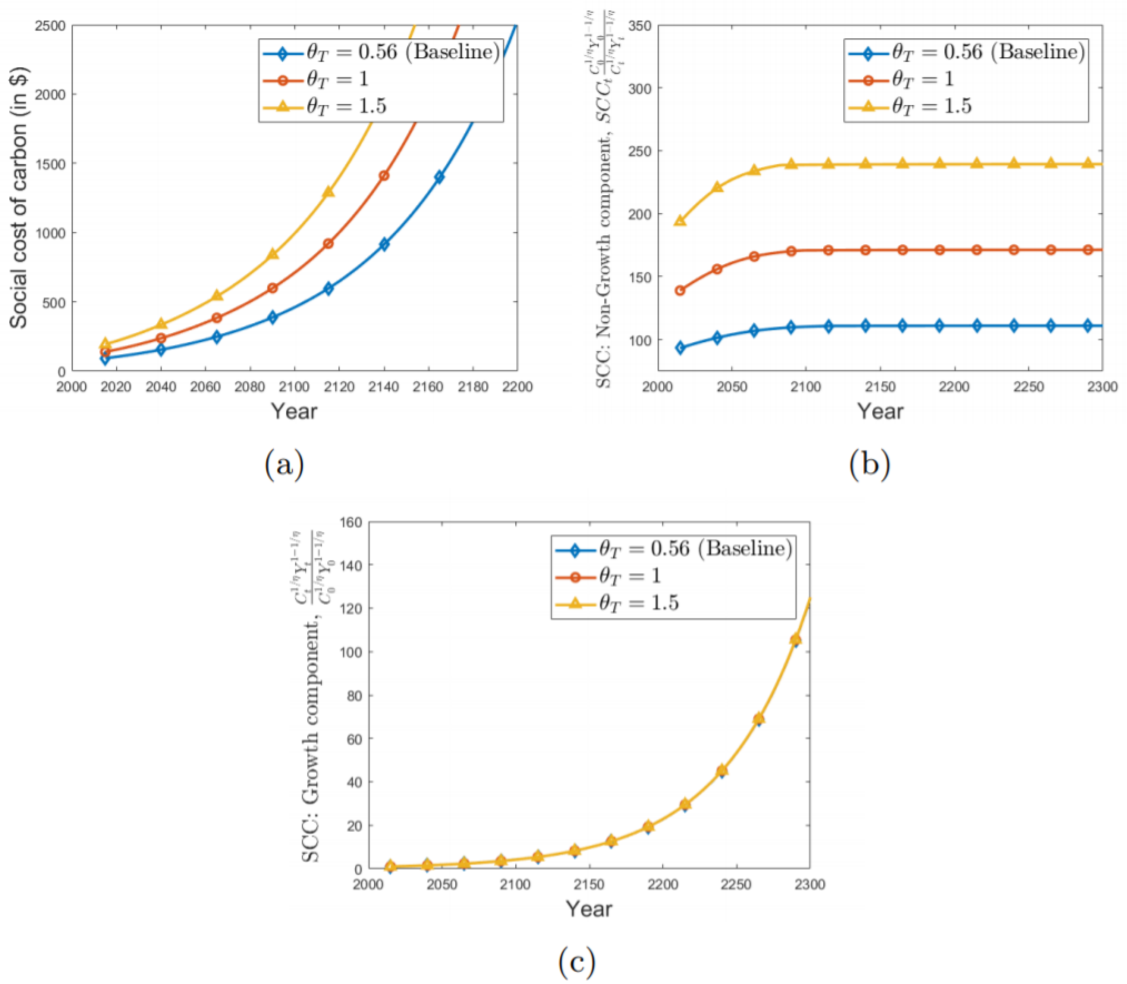

Fig. B3. A more convex relationship between the damage ratio and temperature. More convex relationships between the damage ratio and temperature lead to higher paths of the CO_2 price. This is most visible in the non-growth-related component of the price (panel (b)) rather than the growth-related component of the price (panel (c)).

