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Abstract

Asset-based lending, the supply of loans based on floating collateral, is an important source of funding for small firms. We analyze the effect of competition on asset-based loan markets on interest rate distributions and the mobility of small firms. Close monitoring of collateral by lenders results in an informational advantage for the incumbent lender and third-degree price discrimination. We find that adverse selection results in a unique equilibrium in which lenders randomize interest rates and firms switch lender with positive probability. Increased competition between lenders does not benefit firms through lower expected interest rates, neither does it improve their mobility.

Keywords: asset-based lending, floating collateral, adverse selection
JEL code: D53, D82

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1 Introduction

Small business lending was unequally hit by the 07-09 financial crisis (OECD, 2013, ECB, 2013). Small businesses (small- and medium-sized enterprises or SMEs) are the main drivers of innovation and employment, and depend primarily on bank funding.[1] Even though after the 07-09 financial crisis the number of lenders and the amount of loans for small business have increased, the ECB survey (2014) and Chen *et al.* (2017) report that the interest rates for small businesses in the U.S. and Europe remain elevated in contrast to the interest rates for larger firms. Asset-based lending is one form of transaction-based lending and an important source of (bank) funding for small businesses (Berger and Udell, 2006). In 2012 the asset-based loan market was estimated to consist of \$620 billion outstanding loans in the U.S.,[2] \$27 billion in the U.K., and \$1.1 trillion in continental Europe.[3] Asset-based loans are loans that are based on floating collateral or liens, like inventory and accounts receivable.

In this paper we analyze how competition on the asset-based loan market affects interest rates and the mobility of SMEs between lenders. We study a loan market consisting of high-risk and low-risk SME borrowers. We assume the ratio of high-risk to low-risk SMEs in the portfolio of each lender is common knowledge.[4] But the risk profile of each specific SME is private knowledge of the (informed) lender, who has served the client in the previous period. As a result third-degree price discrimination by the informed lender takes place. Each period firms roll over their debt. The informed and the uninformed lenders offer interest rates to these SMEs simultaneously. We assume that daily monitoring of collateral keeps the collateral value fixed in comparison to the loan amount.[5]

One key result of our paper is that there is a unique equilibrium distribution of interest rates in a market with two or more lenders. Perhaps somewhat surprisingly, an increase in competition[6] does not affect the expected lowest loan rate

[1] OECD (2013), European Central Bank survey for SMEs (2014)

[2] This number is not specific for small businesses. It is hard to estimate the size of the asset-based loan market, because syndicated loans, bank loans and other debt products frequently also use floating liens. Additionally factoring and supply chain financing are also forms of asset-based lending. Source: http://www.oas.org/en/sla/dil/docs/gm_jamaica_feb_2015_presentations_Richard_Hawkins_1.pdf,

[3] https://mazarsledger.com/article/the-appeal-of-asset-based-lending-in-the-uk-and-throughout-europe/

[4] Loan loss provisions in year reports and financial analysts give a fair view on the quality of loan portfolios of lenders and are public knowledge. In Section 6 we check the robustness of this assumption.

[5] Mester *et al.* (2007) give a more extensive description on how asset-based lenders keep this ratio fixed. In practice this ratio is referred to as the borrowing base or the collateral coverage ratio, see Cerqueiro *et al.* (2016).

[6] For our model it does not matter if competition is caused by more or less concentration of

SMEs can get in the market, neither does it affect their probability of switching between lenders. We find that adverse selection results in a unique equilibrium in which lenders randomize interest rates. Close monitoring of collateral by lenders results in an informational advantage for the incumbent lender and third-degree price discrimination. If the number of uninformed lenders increases, the uninformed lenders adapt their interest setting strategies in such a way that the lowest average interest rate offered to SMEs is not affected. Hence the lowest expected interest rate for SMEs is the same in every equilibrium, but the interest setting strategy of the individual uninformed lenders may differ. The informed lender also does not adjust its interest setting strategy to more competition. The informed lender earns positive informational rents, that are unaffected by a change in competition. Thus SMEs do not benefit from increased competition through lower expected interest rates or increased mobility. This matches the empirical evidence of Chen *et al.* (2017) that an increase of lenders on the SME loan market and an increase in the flow of credit after the crisis, did not decrease the loan rate.

We also find that low-risk SMEs are worse off when the ratio of low-risk SMEs to high-risk SMEs deteriorates as during a recession. In contrast to the SME borrowers, informed lenders in our model are better off. We show that the increase in adverse selection induces the uninformed lenders to quote on average higher interest rates. These less attractive quotes result in a lower probability of switching for low-risk SMEs. Hence, this gives the opportunity for informed lenders to quote higher interest rates to their low-risk SMEs and receive higher informational rents. Thus if the ratio of high-risk SMEs increases, low-risk SMEs are worse off, even if their own risk profile remains unaffected.

In Section 5 and 6 we investigate the robustness of our results. In Section 5 we reconsider our assumption on loan costs. We show that even if lenders have heterogeneous costs, but do not differ too much in their (marginal) loan costs, the mixed strategy equilibrium does not change qualitatively. But if the difference in marginal loan costs, by for example online lenders with web-based platforms that exploit the availability of electronic transaction data, is substantial then SMEs might benefit through lower interest rates.[7] In Section 6 we investigate our assumption regarding public information of the ratio of low-risk and high-risk firms of competing lenders. We consider an incomplete information setting, in which uninformed lenders only know the distribution but not the ratio of firm types of their competitors. We show that the uninformed lender still randomizes its interest rates and incumbent lenders earn informational rents, similar to the complete information setting.

In 2018 the most relevant source of funding for small businesses is still bank

lenders or by entry of more or less lenders on the market.

[7] https://www.economist.com/leaders/2015/05/09/the-fintech-revolution

funding, and interest rates on bank loans seem to be on the rise again (ECB, 2018). European banks are being urged to cut costs through digitalization and push for consolidation.[8] Our paper hypotheses that (substantial) cutting of costs by lenders with floating liens benefits small enterprises, whereas consolidation would not hurt. IMF's call for policy measures that restrict mergers, might be less urgent for the asset-based loan market.[9] Rapid technological progress offers a cheaper way to extensively monitor firms for new asset-based lenders and web-based platforms, alike bankers.[10] This trend does not only foster the use of asset-based loans, but also decreases the dependence of small businesses on bank funding. But our model shows that when another crisis hits and adverse selection again rises, small business borrowers will pay through higher interest rates. This also applies for those small firms whose risk profile has not altered.

The role of collateral based on fixed assets for the functioning of loan markets is well analyzed in theory (*e.g.* Bester, 1985, Aghion and Bolton, 1992, Manove *et al.*, 2001). The effect of collateral based on floating assets on competition in loan markets has received less attention in academic literature,[11] and differs for two main reasons. First, because the collateral value of floating assets is highly volatile, it cannot be contracted upon unlike collateral based on fixed assets (Aghion and Bolton, 1992). Second, interest rates in combination with collateral based on floating assets, can therefore also not be used as a screening device for risky borrowers (as in *e.g.* Bester, 1985, and Besanko and Thakor, 1987). Because floating assets are used for everyday business within small businesses, the value of the assets fluctuates on a daily basis. As a result, floating assets can also not be used to counter moral hazard without monitoring, compare Boot *et al.* (1991), Boot and Thakor (1994). As a result asset-based lenders monitor extensively. The daily monitoring of collateral ensures that the ratio of the floating collateral value to the loan amount remains fixed.[12]

The role of monitoring on loan markets with floating liens also differs from loan markets with collateral based on fixed assets. Cerqueiro *et al.* (2016) show empirically that monitoring is not a substitute for collateral on a market with floating liens, but functions as a complement.[13] Asset-based lenders apply close

[8] https://www.economist.com/leaders/2019/04/06/fixing-europes-zombie-banks

[9] https://www.economist.com/finance-and-economics/2019/04/04/the-imf-adds-to-a-chorus-of-concern-about-competition

[10] https://www.economist.com/leaders/2015/05/09/the-fintech-revolution

[11] Notable exceptions in empirics are Carey *et al.* (1998), Klapper (1999), and Cerqueiro *et al.* (2016).

[12] Mester *et al.* (2007)

[13] Carey *et al.* (1998) also report that asset-based lenders monitor more extensively. Rajan and Winton (1995) and Ono and Uesugi (2009) analyze collateral as a complement for monitoring effort. In contrast Manove *et al.* (2001) consider the case where monitoring is a substitute for collateral.

monitoring for two reasons. The first motive is to counter the potential for moral hazard on part of the borrower with regard to the floating assets. The second motive is that the flow of business (merchandise) and transaction accounts offers lenders information on the risk profile of their borrowers, as also found in Picker (1992), Rajan and Winton (1995), Mester *et al.* (2007), and Norden and Winter (2010). The accumulated transaction information gives incumbent lenders an informational advantage over their competitors. Dell'Ariccia (2001) and Marquez (2002) refer to this effect as learning by lending. This learning by lending effect is similar to the 'learning by holding' studied by Plantin (2009). As a result this learning effect creates adverse selection at the time of contract renewal or rollover of the credit line, since the incumbent lender has better information regarding his borrower than a competing lender.

Our paper is most closely related to Dell'Ariccia *et al.* (1999) and Marquez (2002). But in contrast to these paper we use a setup, in which informed and uninformed lenders offer interest rates simultaneously. The simultaneous loan offers in our model allow us to focus on the effect of adverse selection on competition and interest setting behavior.[14] In this respect our model is closer in nature to the simultaneous setup in Varian (1980) and Baye *et al.* (1993). When informed lenders can counter loan offers from competitors in a sequential setup, a lock-in of low-risk firms results (Dell'Ariccia *et al.*, 1999, and Marquez, 2002). This lock-in of firms is not a feature of the asset-based loan market, where businesses repeatedly switch.[15] Thus we explicitly analyze the equilibrium switching behavior of businesses. Dell'Ariccia *et al.* (1999) also obtain a two-lender mixed strategy equilibrium, and Marquez (2002) extends this to a market with more symmetric lenders. In these papers the lock-in of low-risk businesses and the inability of lenders to distinguish between switching (high-risk) businesses and new businesses (of both risk profiles) deters entry. We find that in our model, entry can occur, but does not affect the expected minimum interest rate. We also show that entry of new (symmetric or asymmetric) lenders does however affect the individual equilibrium strategies of the non-incumbents.

Less related, but also relevant are the two papers by Hauswald and Marquez (2003, 2006). They consider the effect of screening on competition in credit markets. Because screening costs are a function of geographical distance in their model, only the two (nearest) lenders participate in setting an interest rate for the specific borrower.[16] The volatile nature of floating collateral in the asset-based loan

[14] A sequential setup cannot disentangle the influence of the informed lender's bargaining power from the effect of adverse selection.

[15] Also see DeGryse and Ongena (2005) who find that transaction-based borrowers (of which asset-based lending is a subset) positively switch lenders. Berger and Udell (1995) and Jimenez *et al.* (2006) also show that collateralized loans have shorter maturities.

[16] DeGryse and Ongena (2005) show in Table VI that geographical distance between a bank

market makes screening worthless, but close monitoring prevents moral hazard. As a result and in contrast to these papers, we assume that adverse selection is the dominant feature of the asset-based loan market.

The paper proceeds as follows. Section 2 outlines the model and Section 3 analyzes the case of two lenders. The effect of increased competition is analyzed in Section 4. The extension to cost heterogeneity is in section 5. The case of incomplete information regarding the ratio of high and low-risk borrowers is analyzed in Section 6, Section 7 concludes. Appendix A provides the detailed proofs, except for the proof of Proposition 3 which claims uniqueness. The proof of Proposition 3 is in Appendix B.

