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Demography and Provisions for Retirement: The Pension Composition, an Equilibrium Approach

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Abstract

Pensions may be provided for in a modern society by several methods, viz., voluntary individual savings, mandatory fully funded occupational pension systems, and mandatory social security financed by pay-as-you-go. The specific mixture of the three systems we will call the *pension composition*. We assume that individual workers decide about their own individual savings, that the fully funded occupational system is decided upon by the age cohort of the median worker and that social security is decided upon by the median voter. For a given demography and interest rate the joint result of those decisions is a Pareto-equilibrium. Nowadays most of capital supply stems from individual and institutionalised pension savings. For ease of exposition we will assume that individual and collective pension savings are the only source of capital supply. When capital supply equals demand from industry there is equilibrium on the capital market with a corresponding equilibrium interest rate. In this paper we assume a demography with hundred age brackets and we investigate how changes in the birth and survival rates affect the pension composition and the capital market equilibrium. Our conclusion is that the demographic effects are considerable not only for the resulting pension composition, but also for macro-economic variables as the wage rate, the interest rate and the capital-income ratio. It follows that the pension composition in general and social security in particular is determined by the demography and cannot be used as long-term political instrument. We find that this is relevant for the present century, where birth and mortality rates in most western countries are steeply declining.

Keywords: demography, funded pensions, unfunded pensions, social security, interest rate, overlapping generations, individual savings.

JEL codes: H55, H75, J1, J26

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1. Introduction.

Nowadays there is much concern on pension systems and more in particular on the question whether the present systems by which old age support is given can be maintained in the future, given the fact that populations are ageing. In many countries, e.g. USA, France, the United Kingdom, Germany, Spain, Italy, and The Netherlands there are lively political debates, the majority of those is tending to proposals to increase occupational mandatory pensions premiums and/or private savings and to reduce the mandatory public pension system run on a pay-as-you-go basis, and moreover to increase the retirement age. There is a large variety of proposals and measures taken. There are also some proposals to increase the PAYG-part and as a consequence to reduce savings.

Our view on the problem is rather fatalistic. We believe that there is very little room for long-term economic policy as the mixture of public pensions, funded occupational pensions, private pension insurance, with perhaps additional hoarding in cash, a mixture which we call the *pension composition* for short, is the equilibrium outcome of a rather complex system with many players. The players are Parliament, represented by the Median Voter, as far as social security is concerned, the trade union[1], represented by the Median Worker, deciding on the existence and the level of mandatory occupational pensions and the individual households as far as it concerns their private savings. We do not assume a social planner who determines the pension system with the intention to optimize a social welfare function. Rather we will assume a Pareto-type equilibrium where Median Voter, Median Worker and the individual private savers try to optimize their decisions simultaneously. Under *ceteris paribus* (c.p.) conditions the system tends to an equilibrium, i.e., a *pension composition*. We assume a closed economy. Hence, national capital supply is assumed to equal the sum of individual savings and the reserves of the pension funds. Equating the capital supply with the capital demand from firms we find an equilibrium interest rate on the capital market and this closes the model.

A specific part of those c.p. conditions is the demography of the country. When the demography changes the pension composition will change as well, and this is what we perceive in reality. Policy makers think that they are independently acting but in the end the external observer sees they are puppets on a string, steered by the changes in demography towards a specific outcome.

The structure of this paper is as follows. In Section 2 we have a look on the existing literature. In Section 3 we specify the general model. We describe our demographic

[1] In some countries the employers are also formally involved, but since employers are primarily interested in the total wage costs, the division of those costs between present net income and future pension income is irrelevant for the employers and left to labor representatives.

model, the behavior of the different players, and the different pension systems, i.e., public pensions, funded occupational pensions, private pensions and hoarding.
We are unable to give analytical solutions, but we are able to calculate solutions in the spirit of the seminal contribution by Auerbach and Kotlikoff (1987) when we functionally specify the model and assume not improbable values for some fundamental parameters. The model will be functionally specified in Section 4. In this paper we focus on three sets of parameters, viz., the age-differentiated birth rates, the age-differentiated survival rates, and the retirement age. In order to get an idea of the effect of changes in those parameters we calculate solutions for a variety of parameter sets.
In Section 5 we describe our solution method. In Section 6 we consider and evaluate the outcome, of our model. Those outcomes include pension and social security premiums and benefits, wage rates, interest rates, and capital per workplace. In Section 7 we consider the political relevance of our findings and position our approach within the literature. The main novelty of our study seems to be the Pareto-approach, according to which we find an equilibrium pension composition, which explains the simultaneous co-existence of social security, mandatory occupational pensions, individual savings, and hoarding and the ensuing importance of the demography with respect to wages, interest and capital.
Obviously, our model is a simplification of reality in several aspects. First, we consider the final solution of a dynamic model. In reality we are never in such an equilibrium situation. In this case, for instance, the demographic parameters vary so fast that the situation of a stable age distribution with a constant population growth rate is nearly never reached in practice. Typically, from an arbitrary position a demographic process needs several hundred years to converge to the demographic equilibrium. However, the latent equilibrium situation is relevant to know as the real behavior of the system may be assumed to *tend* to the equilibrium in the long run. Although we did our best to choose more or less realistic parameter values and functional specifications, it is not difficult to suggest other values and specifications as being more realistic alternatives. For instance, the demographic sub-model we use is found in the literature, but due to its stylisation it does not equal any specific national demography. This is of course the price to be paid when one wants to analyze the *structural* properties. Actually, our model is very flexible, and can be easily implemented in practice to make predictions about dynamic developments for real national economies.

2. A look at the literature.

There is a lot of literature on the subject of pensions. A first early strand of the literature focuses on the conditions of dynamic efficiency, e.g. Samuelson (1954) and Aaron (1966).

A second strand of the literature focuses on risk sharing and the effect of aging. Examples include Gordon and Varian (1988), Bohn (2003), Ball and Mankiw (2007), Beetsma and Bovenberg (2007), Matsen and Thøgerson (2004) and Gollier (2008). In this paper we are focusing on the role of the demography and on the question what determines the pension composition. In the literature we find various approaches to the relation between demography and economics. It is beyond the scope of this paper to consider the hundreds of articles written. Those studies differ in many ways. We may distinguish between more theoretical and more applied papers. In the theoretical papers one looks for a dynamical equilibrium, where the demography consists of a few age brackets. In the applied papers one looks mostly at a specific realistic setting, where the model is calibrated to reality and tries to predict developments for a specific country. In most papers two sources of old-age provision are considered, individual voluntary savings (IVS) and unfunded social security (SS). Sometimes the interest rate is taken as exogenous, while others take it to be endogenous.

In the theoretical analyses like Samuelson (1954), Aaron (1966), Cooley and Soares (1999a,b), Galasso (1999), Casamatta, Cremer and Pestieau (2000), Galasso and Profeta (2004), Galasso (2008), Gonzalez-Eiras and Niepelt (2007), Mateos-Planas (2008), Cremer et al (2009) there is a two- or three-period overlapping generation population. Individuals are assumed to save within the constraint of a life budget, while the government is assumed by means of taxation and social security to affect savings behavior in order to reach some optimal outcome according to a social welfare function. Matters are becoming more complex if we assume three sources for old-age provisions, viz., social security, mandatory funded occupational pensions, and voluntary individual savings. This difference is stressed by Lindbeck and Perssons (2003). In the more applied papers simulations are performed on real populations with many age cohorts in order to predict the development of the pension system for specific economies (e.g. the seminal Auerbach, Kotlikoff (1987), Miles (1999), Poterba (2001), Barr and Diamond (2006), Krueger and Ludwig (2007), Beetsma and Bovenberg (2009), Bovenberg and Nijman (2009), Boersch-Supan and Ludwig (2010), Lee and Mason (2010), Bell and Hill (1984) and Auerbach and Lee (2011)). For such more realistic worlds analytical results are difficult to find, and one has to rely on model simulations, which we will use as well. Recently, there are some authors who

