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Funding Shocks and Credit Quality

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Abstract

Some credit booms, though by no means all, result in financial crises. While risk-taking incentives seem a plausible cause, market participants do not appear to anticipate increasing risk. We show how credit expansions driven by credit supply shocks may be misunderstood as productivity driven, due to the opacity of bank balance sheets. Large funding shocks may induce some intermediaries to scale up speculative lending, distorting price signals. Other banks and firms may misjudge actual profitability, reinforcing the credit expansion. Similarly, at times of low saving supply credit may be inefficiently low, and speculative assets underpriced.

1. Introduction

In the classic view of business cycle the volume of credit is driven by real shocks, and financial intermediation affects the cycle only via contractual frictions. However, recent evidence points to an independent role of credit supply in explaining output fluctuations (Krishnamurthy and Vissing-Jorgensen, 2012a; Mian et al., 2017a). Credit booms with weak productivity are more likely to end in financial crises (Gorton and Ordoñez, 2016; Mian et al., 2017b), suggesting instability may stem from excess lending relative to productive demand.

While a diffused view holds that the recent credit crisis was caused by deliberate risk taking, excess credit may also have resulted from economic agents overlooking the build up of risk. In the credit boom of the early 2000s, financial prices such as bank equity returns and credit spreads suggest that investors did not anticipate rising losses (Baron and Xiong, 2017; Krishnamurthy and Muir, 2016). Studies on historical credit booms confirm that risk accumulation is often not recognized, so financial instability comes as a surprise (Gennaioli et al., 2015; Reinhart and Rogoff, 2009; Richter et al., 2017). A natural question is why most investors and banks do not recognize increasing risk.

While behavioral drivers likely play a role, even rational agents may be unable to correctly assess the state of the economy. This paper offers a rational explanation for episodes of excess credit, in which deliberate risk taking by some agents may be amplified via an imperfect inference of underlying fundamentals by other players. It thus reconciles the risk shifting view with the evidence of market participants underestimating the underlying risk during booms.

In the model strong funding supply can induce banks to engage in speculative lending that distorts asset prices. Because bank balance sheets are opaque other agents are unable to disentangle demand and supply effects and may thus misjudge high prices as reflecting strong economic fundamentals. As a result, they may overestimate the quality of investment opportunities, amplifying the credit expansion. Likewise, when funding supply is weak, agents may underestimate the quality of opportunities and hence underinvest.

We study the investment choices of two types of agents: large intermediaries informed about aggregate productivity, and small uninformed firms. We think of the former as global intermediaries with access to a broader set of information on international capital markets. global intermediaries receive insured debt funding and privately choose the levels of productive lending and speculative investment in a speculative asset. An aggregate shock determines the level of available bank funding. Uninformed firms cannot observe the realization of this shock nor the level of aggregate productivity. However, as the publicly observed price of the speculative asset depends on the productivity, it conveys some information to the less informed agents.

In equilibrium each bank chooses between a solvent or a risk shifting strategy. The latter entails a higher exposure to the speculative asset relative to an optimal choice of an unlevered investor and may lead to bank default. The solvent strategy involves less risky investment, so that losses never compromise debt repayment.

Under low funding supply the speculative asset is under-priced, it reflects the available cash in the market and the relative marginal returns of the two investment opportunities. Under moderate funding supply all global intermediaries follow the solvent strategy and their exposure to the risky asset is justified by its fundamental value. Consequently, the price of the asset reflects the aggregate productivity. Under large funding supply, banks are able to reach high leverage, which encourages risk shifting. The resulting speculative investment results in the asset being overpriced.

The same asset price may result from speculation investment in a low productivity and high funding state and from a solvent equilibrium in a high productivity and moderate funding state. Since the firms cannot observe the funding shock, their inference is distorted. They may be unable to determine whether the observed price indicates a good boom, characterized by high productivity and solvent investment by global intermediaries or a bad boom, with a low productivity and excessive risk taking driven by an abundant supply of bank funding. If the expected payoff of the speculative investment is sufficiently sensitive to changes in aggregate productivity, firms underestimate productivity when funding supply is moderate and overestimate it when supply is abundant.

If uninformed agents are unlevered, the inference problem leads to a symmetric amplification: they over-invest when funding supply is high, and under-invest when it is low. In contrast, if the less informed agents are also levered, high uncertainty regarding the true productivity may alter their incentives. We study this possibility by extending our setting to include local banks, which similarly to the uninformed firms are unable to observe the aggregate productivity and funding flows towards global intermediaries. The distorted inference may induce them to invest excessively, as they do not internalize the losses in case of default. The result is a form of *uninformed risk shifting* where lending decisions are efficient only if productivity is high, but lead to widespread default when high asset prices are caused by supply-driven speculation. Their risk choice represents an induced rather than fully deliberate form of risk taking, and amplifies the direct effect of excess funding on large banks.

We next consider how a national regulator may promote better lending by local banks. This requires increasing capital requirement or imposing liquidity reserves when speculative prices are so high as to induce local bank risk-shifting. The key trade off for the regulator is between allowing for excessive lending by local banks and inducing an inefficient contraction. The efficient solution cannot be implemented when uncertainty is too high.

The inference problem faced by investors in the model is analogous to the identification problem faced by an empirical researcher seeking to disentangle supply and demand effects. Quite unfairly to empirical researchers, rational agents are assumed to know all probability distributions and most parameters. In an extension we allow the uninformed agents to also observe the total investment by banks. We show that once agents observe both asset prices and credit volume they can precisely infer the underlying credit demand and supply. In reality the opacity of bank assets, imprecise measurement and gradual learning is likely to allow confusion to persist even when both price and volume signals are observed. We show that if uninformed agents face additional uncertainty regarding banks' risk shifting incentives (for instance, if they cannot observe bank's funding and it's exposure to legacy assets), the imperfect inference may persist even when agents observe both credit quantity and speculative price.

The next section discusses the related evidence and theoretical contributions. Section 3 introduces the baseline model and offers the main results. Section 4 discusses the investment problem by uninformed local banks and the optimal policy response of a local regulator. Section 5 introduces the additional signal and additional source of balance sheet opacity. Section 6 concludes

2. Related Literature

Recent evidence on credit booms suggests that abundant bank funding is more likely to lead to a crisis. High credit growth is one of the best predictors of bank distress (Borio, 2014; Reinhart and Rogoff, 2009; Schularick and Taylor, 2012). The quality of credit tends to deteriorate during booms with weak productivity growth, suggesting endogenous build-up of risk in the expansion phase (Bolton et al., 2016; Dell'Ariccia et al., 2012; Greenwood and Hanson, 2013). Credit booms characterized by surging house prices and loan-to-deposit ratios suggest an imbalance between new investment and funding volumes, and are more likely to end in crises (Richter et al., 2017). In contrast, credit expansions accompanied by high and sustained productivity growth are unlikely to be followed by a crisis (Gorton and Ordoñez, 2016).

While credit supply may be boosted by demographic trends, safety demand and capital inflows, it is also enhanced by financial liberalization and bank deregulation. These trends were believed to attenuate financial frictions, enhancing access to finance and boosting growth. Yet evidence on the recent credit expansion suggests it was driven by abundant bank funding rather than a relaxation of borrowers' collateral constraints (Justiniano et al., 2015).

US evidence suggests that local credit supply shocks boosted house prices rather than local productivity (Mian and Sufi, 2009; Mian et al., 2017b), and led to a sharper cycle correction in the bust (Di Maggio and Kermani, 2015). The result is an amplified business cycle, with higher growth in output, employment and house prices in good times and sharper declines in recessions. The model offers a rationalization of recent evidence on the quality of credit boom, in particular on the combination of rapid credit expansion with weak productivity growth at times of high asset values.

A prominent view why rising risk may not be properly anticipated is that a collective misjudgement reflects diffused irrationality. While models with overconfident agents can produce rising leverage and asset prices under marked differences in opinions (Geanakoplos, 2010), there is little evidence of wide skepticism over risk during the recent boom, even among bankers (Baron and Xiong, 2017). The nonfictional book "The Big Short" (Lewis, 2011) describes how hardly any investor realized even late in the boom how risky lending had become. In fact, many of the few who did had autistic traits that may have made them less prone to social herding.

Behavioral rules of posterior belief formation such as representativeness heuristics Gennaioli et al. (2015) do explain how market participants may underestimate the chance of a crisis in an economic expansion. Without denying any role for limited rationality, we seek to offer a rational benchmark on why public inference may be confused.

Our work relates to the literature on overconfidence arising via rational learning (Pástor and Veronesi, 2003; Biais et al., 2015). Since market participants who observe the improving economic conditions during the boom phase may be unable to recognize the growing risk, misinterpreting high asset prices as reflecting buoyant productivity. Our insight is that opacity of bank balance sheets (perhaps justified by optimal risk sharing (Dang et al., 2017)) may be an important factor obstructing the interpretation of price and quantity signals by market participants. A related effect is suggested by Thakor (2016), who shows that uncertainty over banker quality may result in under- or overestimation of risk, and by Lee (2016) where investors are unable to recognize the quality of bank lending by observing bank funding demand alone.

Abundant credit supply may result from capital inflows driven by safety needs (Caballero and Krishnamurthy, 2008), demographic trends or secular stagnation (Eichengreen, 2015; Doettling et al., 2017). A relaxed regulatory stance may also contribute, though this view raises the question why regulators become more confident about risk.

Our work is related to the literature suggesting a link between low interest rates and risk-taking (Dell'Ariccia et al., 2017; Jiménez et al., 2014). Recent work explains possible channels when rates are exogenously low (Acharya and Plantin, 2017; Dell'Ariccia et al., 2014; Martinez-Miera and Repullo, 2017). Our setup suggests low rates may reflect abundant funding supply, which affects risk incentives directly.

