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The economics of participatory value evaluation

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Abstract

This paper develops a novel approach to the economic evaluation of public policies: participatory value evaluation (PVE). PVE involves citizens directly in decisions of the government, taking into account governmental and individual budget constraints. Citizens receive reliable information on social impacts and can choose the best portfolio of projects according to their social preferences. This paper develops the economic and econometric theoretical framework for fixed budget and flexible budget PVE experiments which allows us to directly measure the change in social welfare for investments in water infrastructure in The Netherlands.

Keywords: participatory value evaluation; cost-benefit assessment; welfare analysis; discrete-continuous choice models

JEL codes: H43, C35, C91, D63, D71

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1. Introduction

Cost-Benefit Analysis (CBA) is widely adopted by policy makers for economic appraisal of public sector projects. Monetary valuation, i.e. people's willingness-to-pay and willingness-to-accept for changes in the provision of public goods, is used to evaluate and compare the aggregate cost and benefits of public sector projects. Nyborg [13] convincingly argues that CBA measures total net willingness-to-pay '*neither more, nor less*'. CBA has, however, often been connected to social welfare and efficiency analysis. For this connection to hold controversial value judgements need to be made. These value judgements primarily relate to the conversion of utility into money, the assumptions on what kinds of value to include in the social welfare function and which distributional and moral concerns to account for [12, 15, 11].

This paper introduces a new economic evaluation framework avoiding some of these controversial value judgements. Instead of quantifying net willingness-to-pay, Participatory Value Evaluation (PVE) quantifies and compares the attractiveness of public sector projects in terms of utility and directly uses the social welfare function (SWF) to inform public policy makers on the ranking of alternative policy portfolios, and thus the selection of the best public sector policies, given the available budget. PVE estimates the individual direct utility function based on observed portfolio choices in a fit-for-purpose experiment. A representative sample of citizens uses a web-tool to select a portfolio of public sector projects given a governmental budget constraint (fixed budget PVE) or selects a portfolio of public sector projects and adjusts the governmental budget (flexible budget PVE). The public sector projects are characterized by a diverse range of social impacts described in qualitative and quantitative terms. The estimated direct utility function can then be aggregated and implemented into the SWF providing relevant information to policy makers, including the optimal policy portfolio.

The key contribution of this paper is the development of a novel economic and econometric framework for PVE. Specifically, extensions of existing econometric approaches on discrete-continuous choices are provided to accommodate that the utility of all options is random, including the outside 'good', the consumption of some alternatives is purely discrete and that overall consumption is limited by public and private budget constraints (see discussions in Bhat [4], Phaneuf et al. [14], Kim et al. [9]). PVE experiments show i) whether individual projects are more attractive than not spending the required budget, and thus whether the social benefits of public sector measures outweigh the costs; ii) what the optimal composition of the project portfolio is given a restrictive budget. Since citizens make direct choices about the spending of money, PVE gives clear evidence about the acceptance of public sector policies - especially when the quantitative information is combined with qualitative information. Such information is not available when indirect willingness-to-pay estimates, such as the value of time, are used in economic appraisal. This paper presents the results of a PVE experiment on public decisions for investments in flood risk protection in the Netherlands.

Section 2 starts with introducing the policy decision and the social welfare function. Section 3 presents the underlying individual utility functions, the econometric approach to estimate preferences for portfolios of projects taking into account the private and public budget constraints. Section 4 revisits the policy analysis based on the SWF for the adopted utility function. Section 5 presents the setup of the experiment and the empirical results. Section 6 concludes with a discussion of our approach.

2. PVE and social welfare

Suppose a policy maker is faced with the decision to spend a limited budget B on a portfolio of public sector projects. The policy maker has to decide which of the projects $j = 1, \ldots, J$ should be included in the policy portfolio p. The set of possible portfolios may comprise all 2^J project combinations, including the null-portfolio which is empty.

Portfolios for which the total costs exceed the budget are excluded. Any remaining budget can be either shifted forward to the next period (or to another department) or given back to the public in the form of a tax reduction. The former option is denoted by y_0 and the latter by $-\tau$. Similarly, the budget (and thus the set of feasible portfolios) can be expanded by levying a uniform tax of size τ resulting in a budget increase of $\tau \cdot Q$, where Q is the total number of households affected by the tax.[4] This results in the following budget constraint at the level of the policy maker:

$$B + \tau \cdot Q = y_0 + \sum_{j=1}^{J} c_j \cdot y_j \tag{1}$$

In Eq. (1), c_j represents the costs of the respective project. Inclusion of project j in the portfolio is denoted by $y_j = 1$ and when the project is not selected $y_j = 0$ applies. The budget constraint ensures that only a limited set of projects can be included in the portfolio. The policy maker maximises the following expected social utility function:

$$\mathbb{E}SU_p = \sum_{g=1}^{G} Q_g \cdot \mathbb{E}U_{pg} \tag{2}$$

In Eq. (2), society is expected to be split into $g = 1, \ldots, G$ socio-economic groups each with their own (expected) utility function $\mathbb{E}U_{pg}$ for portfolio p . Q_g denotes the number of households in group g, with $Q = \sum_{g=1}^{G} Q_g$. The utility functions include both private preferences as well as social or altruistic preferences. When citizens vote altruistically, they give up private consumption in the form of tax reductions and/or private benefits from public sector projects and vote to implement other public sector projects. A reduced form social welfare function is adopted where each household receives the same weight, an assumption which is in line with (representative) democratic decision making.

[4] The tax payers are assumed to be the beneficiaries of the proposed projects.

Once the parameters of the utility function are estimated, selecting the optimal portfolio conditional on B and τ involves enumerating Eq. (2) for all feasible project portfolios. The optimal tax conditional on a selected portfolio is determined by the following first-order condition:

$$\frac{\partial \mathbb{E} SU_p}{\partial \tau} = \sum_{g=1}^{G} Q_g \cdot \frac{\partial \mathbb{E} U_{pg}}{\partial \tau} = 0 \tag{3}$$

Increasing taxes will reduce private consumption and therefore implementing more or even all public sector policies is not necessarily beneficial. Section 3 provides more details on the specification of the citizens' utility functions before revisiting the implications of the adopted functional form for policy evaluation in Section 4.

3. PVE and individual utility

In PVE experiments, citizens are presented with the decision problem faced by the policy maker and asked to identify their optimal policy portfolio. An explicit distinction is made between *fixed* and *flexible* budget experiments. In the flexible budget experiment citizens can adjust the tax, whereas $\tau = 0$ for the fixed budget experiment. In both experiments any remaining budget is allocated to alternative projects in the near future (or to an alternative department) ensuring the budget constraint is satisfied. Shifting budget to the near future can be desirable to provide a buffer for future cost overruns or for investment in upcoming public sector projects.

The multiple discrete continuous consumption framework [e.g. 3, 5] provides a suitable econometric framework for this decision problem, because i) citizens are faced with multiple decision variables of which some are continuous (private consumption and shifting budget to a future decision periods) and others are discrete (inclusion or exclusion of public sector projects in the project portfolio); ii) consumption is restricted by private and public budget constraints; and iii) there is a need to account for potential satiation effects associated with the continuous decision variables.[5] The presence of purely discrete alternatives requires a number of model extensions along the lines of Bhat [4] which will be outlined below.

3.1. Utility maximisation problem

In what follows, citizens are denoted by $n = 1, \dots, N$, public sector projects are indexed by $j = 1, \dots, J$ and their inclusion (exclusion) from the project portfolio captured by the binary indicators $y_{nj} = 1$ ($y_{nj} = 0$). Following Bhat [3], the utility derived from the selected project portfolio, the change in future budget y_{n0} and current private consumption $y_{n,J+1}$ is described by:

$$U_n = \frac{\gamma_0}{\alpha_0}\left[\left(\frac{y_{n0}}{\gamma_0}+1\right)^{\alpha_0}-1\right]\Psi_{n0} + \sum_{j=1}^{J} y_{nj}\Psi_{nj} + \frac{1}{\alpha_{J+1}} y_{n,J+1}^{\alpha_{J+1}}\Psi_{n,J+1} \tag{4}$$

The governmental budget can be fully exhausted and therefore corner solutions should be allowed for y_{n0}. Any positive value for γ_0 allows for such corner solutions. The parameter α_0 controls for satiation effects associated with shifting budget forward. Ψ_{n0} denotes the marginal utility of the first unit of y_{n0}. Since only a single unit can be consumed for the projects, Ψ_{nj} represents the direct utility of including project j in the portfolio. The level of private consumption $y_{n,J+1} > 0$, takes the role of an outside good which will always be consumed and hence is not associated with a translation parameter. Satiation effects are taken into account via the parameter α_{J+1}. Finally, $\Psi_{n,J+1}$ is the marginal utility of private consumption at the first unit of consumption ($y_{n,J+1} = 1$).