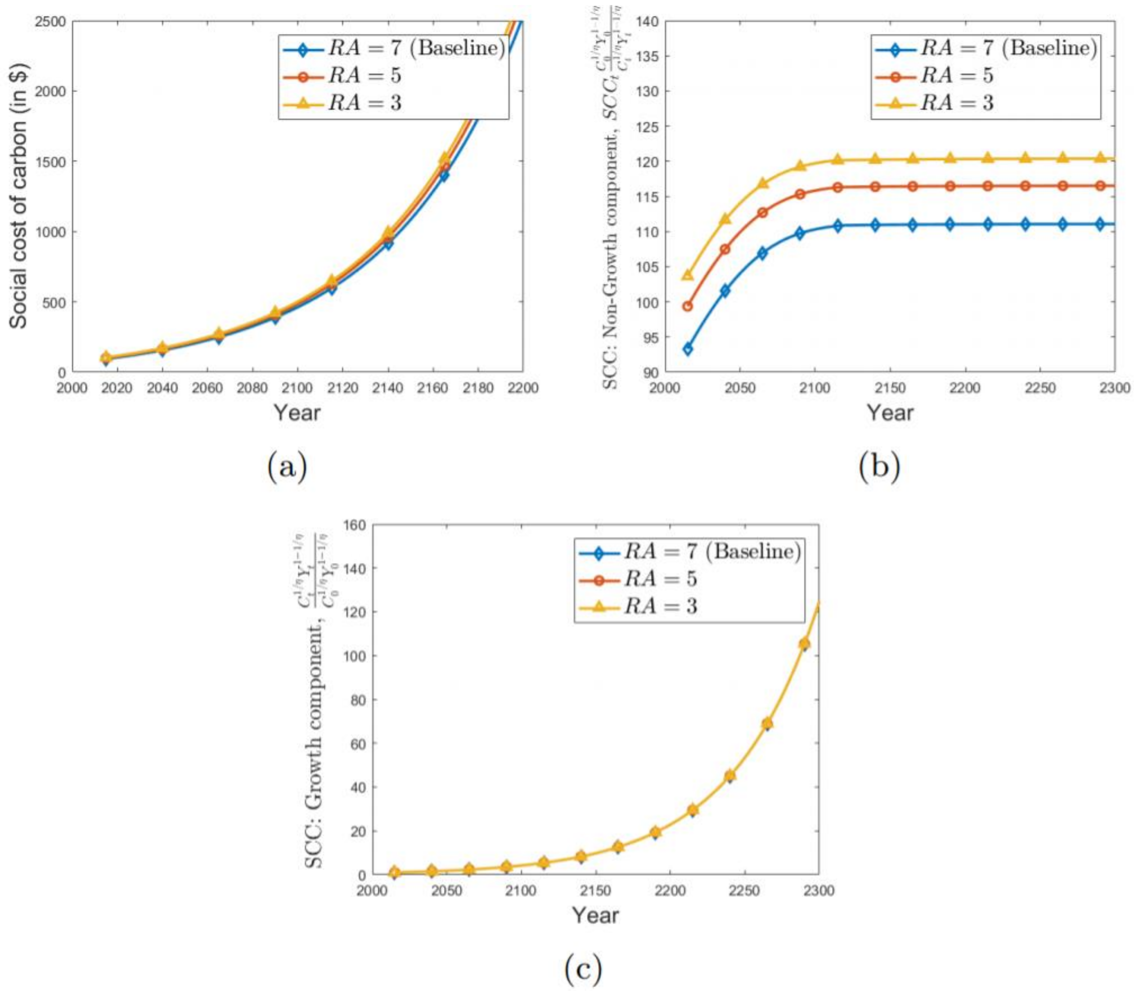

Fig. B4. Impact of Changing the Degree of Risk Aversion (γ). Lower risk aversion decreases the risk-corrected discount rate as in our calibration $EIS > 1$, hence increases the ratio of the optimal carbon price (or SCC) to the size of the endowment (panel (b)). This effect on the SCC is very modest compared with the effect of the growth of the endowment (panel (a)).