looked more systematically to the relationship between demography and economics *per se*. They are mostly working in continuous time. The problem with this approach is that unless we use some tractable functional specifications, it becomes impossible to find nice formulas for the solutions. We mention a.o. d' Albis (2007), Bruce and Turnovsky (2013), Heijdra and Mierau (2011), Cipriani (2016).
A third, political-economic stream emphasizes the importance of aging on election outcomes. Examples include Conesa and Krueger (1999), Cooley and Soares (1999a,b), Rangel and Zeckhauser (1999), Boldrin and Rustichini (2000), Breyer and Stolte (2001), Demange (2005), Gonzalez-Eiras and Niepelt (2007) and D'Amato and Galasso (2010). See Galasso and Profeta (2002) for a still relevant overview of the political economy of social security. The main result of this literature goes back to the seminal paper of Browning (1975), who shows in a static 3-period OLG-model why a democracy may overspend on social security. More general and influential opinion papers include Feldstein (1997) and Sinn (2000). See for an up to date survey on macro-economics and ageing Lee (2016). That survey does not deal with pensions in particular. In this study we differentiate between three age-providing systems, viz. social security on a PAYG-basis, occupational pensions and individual savings. There are only a few studies where the three systems are looked at simultaneously. We mention Knell (2010). In reality the three systems will mostly exist side by side and the relative sizes of the system are endogenously determined. We will distinguish between 45 working age cohorts, where each cohort determines its future individual savings, and where two cohorts have a specific additional role. The Median Worker has the choice to save either individually or via a mandatory occupational pension. If he chooses for the mandatory system he binds all other working cohorts as well to participate in the mandatory pension system. Similarly the Median Voter may choose between individual saving or providing for old age via a mandatory social security system. In the latter case he forces the other cohorts to participate as well. The aggregate capital supply from individual and mandatory savings may sometimes exceed the demand for capital, even at interest zero. In that case we leave the possibility open that part of individual savings are held in cash, as hoarding is less costly than keeping deposits at the bank at a negative interest rate would be.

3. Structure of the equilibrium model.

In this section we describe the model firstly in general terms. In the next section we focus on the functional specifications. We consider a homogeneous population with N age cohorts $n = 0,1, \dots, N$. The population is assumed to be stable, i.e., the age distribution $p = (p_0, \dots, p_N)$ is constant and the population grows at a constant growth rate $1+\nu$.

The demographic process depends on a birth pattern $\beta = (\beta_0, \dots, \beta_N)$ and a survival pattern $\mu = (\mu_0, \dots, \mu_N)$. It follows that $p = p(\beta, \mu)$ and $\nu = \nu(\beta, \mu)$. In most studies the demography is succinctly described by its growth rate ν only without looking at the underlying birth and survival process. Making a distinction between both opens up the possibility to investigate the effects of a declining birth rate or increasing survival rates separately.

Four age cohorts are pivotal for the analysis. First, the age SW (taken in our numerical analysis at 20) at which one starts working and saving, second, the final working age EW (taken in our numerical analysis at 64) at which one ends his/her working period after which retirement starts. For our analysis we consider mostly the population of adults only, where adults stand for all cohorts with $n \geq SW$. We denote the corresponding conditional adult population shares by $p_{n|SW}$. Similarly, the conditional distribution of workers only will be denoted by $p_{n|SW,EW}$. The (adult) population share of the workers is denoted by $P_{work|20}$. The population share of the retired is $P_{ret|20}$. The third pivotal cohort is that of the median workers (MW), and the fourth pivotal cohort is that of the median voters (MV).

We assume there are four methods of providing for old-age:

Individualistic voluntary

a. *Voluntary participation in a pension insurance contract* (IVS)

b. *Hoarding in cash* (HO)

Collectivistic mandatory

c. *Fully Funded occupational pension* (FF)

d. *Pay-as-you-go social security* (SS)

Participation in an individual voluntary pension insurance contract is understood to mean that one voluntarily promises at age n to pay an annual premium $S_n^{(IVS)}$ to the insurance company until retirement in return for which the insurance promises to pay an annual benefit $B_n^{(IVS)}$ as pension income to the retired until death. Entering the voluntary pension insurance contracts may be at any working age n. Crucial is the

pension/premium ratio $B_n^{(IVS)} / S_n^{(IVS)} = G_n^{(IVS)}$ of the arrangement, which depends on the age at which the insurance contract is initiated. We denote those revenue rates by $G_{SW}^{(IVS)}, \ldots, G_{EW}^{(IVS)}$, where we assume a pension insurance policy may start in any working year; consequently an individual may enter successively into a cascade of insurance contracts. Similarly, for hoarding in cash we assume that if the individual at age n decides to hoard from now on each coming working year an amount $S_n^{(HO)}$, while the collected cash will be consumed in equal parts $B_n^{(HO)}$ during retirement. The hoarding contract is similar to the pension insurance contract but with a zero interest rate. Moreover, citizens have to participate in a mandatory fully funded pension fund (if it exists) with a premium $S^{(FF)}$ and an old–age benefit $B^{(FF)}$ and similarly for social security. The revenue rates for the collectivistic arrangements are taken to be uniform with respect to age. They are denoted by $G^{(FF)}$, $G^{(SS)}$, respectively.
A second distinction between the four arrangements is whether the arrangement is interest–bearing or not. This yields a classification as in Figure 1.

	Individual	**Collective**
Interest bearing	Pension insurance (IVS)	Mandatory occ. pens. (FF)
Not interest bearing	Hoarding	Social security (SS)

Fig. 1. Classification of pension arrangements.

The decisions about participation in those arrangements are made by different parties/actors. For the individualistic arrangement it is obvious that the different individuals make their own decisions. For the fully funded occupational pensions the decision is assumed to be in the hands of the trade union and we assume that within the trade union the cohort MW of median workers is decisive. They decide both on whether there will be a mandatory pension arrangement or not, and, if that decision is positive, how large the premium and the resulting benefit will be. Similarly, the existence and the size of social security is decided by Parliament and there the cohort MV of median voters is assumed to be decisive.
The revenue rates G differ per arrangement. It is a technical question how they are calculated, but here we can already see that they are not all affected by the same set of variables. The dependencies are laid out in Figure 2.

	n	β	μ	i
$G_n^{(IVS)}$	x		x	x
$G^{(FF)}$		x	x	x
$G^{(SS)}$		x	x	
$G_n^{(HO)}$	x		x	$(i=0)$

Fig.2. Determinants of *G.*

The revenue rate of individual policies depends on the age n of the individual when the policy is started, on the expected survival rates μ and the resulting longevity, and on the interest rate i at which the premiums are invested. For the fully-funded mandatory pension individual ages do not count, but the demography, characterized by birth and survival rates β and μ, is a determinant together with the interest rate i. For social security the demography, that is (β,μ), is relevant but the interest rate is irrelevant. For hoarding the age n at which the hoarding arrangement starts and the survival pattern μ count. Details are described in the next section.

Decision making.

Decisions on the four arrangements are made by the different actors by optimizing their remaining life utility functions, where they take into account their wage income w, their already standing other obligations OO_n like pension premiums agreed on in earlier contracts, mandatory pension premiums, and social security contributions, and their already secured other retirement benefits OB_n.

Decision making by the individual worker.

We start by looking for the individual savings pattern $S_{20}^{(IVS)}, S_{21}^{(IVS)}, ..., S_{EW}^{(IVS)}$, where we assume that individuals start working at the age of twenty and where we set *EW*=64, and where we also assume that negative saving is impossible. If one buys at the age of 20 and again at the age of 21, he will pay at the age of 21 a total premium amount of $S_{20}^{(IVS)} + S_{21}^{(IVS)}$. For simplicity we assume that the individual departs from the assumption that his wage *w* and savings will not change over the years to come. Of course, this can be replaced by assuming a variable wage profile, but this will not substantially change the results of our paper. Moreover, for most individuals the constant-wage-assumption seems to be a plausible behavioral assumption. On the other hand each succeeding year the remaining lifetime utility function will change, at least due to the fact that the period till retirement is reduced by one year.The remaining lifetime utility function $\bar{U}_n$ of the decision maker of age n looks like

$$\bar{U}_n = W_n.U(w - S_n^{(IVS)} - OO_n) + (1 - W_n).U(S_n^{(IVS)}.G_n^{(IVS)}(i) + OB_n) \qquad (3.1)$$

where $U(.)$ stands for the instantaneous utility function, with W_n the weight attached to remaining working life and $(1-W_n)$ the weight attached to the retirement period. The utility when working is $U(w - S_n^{(IVS)} - OO_n)$, where the argument stands for net consumption of workers and $S_n^{(IVS)}$ the decision variable. The amount $S_n^{(IVS)}.G_n^{(IVS)}$ is the annual pension benefit derived from the new pension insurance. Clearly, the premium/benefit ratio $G_n^{(IVS)}(i)$ is an increasing function of the prevailing interest rate i. The individual maximizes (3.1) with respect to $S_n^{(IVS)}$.Since $G_n^{(IVS)}$ increases with the interest rate, savings are a decreasing function in interest. The working individual has to take into account that there may be a mandatory occupational pension premium $S^{(FF)}$ and a mandatory social security contribution $S^{(SS)}$ to be paid as well and perhaps premiums on voluntary pension insurance contracts closed in previous years, say $S_{20}^{(IVS)},...,S_{n-1}^{(IVS)}$. We call these amounts *other obligations* OO_n for short. Similarly, we define *other benefits* OB_n, consisting of occupational pension $S^{(FF)}.G^{(FF)}$, social security benefit $S^{(SS)}.G^{(SS)}$ and voluntary pensions $S_{20}^{(IVS)}.G_{20}^{(IVS)},...,S_{n-1}^{(IVS)}.G_{n-1}^{(IVS)}$ stemming from earlier pension contracts.