The analysis adopts a partial equilibrium setting to illustrate a possible channel for distorted inference from financial prices. A simple framing enables to interpret some empirical finding on credit cycles. We abstract from bank funding costs, and focus on the volume of available funds. We assume suppliers of bank funding are insured or able to avoid losses (running is safe if there is adequate insured deposit funding). Savings appear fairly inelastic to interest rates, and a segmented demand for safe assets may eliminate an important price signal (Krishnamurthy and Vissing-Jorgensen, 2012b; Gorton et al., 2012).

A more complete model may enable to study welfare issues related to long term speculative assets that reflect pure rents, such as land or real estate (net of their productive component). When funding supply and profitable demand are not too far the asset is fairly priced, revealing the true value of productivity and contributing to efficient lending. When risk shifting triggered by excess or scarce credit supply causes mispricing, it distorts both incentives and rational inference on productive investment.

3. The Model

There are two dates ($t = 0, 1$) and two types of active agents: a unit mass of large (global), informed banks and a unit mass of uninformed firms (later we consider uninformed local banks). At $t = 0$ there is uncertainty about productivity and savings supply.

Assets

There are two assets: productive investment, available to all agents, and a speculative asset in fixed supply (eg. land and real estate), available only to large banks [1]. Each unit x_i invested by an agent i at $t = 0$ into the productive technology yields $f(x_i) = \alpha x_i^{\gamma}$ at $t = 1$, where productivity α is drawn from a uniform distribution $\alpha \sim U[\underline{\alpha}, \overline{\alpha}]$. Large banks receive a precise signal about the realized productivity α at $t = 0$, other agents know only the distribution. We focus on the case when $\gamma = \frac{1}{2}$ (in the appendix we discuss the general formulation with $\gamma \in (0, 1)$)

The payoff of the speculative asset at $t = 1$ depends on the aggregate state of the economy. The probability of the good state of the economy increases in the $t = 0$ aggregate productivity. An interpretation is that the end date return represents the long term value of the durable asset. From ex-ante perspective, the asset value is uncertain and correlated with current productivity (long term value depends on future prospects, which are more likely to be high when the current fundamentals are strong). The payoff is drawn from the following binomial distribution:

$$y \rightarrow \begin{cases} Ry & \text{with prob. } q(\alpha) \\ 0 & \text{with prob. } 1 - q(\alpha) \end{cases}$$

Where $q'(\alpha) > 0$ a proxy for the persistence of the productivity shock. We assume that the upside payoff of the asset is not too high relative to the productivity of the technology.

We denote the amount of the asset purchased at $t = 0$ by bank i as y_i. He buys it from the initial asset owners, who are uninformed and derive utility only from $t = 0$ consumption, so that they accept any positive price (thereafter we refer to it as "investment in the speculative asset"). Thus, the asset price p is determined by banks' demand at $t = 0$. Uninformed firms can observe the price and may use it to infer the realized productivity.

Funding supply

Available bank funding s represents aggregate savings from global capital markets. Large banks choose how much to accept as deposits, with any residual placed in private storage by the savers.

[1] Allowing uninformed players to invest in the speculative asset does not affect the main results. Due to asymmetric information about asset quality, smaller agents are unable to assess whether the asset is fairly priced. Since in equilibrium, the risky asset is overpriced with positive probability, the expected return of the asset is always lower than the return on storage for the uninformed agents.

We assume that bank deposits are insured. The bank funding cost, thus equals the return of private storage, which is normalized to 1. The amount of funding available to large banks is drawn at $t = 0$ and is private information/unverifiable (??):

$$s = \begin{cases} s^L & \text{with prob. } \rho \\ s^H & \text{with prob. } 1 - \rho \end{cases}$$

We assume $s^L > 1$, so that the lowest feasible funding supply is always larger than the speculative asset supply. Uninformed firms are endowed with a fixed amount of own capital $c_j = c$.

Impaired assets

Global intermediaries enter date $t = 0$ holding λ of unreported impaired assets, which generate certain losses at $t = 1$. For simplicity we assume that banks are homogeneous with respect to their holdings of this legacy asset. In the subsequent Sections we introduce uncertainty over the size of future losses to study the additional source of balance sheet opacity.

To ensure limit the cases that we need to consider in the solution the following restrictions are introduced on the parameters.

Assumption 1. *The relation between the probability of the good state of the economy at $t = 1$ and aggregate productivity at $t = 0$ satisfies the following restrictions:*

A. $q^3(\alpha) - 2q(\alpha) + 1 - \frac{4\lambda}{\alpha} > 0 \ \forall \alpha, \lambda$

B. $1 + q(\alpha)q(\alpha) < q'(\alpha)\frac{\alpha}{2} \ \forall \alpha$

C. $1 - q(\alpha) < q'(\alpha)(\frac{\alpha}{2} - \lambda) \ \forall \alpha, \lambda$

D. $R < (\frac{\alpha}{2})^2 \ \forall \alpha$

Assumption 1.A requires that the probability of a good state of the economy is not too large. Assumptions 1.B and 1.C ensure that the persistence of the productivity shock is sufficiently high. Assumption 1.D limits the upside return of the speculative asset.

Timing

The timing is as follows:

- At $t = 0$:
  - Aggregate productivity α is realized and observed by large banks
  - Large banks face funding supply s and choose their portfolio
  - All funding not used by banks is stored by savers
  - Uninformed firms observe the asset price p, and choose their productive investment
- At $t = 1$:
  - The state of the economy is realized
  - All assets pay off
  - Banks or deposit insurance pay back depositors

3.1. Investment by large banks

After observing aggregate productivity and his funding, each large bank chooses an investment into productive lending (x_i) and the long term asset (y_i) to maximize expected profits subject to funding and no-short-selling constraints.

Unlevered investment benchmark

We here define as benchmark the optimal investment by an unlevered informed investor with endowment s who face unlimited liability. Unlevered investors can store (equivalent to a levered bank refusing some available funding). Their productive investment x_i, speculative asset purchases y_i and storage z_i is obtained by solving:

$$max_{x_i,y_i,z_i}\ \alpha\sqrt{x_i} + q(\alpha)Ry + z - s - \lambda$$

subject to:

$$x_i + py_i + z_i = s \qquad \text{(budget constraint)}$$

$$x_i \geq 0,\ y_i \geq 0,\ z_i \geq 0 \qquad \text{(no-short-selling constraint)}$$

The optimal investment into productive technology equalizes the marginal return with the opportunity cost of capital. In this setting, the relevant opportunity cost depends on the price of the speculative asset, taken as given by atomistic investors.

$$x_i^* = \begin{cases} \min[(\frac{\alpha p}{2q(\alpha)R})^2, s] & \text{if } p < q(\alpha)R \\ \min[(\frac{\alpha}{2})^2, s] & \text{if } p \geq q(\alpha)R \end{cases}$$

The allocation of the endowment in excess of the optimal productive investment (thereafter referred to as residual endowment) depends on the speculative return. An unlevered investor has no incentives to take excessive risk, so if the expected speculative return is higher than the return on storage $p < q(\alpha)R$, he allocates all residual endowment into the speculative asset ($y_i^* = \frac{s-x_i^*}{p}$, $z_i^* = 0$). If the returns of storage and speculative investment are equal ($p = q(\alpha)R$), he is indifferent between the two ($y_i^* + z_i^* = \frac{s-x_i^*}{p}$). Otherwise (when $p > q(\alpha)R$), the investor chooses storage over the risky investment ($y_i^* = 0$, $z_i^* = \frac{s-x_i^*}{p}$).

Bank problem: solvent and risk-shifting strategy

We next study the investment choice of a levered bank subject to limited liability and deposit insurance:

$$\max_{x_i, y_i, s_i} q(\alpha)(\alpha\sqrt{x_i} + R(y) - s_i - \lambda) + (1 - q(\alpha))max[\alpha\sqrt{x_i} - s_i - \lambda, 0]$$

subject to:

$$x_i + py_i = s_i \qquad \text{(budget constraint)}$$

$$s_i \leq s \qquad \text{(funding constraint)}$$

$$x_i \geq 0, \; y_i \geq 0 \qquad \text{(no-short-selling constraint)}$$

The bank's choice is between a solvent strategy and a risk-shifting strategy. A solvent strategy ensures limited leverage and scale of speculative investment is such to ensure deposit repayment. A risk-shifting strategy involves default in the bad state.The optimal strategy choice is determined by the level of available funding supply.

Lemma 1. *There exists an individual risk-shifting threshold of funding supply $\hat{s}(\alpha, p, \lambda)$, such that:*

- *When a bank's funding supply is below $\hat{s}(\alpha, p, \lambda)$ it chooses a solvent strategy, given by:*

$$x_s^* = \begin{cases} \min[(\frac{\alpha p}{2q(\alpha)R})^2, s] & \textit{if } p < q(\alpha)R \\ \min[(\frac{\alpha}{2})^2, s] & \textit{if } p \geq q(\alpha)R \end{cases} \qquad (y_s^*, s_s^*) = \begin{cases} (\frac{s - x_s^*}{p}, s) & \textit{if } p < q(\alpha)R \\ (0, x_s^*) & \textit{if } p > q(\alpha)R \end{cases}$$

if $p = q(\alpha)R$, any (y_i, s_i) that satisfies $y_i + s_i = x_s^*$ is part of the solvent strategy.

- *When bank's funding supply is above $\hat{s}(\alpha, p, \lambda)$, it chooses a risk-shifting strategy given by:*

$$x_r^* = (\frac{\alpha p}{2R})^2 \qquad (y_r^*, s_r^*) = (\frac{s - x_r^*}{p}, s) \qquad \forall p$$

- *When bank's funding supply is equal to $\hat{s}(\alpha, p, \lambda)$, it is indifferent between the two strategies.*

Proof. In Appendix □

In a solvent strategy, bank's portfolio corresponds to the unlevered agent's choice. If asset's expected return is negative bank excludes any investment in a speculative asset and chooses productive investment so that to equalize marginal return with the cost of funding, declines any additional savings (thereafter referred to as residual funding).