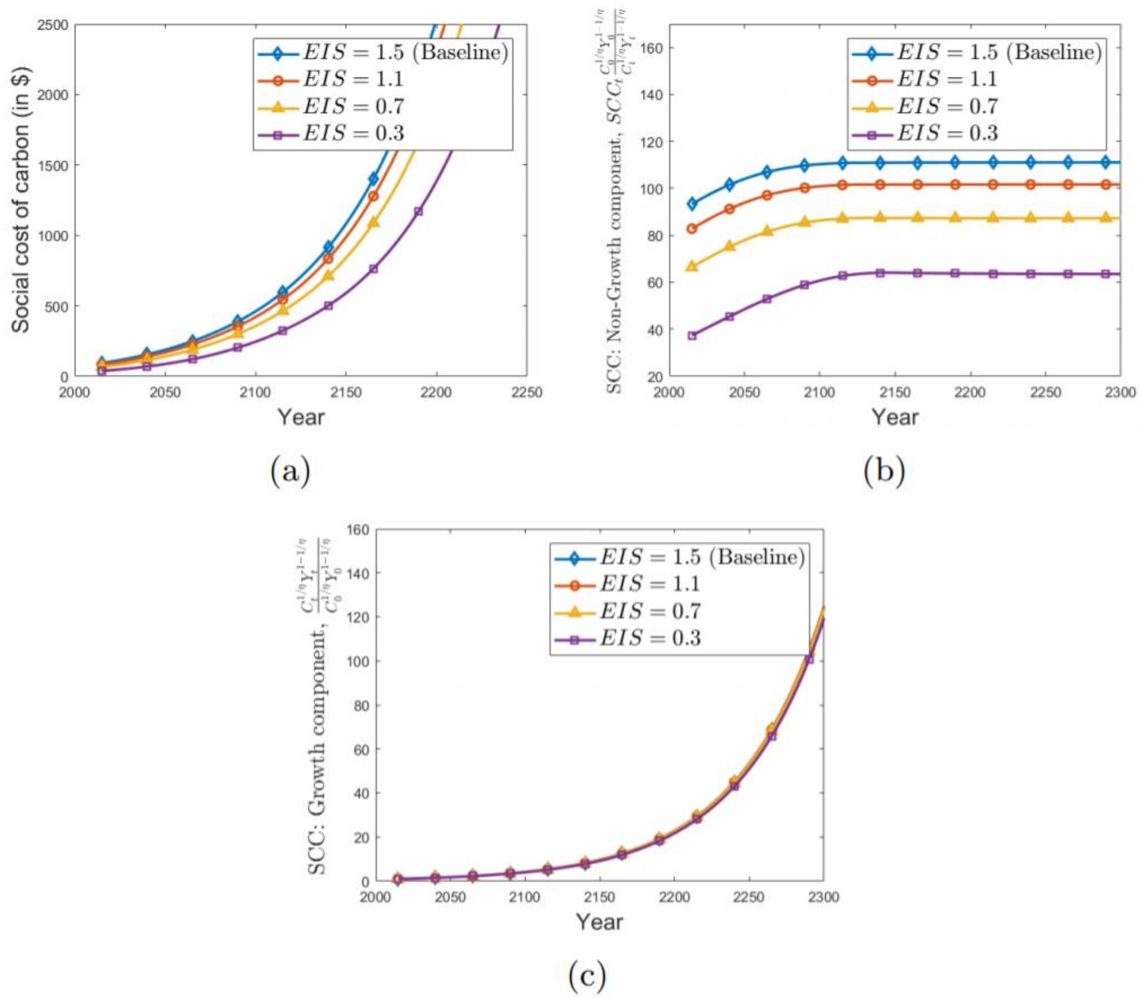

Fig. B5. Impact of Changing the Degree of Intergenerational Inequality Aversion (i.e. the Inverse of the Elasticity of Intertemporal Substitution). A lower *EIS* implies a higher degree of intergenerational inequality aversion. It reduces the ratio of the SCC to the size of the endowment, since it implies less willingness to sacrifice consumption today to curb global warming in the future provided future generations are richer than current generations. This has a bigger impact on the time path of the SCC than a lower RA shown in Fig. B4. Note that a lower IES also implies that the negative effect of growth uncertainty on the risk-adjusted discount rate is higher and thus that the positive effect of growth uncertainty on the SCC is bigger. This explains why the non-growth-related component of the SCC in Eq. [7] is bigger (panel (b)). The growth-related component of the SCC rises much more steeply (panel (c)) and swamps the non-growth-related component of the SCC.

