TI 2018-054/VIII
Tinbergen Institute Discussion Paper

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Xiaojuan Yu[1]
Vincent van den Berg[1]
Erik Verhoef[1]

[1] VU Amsterdam

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3062 PA Rotterdam
The Netherlands
Tel.: +31(0)10 408 8900

Carpooling with heterogeneous users in the bottleneck model

Xiaojuan Yu*[a,b], Vincent A.C. van den Berg[b,c], Erik T. Verhoef[b,c]

[a] *School of Management, Huazhong University of Science and Technology, Wuhan 430074, China*

[b] *Department of Spatial Economics, VU Amsterdam, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands*

[c] *Tinbergen Institute, Gustav Mahlerplein 117, 1082 MS Amsterdam, The Netherlands*

Abstract

When drivers opt for carpooling, road capacity will be freed up, and this will reduce congestion. Therefore, carpooling is interesting for policy makers as a possible solution to congestion. We investigate the effects of carpooling in a dynamic equilibrium model of congestion, which captures various dimensions of heterogeneity: heterogeneity in preference for carpooling, "ratio heterogeneity" between the values of time and the values of schedule delay, and "proportional heterogeneity" that scales all values equally. We investigate three policy scenarios: no-toll, first-best pricing, and subsidization of carpooling. The optimal second-best subsidy equals each type's heterogeneous marginal external benefit (MEB) of switching to carpooling. If such differentiation is impossible, the third-best subsidy is a weighted average of the MEBs, where the weights depend on the number of each type and their sensitivity to the subsidy. In our numerical example, we find that when increasing the degree of "ratio heterogeneity", the relative efficiency of the second-best subsidization first increases and then falls with the degree of heterogeneity and L type carpoolers benefit more than H type carpoolers. However, when increasing the degree of "proportional heterogeneity", H type users benefit more than L types for both solo drivers and carpoolers. Moreover, the relative efficiency of the second-best subsidization decreases throughout.

Keywords: Carpooling; Heterogeneity; Bottleneck model; Welfare effects; Distributional effects.

*Corresponding author.

Email addresses: xiaojuan_yu@hust.edu.cn (Xiaojuan Yu), v.a.c.vanden.berg@vu.nl (Vincent van den Berg), e.t.verhoef@vu.nl (Erik Verhoef)

1 Introduction

Many cities face increasing traffic flows and road congestion, and certainly so in the morning peak at arterial roads. This forces municipalities to consider alternatives for conventional car use, such as public transport, bicycles and carpooling. Because of its flexibility and the more personal atmosphere than transit, carpooling is an interesting alternative for solo car use (Ferguson (1997), Caulfield (2009)). Recently, with the proliferation of technology-enabled ride matching, carpooling has got more attention than before (Masoud and Jayakrishnan (2017a,b), Wang et al. (2017)). Various policy measures have been proposed to encourage carpooling, including carpool lanes, free carpool parking, ride parking and carpool matching platforms. Also a subsidy may be an efficient policy to attract people to switch to carpooling. For example, many ridesharing platforms like Didi, Uber and Meituan-Dianping in China competed to provide subsidies to users and this made the number of carpoolers soar[1]. This raises an interesting and important issue: how to design a subsidization scheme to attract more solo drivers to switch to carpooling and thus increase the social welfare. This question becomes particularly complex when the commuters are heterogeneous and differ in their value of time, schedule delay early and/or schedule delay late. How does this heterogeneity influence the welfare effects and the distributional effects of subsidization? That is one of the main questions we will address in this paper.

We define carpooling as the sharing of a car between people on a trip from a certain origin to a specific destination. The monetary cost savings results from sharing, for example, fuel cost, tolls and parking charges. But extra gathering time and inconvenience costs negatively influence the intention to switch to carpooling (e.g., Kocur and Hendrickson (1983)). These may differ substantially across travellers. To allow for this type of heterogeneity in carpool preferences, we will assume that travel mode choices are based on random utility maximization. Our model therewith also ensures interior equilibria, where some people carpool and some do not, as is also seen in reality.

Several earlier studies have studied carpooling behavior in the morning commute traffic equilibrium. Yang and Huang (1999) use a deterministic equilibrium model to discuss carpooling behavior and the optimal congestion pricing in a multilane highway with or without HOV (High-Occupancy Vehicle) lanes and find that in the presence of HOV lanes, first-best pricing for a social optimum requires differentiating the toll per vehicle across segregated lanes. When toll differentiation cannot be applied, the optimal uniform toll is a weighted average of the marginal external congestion costs between non-carpooling and carpooling commuters. Huang et al. (2000) present deterministic and stochastic

[1] https://www.caixinglobal.com/2018-01-25/didis-new-subsidies-augur-second-price-war-101202995.html (accessed on 25 January 2018).

models to investigate the shifting behavior of work commuters between carpooling and driving alone modes. Through solving each model for both the no-toll equilibrium and the social optimum, they find that carpooling is sensitive to traffic congestion reduction only when a congestion externality-based tolling scheme is implemented. Qian and Zhang (2011) analyze the interactions among transit, driving alone and carpool with identical commuters, addressing that parking availability at the destination is another factor stimulating carpooling. Through investigating carpooling behavior under a parking space constraint, Xiao et al. (2016) find that the best system performance can be realized with joint consideration of total travel cost and vehicle emission cost through optimizing the lane capacity allocation and the parking supply. Based on the work of Xiao et al. (2016), Ma and Zhang (2017) studies the traffic flow patterns in a single bottleneck corridor with a dynamic ridesharing mode and dynamic parking charges. Liu and Li (2017) propose a time-varying compensation scheme to maintain a positive ridesharing ridership at user equilibrium with considering the congestion evolution over time.

These studies usually assume that all commuters are homogeneous, and thus the effects of preference heterogeneity are not considered. Still, various studies have found that heterogeneity in travel mode and departure timing selection is important, and that heterogeneous commuters may exhibit large behavioral differences in departure time choice during peak hours, and in response to congestion tolls. Ignoring preference heterogeneity may cause a biased estimation of the efficiency and welfare impacts of policies. It is, thus, of great importance to incorporate preference heterogeneity of commuters.

Dynamic models of peak hour congestion have considered different forms of heterogeneity. In particular when heterogeneity concerns both heterogeneity in value of time and in values of schedule delay, different possibilities arise. Three ideal types can be distinguished: ratio heterogeneity as in de Palma and Lindsey (2002) and Van den Berg and Verhoef (2011a,b, 2014); proportional heterogeneity as introduced by Vickrey (1973) and Van den Berg and Verhoef (2011b), and general heterogeneity as in Newell (1987), Lindsey (2004), Wu and Huang (2015), Liu et al. (2015), Chen et al. (2015), Li et al. (2017), Takayama and Kuwahara (2017), and Börjesson and Kristoffersson (2014). Ratio heterogeneity refers to heterogeneity in the ratio of the value of time over the value of schedule delay, or α_i/β_i, in the conventional notation. It reflects the willingness to accept greater schedule delays in order to reduce travel time. It hence measures differences in arrival time flexibility, and could stem from differences in job type, trip purpose, family status and age. Proportional heterogeneity refers to the case where the values of time α_i and schedule delay β_i vary over individuals, but in fixed proportions, so that the ratio is the same for everybody. It could stem from differences in incomes. General heterogeneity, finally, occurs when the two types heterogeneity would jointly lead to an unrestricted bivariate distribution. We will consider separately "ratio heterogeneity" and "proportional heterogeneity", as well as general heterogeneity. We will consider discrete distributions, with 2 groups of drivers with these three types of heterogeneity.

Such heterogeneity affects the welfare gain of policies, and is naturally an important determinant of distributional effects of policies (see, e.g., Arnott et al. (1988), Small and Yan (2001), Verhoef and Small (2004), Van den Berg and Verhoef (2011a,b)). These distributional effects are important, if only because they are a major reason for resistance against a new policy. Moreover, if one would like to compensate those who lose disproportionately due to a new policy, one needs to know which types of drivers lose, and by how much. We will see that carpooling imposes a positive externality and allows travellers to share monetary costs, alongside the discomfort and extra travel time to drivers it may cause. The positive externality makes it worthwhile to provide a subsidy, to make carpooling more attractive. We therefore examine three policy schemes: no tolling, first-best tolling, and carpool subsidization. To the best of our knowledge, the effects of subsidization of carpooling on welfare and the distributional effects have not been analysed before for heterogeneous users with dynamic congestion.

This paper studies carpooling behavior in the bottleneck model with general heterogeneity. We consider various dimensions of heterogeneity: heterogeneity in preferences for carpooling, in values of time and in values of schedule delay.[2] This paper makes three main contributions to the literature. *First*, we investigate the welfare effects and distributional effects of introducing carpooling as well as the effects of policies on carpooling behavior. Our study shows that the introduction of voluntary carpooling itself makes all users better off, making it a politically attractive option. With an increasing of proportional heterogeneity, the group with the high value of schedule delay benefits more than other group for all three pricing schemes. However, when increasing the degree of ratio heterogeneity, the group with the low value of time benefits more than other group when tolling is implemented. The relative efficiency of a subsidy on carpooling decreases with the degree of proportional heterogeneity, and first increases and then decreases for most of the range with the degree of ratio heterogeneity. *Second*, we derive the analytical second-best optimal subsidies on carpooling, maximizing the social welfare. The result shows that the second-best subsidy should be set to equal the marginal external benefit (MEB) for each user type. When the subsidy cannot be differentiated, the third-best subsidy is an average of each type's MEB, with the weight reflecting the relative sensitivity of the group size to the subsidy. *Third*, heterogeneity in the preference for carpooling is incorporated in our model, and allows for interior equilibria with each type choosing both solo driving and carpooling with positive

[2]We only consider car travel and not also public transport or mobility services such as taxis or Uber (e.g., Djavadian and Chow (2017), Wang et al. (2017), Masoud and Jayakrishnan (2017a)). We only consider a single road, there are no HOV lanes (e.g.,Yang and Huang (1999)) and no parking (e.g., Xiao et al. (2016), Ma and Zhang (2017)).

probability. Naturally, our results confirm that first-best tolling and second-best subsidization could help enhance the fraction of carpoolers. This is true especially for subsidization, which may lead to almost 99% carpoolers.

The remainder of this paper is organized as follows. Section 2 briefly explains the model set-up and the equilibria under homogeneity. Section 3 introduces heterogeneity, including ratio heterogeneity, proportional heterogeneity and a general heterogeneity. Section 4 develops a numerical example, and provides sensitivity analyse. Section 5 concludes.

For ease of reference, Table 1 below summarises the notation. The notation will also be introduced in the text.

2 The model

2.1 Set-up

We begin our exposition with listing our assumptions, and in passing, introducing the notation.

We assume that the total number of drivers N is fixed. We also ignore other transport modes such as public transport. Everybody travels by car; either solo or in a 2-person carpool.

Travel time cost equals travel time multiplied by the value of time (VOT). The VOT is denoted α. The travel time, TT, is the sum of free-flow travel time and the delay from queuing at the bottleneck. For driving alone, the free-flow travel time is TT_{ff}; for carpoolers, it is $(TT_{ff} + TT_{ff}^{p})$, where TT_{ff}^{p} is the extra time cost of gathering the people together for pooling, which is assumed to be equal for the two carpoolers. In the analysis the free flow travel time TT_{ff} is normalized to 0; the numerical study will consider a positive value. The queuing delay equals the number of cars in the queue when entering it, divided by the capacity of the bottleneck during the queuing time. As a carpool has two persons in it, carpooling raises the effective capacity of the bottleneck, when expressed in passengers per unit of time.

A person's bottleneck cost equals the queuing time cost plus the schedule delay cost. The schedule delay cost is the cost due to arriving at a different time than the most preferred arrival moment, t^*, which is assumed to be identical for all and which will be normalized to 0. We follow Small (1982) and Arnott et al. (1993), and use schedule delay costs that are linear in the time difference between t^* and the actual arrival time, t. The shadow cost per hour for arrivals earlier than t^* is β; for hours late it is γ. The schedule delay costs thus equals $max[-\beta t, \gamma t]$. An individual's travel cost equals the free-flow travel time cost plus the bottleneck cost.