If the speculative asset has a positive expected return ($p < q(\alpha)R$), the bank invests in the productive technology until its return matches the speculative return and invests all residual funding into the speculative asset. As long as available funding is below the risk-shifting threshold, the potential losses are not large enough to compromise the repayment of deposits.

In a risk-shifting strategy the bank defaults with probability $1 - q(\alpha)$, thus it maximizes profits conditional on the good state. Its portfolio choice equalizes the marginal productive return with the opportunity cost: the speculative return in the good state ($\frac{R}{p}$). Higher opportunity cost implies that risk-shifting banks invest less into productive technology (compared to solvent banks or unlevered agents). They allocate all residual funding to the speculative asset.

A basic result is that risk-shifting is preferred whenever the bank can achieve a sufficient scale of speculation, namely whenever the supply of available funding exceeds the risk-shifting threshold $\hat{s}(\alpha, p, \lambda)$. The strategic shift occurs as a large supply shock implicitly enables higher leverage, so the expected return on a larger risky portfolio exceeds the solvent strategy return on a smaller investment scale.

Lemma 2. *The individual risk-shifting threshold is given by:*

$$\hat{s}(\alpha, p, \lambda) = \begin{cases} (\frac{\alpha}{2})^2 \frac{p}{q(\alpha)R}[1 + q(\alpha)] - \lambda & \textit{if } p < q(\alpha)R \\ (\frac{\alpha}{2})^2 \frac{1-q(\alpha)\frac{p}{R}}{q(\alpha)(\frac{R}{p}-1)} - \frac{1-q(\alpha)}{q(\alpha)(\frac{R}{p}-1)}\lambda & \textit{if } p \geq q(\alpha)R \end{cases}$$

The threshold:

- *decreases in the losses on existing assets* $\frac{\partial \hat{s}}{\partial \lambda} < 0$
- *increases in the price of the speculative asset,* $\frac{\partial \hat{s}}{\partial p} > 0$*, whenever assumption 1.A holds (probability of th good state is not too high)*
- *decreases in the productivity,* $\frac{\partial \hat{s}}{\partial \alpha} > 0$ *whenever assumption 1.B and 1.C hold (persistence of the productivity shock is sufficiently high)*

Proof. In the Appendix □

Higher losses on pre-existing assets, increase leverage and thereby make the risk-shifting more attractive. The expected return of the speculative asset affects the risk shifting threshold in the opposite direction, as higher price leads to lower speculative profits shifting the threshold upwards [2].

An increase in α implies higher return on productive technology and an increase in probability of a high speculative payoff. If the latter effect is strong (the measure of persistence of productivity $q'(\alpha)$ is sufficiently high as assumed in 1.B and 1.C), an increase in productivity disproportionately increases the profits from risk shifting, moving the risk shifting threshold downwards. If the persistence of the productivity shock is low, the risk shifting threshold will increase as a result of a rise in productivity.

Since the individual risk-shifting threshold of funding is monotonically increasing in the asset price, we can reformulate Lemma 2 in terms of a price threshold. At any given level of funding s, there exists a risk-shifting price threshold $\hat{p}(\alpha, s) = \hat{s}^{-1}(\alpha, s)$, such that when the price is at the threshold, the bank is indifferent between risk-shifting and a solvent investment. He prefers the risk-shifting (solvent) strategy if the price is below (above) the threshold.

[2] Assumption 1.A is a sufficient conditions to ensure that the relationship is satisfied even for very high ranges of equilibrium price

3.2. Equilibrium investment and prices

The individual decision rule of the banks determines the equilibrium investment and prices as a function of productivity and funding supply.

Proposition 1. *There is an equilibrium risk shifting threshold of funding supply given by:*

$$\Sigma(\alpha, \lambda) = \hat{s}(\alpha, q(\alpha)R, \lambda)$$

When funding supply is above $\Sigma(\alpha, \lambda)$ a positive fraction of banks engages in risk-shifting, else all banks choose the solvent strategy.

Proof. In the Appendix $\square$

Consider the case when all banks invest according to the solvent strategy. The resulting price is $p_s^* = min[s - (\frac{\alpha p}{2q(\alpha)R})^2, q(\alpha)R]$, thereafter referred to as the solvent price. If the solvent price is weakly higher than the risk-shifting price-threshold, $p_s^* \geq \hat{p}(\alpha, s)$, banks have no risk-shifting incentives and all banks choosing the solvent strategy constitutes the equilibrium. If the solvent price is below the price-threshold, $p_s^* < \hat{p}(\alpha, s)$, risk-shifting becomes a dominant strategy. Thus, banks switch from the solvent to the risk-shifting strategy up until the price equals the risk-shifting price-threshold $p^* = \hat{p}(\alpha, s, \lambda)$. The graph below plots the solvent price, the risk-shifting threshold and the price that would result if all banks chose the risk-shift strategy (the risk-shifting price) for some fixed productivity α and losses λ as a function of funding supply.

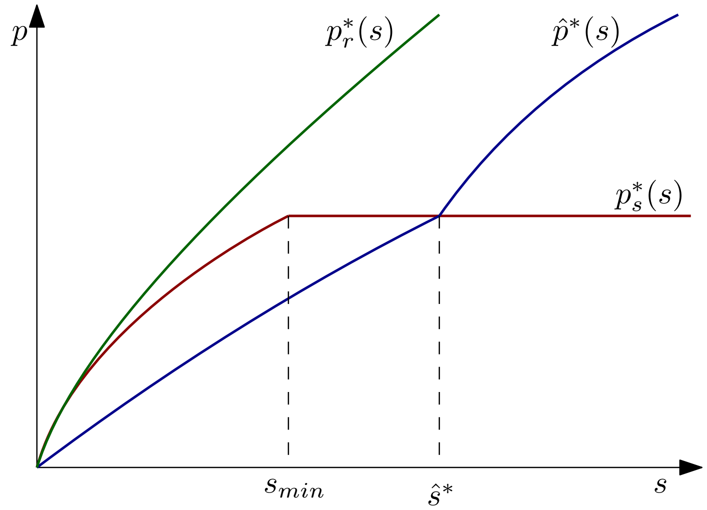


Figure 1: Prices as a function of funding supply if all banks invest solvently $p_s^*(s)$, red curve, and if all banks risk-shift $p_r^*(s)$, green curve. The blue curve corresponds to the solvent investment lower bound.

If the supply funding is below $s_{con}(\alpha) = (\frac{\alpha}{2})^2 + 1$ (thereafter refered to as constrained funding threshold), it is insufficient for the banks to utilize all the investment opportunities. Cash in the market determines the asset price in this region. Higher level of funding allows the bank to invest more in both the productive technology and the speculative asset. Due to the diminishing marginal returns to scale of the productive technology, the solvent price increases steeply in s. The increase in the risk-shifting price threshold is linear and, due to Assumption 1.D which introduces an upper bound on the upside payoff of the asset, it is more gradual than the change in the solvent price. Because banks investing solvently demand the asset only until $p = q(\alpha)R$, the solvent price is fixed as the funding supply increases above the constrained funding threshold $s_{con}(\alpha)$. The the risk-shifting price threshold continues to increase in the funding supply, thus the risk-shifting equilibrium can only occur if the supply exceeds the individual risk-shifting threshold evaluated at at highest solvent price ($p = q(\alpha)R$). Thereafter we refer to a funding level that is above the constrained threshold and below the individual risk-shifting threshold evaluated at the fair price as balanced, while any funding above the risk-shifting threshold is labeled as excessive

Lemma 3. *For a given level of productivity*

- *if funding supply is* **constrained**, $s < s_{con}(\alpha)$, *all banks choose the solvent strategy* $(x^* = (\frac{\alpha p^*}{2q(\alpha)R})^2$, $y^* = s - (\frac{\alpha p^*}{2q(\alpha)R})^2)$ *and the asset is underpriced* $(y^* = p^* < q(\alpha)R)$

- *if funding supply is* **balanced**, $s_{con}(\alpha) < s < \Sigma(\alpha)$, *all banks choose the solvent strategy* $(x^* = (\frac{\alpha}{2})^2$, $y^* = q(\alpha)R)$ *and the asset is fairly priced* $(y^* = p^* = q(\alpha)R)$

- *if funding supply is* **excessive** $(s > \Sigma(\alpha))$: *fraction* $\psi^*(\alpha, s) = \hat{s}(\alpha)/[s - (\frac{\alpha p^*}{2R})^2]$ *of banks is risk-shifting* $(x^*_r = (\frac{\alpha p^*}{2R})^2$, $y^*_r = s - (\frac{\alpha p^*}{2R})^2)$ *and the remaining banks choose the solvent strategy* $(x^*_s = (\frac{\alpha}{2})^2$, $y^*_s = 0)$, *the asset is overpriced* $(p^* = \hat{p}(\alpha, s) > q(\alpha)R)$

Proof. In the Appendix □

When the funding supply is **constrained**, none of the banks is willing to risk shift and the demand for the speculative asset is insufficient to drive the asset price up to its fundamental value. If the funding is **balanced** the solvent investors demand the speculative asset only until its' price is equal to the expected payoff. Thus, any funding above the constrained threshold $s_{cons}(\alpha)$ will be rejected by banks (or equivalently: invested in storage). When funding supply is **excessive**, possibility of raising high leverage makes excessive speculation attractive, that in turn increases the equilibrium price. A mixed equilibrium emerges in which some banks risk-shift while others invest solvently so that the equilibrium price equals the risk-shifting price threshold.

