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Underrepresentation of women in high level positions is widespread and persistent. We analyze the consequences of labor market quotas for the wages of women in high level positions. The key point of our paper is that quotas cause asymmetric information about why women work in high level positions. Firms know why they have assigned their own female employees to high level positions, but do not know why women at other firms have been assigned to those positions. A winner's curse, reducing competition for women in high level positions, results. This widens the gender pay gap. We show that ex ante women are better-off without quotas. Next, we investigate how quotas affect incentives for employers to learn the abilities of women to make better job-assignment decisions. Then, under specific conditions women may benefit from quotas.

Keywords: labor market quota, winner's curse, screening
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1 Introduction

There is ample evidence that in a variety of markets, women are underrepresented in high level positions. Academia is an illuminating example. In Europe, almost 50% of the PhD students, approximately 30% of the associate professors, but a mere 20% of the full professors are female.[1] To improve representation of women in high level positions, several countries consider some form of affirmative action, ranging from offering women programs to better promote themselves to imposing labor market quotas. Usually, affirmative action is motivated by an obstacle women face when competing for high level jobs. This obstacle can be explicit discrimination.[2] However, it can also have more subtle causes. For instance, psychologists have found evidence for the “out-group homogeneity hypothesis”, meaning that individuals of one group tend to perceive other groups as homogenous (Quattrone and Jones, 1980). Bjerk (2008) shows that when predominantly men decide on promotions, this tendency may explain underrepresentation of women in high level positions. There is also evidence that women tend to comply to a norm of modesty. This would make them less visible, damaging their chances of promotion (Rudman, 1998, and Heilman, 2001).[3]

In this paper, we analyze theoretically the consequences of an affirmative action program on the wages of women in high level positions and on firm profit. Following Milgrom and Oster (1987), this program consists of a quota for women in high level positions, and a lower bound on the wages for those women. We also examine the effects of the affirmative action program on firms' incentives to learn the abilities of women in order to make better job-assignment decisions. Point of departure of our analysis is a labor market

[1] We borrowed this striking example from Bagues et al. (2014), who based themselves on a report of the European Commission in 2013.

[2] See Becker (1957) who provides a taste-based explanation for discrimination. Another strand of the literature considers models which focus on statistical discrimination, like Coate and Loury (1993), Fryer (2007) and Bjerk (2008). Bertrand and Duflo (2016) review the literature on field experiments with regard to discrimination.

[3] An alternative explanation for the underrepresentation of women in top positions is that women shy away from competitive environments (Niederle and Vesterlund, 2007). Balafoutas and Sutter (2012) and Niederle et al. (2013) show that affirmative action policies (e.g. quotas) make women more likely to enter competitive environments. Scotchmer (2008) shows that a difference in risk-taking behavior between men and women might also effect promotion decisions. In our paper, however, we do not investigate the decision of women to enter a competitive environment.

in which underrepresentation of women in high level positions is caused by firms being better able to assess the abilities of men than the abilities of women.[4] The affirmative action program can be seen as a response to this underrepresentation.

Our paper combines two fundamental insights from earlier research. First, Waldman (1984) points out that promotion decisions contain information about employees' abilities. In our model, in the absence of a quota, firms promote women with known high abilities. A quota forces firms to also promote other women. Quotas therefore cause asymmetric information. Other firms do not know whether a woman has been promoted for her ability or for meeting the quota. The second insight originates from common-value auction theory. Bidders in common-value auctions are subject to a winner's curse if they bid their own estimated values (Stiglitz and Weis, 1981, and Milgrom and Weber, 1982). If a bidder wins the auction, it is likely that he was too optimistic about the value of the good. Rational bidders respond to the winner's curse by bidding less than their estimates or by abstaining from making a bid. In our model, a firm that has attracted a woman who was promoted at another firm is also subject to a winner's curse. The fact that the other firm offered no higher wage, makes it more likely that she has been promoted because of the quota instead of her ability.

We derive three sets of results. The first set of results concerns the effect of a quota on female employees and firms, holding screening - the probability with which firms observe the ability of a woman - constant. Initially, we assume a labor market where firms simultaneously make wage offers to their own employees and those of the competitor. In our model, by definition, a quota causes a shift from women in lower level positions to women in higher level positions. A direct implication is that the average ability of women in high level positions declines. This increases the gender wage gap. In addition, uncertainty about women's abilities in high level positions at other firms creates a winner's curse. This reduces competition for those women, further widening the gender wage gap. Due to the latter, women suffer from a quota from an ex ante point of view. Firms in contrast receive an informational rent. Firms may benefit from a quota if the lower bound on

[4]Sarsons (2017) finds that female surgeon's performance is interpreted differently than male surgeon's performance. Women (Men) are rated less (more) favorably after bad (good) patient outcomes. Mengel et al. (2017) and Boring (2017) find a similar result in academia, where female professors are rated lower than male professors in student evaluations, particularly by male students.

the wages for women in high level positions is sufficiently small.

Our second set of results concerns the effect of an affirmative action program on firms' incentives to screen women. As discussed above, the introduction of a quota gives firms an informational advantage. Each firm knows whether the promotions of women in it's own firm are ability or quota-based. In equilibrium, the firm makes a profit on promoted women with high ability. This provides an incentive to firms to detect high ability women. From this point of view alone, a quota strengthens firms' incentives to screen. There is a counterforce, however. With perfect screening, only high ability women would be promoted. In equilibrium, other firms know this. As a result, the informational advantage vanishes and firms' rents would go to zero. Hence, a quota may alleviate the cause of underrepresentation of women in high level positions, but only partially.

Our final set of results concerns the sensitivity of our results to our assumption of a simultaneous wage bidding process. We show that the severity of the winner's curse depends on the precise nature of the wage setting process. However, qualitatively, the nature of the bidding process is not important for the effects of quotas on wages for women, given screening. For a certain level of screening, quotas generally impair women in expected wage terms. The effect of a quota on screening, however, does depend on the exact nature of the bidding process. When firms simultaneously make wage offers, competition for promoted women is relatively fierce. In that setting, if screening improves, women may benefit from quotas. When the wage bidding process is sequential, the firm that can make the last offer has full bargaining power and receives all informational rents. If the raiding firm receives the rents, incentives to screen are weak, while if the current firm receives the rents, incentives to screen are strong.

Our model studies the effects of affirmative action policies in environments where those who are responsible for job assignment decisions are better able to identify talented men than talented women. We are aware that this is only one cause of underrepresentation of women in high level positions. Our paper is nevertheless relevant. In the last decades, the skills of women have improved considerably relative to the skills of men in many modern countries. In the US, the gender gap in college attendance is even reversed (Blau and Kahn, 2017). With traditional explanations for underrepresentation of women in high level positions becoming less relevant, and against the background of the persistence of underrepresentation, it is natural to turn attention to alternative explanations.