Here we notice that we do not assume that an individual optimizes over a 60-period budget set, that is, he would program at the age of 20 his future consumption and savings at ages ten or thirty years ahead. We stick to the relatively more realistic assumption that the individual will continue savings at the same rate for the years ahead. On the other hand the individual revises his/her savings pattern each working year, where his utility function changes with age n.

If the instantaneous utility function is concave, i.e. the second-order derivative U'' <0, then it follows that the remaining lifetime utility function (3.1) is concave as well in $S_n^{(IVS)}$ and consequently has a unique maximum. Since we will exclude negative savings the optimum may be a corner solution with $S_n^{(IVS)}=0$, in words, zero savings. There is still another instance where engaging in a voluntary pension insurance is not the first choice. If $i<0$, a case which nowadays is not merely hypothetical, the individual is better-off by hoarding the savings in cash than investing the savings in a voluntary pension contract on a negative interest rate. In that case hoarding will yield $G_n^{(HO)} = G_n^{(IVS)}(0)$ per dollar saved. In that case the individual will maximize

$$\bar{U}_n = W_n.U(w - S_n^{(HO)} - OO_n) + (1 - W_n).U(S_n^{(HO)}.G_n^{(HO)} + OB_n) \qquad (3.1a)$$

We see from (3.1) that both an increase in OO_n and in OB_n will have a negative effect on the new savings.
The individual differentiates between four situations:

a. If $i > 0$ and $\left[\dfrac{d\bar{U}_n}{dS_n^{(IVS)}}\right]_{S_n^{(IVS)}=0} \geq 0$, then $S_n^{(IVS)} \geq 0$ and zero hoardings
b. If $i > 0$ and $\left[\dfrac{d\bar{U}_n}{dS_n^{(IVS)}}\right]_{S_n^{(IVS)}=0} < 0$, then $S_n^{(IVS)} = 0$ and zero hoardings
c. If $i = 0$ and $\left[\dfrac{d\bar{U}_n}{dS_n^{(HO)}}\right]_{S_n^{(HO)}=0} \geq 0$, then $S_n^{(HO)} \geq 0$
d. If $i = 0$ and $\left[\dfrac{d\bar{U}_n}{dS_n^{(HO)}}\right]_{S_n^{(HO)}=0} < 0$, then $S_n^{(HO)} = 0$

Cases b. and d. may occur if there is already enough mandatory pension (FF and/or SS).

Finally, there is the border case $i = 0$, where the individual is indifferent between saving in pension contracts or hoarding in cash. These conditions have to be satisfied for each working cohort. For instance, if individuals start working at 20 and stop at 65, it implies 45 conditions to be met.

The median worker (*MW*)
Consider now the special position of the median worker (MW). As an individual the median worker may either save individually or hoard according to the behavioral rules just specified. However, as a median worker he is also the deciding cohort in the population of workers, that is, the age cohorts from 20 to 65. The decision is here whether there should be a mandatory funded occupational pension or not, and if so, what should be the size of that pension premium $S^{(FF)}$. The pension-premium ratio for an individual pension starting at working age n is denoted by $G_n^{(IVS)}$. If the median worker decides for a fully-funded mandatory occupational pension the corresponding pension-premium ratio is denoted by $G^{(FF)}$. If the mandatory pension

covers all workers, it is identical to a pension contract starting at the beginning of the working period, i.e., at 20. Therefore, the corresponding pension/premium-ratio is then $G^{(FF)} = G_{20}^{(IVS)}$. Now it is obvious that $G_n^{(IVS)}$ is decreasing in n. More specifically, there holds $G_{MW}^{(IVS)} < G_{20}^{(IVS)}$. Hence, it follows that the median worker's first choice, if he is inclined to make additional pension savings, will be in favor of the mandatory occupational pension framework, since this presents better value for money than the individual contract would give. Hence, the median worker will not close an individual pension contract. However, it may be that the median worker feels he has already enough pension contracts collected anyhow, and then he will abstain from the new mandatory contract as well. As a consequence that negative decision would imply that there would be no mandatory occupational pension fund, because the median worker is the one who decides about its existence. If he goes for the mandatory pension fund by choosing a (for him) optimum premium $S^{(FF)}$, then every worker, old and young, will have to participate in it, because it is a *mandatory* pension arrangement. If i >0, the optimal premium is found by optimizing the remaining lifetime utility (3.1) with $n = MW$ and $G_n^{(IVS)}$ replaced by $G_{20}^{(IVS)}$. If i <0, the median worker will hoard as well and optimize (3.1a).

The median voter

For the behavior of the median voter the situation is a bit different. If individual saving yields a better pension, that is when $G_{MV}^{(IVS)} > G^{(SS)}$, then the median voter will opt for the individual arrangement either *IVS* or hoarding if $i < 0$, if he wants to create an additional pension. Then, there will not exist social security. If, on the contrary, $G_{MV}^{(IVS)} < G^{(SS)}$, he will choose for an additional social security pension, if he wants to create additional pension. The optimal premium is found by optimizing the remaining lifetime utility (3.1) with $n = MV$ and $G_n^{(IVS)}$ replaced by $G^{(SS)}$. We notice that social security functions according to a PAYG-system. We have

$$S^{(SS)}.P_{work|20} = B^{(SS)}.P_{ret|20}$$

Consequently $G^{(SS)} = \dfrac{B^{(SS)}}{S^{(SS)}} = \dfrac{P_{work|20}}{P_{ret|20}}$ is the old-age dependency ratio.

The inner equilibrium.

Since the joint optimization model just sketched consists of 2(65-20)+2(2)=94 interdependent first-order conditions in 94 unknowns and corner solutions are possible, analytical solution of this system is out of the question. Combining the conditions for $S^{(IVS)}$, $S^{(FF)}$, $S^{(SS)}$.and $S^{(HO)}$ the question is whether there is a numerical solution to the system for a given value interest rate i.

$$\begin{aligned} S^{(IVS)} &= f_{IVS}(S^{(FF)},S^{(SS)},S^{(HO)}\,|\,w,i) \\ S^{(FF)} &= f_{FF}(S^{(IVS)},S^{(SS)},S^{(HO)}\,|\,w,i) \qquad \forall S \geq 0 \\ S^{(SS)} &= f_{SS}(S^{(IVS)},S^{(FF)},S^{(HO)}\,|\,w,i) \end{aligned} \tag{3.2}$$

where the $f(.)$'s are short-hand notation for the optimization outcomes above. It appears that it is possible to find an equilibrium by iteration. In practice, as we will see later on, we found always a unique equilibrium. This equilibrium we call the 'inner' equilibrium. It depends on the interest rate i.

Capital, interest, and wages; the outer equilibrium.