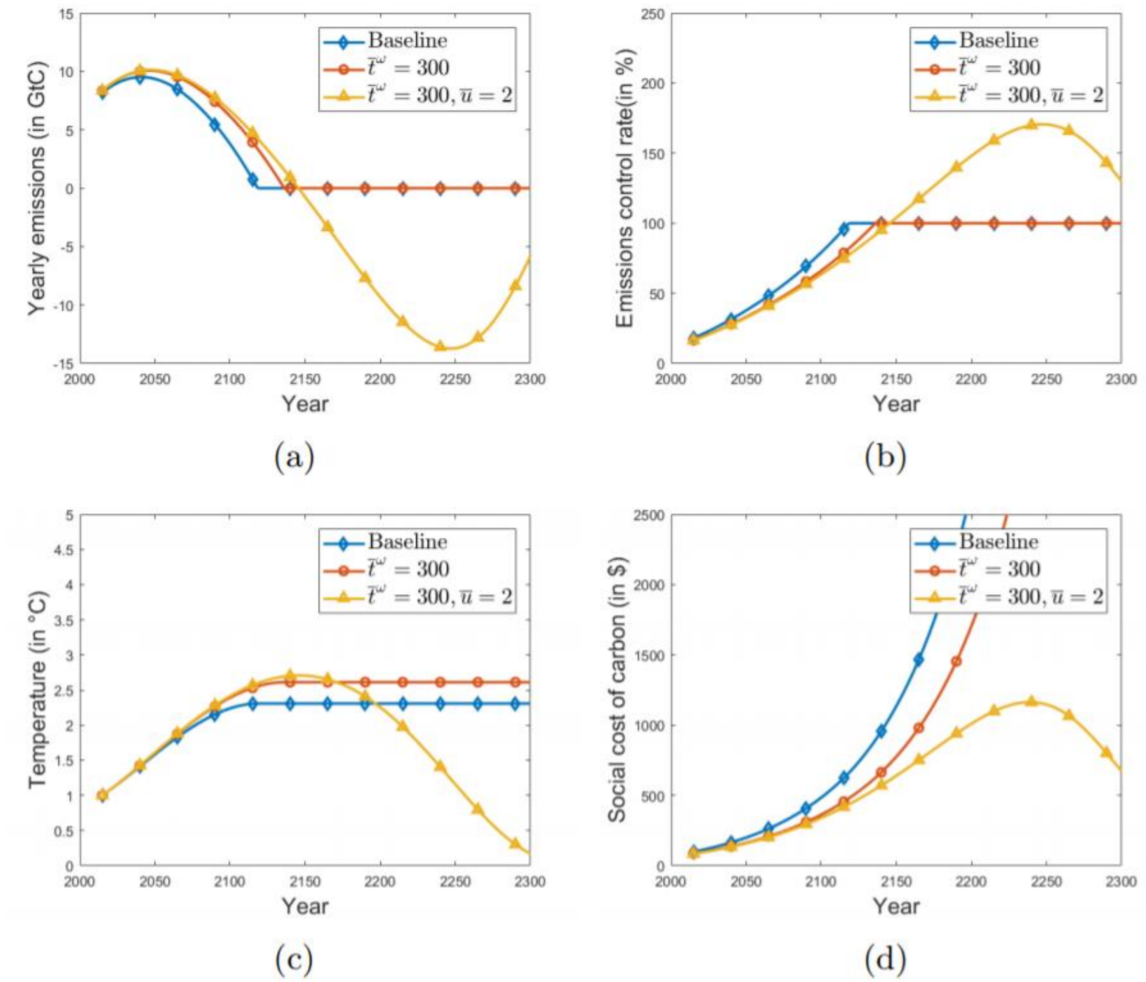

Fig. B6. Policy simulations with ethics-based preferences: risk aversion = γ = 2, elasticity of intertemporal substitution = η = 0.5, rate of impatience = β = 0.1% per year. The outcomes are very similar to the market-based calibration and therefore the conclusions of the main model are robust to this calibration. The reason for this is that the growth adjusted discount rate (i.e. the interest rate *plus* risk premium *minus* expected growth rate) with ethics-based preferences is close to that of the outcome in the market-based calibration.

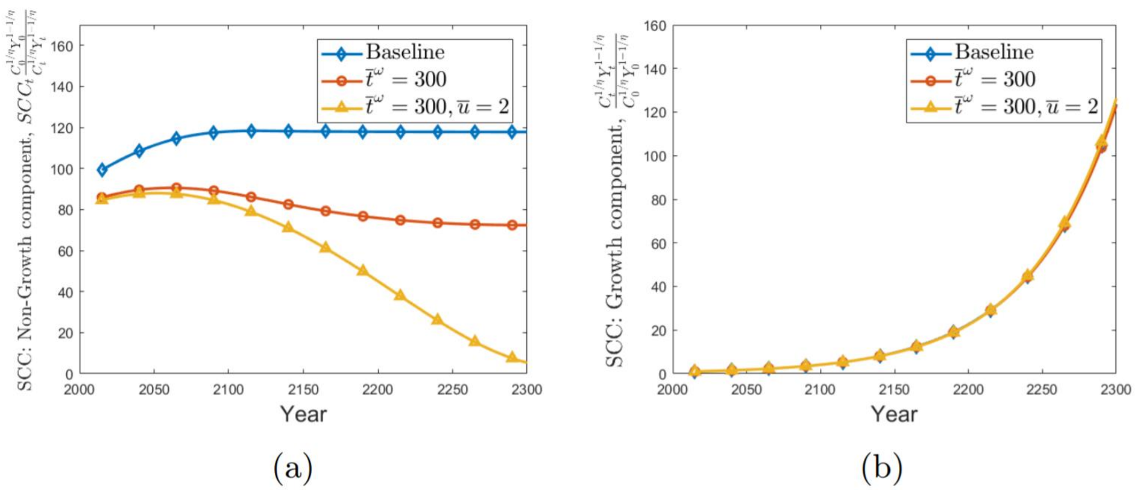

Fig. B7. Decomposition of optimal CO_2 price with ethics-based preferences. The same messages apply as with the market-based calibration. Panel (b) shows that the optimal carbon price rises under all three variants, because the endowment component dominates the temperature-related component in [7]. If the optimal carbon price is detrended for growth in economic activity, then it rises slightly over time when volatility is constant, it temporarily rises and then falls but not by much for declining volatility, and only falls and drops off to zero when volatility declines and carbon removal is allowed to unrealistically fully remove the temperature anomaly.

