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Need to Know? On Information Systems in Firms

Suzanne Bijkerk[1]
Josse (J.) Delfgaauw[1]
Vladimir (V.A.) Karamychev[1]
Otto (O.H.) Swank[1]

[1] Erasmus School of Economics, The Netherlands

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Suzanne H. Bijkerk[1] Josse Delfgaauw Vladimir Karamychev
Otto H. Swank

Erasmus School of Economics
Tinbergen Institute

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Abstract

In the context of firm decision-making, several motives for acquiring and conveying information exist. Information serves to make better decisions, to persuade, and to impress. In this paper, we study how these motives shape incentives to acquire and communicate information. We employ a cheap-talk model with information acquisition and communication by a firm's executive. The executive wants to accurately inform an internal decision-maker regarding the value of an opportunity, but has an incentive to overstate this value to persuade or impress external parties. We show that information acquisition and communication interact. The executive's impression and persuasion motives yield limited distortions in communication, if any. Instead, they reduce information acquisition. Furthermore, we find that for firms, transparent communication is a necessary evil. Transparency allows for influential communication to external parties, but constrains internal communication. Theoretically, we contribute by showing that the forward induction refinement excludes babbling as an equilibrium outcome if non-babbling equilibria exist.

Keywords: Information system, Information acquisition, Cheap-talk, Transparency, Firm behavior

JEL code: D21, D83

[1]Corresponding author. Department of Economics, Erasmus University Rotterdam, Burgemeester Oudlaan 50, 3062 PA Rotterdam, e-mail: bijkerk@ese.eur.nl

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1 Introduction

Ann is an executive at a US restaurant chain. She sees an opportunity to expand business towards The Netherlands. To explore and realize this opportunity, two items require specific attention. First, establishing a new restaurant in a particular Dutch town requires approval of the town council. The town council only approves if it is convinced that the restaurant will hire a sufficient number of local employees. Second, to determine the viability of the restaurant, she needs information on the local demand for US cuisine. This information can also be used to persuade the town council. Ann has delegated the final decision on the expansion to John, a loyal employee of the firm who collects information about local suppliers, possible locations, local health and safety regulations, *et cetera*. Market research gives Ann information on potential customers. The more she spends on market research, the more detailed information she gets. As executive in charge of exploring this opportunity, Ann has to make several decisions. How much market research to do? What information to convey to the town council? What information to convey to John? Does it make sense to give the town council and John the same information?

In organizations, major decisions are rarely made by single individuals. As in the above restaurant example, the involvement of multiple parties in decision making raises questions regarding information collection and communication. In the context of firm decision-making, several motives for acquiring and conveying information exist. First, information serves to make better decisions. Second, information acquisition and communication serve to persuade. If the execution of a firm's plans requires approval of external stakeholders, information is collected and conveyed to gain approval. Third, collecting and transmitting information serve to impress. A firm may benefit from favorable perceptions held by outsiders regarding the firm's prospects.[2]

In this paper, we study theoretically how informational, persuasion, and impression motives shape firms' incentives to acquire information and to convey it internally and externally.[3] We employ a cheap-talk model à la Crawford and Sobel (1982) with information acquisition and multiple audiences. We consider a firm that sees an opportunity with uncertain payoff. The firm's executive decides about acquisition and communication of information on the value of the opportunity.[4]

[2] Besides the three motives discussed here, monitoring and evaluation are prominent motives for acquiring and communicating information. We abstract from these. They have been analyzed extensively in the literature on incentive pay following Holmström (1979).

[3] There are many firm decisions where these three motives play a role. In developing a new drug, pharma companies require approval from the FDA to start clinical trials. Similarly, developing real estate projects require approval from the municipality. These external parties only approve the new projects if the option value of the projects is sufficiently high.

[4] Recently, Bandiera *et al.* (2017) observed that CEOs spend on average 70% of their time in meetings, and serve as a linking pin between insiders and outsiders of their firms. Almost

Exploring the opportunity requires the approval of an external party, which is provided only if the expected value is sufficiently high. Through this channel, we model the persuasion motive. The implementation decision is made by an internal party, who acts in the firm's interest and has local information on the opportunity that is relevant for the implementation decision. This generates an informational motive. Finally, the executive cares about the firm's stock price, which depends on external parties' perceptions of the firm's value. This creates an impression motive.[5]

The executive makes three decisions. First, she decides how much information to generate. Second, she decides about the transparency of the firm's communication. She either sends a public message to all parties involved or she sends private messages. Finally, the executive decides what information to convey.[6] According to Cyert and March (1963, p.80 and p.127), these decisions together comprise a firm's information system.[7]

The persuasion and impression motive both give the executive an incentive to exaggerate the value of the opportunity in communication towards external parties. In contrast, the informational motive gives an incentive to communicate truthfully to the internal party. Hence, if the executive sends a public message, she faces a trade-off. Exaggerating may raise the firm's stock price or may persuade an external party to approve, but it distorts internal decision making.

The recurrent theme of this paper is that information acquisition and communication are intertwined. One key result is that in equilibrium, the executive's

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50 years ago, Mintzberg (1971) analyzed daily activities of top managers and reached a similar conclusion: top managers are pre-dominantly involved in collecting and sharing information. Hence his characterization of top managers as the 'nerve center' of their organizations.

[5] The persuasion and impression motive both originate from beliefs held by external parties. We use persuasion to refer to situations where the executive's payoff depends on actions taken by people with non-aligned preferences in the decision-making process. The impression motive captures settings where the executive directly benefits from more favorable beliefs held by people not involved in the decision-making process.

[6] Our paper considers acquisition and communication of strategic, forward-looking information. Measurement and reporting of firms' past performance is heavily regulated. Still, the accounting literature documents substantial earnings management in reporting, often linked to managerial incentives (Watts and Zimmermann 1986, Habib and Hansen 2008). Information gathered and reported for decision-making is far less subject to regulation. For instance, in the US, the Private Securities Litigation Reform Act of 1995 shelters firms from possible litigation if forecasts turn out to be ill predictions ex post.

Theories on disclosure and reporting typically assume that firms or managers possess, rather than acquire, private information, see e.g. Diamond (1985), Dye (1985), Stocken and Verrecchia (2004), Goldman and Slezak (2006), Crocker and Slemrod (2007), and Hermalin and Weisbach (2012). Notable exceptions are Pae (1999), Hughes and Pae (2004), and Einhorn and Ziv (2007).

[7] In Cyert and March (1963, p.80 and p.127), a firm's information system describes how a firm generates and condenses information, and how and what information is distributed internally and externally.

impression motive leads to limited distortions in communication, if any. Instead, it reduces information acquisition, even if information acquisition is costless. In our model, the cost of overstating firm value, *i.e.*, the distortion in internal decision making, is inversely related to the amount of information acquired. By acquiring less information, the executive effectively commits herself to communicate truthfully. Still, the reduction in information acquisition yields sub-optimal internal decision-making.[8]

We further show that the impression motive and the persuasion motive are imperfect substitutes in hindering communication and information acquisition. The impression motive gives the executive an incentive to overstate firm value for any actual firm value. The persuasion motive gives an incentive to exaggerate only if actual firm value is low. If firm value is sufficiently high, exaggeration is not necessary to gain approval, and the executive prefers to share her information. This result stems from the binary nature of the approval decision. We show that in equilibrium at most one report can induce the external party to deny approval. The executive has a strong incentive not to send this report.[9] Generally, the

[8]The Enron scandal illustrates the impression motive at work at an extreme. In the years leading up to their demise in 2001, Enron's executives obsessively focused on raising the stock price. They convinced analysts and investors that Enron's prospects were glorious; Fortune named Enron the most innovative company for six consecutive years up to 2000. Internally, they demanded ever-higher revenues, which led to a series of bad investment decisions. Despite such setbacks, Enron's executives kept expressing confidence in the firm's value and prospects to the outside world. Besides stock-based incentives, observers attribute the executives' behavior largely to their desire to impress others (McLean and Elkind 2003; Eichenwald 2005). Employees who critized projects were removed from these projects, and internal warnings on malpractice were ignored (Behr and Witt 2002a, 2002b; Free and Macintosh 2008). Recalling how CEO Kenneth Lay handled internal warnings, a former CFO of one of Enron's units noted "[he] has always been hands off even in his best days. ... My surmise is he didn't want to be informed. His attitude was, 'I don't want to know' " (Behr and Witt 2002b). In Enron's case, reputational concerns led to decision makers who are poorly informed.

Our model also speaks to less extreme situations. For instance, an increasing number of firms provide forward-looking statements, such as management earnings forecasts, often based on non-verifiable information (Bozanic *et al.* 2018). Despite the widespread concern that such statements can deliberately be misleading, investors and analysts do respond to this information (Patell 1976, Penman 1980, Waymire 1986, and Jennings 1987). Investors primarily respond to credible information (Bamber and Cheon 1998, Hutton *et al.* 2003, Dzieliński *et al.* 2017). In line with our results, Graham *et al.* (2005) present survey evidence suggesting that managers are willing to make decisions that reduce project quality if this prevents a negative financial report.

[9]The persuasion motive played an important role in the Volkswagen scandal. Increased emission standards forced Volkswagen to improve its diesel engines. Experts were doubtful about the possibility to meet the standards, but Volkswagen kept exploring and ultimately claimed to have found a solution. The new engines received regulatory approval, and Volkswagen's executives expressed their confidence in the new technology (Volkswagen Group 2012). In 2015, fraudulent software was exposed. The software made engines appear cleaner during regulatory tests than during regular driving.

executive's response to her persuasion motive is to reduce information acquisition, and to distort communication if firm value is low but less so if firm value is high.

Our paper also highlights the role of transparency in communication for information systems. If the executive can send private reports, she shares all her information privately with insiders. This serves the informational motive. This also eliminates the costs of sending distorted messages to outsiders. Hence, in equilibrium, outsiders receive only non-informative messages.[10] In the absence of the persuasion motive, the possibility of privately informing insiders typically leads to proper decision making based on the optimal amount of information. However, if the external party needs to be persuaded, the executive must impose transparent reporting, despite the resulting reduction in information acquisition and, hence, in the quality of internal decision making. For a firm, transparency is a necessary evil.

We view our analysis as a first step towards a theory of information systems. A first *step*, because, to the best of our knowledge, our paper is the first that examines in a single setting how internal and external forces shape decisions in firms regarding transparency of communication, how much information is generated, and which information is conveyed internally and externally. A *first* step, because, the model is setup with a view of readily distinguishing three motives of acquiring and conveying information. This enables us to identify the effects of changes in the relative importance of these three motives on a firm's information system. In practice, however, these three motives are more blurred than in our model. In particular, the persuasion and impression motives may be more important internally than we assume. Our model is also only a *first* step, as it does not capture all aspects of an information system. For example, our analysis abstracts from the time dimension. If delaying decision-making is costly, this affects the duration of the search for information. Implicitly, we assume that the cost of information acquisition also includes the cost of delay. Relatedly, if information is collected over time, a relevant question is *when* to communicate. Recently, Grenadier *et al.* (2016) and Orlov *et al.* (2018) consider timing of communication in dynamic frameworks, analyzing how the release of information depends on the

[10] If the executive could make the implementation decision herself rather than the insider, communication to the external party and the public is also non-informative. Hence, as anticipated by Cyert and March (1963), the effects of an information system depend on the decision-making process. Aghion and Tirole (1997) model the interaction between the decision-making structure and information acquisition in firms, and Dessein (2002) models the interaction between decision-making and communication. More recently, several papers study (de-)centralization and communication in situations where local units possess private information and potential benefits of coordination and adaptation exist (see, for example, Alonso *et al.*, 2008, Rantakari, 2008, and Swank and Visser, 2015). A key difference between these studies and ours is that they take the distribution of information as given, whereas in our model part of the information has to be acquired.

alignment of the preferences of the sender and receiver. Furthermore, we do not explicitly model how and by whom information is collected. In practice, many parties in firms are involved in gathering, recording, and processing information. Team theory, starting with Marschak and Radner (1972), analyzes how firms handle information when processing is costly. Crucially, team theory assumes everyone shares the same objective. Bolton and Dewatripont (1994) show that to handle flows of information most effectively, firms create networks of individuals that resemble classic forms of organizations. Sah and Stiglitz (1986) take the network as given and analyze the effects of alternative decision processes if individuals can make mistakes. Our paper is influenced by team theory in placing information at the heart of the analysis of organizations. However, we take the network, or organizational form, as given. Instead, we focus on how divergence of individuals' objectives affects how much information is collected, what information is conveyed and to whom it is conveyed.

Even though we regard our paper as applied theory, we also make a theoretical contribution. We show that forward induction as an equilibrium refinement excludes babbling as an equilibrium outcome if non-babbling equilibria exist. Loosely speaking, forward induction requires previous actions to be rational. In our model this implies that if babbling is an equilibrium outcome, the executive has not acquired any information. It would be a pure waste. Yet, this also implies that by acquiring some information, the executive can avoid a babbling equilibrium. This result is interesting in itself, in the sense that virtually all papers that use a cheap-talk model à la Crawford and Sobel (1982) acknowledge that one equilibrium of their models is the babbling one. Argenziano *et al.* (2016) even use the babbling equilibrium as an off-equilibrium-path punishment by the receiver, which induces the sender to overinvest in information collection. If forward induction is imposed, babbling is no longer a credible threat. We derive conditions under which forward induction selects a unique equilibrium.

Our results are derived from a cheap-talk model with information acquisition and multiple receivers. As one of our objectives is to better understand executives' incentives to manipulate information - one of the observations by Cyert and March - our choice for a cheap-talk model seems natural. A Bayesian Persuasion model à la Gentzkov and Kamenica (2011 and 2014) is more suitable for studying settings where firms are legally obliged to reveal all information gathered. One way of looking at our results is that in a cheap-talk setting the need to persuade or the desire to impress is costly for firms. It leads to internal decisions based on too little information. As a result, firms may look for other ways of making messages to outsiders credible, for example by hiring auditors. To examine those settings, the model of Dewatripont and Tirole (1999) seems appropriate. Following Milgrom and Roberts (1986), they assume verifiable information that can be concealed, but not manipulated. Then, the need to persuade strengthens incentives to gather information.

Farrell and Gibbons (1989) were the first to analyze a cheap-talk model with multiple receivers. Comparing public and private communication, they showed that a public message can be more informative than separate, private messages if preferences are sufficiently misaligned. This carries over to our model, where informative communication to the public and the external party requires the informational motive of communication to the insider. Goltsman and Pavlov (2011) generalize Farrell and Gibbons (1989) by allowing for a more general distribution of sender types. By adding an information acquisition stage, the number of sender types is endogenous in our model. Taking information acquisition into account, we show that public communication is more informative than private communication if the sender needs to persuade, but may lead to less information acquisition if the sender only wants to impress.[11]

Di Pei (2015) and Argenziano *et al.* (2016) consider information acquisition in a cheap-talk model with one receiver. In Di Pei (2015), the sender can first segment the state space in any arbitrary way, and subsequently learns in which segment the true state lies. Finer segmentation is more costly. The main result is that the sender never collects more precise information than she can communicate in equilibrium. In our model, the sender also segments the state space, but it is assumed that all segments are equally large. In Argenziano *et al.* (2016), the sender chooses the accuracy of information by deciding how many Bernoulli trials to conduct. Their way of modeling information acquisition can be regarded as a micro-foundation of the technology we assume.[12]

The next section describes the model. In Section 3, we present the analysis and results, and in Section 4, we apply the forward induction refinement. In Section 5, we extend our model to discuss the pros and cons of transparency. We conclude by discussing the implications of our findings for understanding firms' information systems in Section 6.