We notice that wage w and interest rate i have been taken to be exogenous up to this point. If we assume a small and open economy we may accept this exogeneity. However, at the moment pension funds and other institutional investors are the main suppliers of capital in most countries. Hence, we have to take wage and interest rates as endogenous. This suggests that we introduce a capital market. We assume for simplicity that capital supply is only provided by individual savings and by the reserves of the occupational pension fund. Hence, we may write for the individual and collective accumulated savings

$$\begin{aligned} K^{(IVS)} &= K^{(IVS)}.(S^{(IVS)};i,D) \\ K^{(FF)} &= K^{(FF)}.(S^{(FF)};i,D) \end{aligned} \tag{3.3}$$

where we summarize the demographic variables by D for the moment. This total capital supply yields the capital per workplace of

$$k_{\text{sup}} = \left(K^{(IVS)} + K^{(FF)}\right) / N.P_{work|20} \tag{3.4}$$

We notice that this capital supply is a function of the interest rate i, since $G^{(IVS)}$ and $G^{(FF)}$ depend on i. Looking at (3.1) it is seen that an interest increase leads to a decline in voluntary and mandatory savings. It follows that capital supply is decreasing in interest. We close the model by introducing a capital demand function per workplace, denoted by $k_{dem}(i)$, standing for the demand of an optimizing firm owner. The function $k_{dem}(i)$ is monotonically decreasing in i as well. For a stable full employment equilibrium $k_{dem}(i)$ is derived by maximizing the profit per workplace, which is

$$f(k) - w - (i + \delta + \nu).k \tag{3.5}$$

where we assume a production per workplace function $f(k)$. Capital costs consist of three components, viz., interest, depreciation, and new investment to cope with

population growth (or decline) to ensure a constant capital per head of the population.
In our specified model to be specified hereafter we find that both capital supply and demand fall with increasing interest, but that demand for zero interest is much higher than supply, while the demand curve is much steeper falling than the supply curve with increasing interest. Consequently, there is one point of intersection where $k_{dem}(i) = k_{\sup}(i)$; the equalizing value of i is the equilibrium interest rate. An example is sketched in Fig.3. For the explanation of the parameters see below. This equilibrium is called the 'outer' equilibrium.

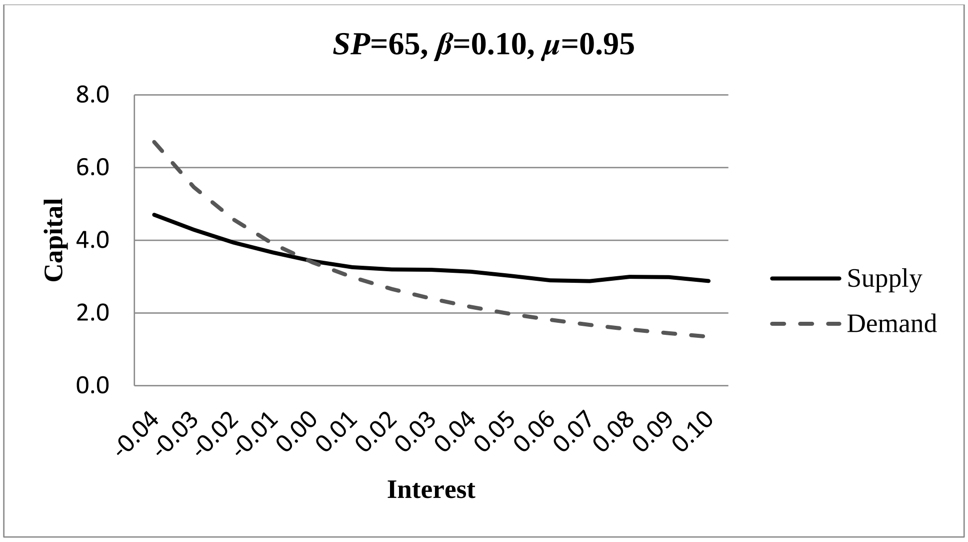

Fig.3. Capital Demand and Supply curve

It stands to reason that this solving for the equilibrium interest on the capital market requires an iterative solution. It follows that we have two sequential iteration processes: the inner loop which gives the total capital supply function per workplace as a function of the interest rate, and the outer loop in which the equilibrium interest rate is found at which supply and demand curves intersect each other.
In the next section we will specify the model by choosing specific functions and parameter values. If the supply and demand curves intersect each other for a negative interest rate i, hoarding of part of the supply becomes relevant. The interest rate t is reaches its lower bound at $i = 0$ and the difference between supply and demand at $i = 0$ is hoarded. It is not determined whether the hoarding is done wholly by individuals or by the pension fund or by a mixture of both.

4. Specifications.

When we like to get insight in the effects of changing demographics we have to specify the above model. That means that we have to assume a specific demography, a specific instantaneous utility function, and a production function, and that we have to calculate the multiplication factors G. But as soon as we specify these ingredients we may meet the objections that the demography studied is not realistic, that utility functions and production functions should be replaced by others, etc. Let us say here explicitly that the model we will use is not intended to be realistic in the sense that it predicts the development of a specific country. This is also impossible, because in no country there is a stable population, that is, in which birth rates and survival rates are constant over time. However, the stable population reflects a population towards which the present population would tend if present birth rates and survival rates would remain constant from now on into the future. The same holds for the choice of production functions and utility functions. There are many different estimates of those functions. We shall make a choice such that the resulting model is plausible. We shall not make an attempt to calibrate the model resembling a specific country economy, but we chose some parameter values in order to get outcomes not too far away of what we observe in developed western economies. If one wants to use different parameter values or functional specifications one is free to do so. The theoretical model and the computer program are easily adaptable.

Demography.

The population at time t is described by a vector $N_t^{'} = (N_{0,t},...,N_{100,t})$ where $N_{n,t}$ stands for the number of people of age n at time t. The population develops according to an equation system

$$\begin{aligned} N_{0,t+1} &= \beta' N_t \\ N_{-,t+1} &= MN_{-,t} \end{aligned} \tag{4.1}$$

where $N_{0,t}$ stands for the number of newborn at time t, and $N'_{-,t} = (N_{1,t}, \ldots, N_{100,t})$ stands for the vector of age cohorts from 1 to 100, where β stands for a vector of (age-differentiated) birth rates and where M stands for a diagonal (100×100)-matrix of (age-differentiated) survival rates. The diagonal elements are also denoted as the vector μ. We assume that there is a fertility period during which individuals get children. This fertility period starts at the age of 25 and ends at 34. During that period the annual birth rate is taken to be constant at β. Since there is made no difference between males and females, it turns out that at β=10% a couple is just reproducing (the expected number of children is 2(=20β), if we exclude child mortality as we do, and, consequently, the population growth rate is ν=0%. This parameter choice makes our model to resemble the demographic situation in many western countries at this moment. In order to investigate the effect of changes in the birth rate we simulate the model for β =7.5%, 10%, ..,30% where β ==7.5% stands for 1.5 child per couple and β =30% stands for 6 children per couple. It is well-known from demographic theory that along the equilibrium path the population is growing at a constant rate $1+\nu$ and has a stationary age distribution $p=(p_0,\ldots,p_{20},\ldots,p_{65},\ldots,p_{100})$. We shall assume for ease of exposition no mortality $\mu=1$ before 65 and a constant annual survival rate $\mu<1$ from the age of 65 onwards. We will vary the annual survival rate from 0.93 to 0.98.

We assume that individuals start working (SW, StartWork) at the age of 20 and retire when they have reached the retirement age SP (StartPension). Their last working year is EW (EndWork= $SP-1$) Silently assuming that individuals younger than 20 do not work, from here on we denote the population share of the workers in the age interval $[SW,EW]$ by P_{work} and the share of the retired in the age interval $[SP,100]$ by P_{ret}, where we normalize such that $P_{work}+P_{ret}=1$. We will vary SP from 63 to 72.

In order to get insight in the effects of demographic key parameters we present Table 1. It is obvious that we can opt for a more sophisticated demographic model where the birth rate and survival patterns vary continuously with age; computationally this is no problem. However, it implies that birth and survival patterns cannot be easily characterized by only one parameter each, which would obscure our analysis.