Riding with a stranger decreases the comfort and privacy for the drivers. Therefore, carpoolers

Table 1

Nomenclature

α	Value of time (VOT).
β	Value of schedule delay early.
γ	Value of schedule delay late.
δ	Compound preference parameter: $\delta = \beta\gamma/(\beta+\gamma)$.
TT_{ff}	Free-flow travel time.
TT_{ff}^{p}	Extra time cost of gathering the people together for carpoolers.
c_{fuel}	Fuel cost for each car per trip.
θ	Inconvenience cost due to carpool.
s	Bottleneck capacity.
N	Total number of users.
c_i^j	Generalized cost for each travel mode $j = a, p$ of group i, where superscript a denotes driving alone and superscripts p denotes carpooling.
p_i^j	Generalized price for each travel mode $j = a, p$ of group i. It equals the generalized travel costs plus the toll or minus the subsidy.
ϕ	The scale of utility.
ϵ_i^j	The random mode preference utility.
U_i^j	$j = a, p$. The random utility with mode $j$of group i.
N_i^j	$j = a, p$. The number of users with mode j of group i.
F_i^j	$j = a, p$. The fraction of mode j chosen by group i.
Δc_i	Change in per user generalized cost when switching from driving alone to carpooling of group i.
Δp_i	Change in per user generalized price when switching from driving alone to carpooling of group i.
τ	Time-varying toll with the first-best case.
MEB_i	Marginal external benefit of group i incurred by switching to carpooling.
S_i^*	Optimal second-best subsidy for type i with social welfare maximization, $i = H, L$.
S^*	Optimal third-best subsidy with social welfare maximization.
cs_i	Consumer surplus per user of group i.
$cs^{\#}$	Arbitrary constant of the integration in consumer surplus.
Δcs_i	The change of consumer surplus per user due to the introducing of carpool.
SW	Social welfare.
t^*	Preferred arrival time.

inevitably undergo inconvenience cost θ, which is assumed to be a fixed amount per trip and per person. This adds to the time loss TT^p_{ff} introduced earlier. The fuel cost per trip, c_{fuel}, is also assumed to be a fixed amount per trip. For carpoolers, the drivers will share the fuel cost equally.

The generalised travel cost per trip per person is then the sum of the travel cost, the inconvenience cost and the fuel cost:

$$c^a[t] = max(-\beta t, \gamma t) + c_{fuel} \qquad \text{when driving alone;} \tag{1}$$

$$c^p[t] = max(-\beta t, \gamma t) + \alpha TT^p_{ff} + c_{fuel}/2 + \theta \qquad \text{when carpooling.} \tag{2}$$

where superscript ^{a} indicates driving alone, and superscript ^{p} indicates carpooling. Note that there is a fixed cost difference of $\alpha TT^p_{ff} + \theta - c_{fuel}/2$ for carpoolers vs solo drivers. We assume that with carpooling two people share the car, ignoring the possibility to share with even more people.

In equilibrium, for both modes, the travel cost for a specific group of users needs to be constant over the arrival times used by those drivers. We consider the case where the discrete choice behaviour between driving alone and carpooling is characterized by random utility maximization, representing unobserved idiosyncratic preferences for carpooling versus driving alone. The random utility function of user i with mode j is:

$$U^j_i = -c^j_i + \frac{\epsilon^j_i}{\phi}, \quad j = a, p, \tag{3}$$

where U^j_i depends on a deterministic utility component $-c^j_i$, and on a random mode preference utility ϵ^j_i. ϵ^j_i is assumed to be i.i.d. Gumbel distributed. The parameter ϕ defines the scale of systematic utility: the larger ϕ, the less important idiosyncratic preferences, and hence the more deterministic the choices are (i.e., the stronger these are determined by systematic utility $-c^j_i$). At equilibrium, the mode choice probabilities F^j_i are governed by the following logit formulae:

$$F^a_i = \frac{e^{-\phi c^a_i}}{e^{-\phi c^a_i} + e^{-\phi c^p_i}}, \quad F^p_i = \frac{e^{-\phi c^p_i}}{e^{-\phi c^a_i} + e^{-\phi c^p_i}}. \tag{4}$$

We use the familiar log-sum formula (e.g., Train (2009)) to express the consumer surplus,

$$cs_i = \frac{\log(e^{-\phi c^a_i} + e^{-\phi c^p_i}) + cs^{\#}}{\phi}, \tag{5}$$

where $cs^{\#}$ is the arbitrary constant of the integration.

Each individual needs to decide on a departure time from home to minimize the total travel cost of the entire car. In doing so, she makes a trade off between travel time cost and schedule delay cost. Equilibrium is achieved when no individual can reduce his travel cost by altering the departure time. The three subsections look at three cases, namely the no-toll equilibrium, the first-best equilibrium, and the second-best subsidization on carpooling. It is instructive to start our exposition with the homogeneous preferences model, as the effects of carpooling are easier to understand in this case.

2.2 No-toll equilibrium with homogeneous users

In the dynamic equilibrium with homogeneous users but with carpooling, the generalised travel cost should be constant over time as long as arrivals occur. Fig.1 illustrates the equilibrium. The solid line shows the queuing times. It indicates that the carpoolers and solo drivers are travelling jointly over the peak as the two groups have the same ratio β/α per vehicle: both α and β in a carpool are twice that of a solo-drive car (see Arnott et al. (1988), Van den Berg and Verhoef (2011a,b)).

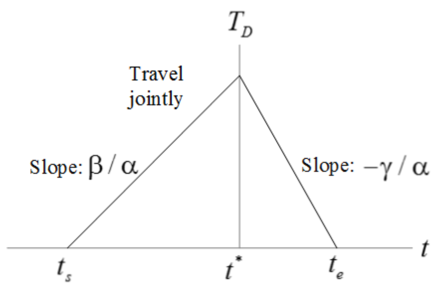


Fig. 1. No-toll equilibrium with homogeneity

The per person equilibrium costs for driving alone and carpooling are:

$$c^a_{NT} = \delta\frac{N^a + N^p/2}{s} + c_{fuel}, \tag{6}$$

$$c^p_{NT} = \delta\frac{N^a + N^p/2}{s} + \alpha TT^p_{ff} + c_{fuel}/2 + \theta, \tag{7}$$

where δ is the conventional composite scheduling preference parameter: $\delta = \frac{\beta\gamma}{\beta+\gamma}$. N^a is the number of solo drivers, N^p is the number of carpoolers. The bottleneck costs (the first terms in Eqs.(6-7) are straightforward to derive from the conventional bottleneck model, where these costs are equal to $\delta N/s$ (e.g., Arnott et al. (1987)).

A user who switches from driving alone to carpooling will experience a change in travel cost that we denote Δc_{NT}, where NT denotes the no-toll equilibrium:

$$\Delta c_{NT} = c^a_{NT} - c^p_{NT} = -\alpha TT^p_{ff} - \theta + c_{fuel}/2. \tag{8}$$

Note that, while α can differ between drivers, θ and c_{fuel} will be assumed identical across drivers, so that in fact only the sum $-\theta + c_{fuel}/2$ matters. Also note that a positive value of Δc_{NT} reflects a positive incentive to form a carpool.

Eq.(8) shows that with deterministic preferences, a corner solution with only carpooling or solo driving would prevail, as the cost difference is independent of N^a and N^p. With random utility, choice probabilities for users to choose carpooling (p) and driving alone (a) are F^p and F^a.

Let N denote the total number of users, i.e., $N = N^a + N^p$. At equilibrium, using the logit model, the number of carpoolers and solo drivers can be endogenously determined as:

$$N^p_{NT} = \frac{N}{1 + e^{\phi(\alpha TT^p_{ff} + \theta - c_{fuel}/2)}}, \quad N^a_{NT} = \frac{N}{1 + e^{\phi(-\alpha TT^p_{ff} - \theta + c_{fuel}/2)}}. \tag{9}$$

2.3 First-best equilibrium with homogeneous users

Now we turn to the first-best case, where tolling removes all queuing (Arnott et al. (1987)). This requires the toll to increase at a rate β for early arrivals, and to fall at a rate γ for late arrivals, since this ensures that zero travel time delays constitute the dynamic equilibrium. Because there are two persons in each carpool, the total value of schedule delays in a carpool is twice as large, and therefore the toll should also grow or shrink at a double rate to maintain equilibrium. As a consequence, the carpools travel in the center of the peak and the solo drivers travel away from the center: carpools will find an arrival moment closer to t^* more attractive than arriving at moments where solo drivers arrive, since the gain in schedule delay cost outweighs the increase in toll. The opposite applies for solo drivers in the time window where carpoolers arrive: they would prefer arriving further from t^* as the toll savings exceed the additional schedule delay cost. Hence the temporal separation of traffic that is shown in Fig.2 illustrates this optimum.

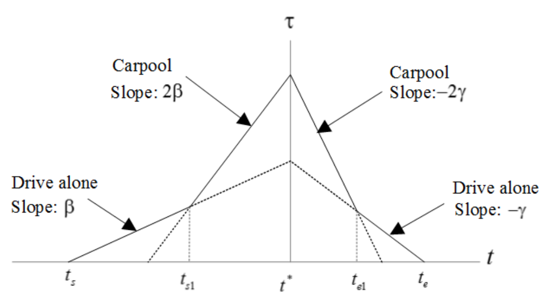


Fig. 2. First-best equilibrium with homogeneity

The generalized price, when arriving at t, now includes generalized travel cost and the toll. For the carpoolers, the toll is equally shared by the two travellers in the car. We find the following prices:

$$p^a_{FB} = \delta \frac{N^a + N^p/2}{s} + c_{fuel}, \tag{10}$$

$$p^p_{FB} = \delta \frac{N}{2s} + \alpha TT^p_{ff} + c_{fuel}/2 + \theta. \tag{11}$$

The generalized price of carpooling, and also the bottleneck cost, is different from that of driving alone. For the carpoolers, the toll is shared by the two drivers, and the separation of type over time mean that carpoolers benefit from the relatively flat toll scheme for solo drivers. (See also Van den Berg and Verhoef (2011a,b)). At the same time, they suffer from the fact that solo drivers impose a higher demand, per traveller, on bottleneck capacity. The solo drivers travel in $[t_s, t_{s1}]$ and $[t_{e1}, t_e]$ while the carpoolers travel in $[t_{s1}, t_{e1}]$. The toll at t_{s1} equals $\frac{\delta N^a}{s}$, so for the carpoolers there is a $\frac{\delta N^a}{2s}$ toll reduction due to sharing in Eq.(11), compared to the case where the same amount of cars would have been occupied by carpoolers.

For a user who switches from driving alone to carpooling, the price now changes:

$$\Delta p_{FB} = p^a_{FB} - p^p_{FB} = \delta N^a/(2s) - \alpha TT^p_{ff} + c_{fuel}/2 - \theta. \tag{12}$$

Again, a positive value reflects an advantage for carpooling and vice versa.

The number of carpoolers and solo drivers are respectively:

$$N^p_{FB} = \frac{N}{1 + e^{-\phi \Delta p_{FB}}}; N^a_{FB} = \frac{N}{1 + e^{\phi \Delta p_{FB}}}. \tag{13}$$

As Δp_{FB} is determined by N^a_{FB} and N^a_{FB}, which are determined by Δp_{FB}, we cannot obtain analytical solutions. The Method of Successive Average (MSA)[3] is used to find the equilibrium in the numerical study. Finally, social welfare in the FB case is the sum of the total consumer surpluses and the toll revenues.

2.4 Second-best subsidization with homogeneous users

We now turn to an interesting second-best policy (SB): a flat time-invariant subsidy for carpooling. The generalized cost of driving alone follows the same expressions as for the NT case, while for carpooling, a fixed subsidy S is subtracted from the generalized price. For that reason, and because N^a and N^p will change, equilibrium costs and prices will change as well. The two groups, however, keep travelling jointly in time. We optimize the subsidy by maximizing social welfare (SW) with respect to S. Note that S is per passenger in a carpool; the subsidy per carpool is therefore $2S$.