Comparative statics

The relationship between equilibrium prices and funding supply is reflected in Figure **??**. In the graph, the equilibrium price of the speculative asset is given by the maximum of the solvent price and the risk-shifting price-threshold: $p^* = \max[p^*_s(s), \hat{p}^*(s)]$. We never observe all banks risk-shifting in the equilibrium, because the resulting price would be too high for the risk-shifting to remain optimal for all: $p^*_r(s) > \hat{p}^*(s)$.

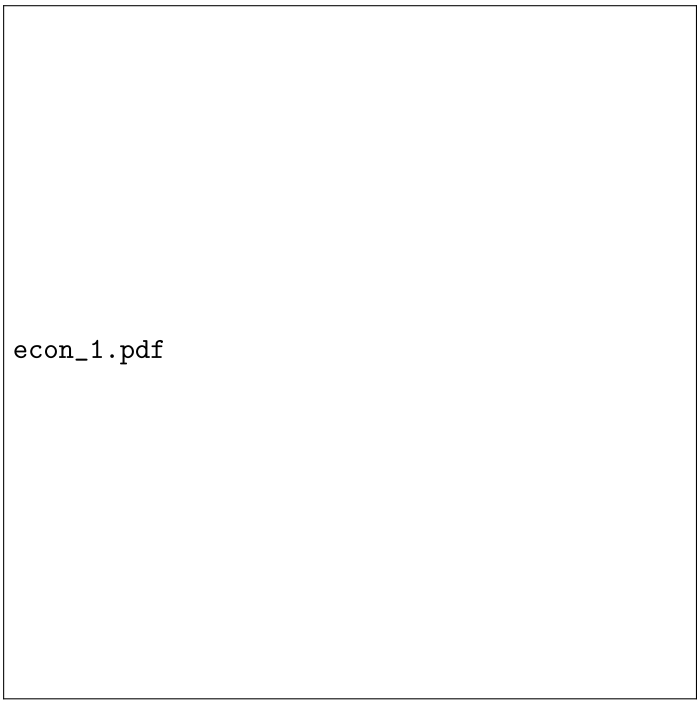
Corollary 1. *For a given level of productivity, the equilibrium price is a non-decreasing function of the funding supply. The equilibrium price is constant in the* **balanced** *funding range and strictly increasing in the funding supply in* **constrained** *and* **excessive** *funding range.*

The relationship between price and productivity depends on persistence of the productivity shock, $q'(\alpha)$.

Corollary 2. *The equilibrium price increases in productivity whenever the funding supply is* **balanced**. *When funding is in the* **constrained** *or* **excessive** *range, price increases in productivity* *whenever Assumptions 1.B and 1.B are satisfied (the persistence of the productivity shock, $q'(\alpha)$, is high enough).*

A rise in productivity increases the return on both, the productive and speculative investment. When bank funding is **balanced**, the price of the speculative asset reflects its' expected payoff and is thus increasing in the aggregate productivity. In the case of **constrained** funding supply, the equilibrium price equates the marginal return on the productive technology with the speculative return, both of which increase as the aggregate productivity rises. If the persistence of the productivity shock is high, the impact on the speculative payoff dominates and consequently the equilibrium price increases in productivity. If funding is **excessive**, the price is at the risk-shifting price threshold, which is the level that leaves banks indifferent between the two investment strategies. An increase in productivity increases the profits for both. The effect on the risk-shifting gains dominates if the payoff of the speculative asset is sufficiently affected, which occurs under the assumption of a high persistence of the productivity shock.

Our setting gives rise to four different states of the global economy: a constrained growth, a stagnation, a good boom, a bad boom.



In a constrained growth, available funding is insufficient for the banks to utilize all of the available investment opportunities. In the stagnation and the good boom, the funding is sufficient for all of the good opportunities to be exploited but not too high, so that bank's leverage remains limited and their incentives are sound. The two differ in the quality of investment opportunities. In the bad boom, the funding supply is excessive relative to the quality of the investment opportunities. This gives rise to risk shifting incentives as global intermediaries invest in an asset with a negative expected return, hoping to benefit from its' upside. Their exorbitant demand for the speculative investment comes at a cost of an inefficiently low productive lending. Thus, in the bad boom the total investment is excessive and the allocation of the funds to the available opportunities is misguided. The bad boom is associated with a positive probability of bank default.

Inference by uninformed

Since the uninformed agents do not observe the level of funding supply nor the, they may be unable to recognize whether a given price follows from a stagnation, a good or a bad boom. Thus, precise inference of the productivity from asset prices may be impossible.

Definition 1. *Define $\hat{\alpha}(s^H)$ such that $s^H = \Sigma(\hat{\alpha}, \lambda)$, as a risk-shifting productivity threshold (RSPT) in the high funding state. Define $\alpha_c(s^L)$ such that $s^L = s_{con}(\alpha_c)$, as a constrained-growth productivity threshold (CGPT) in the low funding state*

Proposition 2. *The uninformed infer two productivity values when the equilibrium price is in the* **confused inference set**, *$\mathcal{P}$, and make a correct inference of one productivity value when the equilibrium price is outside of this set. The confused inference set is non empty if*

$$\hat{\alpha}(s^H) \in (\underline{\alpha}, \overline{\alpha}) \text{ or} \alpha_c(s^L) \in (\underline{\alpha}, \overline{\alpha})$$

Proof. In the Appendix □

Whenever funding is balanced, the speculative price reflects the expected payoff of the asset and thus the productivity. In both the constrained and the excessive funding range the price of the asset is pinned down by a form of cash in the market pricing. In the former case, the price equalizes marginal returns of the assets for a given funding constraint. In the latter, the price is set so that to ensure indifference between the two investment strategies. If for the range of pr

- Both constrained and balanced funding

The figure below plots feasible equilibrium prices as a function of productivity for the two possible realizations of funding supply.

Lemma 4. *The confused inference range is a union of four disjoint subsets:*

- *Risk: level of funding*
- *Risk vs solvent*
- *Solvent vs constrained*
- *Constrained: level of funding*

To illustrate the possibility of confused inference in the simplest setting that is relevant for our inquiry into good and bad booms, let us choose the parameters which ensure that bank investment is never constrained and the bad boom may occur if the supply of funding is high: $\hat{s} > s^L > s_{min}(\overline{\alpha})$ and $s^H = \hat{s}^*(\hat{\alpha})$ for some $\hat{\alpha} > \underline{\alpha}$. Furthermore, we assume that the probability of high payoff of the asset increases steeply in productivity, so that the equilibrium price increases in productivity for all values of α and s[3]. The graph below plots the equilibrium price as a function of productivity for the two feasible levels of available funding.

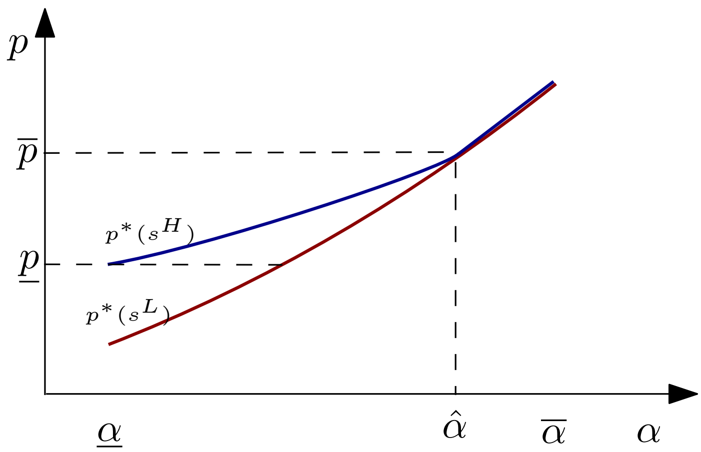


Figure 2: Prices as a function of productivity for the high and low supply realization if $s^L > s_{min}(\alpha)$, $p^{*\prime}(\alpha) > 0$ and $s^H = s^*(\hat{\alpha})$. Blue curve corresponds to the high supply shock, red curve corresponds to the low supply shock.

[3]These assumptions are not necessary for the confusion to arise. We can immediately observe from Figure **??**, that a confusion may arise also if prices are decreasing in the productivity in some range

Under our assumptions on the model parameters whenever a low funding supply is realized all banks invest solvently and the equilibrium price is equal to the expected payoff of the asset. As the high funding supply is equal to the equilibrium risk-shifting threshold evaluated at $s_H = \hat{s}^*(\hat{\alpha})$, the risk-shifting equilibrium arises whenever the productivity is below $\hat{\alpha}$. Thus, can define the minimum price under the high funding shock as: $p_{min}(s_H) = \hat{p}(\underline{\alpha}, s_H)$. When the observed price is below that level, it can only emerge from a solvent equilibrium. This allows the uninformed agents to infer the productivity using $p^* = q(\alpha)R$.

If the productivity is above $\hat{\alpha}$ all banks invest solvently even under the high funding shock and the equilibrium price is given by $p = q(\alpha)R$. Such prices also allow a perfect inference using the solvent equilibrium price formula.