Private consumption is not included in the governmental budget constraint, but is

[5]The Multiple Discrete Continuous Extreme Value (MDCEV) model of Bhat [3] is rooted in the economic literature (see Dekker et al. [6]) and provides extensions to earlier work by Wales and Woodland [18], Hanemann [7], Phaneuf et al. [14], Kim et al. [9], von Haefen et al. [17], von Haefen and Phaneuf [16].

governed by the private budget constraint (see Eq. 5), where the change in taxes τ_n ensure the connection between the two constraints. Y_n represents gross income and t_n the income tax rate and c_{J+1} the cost of private consumption. The latter term will be normalised to unity and assumed equal across all citizens.

$$Y_n \cdot (1 - t_n) - \tau_n = c_{J+1} \cdot y_{n,J+1} \tag{5}$$

Assume, $y_{n0} \geq 0, y_{n,J+1} > 0$, $y_{nj} = 0$ or $y_{nj} = 1$ $\forall j = 1, \ldots, J$; $\alpha_0, \alpha_{J+1} \leq 1$ and $\gamma_0 > 0$. The Lagrangian multipliers λ and θ refer to the marginal utility of additional governmental budget and private net income, respectively. This results in the following Kuhn Tucker (KKT) conditions:

$$\begin{aligned}
\mathcal{L} &= U_n + \lambda \left(B + \tau_n \cdot Q - y_{n0} - \sum_{j=1}^{J} y_{nj} \cdot c_{nj} \right) && (6)\\
&+ \theta \left(Y_n \cdot (1 - t_n) - \tau_n - y_{n,J+1} \right) && (7)\\
\frac{\partial \mathcal{L}}{\partial y_{n0}} &= \left(\frac{y_{n0}}{\gamma_0} + 1 \right)^{\alpha_0 - 1} \Psi_{n0} - \lambda = 0 \text{ for } y_{n0} > 0 && (8)\\
\frac{\partial \mathcal{L}}{\partial y_{n0}} &= \Psi_{n0} - \lambda < 0 \text{ for } y_{n0} = 0 && (9)\\
\frac{\partial \mathcal{L}}{\partial y_{nj}} &= \Psi_{nj} - \lambda \cdot c_{nj} \geq 0 \text{ for } y_{nj} = 1, \forall j =, 1 \ldots, J && (10)\\
\frac{\partial \mathcal{L}}{\partial y_{nj}} &= \Psi_{nj} - \lambda \cdot c_{nj} < 0 \text{ for } y_{nj} = 0, \forall j =, 1 \ldots, J && (11)\\
\frac{\partial \mathcal{L}}{\partial y_{n,J+1}} &= y_{nJ+1}^{\alpha_{J+1} - 1} \Psi_{n,J+1} - \theta = 0 && (12)\\
\frac{\partial \mathcal{L}}{\partial \tau_n} &= \lambda Q - \theta = 0 && (13)
\end{aligned}$$

Because the project utilities Ψ_{nj} and the baseline marginal utilities Ψ_{n0} and $\Psi_{n,J+1}$ are assumed to be stochastic the KKT conditions lead to portfolio choice probabilities.

3.2. *Derivation of choice probabilities*

This subsection describes how to derive the choice probabilities for the observed choice vector y_n^* (i.e. the optimal policy portfolio) for citizen n. Four cases can be distinguished aligning with the fixed and flexible budget experiments and whether the entire governmental budget will be spend on projects or not.

3.2.1. *Case 1: Fixed governmental budget, $y_{n0}^* > 0$*

Case 1 assumes a fixed governmental budget ($\tau_n = 0$) and $y_{n0}^* > 0$. KKT conditions Eq. (12)-(13) do not apply since private consumption cannot change. Eq. (8) does apply and results in a marginal utility of governmental budget of $\lambda^* = \left(\frac{y_{n0}^*}{\gamma_0} + 1\right)^{\alpha_0 - 1} \Psi_{n0}$. Implementing this solution into Eq. (10) and Eq. (11) and log-linearising provides:

$$
\begin{aligned}
\ln[\Psi_{nj}] - \ln[c_{nj}] &\geq (\alpha_0 - 1) \cdot \ln\left[\frac{y_{n0}}{\gamma_0} + 1\right] + \ln[\Psi_{n0}] \text{ for } y_{nj} = 1, \forall j =, 1 \ldots, J \\
\ln[\Psi_{nj}] - \ln[c_{nj}] &< (\alpha_0 - 1) \cdot \ln\left[\frac{y_{n0}}{\gamma_0} + 1\right] + \ln[\Psi_{n0}] \text{ for } y_{nj} = 0, \forall j =, 1 \ldots, J
\end{aligned}
\tag{14}
$$

Assume $\Psi_{nj} = \exp(\delta_j + X_{nj} \cdot \beta - \epsilon_{nj})$ and $\Psi_{n0} = \exp(\delta_0 - \epsilon_{n0})$, where X_{nj} is a row vector of policy attributes, β a column vector of corresponding parameters assumed to be homogeneous for all respondents, δ_j the project constant and ϵ_{nj} denotes an Extreme Value Type I i.i.d. error term.[6] Furthermore, define $V_{nj} = \delta_j + X_{nj} \cdot \beta - \ln[c_{nj}]$ and $V_{n0} = \delta_0 + (\alpha_0 - 1) \cdot \ln\left[\frac{y_{n0}^*}{\gamma_0} + 1\right]$such that:

$$
\begin{aligned}
V_{nj} - \epsilon_{nj} &\geq V_{n0} - \epsilon_{n0}, \text{ for } y_{nj} = 1, \forall j = 1, \ldots, J \\
V_{nj} - \epsilon_{nj} &< V_{n0} - \epsilon_{n0}, \text{ for } y_{nj} = 0, \forall j = 1, \ldots, J
\end{aligned}
\tag{15}
$$

Denote the non-chosen alternatives by $k = 1...K_n$ and the chosen alternatives by $m = K_n+$

[6] Consequently, $exp(-\epsilon_{nj})$ follows a Weibull distribution with scale parameter 1 and shape parameter $\frac{1}{\sigma}$, where σ is the scale parameter of the underlying extreme value density.

1...J. The choice probability in Eq. (16) then comprises a product over 1-CDF expressions for the non-chosen alternatives and CDF expressions for the chosen alternatives:

$$\mathbb{P}_{n1}(y_n^*) = \int_{\epsilon_{n0}=-\infty}^{\infty} \prod_{k=1}^{K_n} (1 - F\left[V_{nk} - V_{n0} + \epsilon_{n0}\right]) \prod_{m=K_n+1}^{J} F\left[V_{nm} - V_{n0} + \epsilon_{n0}\right] f\left[\epsilon_{n0}\right] d\epsilon_{n0} \tag{16}$$

Given the assumed i.i.d. EV Type 1 densities with common scale parameter σ, let $|S_n| = 2^{K_n}$ represent the number of elements in the set S_n. The set S_n comprises all possible combinations of $W_{nk0} = e^{-\frac{V_{nk}-V_{n0}}{\sigma}}$ for the non-chosen alternatives including the empty set. Let S_{ns} be element s of the set S_n. Then, following Bhat [4], the closed form choice probability is given by (see Appendix A for the derivations):

$$\mathbb{P}_{n1}(y_n^*) = \sum_{s \in S_n}^{|S_n|} (-1)^{|S_{ns}|} \frac{1}{1 + \sum_{m=K_n+1}^{J} e^{-\frac{V_{nm}-V_{n0}}{\sigma}} + \sum_{q \in S_{ns}}^{|S_{ns}|} S_{nsq}} \tag{17}$$

3.2.2. Case 2: Fixed governmental budget, $y_{n0}^* = 0$

Case 2 applies when no remaining budget is shifted forward, i.e. $y_{n0}^* = 0$. Hence, Eq. (8) is replaced by Eq. (9) such that only the bounds on the marginal utility of governmental budget λ_n can be obtained:

$$\begin{gathered}
\max_k \left[\frac{\Psi_{nk}}{c_{nk}}\right] < \lambda_n \leq \min_m \left[\frac{\Psi_{nm}}{c_{nm}}\right], \; \forall k = 0, 1, \ldots, K_n; m = K_n + 1, \ldots, J \\
\min_k \left[-V_{nk} + \epsilon_{nk}\right] > \max_m \left[-V_{nm} + \epsilon_{nm}\right], \; \forall k = 0, 1, \ldots, K_n; m = K_n + 1, \ldots, J
\end{gathered} \tag{18}$$

Note that here the set of non-chosen goods is extended by alternative 0. The i.i.d. Extreme Value Type I error distribution ensures that $\max_m \left[-V_{nm} + \epsilon_{nm}\right]$ follows an Extreme Value Type I distribution [2] such that:

$$\epsilon_{nk} > LS_n + V_{nk} + \epsilon_{m*}, \; \forall k = 0, \ldots, K_n, \tag{19}$$

where $LS_n = \sigma \ln \left[\sum_{m=K_n+1}^{J} e^{\left(\frac{-V_{nm}}{\sigma}\right)} \right]$. This provides the closed form choice probability:

$$\begin{aligned}
\mathbb{P}_{n2}(y_n^*) &= \int_{\epsilon_{m^*}=-\infty}^{\infty} \prod_{k=0}^{K_n} \left(1 - F\left[LS_n + V_{nk} + \epsilon_{m^*}\right]\right) \cdot f\left[\epsilon_{m^*}\right] d\epsilon_{m^*} \\
&= \sum_{s \in S_n}^{|S_n|} (-1)^{|S_{ns}|} \frac{1}{1 + \sum_{q \in S_{ns}}^{|S_{ns}|} S_{nsq}}
\end{aligned} \tag{20}$$

The last step again relies on the enumeration of all possible combinations of non-chosen alternatives in K_n. Define the set S_n as the set with all possible combinations of $W_{nLSk} = e^{-\frac{LS_{nm}+V_{nk}}{\sigma}}$ of the non-chosen alternatives (which also covers y_{n0}), including the empty set. Let $|S_n| = 2^{K_n+1}$ be the number of elements in S_n and S_{ns} be element s of the set S_n. See Appendix B for the derivations.

