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DSGE Models with Observation-Driven Time-Varying parameters

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Abstract

This paper proposes a novel approach to introduce time-variation in structural parameters of DSGE models. Structural parameters are allowed to evolve over time via an observation-driven updating equation. The estimation of the resulting DSGE model can be easily performed by maximum likelihood without the need of time-consuming simulation-based methods. An application to a DSGE model with time varying volatility for structural shocks is presented. The results indicate a significant improvement in forecasting performance.

Keywords: DSGE models, score-driven models, time-varying parameters
JEL Codes: C32, C5

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1 Introduction

There is an expanding literature on estimating DSGE models with time-varying structural parameters. Castelnuovo (2012), Canova and Sala (2009) show evidence of time-variation in parameters by estimating DSGE models over rolling samples. Justiniano and Primiceri (2008) and Fernández-Villaverde et al. (2007) specify a stochastic process for a subset of the structural parameters. Galvão et al. (2016) recently proposes a Bayesian method that introduces time variation in the estimation of the model. As argued in Galvão et al. (2016), time variation in structural parameters can be interpreted as cultural and technological shifts, or other forms of misspecification, that DSGE models are unable to capture. Accounting for time variation is useful since DSGE models are widely used for forecasting and relying on local structural parameters can enhance forecasting accuracy.

In this paper, we propose a new approach to account for time-variation in structural parameters of DSGE models. We allow the structural parameters to follow an autoregressive process with innovation given by the score of the predictive likelihood. This method is based on the Generalized Autoregressive Score (GAS) framework of Creal et al. (2013) and Harvey (2013). Dynamic models with score-driven parameters have been successfully employed in economic and financial studies, see for instance Lucas et al. (2017), Blasques et al. (2016b), and Harvey and Luati (2014).

The resulting DSGE models with score-driven parameters are easy-to-implement and deliver more accurate forecasts compared to static DSGE models. In particular, we implement a dynamic version of the DSGE of An and Schorfheide (2007) with time-varying volatility for the innovation components of interest rate, supply and demand shocks. We show that our approach improves significantly the performance of the DSGE model in-sample as well as out-of-sample.

2 DSGE with score-driven structural parameters

Let $Z_t = (Z_{1,t}, Z_{2,t}, \cdots, Z_{n,t})'$ be an $n_z \times 1$ vector of endogenous variables and assume that, after a log-linearization, the economy is described by the following structural model:

$$\Gamma_{0,t} Z_t \quad = \quad \Gamma_{f,t} E_t Z_{t+1} + \Gamma_{b,t} Z_{t-1} + \Pi_t \varepsilon_t, \tag{1}$$

where $\Gamma_{i,t} = \Gamma_i(\theta_t)$, $i \in \{0, b, f\}$ are $n_z \times n_z$ and Π_t is a $n_z \times n_\varepsilon$, whose elements depend on the $n_\theta \times 1$ vector of time-varying structural parameters θ_t, ε_t is a $n_\varepsilon \times 1$ fundamental white noise term with covariance matrix $\Sigma_{t,\varepsilon}$. The matrix $\Gamma_{t,0}$ is assumed to be non-singular, while $\Gamma_{t,f}$ and $\Gamma_{t,b}$ can be singular and $\Gamma_{t,b}$ possibly zero. Assuming that there exists an unique stable solution of the system, one way to express the reduced form solution associated with the system (1) is:

$$\underset{n_m\times 1}{Z_{m,t}} = \underset{n_m\times n_m}{A(\theta_t)}\,\underset{n_m\times 1}{Z_{m,t-1}} + \underset{n_m\times n_\varepsilon}{B(\theta_t)}\,\underset{n_\varepsilon\times 1}{\varepsilon_t} \tag{2}$$

$$\underset{n_y\times 1}{y_t} = \underset{n_y\times n_m}{C(\theta_t)}\,\underset{n_m\times 1}{Z_{m,t-1}} + \underset{n_y\times n_\varepsilon}{D(\theta_t)}\,\underset{n_\varepsilon\times 1}{\varepsilon_t} \tag{3}$$

where $Z_{m,t}$ is the n_m-dimension sub-vector of Z_t that contains the candidate minimal states of the system, $A(\theta_t), B(\theta_t), C(\theta_t)$ and $D(\theta_t)$ are time-varying matrices of parameteres that depend non-linearly on θ_t through a set of Cross Equation Restrictions (CER), see Castelnuovo and Fanelli (2015) for more details about the derivation of the CER.