Whenever the observed price is between the minimum feasible price under risk-shifting and the fair price evaluated at $\hat{\alpha}$, uninformed agents are unable to determine whether the price results from a good boom (an all-solvent equilibrium under the low funding supply) or a bad boom (a risk-shifting equilibrium under the high funding). In this case they will form two estimates of the productivity: $\tilde{\alpha}(p, s^H) = \hat{p}^{-1}(s^H, p)$ and $\tilde{\alpha}(p, s^L) = q^{-1}(\frac{p}{R})$.

$$\tilde{\alpha} = \begin{cases} \tilde{\alpha}(p, s^H) \text{ with prob. } \rho \\ \tilde{\alpha}(p, s^L) \text{ with prob. } 1 - \rho \end{cases} \tag{1}$$

Lemma 5. *If the parameters are such that the equilibrium price is increasing in the productivity, then whenever the asset price is in the confused inference range $p^* \in (\underline{p}, \overline{p})$, the expected value of the productivity according to the uninformed agents is:*

- *Higher than the actual productivity if the funding supply is low*
- *Lower than the actual productivity if the funding supply is high*

Proof. In the Appendix □

For given productivity the high funding price is weakly higher than the low funding price. If the asset price increases in productivity, then whenever the distribution of the funding shock is such that two different levels of productivity can result in the same price, the productivity level that justifies the price conditional on high funding supply realization is lower than the productivity level under the low funding supply ($\tilde{\alpha}(p, s^H) < \tilde{\alpha}(p, s^L)$).

Investment by uninformed firms

Let A denote the set of productivity estimates $\tilde{\alpha}_e$ inferred by the uninformed. They maximize the profits, subject to the budget constraint.

$$\max_{x_j} \sum_{\tilde{\alpha} \in A} P(\tilde{\alpha})[\tilde{\alpha}\sqrt{x_j} - x_j]$$

$$st.\ x_j \leq c$$

As far as firm capital endowment is sufficient, their investment choice reflects expected productivity. Let us focus on the parameters such that the local firm endowment is sufficient for any $\alpha < \overline{\alpha}$. In this case $x_j^* = (\frac{E(\alpha)}{2})^2$. Thus, under the conditions of Lemma 5 uninformed firms over-invest from ex-post perspective when the funding supply is high. They under-invest from the ex-post perspective when the funding supply is low.

4. Prudential Regulation

This section study how a prudential supervisor can address the consequences of excess credit supply on risk taking and distorted decisions by uninformed local banks. Let each bank have access to a fixed deposit base $s_k = s_u$ of adequate scale to rule out corner solutions.Local banks may well be informed about local projects, but less so about aggregate productivity.

As deposits are insured, the local bank cost of funding equals the unit return of storage.[4] As before, local banks do not have access to the speculative asset market. We now show that as long as uninformed bank funding is not constrained. the setup results in either an unconstrained solvent or risk-shifting equilibrium.

The local bank maximizes profits subject to its funding constraint.

$$\max_{x_k} \sum_{\tilde{\alpha} \in A} P(\tilde{\alpha}) \max[\tilde{\alpha}\sqrt{x_j} - x_k, 0]$$

$$st.\ x_k < s_u$$

When productivity values cannot be precisely inferred, its expected value is given by (1). In this case local bank choice differs from the choice of uninformed unlevered firms.

[4]Our results persist if local banks were to raise risky funding before global asset prices are known.

Lemma 6. *Consider the case when confused inference yields two productivity estimates: $\tilde{\alpha}_L < \tilde{\alpha}_H$. If the dispersion of the productivity estimates is not too high ($2\tilde{\alpha}_L > E(\alpha)$), local uninformed banks invest like firms: $x_k^* = (\frac{E(\alpha)}{2})^2 = x_j^*$. Otherwise, they risk-shift on the productive technology: $x_k^* = (\frac{\tilde{\alpha}_H}{2})^2 > x_j^*$.*

Proof. In the Appendix. □

High uncertainty about the productivity of the technology may result in risk-shifting by local banks. If the investment based on the expected productivity would result in losses at low productivity values ($2\tilde{\alpha}_L > E(\alpha)$), it is optimal for banks to bet on the high productivity estimates. As a result, local banks invest more than the uninformed firms. If the true productivity is low, this results in larger losses than those faced by firms and a default by local banks. If the true productivity is high, banks' investment is closer to the ex-post optimal level than that of the firm's. Thus local banks engage in a form of risk-shifting induced by the confused inference

If the conditions of Lemma 5 are satisfied, so that the equilibrium prices increase in productivity in the risk-shifting range of funding supply, the low productivity estimate corresponds to the high funding state. In this setting, risk shifting by local banks results in their default whenever the global economy is the bad boom. Thus, uncertainty among local banks may further amplify the "cycle". At $t = 0$, the supply driven over-investment by global intermediaries may come hand in hand with confusion driven over-investment by local banks. As a consequence at $t = 1$, we will observe default of some global and all local banks.

4.1. Local regulator

The possibility of risk-shifting by local banks implies a scope for regulation by local authorities. In this section we discuss the optimal choice of the local regulator, who observes the global prices like other uninformed agents and may impose requirements on local banks accordingly.

We assume that the regulator acts like a local social planner, aiming to maximize the total output. As means of macro-prudential policy the regulator may impose a proportional tax on the investment by local banks: τx_k and redistribute the proceeds to all banks at the final date. The policy operates in via two channels. On one hand, the tax increases the cost of investment allowing the regulator to curb potential over-investment. On the other hand, by redistributing the proceeds at date $t = 1$ the policy maker improves bank's incentives. Thus, this reduced form definition of the policy tool could represent a reduced form capital or reserve requirement.

In the presence of the macro-prudential policy, the problem of the local bank can be expressed as:

$$\max_{x_k} \sum_{\tilde{\alpha} \in A} P(\tilde{\alpha}) \max[\tilde{\alpha}\sqrt{x_k} - (1+\tau)x_k + \tau x, 0]$$

$$st.\ x_k < s_u$$

$$x = \int_k x_k$$

In the presence of the requirement the optimal investment of the local bank is given by $x_k^* = (\frac{E(\alpha)}{2(1+\tau)})^2$ if the productivity estimates are not too dispersed $2\tilde{\alpha}_L > (1+\tau)E(\alpha)$. Otherwise, the investment into productive technology is $x_k^* = (\frac{\tilde{\alpha}_H}{2(1+\tau)})^2$.

The problem of the regulator can be expressed as:

$$\max_{\tau} \sum_{\tilde{\alpha} \in A} P(\tilde{\alpha}) \max[\tilde{\alpha}\sqrt{x} - (1+\tau)x + \tau x, 0]$$

Where: $x = \begin{cases} (\frac{E(\alpha)}{2(1+\tau)})^2 \text{ if } 2\tilde{\alpha}_L(1+\tau) > E(\alpha) \\ (\frac{\tilde{\alpha}_H}{2(1+\tau)})^2 \text{ if } 2\tilde{\alpha}_L(1+\tau) \leq E(\alpha) \end{cases}$.

It is straightforward to see that if the productivity estimates are not too dispersed, so that local banks have no risk-shifting incentives and the private solution is efficient, the optimal tax set by the regulator is equal to zero. If the local banks have risk-shifting incentives the regulator prefers to set macro-prudential tax to either limit the over-investment or eliminate the risk-shifting incentives.

Lemma 7. *If the productivity estimates are such that the local banks have risk-shifting incentives, the regulator sets a positive macro-prudential tax $\tau^* > 0$. The resulting investment may remain inefficient.*

Proof. In the Appendix □

To illustrate this result it is useful to define two intuitive benchmark levels of the macro-prudential tax. First let us label as τ_p the level of tax at which the investment of a risk-shifting local bank would equal the efficient level of investment: $\frac{\tilde{\alpha}_H}{2(1+\tau_p)} = \frac{E(\alpha)}{2}$. This policy is optimal only if local banks continue to have risk-shifting incentives when $\tau = \tau_p$. To establish whether this is the case one needs to evaluate the second benchmark, τ_r, the highest tax rate at which local banks have risk-shifting incentives: $\tilde{\alpha_L} = \frac{E(\alpha)}{2(1+\tau_r)}$.

Lemma 8. *The optimal macro-prudential tax set by the regulator depends on the relative size of inferred productivity parameters.*

- *If $2\tilde{\alpha}_L > E(\alpha)$: the private solution coincides with the efficient solution, so $\tau^* = 0$*
- *If $2\tilde{\alpha}_L < E(\alpha)$ and $2(\tilde{\alpha}_L\tilde{\alpha}_H) < E(\alpha)^2$, the regulator can achieve the efficient investment by setting $\tau^* = \tau_p = \frac{\tilde{\alpha}_H}{E(\alpha)} - 1$*
- *If $2\tilde{\alpha}_L < E(\alpha)$ and $2(\tilde{\alpha}_L\tilde{\alpha}_H) > E(\alpha)^2$, the efficient solution is unfeasible*
  - *if $2(\tilde{\alpha}_L\tilde{\alpha}_H)^2 > E(\alpha)^2[E(\alpha)^2 - 2\tilde{\alpha}_L^2]$, eliminating the risk-shifting is the best the regulator can do, so $\tau^* = \tau_r + \epsilon = \frac{E(\alpha)}{2\tilde{\alpha}_L} - 1 + \epsilon$*
  - *if $2(\tilde{\alpha}_L\tilde{\alpha}_H)^2 < E(\alpha)^2[E(\alpha)^2 - 2\tilde{\alpha}_L^2]$ minimizing the over-investment under risk-shifting is the best the regulator can do, so $\tau^* = \tau_r = \frac{E(\alpha)}{2\tilde{\alpha}_L} - 1$*

Proof. In the appendix □

The regulator is able to ensure efficient investment only if $\tau_p \leq \tau_r$. Otherwise, any policy rate above τ_r would be dominated, since it would lead to an inefficient fall in the local bank's investment. In this case the regulator compares two inefficient solutions: setting the requirement at τ_r so that to minimize the over-investment of risk-shifting local banks ($x_j = (\frac{\tilde{\alpha}_H}{2(1+\tau_r)})^2$) or just above that level to ensure that local banks do not risk-shift (in this case $x_j \approx (\frac{E(\alpha)}{2(1+\tau_r)})^2$). Since the objective of the regulator is to maximize the expected profits of the local banks, he imposes the requirement that results in the lowest deviation from the efficient solution.