[3] The Method of Successive Average (MSA): Step 1, Initialize. Calculate the initial free-flow travel price, $p^a(0), p^p_H(0), p^a_L(0), p^p_L(0)$, set n=0; Step 2, Calculate the augmented flow with logit model, $N^a(n+1) = \frac{e^{p^a(n)}}{e^{p^a(n)}+e^{p^p(n)}}N$, $N^p(n+1) = \frac{e^{p^p(n)}}{e^{p^a(n)}+e^{p^p(n)}}N$; Step 3, Use MSA to update the flow, where $N^a(n+1) = (1-\frac{1}{n})N^a(n) + \frac{1}{n}N^a(n+1)$ and $N^p(n+1) = (1-\frac{1}{n})N^p(n) + \frac{1}{n}N^p(n+1)$; Step 4, Termination check. If $|N^j(n+1) - N^j(n)| < 10^{-6}, j = a, p$, terminate and output the optimal solution $N^j(n+1)$. Otherwise, set $n = n+1$ and go to step 2.

The generalized prices for driving alone and carpooling are respectively:

$$p_{SB}^a = \delta \frac{N^a + N^p/2}{s} + c_{fuel}, \tag{14}$$

$$p_{SB}^p = \delta \frac{N^a + N^p/2}{s} + \alpha TT_{ff}^p + c_{fuel}/2 + \theta - S, \tag{15}$$

reflecting that the type of equilibrium will qualitatively resemble the one depicted in Fig.1, although the equilibrium share of carpoolers will be different.

Using the same logic as for NT and FB, the numbers of carpoolers and solo drivers can be found as:

$$N_{SB}^p = \frac{N}{1 + e^{\phi(\alpha TT_{ff}^p + \theta - c_{fuel}/2 - S)}}; \quad N_{SB}^a = \frac{N}{1 + e^{\phi(-\alpha TT_{ff}^p - \theta + c_{fuel}/2 + S)}}. \tag{16}$$

To find the second-best subsidy, we maximize social welfare, which is the total consumer surplus again by the log-sum measure, minus the total subsidy. Combining Eq.(5) and Eqs.(14)-(16), social welfare (SW) is therefore:

$$\begin{aligned} SW &= \frac{N \log(e^{-\phi p_{SB}^p} + e^{-\phi p_{SB}^a}) + Ncs^{\#}}{\phi} - SN_{SB}^p \\ &= \frac{N \log(1 + e^{\phi(-\alpha TT_{ff}^p - \theta + c_{fuel}/2 + S)}) + Ncs^{\#}}{\phi} + \frac{N(\delta N/2s - S)}{1 + e^{\phi(\alpha TT_{ff}^p + \theta - c_{fuel}/2 - S)}} - \frac{\delta N^2}{s} - Nc_{fuel}. \end{aligned} \tag{17}$$

The derivative of social welfare with respect to S is:

$$\frac{\partial SW}{\partial S} = \frac{e^{\phi(-c_{fuel/2 - S + \alpha TT_{ff}^p + \theta})} N}{(1 + e^{\phi(-c_{fuel/2 - S + \alpha TT_{ff}^p + \theta})})^2} (-S + \delta N/2s). \tag{18}$$

The first part of the right side of Eq.(18) is always positive. When $S < \delta N/2s$, social welfare increases with S; when $S > \delta N/2s$, social welfare decreases with S. The optimal subsidy S^* can thus be expressed as

$$S^* = \frac{\delta N}{2s}. \tag{19}$$

Note, from Eqs.(14) and (15), that each user, irrespective of whether she is a carpooler or a solo driver, benefits with $\frac{1}{2}\frac{\delta}{s}$ decrease in price when one marginal traveller transfers from solo driving to carpooling. The optimal subsidy in Eq.(19) is thus naturally interpreted as the reduction in total social cost following a marginal change from solo-driving to carpooling, on top of the change in cost for that marginal driver. This benefit is therefore external to the choice of an individual, and thus the subsidy equals the marginal external benefit (MEB).[4] Naturally, with homogeneous users, the MEB is also homogeneous.

[4]Note that instead of maximizing with respect to S, we could also have maximized with respect to the number of solo drivers and carpoolers as in Huang et al. (2000).

3 Heterogeneity

Now we turn to the case with heterogeneous users. When taking heterogeneity into consideration, we assume that users are separated into two discrete groups, which we can denote for each type of heterogeneity as type high (H) and type low (L), where the exact interpretation differs between the cases. To simplify, we assume up-front that carpoolers share the carpool with the same type; i.e., the H type share with the H type and the L type share with the L type. This is, however, consistent with the idea that joint optimization with someone who has the same preferences leads to a better outcome than with someone who has partly conflicting preferences. Therefore, if θ is the same for mixed and homogeneous carpools, the assumption is not harmful. This assumption is furthermore not essential for our results, but it helps in restricting the number of groups travelling on the road to four: solo drivers of H type, solo drivers of L type, carpoolers of H type, carpoolers of L type. We will first be considering two types of heterogeneity: ratio heterogeneity and proportional heterogeneity, and next consider general heterogeneity. With ratio heterogeneity, the groups differ because their ratios of value of time over value of schedule delay differ. With proportional heterogeneity, these ratios are the same between the groups, but the values themselves differ.

3.1 Ratio heterogeneity

Ratio heterogeneity refers to the case where there is heterogeneity in the value of time α, where there are uniform values of schedule delay β and γ. The ratio for group i is denoted as $\mu_i = \alpha_i/\beta$ (Arnott et al. (1987), de Palma and Lindsey (2002) and Van den Berg and Verhoef (2011a)). Users with a high ratio are less willing to queue (or, alternatively, they are more willing to adjust when to arrive), as a higher travel time is relatively more costly for them than a lower schedule delay. We suppose the High group has a higher ratio $\mu_H = \alpha_H/\beta$, and the Low group has a lower ratio $\mu_L = \alpha_L/\beta$. The different types of users self-select travel mode and arrival moment, to maximize their utility.

3.1.1 No-toll equilibrium with ratio heterogeneity

The no-toll equilibrium requires travel times by arrival time to grow at a rate $1/\mu_i = \beta/\alpha_i$ when group i arrives ($i = H, L$). Travellers with a high μ_H will choose to arrive relatively early or late, to avoid long travel times. The reverse applies to the L group, which thus leads to separate travelling for H and L: group L arrives closest to t^*, and group H arrive further from t^*. Due to the same ratios applying to solo drivers and carpools within a group, solo drivers and carpools of the same type will travel jointly. Fig.3 illustrates this equilibrium. The red solid line represents the H group, and the

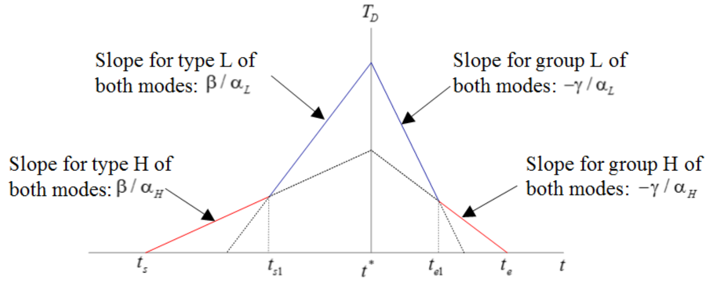


Fig. 3. No-toll equilibrium with ratio heterogeneity

blue solid line represents the L group.

Following Van den Berg and Verhoef (2011a,b), the group-specific generalized travel costs for driving alone and carpooling can be shown to be:

$$c_H^a = \delta \frac{N_H^a + N_H^p/2 + N_L^a + N_L^p/2}{s} + c_{fuel}, \tag{20a}$$

$$c_H^p = \delta \frac{N_H^a + N_H^p/2 + N_L^a + N_L^p/2}{s} + c_{fuel}/2 + \theta + \alpha_H TT_{ff}^p, \tag{20b}$$

$$c_L^a = \delta \frac{\alpha_L}{\alpha_H} \frac{N_H^a + N_H^p/2}{s} + \delta \frac{N_L^a + N_L^p/2}{s} + c_{fuel}, \tag{20c}$$

$$c_L^p = \delta \frac{\alpha_L}{\alpha_H} \frac{N_H^a + N_H^p/2}{s} + \delta \frac{N_L^a + N_L^p/2}{s} + \alpha_L TT_{ff}^p + c_{fuel}/2 + \theta, \tag{20d}$$

where the subscript $_H$ denotes H type users and subscript $_L$ denotes L type users. $N_H^a, N_H^p, N_L^a, N_L^p$ thus denote for both type the number of solo drivers and carpoolers, respectively. As these values are all endogenously determined, ratio heterogeneity does affect the generalized price of both types for a given total number of travellers. The higher a user's VOT is, relative to the values of schedule delay, the less queuing this user causes, and the lower this user's congestion externality. Hence the NT bottleneck cost for the L group of each travel pattern is less than what it would have been if all H drivers were replaced by L drivers, and also less than that of the H group. The equilibrium cost for the H group is constant, does not depend on the shares of L and H drivers. Note that this can be seen from Fig.(3) by determining equilibrium cost for a group as the distance between the intersection of the (extra plotted) queuing time function with the x-axis, multiplied by the appropriate β or γ.

For a user who switches from driving alone to carpooling, the travel cost drops by:

$$\Delta c_H = c_H^a - c_H^p = -\alpha_H TT_{ff}^p - \theta + c_{fuel}/2, \quad \Delta c_L = c_L^a - c_L^p = -\alpha_L TT_{ff}^p - \theta + c_{fuel}/2. \tag{21}$$

As α_H is larger than α_L, the price drop between driving alone and carpooling for H type is lower than for L type. Carpooling is more attractive for the L type than the H type, reflecting the lower penalty from additional time lost in forming a carpool. Using the logit model, we can next determine the number of carpoolers and solo drivers for each type users as:

$$N_i^a = \frac{N_i}{1+e^{\phi(-\alpha_i TT_{ff}^p - \theta + c_{fuel}/2)}}, \quad N_i^p = \frac{N_i}{1+e^{\phi(\alpha_i TT_{ff}^p + \theta - c_{fuel}/2)}}, \quad (i = L, H), \tag{22}$$

where N_i is the number of type i drivers, which is assumed to be given.

The total travel cost is:

$$TC = N_H^a c_H^a + N_H^p c_H^p + N_L^a c_L^a + N_L^p c_L^p. \tag{23}$$

Switching to carpooling imposes a positive externality by decreasing the travel time of other drivers. We find MEB's by using the cost function implied by Eq.(23) and taking the difference between the marginal social cost and the privately incurred cost from switching:

$$MEB_H = \Delta c_H - (\frac{\partial TC}{\partial N_H^p} - \frac{\partial TC}{\partial N_H^a}) = \frac{\delta N_H}{2s} + \frac{\delta \alpha_L}{\alpha_H}\frac{N_L}{2s}; \tag{24}$$

$$MEB_L = \Delta c_L - (\frac{\partial TC}{\partial N_L^p} - \frac{\partial TC}{\partial N_L^a}) = \frac{\delta N}{2s}. \tag{25}$$

Van den Berg and Verhoef (2011a) has shown that the external marginal benefit decreases with its VOT. Now with the introduction of carpooling, this conclusion still holds, i.e., $MEB_H < MEB_L$.

3.1.2 First-best equilibrium with ratio heterogeneity

FB tolling removes all the queuing and this requires the toll to increase at a rate β for early arrivals and decrease at a rate γ for late arrivals (de Palma and Lindsey (2002)). Early travellers are ordered by increasing values of β, for the same sort of self-selection mechanism as described before. The carpoolers arrive in the center of the peak due to the doubled value of β, induced by having two persons in each carpool. Because there is no difference of the values of schedule delay between H type and L type, all solo drivers and all carpoolers will travel jointly, as was the case under homogeneity. Fig.4 illustrates this equilibrium. The red solid line represents solo drivers and the blue solid line represents carpoolers. At equilibrium, the generalized prices for each group i $(i = H, L)$ drops by:

$$p_i^a = \delta \frac{N_L^a + N_L^p/2 + N_H^a + N_H^p/2}{s} + c_{fuel}, \tag{26}$$

$$p_i^p = \delta \frac{N}{2s} + \alpha_i TT_{ff}^p + c_{fuel}/2 + \theta, \qquad i = H, L. \tag{27}$$

The expression for the FB price of solo drivers of both type replicates the price expression for H type users in the NT situation. However, because the numbers of solo drivers and carpoolers are endogenous, the FB price may still be expected to be different from that in the NT case.