3.2.3. Case 3: Flexible governmental budget, $y_{n0}^* > 0$

Tax adjustments give the opportunity for citizens to adjust their private consumption. Since $y_{n,J+1}$ is always consumed, the marginal utility of net income is always defined and equal to $\theta_n^* = y_{n,J+1}^{\alpha_{J+1}-1} \Psi_{n,J+1}$, where $\Psi_{n,J+1} = e^{-\epsilon_{n,J+1}}$. Moreover, when both y_{n0} and $y_{n,J+1}$ are consumed in optimal amounts then (Eq. (13)) implies:

$$\lambda_n^* = \frac{\theta_n^*}{Q} = \frac{(y_{n,J+1}^*)^{\alpha_{J+1}-1} \Psi_{n,J+1}}{Q} \tag{21}$$

That is, the marginal utility of governmental budget is equal to the marginal utility of net income divided by the number of households Q. For positive consumption of y_{n0} the equality condition Eq.(8) applies. For the discrete projects, the solution for the marginal utility of budget is substituted into the inequality conditions Eq.(10) and Eq.(11) resulting in the following inequality conditions for the public projects:

$$\begin{aligned}
&\epsilon_{nj} \leq V_{nj} - V_{nJ+1} + \epsilon_{n,J+1}, \text{ for } y_{nj} = 1, \forall j = 1, \ldots, J \\
&\epsilon_{nj} > V_{nj} - V_{nJ+1} + \epsilon_{n,J+1}, \text{ for } y_{nj} = 0, \forall j = 1, \ldots, J
\end{aligned} \tag{22}$$

with $V_{n,J+1} = (\alpha_{J+1} - 1) \ln \left[\frac{y^*_{n,J+1}}{Q}\right]$. The choice probability can then be obtained by integrating over $\epsilon_{n,J+1}$:

$$\begin{aligned}
\mathbb{P}_{n3}(y^*_n) = \big||G|\big| \int_{\epsilon_{n,J+1}=-\infty}^{\infty} \prod_{k=1}^{K_n} \left(1 - F\left[V_{nk} - V_{n,J+1} + \epsilon_{n,J+1}\right]\right) \prod_{m=K_n+1}^{J} F\left[V_{nm} - V_{n,J+1} + \epsilon_{n,J+1}\right] \\
\cdot f\left[V_{n0} - V_{n,J+1} + \epsilon_{n,J+1}\right] \cdot f\left[\epsilon_{n,J+1}\right] d\epsilon_{n,J+1},
\end{aligned} \tag{23}$$

where $\big||G|\big| = \left|\frac{\partial\left(V_{n0} - V_{n,J+1} + \epsilon_{n,J+1}\right)}{\partial y^*_{n0}}\right| = \left|(\alpha_0 - 1)\frac{1}{y^*_{n0} + \gamma_0} - (\alpha_{J+1} - 1)\frac{1}{y^*_{n,J+1}}\frac{1}{Q}\right|$ is the absolute value of the determinant of the Jacobian matrix. The latter is needed because of the change of variable for ϵ_{n0}. It ensures that changes in equilibrium private consumption ($y^*_{n,J+1}$) due to changes in shifting money (y^*_{n0}) are properly accounted for. Define the set S_n as the set with all possible combinations of $W_{nk(J+1)} = e^{-\frac{V_{nk} - V_{n,J+1}}{\sigma}}$ of the non-chosen alternatives including the empty set. Let $|S_n| = 2^{K_n}$ the number of elements and let S_{ns} be element s of the set S_n. Then the choice probability is given by (see Appendix C for the derivations):

$$\mathbb{P}_{n3}(y^*_n) = \frac{1}{\sigma}\big||G|\big| e^{-\left(\frac{V_{n0} - V_{n,J+1}}{\sigma}\right)} \sum_{s \in S_n}^{|S_n|} (-1)^{|S_{ns}|} \frac{1}{\left(1 + \sum_{m=0,K_n+1}^{J} e^{-\frac{V_{nm} - V_{n,J+1}}{\sigma}} + \sum_{q \in S_{ns}}^{|S_{ns}|} S_{nsq}\right)^2} \tag{24}$$

3.2.4. Case 4: Flexible governmental budget, $y^*_{n0} = 0$

The last case describes the outcome of the flexible budget experiment when no budget is shifted to the next year implying, i.e. $y^*_{n0} = 0$. The marginal utility of net income can still be pinned down using Eq (12). The equality condition Eq (8) is replaced by the inequality condition Eq (9). Denote $k = 0...K_n$ as the non-chosen alternatives and $m = K_n + 1...J$ as the chosen alternatives. The choice probability is then given by:

$$\mathbb{P}_{n4}(y^*_n) = \int_{\epsilon_{n,J+1}=-\infty}^{\infty} \prod_{k=0}^{K_n} \left(1 - F\left[V_{nk} - V_{n,J+1} + \epsilon_{n,J+1}\right]\right) \prod_{m=K_n+1}^{J} F\left[V_{nm} - V_{n,J+1} + \epsilon_{n,J+1}\right] \cdot f\left[\epsilon_{n,J+1}\right] d\epsilon_{n,J+1}, \tag{25}$$

Similar to the other cases, the set S_n is defined as the set with all possible combinations of $W_{nk(J+1)} = e^{-\frac{V_{nk}-V_{n,J+1}}{\sigma}}$ of the non-chosen alternatives including the empty set. Let $|S_n| = 2^{K_n+1}$ denote the number of elements and let S_{ns} be element s of the set S_n. Then the choice probability is given by (see Appendix D for the derivations):

$$\mathbb{P}_{n4}(y^*_n) = \sum_{s \in S_n}^{|S_n|} (-1)^{|S_{ns}|} \frac{1}{1 + \sum_{m=K_n+1}^{J} e^{-\frac{V_{nm}-V_{n,J+1}}{\sigma}} + \sum_{q \in S_{ns}}^{|S_{ns}|} S_{nsq}}. \tag{26}$$

Finally, denote I_{n1}, I_{n2}, I_{n3} and I_{n4} as indicator functions taking the value of 1 when the respective case applies to individual n and 0 otherwise. The choice probability for individual n is then given by:

$$\mathbb{P}_n(y^*_n) = \mathbb{P}_{n1}(y^*_n)^{I_{1n}} \mathbb{P}_{n2}(y^*_n)^{I_{n2}} \mathbb{P}_{n3}(y^*_n)^{I_{n3}} \mathbb{P}_{n4}(y^*_n)^{I_{n4}} \tag{27}$$

3.3. Delegation and the log-likelihood function

In the experiments citizens have the option to delegate their decision either to an expert, to the average citizen in the sample or to the average citizen living in the affected region where specific policies are taking place. Giving citizens the option to delegate ensures they are not forced to make a decision themselves. For example, delegating to an expert can be a sensible approach when citizens feel insufficiently qualified to make a decision. The option to delegate the decision to the average citizen living in the region where the policies take place enables citizens to make a private judgement where they find that these individuals are the best to judge about the public policies. When citizens want to go with the average behaviour in society, they can delegate their decision to the average citizen in the sample.

Denote N_A as the number of citizens in the sample living in the affected region, and N_B as the number of other citizens (where $N = N_A + N_B$). Let ρ_{n_A} and ρ_{n_B} denote a binary indicator which equals 1 when a given citizen n in region A or B delegates his (her) decision respectively. These indicators can be split up using indicators for the three alternative delegation options such that $\rho_{n_A} = \rho_{n_A|e} + \rho_{n_A|A} + \rho_{n_A|A+B}$ and $\rho_{n_B} = \rho_{n_B|e} + \rho_{n_B|A} + \rho_{n_B|A+B}$. The weighted log-likelihood in Eq (28) only takes into account the individuals that have not delegated, through $(1 - \rho_{n_A})$ and $(1 - \rho_{n_B})$, and the decisions of the experts $e \in E$. Where $\rho_{n_e} = \rho_{n_A|e} + \rho_{n_B|e}$ measures whether an individual has delegated to an expert or not. Thus when citizens delegate their decision to an expert e, the weight of the experts' choice increases with 1 in the log-likelihood function. When the decision is delegated to the average person in the sample, the weight of all other individuals that have made a decision, but not the experts, increases by $\frac{1}{N - \sum_{n_A=1}^{N_A} \rho_{n_A} - \sum_{n_B=1}^{N_B} \rho_{n_B}}$. When citizens delegate their decision to citizens in the affected regions, the weight of those citizens increases with $\frac{1}{N_A - \sum_{n_A=1}^{N_A} \rho_{n_A}}$.