It is worth emphasizing that a quota does not require a rationale in terms of improving labor outcomes. Diversity in higher positions can be a goal in itself. From this perspective, our analysis examines the consequences of enforcing diversity for labor-market outcomes. It identifies costs, such as lower expected wages, and possible benefits, such as better screening, of having more gender diversity in high level positions.

Our paper is organized as follows. The next section discusses related literature. Section 3 describes the model. Section 4 presents the unique equilibrium of the model and analyzes the consequences of affirmative action policies on wages and firm profit. We discuss how affirmative action policies affect firms' incentives to screen employees in Section 5. Section 6 shows how our results depend on the exact nature of the wage bidding process. Section 7 concludes.

2 Related Literature

Our paper is closest related to Milgrom and Oster (1987), henceforth MO. Like ours, MO's analysis builds on the idea that abilities of members of advantaged groups, here men, are better recognized than abilities of members of disadvantaged groups, here women. They apply this idea by assuming an environment of asymmetric information. Employers know the abilities of their own employees, but not the abilities of employees at other firms. In MO, as in Waldman (1984), potential new employers may infer information about the abilities of females from promotion decisions. MO show that in such environments, employers have incentives to hide talented women. When able women have a comparative advantage in high level jobs, not promoting them is inefficient from a social point of view. MO also show that in the long run appropriate quotas may weaken employers' incentives to hide able women.[5]

Like MO, we employ a promotion model in the spirit of Waldman (1984). In our model, underrepresentation of women in high level positions does not result from employers hiding able women for potential employers, but directly results from the assumption that women whose abilities are not known are in expectation less productive in the high-level position than in the lower-level position. Therefore, given that employers cannot always distinguish

[5] See Cassidy et al.(2012) and DeVaro et al.(2012) for extensions of the MO model and empirical evidence that is consistent with the model's main predictions.

more talented women from less talented women, job assignment decisions are efficient in the absence of a quota. Where the focus of MO is on promotion decisions (in order to explain underrepresentation of women), our focus is on the consequences of a quota for the wage-setting process after the promotion decisions.

Our analysis also differs from MO in that we examine various bidding processes. MO assume that current employers can match outside offers. They admit that this bidding process is not an accurate description of reality. We show that in our model, qualitatively, the exact nature of the bidding process is not important for the effect of a quota on the expected wage of promoted women. However, the exact nature of the bidding process is qualitatively important for employers' incentives to learn females' abilities.

Our variation on the Waldman model has been widely used in the banking literature.[6] As in our model, key in this literature is a winner's curse, which reduces competition and enables banks to make positive informational rents. Hauswald and Marquez (2003) analyze banks' incentives to acquire private information about their clients' credit quality. Their analysis resembles ours in Section 5, which deals with firms' incentives to screen women better. In Hauswald and Marquez possessing more information about a client is always an improvement. In our model, more information also has a cost. It relaxes the winner's curse as more information also means that promotion decisions contain more information. If a firm has no uncertainty about the ability of the employees it promotes, it's informational rents equal zero.

Arcidiacono and Lovenheim (2016) review the contentious literature on the effect of affirmative action in school admission standards for racial minorities on student performance and earnings. Sander (2004) is the first to reason that affirmative action policies have the same effect on schools, as in our model quotas have on firms. These policies lead to a mismatch. Sander (2004) shows that students, who attended high-ranked schools because of affirmative action policies, would have been better off at lower-ranked schools. They lack the academic training to benefit from the education in high-ranked schools. In our model, women who due to a quota end up in high level posi-

[6] These studies model competition for financial services with private information about the credit quality of portfolio clients. This literature started with Stiglitz and Weis (1981). Sharpe (1990), Marquez (2002), Hauswald and Marquez (2003), Von Thadden (2004) and Bijkerk and de Vries (2012) use models in which banks simultaneously choose interest rates as in Section 4. Dell'Ariccia et al. (1999), Boot and Thakor (2000) and Bouckaert and Degryse (2004) use a sequential setting as in Section 6.

tions, are in expectation more productive in lower positions. Sander (2004) also finds evidence indicating that the effects of affirmative action policies on earnings are negative. Although this evidence is consistent with the predictions of our model, it is not sufficiently specific to view it as supportive for our model. The result that by admitting academically weaker students, average earnings decline, is not very surprising. Our model predicts that by assigning less productive women to high level positions, the wages of the *high ability* women in high level positions decline. Moreover, a key feature of our model is that no verifiable information exists about an employee's productivity. In an educational setting, grades contain at least some information. This makes an educational setting less suitable for testing the predictions of our model.[7]

Our paper also contributes to a broad literature on the gender pay gap. In the US, the gender pay gap fell dramatically in the eighties, and has slowly, but not fully, converged in the last 25 years. Blau and Kahn (2017) give a comprehensive review of this literature. Convergence of human capital skills has significantly contributed to the decrease of the gender pay gap. However, the gender pay gap is still large at the top of the wage distribution. In this respect the emphasis of our paper on wages of women in high positions seems relevant. In our model, without affirmative action policies a gender pay gap exists because fewer women are promoted to high level positions. For each position, however, wages are equal for men and women. Affirmative actions policies widen the gender pay gap and lead to a decrease in expected wages for high ability women in higher positions.

3 The Model

We consider a labor market with a continuum of employees and two identical firms, F and $-F$. Each employee i has either high ability or low ability $a_i \in \{l, h\}$, with $h > l > 0$. The probability that $a_i = h$ equals α.[8] We denote the ex ante expected ability of an employee by $\underline{a} \equiv \alpha h + (1-\alpha)\, l$. We

[7] A recent strand of literature investigates the impact of gender quotas for the board of directors on firm valuation (Ahern and Dittmar, 2012), on the gender wage gap, and on spillover effects to other women working in the same firm (Bertrand et al., 2017).

[8] In our model it is not important whether or not an employee knows its own ability. It is important that the employee cannot send informative signals about its ability. We assume that ability is not verifiable.

employ a two-period model. In the first period, each firm hires a continuum of employees of measure 1. Each firm learns the ability of an employee with probability ρ. With probability $1-\rho$, the firm does not learn the ability of that employee. The composition of each firm's labor force consists of three types: the (known) high ability employees, with share $\rho\alpha$, the (known) low ability employees, with share $\rho(1-\alpha)$, and the employees with unknown ability, with share $1-\rho$.