2 Model

Consider a loan market with two risk-neutral lenders, lender i and lender j, say. Lender i has a loan portfolio consisting of l_i low-risk SME borrowers and h_i high-risk SME borrowers, where $l_i, h_i > 0$.[17] Lender j has l_j low-risk SMEs and h_j high-risk SMEs in his portfolio, $l_j, h_j > 0$. Our model starts at the time the loan has to be renewed (rollover). By that time lenders have learned the risk profile of the SMEs in their own portfolio, but do not know the risk profile of the SMEs in the portfolio of their opponent. The ratio of SMEs, l_i/h_i or l_j/h_j, in the portfolio of each of the lenders is common knowledge. Lenders only offer one-period loans of size 1 to SMEs and set net interest rates $R_i \in \mathbb{R}^+$ simultaneously. The net marginal costs per unit loan for each lender are equal to $K > 0$.[18] Lenders maximize the expected return on loans.[19]

All SMEs need a loan of size 1 and offer floating assets as collateral of size C.[20]

and borrower is not a determinant for the pricing of collateralized loans.

[17]Borrowers cannot credibly reveal their risk profile, either because the information with regard to their risk profile consists primarily of soft information, or because they do not know their own risk profile. We consider these borrowers to have an inelastic demand for loans. These assumptions reflect the informational opacity of SMEs and their limited options for funding, cf. Berger and Udell (2006).

[18]We assume lenders to have unlimited resources for funding at the current rate of $K > 0$. In Section 5 we analyze the situation where lenders have different marginal costs for loans or when there are fixed costs associated with a loan.

[19]We assume there are no long-term contracting options for lenders, as do e.g. Sharpe (1990) and Von Thadden (2004).

[20]We assume the size of collateral is fixed relative to the loan amount. Asset based lenders monitor on a daily base the accounts receivable and inventory of the borrower to determine the timely value of collateral. On an instant base lenders adapt the loan size to the changing value of the collateral, to keep the ratio between the value of collateral and the loan size equal. In practice this is referred to as the borrowing base or the collateral coverage ratio, see Cerqueiro *et al.* (2016). The borrowing base that asset based lenders use, thus directly links the size of the

The survival probability for low-risk SMEs is equal to p, whereas this probability for high-risk SMEs is equal to q, where $0 < q < p < 1$. Default rates are uncorrelated across SMEs. If the borrower survives, she pays back the loan including the interest rate set by the lender, $(1 + R_i)$. If the borrower defaults, the lender receives the value of the pledged collateral. Thus the expected return of a loan to a low-risk borrower is equal to

$$p\,(1 + R_i) + (1 - p)\,C - (1 + K)\,. \tag{1}$$

Define $r_i \equiv 1 + R_i - C$ and $c \equiv 1 - C + K$. The variable r_i is the strategic choice variable of a lender as it varies one-on-one with the quoted interest rate R_i. Here c stands for the marginal costs of a loan. We keep the required collateral value C fixed in comparison to the loan amount for simplicity. Thus the expected return on a loan to a low-risk borrower is equal to

$$v\,(r_i) \equiv pr_i - c$$

Similarly, the expected return on a loan to a high-risk borrower is equal to

$$w\,(r_i) \equiv qr_i - c$$

which only differs in the survival rate. SMEs maximize their utility by choosing a loan at the lender that offers the lowest interest rate r_i.[21]

Assumption 1 (Tie Rule) *If two lenders offer the same interest rate, the borrower rolls over his loan at his current lender. If both lenders are uninformed, the borrower chooses a loan at either lender based on a coin flip.*

Lender i can price discriminate and may offer a different interest rate to her low-risk SMEs l_i and her high-risk SMEs h_i. But it can only offer a single rate to vie the SMEs from lender j, as it has no information regarding their types. The interest setting strategy for lender i with regard to her SMEs l_i is denoted as $L\,(r_i)$, her SMEs h_i as $H\,(r_i)$ and the $l_j + h_j$ SMEs of lender j as $U\,(r_i)$. Hereafter we also refer to lender i as the informed lender. We do not subscript the low-risk SMEs l and high-risk SMEs h of lender i, when this does not cause confusion. The probability that lender j, hereafter the uninformed lender, offers the SMEs of

loan to the value of collateral. We assume a ratio of 1 for simplicity, but the analysis does not change for any other ratio.

[21] In conformity with Rajan (1992), Diamond and Rajan (2000) and Von Thadden (2004), we assume that borrowers have no alternative funding options and their reservation utility is equal to zero.

lender i a lower interest rate than lender i is equal to $\Pr\{r_j \leq r_i\} = U(r_i)$. Hence the probability that the SMEs of lender i remain at lender i is equal to $1 - U(r_i)$, in case $U(\cdot)$ is continuous.

In the two lenders case, a Nash equilibrium consists of strategy $L^*(r_i)$ for lender i to quote interest rates to its low-risk SMEs l, that maximizes

$$\max_{r_i} (1 - U(r_i))\, v(r_i)\, l; \tag{2}$$

and interest rate strategy $H^*(r_i)$ for lender i's high-risk SMEs h, that maximizes

$$\max_{r_i} (1 - U(r_i))\, w(r_i)\, h. \tag{3}$$

Additionally lender i uses interest rate strategy $U^*(r_i)$ for the SMEs of lender j that maximizes

$$\max_{r_i} (1 - L(r_i))\, v(r_i)\, l_j + (1 - H(r_i))\, w(r_i)\, h_j \tag{4}$$

The interest rate setting strategies for lender j are equivalent and the analysis is analogous. All proofs are in Appendix A and B.

3 Benchmark Analysis

We first obtain the Nash equilibrium and subsequently analyze the economic implications. Consider lender i and the SMEs in its portfolio, l and h. Lender i is perfectly informed about its own client base, but competing lender j only knows the ratio l/h of SMEs. Denote the break-even interest rate for the high-risk types h as $\bar{r} = c/q$, and denote the break-even interest rate for all SMEs of lender i when types cannot be identified by j as

$$\underline{r} = \frac{(l+h)\,c}{pl + qh}$$

An uninformed lender never quotes below $\underline{r}$, as this implies a loss for sure, see (4). The prevailing Nash equilibrium is stated in Proposition 1.

Proposition 1 *Consider the loan portfolio of lender i. Lender i quotes interest rates for its low-risk SMEs l according to the mixed strategy*

$$L^*(r_i) = 1 + \frac{w(r_i)\,h}{v(r_i)\,l} \quad \text{on the support } r_i \in [\underline{r}, \bar{r}]\,;$$

where $w(r_i)\,h/v(r_i)\,l < 0$ *on the support. The interest setting strategy for the high-risk SMEs* h *is a pure strategy, as lender* i *quotes them* c/q *with certainty; i.e.* $H^*(r_i < c/q) = 0$ *and* $H^*(r_i = c/q) = 1$*. The uninformed lender* j *quotes an interest rate for the SMEs* l *and* h *according to the mixed strategy*

$$U^*(r_j) = 1 - \frac{v(\underline{r})}{v(r_j)} \text{ on the support } r_j \in [\underline{r}, \bar{r}) \text{ and } U(\bar{r}) = 1$$

The Nash equilibrium consists of the strategies $(L^*(r_i), H^*(r_i), U^*(r_j))$*. In equilibrium lender* j *has zero expected return* $E[\pi_j(r_j)] = 0$ *and lender* i *has a positive expected return equal to* $E[\pi_i(r_i)] = v(\underline{r})\,l$*.*

A pure strategy equilibrium does not exist. Two forces block a pure strategy equilibrium. The informed lender would like to quote high interest rates to his low-risk SMEs l to extract rents, but this comes at the cost of a higher probability of losing them to an uninformed lender. The uninformed lender prefers to quote low interest rates to the SMEs of lender i to capture the low-risk SMEs l, but this comes at the cost of making a larger loss on the high-risk SMEs h he receives from lender i.[22]

In equilibrium lender i quotes her low-risk SMEs l interest rates according to a mixed strategy, as does lender j. The strategy of lender j is characterized by a mass point of size $1 - U^*(\bar{r}) = hq/(lp + hq)$ at the upper bound of the support. This implies that in every equilibrium lender j quotes the SMEs of lender i interest rate $\bar{r}$ with positive probability. This 'guarantees' that lender i earns positive informational rents in equilibrium on her low-risk SMEs. As a result, the high-risk SMEs h switch to lender j in equilibrium with probability $U^*(\bar{r}) = lp/(lp + hq)$. But not only may the high-risk SMEs transit to the uninformed lender j, also the low-risk types may switch lender.

Corollary 1 *In equilibrium low-risk SMEs switch with positive probability*

$$\Pr\{r_j \leq r_i\} = \frac{1}{2}\left(\frac{lp}{lp + hq}\right) > 0$$

In the asset-based loan market low-risk SMEs switch with positive probability in equilibrium, but are only half as likely to switch as high-risk SMEs. This is in contrast to Dell'Ariccia *et al.* (1999) and Marquez (2002), where the market is characterized by lock-in of low-risk SMEs.

[22] The lack of a pure strategy equilibrium is standard in this setting, see Varian (1980), Narasimhan (1988), Baye *et al.* (1993), Marquez (2002), and Von Thadden (2004).

Corollary 2 *An increase in the relative amount of high-risk SMEs h in the portfolio of lender i increasing adverse selection, lowers the switching probability of the low-risk SMEs l, increases the average interest rate quoted and increases the informational rents of lender i.*

Two aspects stand out with regard to Corollary 2. First, if the amount of high-risk SMEs increases, the low-risk SMEs have a lower probability of switching. This effect is caused by the uninformed lenders as they internalize the 'winner's curse'. When the adverse selection of high-risk SMEs gets worse, lender j sets less competitive interest rates for the SMEs of lender i as it stands to lose more from winning over only the high-risk agents. This implies that the low-risk SMEs are less likely to switch to lender j. Second, this implies that in equilibrium lender i quotes on average higher interest rates to his low-risk SMEs. Higher average interest rates result in higher informational rents for lender i. If the ratio of high to low-risk SMEs varies over the business cycle, Corollary 2 demonstrates the implications of these business cycle changes for the mobility and interest rates of SMEs and rents of lenders.

4 Effect of Increased Competition

The benchmark analysis in Section (3) considers a duopoly of lenders. In this section we assume a market with $n \geq 2$ lenders, where lender $i, j \in I = \{1, ..., n\}$. We denote the number of lenders that actively offer interest rates to the SMEs of lender i as $m \geq 2$.[23] Besides the number of lenders, the setup of the model remains as in Section (2). We assume lenders can distinguish between the SMEs from different competitors. $U_{\min}(r_{-i})$ denotes the distribution of the minimum interest rate offers made by active uninformed lenders. The following proposition gives the equilibrium strategies.