The weighted log-likelihood function is then given by:

$$\begin{aligned}
WLL = &\sum_{n_A=1}^{N_A}(1-\rho_{n_A})\left(1+\frac{\sum_{n_A=1}^{N_A}\rho_{n_A|A}+\sum_{n_B=1}^{N_B}\rho_{n_B|A}}{N_A-\sum_{n_A=1}^{N_A}\rho_{n_A}}+\frac{\sum_{n_A=1}^{N_A}\rho_{n_A|A+B}+\sum_{n_B=1}^{N_B}\rho_{n_B|A+B}}{N-\sum_{n_A=1}^{N_A}\rho_{n_A}-\sum_{n_B=1}^{N_B}\rho_{n_B}}\right)\ln\left[\mathbb{P}_{n_A}(y^*_{n_A})\right] \\
&+\sum_{n_B=1}^{N_B}(1-\rho_{n_B})\left(1+\frac{\sum_{n_A=1}^{N_A}\rho_{n_A|A+B}+\sum_{n_B=1}^{N_B}\rho_{n_B|A+B}}{N-\sum_{n_A=1}^{N_A}\rho_{n_A}-\sum_{n_B=1}^{N_B}\rho_{nB}}\right)\ln\left[\mathbb{P}_{n_B}(y^*_{n_B})\right] \\
&+\sum_{e=1}^{E}\left(\sum_{n=1}^{N}\rho_{n_e}\right)\ln\left[\mathbb{P}_e(y^*_e)\right],
\end{aligned} \tag{28}$$

where n_A and n_B are the indicators used to denote citizens from the sample living in the regions A and B respectively. The weighted log-likelihood is used to find the parameters that best describe the observed choice vector y^* for citizens and experts in the sample.

4. Policy analysis - the optimal portfolio and budget

When conducting policy analysis based on the estimated individual utility functions two situations can be considered depending on the institutional possibilities. First, the budget can be fixed and equal to the budget presented in the experiment (B). Any remaining budget will be shifted to the next period leading to a positive value for y_0. Second, budget can be optimised. This can be done by adjusting the tax level τ in such a way that marginal benefits are equal to the marginal costs of a tax adjustment. An increase in the tax leads to additional budget and gives the opportunity to choose portfolios with more projects but leads to a reduction in private consumption.

Each socio-economic group g in the population has a different utility function as private income levels differ. This will lead to different levels of optimal private consumption. The expected value of the estimated random utility function for individual n belonging to group g is given by Eq. (29):

$$\mathbb{E}U_g \equiv \mathbb{E}U_{ng} = \frac{\gamma_0}{\alpha_0}\left[\left(\frac{y_0}{\gamma_0}+1\right)^{\alpha_0}-1\right]\mathbb{E}\Psi_{n0} + \sum_{j=1}^{J} y_j \mathbb{E}\Psi_{nj} + \frac{y_{g,J+1}^{\alpha_{J+1}}-1}{\alpha_{J+1}}\mathbb{E}\Psi_{n,J+1,}, \tag{29}$$

with $y_j = 1$ if project j is included in a portfolio, $y_0 = B + \tau Q - \sum_{j=1}^{J} y_j c_j$ and $y_{J+1,g} = Y_g(1-t_g) - \tau$. Due to the public good nature of the decision problem, the costs of the portfolio and the additional tax τ are assumed to be equal for all citizens. The distributional assumptions on ϵ_{nj} imply:

$$\mathbb{E}\Psi_{nj} = \Gamma\left[1+\sigma\right] e^{\delta_j + \beta X_j} \ \forall j = 0, 1, \ldots, J+1 \tag{30}$$

In Eq. (29), $\Gamma(\cdot)$ denotes the Gamma function which acts as a scalar on expected utility. The ranking of the portfolios is therefore independent of the scale parameter σ. Note that when the the tax $\tau = 0$ and the budget is set at B, group income levels have no impact on the ranking of the portfolios as private consumption does not change across the different policy portfolios.

5. Experimental results

5.1. PVE experiment on flood risk reductions in the Netherlands

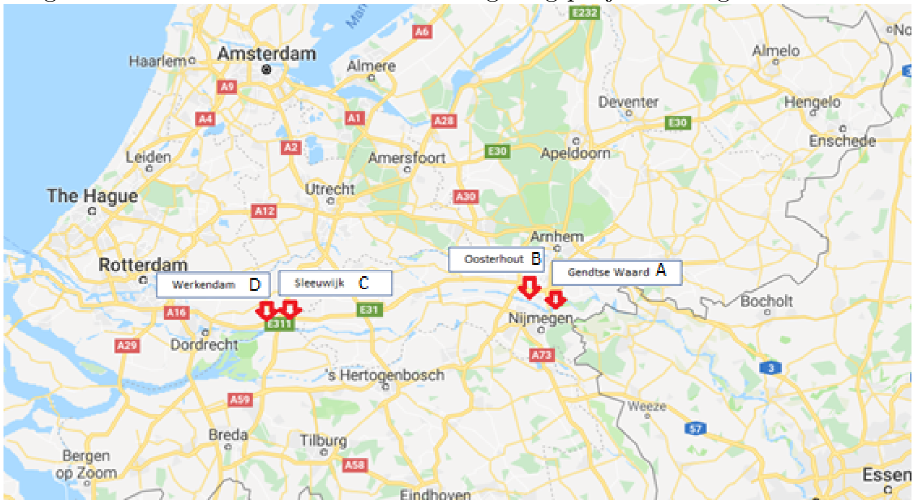
The Netherlands has a long history in flood protection measures and invests about 1% of its GDP in water management. About 60% of the territory is below sea level and around 70% of GDP is earned in flood prone areas [8]. The Delta Committee is the policy board responsible for future water management strategies and argues to give more space to rivers to accommodate increased river inundation due to climate change (see Winsemius et al. [19]). The threat of an increased risk of flooding and loss of biodiversity, has sparked a heated policy debate as investment costs are substantial [8]. Against this backdrop, the first ever application of a PVE experiment on public decisions for investments in flood risk protection was conducted for the Dutch Ministry of Infrastructure and Water Management.

The PVE experiment focused on a trade-off between two types of projects mitigating flood risks at locations along the Dutch river 'de Waal' where prescribed safety standards are not met. The Ministry's objective of the PVE experiment was to investigate societal support for investments to be made. The simplest type of project considered strengthens the dikes (henceforth: 'classical project'). The second type of project involves some strengthening of the dikes alongside measures providing the river space to flood safely (henceforth: 'combination project'). The two types of projects are mutually exclusive and have an equal impact on mitigating increases in flood risks. They are, however, characterized by different costs and societal impacts (e.g. impact on biodiversity, impact on recreational activities and number of households that need to relocate). The combination project increases recreation opportunities and biodiversity but is more expensive. On four locations alongside the river 'de Waal' citizens must choose between the mutually exclusive 'classical project' and 'combination project' (Figure 1 depicts the four locations). The governmental budget can also be spend on six other projects that fall within the remit of the Dutch Ministry of Infrastructure and Water Management (two road projects, two projects mitigating damage from heavy rainfall, and two projects reducing flood risks beyond current safety standards).

Experimental software[7] was developed for the PVE experiments. An introduction page explained the purpose of the experiment and citizens could watch a video in which the software tool was explained. The video also showed how citizens could obtain quantitative and qualitative information about the different public sector projects. Within

[7] http://ienw.participatie-begroting.nl/

Figure 1: Location of the flood risk mitigating projects along the Waal river



the experiment, citizens could highlight projects and compare their impacts. Maps were provided showing the spatial area where the project was planned. Table 1 describes the different projects and their corresponding minimum and maximum social impacts.[8] The set of projects is heterogeneous in that the social impact variables vary across projects, ranging from improvements in biodiversity to reductions in traffic injuries. Even when the same social impacts apply to multiple projects their levels vary significantly. For example, the size of additional space made available for biodiversity and recreational purposes varies substantially across the projects.

The fixed budget experiment assumed a budget of $B = 700$ million euros. In the flexible budget experiment that same value was used as the reference value, but citizens could change the budget by lowering or increasing household taxes. The additional tax needed/received was distributed equally over all eight million households (Q) in The Netherlands.

[8]Individuals were presented one of 40 design versions each using different costs and social impacts.