At the beginning of the second period, each firm assigns employees to positions. We do not consider layoffs and new hires. The firms have two positions available, $P \in \{R, H\}$. We refer to R as the routine position, and to H as the high level position. In position R employee i produces output $y_i = r$, independent of ability. In position H employee i produces output $y_i = a_i$. We assume that $h > r > \underline{a} > l$, implying that from a productivity point of view each firm wants to assign high ability employees to position H, and known low ability employees and employees with unknown ability to position R.[9]

After the firms have assigned their employees to positions, they simultaneously make wage offers to their own employees and to each other's employees. Wage offers are determined prior to second period production and cannot be conditioned on the realization of output. The assignment of employees to positions is common knowledge. Additionally when making wage offers, firms have private information on the ability types of their own employees. In accordance with Waldman (1984), one of our key assumptions is that firm F does not know the abilities of the employees of firm $-F$. However, firm F may infer information about the abilities of firm $-F$'s employees from $-F$'s assignment decisions. When making wage offers, each firm tries to maximize the expected profit on each employee i, that is the output employee i produces minus the wage. Each employee chooses the firm that offers the highest wage. In case of a tie, an employee chooses to work for the competing firm.[10]

As discussed in the introduction, a well-known source of underrepresentation of women in high level positions is that those who assign jobs, are better able to assess the abilities of men than the abilities of women. Applied to our model, this can be captured by the assumption that ρ can take on two values $\rho \in \{\rho_M, \rho_W\}$ with, say, $\rho_M = 1$ for men, and $\rho_W < 1$ for women. Because

[9]Since the routine job an employee's output does not depend on its ability, firms do not have incentives to conceal able workers, as in Milgrom and Oster (1987).

[10]This tie-breaking rule does not affect our main results.

of our assumption that $r > \underline{a}$, firms do not want to assign employees with unknown ability to position H. Underrepresentation of women in $P = H$ results: the share of women in $P = H$ equals $\rho_W \alpha$, while the share of men equals α.

In an attempt to improve representation of women in high level positions, the government implements an affirmative action program. Following Milgrom and Oster (1987), this program consists of a quota for women in position H, of size q, and a wage standard, $m \geq r$, that women in $P = H$ should at least receive.[11] Specifically, we assume that the government compels each firm to assign a proportion $q > \alpha\rho_W$ of their female employees to $P = H$.[12] Throughout, we assume that $\rho_W \alpha < q \leq \alpha$, implying that to meet the imposed quota, the firm needs to assign a share of its employees with unknown ability to $P = H$, but does not need to assign known low ability employees to $P = H$. Given these assumptions the expected ability of an employee assigned to $P = H$ under a quota q equals

$$\overline{a}(q) \equiv E(a|P = H) = \left(\frac{q - \alpha\rho_W}{q}\right)\underline{a} + \left(\frac{\alpha\rho_W}{q}\right)h \tag{1}$$

We assume that $m < \bar{a}(q)$, implying that the affirmative action policies do not force firms to make an expected loss on employees in $P = H$. In the next section, we examine the consequences of q and m for the wages employees receive in $P = H$, and the expected profit for the firms.

The analysis concerns the wage setting process after the firms have assigned their employees to positions. Each firm makes three kinds of wage offers. First, each firm offers a wage to all employees who are assigned the routine job. Competition for those employees drive wages to r. In the remainder of this paper we ignore competition for employees in $P = R$. Second, each firm offers a wage to each of its own employees being assigned position H. These wage offers can be conditioned on the expected ability of employees. We denote by $w_F^h \geq m$ and $w_F^{\underline{a}} \geq m$ the wage firm F offers to an employee in $P = H$ with ability $a = h$ and expected ability $a = \underline{a}$, respectively. Finally,

[11] An alternative interpretation of $m \geq r$ is that employees in $P = H$ should at least receive the wage of employees in $P = R$.

[12] In our model, firms hire a continuum of employees, whom are identical and in equilibrium follow the same strategies. Therefore, the shares of employees in $P = H$ are equal before and after employees can transfer from one firm to the other. Consequently, the precise timing of the implementation of the quota is not important for our results.

firm F may offer a wage, $w_F \geq m$, to each employee of firm $-F$ who has been assigned position H.

In the next section we solve the model under the assumption that firms make offers simultaneously. In this setting, neither firm has a clear bargaining advantage, besides the informational one. In Section 6, we discuss how alternative bidding processes affect our results. We show that quantitatively the nature of the bargaining process is important for the effects of affirmative action policies on the expected wage for employees in $P = H$. However, we also show that our qualitative results depend far less on the nature of the bargaining process. Qualitatively, the nature of the bidding process is key for the effects of a quota on firms' incentives to screen employees.

We solve the model by identifying Perfect Bayesian Nash equilibria, hereafter PBE. Firm F's bidding strategy consists of (i) three distribution functions over wage offers to employees in position H, and (ii) the probability η that it does not bid on a promoted employee of $-F$. Firm F makes wage offers

- w_F^h (with cumulative distribution function $H(w)$) for an own employee in $P = H$ of known high ability;
- w_F^a (with cumulative distribution function $L(w)$) for an own employee in $P = H$ of unknown ability.

Moreover, with probability $1 - \eta$, Firm F makes an offer to an employee of $-F$ in $P = H$. In that case, F offers

- w_F (with cumulative distribution function $U(w)$) for an employee of $-F$ in $P = H$.

Before proceeding with the analysis of the consequences of affirmative action policies on wages and profits, we first discuss the outcomes of our model in the absence of affirmative action policies. In the absence of a quota, firms assign all employees with known high ability to $P = H$ and all other employees to $P = R$.[13] Firm $-F$ infers from firm F's assignment decisions the abilities of the employees in $P = H$. Competition for those employees drive their wages to h, leaving no profits for the firms. Without a quota

[13] We assume that firms benefit at least marginally from assigning workers to the task in which they are expected to be most productive. These benefits might result for example from a time difference between the allocation decision and the wage setting process.

underrepresentation of women in position $P = H$ is present: $\alpha\rho_W$ women work in this position and α men.

Of course, these results hinge on the specific assumptions we have made. For example, if the output of employees in position R were to depend on their ability, firms might have an incentive to hide employees with known high ability by assigning them to $P = R$ (see Milgrom and Oster, 1987). We eliminate such effects from our model to focus on a direct implication of a quota: asymmetric information about the characteristics of those who have been assigned to $P = H$. In case of a quota, firm $-F$ does not know whether an employee of firm F has been assigned to $P = H$ because of her ability or because of the quota. This uncertainty drives most results in our paper.