Proposition 2 *Consider the loan portfolio of lender i. In equilibrium lender i quotes interest rates to its low-risk SMEs l according to the strategy*

$$L^*(r_i) = 1 + \frac{w(r_i)\, h}{v(r_i)\, l} \text{ on the support } r_i \in [\underline{r}, \bar{r}] \, ;$$

and to its high-risk SMEs h according to the strategy $H^*(r < \bar{r}) = 0$ and $H^*(\bar{r}) = 1$. Regardless of the equilibrium strategies employed by the $-i$ competitors, the

[23] We show in Proposition 2 that uninformed lenders have a zero return in equilibrium, and hence are indifferent between actively offering interest rates and remaining passive. As a consequence m is a subset of n, and $m = n$ need not hold.

distribution of the lowest quoted interest rates by these uninformed competitors $-i$ is equal to

$$U^*_{\min}(r_{-i}) = 1 - \frac{v\left(\underline{r}\right)}{v\left(r_{-i}\right)} \text{ on the support } r_{-i} \in [\underline{r}, \bar{r}) \text{ and } U^*_{\min}(\bar{r}) = 1 \qquad (5)$$

At least one competitor j randomizes over the support including a mass point at the upper bound of the support. Other competitors may also offer interest rates on the entire support, part of the support, or abstain. The uninformed lenders have a zero expected return, while the informational rents of the i-th lender are as in the benchmark case in Section 3.

We include below the proof that these strategies constitute a Nash equilibrium to provide intuition as to why competition has no effect on the distributions of interest rates that are contracted upon.

Proof. Consider the above strategies. We first determine whether uninformed lenders have an incentive to deviate given $L^*(r_i)$, $H^*(r_i)$, and $U^*_{\min}(r_{-i})$. Offering an interest rate $r_j < \underline{r}$ would result receiving all the SMEs of lender i, but also implies a loss since

$$EU\left(r_j\right) = v\left(r_j\right) l + w\left(r_j\right) h < 0 \text{ for } r_j < \underline{r}$$

on the support. Charging an interest rate $s \in [\underline{r}, \bar{r})$ results in an expected return of

$$EU\left(s\right) = \left(1 - U^*_{\min}\left(s\right)\right)\left(\left(1 - L^*\left(s\right)\right) v\left(s\right) l + \left(1 - H^*\left(s\right)\right) w\left(s\right) h\right)$$

Substituting $L^*(r_i), H^*(r_i)$, and $U^*_{\min}(r_{-i})$ gives

$$\begin{aligned} EU\left(s\right) &= \left(\frac{(p-q)\,h}{lp+hq}\left(\frac{c}{ps-c}\right)\right) \times \\ &\quad \left(\left(\frac{h}{l}\left(\frac{c-qs}{ps-c}\right)\right)(ps-c)\,l + (qs-c)\,h\right) \\ &= 0 \end{aligned}$$

Quoting an interest rate $r_j > \bar{r}$ results in receiving no SMEs from lender i and hence a zero expected return.

Now consider lender i, quoting an interest rate $r_i > \bar{r}$, results in losing all SMEs and hence a zero return. Quoting an interest rate below the support to its low-risk SMEs l, for example $\underline{r}-\varepsilon$, results in an expected return

$$\begin{aligned} E\left[\pi_i\left(\underline{r}-\varepsilon\right)\right] &= \left(1-U\left(\underline{r}-\varepsilon\right)\right)\left(p\left(\underline{r}-\varepsilon\right)-c\right)l \\ &= \left(\frac{\left(p-q\right)lhc}{pl+qh}\right)-lp\varepsilon < v\left(\underline{r}\right) \end{aligned}$$

Similarly, quoting $r_i < \bar{r}$ to the high-risk SMEs h results in an unnecessary loss as $w\left(r_i\right) < w\left(\bar{r}\right) = 0$. Thus the informed lender has no incentive to deviate to a point below the support. In accordance with the uninformed lender's strategy, one shows that deviating on the support results in the same expected return $v\left(\underline{r}\right)l$. Thus the informed lender also has no incentive to deviate. We conclude that the strategies in Proposition 2 entail a Nash equilibrium. ■

The above mixed strategy equilibria set $(L^*(r_i), H^*(r_i), U^*_{\min}(r_{-i}))$ is unique.[24] The proof is in Appendix B.

Proposition 3 *The strategy configuration* $(L^*(r_i), H^*(r_i), U^*_{\min}(r_{-i}))$ *from Proposition 2 is the unique Nash equilibrium.*

Proposition 2 and 3 extend the duopoly equilibrium to a loan market with multiple lenders. Two aspects of this equilibrium are interesting. First, a unique symmetric equilibrium exists in which all lenders participate in quoting to low-risk SMEs l and high-risk SMEs h. In this equilibrium each uninformed lender quotes according to the strategy

$$U^*(r_{j\neq i}) = 1-\left(\frac{v\left(\underline{r}\right)}{v\left(r_j\right)}\right)^{\frac{1}{n-1}}$$

In Appendix B we discuss some examples of asymmetric equilibria. One simple case is where only some of the lenders are actively quoting, while others abstain from offering an interest rate. In each equilibrium at least one uninformed lender and the informed lender i quote interest rates on the support $[\underline{r}, \bar{r})$. The equilibrium in Proposition 1 is also a subset of the equilibria in Proposition 2. Regardless of the number of uninformed lenders that quote an interest rate, or their equilibrium strategy, the distribution of their lowest quotes is, however, in every equilibrium always equal to $U^*_{\min}(r_{-i})$.

[24] In our model not all lenders are symmetric. As a result the Baye and Morgan (1999) equilibria with unbounded support do not exist. The information asymmetry offers the informed lender a beneficial deviation from the symmetric Baye and Morgan (1999) type equilibrium interest rate setting strategies. The proof is in Appendix B.

Second, lender i does not change her strategy for quoting interest rates to her l (and h) SMEs, if the number of competitors increases or if their strategies change. For SMEs only the minimum interest rate they receive from a lender is of interest. The distribution of the lowest interest rates is therefore independent from the number of uninformed lenders who actively participate.[25] Corollary 3 offers the main economic implications of Proposition 2.

Corollary 3 *The minimum expected interest rate uninformed lenders quote and the expected interest rate the informed lender quotes is independent of the number of (actively quoting) lenders, $m \geq 1$. The informational rents of lender i and the probability of switching of the low-risk SMEs l and the high-risk SMEs h are also independent from the number of lenders.*

Competition does not influence the expected interest rate quoted by the informed lender and it's expected rents, nor does it affect the minimum expected interest rate set by the uninformed lenders. The intuition for this result is as follows. If there are few lenders in the market, uninformed lenders quote competitive interest rates to the SMEs of their opponent. As soon as more lenders quote interest rates to these SMEs, the probability of receiving the low-risk SMEs l goes down for a specific uninformed lender. They respond by quoting the upper boundary of the support $\bar{r}$ more frequently and hence offering less competitive interest rates to circumvent adverse selection. Because uninformed lenders strategically take into account the impact of other uninformed lenders, the distribution of lowest quoted interest rates, $U^*_{\min}(r_{-i})$ remains as it is. Hence the interest quoting strategy for informed lender i is unaffected by competition, as are her informational rents.

Because the minimum interest rate uninformed lenders quote is unaffected by the amount of competition, the probability of switching does not change either. Chen *et al.* (2017) empirically analyze small business lending during and after the financial crisis (2006-2014). They find that when competition increased after the crisis, interest rates remained elevated. This result matches our model, where interest rates increase due to an increase in adverse selection but are not influenced by competition.

The main result of this section is that informational rents and expected interest rates in our model are determined by the information asymmetry between lenders. Adverse selection results in informational rents for incumbent lenders. SMEs do not benefit through lower interest rates from more competition. Increasing competition, for example through policy measures that stimulate entry of lenders, does not benefit SMEs.

[25] In a first-price auction Engelbrecht-Wiggans *et al.* (1983) also find that competition does not influence the maximum bid of the players.

Do SMEs benefit from a merger through lower average interest rates? The low-risk SMEs l and high-risk SMEs h of lender i only benefit from a merger if the ratio low-risk SMEs to high-risk SMEs of the opponent lender j that merges is higher than that of their current lender, hence if $l_j/h_j > l/h$. This automatically implies that the SMEs of lender j are worse off because of the merger. Because the average interest rate uninformed lenders quote does not depend on the number of lenders in the market if $n \geq 2$, competition after a merger is only affected through the l/h ratio.[26]

5 Effect of Cost Heterogeneity

While the asset-based loan market is quite competitive, some differences between lenders exist. Chen *et al.* (2017) suggest in their empirical analysis that the persistence of elevated interest rates on the SME loan market after the financial crisis, might be caused by the entry of higher cost providers of credit. In this section we investigate how our results are affected by cost heterogeneity. We analyze the effect of heterogeneity in marginal costs and separately consider the effect of fixed costs. First, consider heterogeneity in marginal costs (monitoring or funding costs) between lenders. Assume lender i and j differ in their marginal costs $c_i, c_j > 0$. Proposition 4 states our result.

Proposition 4 *Consider the market for low-risk SMEs l and high-risk SMEs h. Under the condition*

$$\frac{c_i}{p} < \frac{(l+h)\, c_j}{lp + hq} < \frac{c_i}{q} \tag{6}$$

a mixed strategy Nash equilibrium exists where lender i earns informational rents and has efficiency gains if $c_i < c_j$ or losses if $c_j < c_i$. If condition (6) is not met, at least a pure strategy Nash equilibrium exists where all l and h SMEs switch to lender j or no SMEs switch.

If the difference between the monitoring costs of lender i and lender j are sufficiently small and condition (6) is met, lender i still uses a mixed strategy

$$L^*(r_i) = 1 - \frac{h}{l}\left(\frac{c_j - qr_i}{pr_i - c_j}\right) \text{ on the support } r \in \left[\frac{(l+h)\, c_j}{lp + hq}, \frac{c_j}{q}\right]$$

[26] Empirical research on the SME loan market by Scott and Dunkelberg (2003) shows a similar result with regard to mergers.

in equilibrium. The monitoring costs of uninformed lender c_j determine the support and the mixed strategy of lender i. The informed lender still earns informational rents in equilibrium, but additionally earns efficiency gains or losses. In equilibrium the rents for lender i are equal to

$$\frac{(l+h)\,plc_j}{lp+hq} - lc_i$$

When condition (6) is not met and the costs are $(l+h)\,c_j/\,(lp+hq) < c_i/p$, lender i is too inefficient. Hence lender j wins over all SMEs of lender i and has an efficiency gain in equilibrium. Apart from the pure strategy equilibrium, it is shown in Appendix A that there are also mixed strategy equilibria in which both lenders earn positive rents. When condition (6) is not met and $\frac{c_i}{q} < \frac{(l+h)c_j}{lp+hq}$, lender i is too efficient to be a match for lender j. In equilibrium all SMEs remain at lender i, and lender i earns positive informational and efficiency rents on his l and h SMEs. If lenders do not differ so much regarding their marginal loan costs, there exists a single mixed strategy equilibrium. In such cases, uninformed lenders still have an expected return of zero.

In the near future there may be some disruptive effects from cost differences due to entry of new types of lenders. In the European Union large internet-retailers, like Amazon, may in the future possess similar borrower information as asset-based lenders through the implementation of the payment service directive PSD 2 by the European Commission. Although increased competition as shown in section (5) does not benefit SMEs, lower marginal costs would change market conditions and might be beneficial to SMEs.

The cost price of an asset-based loan comprises variable costs and fixed costs. In order to determine the cost price of a loan, fixed costs are allocated to each loan.[27] Regardless of the allocation method used for fixed costs, allocated fixed costs $f > 0$ influence the expected return on a low-risk borrower. The allocated fixed costs change the payoff for a low-risk borrower in the following way: $pr_i - (c+f)$. Similarly, the expected return on a high-risk borrower changes to $qr_i - (c+f)$. Corollary 4 gives the effect of the introduction of fixed costs in our model.