Table 1: Overview of projects and range of changes in impacts

Label	Project name	Project type	Size of nature *hectares*		Biodiversity *Categorical 1-5*		Recreation *Categorical 1-5*		Affected *# relocating hh*		Flood prev *X per 25 years*		Costs *million euros*	
			min	max	min	max	min	max	min	max	min	max	min	max
I	Gendtse Waard	classical project	0	0	0	0	0	0	-	-	-	-	35	90
I*		combination project	30	70	1	4	0	5	-	-	-	-	40	97
II	Oosterhout	classical project	0	0	0	0	0	0	0	0	-	-	20	35
II*		combination project	80	120	1	4	0	5	1	30	-	-	55	90
III	Sleeuwijk	classical project	0	0	0	0	0	0	-	-	-	-	70	115
III*		combination project	120	180	1	5	1	5	-	-	-	-	145	280
IV	Werkendam	classical project	0	0	0	0	0	0	-	-	-	-	70	110
IV*		combination project	140	200	1	5	0	5	-	-	-	-	105	255
V	Hooge Boezem	Water management	35	55	1	3	1	3	-	-	2.5	25	1	10
VI	Driemanspolder	Water management	300	400	2	4	1	3	-	-	1	5	60	100

Label	Project name	Project type	Affected *# relocating hh*		Flood safety *Categorical 1-3*		Users *# of daily users*		Time saved *Minutes per trip*		Traffic safety *Less injuries/yr*		Costs *million euros*	
			min	max	min	max	min	max	min	max	min	max	min	max
VII	Joure (A6/7)	Road Project	-	-	-	-	30,000	45,000	1	8	1	7	55	95
VIII	Kerensheide (A2)	Road Project	1	27	-	-	10,000	22,000	1	6	0	7	205	305
IX	Moerdijk	Flood risk reduction	-	-	1	3	-	-	-	-	-	-	8	20
X	Venlo	Flood risk reduction	-	-	1	3	-	-	-	-	-	-	2	10

Categorical variables

	Base level	Level 1	Level 2	Level 3	Level 4	Level 5
Biodiversity	No improvement	Small improvement	Improvement	Substantial improvement	Large improvement	Very large improvement
Recreation	No improvement	Small improvement	Improvement	Substantial improvement	Large improvement	Very large improvement
Flood safety	Below safety standard	At standard	Above standard	Well above standard		

In the Fixed budget experiment citizens could delegate their decision to a programme manager working at the government, to a member of the Deltacommittee on water policy or to an environmental scientist. These experts filled in the experiment which had an underlying design with the average value of the social impacts across the designs. In the Flexible budget experiment citizens could in addition delegate their decision to the average participant in the experiment or to people living in the area where the water projects take place. To stimulate participation in the experiment, a reduction in the reward (6 instead of 17 survey company tokens) was associated with delegating the decision.

5.2. Sample characteristics and descriptive results

The survey was administered online by Kantar Public and took place in February 2018 (Fixed budget experiment) and April 2018 (Flexible budget experiment) respectively. A total of 2900 respondents participated and 2793 completed the full experiment (Fixed: $n = 1826$; Flexible: $n = 805$). Table 2 displays the share of portfolios in which each project was included for both samples; the choices of the three experts; and the average choices of the residents living in the local areas I,II,III and IV. The number of observations in each sample and the delegations are reported in the bottom line of Table 2. The majority of respondents and experts opt to include the combination projects in the optimal policy portfolio instead of the classical projects. The combination project is, however, least popular in Oosterhout (II,II*). With regards to the last six projects, the road projects are least popular and in particular Kerensheide (VIII). The latter project is also never selected by the experts. Finally, a large degree of similarity can be observed between the choices in the overall sample and those of the local population.

5.3. Estimation results

The δ's in Table 3 represent the attractiveness of each option relative to adjusting the tax. That is, $\delta_{J+1} = 0$ for identification purposes [3]. In general, a higher δ_j implies

Table 2: Choice shares in the sample and of choices of delegates

Project type	Project	Sample shares Fixed	Flex.	Expert decisions Prog. Man.	Com. Mem.	Env Sci.	Shares Locals†
Flood risks along 'de Waal'	I	21.7%	20.1%	0	0	0	18.2%
	I*	78.3%	79.9%	1	1	1	81.8%
	II	39.8%	35.8%	0	0	1	27.3%
	II*	60.2%	64.2%	1	1	0	72.7%
	III	30.5%	32.8%	0	0	0	36.4%
	III*	69.5%	67.2%	1	1	1	63.6%
	IV	30.6%	32.4%	0	0	0	54.5%
	IV*	69.4%	67.6%	1	1	1	45.5%
Mitigating damage	V	71.0%	72.0%	1	1	1	72.7%
from rainfall	VI	63.4%	61.1%	1	1	1	54.5%
Road Projects	VII	57.9%	59.8%	0	0	1	36.4%
	VIII	19.1%	25.7%	0	0	0	36.4%
Flood risk reductions	IX	73.5%	73.9%	0	1	1	63.6%
	X	70.9%	73.4%	0	1	1	63.6%
Respondents (delegating to experts / locals)		1561	805	85	143	167	32

* denotes a combination project which is mutually exclusive from a standard dike strengthening project.
† The locals (living in area I,II,III and IV) are included in the presented sample shares.

that the project is more attractive. The level is, however, not independent of the costs of the project c_{nj}, as reflected by the first order conditions in Eq. (10)- (11). For example, the high δ_{VIII} is corrected for by its high costs.[9] The cheap nature of public money causes $\delta_0 > 0$. Respondents prefer to shift any remaining budget forward to the next period than reduce taxes. This is not surprising because the uniform tax implies that one additional euro of taxes increases the governmental budget by 8 million euros. The β's reflect the impact of the policy attributes on the composition of the policy portfolio.[10] The biodiversity and recreation attributes have a positive and significant impact on the attractiveness of the combination projects. For example, increasing both from level 0 to level 3 for combination project I* (i.e. substantial improvement) results in an increase

[9]The *difference* in costs and impact characteristics between the classical and combination projects is used for estimation and policy analysis.

[10]Define $\Psi_{nj}^{old} = exp(\delta_j - \epsilon_{nj})$ and $\Psi_{nj}^{new} = exp(\delta_j + \beta \cdot X_{nj} - \epsilon_{nj})$ such that $\frac{\Psi_{nj}^{new}}{\Psi_{nj}^{old}} = exp(\beta \cdot X_{nj})$ denotes a scalar on the utility of an alternative and thus whether the alternative becomes more ($\beta \cdot X_{nj} > 0$) or less attractive ($\beta \cdot X_{nj} < 0$) than $exp(\delta_j)$ due to its characteristics.

in attractiveness - over the classical project I - by a factor 1.77 ($exp(0.087 \cdot 3 + 0.103 \cdot 3) \approx 1.77$).[11] Results are only presented for the policy characteristics Biodiversity and Recreation, because four characteristics were insignificant (total minutes saved, prevented injuries, number of floods prevented every 25 years and level of flood protection) and two characteristics displayed an incorrect sign (size and the number of household that had to relocate). The decision was made to exclude the latter two characteristics in order to facilitate the policy analysis. For reassurance, these excluded policy attributes have a limited impact on the policy analysis and the issue is revisited in the conclusions section.

Finally, like in Bhat [3], the joint identification of γ_0 and α_0 is problematic, such that γ_0 is normalised to 1 and α_0 and α_{J+1} are estimated. The former parameter approached its upper bound of $\alpha_0 = 1$, i.e. non-satiation, whereas α_{J+1} approaches 0 and takes the form of the linear expenditure function on y_{J+1} [18]. Accordingly, these values are also assumed to be fixed in estimation without loss of model fit.

Table 3: Estimation results

Coefficient	mean	t-val
$\delta_{y_{n0}}$	1.337	30.11
δ_{I*}	4.177	48.33
δ_{II*}	5.301	70.74
δ_{III*}	6.680	77.74
δ_{IV*}	6.315	76.02
δ_{V}	3.805	51.40
δ_{VI}	6.107	76.98
δ_{VII}	6.122	117.47
δ_{VIII}	6.302	128.46
δ_{IX}	5.087	79.61
δ_{X}	4.180	67.42
$\beta_{Biodiversity}$	0.087	5.26
$\beta_{Recreation}$	0.103	7.34
σ	1.013	37.47
α_0	1	-
α_{J+1}	0	-
wLL	-16933.5	

[11] This is conditional on a cardinal interpretation of utility

5.4. *Policy evaluation based on estimation results*

When the government chooses a portfolio, the public good nature ensures that project costs and consumption of y_0 and $y_1, ..., y_J$ are equal for all citizens. Only one portfolio can be chosen rendering the supply of projects to be deterministic. This results in an evaluation of corner solutions of all feasible portfolios at a given budget to select the optimal policy portfolio.

5.4.1. *Attractiveness of individual projects*

In the estimated utility function (Table 3), shifting budget to the next period is associated with a constant marginal utility $\Psi_0 = exp(\delta_0 - \epsilon_0)$, since $\alpha_0 = 0$ and γ_0. This property allows evaluating whether an individual project is considered more attractive than shifting budget to the next period or not. Specifically, the (logit) probability that spending money on the project increases individual utility more than saving this money for future budget periods can be derived, i.e. $P(\Psi_j \geq \Psi_0 \cdot c_j)$.[12] The final column of Table 4 shows that all project but project VIII (road project at Kerensheide) have a more than 50% probability of being better than saving money for future budget periods - with most projects scoring well above 70%. In the same way, $exp(\delta_j + X_j\beta) > exp(\delta_0 + ln(c_j))$ (based on Eq.(30)) can be used to evaluate whether project j should (not) be considered for inclusion in the optimal portfolio (measured in expected utility). The results indicate that project VIII will never be included in the optimal portfolio and is thus not considered value for money.

5.4.2. *Optimal budget*

When adjusting taxes is more attractive than shifting budget forward, less alternatives may be considered for inclusion in the optimal portfolio than presented in Section 5.4.1. For the estimated utility function, the optimal tax for income group g is given by: $\tau_g^* =$

[12] Since Ψ_0 and $\Psi_j, \forall j = 1, \ldots, J$ are independent of private consumption, this probability is equal for all income groups in society.