2 The Model

We consider a firm that sees an opportunity, called the project. The profitability of the project depends on the value of the firm's ongoing activities, represented

[11] Using the Bayesian Persuasion framework, Michaeli (2017) shows that the sender may acquire more information if only a subset rather than all receivers obtain the information acquired.

[12] In Dur and Swank (2005), the sender exerts costly effort that increases the quality of her signal about the state of the world. They show that the receiver benefits from a sender whose preferences deviate from his own preferences, as this increases the incentive to exert effort. Che and Kartik (2009) derive a similar result for the case that the sender and receiver have different priors.

by random variable v, and on the idiosyncratic characteristics of the project, represented by random variable z. The incremental value of the project to the firm is $\gamma(v-z)$, where γ measures the importance of the project relative to the ongoing activities. Both v and z are independently and uniformly distributed on the interval $[0,1]$.

Our model is designed to investigate the incentives of an executive, X, who has three motives for acquiring and conveying information about v. The first motive we consider is a persuasion motive. We model this motive by assuming that exploration of the project, through which z is learnt, requires the approval of an External party, E. Second, we model an informational motive. We assume that the final decision on the project is made by an Insider, I, whose preferences are perfectly aligned with those of X. Finally, to model the impression motive we assume that X is concerned with the Public's, P, perception of firm value.

At the beginning of the game, X acquires information about v. The accuracy of information is reflected by $a \in \mathbb{N}$. For any given a, the interval $[0,1]$ is split into a subintervals of equal length. Let $k \in \{1,..,a\}$ denote the subinterval $\left[\frac{k-1}{a}, \frac{k}{a}\right]$. X observes to which subinterval v belongs. We refer to k as X's type. Clearly, the higher is a, the more accurate is X's information about v. The cost of acquiring information is $(a-1)c$. After learning her type k, X's expectation of v is denoted by v_k:

$$v_k \equiv \mathrm{E}[v|k] = \frac{2k-1}{2a} \tag{1}$$

X's choice of a is publicly observed.[13] Her type, however, is private information. After X learns her type, she sends cheap-talk reports to E, I, and P. We consider two communication regimes: transparency and non-transparency. We start with the analysis of the case where communication is transparent: X sends a *public* report, r, to E, I, and P. In Section 5, we analyze the case where communication is non-transparent: X sends a private report r^I to I and another private report r^E to E and P. Under both regimes, reports can take values from any sufficiently large report space. For brevity, in the remainder of this section we denote X's reports by r. It is straightforward to adjust this to the non-transparency regime.

As mentioned above, exploration and implementation of the project requires E's approval. We denote E's approval decision by $d^E \in \{0,1\}$, where $d^E = 1$ denotes approval and $d^E = 0$ denotes rejection. If $d^E = 0$, the game ends. If $d^E = 1$, I explores the project, observes z, and makes the implementation decision, $d^I \in \{0,1\}$, where $d^I = 1$ denotes that the project is implemented, and $d^I = 0$ denotes that it is not. Figure 1 summarizes the timing of our model.

[13] This is a strong, but not unrealistic assumption. Reporting regulation requires firms to specify their investments in software (IAS 38) and hardware (IAS 16) in their (public) year reports. These investments in information technology can be used to infer the extent of information

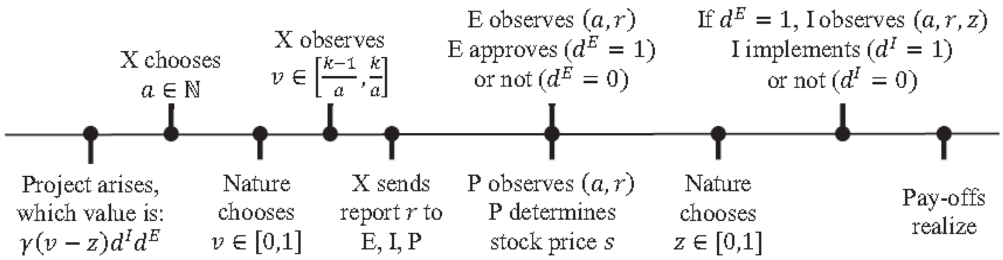

Figure 1: Timeline.

We denote firm value by w:

$$w = v + \gamma (v - z) d^E d^I - (a - 1) c \tag{2}$$

Following Stein (1989), we assume that X is concerned with w and with the firm's stock price, s. Hence X's payoff is equal to

$$u^X = (1 - \lambda) w + \lambda s \tag{3}$$

with

$$s = \mathrm{E}\left[w | a, r, d^E\right] \tag{4}$$

Note that stock price s equals P's perception of firm value w, conditional on r and d^E. Hence, X cares about the stock price that arises after E has made his approval decision, but before the implementation decision is made.[14] The parameter $\lambda \in [0, 1]$ denotes the relative weight on the stock price in X's utility, which can be interpreted as the strength of stock-based remuneration. In our model, it reflects the strength of the impression motive.

E's payoff is

$$u^E = \left(\gamma (v - z) d^I - h\right) d^E \tag{5}$$

Threshold h is the cost, borne by E, of allowing X to explore the project. Equation (5) captures that E approves exploration if the option value of the project exceeds threshold h. Through h we model the persuasion motive.

Lastly, I's payoff is equal to X's payoff, $u^I = u^X$. We abstract from agency problems between X and I to model the informational motive. X wants to share

collection.

[14] This could, for instance, reflect short-term financial incentives. Stein (1989) discusses several other reasons why executives may care about current stock prices, as implied by (3). None of our results under public reporting change if X cares about the stock price that realizes after P observes decision d^I, instead of before.

her information with I.[15] As is clear from (2), the importance of the project for firm value and, hence, the importance of I's decision is given by γ. Hence, γ reflects the strength of the informational motive.

Our model is stylized, but captures in a natural way the trade-offs faced by an executive who wants to persuade, impress, and inform. By sending r, X wants E to approve, s to be high, and I to make the proper decision on the project.

We solve the model for Sequential Equilibria (SEQ). In the main text below, we offer a relatively informal analysis and discussion. In the Appendix, we provide formal results and proofs. We use the following notation regarding players' strategies and beliefs. A SEQ consists of a collection $(\alpha, \rho(k,a))$ of behavioral strategies of X, an approval strategy $\delta^E(r,a)$ of E, a decision strategy $\delta^I(z,r,a)$ of I, and beliefs $G(k|r,a)$ of I, E, and P about X's type such that:

1. For any a, z, and r, decision $d^I = \delta^I(z,r,a)$ maximizes I's expectation of (3) given belief $G(k|r,a)$;
2. For any a and r, approval decision $d^E = \delta^E(r,a)$ maximizes E's expectation of (5) given belief $G(k|r,a)$;
3. For any a and type k, report $r = \rho(k,a)$ maximizes X's expectation of (3);
4. Information accuracy, $a = \alpha$ maximizes X's expectation of (3).
5. Beliefs $G(k|r,a)$ follow Bayes' rule on all information sets.

By $\Gamma(a)$, we denote the continuation game that is played after a is chosen and observed. In a Sequential Equilibrium, a perfect Bayesian equilibrium (PBE) is played in $\Gamma(a)$ for any $a \in \mathbb{N}$. In the remainder, for brevity we omit variable a from argument lists of functions and expectations whenever it does not lead to confusion.

As is usual in cheap-talk models, the 'language' of the reporting strategy $\rho(k)$ is defined only in equilibrium. Multiple reporting strategies can lead to the same beliefs and, hence, to the same equilibrium outcome. We ignore this type of equilibrium multiplicity. Therefore, we construct equilibrium sets by placing all equilibria with outcome-equivalent reporting strategies into one set. We refer to such an equilibrium set as an 'equilibrium'.

Cheap-talk games are also plagued by non-outcome-equivalent equilibrium multiplicity. The babbling equilibrium always exists. Hence, any equilibrium with

[15] Identical payoffs of X and I is a straightforward way of creating an informational motive, but not the only way. For instance, as s is determined before I makes a decision, λ is irrelevant for I´s decision. Hence, none of our results change if the level of λ differs between X and I. Similarly, I could maximize firm value (2) or project value $\gamma(v-z)d^I$.

influential communication is never the unique equilibrium. In Section 4, we show that if an equilibrium with influential communication exists, the forward induction refinement eliminates the babbling equilibrium. Furthermore, for some range of parameter values, forward induction yields a unique equilibrium outcome, in which influential communication does take place.

Before turning to the analysis of the game, we first determine the accuracy of information if X were to choose a in the absence of a persuasion and impression motive ($h = 0$ and $\lambda = 0$), and X reveals her type to I, $r = k$. For ease of exposition, we present the optimal a as a continuous variable. X anticipates that I implements whenever she learns that $z < \mathrm{E}(v|k)$, which happens with probability $\frac{2k-1}{2a}$. The expected value of the project, conditional on k and $d^E = 1$, equals

$$\begin{aligned}\mathrm{E}\left[\gamma(v-z)\, d^I d^E | k, d^E = 1\right] &= \gamma \Pr(z < \mathrm{E}(v|k)) \left[\mathrm{E}(v|k) - \mathrm{E}(z|z < \mathrm{E}(v|k))\right] \\ &= \gamma \frac{2k-1}{2a}\left(\frac{2k-1}{2a} - \frac{2k-1}{4a}\right) = \gamma \frac{(2k-1)^2}{8a^2}\end{aligned}$$

When choosing a, X's expectation of project value equals

$$\frac{1}{a}\sum_{k=1}^{a} \gamma \frac{(2k-1)^2}{8a^2} = \frac{\gamma}{24}\left(4 - \frac{1}{a^2}\right) \tag{6}$$

The marginal benefit of a to project value is given by the derivative of (6). Equating this to marginal cost c yields the optimal accuracy a^{opt}:

$$a^{opt} \equiv \sqrt[3]{\frac{\gamma}{12c}} \tag{7}$$

The value a^{opt} measures the accuracy of information that maximizes firm value in the absence of persuasion and reputational motives, provided the implementation decision is optimal for every type of X. We refer to underinvestment (overinvestment) in information acquisition if X chooses $a < a^{opt}$ ($a > a^{opt}$).

3 Analysis

We begin the analysis by considering a continuation game $\Gamma(a)$ that follows a choice of accuracy a, given transparency in communication. As insider I's and executive X's preferences are perfectly aligned, maximizing (3) yields that I chooses to implement the project, $d^I = 1$, if his expectation of project value given report r is positive, *i.e.*, if $\mathrm{E}[v|r] > z$, and chooses $d^I = 0$ otherwise. When making the approval decision, external party E anticipates I's strategy. Maximizing (5) yields that E approves if the option value of the project given r exceeds threshold h. Hence, $d^E = 1$ if $\gamma \, \mathrm{E}\left[(v - z)\, d^I | r\right] > h$ and $d^E = 0$ otherwise.

Lemma 1 characterizes X's equilibrium communication strategy in $\Gamma(a)$ and presents two immediate consequences.

Lemma 1 *Consider a continuation game* $\Gamma(a)$. *In any equilibrium of* $\Gamma(a)$*:*

(i) there is a number of distinct reports $N \in \{1, \ldots, a\}$ *and a set of* N *marginal types* $\{k_n\}$*, with* $k_{n-1} < k_n$*,* $k_0 = 0$ *and* $k_N = a$*, so that for all* $n \in \{1, \ldots, N\}$*, all types* $k \in \{(k_{n-1} + 1), \ldots, k_n\}$ *send report* r_n*;*

(ii) if $\lambda \geq \frac{1}{2}$*, then* $N = 1$*, and the pooling equilibrium is the unique equilibrium of* $\Gamma(a)$*;*

(iii) only report r_1 *may lead to E's disapproval,* i.e., $\delta^E(r_1) = \{0, 1\}$ *and* $\delta^E(r_n) = 1$ *for* $n \geq 2$.

Item (i) of Lemma 1 states that in a PBE, every report is sent by a subset of adjacent executive types k. This is illustrated in Figure 2. The top drawing depicts a separating equilibrium following a choice of $a = 4$. Each of the four types k sends a different report r_n, and the total number of reports $N = 4$. The bottom drawing depicts a semi-pooling equilibrium following $a = 14$, with $N = 5$. Types $k \in \{1, 2, 3\}$ send report r_1, types $k \in \{4, 5, 6, 7, 8\}$ send report r_2, types $k \in \{9, 10\}$ send r_3, and so on. Marginal types k_n are the highest types that send reports r_n, *i.e.*, $k_1 = 3$ and $k_2 = 8$, etc. As there are a different types, $k_N = a$ in any equilibrium. Furthermore, each marginal type k_n is the highest type that (weakly) prefers sending report r_n over sending report r_{n+1}. *E.g.*, in the bottom drawing of Figure 2, type $k = 3$ must prefer sending r_1 over r_2, whereas type $k = 4$ must prefer sending r_2 over r_1.[16]

In the communication stage, the informational motive meets the persuasion and impression motives. Consequently, X faces a dilemma. On the one hand, she wants to inform I about value v in order to maximize project value. On the other hand, she has an incentive to overstate v for two reasons. First, overstating v may persuade E to approve. Second, overstating v increases P's expectation of firm value and, hence, increases stock price s. The relative strength of the impression motive depends on how much X cares about the firm's stock price, λ. This drives

[16] The communication strategy presented in item (i) of Lemma 1 is almost equivalent to the communication strategy in a cheap-talk game à la Crawford and Sobel (1982). If $a \to \infty$, as in Crawford and Sobel (1982), marginal type k_n is indifferent between sending reports r_n and r_{n+1}. For finite a, however, marginal types are generally not indifferent, so that k_n strictly prefers report r_n over report r_{n+1}. This feature is also present in Argenziano *et al.* (2016) and is caused by the information collection technology used.

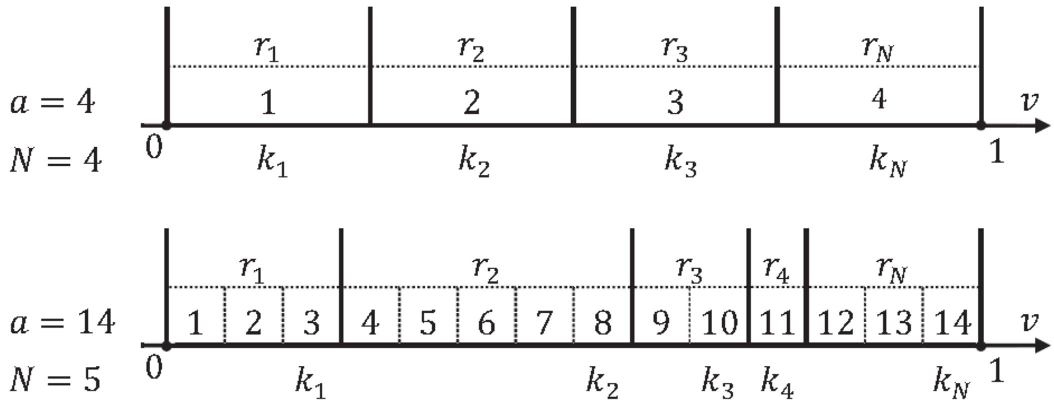

Figure 2: Illustrations of equilibria.

item (ii) in Lemma 1. If X cares too much about s, she cannot credibly communicate her value. Anticipating this, she does not invest in acquiring information, and chooses $a = 1$. If λ is sufficiently small, X's incentive to overstate v is offset by the distortion it induces in I's decision. Then, the informational motive makes informative communication possible, by imposing a cost of overstating.