Corollary 4 *The introduction of fixed costs $f > 0$, shifts the support of the equilibrium strategies of the informed and uninformed lender upwards $r_i, r_j \in \left[\frac{(l+h)(c+f)}{hq+lp}, \frac{c+f}{q}\right)$. Both lenders still use mixed strategies in equilibrium and the*

[27] One method to allocate fixed costs to a loan is absorption costing, see Solomons (1968). This method allocates fixed costs to a loan in accordance with normal loan capacity. The normal loan capacity takes into account fluctuations in capacity over time and is estimated over multiple periods. Activity Based Costing is another allocation method for fixed costs.

informed lender earns higher expected informational rents in equilibrium.

From Proposition 1 it follows that $\partial \overline{r}/\partial c, \partial \underline{r}/\partial c > 0$ and $\partial E[\pi_i]/\partial c > 0$, which essentially proves the Corollary 4. Fixed costs do not cause lower expected returns for uninformed lenders and in that sense also do not deter entry, but the informed lenders are better off. The elevated interest rates Chen *et al.* (2017) observe could be caused by uninformed lenders with higher monitoring costs.

6 Effect of Incomplete Information Asymmetry

In the previous sections we assumed that lender j has complete information regarding the ratio of l and h SMEs in the portfolio of lender i. In this section we consider the case where the number of low-risk SMEs l is common knowledge, but lender j only knows the distribution of h high-risk SMEs.[28] Lender i receives a perfect signal t from distribution $G(t)$, say, where $t \in [k, u]$ with regard to the number of her high-risk SMEs h. Suppose that the strategy $L(t)$ of lender i is monotone and increasing in signal t. The distribution $G(t)$ of the high-risk SMEs h is common knowledge. We denote the uninformed expectation of the number high-risk SMEs h in the portfolio of lender i as $E[h]$.

Proposition 5 *A Nash equilibrium exists in which lender i sets interest rates for her low-risk SMEs l, according to the pure strategy*

$$r_i(t) = \frac{(1-G(t))\,l + E[h]}{(1-G(t))\,pl + qE[h]}c \text{ on the support } t \in [k,u]$$

and to the high-risk SMEs h according to $H^*(r < c/q) = 0$ and $H^*(r = c/q) = 1$. Lender j quotes interest rates to the SMEs of lender i according to strategy

$$U^*(r) = 1 - \frac{(p-q)\,E[h]}{pl + qE[h]}\left(\frac{c}{pr-c}\right) \text{ on the support } r \in \left[\frac{l+E[h]}{pl+qE[h]}c, c/q\right)$$

Proposition 5 shows that lender i quotes her low-risk SMEs l an interest rate according to a pure strategy based on the signal t she receives. Lender i shades her interest rate bid to the low-risk SMEs l upwards in order to earn informational rents. If the signal is for example uniformly distributed $t \sim U[0, 2h]$, lender i's informational rents are equal to the benchmark case in Section 3.

[28] The reverse case in which only the distribution of a_i borrowers is common knowledge, is equivalent.

Adverse selection again hampers lender j when setting an interest rate for the SMEs of lender i. Lender j randomizes her interest rate over an interval and quotes $\bar{r}$ with positive probability. But in this case the strategy of lender j and the support depend on the first moment of the signal distribution t. We show that lender j uses a mixed strategy in equilibrium, even when information asymmetry increases. The intuition for this result is no different than the one described in Section 3: Competition on loan markets with adverse selection results in an equilibrium where at least one lender mixes her interest rates.

7 Concluding remarks

Asset-based lending is an important source of funding for SMEs.[29] Asset-based lenders supply loans based on floating assets, like accounts receivable and inventory. The volatile nature of these assets and the use of these in daily business activities by SMEs requires intensive monitoring by lenders.[30] Intensive monitoring of these floating assets offers an informational advantage for the incumbent lender at the time of contract renewal. As a result adverse selection arises in these markets.

We analyze the asset-based loan market and an inelastic demand for loans, where the ratio of SMEs in a portfolio is common knowledge. We first show that in a two-lender market lenders use a randomized strategy in equilibrium to set interest rates for SMEs. An increase in adverse selection, for example due to a downturn in the business cycle that raises the proportion of high-risk SMEs, decreases the probability that low and high SMEs switch lenders. The only beneficiaries are the asset-based lenders with an informational advantage, who earn higher informational rents.

Our main result states that an informed lender does not adjust its interest rate strategy to the number of uninformed lenders that are active in the market. We show that the unique equilibrium distribution of minimum interest rates uninformed lenders quote is also invariant to the number of competitors. Competition has no effect on the expected interest rates for SMEs, neither does it affect the informational rents of the incumbent lender. An equilibrium in which the uninformed lender uses a mixed strategy and the incumbent lender earns informational rents, is also robust to (small) differences in marginal costs between lenders and incomplete information. Fixed costs result in a mark-up for SMEs, but do not qualitatively change the equilibrium strategies.

The European Union has issued a new payment service directive, PSD 2 in 2018, which intends to offer consumer data integrity and increase innovation. A majority of these recent policy measures and recommendations, like PSD 2, (at

[29] Berger and Udell (2006)

[30] Carey *et al.* (1998)

least partly) aim to increase banking competition, and disclosure requirements for banks. But Chen *et al.* (2017) show that an increase in competition on the market for SME lending and an increase in credit flows after the recent financial crisis (2008-2014) did not decrease interest rates for SMEs in comparison to the crisis period. Our results do not only show *why* this empirical result may arise, but also indicate that policy measures aimed at increasing competition (or the disclosure of loan portfolio composition) may not benefit SMEs on the asset-based loan market through lower expected interest rates or increased mobility.

Appendix A

Proof of Proposition 1.
It is straightforward to verify that it does not pay to deviate unilaterally from the postulated equilibrium strategies $(L^*(r_i), H^*(r_i), U^*(r_j))$ inside or outside the support, and hence these strategies constitute a Nash equilibrium. The Nash equilibrium in the two-lender market is a subset of the Nash equilibria in an n-lender market from Proposition 2. Uniqueness of this equilibrium follows from the proof of Proposition 3 for the n-lender market. □

Proof of Corollary 1.
The probability of switching for a low-risk borrower is equal to

$$\begin{aligned}
&\Pr\{r_j \leq r_i\} \\
&= E_i\left[\Pr\{r_j \leq r | r_i = r\}\right] = \int_{\underline{r}}^{c/q} U^*(r) \cdot l^*(r) dr
\end{aligned}$$

$$\begin{aligned}
&= \frac{(p-q)\,hc}{l} \int_{\underline{r}}^{c/q} \frac{1 - \left(\frac{(p-q)h}{lp+hq}\right)\left(\frac{1}{pr-c}\right)}{(pr-c)^2} dr \\
&= \frac{1}{2}\left(\frac{lp}{lp+hq}\right) > 0
\end{aligned}$$

as $p, q, l, h > 0$. It directly follows that

$$\partial \Pr\{r_j \leq r_i\} / \partial l = pqh/2\,(pl+qh)^2 > 0,$$

and

$$\partial \Pr\{r_j \leq r_i\} / \partial h = -pql/2\,(pl+qh)^2 < 0.$$

The probability of switching is increasing in the number of low-risk SMEs and decreasing in the number of high-risk SMEs. □

Proof of Corollary 2.
The average interest rate lender i quotes its low-risk SMEs l is

$$\begin{aligned} E\left[r_i|L^*(r_i)\right] &= \int_{\underline{r}}^{\overline{r}} r_i \cdot l^*(r_i)dr_i = c\left(p-q\right)\frac{h}{l}\int_{\underline{r}}^{\overline{r}} \frac{r_i}{\left(pr_i-c\right)^2}dr_i, \\ &= \frac{c}{p} + \frac{\left(p-q\right)hc}{lp^2}\ln\left(\frac{lp+hq}{hq}\right) \end{aligned} \tag{7}$$

and

$$\partial E\left[r_i|L^*(r_i)\right]/\partial h_i = \frac{c}{p}\left(p-q\right)\left[\frac{1}{lp}\ln\left(\frac{lp+hq}{hq}\right) - \frac{1}{lp+hq}\right] > 0.$$

The average interest rate lender j offers to the SMEs of lender i is

$$\begin{aligned} E\left[r_j|U^*(r_j)\right] &= \int_{\underline{r}}^{\overline{r}} r_j \cdot u^*(r_j)dr_j + \left(1-U^*(\overline{r})\right)\cdot\overline{r} \\ &= \left(\frac{l+h}{lp+hq}\right)c + \left(\frac{h\left(p-q\right)}{p\left(lp+hq\right)}c\right)\ln\left(\frac{lp+hq}{hq}\right) \end{aligned} \tag{8}$$

and

$$\partial E\left[r_j|U^*(r_j)\right]/\partial h = \left(\frac{l\left(p-q\right)}{\left(pl+qh\right)^2}c\right)\ln\left(\frac{lp+hq}{hq}\right) > 0$$

By comparing (7) with (8) we get

$$\frac{c}{p} + \frac{\left(p-q\right)hc}{lp^2}\ln\left(\frac{lp+hq}{hq}\right) \leq \frac{l+h}{lp+hq}c + \left(\frac{h\left(p-q\right)}{p\left(lp+hq\right)}c\right)\ln\left(\frac{lp+hq}{hq}\right)$$

or

$$\ln\left(\frac{lp}{hq}+1\right) \leq \left(\frac{lp}{hq}\right) : \forall l,h,p,q > 0$$

Thus $E\left[r_i|L^*(r_i)\right] < E\left[r_j|U^*(r_j)\right]$, that is the informed lender i offers on average lower interest rates to its low-risk agents than the outside uninformed lenders do. The informational rents for lender i are

$$E\left[\pi_i\left(r_i\right)\right] = lv(\underline{r})$$

Since

$$\partial E\left[\pi_i\left(r_i\right)\right]/\partial h = \frac{\left(p-q\right)l^2p}{\left(lq+hp\right)^2}c > 0$$

the informational rents are increasing in h. This concludes the proof. $\square$

Proof of Corollary 3.
The claim directly follows from the proof of Proposition 2 and **3** in Appendix B.
□

Proof of Proposition 4.
Consider the SMEs in the portfolio of lender i. Lender i has marginal cost equal to c_i and lender j equal to c_j. Denote the break-even rates for the high-risk SMEs of lender i as $\overline{r}_i = c_i/q$ for lender i and $\overline{r}_j = c_j/q$ for lender j, respectively. The break-even rates of i and j for the low-risk SMEs of lender i are respectively $s_i = c_i/p$ and $s_j = c_j/p$. The break-even rate for all the SMEs in the portfolio of lender i is $\underline{r}_i = \left(\frac{l+h}{lp+hq}\right) c_i$ and $\underline{r}_j = \left(\frac{l+h}{lp+hq}\right) c_j$ in the case j captures both types of clients form lender i. We consider four different cases:

1. **Lender j is slightly more efficient than lender i: $s_i < \underline{r}_j < \overline{r}_j < \overline{r}_i$.** Lender i does quote its high-risk clients not lower than $\overline{r}_i$, so that in equilibrium the h_i SMEs always switch (we show that the support is $\left[\underline{r}_j, \overline{r}_j\right]$). Moreover, i can at least make $\underline{r}_j$ on its low-risk SMEs. Consider

$$E\left[\pi_j\left(r_j\right)\right] = \left(1 - L(r)\right)\left(pr - c_j\right) l + \left(qr - c_j\right) h \tag{9}$$

and

$$E\left[\pi_i\left(r_i\right)\right] = \left(1 - U\left(r\right)\right)\left(pr - c_i\right) l. \tag{10}$$