$Y_g(1-t_g) - \frac{1}{exp(\delta_0)\cdot Q}$. Since $\delta_0 > 0$ and $Q = 8 \cdot 10^6$, shifting budget is much more attractive than reducing taxes. In fact, citizens would prefer to sacrifice nearly all their private consumption. As such, the conclusions from Section 5.4.1 hold and all projects but project VIII are attractive for inclusion in the optimal policy portfolio. Implementing these nine projects would require increasing the budget from 700 to (at least) 716 mln euros. Although the model indicates the budget should be increased further, that is not considered a desirable outcome from a policy perspective.

Table 4 present the Top 10 portfolios at the optimal budget of 716 mln euros. As discussed, the optimal portfolio comprises all projects but project VIII (which is never included). Lower ranked portfolios comprise fewer alternatives and have a lower budget requirement. The combination projects II*, III*, IV* and regular projects VI and VII are always included in the Top 10 portfolios. The ranking reveals that projects V and X would be the first to be sacrificed if the available budget is limited. Dropping these two alternatives is better than dropping the combination project I*, for example. Finally, the bottom row compares each portfolio in the Top 10 with the null portfolio, which comprises only the four classical projects. When a combination project is selected the cost difference is shifted to the next budget period. Simulation is used to evaluate the extent to which the selected portfolio is better than the null portfolio. With a probability of around 87%, policy makers can have good confidence that either of the portfolios in the portfolios in the Top 10 would have a high level of acceptance amongst the citizens.

5.4.3. Fixed budget

An alternative approach to the policy evaluation would be to restrict the budget to the 700 mln euros as presented in the fixed budget experiment. Table 5 summarizes the Top 10 of portfolios under that assumption. Combinations projects III* and IV* are still always included in the Top 10 portfolios. Combination projects I* and II* are included in all but

Table 4: Ranking and composition of top 10 portfolios at an optimal budget of 716 mln euros

Top 10 portfolios:	1	2	3	4	5	6	7	8	9	10	$\mathbb{P}(\Psi_j > \Psi_0 \cdot c_j)$
I*	1	1	1	1	0	1	0	0	1	1	0.86
II*	1	1	1	1	1	1	1	1	1	1	0.67
III*	1	1	1	1	1	1	1	1	1	1	0.75
IV*	1	1	1	1	1	1	1	1	1	1	0.75
V	1	1	0	0	1	1	1	0	1	0	0.74
VI	1	1	1	1	1	1	1	1	1	1	0.70
VII	1	1	1	1	1	1	1	1	1	1	0.61
VIII	0	0	0	0	0	0	0	0	0	0	0.36
IX	1	1	1	1	1	0	1	1	0	0	0.75
X	1	0	1	0	1	1	0	1	0	1	0.74
Costs (in mln euros)	716	710	710	704	711	702	705	705	696	696	
$\mathbb{P}(SU_p > SU_{null})$	0.88	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87	

two of the Top 10 project portfolios, which is consistent with the general choice shares in Table 2. Moreover, the required budget for each of the ten portfolios indicates it is not necessarily best to spend as much budget as possible on projects. For example, Rank 3 does not implement VII (costing 75 mln), but saving 59 mln for the next budget period is better than spending 699 mln euros and sacrificing projects I*, V and X to implement road project VII. Thus, the budget constraint prevents all 9 attractive projects from being included in the optimal policy portfolio. Given the budget, an evaluation of all feasible portfolios is, however, required to determine the optimal portfolio as citizens are willing to make trade-offs across projects and shifting budget to future periods.

Table 5: Ranking and composition of top 10 portfolios at a budget of 700 mln euros

Rank of portfolio:	1	2	3	4	5	6	7	8	9	10	$\mathbb{P}(\Psi_j > \Psi_0 \cdot c_j)$
I*	1	1	1	0	1	1	0	1	1	1	0.86
II*	1	1	1	1	0	1	1	1	1	0	0.67
III*	1	1	1	1	1	1	1	1	1	1	0.75
IV*	1	1	1	1	1	1	1	1	1	1	0.75
V	1	0	1	0	1	0	1	1	0	1	0.74
VI	1	1	1	1	1	1	1	1	1	1	0.7
VII	1	1	0	1	1	1	1	0	0	1	0.61
VIII	0	0	0	0	0	0	0	0	0	0	0.36
IX	0	0	1	1	1	0	0	1	1	1	0.75
X	0	1	1	0	1	0	1	0	1	0	0.74
Costs (in mln euros)	696	696	641	699	671	690	697	635	635	665	
$\mathbb{P}(SU_p > SU_{null})$	0.87	0.87	0.88	0.87	0.87	0.87	0.87	0.88	0.88	0.87	

5.4.4. Sensitivity analyses: project costs and impacts

Section 5.4.3 illustrated how the governmental budget constraint affects the optimal portfolio ranking and thereby the need to make trade-offs across projects. In this subsection two additional sensitivity analyses are presented in order to investigate changes in the portfolio ranking due to changes in: (i) project costs and (ii) project impacts. In both cases the budget is set at 700 mln. First, the attractiveness of the combination projects I*-IV* is reduced by limiting their biodiversity and recreation improvements to level 2. The final column and bottom row in Table 6 highlights that the reduced policy impacts reduce the probability that spending the money on the combination projects is better than saving the money for the next budget period and that an increase in SWF is obtained. Moreover, the portfolio that was ranked fifth in Table 5 is now ranked first. Particularly combination projects I* and II* are included less often in the Top 10 portfolios, whereas projects VII and X are included more often.

Table 6: Top 10 portfolios under alternative biodiversity and recreation impacts in I*-IV*

Rank of portfolio:	1	2	3	4	5	6	7	8	9	10	$\mathbb{P}(\Psi_j > \Psi_0 \cdot c_j)$
I*	1	1	1	0	1	1	1	0	1	0	0.83
II*	0	1	1	1	0	0	1	1	1	0	0.63
III*	1	1	1	1	1	1	1	1	1	1	0.71
IV*	1	1	1	1	1	1	1	1	1	1	0.72
V	1	1	0	0	1	0	1	1	0	1	0.74
VI	1	1	1	1	1	1	1	1	1	1	0.7
VII	1	1	1	1	1	1	0	1	1	1	0.61
VIII	0	0	0	0	0	0	0	0	0	0	0.36
IX	1	0	0	1	1	1	1	0	0	1	0.75
X	1	0	1	0	0	1	1	1	0	1	0.74
Costs (in mln euros)	671	696	696	699	665	665	641	697	690	666	
$\mathbb{P}(SU_p > SU_{null})$	0.85	0.84	0.84	0.84	0.85	0.85	0.86	0.84	0.84	0.85	

Finally, policy makers can evaluate the impacts of alternative costs estimates. In this sensitivity test, the costs of the two road projects VII and VIII are reduced to respectively 60 mln and 150 mln. Again, the optimal portfolio changes in Table 7 where, compared to Table 5 project IX is now added to the optimal portfolio due to the budget that has become available. Despite the significant reduction in costs for project VIII, the project is still insufficiently attractive to be included in the optimal policy portfolio.

Table 7: Top 10 portfolio for reduced costs for projects VII and VIII

Rank of portfolio:	1	2	3	4	5	6	7	8	9	10	$\mathbb{P}(\Psi_j > \Psi_0 \cdot c_j)$
I*	1	1	1	0	1	0	0	1	1	0	0.86
II*	1	1	1	1	1	1	1	1	1	1	0.67
III*	1	1	1	1	1	1	1	1	1	1	0.75
IV*	1	1	1	1	1	1	1	1	1	1	0.75
V	1	0	0	1	1	1	0	1	0	0	0.74
VI	1	1	1	1	1	1	1	1	1	1	0.7
VII	1	1	1	1	1	1	1	1	1	1	0.67
VIII	0	0	0	0	0	0	0	0	0	0	0.49
IX	1	1	1	1	0	1	1	0	0	1	0.75
X	0	1	0	1	1	0	1	0	1	0	0.74
Costs (in mln euros)	695	695	689	696	687	690	690	681	681	684	
$\mathbb{P}(SU_p > SU_{null})$	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.87	

6. Conclusions and discussion

The present paper developed an economic framework allowing straightforward policy evaluation based on novel Particapatory Value Evaluation (PVE) experiments. Building on recent literature of Kuhn-Tucker models, particularly the MDCEV model, a range of methodological and econometric contributions are provided facilitating model estimation and policy evaluation. The empirical application in the context of Dutch flood risk policies illustrates the potential of PVE experiments as a promising alternative to CBA.

The key strength of PVE is the assumption of one-person-one-vote (OPOV) in policy analysis. Citizens have an equal vote for the public budget irrespective of their willingness or ability to pay. Although possible, (see [10]), there is no need to derive willingness to pay for policy attributes, or demand functions for projects. PVE only relies on the estimated direct utility functions and contrary to CBA gives all citizens the same weight in the social welfare function. Potential distributional and altruistic considerations of the citizens are already an integral part of these estimated utility functions. This aligns with the non-paternalistic assumption in democratic voting that individuals are the best to judge on their own social welfare function. The possibility of delegation gives citizens who do not agree with this assumption the option to abstain from making a decision. The governmental budget constraint takes care of the fact that governmental project costs decrease social and individual welfare either through limiting investments in alternative projects or by limiting private consumption.