4 Simultaneous Wage Offers

This section presents two main results. First, Proposition 1 presents the unique PBE of our game with affirmative action policies. Next, Proposition 2 shows how affirmative action policies affect the expected wages of female employees in $P = H$, and firm profit. In this section, the term employee(s) refers to employee(s) in position $P = H$. All proofs are in the Appendix.

Proposition 1 *Our model with affirmative action policies has a unique PBE in mixed strategies, in which firm F offers each employee with unknown ability $w_F^a = m$, and $L^*(m) = 1$. Firm F offers wages to each employee with known high ability w_F^h according to the distribution function*

$$\begin{aligned} H^*(w) &= \left(\frac{q-\alpha\rho_W}{\alpha\rho_W}\right)\frac{w-\underline{a}}{h-w} \text{ on the support } w \in (m, \overline{a}(q)] \\ \text{with } H^*(m) &= \left(\frac{q-\alpha\rho_W}{\alpha\rho_W}\right)\left(\frac{m-\underline{a}}{h-m}\right) > 0 \end{aligned}$$

and offers a wage to each employee of firm $-F$, w_F with probability $1-\eta^*$ according to the distribution function

$$U^*(w) = \left(\frac{h-\overline{a}(q)}{h-w}\right)\left(\frac{w-m}{\overline{a}(q)-m}\right) \text{ on the support } w \in [m, \overline{a}(q)]$$

and does not offer a wage to each employee of firm $-F$ with probability

$$\eta = \frac{h-\overline{a}(q)}{h-m}$$

Two aspects of Proposition 1 are worth emphasizing. First, no equilibrium in pure strategies exists. The explanation is as follows. Firm F is informed about the abilities of it's employees. Firm $-F$ is not informed about the abilities of the employees of firm F, but it knows that there are employees with a lower expected ability at position $P = H$. Consider a pure strategy equilibrium and consider the case that F knows that the employee has high ability. If $w_F < \overline{a}(q)$ then the unique best reply of $-F$ is to slightly outbid F. But then F regrets not bidding $\overline{a}(q)$ as that would guarantee a profit equal to $h - \overline{a}(q) > 0$. If instead $w_F \geq \overline{a}(q)$, then the unique best reply of $-F$ is not to bid. Otherwise it is sure to obtain the employee if and only if the employee has unknown ability. That would result in a negative return. But if $-F$ does not bid, F regrets not bidding m. So F cannot have a pure strategy in equilibrium if it knows that the employee is of high ability. Similarly, if $-F$ has a pure strategy, F has a unique best reply to slightly outbid $-F$. Therefore no pure strategy equilibrium exists. Note that F is willing to bid m because $-F$ refrains from bidding with positive probability. By doing so it trades off a lower probability that the high ability employee will leave, against a larger profit in case she stays.

Second, firm F makes an offer equal to the wage standard, m, with positive probability when facing a high ability employee.[14] More formally, the distribution function of w_F^h, $H(w)$, has a mass point at $w = m$. This outcome goes hand in hand with the outcome that with positive probability ($\eta^* > 0$) firm $-F$ abstains from making a wage offer. Firm $-F$'s strategy allows firm F to keep an high ability employee at the lowest cost with positive probability.

Proposition 2 *The expected profit of firm F equals*

$$E[\pi(q)|P = H] = \left(\frac{q - \alpha\rho_W}{q}\right)\left(\frac{h - \underline{a}}{h - m}\right)[\alpha\rho_W(h - \underline{a}) - q(m - \underline{a})] > 0. \tag{2}$$

The expected wage of an employee in $P = H$ equals

$$E[w(q)|P = H] = \left(\frac{\alpha\rho_W}{q}\right)h + \left(\frac{q - \alpha\rho_W}{q}\right)\underline{a} - E\pi(q) < h$$

[14] In the absence of an effective minimum wage, the lowest wage either firm offers is equal to $\underline{a}$. To see this, note that $H^*(w) > \underline{a}$ for all $w > \underline{a}$, while $H^*(\underline{a}) = 0$. The lower bound on H^* in that case does not have a mass point. The reason is that $-F$ is willing to bid the expected productivity of an unknown worker even if he is sure that he will not attract a good worker. In Section 7 we discuss our assumption of a minimum wage.

If $q > \alpha\rho_W$ and $m < \bar{a}(q)$, then the expected wage of employees, ***before*** *firms learn employees' abilities, is lower with a quota than without a quota.*

Proposition 2 presents three results. First, the introduction of a quota reduces the expected wage employees in $P = H$ receive. The wage standard m only partially compensates. This result is not surprising. In our model, a quota forces firms to assign employees with unknown ability to $P = H$. This reduces the expected productivity of employees in position H. Moreover, by creating asymmetric information between firms, a quota reduces competition for employees in position H. This also reduces their expected wage. A bit more surprising is the second result that states that firms benefit from a quota. The increase in expected profit due to less competition over employees exceeds the loss in production resulting from being forced to promote less productive employees.[15] Of course, our assumption that $m < \bar{a}(q)$ is responsible for this result.[16] For example, if firms were obliged to pay employees the wage they would receive without a quota, firms would suffer from the introduction of a quota.

Finally, the ex ante expected wage of an employee, that is her expected wage before positions have been assigned, decreases because of the introduction of a quota. Though this result may seem surprising at first blush, the intuition is straightforward. Without a quota, all rents end up with the employees. With a quota, some rents go to the firms. As a result ex ante, employees receive a lower wage, unless a quota leads to better allocation of employees. A quota, however, leads to a distortion in the allocation of employees. By assumption, women with unknown abilities should be assigned to the routine job. Hence, ex ante, employees suffer from a quota.

5 The Effect of a Quota on Screening

In the previous section, we have shown that in expected terms a quota for women in high level positions benefits firms and impairs female employees. In this section, we provide a rationale for a quota. We show that it might provide an incentive for firms to better screen their female employees.

[15] Firm F will also attract some employees from firm $-F$. Firm F's expected profit with regard to these employees equals zero.

[16] The assumption that $m < \bar{a}(q)$ implies that the term in squared brackets in (2) is higher than zero. When firms makes profits on women and not on men, women's relative intial wage increases in the long-run. We ignore these long-run effects in the present paper.