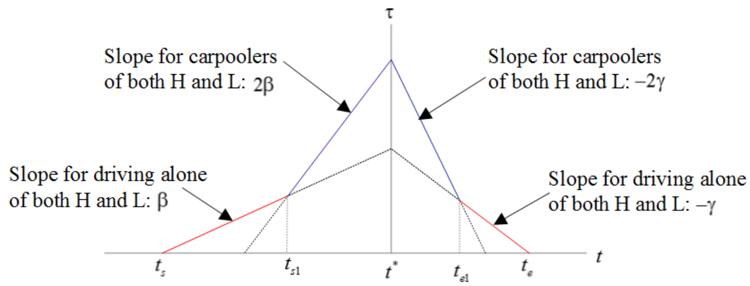


Fig. 4. First-best equilibrium with ratio heterogeneity

For a user who switches from driving alone to carpooling, the generalized price drops by:

$$\Delta p_i = p_i^a - p_i^p = \delta \frac{N_L^a + N_H^a}{2s} - \alpha_i TT_{ff}^p + c_{fuel}/2 - \theta, \quad i = H, L. \tag{28}$$

Because N_L^a and N_L^p are determined by Δp, and Δp is also determined by N_L^a and N_L^p, we can not obtain an analytical solution, and we will use Method of Successive Average (MSA) to get numerical results. Social welfare in the FB equilibrium is again the sum of consumer surplus and the toll revenues.

3.1.3 Second-best subsidization with ratio heterogeneity

A flat subsidy policy maximizes the total social welfare by finding the optimal subsidy, S^*. At equilibrium, the generalized price of solo drivers has the same expression as that for the NT case, and is given in Eqs.(29a) and (29c). The generalized prices for carpoolers are lower than in the NT case, due to the subsidy S. The generalized price reduction, from switching to carpooling is increased by the subsidy, or the price increase is decreased. We first consider the case where the subsidy on carpooling can be differentiated between the two groups, which means we can first put aside the complication of finding the best compromise of the differentiated subsidies. The generalized prices then become:

$$p_H^a = \delta \frac{N_H^a + N_H^p/2 + N_L^a + N_L^p/2}{s} + c_{fuel}; \tag{29a}$$

$$p_H^p = \delta \frac{N_H^a + N_H^p/2 + N_L^a + N_L^p/2}{s} + c_{fuel}/2 + \theta + \alpha_H TT_{ff}^p - S_H; \tag{29b}$$

$$p_L^a = \delta \frac{\alpha_L}{\alpha_H} \frac{N_H^a + N_H^p/2}{s} + \delta \frac{N_L^a + N_L^p/2}{s} + c_{fuel}; \tag{29c}$$

$$p_L^p = \delta \frac{\alpha_L}{\alpha_H} \frac{N_H^a + N_H^p/2}{s} + \delta \frac{N_L^a + N_L^p/2}{s} + \alpha_L TT_{ff}^p + c_{fuel}/2 + \theta - S_L. \tag{29d}$$

The numbers of carpoolers and solo drivers for both types of drivers are given by:

$$N_i^a = \frac{N_i}{1 + e^{\phi(-\alpha_i TT_{ff}^p - \theta + c_{fuel}/2 + S_i)}}, \quad N_i^p = \frac{N_i}{1 + e^{\phi(\alpha_i TT_{ff}^p + \theta - c_{fuel}/2 - S_i)}}, \quad (i = L, H). \tag{30}$$

The total social welfare is again the total consumer surplus minus the total subsidy, which can now be expressed as:

$$SW = \frac{N_H \log(e^{-\phi p_H^a} + e^{-\phi p_H^p}) + N_L \log(e^{-\phi p_L^a} + e^{-\phi p_L^p}) + Ncs^{\#}}{\phi} - S_L N_L^p - S_H N_H^p. \tag{31}$$

Let $\frac{\partial SW}{\partial S_H} = 0$, $\frac{\partial SW}{\partial S_L} = 0$, we obtain

$$S_H^* = \frac{\delta N_H}{2s} + \frac{\delta \alpha_L}{\alpha_H}\frac{N_L}{2s}, \qquad S_L^* = \frac{\delta N}{2s}. \tag{32}$$

The optimal subsidies in Eq.(32) are equal to the marginal external benefits (MEB) of switching to carpooling for both groups, in Eqs.(24-25). This is consistent with what was found for homogeneous drivers. We can also find that S_H^* decreases with α_H/α_L, i.e., the reduction in total social cost from solo-driving to carpooling by H type users decrease with the degree of ratio heterogeneity.

3.1.4 Third-best subsidization with ratio heterogeneity

When the subsidy cannot be differentiated, (i.e., $S_H = S_L$), to maximize the total social welfare, a third-best (TB) subsidy S^* can be derived as a weighted average of the MEB's. It amounts to: [5]

$$S^* = \lambda_H\left(\frac{\delta N_H}{2s} + \frac{\delta \alpha_L}{\alpha_H}\frac{N_L}{2s}\right) + \lambda_L \frac{\delta N}{2s}, \tag{33}$$

with

$$\lambda_H = \frac{F_H^{'} N_H}{F_H^{'} N_H + F_L^{'} N_L}, \qquad \lambda_L = \frac{F_L^{'} N_L}{F_H^{'} N_H + F_L^{'} N_L} = 1 - \lambda_H, \tag{34}$$

where $F_H^{'}$ and $F_L^{'}$ are the derivative of F_H, F_L (i.e., the fraction of carpoolers of type H and type L) with respect to S, respectively. The weights λ_H and λ_L hence depend on the numbers of users of both types and their sensitivity to subsidy. In this condition and for given N's, the optimal subsidy S^* is between S_H^* and S_L^*; i.e., $MEB_H < S^* < MEB_L$. As it cannot be solved analytically, the specific solution will be demonstrated by numerical examples in Section 4. We define the differentiated subsidy in Eq.(32) as second-best subsidization and the undifferentiated subsidy in Eq.(33) as third-best subsidization.

3.2 Proportional heterogeneity

Now we turn to the case of proportional heterogeneity. This refers to heterogeneity where the ratio of values of time and schedule delay (α_i/β_i) is uniform, but all values vary in fixed proportions following

[5] The first-order and second-order conditions of maximization hold for an interior solution and there is one unique optimum. We have also tested the expressions in Eqs.(32) and (33) numerically by maximizing the social welfare, and the result is consistent with the subsidy in (33).

the scalar k_i: $\alpha_i = k_i\alpha$, $\beta_i = k_i\beta$ and $\gamma_i = k_i\gamma, (i = L, H, k_H > k_L)$. This type of heterogeneity may well stem from income differences, with apart from the marginal utility of income otherwise identical preferences: all three values α, β, γ depend linearly on the inverse of the marginal utility of income, which decreases with income.

3.2.1 No-toll equilibrium with proportional heterogeneity

Without tolling, travel times follow the same pattern as with homogeneity: all users travel jointly. This is because the ratios β_i/α_i and γ_i/α_i measure the willingness to queue, and hence determine the arrival order of drivers. Under proportional heterogeneity, these ratios are the same for all users. Fig.5 illustrates this equilibrium. The following costs levels apply:

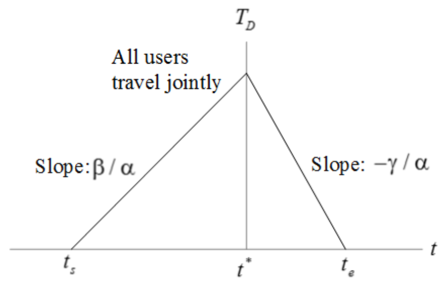


Fig. 5. No-toll equilibrium with proportional heterogeneity

$$c_i^a = \delta_i \frac{N_H^a + N_H^p/2 + N_L^a + N_L^p/2}{s} + c_{fuel}; \tag{35a}$$

$$c_i^p = \delta_i \frac{N_H^a + N_H^p/2 + N_L^a + N_L^p/2}{s} + c_{fuel}/2 + \theta + \alpha_i TT_{ff}^p. \quad i = H, L. \tag{35b}$$

For a user who switches from driving alone to carpooling, the travel cost decreases:

$$\Delta c_i = c_i^a - c_i^p = -\alpha_i TT_{ff}^p - \theta + c_{fuel}/2, \qquad i = H, L. \tag{36}$$

The higher α_i is, the smaller the cost difference in favor of carpooling. The total travel cost is:

$$TC = N_H^a c_H^a + N_H^p c_H^p + N_L^a c_L^a + N_L^p c_L^p, \tag{37}$$

where $N_H^a, N_H^p, N_L^a, N_L^p$ are determined by the logit model in Section 2.1.

And, just as in the previous section, the marginal external benefits due to switching to carpooling for each type are:

$$MEB_H = \Delta p_H - (\frac{\partial TC}{\partial N_H^p} - \frac{\partial TC}{\partial N_H^a}) = \frac{\delta_H N_H + \delta_L N_L}{2s}, \tag{38}$$

$$MEB_L = \Delta p_L - (\frac{\partial TC}{\partial N_L^p} - \frac{\partial TC}{\partial N_L^a}) = \frac{\delta_H N_H + \delta_L N_L}{2s}. \tag{39}$$

With proportional heterogeneity, the marginal external benefits are therefore the same for all users.

3.2.2 First-best equilibrium with proportional heterogeneity

Queuing is again a pure loss, and the FB toll eliminates it. Each commuter chooses his departure time and travel mode. Types will then arrive in order of their β_i. The drivers with the highest values choose to arrive closest to the preferred arrival time, as for them schedule delays are most costly and they are thus most willing to pay the toll to avoid these. The L type arrives the furthest from t^*, as they care least about schedule delays. The first-best toll thus fully separates the types by self-selection of drivers, and it not only removes the queuing but also reduces total schedule delay cost. We still assume that carpoolers share the carpool with the same type, so that the compound travel delay for each carpool car should be $2\beta_i, (i = L, H)$. The assumption of $\alpha_H > \alpha_L$ ensures $\beta_H > \beta_L$ holds. The departure order contains 2 cases, based on the relative values of β_H and $2\beta_L$.

- Case 1: $\beta_H < 2\beta_L$

  If the value of schedule delays of a L type carpool is larger than for a H type solo car, the order for early arrivals will be: L type solo drivers, H type solo drivers, L type carpoolers, H type carpoolers.

- Case 2: $\beta_H > 2\beta_L$

  If the value of schedule delays of a H type solo car is larger than for a L type carpool, the departure order will be: L type solo drivers, L type carpoolers, H type solo drivers, H type carpoolers.

Fig.6 gives examples of the FB equilibrium for these two cases. FB equilibrium prices can again be written as the schedule delay cost at the moment that the relevant iso-price line intersects the horizontal axis.