To see why only report r_1 may lead to E's disapproval (item iii), suppose that E does not approve after receiving either of two distinct reports which lead to different stock prices. As neither report is used to decide about project implementation, X faces no cost of overstating. Absent the informational motive, the impression motive induces X to always send the report that leads to a higher stock price. Hence, in equilibrium, at most one report can lead to rejection by E.

The informational motive implies that X wants I to have an accurate view of value v after receiving the report. It is useful to distinguish between two factors that together determine the effectiveness of communication. The first factor is the number of distinct reports N used. More distinct reports imply that I can receive more precise information. Let $\overline{N}$ denote the *maximum* number of reports over all equilibria of all continuation games for all a:

$$\overline{N}(\gamma, \lambda, h) \equiv \max \{N \in \mathbb{N} : \exists\ a \in \mathbb{N} \text{ such that } \Gamma(a) \text{ has a PBE with } N \text{ reports}\}$$

Lemma 2 shows that the informational motive facilitates using more distinct reports, whereas the persuasion and impression motives hinder this.

Lemma 2 *The maximum number of reports $\overline{N}(\gamma, \lambda, h)$ over all equilibria of all continuation games is weakly increasing in γ and weakly decreasing in λ and in h.*

The second factor that determines the effectiveness of communication is the *relative* precision of the reports. Given N, communication is most effective if all reports are equally precise, *i.e.,* if each report is sent by the same number of types. For a given a, communication is most effective in the separating equilibrium, provided it exists. Hence, if the separating equilibrium with $a = \overline{N}$ exists, this equilibrium yields the most accurate communication. However, if the separating equilibrium with $a = \overline{N}$ does not exist, a trade-off arises. Then, a higher number of reports can lead to less effective communication, if it comes at the expense of the relative precision of reports.[17] As shown below, this implies that X can prefer an equilibrium with $N < \overline{N}$ reports over all equilibria with $\overline{N}$ reports.

3.1 Informing versus Impressing

In this section, we discuss equilibria where external party E always approves exploration of the project, $d^E = 1$, so that the persuasion motive plays no role. This requires h to be sufficiently small. One of the novel insights of our paper is that if E's approval strategy imposes no constraints, executive X's incentive to overstate value v hardly leads to distortions in her reports. Instead, the impression motive weakens her incentives to acquire information. We discuss this result in two steps. First, we show that acquiring finer information, *i.e.*, a higher level of a, narrows the range of λ for which a separating equilibrium exists. Second, we show that X prefers to avoid distorting communication. This implies that the impression motive induces X to acquire more coarse information.[18]

Lemma 3 *Consider a continuation game* $\Gamma(a)$*. A separating equilibrium of* $\Gamma(a)$ *in which all types receive approval of* E *exists if and only if*

$$a \leq \overline{a} \equiv \left\lfloor \frac{\gamma}{1+\gamma} \frac{1+\lambda}{2\lambda} \right\rfloor \tag{8}$$

and

$$h \leq \overline{h}(a) \equiv \frac{\gamma}{8a^2} \tag{9}$$

Lemma 3 implies that acquiring too precise information makes sharing all information impossible. In other words, too much information acquisition necessitates distorting communication. To understand this result, note that a separating equilibrium requires that for all types k, the net benefit of overstating v by sending

[17] The two equilibria in Figure 2 illustrate this. Assuming that all reports lead to approval, expected project value is higher in the equilibrium in the top drawing than in the equilibrium in the bottom drawing, despite the higher number of reports used in the latter. Typically, in applications of cheap-talk models à la Crawford and Sobel (1982) without information acquisition, the number of reports sent in equilibrium is a sufficient measure of the effectiveness of communication.

[18] In Lemma 3, $\lfloor x \rfloor$ denotes the floor function, which gives the largest integer not exceeding x: $\lfloor x \rfloor = \max\{n \in \mathbb{Z} | n \leq x\}$. This ensures that $\bar{a}$ is an integer.

report r_{k+1} ("my type is $k+1$") instead of r_k should be negative. If all reports lead to E's approval, as ensured by (9), the cost of overstating (distorting insider I's implementation decision) is constant in type k, whereas the benefit (higher stock price s) is increasing in k. Consequently, type $k = a - 1$ has the largest incentive to deviate from truth-telling.

Crucially, as long as the informational motive is sufficiently important, X prefers exaggerating to a limited extent over exaggerating a lot. Expected project value decreases quadratically in the difference between the actual v and I's expectation of v after receiving the report. Hence, X abstains from making too large overstatements. Coarse information (*i.e.*, low a) only allows X to overstate heavily, whereas fine information enables X to overstate v by a limited amount. As a result, the maximum level of accuracy at which X can credibly communicate her type is limited, as given by (8). This maximum level is decreasing in the strength of the impression motive λ and increasing in the strength of the informational motive γ.

In a separating equilibrium, the number of distinct reports N equals a. Thus, $\overline{a}$ in equation (8) can be interpreted as an upper bound on the number of reports in all separating equilibria. Lemma 4 gives the conditions for which $\overline{a}$ is the actual upper bound of the number of reports in *all* equilibria, for $\gamma \leq 1$.[19].

Lemma 4 *Suppose $\gamma \leq 1$. Consider the maximum number of reports $\overline{N}$ over all equilibria of all continuation games:*

(i) if $\lambda \leq \frac{\gamma}{4+3\gamma}$ so that $\overline{a} \geq 2$, then $\overline{N}(\gamma, \lambda, h) = \overline{a}$;

(ii) if $\lambda \in \left(\frac{\gamma}{4+3\gamma}, \frac{1}{2}\right)$ so that $\overline{a} = 1$, then $\overline{N}(\gamma, \lambda, h) \leq 2$.

Case (i) implies that if the impression motive is not too strong, X cannot use more reports in any equilibrium than in the most informative separating equilibrium. Hence, increasing a beyond $\bar{a}$ does not lead to more messages, but only to distorted communication and, hence, to less informed decisions. To understand the intuition behind this result, suppose that $a > \bar{a} \geq 2$.[20] According to Lemma 3, the separating equilibrium of $\Gamma(a)$ does not exist because type $k = a - 1$ wants to deviate. Consequently, at least the top two types pool and send the same report. However, as types pool, sending N reports requires $a > N$. As a higher a further strengthens the incentive to exaggerate, this yields even more pooling. As a result, for $a > \bar{a}$ the maximum number of messages (or partitions) in the communication stage does not increase with a.

[19] We discuss the case $\gamma > 1$ after Corollary 1.

[20] We discuss case (ii) in Corollary 1 below.

Using Lemma's 1 to 4, we can characterize the information acquisition and communication strategy of X in equilibrium. If h is small such that E approves after receiving any report (in equilibrium), the communication continuation game is akin to a cheap talk game as in Crawford and Sobel (1982). Typically, such games are characterized by multiplicity of equilibria. Ours is no exception. In Proposition 1, we describe the equilibrium path of the sender-optimal equilibrium, *i.e.*, the equilibrium that is optimal for X.[21] In Section 4 we show that this equilibrium path is the unique forward induction outcome.

Proposition 1 *Let a^* be*

$$a^* \equiv \min\left\{\overline{a}, a^{opt}\right\} \tag{10}$$

Suppose that $h \leq \overline{h}(a^*)$, $\gamma \leq 1$ and $\lambda \leq \frac{\gamma}{4+3\gamma}$ so that $\overline{a} \geq 2$. The unique equilibrium outcome that maximizes the ex-ante *expected utility of X consists of accuracy $\alpha = a^*$ followed by the separating equilibrium of the continuation game.*

Proposition 1 presents two results. First, X's impression motive to exaggerate v in the hope of influencing the firm's stock price does not lead to distorted communication. Reports do not contain pooled information. Instead, the impression motive leads to less information acquisition. The number of distinct reports that X can credibly send is limited. These messages are most efficiently used when each message is equally likely to be sent in equilibrium. The separating equilibrium achieves this, at lowest cost. Second, X's choice of accuracy is either driven by the cost of information c or by the relative weight X attributes to the firm's stock price λ. If information is costly and the impression motive relatively weak, X chooses the level of accuracy that maximizes firm value, a^{opt}. If, instead, information is cheap and the impression motive strong, such that $\overline{a} < a^{opt}$, X's incentive to overstate v leads to underinvestment in information.

Proposition 1 assumes $h \leq \overline{h}(a^*)$, $\gamma \leq 1$ and $\bar{a} \geq 2$. We discuss these assumptions in reversed order. First, if $\bar{a} = 1$, the impression motive is so strong that no separating equilibrium exists for any a (Lemma 3) and that any non-pooling equilibrium of the continuation game is a semi-pooling equilibrium with $N = 2$ messages, which requires $a \geq 3$ (Lemma 4). Corollary 1 immediately follows.[22]

Corollary 1 *Let $\gamma \leq 1$, $h = 0$, and $\lambda \in \left(\frac{\gamma}{4+3\gamma}, \frac{1}{2}\right)$ so that $\overline{a} = 1$. The unique equilibrium outcome that maximizes* ex ante *expected utility of X:*

[21] Characterizing the full equilibrium instead of the equilibrium path requires the addition of the sender-optimal communication strategy following any (sub-optimal) choice of a. This adds little, in particular since for $a > \bar{a}$, it cannot be expressed in closed-form (but can be computed numerically).

[22] We assume $h = 0$ in Corollary 1 to ensure that E always approves, to allow for the proper comparison with Proposition 1.

(i) *either has accuracy* $\alpha = 1$ *followed by the pooling equilibrium of the continuation game, or*

(ii) *has accuracy* $\alpha \geq 3$ *followed by a semi-pooling equilibrium of the continuation game with* $N = 2$ *reports. Moreover,* $\alpha \to \infty$ *if and only if* $\lambda \nearrow \frac{\gamma}{2(2+\gamma)}$ *and* $c \to 0$*.*

If c is sufficiently small, X optimally chooses $a > \bar{a}$ to enable some information transmission. Moreover, α, *i.e.*, the optimal value of a, converges to infinity only if this is necessary for having two distinct reports in equilibrium, *i.e.*, if λ converges to $\frac{\gamma}{2(2+\gamma)}$ from below. Note that if the impression motive induces some pooling in equilibrium, pooling takes place at the top. This is typical in cheap-talk games.

Now, consider the case where $\gamma > 1$. The value of γ can be interpreted as the importance of the informational motive relative to the impression motive. Above we have shown that if the impression motive is relatively important ($\gamma \leq 1$), acquiring precise information leads to a semi-pooling equilibrium where reports are used too inefficiently. By acquiring less information, X reduces her incentive to overstate and, thereby, the need for pooling information to communicate credibly. For higher values of γ, the incentive to overstate firm value is weaker. This means that if in equilibrium some pooling occurs, reports are still used relatively efficiently, so that the cost of pooling is small. This allows for continuation game equilibria with a higher number of reports used efficiently enough to outperform the best-possible separating equilibrium. We cannot fully characterize the sender-optimal equilibria for $\gamma > 1$. However, numerical simulations show that the differences with the equilibrium described in Proposition 1 are small. We find that sender-optimal equilibria with $\alpha > \bar{a} \geq 2$ exist only if $\bar{a}$ is sufficiently small. In these equilibria, only the top two types pool, so that $\alpha = \bar{a} + 2$ and $N = \bar{a} + 1$. This holds even if $c = 0$. Hence, there can be a limited amount of distorted communication if $\gamma > 1$, but X's incentive to overstate v is still pre-dominantly reflected by less information acquisition.

Figure 3 depicts numerically computed α in a sender-optimal SEQ as a function of c for various levels of γ under the assumptions $\lambda = 0.1$ and $h = 0$. The graph for $\gamma = 1$ gives α as described in Proposition 1. If $\gamma = 1$ and c is sufficiently small, X optimally chooses $\alpha = \bar{a} = 2$. The graph also shows that for $\gamma = 3$ and low values of c, $\alpha = \bar{a} = 4$. For higher values of γ, *e.g.*, for $\gamma = 100$, the graph shows that for low values of c, $\alpha = 7$ whereas $\bar{a} = 5$. Hence $\alpha = \bar{a} + 2$, but only when c is sufficiently small. The graph also shows that α is decreasing in c.

Figure 4 also depicts α, but now for $\lambda = 0.01$. As compared to Figure 3, $\bar{a}$ is higher for each value of γ. Now, the only effect of an increase in γ is an increase in $\bar{a}$. In any sender-optimal equilibrium, $\alpha = a^*$, as in case of $\gamma \leq 1$ (Proposition 1).

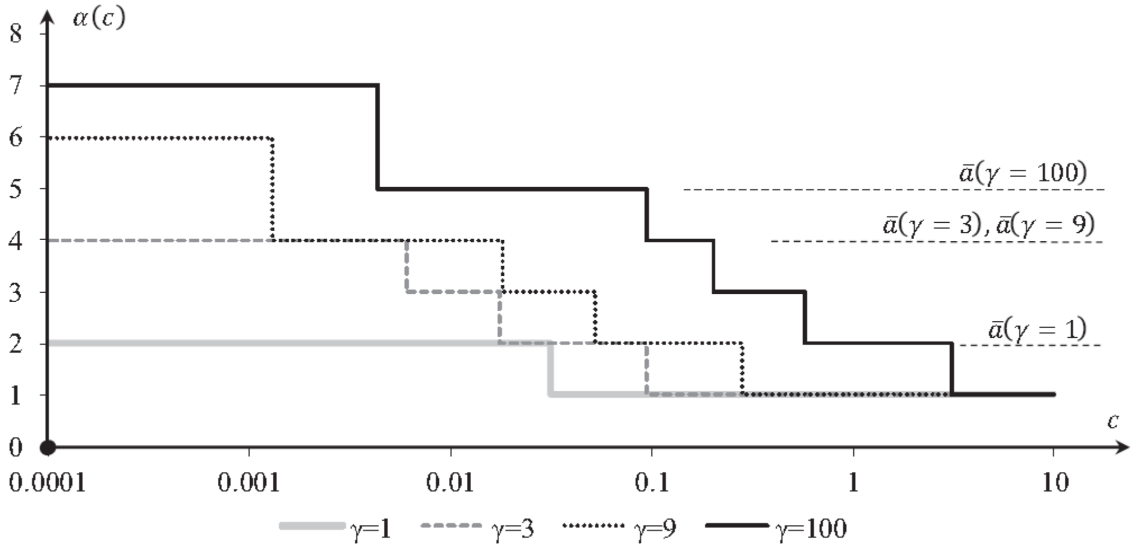

Figure 3: Optimal a as a function of c for $\gamma \in \{1, 2, 9, 100\}$, and $\lambda = 0.1$.