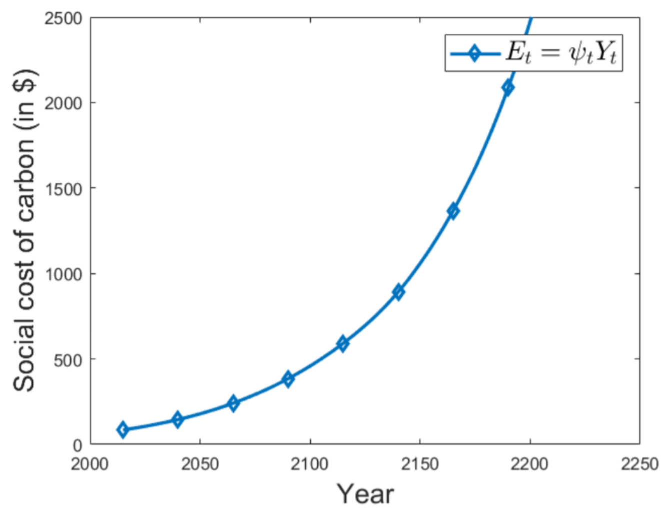

Fig. B8. Effects of Emissions Reacting Directly to Shocks in the Endowment. Here we relax the assumption that the time path of business-as-usual emissions, E_t, is exogenous. We assume that these emissions are proportional to the endowment, $E_t = \psi_t Y_t$, where ψ_t is the carbon intensity of endowment (in GtC per trillion $). We use the initial value $\psi_0 = 0.125$ and let the carbon intensity decline at a rate $\delta_t^{\psi} = -0.017 + 0.01\exp(-0.015t)$. This implies that initial carbon emissions are $E_0 = 0.0125 \times 80 = 10$ GtC per year. Despite that the SCC no longer decomposes as in Eq. [7], the qualitative nature of the time paths of the SCC and the ratio of the SCC to the endowment are very similar to those in the first variant of panel (d) of Fig. 1.

Appendix C: Learning by Doing in Mitigation and the Optimal Carbon Price

Learning by doing in renewable energy production is captured by letting the cost of renewable energy production decline in cumulative use of renewable energy. To make the point that a temporary renewable energy is needed alongside a growing carbon price (cf. (12)), we abstract from all uncertainty about the rate of economic growth and the damage ratio and from the possibility of carbon removal (i.e. $\bar{u}=1$). Instead of exogenous technical progress in abatement, we assume endogenous technical progress driven by learning by doing and therefore set the marginal cost mitigation to $c_0 e^{-c_1 X_t} u_t^{c_2} Y_t$, with X_t denoting the accumulated stock of knowledge in renewable energy production. The problem is thus to maximize $V_0 = \int_0^\infty e^{-\beta t} \frac{C_t^{1-1/\eta}}{1-1/\eta} dt$ subject to the equation for private consumption $C_t = \left(\frac{1-c_0 e^{-c_1 X_t} u_t^{c_2}}{1+T_t^{1+\theta_T} \omega_t^{1+\theta_\omega}} \right) Y_t$, the dynamics of temperature, $\dot{T}_t = \chi(1-u_t)E_t$, and the dynamics of learning by doing, $\dot{X}_t = u_t E_t$, where Y_t, ω_t and E_t follow exogenous time paths. The Hamiltonian function for this deterministic optimal control problem is

$$H_t \equiv \frac{1}{1-1/\eta} \left[\left(\frac{1-c_0 e^{-c_1 X_t} u_t^{c_2}}{1+T_t^{1+\theta_T} \omega_t^{1+\theta_\omega}} \right) Y_t \right]^{1-1/\eta} + \lambda_t \chi (1-u_t) E_t + \mu_t u_t E_t ,$$

where λ_t denotes the co-state for temperature and μ_t denotes the co-state for accumulated learning by doing knowledge. The first-order optimality conditions are

$$\frac{\partial H_t}{\partial u_t} = -\left(\frac{c_0 e^{-c_1 X_t} c_2 u_t^{c_2-1}}{1+T_t^{1+\theta_T} \omega_t^{1+\theta_\omega}} \right) Y_t C_t^{-1/\eta} - \lambda_t \chi E_t + \mu_t E_t = 0,$$