Table 1, Effect of demography on the demographic key variables

birth rate (β)	survival rate (μ)	retirement rate (SP)	population growth %	life expectancy	median worker	median voter	Dependen-cy ratio
0.075	0.95	65	-0.9	81	44	55	1.92
0.10	0.95	65	0.0	81	42	50	2.67
0.15	0.95	65	1.3	81	39	44	4.29
0.20	0.95	65	2.3	81	36	40	6.05
0.25	0.95	65	3.0	81	35	38	7.94
0.30	0.95	65	3.6	81	34	36	9.95
0.10	**0.93**	65	0.0	77	42	49	3.40
0.10	**0.94**	65	0.0	79	42	49	3.03
0.10	**0.95**	65	0.0	81	42	50	2.67
0.10	**0.96**	65	0.0	83	42	52	2.34
0.10	**0.97**	65	0.0	87	42	53	2.03
0.10	**0.98**	65	0.0	90	42	55	1.74
0.10	0.95	**63**	0.0	81	41	50	2.28
0.10	0.95	**64**	0.0	81	41	50	2.47
0.10	0.95	**65**	0.0	81	42	50	2.67
0.10	0.95	**66**	0.0	81	42	50	2.90
0.10	0.95	**67**	0.0	81	43	50	3.15
0.10	0.95	**68**	0.0	81	43	50	3.42
0.10	0.95	**69**	0.0	81	44	50	3.71

0.10	0.95	**70**	0.0	81	44	50	4.02
0.10	0.95	**71**	0.0	81	45	50	4.36
0.10	0.95	**72**	0.0	81	45	50	4.72
0.075	**0.98**	**70**	-0.9	90	47	61	1.65

Pension systems.
As said before we distinguish between four pension systems:

a. Individual Voluntary Savings (IVS)
b. Mandatory Fully Funded pension (FF)
c. Mandatory Social Security (SS)
d. Hoarding (HO)

In modern societies we mostly find a mixture of those systems simultaneously present, although the sizes differ between economies. The pension composition (PC) may be described by a vector

$$(S_{SW}^{(IVS)},...,S_{EW}^{(IVS)},B_{SW}^{(IVS)},...,B_{EW}^{(IVS)},S^{(FF)},B^{(FF)},S^{(SS)},B^{(SS)},S_{SW}^{(HO)},...,S_{EW}^{(HO)},B_{SW}^{(HO)},...,B_{EW}^{(HO)}).$$

or more shortly $(S^{(IVS)},S^{(FF)},S^{(SS)},S^{(HO)})$. We assume that under voluntary individual saving the individual at age n $(SW,...,EW)$ buys a pension insurance contract according to which he agrees to pay a premium $S_n^{(IVS)}$ for the rest of his working life in exchange for an annual pension of $B_n^{(IVS)}$, starting at the retirement age. The link between premiums and benefits is given by the actuarial balance equation

$$S_n^{(IVS)}.\left[1+\frac{1}{1+i}+...+\left(\frac{1}{1+i}\right)^{EW-n}\right]=B_n^{(IVS)}.\left[\left(\frac{1}{1+i}\right)^{SP-n}.\mu+...+\left(\frac{1}{1+i}\right)^{EP-n}.\mu^{EP-EW}\right] \quad (4.2)$$

We define the benefit-premium ratio $G_n^{(IVS)}$ by $B_n^{(IVS)}=G_n^{(IVS)}.S_n^{(IVS)}$. Benefits are proportional to the premium paid. The sum of those benefits at the start of retirement, that is total individual pension, will be denoted by $\widehat{B}_{EW}^{(IVS)}=\sum_{n=SW}^{EW}B_n^{(IVS)}$. For the hoarding benefits we get, similarly, $\widehat{B}_{EW}^{(HO)}=\sum_{n=SW}^{EW}B_n^{(HO)}$. In a similar way we denote the mandatory funded pension by its premium $S^{(FF)}$ and the corresponding benefit by $B^{(FF)}$, and the social security contribution by $S^{(SS)}$ and the corresponding benefit by $B^{(SS)}$. Since all age groups from $SW=20$ are obliged to participate in the mandatory system, this mandatory insurance is identical with the voluntary insurance in which we may participate at the age of 20. It follows that $G^{(FF)}=G_{20}^{(IVS)}$. In Figure 4 we sketch the behavior of the $G's$ as functions of the interest rate i.

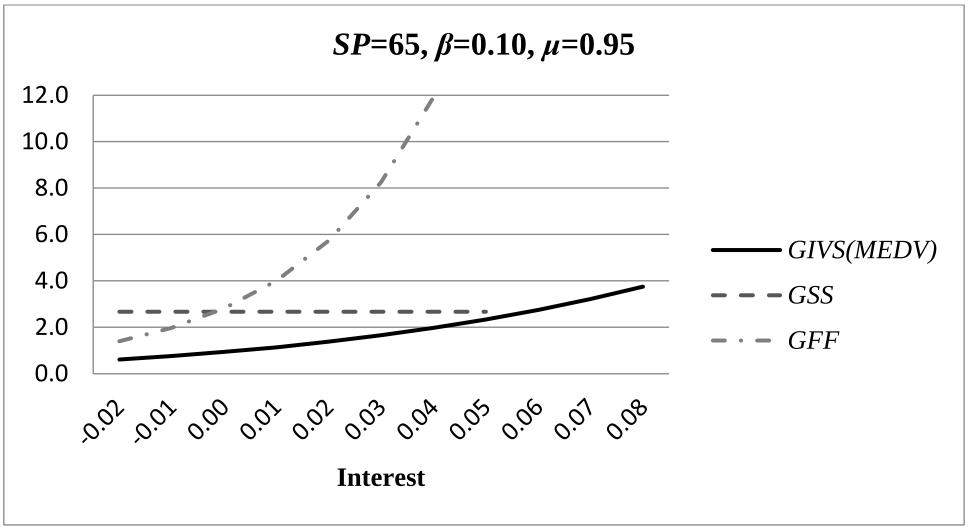

Fig. 4. The behavior of $G_{MV}^{(IVS)}, G^{(SS)}, G^{(FF)}$ as a function of the interest rate.

The median voter may make a choice between IVS and social security. If $G^{(SS)} < G_{MV}^{(IVS)}$, he will prefer to save individually instead of contributing to a social security system. This may be the case for high interest rates. In Fig.4. this occurs if the interest rate exceeds about 5.5%. If the median voter prefers the individual pension, this will entail that there will not be a majority for a social security system in the society.

In some societies the institutional structure may be such that not all three systems are at work. For instance, in Chile there is no social security arrangement for old-age pensions on a PAYG-basis. In other countries occupational pensions are mostly run on a pay-as-you-go basis. Hoarding in cash is a primitive last method of saving for old-age. We refer to OECD (2017) for an international survey. In countries with a lacking banking system individual saving may be nearly impossible. In this paper we will start to assume that all three pension schedules are accessible, even if some of those schedules are not actually used. We assume that all voluntary and mandatory savings by individuals are eventually aimed at safeguarding an old-age pension. The retirement age SP is fixed here at 65. Later on we shall vary the retirement age as well.

Capital supply.

The resulting aggregate of individual saving reserves per working adult of age $n \le EW$, where EW is set at 64, is

$$RES_n^{(IVS)} = \sum_{j=20}^{n} S_j^{(IVS)} . \sum_{m=j}^{n} (1+i)^{m-j} \qquad (4.3a)$$

The individual reserves for a retiree at age $n \geq 65$ are the present values of the future benefit flow

$$RES_n^{(IVS)} = \widehat{B}^{(IVS)}.\sum_{t=n}^{99}\left(\frac{\mu}{1+i}\right)^{t-n} \qquad n \geq 65 \tag{4.3b}$$

It follows that the average IVS-reserve per head in the adult population is

$$RES^{(IVS)} = \sum_{n=20}^{64} p_{n|20}.RES_n^{(IVS)} + \sum_{n=65}^{100} p_{n|20}.RES_n^{(IVS)} \tag{4.3}$$

The per capita reserve in the mandatory FF-system is calculated likewise. It equals the individual pension contract for $n = 20$, where the premium $S^{(FF)}$ is determined by the median worker. Hence, using the formulas (4.3) we get

$$RES^{(FF)} = S^{(FF)}\sum_{j=SW}^{EW} p_{j|SW}\cdot\sum_{m=j}^{EW}(1+i)^{m-j} + S^{(FF)}.G^{(FF)}.\sum_{j=SP}^{EP} p_{j|SW}\cdot\sum_{t=j}^{EP-1}\left(\frac{\mu}{1+i}\right)^{EP-j-1} \tag{4.3a}$$

For other values of the retirement age EW the formulas have to be slightly changed because mortality may start before retirement when individuals are still at work or after retirement.

The total capital supply per worker is the sum of individual and collective savings. We have

$$k_S(i) = \left(RES^{(IVS)}(i) + RES^{(FF)}(i)\right) / P_{work} \tag{4.4}$$

Since social security is run on a pay-as-you-basis it does not generate a reserve. The same holds for hoarding.

Parameter values.