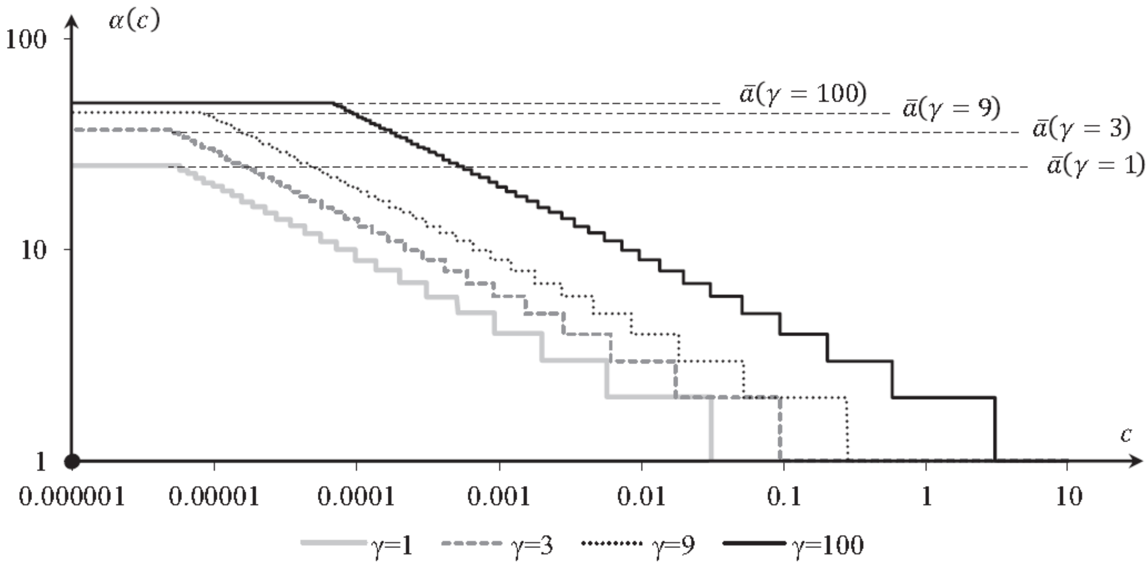

Figure 4: Optimal a as a function of c for $\gamma \in \{1, 2, 9, 100\}$, and $\lambda = 0.01$.

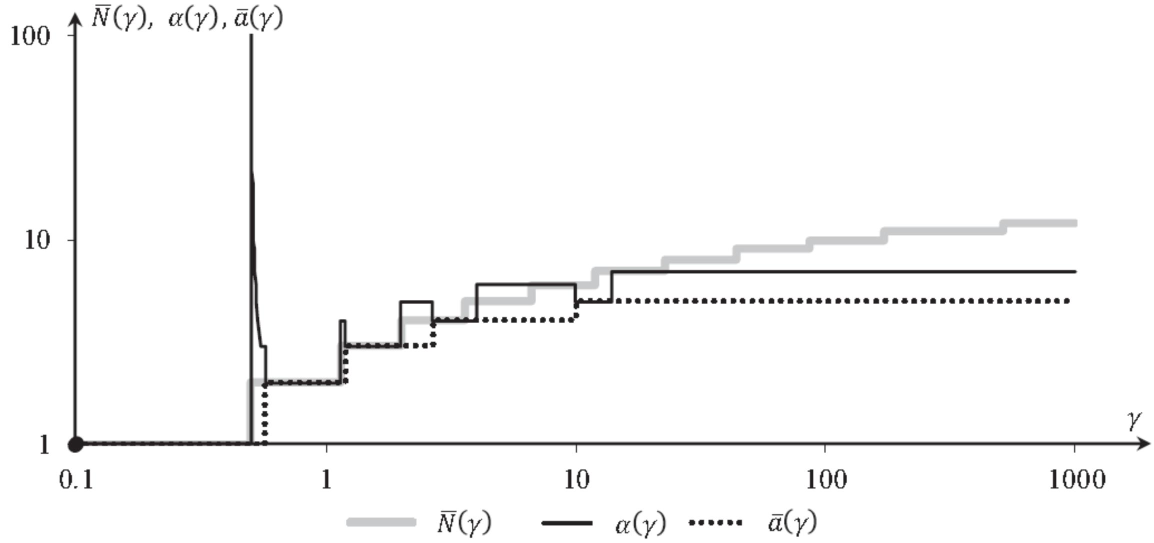

Figure 5: $\overline{N}$, $\bar{a}$, and optimal a as a function of γ for $\lambda = 0.01$, and $c = h = 0$.

Lastly, Figure 5 shows that α in a sender-optimal SEQ is non-monotonic in γ. Starting from $\alpha = N = \bar{a}$ for small values of γ, an increase in γ can sustain a semi-pooling equilibrium with $\alpha = \bar{a} + 2$ and $N = \bar{a} + 1$. If γ increases further, $\bar{a}$ increases by 1, which renders this separating equilibrium optimal.

Figure 5 also shows that when the information motive becomes more important, the maximum number of reports in any equilibrium, $\overline{N}$, increases. However, when γ is sufficiently high, it is not optimal for X to opt for an equilibrium with $\bar{N}$ reports. In these semi-pooling equilibria, reports are used inefficiently. X is better off acquiring less information (even when $c = 0$) which either prevents pooling altogether, or allows for very limited pooling (only the two highest types pool, and $\alpha = \bar{a} + 2$). Despite fewer reports, communication is more effective as (almost) all reports are equally precise.

The condition $h \leq \overline{h}(a^*)$ in Proposition 1 ensures external party E's approval in equilibrium, even if X sends r_1. If h slightly exceeds $\overline{h}(a^*)$, X can induce E to always approve the project by choosing $a < a^*$. The cost of reducing a is, as before, an implementation decision based on less information. Through this channel, the impression motive may lead to a further underinvestment in information collection. Alternatively, X accepts that E may reject the project, as analyzed in the next section.

3.2 Informing versus Persuading (and Impressing)

Now suppose that threshold h is sufficiently large, such that it is not possible or not optimal for X to choose an a that ensures E's unconditional approval of exploration of the project. Hence, a persuasion motive is present. At least one type does not receive E's approval in equilibrium. This hurts X, who prefers to minimize this probability. The optimal equilibrium outcome for X differs in two ways from the outcome stated in Proposition 1. It is possible that some reports are sent by multiple types and, therefore, the equilibrium of the continuation game is a semi-pooling equilibrium. As a result, the optimal value of a for X is affected by all parameters (c, h, γ, λ).

To highlight the effects of X's need to persuade E to get approval, we consider the case where c is infinitely small. Furthermore, we first assume that λ is also infinitely small. This eliminates the impression motive. Then, in the absence of the need to persuade ($h = 0$), the optimal outcome would be a choice of $a \to \infty$ followed by the separating equilibrium. The key result of this section is that $h > 0$ limits credible communication, which, in turn, induces X to acquire only a limited amount of information. Remarkably, if pooling occurs in equilibrium, we have pooling by the lowest types.

Lemma 5 shows the upper bound of the maximum number of reports in equilibrium.

Lemma 5 *Consider the maximum number of reports $\overline{N}$ over all equilibria of all continuation games. In the limit when $\lambda \to 0$, $\overline{N}$ has the following upper-bound:*

$$\overline{N} \leq \max\left\{\left(\frac{\gamma}{4h} + \sqrt{\frac{\gamma}{8h}} + \frac{1}{2}\right), \left(6\sqrt{\frac{\gamma}{8h}} - 2\right), 2\right\}$$

Lemma 5 implies that h limits communication, akin to the role of λ in Lemma 4. Given $\lambda \to 0$, each type who receives approval in equilibrium would like to reveal its type. However, for types that do not receive approval, the incentive to misreport is very strong, as revealing one's type implies losing the option value of the project. As only the first report receives no approval (Lemma 1), the incentive to misreport v is largest for type k_1, the highest type in the first partition that reports r_1 and gets no approval. To prevent this type from misreporting, sending r_2 must lead to a negative expected project value for type k_1. This requires that the second partition (*i.e.*, the number of types that send r_2) must be sufficiently wide, which, in turn, requires that the third partition is also sufficiently wide, and so on. As an increase in a brings $\mathrm{E}[v|k_1]$ closer to the border between the first and second partition, the width of the second partition must also increase in a.

In choosing a, X faces the following trade-off: a higher a leads to better project decisions when k is sufficiently large, but to worse decisions when k is small. Furthermore, X prefers to choose an a such that the first partition is small. The effect of a on the length of the first partition can be erratic, especially for small values of a.[23] This prevents us from making precise analytical statements concerning α, the optimal value of a, except that the persuasion motive induces X to choose a finite α even though she could observe v for free by taking $a \to \infty$.

Proposition 2 *Consider the case where $\lambda \to 0$ and $c \to 0$. An equilibrium that maximizes the* ex ante *expected utility of X always exists and is generically unique. There exists a finite number $\widehat{a}$ such that in this equilibrium, X chooses $a \leq \widehat{a}$.*

Proposition 1 states that for small values of h, X chooses a finite a, even if $c \to 0$. Proposition 2 states that this also holds for high values of h. The incentive for X to persuade E to approve limits communication, in particular after acquiring precise information. Acquiring less information improves the effectiveness of communication.

Proposition 2 does not exclude partially pooling equilibria, in contrast to Proposition 1. To understand why, consider a separating equilibrium in which $d^E(r_1) = 0$. As discussed above, a should be sufficiently small to prevent type $k = 1$ of X from sending r_2. The benefit of coarse information acquisition only realizes when $k = 1$. However, the cost of coarse information is poor decision-making by I, which realizes for all types $k > 1$. Hence, X may prefer to choose a value of a such that some pooling occurs. If so, pooling occurs pre-dominantly for low types. In the absence of the impression motive, the width of partitions is decreasing beyond the second partition. Table 1 lists the sender-optimal equilibria for various levels of h, obtained numerically, showing how the persuasion motive can yield pooling at the bottom.[24]

h	α	N	Communication
0.25	2	2	$\{1, 1\}$
0.1	12	3	$\{3, 5, 4\}$
0.09	8	4	$\{2, 3, 2, 1\}$
0.08	5	4	$\{1, 2, 1, 1\}$
0.019	28	16	$\{3, 5, 4, 3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$
0.018	18	14	$\{2, 3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$
0.006	50	38	$\{3, 5, 4, 3, 2, 1, 1, \ldots, 1\}$
0.005	35	31	$\{2, 3, 2, 1, 1, \ldots, 1\}$

Table 1. Values of α and N in sender-optimal SEQ for various levels of h, when $\lambda \to 0$, $c \to 0$, and $\gamma = 1$. Column 'Communication' shows the number of types sending identical reports (pools).

[23] For example, it is possible that for $a = 2$, $v \in \left[0, \frac{1}{2}\right]$ leads to $d^E = 0$, and that $a = 3$, $v \in \left[0, \frac{1}{3}\right]$ leads to $d^E = 0$, and for $a = 5$, $v \in \left[0, \frac{2}{5}\right]$ leads to $d^E = 0$.

[24] Table 1 also shows that the sender-optimal a is non-monotone in h, as a result of the discrete nature of a.

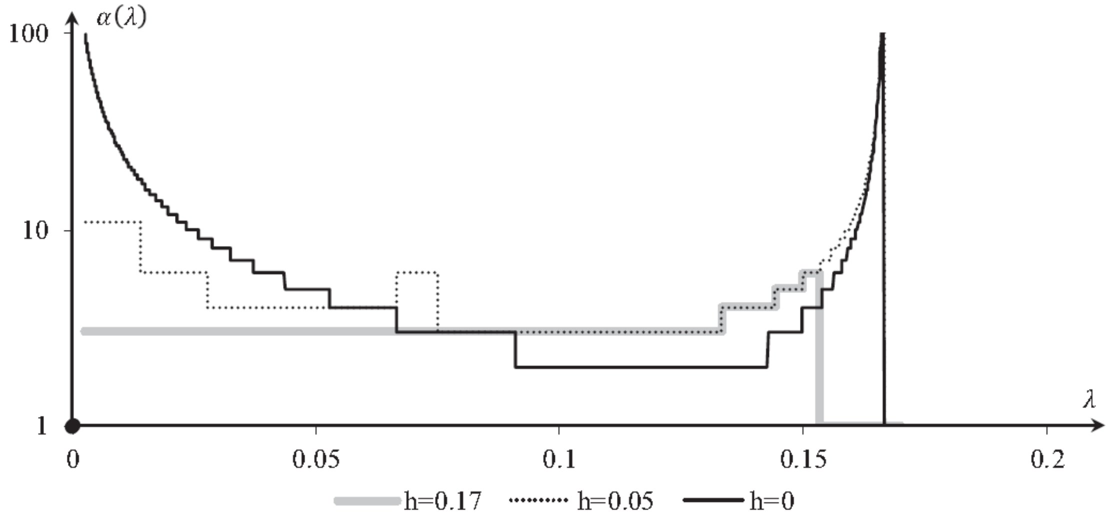

Figure 6: Optimal a as a function of λ for $h \in \{0, 0.05, 0.17\}$, $\gamma = 1$, and $c = 0$.

This highlights another difference between the effects of the impression motive and the persuasion motive. If the impression motive leads to distortions in communication (*e.g.*, if $\gamma > 1$), we obtain pooling at the top, as is typical in cheap-talk games. The persuasion motive, however, leads to pooling at the bottom, as it induces an incentive to overstate v that is stronger for low types than for high types. This also implies that adding an impression motive to the persuasion motive, hence allowing $\lambda > 0$, further restricts communication. Figure 6 depicts α in sender-optimal SEQ as a function of λ for various levels of h, given $c = 0$ and $\gamma = 1$, obtained numerically.

If $h = 0$, Proposition 1 implies that $\alpha = \bar{a}$, which is decreasing in λ provided $\bar{a} \geq 2$. For $\lambda > \frac{1}{7}$, $\bar{a} = 1$ and $N = 2$, and Corollary 1 states that $a \geq 3$ can be optimal. Figure 6 depicts this: α keeps increasing in λ up to the spike at $\lambda = \frac{1}{6}$. A positive value of h generally reduces α if λ is small, as illustrated by graphs for $h = 0.05$ and $h = 0.17$.[25] This shows that the persuasion motive and impression motive are imperfect substitutes in hindering communication and, consequently, in reducing optimal information acquisition.

For $h = 0.17$, E does not approve after report r_1. Hence, X needs to take into account the positive probability of being denied approval when determining α. If λ gets larger, the impression motive interferes with the motive to persuade E. For small values of λ, X responds by increasing a, but as λ becomes larger, X can no longer credibly send a report that secures approval. As a consequence, $\alpha = 1$ for large values of λ, and only the babbling equilibrium remains.

[25] Around $\lambda = 0.07$, the effect of h on α is not monotone. Here, choosing $a > \bar{a}$ allows X to increase the probability of approval if $h = 0.05$, but not if $h = 0.17$.

4 Forward Induction Refinement

Proposition 1 gives the sender-optimal equilibrium when h is small. It is well-known that cheap-talk models à la Crawford and Sobel (1982) generally have multiple equilibria. In particular, if any equilibrium with influential communication exists, the equilibrium without influential communication (so called 'babbling' equilibrium) also exists. In our model, the communication game is preceded by an information acquisition stage. This enables us to apply a forward induction refinement. Loosely speaking, forward induction assumes that in equilibrium past actions have been rational. Proposition 3 shows that the sender-optimal equilibrium is the *unique* equilibrium selected by the forward induction refinement, given $c > 0$ and the conditions under which Proposition 1 holds.