As in the symmetric case $E\left[\pi_j\left(r_j\right)\right] = 0$, since $\underline{r}_j$ is the lower bound of the support. Also note that at the upper bound, j makes zero on the high-risk types who switch over. Rewriting equation (9) gives

$$L^*(r) = 1 - \frac{\left(c_j - qr\right) h}{\left(pr - c_j\right) l} \text{ on the support } \left[\underline{r}_j, \overline{r}_j\right]$$

Suppose $E\left[\pi_i\left(r_i\right)\right] > 0$. Substitution in equation (10) gives

$$U\left(r\right) = 1 - E\left[\pi_i\left(r_i\right)\right] / \left(\left(pr - c_i\right) l\right)$$

In accordance with the proof of Proposition (2), $U\left(\underline{r}_j\right) = 0$. Substitution of $U\left(\underline{r}_j\right) = 0$ into equation (10) results in

$$E\left[\pi_i\left(r_i\right)\right] = \frac{\left(l + h\right) pl}{lp + hq} c_j - lc_i.$$

The equilibrium strategy of the uninformed lender follows as

$$U^*(r) = 1 - \left(\frac{(c_j - c_i)\, lp + (pc_j - qc_i)\, h}{(lp + hq)\,(pr - c_i)}\right) \text{ on the support } \left[\underline{r}_j, \overline{r}_j\right)$$

with a mass point equal to $1 - U(\overline{r}_j) = 1 - \frac{plc_j(p-q)}{(lp+hq)(pc_j - qc_i)}$. The equilibrium strategy for the high-risk SMEs remains $H(r < \overline{r}_i) = 0$ and $H(r = \overline{r}_i) = 1$. Lender i earns lower informational rents in comparison to equal marginal costs case, i.e. in comparison to the when c_i is lowered to c_j.

2. **Lender j is much more efficient: $\underline{r}_j < s_i < \overline{r}_j < \overline{r}_i$.** In this case there are multiple equilibria. There is one pure strategy ε-equilibrium and host of mixed strategy equilibria. The pure strategy equilibrium is for the uninformed to bid $r_j = s_i - \varepsilon$, while the informed bids $r_i = s_i$. The informed lender makes zero as it loses all SMEs to j. For ε sufficiently small, the uninformed makes

$$\pi_j(s_i - \varepsilon) = (lp + hq)(s_i - \varepsilon) - (l + h)\, c_j > 0$$

The informed lender clearly does not gain by deviating upwards, while deviating downwards with a interest rate offer to its low-risk clients below $s_i - \varepsilon$, would be loss making. The uninformed lender would not receive the low-risk clients l, if it raised its price slightly to s_i, given assumption (1). Thus this would yield a discrete drop in profits. By the linearity of $(pr - c_j)$ and $(qr - c_j)$, it does not pay j to lower its interest quote below $s_i - \varepsilon$. There are, however also mixed strategy equilibria in which both lenders make a profit. One such an equilibrium involves the following mixed strategies:

$$L(r) = 1 - \left(\frac{(c_i - c_j)\, h - (qr - c_j)\, h}{(pr - c_j)\, l}\right)$$

and

$$U(r) = 1 - \left(\frac{p\,(c_j + hc_i)}{lp + hq} - c_i\right)\left(\frac{1}{pr - c_i}\right)$$

on $[\frac{lc_j + hc_i}{lp+hq}, \frac{c_i}{q})$. Note that $U(.)$ has a mass point at the upper bound $\overline{r}_i$. The informed offers $\overline{r}_i$ to its high-risk types, who subsequently always switch. Furthermore, the payoffs are

$$E[\pi_i] = \left(\frac{lp}{lp + hq}\right)(c_j + hc_i) - lc_i$$

and

$$E\left[\pi_j\right] = \left(c_i - c_j\right) h$$

Other mixed strategy equilibria have a support with an upper bound below $\overline{r}_i$.

3. **Lender i is much more efficient than j: $s_i < \overline{r}_i < \underline{r}_j < \overline{r}_j$.** There is a pure strategy equilibrium whereby both lenders quote $\underline{r}_j$ to the SMEs from i. In this case both types stay with lender i, given the assumption (1). The uninformed makes zero as it gets no clientele, but it can also not deviate downwards to capture clients, as offering less than $\underline{r}_j$ and receiving both types is loss making. Note that i makes positive returns on both types since $\underline{r}_j > \overline{r}_i$. Hence, i likes to retain both types and has no incentive to deviate upwards as it would lose all SMEs. Deviating downwards would lower returns on both types.

4. **Lender i is only slightly more efficient: $s_i < \underline{r}_j < \overline{r}_i < \overline{r}_j$.** The following is a mixed strategy Nash equilibrium. Lender i offers the high-risk SMEs h $\overline{r_j}$. Consider the expected profit for uninformed lender j on the SMEs of lender i

$$E\left[\pi_j\left(r_j\right)\right] = \left(1 - L(r)\right)\left(pr - c_j\right) l + \left(qr - c_j\right) h$$

Substituting $E\left[\pi_j\left(r_j\right)\right] = 0$, results in the strategy for lender i for her low-risk SMEs

$$L^*\left(r\right) = 1 - \frac{h}{l}\left(\frac{c_j - qr}{pr - c_j}\right) \text{ on the support } r \in \left[\underline{r}_j, \overline{r}_j\right]$$

The expected profit for lender i on her low-risk SMEs is equal to

$$E\left[\pi_i^l\left(r_i\right)\right] = \left(1 - U\left(r\right)\right)\left(pr - c_i\right) l$$

From this we get

$$U\left(r\right) = 1 - E\left[\pi_i^l\left(r_i\right)\right] / \left(\left(pr - c_i\right) l\right)$$

Substituting $U\left(\underline{r}_j\right) = 0$, gives the equilibrium payoff

$$E\left[\pi_i^l\left(r_i\right)\right] = \frac{\left(l + h\right) p c_j l}{lp + hq} - c_i l \tag{11}$$

Hence lender j's strategy follows as

$$U^*(r) = 1 - \frac{(c_j - c_i)\,lp + (pc_j - qc_i)\,h}{(lp + hq)\,(pr - c_i)} \text{ where } r \in \left[\underline{r}_j, \overline{r}_j\right)$$

where $U^*(\overline{r}_j) < 1$, so that there is a mass point. In order for this combination of strategies $(H^*(r_i), L^*(r_i), U^*(r_j))$ to be an equilibrium neither of the lenders should have an incentive to deviate. Let's start with lender j. If he offers an interest rate above $\overline{r}_j$, he receives none of the SMEs of lender i and has a zero expected return. If he offers an interest rate below $\underline{r}_j$, he receives all of the SMEs but has a negative expected return (as this is below his break-even interest rate $\underline{r}_j$). Hence lender j weakly prefers offering interest rates according to strategy $U^*(r_j)$. If lender i offers her low-risk SMEs an interest rate above $\overline{r}_j$, she loses her low-risk SMEs and has a zero expected return. If she offers her low-risk SMEs an interest rate below $\underline{r}_j$, her expected return is

$$\begin{aligned}&\left(p\left(\underline{r}_j - \varepsilon\right) - c_i\right) l = \\ &\left(\frac{(l_i + h_i)\,pc_j l}{lp + hq}\right) - lc_i - pl\varepsilon\end{aligned}$$

which is smaller than $E\left[\pi_i^l(r_i)\right]$ in equation (11). Thus informed lender i does not want to deviate, and $L^*(r_i)$ is an equilibrium strategy. Now consider the high-risk SMEs of lender i. If lender i sets an interest rate according to the pure strategy with all mass at $\overline{r_j}$, her expected return is

$$E\left[\pi_i^h(\overline{r}_j)\right] = \left(\frac{(c_j - c_i)^2\,lp}{(pc_j - qc_i)} + (c_j - c_i)\,h\right)\left(\frac{hq}{lp + hq}\right) > 0 \tag{12}$$

If lender i offers her high-risk SMEs an interest rate above $\overline{r}_j$, all high-risk SMEs switch and she makes a zero return. If lender i offers an interest rate below $\overline{r}_j$, her expected return is equal to

$$(1 - U^*(\overline{r}_j - \varepsilon))\,(q\,(\overline{r}_j - \varepsilon) - c_i)\,h$$

Substitution gives

$$\left(\frac{(c_j - c_i)\,lpq + (pc_j - qc_i)\,hq}{(lp + hq)\,(pc_j - pq\varepsilon - qc_i)}\right)(c_j - q\varepsilon - c_i)\,h$$

Because this expected return is smaller than $E\left[\pi_i^h(\overline{r}_j)\right]$ in equation (12), she does not want to deviate.[31] Thus the strategies

$$\left(H^*\left(r_i\right), L^*\left(r_i\right), U^*\left(r_j\right)\right)$$

constitute a Nash equilibrium.□

Proof of Proposition 5.

Consider the benchmark case of Section (3) with just two competitors. Suppose that the number of high-risk types in the portfolio of the informed bank i is a random variable h. Assume that the informed lender i observes this number perfectly, whereas the other lender j is not informed, but knows the distribution of h. Let the distribution of the number of high-risk types be $G(t)$, where $G(t) = \Pr\{h \leq t\}$ with bounded support $[k, u]$. The expectation of signal t is $E[h] = \int_k^u t dG(t)$. The uninformed lender only knows the distribution $G(t)$ and hence $E[h]$. The random payoff to lender j is

$$\pi_j\left(r_j\right) = \Pr\{j \text{ wins}\}\left[pr_j - c\right] l + \left[qr_j - c\right] h$$

The probability that j wins is the probability that j bids below the interest rate that i offers to its low-risk types (the high-risk types again always switch to j as i prices at c/q for these types), i.e. this amounts to $\Pr\{r_j < r_i\}$. The offer that i makes is a function of the number of high-risk types h that occur in his portfolio, $r_i = r_i(t)$. Suppose that this interest rate offer function is monotonically increasing in t. Upon inverting $r_j < r_i(t)$, we get that j wins if $r_i^{-1}(r_j) < t$. This happens with probability

$$\int_{r_i^{-1}(r_j)}^{u} dG(t) = 1 - G(r_i^{-1}(r_j))$$

Hence

$$E\left[\pi_j\left(r_j\right)\right] = \left[1 - G(r_i^{-1}(r_j))\right]\left[pr_j - c\right] l + \left[qr_j - c\right] E[h]$$

Suppose that j can be held down to making zero profits on average, then

$$1 - G(r_i^{-1}(r_j)) = \frac{c - qr_j}{pr_j - c}\frac{E[h]}{l} \tag{13}$$

[31] One shows that lender i deviates when $E\left[\pi_i^{b_i}\left(\bar{r}_j - \varepsilon\right)\right] > E\left[\pi_i^{b_i}\left(\bar{r}_j\right)\right]$, which is the case when

$$\frac{pc_j - qc_i}{pc_j - pq\varepsilon - qc_i} > \frac{c_j - c_i}{c_j - q\varepsilon - c_i}$$

But this requires

$$-\left(p - q\right) qc_i\varepsilon > 0$$

which violates the assumption that $p > q$. Hence the informed lender i has no incentive to deviate.