$$\beta \lambda_t - \dot{\lambda}_t = \frac{\partial H_t}{\partial T_t} = -(1+\theta_T) T_t^{\theta_T} \omega_t^{1+\theta_\omega} \left(\frac{1-c_0 e^{-c_1 X_t} u_t^{c_2}}{\left(1+T_t^{1+\theta_T} \omega_t^{1+\theta_\omega}\right)^2} \right) Y_t C_t^{-1/\eta} \text{ and}$$

$$\beta \mu_t - \dot{\mu}_t = \frac{\partial H_t}{\partial X_t} = \left(\frac{c_0 c_1 e^{-c_1 X_t} u_t^{c_2}}{1+T_t^{1+\theta_T} \omega_t^{1+\theta_\omega}} \right) Y_t C_t^{-\eta}.$$

Hence, policy makers set the marginal cost of mitigating emissions to the *SCC* defined by $P_t \equiv -\chi \lambda_t C_t^{1/\eta}$ *plus* the social benefit of learning or *SBL* defined by $S_t \equiv \mu_t C_t^{1/\eta}$, i.e.

$$c_0 e^{-c_1 t} c_2 u_t^{c_2-1} Y_t = P_t E_t + S_t E_t .$$

The *SCC* or carbon price follows from integrating $\dot{P}_t = r_t P_t - \chi(1+\theta_T) T_t^{\theta_T} \omega_t^{1+\theta_\omega} \left(\frac{1-c_0 e^{-c_1 X_t} u_t^{c_2}}{\left(1+T_t^{1+\theta_T} \omega_t^{1+\theta_\omega}\right)^2} \right) Y_t$ backwards in time and yields the social cost of carbon (cf. Eq. [7]):

$$P_t = SCC_t = \chi\left(\int_t^{\infty} e^{-\int_t^s (r_{s'} - \dot{C}_{s'}/C_{s'})ds'}\left(\frac{\partial D_s / \partial T_s}{1+D_s}\right)ds\right)C_t, \qquad [S1]$$

where the rate of interest follows from the Keynes-Ramsey rule as the rate of time impatience plus the growth rate of consumption divided by the *EIS*, i.e.

$$r_t = \beta + \eta^{-1}(\dot{C}_t / C_t).$$

The *SBL* follows from integrating $\dot{S}_t = r_t S_t - \left(\frac{c_0 c_1 e^{-c_1 X_t} u_t^{c_2}}{1+T_t^{1+\theta_T}\omega_t^{1+\theta_\omega}}\right)Y_t$ backwards in time and yields

$$S_t = SBL_t = \left(\int_t^{\infty} e^{-\int_t^s (r_{s'} - \dot{C}_{s'}/C_{s'})ds'}\left(\frac{-\partial A_s / \partial X_s}{1-A_s}\right)ds\right)C_t. \qquad [S2]$$

The renewable energy subsidy or *SBL* thus corresponds to all the present and future marginal benefits in terms of lower mitigation costs resulting from using one unit of mitigation more today. Like the *SCC*, it is also proportional to aggregate consumption. The main insight is that in more disaggregated models of energy use and climate change emissions should be priced at the *SCC* whilst mitigation should be subsidized at the *SBL*. The *SBL* typically shows a temporary spike to quickly move the economy to green production. If the *SBL* is added to the *SCC* as in (3) to obtain a "carbon price", this gives the misleading impression that the carbon price has a spike and then starts declining. In fact, what happens is that the carbon price and the renewable energy subsidy are intermingled. They must be separately implemented for the right economic incentives.

Numerical simulations of a comparable integrated assessment model suggest that it is optimal to have a temporary spike for the renewable energy subsidy for a few decades and a carbon tax that rises steadily for more than a century (see Figure 2 in (12)). For our model we also get a slowly rising carbon tax and a temporary renewable energy subsidy. If one examines the time path of the carbon tax plus the renewable energy subsidy, then one might get the erroneous impression that the carbon tax rises before it falls.