Proposition 3 *Suppose that $c > 0$, $h \leq \overline{h}(a^*)$, $\gamma \leq 1$ and $\lambda \leq \frac{\gamma}{4+3\gamma}$ so that $\overline{a} \geq 2$. Then, Proposition 1 characterizes the unique forward induction equilibrium outcome.*

To illustrate how forward induction selects a unique equilibrium, suppose $a^* \geq 2$ and $h \leq \overline{h}(a^*)$. If X has chosen $a = 1$, no information can be conveyed in the communication game. Now suppose $a = 2$. In the continuation game $\Gamma(2)$, a pooling equilibrium as well as a separating equilibrium exist. However, for $c > 0$, the pooling equilibrium does not satisfy forward induction: choosing $a = 1$ yields a higher payoff to X than choosing $a = 2$ followed by pooling. Hence, after observing $a = 2$, forward induction implies that neither E nor I expect themselves to play the pooling equilibrium in $\Gamma(2)$. Therefore, $a = 2$ followed by pooling does not satisfy the forward induction refinement.

This line of reasoning extends to higher levels of information acquisition too. Suppose that $a^* \geq 3$ and that X has chosen $a = 3$. Again, pooling and semi-pooling equilibria exist in $\Gamma(3)$, but $a = 2$ followed by the separating equilibrium would yield a higher payoff to X than $a = 3$ followed by pooling or partially pooling. Consequently, for $a = 3$, none of the (partially) pooling equilibria satisfy forward induction. This process of eliminating equilibria ends when $a = a^* = \min\{\overline{a}, a^{opt}\}$. Choosing $a > a^*$ reduces X's payoff either because acquiring more information is too expensive ($a > a^{opt}$) or because more information does not lead to more informative communication ($a > \overline{a}$). Hence, the forward induction refinement selects the sender-optimal equilibrium as the unique equilibrium outcome.[26]

[26] If $h > \bar{h}(a^*)$, forward induction may not select a unique equilibrium. If message r_1 does not lead to approval, an increase in a affects the probability of receiving approval as well as the possible equilibria of $\Gamma(a)$. As a result, after choosing some $a > 2$, there can be more than one equilibrium of $\Gamma(a)$ that leads to a higher payoff for X than the highest possible payoff after

The same argument implies that forward induction excludes the babbling equilibrium outcome whenever there exists an a and an equilibrium of $\Gamma(a)$ that yields a higher payoff to X than choosing $a = 1$.

Corollary 2 *If for some $\hat{a} > 1$, the continuation game $\Gamma(\hat{a})$ has an equilibrium that yields a higher payoff to X than $\Gamma(1)$, then $\alpha \leq \hat{a}$ followed by the pooling equilibrium of $\Gamma(a)$ is not a forward induction equilibrium outcome.*

For forward induction to be applicable, it is crucial that information acquisition is observable. As discussed in Footnote 13, in the context of the situations of our model, this assumption is often met. In other settings, however, information acquisition is not observable. See Argenziano *et al.* (2016) for a thorough discussion of 'covert' information acquisition.

5 Transparency

This section discusses the role of transparency in our model. In the previous sections, communication by X was fully transparent: insider I, external party E, and public P received the same report. Firms can also choose to be less transparent. Proposition 4 states how much information executive X collects and what she communicates if she can send a private report to I and another private report to E and P.[27]

Proposition 4 *Suppose X sends private report r^I to I and private report r^E to E and P. An equilibrium in which X truthfully reports its type to I always exists. In this equilibrium:*

(i) *I chooses $d^I = 1$ if $z < \mathrm{E}[v|k]$ and $d^I = 0$ otherwise;*

(ii) *the report of X to E and P is uninformative;*

(iii) *E chooses $d^E = 1$ if $h < \frac{\gamma}{24}\left(4 - \frac{1}{a^2}\right)$ and $d^E = 0$ otherwise;*

(iv) *X chooses $a = a^{PR}$, where*

$$a^{PR} = \begin{cases} a^{opt}, & \text{if } h < \frac{\gamma}{8} \\ \max\left\{a^{opt}, \frac{1}{2}\sqrt{\frac{\gamma}{\gamma - 6h}}\right\}, & \text{if } \frac{\gamma}{8} < h < \frac{\gamma}{6} \text{ and } c \leq \bar{c} = \frac{2h}{\sqrt{\frac{\gamma}{\gamma - 6h}} - 2} \\ 1, & \text{if } h > \frac{\gamma}{6} \text{ or if } \frac{\gamma}{8} < h < \frac{\gamma}{6} \text{ and } c > \bar{c} \end{cases}$$

choosing $a - 1$. Forward induction then selects all these equilibria of $\Gamma(a)$, implying that there can be no unique forward induction equilibrium outcome.

[27] We have also considered the case where on top of a public report, X can send a private report to I. The equilibrium presented in Proposition 4 carries over to that setting.

Non-transparency induces complete information sharing within the firm, but renders any communication with outsiders non-informative. If approval is provided nonetheless, X chooses the level of information acquisition that maximizes project value. If, instead, E does not approve in the absence of an informative report, information acquisition is worthless.

The strategy of I, item (i), is the same as before. As r^I is only received by I, only the informational motive affects the content of r^I. The preference alignment of X and I allows for sharing all information. Item (ii) in Proposition 4 is a direct consequence of the misalignment of preferences between X on the one hand and E and P on the other. As the decision by I is not affected by r^E, the informational motive is absent in determining r^E. In isolation, the persuasion motive and the impression motive obstruct influential communication.

Anticipating I's strategy, E infers that the expected project value is given by (6). Lacking any further information, item (iii) implies that E approves if the expected project value is greater than threshold h. Clearly, E is more willing to approve if accuracy a is higher. A more accurate information system implies that I makes a better decision, leading to higher expected project value. The condition in item (iii) shows that, independent of a, E never approves if $h > \frac{\gamma}{6}$, whereas E always approves if $h < \frac{\gamma}{8}$.

Item (iv) in Proposition 4 shows that if X is not constrained by E's approval decision, she chooses the level of accuracy that maximizes firm value, a^{opt}. If $\frac{\gamma}{8} < h < \frac{\gamma}{6}$, however, X may need to increase a to meet E's approval constraint given in item (iii). In other words, X increases the level of accuracy to persuade E to approve. Of course, increasing a is optimal for X only if the cost of information is sufficiently small, $c \leq \bar{c}$. Finally, if E never approves ($h > \frac{\gamma}{6}$), X sets $a = 1$, as information acquisition would be a pure waste.[28] Remarkably, Proposition 4 shows that information acquisition is not affected by the impression motive, λ. The ability to send private reports to insiders and outsiders decouples the informational and impression motives, rendering the latter irrelevant for internal decision-making.

The main take-away is that lack of transparency works well for internal decision-making, but poorly for external decision-making. In the absence of a need to persuade, information acquisition and the implementation decision maximize firm value. If the impression motive is sufficiently strong, adopting transparent reporting would reduce firm value. At the same time, the outsider's approval decision is sub-optimal, as it cannot be based on the information on v that is available inside the firm. E may approve projects that he would have rejected if informed fully.

[28] Other equilibria may exist. However, given the preference alignment between X and I, this is a natural equilibrium to consider. Furthermore, similar to Proposition 3, we can show that the equilibrium described in Proposition 4 is selected by the forward induction refinement if $c > 0$ and $h < \frac{\gamma}{8}$.

Hence, it is not surprising that often firms are reluctant to give in to calls for more transparency.

Yet, if maximizing firm value does not lead to outsider's approval, the firm is forced to alter its information system to persuade E. One way is to over-invest in information acquisition. If that is not sufficient, the firm needs to commit to transparent communication, despite the negative consequences for internal decision-making. Hence, from the firm's perspective, transparency is a necessary evil.[29]

6 Discussion

We view this paper as a first step towards an integral theory of the design and use of (formal and informal) information systems in firms. Various subsequent steps are required to capture all aspects of an information system highlighted by Cyert and March (1963). We have focused on how the main motives of informing, persuading, and impressing affect information acquisition and communication. To do so, we have assumed a given, static decision-making procedure, a given set of stakeholders, a given, simplified incentive structure, and observable expenditures on information acquisition. The effects of each of these assumptions deserve to be explored. Below, we discuss several implications and extensions of our results.

Information often gets stored or processed in categories, even if the underlying data is continuous. Employee performance evaluation forms often have only three or five distinct categories of performance. Credit rating agencies assign firms to one of a limited number of categories based on their assessment of the (continuous) probability of default. Our model provides a rationale. If information is to be stored or processed in a too precise manner, this strengthens potential incentives to manipulate information. By using broad categories, more reliable information can be obtained and retained.

We have shown that if firms need to persuade outsiders, voluntary commitment to transparent reporting can be optimal. If such commitment is difficult to sustain, regulation that requires transparency can be beneficial for companies. At the same time, such regulation hurts companies that do not need to persuade outsiders and prefer private reporting to avoid the negative consequences of the impression motive. Hence, regulation that imposes transparency can have differential effects on firms. Relatedly, we have shown that the possibility to manipulate information can backfire if firms need to persuade outsiders. If transparent reporting does not

[29] Durnev and Kim (2005) find that in countries where investor protection is low, so that investors need more persuasion to supply funds, the relation between firms' valuation and transparent reporting is stronger. Similarly, Lang et al. (2012) show that firm-level transparency matters more for firms' valuation if investor uncertainty is higher.

suffice, firms can attempt to make their reports verifiable, for instance through hiring external auditors. However, this increases the cost of information acquisition, which also reduces the amount of information available.

Our model can also be used to illustrate the interaction between the decision-making process and the information system. In our model, the final decision on the project is made by I. At first glance, there is no reason in our model for X to delegate the final decision. Since the preferences of X and I are aligned, I would be willing to share his information about z with X. Note, however, that if we had assumed that X instead of I would make the final decision, X would lose the possibility of sending influential messages to E. Delegation of this decision, in combination with transparency in communication, thus creates the possibility to persuade outsiders.

Similarly, our model can illustrate how incentive pay affects the use of a firm's information system. Above, we have shown that strong stock-based incentive pay for executives hinders communication to outsiders. Now suppose that X, but not I, receives a long-term incentive plan (LTIP), which makes that X cares about the stock price that arises *after* public P has observed I's decision d^I. Under private reporting, a positive implementation decision is interpreted by P as a positive signal regarding firm value. Thus the LTIP gives X an incentive to overstate v in communicating with I. By reducing the alignment of interests between X and I, the LTIP hinders internal communication, which in turn negatively affects information acquisition.[30]

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[30] We do not explicitly model a rationale for providing incentive pay to executives. Benmelech et al. (2010) analyze the effects of various incentive pay structures on executives' effort and communication of exogenously obtained information on growth prospects.

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Appendix

Proof of Lemma 1.
Consider an equilibrium of the continuation game $\Gamma(a)$. Using (3) and noting that s is not affected by d^I, the equilibrium strategy of I is:

$$\delta^I(r) = 1 \text{ if } \mathrm{E}[v|r] > z \text{ and } \delta^I(r) = 0 \text{ otherwise} \tag{11}$$

Hence, $\mathrm{E}\left[d^I|r\right] = \mathrm{E}[v|r]$ and $\mathrm{E}\left[z|r, d^I = 1\right] = \frac{1}{2}\mathrm{E}[v|r]$.

Consider type k. Substituting (1), (2), and (4) into (3) and using (11), we write the expected utility of X of type k from reporting r as follows:

$$\begin{aligned} \mathrm{E}\left[u^X|k,r\right] &= (1-\lambda)\left(v_k + \gamma\,\mathrm{E}[v|r]\left(v_k - \tfrac{1}{2}\mathrm{E}[v|r]\right)\delta^E\right) \\ &\quad +\lambda E[v|r]\left(1 + \tfrac{\gamma}{2}\mathrm{E}[v|r]\,\delta^E\right) - c(a-1) \end{aligned} \tag{12}$$

Suppose two reports r_1 and r_2 with $\mathrm{E}[v|r_2] \geq \mathrm{E}[v|r_1]$ are used in an equilibrium. Then, it must be one of the four cases below:

1. If $\delta^E(r_1) = \delta^E(r_2) = 0$ then it must be that $\mathrm{E}[v|r_2] > \mathrm{E}[v|r_1]$ (otherwise the reports must be identical) so that

   $$\mathrm{E}\left[u^X|k,r_2\right] - \mathrm{E}\left[u^X|k,r_1\right] = \lambda\left(\mathrm{E}[v|r_2] - \mathrm{E}[v|r_1]\right) > 0$$

   for all k. Then, no type uses r_1, a contradiction.

2. If $\delta^E(r_1) = \delta^E(r_2) = 1$, then the difference $D = \mathrm{E}\left[u^X|k,r_2\right] - \mathrm{E}\left[u^X|k,r_1\right]$ is strictly increasing in v_k:

   $$D = \gamma\left((1-\lambda)\,v_k - \tfrac{(1-2\lambda)}{2}\left(\mathrm{E}[v|r_2] + \mathrm{E}[v|r_1]\right) + \lambda\right)\left(\mathrm{E}[v|r_2] - \mathrm{E}[v|r_1]\right)$$

   and, hence, in k as well (the so-called 'single-crossing' property). As a result, if some type k_2 prefers reporting r_1 to reporting r_2, all types $k > k_2$ do so as well. Thus, the set of types reporting r_n is necessarily a set of consecutive types $\{(k_{n-1}+1), \ldots, k_n\}$.

3. If $\delta^E(r_2) = 1$ and $\delta^E(r_1) = 0$, then then the difference $D = \mathrm{E}\left[u^X|k,r_2\right] - \mathrm{E}\left[u^X|k,r_1\right]$ is strictly increasing in k:

   $$D = \gamma\,\mathrm{E}[v|r_2]\left((1-\lambda)\,v_k - \tfrac{(1-2\lambda)}{2}\mathrm{E}[v|r_2]\right) + \lambda\left(\mathrm{E}[v|r_2] - \mathrm{E}[v|r_1]\right)$$

   The single-crossing property holds and combining it with the result from case 1, we conclude that the set of types reporting r_1 is a set of lowest types $\{1, \ldots, k_1\}$.

4. If $\delta^E(r_2) = 0$ and $\delta^E(r_1) = 1$, then $\mathrm{E}\left[u^X|k, r_2\right] - \mathrm{E}\left[u^X|k, r_1\right]$ is decreasing in k, types reporting r_2 are lower than types reporting r_1 so that $\mathrm{E}[v|r_2] < \mathrm{E}[v|r_1]$, a contradiction.

We conclude that each report r_n is sent by a subset of types $\{(k_{n-1}+1), \ldots, k_n\}$, which is fully characterized by the set of marginal types $\{k_n\}$, $n = 1, \ldots, N$, with $k_n \geq k_{n-1}$ and $k_{n-1} \equiv 0$. In this case,

$$\mathrm{E}[v|r_n] = \tfrac{1}{2}\left(v_{k_{n-1}+1} + v_{k_n}\right) = \tfrac{1}{2a}\left(k_{n-1} + k_n\right) \tag{13}$$

Moreover, the single-crossing property implies that if type $k = k_n$ prefers reporting r_n to reporting r_{n+1}, all types $k \leq k_n$ also prefer r_n to $r_{n+\tau}$ for any $\tau \geq 1$, and if type $k = k_{n-1}+1$ prefers reporting r_n to reporting r_{n-1}, all types $k \geq k_{n-1}+1$ also prefer r_n to $r_{n+\tau}$ for any $\tau \leq -1$ (the absence of local deviations implies the absence of global deviations). Thus, the necessary incentive compatibility constraints

$$\mathrm{E}\left[u^X|k_n, r_n\right] - \mathrm{E}\left[u^X|k_n, r_{n+1}\right] \geq 0 \tag{14}$$

$$\mathrm{E}\left[u^X|k_n+1, r_{n+1}\right] - \mathrm{E}\left[u^X|k_n+1, r_n\right] \geq 0 \tag{15}$$

are also the sufficient equilibrium conditions. This proves item (i) of the proposition.