Note that consistent with the assumption of a monotonic interest rate offer function

$$\frac{dG(r_i^{-1}(r_j))}{dr_j} = g(r_i^{-1}(r_j))\frac{1}{r_i'(r_i^{-1}(r_j))} = \frac{p-q}{(pr_j-c)^2}c\frac{E[h]}{l} > 0$$

and by assumption $r_i'(.) > 0$. We can now solve for the (deterministic) bid function $r_i(t)$ given that i observes t. Suppose that j quotes t to the SMEs of i, i.e. $r_j = t$. Then

$$r_i^{-1}(r_j = t) = t$$

since in equilibrium

$$t = r_i\left(r_i^{-1}(t)\right) = r_i(t)$$

Then by substituting $r_i^{-1}(r_j = t) = t$ and $r_j = t$ into (13), gives

$$1 - G(t) = \frac{c - qr_i}{pr_i - c}\frac{E[h]}{l}$$

Inverting this expression gives

$$r_i(t) = \frac{l[1-G(t)] + E[h]}{pl[1-G(t)] + qE[h]}c$$

Note that indeed

$$\frac{dr_i(t)}{dt} = \frac{(p-q)\,lg(t)E[h]}{\{pl[1-G(t)] + qE[h]\}^2}c > 0$$

Turn to the expected profit of lender i:

$$E[\pi_i] = \Pr\{i \text{ wins if bidding } x\}[px - c]\,l$$

On the support the mixing strategy of j must be such that $E[\pi_i]$ is constant. Thus the mixing strategy follows as

$$\Pr\{i \text{ wins if bidding } x\} = \frac{E[\pi_i]}{[px - c]\,l}$$

Since $G(r_i^{-1}(k)) = 0$ and $G(r_i^{-1}(u)) = 1$, we get from the bidding strategy of i

$$r_i(k) = \frac{l[1-G(k)] + E[h]}{pl[1-G(k)] + qE[h]}c = \frac{l + E[h]}{pl + qE[h]}c$$

Suppose that i wins with probability one if it bids k. Subsequently, plugging $r_i(k)$

into $E[\pi_i] = \Pr\{i \text{ wins}\}[pr_i - c]\, l$ gives

$$E[\pi_i] = [pr_i(k) - c]\, l = pl\frac{l + E[h]}{pl + qE[h]}c - lc = \frac{(p-q)\, E[h]}{pl + qE[h]}lc > 0$$

Also note that by offering at the upper end of the support u, $G(u) = 1$ so that

$$r_i(u) = \frac{l[1 - G(u)] + E[h]}{pl[1 - G(u)] + qE[h]}c = \frac{c}{q}$$

It follows that

$$\begin{aligned} \Pr\{i \text{ wins if bidding } u\} &= \frac{(p-q)\, E[h]}{pl + qE[h]}lc\frac{1}{p\frac{c}{q} - c}\left(\frac{1}{l}\right) \\ &= \frac{qE[h]}{pl + qE[h]} < 1 \end{aligned}$$

So there is a mass point

$$\frac{pl}{pl + qE[h]}$$

at the upper end. Moreover one verifies that deviations from the strategies $r_i(t)$ and $E[\pi_i] / [px - c]\, l$ do not pay.□

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Appendix B

The purpose of this appendix is to obtain uniqueness regarding the pricing strategies of Proposition 2. The proof of Proposition 3 also implies that the strategies can only have bounded supports.

Preliminaries for the proof of Proposition 3.
Consider the competition over the portfolio of the specific informed lender $i \in I = \{1, 2, ..., n\}$, the portfolios of lenders $-i$ being isomorphic. Denote $\delta_i(r)$ as the mass placed by lender i's strategy at r. Recall $\underline{r} = ((l+h)c)/(pl+qh)$, and $\bar{r} = c/q$. Denote the strategy of lender i for its low-risk SMEs l by $L(r)$ and the strategy for her high-risk SMEs h by $H(r)$. The strategy of uninformed lender j who offers an interest rate for the SMEs of lender i is $U_j(r)$.

The uninformed lenders may differ regarding their interest rate strategies. An uninformed lender for example has the choice to participate or not. In any equilibrium there are $m \in M \subset I$ active uninformed lenders who quote an interest rate to the l and h SMEs of lender i, where $1 \leq m \leq n-1$. The minimum poaching rate offered by the uninformed lenders follows the distribution

$$U_{\min}(r) = 1 - \prod\nolimits_{j \neq i}^{m} [1 - U_j(r)]$$

By assumption, SMEs go for the lowest rate on offer. The distribution of the best outside offer is $U_{\min}(r)$. Furthermore, define

$$U_{\min}^{k}(r) = 1 - \prod\nolimits_{k \neq i,j}^{m} [1 - U_k(r)]$$

to be the distribution of the lowest rate offered by uninformed lenders other than uninformed lender j. Furthermore, for another uninformed lender k, define

$$U_{\min}^{j \neq k} = 1 - \prod\nolimits_{j \neq i,k}^{m} [1 - U_j(r)]$$

The expected return for lender i on her high-risk SMEs h is

$$E\left[\pi_i^h(r)\right] = [1 - U_{\min}(r)]\, w(r) h$$

Lender i expects to make on her low-risk SMEs l

$$E\left[\pi_i^l(r)\right] = [1 - U_{\min}(r)]\, v(r)\, l \tag{14}$$

The expected return for uninformed lender j on the SMEs of lender i is

$$E\left[\pi_j\left(r\right)\right] = \left(1-U_{\min}^{k}\left(r\right)\right)\left[1-L\left(r\right)\right]v\left(r\right)l + \left(1-U_{\min}^{k}\left(r\right)\right)\left[1-H\left(r\right)\right]w\left(r\right)h \tag{15}$$

Recall that both $v(r)$ and $w(r)$ are linear and increasing in the interest rate r. Denote by r_j the interest rate offered by the j-th uninformed lender. Let r_l and r_h represent the interest rates the informed lender quotes to its l and h SMEs. The proof of Proposition 3 is structured as follows. We first determine the support of the equilibrium strategies in Lemma's 1 - 10. Next, we characterize the equilibrium strategies and proof uniqueness in Lemma 11 - 12.

Proof of Proposition 3.

Lower Bound of the Support

Lemma 1 *Informed lender i does not quote interest rates below $\bar{r}$ to its high-risk SMEs h, that is $H\left(r_i<\bar{r}\right)=0$.*

Proof. Since the informed lender i knows its h clientele and $w\left(r<\bar{r}\right)<0$, it follows that $H\left(r<\bar{r}\right)=0$. ■

Lemma 2 *Below $\bar{r}$, if r_j is such that j attracts the l, it also attracts the h.*

Proof. Since uninformed lenders cannot discriminate between borrower types and if the l switch to j, it means that j has set the lowest rate among the uninformed lenders. Thus the h have no incentive to switch to an uninformed lender other than j. By Lemma 1 the informed lender does not compete over the h below $\bar{r}$. ■

Lemma 3 *No uninformed lender offers an interest rate below $\underline{r}$.*

Proof. If $r_j\leq\underline{r}$ is such that j only attracts the h, this is loss making, since $w(r_j)<0$ for $\forall r_j<\bar{r}$. If r_j is such that j attracts the low-risk SMEs l, then by Lemma 2 j also gets the high-risk SMEs h. But for $r_j<\underline{r}$, by the definition of $\underline{r}$ the sum $v\left(r_j\right)+w\left(r_j\right)<0$. Hence lender j has no incentive to quote $r_j<\underline{r}$. ■

Lemma 4 *Informed lender i sets interest rates r_i for its low-risk clientele such that $r_i\geq\underline{r}$.*

Proof. At $r_i=\underline{r}$ lender i retains all its low-risk SMEs with certainty. This holds as no agents switch if this quote is matched by some uninformed lender, due to assumption (1), and because no uninformed lender quotes interest rates below $\underline{r}$ by Lemma 3. Thus lender i can guarantee itself $v\left(\underline{r}\right)$. Because $v\left(r\right)$ is increasing in r, lender i has no incentive to set $r_i<\underline{r}$. ■

Upper Bound of the Support

We first show that strategies with mass points above $\bar{r}$, including pure strategies, cannot exist in equilibrium.

Suppose a distribution function $U_j(s)$ of some j has some probability mass at x denoted by $\delta_j(x)$. Due to this $U_{\min}(s)$ jumps up at x. By convention the distribution function is right continuous at x. Due to assumption (1) by which existing customers do not switch if bank i quotes the same rate x as an uninformed lender, we need the left limit of the distribution function to identify the probability that customers stay with bank i. Thus, let

$$U_{\min}(x^-) \equiv \lim_{s \uparrow x} U_{\min}(s)$$

Lemma 5 *The strategies $U_j(r)$ for any $j \epsilon M$, and by implication $U_{\min}(r)$ contain no atoms in the open interval $(\bar{r}, \infty)$.*

Proof. Note that both $v(r) > 0$ and $w(r) > 0$ are continuous on $r > \bar{r}$ and increase monotonically. Suppose $U_j(r)$ has a mass point $\delta_j(x) > 0$ at some $x > \bar{r}$. Then $[1 - U_{\min}(x)]$ has a downward jump at x. The payoff to the informed lender i at x is

$$\left[1 - U_{\min}\left(x^-\right)\right]\left[v(x) l + w(x) h\right]$$

since whatever the mass that the uninformed places at x, does not affect the probability that customers switch due to assumption (1). We deal with two cases: a) x is not an endpoint $U_{\min}(x) < 1$ and b) x is an endpoint $U_{\min}(x) = 1$. Start with case a). For the no endpoint case, there exists $\varepsilon > 0$ such that

$$\begin{aligned} & \lim_{\varepsilon \downarrow 0} \left[1 - U_{\min}(x + \varepsilon)\right]\left[v(x + \varepsilon) l + w(x + \varepsilon) h\right] \\ < \ & \left[1 - U_{\min}\left(x^-\right)\right]\left[v(x) l + w(x) h\right] \end{aligned}$$

by continuity of $v(r) l + w(r) h$ and the discontinuity in $U_{\min}(r)$ at x. Therefore the informed lender i does not place any mass in some ε-neighborhood above x. But then it is not an equilibrium strategy for uninformed lender j to place mass at x, as it could make more by moving the mass point slightly to the right (and have the same probability of winning over the clientele from i).[32] Next, consider case b), where x is the endpoint of the support of $U_{\min}(x)$, so that $U_{\min}(x) = 1$,

[32] Other uninformed lenders also remove mass above x. Consider uninformed lender $k \neq i, j$, and denote

$$U_{\min}^{j}(r) = 1 - \prod_{j \neq i,k}^{m} \left[1 - U_j(r)\right]$$

but $U_{\min}(x^-) < 1$. In that case for the $j-$th lender

$$\left[1 - U^k_{\min}(x)\right]\left[(1 - L(x))\, v(x)\, l + (1 - H(x))\, w(x)\, h\right] = 0$$

If over some interval $(d, e)\,\epsilon\,(\bar{r}, x)$ it holds that $U^k_{\min}(r) < 1$, and at least $L(r) < 1$ or $H(r) < 1$ for $r\epsilon\,(d, e)$, then

$$\left[1 - U^k_{\min}(r)\right]\left[(1 - L(r))\, v(r)\, l + (1 - H(r))\, w(r)\, h\right] > 0$$

Thus the j-th lender would be better off by moving the mass from x to the lower interval (d, e). ■

Lemma 6 *The strategies $L(r)$ and $H(r)$ contain no atoms on the open interval $(\bar{r}, \infty)$.*