For Case 1 ($\beta_H < 2\beta_L$), the generalized prices are given by:

$$p_L^a = \delta_L \frac{N_L^a + N_H^a + N_L^p/2 + N_H^p/2}{s} + c_{fuel}; \tag{40a}$$

$$p_L^p = \frac{\delta_L N}{2s} + \delta_L(1 - \frac{\beta_H}{\beta_L})\frac{N_L^p + N_H^p}{4s} + \delta_H(1 - \frac{\beta_L}{\beta_H})\frac{N_H^a + N_L^p/2 + N_H^p/2}{2s} + \alpha_L TT_{ff}^p + c_{fuel}/2 + \theta; \tag{40b}$$

$$p_H^a = \delta_H(1 - \frac{\beta_L}{\beta_H})\frac{N_H^a + N_L^p/2 + N_H^p/2}{s} + \delta_L \frac{N_L^a + N_H^a + N_L^p/2 + N_H^p/2}{s} + c_{fuel}; \tag{40c}$$

$$p_H^p = \frac{\delta_L N}{2s} + \delta_L(1 - \frac{\beta_H}{\beta_L})\frac{N_L^p + N_H^p}{4s} + \delta_H(1 - \frac{\beta_L}{\beta_H})\frac{N_H^a + N_L^p/2 + 3N_H^p/2}{2s} + \alpha_H TT_{ff}^p + c_{fuel}/2 + \theta. \tag{40d}$$

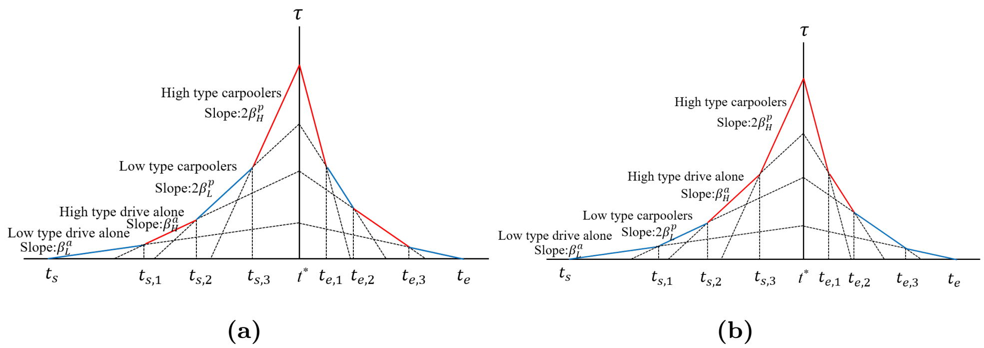


Fig. 6. First-best equilibrium with proportional heterogeneity.
(a) Case 1; (b) Case 2.

For Case 2 ($\beta_H > 2\beta_L$), the generalized prices are given by:

$$p_L^a = \delta_L \frac{N_L^a + N_L^p/2 + N_H^a + N_H^p/2}{s} + c_{fuel}; \tag{41a}$$

$$p_L^p = \delta_L \frac{N_L^p/2 + N_L^a/2 + N_H^a + N_H^p/2}{s} + c_{fuel}/2 + \alpha_L TT_{ff}^p + \theta; \tag{41b}$$

$$p_H^a = \delta_H (1 - \frac{2\beta_L}{\beta_H}) \frac{N_H^a + N_H^p/2}{s} + \delta_L \frac{N + N_H^a}{s} + c_{fuel}; \tag{41c}$$

$$p_H^p = \delta_H (\frac{N_H}{2s} - \frac{\beta_L}{\beta_H} \frac{N_H - N_H^p/2}{s}) + \delta_L \frac{N + N_H^a}{2s} + \alpha_H TT_{ff}^p + \theta + c_{fuel}/2. \tag{41d}$$

The generalized prices and numbers of drivers for carpooling and driving alone for each type have no closed-form solution and in the numerical model again obtained by the Method of Successive Average. Van den Berg and Verhoef (2011b) showed that with only proportional heterogeneity, first-best (FB) tolling reduces the 'generalised price' (i.e. toll plus travel costs) for all users, except for those with the very lowest values, who are unaffected. This also means that the gain of first-best tolling increases with the degree of proportional heterogeneity. We will use a numerical example in Section 4 to illustrate that this result also applies in the current context.

3.2.3 Second-best subsidization with proportional heterogeneity

As noted, with proportional heterogeneity the MEB is the same for all, and as we will see, hence so is the optimal carpool subsidy.

The government maximises social welfare, which again equals the total consumer surplus minus the subsidy to carpoolers:

$$SW = \frac{N_H \log(e^{-\phi p_H^a} + e^{-\phi p_H^p}) + N_L \log(e^{-p_L^a} + e^{-p_L^p}) + Ncs^{\#}}{\phi} - S_L N_L^p - S_H N_H^p, \tag{42}$$

where $p_i^a = c_i^a, p_i^p = c_i^p - S_i$ by Eq.(35), N_i^a and N_i^p are derived from the logit model.

Let $\frac{\partial SW}{\partial S_H} = 0$ and $\frac{\partial SW}{\partial S_L} = 0$, respectively. We then obtain a uniform subsidy:[6]

$$S^* = S_H^* = S_L^* = \frac{\delta_H N_H + \delta_L N_L}{2s}. \tag{43}$$

The uniform subsidy is consistent with the marginal external benefit in Eqs.(38-39). The optimal subsidy thus again equals the marginal external benefit from switching to carpooling. The third-best undifferentiated subsidy in this case replicates second-best subsidization.

3.3 Generalizing the model: both ratio and proportional heterogeneity

Now we turn to the full setting, in which both ratio heterogeneity and proportional heterogeneity bring changes in the carpooling behavior and welfare effects. For convenience, we assume that each type has a different ratio $\mu_i = \alpha_i/\beta_i$ and different values of β_i and γ_i, so that all groups travel separated in time in the no toll equilibrium as well as in the first best equilibrium. We denote the labels H and L such that $\alpha_H > \alpha_L$. We furthermore impose a common ratio $\eta_i = \gamma_i/\beta_i = \eta$ for both types, to ensure symmetry across groups. The effects will prove to be a combination of those in the previous two subsections.

In the NT and SB equilibrium, the orders are determined by the ratio of μ_i, which is the same for solo drivers vs carpoolers to type i drivers. Consequently, the departures of solo drivers and carpoolers of each type are mixed. Besides, in contrast to the earlier case of ratio heterogeneity where $\alpha_H/\beta_H > \alpha_L/\beta_L$, and proportional heterogeneity, for which $\alpha_H/\beta_H > \alpha_L/\beta_L$, there are now the two possibilities of $\alpha_H/\beta_H > \alpha_L/\beta_L$ and $\alpha_H/\beta_H < \alpha_L/\beta_L$. When $\alpha_H/\beta_H > \alpha_L/\beta_L$, H type users depart from home at the center of peak hours, where the opposite occurs when $\alpha_H/\beta_H < \alpha_L/\beta_L$, L type depart from home at the center of peak hours.

Similarly, in the FB case, the orders are determined by β_i and $2\beta_i$. By comparing the value of travel delay for each type users, we can determine the departure order at user equilibrium. Because β_H can now be less than β_L even though $\alpha_H > \alpha_L$, the departure order can be separated into 3 cases.

- Case 0: when $\beta_H < \beta_L < 2\beta_H < 2\beta_L$, the departure order is H type solo drivers, L type solo drivers, H type carpoolers, L type carpoolers.
- Case 1: when $\beta_L < \beta_H < 2\beta_L < 2\beta_H$, the departure order is L type solo drivers, H type solo drivers, L type carpoolers, H type carpoolers.

[6] The first-order and second-order conditions of maximization holds for an interior solution and there is one unique optium.

- Case 2: when $\beta_L < 2\beta_L < \beta_H < 2\beta_H$, the departure order is L type solo drivers, L type carpoolers, H type solo drivers, H type carpoolers.

We use a numerical example to illustrate the results for general heterogeneity in Section 4, where the NT equilibrium is given analytically, FB is obtained by Method of Successive Average, and the optimal subsidy provided by the government is also given numerically.

4 Numerical example

This section presents numerical results for two base calibrations of the model developed above: one for homogeneity and one for heterogeneity, which includes ratio heterogeneity, proportional heterogeneity and a general heterogeneity. The differences between the types of heterogeneity will turn to be important. We will find that the introduction of carpooling makes all users better off, but a gap remains between different types of drivers' benefits from different policies.

After discussing the base cases, we therefore investigate heterogeneity and its impact on efficiency. Apart from welfare gains, policy makers are also interested in distributional effects of polices. We therefore also investigate heterogeneity and distributional effects. For simplicity, we there only consider ratio heterogeneity and proportional heterogeneity. After this, we turn to the sensitivity analyses. The model outcomes are sensitive to the parameterisations, and hence it is important to present these results.

4.1 Calibration of the numerical models

As in Van den Berg and Verhoef (2011a,b), we use $N = 9000$ and $s = 3600$. We consider a trip of 30 km with a free-flow travel time TT_{ff} of 30 minutes and an extra travel time for carpooling TT_{ff}^{p} of 6 minutes. Fuel costs per trip c_{fuel}, are 7.30€[7]. The inconvenience costs for carpooling θ are supposed to be €4 per trip. Our base value of the utility scale ϕ is set to be 1. As the constant $cs^{\#}$ in consumer surplus dose not influence the travel mode behavior, we arbitrarily set $cs^{\#} =$ €10. We suppose H type and L type users are evenly distributed, i.e., $N_H = N_L = 4500$.

With homogeneity, we use a VOT α of 10 (Van den Berg (2014)). The schedule delay parameters follow the ratios $\beta/\alpha = 39/64$ and $\gamma/\alpha = 1521/640$, established in Small (1982). This implies that we adopt $\beta = 6$, $\gamma = 23.8$. For ratio heterogeneity, to make sure the average value of time is 10, we use $\alpha_H = 12.5$, and $\alpha_L = 7.5$. For proportional heterogeneity, we use $k_H = 1.2$ and $k_L = 0.8$. Hence,

[7] The monetary unit in this paper is €.

$\beta_L = 4.8$, $\beta_H = 7.2$, $2\beta_L = 9.6$, $2\beta_H = 14.4$. The travel order in the FB equilibrium for early arrivals is therefore: L type solo drivers, H type solo drivers, L type carpoolers, H type carpoolers (Case 2).

For the general case, with both types of heterogeneity, we use $\alpha_H = 12.5$, $\alpha_L/\alpha_H = 0.6$, $\alpha_H/\beta_H = 2.4, \alpha_L/\beta_L = 1.2, \gamma_L/\beta_L = \gamma_H/\beta_H = 1521/390$. Thus $\alpha_H = 12.5, \alpha_L = 7.5, \beta_H = 5.21, \beta_L = 6.25, \gamma_H = 20.28, \gamma_L = 24.375$. In the no-toll and second-best as well as third-best equilibrium, L type users will choose to travel in the center and H type users travel away from the peak. In the FB equilibrium, as $\beta_H < \beta_L < 2\beta_H < 2\beta_L$ holds, the departure orders are H type solo drivers, L type solo drivers, H type carpoolers, L type carpoolers (Case 0).

Table 2 shows the outcomes[8] for the base parameters with homogeneity under different policies. Table 3 shows the outcomes for three types of heterogeneity: ratio heterogeneity, proportional heterogeneity and general heterogeneity. Δcs_i is used to measure the effect of the introduction of carpooling compared to only solo driving on type i drivers' consumer surplus. Again a positive value denotes a net benefit from carpooling. It can be calculated from the logsum function by replacing the number of carpoolers with 0, and the number of solo drivers with the total number of each type. Relative efficiency ω is the total social welfare gain of a policy relative to the NT policy, divided by the gain of FB policy relative to the NT policy.

4.2 Base case

As a benchmark, Δcs in Table 2 indicates that the introducing of carpooling increases the consumer surplus. Even in the NT case, users are better off with than without carpooling. Of course, users can never be worse off than before, as they can always all choose to stick to driving alone and benefit from other drivers forming carpools. Compared to the NT equilibrium, the SB policy provides an optimal second-best subsidy of 5.99 to each carpooler to maximize the total social welfare, while the FB policy eliminates all queuing by tolling. This does not only affect the generalized price for each user, but also the price difference between the two modes. As a result, the generalized price (cost) for SB is the lowest, then FB, and NT is the highest. Besides, because the price difference between driving alone (20.31) and carpooling (19.63) is 0.68 in the FB case, the fraction of carpooling will be larger than driving alone (0.66 vs 0.34). For the SB case, due to the subsidy it provides to carpoolers, the generalized price for carpooling (13.71) is much less than that of driving alone (18.35), which leads

[8]The results were calculated in Matlab R2017b. For NT equilibrium, we use analytical solutions. For FB equilibrium, we use the method of Successive Average to get numerical solutions. For SB equilibrium, we use analytical condition for homogeneity and separate heterogeneity and verified the result numerically; we use the command 'maximize' to find the optimal subsidy in general heterogeneity.

Table 2

Outcomes under the base case with homogeneous users.