Following the GAS framework of Creal et al. (2013) and Harvey (2013), the specification of the time-varying structural parameter vector θ_t in (2) and (3) is:

$$\theta_t = g(f_t), \quad f_{t+1} = \omega + Bf_t + As_t, \tag{4}$$

where f_t is a $n_\theta \times 1$ vector, $g(\cdot)$ is a link function, ω, B and A are matrices containing static parameters to be estimated and s_t is the score innovation of the dynamic equation. More specifically, s_t is specified as

$$s_t = \frac{\partial \log p\,(y_t|\mu_t, \Sigma_t)}{\partial f_t}.$$

where $p\,(\cdot|\mu_t, \Sigma_t)$ denotes the density function of a multivariate normal with mean μ_t and covariance matrix Σ_t. The mean μ_t and covariance Σ_t are the conditional mean and covariance matrix of y_t obtained from the Kalman filter for the state space model in (2) and (3). We refer the reader to Delle Monache et al. (2016) and Buccheri et al. (2017) for further applications of GAS time-varying parameters in the context of Kalman filtering. We note that in practice the score innovation s_t is typically not available in closed form and it can be computed using numerical differentiation. The estimation of the static parameters of the model can be performed by standard maximum likelihood through the Kalman filter.

3 Empirical illustration

The empirical analysis is based on the DSGE model of An and Schorfheide (2007) in which we allow for time-variation in the variances of structural shocks. The model is described by the following equations:

$$\begin{aligned}
\tilde{x}_t &= E_t\tilde{x}_{t+1} + g_t - E_t g_{t+1} - \tau^{-1}(r_t - E_t\pi_{t+1} - E_t z_{t+1}), &(5)\\
\pi_t &= \beta E_t\pi_{t+1} + \kappa(\tilde{x}_t - g_t), &(6)\\
r_t &= \rho_r r_{t-1} + (1-\rho_r)\psi_1\pi_t + (1-\rho_r)\psi_2(\tilde{x}_t - g_t) + \varepsilon_{r,t}, &(7)\\
g_t &= \rho_g g_{t-1} + \varepsilon_{g,t}, &(8)\\
z_t &= \rho_z z_{t-1} + \varepsilon_{z,t}, &(9)
\end{aligned}$$

where $\varepsilon_{i,t} \sim WN(0, \sigma^2_{i,t}), i = r, g, z$, (5) is a forward-looking output-gap equation where $\tilde{x}_t$ is the output gap, (6) is a forward-looking New-Keynesian Phillips Curve (NKPC) with inflation rate π_t, (7) is the monetary policy rule with policy rate r_t, while (8)-(9) define two autoregressive processes of order one for the aggregate supply (g_t) and demand (z_t) disturbances (see An and Schorfheide (2007) for a discussion of the system in (5)-(9)). In the notation of (2)-(3), we have $Z_{m,t} = (r_t, g_t, z_t)'$ and $y_t = (y_t, \pi_t, r_t)'$. The variances $\sigma^2_{i,t},\ i = r, g, z$, are given by

$$\sigma^2_{i,t} = \exp(f_{i,t}), \quad f_{i,t+1} = \omega_i + \beta f_{i,t} + \alpha s_{i,t},$$

where the score innovation $s_t = (s_{r,t}, s_{g,t}, s_{z,t})'$ is specified as described in Section 2. The static parameters β, α and w_i, $i = r, g, z$, are estimated by maximum likelihood, instead, the other parameters of the DSGE are calibrated to the values given in Komunjer and Ng (2011). The static DSGE model is obtained by setting β and α to zero.

We consider U.S. quarterly data from the first quarter of 1984 to the third quarter of 2017. Figure 1 shows the estimated time-varying standard deviations $\sigma_{r,t}$, $\sigma_{g,t}$ and $\sigma_{z,t}$. The plots illustrate that the three variances are not constant over time assuming, for example, higher values during the recent financial crisis. The confidence bands are computed following the procedure in Blasques et al. (2016a). We perform an in-sample and out-of-sample comparison between our approach and the static DSGE model. The out-of-sample forecasting study is based on the last 5 years of the sample (from 2012 to 2017) and the log-score criterion for density forecasts is employed as means of comparison, see Geweke and Amisano (2016). Table 1 reports the AIC criterion and the difference in log-score criterion between the dynamic and the static DSGE. The difference in log-scores is reported separately for density forecasts of inflation, output gap and interest rates but also jointly for the forecast of the joint density of the three variables. From Table 1, we can see that our model clearly has a better in-sample fit according to the AIC. As concerns the out-of-sample study, we note that our model is significantly better in forecasting inflation and the joint distribution of the three variables.