It follows from cases 1 and 3 above that only one report, namely r_1, may lead to no approval, so that $\delta^E(r_n) = 1$ for all $k \geq 2$. This proves item (iii) of the proposition.

Next, consider the approval decision d^E by E. Using (5) and I's strategy, the optimal strategy for E is to choose $d^E = 1$ if and only if $\mathrm{E}\left[u^E|r\right] > 0$. Hence, $\delta^E(r) = 1$ if $\frac{\gamma}{2}\left(\mathrm{E}[v_k|r]\right)^2 > h$, and $\delta^E(r) = 0$ otherwise, which can be written as follows:

$$\delta^E(r_n) = 1 \text{ if } \tfrac{\gamma}{8a^2}\left(k_{n-1} + k_n\right)^2 > h, \text{ and } \delta^E(r_n) = 0 \text{ otherwise} \tag{16}$$

To prove item (ii), we rewrite the ICC in (14) for $\delta^E(r_n) = 1$ and $\delta^E(r_1) = 0$ correspondingly

$$\begin{aligned}&\gamma\left(\tfrac{1}{2}(1-2\lambda)\left(\mathrm{E}[v|r_n] + \mathrm{E}[v|r_{n+1}]\right) - (1-\lambda)v_k\right) - \lambda \geq 0, \text{ and}\\ &\tfrac{\gamma}{2}(1-2\lambda)\left(\mathrm{E}[v|r_{n+1}]\right)^2 - \tfrac{\lambda}{2a}\left(k_{n+1} - k_{n-1}\right) - (1-\lambda)\gamma\,\mathrm{E}[v|r_{n+1}]\,v_k \geq 0\end{aligned}$$

These inequalities never hold when $\lambda \geq \frac{1}{2}$. Hence, if $\lambda \geq \frac{1}{2}$, the pooling equilibrium is the unique equilibrium of the continuation game, which ends the proof. ■

Proof of Lemma 2.
To determine the properties of $\overline{N}(\gamma, \lambda, h)$, it is helpful to consider an auxiliary function $\underline{a}(N, \lambda, \gamma, h)$, defined as the lowest value of a for which the continuation game $\Gamma(a)$ has an equilibrium with N reports:

$$\underline{a}(N, \lambda, \gamma, h) = \min\{a \in \mathbb{N} : \Gamma(a) \text{ has a PBE with } N \text{ reports}\}$$

If, for some N, all equilibria for all a have less than N reports, we set $\underline{a} = \infty$. Function $\underline{a}(N, \lambda, \gamma, h)$ is monotonically increasing in N (as is shown below), so that $\bar{N}$ can be written as

$$\overline{N}(\lambda, \gamma, h) = \max\{N \in \mathbb{N} : \underline{a}(N, \lambda, \gamma, h) < \infty\} \tag{17}$$

We show below that $\underline{a}(N, \lambda, \gamma, h)$ is monotonically increasing in λ and h, and is decreasing in γ. Using this monotonicity property of $\underline{a}$, the desired monotonicity property of $\overline{N}$ can be shown as follows (for brevity, we omit unnecessary arguments form argument lists). Take γ and γ' such that $\gamma' < \gamma$, so that $\underline{a}\left(\overline{N}(\gamma), \gamma\right) \leq \underline{a}\left(\overline{N}(\gamma), \gamma'\right)$ and $\underline{a}\left(\overline{N}(\gamma)+1, \gamma\right) \leq \underline{a}\left(\overline{N}(\gamma)+1, \gamma'\right)$. By the definition of $\overline{N}(\gamma)$, $\underline{a}\left(\overline{N}(\gamma), \gamma\right) < \underline{a}\left(\overline{N}(\gamma)+1, \gamma\right) = \infty$. Therefore,

$$\underline{a}\left(\overline{N}(\gamma), \gamma\right) \leq \underline{a}\left(\overline{N}(\gamma), \gamma'\right) \leq \underline{a}\left(\overline{N}(\gamma)+1, \gamma\right) = \underline{a}\left(\overline{N}(\gamma)+1, \gamma'\right) = \infty$$

Then, if $\underline{a}\left(\overline{N}(\gamma), \gamma'\right) < \infty$ then $\underline{a}\left(\overline{N}(\gamma), \gamma'\right) \leq \underline{a}\left(\overline{N}(\gamma)+1, \gamma'\right) = \infty$ implies $\overline{N}(\gamma') = \overline{N}(\gamma)$, and if $\underline{a}\left(\overline{N}(\gamma), \gamma'\right) = \infty$ then it implies $\overline{N}(\gamma') \leq \overline{N}(\gamma) - 1$. The proof of monotonicity of $\overline{N}$ w.r.t. λ and h is similar and is, therefore, omitted.

The remaining part of the proof shows the monotonicity of $\underline{a}$. We define l_n as the number of types sending report r_n:

$$l_n \equiv k_n - k_{n-1} \tag{18}$$

Using (13) we express the ICCs (14) and (15) as follows, respectively:

$$(l_{n+1} - l_n) \geq G(k_n, a) + \left(1 - \delta^E(r_n)\right)\underline{H} \tag{19}$$

$$(l_{n+1} - l_n) \leq G(k_n, a) + \tfrac{4(1-\lambda)}{(1-2\lambda)} + \left(1 - \delta^E(r_n)\right)\overline{H} \tag{20}$$

where

$$G(k, a) = \frac{4\lambda(\gamma k + a) - 2\gamma(1-\lambda)}{\gamma(1-2\lambda)} \tag{21}$$

$$\begin{aligned}
\underline{H} &= \tfrac{(2k_n - l_n)}{(l_{n+1}+l_n)}\left(2\tfrac{k_n-(1-\lambda)}{(1-2\lambda)} + l_n\right) > 0 \\
\overline{H} &= \tfrac{(2k_n - l_n)}{(l_{n+1}+l_n)}\left(2\tfrac{k_n+(1-\lambda)}{(1-2\lambda)} + l_n\right) > \underline{H} > 0
\end{aligned}$$

Using (16), we write the equilibrium condition $\delta^E(r_2) = 1$ as

$$k_1 + k_2 > 2a\sqrt{\frac{2h}{\gamma}} \tag{22}$$

Consequently, conditions (19), (20), and (22) constitute all the necessary and sufficient conditions that any arbitrary sequence $\{l_n\}$ of size N must satisfy to represent a PBE of $\Gamma(k_N)$ with N reports. Conditions (19) and (20) can jointly be written as the following double inequality

$$l_L(k_n, l_n, a, n) \leq l_{n+1} \leq l_H(k_n, l_n, a, n) \tag{23}$$

In the following Lemma, we establish properties of l_L and l_H that we use in the rest of the proof.[31]

Lemma 6 *Functions $l_L(k, l, a, n)$ and $l_H(k, l, a, n)$ have the following properties:*

(i) For $n \geq 2$, for $n = 1$ and $l > 2\sqrt{\frac{2h}{\gamma}}a$, and for $n = 1$ and $l < 2\sqrt{\frac{2h}{\gamma}}a$, l_L and l_H are increasing in (k, l, a, λ), decreasing in γ, and independent of h;

(ii) For for $n = 1$ and $l = 2\sqrt{\frac{2h}{\gamma}}a$, $l_L(l, l, a, 1)$ and $l_H(l, l, a, 1)$ are discontinuous, increasing in (h, a), and decreasing in (l, γ);

(iii) For $n \geq 2$, $l_H - l_L > 4$, and $l_H(l, l, a, 1) - l_L(l, l, a, 1) > 4$;

(iv) $l_H(l, l, a, 1) - l_L(l + 1, l + 1, a, 1) > 2$;

(v) Let, for some integers a, $l_1 \geq 1$, and $l_2 > l_1$, it holds that $l_H(l_1, l_1, a, 1) < l_2$. Then, there exists an integer $x \geq 1$ so that $l_L(l_1 + x, l_1 + x, a, 1) \leq l_2 - x \leq l_H(l_1 + x, l_1 + x, a, 1)$.

We also define $\underline{l}$ as the smallest integer for l_{n+1} that satisfies (23):

$$\underline{l} \equiv \lceil l_L \rceil = \min\{z \in \mathbb{Z} : z \geq l_L\}$$

To prove that $\underline{a}(N)$ increases in N we take an arbitrary N such that $\underline{a}(N) < \infty$ and assume, to the contrary, that $\underline{a}(N+1) < \underline{a}(N)$. Consider an equilibrium of the continuation game $\Gamma(a)$ for $a = \underline{a}(N+1)$ with $N + 1$ reports. We will show that there exists an $a' < \underline{a}(N+1) < \underline{a}(N)$ and there exists an equilibrium of the continuation game $\Gamma(a')$ with N reports. This will contradict the definition of $\underline{a}(N)$. We construct this equilibrium iteratively in the following steps. We use the iteration number t as a superscript (t).

[31] The proof of this Lemma uses standard extensive algebraic transformations, and is available upon request.

Step 1. We begin with the set $\{l_n\}$, $n \in \{1, \ldots, N+1\}$, from the equilibrium for $a = \underline{a}(N+1)$ with $N+1$ reports. At first iteration, we set $l_n^{(1)} = l_n$ for $n \in \{1, \ldots, N\}$ (we just truncate $\{l_n\}$ at N) to obtain $\left\{l_n^{(1)}\right\}$, and proceed to Step 2.

Step 2. Using $k_{n+1}^{(t)} = l_{n+1}^{(t)} + k_n^{(t)}$, we obtain $\left\{k_n^{(t)}\right\}$ and $a^{(t)} = k_N^{(t)}$. The set $\left\{k_n^{(t)}\right\}$ may represent no equilibrium because (23) may fail. However, condition (22) holds by its monotonicity. We proceed to Step 3.

Step 3. If the sufficient equilibrium condition (23) holds for all $n \in \{1, \ldots, N\}$, then we have constructed an equilibrium with N reports for $a < \underline{a}(N)$. Otherwise, by the monotonicity of l_L and l_H, it must be the condition $l_{n+1}^{(t)} \leq l_H\left(k_n^{(t)}, l_n^{(t)}, a^{(t)}, n\right)$ that fails (the other condition $l_L\left(k_n^{(t)}, l_n^{(t)}, a^{(t)}, n\right) \leq l_{n+1}^{(t)}$ may only become weaker). We proceed to Step 4 to adjust $\left\{l_n^{(k)}\right\}$.

Step 4. If (23) fails for $n = 2$, we proceed to step 5. Otherwise, let $n^* \geq 3$ be the lowest n for which (23) fails. We set $l_n^{(t+1)} = l_n^{(t)}$ for $n < n^*$ and $l_n^{(t+1)} = \underline{l}\left(k_n^{(t+1)}, l_n^{(t+1)}, a^{(t)}, n\right)$ for $n \geq n^*$ and proceed to the next iteration $(t+1)$ in Step 2. By the monotonicity of $\underline{l}$, $l_n^{(t+1)} \leq l_n^{(t)}$ for all n, and condition (22) holds.

Step 5. If $l_2^{(t)} > l_H\left(l_1^{(t)}, l_1^{(t)}, a^{(t)}, 1\right)$, then, according to item (v) of the Lemma), there exists an integer $x \geq 1$ such that $l_L\left(l_1^{(t)} + x, l_1^{(t)} + x, a^{(t)}, 1\right) \leq l_2^{(t)} - x \leq l_H\left(l_1^{(t)}, l_1^{(t)}, a^{(t)}, 1\right)$. We set $l_1^{(t+1)} = l_1^{(t)} + x$ and $l_2^{(t+1)} = l_2^{(t)} - x$. As a result, $k_2^{(t+1)} = k_2^{(t)}$, $k_1^{(t+1)} > k_1^{(t)}$ so that condition (22) holds by its monotonicity. Then, we set $l_n^{(t+1)} = \underline{l}\left(k_n^{(t+1)}, l_n^{(t+1)}, a^{(t)}, n\right)$ for $n \geq 3$ and proceed to the next iteration $(t+1)$ in Step 2. By the monotonicity of $\underline{l}$, $l_n^{(t+1)} \leq l_n^{(t)}$ for all $n \geq 2$.

After each iteration, either $a^{(t+1)} < a^{(t)}$ or $l_1^{(t+1)} > l_1^{(t)}$, so that after a finite number of iterations, we obtain an equilibrium with N reports following a choice of $a < \underline{a}(N)$, a contradiction.

To prove that $\underline{a}$ increases h, we take arbitrary h, $h' > h$, and N such that $\underline{a}(N, h') < \infty$. Consider an equilibrium of $\Gamma(a)$ for $a = \underline{a}(N, h')$. We will show that there exists an $a' < a = \underline{a}(N, h')$ and the corresponding equilibrium of $\Gamma(a')$ with N reports. As the first iteration, we take $l_n^{(1)} = l_n$ from the equilibrium for $a = \underline{a}(N, h')$ and proceed to Step 2 above (we use h in all the steps). Since $h < h'$, due to the monotonicity, only the condition $l_{n+1}^{(t)} \leq l_H$ may fail in Step 3 so that the iterative procedure necessarily results in an equilibrium for h and $a' \leq a = \underline{a}(N, h')$ with N reports. This implies that $\underline{a}(N, h') \leq \underline{a}(N, h)$. Similarly, the monotonicity of $\underline{a}$ in γ (by taking $\underline{a}(N, \gamma') < \infty$ and $\gamma > \gamma'$) and in λ (by taking $\underline{a}(N, \lambda') < \infty$ and $\lambda' > \lambda$) can be shown. This ends the proof. ■

Proof of Lemma 3.
For $\lambda \geq \frac{1}{2}$, no separating equilibrium exists according to Lemma 1. Since $\bar{a} \leq 1$ in this case, the Lemma holds for $\lambda \geq \frac{1}{2}$. In the reminder of the proof, $\lambda < \frac{1}{2}$ is assumed.

In any equilibrium, the strategy of E is given by (16). In a separating PBE, $\mathrm{E}[v|r_k] = v_k$. Hence, $\delta^E(r_k) = 1$ if

$$h < \tfrac{\gamma}{2} \mathrm{E}\left[(v_k)^2 | r_k\right] = \tfrac{\gamma}{2}(v_k)^2$$

and $\delta^E(r_k) = 1$ for all k when $h < \frac{\gamma}{2}(v_1)^2 = \overline{h}(a)$. In this case, the sufficient equilibrium conditions (19) and (20) become:

$$2\lambda k \leq (1-\lambda) - 2a\tfrac{\lambda}{\gamma} \text{ and } 2\lambda k \geq -(1-\lambda) - 2a\tfrac{\lambda}{\gamma}$$

The second ICC always holds whereas the first ICC holds for all $k = 1, \ldots, (a-1)$ if and only if it holds for $k = a - 1$, *i.e.*, if $a \leq \frac{\gamma}{1+\gamma}\frac{(1+\lambda)}{2\lambda}$. For integer a it is equivalent to $a \leq \bar{a}$, which ends the proof. ■

Proof of Lemma 4.
Consider a PBE of $\Gamma(a)$. Since $N \leq a$, when $a \leq \bar{a}$ the proof is straightforward. Suppose that $a > \bar{a}$ and suppose a PBE exists with $N > \bar{a}$ reports. For expositional clarity, we introduce the following notation. Let integers x and q be

$$x \equiv a - \bar{a} \geq 1 \text{ and } q \equiv N - \bar{a} \geq 1$$

According to (8), $\bar{a} + 1 > \frac{\gamma}{1+\gamma}\frac{1+\lambda}{2\lambda}$, and we define δ to be

$$\delta \equiv 2\lambda\left((1+\gamma)\bar{a} + 1\right) > \gamma(1-\lambda) \tag{24}$$

We also use l_n as the number of types sending report r_n, as given by (18).