Aggregation of the estimated utility functions in PVE is possible because the adopted specification of the individual's utility function is cardinal in nature and rooted in random utility modelling. Ordinal project rankings alone lead to aggregation problems as the discrete choices of citizens cannot be made comparable [1]. The intensity of preferences in our model is estimated empirically through a series of PVE experiments where a representative sample of citizens selects their most preferred policy portfolio given a fixed or flexible policy portfolio. Potential differences in the unobserved utility between citizens are captured by the distribution of ϵ_{nj} for which we estimate the degree of heterogeneity with the scale parameter σ. This degree of heterogeneity does not influence the ranking of the available portfolios. A remaining challenge is to examine whether the ranking of portfolios is sensitive to any concave monotonic transformation of the utility function. The latter requires satisfaction of second order stochastic dominance, which falls outside the scope of the current paper. The reported probabilities in the policy analysis are, however, invariant to concave monotonic transformations.

A second key benefit of PVE is the ease by which it allows for policy evaluation and sensitivity tests regarding the available government budget, the project costs and characteristics. Notably, the null portfolio is part of the set of feasible portfolios. The null portfolio will result in a substantial shift of money for the case of a fixed budget experiment and in an increase of net income for the flexible budget case. This is different from voting experiments where only the ranking of projects is measured as these rankings do not take into account the governmental and private budget constraints in the evaluation stage. Of the key outcomes from the presented PVE analysis is that the optimal portfolios have a more than 85% chance of being better than the null portfolio. That is, there is significant societal support for implementing a range of public poliy projects. The composition of the optimal policy portfolio is, however, determined by the available budget constraint, projects costs and characteristics. Moreover, PVE allows to judge whether individual projects are value for money or not.

Finally, PVE experiments provide citizens with an opportunity to display their value judgements when it comes to public policy decisions. Posner and Sunstein [15] have argued that viewing these concerns of citizens as valueless in the evaluation stage can be viewed as a violation of the law. Measuring preferences for public policies at the level of the actual policy decision rather than silently assuming that preferences obtained from private choices reflect public value is therefore a key benefit of PVE.

Estimating the marginal valuations of the different policy impacts proved to be challenging. First, it was observed that in terms of fit and policy implications they only have a limited impact. Second, most characteristics turned out to be insignificant and (or) have the incorrect sign. The most likely cause for this is the heterogeneous set of alternatives that could be included in the policy portfolio assigning a high importance to the δ's. Moreover, the set of included policy characteristics varies across policies including their levels. For example, the size variable showed an incorrect sign due to the small project I* being included in the portfolio by many respondents (79%). The biodiversity and recreation variables were not included for 4 projects, whereas time savings were only included for 2 projects making it much harder to identify their impacts on the overall decision. This is clearly an issue to be addressed in future research by making experimental adjustments that can lead to more knowledge about marginal valuations of impacts. This should, however, not hamper the empirical uptake of PVE experiments as a suitable alternative to CBA.

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policies".

Appendix A. Derivation of choice probabilities - Case 1

The choice probability is given by:

$$\mathbb{P}_{n1}(y_n^*) = \int_{\epsilon_{n0}=-\infty}^{\infty} \prod_{k=1}^{K_n} \left(1 - F\left[V_{nk} - V_{n0} + \epsilon_{n0}\right]\right) \prod_{m=K_n+1}^{J} F\left[V_{nm} - V_{n0} + \epsilon_{n0}\right] f\left[\epsilon_{n0}\right] d\epsilon_{n0} \tag{A.1}$$

Substituting the cumulative density $F(.)$ and the probability density function $f(.)$ of the EV Type 1 distribution gives:

$$\mathbb{P}_{n1}(y_n^*) = \frac{1}{\sigma} \int_{\epsilon_{n0}=-\infty}^{\infty} \prod_{k=1}^{K_n} \left(1 - e^{-e^{-\frac{V_{nk}-V_{n0}+\epsilon_{n0}}{\sigma}}}\right) \prod_{m=K_n+1}^{J} e^{-e^{-\frac{V_{nm}-V_{n0}+\epsilon_{n0}}{\sigma}}} \cdot e^{-\frac{\epsilon_{n0}}{\sigma}} e^{-e^{-\frac{\epsilon_{n0}}{\sigma}}} d\epsilon_{n0} \tag{A.2}$$

Use the substitution $t = e^{-\frac{\epsilon_{n0}}{\sigma}} \Leftrightarrow d\epsilon_{n0} = -\frac{\sigma}{t} \cdot dt$ to obtain:

$$\mathbb{P}_{n1}(y_n^*) = \int_{t=0}^{\infty} \prod_{k=1}^{K_n} \left(1 - e^{-t \cdot e^{-\frac{V_{nk}-V_{n0}}{\sigma}}}\right) \prod_{m=K_n+1}^{J} e^{-t \cdot e^{-\frac{V_{nm}-V_{n0}}{\sigma}}} \cdot e^{-t} dt \tag{A.3}$$

Next, use the substitution $v = e^{-t} \Leftrightarrow dt = -v^{-1} \cdot dv$:

$$\begin{aligned} \mathbb{P}_{n1}(y_n^*) &= \int_{v=0}^{1} \prod_{k=1}^{K_n} \left(1 - v^{e^{-\frac{V_{nk}-V_{n0}}{\sigma}}}\right) \prod_{m=K_n+1}^{J} v^{e^{-\frac{V_{nm}-V_{n0}}{\sigma}}} dv \\ &= \int_{v=0}^{1} \prod_{k=1}^{K_n} \left(1 - v^{e^{-\frac{V_{nk}-V_{n0}}{\sigma}}}\right) v^{\sum_{m=K_n+1}^{J} e^{-\frac{V_{nm}-V_{n0}}{\sigma}}} dv \end{aligned} \tag{A.4}$$

The last step can be done using enumeration of all possible combinations of alternatives given the value of K_n. Define the set S_n as the set with all possible combinations of $W_{nk0} = e^{-\frac{V_{nk}-V_{n0}}{\sigma}}$, including the empty set. Let $|S_n| = 2^{K_n}$ denote the number of elements and S_{ns} be element s of the set S_n. Then, following Bhat [4], a closed form choice probability results:

$$\mathbb{P}_{n1}(y_n^*) = \sum_{s \in S_n}^{|S_n|} (-1)^{|S_{ns}|} \frac{1}{1 + \sum_{m=K_n+1}^{J} W_{nm0} + \sum_{q \in S_{ns}}^{|S_{ns}|} S_{nsq}} \tag{A.5}$$

Appendix B. Derivation of choice probabilities - Case 2

The choice probability for case 2 is given by:

$$\mathbb{P}_{n2}(y_n^*) = \int_{\epsilon_{m^*}=-\infty}^{\infty} \prod_{k=0}^{K_n} (1 - F\left[LS_{nm} + V_{nk} + \epsilon_{m^*}\right]) \cdot f\left[\epsilon_{m^*}\right] d\epsilon_{m^*}. \tag{B.1}$$

Define $LS_n = \sigma \ln \left[\sum_{m=K_n+1}^{J} e^{\left(\frac{-V_{nm}}{\sigma}\right)}\right]$ as the logsum measure over the chosen alternatives, and substitute the Gumbel density functions to obtain:

$$\mathbb{P}_{n2}(y_n^*) = \frac{1}{\sigma} \int_{\epsilon_{m^*}=-\infty}^{\infty} \prod_{k=0}^{K_n} \left(1 - e^{-e^{-\frac{LS_{nm}+V_{nk}+\epsilon_{m^*}}{\sigma}}}\right) \cdot e^{-\frac{\epsilon m^*}{\sigma}} e^{-e^{-\frac{\epsilon m^*}{\sigma}}} d\epsilon_{m^*} \tag{B.2}$$

Using the substitution of variable $t = e^{-\frac{\epsilon m^*}{\sigma}} \Leftrightarrow d\epsilon_{m^*} = -\sigma \frac{1}{t} dt$ results in:

$$\mathbb{P}_{2}(y_n^*) = \int_{t=0}^{\infty} \prod_{k=0}^{K_n} \left(1 - e^{-t \cdot e^{-\frac{LS_{nm}+V_{nk}}{\sigma}}}\right) \cdot e^{-t} dt \tag{B.3}$$

Next, use the substitution $v = e^{-t} \Leftrightarrow dt = -v^{-1} dv$:

$$\mathbb{P}_{n2}(y_n^*) = \int_{v=0}^{1} \prod_{k=0}^{K_n} \left(1 - v^{e^{-\frac{LS_{nm}+V_{nk}}{\sigma}}}\right) dv \tag{B.4}$$