Proof. Similarly, if $L(r)$ or $H(r)$ have a mass point, one shows that the uninformed lenders have no incentive to place any mass in the neighborhood above of this mass point. So supose that either $L(r)$ or $H(r)$, or both have a mass point at x. Due to assumption (1), we can now directly use the right continuity of $L(x)$ and $H(x)$. Thus the expected payoff to an uninformed lender j offering x is

$$\left[1 - U^k_{\min}(x)\right]\left[(1 - L(x))\, v(x)\, l + (1 - H(x))\, w(x)\, h\right]$$

Because for some $\varepsilon > 0$

$$\begin{aligned}&\left[1 - U^k_{\min}(x)\right]\left[(1 - L(x))\, v(x)\, l + (1 - H(x))\, w(x)\, h\right] < \\ &\left[1 - U^k_{\min}(x-\varepsilon)\right]\left[(1 - L(x-\varepsilon))\, v(x-\varepsilon)\, l + (1 - H(x-\varepsilon))\, w(x-\varepsilon)\, h\right]\end{aligned}$$

due to the discrete jump downwards at x of $1 - L(r)$ or $1 - H(r)$, or both, while $v(r)$ and $w(r)$ are continuous at x. Hence, the uninformed lenders do not place any mass in some right neighborhood of x, and the informed would be better of to move mass to the right. ■

The expected payoff to k is

$$\begin{aligned} E[\pi_k(r)] &= \left[1 - U^j_{\min}(r)\right]\left[1 - L(r)\right] v(r)\, a \\ &\quad + \left[1 - U^j_{\min}(r)\right]\left[1 - H(r)\right] w(r)\, b \end{aligned}$$

due to the discrete jump upwards at x in $U_j(r)$, the term $1 - U^j_{\min}(r)$ jumps downwards. As a result $\exists\varepsilon, \delta > 0$ for which

$$E[\pi_k(r+\varepsilon)] < E[\pi_k(r-\delta)]$$

hence uninformed lender k also removes any mass in some right-hand side ε neighborhood.

Combining the two cases in Lemma 5 and Lemma 6, it follows that there cannot occur any mass point in the potential mixing strategies, i.e. $\delta(r) = 0$ for $r > \bar{r}$.

The following Lemma's prove that no continuous mixing equilibrium exists where $L(r_i > \bar{r}) > 0$, $H(r_i > \bar{r}) > 0$ or $U_{\min}(r_i > \bar{r}) > 0$.

Lemma 7 *For $r > \bar{r}$ the informed lender does not quote the same interest rate to its two types of SMEs.*

Proof. Suppose $U_{\min}(r_j)$ is continuous on the open interval (d, e), where $d > \bar{r}$. The expected payoff to the informed lender if it quotes rate t to its low-risk SMEs l and s to its high-risk SMEs h is

$$E[\pi_i(t;s)] = [1 - U_{\min}(t)]\, v(t)\, l + [1 - U_{\min}(s)]\, w(s)\, h \tag{16}$$

Suppose $t = s \in (d, e)$. Then by the assumed continuity of $U_{\min}(r)$ on (d, e), payoffs must be constant, equal to $E[\pi_i]$ say. Hence for every $r \in (d, e)$ it holds that

$$[1 - U_{\min}(r)]\, v(r)\, l + [1 - U_{\min}(r)]\, w(r)\, h = E[\pi_i]$$

as a result we can solve for the mixing strategy of the minimum interest rate quoted by the uninformed

$$1 - U_{\min}(r) = \frac{E[\pi_i]}{v(r)\, l + w(r)\, h} \tag{17}$$

Substituting (17) in (16) gives

$$E[\pi_i(t;s)] = \frac{E[\pi_i]}{v(t)\, l + w(t)\, h} v(t) + \frac{E[\pi_i]}{v(s)\, l + w(s)\, h} w(s)$$

Thus $E[\pi_i(t = r; s = r)] = E[\pi_i]$. Consider a deviation $t \neq s$ in such a way that $t = r - \varepsilon$ and $s = r + \varepsilon$ for some $\varepsilon > 0$. Then

$$\frac{E[\pi_i]}{v(r-\varepsilon)\, l + w(r-\varepsilon)\, h} v(r-\varepsilon) > \frac{E[\pi_i]}{v(r)\, l + w(r)\, h} v(r)$$

or

$$[v(r)\, l - lp\varepsilon]\, [v(r)\, l + w(r)\, h] > v(r)\, l\, [v(r)\, l - lp\varepsilon + w(r)\, h - hq\varepsilon]$$

which reduces to

$$lh\varepsilon\, (p - q)\, c > 0$$

and which holds by assumption. In a similar way one shows that

$$\frac{E[\pi_i]}{v(r+\varepsilon)\, l + w(r+\varepsilon)\, h} w(r+\varepsilon)\, h > \frac{E[\pi_i]}{v(r)\, l + w(r)\, h} w(r)\, h$$

which also reduces to $lh\varepsilon\,(p-q)\,c > 0$. It follows that lender i does not want to quote the same rate $s = t = r$ to both type of SMEs. ■

Note that the Lemma 7 does not rule out that the strategies $L(s)$ and $H(t)$ have an identical upper bound $\bar{r}$. If the two mixing distributions are independent and continuous over the same support, then the probability of quoting the same interest rate to both types is zero.

Lemma 8 *Strategies $L(\cdot)$, $H(\cdot)$, $U(\cdot)$ with a bounded endpoint x, where $x \in (\bar{r}, \infty)$, cannot be part of a Nash equilibrium if $U(\bar{r}) < 1$ and at least $L(\bar{r}) < 1$ or $H(\bar{r}) < 1$.*

Proof. Assume the upper bound $x > \bar{r}$ is finite and $U_j(x) = 1$. In this case the informed lender does not place any mass above x, as this generates no clientele from either type of borrower for sure. So the upper bound for $H(\cdot)$ and $L(\cdot)$ is not above x. At x, the payoff for the j-th uninformed is

$$\left[1 - U^k_{\min}(x)\right]\{[1 - L(x)]\,v(x)\,l + [1 - H(x)]\,w(x)\,h\} = 0$$

since $L(x) = H(x) = 1$, regardless of whether or not the upper boundaries for $L(\cdot)$ and $H(\cdot)$ are at or below x. Suppose first that $L(\cdot)$ and $H(\cdot)$ share this upper bound x. Then by continuity $L(x-\varepsilon) < 1$, $H(x-\varepsilon) < 1$ and $U^k_{\min}(x-\varepsilon)$ for any $\varepsilon > 0$. But then

$$\left[1 - U^k_{\min}(x-\varepsilon)\right] \times$$
$$\{[1 - L(x-\varepsilon)]\,v(x-\varepsilon)\,l + [1 - H(x-\varepsilon)]\,w(x-\varepsilon)\,h\} > 0$$

Thus it is optimal for j to move the upper bound of its strategy down from x.

The same argument holds for uninformed lender k. If $U_j(x) = 1$, the expected return for lender k at x is

$$\left[1 - U^{j\neq k}_{\min}(x)\right][(1 - L(x))\,v(x)\,l + (1 - H(x))\,w(x)\,h] = 0$$

where $U^{j\neq k}_{\min}(x)$ is the lowest interest rate offered by the other uninformed lenders, not being lender k. Because $U_j(x) = 1$, and hence $U^{j\neq k}_{\min}(x) = 1$, the expected return for uninformed lender k is zero. If uninformed lender k quotes an interest rate $x - \varepsilon$, his expected return is

$$\left[1 - U^{j\neq k}_{\min}(x-\varepsilon)\right] \times$$

$$\{(1 - L(x-\varepsilon))\,v(x-\varepsilon)\,l + (1 - H(x-\varepsilon))\,w(x-\varepsilon)\,h\} > 0$$

for some $\varepsilon > 0$ if $L(x-\varepsilon) < 1$ or $H(x-\varepsilon) < 1$, and $U^{j\neq k}_{\min}(x-\varepsilon) < 1$. As a result, uninformed lender k is better off deviating to a lower interest rate than x.

If the strategies $L(\cdot)$ and $H(\cdot)$ have an upper bound (possibly different) lower than x, then it is in the interest of j to move x down to at least just below the highest of these two upper bounds. Suppose for example that the highest upper bound is for $H(\cdot)$, say at $s < x$, so that $H(s) = 1$, then the expected return for uninformed lender j is

$$\left[1 - U_{\min}^k(s)\right][1 - H(s)]\,w(s)\,h = 0$$

but

$$\left[1 - U_{\min}^k(s-\varepsilon)\right][1 - H(s-\varepsilon)]\,w(s-\varepsilon)\,h > 0$$

if $U_{\min}^k(s-\varepsilon) < 1$, i.e. if the other uninformed also have the upper bound at s. In either of these cases as the j-th uninformed lender moves the upper bound of its strategy down, the informed lender wants to follow suit. As a result of this leap frogging, any finite $x > \bar{r}$ is not part of a Nash equilibrium. A similar reasoning applies if the highest upper bound is for $L(\cdot)$. ■

Lemma 9 *Consider the case where $U_{\min}(\cdot)$ and $H(\cdot)$ are unbounded on the open interval $r \in (\bar{r}, \infty)$, but $L(x) = 1$, where $\bar{r} < x < \infty$. This configuration cannot be part of a Nash equilibrium.*

Proof. Suppose first that $L(\bar{r}) = 1$. Then the expected payoffs for $r > \bar{r}$ are respectively

$$E[\pi_j] = \left[1 - U_{\min}^k(r)\right][1 - H(r)]\,w(r)\,h$$

for the j-th uninformed. For the informed we have

$$E\left[\pi_i^h\right] = \left[1 - U_{\min}^k(r)\right][1 - U_j(r)]\,w(r)\,h$$

and $E\left[\pi_i^l\right] = v(\bar{r})$ for sure if $U_{\min}(\bar{r}) = 0$. Suppose that $H(x) = U_j(x) = 0$ for some $x > \bar{r}$ and $H(r) > 0$, $U_j(r) > 0$ for $r > x$. Then

$$E[\pi_j] = E\left[\pi_i^h\right] = \left[1 - U_{\min}^k(x)\right] w(x)h$$

and

$$1 - H(r) = 1 - U_j(r) = \frac{\left[1 - U_{\min}^k(x)\right] w(x)}{\left[1 - U_{\min}^k(r)\right] w(r)}$$

This can be further simplified, since $U_{\min}^k(x) = 0$ for any k. Since if another uninformed lender than j has a lower bound below x, say at $x - \varepsilon$, but still above $\bar{r}$, it would make for sure (capturing all h-type SMEs) $w(x-\varepsilon)h$. This, however, is less than if this lender raises its lower bound also to x, as then

$$E[\pi_j] = E\left[\pi_i^h\right] = \left[1 - U_{\min}^k(x)\right] w(x)h = w(x)h$$

A similar argument can be made for the informed lender i. If all informed have the lower bound of their $U(.)$ at x, while i chooses a lower bound $x-\varepsilon$ for its strategy $H(\cdot)$, i could make more by moving its lower bound up to x. It follows that for i and all j

$$1-H(r)=1-U_j(r)=\frac{w(x)}{w(r)}$$

This appears to resemble the Baye and Morgan (1999) type of unbounded Bertrand equilibria, since it is in all lender's interest to choose x as high as possible. In this case, however, with two types of SMEs, if $x>\bar{r}$, it is no longer in the interest of lender i to keep $L(\bar{r})=1$. Instead, the informed wants to move all mass from the upper bound of $L(\cdot)$ up to at least x, as this increases profits on his low-risk SMEs l from $v(\bar{r})$ to $v(x)$. So suppose that the informed has $L(x)=1$. But then it is in the interest of an uninformed to move all mass down to ε below x, directly violating the supposition of this Lemma. The uninformed then captures the l type SMEs for certain, implying a gain of

$$v(x-\varepsilon)l+[w(x-\varepsilon)-w(x)]\,h>0$$

for some ε sufficiently small. ■

To conclude, the configurations where $L(r)$ has an endpoint $x\in(\bar{r},\infty)$, but $H(r)$ and $U(r)$ do not, cannot be part of a Nash equilibrium according to Lemma 9. Lastly the configuration where $U(r)$ has a finite endpoint, but $L(r)$ and $H(r)$ are unbounded cannot hold according to Lemma 8. Note that by Lemma 7 the configuration in which $H(r)$ has an endpoint, while $L(r)$ and $U(r)$ do not, cannot be part of a Nash equilibrium. So all three distributions $L(\cdot)$, $H(\cdot)$ and $U(\cdot)$ must have unbounded supports if there exists a valid mixing equilibrium for r in the interval $(\bar{r},\infty)$.