The choice of specific parameter values is a delicate one. There are many different estimates and they vary also between countries, between moments of estimation and between the empirical estimation methods used. Since we are developing a general theory and our numerical simulations are only intended to get qualitative insights, we abstain from calibrating our parameter values in order to fit one specific country at a specific moment in time.

For the instantaneous utility function we take the well-known Constant Relative Risk Aversion (CRRA) specification $U(y) = y^{1-\gamma}/(1-\gamma)$, where we take $\gamma = 3$. In the literature there are many estimates for γ, but they vary over a great range. See e.g. Gandelman and Hernandez-Murillo (2015) and the recent survey by Outreville (2015), see also Booij and Van Praag (2009). The value of 3 is somewhere in the middle of recent empirical estimates, but there is much uncertainty about it. The time weights are assumed to be

$$W_n = \sum_{m=n}^{EW} \rho^{m-n} \Big/ \sum_{m=n}^{EP} \rho^{m-n}$$

where the subjective time discount rate is set equal to 0.89. There is a host of different estimates for ρ as well, but for macro-economic long-term decision settings this value seems to be in the middle of the range. (see Shane, Loewenstein, O'Donoghue (2002)). It appears that outcomes of the model are very sensitive with respect to the value of ρ. We tried several values.

For the production function we take the traditional Cobb-Douglas function $Y = C.K^{\alpha} L^{1-\alpha}$ where we set α =0.25. This value is debatable as well, since the capital elasticity varies a lot between industries and it seems to increase over the years (see Piketty (2014), Karabarbounis, Neiman (2014)). Nevertheless, as an average it is still a plausible value. Finally, we assume the depreciation rate to be δ =10%. Also here the value of the macro-depreciation rate is rather uncertain. We refer to Nadiri and Prucha (1996) and a recent very down-to-earth but detailed catalogue of depreciation rates as prescribed by the New Zealand tax authorities (Taake (2017)).

5. Description of the numerical solution.

We start to solve the system (3.2) by iteration according to the schedule

$$\begin{aligned} S^{(m,IVS)} &= f_{IVS}(S^{(m-1,FF)}, S^{(m-1,SS)} | w,i) \\ S^{(m,FF)} &= f_{FF}(S^{(m,IVS)}, S^{(m-1,SS)} | w,i) \\ S^{(m,SS)} &= f_{SS}(S^{(m,IVS)}, S^{(m-1,FF)} | w,i) \end{aligned} \tag{5.1}$$

where m *is the iteration step.* We start with $S^{(0,FF)}$, $S^{(0,SS)}$=0, $i^{(0)} = -(\nu + \delta)$ and w, as defined below by (5.2), (5.3) for $i^{(0)} = -(\nu + \delta)$. In practice, the system (5.1) always converges to a unique equilibrium for every value of i, although we were unable to prove this analytically. Mostly, the iteration process takes about six rounds. For a given i we find hence $S^{(m,IVS)}(i)$, $S^{(m,FF)}(i)$, $S^{(m,SS)}(i)$. Finally, we calculate the capital supply $k_S(i)$ per worker according to (4.2), (4.3) and (4.4). We call this iteration process (5.1) the 'inner loop', and the resulting equilibrium the ' inner' equilibrium. It depends on the interest rate i.

Assuming a Cobb-Douglas production function and capital costs consisting of interest, depreciation and net investment in order that the capital per worker keeps pace with population growth ν, the demand per worker for capital by a profit-maximising firm for a given i is found by solving the first-order-condition $f'(k_D(i)) = \left(i + \nu + \delta\right).k_D(i)$. We notice that there has to hold $\left(i + \nu + \delta\right) > 0$. It follows that a negative interest is possible, where $i > -(\nu + \delta)$. We get a capital demand function

$$k_D(i) = \left(\frac{\alpha}{i + \nu + \delta} \right)^{1/(1-\alpha)} \tag{5.2}$$

while the corresponding wage rate is

$$w(i) = f(k_D(i)) - (i + \nu + \delta).k_D(i) \tag{5.3}$$

Now we have to confront capital demand and supply on the capital market. There is equilibrium on the capital market if $k_S(i) = k_D(i)$. This equilibrium rate of interest is also found by iteration, which we call the 'outer loop'. It takes normally only a few rounds. The corresponding value of i, say $\bar{i}$, is the equilibrium interest, and departing from that value (5.1) provides us with the $\bar{S}^{(IVS)}(i)$, $\bar{S}^{(FF)}(i)$, $\bar{S}^{(SS)}(i)$. This is called the 'outer' equilibrium. In fig.3 above we sketched the demand and supply curve for β =0.10, μ =0.95, SP =65 and ρ =0.89 ,and $\delta = 0.10, \alpha = 0.25$.

We will see from our numerical examples in the next section that the equilibrium interest rate thus found may turn out to be negative. There are examples for 'old' populations, that is, with a low birth rate and/or a high life expectancy, where the equilibrium rate would be negative. We give one example in Table 2. This is, of course, not attractive for savers. In such a situation hoarding at an effective rate of interest of 0% is favored above bringing the money to the bank or the capital market where the revenue would be negative. Hence, there is an effective lower bound on the interest rate at 0%. It implies that there may be an oversupply of capital, where part of the savings are hoarded, since not all capital supply can be invested at a non-negative interest rate. Whether this hoarding is done by individuals or by pension funds or both is irrelevant. We will find one instance in the numerical results below.

6. Outcomes for a closed economy.

In this section we present the equilibria for four basic sequences. We take as point of departure a birth rate of β =0.10, equivalent to on the average two children per couple and zero population growth, a survival rate of μ =0.95 and a retirement age EW *=64*, i.e., pension payments start at 65. The subjective time preference rate ρ will be taken at 0.89. The depreciation rate $\delta = 10\%$ and capital productivity $\alpha = 0.25$.

The subjective time preference rate.

We start by varying ρ from 0.88 up to 0.92, while setting β =0.10, μ =0.95 and SP =65.

Table 2. The effect of subjective time preference on the pension composition

Sub-jective time preference rate (ρ)	Indiv. savings start/ finish %	Private pen-sion %	Pre-mium fully funded %	Fully funded pen-sion %	Pre-mium social secu-rity %	Social secu-rity pen-sion %	Net wage %	Total pen-sion %	Inte-rest rate %	Gross wage	Capital-income ratio	Capital	Hoar-ding capital %
	1	2	3	4	5	6	7	8	9	10	11	12	13
ρ =0.88	0.7 11.1	23.6	2.7	15.5	2.2	5.8	84.1	44.8	2.0	0.957	2.774	2.656	-
ρ =0.89	1.0 8.5	23.0	3.0	13.8	3.0	8.1	85.5	45.0	1.4	0.974	2.916	2.839	-
ρ =0.90	1.7 8.1	22.7	3.3	12.2	3.9	10.4	84.7	45.4	0.8	0.993	3.092	3.069	-
ρ =0.91	2.6 9.5	22.6	3.5	10.6	4.8	12.8	82.2	46.0	0.2	1.012	3.277	3.317	-
ρ =0.92	3.8 11.1	22.5	3.8	9.2	5.6	15.0	79.4	46.6	-0.4	1.033	3.486	3.602	-
	4.0 10.8	25.3	3.7	10.4	4.9	13.1	80.5	48.9	0.0	1.018	3.333	3.393	10.5

Birth rate (β)=0.10, survival rate (μ)=0.95 and age of retirement (*SP*)=65. For legend see Appendix.

An increase in ρ implies that all parties put more weight to the retirement period. This will result in more capital and/or more social security. More savings will be reflected in more capital per worker and indeed we see that capital per worker increases from 2.656 to 3.393. This is a capital elasticity with respect to ρ in the order of 5. It seems to imply, intuitively not implausible, that the outcomes are rather sensitive with respect to ρ . We see that with an increase in ρ the equilibrium interest rate falls from 2% to 0%. In the fifth line of the table we see that interest rate would become negative, if we exclude the possibility of hoarding. At the last line the outcomes are presented when hoarding is applied, i.e., when the interest rate is fixed at a lower limit of 0%. The amount which is hoarded in the last situation is about 10% according to the last column[2]. The net benefit–ratio is about 48.9/80.5, that is about 60%. The total savings ratio is about 19.4% of which 4.9% is spent on social security. Finally, we have a look on voluntary savings behavior over life. The individual with ρ =0.88 starts at twenty with a tiny individual savings ratio of 0.7% which increases over life to 11.1% just before retirement. For an individual with a higher time preference of 0.92 the corresponding ratios are 4% and 10.8%, respectively.