The last step can be done using enumeration of all possible combinations of non-chosen alternatives in K_n. Define the set S_n as the set with all possible combinations of $W_{nmk} = e^{-\frac{LS_{nm}+V_{nk}}{\sigma}}$ of the non-chosen alternatives (including alternative 0) including the empty set. Let $|S_n| = 2^{K_n+1}$ be the number of elements. Let S_{ns} be element s of the set S_n. Then, similar to Case 1, a closed form choice probability results:

$$\mathbb{P}_{n2}(y_n^*) = \sum_{s\in S_n}^{|S_n|} (-1)^{|S_{ns}|} \frac{1}{1 + \sum_{q\in S_{ns}}^{|S_{ns}|} S_{nsq}} \tag{B.5}$$

Appendix C. Derivation of choice probabilities - Case 3

The choice probability for case 3 is given by:

$$\begin{aligned}\mathbb{P}_{n3}(y_n^*) = \big||G|\big| \int_{\epsilon_{n,J+1}=-\infty}^{\infty} \prod_{k=1}^{K_n} \left(1 - F\left[V_{nk} - V_{n,J+1} + \epsilon_{n,J+1}\right]\right) \prod_{m=K_n+1}^{J} F\left[V_{nm} - V_{n,J+1} + \epsilon_{n,J+1}\right] \\ \cdot f\left[V_{n0} - V_{n,J+1} + \epsilon_{n,J+1}\right] \cdot f\left[\epsilon_{n,J+1}\right] d\epsilon_{n,J+1},\end{aligned} \tag{C.1}$$

where $\big||G|\big| = \frac{\partial\left(V_{n0} - V_{n,J+1} + \epsilon_{n,J+1}\right)}{\partial y_{n0}} = (\alpha_0 - 1)\frac{1}{y_{n0}+\gamma_0} - (\alpha_{J+1} - 1)\frac{1}{y_{n,J+1}}\frac{1}{Q}$ is the absolute value of the Jacobian determinant. The latter is required because of the change of variable for ϵ_{n0}. Substituting the density functions gives:

$$\begin{aligned}\mathbb{P}_{n3}(y_n^*) = \frac{1}{\sigma^2}\big||G|\big| \int_{\epsilon_{n,J+1}=-\infty}^{\infty} \prod_{k=1}^{K_n} \left(1 - e^{-e^{-\frac{V_{nk} - V_{n,J+1} + \epsilon_{n,J+1}}{\sigma}}}\right) \prod_{m=K_n+1}^{J} e^{-e^{\frac{V_{nm} - V_{n,J+1} + \epsilon_{n,J+1}}{\sigma}}} \\ \cdot e^{-\frac{V_{n0} - V_{n,J+1} + \epsilon_{n,J+1}}{\sigma}} e^{-e^{-\frac{V_{n0} - V_{n,J+1} + \epsilon_{n,J+1}}{\sigma}}} \cdot e^{-\frac{\epsilon_{n,J+1}}{\sigma}} e^{-e^{-\frac{\epsilon_{n,J+1}}{\sigma}}} d\epsilon_{n,J+1}\end{aligned} \tag{C.2}$$

Using the substitution of variable $t = e^{-\frac{\epsilon_{n,J+1}}{\sigma}} \Leftrightarrow d\epsilon_{n,J+1} = -\sigma\frac{1}{t}dt$ we obtain:

$$\mathbb{P}_{n3}(y_n^*) = \frac{1}{\sigma}||G||e^{-\frac{V_{n0}-V_{n,J+1}}{\sigma}} \int_{t=0}^{\infty} \prod_{k=1}^{K} \left(1 - e^{-t \cdot e^{-\frac{V_{nk}-V_{n,J+1}}{\sigma}}}\right) \prod_{m=K+1}^{J} e^{-t \cdot e^{-\frac{V_{nm}-V_{n,J+1}}{\sigma}}}$$
$$\cdot e^{-t \cdot e^{-\frac{V_{n0}-V_{n,J+1}}{\sigma}}} \cdot e^{-t} \cdot t \cdot dt \tag{C.3}$$

Next, use the substitution $v = e^{-t} \Leftrightarrow dt = -v^{-1}dv$ to obtain:

$$\begin{aligned}\mathbb{P}_{n3}(y_n^*) &= \frac{1}{\sigma}||G||e^{-\frac{V_{n0}-V_{n,J+1}}{\sigma}} \int_{v=0}^{1} \prod_{k=1}^{K_n} \left(1 - v^{e^{-\frac{V_{nk}-V_{n,J+1}}{\sigma}}}\right) \prod_{m=K+1}^{J} v^{e^{-\frac{V_{nm}-V_{n,J+1}}{\sigma}}} \cdot v^{e^{-\frac{V_{n0}-V_{n,J+1}}{\sigma}}} \left(-\ln[v]\right) dv \\ &= \frac{1}{\sigma}||G||e^{-\frac{V_{n0}-V_{n,J+1}}{\sigma}} \int_{v=0}^{1} \prod_{k=1}^{K_n} \left(1 - v^{e^{-\frac{V_{nk}-V_{n,J+1}}{\sigma}}}\right) v^{\sum_{m=0,K_n+1}^{J} e^{-\frac{V_{nm}-V_{n,J+1}}{\sigma}}} \left(-\ln[v]\right) dv.\end{aligned} \tag{C.4}$$

Here, the set of chosen goods is extended with good 0.Define the set S_n as the set with all possible combinations of $W_{n(J+1)m} = e^{-\frac{V_{nm}-V_{n,J+1}}{\sigma}}$ of the chosen alternatives including the empty set. Let $|S_n| = 2^{M_n}$ the number of elements. Let S_{ns} be element s of the set S_n. Then the choice probability is given by:

$$\mathbb{P}_{n3}(y_n^*) = \frac{1}{\sigma}||G||e^{-\left(\frac{V_{n0}-V_{n,J+1}}{\sigma}\right)} \sum_{s \in S_n}^{|S_n|} (-1)^{|S_{sn}|} \frac{1}{\left(1 + \sum_{m=0,K_n+1}^{J} W_{n(J+1)k} + \sum_{q \in S_{ns}}^{|S_{ns}|} S_{nsq}\right)^2} \tag{C.5}$$

Appendix D. Derivation of choice probabilities - Case 4

The choice probability for case 4 is given by:

$$\mathbb{P}_{n4}(y_n^*) = \int_{\epsilon_{n,J+1}=-\infty}^{\infty} \prod_{k=0}^{K_n} (1 - F\left[V_{nk} - V_{n,J+1} + \epsilon_{n,J+1}\right]) \prod_{m=K_n+1}^{J} F\left[V_{nm} - V_{n,J+1} + \epsilon_{n,J+1}\right] \cdot f\left[\epsilon_{n,J+1}\right] d\epsilon_{n,J+1}, \tag{D.1}$$

Substituting the density functions gives:

$$\mathbb{P}_{n4}(y_n^*) = \frac{1}{\sigma} \int_{\epsilon_{n,J+1}=-\infty}^{\infty} \prod_{k=0}^{K_n} \left(1 - e^{-e^{-\frac{V_{nk}-V_{n,J+1}+\epsilon_{n,J+1}}{\sigma}}}\right) \prod_{m=K_n+1}^{J} e^{-e^{-\frac{V_{nm}-V_{n,J+1}+\epsilon_{n,J+1}}{\sigma}}} \cdot e^{-\frac{\epsilon_{n,J+1}}{\sigma}} e^{-e^{-\frac{\epsilon_{n,J+1}}{\sigma}}} d\epsilon_{n,J+1}. \tag{D.2}$$

Using the substitution of variable $t = e^{-\frac{\epsilon_{n,J+1}}{\sigma}} \Leftrightarrow d\epsilon_{n,J+1} = -\sigma \frac{1}{t} dt$ we obtain:

$$\mathbb{P}_{n4}(y_n^*) = \int_{t=0}^{\infty} \prod_{k=0}^{K_n} \left(1 - e^{-t \cdot e^{-\frac{V_{nk}-V_{n,J+1}}{\sigma}}}\right) \prod_{m=K_n+1}^{J} e^{-t \cdot e^{-\frac{V_{nm}-V_{n,J+1}}{\sigma}}} \cdot e^{-t} dt \tag{D.3}$$

Next, use the substitution $v = e^{-t} \Leftrightarrow dt = -v^{-1} dv$ to obtain:

$$\mathbb{P}_{n4}(y_n^*) = \int_{v=0}^{1} \prod_{k=0}^{K_n} \left(1 - v^{e^{-\frac{V_{nk}-V_{n,J+1}}{\sigma}}}\right) v^{\sum_{m=K_n+1}^{J} e^{-\frac{V_{nm}-V_{n,J+1}}{\sigma}}} dv \tag{D.4}$$

Define the set S_n as the set with all possible combinations of $W_{n(J+1)nm} = e^{-\frac{V_{nm}-V_{n,J+1}}{\sigma}}$ of the chosen alternatives with the empty set included as well. Let $|S_n| = 2^{K_n+1}$ the number of elements of S_n. Let S_{ns} be element s of the set S_n. Then the choice probability is given by:

$$\mathbb{P}_{n4}(y_n^*) = \sum_{s \in S_n}^{|S_n|} (-1)^{|S_{ns}|} \frac{1}{1 + \sum_{m=K_n+1}^{J} W_{n(J+1)m} + \sum_{q \in S_{ns}}^{|S_{ns}|} S_{nsq}}. \tag{D.5}$$

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