Instead, the forecasts of output gap and interest rates are not significantly different. We can conclude that overall the empirical results underline how our time-varying DSGE model can outperform the static DSGE.

4 Conclusion

In this paper we have proposed a new way for incorporating time-varying parameters in DSGE modeling. Our approach is easy to implement and the empirical results suggest a better in-sample and out-of-sample performance compared to DSGE model with constant parameters. Future research may focus on exploring the performance of the proposed score-driven approach to more complex DSGE models.

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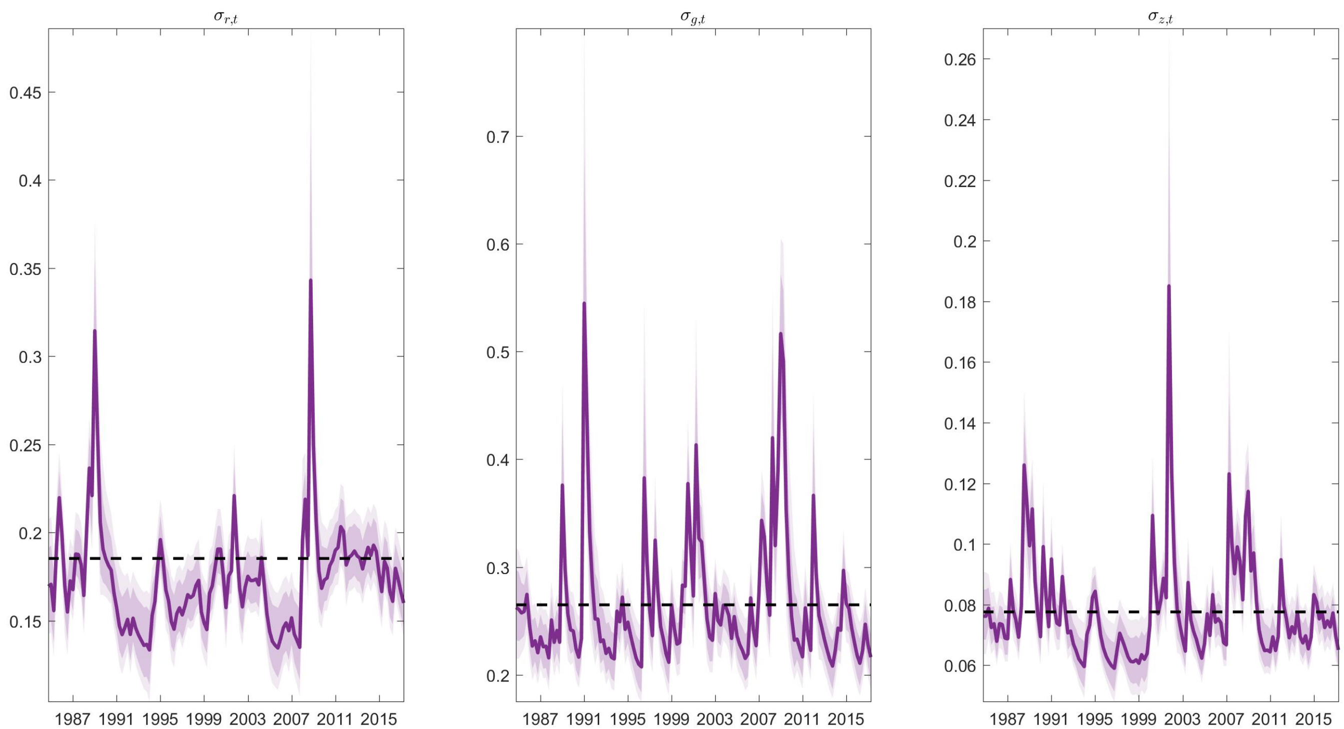

Figure 1: Estimation of the time-varying standard deviations $\sigma_{r,t}$, $\sigma_{g,t}$ and $\sigma_{z,t}$ based on U.S. quarterly data in the period 1984-2017. Shaded purple areas denote the 80% and 95% confidence bands computed using the approach of Blasques et al. (2016a). Black dashed lines represent the estimated constant variances.

	In-sample	
	Constant variances	Time-varying variances
Log-likelihood	115.35	130.54
AIC	-224.69	-251.09
	Out-of-sample, log-score	
$\tilde{x}_t$	0.07	
π_t	0.08*	
r_t	-0.04	
Overall	0.07*	

Table 1: In-sample and out-of-sample results. The forecasting exercise is based on the last 5 years of the sample (from 2012 to 2017). '*' denotes statistically significance at 5% level using the Diebold-Mariano test.