	NT	FB	SB
Number of driving alone N^a	7147	3040	86
Number of carpooling N^p	1853	5960	8914
Fraction of the carpoolers F^p	0.26	0.66	0.99
Generalized cost (price) for driving alone (€)	23.05	20.31	18.35
Generalized cost (price) for carpooling (€)	24.40	19.64	13.71
Optimal subsidy S^* (€)	-	-	5.99
Consumer surplus for each person cs (€)	-12.82	-9.23	-3.69
Consumer surplus change$^\#$ Δcs (€)	1.46	5.05	10.58
Total toll revenue (ten thousand €)	-	3.00	-5.34
Total social welfare SW (ten thousand €)	-11.54	-5.30	-8.67
Relative efficiency* (ω)	0	1	0.46

Note: cs is negative as we only consider the cost; $^\#\Delta cs$ is the consumer surplus change between with carpooling and without carpooling; ω is the welfare gain of a policy relative to the NT policy divided by the gain of FB policy relative to the NT policy.

to a lager probability for drivers to choose carpooling; nearly 100%. The relative efficiency of the second-best subsidization is still below 1, despite the 99% carpoolers. The difference is, obviously, due to the cost of queuing.

Table 3

Outcomes under the base case with heterogeneous drivers.

	Ratio				Proportional			General			
	NT	FB	SB	TB	NT	FB	SB	NT	FB	SB	TB
Fraction of carpoolers H	0.17	0.61	0.96	0.97	0.18	0.75	0.99	0.17	0.52	0.93	0.97
Fraction of carpoolers L	0.25	0.72	0.99	0.98	0.24	0.55	0.99	0.25	0.80	0.99	0.98
Generalized price for driving alone H	24.28	21.57	19.68	19.66	26.18	21.93	20.56	22.83	20.50	18.94	18.87
Generalized price for carpooling H	25.88	21.14	16.49	16.01	27.73	20.84	16.13	24.43	20.43	16.39	15.51
Generalized price for driving alone L	19.58	19.07	15.94	15.93	19.89	17.76	16.14	19.34	18.88	15.86	15.83
Generalized price for carpooling L	20.68	18.14	11.05	11.79	21.04	17.54	11.30	20.44	17.48	11.26	11.97
Optimal subsidy H	-	-	4.79	5.25	-	-	5.99	-	-	4.15	4.96
Optimal subsidy L	-	-	5.99	5.25	-	-	5.99	-	-	5.70	4.96
Δcs_H	1.44	4.89	9.08	9.54	1.69	4.73	11.56	1.27	4.14	7.60	8.44
Δcs_L	1.34	5.22	9.60	8.87	1.27	3.93	9.60	1.30	11.40	9.13	8.43
Consumer surplus for each person H	-14.09	-10.64	-6.44	-5.99	-15.99	-10.55	-6.12	-12.65	-9.77	-6.32	-5.48
Consumer surplus for each person L	-9.30	-7.81	-1.04	-1.77	-9.61	-6.95	-1.29	-9.05	-7.26	-1.25	-1.95
Total toll revenue (*10000)	-	3.00	-4.75	-4.61	-	2.78	-5.34	-	2.68	-4.27	-4.35
Total social welfare (SW) (*10000)	-10.53	-5.30	-8.11	-8.12	-11.52	-5.10	-8.67	-9.76	-4.98	-7.68	-7.69
Relative efficiency* (ω)	0	1	0.46	0.46	0	1	0.44	0	1	0.43	0.43

Note: cs_H and cs_L are negative as we only consider costs; $^{\#}\Delta cs_H$ and Δcs_L are the consumer surplus change between with carpooling and without carpooling;$^{*}\omega$ is the welfare gain of a policy relative to the NT policy divided by the gain of FB policy relative to the NT policy.

The effect of heterogeneity is shown in Table 3. As noted before, we consider ratio heterogeneity, proportional heterogeneity, and general heterogeneity under no-toll, first-best tolling and subsidization policies. We get the following insights. First, in most cases, the fraction of L type carpoolers is more than that of H type carpoolers, except for FB under proportional heterogeneity. There, surprisingly, the share of H type carpoolers is 25 percent higher than the L type carpoolers. Because by switching to carpooling, the H type can not only drive in the peak enjoying the elimination of delays, but also sharing tolling with the other carpooler.

Second, compared to no tolling, first-best tolling has different impacts on H and L type users. Specifically, under both ratio and proportional heterogeneity, the carpoolers of H type benefit most, followed by the H type solo drivers, L type carpoolers and L type solo drivers. The H type users benefit more because they value time savings more. And carpoolers benefit more because they share the toll between two, while enjoying the same travel time gains. When switching to a two-person carpool, users can cut the congestion toll in half and therefore come out almost even on a time-plus-money basis, given the assumed carpooling inconvenience. Under general heterogeneity, L type carpoolers benefit more than H type solo drivers due to the calibration of $\beta_L > \beta_H$.

When the government provides a second-best subsidy to attract carpoolers, ratio heterogeneity and proportional heterogeneity also lead to different results. Under ratio heterogeneity, L type carpoolers benefit the most since the extra time cost of carpooling is lower while the benefit of queue elimination, especially enjoyed by H type, no longer exists. Naturally, carpoolers benefit more because they receive the subsidy, followed by H type solo drivers and L type solo drivers. Conversely, under proportional heterogeneity, H type carpoolers again benefit the most.

Third, recall that Δcs is used to measure the change of consumer surplus due to the introduction of carpooling. Interesting results happen in the third-best subsidization and FB with proportional heterogeneity, where the H type of users again benefit more than L type users. For third-best subsidization, it is because the third-best subsidy H type carpoolers gain is more than their MEB_H (see Eqs.(32-34)). For FB with proportional heterogeneity, the H type benefit more because they drive in the peak enjoying the elimination of delays and sharing tolling with the other carpooler with carpooling.

Finally, the relative efficiency of the second-best subsidization under ratio heterogeneity is roughly the same as under homogeneity (0.46), and a modest 2 percent point lower under proportional heterogeneity. With general heterogeneity, the relative efficiency is about 43%. Theses indicate that ignoring heterogeneity may slightly overestimate the relative efficiency of the subsidization, but only mildly so. We can also find that although second-best and third-best subsidization result in different generalized prices, the welfare gains and the relative efficiency of second-best and third-best subsidization nearly keep the same (0.46 with ratio heterogeneity and 0.43 with general heterogeneity).

4.3 Heterogeneity and Efficiency

As heterogeneity will greatly influence the carpooling behavior and departure timing, we summarize the departure order in the numerical model in Table 4. We investigate 3 cases: heterogeneity in α_H/α_L, heterogeneity in β_H/β_L, and the combination of these two.

Fig.7 shows what happens to the relative efficiency of the second-best subsidization, where the subsidy can differ over types, when the two groups become more different in terms of value of time and schedule delay value; in other words, as the degree of heterogeneity (defined as $\frac{\alpha_H}{\alpha_L}$ or $\frac{\beta_H}{\beta_L}$) increases. To

Table 4

Departure order with heterogeneity.

Heterogeneity	NT and SB	FB
Separate α_H/α_L	$H^p\&H^a, L^p\&L^a$	$H^a\&L^a, H^p\&L^p$
General α_H/α_L	$H^p\&H^a, L^p\&L^a$	H^a, L^a, H^p, L^p
Separate β_H/β_L	$H^p\&H^a\&L^p\&L^a$	Case 1, and Case 2
General β_H/β_L	$H^p\&H^a, L^p\&L^a$	
Both	if $\frac{\beta_H}{\alpha_H} > \frac{\beta_L}{\alpha_L}$, $L^a\&L^p, H^a\&H^p$ if $\frac{\beta_H}{\alpha_H} < \frac{\beta_L}{\alpha_L}$, $H^a\&H^p, L^a\&L^p$	Case 0, Case 1, and Case 2

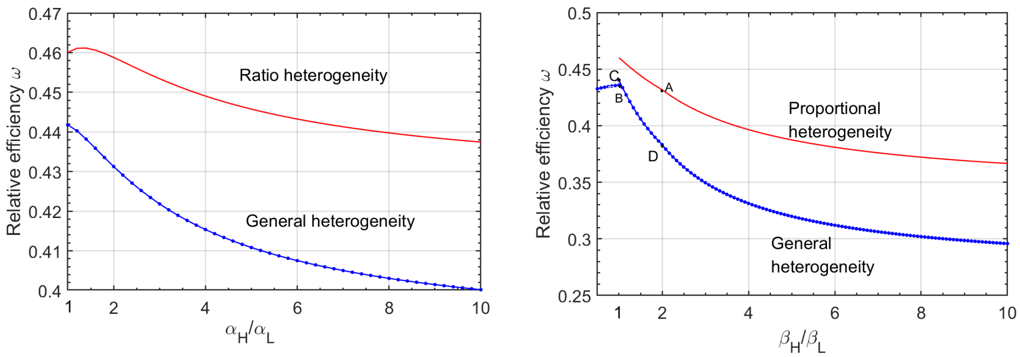
Note:As previously noted, in Case 0, the departure orders are: H^a, L^a, H^p, L^p; in Case 1, the departure orders are: L^a, H^a, L^p, H^p; in Case 2, the departure orders are: L^a, L^p, H^a, H^p.

understand the relative efficiency, recall that the FB scheme has an efficiency of 1 and the NT scheme has an efficiency of 0. When $\frac{\alpha_H}{\alpha_L} = 1$ or $\frac{\beta_H}{\beta_L} = 1$, both types are the same, ratio and proportional heterogeneity result in the same equilibrium, and the relative efficiency of the second-best subsidization stays at 0.46, the same value as under homogeneity. The general case shows a relative efficiency slightly below 0.46 at the starting point, because also when $\alpha_H/\alpha_L = 1$ or $\beta_H/\beta_L = 1$, the other values are not at the same levels as in the homogeneous base case for calibration purpose.[9]

Fig.7(a) shows that with more heterogeneity in α_H/α_L, ratio heterogeneity leads the relative efficiency of second-best subsidization to first increase over a small range, and then start to slightly fall. For general heterogeneity, the relative efficiency decreases throughout, and strongly more than under ratio heterogeneity. From Eqs.(22), (28) and (30), a larger range of α_H/α_L lowers the fraction of H type carpoolers and increases the fraction of L type carpoolers, except the L type in the SB case. Still, both NT and SB leads to relatively higher social welfare improvements with α_H/α_L increasing, while FB's welfare gain is not sensitive to α_H/α_L. The relative efficiency of the second-best subsidization hence depends on whether the change in NT surplus dominates, or the change in SB surplus dominates. Our numerical results demonstrate that when α_H/α_L is small, the effect in SB is stronger, and when α_H/α_L is large, the effect of NT becomes stronger. As a result, the relative efficiency of the second-best subsidization first increases and then decreases. Besides, the relative efficiency in general heterogeneity decreases more than under separate ratio heterogeneity because the different values of

[9]For general case, when changing α_H/α_L, the heterogeneity of β_H and β_L still exist as we keep $\alpha_H/\beta_H = 2.4$ and $\alpha_L/\beta_L = 1.2$; when changing β_H/β_L, the heterogeneity of α_H and α_L also exist as we keep the average value of α at e 10.

β_H and β_L in general heterogeneity make drivers benefits more from time-varying tolling.



(a) **(b)**

Fig. 7. Effects of heterogeneity on relative efficiency.

(a) Changing ratio heterogeneity; (b) Changing proportional heterogeneity.

With more heterogeneity in β_H/β_L, in the right panel, proportional heterogeneity leads to a decreasing relative efficiency of second-best subsidization. In this case, all policies lead to relatively higher social welfare improvement with an increasing β_H/β_L, but different from the insensitivity to α_H/α_L, FB welfare gains now play a key role in the results. Consistent with the earlier discussion around Table 4, there are two departure orders with changing proportional heterogeneity in FB pricing: Case 1 before point A, and Case 2 after point A. In Case 1, the fraction of L type carpoolers increases with β_H/β_L and that of H type carpoolers decreases with β_H/β_L, and H type users dominate L type users in the computation of social surplus. In Case 2, in contrast, the fractions of carpoolers of both types decrease with β_H/β_L. As a result, the gain of first-best pricing increases with β_H/β_L, and increases more in Case 2. This explains why the kink at point A exists, and explains why the relative efficiency of second-best subsidization drops and slightly quicker from point A. Because carpoolers are always from the same type, this happens exactly at $\beta_H/\beta_L = 2$.