[2] Notice that in this model direct hoarding by the individuals or deposits at 0% in the bank yield the same result. If we assume that banks will charge for hoarding costs, that is tantamount to a negative interest, e.g. -1%, individuals will prefer to hoard at home.

Increasing longevity.

We will now consider in Table 3 how the equilibrium changes if the survival rate μ is varied from 0.93 up to 0.98, keeping β =0.10, SP =65 and ρ =0.89.

Table 3. The effect of ageing on the pension composition

Survival rate (μ)	Indiv. savings start/ finish %	Private pen-sion %	Pre-mium fully funded %	Fully funded pen-sion %	Pre-mium social secu-rity %	Social secu-rity pen-sion %	Net wage %	Total pen-sion %	Inte-rest rate %	Gross wage	Capital-income ratio	Capital
	1	2	3	4	5	6	7	8	9	10	11	12
μ=0.93	1.5 11.8	33.9	2.5	19.1	0.6	2.1	85.0	55.1	2.1	0.955	2.751	2.627
μ=0.94	1.5 11.7	28.6	2.8	15.5	1.7	5.2	83.8	49.3	1.6	0.969	2.871	2.781
μ=0.95	1.0 8.5	23.0	3.0	13.8	3.0	8.1	85.5	45.0	1.4	0.974	2.916	2.839
μ=0.96	0.3 6.2	17.9	3.1	13.3	4.9	11.4	85.8	42.7	1.6	0.970	2.881	2.793
μ=0.97	0.0 6.3	14.0	3.3	11.5	6.7	13.5	83.6	39.0	1.3	0.976	2.943	2.873
μ=0.98	0.0 5.3	9.9	3.5	11.1	9.1	15.9	82.0	36.9	1.5	0.972	2.904	2.824

Birth rate (β)=0.10, age of retirement (SP)=65 and time discount (ρ)=0.89. For legend see Appendix.

Similarly, we will look at the effects for values β =0.075, 0.10, 0.15,..., 030 and the effects when the retirement age is increased from 63 up to 72. Here the interesting changes are seen in savings behavior. Individual savings dwindle when life expectation increases while the mandatory schedules gain in weight. The occupational pension premium increases from 2.5% to 3.5% and social security from 0.6% to 9.1%. While the ratio of fully funded pension to the social security benefit 19.1/2.1=9.1 for μ =0.93 (life expectancy 77) that ratio changes into 0.70 for μ =0.98 (life expectancy 90). Or in other words, individual pensions amount to 61% of total pension, the mandatory occupational pension to 35% and social security to a meager 4% of total pension for μ =0.93. For a rather old population these fractions are 27%, 30% and 43%, respectively. Total pension as a fraction of gross wage falls from 55.1% to 36.9% and the net-benefit ratio falls rather dramatically from about 65% to 45%. The situation of workers does not change dramatically, but the situation of pensioners deteriorates dramatically. If we may believe these figures, at least qualitatively, the future for an ageing society seems to be bleak.

Increasing birth rate

The effect of a varying birth rate is not so straightforward. A consequence of a rising birth rate is a strongly growing labor force. The effect when capital is unchanged is that capital per worker becomes scarcer. It follows that the interest rate will increase while gross wage will fall. Indeed we see that the interest rate increases from a moderate 1.4% to 15.2% when the number of children increases from 1.5 to 6 per couple. Actually, the interest rate grows much faster than the population growth rate. Gross wages fall from 1.002 to 0.715 and capital per job falls from 3.186 to 0.826. This capital thinning is due to the fact that the ratio of workers to retired increases (see Table 1) from 1.92 to 9.95.

The tremendous increase in the interest rate to about 15% makes voluntary and mandatory saving very profitable with as a result very tiny savings, while social security vanishes. When the birth rate rises the situation of the retired relative to that of the workers improves very much to such an extent that retirees' pensions are much larger than net wages, surprisingly indeed. For western countries where the birthrate hovers around 0.10 or below we get rather low interest rates. Countries where the birth rate is still 0.20 or above are nowadays the less developed economies.

Table 4. The effect of changes in the birth rate on the pension composition

Birth rate (β)	Indiv. savings start/ finish %	Private pen-sion %	Pre-mium fully funded %	Fully funded pen-sion %	Pre-mium social secu-rity %	Social secu-rity pen-sion %	Net wage %	Total pen-sion %	Inte-rest rate %	Gross wage	Capital-income ratio	Capital
	1	2	3	4	5	6	7	8	9	10	11	12
β=0.075	0.0 6.6	19.2	3.4	15.8	5.4	10.4	84.6	45.5	1.4	1.002	3.179	3.186
β=0.10	1.0 8.5	23.0	3.0	13.8	3.0	8.1	85.5	45.0	1.4	0.974	2.916	2.839
β=0.15	1.8 14.6	30.4	2.3	13.3	1.2	5.3	81.9	49.0	2.1	0.923	2.486	2.295
β=0.20	2.1 15.8	40.3	1.6	13.9	0.3	1.7	82.3	55.9	3.2	0.880	2.156	1.898
β=0.25	0.0 0.0	0.0	0.2	139.7	0.0	0.0	99.8	139.7	14.8	0.724	1.197	0.866
β=0.30	0.1 0.1	81.6	0.1	81.5	0.0	0.0	99.8	163.1	15.2	0.715	1.155	0.826

Survival rate (μ)=0.95, age of retirement (SP)=65 and time discount (ρ)=0.89. For legend see Appendix.

In those countries the whole pension system is clearly different from the one in our model as frequently there is not a well-developed IVS, FF, and/or SS-system.

Moreover, the demography is rather different from ours with respect to the survival rate and the same probably holds for the subjective time discount rate ρ.

Increasing retirement age

Finally, let us consider the effect of the retirement age. We assume here that individuals of 72 are as efficient workers as those of 63, which is improbable in reality. We see a similar phenomenon as when the birth rate increases. Capital has to be spread over more workers with the effect that gross wages decrease and interest increases. The inequality between workers and retired decreases when the retirement age increases. If the retirement age increases to 71 we find that retired become even better-off than the workers.

Table 5. The effect of changes in the retirement age on the pension composition

Retire-ment age (SP)	Indiv. savings start/ finish %	Private pen-sion %	Pre-mium fully funded %	Fully funded pen-sion %	Pre-mium social secu-rity %	Social secu-rity pen-sion %	Net wage %	Total pen-sion %	Inte-rest rate %	Gross wage	Capital-income ratio	Capital
	1	2	3	4	5	6	7	8	9	10	11	12
SP=63	0.7 7.3	18.7	3.3	11.9	4.9	11.3	84.4	42.0	1.2	0.980	2.977	2.919
SP=64	1.0 6.7	20.7	3.0	13.0	4.1	10.1	86.2	43.7	1.4	0.973	2.914	2.837
SP=65	1.0 8.5	23.0	3.0	13.8	3.0	8.1	85.5	45.0	1.4	0.974	2.916	2.839
SP=66	1.1 9.9	25.9	2.7	15.2	2.1	6.0	85.4	47.1	1.7	0.965	2.841	2.741
SP=67	1.2 10.9	28.9	2.6	16.1	1.3	4.1	85.1	49.1	1.7	0.965	2.844	2.746
SP=68	1.3 11.5	33.2	2.3	17.6	0.4	1.5	85.7	52.3	2.1	0.956	2.763	2.642
SP=69	0.2 2.5	28.9	1.6	35.5	0.0	0.0	95.8	64.4	4.5	0.899	2.300	2.069
SP=70	0.0 0.0	0.0	0.5	96.5	0.0	0.0	99.5	96.5	9.5	0.815	1.709	1.393
SP=71	0.0 0.0	0.0	0.5	108.0	0.0	0.0	99.5	108.0	9.4	0.816	1.716	1.400
SP=72	0.0 0.0	0.0	0.2	137.1	0.0	0.0	99.8	137.1	11.7	0.786	1.535	1.206

Birth rate (β)=0.10, survival rate (μ)=0.95 and time discount (ρ)=0.89. For legend see Appendix.

6. Discussion and evaluation.

In this section we want to answer the following questions:

a. In how far is the model realistic?
b. What is the political relevance?
c. How does it fit in the economic literature? What is new?

Realism.