Thus, in our simple setting the macro-prudential regulation may be unable to ensure that local banks invest efficiently. Interestingly, Lemma 8 shows that the policy may result in both over- and under-investment relative to the ex-ante efficient level. If ensuring the efficient investment level is impossible regulator may allow the local banks to risk-shift or tighten the macro-prudential policy so that to eliminate risk-shifting at a cost of inefficient under-investment. His preference over these two depends solely on the distance from the efficient investment level. This reflects the fact that we assumed no dead-weight loss from local bank's default, so that the only consideration of the social planner (and the regulator) is for the level of investment.

5. Extension: more signals vs more opacity

While in general prices offer some information on supply and demand, in our setup agents may observe only the asset price, as the cost of bank funding is unobservable as well as inelastic. [5] Less informed agents may have also non price information, and the natural variable is total amount of credit. This indicator may not be readily available nor easily measured (not least since banks report risk weighted rather than total assets). On the other hand, it has received considerably more attention since the crisis, and its realization may affect regulatory actions (with some delay) in the context of the countercyclical buffer in the Basel III framework.

This section studies how the inference process is improved when uninformed agents can observe total investment of large banks ($V = X+pY$). It first assesses as a benchmark the inference based solely on total investment, then considers the case when uninformed agents observe both quantity and price signals. For simplicity we assume the signal is precise. Next, we introduce additional uncertainty about banks' leverage, by assuming that large banks may be holding impaired assets on their balance sheets. We first study the investment choice of a bank holding a legacy asset. Then we analyze the effect on the inference: under price signal and both price and quantity signals.

5.0.1. Observable total investment

The total investment by large banks is given by:

$$V(s,\alpha) = \begin{cases} (\frac{\alpha}{2})^2 + q(\alpha)R & \text{if } s = s^L \\ (1-\psi(\alpha,s))(\frac{\alpha}{2})^2 + \psi(\alpha,s)s^H & \text{if } s = s^H \end{cases} \tag{2}$$

When the available funding is low the total investment reflects the quality of the available opportunities. If the funding supply is abundant, some large banks are risk-shifting, using up all the available funding to scale up their investment, while others invest only in the productive technology.

When uninformed agents observe only the quantity signal (and no prices), they infer the productivity level from the total investment equation (2). It is straightforward to show that in this case their inference may be confused.

[5] Even if bank shares were traded, in a context without short sales their pricing may still not be informative, as their value rises both with lending quality and risk shifting.

This inference error is resolved once uninformed agents observe both total investment and the asset price. Inference based on each of the variables will give two estimates of productivity. If bank lending is driven by demand factors, the estimates of "safe" productivity level will match $\hat{\alpha}(V, s^L) = \hat{\alpha}(p, s^L)$. If the decisions by banks are driven by abundant funding supply, the estimates that assume risk-taking by large banks will overlap: $\hat{\alpha}(V, s^H) = \hat{\alpha}(p, s^L)$. Thus, if uninformed agents have access to precise information about the price of the risky asset and the total investment by global intermediaries, they are able to correctly infer the aggregate productivity. Ultimately, observing enough variables enables to solve the system of unknown economic conditions.[6]

5.1. Additional balance sheet opacity

Inference problem by uninformed

The inference problem of the smaller players takes a different form depending on the maximum potential losses from the impaired assets ($\overline{\lambda}$).

If the maximum losses are not too large ($\overline{\lambda} < \hat{s}^*(\alpha) - s^L$ for all $\alpha \in [\underline{\alpha}, \overline{\alpha}]$) banks always invest safely in a low funding state. Thus, when uninformed agents observe a price such that $p \in (p^*(\underline{\alpha}, s^L, \lambda), p^*(\overline{\alpha}, s^H, \bar{\lambda}))$, they know that it can result from:

- A solvent equilibrium with high productivity and low funding supply (and either high or low losses from the legacy asset), which occurs with probability $1 - \rho$
- A risk-shifting equilibrium with low productivity, high funding supply and low future losses from the legacy asset, which occurs with probability $\rho\kappa$
- A risk-shifting equilibrium with low productivity, high funding supply and high future losses from the legacy asset, which occurs with probability $\rho(1 - \kappa)$

Thus, the posterior beliefs about the productivity of the uninformed agents can be expressed

[6]In all fairness to empirical researchers, uninformed agents in our model may come to such precise estimates because the postulated view of economic rationality involves not just rational updating, but a precise knowledge of all probability distributions and precise value of all economic parameters.

as:

$$P(\alpha|p) = \begin{cases} \rho(1-\kappa) & \text{if } \hat{\alpha}(p, s^H, \overline{\lambda}) \\ \rho\kappa & \text{if } \hat{\alpha}(p, s^H, 0) \\ 1-\rho & \text{if } \hat{\alpha}(p, s^L, \overline{\lambda}) \end{cases}$$

The uncertainty about the initial leverage of large banks additionally confuses the inference from prices. However, if uninformed agents also observe the signal on the quantity of total investment by banks, their inference problem is resolved. They can now recognize whether large banks invested safely (as the price- and quantity-derived estimates should match). Moreover, if some of the large banks are risk-shifting, only the true productivity and size of future losses from the legacy asset are consistent with both the price and the quantity equations.

If the maximum value of legacy asset is sufficiently large ($\overline{\lambda} > \hat{s}^*(\alpha) - s^L(\alpha)$), banks may be risk-shifting even when the available funding is low. Let's assume that the above condition is satisfied for all productivity values in some subset of the range $\alpha \in [\alpha_L, \alpha_H]$ (where $[\alpha_L, \alpha_H] \subset [\underline{\alpha}, \overline{\alpha}]$). Then, we can define a range of prices $p \in (p^c_{min}, p^c_{max})$ for which the uninformed agents form four estimates of the productivity using the following as the bounds:

$$p^c_{min} = \max[p^*(\alpha_L, s^L, \overline{\lambda}), p^*(\underline{\alpha}, s^H, \overline{\lambda}],\ p^c_{max} = \min[p^*(\alpha_H, s^L, \overline{\lambda}), p^*(\overline{\alpha}, s^L, 0)]$$

If the observed price is in these bounds, the uninformed agents know that it can result from:

- A solvent equilibrium with high productivity, low funding supply, and low future losses from the legacy asset, which occurs with probability $(1-\rho)\kappa$
- A risk-shifting equilibrium with moderate productivity, low funding supply and high future losses from the legacy asset, which occurs with probability $(1-\rho)(1-\kappa)$
- A risk-shifting equilibrium with low productivity, high funding supply and low future losses from the legacy asset, which occurs with probability $\rho\kappa$
- A risk-shifting equilibrium with low productivity, high funding supply and high future losses from the legacy asset, which occurs with probability $\rho(1-\kappa)$

Thus, the posterior beliefs about the productivity of the uninformed agents can be expressed as:

$$P(\alpha|p) = \begin{cases} \rho(1-\kappa) & \text{if } \hat{\alpha}(p, s^H, \overline{\lambda}) \\ \rho\kappa & \text{if } \hat{\alpha}(p, s^H, 0) \\ (1-\rho)(1-\kappa) & \text{if } \hat{\alpha}(p, s^L, \overline{\lambda}) \\ (1-\rho)\kappa & \text{if } \hat{\alpha}(p, s^L, 0) \end{cases}$$

The possibility of risk-shifting at the low funding supply leads smaller players to form an additional estimate of productivity. Also in this setting, observing an additional signal about the total investment allows to perfectly infer the underlying productivity if the economy is in a solvent equilibrium. That's because the price and quantity equations in the solvent equilibrium do not depend on the size of the funding shock or the legacy asset. However, because smaller agents are now unsure whether a risk shifting equilibrium is driven by high supply or large future losses on legacy asset, precise inference may be impossible. In particular, the inference will be confused when the price and total investment levels are such that:

$$p = p^*(\hat{\alpha}_U, s^L, \overline{z}) = p^*(\hat{\alpha}_D, s^H, 0)$$
$$V = V^*(\hat{\alpha}_U, s^L, \overline{z}) = p^*(\hat{\alpha}_D, s^H, 0)$$

In this case, the posterior beliefs of small players about the productivity are given by:

$$P(\alpha|p, V) = \begin{cases} \frac{(1-\rho)*\kappa}{(1-\rho)*\kappa+\rho*(1-\kappa)} & \text{if } \alpha = \hat{\alpha}_U \\ \frac{\rho*(1-\kappa)}{(1-\rho)*\kappa+\rho*(1-\kappa)} & \text{if } \alpha = \hat{\alpha}_D \end{cases}$$

We establish that such combinations may exist in our setting by solving the model numerically.

Thus, the additional source of opacity on bank's balance sheet's further complicates the inference, so that even if additional signals are available to the uninformed agents confusion may remain. This example highlight that in reality, the high degree of complexity and opacity of bank balance sheets is likely to distort the inference of market participants, econometricians and policymakers alike. Comparing the cases with three and four productivity estimates we can notice that a core to ensuring confusion with additional signals is for the element of the balance sheet that is assumed to be opaque be able to influence the risk-shifting incentives and the outcomes under risk-shifting.

6. Conclusions

We argue that opaqueness of intermediary balance sheets (in terms of asset values, funding volume and actual leverage) may add noise to asset prices, and explain why rational market participants underestimate risk in credit booms (and may overestimate it when bank funding is scarce). In our setting, a large expansion of available funding boosts banks' incentives to speculate. Other agents seeking to infer the underlying credit quality from asset prices may be unable to interpret it correctly. As a consequence whenever the price in a risk-shifting equilibrium increases in productivity, uninformed market participants over-invest precisely at the time when large banks are risk-shifting, potentially adding to financial instability.