To investigate the effects of β_H/β_L in general heterogeneity (the blue line in Fig.7(b)), we increase and decrease β_H/β_L, α_H/α_L, γ_H/γ_L by the same percentages, while keeping the ratio $\frac{\alpha_H}{\beta_H} = \frac{12.5}{5.21} = 2.4$, $\frac{\alpha_L}{\beta_L} = \frac{7.5}{6.25} = 1.2$, $\frac{\gamma_H}{\beta_H} = \frac{\gamma_L}{\beta_L} = \frac{20.28}{5.21} = 3.89$ constant. As β_H can now be lower than β_L, the minimal value of β_H/β_L must exceed 0.6, to make sure that the assumptions $\alpha_H > \beta_H$ and $\alpha_L > \alpha_L$ remain satisfied. The no-toll and the second-best policy are the same as under proportional heterogeneity. But with FB tolling, there is a peak at $\beta_H = \beta_L$. When $\beta_H < \beta_L$, the welfare gains from tolling is decreasing with the degree of heterogeneity, as the two groups then become more similar, which of course differs from the increasing tendency for $\beta_H > \beta_L$. This leads to the relative efficiency of second-best subsidization increasing for β_H/β_L to the left of B. Then a jump from point B to point C happens, when the departure order changes from case 0 to case 1 at $\beta_H = \beta_L$. From this point onward, the relative efficiency decreases smoothly until another departure order appears at $\beta_H/\beta_L = 2$, where there is again a kink at point D. The relative efficiency of SB in Case 2 is again decreasing more strongly than in Case 1. In the Appendix, we discuss the general heterogeneity case in some more detail.

4.4 Heterogeneity and Distributional Effects

Apart from concerns over welfare gains, policy makers are also interested in the distributional effects of pricing, not in the least place because this has a strong impact on the social and political feasibility. A policy is naturally more likely to meet resistance from travellers if they are made worse off. Fig.(8) indicates that heterogeneity has a strong impact. Fig.8 (a-b) shows the consumer surplus changes (Δcs) from the situation without carpooling to the situation with, by separately varying α_H/α_L and β_H/β_L. Fig.8(c-d) shows the generalized prices change (Δp) from no tolling to second-best subsidization.[10] A positive value is therefore "good" in panels (a) and (b), and "bad" in panels (c) and (d).

As Fig.8(a-b) shows that, although all users are better off by introducing carpooling, the benefits for different types of users and heterogeneity under different policies greatly differ. For all of the policies, heterogeneity raises the generalized price of H type users and lowers that of L type users both with and without carpooling, and therefore cs_H decreases and cs_L increases with the degree of heterogeneity. But Δcs depends on whether the effects of heterogeneity on cs with carpooling dominate over the effect on without carpooling. In line with this, specifically, first-best tolling and second-best as well as third-best subsidization show interesting results. First, for the FB policy, in Fig.8(a), L type users benefit more than H type users, and this benefit increases with α_H/α_L for H type and decreases for L type. This is because the increasing of cs with carpooling dominates that of without carpooling for type H and conversely for type L. But for proportional heterogeneity in Fig.8(b), H type users benefit more than L type users, and as the two groups become more different in β_H/β_L, both of them will benefit less with an increasing of β_H/β_L.

Second, in terms of subsidization, different types of heterogeneity also result in different effects from introducing carpooling. The first intuitive comparison is between second-best and third-best subsidization in Fig.8(a). As we have discussed in Section 4.2, with the second-best subsidization, L type users benefit more than H type users, and with the third-best subsidization, H type users benefit

[10] In panels (a) and (b), $\Delta cs = cs(\textit{with carpooling}) - cs(\textit{no carpooling})$; in panels (c) and (d), $\Delta p = p(SB) - p(NT)$.

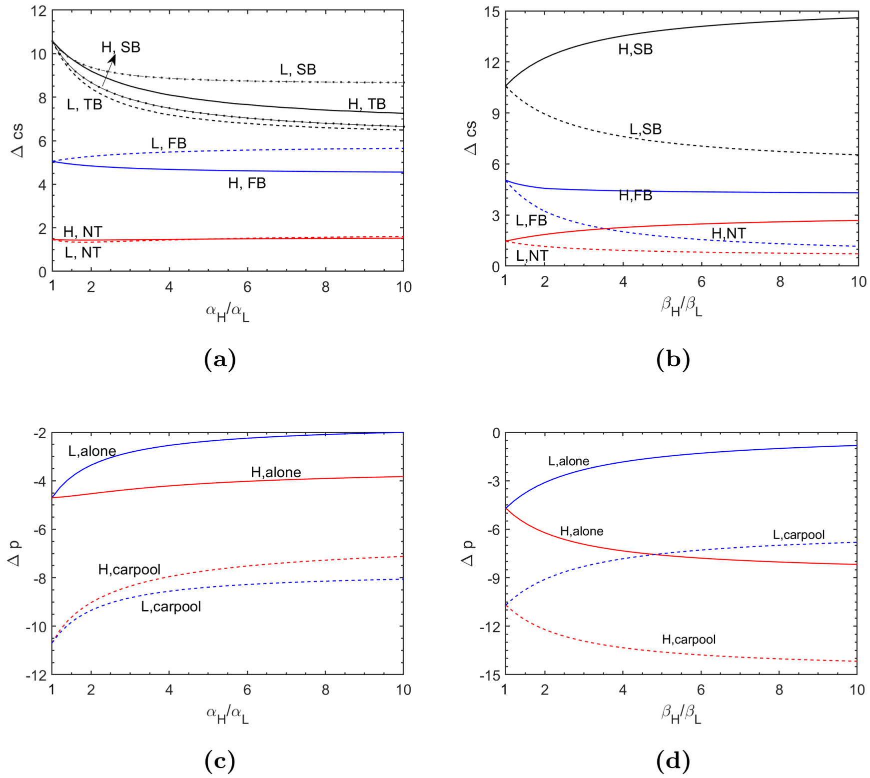


Fig. 8. Heterogeneity and distributional effects.

(a) and (c) Ratio heterogeneity; (b) and (d) Proportional heterogeneity. Panels (a) and (b) show change in consumer surplus from situation without to with carpooling; Panels (c) and (d) show change in generalized price from no tolling to second-best subsidization.

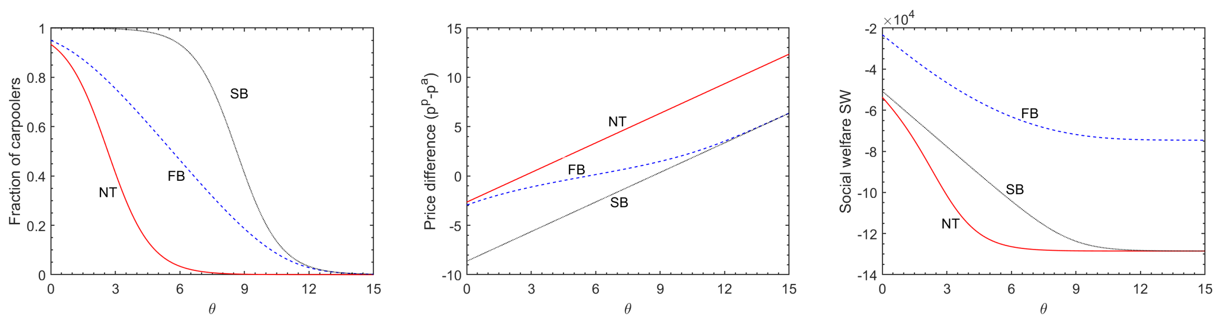
more. We can also find that the benefits of both type users are dropping as the two groups become more heterogeneous. Specifically, for H type carpoolers, it is because the optimal second-best subsidy decreases with α_H/α_L (see Eq.(32)). In contrast, in Fig.8(b), H type users benefit more than L type users for all of the three policies. But as β_H/β_L increases, the benefits of L type users still decrease with the degree of heterogeneity and those of H type users increase with β_H/β_L, because H type users value their time savings more.

Fig.8(c-d) shows that, all users gain from second-best subsidization, but the distributional effects differ with the types of uses and heterogeneity. First, consistent with the earlier discussion, with ratio heterogeneity in Fig.8(c), L type carpoolers benefit more than H type carpoolers. And with proportional heterogeneity in Fig.8(d), H type users benefit more than L type users for both solo drivers and carpoolers. Second, as the degree of ratio heterogeneity increases in Fig.8(c), all users benefit less. In contrast, in increasing β_H/β_L in Fig.8(d), L type's benefits from second-best subsidization decrease and H type's benefits increase with the degree of proportional heterogeneity. Indeed, under second-best subsidization, ratio heterogeneity raises all group's generalized prices, thus leading to a decreasing benefit for all of the users. Under proportional heterogeneity, although the second-best subsidization lowers the price for L type users and raises that for H type users as the proportional heterogeneity increases, combining with the effects of no-tolling, L type's benefits from second-best subsidization still decrease and H type's benefits still increase with β_H/β_L.

4.5 Sensitivity analysis

There is little to no guidance from the literature on the values of θ and the scale of utility ϕ. Therefore, it is vital to do extensive sensitivity analyses. The effects of TT^p_{ff} and c_{fuel} are presumably similar to those for θ. Hence, these parameters will not be discussed further here.

4.5.1 Sensitivity analysis with respect to θ



(a) (b) (c)

Fig. 9. Effect of θ under homogeneity. (a) Fraction of carpoolers; (b) Price difference; (c) Social welfare.

In this section we vary the inconvenience cost of carpooling, θ, from 0 to 15. Fig.9 shows the equilibrium share (a) of carpoolers, the price difference (b) between driving alone and carpooling, and social welfare SW (c) under different policies with homogeneity. For all the three polices, the fraction of carpoolers naturally and nonlinearly decreases with an increasing θ, consistent with the logit preferences. It is not surprising that the share of carpoolers with the SB subsidy is the largest, followed by FB and finally NT. When θ increases to 12, the fraction of carpoolers goes towards 0. But the curve for NT and SB is steeper than that for the FB case. The numbers of carpoolers and solo drivers are determined by the price difference between driving alone and carpooling.

Fig.9(b) shows this price difference. It can be seen that the curves of NT and SB both increase with the same slope 1, reflecting that θ does not affect the marginal external benefit and hence the subsidy, but the intercepts are different due to the subsidy provided by the SB policy. For the FB, the price difference is not only determined by θ but also the tolling reduction $\frac{\delta N^a}{2s}$. When θ is small, the number of solo drivers N^a is almost 0 so that the price difference under FB is close to that under NT. When θ is large, almost all users will choose to drive alone, and N^a is almost N, so that the price difference in FB is close to that in SB. For θ between 0 and 12, the price difference of FB shows a less steep trend.

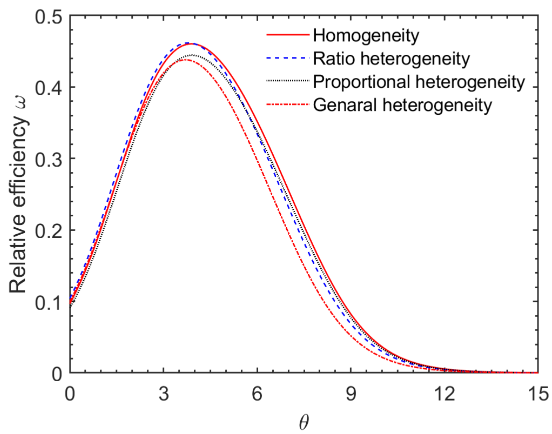


Fig. 10. Effect of θ on relative efficiency

Fig.9(c) shows the relationship between inconvenience cost θ and social welfare (SW). It can be seen that SW decreases with θ for all the three policies. However, the decline gradient for FB is flatter than that of SB and NT, and SB shows a relative uniform decrease, while NT first decreases rapidly when θ is less than 6 and then declines uniformly to stabilization. This is caused by the fact that there are no carpoolers to be "lost" with further increasing of θ. Specifically, when θ is 0 and 10, the social welfare under NT and SB cases are very close and when theta is about 4, the gap is the largest. The reason is that the subsidy is relatively ineffective when carpooling is intrinsically very popular (when θ is low), or so unattractive in terms of private disutility that the marginal external benefit becomes negligible and is insufficient to induce behavior change (when θ is high).