The proof is conducted as follows. In Part 1, we show that l_n weakly increases in n. Part 2 is by induction. We show that if $(l_{n+1} - l_n) \geq y$ for some $y \geq 0$ and all $n \leq N - 1$, then there is a lower-bound on a, $a \geq a^{LB}$. Using this lower bound, we show that ICC (19) implies $(l_{n+1} - l_n) \geq y + 1$ for all all $n \leq N - 1$. By induction, it follows that $(l_{n+1} - l_n)$ is unbounded, a contradiction. This result holds when $N > \bar{a} \geq 2$, and when $\bar{a} = 1$ and $N \geq 3$. Items (i) and (ii) of the lemma then follow.

Part 1. Consider the ICC (19). Since $G(k,a)$ increases in k and $\underline{H} > 0$, it follows that

$$(l_{n+1} - l_n) \geq G(1,a) > \tfrac{\lambda}{\gamma(1-2\lambda)}\left(3\gamma + 2(1-\gamma)\bar{a} + 2(2x-1)\right) - 1 > -1$$

due to $\gamma \leq 1$. Since $(l_{n+1} - l_n)$ is integer, it must be that $(l_{n+1} - l_n) \geq 0$ for all $n \leq N-1$.

Part 2. Suppose (induction assumption) that $(l_{n+1} - l_n) \geq y$ for all $n \leq N-1$ and some $y \in \mathbb{Z}_+$. This assumption holds for $y = 0$. We obtain the following lower-bounds on a and k_{N-1}:

$$\begin{aligned} a &= \sum_{n=1}^{N} l_n \geq \sum_{n=1}^{N}\left(l_1 + (n-1)y\right) \geq N\left(1 + \frac{1}{2}y(N-1)\right) \equiv a^{LB} \\ k_{N-1} &= \sum_{n=1}^{N-1} l_n \geq (N-1)\left(1 + \frac{1}{2}y(N-2)\right) \equiv k^{LB} \end{aligned}$$

Using these lower bounds and (24), we evaluate $G(k_{N-1},a) - y$:

$$G(k_{N-1},a) - y > \tfrac{1}{\gamma(1-2\lambda)}\left(\left(4\left(\gamma k^{LB} + a^{LB}\right) + y\gamma\right)\lambda - (2+y)\delta\right) > 0$$

Hence, $G(k_{N-1},a) > y$ and, therefore, $(l_N - l_{N-1}) \geq (y+1)$. As a result, $a \geq a^{LB} + 1$. We consider cases $\overline{a} \leq 2$ and $\overline{a} \geq 3$ separately.

1. Let $\overline{a} \leq 2$ and $N \geq 3$. Then,

   $$a = \sum_{n=1}^{N} l_n \geq l_1 + 2(l_1 + y) + (y+1) = 3l_1 + 3y + 1 \geq 4 + 3y$$

   which, together with (21) and (24), implies

   $$G(1,a) - y \geq \tfrac{1}{\gamma(1-2\lambda)}\left(\left(4(\gamma + 4 + 3y) + y\gamma\right)\lambda - (2+y)\delta\right) > 0$$

   Thus, $(l_{n+1} - l_n) \geq (y+1)$ for all $n \leq N-1$.

2. Let $\overline{a} \geq 3$ and $N \geq 4$. In this case, we use another induction argument. Suppose $(l_{N-j+1} - l_{N-j}) \geq (y+1)$ for all $j \in \{1,\ldots,r\}$. This assumption holds for $r = 1$. We define (new) lower bounds on a and on k_{N-r-1} as follows:

   $$\begin{aligned} a &= \sum_{n=1}^{N} l_n \geq \sum_{n=1}^{N}\left(l_1 + (n-1)y\right) + \sum_{n=N-r+1}^{N}\left(n - (N-r)\right) \\ &\geq N\left(1 + \tfrac{1}{2}y(N-1)\right) + \tfrac{1}{2}r(r+1) \equiv a^{NLB} \\ k_{N-r-1} &= \sum_{n=1}^{N-r-1} l_n \geq (N-r-1)\left(1 + \tfrac{1}{2}y(N-r-2)\right) \equiv k^{NLB} \end{aligned}$$

Using (21) we show that $G(k_{N-r-1}, a) > y$. We consider cases $y = 0$ and $y \geq 1$ separately.

(a) When $y = 0$, we define $J_1(r)$ as the following lower-bound on $(\gamma k_{N-r-1} + a)$:

$$(\gamma k_{N-r-1} + a) \geq \left(\gamma k^{NLB} + a^{NLB}\right) \geq (\gamma + 1) N + \left(\tfrac{1}{2}r - \gamma\right)(r + 1) \equiv J_1(r)$$

Since $J_1(r)$ is a second degree convex polynomial, which increases for $r > \left(\gamma - \frac{1}{2}\right)$, it follows that:

$$J_1(r) > J_1(1) = (\gamma + 1) N + (1 - 2\gamma)$$

and using (24), we get

$$G(k_{N-r-1}, a) > \tfrac{1}{\gamma(1-2\lambda)} (4\lambda J_1(r) - 2\delta) \geq 0 = y$$

(b) When $y \geq 1$, we define $J_2(r)$ as the following lower-bound on $(\gamma k_{N-r-1} + a)$:

$$(\gamma k_{N-r-1} + a) \geq \gamma + \tfrac{1}{2}\gamma y (N - r - 2)^2 + N + \tfrac{1}{2}N(N-1) y + \tfrac{1}{2}\left(r^2 + 1\right) \equiv J_2(r)$$

Since $J_2(r)$ is a second degree convex polynomial, it attains its minimum at $r = r^*$, where $r^* = \frac{\gamma y}{\gamma y + 1}(N - 2)$. Therefore,

$$J_2(r) \geq J_2(r^*) = \gamma + \tfrac{\gamma y}{2(\gamma y + 1)}(N - 2)^2 + N\left(1 + \tfrac{1}{2}(N - 1) y\right) + \tfrac{1}{2}$$

Using $(\gamma k_{N-r-1} + a) \geq J_2(r^*)$, we evaluate $G(k_{N-r-1}, a) - y$:

$$G(k_{N-r-1}, a) - y \geq \tfrac{1}{\gamma(1-2\lambda)} ((4J_2(r^*) + y\gamma)\lambda - (2 + y)\delta) > 0$$

Hence, finally, $G(k_{N-r-1}, a) > y$.

Cases (a) and (b) imply $G(k_{N-r-1}, a) > y$ so that $(l_{N-j+1} - l_{N-j}) \geq (y + 1)$ holds for $j = (r + 1)$. By induction, it holds for all $j \leq N - 1$. That is, $(l_{n+1} - l_n) \geq (y + 1)$ for all $n \leq N - 1$.

In both cases 1 and 2, $(l_{n+1} - l_n) \geq (y + 1)$ for all $n \leq N - 1$. By induction, $(l_{n+1} - l_n) \geq y$ for any $y \in \mathbb{N}$, a contradiction. Therefore, $N \leq \bar{a}$ if $\bar{a} \geq 3$ and $N \leq 2$ if $\bar{a} \leq 2$. Since for $\bar{a} \geq 2$, the separating subgame equilibrium always exists, it follows that $\overline{N} = \bar{a}$ if $\bar{a} \geq 2$. This occurs when $\frac{\gamma}{1+\gamma}\frac{(1+\lambda)}{2\lambda} \geq 2$, *i.e.*, when $\lambda \leq \frac{\gamma}{4+3\gamma}$. When $\bar{a} = 1$, $N \leq 2$ so that $\overline{N} \leq 2$ as well. This ends the proof. ■

Proof of Proposition 1.
Consider a PBE of $\Gamma(a)$. Let $U^X(a)$ be the *ex-ante* expected utility of X. Taking expectations of (12) yields:

$$U^X(a) \equiv \mathrm{E}\left[\mathrm{E}\left[u^X|k, r_n\right]\right] = \tfrac{1}{2} + \mathrm{E}\left[\gamma(v-z)\,\delta^I\delta^E\right] - c(a-1) \tag{25}$$

First, we show that over all equilibria with N reports of all continuation games, the separating equilibrium of $\Gamma(N)$ maximizes $U^X(a)$. Second, we maximize (25) with respect to a over $a \in \{2, \ldots, \overline{a}\}$, according to Lemma 4.

Fix the number of reports $N \in \{2, \ldots, \overline{a}\}$. Using (11) and (13), we write the *ex-ante* expected project value as follows:

$$\mathrm{E}\left[\gamma(v-z)\,\delta^I\delta^E\right] = \frac{\gamma}{8a^3}\sum_{n=1}^{N}(k_n - k_{n-1})(k_n + k_{n-1})^2 - \frac{\gamma}{8a^3}\left(1-\delta^E(r_1)\right)(k_1)^3 \tag{26}$$

We maximize (26) w.r.t. $\{k_n\}$ assuming k_n are real numbers. For $\delta^E(r_1) = 1$, the first-order conditions are:

$$\tfrac{\gamma}{4a^3}\left(\tfrac{1}{2}(k_{n-1} + k_{n+1}) - k_n\right)(k_{n+1} - k_{n-1}) = 0$$

Hence, (26) attains its global maximum over $\{k_n\}$, $k_n \in \mathbb{R}$, when $k_n = \frac{1}{2}(k_{n-1} + k_{n+1})$, *i.e.*, when intervals $(k_n - k_{n-1})$ are of equal length for all n. It follows that (26) is maximized over a set of natural numbers $\{k_n\}$, $k_n \in \mathbb{N}$, when $k_n = nt$ for some $t \in \mathbb{N}$. In this case, $a = Nt$ and

$$\mathrm{E}\left[\gamma(v-z)\,\delta^I\delta^E\right] = \frac{\gamma}{8N^3}\sum_{n=1}^{N}(2n-1)^2 = \frac{\gamma}{24}\left(4 - \frac{1}{N^2}\right) \tag{27}$$

When $t = 1$, we have a separating equilibrium of $\Gamma(N)$. Hence,

$$U^X(N) = \tfrac{1}{2} + \tfrac{\gamma}{24}\left(4 - \tfrac{1}{N^2}\right) - (N-1)\,c \tag{28}$$

Maximizing (28) yields $N = a^{opt}$ given by (7). Moreover, E's strategy (16), $h \leq \overline{h}(a^*)$ assures E approves after receiving r_1, $\delta^E(r_1) = 1$.

The above arguments are based on the assumption $\delta^E(r_1) = 1$ for all a. However, if $\mathrm{E}\left[\gamma(v-z)\,\delta^I|r_1\right] < h$ for some $a > a^*$, so that $\delta^E(r_1) = 0$ for these a, then the *ex-ante* expected utility $U^X(a)$ of X will be lower than in the above derivations. Therefore, $a = N = a^*$ maximizes $U^X(a)$ for all $a \in \mathbb{N}$, including $a > a^*$. This ends the proof. ■

Proof of Corollary 1.
According to Lemma 4, when $\overline{a} = 1$ then either $N = 1$ so that $a = 1$ is optimal, or $N = 2$ so that $a \geq 3$. The limiting property of optimal a can be shown as follows. For $N = 2$, $l_2 = a - l_1$, and $\delta^E(r_1) = 1$, the ICC (19) is

$$l_1 \leq \left(\tfrac{1}{2} - \tfrac{2+\gamma}{\gamma}\lambda\right) a + (1 - \lambda) \equiv \overline{l_1}(a) < \tfrac{1}{2}a$$

According to (25) and (26), the expected utility of X in equilibrium, as a function of (l_1, a) is

$$U^X(l_1, a) = \tfrac{1}{2} + \tfrac{\gamma}{32}\left(5 - \left(1 - \tfrac{2l_1}{a}\right)^2\right)$$

which monotonically increases in l_1 over $\left[0, \overline{l_1}(a)\right]$. Therefore, taking the highest value $l_1 = \left\lfloor \overline{l_1}(a) \right\rfloor$ is optimal for X. The necessary (and sufficient) condition for this is $\overline{l_1}(a) \geq 1$, *i.e.*, $\left(\tfrac{1}{2} - \tfrac{2+\gamma}{\gamma}\lambda\right) a \geq \lambda$. When $\lambda \geq \frac{\gamma}{2(2+\gamma)}$, this condition does not hold for any a, and $a = 1$ is optimal. If, on the other hand, $\lambda < \frac{\gamma}{2(2+\gamma)}$, this condition only holds if $a \geq \frac{2\lambda\gamma}{(\gamma - 2(2+\gamma)\lambda)}$. Hence, optimal a unboundedly increases if $\lambda \to \frac{\gamma}{2(2+\gamma)}$. This ends the proof. ■

Proof of Lemma 5.
In the limit $\lambda \searrow 0$, the ICCs (19) and (20) can be written as follows

$$\left(2 - \delta^E(r_n)\right) l_n - 1 \leq l_{n+1} \leq \left(2 - \delta^E(r_n)\right) l_n + 2 \tag{29}$$

Let there be $N \geq 2$ reports in a PBE of $\Gamma(a)$ for $a \geq N$. Suppose, first, that $\delta^E(r_1) = 1$. In this case, $l_{n+1} = l_n = 1$ satisfies (29) and the largest N is achieved in the separating equilibrium when $N = a$. Using Lemma 3, the largest N among the equilibria where $\delta^E(r_1) = 1$ is achieved when $a = a_0$ where $\bar{h}(a_0 + 1) < h \leq \bar{h}(a_0)$.

When a increases by 1, $a = (a_0 + 1)$, then $\delta^E(r_1) = 0$ and $\delta^E(r_2) = 1$ even when $l_2 = l_1 = 1$. The latter follows from E's strategy (16). As $l_2 = l_1 = 1$ still satisfies (29), N also increases by 1, $N = (a_0 + 1)$. Hence, the largest N arises when $\delta^E(r_1) = 0$.

Now, we compute upper-bounds for N when $\delta^E(r_1) = 0$. We consider two cases.