Appendix D: Efficient Carbon Pricing with a Temperature Cap

To show the effects of a temperature cap, we for simplicity also abstract from all uncertainty. Furthermore, we will abstract from the damage ratio so that all concerns about global warming are captured by the temperature cap. We will abstract from learning by doing in mitigation. The problem is thus to maximize $V_0 = \int_0^{\infty} e^{-\beta t}\frac{C_t^{1-1/\eta}}{1-1/\eta}dt$ subject to $C_t = (1 - c_0 e^{-c_1 t} u_t^{c_2})Y_t$ and $\dot{T}_t = \chi(1-u_t)E_t$, where Y_t and E_t follow exogenous time paths. The optimality conditions for this optimal control problem are $-C_t^{-1/\eta} c_0 e^{-c_1 t} c_2 u_t^{c_2 - 1} Y_t - \chi E_t \lambda_t = 0$ and $\beta\lambda_t - \dot{\lambda}_t = 0$, where λ_t denotes the co-state variable corresponding to temperature. It follows that policy makers set the marginal cost of mitigating emissions to the carbon price defined by $P_t \equiv -\chi \lambda_t C_t^{1/\eta}$, i.e.

$$c_0 e^{-c_1 t} c_2 u_t^{c_2 - 1} Y_t = P_t E_t .$$

The carbon price is *TCRE* = χ times the disutility of emitting one ton of carbon, converted from utility units into dollars by dividing by marginal utility $C_t^{-\eta}$. It follows that the efficient price of carbon must rise at a rate equal to the rate of interest, i.e. $\dot{P}_t / P_t = r_t$, where the rate of interest is as in Appendix C. The carbon price is pinned down by the condition that at the end of the fossil period, say at time $\bar{t}$, it must equal to the cost of full de-carbonization, i.e.,

$$P_{\bar{t}} = c_0 e^{-c_1 \bar{t}} c_2 Y_{\bar{t}} / E_{\bar{t}} \quad \text{and} \quad P_t = \exp\left(-\int_t^{\bar{t}} r_s ds\right) P_{\bar{t}}, \quad \forall 0 \le t \le \bar{t}. \qquad \text{[S3]}$$

This is called the Hotelling rule for the efficient carbon price under a temperature cap. It is straightforward to show that, if the damage ratio is an increasing function of temperature, the efficient growth rate of the carbon price is somewhere in between the growth rate of the economy and the interest rate (15). With a temperature cap the optimal carbon price grows even faster as the interest rate is bigger than the rate of economic growth and, furthermore, the energy transition is consequently more quickly completed.

Progress has been made by Gollier (16) on extending this Hotelling rule for uncertainty both about the future growth rate of the economy and emissions and for uncertainty about future marginal abatement costs. The growth rate of carbon prices is then equal to the safe interest rate plus ϕ times the risk premium, where ϕ is income elasticity of marginal abatement costs (correlation between marginal abatement costs and consumption growth). Gollier (16) argues that ϕ > 0, so that prosperity is the main source of uncertainty: in future states of nature where prosperity and emissions are higher than expected, the economy must abate more to stay below the cap and therefore marginal abatement cost is higher than expected and beta must be positive. He therefore recommends an *even higher rate of growth in the carbon price* than the safe rate of interest (around 1% per year), i.e. 3.8% per year (excluding the annual correction for inflation). This rate is lower than the rate used in many integrated assessment models (5% to even 12% year) but high enough to satisfy dynamic efficiency as it exceeds the rate of economic growth. Hence, carbon pricing takes much place much more upfront than in these integrated assessment models. The rate of growth of carbon prices is still higher than the rate of economic growth (say 2% per year) that it would be if welfare is maximized subject to global warming damages as in Eq. [7].