Recall that in our model, the source of underrepresentation of women in $P = H$ is that firms are less able to determine the abilities of women than the abilities of men ($\rho_W < 1$). So far, we have considered ρ_W to be an exogenous parameter. In order to investigate firms' incentives to screen employees, we allow firms to choose ρ_W in this section. Specifically, we assume that at the beginning of our game each firm chooses the probability with which it learns the ability of each employee it has hired. Then firms hire a continuum of employees, receive information on their employees, and allocate them to positions. Finally they make wage offers to the employees. As before, only the information on the ability of their employees is private. In a model without affirmative action policies, firm profit does not depend on ρ_W. A higher value of ρ_W would lead to a higher share of employees in $P = H$. However, competition for those employees drives their wages to h. The employees would benefit from better screening, not the firms. We show that affirmative action policies affect firms' incentives to screen employees. The analysis consists of two parts. First, we determine firm F's decision on ρ_W, and analyze how this decision depends on q and m. Next, we identify under which condition a quota makes employees in expected terms better off. In the analysis below, the value of ρ_W in the absence of a quota plays a role. From now on, we refer to this value of ρ_W as ρ_W^{AQ}.[17]

Consider firm F's decision on ρ_W. When choosing ρ_W, firm F anticipates how ρ_W influences its profit. Maximizing (2) with respect to ρ_W yields

$$\rho_W^* = \left(\frac{(h - \underline{a}) + (m - \underline{a})}{2\,(h - \underline{a})} \right) \frac{q}{\alpha} \tag{3}$$

Equation (3) shows that ρ_W^* increases in both m and q. The intuition on why ρ_W^* is increasing in q and m is straightforward. Better screening leads to detecting more able employees. On these employees firms make a profit. A higher value of m leads to higher expected losses on employees with unknown ability. Better screening increases the chances of avoiding these losses. One can verify that $\rho_W^* < \frac{q}{\alpha}$, meaning that screening never eliminates uncertainty about the abilities of employees in position $P = H$. Otherwise, firm F assignment decisions would perfectly inform firm $-F$ about those employees'

[17] We consider the value of ρ in the absence of a quota, ρ_W^{AQ}, to be exogenous in our model. In practice this value will most likely depend on many aspects we do not include in our model, like the costs of screening, regulation, and other information that might reveal or signal an employee's ability (education and working experience for example).

abilities. Competition for employees would increase, driving wages to h and expected profits to zero. Hence, $\rho_W^* < \frac{q}{\alpha}$.

Proposition 3 *In our extended model, if $\rho_W^{AQ} < \overline{\rho_W}(q)$, where*

$$\overline{\rho_W}(q) \equiv \frac{q}{4}\left(\frac{2}{\alpha} + \frac{2(m-r) - q(h-m)}{\alpha(h-r)}\right)$$

the affirmative action program makes employees better off from an ex ante point of view, because it induces firms to screen better. Furthermore, $\overline{\rho_W}(q)$ is increasing in q, thus providing a rationale for a quota equal to $q = \alpha$.

Proposition 3 provides a potential rationale for a quota. By inducing firms to screen better, a quota may make employees in expected terms better off. The condition in Proposition 3 shows that this rationale requires that ρ_W^{AQ} is sufficiently small. Furthermore, a larger value of q widens the range of ρ_W^{AQ} for which a quota makes employees better off, providing a rationale for having a quota of $q = \alpha$. It is worth emphasizing that a quota in combination with a sufficiently high wage standard always benefits employees. In fact, $q = \alpha$ and $m = h$ induce firms to screen perfectly, such that the market for women is identical to that of men. A caveat is in order, however. We have assumed that for the firms there are no costs attached to screening. Obviously, costs of screening reduce the potential benefits of affirmative action policies on screening.

6 Sequential Wage Offers

So far, we have assumed that firms simultaneously make wage offers to employees. In this section we examine to what extent our results hinge on this assumption. To this end, we consider two simple sequential bidding processes. One in which first firm $-F$ makes offers to employees of firm F and firm F can match these wage offers afterwards, and one in which firm F first makes a wage offer to its own employees and firm $-F$ can make the final wage offer. In the Appendix we show that our result that affirmative action policies impair high ability employees in $P = H$ positions, holds for any bidding process.

Consider the first sequential bidding process where firm F can match any offer of firm $-F$ to F's employees. In this case, firm $-F$ suffers from an extreme winner's curse and firm F has all the bargaining power. Firm F matches any offer of firm $-F$, that is worth matching. As a result, firm $-F$ would acquire employees only when it offered more than they are worth. In equilibrium, firm $-F$ abstains from making offers, and firm F offers the lowest wage possible. In this equilibrium, firm F's expected profit equals

$$E\left[\pi\left(q,\rho_W\right)|P=H\right]=\alpha\rho_W\left(h-m\right)+\left(q-\alpha\rho_W\right)\left(\underline{a}-m\right) \tag{4}$$

Equation (4) shows that firm F has strong incentives to screen. A higher value of ρ_W increases the profit firm F makes on high ability employees and decreases the losses it runs on employees with unknown abilities. But uncertainty about the employees' ability should remain, to maintain the winner's curse. For $\rho_W \geq \frac{q}{\alpha}$, competition on high ability employees would increase their wages to h, eliminating profits. We conclude that with a bidding process in which F can match firm $-F$'s offers, competition for high ability women vanishes. Firm F receives high rents at the expense of women's wages. The incentives for firms to screen are strong.

Now consider the opposite sequential bidding process where firm F first makes offers to its own employees, and firm $-F$ can make counteroffers. First, notice that in this case a similar equilibrium exists as in the previous case: Firm $-F$ matches firm F's offer, and firm F always offers m. Of course, the existence of this equilibrium requires out-of-equilibrium beliefs that prevent firm F from offering higher wages. Note that in this equilibrium, firm F makes a profit on firm $-F$'s employees, but not on his own employees. This eliminates firm F's incentives to screen employees.

Besides the equilibrium in which firm F offers m and firm $-F$ matches, many other equilibria exist. In all equilibria, firm $-F$ matches firm F's offer. No equilibrium exists in which all high ability employees receive a wage equal to h. Moreover, in all equilibria, firm F does not make a profit on its own employees. The reason for this last result is that if a profitable offer for firm F were to exist, it would be matched by firm $-F$. The bargaining power of firm $-F$ eliminates the informational rents for firm F. Because multiple equilibria exist, high ability employees might be better off in comparison to the previous sequential equilibrium. Firm F does not make a profit, regardless of ρ_W, and thus it has no incentive to screen its employees. Regarding the incentives for firms to screen, the exact nature of the bidding process is important.

The discussion above shows that replacing a simultaneous bidding process by a simple sequential one has no qualitative effects on our result that high ability employees suffer from affirmative action policies. However, the bidding process does affect firms' incentives to screen its employees. In case of a simultaneous bidding process, employees may benefit from better screening. Under sequential bidding processes, by contrast, employees do not benefit from screening, only firms do. The intuition is that simultaneous bidding processes foster competition. Sequential bidding processes give power to either firm F or $-F$. This weakens or eliminates competition. Without competition, only firms benefit from more information. The next proposition summarizes and generalizes our results.