1. Suppose $l_2 \geq (N - 1)$. Then $l_{n+1} \geq l_n - 1$ for $n \in \{2, \ldots, N\}$, so that

$$a = l_1 + \sum_{n=2}^{N} l_n \geq l_1 + \sum_{n=2}^{N} (l_2 - (n - 2)) > l_1 + \frac{1}{2} N l_2$$

Using $\delta^E(r_2) = 1$ (see Lemma 1) and (16) we write $h \leq \frac{\gamma}{8a^2}(2l_1 + l_2)^2$ so that $\sqrt{\frac{\gamma}{8h}}(2l_1 + l_2) \geq a > l_1 + \frac{1}{2}Nl_2$. This can be rewritten as the following upper-bound on N:

$$N < 2\left(2\sqrt{\tfrac{\gamma}{8h}} - 1\right)\frac{l_1}{l_2} + 2\sqrt{\tfrac{\gamma}{8h}} \leq 6\sqrt{\tfrac{\gamma}{8h}} - 2$$

2. Suppose $l_2 \leq (N-2)$. Then $l_{n+1} \geq l_n - 1$ for $n \in \{2, \ldots, l_2+1\}$ and $l_n \geq 1$ for $n \in \{l_2+2, \ldots, N\}$, so that

$$a \geq l_1 + \sum_{n=2}^{l_2+1}(l_2 - (n-2)) + \sum_{n=l_2+2}^{N} 1 = l_1 + \frac{1}{2}(l_2-1)\,l_2 + N - 1$$

Using $h \leq \frac{\gamma}{8a^2}(2l_1+l_2)^2$ and $l_1 \leq \frac{1}{2}(l_2+1)$ from (29) yields:

$$N \leq \frac{1}{2}\left(4\sqrt{\tfrac{\gamma}{8h}} - l_2\right)l_2 + \sqrt{\tfrac{\gamma}{8h}} + \frac{1}{2}$$

The right-hand side of this inequality is a second-degree polynomial in l_2, which attains its maximum at $l_2 = 2\sqrt{\frac{\gamma}{8h}}$. Hence,

$$N \leq \tfrac{\gamma}{4h} + \sqrt{\tfrac{\gamma}{8h}} + \frac{1}{2}$$

Combining the two cases above results in $N \leq \overline{N} \equiv \max\left\{\left(\frac{\gamma}{4h} + \sqrt{\frac{\gamma}{8h}} + \frac{1}{2}\right), \left(6\sqrt{\frac{\gamma}{8h}} - 2\right), 2\right\}$, which ends the proof. ■

Proof of Proposition 2.
Let, for each $N \leq \overline{N}$ consider a set A_N of values of a such that for any $a \in A_N$, the corresponding continuation game $\Gamma(a)$ has a PBE (separating or partially separating) with N reports. First, we show for each $N \leq \overline{N}$, there is an $a^*(N) \in A_N$ such that choosing $a > a^*(N)$ is not optimal for X for any c. Second, we define

$$\widehat{a} \equiv \max_{N \leq \overline{N}} a^*(N)$$

This maximum always exists and, by construction, $a > \widehat{a}$ is not optimal for X for any N.

If A_N is finite, $a^*(N)$ is also finite. Let A_N be infinite. For any $a \in A_N$, we consider a PBE Ω_a of $\Gamma(a)$ with N reports (if there are multiple equilibria, we choose Ω_a arbitrarily). We define

$$x_n \equiv \frac{k_n}{a} \text{ and } y_n \equiv \frac{l_n}{a} = x_n - x_{n-1}$$

The *ex-ante* expected utility $U^X(a)$ of X in PBE Ω_a is given by (25). It follows that a only affects $U^X(a)$ through the *ex-ante* expected project value, which can be written as follows:

$$\mathrm{E}\left[\gamma(v-z)\delta^I\delta^E\right]=\frac{\gamma}{8}\sum_{n=12}^{N}(x_n-x_{n-1})(x_n+x_{n-1})^2-\frac{\gamma}{8}\left(1-\delta_a^E(r_1)\right)x_1^3 \quad (30)$$

where δ_a^E is the approval strategy of E in Ω_a. Next, we write the equilibrium condition (29) for Ω_a as follows:

$$y_{n+1}-\left(2-\delta_a^E(r_n)\right)y_n\in\left[-\tfrac{1}{a},\tfrac{2}{a}\right] \quad (31)$$

Consider a limit $a\to\infty$ in which a takes on increasing values from A_N. The interval $\left[-\frac{1}{a},\frac{2}{a}\right]$ converges to a point $\{0\}$. As $\delta_a^E(r_1)\in\{0,1\}$, the sequence $\left\{\delta_a^E(r_1)\right\}$ either has a limit of 0 or 1, or it oscillates between these two values. The following three cases are possible.

1. Let $\left\{\delta_a^E(r_1)\right\}$ converge to 1. Then, $\delta_a^E(r_1)=1$ and (31) implies $y_{n+1}=y_n$ for all n in the limit. Hence, $x_n\to\frac{n}{N}$ and (30) converges to:

$$\lim_{a\to\infty}\mathrm{E}\left[\gamma(v-z)\delta^I\delta^E\right]=\frac{\gamma}{8N^3}\sum_{n=1}^{N}(2n-1)^2=\frac{\gamma}{24}\left(4-\frac{1}{N^2}\right)$$

   which is identical to the project value in the separating equilibrium with $a=N$, as given by (27). This implies that by choosing unboundedly large a, X cannot get higher project value than by choosing $a=N$. Hence, (30) attains its maximum at some finite value of a, $a^*(N)$.

2. Let $\left\{\delta_a^E(r_1)\right\}$ converge to 0. Then, $\delta_a^E(r_1)=0$ and (31) implies $y_{n+1}=y_n$ for $n\geq 2$ and $y_2=2y_1$ in the limit. Hence, $x_n\to\frac{2n-1}{2N-1}$ and (30) converges to:

$$\lim_{a\to\infty}\mathrm{E}\left[\gamma(v-z)\delta^I\delta^E\right]=\frac{\gamma}{(2N-1)^3}\sum_{n=1}^{N}(2n-2)^2=\frac{2}{3}\gamma\frac{N(N-1)}{(2N-1)^2}$$

   which is identical to the project value obtained in the equilibrium where $a=2N-1$ and $k_n=2n-1$. Hence, again, by choosing unboundedly large a, X cannot get higher project value than by choosing $a=2N-1$, implying that (30) attains its maximum at some finite value of a, $a^*(N)$.

3. Let $\left\{\delta_a^E(r_1)\right\}$ have no limit. The maximum of $a^*(N)$ from the two previous cases becomes the upper-bound on optimal a.

Summarizing, choosing $a > \widehat{a}$ is not optimal to X for any N. Consequently, there exists an optimal value of N, $N \leq \overline{N}$, an optimal $a \leq a^*(N)$, and an optimal equilibrium Ω_a of $\Gamma(a)$ that maximizes $U^X(a)$, so that the sender-optimal equilibrium exists. The uniqueness is generic and follows from continuous dependence of $U^X(a)$ on parameters γ and c. This ends the proof. ■

Proof of Proposition 3.
When $h \leq \overline{h}(a^*)$, E's strategy (16) implies $d^E = 1$ in all continuation games $\Gamma(a)$ for all $a \leq a^*$. The *ex-ante* expected utility $U^X_{sep}(a)$ of X from choosing a and playing the separating equilibrium in $\Gamma(a)$ is given by (28) $N = a$:

$$U^X_{sep}(a) \equiv \frac{1}{2} + \frac{\gamma}{24}\left(4 - \frac{1}{a^2}\right) - (a-1)c \tag{32}$$

When $a = 1$ only the pooling equilibrium of $\Gamma(a)$ exists. When $a = 2$, $\Gamma(a)$ has two equilibria: the pooling equilibrium and the separating equilibrium. When $c > 0$, the pooling equilibrium following $a = 2$ does not satisfy the forward induction refinement: choosing $a = 1$ yields a higher payoff than choosing $a = 2$ followed by the pooling equilibrium. When $a^* \geq 2$ so that $U^X_{sep}(2) > U^X_{sep}(1)$, the unique forward induction equilibrium of $\Gamma(2)$ is the separating equilibrium.

Suppose, as an induction assumption, that for some $t \leq a^* - 1$, for each $a \in \{2, \ldots, t\}$ the unique forward induction equilibrium of $\Gamma(a)$ is the separating equilibrium yielding $U^X_{sep}(a)$. Then, the separating equilibrium of $\Gamma(t+1)$ is the unique forward induction equilibrium. Indeed, (i) all semi-pooling equilibria have $N \leq t$ reports and, therefore, yield a strictly lower payoff to X than $U^X_{sep}(t)$ (as is shown in the proof of Proposition 1), and (ii) as $U^X_{sep}(a)$ increases in a over $a \in \{2, \ldots, a^*\}$ (by the definition of a^*). Hence, for $a = t+1$, $U^X(a) > U^X_{sep}(t)$ holds only when the separating equilibrium is played in $\Gamma(a)$. It follows that choosing $a = a^*$ dominates choosing any $a < a^*$. What remains to be shown is that choosing $a = a^*$ also dominates choosing any $a > a^*$. If $a^* = \overline{a}$, then for any $a > a^*$ only semi-pooling equilibria with $N \leq a^*$ reports exist, and they all yield lower utility to X than $U^X_{sep}(a^*)$. If, on the other hand, $a^* < \overline{a}$, then $a^* = a^{opt}$. Since $U^X_{sep}(a)$ decreases in a for all $a > a^{opt}$, choosing $a > a^*$ is dominated by choosing $a = a^*$. Thus, choosing $a = a^*$ is the unique optimal choice of X. This ends the proof. ■

Proof of Proposition 4.
Consider an equilibrium the game with private communication in which X sends truthful internal reports, *i.e.*, in which $\rho^I(k)$ is one-to-one. Let r^I_k be the internal message send by type k. Accordingly, $\mathrm{E}\left[v|r^I_k\right] = v_k$, as defined in (1). Using backward induction, consider I. It follows from (3) that I chooses $d^I = 1$ if $z < v_k$ and $d^I = 0$ otherwise (hereinafter, without loss of generality, we use strict

inequalities in the constraints). This proves item (i) of the proposition. As a result, given k, the expected project value equals $\mathrm{E}\left[\gamma(v-z)\,d^I\right]=\frac{\gamma}{2}(v_k)^2$.

Next, consider the approval decision d^E by E. Using (5) and I's strategy, the optimal strategy for E is to choose $d^E=1$ if and only if $\mathrm{E}\left[u^E|r^E\right]>0$. Hence ,

$$\delta^E\left(r^E\right)=1 \text{ if } \tfrac{\gamma}{2}\,\mathrm{E}\left[(v_k)^2|r^E\right]>h, \text{ and } \delta^E\left(r^E\right)=0 \text{ otherwise} \tag{33}$$

Next, consider communication between X and E. Stock price (4) becomes

$$s\left(r^E\right)=\mathrm{E}\left[v|r^E\right]+\tfrac{\gamma}{2}\,\mathrm{E}\left[(v_k)^2|r^E\right]\delta^E\left(r^E\right)-c(a-1)$$

and the expected utility $\mathrm{E}\left[u^X|k,r^E\right]$ of X of type k when he reports r^E to E is:

$$\mathrm{E}\left[u^X|k,r^E\right]=(1-\lambda)\left(v_k+\tfrac{\gamma}{2}(v_k)^2\delta^E\left(r^E\right)\right)+\lambda s\left(r^E\right)-c(a-1) \tag{34}$$

We proof item (ii) by contradiction. Suppose that in equilibrium, two reports r_1^E and r_2^E are used such that $s\left(r_1^E\right)\neq s\left(r_2^E\right)$ or $\delta^E\left(r_1^E\right)\neq\delta^E\left(r_2^E\right)$ (or both). If $\delta^E\left(r_2^E\right)=1$ and $\delta^E\left(r_1^E\right)=0$, then the difference

$$\mathrm{E}\left[u^X|k,r_2^E\right]-\mathrm{E}\left[u^X|k,r_1^E\right]=(1-\lambda)\tfrac{\gamma}{2}(v_k)^2+\lambda\left(s\left(r_2^E\right)-s\left(r_1^E\right)\right)$$

is increasing in v_k. This implies that if a type $\widetilde{k}$ prefers reporting r_2^E to reporting r_1^E, all types $k>\widetilde{k}$ do so as well, and, therefore, $s\left(r_2^E\right)>s\left(r_1^E\right)$ so that $\mathrm{E}\left[u^X|k,r_2^E\right]>\mathrm{E}\left[u^X|k,r_1^E\right]$ for all k. Hence, no types report r_1^E. If, on the other hand, $\delta^E\left(r_1^E\right)=\delta^E\left(r_2^E\right)$ and $s\left(r_2^E\right)>s\left(r_1^E\right)$, then $\mathrm{E}\left[u^X|k,r_2^E\right]>\mathrm{E}\left[u^X|k,r_1^E\right]$ for all k, and no types report r_1^E. Thus, in any equilibrium, only one report r^E can be used, which proves item (ii) of the proposition.

Since r^E is independent of k, the *ex-ante* expected value of the project conditional on approval is:

$$\tfrac{\gamma}{2}\,\mathrm{E}\left[(v_k)^2|r^E\right]=\tfrac{\gamma}{2}\,\mathrm{E}\left[(v_k)^2\right]=\tfrac{\gamma}{8a^3}\sum_{k=1}^{a}(2k-1)^2=\tfrac{\gamma}{24}\left(4-\tfrac{1}{a^2}\right) \tag{35}$$

Substituting (35) into E's strategy (33) yields item (iii) of the proposition.

Lastly, we consider the choice of a by X. Let $U^X(a)$ denote the *ex-ante* expected utility of X. Taking expectations of (34) over k and using (35) and $\mathrm{E}\left[v|r^E\right]=\mathrm{E}\left[v_k\right]=\frac{1}{2}$ yields

$$U^X(a)=\tfrac{1}{2}+\tfrac{\gamma}{24}\left(4-\tfrac{1}{a^2}\right)d^E-(a-1)\,c \tag{36}$$

According to item (iii), if $h\geq\frac{\gamma}{6}$, then E chooses $d^E=0$ for any a. Choosing $a=1$ is optimal in this case. If $h<\frac{\gamma}{8}$ then E approves for any a. Maximizing (36) yields

$$a^{\mathrm{PR}}=a^{opt}=\sqrt[3]{\tfrac{\gamma}{12c}}$$

When $h \in \left(\frac{\gamma}{8}, \frac{\gamma}{6}\right)$, E only approves if $a > \underline{a}$, where

$$\underline{a} \equiv \frac{1}{2}\sqrt{\frac{\gamma}{\gamma-6h}}$$

If $a^* \geq \underline{a}$, choosing $a = a^*$ is optimal, as this maximizes (36). Suppose $a^* < \underline{a}$. This occurs when $c > \underline{c}$, where

$$\underline{c} \equiv \frac{2}{3}\gamma\left(1 - 6\frac{h}{\gamma}\right)^{\frac{3}{2}}$$

In this case, X either chooses $a = \underline{a} > a^*$or chooses $a = 1$. Using (36), the first option yields a higher payoff if and only if $c < \overline{c}$, where

$$\overline{c} \equiv \frac{h}{\underline{a}-1}$$

This ends the proof of item (iv) of the proposition.

Finally, consider X's internal reporting strategy $\rho^I(k)$. It follows from (34) that r^I only affects X's payoff through the expected project value $\mathrm{E}\left[\gamma\left(v-z\right)\right]$. Given I's strategy and using (1), type k prefers reporting r_k^I to $r_{k+\tau}^I$ if

$$\frac{\gamma}{2}\left(v_k\right)^2 - \gamma\left(v_{k+\tau}\left(v_k - \frac{1}{2}v_{k+\tau}\right)\right) = \frac{1}{2a^2}\tau^2\gamma \geq 0$$

which holds for any τ. Hence, $\rho^I(k) = r_k^I$ maximizes X's payoff for any type k. ■