Lemma 10 *There does not exist a configuration of interest rates s,t, where the informed lender quotes t to its low-risk SMEs l and s to the high-risk SMEs h, where $t\in[d_1,\infty)$ and $s\in[d_2,\infty)$ and $d_1>d_2$, and where $E\left[\pi_i^l\right]$, $E\left[\pi_i^h\right]$ and $E\left[\pi_i(t,s)\right]$ are constant.*

Proof. Suppose that expected profits for the informed are indeed constant $E\left[\pi_i(t,s)\right]=\pi>0$. From (16) and by Lemma 7 we have for $s>t$

$$(1-U_{\min}(t))\,v(t)l+(1-U_{\min}(s))\,w(s)h=\pi \tag{18}$$

where t,s are unbounded from above (given Lemma's 9 and 8). For (18) to hold, it must be the case that

$$(1-U_{\min}(t))\,v(t)l=h(t)\geq 0 \text{ and } (1-U_{\min}(s))\,w(s)h=D(s)\geq 0$$

where $h(t) + D(s) = \pi(t,s) > 0$. The h and D may not be independent. Suppose first h and D are *not* constant and $(1 - U_{\min}(t))\, v(t)$ varies with t and $(1 - U_{\min}(s))\, w(s)$ varies with s. Then varying (t,s) together such that both t and s increase, would require one of the two parts to decrease and the other part to increase, otherwise the sum $\pi(t,s)$ cannot be constant and equal to π. But then it would be optimal to hold one of the two, t or s, fixed for the part that is decreasing and solely increase the other part. This would raise the expected payoff locally above π, contradicting an equilibrium. Thus both h and D are necessarily constant. Then we can express the conjectured strategy of the uninformed lender as

$$1 - U(t) = \frac{h}{v(t)l} \text{ and } 1 - U(s) = \frac{D}{w(s)h}$$

Note that this is consistent with the unbounded upper bound required by Lemma 9. But as long as $h, D > 0$, the two are incompatible. The reason is that eventually for t sufficiently above the lower bound d_1, t rises above the lower bound of s, d_2. Denote such a t by $t^* > d_2$. Thus there is an s such that

$$(1 - U(s))\, w(s)h = D$$

and for $t^* = s$, it also holds that

$$(1 - U(t^*))\, v(t^*)\, l = B$$

But then

$$\frac{v(t^*)\, l}{w(t^*)\, h} = \frac{B}{D}$$

Solving for t^* results in a unique solution due to the linearity of $v(.)$ and $w(.)$. But this contradicts that t^* must run up to infinity and that the ratio $v(s)/w(s)$ is continuously declining, hence a contradiction. ■

We have shown that there does not exist a pure strategy or mixed strategy equilibrium with unbounded support in which both lenders earn positive returns. We turn to the interval $[\underline{r}, \bar{r}]$. Note that we already have established that $H(r < \bar{r}) = 0$ and $H(\bar{r}) = 1$ in Lemma 1.

Equilibrium Strategy and Uniqueness

Lemma 11 *(i)* $\delta_L(\underline{r}) = 0$, $\delta_L(\bar{r}) = 0$ *for* $L(r)$ *and* $\delta_j(\underline{r}) = 0$ *for* $U_{\min}(r)$; *and (ii)* $\delta_j(\bar{r}) > 0$ *for* $U_{\min}(r)$

Proof. (i) Suppose $\delta_j(\underline{r}) > 0$. Then there exists an $\varepsilon > 0$ such that for lender i

$$\begin{aligned} & \lim_{\varepsilon \downarrow 0} \left[1 - U_{\min}\left(\underline{r} + \varepsilon\right)\right] v\left(\underline{r} + \varepsilon\right) l \\ < \quad & \left[1 - U_{\min}\left(\underline{r}^{-}\right)\right] v\left(\underline{r}\right) l \end{aligned}$$

by continuity of $v(r)\,l$ and the discontinuity in $U_{\min}(r)$ at $\underline{r}$. Therefore the informed lender i does not place any mass in some ε-neighborhood above $\underline{r}$. But then it is not an equilibrium strategy for the uninformed lender j to place mass at $\underline{r}$, as it could make more by moving the mass point slightly to the right. Thus $\delta_j(\underline{r}) > 0$ is not part of an equilibrium. If lender i has a mass point at the lower bound $\delta_L(\underline{r}) > 0$, lender j prefers to remove mass, in an ε-neighborhood to the right of $\underline{r}$, since

$$\begin{aligned} & \lim_{\varepsilon \downarrow 0} \left\{\left[1 - L\left(\underline{r} + \varepsilon\right)\right] v\left(\underline{r} + \varepsilon\right) l + w\left(\underline{r} + \varepsilon\right) h\right\} \\ = \quad & \left[1 - L\left(\underline{r}\right)\right] v\left(\underline{r}\right) l + w\left(\underline{r}\right) h < 0 \end{aligned}$$

as $1 - L(\underline{r}) < 1$. But if j puts no mass in this right side neighborhood, i wants to move mass up, a contradiction. If $\delta_L(\bar{r}) > 0$, it must be that $\lim_{\varepsilon \downarrow 0}\left[1 - L\left(\bar{r} - \varepsilon\right)\right] > 0$, so that in some ε-neighborhood to the left of $\bar{r}$ lender j can make positive profits

$$\begin{aligned} & \left[1 - L\left(\bar{r} - \varepsilon\right)\right] v\left(\bar{r} - \varepsilon\right) l + w\left(\bar{r} - \varepsilon\right) h = \\ & \left[1 - L\left(\bar{r} - \varepsilon\right)\right] \left[lp\bar{r} - lp\varepsilon - cl\right] - hq\varepsilon \end{aligned}$$

By taking limits, gives

$$\lim_{\varepsilon \downarrow 0} \left[1 - L\left(\bar{r} - \varepsilon\right)\right] v\left(\bar{r}\right) l > 0$$

So lender j would remove all probability mass from $\bar{r}$, and lender i no longer wants to put mass at $\bar{r}$.

(ii) Because the distribution of the lowest interest rate of the uninformed lenders has support on $r \geq \underline{r}$, and because of assumption (1), lender i earns at least $v(\underline{r}) > 0$ on his low-risk SMEs l. This implies lender i has to outbid the uninformed lenders with positive probability. This can only occur at the upper bound $\bar{r}$ if the distribution of the minimum interest rate of the uninformed lenders has probability mass at the upper bound of the support, thus $\delta_j(\bar{r}) > 0$. ■

Analogous to the Lemma's 5 and 6 one shows that $U^{*}_{\min}(r)$ and $L(r)$ cannot have mass points. Moreover, given Lemma 11 the $U^{*}_{\min}(r)$ and $L(r)$ are continuous and monotonically increasing on the half open interval $[\underline{r}, \bar{r})$. Thus both lenders must make their equilibrium expected profits on this interval. As $\underline{r}$ is in both supports, $E[\pi_j(r)] = 0$ and $E\left[\pi_i^l(r)\right] = v(\underline{r})$. Given this, $1-L(r)$ and $1-U_{\min}(r)$ are uniquely determined by (14) and (15).

Lemma 12 *The equilibrium interest rate strategies* $U_{\min}(r) = 1- v(\underline{r})/v(r)$ *and* $L(r) = 1 + w(r)h/v(r)l$ *are unique.*

Proof. Given that $E\left[\pi_i^l(r)\right] = v(\underline{r})$; this combined with (14) implies

$$U_{\min}(r) = 1 - \frac{v(\underline{r})}{v(r)}$$

Turning to $L(r)$, consider the expected return for uninformed lender j

$$E[\pi_j] = \left[1 - U_{\min}^k(r)\right]\{(1 - L(r))\, v(r)\, l + w(r)\, h\} \tag{19}$$

Using that $w(\overline{r}) = 0$, we get

$$E[\pi_j(\overline{r})] = [1 - L(\overline{r})]\left[1 - U_{\min}^k(\overline{r})\right] v(\overline{r})\, l$$

But at $\overline{r}$, $L(\overline{r}) = 1$ and $U_{\min}^k(\overline{r}) = 1$ so that $E[\pi_j(\overline{r})] = 0$. Substitution of $E[\pi_j] = 0$ in equation (19), gives the unique solution for the strategy for the l-type SMEs of the informed

$$L(r) = 1 + \frac{w(r)\, h}{v(r)\, l}$$

where $w(r)/v(r) < 0$. ■

It is also evident that at least one uninformed lender quotes interest rates to the SMEs of lender i, i.e. $M \neq \varnothing$. If no uninformed lenders are active, $m = 0$, then it is in the interest of the monopolist lender i to quote interest rates as high as can be absorbed. Suppose in such a case at least the interest rates offered to the clientele of i are above $\underline{r}$. But then any uninformed lender has an incentive to enter, undercut and obtain all SMEs l and h.

Examples of Asymmetric Equilibria

We showed that the distribution of lowest interest rate on offer by the uninformed lenders $U_{\min}(r)$ is unique. The strategies of the individual uninformed lenders, however, is not unique. In a symmetric equilibrium with n uninformed lenders participating, the strategies are

$$U_j^*(r_j) = 1 - \left(\frac{v(\underline{r})}{v(r)}\right)^{\frac{1}{n-1}}$$

A first variation is that not all uninformed lenders do necessarily participate. This would reduce the number n in the $U_j^*(r_j)$. But at least $m \geq 1$ uninformed lenders participate in offering an interest rate. Interest rate setting strategies for m uninformed lenders can also differ from the symmetric case where $U_j^*(r) = U_k^*(r) = 1 - (v(\underline{r})/v(r))^{1/m}$, to asymmetric strategies where only lender j and k participate and use the following strategies $U_j^*(r) = 1 - (v(\underline{r})/v(r))^{1-\rho}$ and $U_k^*(r) = 1 - (v(\underline{r})/v(r))^{\rho}$ for some $\rho\epsilon(0,1)$. But here, in contrast to Baye *et al.* (1993), the asymmetric strategies have no bite as SMEs go for the lowest interest rate in the market. Thus regardless of the number of lenders participating in setting interest rates or the strategies they use, in equilibrium the minimum interest rate offered by the uninformed lenders is distributed in accordance with $U_{\min}^*(r)$. The latter distribution is unique.