We study the consequences of the imperfect inference when the uninformed agents are local banks. The limited liability combined with high uncertainty about the productivity values may encourage risk-shifting by local banks. In this case they invest excessively precisely when the the global economy is in the bad boom, amplifying the over-investment as well as losses. We show that in some circumstances a local regulator can restore efficiency by imposing a high macro-prudential requirement.

Finally, we show that making more realistic assumptions about the bank balance sheet opacity (for example assuming unobservability of bank capitalization) further complicates the inference. Thus, observing additional signals on the economy may be insufficient to resolve the inference problem.

The analysis offers a broader classification of forms of risk shifting in terms of awareness (deliberate versus confused) and in terms of channels (excessive lending or speculation) some rational foundations to understand the emerging evidence on underestimation of risk in credit booms.

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A. Appendix

A.1. Lemma 1 and 2

Optimal investment strategy The two investment strategies follow directly from the optimization. If the bank remains solvent in the bad state the FOC's yield

$$\alpha\frac{1}{2}x^{-\frac{1}{2}} - \frac{q(\alpha)R}{p} = 0 \;\Rightarrow\; x = (\frac{\alpha p}{2q(\alpha)R})^2,\; y = \frac{s-x}{p} \; if \; p > q(\alpha)R$$
$$\alpha\frac{1}{2}x^{\frac{-1}{2}} - 1 = 0 \;\Rightarrow\; x = (\frac{\alpha}{2})^2,\; y = 0 \; if \; p \geq q(\alpha)R$$

In order to ensure solvency bank he cannot invest more than $y = \frac{\alpha x_s^{\frac{1}{2}}}{p}$ into the risky asset. A higher investment would result in a risk of default.

If bank risks default, the FOC yields

$$\alpha\frac{1}{2}x^{\frac{-1}{2}} - \frac{R}{p} = 0 \;\Rightarrow\; x_r = (\frac{\alpha p}{2R})^2,\; y_r = \frac{s-x_r}{p}$$

The above investment results in a risk of default only if $\frac{\alpha}{2}x_r - x - \lambda < 0$. So we need $s > (\frac{\alpha}{2})^2 * \frac{p}{qR}(2 - \frac{p}{qR}) - \lambda = s_r(\alpha, p)$ for this strategy to be feasible.

If both strategies are feasible ($s > s_r(\alpha, p)$), the bank prefers a strategy that gives him a higher expected profit. We consider two cases: when $p \geq (\alpha)R$ and when $p < q(\alpha)R$. The level of funding at which the two strategies would yield equal profits follows directly from plugging in the investment levels and yields:

$$\tilde{s} = \begin{cases} (\frac{\alpha}{2})^2 \frac{1-q\frac{p}{R}}{(\frac{R}{p}-1)q} - \lambda \frac{1-q}{(\frac{R}{p}-1)q} \ if \ p < q(\alpha)R \\ (\frac{\alpha}{2})^2 \frac{p}{qR}(1+q) - \lambda \ if \ p \geq q(\alpha)R \end{cases}$$

The risk shifting threshold is given by $\hat{s} = \max[s_r, \tilde{s}]$. For $p < q(\alpha)R$ this is s_r, following from:

$$\begin{aligned} s_r &> \tilde{s} \\ (\frac{\alpha}{2})^2 \frac{p}{qR}(2 - \frac{p}{qR}) - \lambda > (\frac{\alpha}{2})^2 &\frac{1 - q\frac{p}{R}}{(\frac{R}{p} - 1)q} - \lambda \frac{1-q}{(\frac{R}{p} - 1)q} \\ (1 - \frac{p}{R})(2 - \frac{p}{qR}) > 1 - q\frac{p}{R} &= 1 - \frac{p}{R} + (1-q)\frac{p}{R} \end{aligned}$$

using that $1 - q > \frac{qR}{p} - q$ for $p < qR$.

$$\begin{aligned} (1 - \frac{p}{R})(1 - \frac{p}{qR}) &> (1-q)\frac{p}{R} \\ 1 - \frac{p}{qR} &> \frac{p}{R}(2 - q - \frac{p}{qR}) \end{aligned}$$

using that $1 - \frac{p}{qR} > \frac{p}{R}(1 - \frac{p}{qR})$, we need to show that $\frac{p}{R}(1 - \frac{p}{qR}) > \frac{p}{R}(2 - q - \frac{p}{qR})$, which is the case whenever: $1 > \frac{p}{R}(2-q)$, since the highest possible $p = qR$, this always holds as $1 > q(2-q) \iff (1-q)^2 > 0$.

For $p \geq q(\alpha)R$:

$$\begin{aligned} s_r &< \tilde{s} \\ (\frac{\alpha}{2})^2 \frac{p}{qR}(2 - \frac{p}{qR}) &< (\frac{\alpha}{2})^2 \frac{p}{qR}(1+q) - \lambda \\ 2 - \frac{p}{qR} &< 1 + q - \epsilon \\ 1 - \frac{p}{qR} &< q - \epsilon \end{aligned}$$

where $\epsilon = \frac{4\lambda qR}{\alpha^2 p}$. Since $p \geq qR$, $s_r < \tilde{s}$ if $q - \epsilon > 0$, which hols for sufficiently low λ.

Therefore the risk-shifting threshold is given by:

$$\hat{s}(\alpha, p, \lambda) = \begin{cases} (\frac{\alpha}{2})^2 \frac{p}{qR}(2 - \frac{p}{qR}) - \lambda & \text{if } p < q(\alpha)R \\ (\frac{\alpha}{2})^2 \frac{1 - q(\alpha)\frac{p}{R}}{q(\alpha)(\frac{R}{p}-1)} - \frac{1-q(\alpha)}{q(\alpha)(\frac{R}{p}-1)}\lambda & \text{if } p \geq q(\alpha)R \end{cases}$$

Comparative statics on the individual risk-shifting threshold

The partial derviative of the individual risk-shifting threshold with respect to prices is positive if

$$1-\gamma > q(\frac{p}{R})^{\frac{\gamma}{1-\gamma}}(1-\frac{p}{R}\gamma)$$

Let us define the right-side of the inequality: $q(\frac{p}{R})^{\frac{\gamma}{1-\gamma}}(1-\frac{p}{R}\gamma = z(p)$ and show that it is increasing in p:

$$\frac{\partial z(p)}{\partial p} = q(\frac{p}{R})^{\frac{\gamma}{1-\gamma}}\frac{\gamma}{(1-\gamma)p}(1-\frac{p}{R}\gamma) - \frac{\gamma}{R}q(\frac{p}{R})^{\frac{\gamma}{1-\gamma}} = \frac{q}{(1-\gamma)p}\gamma(\frac{p}{R})^{\frac{\gamma}{1-\gamma}}(1-\frac{p}{R}) > 0$$

Thus, the risk shifting threshold is increasing in price if $1-\gamma > z(p)$ for highest feasible p. Since when price exceeds the upside return of the risky asset the risk-shifting threshold is infinitely large, we focus on $p = R$ as the maximum price. If $1-\gamma > z(R)$ then $1-\gamma > z(p)\forall p$:

$$1-\gamma > z(R) = q(1-\gamma) \Rightarrow \frac{\partial \hat{s}}{\partial p} > 0\forall p$$

For $p < q(\alpha)R$ the relation is straightforward. The relationship between the productivity and the risk-shifting threshold follows immediately from the first order derivatives. ($\frac{\partial \hat{s}}{\partial \alpha} > 0$) if

- If $p < q(\alpha)R$, then $1 + q(\alpha) < \frac{\alpha}{2}q'(\alpha)$, or
- If $p \geq q(\alpha)R$, then $q(\alpha)(1 - q(\alpha)\frac{p}{R}) < \frac{\alpha}{2}q'(\alpha)$

)

A.2. Derivation of Proposition 1 and Lemma 3

The equilibrium level of speculative price equalizes its demand and supply: $D_y = S_y$, so it depends on banks' investment choice. Three types of equilibria may emerge:

- A pure solvent equilibrium in which all banks play the solvent strategy and the price is $p = \min[q(\alpha)R, s - (\frac{\alpha\gamma p}{q(\alpha)R})^{\frac{1}{1-\gamma}}]$
- A pure risk-shifting equilibrium in which all banks take risk and the price is $p = s - (\frac{\alpha\gamma p}{R})^{\frac{1}{1-\gamma}}$
- A mixed equilibrium in which fraction $\hat{\psi}(\alpha, s)$ of banks risk shifts while the others choose the solvent strategy. The speculative price ensures all banks are indifferent: p^* is such that $s = \hat{s}(\alpha, p^*)$.

We can use the inverse of the individual risk-shifting threshold: $\hat{p}(\alpha, s)$ to define the equilibrium investment. For any α and s the equilibrium

- If $p_s^* > \hat{p}(\alpha, s)$ all banks prefer solvent investment.
- If $p_r^* > \hat{p}(\alpha, s) > p_s^*$ there is a mixed risk-shifting in equilibrium.
- If $\hat{p}(\alpha, s) > p_r^* > p_s^*$ there is a pure risk-shifting equilibrium.

In the general case with $\gamma \in (0, 1)$, we are unable to find the closed form solutions for this problem and we are only able to implicitly define the equilibrium. As long as for a given productivity α the functions are such that $\hat{p}(s) = p_s^*$ only for one value s, there exists a unique risk-shifting threshold corresponding to that value s. If the functional forms are such that the two functions cross more than once, there may be several thresholds resulting in several ranges of funding supply values for which the solvent or risk-shifting equilibrium occurs.