Proposition 4 *Consider the model of affirmative action policies after replacing the simultaneous, single bid offers with any other bidding structure. Then, in any equilibrium the expected wage of an employee with known high ability is strictly lower than h. Firms' incentives to screen and the extent to which high ability employees benefit from screening crucially depends on the exact nature of the bidding process.*

7 Conclusion

This paper has been framed in terms of gender quotas, but an analysis of quotas for minorities or other disadvantaged groups would lead to similar results. Quotas are debated for a wide range of fields, such as the admission of students to high quality schools, seats in parliament and high level positions in firms and (semi-)governmental bodies. Our analysis thus has a broader scope than gender quotas.

Our labor-market analysis of the costs and benefits of quotas sketches quite a gloomy picture for women. Of course, enforcing quotas helps to solve the problem of underrepresentation of women in high level positions. The key point of our paper is that quotas create uncertainty with regard to the reason why women are in high level positions. This uncertainty causes a winner's curse. The winner's curse reduces competition for women, leading to a wider gender wage gap. Firms benefit, not women. For women, a quota could turn out to be a Pyrrhic victory.

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Appendix

Proof of Proposition 1

First we show that the proposed strategy profile is a PBE. Next, we show that no other PBE exists. For notational simplicity we refer to $\bar{a}(q)$ as $\bar{a}$. Note that $H^*(\bar{a}) = 1 = U^*(\bar{a})$, so any bid of $\bar{a}$ or more is sure to win. Without loss of generality, consider a promoted employee of firm F.

L^*, H^*, U^* and η^* constitute a PBE. We proof by deduction. We first show that any $w \leq \bar{a}$ is a best reply for firm $-F$, if F adopts strategies L^* and H^*. Then we show that $w = m$ is unique best reply for F if the employee is unknown. Finally, we consider a high ability employee. We show that any wage $w \in [m, \bar{a}]$ is a best reply for firm F if $-F$ adopts U^* and η^*.

Consider the strategy of $-F$. By $\eta^* > 0$, the expected payoff of any best reply is equal to zero. Moreover, U^* has full support on $[m, \bar{a}]$. Therefore we need to prove that $E\pi_{-F}(w) = 0$ if $w \in [m, \bar{a}]$ and $E\pi_{-F}(w) \leq 0$ if $w > \bar{a}$. We start with the latter condition. Given L^* and H^*, firm $-F$ is sure to obtain the employee if she has unknown ability. It follows that if $-F$ manages to attract the employee, her expected ability cannot exceed $\bar{a}(q)$. Therefore $E\pi_{-F}(w > \bar{a}) = \bar{a} - w < 0$. Now consider $w \in [m, \bar{a}]$. For any such wage,

$$\begin{aligned} E\pi_{-F}(w_{-F}) &= \frac{\alpha\rho}{q}\frac{q-\alpha\rho}{\alpha\rho}\frac{w-\underline{a}}{h-w}(h-w) - \frac{q-\alpha\rho}{q}(w-\underline{a}) \\ &= \frac{q-\alpha\rho}{q}(w-\underline{a}) - \frac{q-\alpha\rho}{q}(w-\underline{a}) \\ &= 0 \end{aligned}$$

Thus $-F$ plays a best reply against L^* and H^*.

Consider L^*. Suppose the employee has unknown ability, then firm F would like her to switch to firm $-F$. Wage offer $w = m$ minimizes the probability that she stays and minimizes the paid wage, if she does stay. Therefore $w = m$, so $L^*(m) = 1$, is a best reply.

Finally consider H^*. Observe that the probability of F of keeping a worker with offer w is equal to

$$\Pr\left(\text{worker stays} \mid w\right) = \eta^* + (1-\eta^*)\, U^*(w) = \frac{h-\bar{a}}{h-w}.$$

Suppose the employee has high ability and let w be the offer of F. Then $E\pi_F(w=\bar{a}) = (h-\bar{a})$. Therefore we need to show that $E\pi_F(w) = (h-\bar{a})$ for all $w \in [m,\bar{a}]$ and $E\pi_F(w) \leq (h-\bar{a})$ for all $w > \bar{a}$. Both hold, as for $w \in [m-\bar{a}]$ we have $E\pi_F(w) = \frac{h-\bar{a}}{h-w}(h-w)$, and for $w > \bar{a}$ we have $E\pi_F(w) = \bar{a} - w < 0$. It follows that the given strategy profile is a PBE.

Uniqueness of the equilibrium.

Now we prove that no other PBE exists. For easy reference we introduce some terminology and notation regarding U and H. Consider a PBE. We say that w is in the support of U if $-F$ bids w with positive probability. Similarly, w is in the support of H if F offers w with positive probability to a high ability employee. Let the minimum bid of $-F$, u^-, refer to the lowest bid in the support of U. Similarly, let u^+, the maximum bid of $-F$, be the maximum bid in the support of U. If $-F$ is sure to abstain, $\eta = 1$, neither u^- nor u^+ are defined. Finally, we define h^- (respectively h^+), the minimum (maximum) bid of F, as the lowest (highest) bid in the support of H.

We prove the result in a number of steps. First, we show the winner's curse of $-F$. Second, we show a reduction in competition for the employees of F. Third, we show that $-F$ bids with positive probability.

Step 1: Consider any PBE. If the employee is of unknown ability, firm $-F$ is sure to obtain her if $-F$ bids. Suppose not. Then $L^*(m) < 1$. By offering this employee $w = m$, F can increase the probability that she leaves, while reducing its wage bill if she stays. Therefore $L^*(m)$ is not a best reply. The claim follows.

Step 2: In no PBE $-F$ has a higher maximum bid than $\bar{a}$. So $u^+ \leq \bar{a}$. Suppose not. By step 1, the expected productivity of an employee obtained by $-F$ cannot exceed $\bar{a}$. Therefore $E\pi_{-F}(u^+) = \frac{\alpha\rho}{q} H(u^+)\, h + \frac{q-\alpha\rho}{q}\underline{a} - u^+ \leq \bar{a} - u^+ < 0$. As not bidding gives a payoff zero, $u^+ > \bar{a}$ cannot be part of a best reply.

Step 3: In no PBE $-F$ is sure to abstain. Suppose not, so $\eta^* = 1$. Then F is sure to keep the employee. Therefore its only concern is to minimize the wage bill, so $L^*(m) = H^*(m) = 1$. This cannot be a PBE. By bidding $w_{-F} = m$, firm $-F$ is sure to attract the employee, regardless of her type. Therefore $-F$ would expect to earn $\bar{a} - m > 0$, which is better than not bidding.