The dilemma behind economic modeling is that we have to choose between realism and transparency. If we choose for realism we may end up with a jungle of details, actors, relationships and variables. At the other extreme we may have a simple elegant and transparent model, but it is so far simplified and stylized that it cannot be seen as a relevant description of reality. Moreover, by leaving out essential variables we may find strongly biased effects of the remaining variables. We have looked for a compromise. Hence, some of our readers will object that our model is not realistic enough while others will complain that given the multiple simultaneous non-linear relationships in the model we do not always get monotonic clear-cut effects which can be economically interpreted. However, the model in this paper can be easily extended to a more realistic demography, a heterogeneous labor force, a heterogeneous industrial sector, etc. It has to be seen as a first step. The main objective is to present a fresh way of thinking on the pension problem, which may be a stepping-stone to investigate the effects of changing demographies and retirement ages.

In this study we assume a stationary demography, that is, a fixed population growth rate (which is negative for $\beta < 0.10$) and a fixed age distribution. Clearly, this is unrealistic since the demographic parameters, i.e., birth and mortality rates, are never constant over time. However, since all demographic parameters change from one year to another, it is not helpful either to depart from a specific population, say the American or the British, in a specific year, say 2016, and to follow that population over the years, when one wants to get insight into the general effects of demographic changes. The model, which we developed, can be put into a dynamic version, not starting from a stationary equilibrium. But, even if we assume birth and survival rates to stay constant from now on, from disequilibrium to reach the equilibrium path would take many decades or even centuries. The attractiveness and the usefulness of studying an equilibrium model is that one can abstract from the specific peculiarities of different real situations, random shocks, intertemporal changes in values of model parameters, or specifications of behavioral equations. We consider the dynamic equilibrium as the basic structure behind the reality. For country-specific studies that start from an actual demographic disequilibrium we refer a.o. to Krueger and Ludwig (2007), Börsch-Supan and Ludwig (2010), Miles (1999).

One central, and to our knowledge novel, point in our analysis is that we admit for the simultaneous existence of four old-age support arrangements, viz., individual voluntary saving, funded occupational pension, social security on a PAYG-basis, and as a residual hoarding. We call the mix the pension composition. That composition is not exogenously determined, but it is the joint result of the independent decisions of several parties, viz., all individual workers deciding about their individual savings, the median worker (or trade union) as representative for the body of workers deciding about the mandatory occupational pension system, and the median voter as representative for the electorate deciding on the existence and the size of social security (see also e.g. Galasso (2008), Galasso and Profeta (2004), Bruce and Turnovsky (2013)). All decision makers act within a specific demography and this demography determines their decisions in the last resort. And therefore the main macro-economic variables like wages, interest and investments are in this model in the end determined by the demography.

We assume that in the economy the sources for capital investment are voluntary savings and mandatory savings for old age. In our time the weight of institutional pension funds is becoming overwhelming. We refer to Boeri, et al. (2006), Bijlsma, Van Ewijk, and Haaijen (2014), Conference Board (2010), see also Mitchell (2008). It would have been possible within this model to make an extension such that individuals could also save for private investment without the explicit goal of old-age provision, but this would not have changed the essential message of this paper. Moreover, our addition of a mandatory funded occupational pension, where the median worker decides about the existence and the size of the pension is also novel.

With respect to individual saving most authors assume that utility is maximized subject to a budget constraint, where it is assumed that savings from one year may be used for consumption next year in order to smooth consumption and that the citizen plans his savings and dis-savings for each of the future periods over the remaining lifetime. This depicts a perfectly rational individual who has perfect knowledge about his future using the Euler conditions. But is such an assumption realistic when we face a future of about sixty years with sixty decision moments? Apart from the heroic assumption about the deciding capacities of the individual is it possible to know what will be the situation in the future many decades ahead? Instead, we make the rather naïve saving assumption that individuals expect their wage and their annual savings to be constant over the years ahead. Each year to come the individual will revise his savings decision based on the most recent situation. Although both assumptions do not seem perfectly realistic, we think that our assumption might be nearer to the truth in describing the savings behavior of ordinary humans then assuming an individual with perfect foresight on his lifetime 60-period-budget equation. Our model can be generalized to encompass more general savings assumptions. For instance, we may assume that individuals decide each year about their savings for the current year only.

The main results of our study are:

a. The finding that the room for pension policy is rather restricted, because demography is the main determinant for the long-term equilibrium. It seems there are only a few possible political measures which all deal with the structure of old-age provisions: we may exclude one or more of the channels IVS, FF, or SS[3]. In this paper we looked only at the combination IVS+FF+SS. Moreover, we may change the legal retirement age.

b. The demography appears to be a fundamental determinant of macro-economics, having effects on the wage rate, the interest rate and capital per workplace. This suggests that population policy could (or even should) be a powerful instrument for reaching macro-economic targets (cf. Lee and Mason (2010)).

c. Ageing of the population will result in a severe worsening of the net income of the retired.

d. Ageing will also strongly increase the inequality between net wages and pensions to the disadvantage of the retired.

e. Increasing the retirement age will have not much effect on the financial situation of workers but will improve the position of the retired.

f. Fertility increases will have a strong increasing effect on the interest rate.

g. Fertility increases will weaken social security and above a certain fertility rate social security may even vanish.

h. Fertility increases will strongly improve the situation of the retired.

Notwithstanding that this paper is based on a model which is oversimplified with respect to a number of issues, we believe that the way of thinking about the demographic problem in this paper sheds new light on one of the most threatening questions of our time: how do we provide for our old age and what are the possibilities, if we stick to the present institutional setup, where the pension composition is the joint result of the decisions of a number of parties.

[3] It is possible in the same model to assume that only the channels IVS and FF exists or other subsets. Such restrictions exist in reality. For instance, in Chile there is no SS-system. The exception is to assume only SS, for then there is no source of capital in the economy. In this paper we ignored these possibilities to focus on the main message.

Obviously the model may be extended in many ways but already within the present setup its potential political relevance may be demonstrated by looking at a prediction for developed countries when we assume that they will stay at a low under-reproduction fertility level of 1.5 child per couple and a high survival rate of 0.98, that is a life expectancy of about 90 in our model, and that the retirement age becomes 70 in the years ahead. Still one step further would be to increase the highest age in our model from 100 to 120 in order to reflect the increasing longevity in the present century. In table 6 we present in the first line the situation with a maximum age of 100 and in the second line the outcomes of the model when the maximum age is increased to 120.

When the maximum age is kept a 100 the rough prediction of our model would be a *real* interest rate of about 1.6%, an aggregate premium of about 16.4% of gross wage and a pension/net wage ratio of (see table 6). The capital per worker would be high at about 3.12. When we assume a maximum age of 120, which implies a lengthening of the potential retirement period from 30 to 50 years, the interest rate would increase to 2.7% and the aggregate premium to 21.8%. The pension/net wage ratio would be again about $\frac{1}{2}$ while gross wage would decrease by about 3.5%. Capital per job would decrease by about 10%. Individual voluntary savings are nearly non-existent, while social security premium tends to 18%.

Table 6. A look on a bleak future.

SP=70 β=0.075 μ=0.98 ρ =0.89	Indiv. savings start/ finish %	Private pen-sion %	Pre-mium fully funded %	Fully funded pen-sion %	Pre-mium social secu-rity %	Social secu-rity pen-sion %	Net wage %	Total pen-sion %	Inte-rest rate %	Gross wage	Capital-income ratio	Capital
	1	2	3	4	5	6	7	8	9	10	11	12
Max. age=100	0.0 4.9	11.2	3.3	15.1	8.2	13.5	83.6	39.8	1.6	0.997	3.133	3.123
Max. age=120	0.0 0.2	0.3	3.0	18.3	18.7	20.6	78.2	39.2	2.7	0.963	2.822	2.717

For legend see Appendix.

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Appendix

Legend:

1. individual voluntary savings (IVS), initial and final, as percentage of gross wage
2. private pension (IVS) as percentage of gross wage
3. premium mandatory fully funded (FF) as percentage of gross wage
4. mandatory funded pension (FF) as percentage of gross wage
5. premium social security (SS) as percentage of gross wage
6. social security pension (SS) as percentage of gross wage
7. net wage as percentage of gross wage
8. total pension as percentage of gross wage (benefit-income ratio)
9. Interest rate at equilibrium
10. gross wage
11. capital demand as percentage of gross wage (capital-income ratio)
12. capital demand
13. Hoarding capital as percentage of capital demand (only if applicable)