For the case with $\gamma = \frac{1}{2}$, $\hat{p}(s) = p_s^*$ cross only once. We can show that $\hat{s}(p = q(\alpha)R) > s_{min}$ The funding necessary for the fair pricing of the asset is lower than the risk-shifting threshold evaluated at the fair asset price (the highest price feasible under solvent equilibrium) if and only if $(\frac{\alpha}{2})^2 > R$. If this condition is satisfied, there is no scope for risk-shifting when $s < \hat{s}(p = q(\alpha)R)$ so that we know that for some funding supply s $\hat{p}(s) = p_s^* = q(\alpha)R$. Otherwise we can find a threshold price level $p = (1 + q - q(\frac{2}{\alpha})^2 R)qR$ that ensures that $\hat{s} = p + (\frac{\alpha p}{2qR})^2$.

We can show that if $(\frac{\alpha}{2})^2 > R$ then $p_r^*(s) > \hat{p}(s)$, so that it is impossible for all banks to have risk-shifting incentives. Thus, only a mixed risk-shifting equilibrium can emerge in which the fraction of banks risk-shifting is such that the sum of individual demands brings about the price at which banks are indifferent between the two strategies.

PRICE AND PRODUCTIVITY RELATION:

- when funding in constrained ($s < s_{con}(\alpha)$) if and only if the expected payoff of the asset increases steeply in productivity $\frac{\partial p^*}{\partial \alpha} > 0 \iff q'(\alpha)\frac{\alpha}{q(\alpha)} > 1$
- whenever the funding supply is balanced ($s_{con}(\alpha) < s < \hat{s}^*(\alpha)$)
- when funding supply is excessive ($s > \hat{s}$) if and only if the expected payoff of the asset increases steeply in productivity: $\frac{\partial p^*}{\partial \alpha} > 0 \iff q'(\alpha)\alpha > 2q(\alpha)(1 - q(\alpha)\frac{p^*}{R})$.

The panel below illustrates the relationship between price and productivity for the four combinations of parameters.

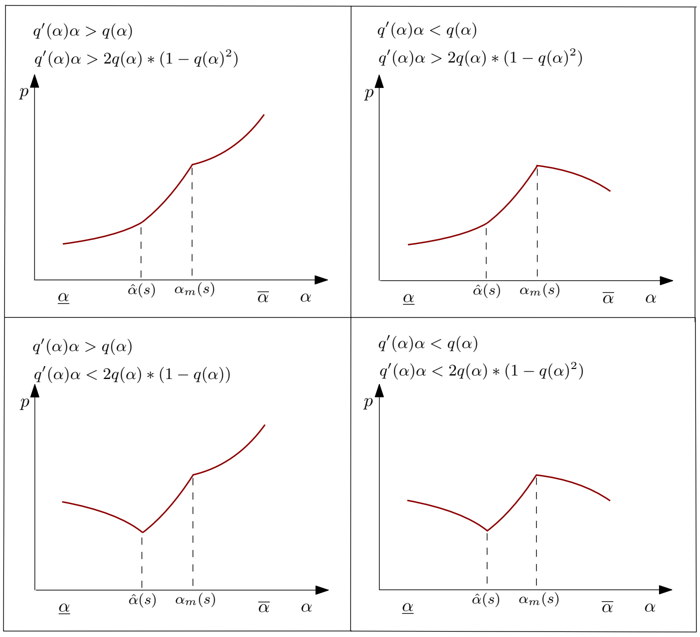

Figure 3: Prices as a function of productivity for different combinations of parameters. A risk-shifting equilibrium emerges when $\alpha < \hat{\alpha}$, all banks invest solvently when $\alpha \geq \hat{\alpha}$, investment by the banks is constrained if $\alpha > \alpha_m$, where $s_{min}(\alpha_m) = s$.

A.3. Derivation of Proposition 2 and Lemma 5

The result on the confused inference follows straightforwardly from the example discussed in the text. In this illustratory case the confusion region of prices is given by: $\underline{p} = \hat{p}(\underline{\alpha}, s^H)$ and $\overline{p} = q(al\hat{p}ha)R$. Since asset prices are assumed to increase in productivity: $\overline{p} > \underline{p}$, so that the confusion region is non-empty.

The confusion may also emerge for other parameter ranges. If the equilibrium price is decreasing in productivity in the risk-shifting equilibrium we will have that $\underline{p} = q(al\hat{p}ha)R$ and $\overline{p} = \min[\hat{p}(\underline{\alpha}, s^H), p_s^*(\overline{\alpha})]$.

The relative size of the inferred productivity values is lower in the high funding state whenever the equilibrium price is an increasing function of productivity. A higher funding shifts parts of the curve of the price as a function of productivity upwards. This upward shift means that for the same value of productivity the equilibrium price is higher. Since equilibrium price increases in productivity, we can only restore the same price with a lower level of productivity.

A.4. Derivation of Lemma 6 and Lemma 7

There are two feasible investment choices by the local bank:

- It can invest like a firm (efficient): $x_k = (\frac{E(\alpha)}{2})^2$, which is a feasible solution as long as it ensures no risk of default by the bank: $\alpha_H \frac{E(\alpha)}{2}) - \frac{E(\alpha)}{2})^2 > 0$ and $\alpha_L \frac{E(\alpha)}{2}) - \frac{E(\alpha)}{2})^2 > 0$

- It can risk-shift, by gambling on the high realization of the productivity $x_k = (\frac{\alpha_H}{2})^2$, this solution is feasible only if at this level of investment banks have risk-shifting incentitves (that is they indeed risk default in the low productivity state): $\alpha_L \frac{\alpha_H}{2}) - \frac{\alpha_H}{2})^2 < 0$

Since the profits under the efficient investment choice are strictly higher than under risk-shifting, risk-shifting is only possible if the efficient investment is unfeasible: $\alpha_L \frac{E(\alpha)}{2}) - \frac{E(\alpha)}{2})^2 < 0 \Rightarrow 2\alpha_L < E(\alpha)$. If this condition is satisfied the feasibility condition for risk-shifting is always satisfied.

The optimal choice of the regulator is interesting only if the local banks have risk-shifting incentives .

$$\max_{\tau} \sum_{\tilde{\alpha} \in A} P(\tilde{\alpha}) \max[\tilde{\alpha}\sqrt{x} - (1+\tau)x + \tau x, 0]$$

Where: $x = \begin{cases} (\frac{E(\alpha)}{2(1+\tau)})^2 \text{ if } 2\tilde{\alpha}_L(1+\tau) > E(\alpha) \\ (\frac{\tilde{\alpha}_H}{2(1+\tau)})^2 \text{ if } 2\tilde{\alpha}_L(1+\tau) \leq E(\alpha) \end{cases}$.

Let's use the above to find the limit τ_r at which the bank switches from risk-shifting to a solvent investment: $\tau_r = \frac{E(\alpha)}{2\alpha_L} - 1$.

Assume that banks have risk-shifting incentives for all τ, the FOC of the regulator problem yields:

$$\frac{\partial SW}{\partial \tau} = \frac{\partial x}{\partial \tau}[\frac{1}{2}E(\alpha)x^{-\frac{1}{2}} - 1] = 0$$

Thus, the optimal choice is such that: $E(\alpha) = 2x^{\frac{1}{2}} = \frac{\alpha^H}{2(1+\tau)} \Rightarrow \tau_p = \frac{\alpha^H}{E(\alpha)} - 1$. This solution is consistent only if at τ_p banks still have risk-shifting incentives:

$$2\tilde{\alpha}_L < (1+\tau_p)E(\alpha)$$

Or simply: $\tau_p < \tau_r$.

If $\tau_r < \tau_p$ we are at a corner solution. If we increase the requirement marginally to $\tau_r + \epsilon$ bank's no longer risk-shift so their investment is given by: $x_j = (\frac{E(\alpha)}{2(1+\tau_r+\epsilon)})^2$. The investment is too low relative to the efficient level $x^* = \frac{E(\alpha)}{2}$. With this investment schedule any further increase in τ would reduce the investment making it even more inefficient. If we set τ_r, banks risk-shift and invest according to: $x_j^* = (\frac{\tilde{\alpha}_H}{2(1+\tau_r)})^2$. Since under risk shifting the investment schedule leads to over investment, in this case we would like to choose a maximum $\tau = \tau_r$ to limit extend of inefficient overinvestment. The thresholds in terms of α_L, α_H and $E(\alpha)$ follow from comparing the Social Planner utility under the two solutions.

A.5. Derivation of Lemma ?? and Proposition ??

The presence of the legacy asset does not affect the FOCs of the problem. Thus, the solvent and the risk-shifting allocations are characterized as before. To derive the individual risk-shifting threshold let us compare the profits under the two allocations: If $p < qR$:

$$\Pi_s = \alpha * \frac{\alpha p}{2qR} + \frac{qR}{p}(s - (\frac{\alpha p}{2qR}^2) - s - \lambda$$
$$\Pi_r = q[\alpha * \frac{\alpha p}{2R} + \frac{R}{p}(s - (\frac{\alpha p}{2R})^2 - s - \lambda]$$

If $p = qR$:

$$\Pi_s = \alpha * \frac{\alpha p}{2qR} - s - \lambda$$
$$\Pi_r = q[\alpha * \frac{\alpha p}{2R} + \frac{R}{p}(s - (\frac{\alpha p}{2R})^2 - s - \lambda]$$

The threshold follows from setting the profits equal to each other.

In the presence of losses on legacy asset, the solvent investment price lower bound is shifted upwards: risk-shifting is preferred at higher prices levels than without the losses.

As before the highest feasible price under the all solvent investment is given by $p = q(\alpha)R$. Evaluating the risk-shifting threshold at the fair price, thus gives us the equilibrium risk-shifting threshold.

Since now the lowest price under which the risk shifting is possible is higher, more banks risk-shift in equilibrium. If the losses are not too large we still have that the resulting equilibrium is mixed as long as $p_r^* > \hat{p}$ for all α.