In the following two steps we use the fact that the minimum bid of both F and $-F$ must be such that they can win in case the employee has known high ability. This is logical: F does not want to lose a high ability employee, and $-F$ wants to bid only if it can acquire a high ability employee. Together this implies that $-F$ abstains with positive probability.

Step 4: In any PBE, a high ability employee stays with F with positive probability. Suppose not. Then F earns zero payoff on the high ability employee. F can then profitably deviate to bidding $\bar{a}$. By step 2, $w_F = \bar{a}$ implies that the employee stays. This is profitable for F as $w_F = \bar{a} < h$. The claim follows.

Step 5: In any PBE the minimum bid of $-F$ is weakly higher than the minimum bid of F, i.e. $u^- \geq h^-$. Suppose not, so $h^- > u^-$. By bidding less than h^- firm $-F$ is sure that it obtains the employee if and only if she has unknown ability. This results in an expected loss. Therefore not bidding is strictly better than bidding u^-. Thus $u^- \geq h^-$.

Step 6: In no PBE $-F$ is sure to bid. Suppose the employee is a high ability employee. By Step 5 we have $h^- \leq u^-$. However, if $-F$ is sure to bid and $h^- \leq u^-$, then F is sure to lose if it bids h^-. This contradicts Step 4. Therefore there is no PBE in which $-F$ is sure to bid.

We have established that the minimum bid of F must be such that it can win a high ability employee, while at the same time $-F$ must be able to win a high ability employee with it's minimum bid. These two conditions hold jointly only if $h^- \leq u^-$ and $\eta > 0$. The following three steps exploit the fact that firm $-F$ is indifferent between bidding and not.

Step 7: In any PBE, $-F$ has an expected profit of zero. By Step 6, it is a best reply for $-F$ not to bid. Therefore $-F$ earns zero expected profits.

Step 8: In any PBE, for any $w \in [m, \bar{a}]$ in the support of U, we have $H(w) = \frac{q-\alpha\rho}{\alpha\rho}\frac{w-\underline{a}}{h-w}$. By step 6, the expected profits of any bid in the support of U gives zero profits. Thus we have $E\pi_{-F}(w) = \frac{\alpha\rho}{q} H(w)(h-w) - \frac{q-\alpha\rho}{q}(w-\underline{a}) = 0$. Rearranging this gives $H(w) = \frac{q-\alpha\rho}{\alpha\rho}\frac{w-\underline{a}}{h-w}$.

Step 9: In any PBE we have $u^+ = h^+ = \bar{a}$. First note that $u^+ = h^+$. The reason is that offering more than the maximum bid of the other firm does not increase the probability of winning over the employee, but does increase the wage bill. Second note that $h^+ \geq \bar{a}$. If not, so $h^+ < a^+$, firm $-F$ would obtain a positive profit by offering $w_{-F} = h^+$, namely $h - h^+$. By step 7, the expected profits of $-F$ are not positive. Third, note that by step 2, we have $u^+ \leq \bar{a}$. Combined this implies $\bar{a} \geq u^+ = h^+ \geq \bar{a}$ and thus $u^+ = h^+ = \bar{a}$.

Now we show that U and H have a full support on $[m, \bar{a}]$. Together with step 8, this proves that in each PBE H is as given in the proposition.

Step 10: In any PBE, there is full support of U or H on $[m, \bar{a}]$. Suppose not. Then there exists $w, w' \in [m, \bar{a}]$ such that $w' > w$ and $H(w) = H(w')$ or $U(w) = U(w')$. We first show that this implies $U(w) = U(w')$. Suppose instead $H(w) = H(w')$. Then $-F$ strictly prefers bidding w to bidding any wage $w'' \in (w, w']$. Therefore $U(w) = U(w')$. Define w^* as the lowest bid higher than w' which is in the support of U. Note that w^* exists as $u^+ = \bar{a}$ (step 9). Then F strictly prefers to bid w rather than any wage $w'' \in (w, w^*]$. So no wage $w'' \in (w, w^*]$ is in the support of H. Consequently $-F$ strictly prefers bidding w to w^*. This contradicts that w^* is in the support of U. The claim follows.

Finally we derive $U(w)$ using step 9, and given the full support of H and U. It follows that the equilibrium given by the proposition is the unique PBE.

Step 11: In any PBE $\eta^* = \frac{h-\bar{a}}{h-m}$ and $U^*(w) = \frac{h-\bar{a}}{h-w}\frac{w-m}{\bar{a}-m}$ on support $w \in$ $[m, \bar{a}]$. By step 9, the expected profit for F of offering $w_F = h^+$ is equal to $(h - \bar{a})$. Therefore for any w in the support of H, the probability of winning should be such that the expected profit is equal to $(h - \bar{a})$. If $w_F = m$, the probability of winning is the probability that $-F$ does not bid. So $\eta^*(h - m) = h - \bar{a}$ and thus $\eta = \frac{h-\bar{a}}{h-m}$. Moreover the expected profit of F for any $w > m$ is equal to $\Pr(\text{worker stays} \mid w)(h - w) = (h - \bar{a})$, giving $\Pr(\text{worker stays} \mid w) = \frac{h-\bar{a}}{h-w}$. Therefore

$$\begin{aligned}\Pr(\text{worker stays} \mid w) &= \eta^* + (1-\eta^*)U^*(w) \\ \frac{h-\bar{a}}{h-w} &= \frac{h-\bar{a}}{h-m} + \frac{\bar{a}-m}{h-m}U(w) \\ U^*(w) &= \frac{h-\bar{a}}{h-w}\frac{w-m}{\bar{a}-m}\end{aligned}$$

Steps 8 to 11 prove uniqueness of the proposed PBE. □

Proof of Proposition 2

The expected profit of firm F from making an offer to employees in $P = H$ is

$$E[\pi_F(q) | P = H] = \alpha\rho_W(h - \overline{a}(q)) + (q - \alpha\rho_W)\eta(\underline{a} - m)$$

The expected profit function of firm F consists of two parts. First, firm F derives an expected profit of $h - \overline{a}(q)$ from known high ability employees. The proportion of known high ability employees equals $\alpha\rho_W$. Second, firm