The inconvenience cost θ can hence affect the welfare gain from a policy regime. Fig.10 further shows this by giving the relative efficiency ω of the second-best subsidization for the different types of heterogeneity, for varying θ. It can be observed that ω first increases and then decreases with the increasing of θ, and is highest when θ is around 4. It does so for different types of heterogeneity. These patterns confirm what was just said: for intermediate values of θ, the SB subsidy is most effective.

4.5.2 Sensitivity analysis with respect to utility scale ϕ

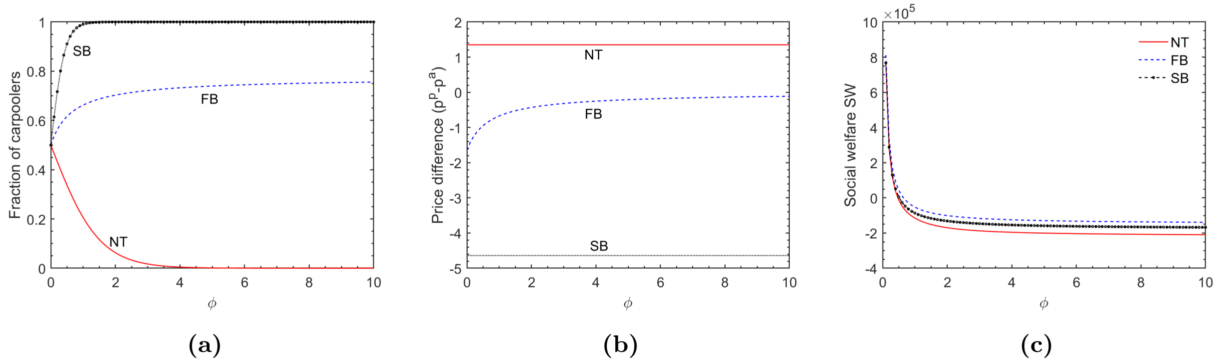


Fig. 11. Effect of utility scale ϕ under homogeneity. (a) Fraction of carpoolers; (b) Price difference; (c) Social welfare.

Next, we look at the impact of the scale of utility, ϕ by varying the value of ϕ from 0 to 10. A larger ϕ means more deterministic preferences. In the base case, the scale of utility is 1. Fig.11(a) depicts the fraction of carpoolers. When $\phi = 0$, the stochastic part of the utility function is very large, resulting in mode choices that are effectively independent of the deterministic part of utility. The probabilities then converge, in the limit, to $1/2$, independent of the policy. When $\phi \to \infty$, NT leads to almost 0% of carpools and SB leads to almost 100% of carpoolers, while FB still has 72% carpoolers.

Fig.11(b) further explains the reason, by showing the price differences between carpooling and driving alone under different policies. As ϕ increases, with no-toll, the price for carpooling is always larger than for driving alone, and with subsidization, the price for driving alone is always lower. This brings corner solutions. For FB, the price difference between carpooling and driving alone is not that large, which results in an interior solution of 0.72. As can be seen from Eq.(13), the benefit of switching from driving alone to carpooling decreases in the number of solo drivers, leading to an interior solution. Also, it can be seen from Fig.11(c) that social welfare decreases with an increasing scale of utility for all three policies. This is due to the increasing importance of idiosyncratic utility when ϕ decreases; see Eq.(3).

Fig.12 shows the combined effects of utility scale and heterogeneity on relative efficiency. An increase in the scale of utility increases the relative efficiency of second-best subsidization for all types of heterogeneity. This is because an increase in ϕ triggers more solo drivers to go carpooling; both under FB tolling and under SB subsidy, but more so under SB. With a stronger systematic utility, the second-best policy therewith becomes a more powerful instrument.

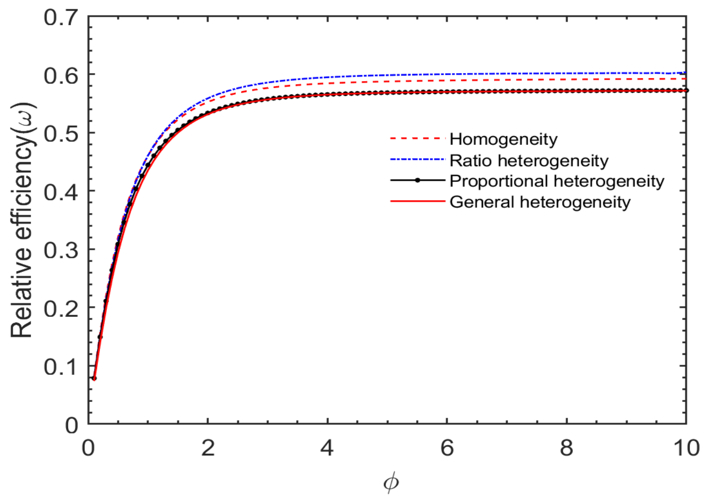


Fig. 12. Effects of ϕ on the relative efficiency

5 Conclusion

We have investigated the effects of carpooling in a dynamic equilibrium model of congestion, that captures various dimensions of users heterogeneity: a distribution of idiosyncratic preferences for carpooling, and heterogeneity of values of time and values of schedule delay, considering both "ratio heterogeneity" and "proportional heterogeneity", as well as the combination of these. The share of users of carpooling is endogenous. We consider four policy scenarios: no-toll, first-best pricing, second-best and third-best subsidization of carpooling.

We find that all the commuters will be better off by introducing a carpooling program. As a large part of the benefits of carpooling goes toward the other drivers, it is worthwhile to provide a subsidy to make carpooling more attractive when no other (road pricing) policy is implemented. The optimal subsidy turns out to be each type's marginal external benefit (MEB) from carpooling. When the subsidy cannot be differentiated (third-best subsidization), the weighted sum reduces to each type's MEB, respectively.

We also evaluated the relative efficiency of the second-best subsidization with differentiated subsidies under different types of heterogeneity, both in terms of social welfare and distributional effects. The relative efficiency first increases and then decreases with the degree of ratio heterogeneity; and decreases more with the degree of proportional heterogeneity. All users gain from second-best subsidization. But surprisingly, with ratio heterogeneity, L type carpoolers benefit more than H type carpoolers. And with proportional heterogeneity, H type users benefit more than L type users for both solo drivers and carpoolers.

Heterogeneity plays an important role in the performance of introducing carpooling, especially when considering tolling and subsidization. Another interesting comparison is between second-best subsidization and third-best subsidization. With second-best subsidization, L type users benefit more than H type users as the L type carpoolers gain for subsidy from the switching to carpooling; and if the subsidy is undifferentiated, (i.e., third-best subsidy), H type users benefit more because the subsidy each L type carpooler gains is less than their MEB_L.

The results are sensitive to the model's crucial parameters. For instance, an intermediate inconvenience cost brings the largest relative efficiency of second-best subsidy, and a larger scale of utility brings a more effective subsidy. There is substantial uncertainty about these parameter values, and this make investigating them an important future research topic.

There are of course some limitations in the model setting. We consider a discrete setting, with only two types of drivers and we ignored alternative transport, elastic demand, and route choice. Therefore, the following possible extensions in the future study are identified. First, an elastic function for the total demand may be considered, so that the reduced travel cost may attract more commuters. Second, interactions among multiple origin-destination pairs may be modeled in a network setting. Third, public transit mode, such as a metro line, may be added to examine a multi-modes transportation system. Fourth, commuters with continuous heterogeneity may be considered. Finally, it will be interesting to study other policies to promote carpooling in our setting, such as HOV lanes and free or preferential parking for carpoolers.

Appendix A Full heterogeneity

All results in Section 4.3 are further confirmed in Fig.13. Fig.13(a-b) shows the departure order in different pricing policies. Fig.13(c-d) shows how the combination of α_H/α_L and β_H/β_L influences the social welfare and the relative efficiency of second-best subsidization. As illustrated in Table 4, there are 2 departure orders for NT and SB equilibrium, and 3 departure orders for FB equilibrium. We first look at the FB equilibrium. Consistent with the earlier discussion, with varying β_H/β_L, welfare gains first decrease in case 0 and then increase in case 1 and case 2, and increase more in case 2. With varying α_H/α_L, welfare gains of FB first increase for a small range ($1 < \frac{\alpha_H}{\alpha_L} < 1.26$) and then start to decrease for the most range. While for NT equilibrium, the change of welfare gains is clearly divided into 2 cases: when $\frac{\alpha_H}{\alpha_L} > \frac{\beta_H}{\beta_L}$, it increases with α_H/α_L, whereas when $\frac{\alpha_H}{\alpha_L} < \frac{\beta_H}{\beta_L}$, it decreases with α_H/α_L. Of course, for both cases, welfare gains of NT decrease with β_H/β_L. Besides, although second-best subsidization has the same departure order as NT, the social welfare curves of SB are not

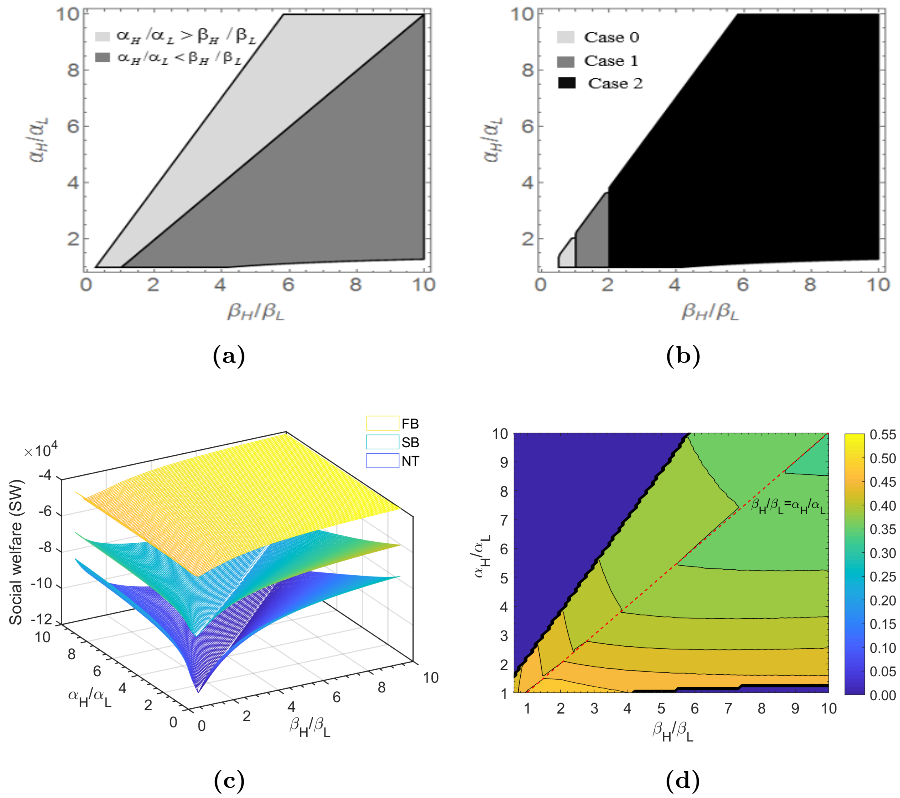


Fig. 13. Relative efficiency and social welfare in the general heterogeneity.
(a) Departure order in NT and SB; (b) Departure order in FB;
(c) Social welfare; (d) Relative efficiency.

exactly downward shifted copies of the NT curves, because the subsidy provided by the government varies over the value of time and schedule delay. Depending on which welfare effects dominate, the relative efficiency of the second-best subsidization decreases with α_H/α_L, and increases with β_H/β_L when $\beta_H < \beta_L$ and start to decrease when $\beta_H > \beta_L$. The red line that separates the contour plot region in Fig.13(d) is $\beta_H/\beta_L = \alpha_H/\alpha_L$, where the departure order changes from H type in the bottleneck center to L type in the bottleneck center in NT and SB.

Acknowledgement

The first author gratefully acknowledges the financial support from China Scholarship Council (CSC) during the one year's visit to VU Amsterdam. Any remaining errors are ours.

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