F is forced to keep employees with unknown ability when firm $-F$ abstains from making a wage offer. These employees will receive a wage equal to $w_F^{\underline{a}} = m$. This results in a return of $\underline{a} - m < 0$. The proportion of employees in $P = H$ with unknown ability equals $(q - \alpha\rho_W)$ and firm $-F$ abstains from making a wage offer with probability η. Finally, there is a third group which is not directly visible in the profit function. Firm F also makes wage offers to employees in $P = H$ in firm $-F$. Note that the expected profit the uninformed firm makes on these employees equals zero. Substituting $\overline{a}(q) = \left(\frac{q-\alpha\rho_W}{q}\right)\underline{a} + \left(\frac{\alpha\rho_W}{q}\right)h$ and rewriting the profit function gives:

$$E\left(\pi_F(q)\,|P = H\right) = \left(1 - \frac{\alpha\rho_W}{q}\right)\left(\frac{h-\underline{a}}{h-m}\right)\left[\alpha\rho_W\left(h-\underline{a}\right) - \left(m-\underline{a}\right)q\right]$$

Thus $E\pi_F\left(q \mid P = H\right) \geq 0$ when $\alpha\rho_W\left(h-\underline{a}\right) - \left(m-\underline{a}\right)q > 0$, *i.e.* if $m < \left(\frac{q-\alpha\rho_W}{q}\right)\underline{a} + \left(\frac{\alpha\rho_W}{q}\right)h = \overline{a}(q)$. Throughout we assume that $m < \overline{a}(q)$.
The expected wage of an employee in $P = H$ is her expected productivity minus the expected profits the firm makes on her. Therefore

$$E\left(w(q)\,|P = H\right) = \frac{\alpha\rho_W}{q}h + \frac{q-\alpha\rho_W}{q}\underline{a} - E\pi\left(q \mid P = H\right) < h$$

where the inequality follows from $\frac{\alpha\rho_W}{q} < 1$, $\underline{a} < h$ and $E\left[\pi(q)\,|P = H\right] > 0$. Furthermore the *ex-ante* expected wage of female employees decreases. The expected wage of employees is the difference between the expected productivity of employees and the profits the firms expect to make. Introducing a quota such that $m < \bar{a}$ lowers the expected productivity of employees (by $(q-\alpha\rho)\left(r-\underline{a}\right)$), while it increases the profits of the firms. Therefore the expected wage of female employees decreases, notwithstanding the fact that more female employees are being promoted. $\square$

Proof of Proposition 3
The ex ante payoff of employees in the extended model equals

$$E\left[w\left(q,\rho_W^*(q)\right)\right] = qE\left[w\left(q,\rho_W^*(q)\right) \mid P = H\right] + (1-q)\,r \tag{5}$$

where the expected wage for an employee in position $P = H$ is equal to difference between the productivity and the expected informational rents for the firm

$$E\left[w\left(q,\rho_W^*(q)\right) \mid P = H\right] = \frac{\alpha\rho_W^*(q)}{q}h + \frac{q-\alpha\rho_W^*(q)}{q}\underline{a} - E\pi\left(q,\rho_W^*(q) \mid P = H\right) \tag{6}$$

and the expected rents for the firm with regard to an employee in position $P = H$ are equal to

$$\begin{aligned} E\left[\pi\left(q, \rho_W^*\left(q\right)\right) \mid P = H\right] &= \left(1 - \frac{\alpha\rho_W^*\left(q\right)}{q}\right)\left(\frac{h - \underline{a}}{h - m}\right) \\ &\quad \cdot \left[\alpha\rho_W^*\left(q\right)\left(h - \underline{a}\right) - \left(m - \underline{a}\right)q\right] \end{aligned} \tag{7}$$

the optimal $\rho_W^*\left(q\right)$ from equation (3) is equal to

$$\rho_W^*\left(q\right) = \frac{\left(h - \underline{a}\right) + \left(m - \underline{a}\right)}{2\alpha\left(h - \underline{a}\right)}q \tag{8}$$

Using the definition of $\underline{a}$ and substitution of equation (6), (7) and (8) into equation (5) yields

$$E\left[w\left(q, \rho_W^*\left(q\right)\right)\right] = \frac{q}{4}\left(\left(2 - q\right)h + \left(2 + q\right)m\right) + \left(1 - q\right)r \tag{9}$$

In the absence of a quota, the ex ante expected wage of each employee equals $\alpha\rho_W^{AQ}h + \left(1 - \alpha\rho_W^{AQ}\right)r$. Therefore, each employee benefits from a quota if

$$\begin{aligned} \alpha\rho_W^{AQ}h + \left(1 - \alpha\rho_W^{AQ}\right)r &< \frac{q}{4}\left(\left(2 - q\right)h + \left(2 + q\right)m\right) + \left(1 - q\right)r \\ \rho_W^{AQ} &< \frac{q}{4}\left(\frac{2}{\alpha} + \frac{2\left(m - r\right) - q\left(h - m\right)}{\alpha\left(h - r\right)}\right) \end{aligned} \tag{10}$$

Thus if (10) is satisfied, then female employees benefit from a quota because firms will choose to become more aware of female talent. Now consider how this depends on the quota. Simple differentiation gives

$$\frac{\partial\rho_W^{AQ}}{\partial q} = \frac{\left(1 - q\right)h + \left(1 + q\right)m - 2r}{2\alpha\left(h - r\right)}$$

which is positive for all $q \in [0, 1)$. This implies that (10) is more likely to be satisfied if q is larger. Hence, setting $q = \alpha$ makes it most likely that employees benefit from a quota. $\square$

Proof of Proposition 4

In this proof we show that in any PBE the expected wage of an employee with known high ability is strictly lower than h. We show this by contradiction. Suppose that this is not the case, then there exists a bidding structure in which any employee i with known high ability will be offered wage h by some firm. Let i be such a worker and F her employer. Now also consider a promoted worker j of unknown ability working for F. We have two cases. In the first, j is sure to stay with F. In the second, j leaves with positive probability. If j is sure to stay, then in equilibrium F will offer wage m to j and not higher. F will not choose to outbid any other firm who bids for j. As j stays, it follows that no other firm bids after F bids m to his promoted worker. Now let F bid m to i. Because no other firm bids after F offers m to i, i will stay and F makes a profit of $(h - m)$ on i. Thus i would not receive wage h but wage m. This is a contradiction. Suppose instead that employee j leaves with positive probability to the other firm, $-F$. This implies that $-F$ bids in equilibrium. However, $-F$ can obtain zero profits by not bidding. If $-F$ bids and obtains j, he obtains a negative expected profit. Therefore $-F$ must also be able to obtain worker on whom he would make a positive expected profits. That implies that with positive probability $-F$ hires i (or another worker of F that F knows to be good) at a wage lower than h. This also is a contradiction. It follows that the expected wage of female workers with known high ability is less than h.